Advances in the Singlet-Triplet Qubit

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Advances in the Singlet-Triplet Spin Qubit

A DISSERTATION PRESENTED
BY
LUCAS AUGUSTÍN ORONA
TO
THE PHYSICS DEPARTMENT
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
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Advances in the Singlet-Triplet Spin Qubit

Abstract

My work on singlet-triplet spin qubits has bridged between the eras of two different semiconductors, gallium arsenide (GaAs) and silicon germanium (SiGe), that have been used in the Yacoby Lab. I have chosen to report my work in chronological order as it fits best with developments in the field but this leaves it with a structure inverted to most experimental physics theses with the main scientific achievements front loaded and the engineering stages at the end.

The first chapter will provide background on spin qubits in semiconductors and will focus on major developments in singlet-triplet qubits that supported and motivated my own work. The second chapter discusses an advancement in two qubit coupling based on using large magnetic field gradients to mitigate charge noise. The third chapter is technical details about a new readout mechanism that allowed for the measurements presented in the second chapter. The fourth chapter discusses the transition from GaAs to SiGe and highlights the main difference between these two materials. The fifth chapter details the Crassula Quantum Dot Simulator that was written to aid in designing devices in SiGe. The sixth chapter discusses the progress made towards qubits in SiGe. The seventh chapter discusses overcoming the difficulties of RF reflectometry that are introduced by SiGe. The thesis concludes with experiment ideas that I was not able to complete before graduation but that I wish to note down so that some later generation of students may try them.
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This thesis is dedicated to the American people who have funded this research through their tax dollars. I wish the best for us all in the face of the COVID-19 pandemic.
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despite their difficulties.
Quantum computation promises improved computational speed based on algorithms that cannot be run on a classical computer. These are already known to include factoring\cite{75}, searching unordered lists\cite{31} and simulating quantum systems\cite{27}. Quantum algorithms are enabled by the ability of quantum bits (qubits) to exist in superposition states that simultaneously have the nature of both computational states and the ability to generate entangled states of multiple qubits\cite{6}. Running these algorithms will require quantum hardware where large numbers of qubits can be physically realized with the required quantum properties. This implementation has lagged significantly behind the theory and is the focus of this work.
A set of requirements for any prospective quantum computation platform was developed by DiVincenzo[21] and has guided the development of a variety of technologies. The first of DiVincenzo's criteria is a quantum system with two distinct levels that can mapped to a qubit’s $|1\rangle$ and $|0\rangle$ states. The second is that the individual qubits must be compatible with scaling up to a system made of many qubits (where "many" will increase in the same way that "many" transistors on a chip is still increasing today). The third is the ability to initialize the qubit in the $|0\rangle$ state so that the computations are always begin from the same state. The fourth is the ability to fully control all qubits, which requires two non-collinear control axes for every qubit. The fifth is the requirement for an interaction between two qubits that depends on both qubits’ states which can be used for a two qubit gate. The sixth is that the qubit has coherence times significantly longer than the time required for computations. As the qubit interacts with its environment it decoheres which causes computation errors. The last requirement is the ability to measure the qubits’ states when the computation is complete.

It is important to evaluate any qubit using all of the DiVincenzo criteria, especially as it is tempting to flaunt the strengths of a qubit while sweeping its weaknesses under the rug. This is especially important as many of these requirements are almost antithetical to each other. For example, it difficult to create a quantum system that is only weakly coupled to its environment so that it has a long coherence time while still allowing it to strongly interact with its fellow qubits for a two qubit gate. These criteria offer a very general framework that makes it easier to understand new qubits and to compare the wide variety of systems that have been proposed and demonstrated. I encourage the reader to examine qubit papers with these in mind so that they can get the most realistic understanding of a given platform. I will strive to provide a very honest and upfront discussion of how the singlet-triplet spin qubit performs on all metrics listed.

The current leading quantum computing technology is the transmon qubit made from superconducting resonators[45, 72]. These qubits have coherence times of hundreds of microseconds and have demonstrated single qubit gate
fidelities of 99.99% and two qubit gates of 99.9%. This has been enabled by engineering the system to have only weak dependence of the qubit energy levels on electric fields, reducing decoherence due to charge noise. However, these qubits are quite large (hundreds of square microns in size) which creates strong capacitive couplings between neighboring devices. This coupling means that controlling a single qubit without effecting its neighbors requires applying compensation so that a single qubit gate is really the application of a complex set of controls on many qubits.

We have chosen to study qubits based on electron spins in semiconductors which have a different set of strengths and weaknesses. Spins in semiconductors have long lifetimes, having been demonstrated as long as 30 seconds in silicon\cite{57}, and are relatively compact (hundreds of square nanometers in size). Single qubit gates of 99.8% fidelity have been demonstrated\cite{56} and in this work we show a two qubit gate of 90\%\cite{59}. Spin qubits can be rapidly initialized by loading electrons from a neighboring reservoir with the proper spin and can be read out through spin to charge conversion in as short as 1 microsecond. Additionally, the fact that these qubits are fabricated in semiconductors should allow them to be fabricated on massive scales using many of the same technologies that have been developed for transistors. The biggest challenge for spin qubits, and the focus of much of this work, is overcoming the decoherence introduced by charge noise\cite{19}. Decoherence is particularly sinister for two qubit gates as they are slower than single qubit gates.

This thesis was completed during an exciting time in quantum computation. Google and IBM have chosen the nearest term bet of superconducting qubits. Intel has chosen to invest in spin qubits in silicon because the technology is closest to its wheelhouse. Microsoft has taken the most speculative bet on majoranna based topological quantum computation. Aside from these technological juggernauts, there are startups focused on quantum computing itself and on designing complementary technology, like control systems that integrate arbitrary waveform generators and data acquisition cards that are specifically designed for qubit feedback. In the last year Google demonstrated
the first coming of quantum supremacy[1]. While their specific demonstration was contrived to be easy for their quantum computer and hard for a classical computer, the demonstration that just 53 qubits can solve problems orders of magnitude more quickly than the most powerful classical computers is still a significant step. Most researchers in the field see quantum supremacy not as a single event but as a gradual transition. It is likely that quantum computers will first become useful for quantum simulations where a quantum system and its Hamiltonian can be mapped onto a collection of qubits and set of gates used to control them. A nearer term imperfect quantum computer that produces approximate answers with calculable error bars may already enable a new realm of possibilities for academic research on large quantum systems and for more applied results like simulating protein folding for pharmaceuticals. Quantum algorithms that need exact solutions, like a number’s factors, will likely be the farthest out as they have more stringent requirements.

I like to say that quantum computation is currently where classical computing was in the 1950s. At that point, vacuum tubes were the dominant technology for classical computing and computers were room sized pieces of equipment housed in government facilities, universities and few large companies. It would have been very challenging to imagine the revolution that semiconductor based computing would drive in the coming decades. It may be that spin qubits prove to be as revolutionary as their semiconductor transistor forefathers or it may be one of the other listed technologies or a platform that has yet to even be considered. In any case, this thesis represents one small step on the long path (whether it proves a dead end or becomes the thoroughfare) towards an attempted quantum computer based on electron spins in semiconductors.

1.1 What’s a qubit?

Before an entire thesis delving into one specific type of qubit, it is prudent to give an introduction to qubits in general[61]. The qubit is the fundamental unit of
quantum computation in the same way that a classical bit is the fundamental unit of classical computation. Information can be stored and manipulated in the wave function of a system of qubits. As previously mentioned, a qubit is a quantum two level system with states generally labeled as $|0\rangle$ and $|1\rangle$. Quantum mechanics allows the qubit to exist partly in both states simultaneously, a property called superposition, and a general single qubit state can be represented as

$$|\Psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\Phi}|1\rangle$$

with $0 \leq \theta \leq \pi$ and $0 \leq \Phi \leq 2\pi$. The set of possible single qubit states can then be thought of as points on a sphere with unit radius where $\theta$ represents the polar angle and $\Phi$ represents the azimuthal angle. This is called the Bloch sphere and is shown in Figure 1. This infinite set of possible single qubit states contrasts with the two possible states for a single classical bit.

The physical significance of $\theta$ is easy to understand as it controls the relative contribution to the state from $|0\rangle$ versus $|1\rangle$, with $\theta=0$ corresponding to $|\Psi\rangle = |0\rangle$, $\theta=\pi$ corresponding to $|\Psi\rangle = |1\rangle$ and every $\theta$ value in between representing some mixture. I have often been asked about the physical meaning of $\Phi$, which is easiest understood in the concrete example of a single spin $\frac{1}{2}$ particle. For this example, $\theta$ and $\Phi$ represent the physical direction of this spin, which can be thought of as a unit vector pointing along the specified direction. It is common to define the spins states along the z axis to be $|0\rangle$ and $|1\rangle$. If we take the specific example of $\theta=\frac{\pi}{2}$, then $\Phi$ controls the direction that the spin points along the equator, with $\Phi=0$ giving the spin pointing along the x axis and $\Phi=\frac{\pi}{2}$ pointing along the y axis. For other qubit systems, the axes of the system will not correspond to physical directions but to more abstract description of the state. In general, $\theta$ and $\Phi$ can still be thought of as controlling where the spin points in this more abstract qubit state space.

Quantum computing requires the ability to map any qubit state onto any other qubit state. We have two real numbers, $\theta$ and $\Phi$ that represent our qubit state and
Figure 1.1.1: The surface of the Bloch sphere represents the possible single qubit states, characterized by the angles $\Phi$ and $\theta$. 

$$\frac{1}{\sqrt{2}} (|0\rangle + i|1\rangle)$$ 

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$
therefore need two control knobs to be able to control these two parameters independently. Qubit manipulations are achieved by allowing the qubit to time evolve with an engineered Hamiltonian. Generally, the Schrödinger Equation is given by

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

which has solution

$$|\Psi(t)\rangle = e^{-iHt/\hbar} |\Psi(0)\rangle$$

For our qubit, let's still refer to the qubit states of $|0\rangle$ and $|1\rangle$ to be along the $z$ axis. We can choose a Hamiltonian to apply to our qubit so that the qubit states are eigenstates with eigenvalues $E_0$ and $E_1$. Then the time evolution of the qubit state can be written as

$$|\Psi(t)\rangle = \cos(\frac{\theta}{2}) e^{-iE_0t/\hbar} |0\rangle + \sin(\frac{\theta}{2}) e^{i\Phi} e^{-iE_1t/\hbar} |1\rangle$$

$$= e^{-iE_0t/\hbar} (\cos(\frac{\theta}{2}) |0\rangle + \sin(\frac{\theta}{2}) e^{i(\Phi - (E_1 - E_0)t/\hbar)} |1\rangle)$$

Recall that the overall phase is nonphysical so we must only concern ourselves with the terms in the parenthesis. We note that the time evolution does not effect the $\theta$ value of the qubit state but that we can reexpress this using $\Phi' = \Phi - (E_1 - E_0)t/\hbar$. In other words, the time evolution does not change the probability of being in $|0\rangle$ versus $|1\rangle$, but it does rotate the state at a rate proportional to the energy difference between the states. To perform a specific rotation about $z$, one must just control the value of $(E_1 - E_0)t$ by controlling the magnitude of the Hamiltonian and how long it is applied for. To perform a rotation about another axis, one must only choose a Hamiltonian that has eigenstates along that given axis. Thus all qubit manipulations can be thought of as rotations of states on the Bloch sphere around a fixed axis determined by the
eigenstates of the Hamiltonian. For full qubit control, one must be able to apply
at least two non-collinear Hamiltonians to every qubit.

To measure $\theta$ and $\Phi$, one needs to repeatedly prepare the same state and
project along the x, y and z axes. This allows the experimenter to extract the
values of $\theta$ and $\Phi$ from the probabilities of measuring the eigenstates in each
direction[80].

1.2 Two qubit gate

Quantum computation relies on the ability to generate entangled states of
multiple qubits from qubits that are initially independent. Entangled states are
superposition states of multi qubit states that are not separable, meaning that
they cannot be written as the product of single qubit states. An example is the
state $|\Psi\rangle = |11\rangle + |00\rangle$. There is no way to express this as a state of the first qubit
tensored with the state of the second qubit.

Entanglement is most often generated by a Hamiltonian term that depends on
the states of two qubits. It is generally easiest to consider the case of a control
qubit and target qubit in which case the target qubit has an evolution that is
dictated by the state of the control qubit. One example of a gate of this type is the
CNOT gate which flips the state of the target qubit if the control qubit is in state
$|1\rangle$ and does nothing if the control qubit is in state $|0\rangle$. When this gate acts on the
originally unentangled state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$, it becomes previously
described entangled state $|11\rangle + |00\rangle$. A particularly useful fact is that any
maximally entangled state can be mapped to any other maximally entangled state
through the application of only single qubit gates. This means that any two qubit
gate along with full control of all single qubits is sufficient for quantum
computation[83].

When qubits enter into entangled states, they can no longer be expressed as
two independent Bloch spheres. Instead, it is often more useful to think about
them in terms of their density matrix, which for brevity we will assume that the
reader has some familiarity with. The density matrix is a mathematical tool that
encompasses both quantum and classical uncertainty. For quantum computation, it is especially useful to discuss the elements of the density matrix that lie on the diagonal compared to those that are off diagonal. The diagonal elements are called populations and describe the probability that the system will be measured to be in a specific state. The off diagonal elements are called coherences and express the “quantumness” of the system. As an example, consider the density matrix for the state \( \frac{1}{\sqrt{2}} (|11\rangle + |00\rangle) \) compared to the density matrix for a system that creates state \( |11\rangle \) with 50\% probability and state \( |00\rangle \) with 50\% probability. The density matrix for the second case has only off diagonal elements equal to zero while the former would have nonzero terms coming from \( |11\rangle \langle 00| \) and \( |00\rangle \langle 11| \). The off diagonal elements reveal whether the probabilistic nature of the measurement is from a classically source or from the uncertainty inherent in quantum superpositions. As we will see in the next section, they also reveal information leakage out of our quantum system.

Measuring the density matrix itself is one way to guarantee that one is able to generate an entangled state. However, this can be challenging as it requires measuring with a full basis of two qubit measurements. Once one has reconstructed the density matrix, measures like concurrence can be used to demonstrate that one has generated a state that is at least partially entangled. A method that is less rigorous but has also been used to demonstrate entanglement is the measurement of a single qubit block vector. Measuring a single qubit from a two qubit state requires us to trace over the states of the unmeasured qubit. For a perfectly entangled state of two qubits, one would see that the Bloch vector has length zero for either of the qubits while the other remains unmeasured. The oscillation of the Bloch vector length therefore has been used to show entanglement because the qubits will entangle and disentangle periodically when interacting. We will see in the next section that this same line of thinking can be used to understand decoherence as well.
1.3 **Decoherence**

Decoherence is the process of a qubit losing information to its environment. To control the qubit, its Hamiltonian must have a dependence upon some physical mechanism like an electric or magnetic field. Fluctuations in these same mechanisms from the environment during qubit manipulation cause errors in the evolution of the qubits. For example, changes in the magnetic field can result from fluctuating nuclear spins and changes in the electric field can result from the motion of charges near the qubit. There are two ways to think of about these decoherence processes. The first is consider only the quantum state of the qubit system under the influence of the controls of the experimental setup and the noise from the environment. These noise terms can be static or have time dependence and typically vary between experiments. To be useful as a qubit, the evolution of the system must be dominated by the qubit controls. This means that the evolution is close to what is desired by the experimenter but that the noise causes divergent evolution of the quantum states created by different experimental runs. The longer that qubits are allowed to evolve, the larger this divergence becomes because the qubit is allowed to evolve in the presence of different Hamiltonians for longer periods. Eventually the qubit is completely decohered and the results of the experiments would produce the set of qubit states in proportion to thermal equilibrium.

The second viewpoint is to consider the quantum state of the system of qubits and the environment. The density matrix for this is generally too large to be of much use. However, if we take a simple example, we can gain some useful intuition from this picture. Let us return to the density matrix for two qubits from the previous section. We define our qubit system to be the first qubit and the environment to be the second qubit. In the presence of an interaction between our qubit and the environment (the two qubit gate from the previous section), we will find that the qubit is equally likely to be in any state when maximally entangled with its environment (the other qubit). We can think of the influence of noise on our system of qubits in a similar fashion. The interactions
with the environment place our qubit into an entangled state with the environment. Since we cannot measure the quantum state of the environment, we trace over all environment states when we measure the qubit. The loss of information about the qubit state is due to the fact that the full quantum state is described partially by the environment we which cannot measure. This second picture is intellectually satisfying in that it looks at the mechanisms of decoherence at a fundamental level rather than as some effective control acting on the qubit. However, it is usually more practically useful to consider the qubit in the previous description when trying to improve qubit results as it is easier to consider correcting for an effective field than the contributions from all members of the environment.

1.4 Spins As Qubits

As previously mentioned, it is very natural to think of a single qubit as a spin 1/2 particle. For this reason Loss and DiVincenzo\cite{48} suggested that one path towards a scalable quantum computer would be based upon electron spins in semiconductors. This is very attractive as an electron’s spin is a two level system whose energies are split by a magnetic field. While rotations could be achieved by applying magnetic fields in different directions, this is experimentally challenging. Instead, it is easy to apply a static magnetic field in a direction which is defined to be along the z direction. A second axis of control can be achieved by applying an oscillating magnetic field directed along the x axis. When the frequency of this oscillating field matches the precession frequency from the static magnetic field, the qubit can be flipped from \( \uparrow \) to \( \downarrow \) and vice versa. This technique was first developed for NMR and is called the rotating frame approximation.

There are two significant challenges presented by the single spin qubit. The first is that it is practically difficult to generate a magnetic field that oscillates with a large magnitude and is localized to only a single qubit. This is most often achieved by oscillating an electron in the presence of of a magnetic field gradient so that in the electron’s frame of reference there is an oscillating magnetic field.
Additionally, the energy splitting between the qubit states depends on the magnetic field which couples the qubit to the nuclear spins of the environment. Fluctuations in the spins of the nuclei cause decoherence as they vary the magnetic field experienced by the qubit. We will solve both of these issues by constructing a qubit out of two electron spins.

1.5 Singlet-Triplet Spin Qubits

The single spin qubit is vulnerable to decoherence from magnetic field fluctuations because the energy difference between the two states depends on the magnetic field. Using two spins allows us to construct two qubit states that have the same zero magnetic moment and thus no dependence of their energy difference on the magnetic field\[65\]. There are four possible states for two spin 1/2. They are

\[
|T_+\rangle = |\uparrow\uparrow\rangle \\
|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\
|T_-\rangle = |\downarrow\downarrow\rangle
\]

The three triplet states can be thought of as a composite spin 1 particle and the lone state can be thought of a composite spin 0 particle. We select \(|S\rangle\) and \(|T_0\rangle\) as our qubit states because they have the same magnetic moment which means that our qubit subspace is protected from decoherence due to magnetic field fluctuations. As we will see in the next sections, this qubit is also electrically controllable, solving both major challenges of the single spin qubit.

One very important difference between the singlet and triplet states is in the symmetry of the spin component of the wave functions. The singlet spin state is anti-symmetric under particle exchange (\(|S\rangle \rightarrow - |S\rangle\)) while the triplet spin
states are symmetric under particle exchange \( (|T_0\rangle \rightarrow |T_0\rangle) \). Electrons are fermions and must have a wave function that is overall anti-symmetric under particle exchange. This will have important ramifications for the allowed spatial states for these two spin states.

In the context of this thesis the singlet-triplet qubit will always refer to a qubit build out of the \( |S\rangle \) and \( |T_0\rangle \) states. A qubit based on the \( |S\rangle \) and \( |T_-\rangle \) states has been demonstrated\(^{[86]}\) but it lacks the same decoherence protection against magnetic field fluctuations.

### 1.6 Quantum Dots

Manipulating two electrons spins is challenging in part because it means isolating two individual electrons. This is achievable in semiconductors using quantum dots. A quantum dot is a system where adding an additional electron requires paying a significant energy cost. As the name dot implies, these systems are typically small and support localized states. The energy cost of adding an additional electron comes from two main sources. The first is the coulomb repulsion between the electrons themselves. Any electron that will be added to the dot must overcoming the potential created by the electrons already in the dot. The second energy cost comes from the Pauli exclusion principle. As electrons are fermions, only two electrons with opposite spins can occupy a single spatial state. This means that subsequent electrons must pay the cost to enter higher energy orbitals as lower orbitals are already filled.

The work in this thesis uses gate defined quantum dots formed in the two dimensional electron gas (2DEG) of GaAs and SiGe. The electrons in the 2DEG are vertically confined so they can only move in a plane parallel to the surface of the crystal. This confinement is achieved by the growth of the crystal itself. An example of a device is shown in Figure 1.6.1. The metal on the surface of the semiconductor is referred to as gates (which can be confusing given that this word is also used for quantum gates, the manipulations applied to qubits). The metal gates are capacitively coupled to the 2DEG and are designed so that when
voltages are applied to the gates, the electrons can be laterally confined into a quantum dot. Designing the gate geometries for these devices is the focus of Chapter 5.

The electrons in the dots can be thought of as particles in a well. For intuition, we will treat this as an infinite square well, which has energies that scale as $\hbar^2/(2m^*L^2)$. Here, $m^*$ is the effective mass of the electron and $L$ is the length scale of the quantum dot. From this, we see that smaller quantum dots should have larger energy splittings between spatial states. The coulomb repulsion also increases with shorted distances between the electrons which means that the energy cost for adding electrons increases as the dot size decreases. Quantum dots are typically designed to be a most a few hundred nanometers in size so that...
the charging energy is significantly larger than the electron temperature in a dilution refrigerator (100 mK).

A double quantum dot is a pair of quantum dots that are capacitively and tunnel coupled. A typical measurement of the charge state of a double quantum dot is shown in Chapter 7. To represent the number of electrons in the double dot, it is standard to use the form (L,R) where L is the number of electrons in the left dot and R is the number of electrons in the right dot. The transitions from states with different numbers of total electrons, L+R, have slopes corresponding to the relative capacitances of the gate to its own dot versus the gate to the other dot. The size of the transitions with the same total number electrons is determined by the strength of the capacitive coupling between the dots. We will make use of the double quantum dot with two electrons for manipulating the singlet-triplet qubit.

1.7 Exchange Energy

As previously discussed, the singlet state has an anti-symmetric component to its wave function and the triplet has a symmetric component to its wave function. Electrons are fermions so their overall wave functions must be anti-symmetric under particle exchange. This means that the spatial components of the wave function of $|S\rangle$ must be symmetric while it is anti-symmetric for $|T_o\rangle$. We can write the full wave functions as follows:

$$|S\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \otimes (\varphi_A(x_1)\varphi_B(x_2) + \varphi_A(x_2)\varphi_B(x_1))$$

$$|T_o\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \otimes (\varphi_A(x_1)\varphi_B(x_2) - \varphi_A(x_2)\varphi_B(x_1))$$

Here $\varphi_A(x)$ and $\varphi_A(x)$ are the spatial states for the two electrons in the system. The symmetry difference between the two states is especially important when trying to place two electrons into the same spatial state, $\varphi_A(x) = \varphi_B(x)$. In this case, the triplet’s spatial wave function is zero which means that this is not an allowed quantum state. Notice that this is still an allowed state for the singlet
state. In our experiments, $\phi(x)$ will be either the ground state orbital of the left dot, $\psi_L(x)$ or the ground state orbital of the right dot, $\psi_R(x)$. The quantum dots are designed so that the excited orbitals are energetically inaccessible. The $|S\rangle$ state can occupy either the $(0,2)$ or $(1,1)$ charge state while the $|T_o\rangle$ is locked in the $(1,1)$ charge state because it cannot have two electrons in the same spatial state.

The charge state difference of the two spin states allows for electrical control of the energy difference between $|S\rangle$ and $|T_o\rangle$. The potential of the two dots are controlled by the metal gates and can tune the potential difference between the two dots. By lowering the potential energy of the right dot relative to the left, the $(0,2)$ charge state has a lower energy than the $(1,1)$ charge state. This means that $|S\rangle$ will have a lower energy as both electrons can enter the right dot while $|T_o\rangle$ will have a higher energy as one of its electrons is locked into the higher energy left dot. This energy difference is referred to as the exchange energy and the magnitude is controlled by the energy difference between the two dots. While this name may make it seem mysterious, it is really nothing more than wave function symmetry combined with an electrical potential difference.

For the rest of this thesis, we will use the following conventions. The detuning voltage between the dots, the potential difference of the right dot relative to the left, will be referred to as $\epsilon$, with $\epsilon=0$ at the transition between $(1,1)$ and $(0,2)$. The exchange energy is written as $J(\epsilon)$ and defined to be along the z axis in our qubit space, meaning that $|0\rangle = |S\rangle$ and $|1\rangle = |T_o\rangle$.

1.8 Full Single Qubit Control - Magnetic Field Gradient

For full single qubit control, we need another energy splitting which we will achieve through the application of a magnetic field gradient. In order to split the qubit subspace ($|S\rangle$ and $|T_o\rangle$) from the other two electron states ($|T_+\rangle$ and $|T_o\rangle$), a static magnetic field is applied. This magnetic field is applied along the z direction in space. Please note that magnetic field is directed along the z axis in real space while the exchange energy is directed along the z axis in the qubit state.
space, which are not the same. We also notice that \( \frac{1}{\sqrt{2}}(|T_o\rangle + |S\rangle) = |↑↓\rangle \) and 
\( \frac{1}{\sqrt{2}}(|T_o\rangle - |S\rangle) = |↓↑\rangle \). In the \((1,1)\) charge state, these states correspond to
definite spins in the left and right dots, and the exchange energy is 0 which causes
all possible states of the qubit to be degenerate. However, this degeneracy is
broken by the application of a gradient of the strength of the magnetic field in the
z direction, \( ΔB_z \). This term has energy eigenstates \(|↑↓\rangle\) and \(|↓↑\rangle\) because the
stronger magnetic field in one of the dots will have an energy preference for \( ↑ \) as
opposed to \( ↓ \). Notice that these states lie on the x axis of the block sphere, as seen
in Figure ???. This means that \( ΔB_z \) drives rotations about the x axis in our qubit
state space.

We can write the Hamiltonian for a single singlet-triplet qubit as

\[
H = J(\varepsilon)σ_z + ΔB_zσ_x
\]

The magnetic field gradient is approximately static during a single experiment
while the exchange interaction can be tuned dynamically with time scales on the
order of a single nanosecond. \( ΔB_z \) can be generated either by growing a
patterned micro-magnet on the surface of the semiconductor or by polarizing the
nuclear spins using dynamic nuclear polarization. Dynamic nuclear polarization
allows for tuning the strength of \( ΔB_z \) by controlling the degree of polarization in
the two dots and allows for gradients as strong as 1 Gauss/nm. However, it does
require using the double quantum dot as both a qubit and a spin pump which
limits the flexibility of the double dot tuning. Using a micro-magnet produces
magnetic field gradients that are an order of magnitude weaker but also works in
the absence on nuclear spins.

Complete control of the singlet-triplet qubit is achieved by controlling \( \varepsilon \) and
the evolution time of the qubit. The magnetic field gradient is limited to be less
than 1 GHz while the exchange interaction can be tuned from 0 to tens of GHz
via \( \varepsilon \) during the experiment. This allows \( \varepsilon \) to control both the magnitude and
direction of energy splitting because it controls the ratio of \( J \) to \( ΔB_z \). This allows
for any arbitrary qubit rotation as the evolution time is also controlled. To perform rotations around the x axis, $\epsilon < 0$ and for rotations around the z axis, $\epsilon > 0$.

To finish this section, I would like to briefly mention two nuances to the magnetic field gradient. Single spin qubits are also controlled by the application of a magnetic field along the z axis and a magnetic field gradient. However, the gradient for these qubit is a $\Delta B_x$, a gradient in the field perpendicular to the main splitting while our gradient is in $\Delta B_z$. Additionally, the energy difference is really the magnetic field difference between the two quantum dots and Professor Halperin was very adamant that this should more accurately be called a magnetic field difference rather than a gradient. The gradient can be converted to a difference by multiplying the gradient by the separation between the two dots.

### 1.9 History of the Singlet-Triplet Spin Qubit

The singlet-triplet qubit had already been in use in the Yacoby and Marcus groups for a decade when I began my PhD. In the case of my work in GaAs, much of the work in this thesis has been built upon this groundwork and in the case of my work in SiGe, the efforts have been to reproduce these results. For this reason, it is prudent to provide the history of the development of the singlet triplet qubit through some of the major developments.

In 2005, Jason Petta published the first paper on the singlet-triplet qubit and demonstrated exchange oscillations. This paper[65] also laid the groundwork for the initialization and readout methods used for singlet-triplet qubits. Both of these process rely on the ability of $|S\rangle$ to exist in $(0,2)$ while $|T_0\rangle$ is locked in $(1,1)$. The qubit is initialized in $|S\rangle$ by waiting in $(0,2)$ near the transition with $(0,1)$ so that electrons can rapidly tunnel in and out of the system, leaving the qubit in the ground state, $|S\rangle$. Readout relies on spin to charge conversion where the $|S\rangle$ enters the $(0,2)$ charge state while $|T_0\rangle$ remains in $(1,1)$. Charge state detection uses a proximal sensor dot (SD) or quantum point contact (QPC) whose resistance is dependent on the charge state of the qubit. In 2009, Sandra Foletti and Hendrik Bluhm were able to demonstrate full control of a
singlet-triplet qubit by using DNP and feedback to set \( \Delta B_Z \)[29].

In 2012, Mikey Shulman and Oliver Dial demonstrated a two qubit gate with singlet-triplet qubits based on the capacitive coupling between the electrons in adjacent double quantum dots[76]. It is easiest to think about this coupling in terms of a control and target qubit. The control qubit has a slightly difference charge state for \( |S\rangle \) compared to \( |T_0\rangle \) which means that the target qubit experiences different electric fields for the two states. The charge state difference is proportional to \( \frac{dJ_c(\epsilon_c)}{d\epsilon_c} \). The target qubit’s sensitivity to the electric field from the control qubit is proportional to \( \frac{dJ_T(\epsilon_T)}{d\epsilon_T} \). However, the target qubit’s sensitivity to noise also scales as \( \frac{dJ_T(\epsilon_T)}{d\epsilon_T} \) which meant that while Mikey and Oliver were able to demonstrate entanglement, they could not improve the entanglement value because any increase in the coupling strength between the qubits also resulted in the same increase in coupling to the environment. They found that charge noise was the limiting factor for two qubit gate performance and further studied the frequency dependence of charge noise in their 2013 publication[20].

In 2015, John, Shannon and Mikey delved further into the process of DNP and discovered that spin orbit coupling competes with hyperfine coupling for flipping spins[60]. This means that only a narrow range of applied magnetic field directions allow for DNP. This work was probably most significant because it primed the group to think deeply about the role that \( \Delta B_Z \) could play in manipulating the qubits. In the next two chapters, we will detail how a large \( \Delta B_Z \) can mitigate the effects of charge noise and how readout can be performed in this regime. At the same time, Shannon began working on using high impedance superconducting resonators to couple two singlet-triplet qubits[33], again with the goal of finding a mechanism that couples qubits to each other differently from how it couples them to their environment. The later chapters of this thesis focus on efforts to realize the singlet-triplet qubit in SiGe and to reproduce the reduce the results shown in 2005 by Jason and in 2009 by Sandra and Hendrik. This has proved challenging due to difference in GaAs and SiGe, as detailed in Chapter 4. While we have not yet demonstrated a singlet-triplet qubit in SiGe, we have made substantial progress on device design, fabrication, charge measurement, and
grounding for device protection. These are detailed in Chapters 5, 6 and 7. In the last Chapter, I discuss experiment ideas that I have not had the time to perform but that I would like to make available to the future generations of singlet-triplet researchers.
High-fidelity entangling gate for double-quantum-dot spin qubits

This chapter is taken from the paper with the same name. Before presenting this, I would like to provide a bit more of a verbose introduction than is allowed for by paper character limits. We found that we could steal the qubits against charge noise by the application of a great magnetic field gradient. The total qubit energy splitting is given by \( \Omega = \sqrt{f(\epsilon) + \Delta B_Z^2} \). The sensitivity of the qubit to charge noise is given by \( \frac{d\Omega}{d\epsilon} \). When \( \Delta B_Z \ll J(\epsilon) \), as was the case in the 2012 two qubit paper, \( \frac{d\Omega}{d\epsilon} \approx \frac{dJ(\epsilon)}{d\epsilon} \). In the opposite limit of \( \Delta B_Z \gg J(\epsilon) \), \( \frac{d\Omega}{d\epsilon} \approx \frac{f(\epsilon)}{\Delta B_Z} \frac{dJ(\epsilon)}{d\epsilon} \). This factor of \( \frac{f(\epsilon)}{\Delta B_Z} \) reduces the coupling of charge noise relative to the coupling between qubits, which when manipulated in synchronous, maintain a coupling of \( \frac{dJ(\epsilon)}{d\epsilon} \). This is responsible for the improved two qubit entanglement observed in
2.1 **Abstract**

Electron spins in semiconductors are promising qubits because their long coherence times enable nearly $10^9$ coherent quantum gate operations. However, developing a scalable high-fidelity two-qubit gate remains challenging. Here, we demonstrate an entangling gate between two double-quantum-dot spin qubits in GaAs by using a magnetic field gradient between the two dots in each qubit to suppress decoherence due to charge noise. When the magnetic gradient dominates the voltage-controlled exchange interaction between electrons, qubit coherence times increase by an order of magnitude. Using randomized benchmarking, we measure single-qubit gate fidelities of approximately 99%, and through self-consistent quantum measurement, state, and process tomography, we measure an entangling gate fidelity of 90%. In the future, operating double quantum dot spin qubits with large gradients in nuclear-spin-free materials, such as Si, should enable a two-qubit gate fidelity surpassing the threshold for fault-tolerant quantum information processing.

2.2 **Introduction**

The quantum phase coherence of isolated spins in semiconductors \cite{25,42,46,49,57,65,67} can persist for long times, reaching tens of milliseconds for electron spins \cite{84} and tens of minutes for nuclear spins \cite{71}. Such long coherence times enable single-qubit gate fidelities exceeding the threshold for fault-tolerant quantum computing \cite{84} and make spins promising qubits. However, entangling spins is difficult because magnetic interactions between spins are weak. For electrons, this challenge can be met by exploiting the charge of the electron for electric-dipole \cite{76} or gate-controlled exchange coupling \cite{65} between spins. In these methods, however, the qubit energy depends on electric fields, and charge noise in the host material limits
single-qubit coherence [20]. Charge noise also affects other qubit platforms. For example, heating due to charge noise is a limiting factor in the coherence of trapped ion qubits [11], and the transmon superconducting qubit was designed to suppress noise from charge fluctuations in superconducting islands [36]. Strategies such as composite pulses [12, 87], dynamical decoupling [20], and sweet-spot operation [7, 54, 69] have been developed to mitigate the effects of charge noise.

In this work, we present a technique to suppress decoherence caused by charge noise. The key idea is to apply a large transverse qubit energy splitting that does not depend on electric fields and therefore suppresses the effects of charge fluctuations. We implement this scheme with two singlet-triplet qubits, each of which consists of two electrons in a double-quantum-dot [65]. In each qubit, the voltage-controlled exchange interaction $J(\epsilon)$, where $\epsilon$ represents the gate voltage, splits the singlet $\langle S \rangle = (\langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle) / \sqrt{2}$ and triplet $\langle T_0 \rangle = (\langle \uparrow \downarrow \rangle + \langle \downarrow \uparrow \rangle) / \sqrt{2}$ states in energy [65], where the left(right) arrow indicates the spin of the left(right) electron. A magnetic gradient $\Delta B_z$ between the two dots lifts the degeneracy between $\langle \uparrow \downarrow \rangle$ and $\langle \downarrow \uparrow \rangle$. These two mechanisms enable universal quantum control of singlet-triplet qubits [29]. Until now, two-qubit gates for singlet-triplet qubits have operated with $J(\epsilon) \gg \Delta B_z$, and charge noise is the limiting factor in two-qubit gate fidelities [65, 76]. However, if $\Delta B_z \gg J(\epsilon)$, the total qubit energy splitting is $\Omega(\epsilon) = \sqrt{\Delta B_z^2 + J(\epsilon)^2} \approx \Delta B_z + \frac{J(\epsilon)^2}{\Delta B_z}$, and the qubit sensitivity to charge noise $\Omega'(\epsilon) = \frac{J(\epsilon)}{\Delta B_z} J'(\epsilon)$ is reduced by a factor of $\frac{1}{\Delta B_z}$, effectively mitigating decoherence due to charge noise.

Intense magnetic field gradients in spin qubits can be created with micromagnets [67, 81, 86]. In GaAs quantum dots, strong magnetic gradients can also be generated via the hyperfine interaction between the electron and Ga and As nuclear spins in the semiconductor [8, 29, 60, 77]. Coherence times for qubit rotations around hyperfine gradients can approach one millisecond [53], which is significantly longer than typical exchange coherence times [20]. Here, we show that when the magnetic gradient in a GaAs singlet-triplet qubit dominates the electrically-controlled exchange interaction, coherence times
increase by an order of magnitude. Through both standard and interleaved randomized benchmarking, we measure average single qubit gate fidelities of approximately 99%. At the same time, this approach maintains a large interaction between adjacent capacitively coupled qubits. We use self-consistent two-qubit state- and measurement tomography to measure a Bell state with a maximum fidelity of 93%. Full process tomography involving 256 tomographic measurements of the two-qubit operation yields an entangling gate fidelity of approximately 90%, consistent with theoretical simulations. In materials without nuclear spins such as silicon, even higher gate fidelities should be possible.

We use two singlet-triplet qubits [65], created in gate-defined double quantum dots similar to those of refs. [20, 76] in a GaAs/AlGaAs heterostructure [Fig. 2.2.1(a)]. Each double quantum dot contains two electrons. The Hamiltonian for each qubit is \( H(\varepsilon) = J(\varepsilon)\sigma_z + \Delta B_z \sigma_x \), in the \( \{ \langle S |, \langle T_0 | \} \) basis. \( J(\varepsilon) \), the exchange interaction between the two spins, depends on \( \varepsilon \), the difference in electrochemical potential between the dots [Fig. 2.2.1(b)]. \( \Delta B_z \), the difference in longitudinal magnetic field between the two dots, results from the wavefunction overlap between each electron and the Ga and As nuclear spins in the heterostructure. Although the nuclear spins are unpolarized in thermal equilibrium, \( \Delta B_z \) can be measured and stabilized up to several hundred mT using feedback [8, 29, 77].

The two adjacent qubits are capacitively coupled, and the interaction Hamiltonian \( H_{int} = J_{12}\sigma_z \otimes \sigma_z \), where \( J_{12} \propto J'_{1}(\varepsilon_1)J'_{2}(\varepsilon_2) \) [76, 82], and the subscripts refer to the different qubits. For the values of \( \varepsilon \) used here, we empirically find that \( J'(\varepsilon) \propto J(\varepsilon) \). This requires that \( J(\varepsilon) > 0 \) to maintain nonzero interqubit coupling.

Figure 2.2.1(c) shows the energy level diagram of the two-electron spin states in a double quantum dot. The qubit states are the \( \langle S \rangle \) and \( \langle T_0 \rangle \) levels in the regime where \( \Delta B_z \gg J(\varepsilon) \) [Fig. 2.2.1(c-d)]. Through dynamic nuclear polarization and feedback, we set \( g^*\mu_B\Delta B_z/h \approx 1 \text{ GHz} \) in all experiments [8, 29]. Here \( g^* = -0.44 \) is the effective electron g-factor in GaAs, \( \mu_B \) is the Bohr magneton.
Figure 2.2.1: Experimental setup. (a) Scanning electron micrograph of a two-qubit device identical to the one used in this work. Red circles indicate approximate positions of electrons in the double-well potentials created by metal depletion gates (gray). Arrows indicate the positions of sensor quantum dots. A voltage difference $\epsilon$ applied to plunger gates adjusts the exchange interaction. (b) The gate-controlled wavefunction overlap between electron spins produces the exchange interaction $J(\epsilon)$. Each electron also interacts with a large number of Ga and As nuclear spins (green and orange circles) via the hyperfine interaction, leading to a difference in the longitudinal magnetic gradient between the dots $\Delta B_z = B_{z,L} - B_{z,R}$. (c) Energy level diagram showing the two-electron spin states of a double quantum dot. We operate the qubit with $\epsilon < 0$ and $J(\epsilon) \ll \Delta B_z$, as indicated with the dashed gray box. (d) Calculated qubit energy splitting $\Omega(\epsilon)$ for the two cases when $\Delta B_z = 0$ and $\Delta B_z \approx 1$ GHz. When $\Delta B_z$ is large, the qubit splitting does not depend on $\epsilon$ and is insensitive to electric fields.
and $h$ is Planck's constant. $\Delta B_z$ is stabilized to within 3 MHz, corresponding to an inhomogeneously broadened coherence time $T_2^* \approx 100$ ns. We initialize the $\langle 0 \rangle$ state through electron exchange with the leads when $\epsilon \gg 0$, where $\langle S \rangle$ is the ground state of the double dot. Then we adiabatically ramp to $\epsilon = \epsilon_o < 0$, where $100$ MHz $< J(\epsilon_o)/2\pi < 300$ MHz $< \Delta B_z$. We measure the qubit state via electron exchange with the leads in a new technique (see Supplementary Information), which is compatible with large magnetic gradients [5].

We drive qubit rotations by adding an oscillating voltage to the plunger gates, such that the total voltage $\epsilon(t) = \epsilon_o + \epsilon_1 \cos(\Omega t)$. For $\epsilon_1 \ll \epsilon_o$, $J(t) \approx J(\epsilon_o) + 2j \cos(\Omega t)$, where $j = \frac{1}{2} J'(\epsilon_o)$ is the Rabi frequency. When the oscillation frequency matches the total qubit splitting $\Omega = \sqrt{\Delta B_z^2 + J^2(\epsilon_o)}$, the time varying component of $J(t)$ drives qubit transitions [Fig. 2.2.2 (a)] [77]. In this regime, $\Omega \approx \Delta B_z$ is analogous to the external magnetic field for a single spin-1/2, while the time varying component of $J(t)$ is analogous to a perpendicular oscillating magnetic field, which drives transitions. We emphasize that when $\Delta B_z \gg J(\epsilon_o)$, $\Omega(\epsilon_o) \approx \Delta B_z + \frac{J(\epsilon_o)^2}{2\Delta B_z}$, and the sensitivity to charge noise $\Omega'(\epsilon_o) = \frac{J(\epsilon_o)}{\Delta B_z} J'(\epsilon_o)$ is smaller by a factor of $\frac{J(\epsilon_o)}{\Delta B_z}$ compared to the case where $\Delta B_z \ll J(\epsilon_o)$ [Fig. 2.2.1 (d)]. However, a key requirement of this technique is that $J(\epsilon_o) > 0$, in order to maintain $J'(\epsilon_o) > 0$ for single-qubit control and two-qubit coupling.

Large magnetic gradients can therefore completely suppress dephasing due to charge noise, although relaxation caused by charge noise at the qubit frequency $\Delta B_z$ still limits the coherence. In our case, however, nuclear spin noise causes the magnetic gradient to fluctuate. To suppress the effects of hyperfine fluctuations, we apply a strong rf drive to the qubit, causing Rabi oscillations. In the reference frame rotating around the qubit splitting $\Omega \sigma_z$, the Hamiltonian is $H_{\text{rot}} = j \sigma_z + \delta \Omega \sigma_x$, where $j$ is the Rabi frequency, and $\delta \Omega$ is a fluctuation in the magnetic gradient. When $j \gg \delta \Omega$, the qubit splitting in the rotating frame $\Omega_{\text{rot}} \approx j + \frac{\delta \Omega^2}{4j}$ is first-order insensitive to fluctuations in the magnetic gradient.
Figure 2.2.2: Single-qubit operations. (a) Time-varying voltage pulses resonant with the qubit splitting $\Omega$ induce Rabi oscillations. (b) Coherence times of driven Rabi oscillations (blue) and rotary echo (red) vs. Rabi drive strength. The solid blue line is a theoretical curve taking into account the measured charge and hyperfine noise levels in our qubit. The data agree with the model. The dashed red line between data points is a guide to the eye. At low drive strengths, hyperfine fluctuations limit the coherence time, and at large drive strengths, charge-noise-induced fluctuations in the Rabi frequency limit the coherence. (c) Randomized benchmarking yields an average gate fidelity of $98.6 \pm 0.2\%$. Error bars are statistical uncertainties. Note that the qubit splitting points along the x direction, and all gates are performed with phase-modulated rf pulses. See Supplementary Information for more details on randomized benchmarking.
2.3 Results

Figure 2.2.2(b) shows the coherence time of driven Rabi oscillations as a function of drive strength for $J(\varepsilon_0)/(2\pi) = 220$ MHz. The maximum coherence time ($\approx 700$ ns) is an order of magnitude larger than that for oscillations around a static exchange splitting with the same $J(\varepsilon_0)$ ($\approx 80$ ns). However, the quality factor of Rabi oscillations is the same as for static exchange oscillations [20], because low-frequency charge noise limits the coherence time in both cases. But because $j \ll J(\varepsilon_0)$, the Rabi coherence time is much longer. It is this improvement in coherence that allows increased two-qubit gate fidelities, as described below. Reversing the phase of the drive halfway through the evolution to perform a rotary echo extends the coherence time by an additional factor of 10 [Fig. 2.2.2(b)]. Rotating-frame echo coherence times are also an order of magnitude longer than static exchange echo [20] dephasing times measured in this device. We do not observe a large increase in coherence with multi-pulse dynamical decoupling, perhaps due to high-frequency charge noise [20].

As the amplitude of the oscillating voltage $\varepsilon$, increases, both the Rabi and echo coherence times reach a maximum [Fig. 2.2.2(b)]. At low drive strengths, hyperfine fluctuations in the detuning limit the coherence. At large drive strengths, charge-noise-induced fluctuations in $J'(\varepsilon)$, which cause the Rabi rates to fluctuate in time, limit the coherence. The observed behavior agrees well with a theoretical simulation based on measured noise levels in our qubit [Fig. 2.2.2(b)] (see Supplementary Information). The simulation correctly predicts the maximum coherence time and corresponding Rabi frequency. Using randomized benchmarking [26, 39, 44, 50–52, 56], we find an average gate fidelity of 98.6 $\pm$ 0.2%, with individual gate fidelities close to the average. [Fig. 2.2.2(c)]. (See the Supplementary Information for further experimental details on randomized benchmarking.) Gate fidelities are likely coherence limited as a result of slow electric-field or hyperfine fluctuations. Given the observed quality factor of Rabi oscillations, which is approximately 5, [Fig. 2.2.2(a)], we would expect roughly 10 coherent $\pi$ rotations within the coherence time. Assuming
Gaussian decay due to low-frequency noise, the fidelity of a π-gate should be approximately $e^{-(t/10)^2} = 0.99$. Because hyperfine or charge fluctuations are slow compared with gate times ($\approx 20$ ns), errors are likely correlated [2], as is the case for most spin qubits. Suppressed low-frequency charge noise or composite pulses [12, 87] would improve gate fidelities.

Next, we take advantage of the long coherence times in the $\Delta B_z$-dominated regime to perform a high-fidelity two-qubit entangling gate. In the lab frame, $H \approx \Omega_1 \sigma_x \otimes I + \Omega_2 I \otimes \sigma_x + J_{12} \sigma_z \otimes \sigma_z$, where $I$ is the identity operator. The single-qubit terms in the Hamiltonian do not commute with the interaction term, and the single-qubit rotations cancel the interaction except when $\Omega_1 = \Omega_2$ (see Supplementary Information), in a manner analogous to the Hartmann-Hahn condition for nuclear double resonance [32]. In this case, the interaction in the rotating frame is

$$H_{\text{int}} \approx \frac{J_{12}}{2} \sigma_z \otimes \sigma_z \cos(\varphi_1 - \varphi_2).$$ (2.1)

Here $J_{12} \propto J'_1(\varepsilon_1)J'_2(\varepsilon_2)$ is the interaction energy, and $\varphi_i$ is the phase of the rf drive on each qubit. When $\Omega_1 = \Omega_2$ and $\varphi_1 = \varphi_2$, the single-qubit rotations constructively interfere, and the interaction is the same as in the lab frame, up to a factor of 1/2. The order-of-magnitude increase in single-qubit coherence discussed above therefore enables a substantially improved two-qubit gate fidelity. This interaction generates an operation equivalent to a controlled phase gate up to single-qubit rotations.

To entangle the qubits, we set $\Omega_1 = \Omega_2 = 960$ MHz and perform a simultaneous rotary echo for varying lengths of time [Fig. 2.3.1(a)], choosing the drive amplitude that maximizes the echo coherence time. Self-consistent two-qubit measurement and state tomography [80] (see Supplementary Information) reveal an oscillating concurrence of the two-qubit state [Fig. 2.3.1(c)]. The concurrence is defined as $C = \lambda_4 - \lambda_3 - \lambda_2 - \lambda_1$, where the
Figure 2.3.1: Entangling gate. (a) To entangle the qubits, we perform a simultaneous rotary echo on both for a varying total length of time $t$, followed by tomographic readout to reconstruct the two-qubit density matrix. (b) Bloch vector length for qubit 1, $l_1$, during the entangling gate as the phase between rf drives varies. Nodes in $l_1$ denote entanglement. The entanglement rate vanishes when the qubits are driven 90° out of phase. (c) Concurrence vs. time for the entangling gate. Positive values of concurrence indicate entangled states. Negative values of concurrence indicate decoherence. The solid red line is a theoretical simulation taking into account hyperfine and low- and high-frequency charge noise (see Supplementary Information). (d) Single-qubit Bloch vector lengths $l_1$ and $l_2$ during joint evolution. When the qubits are detuned from each other, the interaction vanishes.
\( \lambda_i \) are eigenvalues of the two-qubit density matrix, arranged from smallest to largest \([35]\). The concurrence ranges from 1 to -0.5. A value of 0 indicates no entanglement between the qubits, a value of 1 indicates maximal entanglement, and a value of -0.5 indicates complete dephasing. The observed concurrence periodically reaches values above zero, demonstrating repeated entangling and disentangling of the qubits as the interaction time increases. Eventually, the concurrence saturates at a negative value, because both qubits have dephased. We have performed numerical simulations taking into account hyperfine noise and both low- and high-frequency charge noise (see Supplementary Information). The measured concurrence agrees with the simulation \([\text{Fig. 2.3.1(c)}]\). As the concurrence reaches a local maximum, the length of the single-qubit Bloch vectors, \( l = \sqrt{\langle \sigma_x \rangle^2 + \langle \sigma_y \rangle^2 + \langle \sigma_z \rangle^2} \), where \( \langle \cdots \rangle \) indicates a single-qubit expectation value, approach zero, as expected for entangled states \([\text{Fig. 2.3.1(d)}]\).

As equation 2.1 suggests, the interaction strength depends on the relative phase between the rf drives on each qubit. We demonstrate phase control of the two-qubit interaction by measuring the length of the Bloch vector of one qubit as we vary the relative phase between qubits \([\text{Fig. 2.3.1(b)}]\). As the expected, the entangling rate reaches a maximum when the two qubits are driven in phase, and the entangling rate vanishes when the two qubits are out of phase.

The two-qubit interaction also vanishes if \( \Omega_1 \neq \Omega_2 \). To demonstrate frequency control of the two-qubit gate, we turn off the dynamic nuclear polarization \([29]\) on qubit 2, effectively setting \( \Omega_2 \approx 0 \) MHz. However, all gate voltages during the entangling operation remain the same. Measuring the Bloch vector length of qubit 1 as a function of evolution time shows no oscillations, just a smooth decay \([\text{Fig. 2.3.1(d)}]\). This indicates that no entanglement takes place, and hence that the interaction vanishes, when the two qubits are detuned from each other.

To assess the gate fidelity, we perform self-consistent quantum process tomography \([14, 68, 80]\) on the two-qubit gate \([\text{Fig. 2.3.2(a)-(d)}]\), requiring 256 tomographic measurements of the two-qubit operation. We extract a maximum gate fidelity of 90 ± 1% based on a measured tomographically complete set of
input and output states (see Supplementary Information). The extracted process matrix $\chi$ has a few negative eigenvalues, which may result from partially mixed input states. Using a maximum likelihood estimation process to ensure a completely positive process matrix (see Supplementary Information), we extract a gate fidelity of $87 \pm 1\%$, which is consistent with the fidelity obtained by direct inversion.

Figure 2.3.2(e) shows the maximum observed gate and Bell state fidelity as a function of interaction strength, which is varied by adjusting $J(\epsilon_0)$ on each qubit. Similar to the case of single qubit coherence times, the gate fidelity drops at low interaction rates due to hyperfine noise. Gate fidelities are also expected to drop at fast interaction times due to charge noise, but we did not perform this experiment because our dynamic nuclear polarization feedback is not stable in this regime. An additional source of error at large interaction strengths is relaxation of the qubit states during initialization due to increased charge noise. We observe a maximum concurrence of $0.86 \pm 0.02$, corresponding to a Bell state fidelity of $93 \pm 1\%$. Given that the observed Bell state fidelities are equal to or slightly larger than the gate fidelities, it is likely that both decoherence and control errors play a role in overall gate fidelity.

2.4 Discussion

The maximum entangled state fidelity presented here represents a reduction in infidelity of about a factor of 4 over the previous entangling gate between singlet-triplet qubits [76], because the effects of charge noise are reduced when the magnetic gradient dominates the exchange interaction. This gate can be improved in the future by narrowing the hyperfine distribution [43] through rapid Hamiltonian estimation [77], or by using spin qubits in nuclear-spin-free materials such as Si, where strong gradients can be established with micromagnets. We estimate that with laboratory frame coherence times of 1 $\mu$s (instead of $\approx 100$ ns here), rotating frame coherence times could increase by as
Figure 2.3.2: Process tomography for the two-qubit entangling gate. (a) Real component of the measured process matrix. (b) Imaginary component of the measured process matrix. (c) Real component of the ideal process matrix. (d) Imaginary component of the ideal process matrix. (e) Gate fidelity of the measured process matrix and most-likely completely positive process matrix and two-qubit Bell state fidelity as a function of interaction strength. Error bars are statistical errors.
much as 3-4 times. Longer coherence times such as these suggest that two-qubit gate fidelities exceeding 99%, and fault-tolerant quantum computation using spins, are within reach.

2.5 Materials and Methods

The two double quantum dots are fabricated on a GaAs/AlGaAs heterostructure with a two-dimensional electron gas located 91 nm below the surface. The two-dimensional electron gas density $n = 1.5 \times 10^{11}$ cm$^{-2}$ and mobility $\mu = 2.5 \times 10^6$ cm$^2$/Vs were measured at $T = 4$K. Voltages applied to Au/Pd depletion gates define the double-dot potential. The qubits are cooled in a dilution refrigerator to a base temperature of approximately 20 mK. An external magnetic field $B=0.7$ T is applied in the plane of the semiconductor surface perpendicular to the axis of the double quantum dots. This orientation of the magnetic field ensures effective dynamic nuclear polarization [60].

2.6 Acknowledgments

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2.7 Author Contributions

S.F., G.C.G., and M.J.M. grew and characterized the AlGaAs/GaAs heterostructure. S.P.H. fabricated the device. J.M.N. and L.A.O. performed the experiments. All authors discussed and analyzed the data and wrote the manuscript. A.Y. supervised the project.

2.8 Supplemental Materials

2.8.1 Qubit readout

We operate each qubit with $\Delta B_z \gg J(\epsilon_0)$. The qubit eigenstates are approximately $\langle \uparrow \downarrow \rangle$ and $\langle \downarrow \uparrow \rangle$, where the left(right) arrow indicates the spin of the electron in the left(right) quantum dot in the $(1, 1)$ charge configuration, where each electron occupies its own quantum dot. These states can be read out by adiabatic charge transfer of both electrons into the right dot, where $\langle \uparrow \downarrow \rangle \rightarrow \langle (1, 1) T_0 \rangle$, and $\langle \downarrow \uparrow \rangle \rightarrow \langle (0, 2) S \rangle$. The Pauli spin-blockade forces the $\langle T_0 \rangle$ state to occupy the $(1, 1)$ charge configuration. A nearby charge-sensing quantum dot is then used to distinguish $\langle (0, 2) S \rangle$ and $\langle (1, 1) T_0 \rangle$ because they have different charge configurations. However, when $\Delta B_z$ is large, rapid $\langle (1, 1) T_0 \rangle \rightarrow \langle (0, 2) S \rangle$ relaxation occurs, diminishing readout contrast \[5\].

To overcome this challenge, we have developed a new readout technique, which involves transferring $\langle \uparrow \downarrow \rangle \rightarrow \langle (1, 1) T_0 \rangle$ instead of $\langle \uparrow \downarrow \rangle \rightarrow \langle (1, 1) T_0 \rangle$, avoiding the problem of $\langle (1, 1) T_0 \rangle \rightarrow \langle (0, 2) S \rangle$ relaxation. Deep in the $(1, 1)$ charge state, the lowest energy levels of the double dot are, in order of increasing energy, $\{ \langle \uparrow \uparrow \rangle, \langle \downarrow \uparrow \rangle, \langle \uparrow \downarrow \rangle, \langle \downarrow \downarrow \rangle \}$. Here we have assumed the presence of a magnetic gradient that favors spin-up on the right dot. Usually, the lowest-energy $(1, 2)$ charge state, which has a spin-up in the left dot and a singlet in the right dot, $\langle \uparrow S \rangle$, is much higher in energy than the four $(1, 1)$ states. However, we can adjust the electrostatic potential of the left dot such that $\langle \uparrow S \rangle$ becomes lower in energy than $\langle \uparrow \downarrow \rangle$ but still higher than $\langle \uparrow \uparrow \rangle$. In this case, the lowest energy states are now
\{\uparrow\uparrow, \uparrow S, \downarrow\uparrow, \downarrow\downarrow\}. When the double-dot is in the \(\uparrow\downarrow\) state, therefore, thermal equilibration with the right lead will lead to a transition sequence \((1, 1) \uparrow\downarrow \rightarrow (1, 2) \uparrow S \rightarrow (1, 1) \uparrow\uparrow = (1, 1) T_+\), where we have explicitly included the charge configuration for each step. We do not adjust the electrostatic potential of the left dot, so any excited states involving charge transfer to or from the left dot remain inaccessible. The \(\downarrow\uparrow\) state occupation will remain nominally unchanged, because the \(\downarrow S\) state is higher in energy. Following this sequence, we adiabatically transfer both electrons to the right dot and readout with Pauli spin-blockade. The key advantage of this technique is that \((0, 2) S \rightarrow (1, 1) T_+\) relaxation is much slower than \((1, 1) T_0 \rightarrow (0, 2) S\) relaxation at large gradients. A detailed description of this readout procedure will be the subject of a future publication. Based on self-consistent measurement tomography, readout fidelities are approximately 75\% (see section 2.8.5 below). Because this technique relies on thermal equilibration with the leads, the fidelity can be improved in the future by operating at larger magnetic field strengths or lower temperatures.

### 2.8.2 Qubit coherence time

We calculate the inhomogeneously broadened coherence time of driven Rabi oscillations. The amplitude of the total splitting in the rotating frame is, including noise, \(\Omega_{\text{rot}} = \sqrt{(j + \delta j)^2 + \delta\Omega^2}\). \(j\) is the Rabi drive, \(\delta j\) is the noise in the Rabi drive, and \(\delta\Omega\) is the detuning noise.

Assuming that \(j \gg \delta\Omega\), we have

\[
\delta\Omega_{\text{rot}} = \delta j + \frac{1}{2} \frac{\delta\Omega^2}{j}
\]

(2.2)

\(\delta j\) occurs because low-frequency charge noise modulates the value of \(j' (\epsilon)\). Additionally, there are spectral components of the \(\epsilon\)-noise directly at \(\omega\). Set \(\epsilon(t) = \epsilon_o + \epsilon \cos(\Omega t) + \delta\epsilon(t)\). Assume that the noise \(\delta\epsilon(t) \ll \epsilon_o\). Expanding
J(ε), and keeping only terms upconverted by the modulation, we have

\[ J(t) \approx J(ε_o) + (J'(ε_o)ε_1 + J''(ε_o)ε_1δε(t)) \cos(Ωt). \]  \hspace{1cm} (2.3)

Assuming ΔBz ≫ J(εo), we have j = \( \frac{1}{2} J'(ε_o) \), and \( δj(t) = \frac{1}{2} J''(ε_o)ε_1δε(t) \).

In GaAs qubits, noise in Ω arises primarily from fluctuations in ΔBz. Note that δΩ renormalizes the mean value of \( ⟨Ω_{rot}⟩ = j + \frac{ε_1}{2} σ^x_i \). We therefore compute

\[ \left\langle \frac{1}{4j^2} (δΩ^2 - σ^z_{Ω})^2 \right\rangle = \frac{1}{4j^2} (σ^x_Ω - 2σ^z_Ω + 3σ^x_Ω) \]  \hspace{1cm} (2.4)

\[ = \frac{σ^x_Ω}{2j^2}, \]  \hspace{1cm} (2.5)

where we have made use of the fact that a fourth moment of a Gaussian variable is 3 times the standard deviation to the fourth power. Thus, in total, we have

\[ σ^z_{Ω_{rot}} = \left\langle δΩ^2_{rot} \right\rangle = \frac{ε_1}{4} J''(ε_o)σ^z_1 + \frac{1}{2j} \frac{σ^z_{ΔB}}{j}. \]  \hspace{1cm} (2.6)

At low drive strengths, the second term in equation 2.6, which results from hyperfine noise, dominates. At large drive strengths, the first term, which results from charge-noise induced fluctuations in j, dominates. We therefore expect a minimum at intermediate values of ε1, where the inhomogeneously broadened coherence time \( T_1^* = \frac{1}{2πσ_{Ω_{rot}}} \) reaches a maximum.

2.8.3 Randomized benchmarking

Here we describe the details of the randomized benchmarking experiment. Randomized benchmarking \([26, 44, 50–52]\) involves concatenating many quantum gates from the Clifford group to magnify gate errors and average over all noise configurations. Crucially, randomized benchmarking is also insensitive to state preparation and measurement errors. We decompose all Clifford gates into sequences of phase-modulated rf pulses, as described in Ref. [26]. In our work, the qubit splitting is oriented along the x direction, so both y and z gates can be
implemented with rf pulses. In all experiments, \( \pi \)-pulse times were 20 ns, and \( \pi/2 \) pulse times were 10 ns. Identity gates were implemented as a 10-ns wait. After each gate (including the identity) a 5-ns wait was implemented to avoid pulse crosstalk. All pulses were nominally square, although bandpass rf filters in our setup smoothed the pulse edges such that the rise times were a few ns.

The data in Fig. 2(c) in the main text represent the average of 30 different random sequences of Clifford gates. Each of the five datasets in Fig. 2(c) in the main text, corresponding to the reference, \( Z, Y, Z/2, \) and \( Y/2 \) curves, was averaged 1600 times per unique Clifford sequence. To accommodate the required dynamic nuclear polarization, we averaged each of the five data sets (reference, \( Z, Y, Y/2, Z/2 \)) for the same Clifford sequence 16 times before recalibrating the magnetic gradient \( \Delta B_z \). We then repeated the entire sequence of averaging followed by recalibration 100 times per Clifford sequence for 30 different sequences. Pulse amplitudes were recalibrated between different Clifford sequences. The average time for gradient calibration was approximately one second. The statistical uncertainties reported in Fig. 2(c) in the main text refer to the standard error in the mean \( \sigma/\sqrt{N} \) for all experimentally measured data points.

2.8.4 Two-qubit interaction

The Hamiltonian in the lab frame is

\[
H \approx \Omega_1 \sigma_x \otimes I + \Omega_2 I \otimes \sigma_x + J_{12} \sigma_z \otimes \sigma_z,
\]

where for each qubit \( \Omega \approx \Delta B_z \). Transforming into the reference frame rotating around \( \Omega \sigma_x \), for each qubit, \( \sigma_z \rightarrow \sigma_z \cos(\Omega t + \phi) + \sigma_y \sin(\Omega t + \phi) \). Therefore

\[
H_{\text{rot}} = J_{12} (\sigma_z \otimes \sigma_z \cos(\Omega_1 t + \phi_1) \cos(\Omega_2 t + \phi_2) + \\
\sigma_z \otimes \sigma_z \cos(\Omega_1 t + \phi_1) \sin(\Omega_2 t + \phi_2) + \\
\sigma_y \otimes \sigma_z \sin(\Omega_1 t + \phi_1) \cos(\Omega_2 t + \phi_2) + \\
\sigma_y \otimes \sigma_y \sin(\Omega_1 t + \phi_1) \sin(\Omega_2 t + \phi_2)).
\]

(2.7)
\( H_{\text{rot}} \) has a non-zero time averaged value only when \( \Omega_1 = \Omega_2 \). In this case,

\[
\langle H_{\text{rot}} \rangle = \frac{J_{12}}{2} (\sigma_z \otimes \sigma_z + \sigma_y \otimes \sigma_y) \cos(\varphi_1 - \varphi_2) + \\
\frac{J_{12}}{2} (\sigma_z \otimes \sigma_y - \sigma_y \otimes \sigma_z) \sin(\varphi_1 - \varphi_2).
\]

(2.8)

If both qubits are driven in the rotating frame with different Rabi frequencies, they rotate around their \( z \) axes at different rates, and all terms involving \( \sigma_y \) average to zero. Thus

\[
\langle H_{\text{rot}} \rangle = \frac{J_{12}}{2} \sigma_z \otimes \sigma_z \cos(\varphi_1 - \varphi_2).
\]

(2.9)

2.8.5 Measurement, state, and process tomography

We perform self-consistent measurement- and state-tomography \[80\], which requires at minimum state tomography on 4 known input states to reconstruct the positive operator valued measure (POVM) operators characterizing the three tomographic measurements per qubit. However, we can only initialize the qubit in its energy eigenbasis. Furthermore, the qubit state partially depolarizes during a 1.5 \( \mu \)s wait after initialization to let gate voltage stabilize. We load a singlet state in the (0,2) charge configuration with \( > 99 \% \) probability. To assess the depolarization, we measure the amplitude of Rabi oscillations with and without the 1.5 \( \mu \)s wait, and attribute the loss in amplitude to depolarization. At large \( J(\epsilon_0) \), the amplitude diminishes by roughly 20\%, consistent with measurements of \( T_1 \). To generate the required number of states, we follow Ref. \[80\] in evolving the prepared state under two evolution Hamiltonians (rf drives with different phases), performing state tomography at various times, and also fitting for the parameters describing the evolution Hamiltonians. In total, there are 11 unknowns for single-qubit tomography: 2 parameters describing projection fidelities, 9 parameters describing the three measurement axes (3 for each) and 6 parameters describing the evolution Hamiltonians (3 for each). We perform state tomography for 48 different states: 16 measurements of the prepared state, 16 at
different times for one rf drive, and 16 at different times for the other rf drive. All data are fitted simultaneously to calibrate the tomography. Calibrations are consistent from run to run, and typical readout fidelities are $\approx 75\%$. To calibrate measurements for two-qubit tomography, we perform single-qubit calibrations for each qubit on all pairs of single qubit measurements. Based on our measurements, we find the most likely physical density matrix using the expressions derived in Ref. [78].

Process tomography is performed through state tomography on 16 input and 16 output states for our two-qubit gate. Using the measured input and output states, we invert the equation for the process matrix: $E(\rho) = \sum_{m,n=1}^{16} \chi_{mn} B_m \rho B_n^\dagger$. Here $E$ is the map representing our two-qubit gate, $\chi$ is the process matrix, and $B_m = \sigma_i \otimes \sigma_j$ are combinations of the Pauli operators. Our inverted process matrix has some negative eigenvalues, which may result from our mixed input states. We can constrain the process matrix to be completely positive and trace-preserving, by constraining the Choi matrix to be positive semidefinite and requiring that the partial trace over the qubit equal the identity [13]. The maximum-likelihood algorithm is implemented with the Matlab CVX library (www.cvxr.com).

To find the ideal process matrix, $\chi_{\text{ideal}}$, we start with the process matrix generated by the interaction Hamiltonian $H_{\text{int}} = \sigma_z \otimes \sigma_z$, and search through all single-qubit rotations to find the highest fidelity, given by $\text{Tr}(\chi_{\text{ideal}} \chi)$. The largest single-qubit rotations occur around the rf drive axis, because of rise-time effects in the coaxial cables in our cryostat. Bell state fidelities are found by searching through all single-qubit rotations for the Bell state with the largest overlap with the measured state.

Uncertainties in the state and gate fidelities are obtained using the measured experimental uncertainties in our data. We add Gaussian distributed noise to the data with the measured standard deviation and reconstruct noisy density and process matrices and corresponding noisy state and gate fidelities. The quoted uncertainties are the standard deviation of 128 different noisy fidelities. The mean of the noisy fidelities generated in this way agrees with the measured fidelity.
2.8.6 Simulation

To generate the simulated curve in Fig. 2(c) in the main text, we numerically integrated the Schrödinger equation using the laboratory frame Hamiltonian

\[ H = J_1(\epsilon_1(t))\sigma_z \otimes I + \Delta B_z \sigma_x \otimes I + J_2(\epsilon_2(t))I \otimes \sigma_z + \Delta B_z I \otimes \sigma_x + J_{12} \sigma_z \otimes \sigma_z \]

with time-varying voltages \( \epsilon_1(t) \) and \( \epsilon_2(t) \) for each qubit and computed the concurrence of the resulting states. We used \( \Omega_1 = \Omega_2 = 960 \text{ MHz} \) and adjusted the interaction strength to match the observed entanglement frequency. We used the measured functional form of \( J(\epsilon) \) for each qubit. We assumed independent fluctuations in \( \Delta B_z \) corresponding to laboratory frame coherence times of 150 ns, independent low-frequency charge fluctuations with standard deviation 8 \( \mu \text{V} \) relative to the gates, and independent high frequency charge noise with power spectrum \( S(f) \propto f^{-0.7} \) with a magnitude of 0.9 nV/\( \sqrt{\text{Hz}} \) at 1 MHz up to \( f = 1 \) GHz and 0 otherwise for each qubit. These values for charge noise are consistent with previous measurements in GaAs singlet-triplet qubits [20].
Readout of singlet-triplet qubits at large magnetic field gradients

This chapter was also adapted from the paper with the same name and underpinned the work shown in Chapter 2. The reader may at this point wonder why there was a five year gap between the original two qubit paper with small $\Delta B_Z$ and the improvements demonstrated with large $\Delta B_Z$ as it would seem very natural to examine the effect of all experimental parameters on the two qubit gate. However, it was also discovered in 2012 that working with large $\Delta B_Z$ is challenging because Pauli blockade readout fails[5]. The development of the T+ readout method was critical to working in this regime and the details of this method are discussed in this chapter.
3.1 Abstract

Visibility of singlet-triplet qubit readout is reduced to almost zero in large magnetic field gradients due to relaxation processes. Here we present a new readout technique that is robust against relaxation and allows for measurement when previously studied methods fail. This technique maps the qubit onto spin states that are immune to relaxation using a spin dependent electron tunneling process between the qubit and the lead. We probe this readout’s performance as a function of magnetic field gradient and applied magnetic field, and optimize the pulse applied to the qubit through experiment and simulation.

3.2 Introduction

Electron spins in semiconductors\[24, 41, 46, 48, 66\] are one promising path to quantum computing because of their scalability and long coherence times\[57, 71, 84\]. Single qubit gate fidelities exceed 99.8\% in single electron spin qubits\[56\] and 99\% in singlet-triplet(S-T) qubits\[59\]. S-T qubits \[30, 65, 76\] have recently demonstrated two qubit gate fidelities of 90\% by using large magnetic field gradients\[59\], $\Delta B_z$, to diminish the effects of charge noise\[19\] and increase coherence times. However, in the presence of $\Delta B_z > 400$ MHz relaxation through coupling to other states reduces readout visibility to almost zero\[5\].

Here we report a new readout scheme that provides readout contrast at large gradients and demonstrate that it has superior performance to previously published methods \[64, 65, 79\] for $\Delta B_z > 500$ MHz. This method is robust up to at least $\Delta B_z = 900$ MHz, the largest magnetic field we could generate, and should continue to function in much larger $\Delta B_z$. S-T qubits have previously been read out by mapping the qubit states on different charge configurations\[65\]. However, large gradients enable transitions between the qubit states during measurement, leaving both in the same charge configuration and diminishing contrast. Our technique adds a step before measurement that shelves the qubit
states into alternate spin states that do not have relaxation pathways enabled by $\Delta B_z$, restoring the ability to map each spin state onto a distinct charge configuration. This method relies on a spin-dependent tunneling between the qubit and the surrounding two dimensional electron gas (2DEG).

To optimize this process, we have measured the visibility of our readout as a function of $\Delta B_z$, the voltage applied during shelving and its duration, and magnetic field, $B$. We have also developed a simple model for this readout and used it to simulate our experiments, finding strong agreement with the data. The model we introduce is applicable to other varieties of spin qubits, including single spin [22, 55], hybrid qubit [40], and donor based S-T qubit [10] and latched readout methods [34, 58] that also rely on tunneling between the qubit and a Fermi sea. This readout technique is general to any host material, and source of $\Delta B_z$ and to schemes that use S-T readout for single spin qubits [28].

3.3 Device

We study S-T qubits formed from two electrons trapped in an electrostatic gate defined double quantum dot in the 2DEG of GaAs shown in Figure 3.3.1(a). We use the pair of numbers (L,R) to represent the number of electrons in the left and right dots respectively. The logical subspace for the qubit is made up of the singlet, $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$, and triplet, $|T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$, states where the arrows represent the electron spin in the left and right dot respectively. The Hamiltonian for this system is given by $H = \Delta B_z \sigma_x + J(\epsilon) \sigma_z$ [65]. The exchange interaction, $J(\epsilon)$, splits S from $T_0$ and is controlled by the detuning, $\epsilon$, and the energy splitting between $\uparrow\downarrow$ and $\downarrow\uparrow$ is controlled by $\Delta B_z$. We call the magnitude of the Hamiltonian $\Omega(\epsilon) = \sqrt{\Delta B_z^2 + J(\epsilon)^2}$, as shown in Figure 3.3.1(c,d). We note that the nature of the qubit’s ground (excited) state changes from being S ($T_0$) in (0,2) to $\uparrow\downarrow$ ($\downarrow\uparrow$) in (1,1).

For all experiments in this work, $\Delta B_z$ is produced by the hyperfine interaction with the nuclei, which is controlled through dynamic nuclear polarization.
Figure 3.3.1: (a) SEM image of the device. Electron positions are approximated with green circles. The sensor quantum dot is shown with a white arrow. (b) Charge stability diagram of the qubit. In the experiment voltages are either applied equally, $\gamma$, or oppositely, $\epsilon$, to the RF gates. (c) Bloch sphere of the qubit showing the eigenstates of $J$, $\Delta B_z$ and total splitting $\Omega$. (d) The energies of relevant states along the $\epsilon$ curve in b. (e) The energies of relevant states along the $\gamma$ curve in b. In both (d) and (e) black and orange curves represent the energies of two and three electron states respectively.
applied prior to every experimental run. The qubit is manipulated by applying voltage pulses to the gates labeled $\text{RF}_L$ and $\text{RF}_R$ in Figure 3.3.1(a). The total number of electrons in the double dot is controlled by $\gamma = (\text{RF}_L + \text{RF}_R)/2$ and the distribution of these between the right and left dot is controlled by $\varepsilon = \text{RF}_L - \text{RF}_R$, shown in Figure ??(b). We define $\gamma = 0$ to be the transition from the $(1,1)$ to the $(1,2)$ region, as shown in Figure 3.3.1(e). The qubit’s charge state is measured using an additional neighboring quantum dot[4].

### 3.4 Shelving Mechanism

We manipulate our qubits deep at Position $A$, shown in Figure 3.4.1(a), where the two spins are well isolated so that the ground state is $\downarrow\uparrow (1,1)$ and the excited state is $\uparrow\downarrow (1,1)$. In previous work S-T qubits were read out through spin blockade by adiabatically ramping the qubit from deep in $(1,1)$ to the measurement point in the $(0,2)$ region. This point is chosen so that $S$ is in $(0,2)$ but $T_o$ is spin blockaded to remain in $(1,1)$ because excited energy levels of the quantum dot are energetically inaccessible. This readout process maps $\downarrow\uparrow (1,1)$ to $S(0,2)$ and $\uparrow\downarrow (1,1)$ to $T_o(1,1)$ so that the distinct charge configurations can be used to measure the qubit’s spin state. However, this style of readout is vulnerable because at the measurement point $\Delta B_z$ mixes $T_o(1,1)$ with the excited $S(1,1)$ state, which decays to $S(0,2)$ on time scales much shorter than the measurement time[5]. When this transition occurs, there is no readout contrast because both qubit states have the same charge configuration. The rate of transition from $T_o(1,1)$ to the excited $S(1,1)$ state increases with $\Delta B_z$, meaning that this method has a measurement fidelity that decreases with increasing $\Delta B_z$.

To overcome readout failure at large $\Delta B_z$ we developed a new readout technique that shelves the qubit states into readout states which do not have relaxation pathways enabled by $\Delta B_z$. This new method maps $\downarrow\uparrow (1,1)$ to $S(0,2)$ and $\uparrow\downarrow (1,1)$ to $T_+(1,1)$. For the remainder of the work, we will refer to this as the $T_+$ readout method. We achieve the desired mapping by using tunneling
Figure 3.4.1: (a-c) Position in the charge stability diagram and occupation of quantum dot states after (a) manipulation, (b) shelving, and (c) measurement. For (a-c) the qubit’s excited state is shown in blue while the qubit ground state is shown in red. (a) After manipulation, the qubit is in its logical subspace, ↑↓ and ↓↑. (b) Grey arrows represent the transitions required for shelving to occur. Filled circles show states that are occupied at the end of process while dotted circles show states that are empty. (c) State occupation at the measurement position. The T+ and S states cannot be mixed by ΔBz. (d) Pulse sequence. Values of ε and γ during different steps of qubit operation. The shelving position, set by γ* and ramp time, t*, to (1,1) are optimized in Figure 3.6.2.
between the right quantum dot and the 2DEG to change the qubit’s spin state. The qubit is tuned so that the left dot is isolated from the lead and the other dot. The shelving process is shown in Figure 3.4.1 (a-c) and begins deep in $(1,1)$, at Point A. After manipulation, the qubit is brought to Point B, where $\gamma = \gamma_s$, which is chosen so the required transitions are energetically favorable, as shown in Figure 3.4.1 (b). At this point, electrons can only tunnel in and out of the right dot, enabling the transition from $\uparrow\downarrow(1,1)$ to $\uparrow S(1,2)$ by a spin $\uparrow$ electron tunneling in. The transition from $\downarrow\uparrow(1,1)$ to $\uparrow S(1,2)$ is blocked because there is no mechanism to change the spin in the left dot. $\uparrow S(1,2)$ decays to $\uparrow\uparrow(1,1)$ by a spin $\downarrow$ electron tunneling from the right dot to the lead. After allowing the qubit to fully transition, the voltages are adiabatically changed back to Point A over a time $t_r$ and then brought to Point C, the same measurement point as in the spin blockade method. The charge state is then measured with $S(0,2)$ corresponding to the ground state, $\downarrow\uparrow(1,1)$, and $T_+(1,1)$ corresponding to the excited state, $\uparrow\downarrow(1,1)$.

This technique also enables us to measure the direction of $\Delta B_z$. We have described this mechanism assuming a specific directionality for $\Delta B_z$ but it functions with the opposite orientation as well. Flipping the direction of $\Delta B_z$ causes $\uparrow\downarrow(1,1)$ to be the ground state and $\downarrow\uparrow(1,1)$ to be the excited state. This readout still maps $\uparrow\downarrow(1,1)$ to $T_+(1,1)$ while $\downarrow\uparrow(1,1)$ is initially mapped to $T_0(1,1)$ and quickly decays to the $S(0,2)$ charge state through the mechanism previously described. This inverts the charge signal we measure from the qubit ground state, allowing for a direct measurement of the direction of $\Delta B_z$. In these experiments, $\Delta B_z$ is oriented as in the second regime because DNP is more effective when pumping with $T_+$ than $S$, as detailed in the Supplementary Materials.

These readout techniques are sufficient for full qubit state tomography because we are able to pair them with high fidelity single qubit gates. We can measure along any axis by performing the proper rotations so that the states along the desired axis are mapped onto $\uparrow\downarrow(1,1)$ and $\downarrow(1,1)$. 
### 3.5 Theoretical Model

We have constructed a simple model for the $T_+$ method that captures the experimental trends that we observe and offers intuition for this technique's behavior. To determine the equilibrium populations of all the different quantum dot states, we have calculated the transition rates between all pairs of states using Fermi’s golden rule to compute the tunneling rates of electrons between the qubit and the 2DEG. We find the following transition rates, $\Gamma_{ij}$ between the $(1,1)$ states, $i$, and the $(1,2)$ states, $j$, and the reverse, $\Gamma_{ji}$:

$$
\Gamma_{ij} = \frac{2\pi}{\hbar} |\langle j|\tau|i\rangle|^2 f(\Delta E_{ij}, T, \mu) \rho_f
$$

$$
\Gamma_{ji} = \frac{2\pi}{\hbar} |\langle i|\tau|j\rangle|^2 (1 - f(-\Delta E_{ji}, T, \mu)) \rho_f
$$

Here $\hbar$ is the reduced Planck constant, $\tau$ is the tunneling term between the right quantum dot and 2DEG, $f$ is the Fermi-Dirac distribution, $\Delta E_{ij} = E_j - E_i$ is energy difference between $i$ and $j$, $T$ is the electron temperature, and $\mu$ and $\rho_f$ are the chemical potential and density of states of the 2DEG. $\Delta E_{ij}$ is controlled by $\epsilon$, $\gamma$, $\Delta B_z$, and $B$. Transitioning between states with different numbers of electrons requires an electron tunneling to or from the lead with an energy that compensates for any change to the qubit’s energy. The Fermi-Dirac distribution dictates the number of electrons and holes available for $\Gamma_{ij}$ and $\Gamma_{ji}$ respectively, which governs the rates. This means that the transition rates from states with lower energy to higher energy are suppressed because they require an excited electron or hole to donate the energy difference. We note that many rates are 0 due to spin conservation, suppressing transitions between states with incompatible spin configurations. We use these rates to simulate the transitions that occur during $T_+$ readout so that we can perform simulations while varying the same parameters as we do experimentally. Details are included in the Supplementary Materials.
Figure 3.6.1: (a) Measurements of the visibility of the spin blockade and $T_+^+$ readout methods as a function of $\Delta B_z$. Red curve is a simulation of the $T_+^+$ method. The visibility of the $T_+^+$ method is superior at large $\Delta B_z$. (b) Measurement and simulations with varied $B_N$ of the visibility of the $T_+^+$ method as a function of $B_A$. The data does not follow one simulation curve, suggesting that the $B_N$ produced by DNPS is a function of $B_A$.

3.6 Results

We determined the contrast of the readout methods that we tested by finding the measurement fidelity $[4]$ for the ground, $F_G$ and excited states, $F_E$, as detailed in the Supplementary Materials. We used these quantities to calculate the visibility, given by $F_G + F_E - 1$. In Figure 3.6.1(a) we present the measured visibility of spin blockade and $T_+^+$ readout techniques as a function of $\Delta B_z$. In Figure 3.6.1(a) we also present a simulation for the visibility of the $T_+^+$ readout and note the agreement with the data.

We see that the spin blockade readout visibility decreases very quickly with increasing $\Delta B_z$ as we expect from the increasing decay rate from $T_o(1,1)$ to $S(0,2)$ at the measurement point. The $T_+^+$ readout is poor at small $\Delta B_z$ because $J(\varepsilon)$ is comparable to $\Delta B_z$ which gives both qubit states the ability to decay to $\uparrow S(1,2)$. However, the $T_+^+$ method has large visibility for $\Delta B_z > 200$ MHz. We
note also the slow fall off of visibility for $\Delta B_z > 500$ MHz. This is due to $\Delta B_z$ decreasing the energy splitting between the $\uparrow \downarrow$ state and the $\uparrow \uparrow$ state, decreasing the thermodynamic equilibrium occupation of $\uparrow \uparrow$, as can be seen from the energies given in the Supplementary Materials. Flipping the direction of $\Delta B_z$ would give a weak improvement instead because $\Delta B_z$ would increase the energy difference between $\uparrow \uparrow$ and $\uparrow \downarrow$ rather than decrease it. We compare the performance of the $T_+$ and another previously published readout method\[ 79 \] as a function of $\Delta B_z$ in the Supplementary Materials.

In Figure 3.6.1 (b) we present the data for the $T_+$ readout method visibility versus the applied magnetic field, $B_A$. We find only a weak dependence on the $B_A$ while the model predicts a sharp increase. Past measurements have shown that DNP pumps both the difference field, $\Delta B_z$, and sum field, $B_N$, experienced quantum dots due to the polarized nuclei. The magnetic field experienced by the qubit is $B = B_A + B_N$. Pumping with $T_+$ states flips nuclei such that $B_N < 0$ while pumping with $S$ states yields $B_N > 0$. While measuring the data presented in Figure 3.6.1 (b), we observed increasing DNP times required for a given value of $\Delta B_z$ to the extent that it took 10 times longer to stabilize $\Delta B_z$ at $B_A = 1.4$ T than at $B_A = 0.7$ T. This suggests that nuclei are flipped more symmetrically between the dots with increasing $B_A$, yielding larger magnitude $B_N$, because DNP is less efficient at pumping $\Delta B_z$. In Figure 3.6.1 (b) we plot simulations at several different $B_N$ and see that the data transition between curves with increasingly negative $B_N$, consistent with DNP becoming less effective at generating $\Delta B_z$ at larger $B_A$. The magnetic field dependence of DNP pumping rates of $\Delta B_z$ and $B_N$ is a subject of current investigation.

The fidelity of the $T_+$ readout method depends strongly on the readout position because the technique relies on the desired transitions being energetically favorable while the undesired transitions remain unfavorable. The energy spectrum of available states as a function of $\gamma$ is shown in Figure 3.3.1 (e). We select the optimal readout position by repeatedly preparing $\uparrow \downarrow(1,1)$ and immediately attempting to measure at readout positions with different $\gamma$, as
Figure 3.6.2: (a) Measurements and simulation for the probability that the ↑↓ state correctly transitions as a function of $\gamma_\star$. The peak occurs where the required transitions are energetically favorable while still keeping undesirable transitions unfavorable. (b) Measurement and simulation of the visibility of the $T_+\!$ as a function of $t_r$. Longer times allow the qubit to more completely transition to the desired end state.
shown in Figure 3.4.1(d). We plot data and a simulation of the probability that the measurement correctly identified the $\uparrow\downarrow$ state in Figure 3.6.2(a).

When $\gamma_\star \ll 0$ the $\uparrow S(1,2)$ state has far more energy than $\uparrow\downarrow (1,1)$, preventing the first transition required for $T_+^+$ readout. As $\gamma_\star$ approaches zero $\uparrow S(1,2)$ comes into resonance with $\uparrow\downarrow (1,1)$ and we see a dramatic upturn in the probability of transitioning because there are thermally excited electrons that allow for the first transition. When $\gamma_\star > 0$ the probability drops again because the desired end state, $\uparrow\uparrow (1,1)$ is not the lowest in energy during the readout process so it is not the most thermodynamically populated. All other measurements in this paper were performed at the optimal measured readout position.

To optimize the $T_+^+$ readout, we also investigated the dependence of the visibility on $t_r$. Our simulations and experiments showed little dependence on how quickly $\gamma$ was increased to ramp the voltages from Point A to Point B, where $\gamma = \gamma_\star$, but a strong dependence on the time, $t_r$, over which $\gamma$ was varied to change the voltages back from Point B back to Point A. We present measurements and simulations for the visibility as a function of $t_r$ in Figure 3.6.2(b). The visibility sharply improves with increasing $t_r$ because the qubit has time to equilibrate as $\gamma$ is varied resulting in a higher occupation of $T_+^+$. At very short times, $(\uparrow, S)$ is rapidly raised above $\uparrow\downarrow$ state, allowing for undesirable transitions and reducing visibility.

The maximum visibility that we observe is approximately 0.6, corresponding to an average readout fidelity of 80%. This is limited by the equilibrium thermodynamic occupation of the states that the qubit transitions through during the shelving process. This thermodynamic limit can be improved by decreasing the electron temperature or by using $\Delta B_z$ and $B$ to increase the energy splittings between the states used for shelving. As mentioned above, the direction of $\Delta B_z$ can be chosen so that it increases the relevant splittings. While the direction of $\Delta B_z$ in these experiments was governed by using DNP and decreased the relevant splittings, the direction is more flexible when generated by a micromagnet [38, 66, 81, 86] so that visibility can instead be enhanced. Another benefit of using a micromagnet is that $B_N$ will remain fixed, so that we have direct
control of $B$ through $B_A$. We expect to observe the behavior predicted by the simulations in Figure 3.6.1(b), allowing this method to achieve visibilities above 90% by increasing $B_A$.

### 3.7 Conclusion

We demonstrated that the $T_+$ readout method allows for measurements with large $\Delta B_z$, a regime that was previously inaccessible due to low readout visibility. We have also demonstrated that calibrating $t_r$ and $\gamma_\star$ is critical to optimizing the visibility. Additionally, we have identified that using an external source of $\Delta B_z$, such as a micromagnet, should enable higher fidelity readout by the application of larger $B_A$ and prudently selecting the direction of $\Delta B_z$. We expect that these changes should enable visibilities in line with other high quality qubit readouts. The $T+$ readout technique is also applicable to scalable architectures that map a single spin qubit onto S-T states for readout[28].

The concept of using a shelving step before measurement is relevant to any system where readout is limited by decay processes during measurement. We have demonstrated that visibilities can be increased by transferring the qubit into states that are immune to the decay pathway before measurement. We have also developed a method for simulating processes that rely on spin dependent tunneling between a quantum dot and a reservoir. This can be used to optimize the initialization and readout in a wide variety of qubits because they rely on these tunneling processes. Our demonstration of using experiments and simulations to develop the $T_+$ readout method can serve as a guide for other researchers who need to develop readout schemes tailored to their specific experimental requirements.
3.8 Supplemental Materials

3.8.1 Experimental

Our devices were fabricated on GaAs/AlGaAs heterostructure with the 2DEG located 91 nm below the surface. At 4K the electron density $n=1.5 \times 10^{11}$ cm$^{-2}$ and mobility $\mu=2.5 \times 10^{6}$ cm$^2$V$^{-1}$s$^{-1}$. Measurements presented in this work were taken in a dilution refrigerator with a base temperature of approximately 15 mK.

The direction of $\Delta B_z$ in these experiments was dictated by DNP’s ability to more strongly pump with $T_+$ than with $S$. Pumping with $T_+$ flips spins counter to the applied magnetic field, reducing the Zeeman splitting experienced by the qubits. This brings the avoided crossing between $T_+$ and the ground state $S$ deeper into the $(1,1)$ region. The hyperfine term used for pumping only couples $T_+(1,1)$ to $S(1,1)$, meaning that pumping is more effective when the ground state singlet branch has more weight in $S(1,1)$ than $S(0,2)$. Pumping with $S$ achieves the opposite effect, increasing the Zeeman splitting and decreasing the coupling between the ground state singlet branch and $T_+$ and reducing the ability to pump the gradient.

In order to measure the readout fidelity we need to prepare and measure both the excited and ground states. We prepare the ground state by loading two electrons into the double dot deep in $(0,2)$, where $S(0,2)$ is the ground state, and then adiabatically ramp into $(1,1)$ where $\uparrow\downarrow$ is the ground state. We prepare the excited state by first preparing the ground state in $(1,1)$ and then rotating it by using a sinusoidally time varying $\epsilon$ to generate a Rabi $\pi$ pulse about $J$. We have previously reported 99% fidelity randomized benchmarking$^{[17, 23, 47]}$ measurements of $\pi$ pulses of at $\Delta B_z=900$ MHz$^{[59]}$. While the $\pi$ pulse will become slightly worse at lower gradients, these state preparation errors are small compared to our readout errors and only negligibly affect our results. Figure 3.8.1(a) shows histograms of measurements of the excited and ground state of the qubit that were used to determine the readout visibility at $\Delta B_z = 900$ MHz.

For thoroughness, we also discuss the readout method described by
Figure 3.8.1: (a) Histograms of measurements using the T+ method to read-out the qubit’s ground and excited state. Histograms are used to determine the readout visibility, in this case for $\Delta B_z = 900$ MHz. (b) A comparison of the performance of the T+ and Studenikin readout methods versus $\Delta B_z$. Studenikin is preferable for $\Delta B_z < 300$ MHz while the T+ has greater visibility for $\Delta B_z > 400$ MHz.

Studenikin[79] that improves readout contrast by choosing the measurement point in (0,2) so that there is a (1,2) charge state with lower energy than the T0(1,1) state but still higher in energy than the S(0,2) state. The right dot of the double quantum dot is coupled to the lead, allowing a third electron to tunnel into double dot when the qubit is in T0(1,1) state because it is an energetically favorable transition but not for when the qubit is in S(0,2). This improves contrast because the charge sensor now detects different numbers of electrons for the two qubit states as opposed to just different positions of two electrons for both states like the spin blockade readout method. This method also suffers as $\Delta B_z$ increase because the transitions from T0(1,1) to the excited S(1,1) compete with the transitions of T0(1,1) to (1,2) and decrease measurement contrast as $\Delta B_z$ increases, as in the spin blockade readout technique. We note that the Studenikin readout method works up to larger gradients than spin blockade because there is a competition between the decays of T0 to the desired $\uparrow S(1,2)$
and $S(0,2)$ but that the undesirable transitions dominates as $\Delta B_z$ surpasses 400 MHz, as shown in Figure 3.8.1(b).

We also acknowledge two other recently published S-T readout mechanisms that, like the Studenikin method, rely on transitions through a $(1,2)$ state in the region where $(0,2)$ is the ground state. The Broome method\[10\] addresses how to readout a S-T qubit in which both quantum dots are equally coupled to the charge sensor while the Fogarty method\[28\] maps a single electron spin in a large array onto the S-T basis for readout. While these methods solve the problems that they were intended to address, we expect that they should also suffer in large magnetic field gradients for the same reason as the Studenikin method.

3.8.2 Theoretical

The $T_+$ readout technique relies on electrons entering and leaving the quantum dot so that the qubit can relax to lower energy states. To model this, we treat the tunneling term, $\tau$, as a small perturbation to the Hamiltonian that confines the electrons in the quantum dots so that $\tau$ couples the $(1,2)$ states to the $(1,1)$ states. Because we only allow for tunneling into the right quantum dot, $\tau$ can only mix states which have the same spin in the left dot, as shown below:

$$\text{Basis}=\begin{pmatrix} \downarrow\downarrow \\ \downarrow S \\ \downarrow\uparrow \\ \uparrow\downarrow \\ \uparrow S \\ \uparrow\uparrow \end{pmatrix}, \quad \tau = \tau_0 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The term $\tau_0$ controls the strength of the tunneling interaction and should be the same constant between all $(1,1)$ and $(1,2)$ states\[3\].

In the real experiment, the qubit eigenstates are not perfectly $\uparrow\downarrow$ and $\downarrow\uparrow$ because $J \neq 0$. This means that the ground and excited states take the following forms, where $\varphi$ is defined as $\tan(\varphi) = J/\Delta B_z$. 

57
\begin{align*}
|G\rangle &= \cos\left(\frac{\Phi}{2}\right) |\uparrow\downarrow\rangle - \sin\left(\frac{\Phi}{2}\right) |\downarrow\uparrow\rangle \\
|E\rangle &= \sin\left(\frac{\Phi}{2}\right) |\uparrow\downarrow\rangle + \cos\left(\frac{\Phi}{2}\right) |\downarrow\uparrow\rangle
\end{align*}

To simplify notation, we number the relevant states in the following way:
\begin{align*}
\uparrow\uparrow &= 1, \uparrow S = 2, G = 3, E = 4, \downarrow S = 5 \text{ and } \downarrow\downarrow = 6. \text{ The states and the transitions, } \Gamma_{ij}, \text{ are shown in Figure 3.8.2. The states have the following energies, } E_i
\end{align*}

\begin{align*}
E_1 &= -g\mu_B B_o \\
E_2 &= -g\mu_B B_o + K(\gamma) \\
E_3 &= -\Omega/2 \\
E_4 &= \Omega/2 \\
E_5 &= \frac{1}{2}g\mu_B \Delta B_z + K(\gamma) \\
E_6 &= g\mu_B B_o
\end{align*}

The (1,2) states, 2 and 5, have an additional energy the \(K(\gamma)\), the energy difference between the \(\uparrow\uparrow\) and \(\uparrow S\) states, because \(\gamma\) controls the energy of the (1,2) states relative to the (1,1) states. It is defined so that \(K(\gamma) \propto -\gamma\) because \(K\) is positive in (1,1) where \(\gamma\) is negative. Some care must be given when considering the energy difference between quantum dot states with numbers of electrons. Our choice of \(K(\gamma)=0\) at the charge transition allows us to set the chemical potential equal to zero because at this point an electron can tunnel from the fermi level into the quantum dot without paying an energy cost.

From Fermi’s Golden Rule we have equations (1) and (2) in the text. We can simplify these by using \(\Gamma_o = \frac{2\pi}{\hbar} J \rho_j\)
Figure 3.8.2: Diagram of the energies of the states used for readout. The single headed arrows show the allowed transitions between states. Those in black have not dependence on $\varphi$, those in blue scale as $\cos^2(\varphi/2)$ and those in red scale as $\sin^2(\varphi/2)$.
The terms \( \alpha_{ij} = a_{ji} \) are the overlap of the spin states and the nonzero terms are given below.

\[
\alpha_{12} = 1 \\
\alpha_{32} = \cos^2(\phi / 2) \\
\alpha_{42} = \sin^2(\phi / 2) \\
\alpha_{35} = \sin^2(\phi / 2) \\
\alpha_{45} = \cos^2(\phi / 2) \\
\alpha_{65} = 1
\]

Knowing the transition rates allows us to calculate the probability, \( P_i \) (\( P_j \)), that the qubit is in state \( i \) (\( j \)) by solving the following six coupled linear first order differential equations for all six states shown in Figure 3.8.2.

\[
\Gamma_{ij} = \Gamma_0 a_{ij}(\phi)f(\Delta E_{ij}, T, \mu) \\
\Gamma_{ji} = \Gamma_0 a_{ij}(\phi)(1 - f(\Delta E_{ji}, T, \mu))
\]
\[
\frac{dP_i}{dt} = \sum_{j=2,5} \left( -P_i \Gamma_{ij} + P_j \Gamma_{ji} \right)
\]
\[
\frac{dP_j}{dt} = \sum_{i=1,3,4,6} \left( -P_j \Gamma_{ji} + P_i \Gamma_{ij} \right)
\]

The first term on the right hand side states that the probability that the qubit remains in state i decreases because it can transition to state j at a rate that is proportional to how likely the qubit is in state i to begin with and the tunneling rate \( \Gamma_{ij} \). The second term is the opposite, stating that the rate of transitioning to i increases when the other states, j, are more occupied.

We solve these coupled differential equations by turning them into a matrix equation which then has the form \( \frac{d}{dt} \vec{p} = R \vec{p} \) where \( \vec{p} \) is the vector of \( P_i \)'s and \( P_j \)'s and \( R \) is a matrix containing all the \( \Gamma_{ij} \)'s, shown at the top of the page. We project \( \vec{p} \) onto the eigenvectors of \( R \), \( \vec{v}_k \), with eigenvalues \( a_k \), whose time dependence is given by \( \vec{v}_k(t) = e^{a_k t} \vec{v}_k \). The time evolution of \( \vec{p} \) is then just the sum of the time evolutions of the projections onto the eigenvectors. We note that one eigenvector will always be the thermal equilibrium because our rates are inherently thermodynamic due to their dependence on temperature through the Fermi-Dirac distribution. All other solutions will decay to zero with time scales that depend on the \( \Gamma_{ij} \)'s.

In our experiment, \( \gamma \), is a function of time because the qubit is biased from deep in \((1,1)\) to the transition to \((1,2)\). We simulate this by discretizing time and assuming that for each time step of length \( \Delta t \), \( K(\gamma) \) has a constant value \( K(\gamma(t)) \) and that this constant value controls the rates of decay. We make sure that \( \Delta t \ll \Gamma_0^{-1} \) so that time evolution at each step is small. We initialize \( \vec{p}(0) \) with all the weight in either the ground or excited qubit state. At each time step we project \( \vec{p}(t) \) onto the eigenvectors of \( R(t) \) and evolve it for time \( \Delta t \) which yields \( \vec{p}(t+\Delta t) \). We repeat this for each time step to find the total time evolution of our qubit. We have performed these computations while varying the same parameters as have experimentally investigated by changing the state energies and \( \gamma(t) \).

All simulations were performed with \( T=90 \) mK, \( J=50 \) MHz, \( \Delta t=1 \) ns, \( \Gamma_0= \)
7.143 \times 10^7 \text{s}^{-1} \text{ and } \mu = 0 \text{ meV. Unless otherwise specified } \Delta B_z = 900 \text{ MHz, } B_A = 0.7 \text{ T, } t_r = 2 \mu \text{s and } \gamma_\star = -0.065 \text{ mV. For the simulations with } \Delta B_z \text{ and } t_r \text{ varied, } B_N = -0.23 \text{ T. For the simulation with } \gamma_\star \text{ varied } B_N = -0.12 \text{ T which is not unreasonable given that this data set was taken several months before the others when the tuning parameters were different. The individual curves are labeled in the figure for } B_A \text{ varied.}

3.9 NEW REVELATIONS

As often happens in research, discussions about past work in the context of new problems led to interesting realizations. Recently, visitors from Michelle Simmon’s group have been interested in applying this readout mechanism to singlet-triplet qubits in donor based devices. They were able to measure \( |S\rangle \) and \( |T_{-}\rangle \) but had difficulty in measuring a \( |T_0\rangle \) state. They were not certain if this was due to an inability to generate a \( |T_0\rangle \) or a failure of Pauli blockade readout. We had a very fruitful discussion about the differences between applying the \( T_+ \) readout method to gate defined quantum dot devices compared to donor based devices and came to the following realizations.

In our work in GaAs, we always required that the tunneling into the left dot was slow compared to the right so that the spin state of the left would remain unchanged. This blocks undesirable transitions due to the conservation of angular momentum. However, donor based devices often have more symmetric tunnel couplings where both the left and right dot have the same tunneling rate to the lead. Additionally, these tunneling rates are not tunable with gate voltage like they are in gate defined quantum dot devices. However, we realized that tunneling into the left dot would be extremely unlikely given how energetically unfavorable that transition would be. Changing the number of electrons in the left dot would require entering either the \((0,1)\) or \((2,1)\) state, of which the \((0,1)\) state should be closer in energy. At the shelving position, the \((0,1)\) is a very excited state and therefore should not present a problem to the \( T_+ \) readout mechanism. The first transition to change the spin of the left dot is so
energetically intensive that it is practically forbidden. This increases the generality of the $T_+$ mechanism.

Another requirement for the $T_+$ mechanism was thought to be a fast tunneling rate between the right dot and the lead. This allows the qubit to rapidly transfer one of the qubit states to the $T_+$ state so that Pauli blockade readout is still possible. However, donor based devices often have significantly slower tunneling rates (1-10 kHz) so that they can also perform Elzerman style readout. This would make the shelving step slow as the wait time at the shelving position would need to be several times as long as the reciprocal of the tunneling rate. However, in this we realized that one could just use the shelving process itself as the readout in a manner reminiscent of Elzerman style readout. At the shelving position, we rely on one qubit remaining always in $(1,1)$ while the other qubit enters $(1,2)$ for some time. By measuring the charge state while shelving, one would be able to detect this transition directly and therefore readout the qubit. This is very similar to performing Elzerman style readout on the right dot except that the end state is not the same for both qubit states. While this alternate way to use the $T_+$ mechanism is useful for qubits with slow tunnel couplings, it is a much slower readout method, like Elzerman style, because one must have tunneling rates slow enough to ensure that one is able to measure the qubit in the temporary intermediate charge state. This typically requires measurement times 1 ms, much slower than the 1 μs times required when Pauli blockade is used.

Second is if the tunneling rates are slow. Can measure during the shelving process, in which case it is analogous to Elzerman style readout.

3.10 Contributions

This work was performed by myself, John Nichol, Shannon Harvey, Charlotte Böttcher and Amir Yacoby. The sample was fabricated by John and Shannon. The $T_+$ readout mechanism was originally conceived by John and the model was developed by myself. The data was taken by John and myself and I performed the simulations and analysis. The paper was written by all contributors.
We cannot change the cards we are dealt, just how we play the hand.

Randy Pausch

Switching experimental systems is costly because new experiments are fraught with unforeseen pitfalls. Our experience transitioning from GaAs to SiGe was no exception. When I arrived in the group, the material parameters, fabrication recipes and device design were already well established for GaAs. Learning all the nuances of qubit experiments proved to be quite challenging, especially given the significant amount of engineering and coding that goes into running the qubit experiments. Amir rightly said that this was like “jumping on a moving train” which was difficult because the project had so much momentum and all hands were already employed just keeping it moving. I appreciate the time John and Shannon took to get my up to speed with this project and gained a new appreciation for the hard work of previous generations of graduate students when we switched materials. It is far easier to catch a moving
train than it is to get a train up to speed.

For many spin qubits, nuclear spins are a significant source of decoherence. The fluctuations of the nuclear background couple to the qubit through the hyperfine interaction and can induce computation errors. Reducing or removing spinful nuclei is one way to reduce decoherence. GaAs was a natural choice as the first semiconductor for spin qubits because it had already been used in condensed matter physics labs for quantum hall experiments and quantum dot experiments. However, every nuclei in GaAs is spin $3/2$, meaning that each of the approximately one million nuclei that the qubit overlaps with had four possible states that it could flip between. Silicon based materials were a clear contender as a replacement for GaAs as silicon is naturally 92% nuclear spin free and the remaining 8% are only spin $1/2$. Additionally, silicon is more plentiful and less expensive material that is already proven to be compatible with mass production in semiconductor foundries. For this reason, the Laboratory for Physical Sciences decided to fund only silicon based spin qubits. As the reader will probably gather from this chapter, we still believe that there is a place for GaAs and that the supposed benefits of less nuclear spins may not outweigh the other challenges introduced by silicon. I will lay out the main differences between SiGe, the material we have switched to, and GaAs and will highlight how they effect the ability to form spin qubits in general and singlet-triplet qubits in specific.

The most significant difference between GaAs and SiGe from the standpoint of forming a spin qubit is that GaAs has an effective electron mass of $m_{\text{GaAs}}^* = 0.04m_e$ while SiGe has an effective mass of $m_{\text{SiGe}}^* = 0.2m_e$. The quantum dot can be approximated as an infinite square well, with the energy gaps between orbital levels proportional to $\hbar^2/2m^*L^2$. As we have already seen, the excited orbitals for the quantum dots must be energetically inaccessible for the singlet-triplet qubit to function. Energy gaps on the order of tens of gigahertz can be achieved with quantum dots approximately 200 nm across, as seen in the previous two chapters. To accomplish the same thing in SiGe, we must shrink the dimensions of the devices by a factor of $\sqrt{5}$ to keep the energy gaps the same.

GaAs was just on the edge of what could be comfortably fabricated using an
academic cleanroom. Using the Elionix 7000 gives 30 nm width features with more than 100 nm spacing fairly consistently and was sufficient for producing GaAs quantum dot devices. However, fabricating SiGe devices pushes the gate separations to around 50 nm at which distances the proximity effect becomes very appreciable. The proximity effect is difficult to counter because it is most significant where the pattern is densest, which in the case of quantum dots is the active region. While there are tools like Beamer that are designed to help with this by varying the dose to compensate from the proximity effect of nearby features, the changes it made were usually quite minimal and only changed by less than 10%. We found that the best way to compensate was to perform a dose test and observe where the device was overdosed. We would then reduce the feature width at the overdosed location in order to compensate for the proximity effect. As can be imagined, this process is not ideal because it requires iterations to produce the desired geometry but we were able to use it to produce a sufficient device.

Another large difference between GaAs and SiGe is in the mobility of the substrates themselves. It is quite common to see mobilities of ten million in GaAs, though mobilities tend to be in the single millions for qubit devices because the 2DEGs are buried less deeply. The high quality of these materials was driven by quantum hall research where cleaner substrates showed more exotic fraction quantum hall states. By contrast, SiGe has relatively modest mobilities of tens to hundreds of thousands. The best intuition that I have developed for the importance of mobility to spin qubit performance is that high mobility materials have less disorder, which is ideal because disorder is the enemy of gate defined quantum dots. The potential in the 2DEG should be dominated by the gates and not by the random disorder of the substrate. Large disorder can make it challenging to form a quantum dot with even the best gate geometry because electrons can be trapped or repelled by natural variations in the Fermi level in undesirable locations.

The singlet-triplet spin qubit relies on the symmetry under particle exchange of the spin component of the wave function enforcing the opposite symmetry on
the spatial component. This is true under the assumption that the electrons have
access to no other quantum numbers, as is true in GaAs. However, SiGe has an
unwanted complication: it is two-fold valley degenerate. The valley degeneracy is
slightly broken by strain\[37\] but even then, it is only on the order of tens to
hundreds of micro electron volts, or several to tens of gigahertz. If the valley
splitting is small, the triplet state can access the (0,2) state by entering the excited
valley state rather than having to access the excited orbital state. Even more
unfortunately, the valley splitting varies across wafers which means that devices
fabricated just millimeters apart may not have the same performance. While one
would hope for large valley splittings, one is not always so fortunate. One
workaround is to use the (1,3) and (0,4) states so that all valley and spin states
are occupied, blocking the triplet from entering (0,4) unless it enters the excited
orbital. While this is feasible, the extra electrons are undesirable from a simplicity
standpoint.

The energy of electron spins in a magnetic field depends on the g factor of the
material itself. It is typically easier if the g factor is larger because it means that
lower magnetic fields, which require less heat inducing currents, to achieve the
same energy differences. On this front, SiGe is preferable to GaAs. The g factor of
SiGe is 2, approximately the same as the free electron. GaAs has a g factor of -.44,
a comparatively small value\[74\]. While the singlet-triplet qubit is not split by the
overall magnetic field, it is split by the difference in the magnetic field of the two
dots $\Delta B_Z$ which scales linearly with g. One would be tempted to assume that this
would mean it is five times easier to generate large $\Delta B_Z$ but we must also take into
account the size difference of SiGe devices compared to GaAs devices. In reality,
we generate a magnetic field gradient and $\Delta B_Z$ is set by the magnetic field
gradient multiplied by the distance between the dots. This means that SiGe
devices, which are two to three times smaller, only experience a boost of $\Delta B_Z$ by
fifty percent. The g factor of GaAs and SiGe also have opposite signs. The only
significant change that this makes to the qubit physics is that $|T_-\rangle$ is the lowest
energy triplet state in SiGe as opposed to $|T_+\rangle$ in GaAs.

The source of $\Delta B_Z$ is another key difference between SiGe and GaAs. The
presence of roughly a million nuclear spin 3/2 nuclei overlapping with the qubit
electrons of GaAs can either be thought of as a feature or a bug. If the nuclei are
uncontrolled, the hyperfine coupling can serve as a strong decoherence channel.
However, one can use this same coupling to flip nuclear spins by trading spins
with electrons in the qubit. Operating the double dot as a nuclear spin pump as
well as a qubit takes advantage of the ability to load electrons with a definite spin
into the quantum dot quick (order of nanoseconds) and the slow fluctuations of
the nuclear spins due to other sources (order of microseconds). This process,
called dynamic nuclear polarization (DNP), can be used with feedback to
produce magnetic field gradients of at least one gauss per nanometer. The
limiting factor on this was the difficulty in measuring oscillations of the qubit
faster than one gigahertz due to the sampling time of the card. We believe that a
closer data acquisition card would allow for gradients several times stronger. By
contrast, the relatively sparse spin 1/2 nuclei of SiGe will provide a significantly
smaller hyperfine coupling. It is standard to instead use micro-magnets, typically
using cobalt based materials, to achieve magnetic field gradients. These
micro-magnets are an additional step to fabrication and often require using
different evaporation chambers as magnetic materials are unwelcome in many
material processing equipment. Additionally, the magnetic field gradient is
weaker in magnitude than can be achieved using DNP, typically capping out at a
fraction of a gauss per nanometer. While using feedback with DNP can be
technically challenging to set up, it does allow for a tunability of the gradient in
between experiments while the gradient from the micro-magnet is fixed.
Controlling the nuclear spins also means that the noise they add to the qubit is
significantly mitigated, and even still, this noise is low frequency and easily
compensated for with echo pulses. For singlet-triplet qubits with access to DNP,
it is questionable how much of a gain the lack of nuclear spins really is.

While GaAs is typically grown as a depletion mode substrate, SiGe is typically
accumulation mode. In depletion mode devices, the 2DEG is naturally populated
and is depleted by the gates. In accumulation mode devices, the 2DEG is
naturally empty and can be accumulated by gates. Accumulation mode devices
require the addition of dopants locally to regions of the 2DEG that are ohmically contacted. This process of phosphorus doping is done commercially and adds a week to the fabrication process, as well as the steps associated with patterning the doped regions. The more significant change is the requirement for accumulation gates over all current pathways. This creates extra capacitance between the current pathways and ground which makes readout using radio frequency reflectometry challenging. The solution to this issue is presented in Chapter 6.

In GaAs, where the effective mass is small and the 2DEG is relatively uniform, open style quantum dots have long served as stable and controllable qubits. The qubits used for the work in Chapters 2 and 3 were of this style. In SiGe, where confinement needs to be much tighter and the 2DEG is less uniform, open style devices do not provide sufficient control. Instead, overlap style devices, where the entire substrate surface is covered with metal, have proved to be more tunable. The direct coverage of the 2DEG by metal in all locations and the absence of thick gate oxides provides sharper wells. These devices are significantly more fabricationally intensive as they require three layers of electron beam lithography as opposed to one. For typical devices, the alignment of the layers to each other must be within twenty nanometers for the devices to function properly. While this is possible in an academic cleanroom facility, the alignment yield is low, typically about fifty percent.

On top of being more challenging to fabricate, the overlap devices are also significantly more fragile. Fabricating devices without a gate oxide layer, typically ten nanometers of an atomic layer deposition oxide, removes upper gate layers further from the 2DEG, decreasing the sharpness of the voltages in the 2DEG. Instead, it is typical to use aluminum which naturally oxidizes on its surface so that layers of gates can be grown directly on top of each other. This oxidization can be enhanced by heating on a hot plate or by exposing the aluminum to an oxygen plasma. Even still, the oxide layer is still only a few nanometers thick, meaning that it is extremely susceptible to damage from electrostatic discharge (ESD). Protecting these devices from ESD requires careful grounding during fabrication and measurement which, along with the fabrication details, is
included in Chapter 6.

If there is a single attribute that would improve spin qubit performance, it would be the reduction of charge noise. However, charge noise in SiGe is almost identical to charge noise in GaAs\cite{15,20}. One wishes that if the government were to uproot one’s research on the argument of an improved noise background, that it would be the most substantial noise source but alas, one is not always so lucky. It appears that SiGe alone is not the magic bullet to producing better qubit gates but that we will still require techniques to mitigate the effects of charge noise, as we demonstrated in Chapter 2.

Most aspects of SiGe make it more challenging than GaAs. For singlet-triplet qubits that use DNP to control the nuclear background, the absence of nuclear spins is a meager gain given the difficulties in device fabrication, lower mobilities, more challenging charge detection, additional valleys and device fragility. The wins that may be achievable in two qubit gates could both be realized in GaAs where smaller overlap devices can (but are not required to by the effective mass) be fabricated. With the shift to silicon, the field also seems to have shifted from a physics to an engineering one with fewer types of new qubits being demonstrated and the emphasis being on metrics for gate performance. With this shift has come a tendency of groups to specialize in qubits because the challenging fabrication rewards economies of scale. In the coming chapters I will discuss how we have faced the challenges of SiGe and our work towards a silicon based singlet-triplet spin qubit. All of the work after this chapter has been in close partnership with Yinyu Liu who deserves equal credit for all progress that we have made.
Software is like entropy: It is difficult to grasp, weighs nothing, and obeys the Second Law of Thermodynamics; i.e., it always increases.

Norman Augustine

Crassula Quantum Dot Simulator

Iterating sample designs is a slow process, especially when fabricating a single sample can take the better part of a month. I wrote the Crassula Quantum Dot Simulator software so that we could simulate device performance before fabrication. The goal was to rule out sample designs that were obviously flawed and to give an idea for expected performance. The disorder in the substrate is not taken into account because it is random and varies based on the substrate. This software is designed to simulate electrostatic gate defined quantum dots in two dimensional electron gases (2DEG). The core of the simulation is based on the paper “Modeling the patterned two-dimensional electron gas: Electrostatics” by John H. Davies, Ivan A. Larkin and E. V. Sukhorukov[18]. In this paper they find the potential at the 2DEG from a rectangular gate on the surface. This simulation takes DXF files that contain the gates and breaks them into a series of rectangles.
The formula found by Davies, et al can then be applied to the gates by applying them to each of their constituent rectangles.

One key to the performance of the simulation is that the potential at the 2DEG depends on the product of the voltage applied to the gate and a geometric factor. The geometric factor is computationally intensive to compute but only needs to be found once for a given gate geometry. After these factors are computed, the gates can be rapidly tuned because it only requires multiplying the geometric factor times the new voltage. By contrast, programs like COMSOL will recompute the equivalent of the geometric factors every time a voltage is changed. This front loading of the computation is convenient because it means that the geometric factors can be calculated while the operator is free to eat lunch, take a nap or fab a sample. With reasonable parameters the front loading time should take about 15 minutes to an hour but this depends on the computer used to run the simulation.

In this chapter, I will highlight the programs features and limitations while discussing its inner workings. I will show what the expected results should be for each stage and a demonstration of its usefulness to tuning quantum dots. The code is available, with examples, on the Yacoby Lab server or on request by contacting the author.

5.1 DXF files

You will need at least two DXF files to run a simulation. The first file is saved in the structure files.compBox. This is a rectangle that tells the simulator where to compute the voltage. It would be annoying to compute the voltage everywhere the gates are when the active region is quite small. Additionally, every layer of gates should have its own DXF files because the distance from the 2DEG is determined by which DXF file the gate is saved in. Every gate should be stored in its own layer with the layer name set as the gate name. Gate names must start with letters and must be a single word. Setting the z value in the DXF file for the gate with set the initial voltage on the gate. These DXF file names should be
stored as elements of a cell that are stored in files.gateFiles.

You must use the EXPLODE command on your files in autocad immediately before saving them for simulation. The DXF file only recognizes lines, not polygons. The EXPLODE command turns all types of shapes into lines.

5.2  **CRASSULA RUN FILE**

The Crassula program itself is a Matlab function that takes four input structures: files, computation parameters, physical parameters, and options. The fields of each of these structures is described below.

5.2.1  **FILES FIELDS**

- **gateFiles** - This is a cell that contains the file names of the DXF files that contain the designs for the gates. Each layer of gates should be stored in a different file because the distance to the 2DEG is set for all gates within a single file. This also allows for different gates to use different grid size, as detailed in the computation parameters section.

- **compBox** - This is a string that contains the name of a file. The file should contain only one rectangle which denotes the region where one would like to simulate the potential. This allows for fine grained calculations without wasting time on the inactive region of the device.

- **saveFile** - String that contains the file name of a previously saved simulation. You cannot use this option until you have already run at least one simulation. If this option is selected, it will not load the files in gateFiles. When not loading from a saved file, it should be set as the empty cell. This will let you skip computing the geometric factors a second time if you want to play with a simulation later and will let you save specific gate voltages if you have a tuning you like. Note that if you change any parameter, as discussed below, you should not use this option because you will just be loading an old calculation, not rerunning the calculation.
5.2.2  **COMPUTATION PARAMETERS FIELDS**

- **rectGrid** - 1 by n array where n is the number of files in files.gateFiles. This stores the grid size for each gate file. The gates will be converted into rectangles with the width corresponding to their file. Smaller numbers are more accurate but time consuming to compute. 5 nm is a reasonable value. By using multiple layers, larger and more removed features can use a large grid size to improve computation efficiency.

- **vGrid** - This single number is the grid size for the voltage simulation in the 2DEG. It should not be smaller than the smallest value in rectGrid.

- **unitScale** - This single number tells Matlab the units used in the DXF files. Typically we do designs where 1 in the DXF file corresponds to 1 micron, meaning that unitScale would be $10^{-6}$.

5.2.3  **PHYSICAL PARAMETERS FIELDS**

- **m** - Effective mass of the electron in kg. This is a number.

- **hbar** - The reduced Planck’s constant as a number in SI units.

- **q** - The electron charge in SI units.

- **EF** - The Fermi Level in SI units.

- **epsilon** - The permitivity of the material in SI units.

- **zos** - A 1 x n array of the distances of the gates to the 2DEG.

5.2.4  **OPTIONS FIELDS**

- **gpuSwitch** - A single number - 0, 1, or 2. These are different settings for the way in which the computations are performed. The computation can either be run using a parallel for loop on the CPUs or in parallel on the GPUs. You must have a CUDA enabled GPU in order to use the GPU.
option. This option is often faster if one has a decent GPU like those found in standard gaming computers. Setting this value to 1 uses the GPU’s and setting it to 2 uses the CPU’s. Setting it to 0 will run a test using both the GPUs and CPUs and will report which ran faster. It is usually best to run a small DXF for this test to know whether to use the GPUs or CPUs.

- plotRects - This is 0 or 1. If it is set to 1, it will plot the rectangular strips that the gates are approximated by.

- indGatePlot - This is 0 or 1. If this is set to 1, it will plot the potential from all the gates individually. This option is useful in areas where the gates are dense so that their individual effects can be understood.

5.3 DXF TO RECTANGLES

The conversion between DXF files to the rectangles used by Crassula is done by the function dxfExtract. This is done by the functions f_LectDxf, polyGenV, and rectCreator. The function f_LectDxf opens the DXF file and was the only non built in function that I used for this program[73]. The function polyGenV creates Matlab polygons from the DXF information. The function rectCreator checks all of the positions in the region with gates to determine which gate they are within. It combines the rectangles when they share a side because the later computations scale with the number of rectangles. An example of a gate geometry and it converted to rectangles is shown in Figure 5.3.1.

5.4 GEOMETRIC FACTOR CALCULATION

The geometric factors are the most computationally intensive part of the voltage computation. This is performed by either voltageComp or voltageCompGPU depending on the state of gpuSwitch in options. The structure gates is modified by these functions so that it has a field associated with each gate. The value of the
Figure 5.3.1: On the left is the gate geometry and on the right is the gridded version.

field is another structure, the most important field of which is unitVoltageMap. This is the effect of one volt on that gate on all points in the computation area. This geometric factor only needs to be calculated once and can be reused for all voltage simulations which why voltage tuning is faster using this simulator than other programs like COMSOL.

5.5 Voltage Simulation

With the geometric factors calculated, the voltage simulations are fast and straightforward. All that must be done is to multiply each gate's voltage by its geometric factor to get its effect on the 2DEG and then to sum the effects of all the gates. This allows for real time simulations because the calculations are just multiplying matrices by a constant and summing them. This makes tuning devices in software feasible while it would be impractical with simulations that take even five minutes to compute. This computation is performed by the gateTune function. An example is shown in Figure 5.5.1.
Figure 5.5.1: A voltage simulation for the device geometry shown in the previous figure.

5.6 Numerical Schrodinger solver

To solve for the electron ground state of the system, we numerically solve the Schrodinger Equation. The potential from the gates is already stored as a function of position. To solve for the second order spatial derivatives, we use the approximation that

\[
\frac{d^2 \Psi}{dx^2} = \frac{\Psi(x + h) - 2\Psi(x) + \Psi(x - h)}{h^2}
\]

To find the effect of the electrons on each other, we treat each as an effective potential that acts on the others. The probability that an electron can be found within a given grid space is treated as the fraction of the charge that is there and the coulomb potential from each grid space is added together to give the effective potential. To find the ground state number of electrons in the system, the energy of \(n\) electrons plus one at the Fermi energy is compared to \(n+1\) electrons in the...
Figure 5.6.1: A numerical simulation of the wave functions of two electrons in the double quantum dot from the previous figure. The left panel shows the lowest energy wave functions of the left quantum dot while the right panel shows the lowest energy wave functions of the right quantum dots.

system. As long as $n+1$ electrons is lower in energy, the computation is continued. An example of the wave functions found for two electrons in a double quantum dot is shown in 5.6.1

5.7 Charge stability diagrams

With the ability to find the ground state wave functions with a specific set of gate voltages, one is able find the charge stability diagram when two gate voltages are varied. While this could be computed at every point in the diagram, this is relatively slow. Instead, we take advantage of the fact that if only one gate is varied and it is changed from $V_1$ to $V_2$ and the charge state is the same at these two voltages, then the charge state is the same for all voltages between $V_1$ and $V_2$. This allows for a divide and conquer strategy that minimizes the number of calculations that must be performed. An example is shown in 5.7.1 and has the expected honeycomb pattern that a double quantum dot should exhibit.
**Figure 5.7.1:** A simulated charge stability diagram of a double quantum dot. It has the expected honeycomb pattern.

### 5.8 Results

The greatest success of the Crassula Quantum Dot Simulator was with a two qubit GaAs device coupled by a superconducting resonator. Charlotte and Shannon were able to tune it into a triple quantum dot but had difficulty in tuning it into a double quantum dot. By simulating their gate geometry and applied voltages, they found an unwanted third dot appearing between the ideal two quantum dots, as seen in Figure 5.8.1. This revealed that applying a more negative voltage to the nose gate of the device pushed out the unwanted third dot. While the simulations proved useful for GaAs, we found them unnecessary for the overlap style devices where the ideal dimensions have already been demonstrated by other experimental groups.
Figure 5.8.1: A simulation by Charlotte and Shannon using the Crassula simulator. It revealed a triple dot, as observed in the experiment and gave intuition about which gates to tune to form a double dot.
Starting the SiGe spin qubits project has required the retrofitting and redevelopment of the suite of engineering tools necessary for running the qubit experiments. On the experimental setup side these have included rewiring the fridge, establishing new grounding protocols, and designing several generations of new circuit boards. On the sample side this has required developing a new set of sample fabrication and measurement procedures. This chapter will detail these engineering challenges and enabled us to fabricate and measure devices in which we have been able to form and measure quantum dots, as detailed in the next chapter.
6.1 Fabrication Details

While working out the electron beam lithography details took quite a while, the resulting recipes are actually very simple. We receive SiGe chips from the Eriksson Group with the 2DEG already contacted and 20 nm palladium pads already connected to shorted bond pads. To complete the overlap style devices on these prefabricated chips we perform three rounds of electron beam lithography, each associated with its own unique gate layer, and deposit increasingly thick aluminum layers. The details are listed below. A SEM image of a complete device is shown in Figure 6.1.1.

Figure 6.1.1: SEM image of a completed three layer quadruple quantum dot overlap device on SiGe. The quantum dots are formed in the upper half of the device and two sensors are formed in the lower half.
6.1.1 **Spin Recipe**

This is the recipe for the resist stack that we use to evaporate our aluminum gates. The spinning is performed in the CNS facility and we always use the smaller spinner in the bench because, by eye, the spin speed seems to be more consistent.

- Pre-bake 10 minute at 180°C. This is also our oxidation step for the previous layer of aluminum.
- Spin EL-9 at 4000 rpm for 45 s
- Bake 5 min at 180°C
- Spin A3 at 4000 rpm for 45 s
- Bake 5 minutes at 180°C

6.1.2 **Electron Beam Lithography**

Great care must be taken during the lithography step in order to achieve the consistent results necessary for all three gate layers. We have found that the dosing is extremely dependent on the substrate height. The Elionix system is not consistent enough with the automatic height map so the user needs to make a manual map by burning spots on the alignment markers surround the devices. This provides for far more consistent results. We use the following base parameters for dosing but there are additional corrections for the proximity effect from Beamer.

- Fine features - 200 pA - 2000 μC/cm²
- Coarse features - 2 nA - 2000 μC/cm²

6.1.3 **Development**

The development process is done with cold developer to improve the consistency by slowing down the chemical reaction with the resist. We place a 100 mL plastic
cleanroom cup with approximately 40 mL of commercially purchased IPA:MIBK 3:1 into a glass beaker filled with ice. The ice is previously packed down around an empty plastic cup so that there is a hole with the exact right shape ready for the developer cup. Water is added to the same height in the beaker as the height of the developer in the beaker to enhance thermal conductivity. The height of the ice is about half an inch below the rim of the cup to avoid water entering the developer. The developer is allowed to cool for at least 15 minutes before development. The development steps are listed below.

- Prepare the IPA:MIBK 3:1 developer, immerse it in the ice bath and wait for 15 minutes.
- Develop while still in the ice bath for 60 s.
- Rince in IPA for 30 s.
- Descum the chip in oxygen plasma excited by 30 Watt with 20 sccm of O₂ for 20 seconds.

We believe that this descum step also contributes to the formation of the oxide between the gates.

6.1.4 Evaporation

Aluminum liftoff is challenging compared to other metals, like gold, because it forms sidewalls on the edges of the resist that can collapse and ruin the lithography. We have found that using the minimum resist thickness possible reduces the effects of sidewalls. Additionally, we place the chip on the evaporator stage directly above the aluminum boat. To check this, we use a washer tied to a string. We hold the string outside the evaporator chamber so that the string appears to run directly through the chip and ensure that it aligns with the boat. We do this from two angles to ensure that the alignment is good. We always evaporate in the Yacoby Lab thermal evaporator. Our gate layers are 30 nm, 60 nm and 90 nm so that the upper layers have no issues climbing over the lower layers.
6.2 Proper Grounding

The overlap style devices are extremely vulnerable to electrostatic discharge (ESD) due to the use of the aluminum oxide formed on surface of the gates as the gate oxide. This layer is at most a few nanometers thick and the regions of overlap are at most a few square micrometers. This means that the energy density in even a small static shock can completely destroy devices, as seen in Figure 6.2.1. For this reason, it is extremely important to properly protect the sample at every step of the fabrication and measurement process! The protocol that we have developed is described in this section.
6.2.1 On chip grounding

We have found from SEM images that devices would be destroyed by ESD before they were even bonded. To solve this issue, all bond pads are now shorted together during the entire fabrication process, as shown in Figure 6.2.2. Each bond pad has a single ground strap that runs to the nearest corner of the device where all bond pads are shorted together. These ground straps are left until after the device is bonded. The geometry is optimized to make cutting the ground straps require as few scribes as possible. For this geometry, it can be achieved with eight, two for each corner.

6.2.2 Bonding

The bonding parameters themselves for the overlap device are quite flexible because the bond pads are 150 nm gold pads. However, the most challenging aspect of bonding these samples is the tight array of bond pads and ensuring that the sample is properly protected from ESD. The chassis of the Yacoby Lab bonder is grounded to the building ground and is electrically connected to the bonding platform and stage. As an extra precaution, we ground the circuit board that the sample is mounted to directly with an alligator clip. It is imperative that the user also grounds themselves during the bonding process! We have not found an alpha particle emitter to be helpful in ESD protection and find that the extra instrument in an already tight space is not worth the risk it poses to the device. It is important to think while bonding about leaving space between the bonds so that the scribing tips can enter and cut the grounding straps. The second port on the circuit board must be shorted during the bonding process so that all gates of the sample are still tied together after the sample is scribed.

6.2.3 Scribing

Scribing is the process of cutting the on chip ground straps after the sample is bonded. While other groups manage to do this manually, this author is not so
Figure 6.2.2: This is the design for on chip ESD protection that has grounding straps that short together all bond pads during fabrication. These are cut after wire bonding.
dexterous. Instead, we scribe using a modified bonder tip, shown in Figure 6.2.3. This tip is placed into the bonder instead of the normal bonding tip and used to scratch through the ground straps. This tip was made by silver pasting a wire bonding tip to a diamond scribe tip with diameter 0.5 mm. The silver paste is hoped to hold the scribing tip at the same potential as the bonder. For mechanical strength, epoxy was used to further hold the tip to the scribe. The scribe tip was also filed on the sides to further narrow the tip so that it is easier to access the ground straps in between the wire bonds. For scribing, the force is set to 3 and the ultrasonic power is set to 0, as is the ultrasonic pulse time. Cuts should always be away from the bond pads so that accidental slips do not damage the device.

6.2.4 Loading the sample

Once the sample is scribed, it is at its most vulnerable point. Great care must be taken while loading the sample. Before electrically connecting the sample to the measurement setup, it should be checked and double checked that all lines are grounded. The sample must be connected to the setup through the primary connector before the secondary connector that shorts the device is removed. This shorting bar is discussed in greater detail in the next section. The experimenters must ground themselves during the entire process of loading the sample into the fridge and whenever approaching the experimental setup. A perimeter around
the experimental setup must be marked and proper threats issued so that the space is respected as touching the experimental setup can destroy the device.

6.3 Board Design

At this point I have designed four circuit boards for the SiGe project. Two of these were for the open style device and two for the overlap style. The designs themselves are saved on the server and the schematics are too complicated to put into a figure for this thesis. I have shown the front and back of all generations of boards in Figure 6.3.1 and 6.3.2. Below, I will focus on the main changes with each generation of board and will provide some advice for circuit board design.

6.3.1 Open Style Boards

This first generation board was the first SiGe board and the largest difference between it and GaAs boards is 48 DC lines compared to the 24 lines on old boards. It also features a separate RF input for both tank circuits where old boards used a single input for both tank circuits. For open style devices, we had
hoped that we would be able to integrate RF gates in the same manner as GaAs where the high frequency gates were at DC ground. For this reason, we included four RF inputs that route straight to the bond pads. We also included four RF inputs with bias tees so that RF pulses could be applied through gates with a nonzero DC bias. This board also includes the power lines for a red laser diode that can be used to heat the sample. The second generation board is the same as the first except that it includes a second DC connector that allows for the bond pads to be shorted together while connecting to the fridge.

6.3.2 OVERLAP STYLE BOARDS

The first overlap style board is similar to the second generation open style board except that it has eight bias tee RF inputs and no RF inputs without bias tees. Additionally, it has the sample space milled down by 500 μm so that there is a pit for the sample to sit in. This makes bonding significantly easier because the sample surface is almost level with the circuit board bond pads. This board also features RCR filters on the board while previous boards had only RC filters. The second resistor is the blocking resistor discussed in the following chapter. The second generation is largely the same as the first except that the sample area was
increased from a square with side length 5 mm to a square with side length 6 mm to make loading the samples easier. Additionally, the second R of the RCR filters was moved to only 3 mm away from the bonds pads to reduce the effect of capacitance on the board. The ground bar has also been replaced with a small circuit board with thermistors to ground. The thermistor shorting board does not need to be removed when loaded into the fridge because the thermistors act as opens at the operating temperature of the fridge. This gives added protection to the device while it is cooling down. This is the current generation of boards that is still in use.

6.3.3 BOARD DESIGN ADVICE

Having designed several circuit boards for qubit experiments, I would like to pass on as much of the intuition I have gained as possible. Below are some of the most important observations I have made.

- Don’t be afraid to order a board that is not perfect. The first generation of board that you make will definitely have bugs that you will only find when it is really in use. Boards are relatively cheap to buy ( $10 each) so you should be careful but not paranoid when ordering them.

- Take advantage of the SparkFun Eagle tutorials if you choose to design with Eagle. I highly recommend this program because it separates the circuit diagram part of the design from the physical layout of the board. This makes designing the boards fairly straightforward.

- Use the ratsnest command frequently in Eagle. This can fill in the board with ground and will help give you the closest points of two elements that need to be connected.

- Don’t hesitate to make your own layout for a device. Once you get the hang of it, designing a solder pad layout for a new device is relatively straightforward and can be done in under an hour for most devices. Doing
this will give you greater flexibility than just trying to stick with standard layouts.

- Do not have the circuit board company mill the sample space. They will charge ten times what the Harvard machine shop will.

- Remove as much ground near the sample as possible. This is important for RF reflectometry. It is trivial to add more capacitance to the system but it is impossible to reduce the capacitance below the value set by the board itself.

- When designing layouts, make the pads slightly larger than suggest when possible. Some designs suggest pads that are covered exactly by the device which makes soldering very challenging.

- Only use 0402 and 0201 elements when absolutely necessary. While they are very compact, they are very tedious to solder. Remember that you will have to solder the elements to the board so don’t make your life unnecessarily difficult.

- I have found that the best way to solder surface mount elements is to melt a little solder onto only one solder pad of the board. Then the element can be placed with tweezers while using the iron to reflow this small amount of solder. This will hold the element in place so that you have two hands to solder the other leads of the element.

- We have had very good luck ordering from Epec Engineered Technologies. They offer both time and money efficient options. Their online submission system for instant circuit board quotes is also quite useful. Their engineers also have caught mistakes in our submissions before which has been very helpful.
6.4 Fridge Wiring

The GaAs devices that were previously used in the Beast Leiden dry fridge pushed the 24 DC lines to the limit. The additional gates required for accumulation mode devices necessitated the addition of a second loom of 24 DC lines. This second set is currently nearly identical to the first set which is described in detail in the thesis of Mikey Shulman. Additionally, a diagram of the wiring would be obsolete in the near future because the RF filters and RC filters will be replaced with a new filter board I have designed. The schematic of the board will soon be publicly available under the name QAngel, a riff on another group’s for profit filter available under another celestial beings name. The details of this filter are discussed at the end of this section. The beginning of this section will focus on advice for wiring a fridge as this topic is relevant to a wide variety of condensed matter experiments and is often passed down by word of mouth rather than documented.

6.4.1 Advice for Wiring a Fridge

Wiring a fridge is as much an art as a science in that it requires a great deal of mechanical dexterity from the experimenter. In this section I will pass on the lessons that I have learned from this process in hopes that they will help ease the learning curve for later generations of students.

- Slow and steady wins the race. Remember that you and later generations of students will be using this wiring for years to come. It pays to spend the extra time and care to make absolutely sure that all steps are completed with as best as possible, despite the temptation to rush. If you use hacky shortcuts, later generations of students will hate you when connections are inconsistent or shorts sporadically appear.

- Establish a connector convention and stick with it! There is nothing more frustrating when wiring a fridge than having many wires and components with different wiring conventions. Most micro-D connectors have 25 pins
while the loom has 24 lines. You should always use pins 1 through 24 and leave 25 empty. From experience, it is very annoying having to remember which cables belong with which elements because some components use pin 24 while other pin 25. Additionally, establish a convention of which connectors are used on lines versus components. The Beast fridge currently uses female connectors for inputs to components and male connectors for component outputs (sadly with a few exceptions grandfathered in). A better convention would be to only use males connectors on components and female connectors on the lines, as is done in MX-400.

- Check your loom before soldering many cables. In recent years, our group and a variety of other qubit groups have received faulty loom where the individual lines had $\Omega$ leaks between lines that were only discovered when the lines were installed in the fridge. When new loom is received, a connector should be soldered onto the end of it so that leaks in the entire loom can be checked for at once.

- Heat shrink tubing for all lines on connectors is a life saver. The tubing prevents unwanted shorts and provides mechanical protection for the wiring. It is better than expoxy because the heat shrink is easily removed if repairs must be made. Even when space is very tight, tubing should be used on every other line to prevent shorting.

- Be gentle when stripping wires. Being rough with them makes them weaker and more likely to break in the fridge.

### 6.4.2 QAngel Filter

The QAngel filter is an RCRCR filter with three additional stages of low pass filters. The resistor and capacitor packages are 0204 for compactness and provide flexibility on the cutoff frequency desired by the experimenter. The suggest low pass filters are mini circuits LFCN-1450, LFCN-5000, and LFCN-80+. The
Figure 6.4.1: The QAngel filter that will soon be added to the fridge and released for public use.

board design will soon be available for public use as the author was aggravated that comparable filters were being sold for $10,000 when the components and board cost less than $300. The board includes labels for all pin numbers to aid in debugging as well as compatibility with both male and female micro-D connectors for the input and output. A soldered example is shown in Figure 6.4.1.

6.5 Contributions

The fabrication recipes were developed by myself and Yinyu Liu. The Beamer recipe specifically was developed by Yinyu. I created the scribing tips. The circuit board schematics were developed by myself and Yinyu and I designed the boards. The fridge wiring was done jointly between myself and Yinyu. I designed the QAngel filter.
One of the useful features of the singlet-triplet qubit is that its spin state can mapped onto a charge state: T0 to $(1,1)$ and S to $(0,2)$. These charge states are more easily detected than directly sensing the spins themselves. This requires a charge sensor capable of detecting the difference in position of an electron sitting in the left quantum dot versus the right quantum dot, a distance that can be as small as 100 nm. This is equivalent to sensing a dipole as we are not sensing the change in the amount of charge but instead its position. It is standard to use a quantum point contact (QPC) or sensor quantum dot (SD) adjacent to the qubit to detect the charge state. Both the QPC and SD are narrow constrictions with a resistance that depends on the electric field generated by the environment, including the qubit. It is then possible to map the charge states of the qubit to the resistance values of the sensor.
The most straightforward way to measure the resistance of the charge detector is to use a DC or low frequency instrument like a lockin. The difficulty of these techniques is that they often require at least milliseconds of integration times which makes single shot readout of the qubit impossible. Instead, it is preferable to measure the qubit using high frequency techniques that can have measurement times of only a few microseconds. One technique that has been particularly successful in GaAs has been radio frequency (RF) reflectometry which uses an impedance matching inductive capacitive (LC) tank circuit to improve the sensitive to the SD[70].

7.1 Introduction to RF Reflectometry

The circuit diagram for a tank circuit is shown superimposed with a SEM image of a device in Figure 7.1.1 (a). The reflection coefficient off a circuit element with impedance $Z$ is given by $\Gamma = \frac{Z-Z_0}{Z+Z_0}$. In our system, $Z_0=50 \, \Omega$ is the impedance of the RF lines that carry the signals. When the impedance of a circuit element is matched to that of the lines ($Z=Z_0$), there is no reflected signal. The impedance for the tank circuit shown in Figure 7.1.1 (a) is given by $Z = i\omega L + \frac{R}{1+i\omega C_R R_{SD}}$. We
note that this has a resonance at \( f = \frac{1}{2\pi\sqrt{LC_0}} \). On resonance, the impedance of the tank circuit loaded with the SD can be written as \( Z = \frac{L}{R_{SD}C_0} \). We are interested in controlling both the resonant frequency of the tank circuit and the value of \( R_{SD} \) that matches \( Z \) to \( Z_0 \). The resonance frequency is important because it is difficult to buy circuit elements that function below 100 MHz. We want to tune the value of \( R_{SD} \) at which the loaded tank circuit matches to the resistance of the sensor dot, typically in the range of 20-200 kOhms. We have two parameters that we can vary to achieve this, \( L \) and \( C_0 \). However, \( C_0 \) will have a lower bound imposed by the parasitic capacitance of the lines on the circuit board and the wire bonds to ground. This is typically on the order of 1 pF but can be reduced slightly by careful circuit board engineering.

In GaAs RF reflectometry has allowed for measurement times of only a few microseconds with SNR that allows for single shot measurements. This is done with inductances of around 780 nH and capacitances on the board of around 1 pF. In GaAs, the simple model of the tank circuit previously presented is adequate to describe the observed behavior. However, this breaks down in SiGe due to the presence of the extra accumulation gates over the current pathways.

### 7.2 Failure of RF Reflectometry in SiGe

As GaAs is depletion mode, the gates surrounding the current pathway to the SD are biased negatively. This means that the 2DEG is not populated directly under or around the gates which minimizes the capacitance between the populated 2DEG and the gates themselves. In SiGe, the accumulation gates must completely cover any 2DEG that is to be populated, including all current paths required for RF reflectometry. This introduces a capacitance between the 2DEG and the accumulation gate on the order of 0.4 pF. This capacitance is significant because it provides low impedance pathways to ground through the RC filters for the DC lines. These pathways are independent of the resistance of SD and reduce the sensitivity of the tank circuit to the qubit state. The additional capacitance of the 2DEG to the gate, \( C_{gate} \), cannot be included into \( C_0 \) because of the resistance of
Figure 7.2.1: (a) The current response through SD to the voltage on the plunger gate of SD. (b) The reflected signal as a function of the plunger gate voltage and frequency. The lack of response to the plunger gate shows that the reflectometry is not sensitive to SD.

The 2DEG in the lead, $R_L$. The most correct model of this is to treat the resistance of the 2DEG and its capacitance to ground as continuous. However, this model is cumbersome, leaving it challenging to understand or simulate. We instead choose to use the simplified model, shown in Figure 7.1.1 (b), which treats the capacitance of the 2DEG to the gate as a single effective capacitance, $C_{gate}$, and contact resistance, $R_L$. While slightly less correct, this model explains the basic trends that are observed and provides for useful intuition.

The lack of sensitivity of the reflected signal to $R_{SD}$ is shown in Figure 7.2.1. The current through a the sensor dot is shown with 1 mV of excitation voltage in Figure 7.2.1 (a) and demonstrates a clear turn on of current with the plunger gate of the sensor. However, in Figure 7.2.1 (b), we see that the reflected signal is unaffected by the plunger gate’s voltage and hence by $R_{SD}$.

To demonstrate that the reduced sensitivity of the reflected signal to $R_{SD}$ is due to $C_{gate}$, we present the data shown in Figure 7.2.2. For these two measurements the SD is completely depleted so that $R_{SD} = \infty$ and the accumulation gate of the lead is varied. The sharp kink occurs when the 2DEG underneath the gate is
accumulated which adds the effect of $C_{gate}$. When the 2DEG is depleted, it has negligible capacitance to the gate. Figure 7.2.2(a) shows the shift from an open style device while Figure 7.2.2(b) shows the shift from an overlap style device. It is not surprising that the shift is larger for the overlap style device because it has metal covering the entire surface and no ALD oxide layer, both of which increase the capacitance of the 2DEG to the gates. The rest of this chapter will focus on overcoming this capacitive coupling of the 2DEG to ground so that we can demonstrate charge detection in SiGe using RF reflectometry.

7.3 **Blocking Resistors - A partial solution**

The gates serve as an effective short to ground at RF frequencies because they are connected to RC filters designed to cut out high frequency noise by shunting it to ground. To prevent the RF reflectometry signal from being shunted to ground directly through the RC filter, we instead implement an RCR filter. We call the second resistor the blocking resistor with value $R_{block}$. The effect of adding $R_{block} = 100 \, M\Omega$ to all gates is shown in Figure 7.3.1(b) with $R_{block} = 0 \, \Omega$ shown in
Figure 7.3.1: The frequency shift of an overlap style device (a) without blocking resistors and (b) with blocking resistors. The shift is reduced because the impedance between the gate and ground is increased when the path through the RC filter is blocked. Figure 7.3.1 (a) for reference. The frequency shift is reduced from 40 MHz to 20 MHz but cannot be eliminated, even with larger values of $R_{\text{block}}$. We interpret this as alternate capacitive pathways to ground that are in parallel to $R_{\text{block}}$ and are then unaffected by its value. We call this reduced capacitance of the 2DEG to ground $C_{\text{gate}}^\ast$.

7.4 Theoretical Understanding of the Matching Condition

To get a better understanding why best matching is never observed for the previous parameters, we numerically solve for $\Gamma$ as a function of $L$ and $C_0$. The simulations are done with $R_L = 3$ kΩ and $C_{\text{gate}} = 0.5$ pF and the resonance frequency is shown in 7.4.1 (a) and matching $R_{SD}$ is shown in 7.4.1 (b). The startling result is that there is no best matching condition in a large section of parameter space, shown as white. As an example, the measurement taken in 7.3.1 (b) used parameters shown as the blue dots in 7.4.1, where we predict no best matching condition, as seen in the measurement. The matching conditions far from the boundary with the no matching region behave like the naive model. The conclusion we draw from this is that $C_0 \gg C_{\text{gate}}$ allows us to use the simpler
model because $C_o$ is the dominant pathway to ground.

To further explore the behavior of the matching condition, we plot simulations of the dependence of the matching resistance as a function of $L$ and $C_o$ with two different values of $R_L$. In Figure 7.4.2(a) we use $R_L = 3$ k$\Omega$ and in 7.4.2(b) we use $R_L = 5$ k$\Omega$. We note that the larger value of $R_L$ forces the matching conditions to higher values of $C_o$. We understand this by looking at the limiting simplifications of the tank circuit. If $R_L = 0\Omega$ then $C_o$ and $C_{gate}$ are just parallel capacitors that can be combined to give the original model. When $R_L = \infty\Omega$, the system just looks like an oscillator made from $L$ and $C_o$ and has no dependence on any of the other elements. It will not in general be matched to $Z_0$ in this case and will therefor not have a best matching $R_{SD}$. The value of $R_{SD}$ limits the minimum impedance to ground through the gate and SD. Larger values of $R_{SD}$ therefore require a lower impedance pathway in parallel to achieve matching with $Z_0$, which can only be achieved by larger values of $C_o$. We demonstrate that these simulations provide the correct predictions for experiments in the next section.
Figure 7.4.2: Simulated matching conditions dependence on L and $C_0$. The value of $R_L$ is 3 kΩ in (a) and 5 kΩ in (b).

7.5 Experimental Verification of the Simulations

To verify the veracity of the model, we measured the matching conditions of one overlap device repeatedly with different values of $C_0$ and L. In Figure 7.5.1 we use a fixed value of $L=2.7 \, \mu H$ with lumped capacitors varied between $C_0=.8 \, pF$ and $10 \, pF$. There is an additional .8 pF contribution to $C_0$ from the board itself. From these measurements we see that there is no best matching condition for $C_0 \leq 1.6 \, pF$ (.8 pF from the lumped element and .8 pF from the board) and that matching conductance can be tuned by increasing $C_0$ beyond this value.

We also demonstrate that the matching condition can be tuned with L in Figure 7.5.2. Lumped element inductors are not available with as many values as capacitors and for this reason we only demonstrate $L=2.2 \, \mu H$ and $L=2.7\mu H$, both with 2.2 pF lumped element capacitors for a total $C_0=3.0 \, pF$ when the board’s contribution is included. The matching conductance is decreased with increasing L as expected.

While these measurements demonstrated that the model is sufficient for understanding the behavior of the system, they also demonstrated that finding a matching condition above 150 MHz would be impossible with these samples. In order to observe best matching at higher frequencies, new samples were
Figure 7.5.1: Matching conditions dependence on $C_0$. A fixed value of $L = 2.7 \, \mu\text{H}$ was used for all four measurements. The lumped element capacitor has value 0.8 pF for (a), 2.2 pF for (b), 4.7 pF for (c) and 10 pF for (d). There is an additional .8 pF from the board that contributes to $C_0$. 
Figure 7.5.2: Matching conditions dependence on $L$. A fixed value of $C_0=2.2+.8 \text{ pF}$ was used for both measurements. The value of $L$ is $2.2 \mu\text{H}$ in (a) and $2.7 \mu\text{H}$ in (b).

Fabricated with narrower accumulation gates to decrease $C_{\text{gate}}$ to $0.2 \text{ pF}$. This newer generation of samples is more suitable to charge detection, as described in the next section.

### 7.6 Charge Sensing

For the new generation of devices, we used $L=620 \text{ nH}$ and $C_0=0.8 \text{ pF}$ (no lumped element, only contribution from the board) which allows us to infer that $C_{\text{gate}}=0.2 \text{ pF}$ because matching occurs at $205 \text{ MHz}$. In Figure 7.6.1(a) we demonstrate that while we were not able to reach best matching for this sample, we were able to measure a shift in the reflected signal with conductance through the SD. In Figure 7.6.1(b) we trace a coulomb peak in the SD as a function of the sensor gate and the plunger gate of the double quantum dot. Tracing this peak allows us to set the sensor gate voltage so that the SD is at its most sensitive point for each value of the dot plunger voltage.

In Figure 7.6.2 we demonstrate charge sensing of a double quantum dot. The honeycomb pattern is as expected and Figure 7.6.2(a-b) demonstrates that we are
Figure 7.6.1: Demonstration of sensitivity of the reflection to $R_{SD}$. In (a) the reflected signal is shown as a function of frequency and conductance ($1/R_{SD}$). Best matching is not reached but there is sensitivity to $R_{SD}$. In (b) a coulomb peak of SD is measured as a function of the sensor’s plunger gate and the plunger gate of the qubit quantum dot.

able to sense the first through eleventh charge transitions. In Figure 7.6.2(c-d) we show a zoomed in region and show that working in a smaller range allows us to better optimize the sensor for higher signal to noise.

7.7 Outlook

Achieving RF reflectometry is an important step towards qubit manipulations in SiGe because it enables single shot qubit readout. For this reason, it has also been worked on by several other groups in parallel. The Nichol Group has demonstrated very similar findings in their recent publication[16]. The Taruchia Group has also approached the problem, though their solution is likely not compatible with overlap style devices[62]. The Vandersypen and Marcus[85] and groups have researched using $C_{gate}$ as the mechanism to introduce the reflectometry signal into the 2DEG and have shown that it allows them to bypass the large resistance of the 2DEG. The results presented in this chapter are
Figure 7.6.2: Charge stability diagrams of a double quantum dot. In (a,c) we plot the reflection off the tank circuit. In (b,d) we plot the x derivatives of (a,c) respectively to highlight the charge transitions. The observed honeycomb pattern is as expected.
currently being drafted into a joint paper with the Vandersypen Group that compares the two different methods.

7.8 Contributions

All work in this chapter was done jointly between myself and Yinyu Liu.
The greatest story everyone can tell is the story of their life. Just don’t let the climax of your story be unrealized dreams.

Unarine Ramaru

Unrealized experiments

It is not uncommon for a graduate student to have bigger eyes than stomach and to leave some ideas unfinished. Spending the later half of my PhD on redeveloping fabrication has left me with many such ideas that I had hoped to test before graduation. However, the difficulties previously mentioned have meant that some of my ideas are yet untested. I would like to document them though so that future researchers working on the singlet-triplet qubit may attempt them.

8.1 $\Delta B_z$ from wires

Currently, $\Delta B_z$ in SiGe are generated by micro-magnets. It remains to be seen how effective DNP will be at generating $\Delta B_z$ due to the decreased number of nuclei in natural silicon and almost total absence in isotopically pure SiGe. DNP
offered the flexibility of tuning $\Delta B_Z$ in situ through feedback while accomplishing the same thing with a micro-magnet requires fabricating a new sample. Here we explore the idea of using micron scale wires that are nanofabricated on the substrate to provide $\Delta B_Z$. We will find the optimal geometry and discuss heating effects as well.

Consider the geometry shown in Figure 8.1.1 which has two long wires with width $W$ and thickness $T$ separated by a gap $G$ on the surface of semiconductor with a 2DEG buried $D$ below the surface. To give some intuition for the behavior of this system, the magnetic field is dominated by the closest wire which means that driving currents in opposite directions in the wires will result in the magnetic field pointing in opposite directions under the two wires. This can generate $\Delta B_Z$ in a tunable manner because the magnitude should depend on $I$.

To calculate the effects of the wires, we will begin by approximating the wires as infinitely long. The magnetic field from an infinitely long wire is $B = \frac{\mu_0 I}{2\pi r}$ where $r$ is the distance to the wire, $\mu_0$ is the magnetic permeability, and $I$ is the current. The assumption of infinite wires introduces errors less than 10% so long as $r$ is...
less than times times the length of the wire. Our real geometry has finite dimensions, so we will instead integrate over \( B = \frac{\mu_0 J dxdy}{2\pi r} \), where \( J \) is the current density. As we have assumed that the wires are infinitely long, we have translational invariance of the magnetic field along the wire meaning that the magnetic field dependence in the 2DEG only depends on the motion along the \( x \) axis. We will now calculate the magnetic field in the \( x \) direction of the 2DEG as a function of position. Consider the position \((p,0)\) where \( p = 0 \) corresponds to halfway between the two wires.

\[
B_x = \int_0^T dy \int_0^W dx \frac{\mu_0 J dxdy}{2\pi} \frac{1}{\sqrt{(y + D)^2 + (-W - G/2 + x - p)^2}} - \int_0^T dy \int_0^W dx \frac{\mu_0 J dxdy}{2\pi} \frac{1}{\sqrt{(y + D)^2 + (G/2 + x - p)^2}}
\]

The first equation is the unsimplified form that explicitly lists the fractions corresponding to the \( 1/r \) term and the \( \cos(\theta) \) terms. The negative sign comes from the currents flowing in opposite directions. These integrals can be numerically calculated with varied wire parameters (\( G, W \) and \( T \)) given \( D \) and the spacing between the quantum dots. I have calculated using a very conservative \( D = 150 \text{ nm} \) which is significantly thicker than the depth to the 2DEG plus the gate layers. I have also assumed an interdot spacing of \( 120 \text{ nm} \). For these parameters, the parameters that minimize the power dissipation are found to be \( G = 180 \text{ nm} \), \( T = 325 \text{ nm} \), and \( W = 475 \text{ nm} \). We were able to fabricate wires with these dimensions as shown in Figure 8.1.2.

The biggest consideration for this method is that it uses currents to generate...
Figure 8.1.2: This device was fabricated with the parameters discussed in the section.
the magnetic field gradient. The on chip heating was calculated to be 2.3 nW for 10 MHz of $\Delta B_Z$. However, the power dissipated scales quadratically with the current while $\Delta B_Z$ scales only linearly. The largest heat load will arise from the filters for the lines used to drive these currents. We have yet to measure the heat loads effect on the base temperature of the fridge. However, we know that the electron temperature is 100 mK, so if the heating is not significantly above this, the effect should be minimal.

8.2 Double Rotating Frame

In GaAs we were able to generate large enough $\Delta B_Z$ so this term was the dominant one in our Hamiltonian. This meant that charge noise was mitigated as we rotated around $\Delta B_Z$. We still required an oscillating $J$ drive to echo away the errors in our $\Delta B_Z$ setting. I propose the following strategy to achieve a similar result in SiGe where we likely will not be able to achieve such strong $\Delta B_Z$. This strategy will require the use of high frequency $\Delta B_Z$ which we should be able to achieve through currents driven in the wires described in the previous section.

It is assumed that the dominant term in the Hamiltonian is $J$ so that the qubit precesses around $z$. We will drive an oscillating $\Delta B_Z$ at a frequency resonant with the rotations driven by $J$. This allows us to enter the first rotating frame in which charge noise should be mitigated by rotations driven by $\Delta B_Z$. However, errors in the oscillating $\Delta B_Z$ will still be present. We enter a second rotating frame by applying an oscillating $J$ with frequency that is resonant with the precession around $\Delta B_Z$. This second rotating frame serves the same purpose as the drive in our previous rotating frame work and should mitigate the effects of errors in $\Delta B_Z$. We work out the mathematical details for this below.
In the lab frame the two qubit Hamiltonian is as follows.

\[ H = (J_1 + \tilde{J}_1 \sin(\omega_1 t + \varphi_1) + \frac{dj_1}{d\varepsilon_1} \delta\varepsilon) \sigma_z \otimes I \\
+ (J_2 + \tilde{J}_2 \sin(\omega_2 t + \varphi_2) + \frac{dj_2}{d\varepsilon_2} \delta\varepsilon) I \otimes \sigma_z \\
+ (\Delta B_{z,1} + \tilde{B}_1 \sin(\omega_3 t + \varphi_3) + \delta B) \sigma_z \otimes I \\
+ (\Delta B_{z,2} + \tilde{B}_2 \sin(\omega_4 t + \varphi_4) + \delta B) I \otimes \sigma_z \\
+ a \frac{dj_1}{d\varepsilon_1} \frac{dj_2}{d\varepsilon_2} \sigma_z \otimes \sigma_z \]

\( \tilde{J} \) and \( \tilde{B} \) are oscillating drives and \( \delta\varepsilon \) and \( \delta B \) are noise. Limit \( \Delta B_z \ll J \) to work in frame rotating around \( z \) with frequency \( \Omega \), the ideal \( J \) that we may be slightly removed from. \( dJ \equiv J \cdot \Omega \)

\[ \sigma_x \rightarrow \cos(\Omega t + \theta) \tilde{\sigma}_x + \sin(\Omega t + \theta) \tilde{\sigma}_y \]
\[ \sigma_y \rightarrow - \sin(\Omega t + \theta) \tilde{\sigma}_x + \cos(\Omega t + \theta) \tilde{\sigma}_y \]
\[ \sigma_z \rightarrow \tilde{\sigma}_z \]

These substitutions yield

\[ H = (dj_1 + \tilde{J}, \sin(\omega_1 t + \varphi_1) + \frac{dj_1}{d\varepsilon_1} \delta\varepsilon) \tilde{\sigma}_z \otimes I \\
+ (dj_2 + \tilde{J}_2 \sin(\omega_2 t + \varphi_2) + \frac{dj_2}{d\varepsilon_2} \delta\varepsilon) I \otimes \tilde{\sigma}_z \\
+ (\tilde{B}_1 \sin(\omega_3 t + \varphi_3) + \delta B)(\cos(\Omega t + \theta_1) \tilde{\sigma}_x + \sin(\Omega t + \theta_1) \tilde{\sigma}_y) \otimes I \\
+ (\tilde{B}_2 \sin(\omega_4 t + \varphi_4) + \delta B)I \otimes (\cos(\Omega t + \theta_2) \tilde{\sigma}_x + \sin(\Omega t + \theta_2) \tilde{\sigma}_y) \\
+ a \frac{dj_1}{d\varepsilon_1} \frac{dj_2}{d\varepsilon_2} \tilde{\sigma}_z \otimes \tilde{\sigma}_z \]

Drive \( \tilde{B} \) at \( \omega_1 = \Omega \) and choose \( \varphi_3 = \theta_1 + \frac{\pi}{2} \) so the cos term survives. Drop terms that average to zero.
\[ H = (dJ_1 + \tilde{J}_1 \sin (\omega_1 t + \varphi_1) + \frac{dJ_1}{d\varepsilon_1} \delta \varepsilon) \tilde{\sigma}_z \otimes I \]

\[ + (dJ_z + \tilde{J}_z \sin (\omega_z t + \varphi_z) + \frac{dJ_z}{d\varepsilon_z} \delta \varepsilon) I \otimes \tilde{\sigma}_z \]

\[ + \frac{1}{2} \tilde{B}_1 \tilde{\sigma}_x \otimes I + \frac{1}{2} \tilde{B}_1 I \otimes \tilde{\sigma}_x \]

\[ + a \frac{dJ_1}{d\varepsilon_1} \frac{dJ_z}{d\varepsilon_z} \tilde{\sigma}_z \otimes \tilde{\sigma}_z \]

Assume \( \frac{1}{2} \tilde{B}_1 \) is the dominant term in the Hamiltonian. Work in a rotating frame so that \( dB = \frac{1}{2} \tilde{B}_1 - \beta \), where \( \beta \) is the ideal magnetic drive amplitude that we may be slightly removed from.

\[ \tilde{\sigma}_x \rightarrow \tilde{\sigma}_x \]

\[ \tilde{\sigma}_y \rightarrow \cos (\beta t + \theta) \tilde{\sigma}_y + \sin (\beta t + \theta) \tilde{\sigma}_z \]

\[ \tilde{\sigma}_z \rightarrow - \sin (\beta t + \theta) \tilde{\sigma}_y + \cos (\beta t + \theta) \tilde{\sigma}_z \]

This substitution gives

\[ H = (dJ_1 + \tilde{J}_1 \sin (\omega_1 t + \varphi_1) + \frac{dJ_1}{d\varepsilon_1} \delta \varepsilon)(- \sin (\beta t + \theta) \tilde{\sigma}_y + \cos (\beta t + \theta) \tilde{\sigma}_z) \otimes I \]

\[ + (dJ_z + \tilde{J}_z \sin (\omega_z t + \varphi_z) + \frac{dJ_z}{d\varepsilon_z} \delta \varepsilon) I \otimes (- \sin (\beta t + \theta) \tilde{\sigma}_y + \cos (\beta t + \theta) \tilde{\sigma}_z) \]

\[ + d\tilde{B}_1 \tilde{\sigma}_x \otimes I + d\tilde{B}_1 I \otimes \tilde{\sigma}_x \]

\[ + a \frac{dJ_1}{d\varepsilon_1} \frac{dJ_z}{d\varepsilon_z} (- \sin (\beta t + \theta) \tilde{\sigma}_y + \cos (\beta t + \theta) \tilde{\sigma}_z) \otimes (- \sin (\beta t + \theta) \tilde{\sigma}_y + \cos (\beta t + \theta) \tilde{\sigma}_z) \]

Now drive \( \tilde{J}_1 \) at \( \omega_1 = \beta \) and choose \( \varphi_1 = \theta_1 + \frac{\pi}{2} \). Drop terms that average to zero.
\[ H = \frac{1}{2} \tilde{J}_1 \tilde{\sigma}_z \otimes I + \frac{1}{2} \tilde{J}_2 I \otimes \tilde{\sigma}_z \]
\[ + d\tilde{B}_1 \tilde{\sigma}_x \otimes I + d\tilde{B}_2 I \otimes \tilde{\sigma}_x \]
\[ + \frac{1}{2} a \frac{d\tilde{J}_1}{d\xi} \frac{d\tilde{J}_2}{d\xi} (\tilde{\sigma}_y \otimes \tilde{\sigma}_y + \tilde{\sigma}_z \otimes \tilde{\sigma}_z) \]

Allowing \( \tilde{J}_1 \gg d\tilde{B}_1 \), meaning that our drive is larger than the errors in our magnetic field gradient, will give us what looks like our normal Hamiltonian in the lab frame, with reduced charge and magnetic field noise.

### 8.3 Cross Resonant Qubit Drive

This idea of coupling two singlet-triplet qubit at a sweet spot was originally conceived of with the observation that the singlet-triplet qubit’s energy spectrum resembles that of the hybrid qubit when \( \Delta B_Z \) is larger than the tunnel coupling between the dots. As with the hybrid qubit, there is a sweet spot but at this point the singlet-triplet qubit becomes a charge qubit, which is not ideal. However, a well known but previously inaccessible sweet spot is located in \((1,1)\), halfway between \((2,0)\) and \((0,2)\). In SiGe, the stronger lever arms due to the increased capacitance of the plunger gates may allow for operations at this point. If this is indeed true, the following scheme may prove useful.

At the sweet spot, \(|S\rangle\) will have spatial wave function contributions from \((2,0)\), \((1,1)\) and \((0,2)\) while \(|T_0\rangle\) will only have contributions from \((1,1)\). Assume the qubits have different energy splittings of \(h_f L\) and \(h_f R\) at this point and that the energy difference are dominated by \(\Delta B_Z\). A charge noise resistant two qubit gate at a sweet spot can be realized by applying an oscillatory \(\epsilon\) drive to the left qubit at \(h_{f_k}\). This will not drive transitions in the left qubit but will cause the weights of the \((2,0)\) and \((0,2)\) states to oscillate at the same frequency for \(|S\rangle\) with no effect on \(|T_0\rangle\). The right qubit will then experience an oscillating \(\epsilon\) at \(h_{f_k}\) that depends on the state of the left qubit. This will mean that \(|S\rangle\) of the left qubit will drive
transitions on the right qubit while $|T_0\rangle$ on the left qubit will leave the right qubit's state unchanged. This two qubit interaction, combined with single qubit gates, should be sufficient to generate a CNOT gate with the left qubit serving as the control qubit and the right qubit serving as the target qubit. This is called cross resonant qubit drive and has been demonstrated in superconducting qubits\cite{63}.

8.4 Reducing Electron Temperature On Chip

This section is by far the most speculative of this thesis but would also be one of the quickest to test with the proper device. The electron temperature is decoupled from the lattice temperature below 1 K because of the reduction in phonons that couple them. However, if no heat were introduced to the electrons, they should eventually come to the same temperature as the lattice. I propose implementing the same split gate geometry for the accumulation gates for the qubit leads as are currently implemented for the sensors. This was actually the inspiration for the split gates of the sensor and was actually implemented for both the sensors and the qubits in the first iteration of devices. However, due to low gate yield, it was decided to drop these additional gates because every additional gate increased the probability of device failure. The purpose of these split accumulation gates for the qubits would be to allow for accumulation of the lead with a population of electrons that is cut off from the lines. This population of electrons would then only have capacitive couplings, which are nondissipative, to its surroundings, meaning that the electrons should cool down to the temperature of the lattice. This would improve qubit loading and all readout methods that depend on tunneling to the lead. The temperature of the lead could be measured by observing the average population of the quantum dot coupled to the reservoir near a charge transition. This would allow for a direct probing of the fermi function and hence the temperature. If the rate at which the electrons cool is slow, it means that the electrons are only weakly coupled to the lattice. One could then take this a step further by implementing a gate geometry that would
allow for adiabatic expansion of the electron gas or evaporative cooling which could reduce the electron temperature to below that of the lattice temperature. While these last two suggestions would require a very specialized device geometry, just measuring the electron temperature with a choked off population of electrons would only require a single additional gate so that the accumulation gate could be split. It is therefore suggested that this be tried when the device yield is high enough that a single gate does not appreciably decrease the probability of fabricating a successful sample.

8.5 Conclusion

This thesis has represented the culmination of a decade of a singlet-triplet research in GaAs and the birth of the SiGe project in the Yacoby Lab. The success of an improved two qubit gate was built upon the foundation laid by past generations of graduate students and post doctoral fellows as I hope that this work will pave the way for future SiGe progress. While I have been interviewing and writing this thesis, Yinyu has observed the measurement point where $|S\rangle$ and $|T_0\rangle$ have different charge states. She has also recently seen what we believe to be the first signatures of coherent oscillations of singlets and triplets in a SiGe device. While progress has been relatively challenging thus far, I am hopeful that this development shows that the project has finally pushed through to a point where more of the emphasis is on qubit physics rather than device and measurement engineering. I believe that the future is bright for the singlet-triplet qubit and that the ideas proposed in this chapter along with many others leave singlet-triplet qubits in SiGe ripe with opportunity.
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Colophon

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