A Comparison of Inquiry Based, Schema Based and Traditional Teaching Approaches
in Colombian Middle School Classrooms

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Abstract

We assessed the efficiency of three teaching approaches (inquiry-based, schema-based and traditional instruction) with 99 sixth-grade Colombian students and their math teachers. Throughout the intervention’s nine hours of class time, students and teachers worked on elements of combinatorics: tree diagrams, permutations, and combinations. A significant difference in favor of schema-based and traditional teaching approaches was found. We suggest specific implementation details to use inquiry-based learning in Colombian classrooms as well as discussing the limitations of the study.
Dedication

To my brother Jonathan.
Acknowledgments

The present work consists of a research study done with Colombian teachers and students from public schools, so the list of people who made this possible is reasonably large. I did my best to keep this section short.

First, and most importantly, I will always be thankful for working under the supervision and advisory of Andrew Engelward. His guidance was definitive to determine the scope and possibilities of this study. Moreover, the structure of the whole project is largely due to Andy’s precise and pertinent questions, recommendations and insights. From the moment I took my first course in the Mathematics for Teaching program, Andy has been a source of inspiration as a teacher, researcher, and human being.

The influence of Paul Bamberg and Brendan Kelly in this work is enormous. Paul taught me how to introduce real analysis to middle school students with minimum sacrifice to mathematical rigor, a task almost impossible to imagine which became a reality months later. Brendan’s course on group theory using inquiry-based learning still impresses me and made realize how an instructor is indeed an artist.

Implementing the study required a lot of resources, either for looking for potential participants, getting proper classrooms, arranging transportation, preparing food, among others. Everything was possible thanks to the cooperation and immense generosity of Asociación Alianza Educativa and its director, Luisa Pizano. The role of Luisa in
Colombian education is groundbreaking; this research is yet another contribution she helped with.

At Harvard I met some of the most extraordinary people and friends I know today. Maura McGlame’s help and support played a paramount role during my study cycle. My friends Marijana Saraf, Erika Guzman, Kyle Botkin, Manli Nouri, and Emily Siegfriedt showed me how awesome can be the time at Cambridge, MA, regardless the amount of work to do.

The presence and love of my family during this work gave me emotional strength and resilience through a process that featured important drawbacks. I am glad that my parents and my two sisters can read this final document and I hope it serves as a contribution to the school education Santi and Gaby receive in Colombia. Angela’s help was decisive, and Efraín and Soraya showed me they are the most noble people I know.

Finally, this thesis is dedicated to my brother Jonathan. He convinced me to apply to Harvard and has always been the fan who never quits nor loses the hope. Nowadays, I admire him from an intellectual point and love him as the life-long companion he is. Fortunately, I will be under the influence of his good ideas in the future.
# Table of Contents

Dedication ....................................................................................................................... iv

Acknowledgments ........................................................................................................... v

Chapter I. Introduction ..................................................................................................... 1

Chapter II. Education in Latin America, the Caribbean, and Colombia ......................... 4

  Educational sector in Latin America and the Caribbean .............................................. 4
  Participation and progression ....................................................................................... 5
  Financial resources ....................................................................................................... 12
  Inequality ....................................................................................................................... 13
  Lack of trained workforce and innovation ................................................................. 15
  Quality of education ..................................................................................................... 16

  A closer look to Colombia’s results in mathematics ................................................... 20

Chapter III. Teaching Approaches ................................................................................ 23

  Schema Based Instruction ........................................................................................... 23
  Implementation ............................................................................................................. 27

  Inquiry Based Learning ............................................................................................... 31
  Implementation ............................................................................................................. 33

  Traditional Teaching .................................................................................................. 36
  Education in Colombia since 1900 ............................................................................ 37
  Implementation ............................................................................................................ 40

Chapter IV. Methodology ............................................................................................... 44
Chapter I.

Introduction

Latin American countries, and Colombia in particular, face important challenges with respect to the quality of mathematics education at school levels. The region’s consistently low results in international assessments highlight the quality of education as an urgent issue not only in mathematics but also in reading, science, critical thinking, problem-solving and other high order skills.

Participant actors of this scenario include almost every instance of society: government officials, school administrators, departments of education, parents, teachers, and students. Naturally, a myriad of initiatives must come from all participants; the complexity of the problem certainly does not have a simple solution.

From the teachers’ perspective, several important questions arise. What are we doing in our classes? Is there something different we could attempt so that our students get better results?

As we stated regarding the general problem, we believe there is no simple answer to these questions. Indeed, we need to be more specific because the art of teaching depends on so many variables. Different students and different topics require different teaching approaches, so we are certain this issue shouldn’t be oversimplified.

Trying to delimitate the scope, we opted for creating a study with some fixed values. We decided to work for nine hours with groups of sixth-grade Colombian students from public schools, and we focused on teaching introduction to combinatorics:
the multiplication principle, permutations, and combinations. Also, the teachers in charge of the instruction were the same teachers the students have at their schools.

The design responds to our interest on assessing different teaching approaches in short interventions. The mathematical content the students worked with was the same for all the study groups: our interest was assessing only the teaching methodologies by means of fixed problems, exercises, and examples.

The present study serves as an introduction to schema-based instruction and inquiry-based learning, directed at Latin American and Colombian teachers. Furthermore, we want to illustrate how to incorporate said methodologies in classrooms, as well as some limitations we found doing during implementation.

Beyond the results obtained, we do not advocate for any of the teaching approaches studied. Instead, we present some possible explanations for our outcome. This study was aimed at modelling how to incorporate different teaching approaches and assess them. It also serves as a reference for the future studies with a different population and mathematical content.

The structure of this document is linear, but each chapter can be read separately. We start in chapter II presenting some demographic statistics on education in Latin America and the Caribbean, as well as the results showing the quality of education in this region.

In chapter III, we introduce the three teaching methodologies assessed on this study, namely, schema-based instruction, inquiry-based learning, and traditional teaching approaches used in Colombia. We describe each of them and present how they have been
used in different contexts. Then, we describe how we implemented these approaches in our study.

Details about the population, sample, and measuring tools are covered in chapter IV. There, we discuss how we assembled each study group, the source material, the students we worked with, and the tests used as assessments.

Finally, in chapter V we discuss the results of the study, as well as the possible explanations and recommendations for further studies.

We hope this work will motivate scholars in Latin America to do controlled experiments assessing different variables from classroom environments, as well as settle even more questions about how different teaching approaches could be used in schools.
Chapter II.

Education in Latin America, the Caribbean, and Colombia

Educational quality became a pivotal concern for Latin America and the Caribbean region in recent years. From a broad perspective, we may say that in the last two decades this region shows improving tendencies in many statistics assessing education. Said statistics include measures on participation, progression, transition, and financial resources. However, inequality prevails as one of the distinctive features of the region, while the deficient quality of education remains a constant burden.

In this chapter, we bring together some trends about education in Latin America and the Caribbean and look at the region’s outcomes in international tests designed to assess the quality of education. Lastly, we take a closer look to Colombia’s results in mathematics.

Educational sector in Latin America and the Caribbean

Data retrieved from the UNESCO Institute for Statistics (2016) shows positive trends regarding the Latin American and Caribbean education sector. In this section, we mention how the numbers for out-of-school children, government expenditure on education, and the rates for enrollment, transition and survival have changed since 1999. Even though we describe the behavior of average results following a methodology similar to Education for All’s reports (UNESCO, 2015a) but using the most recent data, it is
important to notice that there are marked differences among countries in the region, as well as considerable variation within the countries themselves.

Participation and progression

We should begin by describing how participation levels changed in recent years. To do that, we make use of two statistics describing enrollment rates: the gross enrollment ratio (GER), and the adjusted net enrollment rate (ANER).

The GER is defined by the “number of students enrolled in each level of education, regardless of age, expressed as a percentage of the official school-age population corresponding to the same level of education” (UNESCO, 2016). Because of its definition, the GER may surpass 100%, as it considers students enlisted in a level of education who are older than said level’s age range. Due to this situation, it is useful to introduce another statistic.

The ANER is the “total number of students of the official primary school age group who are enrolled at primary or secondary education, expressed as a percentage of the corresponding population.” (UNESCO, 2016). Therefore, to calculate ANER, we “divide the total number of students in the official primary school age range who are enrolled in primary or secondary education by the population of the same age group” (UNESCO, 2016). With these definitions, the ANER will always be a lower value than the GER.

Figure 1 shows the average GER per level of education in Latin America and the Caribbean from 1999 to 2015. We observe three curves with positive trends for most of the time under analysis. First, the GER for pre-primary education went from 54% in 1999 to 75% in 2015. More moderate growth took place in participation in upper-secondary
education, growing from 68% to 78%. Finally, the GER for tertiary education escalated more drastically, from 22% to 46%, in the same period.

Figure 1. Gross Enrollment Ratio (GER) – Latin America and the Caribbean

Data retrieved from UNESCO (2016)

These increasing measurements could be attributed to the region’s effective transition rate from primary to lower secondary education, as well as the improvement of the survival rate to the last grade of primary during this period (Figure 2).
Additionally, Figure 1 shows how the GERs for primary and lower secondary education take values above 100%. As we mentioned before, this can be explained by the presence of a large number of older students at each education level, resulting from late entry ages or a high percentage of repeaters. In any case, Figure 3 shows how the percentage of repeaters in primary and secondary education declined from 1999 to 2015, another favorable feature of the region. Additionally, this behavior helps explaining the GER’s decrease for primary education between 1999 and 2015.
Figure 3. Percentage of Repeaters – Latin America and the Caribbean.

*Data retrieved from UNESCO (2016).*

Since GER values are greater than 100%, using the ANER allows us to get better understanding of actual participation. Unfortunately, data on the average ANER for secondary and tertiary education from 1999 to 2015 is not available for all Latin American and Caribbean countries, so we can only present in Figure 4 the ANERs for pre-primary and primary education levels.
We notice how the ANER for primary education between 1999 and 2015 ranged from 93% to 97%. UNESCO remarks that even though “the number of children enrolled in primary schools in the region decreased … the ANER has been maintained” (2015b). Furthermore, the average ANER for pre-primary education increased from 81% in 1999 to 94% in 2015.

To summarize, both statistics GER and ANER show increasing levels of participation by children, adolescents, and youth in education in Latin America and the Caribbean.

As we mentioned before, the descriptions of these tendencies should not be taken as applying to all countries. For example, the primary ANER ranges from 75% in Guyana to 97% or above in México and Uruguay (UNESCO, 2015b). Furthermore, “the average
lower secondary education GER of Caribbean countries (73%) is considerably lower than that of Latin America (99%) with a similar difference at upper secondary level with GERs of 47% in the Caribbean sub-region and of 77% for Latin America in 2012.” (UNESCO, 2015b)

Other indicators of participation levels are the number and rate of out-of-school students, displayed on Figures 5 and 6. We observe the most noticeable decrease in the rate of out-of-school youth of upper-secondary school age, which went down from 32% in 1999 to 24% in 2015. Unfortunately, some details are not observable from the graphs. Specifically, “while in Latin America the number of out-of-school children declined [from 1999 to 2012] by 8.9%, in the Caribbean it increased by 11.4%. In 2012, conflict-affected Colombia alone accounted for nearly 16% of the entire region’s out-of-school children.” (UNESCO, 2015b)

As was the case with GER and ANER, the number and rate of out-of-school children, adolescents and youth in the region show noticeably favorable trends.
Figure 5. Out-of-school Children, Adolescents and Youth – Latin America and the Caribbean.

*Data retrieved from UNESCO (2016).*
Figure 6. Rate of Out-of-school Children, Adolescents and Youth – Latin America and the Caribbean.

Data retrieved from UNESCO (2016).

Financial resources

One of the main reasons for progress in the educational sector in the region has been the relocation of larger public funding for education during the last years. Indeed, “in 2012, half of countries with data in Latin America and the Caribbean spent about 5% of gross national product or more on education.” (UNESCO, 2015b)

In general, the expenditure on education has increased in recent years, sometimes even exceeding the country’s economic growth. In fact, the share of national income devoted to education increased between 1999 and 2012 in most of the 22 countries with data… While economic growth for the region averaged 3.5% per annum, growth in public expenditure on education averaged 5.3% per year. In 12 of the 18 countries with data in
the region, progress in public spending on education exceeded economic growth. (UNESCO, 2015b)

Figure 7 shows the expenditure on education for each Latin American country as percentage of its gross domestic product (GDP). As a general tendency, these values increased with time.

![Government expenditure on education as % of GDP (%)](image)

**Figure 7.** Government expenditure on education in Latin American countries.

*Data retrieved from World Bank (2017).*

**Inequality**

Inequality is a rampant feature of Latin America and the Caribbean, and it noticeably affects their educational sector.

In fact, Figure 8 shows the historical GINI indexes since 1999 for Latin American and Caribbean countries with available data (World Bank, 2017). While developed
countries have GINI indexes around 0.30 (UNESCO, 2014), we can see from the graph that the region’s average GINI index is closer to 0.50, representing extremely high levels of inequality. According to UNESCO (2014), “Latin America and the Caribbean is the world region with the most imbalanced income distribution.”

Figure 8. GINI index – Latin America and the Caribbean.

Data retrieved from World Bank (2017).

Economic inequality yields to unequal results in the educational sector. Indeed, UNESCO (2015b) point out that progress towards universal primary education is not always uniform. Poverty, ethnicity and location affect primary school participation and attainment… Inequality in access to secondary education persists; marginalized groups are the most affected… In the Plurinational State of Bolivia, Colombia, and Haiti, the gaps between wealth groups did not
change noticeably since 2000… Access to secondary school has been an issue for marginalized groups, including working children and migrants... Substantial proportions of adolescents of secondary school age continued to work outside of school.”

Lack of trained workforce and innovation

As countries’ economies grow, the officials face the necessity of having trained and specialized personnel to fill the emerging job vacancies. This is precisely the case in Latin American and Caribbean countries, which cannot fill open positions for the technical personnel required by the labor market, nor the considerable gap in skills and innovation in the region.

Education plays the main role developing the so-called higher-order skills, essential for the 21st century’s labor market. These include foundational literacies (literacy, numeracy, scientific literacy, ICT literacy, financial literacy, cultural and civic literacy), competencies (critical thinking/problem solving, creativity, communication, collaboration), and character qualities (curiosity, initiative, persistence/grit, adaptability, leadership, social and cultural awareness) (World Economic Forum, 2015b).

In rigor, the skills gap is defined by the number of open positions that companies cannot fill and the skills they look for, yet have a hard time finding in potential employees. In Latin American countries, indicators that explain this mismatch are the region’s unequal access to education, a perception of low quality and value of the education and training systems, misalignment between education providers and employers on how workers should be trained, and weak performance on international student tests. (World Economic Forum, 2015a)

In a similar way, in Latin America and the Caribbean there are low levels of innovation, which are measured by “the output of innovative activities (e.g. high-tech exports, patents) and innovative capabilities at the firm level (e.g. capacity to innovate,
innovation investment intensity and value-chain breadth), as well as at the individual level, measured by the number of knowledge-intensive workers and opportunity-driven entrepreneurs” (World Economic Forum, 2015a). For the region, indicators that explain the lagging performance in innovation include low levels of research and development (R&D) and innovation investment, particularly from the private sector, and a shortage of scientists and engineers, the low quality of scientific research institutions and firms’ low absorptive capacity (the ability to adopt and use new technologies). (World Economic Forum, 2015a)

In sum, a lack of trained workforce and a lag developing innovative products are another two consequences of weak educational systems in the region.

Quality of education

The Programme for International Student Assessment (PISA) test records the demographic and social data of 15-year-old students from different countries and economies, including the members of the Organisation for Economic Cooperation and Development (OECD) and some volunteer participants. The test assesses students’ performance in reading, mathematics, and science. According to the Inter-American Development Bank (IADB), “PISA is the principal international test, that currently exists to measure and compare knowledge and skills of young people in the education system” (Bos, Elías, Vegas, & Zoido, 2016b).

In the 2015 edition, 72 countries and economies participated in the test. Those included 10 countries from Latin America and the Caribbean: Argentina, Brazil, Chile, Colombia, Costa Rica, the Dominican Republic, Mexico, Peru, Trinidad and Tobago, and Uruguay. For Argentina, we must clarify that its participation was restricted to the Ciudad Autónoma de Buenos Aires (CABA), its capital.
Appendix 1 shows the results of the PISA 2015 test, sorted in descending order according to the country’s total score (Organisation for Economic Cooperation and Development & Programme for International, Student Assessment, 2016); there, we can observe how Latin American and Caribbean countries lie at the bottom half of the table.

With figures 9, 10 and 11, we illustrate the position of Latin America and the Caribbean compared with the rest of participating countries/economies for each of the topics assessed: science, reading and mathematics.

Figure 9 – Mean score in PISA 2015 – Science.

Figure 10 – Mean score in PISA 2015 – Reading.

*Data retrieved from the Organisation for Economic Co-operation, and Development, & Programme for International, Student Assessment (2016).*

Figure 11 – Mean score in PISA 2015 – Mathematics.

*Data retrieved from the Organisation for Economic Co-operation, and Development, & Programme for International, Student Assessment (2016).*
In general, CABA, Chile and Uruguay obtained the region’s best results, whereas Dominican Republic got the lowest score followed by Peru and Brazil.

The IADB points out the following conclusions about the region performance in PISA 2015 (Bos, Elías, Vegas, & Zoido, 2016a, p. 3):

- Compared to the OECD average, the region’s scores are equivalent to 2.5 years less of schooling or less.
- Chile, Uruguay, Trinidad and Tobago and Costa Rica have scores equivalent to 2 years less of schooling on average compared to the OECD.
- Colombia, Mexico, Brazil and Peru obtain results equivalent to three years less of schooling than the OECD.
- The Dominican Republic is 161 points behind OECD countries, which is equivalent to 5 years of schooling.
- The leaders of the ranking, Singapore and Japan, have scores equivalent to two years more of schooling than the OECD average. Compared to the leading countries in the ranking, the region has scores equivalent to almost 5 years less of schooling. Chile has scores equivalent to 4 years less of schooling than Singapore, and in the Dominican Republic this figure increases to 7 years.
- Compared to countries with a similar level of per capita income, the region’s scores are equivalent to three years less of schooling than Vietnam, demonstrating what can be achieved in countries with similar levels of income.
- Compared to countries with similar levels of spending per student, such as Turkey, there are no significant differences in performance.
In sum, Latin American and Caribbean countries’ scores make evident that the quality of education students receive at schools can be significantly improved.

In the specific area of mathematics, which is the area concerned in this study, the IADB affirms that the curricula of the region do not comply with the levels of clarity, alignment, and rigor characteristic of the OECD and required by international standards. Also, even though the majority of teachers have a university degree or certification from a teacher-training institute, they are not mathematically competent and do not have the proper preparation to offer the possibility of improving their mathematical skills to their students (Valverde, & Näslund-Hadley, 2016).

A closer look to Colombia’s results in mathematics

Colombia’s performance in mathematics has been consistently low. In Figure 12 we present the score obtained by Colombian students in the PISA mathematics test since 2006, the first year when Colombia became a participant country. Additionally, in the figure we may see the maximum and minimum scores obtained by PISA participants per year, as well as the average score obtained by OECD countries.
With respect to the most recent test (2015), PISA’s detailed note on Colombia specifies the following results (Programme for International Student Assessment, 2016, p. 1-3):

- On average, students in Colombia score 390 points in mathematics—below the OECD average and the mean score of Chile (423 points) and Mexico (408 points), comparable with that of Indonesia, Lebanon, and Peru, and above that of Brazil (377 points).
- Colombia’s mean performance (in mathematics) has improved 20 score points since 2006, the seventh largest improvement among the 52 education systems with comparable data.
On average, nearly 23% of students in OECD countries do not attain the baseline level of proficiency in mathematics, considered the level of proficiency at which students start solving the kinds of problems that are routinely faced by adults in their daily lives. In Colombia, 66% of students are low achievers in mathematics, and share that has shrunk by 4 percentage points since 2009.

Boys outperform girls in mathematics by an average of 11 score points. This gap in performance has narrowed by 15 score points since 2012.

The percentage of students in Colombia who have repeated a grade is the second largest (only behind Algeria) among all the countries and economies that participated in PISA 2015.

There is nearly one computer for every student in Colombia – a higher ratio than that observed on average across OECD countries; higher than the ones observed in Chile and Peru, and higher than would be expected given Colombia’s level of spending on education.
Chapter III.
Teaching Approaches

The main goal of the present study was assessing three different teaching methodologies for mathematics classes with Colombian middle schoolers. These methodologies are schema-based instruction, inquiry-based learning and traditional teaching approaches. In this chapter, we present each of the methodologies, as well as descriptions on how we implemented them.

Schema Based Instruction

Using schemas as a strategy to help students solve word problems is one of the techniques we used in this research.

Broadly speaking, a schema “is a generalized description of two or more problems, which students use to group problems into types that require similar solution methods” (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004). Based on this description, we may think about schemas as sets of features allowing one to cluster similar problems together. The goal of creating these categories is to differentiate problems according to the techniques required to solve them; to do so, the defining features of a schema must focus on structural characteristics of the problems (e.g. “all these problems correspond to splitting quantities into equal number of parts”) rather than any narrative features (e.g. “these problems are all about fruits”).

Therefore, defining a schema is not just listing an unorganized set of features: it must also include a solution strategy for the type of problems we want to cluster together. In this way, we should also consider that “a schema is a framework, outline, or plan for solving a problem” (Marshall, 1995).

Combining these two ideas, we adhere to a more extensive definition (Jitendra et al., 2009):

schemas are domain or context specific knowledge structures that organize knowledge and help the learner categorize various problem types to determine the most appropriate actions needed to solve the problem.

Finding similarities among apparently unrelated problems provides a better way to tackle a problem, because students first classify it and later use the solving strategy given for the category they chose. In this way, schemas facilitate solving word problems in the sense that the process of recognizing/understanding the problem comes bound to a solution strategy. From the problem-solving perspective, “students can use schemas to organize information from a word problem in ways that represent the underlying structure of a problem type. Pictures or diagrams, as well as number sentences or equations, can be used to represent schemas.” (Powell, 2011)

As an example, let’s consider the following five schemas proposed by Marshall (1995) for a set of arithmetic problems.
Table 1. A group of schemas for arithmetic problems.

<table>
<thead>
<tr>
<th>Schema</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change</td>
<td>A Change situation characterizes a problem in which there is a permanent alteration over time in the measurable quantity of a particular thing.</td>
<td>Stan had 35 stamps in his stamp collection. His uncle sent him 8 more as a birthday present. How many stamps are now in his collection?</td>
</tr>
<tr>
<td>Group</td>
<td>A Group situation exists whenever a number of small groups are combined meaningfully into a larger one.</td>
<td>In Mr. Harrison's third-grade class, there were 18 boys and 17 girls. How many children are in Mr. Harrison's class?</td>
</tr>
<tr>
<td>Compare</td>
<td>A Compare situation exists whenever two things are contrasted to determine which of them is larger or smaller.</td>
<td>Bill walks a mile in 15 minutes. His brother Tom walks the same distance in 18 minutes. Which one is the faster walker?</td>
</tr>
<tr>
<td>Restate</td>
<td>A Restate situation is present if a specific relationship is described between two different things at a fixed point in time</td>
<td>At the pet store there are twice as many kittens as puppies in the store window. There are 8 kittens in the window. How many puppies are also in the window?</td>
</tr>
<tr>
<td>Vary</td>
<td>A Vary situation exists when a specified relationship connecting two things can be generalized over other manifestations of those things.</td>
<td>Mary bought a package of gum that had 5 sticks of gum in it. How many sticks would she have if she bought 3 packages of gum.</td>
</tr>
</tbody>
</table>

Solid literature from recent years supports the use of schemas to help students solve word problems. We mention some of the studies which were highly influential for our research.

In a 2004 study (Fuchs et al.), the researchers worked with 366 third-grade students and 24 teachers on four different types of problems and compared the standard problem-solving instruction taught to the students against SBI. One year later, the use of SBI with third-grade students with learning disabilities showed positive results in their problem-solving outcomes and the process of transfer (Fuchs & Fuchs, 2005).

Using schemas for multiplicative, compare, and proportion problems showed positive results during a 4-session study from 2005 (Yan, Jitendra, & Deatline-Buchman, 2005). Similarly, a comparison between single-strategy versus SBI with 88 third-grade students evinced significant advantages in using the former when solving standardized SAT questions (Jitendra et al., 2007). The detailed explanation from these studies on how to implement SBI was a definitive guide for our research.

Two years later, Jitendra et al. (2009) worked with 148 seventh-grade students and their teachers. In this case, SBI outperformed the control group both at the immediate posttest as well as in a delayed posttest administered four months later. A four-step strategy was implemented (FOPS; F – Find the problem type, O – Organize the information in the problem using the diagram, P – Plan to solve the problem, S – Solve the problem), and the topics the students worked on were ratios and proportions.

An interesting approach is given by Lynn S. Fuchs et al (2010) with respect broadening SBI to different types of problems. They worked with second graders solving word problems and they studied how the students tried to anticipate algebraic thinking.
Most of the initial research using SBI aimed to students from elementary school. One of the most comprehensive studies with middle-schoolers involved problems of ratios, proportion, scale drawings, and percentages (Jitendra, Star, Rodriguez, Lindell, & Someki, 2011) with seventh-graders. Even though SBI presented favorable results in comparison with traditional teaching approaches, these improvements did not last a month later when SBI was no longer used.

The use of schemas to solve word problems was further assessed as a tool for creating prealgebraic knowledge, and it was compared with training on calculations and regular instruction in 2014 (Fuchs et al., 2014), as a continuation to the study by Lynn S. Fuchs et al (2010).

Implementation

The study group using the SBI approach worked with three different types of schemas: tree, list, and grouping. In Table 2 we present a description for each schema and an example.
Table 2. Schemas used in the study.

<table>
<thead>
<tr>
<th>Schema</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree</td>
<td>There are two or more decisions to be made and each of them have a (possibly different) number of options to choose from.</td>
<td>Jeff has five different pairs of socks and three pairs of shoes. How many possible combinations are there?</td>
</tr>
<tr>
<td>List</td>
<td>There are different items. Some of them will be selected and put into an ordered list</td>
<td>The softball coach needs to determine how many different batting lineups she can make out of her first three batters, Able, Baker, and Chan. How many different batting orders are there?</td>
</tr>
<tr>
<td>Grouping</td>
<td>There are different items. Some of them will be chosen regardless the order of choice.</td>
<td>Cesar the dog-walker has 6 dogs but only 2 leashes. How many different ways can Cesar take a walk with all 2 dogs at once?</td>
</tr>
</tbody>
</table>

Schemas corresponding to problems of tree diagrams, permutations, and combinations.

Appendix 2 includes the complete SBI Instructor’s Guide. There, it is summarized the training that participant teachers who worked with this methodology went through and the structure of the classes they taught.

As a summary, a teacher using SBI had to adhere to the following recommendations which were adapted from the works of Jitendra, Star, Dupuis, Rodriguez (2013), and Jitendra et al. (2016).
Instructor’s Guidelines

Please adhere to the following steps when you solve a problem with your students:

1. Prioritize on reading the problems in order to recognize its mathematical structure. Stimulate students’ thinking about how problems within and across types are similar and different.
2. Visually map information in the problem using schematic diagrams.
3. Give explicit instructions about the steps to solve the problem. Use the DISC heuristic:
   a. Discover the problem type.
   b. Identify information in the problem to represent in a diagram.
   c. Solve the problem.
   d. Check the solution.

Accompany with deep-level questions each step in the heuristic (e.g., Why this is a proportion problem? How is this problem similar to or different from one I already solved?)

4. Develop students’ procedural flexibility, including explicit teaching of multiple solution methods.

Each problem set is divided into two sections. The first section (labeled as Sample Problems) is meant to be solved by the teacher on the board. Please do this in a clear and detailed way, at slow pace, following the four steps mentioned above.

Then, let your students work individually on the problems labeled as Practice Problems. You should move through the classroom helping students and encouraging them to use the DISC heuristic on every problem. Students who finish all the problems from the section may discuss it with their peers. Do not advance to the next part of the guide until all students have correctly solved all the problems from the section Practice Problems. If you complete all the problems from all the sections before the the Camp is done, you may use the remaining time to work on the problems from the Entrance Test.

Figure 13. Instructor’s guidelines for SBI.

*Adapted from Jitendra, Star, Dupuis, Rodriguez (2013), and Jitendra et al. (2016).*

Teachers were highly encouraged to use visual representations for the schemas. Specifically, the SBI Instructor’s Guide included Figure 14.
Participant teachers who worked as observers and reviewers had to check the level of adherence to the guidelines of teachers working with the SBI study group and verify the level of fidelity of their practice.
Inquiry Based Learning

Learning by inquiry was a strong educational movement started during the later part of the 20th century, encompassing many publications, research and educational policies worldwide. In this section, we briefly describe some important components of inquiry-based learning (IBL) and how we implemented this teaching approach in our study.

As a broad definition, we may say inquiry-based teaching corresponds to different ways of instruction where students “are presented with questions to be answered, problems to be solved, or a set of observations to be explained” (Prince & Felder, 2006). Hence, IBL is a form of active learning where students explore, search, and question, as they are intended to be in control of their learning process rather than passive recipients of information. With this intention, IBL “promotes the acquisition of new knowledge, abilities, and attitudes through students’ increasingly independent investigation of questions, problems, and issues, for which there often is no single answer” (Lee & Wehlburg, 2012).

Efforts to create class scenarios where students face a disequilibrium experience and define the next steps of their learning paths are commonly traced back to Piaget’s formalization of constructivism. However, even before that some instructional models applying IBL were developed and applied… with the of lack of key elements of metacognitive reflection as a serious drawback (Marshall, Horton, & Smart, 2009).

The use of inquiry as a learning drive force is at least as old as the Socratic method. More recently, in the early 19th century, German philosopher Johann Friedrich Herbart proposed science classes with students exposed to rich experiences and new
ideas, allowing them to find the relationships between both (Herbart, 1894), placing inquiry processes as definitive components of his pedagogy. His instructional model included steps dedicated to exploring and seeking connections, and which were later discussed in Dewey’s reflections on recitation (Dewey, 1909).

For Dewey, the pivotal role of inquiry in learning and instruction was a fundamental principle. In his own words, (Dewey, 1966)

where children are engaged in doing things and in discussing what arises in the course of their doing, it is found, even with comparatively indifferent modes of instruction, that children's inquiries are spontaneous and numerous, and the proposals of solution advanced, varied, and ingenious.

Dewey describes the process of inquiry with his five distinct steps in reflection: a felt difficulty; its location and definition; suggestion of possible solution; development by reasoning of the bearings of the suggestion; further observation and experiment leading to its acceptance or rejection (Dewey, 1909). The most common instructional frameworks developed later to promote IBL in classrooms are based on these five steps (Behrenbruch, 2012).

Bruner goes further in the development of constructivism asserting that people are actively looking for sense from reality and not just receiving information as passive agents. Moreover, he suggests building a curriculum which profits from the intuitive ideas kids grasp at early ages and start building from that point. (Bruner, 1960)

A more precise description of an inquiry-based instruction comes with Vygotsky’s zone of proximal development. There, the interaction of a kid with an adult or the social environment could promote the kid’s thinking, reasoning and development via problem-solving. (Vygotsky, 1978)
Implementation

The design of the IBL classes was influenced by Behrenbruch’s essential elements of an inquiry-based classroom (2012, p. 111-130):

1. Discussion with its greater implications of discourse
2. Social mediation of learning
3. The importance of planning
4. Valuing uncertainty
5. Reflection ~ action
6. Respect

Each of these elements comes with a set of recommendations to help teachers planning for purposes, planning for possibilities, and keeping the inquiry environment alive. Those were the principal components we considered to work with the students in the IBL classroom in this research.

Since IBL environments may have different levels of teacher’s guidance, going from giving a lot of instructions or pseudo-inquiry to scenarios mostly designed by the students, we opted for using a version of inquiry guided learning where teacher’s participation was frequent but never in the form of lectures. Instead, the classes had a feed of assignments the students tried to solve in groups of 4 while the instructor monitored each group’s work asking relevant questions to hint the solutions.

Table 3 shows Lee’s description of IBL scenarios (2011). With this classification, we opted for the option number 5.
Table 3. Selected patterns found in IBL courses.

<table>
<thead>
<tr>
<th>Type</th>
<th>No</th>
<th>Pattern</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudo-IBL</td>
<td>1</td>
<td>K, K, K, K, K, K, I</td>
<td>A very traditional course with a final inquiry-like project, often a research paper for which students have not been prepared.</td>
</tr>
<tr>
<td>Emerging IBL</td>
<td>2</td>
<td>K, i1, K, K, i2, K, K, i3</td>
<td>Instructor experimentation with inquiry by introducing inquiry exercises as in-class activities or assignments.</td>
</tr>
<tr>
<td>Guided inquiry</td>
<td>3</td>
<td>K, i1, i2, I, K, i3, i4, I</td>
<td>A series of units each built around an inquiry experience, structured by the instructor, for which students have been prepared through presentation of relevant content and inquiry skills development.</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>K, K, K, i1, i2, i3, I</td>
<td>A final inquiry experience, perhaps with some opportunity for student choice and design, for which students have been prepared through presentation of relevant content and inquiry skills development.</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Ia, Ib, Ic, Id, Ie, I</td>
<td>The course as a series of feeder assignments, designed by the instructor and on which students receive feedback, leading up to a final inquiry experience.</td>
</tr>
<tr>
<td>Inquiry</td>
<td>6</td>
<td>I, K, i1, i2, I, K, i3, i4, …</td>
<td>A series of inquiry experiences, each designed to address a targeted content area and to develop the skills of inquiry. Typical of problem-based learning.</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>I, K, i1, i2, i3, i4, …</td>
<td>An inquiry experience, perhaps student designed, through which students acquire content relevant to the inquiry and further develop the skills of inquiry.</td>
</tr>
</tbody>
</table>

K: presentation of knowledge/content; I: inquiry; Ia: feeder assignment to inquiry; i1: guided inquiry skill development. (Lee, 2011, p. 156)
Additionally, we used as reference Kuhlthau’s six principles of guided inquiry for the class dynamic (2015, p. 24-28):

1. Children learn by being actively engaged in and reflecting on an experience
2. Children learn by building on what they already know
3. Children develop higher order thinking through guidance at critical points in the learning process
4. Children have different ways and modes of learning
5. Children learn through social interaction with others
6. Children learn through instruction and experience in accord with their cognitive development

Appendix 3 includes the complete IBL Instructor’s Guide. There, it is summarized the training that participant teachers who worked with this methodology went through and the structure of the classes they taught.
Instructor’s Guidelines

- There are no lectures at the beginning of the classes. Students will try to solve the problems and the teacher’s role is limited to give them hints to direct their thinking process.
- Students will work in groups of 3 or 4 (no more than 4 students per group in order to avoid distractions.)
- The teacher walks from group to group asking the students what are they thinking about the problem they are solving and setting small questions which could direct the group toward the final answer.
- The teacher is ready to catch any good idea proposed by the students which happens to be useful to solve the problem. In this situation, writing down the idea in the blackboard helps a lot all the groups to benefit from it.
- There is no rush. It is quite important that students elaborate their own paths to the solution.
- The teacher uses the ideas suggested by the students to give shape to a path to solve the problem. Doing so, he or she helps the students to build up and figure out the final answer.
- Most of the ideas proposed by the students will not be useful to solve the problem. In this situations, the teacher should not dismiss immediately the idea, but rather he or she should ask the rest of the class what they think about it.
- Whenever students seems to accept an incorrect idea and they are unable to reject it, the teacher could ask specific questions to make evident why this idea will not work.
- In some cases, the teacher could take an incorrect idea, slightly change it and transform it into a good idea to solve the problem. Generally, this can be done asking questions in the form “what if...?”.
- The teacher should be quite eloquent with his or her non-verbal language. He or she shows enthusiasm when students are getting closer to the answer. The teacher makes evident his or her amusement when students take a promising track.
- Once the class gets the answer to the problem, the teacher should do a review of the whole process students underwent to find the solution, highlighting the definitive steps which made progress towards the answer. Using the elements, ideas and concepts given by the students, the teacher builds up and explain with detail a polished way to solve the problem.
- In some situations, students will be quiet and they will not propose any ideas at all. Do not be tempted by the possibility of solve the problem by yourself and “teach them” how to do it. Instead of that, you could ask many simple short questions to promote students’ participation. Suggest them to do a drawing, make a list, try with some examples or think about particular cases. In some extreme situations, you may even suggest them an idea in the form of asking them a question.
- The classroom will not be a quiet space. Students will propose, discuss, use the blackboard, and ask questions all the time.

Figure 13. Instructor’s guidelines for SBI.

Adapted from Behrenbruch (2012) and Kuhlthau (2015)

Traditional Teaching

Defining traditional teaching in any determined country is not an easy task. To do so, we describe some historical features of teaching mathematics in Colombia, as well as a recent program implemented by the Colombian Ministry of Education (MEN) aimed at
improving the quality of education in the lowest performing regions. We conclude that both the historical methodologies as well as the quality program by MEN make use of direct instruction (DI) as its foundational method of teaching. At the end, we explain how we designed and implemented Direct Instruction in this study.

Education in Colombia since 1900

The history of education in Colombia is an extensive and complex topic. Because of this, we restrict ourselves to describe some features of primary and secondary education in Colombia during the 20th century and how they determined the process of teaching in general (and classes of mathematics in particular).

During the first half of the 20th century, most Colombian schools only offered the first few years of education, and large amounts of children at school age did not attend regularly to classes. Indeed, elementary schools at urban areas consisted of six years, while at rural areas it was limited to three. However, more than 80% of the whole population lived in rural areas and only half of the children there attended classes at some point (Helg, 1984).

Following directions by the Catholic Church, students had to be separated by gender. While in urban areas this led to the creation of schools for either boys or girls, it was not uncommon for rural populations to have only one classroom. Therefore, boys attended school three days of the week, while girls attended the other three (there were no classes on Sundays). As a result, the three years of elementary education were reduced by half. Additionally, having only one classroom implied teachers were meant to simultaneously teach students from the three different grades, with class sizes varying from 50 to 80 children (Rosario, Scott, & Vogeli, 2014).
Following similar gender policies, only female teachers were allowed to teach girls, while boys could be taught by male or female instructors. This led to teaching becoming (mostly) a job for women, especially in rural areas. Hiring requirements were low, a signed approval from the local priest being the most important one. Whereas teachers in rural areas rarely exceeded an elementary-school level of instruction, close to urban centers 38% of male teachers and 56% of female teachers had some pedagogical instruction (Helg, 1984).

Arithmetic was one of the compulsory subjects at elementary schools. During the first three grades, it consisted of the four basic operations with whole numbers. Advanced topics like fractions, proportions, and basic geometry were taught from fourth to sixth grade. Assessments were oral tests in the presence of the parents, who were rarely involved in what the kids learnt at school. Physical punishment was frequent and, giving the lack of books, notebooks or any other resources, the instruction had a strong emphasis on memorization (Rosario, Scott, & Vogeli, 2014).

An important impulse for teaching instruction took place with the foundation in 1936 of the Escuela Normal Superior (ENS), which joined together the three existing faculties of education in the country and intended to improve the quality of secondary-education teachers. The premises of the ENS tried to move teacher instruction apart from the inductive, scholastic, memory-based methods towards a stronger emphasis on biological and physical sciences, the inclusion of pedagogy as a subject on its own, and interdisciplinary studies with the humanities (Ospina, 1984). By the 1950s, more “normal schools,” whose curriculum took more distance from religious discourses, were established.
With the intention of providing education to more children, schools in the 1960s established two sessions: some of the students attended to schools only in the mornings, while others did that in the afternoons. Even though this plan increased enrolment rates, it brought the high toll of a reduced number of hours of schooling per kid (Rosario, Scott, & Vogeli, 2014). At the time of writing, the Colombian government is still working on implementing a unique, complete session nationwide (Ministerio de Educación Nacional, 2013).

During the aforementioned decades, teaching as a profession had low social recognition in Colombia given the meager salaries and humble origins of most teachers (Rosario, Scott, & Vogeli, 2014). Additionally, the few requirements to be appointed as a teacher set low professional standards that are still nowadays visible. In 21st-century Colombia, the students with the lowest academic performance are the ones more likely to end up at a teaching career: specifically, the probability that a student with low scores in Colombia’s standardized tests becomes a teacher is five times the probability that a student with high scores does the same (Barón, Bonilla, Cardona-Sosa, & Ospina, 2013).

While some attempts at creating standards/school inspections appeared during the 1940s, it was only during the 1970s when the quality of education became a concern for the newly founded German Pedagogical Mission (GPM). They forged teaching guides and class material to be used nationwide, as well as instructing inspectors and teachers on how to use them. Unfortunately, because of its differences with the Ministry of Education, the GPM’s purposes were not concluded, an unfortunate development followed by a noticeable lack of continuity in any quality program started before the 1990s (Molano Camargo, 2011).
In 1990, Professor Alonso Takahashi pointed out that Colombian high-schoolers had deep deficiencies in their mathematical knowledge because the instruction they received was exclusively based on less-relevant issues, such as formal procedures and terminology. He states that teachers lacked basic training on arithmetic, equations, problem solving, statistics, and applying mathematics to other areas, while the curriculums of public schools were designed irresponsibly (Departamento Nacional de Planeación, Bogotá (Colombia) Misión de Ciencia y Tecnología, 1990).

Takahashi’s statements are supported by the IADB’s description of Colombian and Latin American pedagogical models used for teaching mathematics, which focus on teachers delivering content and ignore the development of students’ reasoning (Valverde, & Näslund-Hadley, 2016). The IADB also mentions how Colombian teachers have many shortcomings in mathematics, yet tend to blame institutions or contextual factors for their students’ low results (Agudelo-Valderrama, Clarke, and Bishop 2007).

More recently, in 2012, Professors Albis and Sánchez assert that the situation of mathematics education in Colombia has not improved since Takahashi. According to them, teaching mathematics in the country is completely based on instructing students to stick to procedures and use formulas, devoid of context or justification (Albis & Sánchez, 2012).

Implementation

We adhere to the description of traditional classes of mathematics in Colombian schools proposed by Andrade, Perry, Fernández Hernández, and Guacaneme Suárez (2003). They refer to a study describing how classes of mathematics in Bogotá had the following features (Perry, Valero, Castro, Gómez, & Agudelo, 1998, p.17):
1. Routinely class activities. All the classes follow a clear structure: review of the assignment from the last class, the teacher introduces a new topic and presents some examples, the students get exercises to work by themselves.

2. Interaction between teacher and students. The class talking is rigorously controlled by the teacher following the pattern question-answer-evaluation.

3. Focus on the algorithmic part of mathematics. Mathematics is presented as facts and procedures to be done. Usually, mathematics is just viewed as a set of propositions and algorithms.

4. The authority is centered in the teacher and the textbook.

5. The process of teaching consists of a transfer of information from the teacher to the students.

6. The process of learning is considered accomplished when students can replicate the procedures and get right answers.

Colombia’s current policy about school-age mathematics content is determined by the Basic Standards of Mathematical Skills (Ministerio de Educación Nacional, 2006) and the Basic Learning Rights (Ministerio de Educación Nacional, 2017). Public and private schools must fulfill the topics and skills described in these documents. However, according to Law 115 of 1994, commonly known as General Law of Education, educational institutions have full autonomy to implement the pedagogical settings they prefer (Congreso de la República de Colombia, 1994).

In our study, we call traditional teaching approach to the use of direct instruction (DI) as it is described by Perry, Valero, Castro, Gómez, & Agudelo (1998). Additionally, with the intention of making use of more recent sources on how the teaching of
mathematics in Colombia takes place, we used as reference a current program implemented by the Ministry of Education aimed to improve the quality of mathematics instruction in depressed regions.

Namely, the program Classroom without Borders contains guides for classes, recommendations for teaching, and complete sets of activities to be used with the students. They also have timed and structured formats for classes with specific moments for teacher’s introduction of new topics, explanations, examples, and students work (Ministerio de Educación Nacional, 2016a). We made use of these class formats and designed classes applying DI.

Appendix 4 includes the complete DI Instructor’s Guide. There, we summarized the training that participant teachers who worked with this methodology went through and the structure of the classes they taught. Specifically, we present in Figures 14 the format of classes.
<table>
<thead>
<tr>
<th>Stage</th>
<th>Description of the activity</th>
<th>Comments and suggestions</th>
<th>Class distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>10 min: Present the class’ structure</td>
<td>In Activities, state that you will do an initial explanation followed by some examples, and later the students will work with their Student’s Guides.</td>
<td>Options: Individual, Groups of X, Discussion, Debate, Lecture, etc.</td>
</tr>
<tr>
<td>Explanation</td>
<td>30 min:</td>
<td>Write down in this box just the keyword of the topic or concept you will explain</td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>30 min:</td>
<td>Make a log about what problems the students were able to solve and which ones they could not.</td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td>10 min:</td>
<td>Finish the class writing down the main ideas and concepts you taught.</td>
<td></td>
</tr>
</tbody>
</table>

Figure 14. Class template for the DI group.

Adapted from Ministerio de Educación Nacional (2016a)
Chapter IV.

Methodology

In this chapter we describe how we designed the study aimed to compare the three teaching approaches mentioned before.

Population and sample

First, we define the criteria used in this research for participant students and teachers.

Population

In order to determine the specific population for the study, we had to answer three different questions:

1. Which educational level or grade are we going to work with?
2. Where exactly in Colombia is the study going to take place?
3. What type of schools (private or public) are we going to work with?

To answer the first question, we need to mention how the Colombian education system is organized. As a whole, education in Colombia is regulated by Law 115 from February 8, 1994 (Congreso de la República de Colombia, 1994). In Article 11, Law 115 establishes that Colombian formal school education consists of three different levels: Preschool, which consists of at least one compulsory grade; Basic, which consists of nine grades; and Middle, which consists of two grades.
Years corresponding to Preschool Level do not share the same name across schools, so that the initial grade of the Basic Level is the one regularly named first grade. This implies that the last grade in Colombian formal school education is eleventh grade.

However, we must take into account that, since Preschool Level consists of at least one grade, for Colombian students eleventh grade is at the very least their twelfth year of education.

Additionally, the Basic Level is divided into two sub-levels: Elementary, corresponding to the first five grades, and Secondary, corresponding to the last four. Table 4 illustrates said structure.

Table 4. Structure of formal school education in Colombia

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>SUB-LEVEL</th>
<th>GRADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preschool Level</td>
<td></td>
<td>(at least one grade)</td>
</tr>
<tr>
<td>Basic Level</td>
<td>Elementary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1st Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2nd Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3rd Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4th Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5th Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6th Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7th Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8th Grade</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9th Grade</td>
<td></td>
</tr>
</tbody>
</table>
At the end, we opted for working with the sixth-grade students. This choice was based on the data mentioned in the previous chapter, regarding how the high rates of completion for elementary education are interrupted by considerable dropout rates during the first years of secondary education. As was described in previous paragraphs, elementary education in Colombia goes up to fifth grade, so the first grade of secondary educational level corresponds to sixth grade.

With respect to the second question, we chose to work with students from urban areas, more specifically, from Bogotá; both logistic and administrative restrictions were the main reasons behind this decision. As a result, we strongly encourage researchers to replicate this study at different Colombian cities as well as rural areas. Interestingly, Bogotá has 8.081 million inhabitants, accounting for 16.61% of the country’s population. Regardless of its limitations, the fact that the study targets a relatively large group of people may reassure us.

Finally, answering the third question, we opted to work with students from public schools. This decision was completely based on the marked differences observed between the academic results from private and public schools in Colombia. Indeed, Colombian private schools show significantly better results than public schools in both national and international assessments (Steiner, Cadena, & Pardo, 2002). This disparity can be observed, in a similar fashion, at many Latin American countries (Pereyra, 2008). In case
of Colombia, the difference between academic successes between private and public schools is rooted in environmental factors, such as the parents’ income, the school infrastructure, the presence of laboratories, libraries, and sport facilities, among others (Iregui, Melo, & Ramos, 2007).

To summarize, the population we worked with in this study were sixth grade Colombian students from public schools in Bogotá.

Sample

The participants of the study were 99 sixth-grade students and five teachers of mathematics from three public schools located in Bogotá.

Reviewing the publicly available data regarding the academic results of Colombian schools through the years 2014, 2015 and 2016, we found out the three schools participating in this study were classified as the top three categories of Colombian performers.

To clarify, twice a year, the Colombian Institute for the Evaluation of the Quality of Education (ICFES) classifies Colombian schools based on their results in one of the national standardized tests. This classification depends on each school’s General Index (Gi), which is a value between 0 and 1 calculated using the test scores of each school’s top 80% students, their mean, and variance. Likewise, only schools graduating 9 students or more per year are scored with a General Index. (Ministerio de Educación Nacional, 2016b)

Table 5 and Figure 15 show the categories ICFES uses, as well as the percent of Colombian public schools in each category for 2016.
Table 5. Classification of Colombian schools.

<table>
<thead>
<tr>
<th>Category</th>
<th>General Index (Gi) values</th>
<th>Percentage of public schools in each category (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A+</td>
<td>0.77 &lt; Gi</td>
<td>1.51%</td>
</tr>
<tr>
<td>A</td>
<td>0.72 &lt; Gi ≤ 0.77</td>
<td>8.53%</td>
</tr>
<tr>
<td>B</td>
<td>0.67 &lt; Gi ≤ 0.72</td>
<td>30.98%</td>
</tr>
<tr>
<td>C</td>
<td>0.62 &lt; Gi ≤ 0.67</td>
<td>32.96%</td>
</tr>
<tr>
<td>D</td>
<td>0 ≤ Gi ≤ 0.62</td>
<td>26.02%</td>
</tr>
</tbody>
</table>

Data retrieved from Ministerio de Educación Nacional (2016b, 2016c)

Figure 15. Number and percentage of schools in each category.

The top three categories are highlighted in yellow. Source: Ministerio de Educación Nacional (2016c).

Consistently, the three participating schools in the study were ranked in categories A and B during 2014, 2015 and 2016. In particular, we notice how in 2016 the top three
categories (A+, A and B) accounted for only 32.66% of all public schools. This remark is relevant because, even though we initially targeted a somewhat general population consisting of Sixth-Grade students from public schools located in Bogotá, our sample is not representative of such a large group.

Study groups

Participating students were randomly assigned to one of three different groups: schema-based instruction (SBI), inquiry-based learning (IBL), and direct instruction (DI). One of the teachers was randomly assigned Participant teachers got 10 hours of training on the mathematics content to be taught and the teaching methodologies they were going to use during the experiment. Each teacher was trained in the single teaching methodology he or she would use with their students.

Each group of students got 9 hours of classes covering the same topics, examples, and exercises, the main difference between the three groups being the teaching methodology used by the instructors.

Mathematics Content

The topic we decided to work with in this research was an introduction to counting and probability; more specifically, the multiplication principle, permutations and combinations. These three topics were meant to be presented from a problem-solving perspective: all the participating teachers introduced the ideas as potential solutions for real-life counting problems, a goal around which the whole training was centered.

We started by identifying how the topics mentioned above are meant to be covered and presented according to the current regulations dictated by the Colombian
Ministry of Education. In this aspect, there are two official references: Basic Standards of Mathematical Skills (Ministerio de Educación Nacional, 2006) and Basic Learning Rights (Ministerio de Educación Nacional, 2017).

Standards

The chosen topics are related to the following items from the Basic Standards of Mathematical Skills in the category Probability and Data Thinking (Ministerio de Educación Nacional, 2006):

- Grades 6-7. Use models (for example, tree diagrams) to discuss and predict the possibility of an event.
- Grades 8-9. Calculate the probability of simple events using different techniques (lists, tree diagrams, counting techniques).
- Grades 10-11. Solve problems using basic concepts from counting and probability (combinations, permutations, sample space, random sampling, sampling with replacement).

On the other hand, Basic Learning Rights include the following items related with the topics we worked in our experiment (Ministerio de Educación Nacional, 2017):

- Grade 5. Basic Right of Learning Mathematics 15. Understand the probability of obtaining results in simple events.
- Grade 7. Basic Right of Learning Mathematics 13. Understand the difference between theoretical probability and the outcome of an experiment.
  - Relate probabilities with fractions and percentages.
  - Use tree diagrams to calculate the probability of an event.
• Grade 9. Basic Right of Learning Mathematics 15. Solve problems using basic principles of counting (addition and multiplication).


In practice, students from sixth grade are not required to learn the three topics (multiplication principle, combinations, and permutations) by the end of the year. However, the prerequisites for these topics being introduced are mastering addition and multiplication with whole numbers and some experience solving word problems, both covered in Elementary School.

Related standards used by schools in USA are the following (Common Core State Standards Initiative, 2016):

CCSS.MATH.CONTENT.7.SP.C.8.B. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

CCSS.MATH.CONTENT.7.SP.C.8.A. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

CCSS.MATH.CONTENT.HSS.CP.B.9. Use permutations and combinations to compute probabilities of compound events and solve problems.
Source material

The textbooks, examples, and exercises worked in the study were obtained from the open books from the CK-12 Foundation (2017a; 2017b). Beyond the possibility of free content, these books feature Spanish translations and enough solved examples so these became a pretty standard reference to work with.

Entrance and exit tests

Both entrance and exit tests were designed to contain four questions from each of the topics selected: multiplication principle, permutations and combinations. Therefore, the tests featured 12 questions, meant to be solved in 30 minutes. Said questions were selected from the aforementioned source material; while some of them were solved examples, others were proposed exercises.

Appendixes 5 and 6 include the full entrance and exit tests the students presented. To assess the tests, we used the four-point rubric proposed by Van de Walle (2010) which is presented in Table 6.
Table 6. Van de Walle’s four-point rubric.

<table>
<thead>
<tr>
<th>Got It</th>
<th>Not Yet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence shows that the student understands the target concept or idea.</td>
<td>Student shows evidence of major misunderstanding, an incorrect concept or procedure, or failure to engage in the task.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent: Full Accomplishment</td>
<td>Proficient: Substantial Accomplishment</td>
<td>Marginal: Partial Accomplishment</td>
<td>Unsatisfactory: Little Accomplishment</td>
</tr>
<tr>
<td>Strategy and execution meet the content, process, and qualitative demands of the task.</td>
<td>Could work to full accomplishment with minimal feedback. Errors are minor, so teacher is confident that understanding is adequate to accomplish the objective.</td>
<td>Part of the task is accomplished, but there is a lack of evidence of understanding or evidence of not understanding.</td>
<td>The task is attempted, and some mathematical effort is made. There may be fragments of accomplishment, but there is little or no success.</td>
</tr>
<tr>
<td>Communication is judged by effectiveness, not length. May have minor errors.</td>
<td>Communication is judged by effectiveness, not length. May have minor errors.</td>
<td>Direct input or further teaching is required.</td>
<td>Direct input or further teaching is required.</td>
</tr>
</tbody>
</table>

(Van de Walle, 2010, p. 81)
Chapter V.

Results and Conclusions

In this chapter we present the quantitative results we got with the different assessment tools used in the study.

Analysis of the test results

In Table 7, we present the results of the entry and exit tests for each of the treatment groups: IBL, DI and SBI. For each of the groups, two vertical rows contain the results obtained by each single student in their entry and exit test. All the tests are graded on a 100-point scale.

Table 7. Entry and exit tests results.

<table>
<thead>
<tr>
<th></th>
<th>IBL</th>
<th></th>
<th>DI</th>
<th></th>
<th>SBI</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Exit</td>
<td>Entry</td>
<td>Exit</td>
<td>Entry</td>
<td>Exit</td>
<td>Exit</td>
</tr>
<tr>
<td>test</td>
<td>test</td>
<td>test</td>
<td>test</td>
<td>test</td>
<td>test</td>
<td>test</td>
</tr>
<tr>
<td>29.17</td>
<td>45.83</td>
<td>14.58</td>
<td>62.50</td>
<td>33.33</td>
<td>81.25</td>
<td></td>
</tr>
<tr>
<td>10.42</td>
<td>43.75</td>
<td>2.08</td>
<td>43.75</td>
<td>25.00</td>
<td>91.67</td>
<td></td>
</tr>
<tr>
<td>10.42</td>
<td>29.17</td>
<td>4.17</td>
<td>45.83</td>
<td>33.33</td>
<td>89.58</td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>39.58</td>
<td>16.67</td>
<td>70.83</td>
<td>18.75</td>
<td>56.25</td>
<td></td>
</tr>
<tr>
<td>16.67</td>
<td>20.83</td>
<td>18.75</td>
<td>62.50</td>
<td>22.92</td>
<td>45.83</td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>29.17</td>
<td>0.00</td>
<td>31.25</td>
<td>29.17</td>
<td>89.58</td>
<td></td>
</tr>
<tr>
<td>10.42</td>
<td>22.92</td>
<td>10.42</td>
<td>39.58</td>
<td>4.17</td>
<td>79.17</td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>14.58</td>
<td>33.33</td>
<td>77.08</td>
<td>47.92</td>
<td>68.75</td>
<td></td>
</tr>
<tr>
<td>41.67</td>
<td>62.50</td>
<td>10.42</td>
<td>75.00</td>
<td>22.92</td>
<td>79.17</td>
<td></td>
</tr>
<tr>
<td>16.67</td>
<td>31.25</td>
<td>45.83</td>
<td>39.58</td>
<td>45.83</td>
<td>95.83</td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>35.42</td>
<td>6.25</td>
<td>43.75</td>
<td>8.33</td>
<td>33.33</td>
<td></td>
</tr>
<tr>
<td>18.75</td>
<td>31.25</td>
<td>10.42</td>
<td>68.75</td>
<td>22.92</td>
<td>56.25</td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>33.33</td>
<td>31.25</td>
<td>58.33</td>
<td>27.08</td>
<td>39.58</td>
<td></td>
</tr>
<tr>
<td>10.42</td>
<td>27.08</td>
<td>50.00</td>
<td>93.75</td>
<td>27.08</td>
<td>52.08</td>
<td></td>
</tr>
</tbody>
</table>
**Descriptive statistics on the tests results. Tests are graded on a 100-point scale.**

**Entry test results**

Results from the entry test are considerably low for the three treatment groups.

Indeed, boxplots for the results of the entry test of the whole sample of students and each of the study group are presented in Figure 16.
Figure 16. Entry Test Results.

*Boxplots calculated with data from Table 7.*

Getting low results at the entry test is an expected scenario. In fact, Colombian Sixth-Grade students are not expected to have any familiarity with permutations, combinations nor tree diagrams. The few outliers with scores above 50 developed their own methods to count the results and solve the exercises; most of them simply listed all the possible outcomes of the situation described in the problem.

Additionally, from Table 7 we observe the entry tests of IBL, DI and SBI have mean values of 18.53, 19.54 and 24.17, respectively. This difference comes with significantly high standard deviations: 10.84, 17.19, and 10.97. An ANOVA test for the entry test results of the study groups shows that the variability of the three means can be explained by the variance within groups (tables 8, 9, 10).
Table 8. Summary of the entry test results.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Count</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBL Entry</td>
<td>29</td>
<td>537.5000</td>
<td>18.5345</td>
<td>117.6044</td>
</tr>
<tr>
<td>DI Entry</td>
<td>21</td>
<td>410.4167</td>
<td>19.5437</td>
<td>295.3456</td>
</tr>
<tr>
<td>SBI Entry</td>
<td>20</td>
<td>483.3333</td>
<td>24.1667</td>
<td>120.3399</td>
</tr>
</tbody>
</table>

*Calculated from Table 7.*

Table 9. ANOVA of the entry test.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>399.9292</td>
<td>2</td>
<td>199.9646</td>
<td>1.1664</td>
<td>0.3177</td>
<td>3.1338</td>
</tr>
<tr>
<td>Within Groups</td>
<td>11486.2936</td>
<td>67</td>
<td>171.4372</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11886.2227</td>
<td>69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Calculated from Table 7.*

Table W. Pairwise comparison.

<table>
<thead>
<tr>
<th>Group A</th>
<th>IBL Entry</th>
<th>IBL Entry</th>
<th>DI Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group B</td>
<td>DI Entry</td>
<td>SBI Entry</td>
<td>SBI Entry</td>
</tr>
<tr>
<td>I/Count_A + I/Count_B</td>
<td>0.0821</td>
<td>0.0845</td>
<td>0.0976</td>
</tr>
<tr>
<td>S.E. of A and B</td>
<td>3.7517</td>
<td>3.8057</td>
<td>4.0909</td>
</tr>
<tr>
<td>ANOVA t-test</td>
<td>-0.2690</td>
<td>-1.4799</td>
<td>-1.1301</td>
</tr>
<tr>
<td>Df</td>
<td>67</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>t critical</td>
<td>1.9950</td>
<td>1.9950</td>
<td>1.9950</td>
</tr>
<tr>
<td>Bonferroni t critical</td>
<td>2.4500</td>
<td>2.4500</td>
<td>2.4500</td>
</tr>
<tr>
<td>Null hypothesis (mean_A = mean_B)</td>
<td>Accepted</td>
<td>Accepted</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

*Comparison between each pair of groups. We accept the null hypothesis stating that the means of the entry test results of all three groups were equal. Calculated from tables 8 and 9.*
Therefore, the three study groups showed no statistically significant difference between the means of the entry test results.

Exit test results and analysis of differences

Even though the mean scores of the entry test for the three study groups are not significantly different, we got a different situation with the exit test. Rather than calculating the mean score of the exit test for each group, we opted for calculating the difference between the mean scores of the exit and entry tests. Doing that, we are going to compare the change in the mean score each study group underwent after the intervention.

From Table 7 we may calculate the difference between the exit and entry tests result for all observations. We observe in Table 10 that the mean of the difference between the tests have values of 19.6839, 44.4444, and 42.2917 for IBL, DI, and SBI respectively. It seems that IBL produced the least effect between the entry and exit test.

Table 10. Difference between the exit and entry test for each group.

<table>
<thead>
<tr>
<th>IBL Difference</th>
<th>DI Difference</th>
<th>SBI Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.6667</td>
<td>47.9167</td>
<td>47.9167</td>
</tr>
<tr>
<td>33.3333</td>
<td>41.6667</td>
<td>66.6667</td>
</tr>
<tr>
<td>18.7500</td>
<td>41.6667</td>
<td>56.2500</td>
</tr>
<tr>
<td>27.0833</td>
<td>54.1667</td>
<td>37.5000</td>
</tr>
<tr>
<td>4.1667</td>
<td>43.7500</td>
<td>22.9167</td>
</tr>
<tr>
<td>16.6667</td>
<td>31.2500</td>
<td>60.4167</td>
</tr>
<tr>
<td>12.5000</td>
<td>29.1667</td>
<td>75.0000</td>
</tr>
<tr>
<td>2.0833</td>
<td>43.7500</td>
<td>20.8333</td>
</tr>
<tr>
<td>20.8333</td>
<td>64.5833</td>
<td>56.2500</td>
</tr>
<tr>
<td>14.5833</td>
<td>-6.2500</td>
<td>50.0000</td>
</tr>
<tr>
<td>22.9167</td>
<td>37.5000</td>
<td>25.0000</td>
</tr>
<tr>
<td>12.5000</td>
<td>58.3333</td>
<td>33.3333</td>
</tr>
<tr>
<td>20.8333</td>
<td>27.0833</td>
<td>12.5000</td>
</tr>
</tbody>
</table>
To be certain that the difference obtained by IBL is significantly lower than the differences obtained by SBI and DI, and it is not a product of the variability within groups and the variability of the whole dataset, we apply and ANOVA t-test to the results (tables 11, 12 and 13).

Calculated from Table 7.
Table 11. Summary of the difference of test.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Count</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBL Difference</td>
<td>29</td>
<td>570.8333</td>
<td>19.6839</td>
<td>219.3658</td>
</tr>
<tr>
<td>DI Difference</td>
<td>21</td>
<td>933.3333</td>
<td>44.4444</td>
<td>350.8391</td>
</tr>
<tr>
<td>SBI Difference</td>
<td>20</td>
<td>845.8333</td>
<td>42.2917</td>
<td>479.7606</td>
</tr>
</tbody>
</table>

*Calculated from Table 10.*

Table 12. ANOVA of the entry test.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>9596.5566</td>
<td>2</td>
<td>4798.2783</td>
<td>14.4329</td>
<td>6.13 E-06</td>
<td>3.1338</td>
</tr>
<tr>
<td>Within Groups</td>
<td>22274.4752</td>
<td>67</td>
<td>332.4549</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>31871.0317</td>
<td>69</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Calculated from Table 11.*

Table 13. Pairwise comparison.

<table>
<thead>
<tr>
<th>Group A</th>
<th>IBL Difference</th>
<th>IBL Difference</th>
<th>DI Difference</th>
<th>DI Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group B</td>
<td>DI Difference</td>
<td>SBI Difference</td>
<td>SBI Difference</td>
<td></td>
</tr>
<tr>
<td>1/Count_A + 1/Count_B</td>
<td>0.0821</td>
<td>0.0845</td>
<td>0.0976</td>
<td></td>
</tr>
<tr>
<td>S.E. of A and B</td>
<td>5.2245</td>
<td>5.2997</td>
<td>5.6968</td>
<td></td>
</tr>
<tr>
<td>ANOVA t-test</td>
<td>-4.7393</td>
<td>-4.2659</td>
<td>0.3779</td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>t critical</td>
<td>1.9950</td>
<td>1.9950</td>
<td>1.9950</td>
<td></td>
</tr>
<tr>
<td>Bonferroni t critical</td>
<td>2.4500</td>
<td>2.4500</td>
<td>2.4500</td>
<td></td>
</tr>
<tr>
<td>Null hypothesis (mean_A = mean_B)</td>
<td>Rejected</td>
<td>Rejected</td>
<td>Accepted</td>
<td></td>
</tr>
</tbody>
</table>

*Comparison between each pair of groups. Calculated from tables 11 and 12.*

With these results, we reject the null hypothesis stating that the means of the differences obtained by IBL and DI are the same. In a similar way, we reject the
hypothesis saying that the means of the differences of IBL and SBI are the same. The test also shows that there is no statistical difference between the means obtained by SBI and DI.

In conclusion, we observed that the students using the teaching approaches SBI and DI obtained significantly more gains in comparison with the students trained with IBL. Figure 17 shows a boxplot of the differences between the exit and entry test for all groups.

Figure 17. A Difference between entry and exit tests.

*Calculated from Table 10.*

**Conclusions**

Results from the entry and exit tests showed how the IBL classes produced the least progress in students' learning compared with the SBI and DI. In this section we
describe some hypothesis that could explain this difference observed, as well as intrinsic limitations of the study.

A first limitation to note about this study is the confounded effect of different teachers assigned to different treatment groups. This single feature was unavoidable given the small size of the sample (number of participant students and teachers) and the limited number of hours the study monitored.

According to our assessments, the adherence of teachers to the specific teaching methodology they were asked to follow was not a significant problem at all. Appendix 5 shows the results of the forms evaluating fidelity which were filled by observing participant teachers and researchers. On average, the fidelity level that participant teachers scored was high.

Direct instruction

The favorable results of DI may be explained by a variety of factors. The most obvious ones are teachers' expertise using this methodology and students' familiarity with it. Indeed, some studies show how teachers expertise is positively correlated with students' outcomes and how this variable is more relevant than previous training or exams scores (Harris & Sass, 2011). In this research, teachers had at least five years of experience using the traditional approaches (direct instruction) as all of them assured.

Another explanation of the results obtained by the DI group may be the intrinsic value of this teaching approach by its own. Dewey wondered in 1916 "Why is it, in spite of the fact that teaching by pouring in, learning by a passive absorption, are universally condemned, that they are still so entrenched in practice?" (Dewey, 1966). However, a significant body of evidence suggests that the use of DI produces measurable learning
outcomes with the students. Beyond the empirical support from decades of steady application of this pedagogy worldwide, studies by Klahr and Nigam (2004), and Kirschner, Sweller and Clark (2006) emphasize the structural advantages of DI and question the presumed superiority of discovery approaches. A relevant question in this discussion for our study would be about the long-term effects of different teaching methodologies. In this topic, Dean Jr. and Kuhn concluded that DI is not sufficient nor necessary to obtain favorable long-term results (2007), but we consider relevant to assess long-term results in a future version of our experiment.

One possible difference from the settings organized for this study and the proper traditional teaching approaches used in Colombia is related with the time devoted to planning, preparation and following a precise class structure. Data from the Organisation for Economic Co-operation and Development (2017) show how Colombian instructors devote between 1,000 and 1,200 hours per year to teaching, one of the largest numbers for international standards (Figure 16). Participant teachers in the study described how most of their colleagues from different schools had extra jobs to get a decent remuneration. Sometimes, the extra job is another full-time job. Under these circumstances, planning and class preparation simply cannot be a priority. Our hypothesis is that one of the possible causes of the favorable results of DI was that the application of this teaching approach fulfilled all the required steps of fully prepared classes. Following this train of thought, this possibility escalates to the point that, probably, one of the main reasons behind Colombia's low results in mathematics is not intrinsically the teaching methodology, but the possibility of implementing it with proper preparation and rigor.
Figure 16. Average teaching hours per year for a sample of countries.


Schema-based instruction

One of the study’s most interesting outcomes was a consistent improvement in scores obtained by the students in the SBI group. Indeed, the variability of the exit test results was the lowest one between all the treatment groups, a behavior that cannot be exclusively attributed to the use of SBI. Moreover, many questions arise from this result.

The results on the test scores were consistent with the research mentioned in Chapter III. In fact, SBI presents noticeable results in the ability of students solving specific types of problems, namely, those described on the schemas they learnt. A common feature of the research described in Chapter III and our study was the establishment of clear and well-defined categories for the problems to be worked on. A natural question that comes under these circumstances is, how does the SBI approach work when students must face new kinds of problems?
This question, far from a new one, is tackled by different studies by Fuchs (Fuchs et al., 2003; Fuchs et al., 2004; Fuchs et al., 2008; Fuchs et al., 2010; Fuchs et al., 2014). The problem of transfer is settled in the middle of the discussion given its relevance for teaching mathematics, and it is worked with the extended process of schema-broadening instruction (Powell, 2011). In fact, solving problems using delimited procedures which are determined by previously defined archetypes is only one part of the complexity of mathematics. A strong limitation of our study was not assessing problems where students were required to use transferable skills, like sorting objects with precise restrictions or creating more than one unordered group.

A recent meta-analysis reviews many SBI publications and gives some lights about what are the most influential variables to determine the students’ outcome using schemas. Surprisingly, schema-broadening instruction produced better results for immediate problem-solving tasks while schema-based instruction (the one we used on this study) performed better with transfer problems. Other relevant variables for the efficiency of using schemas are the problem type, the implementer and duration. This meta-analysis reveals how much research is still needed on using schemas in the classroom (Peltier & Vannest, 2017).

Also, a different source of results on SBI comes from using it with college students. Some natural question appearing there are completely valid using SBI in primary and secondary schools. Specifically, how are the results when we move from the controlled scenarios the literature presents to real classrooms (Blissett, Goldszmidt, and Sibbald, 2015)? How different are the results using structured knowledge questions and
factual knowledge questions (Blissett, Cavalcanti, & Sibbald, 2012)? Those are only few of the topics we should address in future studies.

Inquiry-based learning

The study group which used IBL obtained the lowest mean score at the end of the study. Previously, we mentioned a highly cited paper by Kirschner, Sweller and Clark (2006) with strong critics to teaching approaches based on discovery. Their main point was showing data about how students require high levels of previous knowledge before being able to successfully embrace a process of inquiry.

One year later, Hmelo-Silver, Duncan and Chinn (2007) responded to Kirschner, Sweller and Clark’s publication (2006) by explaining how some teaching methodologies like problem-based learning and inquiry learning are not necessarily the same as unguided inquiry. They make a strong point on the process of instructors scaffolding students’ learning and they insist on a more complex description of the process of IBL.

In this direction, how much scaffolding we used was something we considered in the design phase when we opted for option 5 in Table 3 (Chapter III). After deciding we wanted to assess IBL scenarios with minimal guidance, probably we went short in the level of scaffolding these students required. Moreover, according to participant teachers, most of the students had their first exposition to an inquiry-based class with this study.

Taking for granted that students will find correct connections is a common mistake some instructors make when trying IBL. In advance, a successful process of inquiry depends on students’ development of high-order skills facilitating exploration, association and, quite importantly, finding causality. However, an extensive study shows how middle schoolers frequently have incorrect mental models of causality, making very
difficult to obtain results in their inquiry processes (Kuhn, Black, Keselman, & Kaplan, 2000).

The references and guides we incorporated designing the IBL classes mention important aspects we did not implemented in our study. Specifically, starting with the student’s world as base space and building up from there both the classes and curriculum was an option we did not consider in this study. Also, creating communities of learners within the students, as well as three-members instructional teams including teachers, librarians, and learning abilities specialists was far beyond our possibilities. Those considerations may lead to a more cohesive study in the future (Kuhlthau, 2015).

A final point we want to mention is the role of the entry and exit test in this study. Precisely, these tests were just measurement tools and the formative value of assessment was not involved. In the specific case of the IBL study group, we should recall the role of assessments in the process of discovery and how it is important to also evaluate the process of inquiry itself, differing from standard tests (Kuhlthau, 2015).

Final remarks

A comparison study is a first step in the process of assessing teaching approaches involving more variables, conducted for a longer period of time, including more mathematical content, covering a larger population. Making a detailed analysis on how to incorporate explicitly Pólya’s steps for solving mathematical problems in the classes (Pólya, 1957), how to use alternative representations with different forms of pedagogical content knowledge (Schulman, 1986), or how would be the results if the class preparation is solely in charge of the participant teachers, are some of the many ideas we want to assess in the future.
The authors are aware that a problem like the quality of mathematics education, with all its economic and social variables requires far more complex studies, but one of the areas needed to be explored would definitely be the use of one or many teaching approaches.

The reviewed literature shows strong advantages for all the instruction models we used. A reasonable question would be how these methods produce efficient results with different topics in Colombian classrooms.

The study was focused on Bogotá, keeping in mind that Colombia has many sub regions with specific cultural and educational features. The possibility of replicating this study in other regions is always open.

Escalating this study to other Latin American and Caribbean countries can be done with relatively low costs. The authors extend an invitation to replicate the study, and welcome the opportunity to get more insights and data regarding possible approaches for teaching mathematics in our region.
Appendix 1.

PISA 2015 results

Table 14. PISA 2015 results.

<table>
<thead>
<tr>
<th>Country / Economy</th>
<th>Mean score in PISA 2015</th>
<th>Science, reading and mathematics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>OECD average</td>
<td>493</td>
<td>493</td>
<td>490</td>
</tr>
<tr>
<td>Singapore</td>
<td>556</td>
<td>535</td>
<td>564</td>
</tr>
<tr>
<td>Japan</td>
<td>538</td>
<td>516</td>
<td>532</td>
</tr>
<tr>
<td>Estonia</td>
<td>534</td>
<td>519</td>
<td>520</td>
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Countries/economies with a mean performance/share of top performers **above** the OECD average
Countries/economies with a share of low achievers **below** the OECD average
Countries/economies with a mean performance/share of top performers/share of low achievers not significantly different from the OECD average
Countries/economies with a mean performance/share of top performers **below** the OECD average Countries/economies with a share of low achievers **above** the OECD average

*Countries and economies sorted in descending order based on the sum of their scores in science, reading and mathematics. Latin American and Caribbean participants are highlighted with blue font. Source: Organisation for Economic Co-operation, and Development, & Programme for International, Student Assessment (2016).*
Appendix 2.

SBI Instructor’s Guidelines

Schemata

Tree: objects are divided into categories and we must choose a single element from each one.

List: we select some objects from a group and place them in an ordered list.

Grouping: we choose some objects from a groups regardless their order of choice.

We choose them in order

We rearrange them

Source: https://www.d12.org/12/Middle-School-Math-Concepts-Grade-7/
https://www.d12.org/12/Middle-School-Math-Concepts-Grade-7/ CC BY NC 3.0
Instructor’s Guidelines

Please adhere to the following steps when you solve a problem with your students:

1. Prioritize on reading the problems in order to recognize its mathematical structure. Stimulate students’ thinking about how problems within and across types are similar and different.
2. Visually map information in the problem using schematic diagrams.
3. Give explicit instructions about the steps to solve the problem. Use the DISC heuristic:
   a. Discover the problem type.
   b. Identify information in the problem to represent in a diagram.
   c. Solve the problem.
   d. Check the solution.
   Accompany with deep-level questions each step in the heuristic (e.g., Why this is a proportion problem? How is this problem similar to or different from one I already solved?)
4. Develop students’ procedural flexibility, including explicit teaching of multiple solution methods.

Each problem set is divided into two sections. The first section (labeled as Sample Problems) is meant to be solved by the teacher on the board. Please do this in a clear and detailed way, at a slow pace, following the four steps mentioned above.

Then, let your students work individually on the problems labeled as Practice Problems. You should move through the classroom helping students and encouraging them to use the DISC heuristic on every problem. Students who finish all the problems from the section may discuss it with their peers. Do not advance to the next part of the guide until all students have correctly solved all the problems from the section Practice Problems. If you complete all the problems from all the sections before the the Camp is done, you may use the remaining time to work on the problems from the Entrance Test.

1 Adapted from http://dx.doi.org/10.1080/19345747.2012.725804 and http://dx.doi.org/10.1016/j.learninstruc.2010.03.001
Sample Problems

1. Nadia’s soccer team has 2 games to play this weekend. How many outcomes are there for Nadia’s team? (There are only two options: win or lose. In case of draw, they will solve it by penalty kicks). What happens if they play 3 games?

2. To remodel her kitchen, Gretchen has the following choices: Floor: tile or wood. Counter: Granite or formica. Sink: white, steel, stone. How many different choices can Gretchen make?

3. Jeff has five different pairs of socks and three pairs of shoes. How many possible combinations are there?

4. Jessie has three sweaters, two turtlenecks and three jackets. How many possible combinations are there?

Practice Problems

5. Kelly has chocolate or vanilla ice cream, three choices of toppings and four sauces. How many possible outcomes does she have for creating her ice cream sundae?
6. Jessica is excited to attend her Halloween middle school dance. There is no dress code, so her options for the outfit she will wear is wide open. She has limited her choices to the following:
- Tops: a lacy blouse or a graphic t-shirt
- Bottoms: skinny jeans or an A-line skirt
- Shoes: platform sandals or Vans tennis shoes
How many different outfits can Jessica create given these options?

7. A local pizza place offers a pizza with several options. On a basic pizza, you can choose one of each option. The options are a sourdough or whole wheat crust, mozzarella or feta cheese, and one of the following toppings: black olives, green olives, green pepper, garlic, pepperoni, ground beef, mushrooms, or onions.
Based on these possibilities, how many possible outcomes are there?
Sample Problems

1. Consider the word CAT. Clearly, order is important when you spell a word. You can write all of the correct letters, but if you don’t put them in the correct order, you don’t spell CAT. For example, here are some orders of C, A, and T that don’t spell CAT:

2. Tomás wants to know how many 3-digit numbers he can write using the digits 7, 8, and 9 without repeating any of the digits. Does order matter for this problem?

3. The softball coach needs to determine how many different batting lineups she can make out of her first three batters, Able, Baker, and Chan. How many different batting orders are there?

4. How many 3-letter words Brenda can write using the letters E, T, A without repeating letters?

Practice Problems

5. Five different cars entered the race. Painted the following colors: red, orange, blue, white, purple? In how many different ways can the cars finish the race?
6. At the breakfast buffet you can take any three of the following: eggs, pancakes, potatoes, cereal, waffles. How many different 3-item breakfasts can you get?

7. Naima and Michael are in charge of staffing the middle school Sock Hop. They begin with the six 6th-graders who have volunteered to work: Cindy, Justin, Brad, Willy, Donna, and Angelle. They are trying to figure out the order of assignment. In how many different ways can Naima and Michael arrange the six 6th-graders?

8. Doug is going to use the following 5 letters to create his new 3-letter computer password: B, F, G, L, and T. How many different passwords can he create if he doesn't repeat any letters?
Sample Problems

1. Jim and Maralie are in charge of tossing bracelets out into the audience for the school float during the town's Holiday Parade. There are a total of four colors of bracelets: orange, red, green, and blue. They will reach into a basket and toss out two at a time. Does the order matter in this problem? In how many different ways can be tossed two bracelets of different colors?

2. How many different 5-player teams can you choose from a total of 10 volleyball players?

3. You got tickets for the amusement park and you can come with 2 of your friends. Unfortunately, you have 5 friends who want to go to the amusement park. Does the order of choice of 2 of your friends matter for this situation? In how many different ways you can choose 2 of your friends to go to the amusement park?

4. Cesar the dog-walker has 6 dogs but only 2 leashes. How many different ways can Cesar take a walk with all 2 dogs at once?

Practice Problems

5. How many 2-book combinations can be drawn from a bookshelf with 10 books in it?
6. The 5 last people at a movie must compete for the last 3 empty seats. How many different groups of 3 can sit and watch the movie?

7. Nine people want to ride on the banana boat but there are only 4 life jackets. How many different groups can ride on the banana boat at one time?

8. At Dudley’s Dude Ranch there are 6 riders but only 4 horses. How many different ways can a group of 4 go out on ride?
Identifying Schemata

1. How many 3-letter words can Brenda write using the letters A, B, and C without repeating any of the letters?

2. The Triplex Theater has 3 different movies tonight: Bucket of Fun, Bozo the Great, and Pickle Man. Each movie has an early and late show. How many different movie choices are there?

3. Jen's soccer team is playing 4 games next week. How many different outcomes are there for the four games? (There are only two options: win or lose. In case of draw, they will solve it by penalty kicks.)

4. How many different 4-singers groups can be made from a total of 10 singers?

5. A bag has 4 marbles: red, blue, yellow, and green. In how many different ways can you reach into the bag and draw out 2 marbles at once and drop them in a cup?

6. A bag has one blue marble and one red marble. If you randomly choose one marble, put it back into the bag, and then you choose again one marble, how many possible outcomes you can get?

7. Mr. Chen has decided that he’s going to give Nikki, Mickey, and Hickey awards for the essay contest. What he doesn’t know is who will get 1st prize, 2nd prize, and 3rd prize. How many different ways can Mr. Chen give out the prizes?

8. Michelle forgot her 6-letter computer password. She knows she used the letters H, I, J, K, L, and M in the password and that she didn’t repeat any of the letters. How many different passwords must she try before she is sure to hit the correct one?

9. How many 3-flower bouquets can Leah make out of a rose, a tulip, a Daffodil, a lily, and a violet?

Appendix 3.

IBL Instructor’s Guidelines

I nstructor’s Guidelines

- There are no lectures at the beginning of the classes. Students will try to solve the problems and the teacher’s role is limited to give them hints to direct their thinking process.
- Students will work in groups of 3 or 4 (no more than 4 students per group in order to avoid distractions.)
- The teacher walks from group to group asking the students what are they thinking about the problem they are solving and setting small questions which could direct the group toward the final answer.
- The teacher is ready to catch any good idea proposed by the students which happens to be useful to solve the problem. In this situation, writing down the idea in the blackboard helps a lot all the groups to benefit from it.
- There is no rush. It is quite important that students elaborate their own paths to the solution.
- The teacher uses the ideas suggested by the students to give shape to a path to solve the problem. Doing so, he or she helps the students to build up and figure out the final answer.
- Most of the ideas proposed by the students will not be useful to solve the problem. In this situations, the teacher should not dismiss immediately the idea, but rather he or she should ask the rest of the class what they think about it.
- Whenever students seems to accept an incorrect idea and they are unable to reject it, the teacher could ask specific questions to make evident why this idea will not work.
- In some cases, the teacher could take an incorrect idea, slightly change it and transform it into a good idea to solve the problem. Generally, this can be done asking questions in the form ‘what if... ?’.
- The teacher should be quite eloquent with his or her non-verbal language. He or she shows enthusiasm when students are getting closer to the answer. The teacher makes evident his or her amusement when students take a promising track.
- Once the class gets the answer to the problem, the teacher should do a review of the whole process students underwent to find the solution, highlighting the definitive steps which made progress towards the answer. Using the elements, ideas and concepts given by the students, the teacher builds up and explain with detail a polished way to solve the problem.
- In some situations, students will be quiet and they will not propose any ideas at all. Do not be tempted by the possibility of solve the problem by yourself and “teach them” how to do it. Instead of that, you could ask many simple short questions to promote students’ participation. Suggest them to do a drawing, make a list, try with some examples or think about particular cases. In some extreme situations, you may even suggest them an idea in the form of asking them a question.
- The classroom will not be a quiet space. Students will propose, discuss, use the blackboard, and ask questions all the time.

1 Adapted from Behrenbruch, Dancing in the Light - Essential Elements for an Inquiry Classroom (2012) and Dostál, Inquiry-based instruction - Concept, essence, importance and contribution (2015)
Section 1

1. Nadia’s soccer team has 2 games to play this weekend. How many outcomes are there for Nadia’s team? (There are only two options: win or lose. In case of a draw, they will solve it by penalty kicks). What happens if they play 3 games?

2. To remodel her kitchen, Gretchen has the following choices: Floor: tile or wood; Counter: Granite or formica; Sink: white, steel, stone. How many different choices can Gretchen make?

3. Jeff has five different pairs of socks and three pairs of shoes. How many possible combinations are there?

4. Jessie has three sweaters, two turtlenecks and three jackets. How many possible combinations are there?

5. Kelly has chocolate or vanilla ice cream, three choices of toppings and four sauces. How many possible outcomes does she have for creating her ice cream sundae?

6. Jessica is excited to attend her Halloween middle school dance. There is no dress code, so her options for the outfit she will wear is wide open. She has limited her choices to the following:
   Tops: a lacy blouse or a graphic t-shirt
   Bottoms: skinny jeans or an A-line skirt
   Shoes: platform sandals or Vans tennis shoes
   How many different outfits can Jessica create given these options?

7. A local pizza place offers a pizza with several options. On a basic pizza, you can choose one of each option. The options are a sourdough or whole wheat crust, mozzarella or feta cheese, and one of the following toppings: black olives, green olives, green pepper, garlic, pepperoni, ground beef, mushrooms, or onions.
   Based on these possibilities, how many possible outcomes are there?

8. The Triplex Theater has 3 different movies tonight: Bucket of Fun, Bozo the Great, and Pickle Man. Each movie has an early and late show. How many different movie choices are there?

9. Jen's soccer team is playing 4 games next week. How many different outcomes are possible for the four games? (There are only two options: win or lose. In case of draw, they will solve it by penalty kicks.)

10. A bag has one blue marble and one red marble. If you randomly choose one marble, put it back into the bag, and then you choose again one marble, how many possible outcomes you can get?
Section 2

1. Consider the word CAT. Clearly, order is important when you spell a word. You can write all of the correct letters, but if you don’t put them in the correct order, you don’t spell CAT. For example, here are some orders of C, A, and T that don’t spell CAT:

2. Tomás wants to know how many 3-digit numbers he can write using the digits 7, 8, and 9 without repeating any of the digits. Does order matter for this problem?

3. The softball coach needs to determine how many different batting lineups she can make out of her first three batters, Able, Baker, and Chan. How many different batting orders are there?

4. Does order matter for this problem? Solve it.
“How many 3-letter words Brenda can write using the letters E, T, A without repeating letters?”

5. Does order matter for this problem? Solve it.
“How many different ways can the cars finish the race?”

6. Does order matter for this problem? Solve it.
"At the breakfast buffet you can take any three of the following: eggs, pancakes, potatoes, cereal, waffles. How many different 3-item breakfasts can you get?"

7. Naima and Michael are in charge of staffing the middle school Sock Hop. They begin with the six 6th-graders who have volunteered to work: Cindy, Justin, Brad, Willy, Donna, and Angelle. They are trying to figure out the order of assignment. In how many different ways can Naima and Michael arrange the six 6th-graders?

7. Determine if the order matters or not in this situation. Explain your choice and solve the problem.
"Doug is going to use the following 5 letters to create his new 3-letter computer password: B, F, G, L, and T. How many different passwords can he create if he doesn't repeat any letters?"

8. Determine if the order matters or not in this situation. Explain your choice and solve the problem.
"Mr. Chen has decided that he's going to give Nikki, Mickey, and Hickey awards for the essay contest. What he doesn't know is who will get 1st prize, 2nd prize, and 3rd prize. How many different ways can Mr. Chen give out the prizes?"

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10. Michelle forgot her 6-letter computer password. She knows she used the letters H, I, J, K, L, and M in the password and that she didn’t repeat any of the letters. How many different passwords must she try before she is sure to hit the correct one?
Section 3

1. Jim and Maralie are in charge of tossing bracelets out into the audience for the school float during the town's Holiday Parade. There are a total of four colors of bracelets: orange, red, green, and blue. They will reach into a basket and toss out two at a time. Does the order matter in this problem? In how many different ways can be tossed two bracelets of different colors?

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8. At Dudley’s Dude Ranch there are 6 riders but only 4 horses. How many different ways can a group of 4 go out on ride?

9. How many 3-flower bouquets can Leah make out of a rose, a tulip, a Daffodil, a lily, and a violet?

10. How many different 4-singers groups can be made from a total of 10 singers?

11. A bag has 4 marbles: red, blue, yellow, and green. In how many different ways can you reach into the bag and draw out 2 marbles at once and drop them in a cup?
### Class Template

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<th>Comments and suggestions</th>
<th>Class distribution</th>
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<tr>
<td>Introduction</td>
<td>10 min: Present the class' structure</td>
<td>In Activities, state that you will do an initial explanation followed by some examples, and later the students will work with their Student's Guides.</td>
<td>Options: Individual, Groups of X, Discussion, Debate, Lecture, etc.</td>
</tr>
<tr>
<td>Explanation</td>
<td>30 min:</td>
<td>Write down in this box just the keyword of the topic or concept you will explain</td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>30 min:</td>
<td>Make a log about what problems the students were able to solve and which ones they could not.</td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td>10 min:</td>
<td>Finish the class writing down the main ideas and concepts you taught.</td>
<td></td>
</tr>
</tbody>
</table>

1 Adapted from [http://aprende.colombiaaprende.edu.co/es/aulassinfronteras/matematicas-primer-bimestre](http://aprende.colombiaaprende.edu.co/es/aulassinfronteras/matematicas-primer-bimestre)

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description of the activity</th>
<th>Comments and suggestions</th>
<th>Class distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>10 min:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explanation</td>
<td>30 min:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>30 min:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td>10 min:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tree Diagrams

Activity 1: Note Taking

Activity 2: Examples
1. Nadia’s soccer team has 2 games to play this weekend. How many outcomes are there for Nadia’s team? (There are only two options: win or lose. In case of draw, they will solve it by penalty kicks). What happens if they play 3 games?

2. To remodel her kitchen, Gretchen has the following choices: Floor: tile or wood. Counter: Granite or formica, Sink: white, steel, stone. How many different choices can Gretchen make?

3. Jeff has five different pairs of socks and three pairs of shoes. How many possible combinations are there?

4. Jessie has three sweaters, two turtlenecks and three jackets. How many possible combinations are there?

5. Kelly has chocolate or vanilla ice cream, three choices of toppings and four sauces. How many possible outcomes does she have for creating her ice cream sundae?

6. Jessica is excited to attend her Halloween middle school dance. There is no dress code, so her options for the outfit she will wear is wide open. She has limited her choices to the following:
   Tops: a lacy blouse or a graphic t-shirt
   Bottoms: skinny jeans or an A-line skirt
   Shoes: platform sandals or Vans tennis shoes
   How many different outfits can Jessica create given these options?

Activity 3. Practice Problems
7. A local pizza place offers a pizza with several options. On a basic pizza, you can choose one of each option. The options are a sourdough or whole wheat crust, mozzarella or feta cheese, and one of the following toppings: black olives, green olives, green pepper, garlic, pepperoni, ground beef, mushrooms, or onions.
   Based on these possibilities, how many possible outcomes are there?

8. The Triplex Theater has 3 different movies tonight: Bucket of Fun, Bozo the Great, and Pickle Man. Each movie has an early and late show. How many different movie choices are there?
9. Jen’s soccer team is playing 4 games next week. How many different outcomes are possible for the four games? (There are only two options: win or lose. In case of draw, they will solve it by penalty kicks.)

10. A bag has one blue marble and one red marble. If you randomly choose one marble, put it back into the bag, and then you choose again one marble, how many possible outcomes can you get?
Permutations

Activity 1: Note Taking


Activity 2: Examples
1. Consider the word CAT. Clearly, order is important when you spell a word. You can write all of the correct letters, but if you don’t put them in the correct order, you don’t spell CAT. For example, here are some orders of C, A, and T that don’t spell CAT:


2. Tomás wants to know how many 3-digit numbers he can write using the digits 7, 8, and 9 without repeating any of the digits. Does order matter for this problem?


3. The softball coach needs to determine how many different batting lineups she can make out of her first three batters, Able, Baker, and Chan. How many different batting orders are there?


4. Does order matter for this problem? Solve it.
“How many 3-letter words Brenda can write using the letters E, T, A without repeating letters?”


5. Does order matter for this problem? Solve it.
   "Five different cars entered the race painted the following colors: red, orange, blue, white, purple. In how many different ways can the cars finish the race?"

6. Does order matter for this problem? Solve it.
   "At the breakfast buffet you can take any three of the following: eggs, pancakes, potatoes, cereal, waffles. How many different 3-item breakfasts can you get?"

7. Naima and Michael are in charge of staffing the middle school Sock Hop. They begin with the six 6th-graders who have volunteered to work: Cindy, Justin, Brad, Willy, Donna, and Angelie. They are trying to figure out the order of assignment. In how many different ways can Naima and Michael arrange the six 6th-graders?

Activity 3: Practice Problems
7. Determine if the order matters or not in this situation. Explain your choice and solve the problem.
   "Doug is going to use the following 5 letters to create his new 3-letter computer password: B, F, G, L, and T. How many different passwords can he create if he doesn't repeat any letters?"
8. Determine if the order matters or not in this situation. Explain your choice and solve the problem.

'Mr. Chen has decided that he's going to give Nikki, Mickey, and Hickey awards for the essay contest. What he doesn't know is who will get 1st prize, 2nd prize, and 3rd prize. How many different ways can Mr. Chen give out the prizes?'

9. How many 3-letter words can Brenda write using the letters A, B, and C without repeating any of the letters?

10. Michelle forgot her 6-letter computer password. She knows she used the letters H, I, J, K, L, and M in the password and that she didn't repeat any of the letters. How many different passwords must she try before she is sure to hit the correct one?
Combinations

**Activity 1: Note Taking**

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**Activity 2: Examples**

1. Jim and Marisle are in charge of tossing bracelets out into the audience for the school float during the town's Holiday Parade. There are a total of four colors of bracelets: orange, red, green, and blue. They will reach into a basket and toss out two at a time. Does the order matter in this problem? In how many different ways can be tossed two bracelets of different colors?

2. Does order matter for this problem? Solve it.
   "How many different 6-player teams can you choose from a total of 10 volleyball players?"

3. You got tickets for the amusement park and you can come with 2 of your friends. Unfortunately, you have 5 friends who want to go to the amusement park. Does the order of choice of 2 of your friends matter for this situation? In how many different ways can you choose 2 of your friends to go to the amusement park?

4. Does order matter for this problem? Solve it.
   "Cesar the dog-walker has 6 dogs but only 2 leashes. How many different ways can Cesar take a walk with all 2 dogs at once?"

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5. How many 2-book combinations can be drawn from a bookshelf with 10 books in it?


6. The 5 last people at a movie must compete for the last 3 empty seats. How many different groups of 3 can sit and watch the movie?


7. Nine people want to ride on the banana boat but there are only 4 life jackets. How many different groups can ride on the banana boat at one time?


Activity 3: Practice Problems
8. At Dudley’s Dude Ranch there are 6 riders but only 4 horses. How many different ways can a group of 4 go out on ride?
8. How many 3-flower bouquets can Leah make out of a rose, a tulip, a Daffodil, a lily, and a violet?

9. How many different 4-singers groups can be made from a total of 10 singers?

10. A bag has 4 marbles: red, blue, yellow, and green. In how many different ways can you reach into the bag and draw out 2 marbles at once and drop them in a cup?
Appendix 5.

Reports from class observations

Each study group was observed by two different teachers and they assessed, both quantitatively and qualitatively, the fidelity instructors featured with their groups.

In tables 15, 16 and 17, A and B represent different observers who followed this indication: “Please grade from 0 to 5 how much the instructors fulfilled and adhered to each of the following instructions (0: not fulfilled at all; 5: completely fulfilled)”.

Table 15. Fidelity of the IBL classes.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are no lectures at the beginning of the classes. Students will try to solve the problems and the teacher’s role is limited to give them hints to direct their thinking process.</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Students will work in groups of 3 or 4 (no more than 4 students per group in order to avoid distractions.)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>The teacher walks from group to group asking the students what are they thinking about the problem they are solving and setting small questions which could direct the group toward the final answer.</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>The teacher is ready to catch any good idea proposed by the students which happens to be useful to solve the problem. In this situation, writing down the idea in the blackboard helps a lot all the groups to benefit from it.</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>There is no rush. It is quite important that students elaborate their own paths to the solution.</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>The teacher uses the ideas suggested by the students to give shape to a path to solve the problem. Doing so, he or she helps the students to build up and figure out the final answer.</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Most of the ideas proposed by the students will not be useful to solve the problem. In these situations, the teacher should not dismiss immediately the idea, but rather he or she should ask the rest of the class what they think about it.</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Whenever students seem to accept an incorrect idea and they are unable to reject it, the teacher could ask specific questions to make evident why this idea will not work.</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
In some cases, the teacher could take an incorrect idea, slightly change it and transform it into a good idea to solve the problem. Generally, this can be done asking questions in the form “what if…?”.  

The teacher should be quite eloquent with his or her non-verbal language. He or she shows enthusiasm when students are getting closer to the answer. The teacher makes evident his or her amusement when students take a promising track.

Once the class gets the answer to the problem, the teacher should do a review of the whole process students underwent to find the solution, highlighting the definitive steps which made progress towards the answer. Using the elements, ideas and concepts given by the students, the teacher builds up and explain with detail a polished way to solve the problem.

In some situations, students will be quiet and they will not propose any ideas at all. Do not be tempted by the possibility of solving the problem by yourself and “teach them” how to do it. Instead of that, you could ask many simple short questions to promote students’ participation. Suggest them to do a drawing, make a list, try with some examples or think about particular cases. In some extreme situations, you may even suggest them an idea in the form of asking them a question.

The classroom will not be a quiet space. Students will propose, discuss, use the blackboard, and ask questions all the time.

### Table 16. Fidelity of the SBI classes.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prioritize on reading the problems in order to recognize its mathematical structure. Stimulate students’ thinking about how problems within and across types are similar and different.</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Visually map information in the problem using schematic diagrams.</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Give explicit instructions about the steps to solve the problem. Use the DISC heuristic: Discover the problem type. Identify information in the problem to represent in a diagram. Solve the problem. Check the solution.</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Accompany with deep-level questions each step in the heuristic (e.g., Why this is a proportion problem? How is this problem similar to or different from one I already solved?)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Develop students’ procedural flexibility, including explicit teaching of multiple solution methods.</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>The first section (labeled as Sample Problems) is meant to be solved by the teacher on the board. Please do this in a clear and detailed way, at slow pace, following the four steps mentioned above.</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Let your students work individually on the problems labeled as Practice Problems.</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

Graded from (0: not fulfilled at all; 5: completely fulfilled). A and B are different observers.
You should move through the classroom helping students and encouraging them to use the DISC heuristic on every problem

Students who finish all the problems from the section may discuss it with their peers

Do not advance to the next part of the guide until all students have correctly solved all the problems from the section Practice Problems

Graded from (0: not fulfilled at all; 5: completely fulfilled). A and B are different observers.

Table 17. Fidelity of the DI classes.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Split the class time into 4 moments: Introduction, Explanation, Application, Summary.</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Dedicate to each moment of the class 10 minutes, 30 minutes, 30 minutes, 10 minutes, respectively.</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>In the Explanation section, the teacher introduces a new topic and presents some examples.</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>In the Application section, the students get exercises and work by themselves.</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The class talking is rigorously controlled by the teacher following the pattern question-answer.</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Focus on the algorithmic part of mathematics. Mathematics is presented as facts and procedures to be done.</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>The authority is centered in the teacher.</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>The process of teaching consists of a transfer of information from the teacher to the students.</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>The process of learning is considered accomplished when students can replicate the procedures and get right answers.</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Graded from (0: not fulfilled at all; 5: completely fulfilled). A and B are different observers.
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