# Essays in Networks and Macroeconomics

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Essays on Networks and Macroeconomics

A dissertation presented

by

Elisa Rubbo

to

The Department of Economics

in partial fulfillment of the requirements

for the degree of

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in the subject of

Economics

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Essays on Networks and Macroeconomics

Abstract

I develop an analytical framework to study monetary policy in an economy with multiple sectors and a general input-output network.

I derive the Phillips curve and welfare as a function of the underlying production primitives, obtaining important new insights. (i) The presence of input-output linkages flattens the Phillips curve. (ii) There is an endogenous tradeoff between stabilizing output and stabilizing consumer prices. (iii) I construct a novel inflation index that eliminates this tradeoff, which I refer to as the “divine coincidence” index. (iv) Monetary policy faces a tradeoff between stabilizing aggregate output and relative output across sectors. While targeting the “divine coincidence” index stabilizes aggregate output, the optimal policy can be implemented by targeting an alternative inflation index which incorporates this tradeoff.

I calibrate the model to the U.S. economy, providing a quantitative counterpart to (i)-(iv). Mirroring (i), the slope of the consumer price Phillips curve is one order of magnitude smaller in the muti-sector model than in the one-sector benchmark, and matches empirical estimates. The model also predicts a 30% decline over the past 70 years, arising from increased intermediate inputs in production. Validating (ii) and (iii), the divine coincidence inflation rate provides a better fit for Phillips curve regressions than conventional consumer price specifications. The baseline policy of targeting consumer inflation leads to a welfare loss of 1.12% of per-period GDP. Switching to the optimal policy in (iv) reduces this loss to 0.28%, but cannot fully eliminate it. Targeting the output gap almost replicates the optimal policy.
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Introduction

The New Keynesian framework informs the central banks’ approach to monetary policy, and constitutes the theoretical foundation underpinning inflation targeting. The baseline New Keynesian model assumes only one sector of production, whereas in reality an economy has multiple and heterogeneous sectors, which trade in intermediate inputs. There are crucial issues that the baseline model is silent about. What is the correct definition of aggregate inflation, depending on the production structure? How should central banks trade-off inflation in different sectors, based on their position in the input-output network?

I extend the New Keynesian framework to account for multiple sectors, arranged in an input-output network. Sectors have arbitrary neoclassical production functions; they face idiosyncratic productivity shocks and heterogeneous pricing frictions. I solve the model analytically, providing an exact counterpart for the traditional results in the multi-sector framework.

I derive analytical expressions for the two key objects which constitute the “backbone” of the optimal policy problem: the Phillips curve and the welfare loss function. Building on this result, I construct two novel indicators. The first inherits the positive properties of inflation in the one-sector model, and therefore can be viewed as its natural extension to a multi-sector economy. Notably this index yields a well-specified Phillips curve, and it is stabilized together with aggregate output—a property which is referred to as the “divine coincidence”. The second indicator instead serves as an optimal policy target.

Traditionally, consumer price inflation has been taken as the relevant real-world counter-
part of inflation in the one-sector model: it is universally used in Phillips curve regressions and as a policy target.\(^1\) This choice, however, has no theoretical backing.\(^2\) Importantly, I argue that in the multi-sector framework no single statistic inherits all the properties of inflation in the one-sector model. The “divine coincidence” index preserves the positive properties, while the optimal policy target maintains the normative ones. The two indexes are distinct, and both are different from consumer prices.

My representation of production is fully general, therefore the model can be readily calibrated to any input-output structure. The evolution of the economy is characterized by three variables (the output gap, sectoral inflation and productivity) and a set of steady-state parameters which depend on input-output shares and sectoral pricing frictions. I construct time series of these variables and calibrate the parameters for the US economy. The analysis shows that taking into account the disaggregated production structure is important, not just from a theoretical but also from a quantitative point of view.

The exposition is organized around two sets of results, positive and normative. Chapter I presents the positive results. I provide a general expression for the Phillips curves associated with any given inflation index. I show that in general these Phillips curves are misspecified, because they have an endogenous residual which depends on sectoral productivity shocks. I then construct the “divine coincidence” inflation index which instead yields a well-specified Phillips curve. Chapter II presents the normative analysis, focusing on the central bank’s problem. I derive welfare as a function of the output gap and sectoral inflation rates, solve for the optimal monetary policy, and construct an inflation target which implements it.

The Phillips curve describes the joint evolution of aggregate inflation (\(\pi\)) and the output

---

\(^1\)Statistical agencies release several different measures of consumer prices: in the US they are the consumer price index (CPI), personal consumption expenditures (PCE), their core versions (excluding sectors with very volatile prices, such as food and energy) and the GDP deflator. Central banks look at all of these measures, and various others (such as wage inflation, commodity prices, import prices, exchange rates...), but they lack a theoretical framework to aggregate them into a proxy for the output gap or into an interest rate target.

\(^2\)As cited in Footnote ??, consumer price inflation does not preserve the “divine coincidence” in simple extensions of the baseline model. In addition, Gali and Monacelli (2005) and Gali (2008) show that for small open economies the relevant statistic for the Phillips curve and monetary policy is producer price inflation.
where $\rho$ is the discount factor, $\kappa$ is the slope and $u_t$ is a residual. In the multi-sector model the slope and residual depend on the inflation index $\pi_t$ on the left-hand-side, and on the production structure. The residual is endogenously determined by sectoral productivity shocks. I derive $\kappa$ and $u$ for a generic choice of $\pi_t$. This is useful to interpret Phillips curve regressions, as it allows to predict how the estimated slope and residual should vary across specifications and over time. Specifications where the slope changes with the production structure or where productivity fluctuations generate large residuals should be more sensitive to the sample period and have lower R-squareds.

An example of this is the consumer-price Phillips curve: the theory predicts a variable slope and large residuals, which are consistent with the poor fit and unstable estimates from the data. By contrast, the “divine coincidence” inflation index which I construct yields a Phillips curve with no endogenous residual, and whose slope does not depend on the production structure. Consistent with the theory, the Phillips curve estimated with this index has a stable coefficient and higher R-squared.

To build to these results, I first provide formulas relating sectoral inflation rates to the output gap and productivity. I then combine sector-level elasticities with respect to the output gap into the slope of the aggregate Phillips curves, and sector-level elasticities with respect to productivity into the residual.

The slope of the Phillips curve maps fluctuations in aggregate demand, captured by the output gap, into the corresponding price response. A positive output gap means that demand is above the efficient level. This puts upwards pressure on labor demand, thereby raising real wages. In turn, wages are passed-through into marginal costs and prices. The aggregate slope combines sector-level responses, with weights that depend on the inflation index on the left-hand-side. I characterize sectoral elasticities as a function of the input-output structure and sector-level frequencies of price adjustment. I demonstrate that these
elasticities, and therefore the slope of the aggregate Phillips curve, are decreasing in the size of intermediate input flows. The intuition is as follows. While the network structure does not affect the response of real wages to the output gap, it is crucial for the pass-through of wages into prices. Sectors are affected by wage changes directly (if they hire workers) and through intermediate input prices. Because of price rigidities, suppliers do not fully reflect changes in wages into their price. As a result the presence of inputs-output linkages reduces the pass-through of wages into sectoral and aggregate prices, thereby flattening the Phillips curve.

From a quantitative point of view, accounting for intermediate inputs leads to very different predictions with respect to the benchmark model. In Chapter III I calibrate the model to the US economy. I focus on the consumer-price Phillips curve, given the prominence of consumer inflation as the standard measure. The network model predicts a slope of around 0.1, consistent with empirical estimates (usually between 0.1 and 0.3). By contrast, the one-sector model implies a slope of about 1. The multi-sector model also allows to relate the evolution of the input-output structure over time with the slope of the Phillips curve. Empirical estimates suggest that the slope has declined.\(^3\) I fit the model to historical input-output tables for each year between 1947 and 2017\(^4\) and find that the implied slope decreased by about 30\%.\(^5\) Mirroring the slope of the Phillips curve, the network model features significantly more monetary non-neutrality than the baseline, and the predicted response of inflation to monetary shocks also decreased over time.\(^6\)

The residual of the Phillips curve captures a wedge between aggregate output and prices

---

\(^3\)See for example Blanchard (2012).

\(^4\)The BEA provides data at different levels of aggregation for different years. For a clean comparison I convert all the data to the most aggregated level, with 47 industries.

\(^5\)Other authors attribute the decline in the slope of the Phillips curve to a different channel (see Blanchard (2016)): with better monetary policy inflation is more stable, therefore firms adjust prices less often. This dampens the response of inflation and reduces the slope of the Phillips curve. I mute this channel by assuming constant frequencies of price adjustment. For many sectors it is impossible to track their evolution over time, due to lack of data. For sectors where data are available, Nakamura and Steinsson (2013) find that the frequency of price adjustment is stable over time.

\(^6\)This result is consistent with results from quantitative models in the literature (see Carvalho (2006) and Nakamura and Steinsson (2013)).
induced by productivity shocks. In the one-sector benchmark the “divine coincidence” tells us that there is no such wedge. Productivity affects marginal costs directly, and indirectly through a change in equilibrium wages. Intuitively, lower productivity corresponds to a lower marginal product of labor, and therefore lower real wages. With only one sector these two effects exactly offset each other, and marginal costs remain constant. With multiple sectors this is no longer true, because marginal costs are asymmetrically exposed to productivity and wage changes. As a consequence sector-level inflation is not stabilized under zero output gap. The same is true for almost all measures of aggregate inflation, including consumer prices. I derive the elasticities of sector-level prices to productivity shocks, and aggregate them into the residual of the consumer-price Phillips curve. I use BEA-KLEMS data to construct a time series for this residual, using measured sectoral TFP shocks. The series has a standard deviation of 25 basis points, suggesting that endogenous cost-push shocks explain a significant fraction of the variation in consumer inflation.

I construct a (unique) inflation index which restores the “divine coincidence” in the aggregate. This index weights sectoral inflation rates according to sales shares, appropriately discounting more flexible sectors. Intuitively, I show that there is a one-to-one mapping between the aggregate output gap and a weighted sum of sectoral markups. High markups are associated with low aggregate demand and a negative output gap. Sectors are weighted by sales shares, which, different from consumption shares, capture their full contribution to total value added. Markups, in turn, are reflected in inflation rates. In my framework sector-level markups change endogenously, due to incomplete price adjustment. After a cost shock, the producers who are able to adjust their price keep a constant markup. Those who cannot instead need to absorb the shock into their markup. As a result, the same shock generates a larger inflation response and a smaller markup response in sectors with more flexible prices. Therefore these sectors need to be discounted.

Interestingly, in the calibrated model the “divine coincidence” index assigns the highest weight (of 18%) to wage inflation. This is because labor has the highest sales share, and
wages are quite rigid. Previous contributions (Mankiw and Reis (2006), Blanchard and Gali (2007), Blanchard (2016)) also suggest using wage inflation as an indicator. I provide a formal argument, and characterize the correct weight for wages relative to other sectors.

In Chapter III construct a time series of the “divine coincidence” index for the US economy, and run Phillips curve regressions over the years 1984-2017. I compare the results with standard measures of consumer inflation. The baseline OLS specification has an R-squared of about 0.2 with the “divine coincidence” index, as opposed to about 0.05 with consumer price inflation. Rolling regressions over 20 year windows for the same sample period are always significant, versus about 50% of the time with consumer prices; the estimated coefficient is stable over time.

Targeting the “divine coincidence” index closes the output gap, but this does not implement the optimal policy. As discussed in Chapter II, the output gap captures distortions in aggregate demand, but with multiple sectors there are also distortions in relative demand across firms and sectors. Relative demand distortions cannot be fully eliminated, and monetary policy cannot replicate the efficient equilibrium which emerges under flexible prices. These distortions however can be alleviated, at the cost of deviating from the optimal aggregate demand. Closing the output gap therefore is not constrained optimal. In this sense the “divine coincidence” does not hold from a normative point of view, unlike in the baseline model.

I derive an expression for the welfare loss as a function of the output gap and sectoral inflation rates. Distortions in relative demand across firms and sectors are summarized by an appropriate function of sectoral inflation rates. Within each sector, shocks induce price distortions between the firms that adjust their prices and those that do not. Incomplete price adjustment further results in relative price distortions across sectors. The size of within- and cross-sector price distortions depends on how shocks propagate through the input-output network. Their welfare effect depends on the response of quantities demanded, which is governed by the relevant elasticities of substitution in production and consumption.
Monetary policy has only one instrument (interest rates or money supply), therefore it needs to trade off aggregate demand against allocative efficiency. The optimal policy can still be implemented via inflation targeting, that is, by stabilizing an appropriate inflation indicator which incorporates the two sides of this tradeoff. The optimal target combines the “divine coincidence” index, which proxies for the output gap, and a weighted sum of sectoral inflation rates. The weights reflect the relative marginal benefit and marginal cost of distorting aggregate demand to alleviate the misallocation associated with sectoral inflation.

Chapter III discusses the quantitative relevance of these results. Targeting consumer inflation, as prescribed by the baseline model, leads to a welfare loss of 1.12% of per-period GDP with respect to a world without pricing frictions. Switching to the optimal policy brings this loss down to 0.28%, but does not fully eliminate it. Closing the output gap yields a small additional loss with respect to the optimal policy. This means that the output gap is a good target from a quantitative point of view: intuitively, monetary policy is a blunt instrument to correct misallocation, because it is one-dimensional. Therefore it can only change relative prices proportionately to their elasticities with respect to the output gap, which are usually not aligned with the efficient adjustment. Therefore the cost of distorting aggregate demand is larger than the allocative efficiency gain, and in practice the optimal output gap is close to zero.

Related literature  From a conceptual standpoint my work is related with the literature on markup distortions and welfare (especially Baqee and Farhi (2017)). The key difference is that in my setup markup changes are not exogenous, but result from productivity shocks and price rigidities.

Previous works extend the one-sector New Keynesian framework to incorporate realistic elements of the production structure, in two dimensions. Earlier papers focus on simple extensions of the baseline model, maintaining its analytical approach (Aoki (2001), Woodford (2003), Benigno (2004), Blanchard and Gali (2007), Gali (2008), Gali and Monacelli
More recent works consider richer input-output structures, but restrict themselves to a quantitative approach. They mostly focus on the positive implications for monetary non-neutrality (Basu (1995), Carvalho (2006), Carvalho and Nechio (2011), Nakamura and Steinsson (2013) and Pasten, Schoenle and Weber (2016)) and aggregate volatility (Pasten, Schoenle, Weber (2017)). Castro-Cienfuegos (2019) explores the normative implications for the cost of trend inflation, showing that it is higher with input-output linkages.

I combine the benefits of the two approaches, providing a full analytical solution and quantitative results under a general production structure. My results about the flattening of the Phillips curve are closely related with the quantitative literature on monetary non-neutrality. My results hold beyond a specific calibration, and my analytical approach makes them more transparent. I can derive Phillips curves for any aggregate inflation index and relate their parameters with the underlying production structure, thereby highlighting the channels that determine the flattening. On top of this, I can characterize the inflation-output tradeoff as a function of sectoral productivity and study optimal policy.

My normative analysis is related to parallel and independent work by La’O and Tahbaz-Salehi (2019). A key difference is that in their setup price rigidities are microfounded as arising from incomplete information, while production functions are restricted to be Cobb-Douglass. Because of these modeling differences, sectoral weights have different determinants in the optimal targeting rule (the information structure versus substitution elasticities).

The question of optimal indicators has also been explored, both from a theoretical perspective in simpler networks (Benigno (2004), Gali and Monacelli (2005)) and in larger quantitative models (Mankiw and Reis (2003), Eusepi, Hobjin and Tambalotti (2011)). With respect to previous theoretical contributions, I consider a fully general input-output structure; with respect to quantitative models, my optimal inflation target is based on a microfounded objective function, and I provide a clear interpretation of how sectoral weights depend on production primitives.

The empirical limitations of consumer price inflation for Phillips curve regressions and
forecasting are also well documented (Orphanides and Van Norden (2002), Mavroeidis, Plagborg-Muller and Stock (2014)), and many studies seek to construct indicators with better statistical properties (Stock and Watson (1999), Bernanke and Boivin (2003), Stock and Watson (2015)). By contrast, I show that Phillips curve regressions with the “divine coincidence” index which I construct yield stable and significant estimates over time and across specifications.
Chapter 1

Positive analysis

1.1 Setup

This section lays out the key elements of the network model and the assumptions about preferences, timing and policy instruments. Section 1.1.5 introduces the equilibrium concept, which is designed to account for the endogenous evolution of markups under price rigidities.

1.1.1 Timing and policy instruments

In the main text I consider a one-period model. The dynamic version is presented in Appendix D.

The timing is as follows: before the world begins, firms set prices based on their expectations of productivity and money supply; then sectoral productivities are realized, and the central bank chooses the money supply; some firms have the possibility to adjust their price after observing the realized productivity and money supply, while others do not; the world ends after production and consumption take place. Inflation is defined as the change in prices with respect to the pre-set ones.

In the static setup money supply is the only policy instrument (to be replaced with interest rates in the dynamic version). I impose that nominal consumption expenditure
cannot exceed the aggregate money supply $M$, so that with incomplete price adjustment an increase in $M$ raises aggregate demand and output.

### 1.1.2 Preferences

Consumers derive utility from consumption and leisure, with utility function

$$U = \frac{C^{1-\gamma}}{1 - \gamma} - \frac{L^{1+\varphi}}{1 + \varphi}$$

$L$ is labor supply. There are $N$ goods produced in the economy, and agents have homothetic preferences over all of these goods. $C$ is their utility from consumption, defined over bundles $(c_1, ..., c_N)$:

$$C = C(c_1, ..., c_N)$$

Consumers maximize utility subject to the budget constraint

$$PC \leq wL + \Pi - T$$

where $P$ is the price index of the consumption bundle, $w$ is the nominal wage, $\Pi$ are firm profits (rebated to households) and $T$ is a lump-sum transfer from the government.

In addition, nominal consumption expenditure $PC$ cannot exceed the aggregate money supply $M$:

$$PC \leq M$$

### 1.1.3 Production

There are $N$ sectors in the economy (indexed by $i \in \{1, ..., N\}$). Within each sector there is a continuum of firms, producing differentiated varieties.
All firms $f$ in sector $i$ have the same constant returns to scale production function

$$Y_{if} = A_i F_i(L_{if}, \{x_{ijf}\})$$

where $L_{if}$ is the amount of labor hired by firm $f$ in sector $i$, $x_{ijf}$ is the quantity of good $j$ that it uses as input, and $A_i$ is a Hicks-neutral, sector-specific productivity shock. Labor is freely mobile across sectors.

Customers buy a CES bundle of the differentiated varieties. Sectoral outputs are given by

$$Y_i = \left( \int Y_{if}^{\epsilon_i-1} df \right)^{\frac{\epsilon_i}{\epsilon_i + 1}}$$

where $\epsilon_i$ is the elasticity of substitution between varieties within sector $i$.

**Cost minimization and markups**  All producers in sector $i$ solve the cost-minimization problem

$$C_i = \min_{\{x_{ij}\}, L_i} wL_i + \sum_j p_j x_{ij} \quad s.t. \quad A_i F_i(L_i, \{x_{ij}\}) = \bar{y}$$

Under constant returns to scale marginal costs are the same for all firms, and they use inputs in the same proportions.

Before the world begins, all firms set their price optimally based on their expected marginal cost. They solve

$$\max p_i \mathbb{E} D_i (p_i - (1 - \tau_i) m c_i) \left( \frac{p_i}{P_i} \right)^{-\epsilon_i}$$

where $D_i$ and $P_i$ are the sector-level demand and price index, and $\tau_i$ is an input subsidy provided by the government. The subsidies $\tau_i$ are set in order to eliminate the distortions that arise under the CES demand structure, where firms have constant desired markup given

\footnote{Note that this is without loss of generality: factor-biased productivity shocks can be modeled by introducing an additional sector which simply purchases and sells the factor, and letting a Hicks-neutral shock hit this sector.}
by
\[ \mu_i^* = \frac{\varepsilon_i}{\varepsilon_i - 1} > 1 \]

This is inefficient, since there are no fixed costs. The optimal subsidies satisfy
\[ 1 - \tau_i = \frac{\varepsilon_i - 1}{\varepsilon_i} \]

They are set so that the resulting markup over pre-subsidy marginal costs is 1, and firms charge price
\[ p_i^* = \mathbb{E} mc_i \]  \hspace{1cm} (1.1)

Input subsidies cannot change in response to shocks, and are constrained to be the same for all firms within the same sector.

After productivity and money supply are realized, firms in the same sector end up charging different prices. Those who can adjust their price keep a constant markup equal to the desired one. All other firms need to keep constant prices, and must accept a change in markup given by
\[ d \log \mu_i^{NA} = -d \log mc_i \]

### 1.1.4 Government

The government provides input subsidies to firms, financing them through lump-sum taxes on consumers. It runs a balanced budget, so that the lump-sum transfer must equal total input subsidies:
\[ T = \sum_i \tau_i mc_i \]

### 1.1.5 Equilibrium

To reflect the consequences of pricing frictions, I define the competitive equilibrium by imposing market clearing for given sectoral markups (as in Baqae and Farhi (2017)). Since in my
setup markups are endogenously determined, I further need to impose that their evolution is consistent with the realization of productivity and monetary shocks.

For given output gap, sectoral probabilities of price adjustment $\delta_i$ and sectoral productivity shifters, general equilibrium is given by a vector of firm-level markups, a vector of prices $p_i$, a nominal wage $w$, labor supply $L$, a vector of sectoral outputs $y_i$, a matrix of intermediate input quantities $x_{ij}$, and a vector of final demands $c_i$, such that: a fraction $\delta_i$ of firms in each sector $i$ adjust their price; markups are optimally chosen by adjusting firms, while they are such that prices stay constant for the non-adjusting firms; consumers maximize utility subject to the budget and cash-in-advance constraint; producers in each sector $i$ minimize costs and charge the relevant markup; and markets for all goods and labor clear.

1.2 Definitions

In what follows, I work with a log-linearized model around the efficient equilibrium where productivity and money supply are equal to their expected value. The model is fully characterized by a set of equilibrium parameters, which capture the input-output structure and sector-level pricing frictions, and three variables: the output gap, the vector of sectoral inflation rates and the vector of sectoral productivity shifters. These variables and parameters are defined below.

1.2.1 Variables

1.2.1.1 Aggregate output gap

Definition 1. The aggregate output gap $\ddot{y}$ is the log-difference between realized output $y$ and efficient output $y_{nat}$:

$$\ddot{y} = y - y_{nat}$$

The output gap captures distortions in aggregate demand with respect to the natural
(efficient) equilibrium. Lemma 9 in Appendix A1 derives the change in natural output $y_{nat}$ in the efficient equilibrium. While I define the output gap in terms of final output, Lemma 10 in Appendix A1 implies that it can be equivalently expressed in terms of labor supply:

$$
\tilde{y} = d \log Y - d \log Y_{nat} = d \log L - d \log L_{nat}
$$

(1.2)

1.2.1.2 Sectoral inflation rates

The $N \times 1$ vector of inflation rates is denoted by

$$
\pi = \begin{pmatrix}
\pi_1 \\
\vdots \\
\pi_N
\end{pmatrix}
$$

Remark 1. While the output gap captures distortions in aggregate demand, Section 2.1.1 shows that the welfare cost of relative demand distortions across sectors (i.e. sectoral output gaps) can be written as a function of sectoral inflation rates (see Proposition 4).

1.2.2 Steady-state parameters

1.2.2.1 Price rigidity parameters

To model price rigidities, I assume that only a fraction $\delta_i$ of the firms in each sector $i$ can adjust their price after observing money supply and productivity. I collect these price adjustment parameters into a diagonal matrix $\Delta$.

Remark 2. The Calvo assumption, together with the firms’ optimal pricing equation (1.1), yields a mapping between inflation, marginal costs and markups. The fraction $\delta_i$ of firms in each sector $i$ who can adjust prices fully passes-through all changes in sectoral marginal costs $d \log mc_i$\(^2\) The remaining fraction $1-\delta_i$ is constrained to keep its price fixed, therefore it fully

\(^2\)Remember that desired markups are constant under the CES assumption (see Section 1.1.3).
absorbs any cost change into its markup. At the sector level, this implies a markup response equal to $d \log \mu_i = -(1 - \delta_i) d \log mc_i$, and a change in price given by $\pi_i = \delta_i d \log mc_i$. As a result, the following relation holds:

$$\pi = \Delta d \log mc = -\Delta (I - \Delta)^{-1} d \log \mu$$  \hspace{1cm} (1.3)$$

where $d \log mc$ is the vector of sectoral marginal cost changes, and $d \log \mu$ is the vector of sectoral markups.

Remark 3. Wage rigidities can be easily incorporated into this setup, by adding a labor sector which collects labor services and sells them to all the other sectors. While there still is a flexible underlying wage (the price paid by the labor sector), we define the market wage as the price charged by the labor sector, which is sticky.

1.2.2.2 Input-output definitions

The input-output structure is characterized by steady-state consumption, labor and input-output shares. We also introduce two useful derived objects, the Leontief inverse and the vector of sales shares, which are constructed from the input-output matrix and the vector of consumption shares.

Consumption shares  The $N \times 1$ vector $\beta$ denotes sectoral expenditure shares in total consumption, and has components

$$\beta_i = \frac{p_i c_i}{PC}$$

Labor shares  Sector-level labor shares in total sales are encoded in the $N \times 1$ vector $\alpha$, with components

$$\alpha_i = \frac{w L_i}{p_i y_i}$$
**Input-output matrix** The input-output matrix $\Omega$ is an $N \times N$ matrix, with elements $\omega_{i,j}$ given by the expenditure share on input $j$ in $i$’s total sales:

$$\omega_{i,j} = \frac{p_j x_{ij}}{p_i y_i}$$

**Leontief inverse** The Leontief inverse of the input-output matrix $\Omega$ is the matrix $(I - \Omega)^{-1}$.

While $\omega_{i,j}$ is the fraction of sector $i$ revenues directly spent on goods from sector $j$, the Leontief inverse captures the total (direct and indirect) expenditure of sector $i$ on goods from sector $j$ (again as a share of $i$’s revenues). The indirect component comes from the fact that $j$’s product can be embedded in $i$’s intermediate inputs, if $i$’s suppliers, or $i$’s suppliers’ suppliers, etc., use good $j$ in their production function. Formally, the Leontief inverse is given by the geometric sum:

$$(I - \Omega)^{-1} = I + \Omega + \Omega^2 + ...$$

where the $k$-th term captures the expenditure of $i$ on $j$ through paths of length $k$.

**Remark 4.** In my setup labor is the only factor of production. Therefore labor and intermediate input shares must sum to one in every sector:

$$\alpha_i + \sum_j \omega_{i,j} = 1$$

Denoting by $\mathbf{1}$ the column vector with all entries equal to 1, this yields the following relation between labor shares and the Leontief inverse:

$$(I - \Omega)^{-1} \alpha = \mathbf{1} \quad (1.4)$$

Given that labor is the only factor of production, Equation (1.4) tells us that the total (direct plus indirect) labor share must be equal to one in every sector. Total labor shares
are captured by the right hand side in Equation (1.4).

**“Adjusted” Leontief inverse**  I refer to the matrix \((I - \Omega \Delta)^{-1}\) as the “adjusted” Leontief inverse.

We established that the Leontief inverse captures the direct and indirect expenditure of sector \(i\) on goods from sector \(j\), as a share of \(i\)’s revenue. With flexible prices this is also the elasticity of \(i\)’s marginal cost to \(j\)’s marginal cost. With price rigidities this is no longer true, because marginal cost changes are not fully passed-through into prices. In this case the “direct” elasticity of \(i\)’s marginal cost with respect to \(j\)’s is \(\omega_{ij}\delta_j\), which discounts the input share \(\omega_{ij}\) by the fraction \(\delta_j\) of producers in \(j\) that adjust their price. Correspondingly, the total (direct plus indirect) elasticity of \(i\)’s marginal cost with respect to \(j\)’s is given by the \((i, j)\) element of the “adjusted” Leontief inverse.

**Sales shares**  The vector \(\lambda\) of sectoral sales shares in total GDP has components

\[
\lambda_i = \frac{P_iY_i}{P_C}
\]

It is a well known result that sales shares are related with consumption shares and the Leontief inverse as follows:

\[
\lambda^T = \beta^T (I - \Omega)^{-1}
\]

**Elasticities of substitution**  While the log-linearized model only depends on the input-output concepts defined above, the elasticities of substitution in production and consumption matter for the second-order welfare loss derived in Section 2.1. I denote the elasticity of substitution between varieties from sector \(i\) by \(\epsilon_i\), the elasticity of substitution between goods \(i\) and \(j\) in the production of good \(k\) by \(\theta^k_{ij}\), and their elasticity of substitution in consumption by \(\sigma^C_{ij}\); the elasticity of substitution between good \(i\) and labor in the production of good \(k\)
is denoted by $\theta_{iiL}^k$.

### 1.3 The Phillips curve

The Phillips curve is a linear relation between aggregate inflation $\pi^{AGG}$ and the output gap $\tilde{y}$. The standard New-Keynesian Phillips curve is given by (see for example Gali (2008)):

$$\pi_t^{AGG} = \rho \mathbb{E}_{t+1} \pi_{t+1}^{AGG} + \kappa \tilde{y}_t + u_t$$  \hspace{1cm} \text{(1.5)}

where $\mathbb{E}_{t+1} \pi_{t+1}^{AGG}$ is expected future inflation, $\kappa$ is the slope, $\rho$ is the discount factor and $u_t$ is a residual. In the main text I focus on a one-period model, where the Phillips curve has no forward-looking term:

$$\pi_t^{AGG} = \kappa \tilde{y}_t + u_t$$  \hspace{1cm} \text{(1.6)}

The slope $\kappa$ captures the percentage increase in prices when output raises by 1% above the efficient level. Intuitively, when output is above the efficient level labor demand also raises. This puts upwards pressure on wages, so that marginal costs and prices increase. The residual $u_t$ captures a time-varying wedge between fluctuations in output and prices. A key result in the one-sector model is that this wedge cannot arise endogenously from productivity shocks ($u_t \equiv 0$). This result is referred to as the “divine coincidence” (see Blanchard and Gali (2007)).

With multiple sectors there are several possible ways to define aggregate inflation, depending on the weighting of sectoral inflation rates. Accordingly, the slope and residual of the Phillips curve depend on the aggregate inflation index $\pi^{AGG}$ on the left-hand-side of (1.6). Two questions arise at this point. First, for a given aggregate inflation index, what are the parameters of the corresponding Phillips curve (slope and residual), and how do they depend on the production structure? Second, is there a special inflation index which inherits

---

\footnote{The dynamic version of the model is derived in Section \ref{sec:2.2.2}}
the “positive” properties of inflation in the one sector model, and in particular it preserves the “divine coincidence”?

To address the first question, in Section 1.3.1 I derive the elasticity of prices sector-by-sector with respect to the output gap and sectoral productivity shocks, as a function of production primitives. For a given index $\pi^{AGG}$ I then aggregate these elasticities into the slope and residual of the corresponding Phillips curve. In general, for a given indicator the slope varies with the production structure, and productivity shocks generate an endogenous residual. In particular, this is true of the consumer-price Phillips curve. The examples in Section 1.3.2 illustrate why this is the case. In Section 3.3.1 below I calibrate the parameters of the consumer-price Phillips curve to the US economy: the slope implied by the multi-sector model is one order of magnitude smaller than in a one-sector model with the same average price rigidity, and it is consistent with empirical estimates; the endogenous residual is very volatile, and significantly increases the R-squared when added into Phillips curve regressions (see Section 3.5.2).

The second question is addressed in Section 1.3.3. Here I construct an inflation index (the “divine coincidence” index) whose corresponding Phillips curve is well-specified, with no endogenous residual and a slope which does not depend on the production structure. I show that this index is unique.

1.3.1 General expression for a given inflation index

1.3.1.1 Notation and aggregation

I first derive the response of prices to productivity and monetary shocks at the sector level, and then combine them into aggregate Phillips curves. All changes in productivity, marginal costs and prices are relative to the flex-price equilibrium where productivity and money supply are equal to their expected value.
Sectoral prices respond to marginal cost shocks according to equation (1.3):

$$\pi = \Delta d\log mc$$

(1.7)

The change in sector-level prices is proportional to the change in marginal costs, times the fraction of adjusting firms. Intuitively, firms would like to fully pass-through changes in marginal costs into their prices, but only a fraction $\Delta$ of them has the opportunity to do so.

In turn marginal costs depend on wages and productivity, either directly, or indirectly through input prices. While productivity shocks are exogenous, wage changes are determined as an equilibrium result of productivity and monetary shocks. Propositions 1 and 2 below solve for the marginal cost change in (1.7) as a function of these underlying shocks. Here monetary shocks are captured by the output gap, which is related one-to-one with the money supply (see Remark 6).

This allows us to express the vector $\pi$ of sector-level inflation rates as a function of productivity shocks ($d\log A$) and the output gap ($\bar{y}$):

$$\pi \underbrace{\frac{N \times 1}{N \times 1}}_{\mathcal{B}} = \underbrace{\mathcal{B}}_{N \times 1} \bar{y} + \underbrace{\mathcal{V}}_{N \times N} \underbrace{d\log A}_{N \times 1}$$

(1.8)

Here I denote by $\mathcal{B}$ the $N \times 1$ vector whose components $B_i$ are the elasticities of sector $i$’s price with respect to the output gap, and by $\mathcal{V}$ the $N \times N$ matrix whose elements $V_{ij}$ are the elasticities of sector $i$’s price with respect to a productivity shock to sector $j$. The elasticities $\mathcal{B}$ and $\mathcal{V}$ are derived in Propositions 1 and 2.

For a given inflation index, the corresponding Phillips curve is obtained by aggregating both sides of Equation (1.8). An inflation index $\pi^{AGG}$ is characterized by the vector of weights $\phi$ that it assigns to sectoral inflation rates:

$$\pi^{AGG} \equiv \phi^T \pi = \sum_i \phi_i \pi_i$$
Weighting both sides of Equation (1.8) according to \( \phi \) we obtain the Phillips curve:

\[
\pi^{AGG} = \phi^T B \bar{y} + \phi^T \mathcal{V} d \log A
\]

(1.9)

The slope is the aggregate elasticity with respect to the output gap, while the residual is the aggregate elasticity with respect to productivity.

Consumer inflation \( \pi^C \) is a special case, obtained by weighting sectoral inflation rates according to consumption shares \( (\phi = \beta) \). Therefore the slope and residual of the corresponding Phillips curve are given by

\[
\kappa^C = \beta^T B
\]

\[
u^C = \beta^T \mathcal{V} d \log A
\]

Sections 1.3.1.2 and 1.3.1.3 below characterize the elasticities \( B \) and \( V \), and derive the slope and residual of the consumer-price Phillips curve as a corollary.

### 1.3.1.2 Slope of the Phillips curve

Proposition 1 derives the elasticities of prices with respect to the output gap sector-by-sector. Corollary 1 aggregates them into the slope of the consumer-price Phillips curve.

**Proposition 1.** The elasticity of sectoral prices with respect to the output gap is

\[
B = \frac{\Delta (I - \Omega \Delta)^{-1} \alpha}{1 - \bar{\delta}_w} (\gamma + \varphi)
\]

(1.10)

where

\[
\bar{\delta}_w \equiv \beta^T \Delta (I - \Omega \Delta)^{-1} \alpha
\]

(1.11)

is the pass-through of nominal wages into consumer prices.

---

4Propositions 1 and 2 can be seen as an application of Proposition 10 in Baqaee and Farhi (2017), recast in terms of sectoral probabilities of price adjustment and in the special case of an efficient initial equilibrium.
Proof. See Appendix A2

**Corollary 1.** The slope of the consumer-price Phillips curve is given by

\[ \kappa^C = \frac{\bar{\delta}_w}{1 - \delta_w} (\gamma + \varphi) \]  

(1.12)

**Proof.** The result follows immediately from Proposition 1 and Equation 1.9

The vector \( \mathbf{B} \) and the slope \( \kappa^C \) are the elasticities of sectoral and consumer prices with respect to the output gap. Intuitively, if output is above potential then labor demand must increase. This puts upwards pressure on wages and prices. The first component on the right hand side of (1.10) and (1.12) is the effect on real wages. It is governed by the parameters of the labor supply curve (\( \gamma \) and \( \varphi \)), and does not depend on the production structure. The pass-through of real wages into prices instead depends on the network structure. It is captured by the second component in (1.10) and (1.12).

The real wage pass-through can be further decomposed into the nominal wage pass-through (given by \( \Delta \left( (I - \Omega \Delta)^{-1} \alpha \right) \) for sectoral inflation rates and by \( \bar{\delta}_w \) for consumer prices) and a general equilibrium multiplier \( 1 - \bar{\delta}_w \) (which maps changes in real wages into changes in nominal wages, through the equilibrium response of consumer prices).

The nominal wage pass-through \( \bar{\delta}_w \) is the key object. With no intermediate inputs (\( \Omega = \emptyset \), \( \alpha = 1 \)), as in benchmark model, marginal costs have unit elasticity with respect to wages. From Equation (1.3), the price pass-through is simply given by the adjustment frequency \( \text{diag}(\Delta) \), and we have \( \bar{\delta}_w = E_\beta(\delta) \). With input-output linkages instead this pass-through is dampened, as stated in Corollary 2.

**Corollary 2.** As long as some sector uses an intermediate input with sticky prices, the pass-through of wages into marginal costs is less than one:

\[ \exists i, j \text{ such that } \omega_{ij} \delta_j < \omega_{ij} \Rightarrow (I - \Omega \Delta)^{-1} \alpha < 1 \]  

(1.13)
As a result, sectoral price pass-throughs are smaller than the corresponding adjustment frequencies, and the aggregate price pass-through $\bar{\delta}_w$ is less than the average price rigidity $E_\beta(\delta)$:

$$\exists i, j \text{ such that } \omega_{ij}\delta_j < \omega_{ij} \Rightarrow \begin{cases} \Delta ((I - \Omega\Delta)^{-1} \alpha) < diag(\Delta) \\ \bar{\delta}_w < E_\beta(\delta) \end{cases} \quad (1.14)$$

A reduction in labor shares compensated by a uniform increase in input shares also reduces $\bar{\delta}_w$:

$$d\alpha_i < 0, \ d\omega_{ij} = d\omega_{ik} \ \forall j, k, \ \exists j \text{ such that } \omega_{ij}\delta_j < \omega_{ij} \Rightarrow d\bar{\delta}_w < 0 \quad (1.15)$$

Proof. See Appendix A2.

The intuition is as follows. Marginal costs are affected by wages directly, or indirectly through input prices. The direct exposure depends on own labor shares, while the indirect exposure depends on the suppliers’ labor share, the suppliers’ suppliers labor share, etc. Incomplete price adjustment dampens the indirect component of the pass-through, as stated in Corollary 2. Formally, the propagation along the network is characterized by the “adjusted” Leontief inverse $(I - \Omega\Delta)^{-1}$, which discounts sectors by the fraction $\Delta$ of adjusting firms. To compute the effect of wage shocks on marginal costs we multiply this matrix times the vector $\alpha$ of steady-state labor shares, which captures the “impulse” component:

$$\frac{d\log mc}{d\log w} = (I - \Omega\Delta)^{-1} \alpha \quad (1.16)$$

We can then use the pricing equation (1.13) to translate changes in marginal costs into inflation rates:

$$\frac{d\log p}{d\log w} = \Delta \frac{d\log mc}{d\log w} = \Delta (I - \Omega\Delta)^{-1} \alpha \quad (1.17)$$

This yields the pass-through of nominal wages into sectoral inflation rates in equation (1.10). Note that different sectors have different pass-through. The pass-through is higher in sectors with a large direct labor share and flexible prices, whose suppliers have a large direct labor.
share and flexible prices, etc.

To obtain the pass-through into consumer prices \( \delta_w \) we simply aggregate the sectoral responses in (1.17) according to consumption shares:

\[
\delta_w = \beta^T \frac{d \log p}{d \log w} = \beta^T (I - \Omega \Delta)^{-1} \alpha
\]  

(1.18)

Corollary 2 implies that in the presence of intermediate inputs \( \delta_w \) is smaller than the average price rigidity. This in turn lowers the slope of the consumer-price Phillips curve:

\[
\kappa^C = (\gamma + \varphi) \frac{\delta_w}{1 - \delta_w} < (\gamma + \varphi) \frac{E_\beta (\delta)}{1 - E_\beta (\delta)}
\]  

(1.19)

The right hand side of Equation (1.19) is the slope predicted by standard calibrations, which directly map the one-sector model into the data without accounting for input-output linkages. Quantitatively, the difference between the left and right hand sides of Equation (1.19) is important. Section 3.3 evaluates it for the US economy, finding that the left hand side is one order of magnitude smaller (\( \sim 0.1 \) against \( \sim 1 \)).

1.3.1.3 Endogenous cost-push shocks

Proposition 2 derives the elasticities of sectoral prices with respect to productivity shocks. Corollary 3 aggregates them into the endogenous residual of the consumer-price Phillips curve.

**Proposition 2.** The elasticity of sectoral prices with respect to productivity shocks is given by

\[
V = \Delta (I - \Omega \Delta)^{-1} \left[ \frac{\alpha \lambda^T - \beta^T (I - \Omega \Delta)^{-1} A}{1 - \delta_w} \right] - I
\]  

(1.20)

so that

\[
V d \log A = \Delta (I - \Omega \Delta)^{-1} \left[ \frac{1 - \delta_A}{1 - \delta_w} \alpha \lambda^T - I \right] d \log A
\]  

(1.21)
where
\[ \tilde{\delta}_A (d \log A) \equiv \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} d \log A}{\lambda^T d \log A} \quad (1.22) \]

is the pass-through of the productivity shock into consumer prices, relative to the aggregate shock.

**Proof.** See Appendix A2

**Corollary 3.** The residual in the consumer-price Phillips curve is given by
\[ u^C = \frac{\delta_w - \tilde{\delta}_A \lambda^T d \log A}{1 - \delta_w} \quad (1.23) \]

**Proof.** See Appendix A2

The elasticities derived in Proposition 2 reflect two competing effects of productivity shocks on marginal costs. The first is a direct effect: if aggregate productivity falls, marginal costs increase. Productivity shocks also have an indirect effect, through a real wage adjustment. In the efficient equilibrium the real wage is equal to the marginal product of labor, given by aggregate productivity \( \lambda^T d \log A \). When output is at the efficient level (\( \bar{y} = 0 \)) real wages must be the same as in the efficient equilibrium.\(^5\) Thus after a negative shock (\( \lambda^T d \log A < 0 \)) the fall in wages counterbalances the effect of productivity on marginal costs.

In the one sector model these two forces exactly offset each other, because changes in wages and productivity have the same effect on marginal costs.\(^6\) As a result, when output is at the efficient level marginal costs and prices are also stabilized. This is the key intuition behind the “divine coincidence”.

This is no longer true with multiple sectors, because sectors are asymmetrically exposed to wages and productivity. Formally, the direct effect of productivity on sectoral prices is

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\(^5\)This result is derived in the proof of Proposition 2

\(^6\)They have unit elasticity with respect to both.
given by the second term in (1.21):

\[
direct \text{ component} = -\Delta (I - \Omega \Delta)^{-1} d\log A
\]

The propagation of the shock into marginal costs is captured by the adjusted Leontief inverse, following the same intuition as in Section 1.3.1.2. The price response is obtained by multiplying the change in marginal costs times the adjustment probability \(\Delta\), according to the pricing Equation (1.17).

The indirect effect through wages is given by the first term in Equation (1.21):

\[
wage \text{ component} = \Delta (I - \Omega \Delta)^{-1} \alpha \left( \frac{1 - \delta_A^A}{1 - \delta_w^w} \right) \lambda^T d\log A
\]

The term \(\lambda^T d\log A\) is the change in real wages, which equals the aggregate productivity shock. The general equilibrium multiplier \(\frac{1 - \delta_A^A}{1 - \delta_w^w}\) maps real wages into nominal wages. The term \(\Delta (I - \Omega \Delta)^{-1} \alpha\) is the pass-through of nominal wages into sectoral prices, which we derived in Section 1.3.1.2. Note that, while the direct component depends on the full distribution of sectoral productivity shocks, the wage component only depends on the aggregate shock.

At the sector level the wage and productivity pass-through are different in general. Section 1.3.2 provides illustrative examples. As a result inflation is not stabilized sector-by-sector, even if the output gap is closed. Corollary 3 shows that consumer inflation is not stabilized either. Its response depends on the relative pass-through of wages and productivity into consumer prices, given by the difference \(\delta_w^A - \delta_A^A\).

The productivity pass-through \(\delta_A^A\) is defined in Equation (1.22), mirroring the wage pass-through \(\delta_w^w\) introduced in Section 1.3.1.2. Note that \(\delta_A^A\) is scaled by the aggregate shock, and it depends on the full distribution of sectoral productivity shocks (while \(\delta_w^w\) is a constant). From Equation (1.23) we see that for a negative shock \((\lambda^T d\log A < 0)\) consumer inflation
is positive if and only if the productivity pass-through is larger than the wage pass-through ($\delta_A > \delta_w$). This is the case whenever downstream or flexible sectors are hit by a “worse” shock than the average, as the examples in Section 1.3.2 illustrate.

A natural question at this point is whether there are shocks after which prices are stabilized sector-by-sector under zero output gap. Corollary 4 shows that the only shock with this property is an aggregate labor augmenting shock, which in this setup is equivalent to a TFP shock proportional to sectoral labor shares $\alpha$.

**Corollary 4.** It holds that $V\alpha = 0$, and $\alpha$ is the only vector with this property.

**Proof.** See Appendix A2

A consequence of Corollary 4 is that perfect stabilization is impossible not only in the presence of asymmetric sector-level shocks, but also after an aggregate TFP shock - except in the horizontal economy where aggregate TFP shocks and labor augmenting shocks coincide. Quantitatively, aggregate TFP shocks generate a significant inflation-output tradeoff. In the calibrated model a 1% negative shock increases consumer inflation by 0.26% under zero output gap.

### 1.3.2 Examples

The three examples in this section illustrate the main channels through which the “divine coincidence” fails in the multi-sector model. The vertical chain isolates the effect of input-output linkages, while the horizontal economy highlights the role of heterogeneous adjustment frequencies and idiosyncratic shocks. The oil economy combines the two. This last example rationalizes the common wisdom that oil shocks generate a tradeoff between stabilizing output and stabilizing consumer prices (they create an endogenous “cost-push” term in the Phillips curve). The Example highlights the crucial role of wage rigidities and heterogeneous adjustment probabilities in generating this outcome.

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7 That is, an economy without intermediate inputs.
Example 1. Vertical chain

Consider the economy represented in Figure 1. This economy is made of two sectors, which we label $U$ (for “upstream”) and $D$ (for “downstream”). Both sectors use labor, and $D$ also uses $U$ as an intermediate input. Only $D$ sells to final consumers.

Figure 1.1: Vertical production chain

To establish whether the “divine coincidence” holds in this economy we need to check if consumer prices are stabilized under zero output gap for all possible productivity shocks. I show that this is not the case.

In a vertical chain consumer prices coincide with the price of the downstream sector $D$, because this is the only consumption good. Let’s start by considering a negative productivity shock to $D$, $d \log A_D < 0$. The elasticities $\mathcal{V}_{UD}$ and $\mathcal{V}_{DD}$ of $U$ and $D$’s prices with respect to this productivity shock can be derived from Proposition 2. The corresponding price responses
are given by $\mathcal{V}_{UD} d \log A_D$ and $\mathcal{V}_{DD} d \log A_D$:

$$\mathcal{V}_{UD} d \log A_D = \frac{\delta_U}{1 - \delta_D} d \log A_D < 0$$ (1.25)

$$\mathcal{V}_{DD} d \log A_D = \left[ \frac{\delta_w}{1 - \delta_D} - \frac{\delta_D}{1 - \delta_w} \right] d \log A_D > 0$$ (1.26)

where

$$\delta_w = \delta_D \left( \frac{\text{direct pass-through through } U}{\alpha_D} + (1 - \alpha_D) \delta_U \right)$$

Equation (1.25) tells us that under zero output gap prices fall in the upstream sector. Here is the intuition. In the flex-price equilibrium real wages fall to exactly compensate the change in $D$’s productivity. Under zero output gap, the same change in real wages needs to occur in the flex price economy. As a result marginal costs and prices fall in the upstream sector $U$, because this sector had no productivity shock and is facing lower wages.

From Equation (1.26) instead we see that inflation is positive in the downstream sector $D$. In this sector the drop in productivity leads to higher marginal costs, but at the same time wages and input costs fall. As long as there is some price stickiness in $U$ the productivity effect dominates, because input prices do not fully reflect the change in wages. The overall wage pass-through is given by

$$\alpha_D + (1 - \alpha_D) \delta_U < 1$$

The inequality tells us that the fall in wages and input costs is not enough to compensate the fall in $D$’s productivity, and producers want to increase their price. Consumer inflation is entirely determined by the downstream sector, therefore it is positive under zero output gap.
In this example the “divine coincidence” fails because of input-output linkages. This is not merely a result of the asymmetric nature of the shock (it hits only one sector): using Proposition 2 it is immediate to show that after an aggregate Hicks-neutral shock inflation is not stabilized either. The issue is that consumer inflation focuses only on the last stage of the production chain. In our example it would be possible to weight sectoral inflation rates in such a way that the average is stabilized, by giving some weight to upstream prices. Proposition 3 below shows that this is a general result, and the correct sectoral weights do not depend on the underlying productivity shock.

Example 2. Horizontal economy

Consider the horizontal economy in Figure 2: there are $N$ sectors, \{1, ..., $N$\}, with consumption shares $\beta_1, ..., \beta_N$ and adjustment probabilities $\delta_1, ..., \delta_N$. There are no input-output linkages, but sectors face idiosyncratic shocks and heterogeneous pricing frictions.

![Figure 1.2: Horizontal economy](image)

Under zero output gap wages adjust to reflect the “average” change in productivity $\mathbb{E}_\beta (d \log A)$. Sectors are equally exposed to wage changes, but they face different productivity shocks, therefore marginal costs and prices cannot be stabilized everywhere. From
Proposition 2: inflation in each sector \( i \) satisfies

\[
\pi_i = \delta_i \left( \frac{1 - \bar{\delta}_A}{1 - \bar{\delta}_w} \mathbb{E}_\beta (d \log A) \right) - d \log A_i
\]

where

\[
\bar{\delta}_w = \mathbb{E}_\beta (\delta)
\]

\[
\bar{\delta}_A = \frac{\mathbb{E}_\beta (\delta d \log A)}{\mathbb{E}_\beta (d \log A)}
\]

We see from (1.27) that inflation increases in sectors which received a worse shock than the average \( d \log A_i < \frac{1 - \bar{\delta}_A}{1 - \bar{\delta}_w} \mathbb{E}_\beta (d \log A) \), and vice versa.

Consumer inflation is not stabilized either. Equation (1.28) tells us that it is negative if productivity increased more in flexible sectors, and vice versa:

\[
\pi^C = - \frac{\text{Cov}_\beta (\delta, d \log A)}{1 - \mathbb{E}_\beta (\delta)}
\]

In other words, consumer inflation overrepresent flexible sectors. As in the vertical chain, it would be possible to weight sectoral inflation rates in such a way that the average is stabilized. In the horizontal economy this can be achieved by discounting flexible sectors. Proposition 3 shows that this is always true, and the correct sectoral weights do not depend on the underlying productivity shock.

Example 3. Oil shocks and consumer inflation

This example presents a stylized “oil economy”, where negative oil shocks increase consumer prices under zero output gap. Section 3 evaluates the quantitative importance of the channels highlighted here for the US economy. It finds that a 10% negative shock raises consumer prices by 0.22% under zero output gap.
Consider a production network which combines a vertical chain and a horizontal economy, as in Figure 1.3. Labor is the first stage, with sticky wages. Then comes oil, and finally the last stage is broken down into multiple sectors. These sectors have heterogeneous consumption shares ($\beta_i$), oil input shares ($\omega_{i,oil}$) and adjustment frequencies ($\delta_i$).

We can interpret our economy as a vertical chain where the upstream sector is labor, with sticky wages. Since oil prices are very flexible, oil shocks are (almost) fully passed-through to the final goods sector. Oil shocks therefore act like a downstream productivity shock, in spite of the role of oil as an intermediate input.

Two channels determine the response of consumer inflation. First, we know from Example 1 that consumer prices increase in response to negative downstream shocks. Second, oil input shares and adjustment frequencies are positively correlated in the data. Consistent with the intuition from the horizontal economy in Example 2, this further increases the pressure on consumer prices.

Formally, if $\delta_{oil} = 1$ and workers adjust wages with probability $\delta_L$, under zero output gap

---

8See Remark 3 in Section 1.2.2.1 for a discussion of how I model wage rigidities.
consumer inflation is given by

\[
\pi^C = -\frac{\text{horizontal} \, \text{Cov}_\beta (\delta, \omega_{oil}) + (1 - \delta_L) \, \mathbb{E}_\beta (\delta) \, \mathbb{E}_\beta (\omega_{oil})}{1 - \delta_L \, \mathbb{E}_\beta (\delta)} \, d \log A_{oil}
\] (1.29)

It is immediate to see from (1.29) that, for \( d \log A_{oil} < 0 \), consumer inflation \( \pi_C \) increases with wage stickiness \( 1 - \delta_L \) and with the covariance between oil shares and adjustment frequencies.

Table 3.2 in Section 3.4.1 reports the calibrated response of inflation to an oil shock in the US network, under different assumptions about price and wage rigidity. The results show that, even if the full network is much more complex than the economy in this example, our simple model captures well the mechanisms at play.

### 1.3.3 The “divine coincidence” inflation index

Section 1.3.1.3 shows that the consumer-price Phillips curve is misspecified: the slope changes with the input-output structure, and productivity fluctuations result in endogenous cost-push shocks. Proposition 3 constructs an inflation index which fixes both of these issues.

**Proposition 3.** Assume that no sector has fully rigid prices \((\delta_i \neq 0 \, \forall i)\). Then the sales-weighted inflation statistic

\[
DC = \lambda^T (I - \Delta) \, \Delta^{-1} \pi
\]

satisfies

\[
DC = (\gamma + \varphi) \, \bar{y}
\] (1.30)

Moreover, unless prices are fully flexible in all sectors \((\Delta = I)\), \( DC \) is the only aggregate inflation statistic that yields a Phillips curve with no endogenous residual.

**Proof.** Lemma 1 states that the output gap is proportional to a notion of “aggregate” markup, that weights sector level markups according to sales shares.
**Lemma 1.** The following relation holds between the output gap and sector-level markups:

\[(\gamma + \varphi) \bar{y} = -\lambda^T d \log \mu \quad (1.31)\]

*Proof.* See Appendix A3. \(\Box\)

The pricing equation \((1.1)\) allows to infer markup changes from inflation rates and price adjustment probabilities:

\[-d \log \mu = (I - \Delta) \Delta^{-1} \pi \quad (1.32)\]

Together, Equations \((1.31)\) and \((1.32)\) yield the sales-weighted Phillips curve:

\[\lambda^T (I - \Delta) \Delta^{-1} \pi = -\lambda^T d \log \mu = (\gamma + \varphi) \bar{y}\]

Finally, Lemma 2 implies that \(DC = \lambda^T (I - \Delta) \Delta^{-1} \pi\) is the only aggregate inflation statistic such that the corresponding Phillips curve has no endogenous residual.

**Lemma 2.** If \(\Delta \neq I\) then \(\lambda^T (I - \Delta) \Delta^{-1} \pi\) is the only vector \(\nu\) that satisfies

\[\nu^T \nu = 0\]

*Proof.* See appendix A3. \(\Box\)

**Remark 5.** The weights in \(DC\) are all positive. Therefore we can have \(\lambda^T (I - \Delta) \Delta^{-1} \pi = 0\) only if \(\pi_i\) is positive in some sectors and negative in others. In other words, under zero output gap there are always sectors where inflation is positive and sectors where it is negative.\(^9\)

\(^9\)We know from Lemma 1 that in general \(\pi_i\) cannot be zero in every sector.
Examples 1 and 2 show that consumer prices are not stabilized under zero output gap. Nonetheless, marginal costs move in different directions across sectors (some are more affected by the productivity shock, while others are more affected by the equilibrium wage response). Therefore we can expect an appropriate “average” to be stabilized. Remark 5 extends this intuition to the general case. The weighting in consumer prices is not the correct one because it does not capture the contribution of upstream sectors to value added, and it fails to account for the fact that flexible sectors respond more to a given cost shock. Building on this intuition, the “divine coincidence” index constructed in Proposition 3 weights sectors according to sales shares and discounts more flexible sectors.

Formally, the proof of Proposition 3 proceeds in two steps. First, Lemma 1 shows that the output gap is inversely proportional to a sales-weighted sum of sectoral markups.\(^{10}\) Intuitively, when markups are high aggregate demand is low, resulting in a negative output gap. Second, Remark 2 allows to infer changes in sector-level markups from inflation rates, by appropriately discounting them for the relevant adjustment probabilities. As explained in the Remark, changes in prices and changes in markups are driven by the same cost shocks. Producers who can adjust their price fully pass-through the shock, and this is reflected in inflation rates. Those who cannot adjust instead absorb the cost change into their markup. For a given shock inflation is higher in flexible sectors, and the change in average markup is smaller. Therefore these sectors need to be discounted.

The two steps in Proposition 3 are sensitive to some key assumptions. First, sectoral weights (here sales shares) depend on the fact that we are approximating around an efficient steady state. Second, while the relation between the output gap and markups derived in Lemma 1 does not rely on the specific pricing assumptions (ex. Calvo), the mapping between markups and inflation rates in equation (1.32) does depend on the Calvo assumption and on the CES demand structure within sectors.\(^{11}\) Nonetheless, the Calvo-CES model provides

---

\(^{10}\)This argument is closely related to Proposition 3 in Baqee and Farhi (2017).

\(^{11}\)Crucially, in the Calvo-CES framework the wedge between changes in prices and markups is exogenous and constant (it is given by \((I - \Delta) \Delta^{-1}\)). This is no longer true under different pricing models, either because the share of adjusting firms is endogenous (as for example in menu cost models), or because the desired
a useful structure to identify the forces at play in more complex setups, and the empirical results in Section 3.5 show that the “divine coincidence” index based on this model provides a much better fit in Phillips curve regressions than consumer price inflation.

pass-through from marginal costs into prices is endogenous (this happens with fixed menu costs, variable adjustment costs or non-CES demand). In general there is no closed form solution for this endogenous wedge.
Chapter 2

Normative analysis

2.1 Welfare function and optimal policy

The presence of pricing frictions determines three types of distortions. First, distortions in aggregate output correspond to a wedge in the optimal consumption-leisure tradeoff. This is captured by the output gap. Second, distortions in the relative output of different varieties from the same sector arise because adjusting and non-adjusting firms charge different prices, even though they face the same marginal cost. Customers inefficiently substitute towards the cheaper varieties. Third, relative output distortions across sectors arise whenever sectoral prices do not fully adjust to reflect their relative productivities. These three channels are captured by the welfare function derived in Proposition 4.

In the one-sector benchmark there are no cross-sector distortions. The “divine coincidence” implies that stabilizing aggregate output also eliminates within-sector distortions, thereby replicating the efficient allocation. This result no longer holds in the multi-sector model. Monetary policy has one instrument (money supply or interest rates) to address all three distortions. As a result, it cannot replicate the first-best. Even though the “divine

\[1\] The second and third channel are conceptually the same. If we considered a fully disaggregated model, where sectors are identified with individual firms, they could be unified into the cross-sector component. For expositional purposes however it is useful to keep them distinct, to facilitate the comparison with the one-sector benchmark.
coincidence” inflation index is stabilized together with aggregate output, inflation is not stabilized sector-by-sector, and relative prices within and across sectors are distorted. In this sense the “divine coincidence” fails from a normative point of view.

Specifically, targeting the “divine coincidence” index replicates the efficient aggregate output, but it ignores relative price distortions. Section 2.1.2 characterizes the optimal monetary policy response to this tradeoff. Section 2.1.3 shows that the optimal policy can still be implemented by stabilizing an appropriate inflation index, which trades off the “divine coincidence” index against an inflation statistic that captures the effect of monetary policy on relative price distortions.

Remark 6. I derive optimal policy in terms of the aggregate output gap, even though the actual policy instrument is money supply. I can do this because there is a one-to-one mapping between the two, which can be derived from the consumer-price Phillips curve and the cash-in-advance constraint:

$$d \log M = \pi_C + y = \pi_C + \tilde{y} + \frac{1 + \varphi}{\gamma + \varphi} \lambda^T d \log A =$$

$$= (1 + \kappa C) \tilde{y} + u C + \frac{1 + \varphi}{\gamma + \varphi} \lambda^T d \log A$$

2.1.1 Welfare function

Proposition 4 derives a second-order approximation of the welfare loss relative to the efficient equilibrium with flexible prices. The loss function is quadratic in the output gap (which captures distortions in aggregate output) and inflation (which is associated with distortions in relative output within and across sectors).

---

2 Corollary 4 shows that perfect stabilization can be achieved only after an aggregate labor augmenting shock.

3 Interestingly, the loss function does not depend on sectoral productivity shocks directly. Intuitively, misallocation is determined by markup distortions. I derive the welfare function around an efficient steady-state, therefore there is no interaction between the productivity shock and initial misallocation (the envelope
Proposition 4. The second-order welfare loss with respect to the flex-price efficient outcome is

\[ \mathcal{W} = \frac{1}{2} \left[ (\gamma + \varphi) \bar{\theta}^2 + \pi^T \mathcal{D} \pi \right] \] (2.1)

The matrix \( \mathcal{D} \) can be decomposed as \( \mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2 \), where \( \mathcal{D}_1 \) captures the productivity loss from within-sector misallocation and \( \mathcal{D}_2 \) captures the productivity loss from cross-sector misallocation. \( \mathcal{D}_1 \) is diagonal with elements

\[ d_{ii}^1 = \lambda_i \epsilon_i \frac{1 - \delta_i}{\delta_i} \] (2.2)

\( \mathcal{D}_2 \) is positive semidefinite. It can be written as a function of the substitution operators in production and consumption defined below.

Definition 2. The substitution operators \( \Phi_t \) (for sector \( t \)) and \( \Phi_C \) (for final consumption) are symmetric operators from \( \mathbb{R}^N \times \mathbb{R}^N \) to \( \mathbb{R} \), defined as

\[
\Phi_t (X, Y) = \frac{1}{2} \sum_k \sum_h \omega_{tk} \omega_{th} \theta_{kh} (X_k - X_h) (Y_k - Y_h) + \\
\alpha_k \sum_k \omega_{tk} \theta_{kL} X_k Y_k
\]

and

\[
\Phi_C (X, Y) = \frac{1}{2} \sum_k \sum_h \beta_k \beta_h \sigma_{kh}^C (X_k - X_h) (Y_k - Y_h)
\]

The elements of \( \mathcal{D}_2 \) are given by

\[ d_{ij}^2 = \frac{1 - \delta_i}{\delta_i} \frac{1 - \delta_j}{\delta_j} \left( \Phi_C \left((I - \Omega)^{-1}_{(i)} , (I - \Omega)^{-1}_{(j)} \right) + \sum_t \lambda_t \Phi_t \left((I - \Omega)^{-1}_{(i)} , (I - \Omega)^{-1}_{(j)} \right) \right) \] (2.3)

Theorem holds). The welfare loss is entirely driven by the change in markups induced by the shock, which we can infer from sectoral inflation rates (see equation (1.32)).

\( \Phi_C \) and \( \Phi_t \) are the same as in Baqee and Farhi (2018). They apply these operators to sector-level price changes and labor shares around a distorted steady-state, to derive the first-order change in allocative efficiency. I work around an efficient steady-state where markup shocks have no first-order effect on allocative efficiency, while the substitution operators applied to sector-level price changes characterize the second-order loss.
Proof. See appendix B1

In the baseline one-sector model the welfare loss is given by

\[ W = \frac{1}{2} \left[ (\gamma + \varphi) \hat{y}^2 + \epsilon \frac{1-\delta}{\delta} \pi^2 \right] \tag{2.4} \]

Again the loss is quadratic in inflation and the output gap, but here inflation only captures within-sector distortions. For a given price distortion, quantities respond more if the elasticity of substitution \( \epsilon \) is higher. Therefore the welfare cost in Equation (2.4) is increasing in \( \epsilon \). In the network model instead the welfare loss associated with inflation comes from both cross-sector and within-sector price distortions, and the latter need to be appropriately aggregated.

From equation (2.2) we see that the price dispersion loss within each sector is \( \epsilon_i \pi_i^2 \), the same as in the one-sector model. Sector-level losses are then aggregated by sales shares, discounting flexible sectors. The intuition is the same as for the “divine coincidence” index in Proposition 3. Overall, the within-sector component of the total welfare loss is given by

\[ \pi^T D_1 \pi = \sum_i \lambda_i \frac{1-\delta_i}{\delta_i} \epsilon_i \pi_i^2 \]

The welfare loss from cross-sector misallocation in Equation (2.3) can be expressed as a weighted sum of negative sector-level productivity shocks:

\[ \pi^T D_2 \pi = \sum_{t} \lambda_t \sum_{i,j} \Phi_t(i,j) \tag{2.5} \]

Here we treated final consumption as an additional sector with \( \lambda_C = 1 \), and with some abuse
of notation we defined

$$
\Phi_t(i, j) \equiv \Phi_t \left( (I - \Omega)_{(i)}^{-1} \frac{1 - \delta_i}{\delta_i} \pi_i, (I - \Omega)_{(j)}^{-1} \frac{1 - \delta_j}{\delta_j} \pi_j \right)
$$

Intuitively, relative price distortions induce producers in each sector $t$ to substitute towards the inputs whose relative price is lower than in the efficient equilibrium. The welfare consequence of this misallocation is equivalent to a negative TFP shock for sector $t$. The total loss is obtained by aggregating sector-level contributions according to sales shares, as in Hulten’s formula.

To derive the productivity loss for each sector $t$ we proceed in two steps. First, we isolate the distortionary component of sectoral inflation rates and track its propagation across the network, which results in relative price distortions across $t$’s inputs. Second, we translate price distortions into $t$’s productivity loss, resulting from inefficient substitution. This effect is captured by the substitution operators.

Let’s start with the first step. Intuitively, inflation is associated with a distortion because it mirrors an inefficient change in the markup of non-adjusting firms. We want to map inflation rates into markup distortions, and study their propagation through the network into the relative price of $t$’s inputs. I define relative prices with respect to nominal wages. Lemma 3 provides the mapping between inflation rates and relative price distortions.

**Lemma 3.** The distortion in sectoral relative prices with respect to the flex-price outcome is given by

$$
d \log p - d \log w = (I - \Omega)^{-1} (I - \Delta) \Delta^{-1} \pi
$$

**Proof.** See Appendix B1

From Equation (2.6) we see that relative price distortions can be decomposed into a direct
and a propagation effect:

\[
d \log p - d \log w = (I - \Omega)^{-1} (I - \Delta) \Delta^{-1} \mu
\]

Here is the intuition. A distortion in the relative price of a sector \( k \) can come either directly from a change in \( k \)'s markup, or indirectly from a change in the markup of some of its inputs. The Leontief inverse \( (I - \Omega)^{-1} \) captures the propagation of these markup shocks: the price distortion induced in sector \( k \) by a change in \( i \)'s markup is given by \( (I - \Omega)_{ik}^{-1} d \log \mu_i \). From Remark 2, we can then express markup shocks as a function of inflation rates:

\[
-d \log \mu = (I - \Delta) \Delta^{-1} \pi
\]

Using Lemma 3, we can also derive the relative price distortion between each sector pair \((k, h)\) triggered by inflation in sector \( i \). This is given by

\[
d \log p_k - d \log p_h = \left( (I - \Omega)^{-1}_{ki} - (I - \Omega)^{-1}_{hi} \right) \frac{1 - \delta_i}{\delta_i} \pi_i
\]

This allows us to characterize the relative price distortions across \( t \)'s inputs associated with given sectoral inflation rates. The negative effect on \( t \)'s productivity is captured by the corresponding substitution operator \( \Phi_t \) (see Definition 2), and it depends on the interaction between inflation in different sectors. Intuitively, the distortions associated with \( \pi_i \) and \( \pi_j \) reinforce each other if they produce similar relative price changes across input pairs \((k, h)\), especially those with higher input shares or higher elasticity of substitution. Correspondingly, \( \Phi_t(i, j) \) measures the productivity loss of sector \( t \) induced by a 1% increase in \( i \)'s inflation, given that \( j \)'s also increased by 1%. \( \Phi_t(i, j) \) weights each pair \((k, h)\) by the relevant input
shares $\omega_{ti}$ and $\omega_{tj}$, and the substitution elasticity $\theta_{kh}^t$:

$$
\Phi_t(i, j) = \omega_{tk} \omega_{th} \theta_{kh}^t \left( (I - \Omega)^{-1}_{ki} - (I - \Omega)^{-1}_{hi} \right) \frac{1 - \delta_i}{\delta_i} \pi_i \left( (I - \Omega)^{-1}_{kj} - (I - \Omega)^{-1}_{hj} \right) \frac{1 - \delta_j}{\delta_j} \pi_j
$$

When elasticities of substitution are uniform ($\theta_{kh}^t \equiv \theta^t$), the substitution operator is simply given by the covariance between the price distortions induced by $i$ and $j$ across sector pairs $(k, h)$, with probability weights given by $t$’s input shares $\{\omega_{tk}\}_{k=1..N}$:

$$
\Phi_t(i, j) = \theta^t Cov_{\Omega_t} \left( (I - \Omega)^{-1}_{(i)} \frac{1 - \delta_i}{\delta_i} \pi_i, (I - \Omega)^{-1}_{(j)} \frac{1 - \delta_j}{\delta_j} \pi_j \right)
$$

The total productivity loss in sector $t$ is obtained by summing the contributions of all pairs $(i, j)$:

$$
\text{Loss in } t = \sum_{i,j} \Phi_t(i, j)
$$

and the aggregate productivity loss is given by Hulten’s formula (see Equation (2.5)).

2.1.2 Optimal policy

Optimal monetary policy minimizes the welfare loss derived in Proposition 4, subject to the response of inflation to the output gap and productivity shocks.

In the one-sector model the central bank solves

$$
min_{\pi, \bar{y}} W = \frac{1}{2} \left[ (\gamma + \varphi) \bar{y}^2 + \epsilon \frac{1 - \delta}{\delta} \pi^2 \right]
$$

s.t. $\pi = \kappa \bar{y}$

(2.7)

Here the constraint is given by the aggregate Phillips curve. The “divine coincidence” implies that there is no tradeoff between stabilizing output and stabilizing prices, therefore the
optimal policy achieves the first best by setting $\pi = \bar{y} = 0$.

With multiple sectors the optimal policy problem extends this baseline in two dimensions. First, the inflation term is replaced by the more complex misallocation loss derived in Proposition 4, which captures both within- and cross-sector distortions. Second, the constraint is not just the aggregate Phillips curve, but it is given by the full vector of sectoral Phillips curves. Thus the problem becomes:

$$\min_{\bar{y}, \pi} \frac{1}{2} \left[ (\gamma + \varphi) \bar{y}^2 + \pi^T D \pi \right]$$

s.t. $\pi = B\bar{y} + Vd \log A$

(2.8)

Proposition ?? characterizes the solution to the policy problem.

**Proposition 5.** The value of the output gap that minimizes the welfare loss is

$$\bar{y}^* = -\frac{B^T D Vd \log A}{\gamma + \varphi + B^T DB}$$

(2.9)

**Proof.** The result follows immediately from the first order conditions of the minimization problem (2.8).

In the multi-sector model replicating the efficient aggregate output does not eliminate either within- or cross-sector misallocation, and improvements in allocative efficiency come at the cost of distorting aggregate output. The constraint tells us that the effect of monetary policy on misallocation is limited, in that it can only implement relative price changes that are proportional to the vector $B$ of sectoral elasticities with respect to the output gap.

The optimal policy trades off the marginal cost and benefit of deviating from the efficient aggregate output. The denominator in equation (2.9) reflects the marginal cost, and it is always positive. It comes directly from distortions in aggregate demand (whose welfare effect is proportional to the labor supply elasticities $(\gamma + \varphi)$), and indirectly from the relative price distortions caused by the output gap (captured by the term $B^T DB$).
The numerator in \( \pi^T \mathcal{D} \pi \) is the marginal gain. Here \(-\pi^T \mathcal{D} \pi\) is the marginal benefit of inducing inflation \( \tilde{\pi} \) for given current inflation \( \pi \). Since the output gap affects prices proportionately to the elasticity \( \mathcal{B} \), the marginal benefit of increasing the output gap is \(-\mathcal{B}^T \mathcal{D} \pi\).

For \( \tilde{y} = 0 \) the inflation \( \pi \) induced by the productivity shock is

\[
\pi = \mathcal{V} d \log A
\]

so that the overall marginal gain is given by \(-\mathcal{B}^T \mathcal{D} \mathcal{V} d \log A\).

### 2.1.3 Inflation targeting

The optimal output gap in Proposition 5 depends on productivity. In the one sector model instead the optimal output gap is always zero, regardless of productivity. Moreover in this model it is equivalent to target inflation or the output gap, thanks to the “divine coincidence”. This is a useful result from an implementation point of view, because the output gap and productivity are difficult to measure in real time.

Proposition 6 demonstrates that the convenient implementation properties of the one sector model are preserved in the multi-sector framework: the optimal policy can still be implemented by targeting an appropriate inflation index.

**Proposition 6.** Assume that no sector has fully rigid prices. Then there exists a unique vector of weights \( \phi \) (up to a multiplicative constant) such that the aggregate inflation

\[
\pi_\phi = \phi^T \pi
\]

is positive if and only if \( \tilde{y} > \tilde{y}^* \).

This vector is given by

\[
\phi^T = \lambda^T (I - \Delta) \Delta^{-1} + \mathcal{B}^T \mathcal{D}
\]

(2.10)
Proof. See Appendix B2

To build intuition, note that the first order condition from the policy problem (2.8) can be written as

\[(\gamma + \varphi) \ddot{y} + B^T D \pi = 0 \quad (2.11)\]

From Equation (2.11) it is immediate to derive the policy target (2.10): we just need to replace the output gap with the divine coincidence inflation index, using Proposition 3.

Consistent with our discussion in Section 2.1.2, Equation (2.11) highlights that the optimal target weights the output gap against sectoral inflation rates according to the relative marginal benefit \((-B^T D\pi\)) and marginal cost \((\gamma + \varphi)\) of distorting aggregate output to reduce misallocation.

This result extends with minimal modifications to the dynamic setup (see Section 2.2.2). Here the optimal policy can be implemented via a Taylor rule which targets the inflation statistic in Proposition (6), with an additional correction for inflation expectations.

2.1.4 Examples

The examples below illustrate optimal monetary policy in the three simple production networks introduced in Section 1.3.2 (vertical chain, horizontal economy and oil economy).

Example 4. Optimal policy in the vertical chain

Consider a negative downstream shock in a two-stage vertical chain, as in Example 1. Can monetary policy do better than implementing a zero output gap? Which sector shall it seek to stabilize?

Recall from Example 1 that under zero output gap marginal costs and prices increase downstream and fall upstream. A positive output gap raises wages, so that marginal costs increase everywhere. Thus inflation increases even more in the downstream sector, while it gets closer to zero in the upstream sector. I argue that this is optimal, for two reasons. First,
distortions are more costly in the upstream sector; second, the impact of wages is smaller in the downstream sector.

Let’s first analyze cross-sector misallocation. Since there is one consumption good and \( U \) only uses labor as an input, cross-sector misallocation can happen only across \( D \)’s inputs (labor and \( U \)’s product). \( D \) inefficiently substituted between \( U \) and labor whenever \( U \)’s price does not fully adjust to reflect changes in wages. Monetary policy can offset this effect by stabilizing wages, and thereby reducing \( U \)’s desired price adjustment. Since wages fall after a negative productivity shock, wage stabilization requires to implement a positive output gap. This is reflected in the cross-sector component of the optimal output gap:

\[
\frac{\gamma + \varphi}{(1 - \delta_D (1 - (1 - \alpha_D) \delta_U))} \left[ \frac{(1 - \alpha_D) \alpha_D}{\text{input shares}} \left( 1 - \delta_U \right)^2 \left( 1 - \delta_D \right) \right] d \log A_D > 0
\]

The within-sector component of the optimal output gap instead is

\[
\frac{-\epsilon (\gamma + \varphi) (1 - \alpha_D) (1 - \delta_U) (1 - \delta_D)}{(1 - \delta_D (1 - (1 - \alpha_D) \delta_U))^2} \left[ \frac{\delta_U}{\text{benefit for } U} - \frac{\delta_D (1 - (1 - \alpha_D) \delta_U)}{\text{cost for } D} \right] d \log A_D
\]

Here \( \frac{\delta_D}{\delta_U} \) is the relative cost of within-sector price dispersion in \( D \) and \( U \), and \( 1 - (1 - \alpha_D) \delta_U \) is the relative marginal effect of monetary policy on inflation in the two sectors. Note that \( 1 - (1 - \alpha_D) \delta_U < 1 \), reflecting the fact that marginal costs are more sensitive to monetary policy in the upstream sector (because they are more directly exposed to wages). In this sense the upstream sector is easier to stabilize. We argued that stabilizing \( U \) entails implementing a positive output gap. This is optimal for within-sector misallocation if and only if the benefit for \( U \) \( (\delta_U) \) is greater than the loss for \( D \) \( (\delta_D (1 - (1 - \alpha_D) \delta_U)) \), which is always the case when adjustment frequencies are the same in the two sectors.

**Example 5. Optimal policy in the horizontal economy**

Consider the horizontal economy in Example 2. Here the tradeoff between within- and
cross-sector misallocation is particularly clear. The marginal gain of increasing the output
gap for within-sector misallocation is given by

$$B^T D_1 \pi = \epsilon \left( \gamma + \varphi \right) \mathbb{E}_{\beta (1-\delta)} \pi$$

(2.12)

and the corresponding cross-sector effect is

$$B^T D_2 \pi = -\sigma \left( \gamma + \varphi \right) \mathbb{E}_{\beta (1-\delta)} \pi$$

(2.13)

Here I denoted by $\mathbb{E}_{\beta (1-\delta)}$ the expectation computed with probability weights

$$\left\{ \frac{\beta_i (1 - \delta_i)}{\sum_j \beta_j (1 - \delta_j)} \right\}_{i=1,...,N}$$

The two components are proportional, with opposite signs. Reducing within-sector mis-
allocation is optimal if and only if the corresponding substitution elasticity is larger than the
cross-sector one. Intuitively, stabilizing within-sector misallocation would require all firms
to charge the same price. Since some firms cannot adjust, this is achieved only if all prices
remain constant. At the sector level however incomplete price adjustment results in a price
response that is too small relative to the change in productivity. Therefore reducing cross-
sector misallocation requires a larger price adjustment. The sign of optimal output gap is
determined by the within-sector component if and only if $\epsilon > \sigma$.

Let’s now derive the response of inflation in (2.12) and (2.13) as a function of productivity
shocks. As in Example 2 the optimal output gap depends on the covariance between produc-
tivity shocks and adjustment frequencies (although with different weights). This covariance
captures the competing effect of wage and productivity on marginal costs. To see this we
can express inflation in (2.12) and (2.13) as a function of productivity shocks:

$$\mathbb{E}_{\beta (1-\delta)} \pi = \text{Cov}_{\beta (1-\delta)} (\delta, d \log A) = \frac{\sum_i \beta_i (1 - \delta_i) \delta_i (d \log mc_i)}{\sum_j \beta_j (1 - \delta_j)}$$

(2.14)
Even though within- and cross-sector misallocation depend on the same covariance, this happens for different reasons. Within sectors, the marginal welfare loss is proportional to

\[ \beta_i (1 - \delta_i) \delta_i (-d \log mc_i) \]  

(2.15)

Misallocation is highest when the fraction of adjusting firms is closer to \( \frac{1}{2} \), because price dispersion is maximal. To fix this, monetary policy should bring marginal cost changes closer to zero. Since marginal costs have unit elasticity with respect to the output gap, its marginal effect on within-sector misallocation is proportional to (2.15). This component of the optimal output gap is positive if and only if marginal costs decrease more in sectors with large consumption share and intermediate adjustment probabilities.

For cross-sector misallocation instead only the fraction \((1 - \delta_i)\) of non-adjusting firms matters, because these are the firms whose relative price is distorted with respect to producers in other sectors. The marginal contribution of sector \(i\) to the welfare loss is given by

\[ \beta_i (1 - \delta_i) (-d \log mc_i) \]

Monetary policy should amplify the price adjustment by flexible firms, so that sectoral prices better reflect the change in productivity. In this case the relevant elasticity is not the marginal cost one, but the price one, which is given by \(\delta_i\). The marginal effect on cross-sector misallocation is then also proportional to (2.15), but with opposite sign.

**Example 6. Optimal policy in the oil economy**

Let’s go back to the simple oil economy in Example 3. Here the production structure is a combination of a vertical chain and a horizontal economy.

First, consider the sources of cross-sector misallocation. In this economy all the cross-sector misallocation comes from inefficient substitution by consumers across final goods. We
rule out misallocation across inputs in production for two reasons: first, oil uses only labor as an input; second, the oil price is fully flexible, so that it is never distorted relative to wages. As a result the cross-sector component of the optimal output gap is the same as in the horizontal economy in Example 5. It is given by:

$$B^T D_2 \pi = -\sigma (\gamma + \varphi) \mathbb{E}_{\beta(1-\delta)} \pi$$

(2.16)

As in the horizontal economy, the cross-sector component of the optimal output gap is negative if and only if marginal costs decreased relatively more in sectors with large consumption shares and/or intermediate adjustment probabilities.

Within-sector misallocation instead is present both at the final goods stage and at the labor stage. There is no misallocation across oil producers, because we assumed that they have flexible prices. The final good stage is the same as in the horizontal economy, and it is given by

$$(B^T D_1 \pi)_{hor} = \epsilon (\gamma + \varphi) \mathbb{E}_{\beta(1-\delta)} \pi$$

As for the labor stage, from the vertical chain example we know that it is stabilized with a positive output gap. Given the fall in productivity, labor demand also falls under zero output gap and wages decrease. However with wage rigidities ($\delta_L < 1$) not all workers can adjust, so that labor demand is inefficiently low for the workers whose wage remains too high. A positive output gap reduces the desired wage cut, thereby shrinking the gap between adjusting and non-adjusting workers and the resulting misallocation. The corresponding component of the optimal output gap is:

$$(B^T D_1 \pi)_{vert} = \epsilon (\gamma + \varphi) \frac{\delta_L \mathbb{E}_\beta (1-\delta)}{1 - \delta_L \mathbb{E}_\beta (\delta)} \left( 1 - \frac{\delta_L (1 - \mathbb{E}_\beta (\delta))}{1 - \frac{\delta_L \mathbb{E}_\beta (\delta)}} \right) \mathbb{E}_{\beta(1-\delta)} (\omega_{oil}) > 0$$

The response of inflation to the oil shock is different from the horizontal economy (see Equation (??)). Substituting for inflation in Equation (2.16) as a function of the productivity.
shock, we obtain:

\[ B^T D_{2\pi} = \]

\[
= \sigma (\gamma + \varphi) \frac{\delta_L (1 - \mathbb{E}_{\beta} (\delta))}{1 - \delta_L \mathbb{E}_{\beta} (\delta)} \left[ \mathbb{E}_{\beta(1-\delta)} (\delta \omega_{oil}) - \frac{\delta_L (1 - \mathbb{E}_{\beta} (\delta))}{1 - \delta_L \mathbb{E}_{\beta} (\delta)} \mathbb{E}_{\beta(1-\delta)} (\delta) \mathbb{E}_{\beta(1-\delta)} (\omega_{oil}) \right] d \log A_{oil}
\]

Equation (2.17) differs from (??) because we introduced sticky wages. First, the expression in (2.17) is proportional to the wage flexibility \( \delta_L \). When wages are fully rigid (\( \delta_L = 0 \)) monetary policy has no effect on marginal costs and markups, and the optimal output gap is zero. Second, the covariance \( \text{Cov}_{\beta(1-\delta)} (\delta, d \log A) \) from Example 5 is replaced with the expression in square brackets in (2.17). Here sectoral productivity shocks are replaced by oil input shares, which reflect the pass-through of the oil shock into marginal costs. The first term in the expression reflects the effect of productivity on marginal costs, while the second is the effect of the equilibrium change in wages on marginal costs. In the presence of wage rigidities (\( \delta_L < 1 \)) the latter is muted. As a result we have:

\[
\mathbb{E}_{\beta(1-\delta)} (\delta \omega_{oil}) - \frac{\delta_L (1 - \mathbb{E}_{\beta} (\delta))}{1 - \delta_L \mathbb{E}_{\beta} (\delta)} \mathbb{E}_{\beta(1-\delta)} (\delta) \mathbb{E}_{\beta(1-\delta)} (\omega_{oil}) > \text{Cov}_{\beta(1-\delta)} (\delta, \omega_{oil})
\]

Even though the mapping between productivity and inflation is different, the intuition behind the optimal policy remains the same as in the horizontal economy. The cross-sector component of the optimal output gap is negative if and only if marginal costs decreased relatively more in sectors with large consumption shares and/or intermediate adjustment probabilities.

The within-sector component instead combines insights from both the horizontal economy and the vertical chain. The optimal response to within-sector misallocation in the final goods stage is proportional to (2.18) as in the horizontal economy. However we have an additional term coming from misallocation in the labor sector, which plays the same role as the upstream sector in the vertical chain. Overall, the within-sector component of the optimal output gap
is given by

\[ B^T D_1 \pi = \epsilon (\gamma + \varphi) \frac{\delta_L \mathbb{E}_\beta (1 - \delta)}{1 - \delta_L \mathbb{E}_\beta (\delta)} \left[ 1 - \frac{\delta_L (1 - \mathbb{E}_\beta (\delta))}{1 - \delta_L \mathbb{E}_\beta (\delta)} \right] \mathbb{E}_{\beta(1-\delta)} (\omega_{oil}) \]

vertical chain

\[ - \left[ \mathbb{E}_{\beta(1-\delta)} (\delta \omega_{oil}) - \frac{\delta_L (1 - \mathbb{E}_\beta (\delta))}{1 - \delta_L \mathbb{E}_\beta (\delta)} \mathbb{E}_{\beta(1-\delta)} (\delta) \mathbb{E}_{\beta(1-\delta)} (\omega_{oil}) \right] d \log A_{oil} \quad (2.19) \]

horizontal economy

In principle, the sign of this component depends on the “adjusted” covariance in (2.18). Quantitatively, the calibration in Section 3.4.1 shows that this covariance is positive. Given that the within-sector elasticity is larger than the cross-sector elasticity in our calibration, and the vertical chain term is also positive, the optimal output gap is positive.

### 2.2 Extensions

#### 2.2.1 Exogenous cost-push shocks

In this section I extend the model presented in the main text to allow for exogenous sector-level cost-push shocks, which I model as a change in producers’ desired markups. I denote these changes by the \( N \times 1 \) vector \( d \log \mu^D \). Lemma 4 derives sectoral inflation rates and the Phillips curve.

**Lemma 4.** The elasticity of sectoral prices with respect to cost-push shocks is given by

\[ \left( \frac{B \Lambda^T}{\gamma + \varphi} - \mathcal{V} \right) \quad (2.20) \]

The “divine coincidence” Phillips curve is
\[ DC = (\gamma + \varphi) \tilde{y} + \lambda^T d \log \mu^D \]  

(2.21)


\textit{while the consumer-price Phillips curve is}

\[ \pi^C = \kappa \tilde{y} + u + v \]  

(2.22)

\[ u = \frac{\delta_w - \delta_A \lambda^T d \log A}{1 - \delta_w} \lambda^T d \log A \]

\[ v = \frac{\delta_\mu}{1 - \delta_w} \lambda^T d \log \mu^D \]

\[ \delta_\mu = \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} d \log \mu^D}{\lambda^T d \log \mu^D} \]

The two Phillips curves expressed in terms of deviations from steady-state output are

\[ DC = (\gamma + \varphi) y + \lambda^T (d \log \mu^D - d \log A) \]  

(2.23)

\[ \pi^C = \kappa y + \frac{\delta_\mu \lambda^T d \log \mu^D - \delta_A \lambda^T d \log A}{1 - \delta_w} \]  

(2.24)

Similar to the baseline model, the central bank faces a worse trade-off after a cost-push shock than after a negative productivity shock of the same size (i.e. \( d \log A = -d \log \mu^D \)). We can see this from the inequality

\[ \frac{u}{\lambda^T d \log A} < \frac{v}{\lambda^T d \log \mu^D} \]

This is because the change in firms’ desired price is the same for the two shocks, but after the cost-push shock natural output hasn’t changed. In other words, inflation is the same
after the two shocks for a given deviation of output from steady-state, but the output gap is lower under the cost-push shock. Therefore for a given output gap the cost-push shock generates higher inflation. Correspondingly, an additive “aggregate” cost-push term appears in the divine coincidence Phillips-curve.

Lemma 5 solves for the optimal policy response.

**Lemma 5.** The optimal monetary policy response to a cost-push shock $d \log \mu^D$ implements the output gap

$$
\bar{y}_{CP}^* = 
\frac{
B^T \left( \mathcal{D} \left( \frac{B \lambda^T}{\gamma + \varphi} \mathcal{Y} \right) - \mathcal{D}_2 \Delta (I - \Delta)^{-1} \right)
}{(\gamma + \varphi) + B^T DB}
\ d \log \mu^D
$$

**Under the optimal policy the inflation target derived in Proposition 6 takes value**

$$
\pi_\phi = \left( \lambda^T - B^T D_2 \Delta (I - \Delta)^{-1} \right) d \log \mu^D
$$

Comparing (2.25) with (2.9) above, we see that the optimal response to productivity and cost-push shocks has a common component, given by the “propagation” term in (2.25). Monetary policy seeks to address the relative price distortions that arise as the shock propagates through the input-output network. These are the same regardless of whether the inflation response is triggered by fluctuations in productivity or desired markups.

While productivity shocks cause misallocation only through the propagation channel, by definition cost-push shocks directly distort relative markups. The response of monetary policy to this direct effect is captured by the third term in (2.25). Implementing a positive output gap is optimal if it raises marginal costs more in the sectors which faced a larger increase in their desired markup. Whenever this is the case the policy target is positive under the optimal policy, as reflected in the second term of (2.26).

Finally, the first term in equation (2.25) comes from the fact that monetary policy faces
a “worse” trade-off under the cost-push shock than under the productivity shock, because natural output has not fallen. This is also captured by the first term of the policy target (2.26). The intuition is the same as in the one-sector model: in the face of a cost-push shock the central bank trades off the output loss with the increase in inflation. Therefore the optimal output gap is lower than after an equally-sized negative productivity shock, while the output level and inflation should be higher.\footnote{However note that in the multi-sector model this channel is potentially counteracted by the response to the “direct” effect.}

2.2.2 Dynamics

Consumers

Consumers’ preferences are given by

$$U_t = \sum_{i=0}^{\infty} \rho^i \left( \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

where $\rho$ is the discount factor, and $C_t$ and $L_t$ are defined as in Section 1.1.2.

In each period consumers are subject to the budget constraint

$$P_t C_t + B_{t+1} \leq w_t L_t + \Pi_t - T_t + (1 + i_t) B_t$$

where $w_t L_t$ is labor income, $\Pi_t$ are firm profits (rebated lump-sum to households), $T_t$ is a lump-sum transfer (which the government uses to finance input subsidies to firms), $B_t$ is the quantity of risk-free bonds paying off in period $t$ owned by the household and $i_t$ are nominal interest rates.

Consumer optimization yields the Euler equation

$$U_{ct} = \rho (1 + i_{t+1}) \mathbb{E} \left[ U_{ct+1} \frac{P_{ct}}{P_{ct+1}} \right] \quad (2.27)$$
where $P_{ct}$ is the consumer price index at time $t$. Log-linearizing equation (2.27) and imposing market clearing for final goods we find

$$y_t = E[y_{t+1}] - \frac{1}{\gamma} \left( i_{t+1} - E[\pi_{c t+1}] - \log \rho \right)$$

(2.28)

Re-writing equation (2.28) in gaps yields

$$\bar{y}_t = E[\bar{y}_{t+1}] - \frac{1}{\gamma} \left( i_{t+1} - E[\pi_{c t+1}] - r_{nt+1} \right)$$

(2.29)

where $r_{nt+1}$ is the natural interest rate, satisfying

$$r_{nt+1} = \log \rho + \gamma \lambda^T E[\log A_{t+1} - \log A_t]$$

**Policy instruments**

I consider a cashless economy, in which interest rates are the only policy instrument. At each period $t$ the central bank sets the risk-free rate $i_{t+1}$.

**Production**

Within each period the production technology is as described in Section 1.1.3. Sectoral productivity shifters $A_{it}$ vary across periods.

As in the one-period model, I assume that the government sets input subsidies to offset the markup distortions arising from monopolistic competition. Sectoral subsidies are constant over time, and given by

$$1 - \tau_t = \frac{\epsilon_{it} - 1}{\epsilon_{it}^s}$$

as in Section 1.1.3.

All producers minimize costs given wages and input prices. At every time $t$ producers in
sector $i$ solve

\[
c_{it}(\bar{y}) = \min_{x_{ijt}, L_{it}} \ w_tL_{it} + \sum_{j} p_{jt}x_{ijt} \quad s.t. \ A_{it}F_i(L_{it}, \{x_{ijt}\}) = \bar{y}
\]

With constant returns to scale marginal costs are the same for all firms, and all firms use inputs in the same proportions. However not all of them can adjust prices, so that firms within the same sector end up charging different markups.

**Sector-level inflation dynamics**

The firms who can update their price solve

\[
p_{it}^* = \max_{p_i} \ E \left[ \sum_{t} SDF_t (1 - \delta_i)^t Y_{it}(p_i) (p_i - (1 - \tau_i) mc_{it}) \right]
\]

(2.30)

The optimal reset price is

\[
p_{it}^* = \frac{E \sum_{t} \left[ \frac{\delta_i}{\epsilon_i} SDF_t (1 - \delta_i)^t Y_{it}(p_i) mc_{it} \right]}{E \sum_{t} \left[ \frac{\epsilon_i - 1}{\epsilon_i - 1} SDF_t (1 - \delta_i)^t Y_{it}(p_i) \right]}
\]

(2.31)

Log-linearizing equation (2.31) yields the following expression for sector-level inflation rates:

\[
\pi_{it} = \frac{\delta_i (1 - \rho(1 - \delta_i))}{1 - \delta_i} (d \log \mu_{it}^D - \log \mu_{it}) + \rho \mathbb{E}_t [\pi_{it+1}]
\]

(2.32)

where $\mu_{it}$ is the “sector-level” markup, satisfying:

\[
\log \mu_{it} = \log p_{it} - \log mc_{it}
\]

and $\mu_{it}^D$ is the desired markup of firms in sector $i$. 58
Equilibrium

Equilibrium is defined in a similar way as in section 2.5.

For given sectoral probabilities of price adjustment $\delta_i$, sectoral productivity shifters $A_{it}$ and interest rates $i_t$ for each period $t$, general equilibrium is given by a vector of firm-level markups $\mu_{fit}$, a vector of prices $p_{it}$, a nominal wage $w_t$, labor supply $L_t$, a vector of sectoral outputs $y_{it}$, a matrix of intermediate input quantities $x_{ijt}$, and a vector of final demands $c_{it}$ for each period $t$ such that: a fraction $\delta_i$ of firms in each sector $i$ can adjust their price in every period; markups are chosen optimally by adjusting firms (see problem (2.30)), while they are such that prices stay constant for the non-adjusting firms; consumers maximize intertemporal utility subject to the budget constraints; producers in each sector $i$ minimize costs and charge the relevant markup; and markets for all goods and labor clear.

The sales-based Phillips curve

Proposition 7 shows that the “divine coincidence” Phillips curve inherits the properties of the Phillips curve in the one-sector model: it has constant slope, and does not depend on the realization of sectoral productivity shocks; the aggregate cost-push shock enters as an additive residual.

**Proposition 7.** It holds that

$$DC_t \equiv \lambda^T \left( I - \hat{\Delta} \right) \hat{\Delta}^{-1} \pi_t = \rho \mathbb{E} \left( DC_{t+1} \right) + \kappa \bar{y}_t + \lambda^T d \log \mu_{it}^D$$  \hspace{1cm} (2.33)

where

$$\kappa = \gamma + \varphi$$

\[ ^{6}\text{Note that the sales-based Phillips curve in (1.30) does not depend on past markups. This is a consequence of Lemma 12 in Appendix A2.}\]
and \( \hat{\Delta} \) is a diagonal matrix with elements

\[
\hat{\Delta}_{ii} = \frac{\delta_i (1 - \rho(1 - \delta_i))}{1 - \rho \delta_i (1 - \delta_i)}
\]

Response of inflation rates and markups to productivity and monetary shocks

Proposition (8) characterizes the evolution of sector-level inflation, inflation expectations and markups for given productivity shocks \( \log A_t - \log A_{t-1} \), monetary policy \( \bar{y}_t \), and past markups. Note that, different from the one-sector model and the sales-based Phillips curve in Section 2.2.2, the vector of sector-level past markups is a state variable.\(^7\)

**Proposition 8.** Denote by

\[
\mathcal{M} \equiv \left( \frac{\bar{B} \gamma^T}{\gamma + \varphi} - \hat{\mathcal{V}} \right) \left( I - \hat{\Delta} \right) \hat{\Delta}^{-1}
\]

The evolution of sectoral markups and inflation rates is given by the following system of difference equations:

\[
\begin{pmatrix}
\rho \mathbb{E} \pi_{t+1} \\
\log \mu_t
\end{pmatrix}
= \begin{pmatrix}
\mathcal{M}^{-1} & -\mathcal{M}^{-1} \hat{\mathcal{V}} \\
-(I - \hat{\Delta}) \hat{\Delta}^{-1} (I - \mathcal{M}^{-1}) & -(I - \hat{\Delta}) \hat{\Delta}^{-1} \mathcal{M}^{-1} \hat{\mathcal{V}}
\end{pmatrix}
\begin{pmatrix}
\pi_t \\
\log \mu_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
-\mathcal{M}^{-1} \left( \bar{B} \bar{y}_t + \hat{\mathcal{V}} (\log A_t - \log A_{t-1}) \right) - \hat{\Delta} \left( I - \hat{\Delta} \right)^{-1} d \log \mu_t^D \\
-(I - \hat{\Delta}) \hat{\Delta}^{-1} \mathcal{M}^{-1} \left( \bar{B} \bar{y}_t + \hat{\mathcal{V}} (\log A_t - \log A_{t-1}) \right)
\end{pmatrix}
\]

(2.34)

**Proof.** See Appendix D

---

\(^7\)The actual state variables are “relative” past markups \( \mathcal{V} d \log \mu_{t-1} \). Given these, the system is invariant to the “aggregate” past markup \( \lambda^T d \log \mu_{t-1} \).
We can re-write the first equation in (2.34) as

\[
\pi_t = \hat{B}_y + \hat{V} (\log A_t - \log A_{t-1} + \log \mu_{t-1}) + \left( \frac{\hat{B} \lambda T}{\gamma + \varphi} - \hat{V} \right) \left( d \log \mu^D_t + \left( I - \hat{\Delta} \right) \hat{\Delta}^{-1} \rho \hat{H}_{t+1} \right)
\]

(2.35)

The first term contains the elasticities of sectoral inflation rates with respect to productivity and monetary shocks, which are the same as in the static setup (see Propositions 1 and 2). In addition, we now have to account for inherited markup distortions, due to the fact that some producers could not adjust their price in past periods. Lemma 12 in Appendix B2 shows that in the static setting inflation responds in the same way to productivity shocks and to initial markups. This happens because both induce the same desired price change under zero output gap. The last term in equation (2.35) is the response to a “cost-push” shock, which can have an exogenous component \((d \log \mu^D_t)\) and an endogenous component coming from expected future inflation. Inflation expectations act as a “cost-push” shock because they change the desired amount of price adjustment for given output gaps and productivity.

The cost-push shock implied by inflation expectations is given by:

\[
d \log \mu^D = (I - \Delta) \Delta^{-1} \rho \hat{H}_{t+1}
\]

(2.36)

**Consumer price Phillips curve**

We can aggregate sectoral inflation rates in equation (2.35) into the CPI-based Phillips curve, obtaining

\[
\pi_t^C = \kappa \tilde{y}_t + \rho \hat{H}_{t+1} + u_t + v_t
\]

(2.37)

where

\[
u_t = \frac{\tilde{\delta}_w - \tilde{\delta}_A \lambda^T (d \log A_t - d \log A_{t-1}) + \tilde{\delta}_w - \tilde{\delta}_{\mu_{t-1}}}{1 - \delta_w} \lambda^T d \log \mu_{t-1}
\]
\[ v_t = \frac{\tilde{\delta}_w - \tilde{\delta}_w^C}{1 - \delta_w} \rho \mathbb{E} \pi_{t+1}^C + \frac{\tilde{\delta}_n^D}{1 - \delta_w} \lambda^T d \log \mu_t^D \]

\[ \tilde{\delta}_{n-1} = \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} d \log \mu_{t-1}}{\lambda^T d \log \mu_{t-1}} \]

\[ \tilde{\delta}_n^C = \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} (I - \Omega) \mathbb{E} \pi_{t+1}}{\mathbb{E} \pi_{t+1}^C} \]

\[ \tilde{\delta}_n^D = \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} d \log \mu^D}{\lambda^T d \log \mu^D} \]

Equation (2.37) highlights that past markups and inflation expectations create an endogenous cost-push term in the Phillips curve, in the same way as productivity shocks.

**Closing the output gap**

Lemma 6 shows that the central bank can implement zero output gap in all periods by targeting it directly in the Taylor rule, as long as the policy rule is “reactive enough”.

**Lemma 6.** Assume that the productivity shocks follow an AR1 process:

\[ \log A_{t+1} - \log A_t = \eta (\log A_t - \log A_{t-1}) + u_{t+1} \]

with \( \mathbb{E} u_{t+1} = 0 \) and \( \eta < 1 \). Then there is a unique path of inflation rates such that the output gap is constantly zero. This equilibrium can be implemented with the interest rate rule

\[ i_t = r_t^n + \beta^T \mathbb{E} \pi_{t+1}^{zg} + \zeta \tilde{g}_t \quad (2.38) \]

**Proof.** See Appendix D.
Optimal policy

Propositions 9 and 10 characterize the dynamic welfare loss and the central bank’s policy problem.

**Proposition 9.** Given a path \( \{y_t, \pi_t, z_{t+1}\}_{t=0}^{\infty} \) for the output gap, sectoral inflation rates and markups, the expected second-order welfare loss is given by

\[
\sum_{t=0}^{\infty} \rho^t \mathbb{E} \left[ (\gamma + \varphi) \tilde{y}_t^2 + \pi_t^T D_1 \pi_t + z_{t+1}^T D_2 z_{t+1} \right]
\]

where

\[z_t \equiv - \left( I - \hat{\Delta} \right)^{-1} \hat{\Delta} \log \mu_{t-1}\]

The optimal policy problem is

\[
\min_{\{\pi_t, z_{t+1}, \tilde{y}_t\}} \sum_{t=0}^{\infty} \rho^t \mathbb{E} \left[ (\gamma + \varphi) \tilde{y}_t^2 + \pi_t^T D_1 \pi_t + z_{t+1}^T D_2 z_{t+1} \right]
\]

subject to (2.34).

**Proposition 10.** Consider the optimal policy problem without commitment, where the central bank solves

\[
\min_{\{\tilde{y}_t, \pi_t, z_{t+1}\}} (\gamma + \varphi) \tilde{y}_t^2 + \pi_t^T D_1 \pi_t + z_{t+1}^T D_2 z_{t+1}
\]

subject to (2.34). The FOCs yield

\[
\tilde{y}_t^* = - \frac{B^T D V}{(\gamma + \varphi) + B^T D B} [(d \log A_t - d \log A_{t-1}) - d \log \mu_{t-1} + (I - \Omega) \rho \mathbb{E} \pi_{t+1}] \tag{2.39}
\]

**Proof.** See Appendix D

In the same spirit as Proposition 8, Proposition 9 decomposes the optimal output gap into the response to productivity shocks and past markups, which is the same as in the static
setup (see Proposition 5), and the response to inflation expectations, which have a similar effect as a “cost-push” shock (see Section 2.2.1) Lemma 7 and Lemma 8 below show that the optimal policy can be implemented with a targeting rule, in the same way as in the static setup (see Proposition 6).

Lemma 7. Assume that the productivity shocks follow an AR1 process:

$$\log A_{t+1} - \log A_t = \eta (\log A_t - \log A_{t-1}) + u_{t+1}$$

with $E u_{t+1} = 0$ and $\eta < 1$.

Then there is a unique path of inflation rates such that the optimal output gap (2.39) is implemented in each period.

Proof. See Appendix D

Lemma 8. Denote by

$$\phi^T \equiv \frac{\lambda^T (I - \hat{\Delta}) \hat{\Delta}^{-1} + \hat{\beta}^T \hat{D}}{\gamma + \varphi}$$

(2.40)

For $\zeta > \gamma$ the interest rate rule

$$i_{t+1} = \underbrace{r^n_{t+1}}_{\text{real rate under opt policy}} + \underbrace{\gamma [E \tilde{\eta}_{t+1} - \tilde{\eta}_t]^T + \beta E \pi^*_{t+1}}_{\text{nominal rate under optimal policy}} + \underbrace{\zeta \phi^T_i (\pi_t - \rho E \pi_{t+1})}_{\text{inflation target}}$$

(2.41)

implements the optimal policy (2.39).

The nominal rate can be expressed as a function of productivity shocks and current and expected inflation using the relation

$$\gamma [E \tilde{\eta}_{t+1} - \tilde{\eta}_t]^T + \beta E \pi^*_{t+1} = \kappa_i \pi^*_t + \kappa_{t+1} E \pi^*_{t+1} + \kappa_A (d \log A_t - d \log A_{t-1})$$

\(^8\)To facilitate the comparison, use the equality $M = \left( \frac{\beta \lambda^T}{\gamma + \varphi} - \nu \right) (I - \Delta) \Delta^{-1} = I + \nu (I - \Omega)$. 

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where

\[
\begin{align*}
\kappa_t^T &\equiv \gamma B^T D M^{-1} B (\lambda^T (I - \Delta) \Delta^{-1} + B^T D) \\
\kappa_{t+1}^T &\equiv \beta^T - \gamma \rho B^T D M^{-1} B (\lambda^T (I - \Delta) \Delta^{-1} + B^T D) + (I - M^{-1}) \\
\kappa_A &\equiv \gamma \eta B^T D M^{-1} \hat{\gamma}
\end{align*}
\]

Proof. See Appendix D

Comparing equation (2.19) from the static setup with equation (2.40) we see that they yield a very similar inflation target. The key difference is that in the dynamic setup the central bank should not just target current inflation, but also inflation expectations. The intuition is that the welfare-relevant variable are sector-level markups, and the mapping between current inflation and markups depends of inflation expectations. Specifically, from the pricing equation (2.32) we have \( d \log \mu = (I - \Delta) \Delta^{-1} (\pi_t - \rho \mathbb{E} \pi_{t+1}) \).
Chapter 3

Quantitative results

3.1 Data

The log-linearized economy is fully characterized by the parameters introduced in Section 1.2.2. They consist in the input-output parameters ($\Omega$, $\alpha$ and $\beta$), the sectoral frequencies of price adjustment ($\Delta$), and the elasticities of substitution in production and consumption. In addition, computing the expected welfare loss from business cycles (see Section 3.2) requires to calibrate the variance of sectoral productivity shocks.

I calibrate the input-output matrix $\Omega$ and the labor and consumption shares $\alpha$ and $\beta$ from the input-output tables published by the BEA. I use tables for the year 2012, because this is the most recent year for which they are available at a disaggregated level (405 industries). Section 3.3.3 relies on less disaggregated historical input-output data (46 - 71 industries), always from the BEA input-output accounts, to study the slope of the Phillips curve and monetary non-neutrality over time.

I calibrate industry-level frequencies of price adjustment based on the estimates con-

\footnote{The BEA does not provide a direct counterpart to the input-output matrix $\Omega$, however this can be constructed from the available data. The BEA publishes two direct requirement tables, the Make and Use table, which contain respectively the value of each commodity produced by each industry and the value of each commodity and labor used by each industry and by final consumers. In addition the BEA publishes an Import table that reports the value of commodity imports by industry. The Make and Use matrix (corrected for imports) can be combined, under proportionality assumptions, to compute the matrix $\Omega$ of direct input requirements and the labor and consumption shares $\alpha$ and $\beta$.}
constructed by Pasten, Schoenle and Weber (2017). For sectors with missing data I set the adjustment probability equal to the mean. I calibrate the quarterly probability of wage adjustment to 0.25, in line with Barattieri, Basu and Gottschalk (2014) and Beraja, Hurst and Ospina (2016).

I choose values for the elasticities of substitution across inputs and consumption goods based on estimates from the literature. I set the substitution elasticity between consumption goods to $\sigma = 0.9^2$, the elasticity of substitution between labor and intermediate inputs to $\theta_L = 0.5^3$, the elasticity of substitution across intermediate inputs to $\theta = 0.001^4$ and the elasticity of substitution between varieties within each sector to $\epsilon = 8^5$.

I calibrate sectoral TFP shocks and their covariance matrix based on estimates of annual industry-level TFP changes for the period 1988-2016 from the BEA Integrated Industry-Level Production Account data. I refer to the Multifactor Productivity (MFP) measure, and calibrate productivity shocks as the growth rate of this index at the sector level. The MFP is constructed taking into account labor, capital and intermediate inputs from manufacturing and services. Therefore this index captures changes in gross output TFP, which is the correct empirical counterpart of the sector-level TFP shocks in the model.

### 3.2 Welfare loss from business cycles

In the one-sector model productivity fluctuations do not generate a welfare loss with respect to an efficient economy with flexible prices, if the central bank implements the optimal monetary policy. In turn, the well-known Lucas' estimate suggests that in frictionless economies business cycle fluctuations have a very small welfare cost. Lucas estimates it to be about

---

2 Atalay (2017), Herrendorf et al. (2013), and Oberfield and Raval (2014) estimate it to be slightly less than one.
3 This is consistent with Atalay (2017), who estimates this parameter to be between 0.4 and 0.8.
4 See Atalay (2017).
5 This is consistent with estimates of the variety-level elasticity of substitution from the industrial organization and international trade literatures.
6 https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems
0.05% of per-period GDP\footnote{Here the welfare cost comes from the uncertainty generated by fluctuations in consumption.}

In Section 2.1 we argued that in a multi-sector economy monetary policy cannot replicate the flex-price efficient outcome. Thus productivity fluctuations have the potential to generate a larger welfare loss. In this section I calibrate the loss relative to the efficient economy under different policy rules. I assume that productivity shocks are normally distributed with zero mean and covariance matrix $\Sigma$, which I calibrate from BEA-KLEMS data. Quantitatively, the departures from the one sector benchmark are significant. There is a large loss from imperfect stabilization, equal to 0.28% of per-period GDP under the optimal policy. This means that the additional loss induced by price rigidities is one order of magnitude larger than the Lucas’ estimate. The loss increases under suboptimal policy rules. Targeting consumer prices, which is first best in the one sector model, brings it to 1.12% of per-period GDP. Input-output linkages are key in determining these results.

The results for the main calibration are plotted in Figure 3.1. Figure 3.2 reports results for an alternative calibration without input-output linkages. Appendix E1 additionally provides analytical expressions for the welfare loss under different policy rules, and a decomposition of the welfare loss between within- and cross-sector misallocation.
Figure 3.1: Welfare loss from business cycles (main calibration)

The bars correspond to the percentage of per-period GDP that consumers would be willing to forego in exchange of switching from a sticky-price economy to the efficient equilibrium, for a given monetary policy rule. Bars of different colors represent different rules (blue for optimal policy, red for output gap targeting, and yellow for consumer inflation targeting). Each set of bars corresponds to a different assumption about the correlation of sectoral shocks. In the first set the covariance matrix is calibrated from the data, while in the second
set there are only idiosyncratic shocks, and in the third there are only aggregate shocks. I keep the variance of aggregate productivity constant across all calibrations.

### 3.2.1 Optimal policy

The blue bars in Figures 3.1 and 3.2 represent the welfare loss under the optimal policy. In the full calibration business cycles generate a welfare loss of 0.28% of per-period GDP, with respect to an economy without pricing frictions. From the second set of bars in Figure 3.2 we see that the idiosyncratic component of productivity is the main driver.

Input-output linkages are key for the result. Figure 3.2 shows that the welfare loss is much smaller in an economy with the same wage rigidity, productivity shocks and price adjustment frequencies, but without input-output linkages.

### 3.2.2 Targeting the output gap

The red bars in Figures 3.1 and 3.2 show that on average targeting zero output gap yields a small additional loss with respect to the optimal policy. Although monetary policy faces a tradeoff between stabilizing aggregate demand (the output gap) and relative demand across sectors, the fact that it has only one instrument makes it inefficient at correcting relative price distortions. Therefore in practice the optimal policy should focus on aggregate demand.

We reach a similar conclusion when comparing the behavior over time of the “divine coincidence” index DC -our inflation proxy for the output gap- and the optimal policy target, plotted in Figure 3.3.

---

8Here consumption shares are calibrated to replicate relative sales shares.
Figure 3.3: Time series of the “divine coincidence” inflation and the optimal policy target.

The two series move closely together, which means that the optimal target almost coincides with the output gap. The target however is often a few basis points lower than $DC$, suggesting that the optimal policy should be slightly more expansionary than output gap targeting.

### 3.2.3 Targeting consumer inflation

The welfare loss under consumer inflation targeting is represented by the yellow bars in Figures 3.1 and 3.2. While stabilizing consumer inflation is optimal in the baseline model, in the full calibration this policy yields a large welfare loss. The result crucially depends on the input-output structure. Figure 3.2 shows that the loss is much smaller in a calibration without input-output linkages, regardless of the distribution of the shocks.
3.3 Slope of the Phillips curve and monetary non-neutrality

Corollary 2 in Section 1.3 establishes that the presence of intermediate inputs reduces the slope of the Phillips curve. To evaluate the quantitative importance of this result I carry out two exercises. Section 3.3.1 computes the slope of the Phillips curve based on the input-output tables for 2012, under different assumptions about input-output linkages, wage rigidities and pricing frictions. Section 3.3.3 instead studies how the slope implied by the model for the US economy has changed over time, based on the observed evolution of the input-output structure from 1947 to 2017.

The slope of the Phillips curve is also related with monetary non-neutrality, which is a measure of the effectiveness of monetary policy. Here I define monetary non-neutrality based on the response of inflation to a given real output shock. Monetary policy is more effective if it can achieve the same change in real output with a smaller inflation response. Section 3.3.2 derives the response of inflation to real output shocks in the dynamic version of the model, and shows that both input-output linkages and heterogeneous pricing frictions increase monetary non-neutrality.

### 3.3.1 Slope of the Phillips curve

This Section evaluates the relative contribution of input-output linkages, wage rigidity and heterogeneous adjustment frequencies to the flattening of the Phillips curve. Table 3.1 reports the slope implied by several alternative calibrations.

<table>
<thead>
<tr>
<th></th>
<th>full model</th>
<th>no IO, flex w</th>
<th>no IO</th>
<th>( \delta = \text{mean} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>0.09</td>
<td>1.16</td>
<td>0.22</td>
<td>0.08</td>
</tr>
<tr>
<td>slope relative to full calibration</td>
<td>1.00</td>
<td>0.07</td>
<td>0.38</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Table 3.1: Phillips curve slope in the main and alternative calibrations

The results suggest that both input-output linkages and wage rigidities flatten the Phillips curve.
curve, while heterogeneous adjustment frequencies play no role. The first column reports results from the baseline calibration. The implied slope is 0.09, which is in the same ballpark as empirical estimates (see Section 3.5). The second column reports the slope implied by an alternative calibration which directly maps the one-sector model to the data, ignoring input-output linkages and wage rigidities. Here the slope is more than one order of magnitude larger than in the baseline. The third column reports the slope in a calibration with sticky wages, but without input-output linkages. We find that the implied slope more than doubles with respect to the baseline calibration.

Finally, the last column shows that eliminating heterogeneity in adjustment frequencies does not affect the calibrated slope. This is not a general result, but it depends on the specific joint distribution of labor shares and adjustment frequencies that we observe in the data. We find that adjustment frequencies are not correlated with labor shares, so that setting them equal to the mean does not affect the wage pass-through $\delta_w$ which in turn determines the slope of the Phillips curve. Heterogeneity in price stickiness instead matters in the dynamic version of the model, where it increases monetary non-neutrality (see Section 3.3.2).

### 3.3.2 Monetary non-neutrality

This section studies the effect of input-output linkages and heterogeneous pricing frictions for monetary non-neutrality, using the dynamic version of the model. Figure 3.4 plots the impulse response of consumer inflation to a 1% real rate shock. The figure is constructed assuming a Taylor rule with $\varphi_\pi = 1.24$ and $\varphi_y = .33/12$ and a shock persistence of 0.5. It shows that both heterogeneous adjustment frequencies and input-output linkages dampen...

---

10See Section 1.3.1.2 for the definition of $\delta_w$.

11While in the one-sector model the slope of the Phillips curve fully characterizes the impulse response of inflation to real rate shocks, in the multi-sector model this is no longer true. In the dynamic setting the inflation response also depends on the expected speed of adjustment over time, which is decreasing in the variance of adjustment frequencies.
the response of consumer prices.$^{12}$

Figure 3.4 shows that both input-output linkages and heterogeneous adjustment frequencies dampen the response of consumer inflation to real rate shocks. The theory in Section 2.2.2 helps us to better understand this result. The law of motion of sectoral inflation rates is given by

\[ \pi_t = B\bar{y}_t + V\log \mu_{t-1} + \rho ME\pi_{t+1} \]  

(3.1)

We can aggregate both sides of (3.1) to obtain the impact response of consumer inflation:

\[ \pi^C_t = \kappa\bar{y}_t + \beta^T V\log \mu_{t-1} + \rho \beta^T M E\pi_{t+1} \]  

(3.2)

---

$^{12}$My results are consistent previous works, such as Carvalho (2006), Nakamura and Steinsson (2010).
The presence of input-output linkages flattens the Phillips curve, and therefore the response of consumer prices to real rate shocks is also muted. In contrast with the baseline model, however, the response of consumer inflation is not fully characterized by the slope of the Phillips curve, but it also depends on the evolution of sectoral markups. More precisely, heterogeneous adjustment frequencies increase monetary non-neutrality in the dynamic setting (this is in contrast with the static model, as discussed in Section 3.3.1).

To gain intuition consider two scenarios, both with the same average probability of price adjustment across sectors. In the first scenario all sectors have the same adjustment probability, while in the second some sectors are more flexible and some are stickier. As long as the discount factor is large enough, producers reset their prices to be an “average” of the optimal prices over the period before their next opportunity to adjust. If all sectors have the same adjustment probability, the producers who can adjust know that many others will also have changed their price by the time they get to adjust again. Therefore they preemptively adjust more. If instead some sectors can adjust very infrequently, producers in the flexible sectors know that they will likely have another opportunity to reset their price before the stickier sectors also get to change theirs. As a result it is optimal for them to wait. The expectations channel gets muted as the discount factor goes to zero. This is why heterogeneous adjustment frequencies play no role in the static setting.

3.3.3 Phillips curve and monetary non-neutrality over time

The analysis in this Section is based on historical input-output data, which the BEA provides for each year between 1947 and 2017. I study how the slope of the Phillips curve and the impulse responses implied by the calibrated model have evolved over this time period. Due to lack of data, I need to keep the frequencies of price adjustment constant.

Figure 3.5 plots the slope of the Phillips curve computed for each year between 1947 and 2017. The blue solid line depicts the calibrated slope.
We see that it has decreased by about 30% over this time period. This result is consistent with the conventional wisdom that the Phillips curve has flattened (see for example Blanchard (2012)). The total effect captured by the blue line in Figure 3.5 has two distinct components. The first comes from a change in the input-output structure, while the second comes from a shift in consumption shares, away from manufacturing and towards services. To isolate these two components, we can use the results in Section 3.3.1. Corollary 1 shows that the slope is determined by the pass-through of nominal wages into consumer prices, $\delta_w$. This pass-through can be decomposed into a term related with consumption shares, and a

---

13It is difficult to evaluate which fraction of the observed flattening is explained by changes in the input-output structure relative to other factors, given that we do not have consensus estimates of the slope of the Phillips curve at any point in time (see Mavroeidis, Plagborg-Moller and Stock (2013)). The calibration suggests that the input-output structure played an important role. Nonetheless, the fact that the calibrated slope at the beginning of the period is low compared to conventional estimates suggests that other channels, such as the anchoring of inflation expectations, might be relevant as well.
The evolution of these two components is represented by the dashed red and green lines in Figure 3.5. The red line represents the slope implied by a calibration where the input-output matrix is fixed at its 1947 value, and consumption shares evolve as observed in the data. The green line plots the slope of the Phillips curve implied by an alternative calibration where consumption shares remain constant at their 1947 value, while the input-output matrix changes over time as observed in the data. The shift of consumption from manufacturing towards services contributed to the decline after 1980. Service sectors have more rigid prices, therefore a shift towards these sectors increases average price stickiness and flattens the Phillips curve. Pre-1980, however, all of the decline can be attributed to the evolution of the production structure.

I argue that this is driven by a uniform increase in intermediate input purchases, and not by a raise in the input share of rigid sectors. The light blue line depicts the slope implied by a calibration where consumption shares remain constant, and input shares increase uniformly in all sectors. The change in input shares is calibrated to replicate the change in the aggregate value added to output ratio observed in the data. We see that the light blue line tracks the green one closely. A more detailed breakdown of the components highlighted in Figure 3.5 is provided in Appendix E2.

I find similar results in the dynamic setting. Figure 3.6 plots the calibrated impact response of inflation to a 1% shock to the real rate between 1947 and 2017. Consistent with the results in the static setting, the impact response has declined over time. Again, most of the effect can be attributed to changes in the production structure.
3.3.4 Wage Phillips curve vs consumer price Phillips curve

Two pieces of evidence have been found about the wage Phillips curve (see for example Hooper, Mishkin, Sufi (2019)): first, it is steeper than the price Phillips curve; second, it has not flattened over time (or at least not as much as the price Phillips curve).

This evidence is consistent with the predictions of the multi-sector model. While the consumer price Phillips curve is flat in the calibration, and its slope has declined substantially over time (see Sections 3.3.1 and 3.3.3), the wage Phillips curve is much steeper and with constant slope. The calibrated slope was 0.78 in 1947 and 0.77 in 2017.

3.4 Endogenous cost-push shocks

In the one-sector model productivity shocks do not generate a tradeoff between stabilizing inflation and stabilizing output. The only way to generate this tradeoff is via an exogenous shock to producers’ desired markups (which is also called a “cost-push” shock). This aspect of
the one-sector model is somewhat embarrassing: conventional wisdom suggests that shocks to certain sectors, such as oil price shocks, should raise inflation even if output is stabilized. The stylized representation of oil shocks as an increase in producers’ desired markups is not very suitable to study the optimal policy response.

The multi-sector model provides a much more convincing representation of the inflationary effect of oil shocks, and of productivity shocks in general. Section 3.4.1 below demonstrates that oil shocks generate sizable inflation in the calibrated multi-sector economy, even if output is stabilized. It also shows that the optimal policy response is to implement a positive output gap. Section 3.4.2 instead uses measured sectoral productivity shocks to construct a time series of the Phillips curve residual derived in Corollary 3, which captures the endogenous inflation-output tradeoff generated by productivity shocks. Section 3.5.2 below shows that adding this variable to otherwise standard Phillips curve regressions significantly increases the R-squared.

3.4.1 Oil shocks

Example 3 in Section 1.3.2 introduces a simple multi-sector model to show that negative oil shocks can raise consumer inflation, even if output is stabilized. Table 3.2 explores the quantitative relevance of this result in the calibrated US economy, under different assumptions about price and wage rigidity. In the baseline calibration the inflation response is sizable, equal to 0.22 for a 10% negative oil shock.

<table>
<thead>
<tr>
<th></th>
<th>$\delta = \text{actual}$</th>
<th>$\delta = \delta_{mean}, \delta_{out} = 1$</th>
<th>$\delta = \delta_{mean}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sticky wages</td>
<td>0.22</td>
<td>0.07</td>
<td>-0.00</td>
</tr>
<tr>
<td>flexible wages</td>
<td>0.18</td>
<td>0.01</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Table 3.2: Consumer inflation after a 10% negative shock to the oil sector (full model)

Even if the actual US network is more complex than the stylized model in Example 3
this example captures well the mechanisms at play. Our simple model predicts that three elements should contribute to the inflationary pressure of negative oil shocks: the presence of wage rigidities, the presence of a positive correlation between oil shares and adjustment frequencies, and the fact that oil prices are very flexible. Table 3.2 compares the inflation response to oil shocks in the full calibration versus alternative calibrations that shut down each of these channels. The inflation response is indeed smaller when we assume flexible wages ($\delta_L = 1$) or uniform price adjustment frequencies ($\delta_i \equiv \delta_{mean} \forall i$). If in addition we set the adjustment frequency of oil prices equal to the average consumer inflation becomes negative.

To complement the discussion in Example 6, Table 3.3 presents the optimal monetary policy response to a 10% negative oil shock.

<table>
<thead>
<tr>
<th></th>
<th>full model</th>
<th>$\delta = \delta_{mean}, \delta_{oil} = 1$</th>
<th>$\delta = \delta_{mean}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sticky wages</td>
<td>0.11</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>flex wages</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 3.3: Optimal output gap (in percentage points) after a 10% negative oil shock

Here policy is expressed in terms of the optimal output gap (in percentage points). The implied percentage change in output is obtained by adding the log change in natural output, $y_{nat} = -0.69$. The calibration suggests that the central bank should implement a positive output gap in response to negative oil shocks, consistent with the intuition provided in the Example.

### 3.4.2 Time series

In this Section I construct a time series for the endogenous component of the residual $u_C$ in the consumer-price Phillips curve. Corollary 3 in Section 1.3.1.3 derives it as a function of
sectoral productivity shocks:

\[
  u^C = \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} (\alpha \lambda^T - I) d \log A}{1 - \delta_w}
\]  

(3.3)

I proxy for productivity shocks using sector-level measures of yearly TFP growth from the BEA KLEMS data. Figure 3.7 plots the results.

![Figure 3.7: Time series of the endogenous cost-push shock and oil prices](image)

The estimated residual tracks oil prices quite closely, as shown in the figure. It has a mean of $-0.16$ and a standard deviation of $0.25$. Both mean and standard deviation are large relative to the calibrated slope of the consumer-price Phillips curve, which is $0.09$.

This suggests that endogenous “cost-push” shocks coming from TFP fluctuations have significant explanatory power for the behavior of consumer price inflation. I test this more directly in Section 3.5.2. Here I add the endogenous residual $u^C$ in Figure 3.7 into an otherwise standard Phillips curve regression. I find that it significantly increases the R-squared, bringing it close to the “divine coincidence” specification.
3.5 Phillips curve regressions

In this section I run Phillips curve regressions using different inflation measures as left-hand-side variables (various measures of consumer price inflation and the “divine coincidence” inflation index). I compare the estimated coefficients and R-squareds.

The estimation results validate my theoretical framework. First, the R-squared is 2 to 4 times higher when using the “divine coincidence” index on the left-hand-side. This is consistent with Proposition 3: the Phillips curve associated with the “divine coincidence” index is the only one that does not have an endogenous residual. Therefore the explanatory power of the output gap should be maximal for this inflation index. Second, the calibrated model predicts the estimated slopes correctly for both consumer prices and the “divine coincidence” index. Third, controlling for the endogenous cost-push shocks constructed in Section 3.4.2 increases the R-squared of the consumer-price Phillips curve, bringing it in the same ballpark as the “divine coincidence” specification.

Rolling regressions confirm that these results are robust to the choice of a sample period: the estimated coefficient is stable and always significant when using the “divine coincidence” index as left-hand-side variable, in contrast with specifications that use consumer prices.

3.5.1 Data

I construct a time series of the “divine coincidence” index $DC$ for the US economy based on sector-level PPI data from the BLS. I measure inflation as the percentage price change from the same quarter of the previous year. I aggregate sectoral inflation rates based on sales shares implied by the BEA input-output tables, and on sector-level price adjustment frequencies constructed by Pasten, Schoenle and Weber (2018). Appendix E1 describes the sector-level price series more in detail.

Figure 3.8 plots $DC$ against two measures of consumer price inflation (CPI and PCE) and against aggregate producer price inflation (PPI), from 1984 to 2018. Appendix E1 also
reports scatterplots of the output gap against $DC$ and consumer inflation.

Figure 3.8: “Divine coincidence” index, consumer and producer prices (1984-2018)

I refer the reader to Appendix E1 for a more detailed comparison between the weighting of sectoral inflation rates in the PCE and in $DC$. A key difference between the two is that the PCE puts no weight on wage inflation, which instead has a weight of 18% in $DC$. Other important sectors in $DC$ are professional services, financial intermediation and durable goods, whereas the PCE places high weight on health care, real estate and nondurable goods.

In the main text I focus on a regression specification with no lags and a proxy for inflation expectations, which is consistent with the model. I also present results for a specification without inflation expectations. Results for other specifications, which include lagged inflation and other variables, are reported in Appendix E.

I construct a proxy for inflation expectations based on the statistical properties of the inflation process, whose changes are well approximated by an IMA(1,1) (see Stock and Watson (2007)). I estimate the IMA(1,1) parameters and use them to construct a forecast series for each of the inflation measures that I use in the regressions. The forecast series are plotted in Appendix E1. For consumer inflation it has been shown that survey measures of forecasted inflation (such as the SPF) are well approximated by this IMA(1,1) forecast.
3.5.2 Regressions over the full sample period

I run Phillips curve regressions over the period January 1984 - July 2018 using different inflation measures as left-hand-side variables (DC, CPI, PCE, core CPI, core PCE). I take the CBO unemployment gap as a measure of the output gap on the right-hand-side. Appendix E2 shows that the results are robust when using two other measures of the output gap: the CBO output gap and the unemployment rate.

Table 3.4 reports the slope implied by the calibrated model for the consumer-price and the “divine coincidence” Phillips curve. Table 3.5 reports results for a regression specification with just inflation and the output gap:

\[ \pi_t = c + \kappa \tilde{y}_t + u_t \]  \hspace{1cm} (3.4)

Table 3.6 reports results for the preferred specification with inflation expectations:

\[ \pi_t = c + \rho \tilde{\pi}_{t+1} + \kappa \tilde{y}_t + u_t \]  \hspace{1cm} (3.5)

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>consumer prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>-3.00</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

Table 3.4: Calibrated slope of the Phillips curve \( (\gamma = 1, \varphi = 2) \)

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-3.8814***</td>
<td>-0.2832***</td>
<td>-0.1839***</td>
<td>-0.1667**</td>
<td>-0.1007***</td>
</tr>
<tr>
<td></td>
<td>(0.6329)</td>
<td>(0.0729)</td>
<td>(0.0642)</td>
<td>(0.0628)</td>
<td>(0.0565)</td>
</tr>
<tr>
<td>intercept</td>
<td>1.9842***</td>
<td>2.9052***</td>
<td>2.9021***</td>
<td>2.3978**</td>
<td>2.372**</td>
</tr>
<tr>
<td></td>
<td>(0.0475)</td>
<td>(0.1196)</td>
<td>(0.1052)</td>
<td>(0.103)</td>
<td>(0.0926)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2154</td>
<td>0.0991</td>
<td>0.0566</td>
<td>0.0489</td>
<td>0.0227</td>
</tr>
</tbody>
</table>

Table 3.5: Regression results for the CBO unemployment gap
Table 3.6: Regression results for the CBO unemployment gap, with expectations

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-1.1054*</td>
<td>-0.1613*</td>
<td>-0.0344</td>
<td>-0.062</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td>(0.3275)</td>
<td>(0.0809)</td>
<td>(0.052)</td>
<td>(0.0487)</td>
<td>(0.0368)</td>
</tr>
<tr>
<td>inflation expectations</td>
<td>0.8287*</td>
<td>0.4846*</td>
<td>0.5446*</td>
<td>0.6364*</td>
<td>0.6406*</td>
</tr>
<tr>
<td></td>
<td>(0.0383)</td>
<td>(0.1557)</td>
<td>(0.0559)</td>
<td>(0.0621)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3484*</td>
<td>1.3851*</td>
<td>1.3193*</td>
<td>0.5522*</td>
<td>0.8388*</td>
</tr>
<tr>
<td></td>
<td>(0.0789)</td>
<td>(0.5021)</td>
<td>(0.1818)</td>
<td>(0.196)</td>
<td>(0.1228)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8234</td>
<td>0.159</td>
<td>0.4425</td>
<td>0.4635</td>
<td>0.6072</td>
</tr>
</tbody>
</table>

Two results are worth noting. First, the R-squared is much higher when using the “divine coincidence” inflation index on the left-hand-side. This is consistent with Corollary 3 and Proposition 3. According to the theory, the consumer-price Phillips curve has a large and volatile endogenous residual, whereas the “divine coincidence” index yields a Phillips curve with no endogenous residual. Correspondingly, the output gap has higher explanatory power for the “divine coincidence” index, and specifications that use this index as a left-hand-side variable should have higher R-squared.

Second, the calibrated model predicts well the estimated slope, for both consumer prices and the “divine coincidence” index DC. The model predicts a higher slope when using the “divine coincidence” index, consistent with the fact that the weights in this index have a larger sum than for consumer prices (where they always sum to 1). The mapping between sectoral weights and the slope of the corresponding Phillips curve however is non-trivial, and relies on the propagation mechanism described in Section 1.3.1.2. Therefore our result can be viewed as a validation of this mechanism.

Finally, I provide a further validation of my theoretical framework. I consider a specification that augments (3.4) to include the time series of the endogenous residual constructed in Section 3.4.2. The new regression equation is:

$$\pi_t = c + \kappa \bar{y}_t + u_t^C + v_t$$
where $u_t^C$ is the endogenous component of the residual $u_t$ in (??), and $v_t$ is the exogenous component. The results are reported in Table 3.28.

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost-push</td>
<td>0.5627*</td>
<td>2.5545**</td>
<td>0.4886</td>
<td>2.3948**</td>
<td>1.1224**</td>
</tr>
<tr>
<td>gap</td>
<td>-3.7586**</td>
<td>-0.1906**</td>
<td>-0.2175**</td>
<td>-0.0783</td>
<td>-0.0886</td>
</tr>
<tr>
<td>intercept</td>
<td>2.0842**</td>
<td>3.2239**</td>
<td>2.8559**</td>
<td>2.6509**</td>
<td>2.397**</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.3317</td>
<td>0.2782</td>
<td>0.142</td>
<td>0.2558</td>
<td>0.1275</td>
</tr>
</tbody>
</table>

Table 3.7: Regression results for the CBO unemployment gap, with CP shock

Comparing Tables 3.5 and 3.28, we see that including our proxy for the endogenous component of the residual brings the R-squared for consumer price regressions close to the “divine coincidence” specification. The result holds for both CPI and PCE, but not for their core versions. This is consistent with the model, as core inflation excludes flexible sectors (such as food and energy) which are among the main drivers of the residual.

All the results presented in this section are robust: they carry over to other specifications which include lags, use other measures of the output gap, and regress gap levels on inflation changes. Additional specifications and residual plots are reported in Appendix E2.

3.5.3 Rolling regressions

In this Section I test the stability of the results in Section 3.5.2 over time. I run rolling Phillips curve regressions with a 20 year window, over the period January 1984 - July 2018. As in Section 3.5.2 I use different inflation measures as left-hand-side variables (DC, CPI, PCE, core CPI, core PCE).

Here I report results for the preferred specification (3.5) with inflation expectations, using the CBO unemployment gap as right-hand-side variable. Appendix E3 reports results for different measures of the output gap and for other specifications.
Figure 3.9 compares the strength and stability of the estimated relation for different left-hand-side variables.

Figure 3.9: Summary statistics for rolling Phillips curve regressions (unemployment gap)

The left panel reports the average R-squared over the sample period, the middle panel reports the fraction of windows in which the estimated coefficient is significant, and the right panel provides a measure of the stability of the estimated coefficient, given by its standard deviation relative to the mean. The figure shows that DC dominates consumer prices along all three dimensions: the R-squareds are consistently higher, the estimated coefficient is always significant and the variance of the estimate is lower. Plots of the estimated coefficients and confidence intervals are reported in Appendix E3.
Conclusion

This paper extends the New Keynesian framework to incorporate a realistic representation of production, with multiple sectors arranged in a general input-output network. I solve the model analytically, providing an exact counterpart to the traditional results in the multi-sector framework. I construct two novel indicators, which respectively inherit the positive and normative properties of inflation in the one-sector model. Both are different from consumer price inflation, which is the most commonly used indicator. To build to this result, I derive analytical expressions for the Phillips curve and welfare as a function of the underlying production structure.

With respect to the baseline model, the consumer-price Phillips curve is flatter, and productivity shocks generate an endogenous inflation-output tradeoff. Empirical results are consistent with the theoretical predictions, and confirm that the departures from the one-sector benchmark are quantitatively important.

I use the calibrated model to evaluate the performance of the two standard targets in the Taylor rule, the output gap and consumer inflation, against the optimal one. I find that targeting the output gap is close to optimal, while stabilizing consumer prices generates an expected loss of 0.8% of per-period GDP relative to the optimal policy.
References


Stock, J. and M. Watson (1999), Forecasting Inflation. Journal of Monetary Economics,
v44(2,Oct), 293-335.

Appendix

Appendix A: Positive analysis

A1: Natural output and output gap

This Appendix presents two basic results: it derives the elasticity of efficient output with respect to productivity (Lemma 9), and it shows that in the sticky-price economy there is no first-order loss in aggregate productivity due to misallocation (Lemma 10).

Lemma 10 and Equation (3.16) imply that the output gap \( \bar{y} \) can be interpreted equivalently as a deviation of total output or of total labor supply from the efficient level:

\[
\bar{y} = d\log Y - d\log Y^{nat} = d\log L - d\log L^{nat}
\]

(3.6)

Lemma 9. The change in efficient output after a productivity shock \( d\log A \) is given by

\[
y^{nat} = \frac{1 + \varphi}{\gamma + \varphi} \lambda^T d\log A
\]

(3.7)

Proof. The flex-price equilibrium allocation is efficient. Therefore it can be derived as the solution of the planning problem

\[
\max_{L, \{y_i, \{x_{ij}\}\}} \frac{C \left( \{y_i\}_{i=1}^N \right)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \quad \text{s.t.} \quad y_i + \sum_j x_{ij} = A_i F_i (\{x_{ij}\}, L_i) \quad \forall i
\]

(3.8)
The change in natural output is then given by

\[ y_{nat} = \frac{d \log C^*}{d \log A_i} \]

where

\[ C^* \equiv C(\{y_i^*\}_{i=1}^N) \]

is aggregate output under the optimal allocation.

The optimization problem in (3.8) can be solved in two steps: first, we choose \( \{L_i, y_i, \{x_{ij}\}\} \) for given \( L \); then we choose the optimal \( L \). Formally, solving problem (3.8) is equivalent to solving

\[
\frac{C^*(L; A)^{1-\gamma}}{1-\gamma} = \max_{\{L_i, y_i, \{x_{ij}\}\}} \frac{C(\{y_i\})^{1-\gamma}}{1-\gamma} \quad \text{s.t.} \quad y_i + \sum_j x_{ij} = A_i F_i (\{x_{ij}\}, L_i) \quad \forall i \\
\sum_i L_i = L
\]

and

\[
\max_L \frac{C^*(L; A)^{1-\gamma}}{1-\gamma} = \frac{L^{1+\varphi}}{1+\varphi}
\]

The solution of (3.10) must satisfy

\[ C^*(L; A)^\gamma L^\varphi = \frac{\partial C^*}{\partial L} \]

Using the envelope theorem in problem (3.9) we have that

\[ \frac{\partial C^*}{\partial L} = C^{*\gamma} \nu_L(A) \]

where \( \nu_L \) is the Lagrange multiplier associated to the constraint \( \sum_i L_i = L \). Moreover, from the first order condition

\[ L^\varphi = \nu_L(A) \]
we have
\[ \frac{d \log L}{d \log A_i} = \frac{1}{\varphi} \frac{d \log \nu_L}{d \log A_i} \]

Applying again the envelope theorem to problem (3.9), we have
\[ \frac{d \log C^*}{d \log A_i} = C^{*\gamma} \left( \frac{\nu_L L \ d \log \nu_L}{\varphi C^* \ d \log A_i} + \frac{\nu_i F_i (\{x_{ij}\}, L_i)}{C^*} \right) \]  \hspace{1cm} (3.11)

We now re-write the two elements on the right hand side of equation (3.11). First, we show that
\[ C^{*\gamma} \frac{\nu_i F_i (\{x_{ij}\}, L_i)}{C^*} = \lambda_i \]  \hspace{1cm} (3.12)

where \( \lambda_i \) is the share of \( i \)'s sales in GDP; second, we show that
\[ C^{*\gamma} \frac{\nu_L L \ d \log \nu_L}{\varphi C^* \ d \log A_i} = \frac{1}{\varphi} \lambda_i - \frac{\gamma}{\varphi} \frac{d \log C^*}{d \log A_i} \]  \hspace{1cm} (3.13)

Putting these two results together in turn implies that
\[ \frac{d \log C^*}{d \log A_i} = \frac{1 + \varphi}{\gamma + \varphi} \lambda_i \]

which is the result that we set out to demonstrate.

We first prove (3.12). To do this, we show that in the competitive equilibrium \( C^{*\gamma} \nu_i \) is equal to the price of good \( i \) relative to the CPI. It then follows from the definition of the sales share \( \lambda_i \) that
\[ \frac{C^{*\gamma} \nu_i F_i (\{x_{ij}\}, L_i)}{C^*} = \frac{p_i F_i (\{x_{ij}\}, L_i)}{PC^*} = \lambda_i \]

From the FOCs of problem (3.9), we have that \( C_i = C^\gamma \nu_i \), and from consumer optimization
in the competitive equilibrium we have \( \frac{C_j}{C_i} = \frac{p_j}{p_i} \). Thus

\[
\frac{C_j}{C_i} = \frac{\nu_j}{\nu_i} = \frac{p_j}{p_i}
\]

Using the fact that \( C \) is homogeneous of degree one, and normalizing the CPI to 1 (\( \sum_j \frac{p_j y_j}{C} = 1 \)), we have

\[
1 = \sum_j \frac{C_j y_j}{C_i} = \frac{C}{C_i} \Rightarrow p_i = C_i
\]

The FOCs for (3.9) in turn imply that \( p_i = C^\gamma \nu_i \).

Let’s now derive equation (3.13). From the FOCs of (3.9) it holds that \( C^\gamma \nu_L = C^\gamma \nu_i A_i F_i L = p_i A_i F_i L = w \ \forall i \), where the last equality follows from firm optimization in the competitive equilibrium. Moreover, from the consumers’ budget constraint we have that \( w = \frac{C^*}{L} \). Thus

\[
C^\gamma \nu_L \frac{d \log \nu_L}{\varphi C^* \frac{d \log A_i}{d \log A_i}} = \frac{1}{\varphi} \left( \frac{d \log w}{d \log A_i} - \gamma \frac{d \log C^*}{d \log A_i} \right)
\]

To conclude the proof we need to show that

\[
\frac{d \log w}{d \log A_i} = \lambda_i
\]

Using again the consumers’ budget constraint we have

\[
\frac{d \log w}{d \log A_i} = \frac{\partial \log C^*}{\partial \log A_i} + \left( \frac{\partial \log C^*}{\partial \log L} - 1 \right) \frac{d \log L}{d \log A_i} = \lambda_i
\]

The intuition for this result is simple. From Hulten’s theorem, under flexible prices the first-order change in aggregate productivity is a weighted sum of sector-level productivity.
shocks, with weights given by sales shares $\lambda$:

$$d \log A_{AGG} = \lambda^T d \log A$$  \hspace{1cm} (3.14)$$

In the efficient (flex-price) economy, the equilibrium change in labor supply can be derived from the optimal consumption-leisure trade-off. It is equal to

$$d \log L^{nat} = \frac{1 - \gamma}{\gamma + \varphi} \lambda^T d \log A$$  \hspace{1cm} (3.15)$$

Finally, aggregate output can be derived as a function of aggregate labor supply and aggregate productivity:

$$Y = A_{AGG}L$$  \hspace{1cm} (3.16)$$

Log-linearizing equation (3.16) we obtain

$$y^{nat} = d \log L^{nat} + d \log A_{AGG}$$  \hspace{1cm} (3.17)$$

Equation (3.7) follows immediately from (3.14), (3.15) and (3.17).

**Lemma 10.** *Around the undistorted steady-state, the first order change in aggregate productivity in the economy with price rigidities is the same as in the economy with flexible prices.*

**Proof.** The flex-price allocation is efficient. This implies that productivity is maximized by optimally allocating labor both within and across sectors. With sticky prices, instead, after a productivity shock the labor allocation is distorted. This happens because the firms who cannot adjust their price absorb cost changes into their markup. Formally, we can derive

$^{14}$There is a second order productivity loss due to incomplete price adjustment. See Section 4.2
the efficient equilibrium as the solution of the problem

\[
\max_{L, \{L_{if}, y_{ij}, \{x_{ijf} \}, \{\mu_{if} \}} \frac{C \left( \{y_i \}^{N}_{i=1} \right)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi}
\]

\[
y_i + \sum_j x_{ij} = A_i \left[ \int (F_i(\{x_{ijf} \}, L_{if}))^{\epsilon_i} df \right]^{e_i} \forall i
\]

\[
s.t. \quad \frac{F_i(\{x_{ijf} \}, L_{if})}{F_i(\{x_{ijg} \}, L_{ig})} = \left( \frac{\mu_{if}}{\mu_{ig}} \right)^{-\epsilon_i} \forall i, f, g
\]

\[
\sum_{if} L_{if} = L
\]

where \( \mu_{if} \) is the markup of firm \( f \) in sector \( i \). In the efficient equilibrium we have \( \mu_{if}^* = 1 \) \( \forall i, f \). The sticky-price allocation instead solves a modified version of (3.18), where the markups of non-adjusting firms are constrained to be equal to their value in the sticky-price equilibrium. Applying the envelope theorem to problem (3.18) we find that, around the efficient equilibrium, the first-order productivity loss induced by these markup distortions is zero.

A2: Sector-level inflation

Definitions  We first introduce two definitions which will be useful in the proofs to follow.

Definition 3. The cost-based input-output matrix \( \tilde{\Omega} \) is an \( N \times N \) matrix with element \( i, j \) given by the expenditure share on input \( j \) in \( i \)'s cost:

\[
\tilde{\omega}_{ij} = \frac{p_j x_{ij}}{mc_i y_i}
\]

Definition 4. The sector-level steady-state labor shares in marginal costs are encoded in the \( N \times 1 \) vector \( \tilde{\alpha} \) with components

\[
\tilde{\alpha}_i = \frac{w L_i}{mc_i y_i}
\]

In a steady-state with optimal subsidies it holds that \( \Omega = \tilde{\Omega} \) and \( \alpha = \tilde{\alpha} \).
Proof of Propositions 2 and 1

Since the proofs of these two propositions rely on the same calculations, we will merge them together.

Our objective is to derive the elasticities of sector-level prices with respect to productivity and the output gap. To do this, we first solve for the change in marginal costs as a function of the change in prices, wages and productivity. We will then solve for the endogenous response of prices and wages to productivity shocks and the output gap.

The change in marginal costs is given by:

\[ d \log mc_i = \bar{\alpha}_i d \log w + \sum_j \bar{\omega}_{ij} d \log p_j - d \log A_i \]

We can then write the change in sectoral prices as function of the change in marginal costs using the Calvo assumption:

\[ d \log p_i = \delta_i d \log mc_i \quad (3.19) \]

so that

\[ d \log mc_i = \bar{\alpha}_i d \log w - d \log A_i + \sum_j \bar{\omega}_{ij} \delta_j d \log mc_j \]

This allows to solve for the change in marginal cost as a function of the change in wages and productivity:

\[ d \log mc = \left( I - \tilde{\Omega} \Delta \right)^{-1} (\bar{\alpha} d \log w - d \log A) \quad (3.20) \]

The change in consumer prices is

\[ d \log P = \beta^T d \log p = \beta^T \Delta d \log mc = \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} (\bar{\alpha} d \log w - d \log A) \quad (3.21) \]
From the consumption-leisure trade-off, we have
\[
d \log w = d \log P + (\varphi d \log L + \gamma d \log y) = \]
\[
= (\varphi d \log L + \gamma \tilde{y} + \gamma \tilde{y}^{nat} + d \log P) = \]
\[
= ((\gamma + \varphi) \tilde{y} + \lambda^T d \log A + d \log P) \]

We can then use \([3.21]\) to solve for the change in wages as a function of the output gap and productivity shocks. We have:
\[
d \log w - d \log P = \left(1 - \beta^T \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1} \tilde{\alpha}\right) d \log w + \beta^T \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1} d \log A = \]
\[
= (\gamma + \varphi) \tilde{y} + \lambda^T d \log A \]
so that
\[
d \log w = \frac{(\gamma + \varphi) \tilde{y} + \beta^T \left[\left(I - \tilde{\Omega}\right)^{-1} \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1}\right] d \log A}{1 - \beta^T \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1} \tilde{\alpha}} \tag{3.22} \]

Lemma 11 shows that the denominator in \([3.22]\) is always well defined.

**Lemma 11.** \(1 - \beta^T \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1} \tilde{\alpha} > 0.\)

**Proof.** First note that, by definition of labor and input shares, it holds that \(\tilde{\alpha} = (I - \Omega) \mathbf{1},\) where \(\mathbf{1}\) is a \(N \times 1\) vector with all entries equal to 1. Thus we have that
\[
\beta^T \left(I - \tilde{\Omega}\right)^{-1} \tilde{\alpha} = \beta^T \left(I - \tilde{\Omega}\right)^{-1} (I - \Omega) \mathbf{1} = \]
\[
\beta^T \mathbf{1} = \sum_j \beta_j = 1 \]

To prove Lemma 11 it is enough to show that
\[
\beta^T \Delta \left(I - \tilde{\Omega} \Delta\right)^{-1} \tilde{\alpha} < \beta^T \left(I - \tilde{\Omega}\right)^{-1} \tilde{\alpha} \]
A sufficient condition for this to hold is that

$$\Delta \left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij} < (I - \Omega)^{-1}_{ij} \ \forall i, j$$

Note that

$$\Delta \left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij} = \delta_i \left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij} < \left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij}$$

therefore it is sufficient to prove that

$$\left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij} < (I - \Omega)^{-1}_{ij} \ \forall i, j$$

We can do so using the relations

$$\left( I - \tilde{\Omega} \Delta \right)^{-1} = I + \tilde{\Omega} + \left( \tilde{\Omega} \Delta \right)^2 + ...$$

$$\left( I - \Omega \right)^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + ...$$

This yields

$$\left( I - \tilde{\Omega} \Delta \right)^{-1}_{ij} = \mathbb{I}(i = j) + \omega_{ij} \delta_j + \sum_k \omega_{ik} \omega_{kj} \delta_j \delta_k + ... <$$

$$\mathbb{I}(i = j) + \omega_{ij} + \sum_k \omega_{ik} \omega_{kj} + ... = \left( I - \tilde{\Omega} \right)^{-1}_{ij}$$

which proves our result.

To find marginal costs as function of the output gap and productivity shocks, we plug plug (3.22) into (3.20):

$$d \log mc = \frac{(\gamma + \varphi) \left( I - \tilde{\Omega} \Delta \right)^{-1}_{\tilde{\alpha}}}{1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1}_{\tilde{\alpha}}} \bar{y} +$$
\[
(I - \tilde{\Omega} \Delta)^{-1} \left( \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] \frac{1}{1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha}} - I \right) d \log A
\]

From the Calvo assumption \([3.19]\), the price response is

\[
\pi = (\gamma + \varphi) \frac{\Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha}}{1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha}} \tilde{y}^+
\]

\[
\Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \left( \tilde{B} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] \frac{1}{1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha}} - I \right) d \log A \quad (3.23)
\]

The expressions for the elasticities \(B\) and \(V\) in Section 3.2 follow immediately from \([3.23]\).

**Proof of Lemma 2:**

In our setup labor is the only factor of production. Therefore labor and input shares must sum to one:

\[
\alpha + \Omega \mathbf{1} = \mathbf{1}
\]

so that \((I - \Omega)^{-1} \alpha = \mathbf{1}\). The result

\[
\exists i, j \text{ such that } \omega_{ij} \delta_j < \omega_{ij} \implies (I - \Omega \Delta)^{-1} \alpha < \mathbf{1}
\]

follows immediately from the fact that each term in the geometric sum

\[
(I - \Omega \Delta)^{-1} \alpha = (I + \Omega \Delta + (\Omega \Delta)^2 + ...) \alpha
\]

has at least one component that is smaller than in the corresponding term of

\[
(I - \Omega)^{-1} \alpha = (I + \Omega + \Omega^2 + ...) \alpha
\]
It then follows that

\[ \tilde{\delta}_w = \sum_i \beta_i \delta_i \left[ (I - \Omega\Delta)^{-1} \alpha \right]_i < \sum_i \beta_i \delta_i \equiv E_\beta(\delta) \]

Equation 1.15 is obtained by differentiating (1.11).

**Proof of Corollary 4**

We first show that \( V\alpha = 0 \), that is, \( \alpha \) belongs to \( \ker(V) \).

Recall the expression for \( V \):

\[ V = \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \left[ \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \right] \right. \\
\left. \frac{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha}}{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha} - \tilde{\alpha}} - I \right] \]

Thus we have

\[ V\alpha = \left( I - \tilde{\Omega}\Delta \right)^{-1} \left[ \tilde{\alpha} \frac{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \alpha}{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha} - \tilde{\alpha}} \right] = 0 \]

We then prove that \( \tilde{\alpha} \) is the only element of \( \ker(V) \). Note that for every vector \( x \neq 0 \) such that \( Vx = 0 \) it must hold that

\[ \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \right] \frac{x}{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha}} = \left( I - \tilde{\Omega}\Delta \right)^{-1} x \Leftrightarrow \]

\[ \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \right] \frac{x}{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha}} = x \Leftrightarrow \]

\[ \tilde{\alpha}_i \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \right] \frac{x}{1 - \beta^T \Delta \left( I - \tilde{\Omega}\Delta \right)^{-1} \tilde{\alpha}} = x_i \forall i \]

(3.24)
where
\[
\frac{\left(\lambda^T - \beta^T \Delta (I - \tilde{\Omega} \Delta)^{-1}\right) x}{1 - \beta^T \Delta (I - \tilde{\Omega} \Delta)^{-1}} \tilde{\alpha} \in \mathbb{R} \neq 0
\]
otherwise we would have \( x = 0 \). From (3.24) we then have that
\[
\frac{\alpha_i}{\alpha_j} = \frac{x_i}{x_j} \forall i, j
\]
so that \( x \) is proportional to the vector of labor shares \( \alpha \).

A3: Output gap and aggregate inflation

Proof of Lemma 11

From the consumers’ optimal labor supply decision we have:

\[
(\log w - \log w^{nat}) - (\log P - \log P^{nat}) = \gamma (\log C - \log C^{nat}) + \varphi (\log L - \log L^{nat})
\]

From the definition of output gap we have

\[
\tilde{y} = d \log C - d \log C^{nat}
\]

while Lemma 10 implies that

\[
\log L - \log L^{nat} = d \log C - d \log C^{nat}
\]

Therefore we have

\[
\gamma (\log C - \log C^{nat}) + \varphi (\log L - \log L^{nat}) = (\gamma + \varphi) \tilde{y}
\]

so that

\[
(\log w - \log w^{nat}) - (\log P - \log P^{nat}) = (\gamma + \varphi) \tilde{y} \quad (3.25)
\]
We next need to compute the left hand side of (3.25), which corresponds to the change in real wages induced by markup distortions. To solve for real wages as a function of sector-level markups we first need to consider how nominal wages \( w \) impact marginal costs and prices. We have:

\[
d \log mc_i = \bar{\alpha}_i d \log w + \sum_j \bar{\omega}_{ij} d \log p_j - d \log A_i
\]

and

\[
d \log p_i = d \log mc_i + d \log \mu_i \quad (3.26)
\]

\[
\Rightarrow d \log mc = \left( I - \tilde{\Omega} \right)^{-1} \left( \tilde{\alpha} d \log w - d \log A + \tilde{\Omega} d \log \mu \right) \quad (3.27)
\]

\[
\Rightarrow d \log P = \beta^T (d \log mc + d \log \mu) = d \log w + \tilde{\lambda}^T (d \log \mu - d \log A)
\]

It follows that

\[
d \log w - d \log P = \tilde{\lambda}^T (d \log A - d \log \mu) \quad (3.28)
\]

In the natural outcome the productivity change is the same as in the economy with sticky prices, while markups are constant \((d \log \mu = 0)\). Therefore we have

\[
(\log w - \log w^{nat}) - (\log P - \log P^{nat}) = -\lambda^T d \log \mu \quad (3.29)
\]

Equations (3.25) and (3.29) together give the result.

**Proof of Lemma 2**

We need to prove that all the vectors \( x \neq 0 \) satisfying \( x^T \nu = 0 \) are proportional to \((I - \Delta) \Delta^{-1} \lambda\). Proposition 3 implies that \( \lambda^T (I - \Delta) \Delta^{-1} \nu = 0 \).

Consider then all vectors \( x \) such that \( x^T \nu = 0 \). Note that

\[
x^T \nu = 0 \iff \\
x^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \left[ \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] - \left( 1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha} \right) I \right] = 0
\]

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\[ \implies \tilde{x}^T \left[ \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] - \left( 1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha} \right) I \right] = 0 \tag{3.30} \]

where \( \tilde{x}^T \equiv x^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \).

To prove the Lemma we need to show that all vectors \( \tilde{x} \) satisfying (3.30) are proportional to \( \lambda^T \left( I - \Delta \right) \left( I - \tilde{\Omega} \Delta \right)^{-1} \).

From (3.30) we have the relation

\[ \left( 1 - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \tilde{\alpha} \right) \tilde{x}_j = \tilde{x}^T \tilde{\alpha} \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] \quad \forall j \tag{3.31} \]

The product \( \tilde{x}^T \tilde{\alpha} \) is a scalar, and we must have \( \tilde{x}^T \tilde{\alpha} \neq 0 \), otherwise we would get \( \tilde{x}^T = 0 \) (while we imposed that \( \tilde{x} \neq 0 \)). Therefore (3.31) implies the condition

\[ \frac{\tilde{x}_i}{\tilde{x}_j} = \frac{\left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right]_i}{\left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right]_j} \]

The ratio on the RHS is well defined, because

\[ \left[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right]_j > \left[ \lambda^T - \beta^T \left( I - \tilde{\Omega} \right)^{-1} \right]_j = 0 \quad \forall j \]

(see Lemma 11).

Thus, \( \tilde{x}^T \) must be proportional to the vector

\[ \lambda^T - \beta^T \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} = \beta^T \left[ \left( I - \Omega \right)^{-1} - \Delta \left( I - \tilde{\Omega} \Delta \right)^{-1} \right] = \]

\[ = \beta^T \left[ \left( I - \Omega \right)^{-1} \left( I - \Omega \Delta \right) - \Delta \right] \left( I - \Omega \Delta \right)^{-1} = \]

\[ = \beta^T \left( I - \Omega \right)^{-1} \left( I - \Delta \right) \left( I - \Omega \Delta \right)^{-1} = \lambda^T \left( I - \Delta \right) \left( I - \Omega \Delta \right)^{-1} \]
Appendix B: Optimal policy

B1: Welfare function

Proof of Lemma 3:

From equation (3.27) we have

\[ d \log mc = \left( I - \tilde{\Omega} \right)^{-1} \left( \tilde{a} d \log w - d \log A + \tilde{\Omega} d \log \mu \right) \]

so that

\[ d \log p = d \log w + \left( I - \tilde{\Omega} \right)^{-1} (d \log \mu - d \log A) \]

Therefore for each sector \( i \) we have

\[ (d \log p_i - d \log p_{i}^{nat}) - (d \log w - d \log w^{nat}) = (I - \Omega)^{-1} d \log \mu \]

We can then use the pricing equation (1.1) to substitute for markups as a function of inflation rates.

Proof of Proposition 4:

In what follows, I will use the second-order approximation

\[ \frac{Z - Z^*}{Z} \simeq \log \left( \frac{Z}{Z^*} \right) + \frac{1}{2} \log \left( \frac{Z}{Z^*} \right)^2 \]

I denote by

\[ \tilde{z} = \log \left( \frac{Z}{Z^*} \right) \]

I will prove below that, to the second-order, the log change in output with respect to the efficient equilibrium is given by

\[ \hat{y} = \hat{l} - d \]

where \( d \) is a second order term.
Using this result we can approximate the utility function around the efficient outcome as
\[
\frac{U - U^*}{U_cC} \approx \dot{y} + \frac{1}{2} \dot{y}^2 + \frac{1}{2} \frac{U_{cc}}{U_c} \dot{y}^2 + \frac{U_L L}{U_cC} \left( \dot{i} + \frac{1}{2} \frac{U_{il} N \dot{y}^2}{U_i} \right) =
\]
\[
= \dot{y} + \frac{1 - \gamma}{2} \dot{y}^2 - \left( \dot{i} + \frac{1 + \varphi}{2} \dot{i}^2 \right) =
\]
\[
= \dot{y} + \frac{1 - \gamma}{2} \dot{y}^2 - \left( \dot{y} + d + \frac{1 + \varphi}{2} \dot{y}^2 \right) =
\]
\[
= -\frac{\gamma + \varphi}{2} \dot{y}^2 - d
\]
where the last equality follows from the fact that, to the second order, \( \dot{y}^2 = \ddot{y}^2 \) and \( d^2 = \dot{yd} = 0 \).

I will now derive the approximation
\[
\dot{y} = \dot{i} - d
\]
and the explicit expression for the second order component \( d \).

Lemma 10 proves that \( d \log y = d \log L \) to a first order. Therefore we have
\[
\dot{y} = \dot{i} - d + \text{higher order terms}
\]

Intuitively, the second order term is a productivity loss induced by markup distortions. These markup distortions endogenously arise from productivity shocks when prices are sticky, and have two effects. First, the relative price of different firms within the same sector is distorted with respect to the efficient equilibrium, therefore sector-level productivities are lower (i.e. more labor is required to produce one unit of sectoral output). I will denote the
productivity loss from within-sector price distortions by the vector $a$, with components

$$a_i \equiv \log \left( \frac{Y_i}{F(\{x_{ij}\}, L_i)} \right) - \log A_i$$

where

$$x_{ij} = \int x_{ij}(t) dt$$

$$L_i = \int L_i(t) dt$$

and $A_i$ is the TFP of sector $i$. Second, sector-level markups are also distorted, so that the relative price indexes of different sectors are different from the efficient equilibrium. Cross-sector price distortions result in lower aggregate productivity.

I define sector-level markups as

$$\mu_i = \frac{p_i}{mc_i}$$

where $p_i$ is the sectoral price index (note that the marginal cost is the same for all producers in sector $i$). I derive a first-order approximation of the “within-sector” and the “cross-sector” component of the productivity loss, and then compute the second order approximation around the efficient steady-state.

Note that aggregate productivity $\frac{Y}{L}$ can be expressed as a function of real wages and labor shares. Denoting the aggregate labor share by $\Lambda = \frac{wL}{GDP} = \frac{wL}{PY}$, by definition we can write aggregate output as

$$Y = \frac{1}{\Lambda} \frac{w}{P} L$$

In log deviations from steady-state we have:

$$\dot{Y} = \dot{w} - \dot{P} - \dot{\Lambda} + \dot{L}$$

(3.32)

The first order change in real wages $d \log w - d \log P$ is derived in the proof of Lemma
(see equation (3.29)). Combining (3.29) with (3.32) we obtain the first-order approximation

\[ d \log Y - d \log L = \tilde{\lambda}^T (a - d \log \mu) - d \log \Lambda \]  

(3.33)

We then need to compute \( d \log \Lambda \) as function of the change in sectoral markups and productivities.

The consumers’ budget constraint is

\[ PC = wL + \Pi - T \]

where \( \Pi \) are aggregate profits and \( T \) is a lump-sum tax used to finance input subsidies. Dividing both sides by \( PC \) we find

\[
1 = \Lambda + \frac{\Pi - T}{PY} = \Lambda + \lambda^T \left(1 - \frac{1}{\mu} \right)
\]

where \( \mu \) is the vector of sector-level markups defined above. Therefore we have

\[
d \log \Lambda = -\frac{1}{\Lambda} \left( \sum_i d\lambda_i \left(1 - \frac{1}{\mu_i} \right) + \sum_i \lambda_i \frac{d \log \mu_i}{\mu_i} \right)
\]

Using (3.33) we find that, around the efficient steady state (where \( \mu_i = 1 \ \forall i \))

\[
d \log Y - d \log L = \underbrace{\tilde{\lambda}^T a}_{\text{within sector}} + \underbrace{\left(\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T\right)}_{\text{cross-sector}} d \log \mu
\]

(3.34)

As \( \frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T = 0 \) around \( \mu = 1 \), the first-order productivity loss from cross-sector misallocation is zero. To compute the second-order loss we need to take the second derivative of the cross-sector component in equation (3.34).

Note that, since the first order effect on both cross-sector misallocation and sector-level productivities is zero, the second-order terms in \((d \log A)(d \log \mu)\) are also going to be zero.
Therefore we only need to derive the cross-sector component with respect to sector-level markups. We have:

\[ D^2 \left( \left( \frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T \right) d \log \mu \right) = \]

\[ = \frac{1}{\Lambda} \left( - \left( \sum_i \lambda_i \frac{d \log \mu_i}{\mu_i} \right)^2 + 2 \sum_i d \lambda_i \frac{d \log \mu_i}{\mu_i} + \sum_i \frac{\lambda_i}{\mu_i} (d \log \mu_i)^2 \right) - \sum_i d \tilde{\lambda}_i d \log \mu_i = \]

\[ = -\frac{1}{2} \sum_i \sum_j d_{ij}^2 d \log \mu_i d \log \mu_j \tag{3.35} \]

where

\[ d_{ij}^2 = \sum_h \sum_k \beta_h \beta_k \sigma_{hk} \left[ (I - \Omega)^{-1}_{hi} - (I - \Omega)^{-1}_{ki} \right] \left[ (I - \Omega)^{-1}_{hj} - (I - \Omega)^{-1}_{kj} \right] + \]

\[ + \sum_t \lambda_t \sum_h \sum_k \omega_{th} \omega_{tk} \theta_{hk}^t \left[ (I - \Omega)^{-1}_{hi} - (I - \Omega)^{-1}_{ki} \right] \left[ (I - \Omega)^{-1}_{hj} - (I - \Omega)^{-1}_{kj} \right] + \]

\[ + \sum_t \lambda_t \alpha_t \sum_h \omega_{th} \theta_{hL}^t (I - \Omega)^{-1}_{hi} (I - \Omega)^{-1}_{hj} = \]

\[ = \Phi_C \left( (I - \Omega)^{-1}_{(i)}, (I - \Omega)^{-1}_{(j)} \right) + \sum_t \lambda_t \Phi_t \left( (I - \Omega)^{-1}_{(i)}, (I - \Omega)^{-1}_{(j)} \right) \tag{3.36} \]

To derive the welfare loss as a function of sector-level inflation rates we need to solve for the endogenous change in sector-level markups due to price rigidities. The mapping between the two is given by equation (1.1):

\[ d \log \mu = - (I - \Delta) d \log mc = - (I - \Delta) \Delta^{-1} \pi \]

Therefore we can re-write (3.35) as

\[ d^2 \log Y - d^2 \log L = \tilde{\lambda}^T a - \frac{1}{2} \pi^T D_2 \pi \]

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with

\[ d_{ij}^2 = \frac{1 - \delta_i}{\delta_i} \frac{1 - \delta_j}{\delta_j} d_{ij}^2 \]

It remains to compute the “within-sector” component \( \lambda^T a \).

Index by \( t \) the different varieties of product \( i \) and note that, given the CES assumption, sectoral output can be written as

\[ Y_i = A_i F (\{ x_{ij} \}, L_i) \frac{p_i^{-\epsilon_i}}{\int p_i^{-\epsilon_i} dt} \tag{3.37} \]

where

\[ x_{ij} = \int x_{ij}(t) dt \]

\[ L_i = \int L_i(t) dt \]

as above. Using the definition of \( a \) we have

\[ a_i = \log \left( \frac{p_i^{-\epsilon_i}}{\int p_i^{-\epsilon_i} dt} \right) \]

A first order approximation of \( a_i \) is given by

\[ da_i = \epsilon_i \left[ \frac{\int p_i^{-\epsilon_i} d\log p_i dt}{\int p_i^{-\epsilon_i} dt} - \frac{\int p_i^{1-\epsilon_i} d\log p_i dt}{\int p_i^{1-\epsilon_i} dt} \right] \tag{3.38} \]

Given the Calvo assumption, around the efficient steady state we have that

\[ \frac{\int p_i^{-\epsilon_i} d\log p_i dt}{\int p_i^{-\epsilon_i} dt} = \frac{\int p_i^{1-\epsilon_i} d\log p_i dt}{\int p_i^{1-\epsilon_i} dt} = \delta \log mc_i \]

so that \( da_i = 0 \).

Let’s now compute the second-order loss by deriving \( \text{[3.38]} \) a second time with respect to
\{d\log p_{it}\}$. We find\textsuperscript{15}

\[ d^2 a_i = \epsilon_i \left[ \int (\log p_{it} - \log p_i)^2 \, dt - \left( \int (\log p_{it} - \log p_i) \, dt \right)^2 \right] = \]

\[ = \epsilon_i \frac{1 - \delta_i}{\delta_i} \sigma_i^2 \]

We can thus express the second-order welfare loss from within-sector misallocation as

\[ \frac{1}{2} \pi D_1 \pi \]

where

\[ d^1_{ij} = \begin{cases} 
0 & \text{if } i \neq j \\
\lambda_i \epsilon_i \frac{1 - \delta_i}{\delta_i} & \text{if } i = j 
\end{cases} \]

Lemma \textsuperscript{12} below characterizes inflation, welfare and the optimal policy when pre-set prices at the sector-level are not equal to desired prices. This is captured by a deviation of initial markups $\mu_{-1}$ from their optimal level $\mu_{-1} = 1$. This result will be useful to understand the evolution of inflation in the dynamic version of the model, derived in Appendix C2.

**Lemma 12.** Denote the log-deviation of initial sector-level markups by the vector $d \log \mu_{-1}$. The elasticity of sectoral prices with respect to $\mu_{-1}$ is given by the matrix $\mathcal{V}$. The optimal monetary policy implements the output gap

\[ \tilde{y} = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V} d \log \mu_{-1}}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}} \quad (3.39) \]

\textit{Proof.} Sectoral inflation rates are given by

\[ \pi_i = \delta_i (d \log mc_i - d \log \mu_{i-1}) \]

\textsuperscript{15}This is the same as in the traditional NK model (Gali (2008) Ch.4)
The mapping between sector-level inflation and current period markups is not affected by the presence of past markups, and is still given by (1.32). We proceed as in the proof of Propositions 2 and 1 to derive

\[
\pi = \Delta (I - \Omega \Delta)^{-1} (\alpha \log w - \log \mu_{-1})
\]

and

\[
d \log w = \frac{\gamma + \varphi}{1 - \beta^T (I - \Omega \Delta)^{-1} \alpha} (\tilde{y} - \tilde{y}_{-1}) - \frac{\beta^T \Delta (I - \Omega \Delta)^{-1} \alpha}{1 - \beta^T (I - \Omega \Delta)^{-1} \alpha} \alpha d \log \mu_{-1}
\]

We solve for sectoral inflation rates as a function of \( \tilde{y}, \tilde{y}_{-1} \) and \( d \log \mu_{-1} \) following the same steps as in the proof of Propositions 2 and 1.

Welfare is the same function of the output gap and sectoral inflation rates as in (2.1). This is because welfare depends on sector-level markups and on the variance of firm-level prices within sectors, and the mapping between both of these variables and sectoral inflation rates does not change in the presence of past markups. The optimal output gap follows from the first order conditions.

\[\square\]

B2: Policy target

Proof of Proposition 6:

We look for weights \( \phi \) such that

\[
\phi^T \pi = \phi^T (B \tilde{y} + \mathcal{V} d \log A) > 0 \iff \tilde{y} > \tilde{y}^* \tag{3.40}
\]

I will first construct a vector \( \phi \) that satisfies the condition

\[
\phi^T (B \tilde{y} + \mathcal{V} d \log A) = 0 \iff \tilde{y} = \tilde{y}^* \tag{3.41}
\]

and then argue that this vector also satisfies (3.40).
Note that, as long as $\phi^T B \neq 0$, we have

$$\phi^T (B\tilde{y} + \mathcal{V}d \log A) = 0 \iff \tilde{y} = -\frac{\phi^T \mathcal{V}d \log A}{\phi^T B}$$

while the optimal output gap is

$$\tilde{y}^* = -\frac{B^T \mathcal{D} \mathcal{V}d \log A}{\gamma + \varphi + B^T DB}$$

Thus (3.41) is satisfied for all realizations of $d \log A$ if and only if $\phi$ is such that

$$\frac{\phi^T \mathcal{V}d \log A}{\phi^T B} = \frac{B^T \mathcal{D} \mathcal{V}d \log A}{\gamma + \varphi + B^T DB} \forall d \log A$$

In turn, this is true if and only if

$$\phi^T \left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right] \mathcal{V} = 0 \quad (3.42)$$

that is, if and only if $\phi$ is a left eigenvector of the matrix $\left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right]$ relative to the eigenvalue $0$.

We already proved in Lemma 1 that $\lambda^T (I - \Delta) \Delta^{-1}$ is a left eigenvector of the matrix $\mathcal{V}$ relative to the eigenvalue 0 (and it is the only such eigenvector). Therefore, as long as $\left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right]$ is invertible, $\phi^T = \lambda^T (I - \Delta) \Delta^{-1} \left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right]^{-1}$ is the (unique) desired eigenvector of the matrix $\left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right] \mathcal{V}$.

The matrix $\left[I - \frac{B B^T D}{\gamma + \varphi + B^T DB}\right]$ is indeed invertible: it is immediate to see that $\frac{B B^T D}{\gamma + \varphi + B^T DB}$ has only one non-zero eigenvalue, $\frac{B^T DB}{\gamma + \varphi + B^T DB} < 1$, and $B$ is the unique corresponding eigenvector.

Next, to satisfy condition (3.40) we need

$$\phi^T (B\tilde{y} + \mathcal{V}d \log A)$$

to be increasing in the output gap $\tilde{y}$, which is true if and only if $\phi^T B > 0$. To prove this we use
the fact that $B$ is an eigenvector of $\frac{BB^T D}{\gamma + \varphi + B^T DB}$ relative to the eigenvalue $\frac{B^T DB}{\gamma + \varphi + B^T DB}$. Therefore it is also an eigenvector of $\left[I - \frac{BB^T D}{\gamma + \varphi + B^T DB}\right]^{-1}$, relative to the eigenvalue $\frac{\gamma + \varphi + B^T DB}{\gamma + \varphi} > 1$. Thus we have

$$\phi^T B = \lambda^T (I - \Delta) \Delta^{-1} \left[I - \frac{BB^T D}{\gamma + \varphi + B^T DB}\right]^{-1} B =$$

$$= \gamma + \varphi + B^T DB > 0$$

Finally, to obtain the formulation in (2.10) we observe that

$$\left[I - \frac{BB^T D}{\gamma + \varphi + B^T DB}\right]^{-1} = I + \frac{BB^T D}{\gamma + \varphi + B^T DB} + \left(\frac{BB^T D}{\gamma + \varphi + B^T DB}\right)^2 + ...$$

and

$$\left(\frac{BB^T D}{\gamma + \varphi + B^T DB}\right)^n = \left(\frac{B^T DB}{\gamma + \varphi + B^T DB}\right)^{n-1} \frac{BB^T D}{\gamma + \varphi + B^T DB}$$

so that

$$\left[I - \frac{BB^T D}{\gamma + \varphi + B^T DB}\right]^{-1} = I + \frac{BB^T D}{\gamma + \varphi}$$

Moreover, we have that

$$\frac{\lambda^T (I - \Delta) \Delta^{-1} B}{\gamma + \varphi} = \frac{\lambda^T (I - \Delta) (I - \Omega \Delta)^{-1} \alpha}{1 - \beta^T \Delta (I - \Omega \Delta)^{-1} \alpha} = 1$$

so that

$$\lambda^T (I - \Delta) \Delta^{-1} \left[I - \frac{BB^T D}{\gamma + \varphi + B^T DB}\right]^{-1} = \lambda^T (I - \Delta) \Delta^{-1} + B^T D$$

Appendix C: Regressions

Appendix C1: The “divine coincidence” index (time series)

We construct a time series for the “divine coincidence” index $DC$ starting in 1984. This requires to aggregate sector-level price series based on the respective sales shares and ad-
justment frequencies. We compute sales shares from the BEA input-output data, and rely on the price adjustment data collected by Pasten, Schoenle and Weber. The main source for sector-level price series is PPI data from the BLS.

In the BLS dataset the sample period varies across sectors: most manufacturing series are available from the mid-1980s, while most service series are available from 2006 onwards. Out of the 405 sectors in the BEA classification, 172 have an incomplete price series in the BLS dataset, and 67 are missing. Information about the incomplete and missing series (sector names and weights in the $DC$ index) is reported in Appendix C4.

To extend the incomplete price series further back in time we use sector-level data underlying the PCE, which is available from 1960. We run Lasso regressions of each incomplete PPI series on disaggregated (338 sectors) PCE components for the period in which both are available. Summary statistics for the Lasso regressions are reported in Appendix C4. We also use PCE components to make up for 40 missing series, using the concordance table between NAICS sectors and PCE series provided by the BEA.

Figure 3.10 compares the weights assigned to different sectors by the divine coincidence index $DC$ and the PCE (the main indicator used by central banks), at an aggregated 21-sector level. Sectors are ordered by their weight in the “divine coincidence” index. The bars with red borders corresponds to PCE weights. Sectoral weights at a more disaggregated level are reported in Appendix C4.
We see from the figure that wages have the highest weight (of 18%) in $DC$, while they are not part of the PCE. The divine coincidence index also assigns high weight to professional services, durable goods, and IT and administrative services. These sectors have a large input share in production and adjust prices infrequently. By contrast the PCE places the highest weight on healthcare, housing and non-durable goods. These sectors capture a large share of consumer expenditures, but are not important as inputs in production. Therefore their relative consumption share is much larger than their relative sales share, which is why they have a smaller weight in the divine coincidence index relative to the PCE.

Appendix C2: Regressions over the full sample period

This section contains a robustness check for the regressions presented in Section 3.5.2. It shows results for different measures of the output gap on the right hand side, and for different specifications.

Tables ?? and 3.9 below present results for the baseline specification in equation (3.43), using the CBO output gap and the unemployment rate as right hand side variables respectively.

$$\pi_t = c + \kappa \tilde{y}_t + u_t$$  (3.43)
<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
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<td>(0.0618)</td>
<td>(0.055)</td>
<td>(0.0532)</td>
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<tr>
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<td>3.0193**</td>
<td>2.9661**</td>
<td>2.4878**</td>
<td>2.4325**</td>
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<tr>
<td></td>
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<td>(0.1271)</td>
<td>(0.1131)</td>
<td>(0.1095)</td>
<td>(0.0992)</td>
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<td>0.08</td>
<td>0.0407</td>
</tr>
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Table 3.8: Regression results for the CBO output gap

<table>
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<tr>
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<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
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<td>-0.036</td>
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<td>(0.0661)</td>
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<td>2.7595**</td>
<td>2.2906**</td>
<td>2.2514**</td>
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<td></td>
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<td>(0.1259)</td>
<td>(0.1096)</td>
<td>(0.1067)</td>
<td>(0.0945)</td>
</tr>
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<td>R-squared</td>
<td>0.1359</td>
<td>0.0244</td>
<td>0</td>
<td>0.0023</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

Table 3.9: Regression results for the unemployment rate
Tables 3.10, 3.11 and 3.12 below present results for the baseline specification augmented with oil price inflation, as in equation (3.44). The gap measures are given by the CBO unemployment gap, the CBO output gap and the unemployment rate respectively.

\[
\pi_t = c + \kappa y_t + \pi_{oil} + u_t
\]  

(3.44)

<table>
<thead>
<tr>
<th></th>
<th>SW CPI core CPI</th>
<th>PCE core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-3.6385**</td>
<td>-0.2198**</td>
</tr>
<tr>
<td></td>
<td>(0.6294)</td>
<td>(0.0655)</td>
</tr>
<tr>
<td>intercept</td>
<td>1.9532**</td>
<td>2.7286**</td>
</tr>
<tr>
<td></td>
<td>(0.0483)</td>
<td>(0.1099)</td>
</tr>
<tr>
<td>oil prices</td>
<td>0.0032**</td>
<td>0.0185**</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2488</td>
<td>0.2959</td>
</tr>
</tbody>
</table>

Table 3.10: Regression results for the CBO unemployment gap , with oil prices

<table>
<thead>
<tr>
<th></th>
<th>SW CPI core CPI</th>
<th>PCE core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>2.8985**</td>
<td>0.2137**</td>
</tr>
<tr>
<td></td>
<td>(0.5562)</td>
<td>(0.0562)</td>
</tr>
<tr>
<td>intercept</td>
<td>1.9843**</td>
<td>2.8179**</td>
</tr>
<tr>
<td></td>
<td>(0.0536)</td>
<td>(0.1184)</td>
</tr>
<tr>
<td>oil prices</td>
<td>0.0031**</td>
<td>0.0179**</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2199</td>
<td>0.3108</td>
</tr>
</tbody>
</table>

Table 3.11: Regression results for the CBO output gap , with oil prices

<table>
<thead>
<tr>
<th></th>
<th>SW CPI core CPI</th>
<th>PCE core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-2.7813**</td>
<td>-0.0591</td>
</tr>
<tr>
<td></td>
<td>(0.6655)</td>
<td>(0.0684)</td>
</tr>
<tr>
<td>intercept</td>
<td>1.9278**</td>
<td>2.601**</td>
</tr>
<tr>
<td></td>
<td>(0.0516)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>oil prices</td>
<td>0.0033**</td>
<td>0.0196**</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.1707</td>
<td>0.2418</td>
</tr>
</tbody>
</table>

Table 3.12: Regression results for the unemployment rate , with oil prices
Tables 3.26 and 3.27 present results for our preferred specification (3.45) with inflation expectations, using the CBO output gap and the unemployment rate as gap measures.

\[
\pi_t = c + \kappa \bar{y}_t + \rho \hat{\pi}_{t+1} + \epsilon_t
\]  

(3.45)

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>1.0861**</td>
<td>0.1881**</td>
<td>0.0412</td>
<td>0.0881**</td>
<td>0.0084</td>
</tr>
<tr>
<td></td>
<td>(0.2714)</td>
<td>(0.0678)</td>
<td>(0.0449)</td>
<td>(0.0417)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>inflation expectations</td>
<td>0.8297**</td>
<td>0.4412**</td>
<td>0.5398**</td>
<td>0.6231**</td>
<td>0.6365**</td>
</tr>
<tr>
<td></td>
<td>(0.0368)</td>
<td>(0.1515)</td>
<td>(0.0561)</td>
<td>(0.0617)</td>
<td>(0.0455)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3668**</td>
<td>1.6124**</td>
<td>1.3548**</td>
<td>0.6459**</td>
<td>0.8614**</td>
</tr>
<tr>
<td></td>
<td>(0.0772)</td>
<td>(0.4987)</td>
<td>(0.1892)</td>
<td>(0.2005)</td>
<td>(0.1291)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8288</td>
<td>0.1808</td>
<td>0.4442</td>
<td>0.4744</td>
<td>0.6073</td>
</tr>
</tbody>
</table>

Table 3.13: Regression results for the CBO output gap, with expectations

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-0.9404**</td>
<td>-0.0049</td>
<td>0.0781</td>
<td>-0.0505</td>
<td>0.0757**</td>
</tr>
<tr>
<td></td>
<td>(0.3185)</td>
<td>(0.0788)</td>
<td>(0.0499)</td>
<td>(0.0477)</td>
<td>(0.0355)</td>
</tr>
<tr>
<td>inflation expectations</td>
<td>0.8468**</td>
<td>0.6312**</td>
<td>0.5668**</td>
<td>0.6549**</td>
<td>0.6432**</td>
</tr>
<tr>
<td></td>
<td>(0.0372)</td>
<td>(0.1518)</td>
<td>(0.0537)</td>
<td>(0.0608)</td>
<td>(0.0434)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3108**</td>
<td>0.8344*</td>
<td>1.1705**</td>
<td>0.4941**</td>
<td>0.7757**</td>
</tr>
<tr>
<td></td>
<td>(0.0762)</td>
<td>(0.4879)</td>
<td>(0.1711)</td>
<td>(0.1851)</td>
<td>(0.1155)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8202</td>
<td>0.1344</td>
<td>0.4507</td>
<td>0.4616</td>
<td>0.6198</td>
</tr>
</tbody>
</table>

Table 3.14: Regression results for the unemployment rate, with expectations
Finally, Tables 3.15, 3.16 and 3.17 present results for the specification in equation (3.46) with inflation changes on the left hand side (instead of inflation levels). We present results for our three usual gap measures (CBO unemployment gap, CBO output gap and unemployment rate). All of them are in levels.

\[ \pi_t - \pi_{t-1} = c + \kappa \bar{y}_t + u_t \]  

(3.46)

<table>
<thead>
<tr>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-0.6945**</td>
<td>-0.0287</td>
<td>-0.0212*</td>
<td>-0.0169</td>
</tr>
<tr>
<td></td>
<td>(0.3258)</td>
<td>(0.0404)</td>
<td>(0.0119)</td>
<td>(0.0296)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.0182</td>
<td>0.008</td>
<td>-0.003</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(0.0244)</td>
<td>(0.0661)</td>
<td>(0.0195)</td>
<td>(0.0485)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0323</td>
<td>0.0037</td>
<td>0.0227</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

Table 3.15: Regression results for the CBO unemployment gap (inflation changes)

<table>
<thead>
<tr>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>0.7805**</td>
<td>0.0472</td>
<td>0.02*</td>
<td>0.0299</td>
</tr>
<tr>
<td></td>
<td>(0.2768)</td>
<td>(0.0345)</td>
<td>(0.0102)</td>
<td>(0.0254)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.0365</td>
<td>0.0422</td>
<td>0.0046</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.0262)</td>
<td>(0.0713)</td>
<td>(0.0211)</td>
<td>(0.0524)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0552</td>
<td>0.0136</td>
<td>0.0273</td>
<td>0.0101</td>
</tr>
</tbody>
</table>

Table 3.16: Regression results for the CBO output gap (inflation changes)

<table>
<thead>
<tr>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-0.7464**</td>
<td>-0.0317</td>
<td>-0.0238**</td>
<td>-0.0199</td>
</tr>
<tr>
<td></td>
<td>(0.3264)</td>
<td>(0.0405)</td>
<td>(0.0119)</td>
<td>(0.0297)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.0212</td>
<td>0.0113</td>
<td>-0.0002</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0247)</td>
<td>(0.0668)</td>
<td>(0.0197)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.037</td>
<td>0.0045</td>
<td>0.0284</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Table 3.17: Regression results for the unemployment rate (inflation changes)
Appendix C3: Rolling regressions

The figures below provide additional detail for the rolling regressions introduced in Section 3.5.3. They plot estimated coefficients for each 20-year window (with confidence intervals), and average R-squareds over the sample. The years on the x-axis correspond to the middle of the estimation window.

We report results for our preferred specification with inflation expectations, as in equation (3.47). Appendix C5 reports results for alternative specifications and alternative measures of the gap on the right hand side.

\[ \pi_t = \kappa \bar{\gamma}_t + \rho \pi_{t+1} + \epsilon_t \]  

(3.47)
Appendix C4

Sectoral weights

Table 3.18 reports the weights of the top-15 sectors in DC in percentage of the total (at the disaggregated 405 sector level).
<table>
<thead>
<tr>
<th>Industry name</th>
<th>Weight (SW)</th>
<th>Weight (Domar)</th>
<th>Weight (PCE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
<td>18.3221</td>
<td>27.8648</td>
<td>0</td>
</tr>
<tr>
<td>Insurance agencies, brokerages, and related activities</td>
<td>9.23917</td>
<td>1.39786</td>
<td>0</td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td>3.887</td>
<td>1.68309</td>
<td>0</td>
</tr>
<tr>
<td>Architectural, engineering, and related services</td>
<td>2.51957</td>
<td>0.812411</td>
<td>0</td>
</tr>
<tr>
<td>Insurance carriers, except direct life</td>
<td>2.13001</td>
<td>1.04094</td>
<td>2.5369</td>
</tr>
<tr>
<td>Warehousing and storage</td>
<td>2.12367</td>
<td>0.344483</td>
<td>0.0019132</td>
</tr>
<tr>
<td>Accounting, tax preparation, bookkeeping, and payroll services</td>
<td>2.05855</td>
<td>0.53267</td>
<td>0.17815</td>
</tr>
<tr>
<td>Other real estate</td>
<td>2.05001</td>
<td>2.87134</td>
<td>0.057851</td>
</tr>
<tr>
<td>Legal services</td>
<td>1.87954</td>
<td>0.893466</td>
<td>1.0623</td>
</tr>
<tr>
<td>Advertising, public relations, and related services</td>
<td>1.68975</td>
<td>0.415808</td>
<td>0.017779</td>
</tr>
<tr>
<td>Hospitals</td>
<td>1.65114</td>
<td>1.17451</td>
<td>9.6864</td>
</tr>
<tr>
<td>Employment services</td>
<td>1.63912</td>
<td>0.913483</td>
<td>0.012342</td>
</tr>
<tr>
<td>Management consulting services</td>
<td>1.63082</td>
<td>0.569068</td>
<td>0</td>
</tr>
<tr>
<td>Wired telecommunications carriers</td>
<td>1.44281</td>
<td>0.78146</td>
<td>2.0335</td>
</tr>
<tr>
<td>All other miscellaneous professional, scientific, and technical services</td>
<td>1.31821</td>
<td>0.312412</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.18: Weights of top-15 series in DC (in %)

Missing and incomplete series

Tables (3.19) and (3.20) report details of the missing and incomplete series in the PPI dataset. Table (3.21) presents summary statistics from the Lasso regressions used to extend the incomplete series back in time.
<table>
<thead>
<tr>
<th>Activity Description</th>
<th>Weight in SW</th>
<th>Added?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oilseed farming</td>
<td>4.00</td>
<td>0</td>
</tr>
<tr>
<td>Funds, trusts, and other financial vehicles</td>
<td>2.02</td>
<td>1</td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td>0.29</td>
<td>0</td>
</tr>
<tr>
<td>Sound recording industries</td>
<td>0.22</td>
<td>1</td>
</tr>
<tr>
<td>Elementary and secondary schools</td>
<td>0.20</td>
<td>1</td>
</tr>
<tr>
<td>Monetary authorities and depository credit intermediation</td>
<td>0.18</td>
<td>1</td>
</tr>
<tr>
<td>State and local government hospitals and health services</td>
<td>0.13</td>
<td>0</td>
</tr>
<tr>
<td>State and local government passenger transit</td>
<td>0.12</td>
<td>0</td>
</tr>
<tr>
<td>Other educational services</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>Motion picture and video industries</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>Transit and ground passenger transportation</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>Limited-service restaurants</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>Federal general government (nondefense)</td>
<td>0.09</td>
<td>0</td>
</tr>
<tr>
<td>Full-service restaurants</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>Promoters of performing arts and sports and agents for public figures</td>
<td>0.07</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.19: Weights of top-15 missing series in DC (in %)
<table>
<thead>
<tr>
<th>Service</th>
<th>Weight in SW</th>
<th>Initial date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment services</td>
<td>0.85</td>
<td>19940901</td>
</tr>
<tr>
<td>Management consulting services</td>
<td>0.55</td>
<td>20060901</td>
</tr>
<tr>
<td>Insurance agencies, brokerages, and related activities</td>
<td>0.47</td>
<td>20030301</td>
</tr>
<tr>
<td>Architectural, engineering, and related services</td>
<td>0.45</td>
<td>19970301</td>
</tr>
<tr>
<td>Automotive equipment rental and leasing</td>
<td>0.45</td>
<td>19920301</td>
</tr>
<tr>
<td>Custom computer programming services</td>
<td>0.41</td>
<td>20060901</td>
</tr>
<tr>
<td>Specialized design services</td>
<td>0.37</td>
<td>19970301</td>
</tr>
<tr>
<td>Nursing and community care facilities</td>
<td>0.36</td>
<td>20040301</td>
</tr>
<tr>
<td>Services to buildings and dwellings</td>
<td>0.36</td>
<td>19950301</td>
</tr>
<tr>
<td>Environmental and other technical consulting services</td>
<td>0.36</td>
<td>20060901</td>
</tr>
<tr>
<td>Wireless telecommunications carriers (except satellite)</td>
<td>0.31</td>
<td>19930901</td>
</tr>
<tr>
<td>Office administrative services</td>
<td>0.27</td>
<td>19940901</td>
</tr>
<tr>
<td>Satellite, telecommunications resellers, and all other telecommunications</td>
<td>0.23</td>
<td>19930901</td>
</tr>
<tr>
<td>Other computer related services, including facilities management</td>
<td>0.22</td>
<td>20060901</td>
</tr>
<tr>
<td>Internet publishing and broadcasting and Web search portals</td>
<td>0.21</td>
<td>20100301</td>
</tr>
</tbody>
</table>

Table 3.20: Weights of top-15 incomplete series in DC (in %)

<table>
<thead>
<tr>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>127</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 3.21: Number of series in Lasso approximation
Proxy for inflation expectations

Our preferred regression specification controls for inflation expectations. We construct a proxy for the expectations of each of the inflation indexes which are used as left hand side variables, based on the statistical properties of the inflation series (see Stock and Watson (2007)). Inflation changes $\pi_t - \pi_{t-1}$ are well approximated by an IMA(1,1) model. We estimate the parameters of the model for each inflation index, and use it to construct a prediction for future inflation changes, $\mathbb{E}[\pi_{t+1} - \pi_t]$. Inflation expectations are then given by $\mathbb{E}\pi_{t+1} = \pi_t + \mathbb{E}[\pi_{t+1} - \pi_t]$. Figure 3.11 plots the actual inflation series against the expectations series constructed based on the IMA(1,1) model.
Figure 3.11: Time series of actual inflation and expectations
Summary statistics

We report scatterplots of inflation and output gaps for the different inflation and gap measures used in the regressions. Figures (3.12), (3.13) and (3.14) report scatterplots in levels, while Figures (3.15), (3.16) and (3.17) report scatterplots for inflation changes versus gap levels.

Figure 3.12: Scatterplot of inflation and CBO unemployment gap
Figure 3.13: Scatterplot of inflation and CBO output gap
Figure 3.14: Scatterplot of inflation and unemployment rate
Figure 3.15: Scatterplot of inflation changes and CBO unemployment gap
Figure 3.16: Scatterplot of inflation changes and CBO output gap
Figure 3.17: Scatterplot of inflation changes and unemployment rate

Appendix C5

Residual plots

Figures (3.18), (3.19) and (3.20) report residual plots for the baseline specification (3.5) in Section 3.5.2
Figure 3.18: Residual plots for the CBO unemployment gap, with expectations

Figure 3.19: Residual plots for the CBO output gap, with expectations
Other regression specifications

The tables below present results for a regression specification that includes for inflation lags:

\[ \pi_t = c + \kappa \tilde{y}_t + \sum_{s=1}^{4} \gamma_s \pi_{t-s} + u_t \]

Each table is based on a different measure of the output gap (CBO unemployment gap, CBO output gap or unemployment rate).
<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-1.3803**</td>
<td>-0.0696</td>
<td>-0.0232</td>
<td>-0.0285</td>
<td>-0.0069</td>
</tr>
<tr>
<td>intercept</td>
<td>0.6573**</td>
<td>0.7961**</td>
<td>0.367**</td>
<td>0.515**</td>
<td>0.2801**</td>
</tr>
<tr>
<td>lag 1</td>
<td>0.7443**</td>
<td>0.9835**</td>
<td>0.9983**</td>
<td>1.0655**</td>
<td>1.068**</td>
</tr>
<tr>
<td>lag 2</td>
<td>0.1224</td>
<td>-2.051*</td>
<td>-0.0391</td>
<td>-3.083**</td>
<td>-2.537**</td>
</tr>
<tr>
<td>lag 3</td>
<td>-0.1573</td>
<td>-0.0051</td>
<td>-0.068</td>
<td>0.0912</td>
<td>0.1143</td>
</tr>
<tr>
<td>lag 4</td>
<td>-0.0356</td>
<td>-0.0469</td>
<td>-0.0138</td>
<td>-0.0624</td>
<td>-0.0438</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.6954</td>
<td>0.705</td>
<td>0.8465</td>
<td>0.7436</td>
<td>0.8396</td>
</tr>
</tbody>
</table>

Table 3.22: Regression results for the CBO unemployment gap, with lags

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>1.3534**</td>
<td>0.0931**</td>
<td>0.0359</td>
<td>0.0524*</td>
<td>0.0212</td>
</tr>
<tr>
<td>intercept</td>
<td>0.6586**</td>
<td>0.8709**</td>
<td>0.4035**</td>
<td>0.5697**</td>
<td>0.3093**</td>
</tr>
<tr>
<td>lag 1</td>
<td>0.7359**</td>
<td>0.9662**</td>
<td>0.9875**</td>
<td>1.0502**</td>
<td>1.0595**</td>
</tr>
<tr>
<td>lag 2</td>
<td>0.1261</td>
<td>-0.2035*</td>
<td>-0.0362</td>
<td>-0.3064*</td>
<td>-0.2521*</td>
</tr>
<tr>
<td>lag 3</td>
<td>-0.1512</td>
<td>-0.006</td>
<td>-0.066</td>
<td>0.0883</td>
<td>0.1131</td>
</tr>
<tr>
<td>lag 4</td>
<td>-0.0237</td>
<td>-0.0367</td>
<td>-0.012</td>
<td>-0.0522</td>
<td>-0.0397</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.7035</td>
<td>0.7126</td>
<td>0.8484</td>
<td>0.7484</td>
<td>0.8408</td>
</tr>
</tbody>
</table>

Table 3.23: Regression results for the CBO output gap, with lags

<table>
<thead>
<tr>
<th></th>
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<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-1.0051**</td>
<td>-0.0248</td>
<td>0.0067</td>
<td>0.0035</td>
<td>0.0172</td>
</tr>
<tr>
<td>intercept</td>
<td>0.6012**</td>
<td>0.7115**</td>
<td>0.3251**</td>
<td>0.4737**</td>
<td>0.2612**</td>
</tr>
<tr>
<td>lag 1</td>
<td>0.773**</td>
<td>1.002**</td>
<td>1.0058**</td>
<td>1.0747**</td>
<td>1.068**</td>
</tr>
<tr>
<td>lag 2</td>
<td>0.1231</td>
<td>-0.2067*</td>
<td>-0.04</td>
<td>-0.3094**</td>
<td>-0.2536**</td>
</tr>
<tr>
<td>lag 3</td>
<td>-0.1607</td>
<td>-0.0036</td>
<td>-0.048</td>
<td>0.0937</td>
<td>0.1147</td>
</tr>
<tr>
<td>lag 4</td>
<td>-0.0373</td>
<td>-0.0461</td>
<td>-0.0127</td>
<td>-0.0655</td>
<td>-0.0445</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.6854</td>
<td>0.7003</td>
<td>0.8457</td>
<td>0.7423</td>
<td>0.8402</td>
</tr>
</tbody>
</table>

Table 3.24: Regression results for the unemployment rate, with lags
The tables below present results for a regression specification that includes for inflation lags and inflation expectations:

\[ \pi_t = c + \kappa y_t + \rho E_t \pi_{t+1} + \sum_{s=1}^{4} \gamma_s \pi_{t-s} + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-1.1389**</td>
<td>-0.0488</td>
<td>-0.0081</td>
<td>-0.016</td>
<td>0.0073</td>
</tr>
<tr>
<td>inflation expecations</td>
<td>1.0886**</td>
<td>0.0987</td>
<td>0.1086**</td>
<td>0.215**</td>
<td>0.1927**</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3695**</td>
<td>0.5385*</td>
<td>0.2741**</td>
<td>0.2951**</td>
<td>0.1857**</td>
</tr>
<tr>
<td>lag1</td>
<td>-0.3659</td>
<td>0.9746**</td>
<td>0.8858**</td>
<td>0.9401**</td>
<td>0.8238**</td>
</tr>
<tr>
<td>lag2</td>
<td>0.2617</td>
<td>-0.2079*</td>
<td>-0.0063</td>
<td>-0.2927**</td>
<td>-0.1762</td>
</tr>
<tr>
<td>lag3</td>
<td>-0.2055**</td>
<td>-0.0078</td>
<td>-0.0721</td>
<td>0.0827</td>
<td>0.0875</td>
</tr>
<tr>
<td>lag4</td>
<td>0.0353</td>
<td>-0.0509</td>
<td>-0.0068</td>
<td>-0.1163</td>
<td>-0.0043</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8469</td>
<td>0.7072</td>
<td>0.8506</td>
<td>0.7688</td>
<td>0.8645</td>
</tr>
</tbody>
</table>

Table 3.25: Regression results for the CBO unemployment gap, with expectations
<table>
<thead>
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<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>1.0634**</td>
<td>0.0822*</td>
<td>0.0226</td>
<td>0.0383</td>
<td>0.0076</td>
</tr>
<tr>
<td></td>
<td>(0.2599)</td>
<td>(0.0416)</td>
<td>(0.0236)</td>
<td>(0.0284)</td>
<td>(0.0191)</td>
</tr>
<tr>
<td>inflation expectations</td>
<td>1.0744**</td>
<td>0.0618</td>
<td>0.1033**</td>
<td>0.2066**</td>
<td>0.1884**</td>
</tr>
<tr>
<td></td>
<td>(0.0944)</td>
<td>(0.0967)</td>
<td>(0.0378)</td>
<td>(0.0591)</td>
<td>(0.0401)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3678**</td>
<td>0.7088**</td>
<td>0.3114**</td>
<td>0.3494**</td>
<td>0.2103**</td>
</tr>
<tr>
<td></td>
<td>(0.0787)</td>
<td>(0.3064)</td>
<td>(0.1187)</td>
<td>(0.1382)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>lag1</td>
<td>-0.3551**</td>
<td>0.961**</td>
<td>0.8831**</td>
<td>0.9324**</td>
<td>0.824**</td>
</tr>
<tr>
<td></td>
<td>(0.1132)</td>
<td>(0.0866)</td>
<td>(0.0933)</td>
<td>(0.0896)</td>
<td>(0.0948)</td>
</tr>
<tr>
<td>lag2</td>
<td>0.2629**</td>
<td>-0.2053*</td>
<td>-0.0057</td>
<td>-0.2917**</td>
<td>-0.1771</td>
</tr>
<tr>
<td></td>
<td>(0.0758)</td>
<td>(0.1205)</td>
<td>(0.1195)</td>
<td>(0.1205)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>lag3</td>
<td>-0.2004**</td>
<td>-0.0078</td>
<td>-0.0709</td>
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<tr>
<td></td>
<td>(0.0751)</td>
<td>(0.1206)</td>
<td>(0.119)</td>
<td>(0.1205)</td>
<td>(0.1171)</td>
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<tr>
<td>lag4</td>
<td>0.0436</td>
<td>-0.0404</td>
<td>-0.0068</td>
<td>-0.1063</td>
<td>-0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.0566)</td>
<td>(0.0837)</td>
<td>(0.0823)</td>
<td>(0.0824)</td>
<td>(0.0791)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8503</td>
<td>0.7135</td>
<td>0.8515</td>
<td>0.7715</td>
<td>0.8646</td>
</tr>
</tbody>
</table>

Table 3.26: Regression results for the CBO output gap, with expectations

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>-1.0106**</td>
<td>0.0026</td>
<td>0.0214</td>
<td>-0.0048</td>
<td>0.0303</td>
</tr>
<tr>
<td></td>
<td>(0.303)</td>
<td>(0.047)</td>
<td>(0.0265)</td>
<td>(0.032)</td>
<td>(0.0216)</td>
</tr>
<tr>
<td>inflation expectations</td>
<td>1.1128**</td>
<td>0.1424</td>
<td>0.1163**</td>
<td>0.2187**</td>
<td>0.198**</td>
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<td></td>
<td>(0.0958)</td>
<td>(0.0976)</td>
<td>(0.0376)</td>
<td>(0.059)</td>
<td>(0.0396)</td>
</tr>
<tr>
<td>intercept</td>
<td>0.3354**</td>
<td>0.3572</td>
<td>0.2385**</td>
<td>0.2744**</td>
<td>0.1753**</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.2931)</td>
<td>(0.1091)</td>
<td>(0.1283)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>lag1</td>
<td>-0.3745**</td>
<td>0.9843**</td>
<td>0.8803**</td>
<td>0.9422**</td>
<td>0.8131**</td>
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<td></td>
<td>(0.1159)</td>
<td>(0.0871)</td>
<td>(0.0936)</td>
<td>(0.0901)</td>
<td>(0.0945)</td>
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<tr>
<td>lag2</td>
<td>0.2653**</td>
<td>-0.2102*</td>
<td>-0.0047</td>
<td>-0.293**</td>
<td>-0.1736</td>
</tr>
<tr>
<td></td>
<td>(0.0773)</td>
<td>(0.1223)</td>
<td>(0.1196)</td>
<td>(0.1213)</td>
<td>(0.1173)</td>
</tr>
<tr>
<td>lag3</td>
<td>-0.2082**</td>
<td>-0.0076</td>
<td>-0.0724</td>
<td>0.0835</td>
<td>0.0867</td>
</tr>
<tr>
<td></td>
<td>(0.0765)</td>
<td>(0.1224)</td>
<td>(0.1191)</td>
<td>(0.1213)</td>
<td>(0.1163)</td>
</tr>
<tr>
<td>lag4</td>
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<td>-0.0042</td>
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<tr>
<td></td>
<td>(0.0576)</td>
<td>(0.0848)</td>
<td>(0.0824)</td>
<td>(0.0826)</td>
<td>(0.0785)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.8445</td>
<td>0.705</td>
<td>0.8513</td>
<td>0.7684</td>
<td>0.8664</td>
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Table 3.27: Regression results for the unemployment rate, with expectations
The tables below present results for a regression specification that includes the time series of “endogenous” cost-push shocks constructed in Section 3.4.2 as a control:

\[ \pi_t = c + \kappa \bar{y}_t + CP_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost-push</td>
<td>0.5627**</td>
<td>2.5545**</td>
<td>0.4886</td>
<td>2.3948**</td>
<td>1.1224**</td>
</tr>
<tr>
<td></td>
<td>(0.2345)</td>
<td>(0.565)</td>
<td>(0.4768)</td>
<td>(0.4745)</td>
<td>(0.4102)</td>
</tr>
<tr>
<td>gap</td>
<td>-3.7586**</td>
<td>-0.1906**</td>
<td>-0.2175**</td>
<td>-0.0783</td>
<td>-0.0886</td>
</tr>
<tr>
<td></td>
<td>(0.6872)</td>
<td>(0.0758)</td>
<td>(0.064)</td>
<td>(0.0637)</td>
<td>(0.0551)</td>
</tr>
<tr>
<td>intercept</td>
<td>2.0842**</td>
<td>3.2239**</td>
<td>2.8559**</td>
<td>2.6509**</td>
<td>2.397**</td>
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<tr>
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<td>(0.058)</td>
<td>(0.1398)</td>
<td>(0.118)</td>
<td>(0.1174)</td>
<td>(0.1015)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.3317</td>
<td>0.2782</td>
<td>0.142</td>
<td>0.2558</td>
<td>0.1275</td>
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</table>

Table 3.28: Regression results for the CBO unemployment gap, with CP shock

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost-push</td>
<td>0.6059**</td>
<td>2.5472**</td>
<td>0.6387</td>
<td>2.4715**</td>
<td>1.2896**</td>
</tr>
<tr>
<td></td>
<td>(0.2604)</td>
<td>(0.5964)</td>
<td>(0.5145)</td>
<td>(0.4983)</td>
<td>(0.4333)</td>
</tr>
<tr>
<td>gap</td>
<td>2.4282**</td>
<td>0.1363**</td>
<td>0.1176**</td>
<td>0.0369</td>
<td>0.0225</td>
</tr>
<tr>
<td></td>
<td>(0.6496)</td>
<td>(0.0682)</td>
<td>(0.0588)</td>
<td>(0.057)</td>
<td>(0.0495)</td>
</tr>
<tr>
<td>intercept</td>
<td>2.0936**</td>
<td>3.2425**</td>
<td>2.8535**</td>
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<td>2.3802**</td>
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<td>(0.0633)</td>
<td>(0.145)</td>
<td>(0.1251)</td>
<td>(0.1212)</td>
<td>(0.1054)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2458</td>
<td>0.2635</td>
<td>0.0852</td>
<td>0.2484</td>
<td>0.1086</td>
</tr>
</tbody>
</table>

Table 3.29: Regression results for the CBO output gap, with CP shock

<table>
<thead>
<tr>
<th></th>
<th>SW</th>
<th>CPI</th>
<th>core CPI</th>
<th>PCE</th>
<th>core PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost-push</td>
<td>0.6321**</td>
<td>2.8683**</td>
<td>0.8598*</td>
<td>2.6413**</td>
<td>1.3999**</td>
</tr>
<tr>
<td></td>
<td>(0.2357)</td>
<td>(0.5706)</td>
<td>(0.4905)</td>
<td>(0.4722)</td>
<td>(0.4102)</td>
</tr>
<tr>
<td>gap</td>
<td>-3.6783**</td>
<td>-0.0954</td>
<td>-0.1038</td>
<td>0.006</td>
<td>0.0063</td>
</tr>
<tr>
<td></td>
<td>(0.731)</td>
<td>(0.0811)</td>
<td>(0.0697)</td>
<td>(0.0671)</td>
<td>(0.0583)</td>
</tr>
<tr>
<td>intercept</td>
<td>2.0911**</td>
<td>3.1954**</td>
<td>2.8214**</td>
<td>2.6213**</td>
<td>2.3637**</td>
</tr>
<tr>
<td></td>
<td>(0.0594)</td>
<td>(0.1439)</td>
<td>(0.1237)</td>
<td>(0.1191)</td>
<td>(0.1034)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.309</td>
<td>0.2462</td>
<td>0.0706</td>
<td>0.2456</td>
<td>0.1071</td>
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</table>

Table 3.30: Regression results for the unemployment rate, with CP shock
The tables below present results for a regression specification that includes both the time series of “endogenous” cost-push shocks constructed in Section 3.4.2 and oil price inflation:

\[ \pi_t = c + \kappa \bar{y}_t + CP_t + \pi_{oil} + \epsilon_t \]

| Table 3.31: Regression results for the CBO unemployment gap (CP shock and oil prices) |
|--------------------------------------|------------------|------------------|--------------------|------------------|
| | SW | CPI core CPI | PCE core PCE |
| cost-push | 0.2874 | 1.1895** | 0.99* | 1.4128** | 1.3585** |
| gap | -3.8932** | -0.2211** | -0.2062** | -0.1003* | -0.0833 |
| intercept | 2.0185** | 2.8983** | 2.9754** | 2.4167** | 2.4533** |
| oil prices | 0.0034** | 0.0167** | -0.0062* | 0.012** | -0.0029 |
| R-squared | 0.3581 | 0.399 | 0.1692 | 0.3472 | 0.1358 |

| Table 3.32: Regression results for the CBO output gap (CP shock and oil prices) |
|--------------------------------------|------------------|------------------|--------------------|------------------|
| | SW | CPI core CPI | PCE core PCE |
| cost-push | 0.3912 | 1.294** | 1.1824** | 1.5569** | 1.5453** |
| gap | 2.4623** | 0.1454** | 0.1137* | 0.0436 | 0.0207 |
| intercept | 2.0396** | 2.9277** | 2.99** | 2.417** | 2.444** |
| oil prices | 0.0027 | 0.016** | -0.0069** | 0.0116** | -0.0033 |
| R-squared | 0.2632 | 0.3741 | 0.1199 | 0.3346 | 0.1192 |

| Table 3.33: Regression results for the unemployment rate (CP shock and oil prices) |
|--------------------------------------|------------------|------------------|--------------------|------------------|
| | SW | CPI core CPI | PCE core PCE |
| cost-push | 0.3818 | 1.5809** | 1.4042** | 1.7135** | 1.6665** |
| gap | -3.7784** | -0.119 | -0.0938 | -0.011 | 0.0112 |
| intercept | 2.0301** | 2.8814** | 2.9542** | 2.3951** | 2.4287** |
| oil prices | 0.0031* | 0.0161** | -0.0068** | 0.0116** | -0.0033 |
| R-squared | 0.3318 | 0.3584 | 0.1041 | 0.3308 | 0.1181 |
Appendix C6

Alternative specifications

The figures below plot rolling regression coefficients and R-squareds for the baseline specification

\[ \pi_t = \kappa \bar{y}_t + \epsilon_t \]

using different measures of the output gap on the right hand side (CBO unemployment gap, CBO output gap and unemployment rate).
CBO unemployment - average R-squareds

CBO output

SW

CPI

core CPI

PCE

core PCE
The figures below plot rolling regression coefficients and R-squareds for a regression of output gap levels on inflation changes:

\[ \pi_t - \pi_{t-1} = \kappa_y t + \epsilon_t \]

using different measures of the output gap on the right hand side (CBO unemployment gap, CBO output gap and unemployment rate).
Appendix D: Dynamics

Proof of Proposition 8
This lemma characterizes the evolution of sectoral inflation rates and markups as a function of initial markups (which are a state variable), productivity shocks and monetary policy.

Denote by \( \hat{\Delta} \) the diagonal matrix with elements

\[
\hat{\delta}_i \equiv \frac{\delta_i (1 - \rho(1 - \delta_i))}{1 - \rho \delta_i (1 - \delta_i)}
\]

The first step is to solve for the growth rate of sector-level markups, remembering that it is given by the log-difference between the growth rates of prices and marginal costs:

\[
-(\log \mu_t - \log \mu_{t-1}) = \log mc_t = \log mc_{t-1} - \pi_t =
\]

\[
= \alpha (\log w_t - \log w_{t-1}) - (I - \Omega) \pi_t - (\log A_t - \log A_{t-1})
\]

Using the pricing equation (2.32) we can rewrite this as

\[
-(\log \mu_t - \log \mu_{t-1}) = -(I - \Omega) \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} (-\log \mu_t) + \alpha (\log w_t - \log w_{t-1}) +
\]

\[
- \left[(\log A_t - \log A_{t-1}) + (I - \Omega) \left[\rho E \pi_{t+1} + \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} d \log \mu_t^D\right]\right]
\Rightarrow
\]

\[
\left((I - \hat{\Delta}) \hat{\Delta}^{-1} + (I - \Omega)\right) \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} (-\log \mu_t) =
\]

\[
= (-\log \mu_{t-1}) + \alpha (\log w_t - \log w_{t-1}) +
\]

\[
- \left[(\log A_t - \log A_{t-1}) + (I - \Omega) \left[\rho E \pi_{t+1} + \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} d \log \mu_t^D\right]\right]
\]

Denote by

\[
x_t \equiv -\hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} \log \mu_t
\]

\[
x_t^D \equiv \hat{\Delta} \left(I - \hat{\Delta}\right)^{-1} d \log \mu_t^D
\]
We can then re-write equation (3.48) as

\[
\left( \hat{\Delta}^{-1} - \Omega \right) x_t =
\]

\[
= (I - \hat{\Delta}) \hat{\Delta}^{-1} x_{t-1} + \alpha (\log w_t - \log w_{t-1}) - [(\log A_t - \log A_{t-1}) + (I - \Omega) [\rho \mathbb{E} \pi_{t+1} + x_t^D]] \Rightarrow
\]

\[
x_t = \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1} \left[(I - \hat{\Delta}) \hat{\Delta}^{-1} x_{t-1} + \alpha (\log w_t - \log w_{t-1}) + \right.
\]

\[
- [(\log A_t - \log A_{t-1}) + (I - \Omega) [\rho \mathbb{E} \pi_{t+1} + x_t^D]]
\]

From the consumers’ labor-leisure trade-off, wages evolve according to

\[
\log w_t - \log w_{t-1} = (\gamma + \varphi) (\tilde{y}_t - \tilde{y}_{t-1}) + \lambda^T (\log A_t - \log A_{t-1}) +
\]

\[
+ \beta^T (x_t + x_t^D + \rho \mathbb{E} (\pi_{t+1}))
\]

so that

\[
\log w_t - \log w_{t-1} = \frac{\gamma + \varphi}{1 - \beta^T \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1}} (\tilde{y}_t - \tilde{y}_{t-1}) +
\]

\[
+ \frac{\lambda^T - \beta^T \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1} [\log A_t - \log A_{t-1}) + (I - \Omega) [\rho \mathbb{E} \pi_{t+1} + x_t^D]]}{1 - \beta^T \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1}} +
\]

\[
+ \frac{\beta^T \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1} (I - \hat{\Delta}) \hat{\Delta}^{-1} x_{t-1}}{1 - \beta^T \hat{\Delta} \left( I - \Omega \hat{\Delta} \right)^{-1}}
\]

\[
(3.50)
\]

Combining (3.49) and (3.50) we obtain

\[
x_t = \hat{B} (\tilde{y}_t - \tilde{y}_{t-1}) + \hat{V} [(\log A_t - \log A_{t-1}) + (I - \Omega) [\rho \mathbb{E} \pi_{t+1} + x_t^D]] + \mathcal{M} x_{t-1}
\]

\[
(3.51)
\]

Lemma [13] below proves that the matrix \( \mathcal{M} \) is invertible. Denoting by \( z_t \equiv x_{t-1} \), equations (2.32) and (3.51) can then be combined to obtain the following system of difference equations
in $\pi_t$ and $z_t$:
\[
\begin{pmatrix}
\rho \mathbb{E} \pi_{t+1} \\
Z_{t+1}
\end{pmatrix} = \begin{pmatrix}
M^{-1} & -I \\
I - M^{-1} & I
\end{pmatrix} \begin{pmatrix}
\pi_t \\
Z_t
\end{pmatrix} + \\
+ \begin{pmatrix}
-M^{-1} \left( \hat{B} (\bar{y}_t - \bar{y}_{t-1}) + \hat{\gamma} (\log A_t - \log A_{t-1}) \right) - x_t^D \\
M^{-1} \left( \hat{B} (\bar{y}_t - \bar{y}_{t-1}) + \hat{\gamma} (\log A_t - \log A_{t-1}) \right)
\end{pmatrix}
\]
\[(3.52)\]

Finally, it is useful to re-write (3.52) substituting out for the past output gap, using Lemma 1 (see equation (2.34)):  
\[
\begin{pmatrix}
\rho \mathbb{E} \pi_{t+1} \\
Z_{t+1}
\end{pmatrix} = \begin{pmatrix}
M^{-1} & -Z \\
I - M^{-1} & Z
\end{pmatrix} \begin{pmatrix}
\pi_t \\
Z_t
\end{pmatrix} + \\
+ \begin{pmatrix}
-M^{-1} \left( \hat{B} \bar{y}_t + \hat{\gamma} (\log A_t - \log A_{t-1}) \right) - x_t^D \\
M^{-1} \left( \hat{B} \bar{y}_t + \hat{\gamma} (\log A_t - \log A_{t-1}) \right)
\end{pmatrix}
\]
\[(3.53)\]
where
\[
Z \equiv M^{-1} \hat{\gamma} \left( I - \hat{\Delta} \right) \hat{\Delta}^{-1}
\]

To obtain the system in (2.34) just use the definition
\[
z_t \equiv -\hat{\Delta} \left( I - \hat{\Delta} \right)^{-1} \log \mu_{t-1}
\]

**Lemma 13.** As long as no sector has fully flexible prices ($\delta_i < 1 \ \forall i$), the matrix $M$ is invertible. Moreover, all of its eigenvalues have modulus (weakly) smaller than one.

**Proof.** It holds that
\[
M = \left( I + \frac{B\beta^T}{\gamma + \varphi} \right) \Delta (I - \Omega \Delta)^{-1} (I - \Delta) \Delta^{-1}
\]

The matrix $\left( I + \frac{B\beta^T}{\gamma + \varphi} \right)$ has eigenvalues 1 (and all vectors orthogonal to $\beta$ are corresponding
eigenvectors) and 

\[ \frac{1}{1 - \beta^T \Delta (I - \Omega \Delta)^{-1} \Delta^{-1} (I - \Delta) \Delta^{-1}} \alpha, \]

with corresponding eigenvector \( \mathbf{B} \). Therefore it is invertible. The matrix \( \Delta (I - \Omega \Delta)^{-1} (I - \Delta) \Delta^{-1} \) is invertible because we assumed that no sector has fully rigid or fully flexible prices. Thus \( \mathcal{M} \) is invertible.

To prove that all eigenvalues are (weakly) smaller than one in modulus, note that \( \mathcal{M} \mathbf{1} = \mathbf{1} \):

\[
\mathcal{M} \mathbf{1} = \left( I + \frac{\mathbf{B} \beta^T}{\gamma + \varphi} \right) \Delta (I - \Omega \Delta)^{-1} \left( \Delta^{-1} - \Omega - (I - \Omega) \right) (I - \Omega)^{-1} \alpha = \\
= \left( I + \frac{\mathbf{B} \beta^T}{\gamma + \varphi} \right) \Delta \left( (I - \Omega \Delta)^{-1} (I - \Omega)^{-1} - I \right) \alpha = \\
= \left( I + \frac{\mathbf{B} \beta^T}{\gamma + \varphi} \right) \left( (I - \Omega)^{-1} - \Delta (I - \Omega \Delta)^{-1} \right) \alpha = \\
= \mathbf{1} - \Delta (I - \Omega \Delta)^{-1} \alpha + \frac{\mathbf{B}}{\gamma + \varphi} \left( 1 - \beta^T \Delta (I - \Omega \Delta)^{-1} \alpha \right) = \mathbf{1}
\]

In addition \( \mathcal{M} \) has all positive elements, because both \( I + \frac{B \beta^T}{\gamma + \varphi} \) and \( \Delta (I - \Omega \Delta)^{-1} (I - \Delta) \Delta^{-1} \) have positive elements. These two properties imply that all of its eigenvalues must be smaller than one in modulus. \( \square \)

**Proof of Lemma 6**

We want to prove that there is a unique path of inflation rates and markups which remains bounded and where the output gap is zero in every period. We start from the system

\[
\begin{pmatrix}
\mathbb{E} \pi_{t+1} \\
z_{t+1}
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{\rho} \mathcal{M}^{-1} & -\frac{1}{\rho} \mathcal{Z} \\
I - \mathcal{M}^{-1} & \mathcal{Z}
\end{pmatrix}
\begin{pmatrix}
\pi_t \\
z_t
\end{pmatrix} +
\begin{pmatrix}
-\frac{1}{\rho} \mathcal{M}^{-1} \mathcal{V} \left( \log A_t - \log A_{t-1} \right) \\
\mathcal{M}^{-1} \mathcal{V} \left( \log A_t - \log A_{t-1} \right)
\end{pmatrix}
\]

which corresponds to the system (3.53) with the additional condition that \( \bar{y}_t \equiv 0 \). We show that the matrix

\[
\mathcal{A} =
\begin{pmatrix}
\frac{1}{\rho} \mathcal{M}^{-1} & -\frac{1}{\rho} \mathcal{Z} \\
I - \mathcal{M}^{-1} & \mathcal{Z}
\end{pmatrix}
\]

has \( N \) eigenvectors greater than 1, and \( N \) smaller than 1.

This is enough to guarantee that the system has a unique bounded solution for any given
past markups $z_t$ and productivity/markup shocks $\log A_t - \log A_{t+1}$ and $x_t^D$. That is, given an initial condition for $z_t$, imposing that $|\lim_{t\to\infty} \pi_t^*| < \infty \ orall i$ and $|\lim_{t\to\infty} z_t^*| < \infty \ orall i$ pins down a unique initial value for $\pi_t^*$. We will first prove that having $N$ eigenvectors greater than 1, and $N$ smaller than 1 is sufficient to guarantee a unique solution. Then we will demonstrate that this condition is satisfied.

Given our assumption about the productivity process, we have that

$$\mathbb{E}\lim_{t\to\infty} \begin{pmatrix} \pi_t^* \\ z_t^* \end{pmatrix} = \lim_{t\to\infty} A^t \begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} + \lim_{t\to\infty} \left( \sum_{s \leq t} \eta^s A^{t-s} \right) \begin{pmatrix} -\frac{1}{\rho} M^{-1} \hat{V} (\log A_0 - \log A_{-1}) - \frac{1}{\rho} x_0^D \\ M^{-1} \hat{V} (\log A_0 - \log A_{-1}) \end{pmatrix}$$

In turn, we can decompose the as a linear combination of the eigenvectors of $A$, $\{w_1, ..., w_{2N}\}$:

$$\begin{pmatrix} -\frac{1}{\rho} M^{-1} \hat{V} (\log A_0 - \log A_{-1}) - \frac{1}{\rho} x_0^D \\ M^{-1} \hat{V} (\log A_0 - \log A_{-1}) \end{pmatrix} = a_1 w_1 + ... + a_{2N} w_{2N}$$

Denote by $\{\nu_1, ..., \nu_{2N}\}$ the eigenvalues corresponding to $\{w_1, ..., w_{2N}\}$. We then have

$$\lim_{t\to\infty} \left( \sum_{s \leq t} \eta^s A^{t-s} \right) \begin{pmatrix} -\frac{1}{\rho} M^{-1} \hat{V} (\log A_0 - \log A_{-1}) - \frac{1}{\rho} x_0^D \\ M^{-1} \hat{V} (\log A_0 - \log A_{-1}) \end{pmatrix} = C + \lim_{t\to\infty} A^t \sum_{i: \nu_i > 1} \frac{\nu_i}{\nu_i - \eta} a_i w_i$$

where

$$C < \sum_{i: \nu_i < 1} \frac{a_i w_i}{1 - \nu_i} < \infty$$

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To have a unique bounded solution we need the condition

\[ \lim_{t \to \infty} A^t \begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} = -\lim_{t \to \infty} A^t \sum_{i/\nu_i > 1} \frac{\nu_i}{\nu_i - \eta} a_i w_i \] (3.55)

to yield a unique solution \( \pi_0^* \). Let's write \( \begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} \) in components with respect to \( \{w_1, \ldots, w_{2N}\} \):

\[
\begin{pmatrix} \pi_0^* \\ z_0 \end{pmatrix} = \sum_{i=1}^{2N} x_i w_i
\]

For condition (3.55) to be satisfied we need that

\[
\begin{cases}
  x_i = -\frac{\nu_i}{\nu_i - \eta} a_i & \forall i/\nu_i > 1 \\
  \sum_{i/\nu_i < 1} x_i w_i, N+1:2N = z_0 + \sum_{i/\nu_i > 1} \frac{\nu_i}{\nu_i - \eta} a_i w_{i,N+1:2N}
\end{cases}
\] (3.56)

The second line in (3.56) is a system of \( N \) equations, with unknowns the coefficients \( x_i \) for \( i \) such that \( \nu_i < 1 \). The system has a unique solution if and only if there are exactly \( N \) eigenvalues \( \nu_i < 1 \), while the remaining \( N \) are greater or equal than \( 1 \).

Let's then prove that this condition is satisfied. Note that (for \( x_t^D \equiv 0 \)) the two equations in (3.54) yield the optimal reset price equation

\[ \rho E \pi_{t+1} = \pi_t - z_{t+1} \]

It is convenient to substitute this to the first equation and use it together with the second to look for the eigenvectors of the matrix \( A \). Assume that \( \begin{pmatrix} \pi \\ z \end{pmatrix} \) is an eigenvector relative
to the eigenvalue $\nu$. From the optimal reset price equation we find

$$\nu z = (1 - \rho \nu) \pi$$

The second equation in (3.54) yields

$$\nu z = (I - \mathcal{M}^{-1}) \pi + \mathcal{Z} z$$

For $\nu = 0$ these conditions are satisfied for $\pi = 0$ and $z = \mathcal{M}^{-1} \mathcal{B}$.

For $\nu = \frac{1}{\rho}$ the conditions are satisfied for $z = 0$ and $\pi = \left( \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right)$.

Otherwise we can merge the two equations above and substitute out for $\nu z$, to obtain:

$$\rho \nu \pi = \mathcal{M}^{-1} \pi - \frac{1 - \rho \nu}{\nu} \mathcal{Z} \pi$$

(3.57)

It holds that all eigenvectors of $\mathcal{M}$ except $\left( \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right)$ are orthogonal to $\lambda^T (I - \Delta) \mathcal{D}^{-1}$. Therefore if $\pi$ is an eigenvector of $\mathcal{M}$, with corresponding eigenvalue $\xi \neq 0$, then $\mathcal{Z} \pi = \pi$. Thus equation (3.57) becomes

$$\frac{\rho \nu^2 - \rho \nu + 1}{\nu} \pi = \frac{1}{\xi} \pi$$

Now we need to have $\pi \neq 0$ (otherwise we would also have $z = 0$, which cannot be an eigenvector). Therefore it must hold that

$$\frac{\rho \nu^2 - \rho \nu + 1}{\nu} = \frac{1}{\xi}$$

(3.58)

Lemma 13 shows that all eigenvalues $\xi$ of $\mathcal{M}$ have modulus in $(0,1)$. Therefore equation (3.58) has two solutions, $\nu^+$ and $\nu^-$, with $0 < \nu^- < 1$ and $\nu^+ > 1$. Thus we have $N - 1$
couples of solutions (one smaller than 1 and one greater than 1), plus 0 and $\frac{1}{\rho}$. It follows that the matrix $A$ has $N$ eigenvalues greater than 1 and $N$ smaller than 1 in absolute value, as we wanted to show.

It remains to prove that the interest rate rule

$$i_t = r_t^n + \beta^T \mathbb{E} \pi_{t+1}^n + \zeta \bar{y}_t$$

with $\zeta > 0$ implements zero output gap in every period.

Under this rule the system becomes

$$
\begin{pmatrix}
\begin{pmatrix}
\rho \mathbb{E} \pi_{t+1}^n \\
z_{t+1} \\
\mathbb{E} \bar{y}_{t+1}
\end{pmatrix}
\end{pmatrix}
= 
\begin{pmatrix}
\mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1} \hat{B} \\
I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1} \hat{B} \\
0 & 0 & \zeta + 1
\end{pmatrix}
\begin{pmatrix}
\pi_t \\
z_t \\
\bar{y}_t
\end{pmatrix}
+ 
\begin{pmatrix}
-\mathcal{M}^{-1} \hat{Y} \\
\mathcal{M}^{-1} \hat{Y} \\
0
\end{pmatrix}
(\log A_t - \log A_{t-1})
+ 
\begin{pmatrix}
-I \\
0 \\
0
\end{pmatrix}
x_t^D
$$

Note that the solution to the previous system is still a solution of the new system. To prove that there are no additional solutions we will show that the matrix

$$\tilde{A} \equiv 
\begin{pmatrix}
\mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1} \hat{B} \\
I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1} \hat{B} \\
0 & 0 & \zeta + 1
\end{pmatrix}
$$

has the same eigenvalues and eigenvectors as $A$ above, plus the eigenvalue $\nu = \zeta + 1$, with
associated eigenvector \[
\begin{pmatrix}
\pi \\
z \\
\tilde{y}
\end{pmatrix}
\]
such that

\[
\pi = \left( I + \frac{1 - \rho \nu + \rho \nu^2}{\nu} \right)^{-1} \mathcal{B}
\]

\[
z = \frac{1 - \rho \nu}{\nu} \pi
\]

\[
\tilde{y} = (1 - \rho \nu) \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \phi} \pi
\]

This would imply that for \( \zeta > 0 \) the new system has a unique bounded solution, equal to the solution of the original system.

Let’s then study the eigenvalues and eigenvectors of \( \tilde{A} \). Denote the eigenvalues by \( \nu \), and the first \( N \) components of the corresponding eigenvector by \( \pi \). From the first two rows and the definition of eigenvector we derive the conditions

\[
z = \frac{1 - \rho \nu}{\nu} \pi
\]

\[
\tilde{y} = (1 - \rho \nu) \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \phi} \pi
\]

\[
\left( I - \frac{1 - \rho \nu + \rho \nu^2}{\nu} \mathcal{G} \right) \pi = \mathcal{B} \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \phi} \left( I - \frac{1 - \rho \nu + \rho \nu^2}{\nu} \mathcal{G} \right) \pi
\]

The last condition implies

\[
\left( I - \frac{1 - \rho \nu + \rho \nu^2}{\nu} \mathcal{G} \right) \pi = \mathcal{B}
\]

From the last row of \( \tilde{A} \) we derive the relation

\[
(1 + \zeta - \nu) (1 - \rho \nu) \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \phi} \pi = 0
\]

which we know is satisfied by the eigenvalues/eigenvectors of \( A \). In addition, it is also
satisfied for $\nu = 1 + \zeta$ and $\pi = \left( I - \frac{1-\rho \nu + \rho^2 \delta}{\nu} \mathcal{G} \right)^{-1} \hat{\mathcal{B}}$. This proves the result.

**Proof of Proposition 9**

Within each period, the cross-sector misallocation loss is the same function of sector-level markups derived in Section 4. It can be written as

$$x_t^T \mathcal{D}_2 x_t$$

where now $\mathcal{D}_2$ is defined as

$$\mathcal{D}_2 = \left( I - \hat{\Delta} \right) \hat{\Delta}^{-1} \hat{\mathcal{D}}_2 \hat{\Delta}^{-1} \left( I - \hat{\Delta} \right)$$

and the elements of $\hat{\mathcal{D}}_2$ are derived in equation (3.36) (see the proof of Proposition 4).

The within-sector productivity loss is given by

$$\sum_{i=1}^{N} \lambda_i \epsilon_i \left[ \int (\log p_{i ft} - \log p_{it})^2 df - \left( \int (\log p_{i ft} - \log p_{it}) df \right)^2 \right]$$

as derived in Proposition 4.

The following lemma shows how the discounted sum of within-sector losses in the present and future periods can be written as a function of sectoral inflation rates.

**Lemma 14.** It holds that

$$\sum_{s \geq 0} \rho^s \left( \sum_{i=1}^{N} \lambda_i \epsilon_i \left[ \int (\log p_{i ft+s} - \log p_{it+s})^2 df - \left( \int (\log p_{i ft+s} - \log p_{it+s}) df \right)^2 \right] \right) =$$

$$= \sum_{s \geq 0} \rho^s \pi_t^T \mathcal{D}_1 \pi_t + s$$

where $\mathcal{D}_1$ is a diagonal matrix with elements

$$d_{1ii} = \lambda_i \epsilon_i \frac{1 - \hat{\delta}_i}{\hat{\delta}_i}$$
Proof. To prove the lemma it is enough to show that

$$
\sum_{s \geq 0} \rho^s \left[ \int (\log p_{it+s} - \log p_{it})^2 df - \left( \int (\log p_{it+s} - \log p_{it}) df \right)^2 \right] = \frac{1 - \delta_i}{\delta_i} \sum_{s \geq 0} \rho^s \pi_{it+s}^2
$$

Given the Calvo assumption, in each sector $i$ the fraction $\delta_i$ of firms who adjust prices set

$$
\log p_{it} - \log p_{it} = (1 - \delta_i) (\log p_{it}^* - \log p_{it-1}) = \frac{1 - \delta_i}{\delta_i} \pi_{it}
$$

For the remaining fraction $(1 - \delta_i)$ of non-adjusting firms we have

$$
\log p_{it} - \log p_{it} = (-\delta_i) (\log p_{it}^* - \log p_{it-1}) + (\log p_{it-1} - \log p_{it-1}) = (\log p_{it-1} - \log p_{it-1}) - \pi_{it}
$$

Define

$$
D_{it} \equiv \int (\log p_{it+s} - \log p_{it})^2 df - \left( \int (\log p_{it+s} - \log p_{it}) df \right)^2
$$

Around a steady-state where $\log p_{it} - \log p_{it} = 0 \forall f$, we have

$$
D_{it} = (1 - \delta_i) \left( \frac{1 - \delta_i}{\delta_i} \pi_{it}^2 + D_{it-1} \right)
$$

It follows that

$$
\sum_{s} \rho^s D_{it+s} = \sum_{s} \rho^s \frac{1 - \delta_i}{\delta_i} \pi_{is}^2 \left( \sum_{\tau \geq s} (\rho (1 - \delta_i))^{\tau-s} \right) = \frac{1 - \hat{\delta}_i}{\hat{\delta}_i} \sum_{s} \rho^s \pi_{is}^2
$$

\[\square\]

Proof of Proposition 10
Appendix E: Complements to the quantitative analysis

E1: Welfare loss from business cycles

Below I report expressions for the expected welfare loss under different policy rules, as a function of the network primitives (captured by $B, V$ and $D$) and of the covariance matrix of sectoral shocks ($\Sigma$). I further decompose the loss into deviations from zero output gap and misallocation.

Optimal policy

The total welfare loss is

$$\frac{1}{2} \left[ \sum_{i,j} (V^T D V)_{ij} \Sigma_{ij} - \frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)^2} \right]$$

The loss from non-zero output gap is:

$$\frac{1}{2} \left( \gamma + \varphi \right) \frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)^2}$$

The gain in allocative efficiency from non-zero output gap is:

$$\frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)^2} - \frac{1}{2} B^T D B \frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)^2}$$

The net misallocation loss is:

$$\frac{1}{2} \sum_{i,j} (V^T D V)_{ij} \Sigma_{ij} - \frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)} + \frac{1}{2} B^T D B \frac{B^T D \Sigma V^T D B}{(\gamma + \varphi + B^T D B)^2}$$

Loss under zero consumer inflation relative to the optimal policy
The total loss is:

\[
\frac{1}{2} \frac{B^TDB\Sigma V^TDB}{(\gamma + \phi + B^TDB)} + \\
+ \frac{1}{2} \left[ \frac{\gamma + \phi + B^TDB}{(\beta B)^2} \beta^T V - 2 \frac{B^TDB}{\beta^T B} \right] \Sigma V^T \beta
\]

The loss from non-zero output gap is:

\[
\frac{1}{2} \frac{\beta^T \Sigma V^T \beta}{(\beta B)^2}
\]

The loss from misallocation is:

\[
\frac{1}{2} \sum_{i,j} (V^T DV)_{ij} \Sigma_{ij} + \frac{1}{2} \frac{B^TDB}{(\beta B)^2} - \frac{B^TDB}{\beta^T B} \right] \Sigma V^T \beta
\]

Loss under zero output gap relative to the optimal policy

The total loss is:

\[
\frac{1}{2} \frac{B^TDB\Sigma V^TDB}{(\gamma + \phi + B^TDB)}
\]

The total loss from misallocation is:

\[
\frac{1}{2} \sum_{i,j} (V^T DV)_{ij} \Sigma_{ij}
\]

Within- versus cross-sector misallocation

Section 2.1.1 shows that the welfare loss from misallocation has two components, coming from relative price distortions within and across sectors. Figure 3.21 compares the relative
magnitude of these components. The three sets of bars in the figure correspond to different policy rules (optimal policy, output gap targeting and consumer price targeting). Within each group, the bar on the left-hand-side is based on our preferred calibration, which assumes higher substitutability between varieties from the same sector than across goods from different sectors. Unsurprisingly, the within-sector loss dominates in this calibration. The bar on the right-hand-side of each group instead is based on an alternative calibration, which assumes the same elasticity of substitution within and across sectors. In this case we find that the largest contribution to the welfare loss comes from cross-sector misallocation. To construct the Figure we set substitution elasticities to $\epsilon = 8$, $\sigma = 0.9$, $\theta_L = 0.5$ and $\theta = 0.001$ for the main calibration and to $\epsilon = \sigma = \theta_L = \theta = 2$ for the alternative calibration.

Figure 3.21: Losses from within- and cross-sector misallocation

E2: Slope of the Phillips curve

In this Section I further break down the channels that determined the flattening of the Phillips curve.
Section 3.3.1 shows that the slope depends on the nominal wage pass-through $\bar{w}_t$. Formally, $\bar{w}_t$ is an average of sector-level pass-throughs of monetary shocks, with weights given by consumption shares. Thus we can split the overall change in $\bar{w}_t$ into the change in sector-level pass-through for constant consumption shares, and the change in consumption shares for constant pass-through. Sector-level pass-throughs only depend on the production structure, and not on consumption shares. Therefore we obtain the following decomposition:

$$\bar{w}^{2017}_t - \bar{w}^{1947}_t = \frac{\beta^{T}_{1947} + \beta^{T}_{2017}}{2} (PT^{2017}_t - PT^{1947}_t) +$$

$$+ (\beta^{T}_{2017} - \beta^{T}_{1947}) \frac{PT^{1947}_t + PT^{2017}_t}{2}$$

where I used the notation

$$PT \equiv \Delta (I - \Omega \Delta)^{-1} \alpha$$

I find that 79% of the overall decline in $\bar{w}_t$ can be attributed to changes in the input-output structure, while the remaining effect comes from changes in the composition of the consumption basket.

I further break down the effect of changes in consumption and input-output shares into their sector-level components. Figure 3.5.3 provides a graphical representation. The left panel depicts the change in consumption shares and the average wage pass-through for each sector, at an aggregated 15-sector level. The right panel plots the change in pass-through and the average consumption shares.
Figure 3.22: Changes in consumption shares and input output linkages

The grey bars in the two plots respectively represent the average pass-through

$$\frac{1}{2} (PT_{i,1947} + PT_{i,2017})$$

and the average consumption share $\frac{1}{2} (\beta_{i,1947} + \beta_{i,2017})$ for each sector. The bars in color represent changes in sectoral consumption shares $\beta^T_{i,2017} - \beta^T_{i,1947}$ and pass-through $PT_{i,2017} - PT_{i,1947}$. From the left plot we see that consumption shifted away from manufacturing (which has high pass-through) towards services (which has lower pass-through). The right plot shows that the pass-through fell in all sectors, and more so in sectors with high consumption share (such as construction, manufacturing and government). Both channels lead to a flatter Phillips curve, although quantitatively the drop in sectoral pass-through (due to larger intermediate input flows) accounts for most of the effect.