## Essays in Macroeconomics and Finance

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Essays in Macroeconomics and Finance

A dissertation presented

by

Christopher Clayton

to

The Department of Economics

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Economics

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Abstract

This dissertation contains three chapters on macroeconomics and finance. The first chapter, co-authored with Andreas Schaab, studies the scope for international cooperation in macro-prudential policies in the presence of fire sales and other externality problems. It shows that although national governments employing quantity regulations fail to achieve the globally efficient outcome absent cooperation, surprisingly national governments employing Pigouvian taxation can achieve the globally efficient outcome even without cooperation. The second chapter, co-authored with Andreas Schaab, provides a framework to study bail-in regimes for banks. It shows that although bail-ins are an optimal regulatory regime, they can also create refinancing instabilities, leaving a role for fiscal backstop measures such as debt guarantees in the crisis resolution toolkit. The third chapter, co-authored with Andreas Schaab, studies whether a central bank should have flexibility to change its inflation target. It shows that a dynamic inflation target can implement the constrained efficient level of inflation, provided that the target is adjusted sufficiently in advance of it taking effect.
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Introduction

The first chapter, co-authored with Andreas Schaab, studies the scope for international cooperation in macroprudential policies. Multinational banks contribute to and are affected by fire sales in countries they operate in. National governments setting quantity regulations non-cooperatively fail to achieve the globally efficient outcome, under-regulating domestic banks and over-regulating foreign banks, necessitating international cooperation. Surprisingly, non-cooperative national governments using Pigouvian taxation can achieve the global optimum. Intuitively, this occurs because applying taxes, rather than quantity regulations, leads governments to internalize the business value of foreign banks through the tax revenue collected. Our theory not only provides a unified framework to think about international bank regulations, but also yields concrete insights with the potential to improve on the current policy stance.

The second chapter, co-authored with Andreas Schaab, provides a framework to study bail-in regimes for banks. In the presence of a monitoring problem, the optimal bank capital structure combines standard debt, which liquidates the bank, and bail-in debt, which is written down to restore solvency. A bail-in regime, which increases use of bail-in debt, is the optimal regulatory policy when liquidation is socially costly due to fire sales or bailouts. Bail-ins fully replace bailouts. Bail-ins can generate self-fulfilling crises in long-term debt markets, leading to bank runs. Debt guarantees and an expanded notion of lender of last resort prevent these crises, and should complement bail-ins in the crisis resolution toolkit.

The third chapter, co-authored with Andreas Schaab, studies whether central banks’ inflation targets should remain set in stone. We study a dynamic mechanism design problem
between a government (principal) and a central bank (agent). The central bank sets inflation but suffers from a time consistency problem. The central bank also learns about persistent economic shocks affecting optimal inflation, and so influences beliefs of the government and of firms who use that information in price setting (Phillips curve). A “dynamic inflation target” implements the constrained efficient inflation level: the central bank reports its target one period in advance, with a linear incentive scheme for deviations from the target. This mechanism is optimal when the social costs of the incentive scheme are negligible relative to the inflation-output trade-off.
Chapter 1

Multinational Banks and Financial Stability

1.1 Introduction

The banking industry is multinational in its scope: banks that are headquartered in one country lend to, borrow from, and are owned by agents across country borders. More than 30% of global bank claims are on foreign counterparties as of 2019, with more than half of foreign claims being on the non-bank private sector. In the aftermath of the 2008 financial crisis, the scope of global banking has led to concerns that international bank activities can contribute to domestic financial stability risk, with foreign banks both exacerbating and being affected by domestic fire sales. These financial stability concerns have motivated financial regulators to extend post-crisis macroprudential regulatory regimes – such as equity capital and liquidity requirements – to foreign banks operating domestically. They

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1 Co-authored with Andreas Schaab


3 See e.g. Tucker (2016). See e.g. French et al. (2010) for a broader discussion of financial stability concerns and motivations for post-crisis financial regulation.

4 For example, the Intermediate Holding Company requirement in the US applies prudential standards of The Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) to foreign banks with
have also led governments to apply capital control measures – such as residency based
transaction taxes – to manage foreign capital flows. As a result of multinational banking,
the effects of fire sales and macroprudential regulation also extend across country borders.

The global dimension of banking has led to concerns that uncoordinated financial regu-
lation may not be efficient. In the words of former Bank of England Deputy Governor Paul
Tucker, “given the internationalization of finance, the problem of financial system stability is a global
common-resource problem. That means that the standard of resilience needs to be shared and so agreed
internationally” (Tucker (2016)). These concerns have motivated the formation of cooperative
regulatory regimes, in which country regulators coordinate macroprudential policies. Coop-
erative regimes in practice include agreements to common regulatory standards (Basel III)
and explicit common supervisory and resolution regimes (European Banking Union, Single
Point of Entry (SPOE) resolution).5 Despite the prominence of these agreements and their
attention in the policy world, there is relatively limited formal economic analysis studying
the need for macroprudential cooperation in the presence of financial stability concerns
from cross-border banking, or to guide policymakers in forming cooperative agreements.

We provide a simple economic framework to study the regulation of cross-border banks
in the presence of fire sales.6 Despite its simplicity, we show that its insights extend to a
much more general setting. Our model captures essential elements of the global banking
industry and real economy. Banks engage in cross-border investment activities for a variety
of reasons such as comparative advantage or diversification against domestic risk. When
banks experience adverse shocks, they are forced to sell domestic assets, leading to a
domestic fire sale. The model features two sources of cross-border spillovers from fire sales.
First, the domestic fire sale directly spills over to foreign banks investing in the domestic
economy, reducing both recovery values in liquidation and collateral values. Second, the
domestic fire sale leads banks to liquidate foreign assets, exacerbating fire sales in foreign

large operations in the US.

5See BIS (2010) for Basel III, ECB (2018b) for an overview of the EU Single Supervisory Mechanism
responsible for common supervision, and Financial Stability Board (2013) for a discussion of SPOE.

6We frame our model in terms of banks, but it also applies to broader classes of financial intermediaries.
countries. Although fire sales are domestic in origin, cross-border banking leads them to propagate across countries. Domestic financial stability becomes a global concern. We relate the model and its spillovers to several applications, including financial integration in the EU, capital flows between developed economies and emerging markets, and countries with large global banking presences (such as the US).

Fire sale spillovers in the model, which are not internalized by banks, motivate the consideration of macroprudential regulation. Our first contribution is to characterize the globally efficient regulatory policy, which has two important properties. First, the stringency of globally efficient regulation accounts for not only domestic fire sale spillovers, but also for international spillovers that arise through cross-border banking activities. Second, equal regulatory stringency is applied to all banks regardless of their domicile, so that banks can enjoy equally the benefits of international activities.

In practice, regulatory policies are often handled by country level governments, who may engage in cooperative agreements governing policy design. Our second contribution is to provide a theory of the non-cooperative design of macroprudential policies by independent country governments, and to ask whether international cooperation is required to achieve the globally efficient outcome. In practice, countries have regulatory jurisdiction over all activities of domestic banks, as well as the domestic activities of foreign banks operating within their borders. This implies that multiple countries have regulatory jurisdiction over the same bank. In the absence of cooperative agreements, national regulators will set macroprudential policies independently to maximize national welfare. Our framework captures and allows us to study this non-cooperative behavior.

We next provide a theory of non-cooperative design of macroprudential policies under a quantity regulation approach. Non-cooperative quantity regulation does not achieve the globally efficient outcome. Country planners regulate both domestic and foreign banks in equilibrium. However, regulation of domestic banks accounts for domestic fire sale spillovers, but not for international fire sale spillovers. By contrast, country planners “ring fence” foreign banks, that is they ensure foreign banks’ domestic activities are sufficiently
safe that they do not contribute to domestic fire sales, leading to unequal treatment of
domestic and foreign banks. This is consistent with regimes that subject local subsidiaries
of foreign banks to domestic regulatory requirements, often assuming no support will
be provided by the parent organization.\textsuperscript{7} These departures from efficient policy provide
a theory of optimal cooperation, which both increases regulation of domestic banks and
ensures equal regulatory treatment of foreign banks. This resembles and helps to understand
the broad architecture and goals of existing cooperative regimes.

In practice, cooperation can be difficult to sustain when national and international
interests diverge.\textsuperscript{8} One might wonder then whether instruments other than quantity
regulation could improve non-cooperative outcomes and reduce the need for cooperation.
One natural candidate is Pigouvian taxation, which is a common prescription for externality
problems. However, there are two reasons to expect a priori that Pigouvian taxation would
yield similar outcomes to regulation: first, these instruments are typically equivalent in
models lacking standard (Weitzman (1974)) price-quantity trade-offs;\textsuperscript{9} and second, there are
international spillovers associated with fire sales.

The most surprising result of our paper is that non-cooperative Pigouvian taxation can
implement the globally efficient outcome. In particular, country planners set tax rates that
coincide with globally optimal policy, up to a monopolistic revenue extraction distortion.
When countries’ monopoly power is zero due to sufficient substitutability, non-cooperative
Pigouvian taxation is globally efficient. The mobility of global banking assets and the
presence of large offshore financial centers suggests that low monopoly revenues at the
country level is a plausible description of the world.\textsuperscript{10}

\textsuperscript{7}For example, US total loss absorbing capital (TLAC) requirements apply to the US intermediate holding
companies (IHCs) of foreign systemically important banks, rather than to the entire banking group.

\textsuperscript{8}One recent example is the decision by the Italian government to engage in partial bailouts of distressed
Italian banks in 2017, which faced criticisms as undermining the European Banking Union. “Why Italy’s € 17bn

\textsuperscript{9}For example, Erten et al. (2019) argues that “the principle of dualism...implies that every quantity-based
control corresponds to an equivalent price-based control.”

\textsuperscript{10}For example, see the work by Coppola et al. (2019) on global capital flows and tax havens.
The key feature that gives rise to the optimality of Pigouvian taxation is that taxes on foreign banks generate revenues for the domestic planner. When combined with the standard motivation to correct domestic externalities, the revenue generation motive leads to efficient outcomes. The intuition is that a country planner becomes willing to allow foreign banks to engage in socially costly domestic activities because she can collect more tax revenue as a result. This results in an alignment in preferences between country planners and foreign banks. In equilibrium, the marginal tax rates on foreign banks’ domestic activities are equal to the marginal benefit to foreign banks of those activities. Moreover because domestic fire sales reduce the marginal benefit to foreign banks of domestic activities, they also reduce tax revenue collection. The motive to generate tax revenue thus not only leads country planners to internalize the marginal benefit to foreign banks of domestic activities, but also to internalize the spillovers of domestic fire sales onto foreign banks. By contrast, quantity regulation lacks the revenue generation motive, giving no reason for the domestic planner to care about the welfare of foreign banks and leading to inefficient outcomes. Revenue generation from Pigouvian taxation is what aligns incentives. If Pigouvian taxation of foreign banks were revenue neutral, it would lead to the same outcome as quantity regulation.

The efficiency of non-cooperative Pigouvian taxation has implications for both macroprudential policies and capital controls. In the macroprudential context, it implies that giving a larger role in the financial regulatory regime to Pigouvian taxation can reduce or eliminate the need for cooperative arrangements.\textsuperscript{11} Although in practice macroprudential regulation often takes the form of quantity regulation, rather than Pigouvian taxation, we speculate that this may have arisen in part due to a combination of perceived duality between the instruments and political obstacles to taxation. Our results contribute a new argument in favor of a Pigouvian tax approach to macroprudential policies. In the capital controls context, our results imply that price-based capital control measures set non-cooperatively can be globally efficient. This provides further motivation for use of capital control measures

\textsuperscript{11}For example, replacing capital requirements with Pigouvian taxes on debt.
in managing financial stability.

Even when Pigouvian taxation does not yield exact efficiency, it has the potential to simplify cooperation in two ways: first, by restricting the need for cooperation to foreign bank activities; and second, by reducing a multilateral spillover problem to a bilateral monopolist problem. Moreover, it implies that a set of partial equilibrium elasticities can be used to evaluate the need for cooperation.

We show that our simple framework can be generalized to study a number of other salient features of global banking. Our main application is the provision of fiscal backstops such as lender of last resort (LOLR) and deposit insurance. We show that non-cooperative governments under-value fiscal backstops to both domestic and foreign banks, not internalizing the full stability benefits to foreign banks. This motivates common fiscal backstops, such as Common Deposit Insurance in the EU. Because fiscal backstops are chosen by governments rather than by banks, Pigouvian taxation does not result in efficient (non-cooperative) choices of fiscal backstops unless banks are taxed for the bailouts they expect to receive.

Our results are sufficiently general that they hold in a broader class of externality problems featuring multinational agents. We present a general model of these externality problems. We characterize two classes of externalities: local and global. Local externalities, such as local pollution, are externalities that affect domestic agents, but not foreigners. However, foreign agents can contribute to local externalities by conducting domestic activities. Global externalities, such as global pollution or climate change, on the other hand, have costs that are globally diffuse and affect foreign agents. The efficiency of non-cooperative Pigouvian taxation extends to the class of local externalities, even though foreign multinational agents contribute to them, following the same logic as in the main model. By contrast, non-cooperative Pigouvian taxation is not generally efficient for global externalities. While cooperation is not required for local externalities, it is more generally required for global externalities. We show that cooperation tends not to be required when a local externality, such as a fire sale, takes on a global dimension due to cross-border activities, because Pigouvian taxes lead the national government to internalize the externality’s impact on
international agents. By contrast, global externalities that spread even without cross-border activities, such as climate change, generally require cooperation.

**Related Literature:** First, we relate to a large empirical literature on capital flows, retreatment, and home bias, including in the context of banks. The empirical literature documents both home bias and cross-country differences in foreign investment holdings, and suggests that motivations for foreign investment extend beyond diversification. These empirical observations help motivate the assumptions underlying our baseline banking model.

Second, we relate to a smaller literature on optimal regulatory cooperation in international banking and financial markets. Caballero and Simsek (2018, 2019) show fickle capital flows can be globally valuable when they provide liquidity to distressed countries. National regulators do not internalize this benefit and ban capital inflows to mitigate domestic fire sales, generating a scope for cooperation. Korinek (2017) prove a first welfare theorem in a model in which country planners control domestic agents, who interact on global markets. Their welfare theorem does not hold in our model, in which domestic liquidation prices affect foreign agents and where domestic planners are able to affect these liquidation prices. Farhi and Tirole (2018) show that national regulators loosen bank supervision to dilute existing international creditors, generating a time consistency problem that motivates supranational supervision. Bolton and Oehmke (2019) study the trade-off between single- and multi-point-of-entry in bank resolution. Bengui (2014) and Kara (2016) consider regulatory cooperation when banks’ operations are domestic but the asset resale market is global. Our contribution to this literature is to study a global non-cooperative regulatory problem with common agency over cross-border banks, to characterize optimal cooperative policy, and to

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characterize optimality of Pigouvian taxation in the global context.

Third, we relate to a large literature on macroprudential regulation and capital controls.\textsuperscript{14} Both literatures study optimal policy in response to fire sale externalities in the domestic economy, with macroprudential policies mitigating the contribution of domestic agents to fire sales and capital controls mitigating the contribution of foreign agents to fire sales. We study optimal policy in a global economy where banks are exposed to fire sales in all countries they invest in. Domestic planners have controls over both domestic and foreign banks, so that we incorporate both macroprudential policies and capital controls.

Fourth, we relate to the literature on principal-agent problems with common agency.\textsuperscript{15} Our model features a common agency problem between countries regulating multinational banks and externalities between banks, and we differentiate between (sub-optimal) non-cooperative regulation and (optimal) non-cooperative Pigouvian taxation.

Fifth, we relate to the literature on the distinction between quantity and price regulation.\textsuperscript{16} The primary difference between price and quantity regulation in our paper differs from the standard one: Pigouvian taxation generates revenues for the domestic planner, whereas quantity regulation does not. This revenue generation motive is required for the optimality of non-cooperative Pigouvian taxation. If revenues from Pigouvian taxation were remitted to foreigners, rather than to domestic agents, it would be equivalent to quantity regulation in our model.


\textsuperscript{15}See e.g. Bernheim and Whinston (1986) and Dixit \textit{et al.} (1997). In common with much of this literature, we apply an equilibrium concept where all country planners take policies of other countries as given. Another solution concept involves contracting on contracts. See e.g. Szentes (2015).

\textsuperscript{16}See e.g. Weitzman (1974). In the banking context, see Perotti and Suarez (2011).
1.2 Model

There are three dates, \( t = 0, 1, 2 \). The world economy consists of a unit continuum of countries, indexed by \( i \in [0, 1] \). All countries are small and of equal measure, but are not necessarily symmetric or otherwise identical ex ante.

Each country is populated by a representative bank and a representative arbitrageur.\(^{17}\) Banks raise funds from global investors to finance investment in both their home country and in foreign countries. Arbitrageurs are second-best users of bank investment projects, and purchase bank investments that are liquidated prior to maturity. Arbitrageurs and global investors exist in our model to ease solving for the general equilibrium prices that banks face. Accordingly, we make their decision problems as simple as possible.

A global state \( s \in S \) is realized at date 1, at which point uncertainty resolves. The global state \( s \) is continuous with density \( f(s) \). It captures all shocks in the model, including global, regional, and country-level shocks.

In the baseline model presented in this section, we adopt a leading example model of banking with simplifying assumptions on preferences, technology, financing structure, and so on. In Section 1.7, we extend results to a much more general environment.\(^{18}\)

1.2.1 Banks

Banks are risk-neutral and do not discount the future. Banks only consume at date 2, with final consumption denoted by \( c_i(s) \).

\(^{17}\)The core aspects of the model (international activities, home bias, and fire sales) apply to broader financial intermediaries, such as mutual funds. Although for expositional purposes we frame the discussion in terms of banks, the model is also applicable to this broader class of financial intermediaries.

\(^{18}\)For example, the general environment can capture general preferences, technology (e.g. multiple assets), extended liability structures, an internal bank capital market, heterogeneous (within country) banks, additional global goods and markets, and additional local goods and markets.
Bank Activities

Banks engage in activities such as retail banking, lending to small and medium-sized enterprises (SMEs), and investment banking, which we refer to collectively as “projects,” “investment,” or “assets.” Projects are illiquid, and suffer a loss when liquidated (sold) prior to maturity.

In each country, there is an investment project available to banks. We denote by $I_{ij}$ the (date 0) investment by country $i$ banks in the country $j$ project. A core assumption in our model is that bank investment is home biased.\(^{19}\)

**Assumption 1** (Home Bias). A bank investment portfolio is $I_i = \{I_{ij}\}_j$, where

1. Domestic investment $I_{ii} \in \mathbb{R}_+$ is a mass.
2. Foreign investment $I_{ij} : [0, 1] \rightarrow \mathbb{R}_+$ is a density.

Home bias will arise in our model when domestic banks specialize in domestic activities. Fire sales will be a core focus of the model. If domestic banks did not retain a mass exposure to the domestic economy, they would only be marginally affected by domestic fire sales. Home bias ensures that domestic banks are substantially exposed to domestic fire sales. Assuming that banks retain only a marginal exposure to foreign countries is a simplifying assumption to maintain tractability.

Banks operate a technology which uses $\Phi_{ij}(I_{ij})$ units of the numeraire to produce $I_{ij}$, where $\Phi_{ij}$ is increasing and weakly convex. Banks’ total bank investment cost is therefore $\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij}) dj$. Examples of costs are direct lending costs and operational costs.

At date 1, projects in country $j$ experience quality shocks $R_j(s)$. As a result, the scale of projects operated by country $i$ banks in country $j$ changes to $R_j(s)I_{ij}$. Projects do not yield dividends at date 1. When projects are held to maturity at date 2, banks receive $1 + r_{ij} \geq 1$ units of the consumption good per unit of project scale. Intuitively, $R_j(s)$

\(^{19}\)See Caballero and Simsek (2019) for a similar assumption. As highlighted in the introduction, home bias is an empirical regularity, with in the neighborhood of 66% of bank claims being on domestic counterparties (CBS).
captures a common risk exposure from investment in a country, while $r_{ij}$ captures different specializations (comparative advantages) in bank lending. If $r_{ii} > r_{ij}$, then banks specialize in domestic lending, providing one potential source for home bias in lending.

Projects may be liquidated prior to maturity. We denote project liquidations by $L_{ij}$, defined analogously to $I_{ij}$, with $0 \leq L_{ij}(s) \leq R_{ij}(s)I_{ij}$. Projects liquidated at date 1 are sold at price $\gamma_{ij}(s) \leq 1$ to arbitrageurs, with the discount $1 - \gamma_{ij}(s)$ reflecting the degree of illiquidity of a project. The final return $r_{ij}$ is lost when a project is liquidated prior to maturity.

**Bank Budget Constraints**

Banks have an initial endowment $A_i > 0$, and can also raise external debt $D_i$ from risk-neutral global investors at price 1. Given a fixed debt price of 1, the liquidation prices $\gamma$ are the only endogenous prices in the model. The bank uses its total funds to finance its investment portfolio at date 0, so that the date 0 bank budget constraint is

$$
\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij})dj \leq A_i + D_i. \tag{1.1}
$$

At date 1, banks can roll over debt at a price of 1. Consolidating the dates 1 and 2 budget constraints yields

$$
c_i(s) \leq R_{ii}(s) + \int_j R_{ij}(s)dj - D_i, \tag{1.2}
$$

where $R_{ij}(s) = \gamma_{ij}(s)L_{ij}(s) + (1 + r_{ij})(R_{ij}(s)I_{ij} - L_{ij}(s))$ is the total return to investment in country $j$ for country $i$ banks from both date 1 liquidations and date 2 final payoffs.

**Collateral Constraints**

If banks faced no restrictions on rolling over debt, they would never choose to liquidate assets since liquidations always reduce bank value. To introduce a role for liquidations and fire sales, we impose a date 1 collateral constraint, which is a standard method of capturing forced deleveraging (see e.g. Kiyotaki and Moore (1997)).

---

20We consolidate banks’ balance sheets across countries and operations for expositional simplicity.
The date 1 collateral constraint requires banks to back debt issued at date 1 with collateral, and is given by

\[
D_i \leq \gamma_i(s)L_{ii}(s) + \int \gamma_i(s)L_{ij}(s)dj + (1 - h_i(s))C_{ii}(s) + \int (1 - h_j(s))C_{ij}(s)dj,
\]

(1.3)

where \(C_{ij}(s) = \gamma_j(s) [R_{ij}(s)I_{ij} - L_{ij}(s)]\) is the market value of collateral at date 1. The collateral haircut \(h_j(s) \in [0, 1]\) reflects the extent to which investors discount a project’s collateral value, and can reflect economic (e.g. uncertainty) and political (e.g. expropriation) concerns about collateral quality. Banks that cannot roll over their entire liabilities using collateral must liquidate assets to repay investors.

**Bank Optimization**

At date 0, banks choose a contract \((c_i, D_i, I_i, L_i)\) to maximize expected utility

\[
V^B_i = \int c_i(s)f(s)ds
\]

subject to the budget constraints (1.1) and (1.2), and the collateral constraint (1.3). Banks take equilibrium prices \(\gamma\) as given.

Banks in our model choose their entire contract with commitment, including liquidations \(L_i\). The bank liquidation decision outlined in this section and in Section 1.3 is in fact time consistent, so that the assumption is innocuous in this sense. The core advantage of allowing liquidations to be chosen with commitment is that it simplifies the regulatory problems studied in this paper.

**1.2.2 Arbitrageurs and Liquidation Values**

Country \(i\) arbitrageurs are second best users of country \(i\) projects.\(^{21}\) At date 1, they purchase an amount \(L^A_i(s)\) of bank projects and convert them into the consumption good using an increasing and (weakly) concave technology \(F_i(L^A_i(s), s)\). Arbitrageur technology is

\(^{21}\)In Appendix A.4.5, we consider the case where resale markets are also in part global. The baseline model is equivalent to a model where there are global arbitrageurs whose technology is separable across countries.
inefficient in the sense that \( \frac{\partial F_i(L^A_i(s), s)}{\partial L^A_i(s)} \leq 1 \), so that selling projects to arbitrageurs never results in a resource gain.

Arbitrageurs obtain surplus \( c^A_i(s) = F_i(L^A_i(s), s) - \gamma_i(s)L^A_i(s) \) from purchasing projects. Arbitrageurs are price takers, so that the equilibrium liquidation value is

\[
\gamma_i(s) = \frac{\partial F_i(L^A_i(s), s)}{\partial L^A_i(s)} \quad \text{and} \quad L^A_i(s) = L_{ii}(s) + \int_j L_{ij}(s) dj
\]

(1.4)

where \( L^A_i(s) \) is equal in equilibrium to total country \( i \) projects sold by all banks, including foreign ones. There is a fire sale spillover when additional liquidations reduce liquidation values, that is when the marginal product of bank projects in arbitrageur technology is strictly decreasing. The extent of the fire sale spillover reflects the ability of the economy to absorb liquidations by banks, with deeper fire sales arising when limited market depth allocates liquidated bank projects to increasingly less efficient users.

1.3 Competitive Equilibrium

We begin by studying the competitive equilibrium of the global economy to understand the motivations for cross-border banking and map our model into a number of economically important settings in international banking.\(^{22}\) To build concrete intuition, we study the optimal choices of investment and liquidations by banks.

**Lemma 2.** In the competitive equilibrium, the private optimality conditions for investment \( (I_{ij}) \) and liquidations \( (L_{ij}(s)) \), respectively, are

\[
0 \geq -\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial L_{ij}} + E[\lambda_i^1] E \left[(1 + r_{ij})R_j\right] + \text{cov} \left(\lambda_i^1, (1 + r_{ij})R_j\right) + E \left[\bar{\xi}_{ij} R_j\right] + E \left[\Lambda_i^1 (1 - h_j) \gamma_j R_j\right]
\]

\[
\text{Specialization} \quad \text{Diversification} \quad \text{Liquidity} \quad \text{Collateral}
\]

(1.5)

\[
0 = \lambda_i^1 (s) \left( \gamma_j(s) - (1 + r_{ij}) \right) + \frac{\lambda_i^1(s)}{h_j(s)} \gamma_j(s) + \frac{\Lambda_i^1(s)}{\xi_{ij}} (s) - \frac{\bar{\xi}_{ij}}{s}
\]

(1.6)

\(^{22}\)Formally, a competitive equilibrium of the global economy is a vector of allocations \((c, D, I, L, L^A)\) and prices \( \gamma \) such that: (1) the contract \((c_i, D_i, I_i, L_i)\) is optimal for country \( i \) banks, given prices; (2) purchases \( L^A_i \) are optimal for country \( i \) arbitrageurs, given prices; and the markets for liquidations clear.
where the (non-negative) Lagrange multipliers are $\lambda_0^i$ on the date 0 budget constraint (1.1), $\lambda_1^i(s)$ on the date 1 budget constraint (1.2), $\Lambda^i_1(s)$ on the date 1 collateral constraint (1.3), and $\xi_{ij}(s), \overline{\xi}_{ij}(s)$ (respectively) on the constraints $0 \leq L_{ij}(s) \leq R_j(s)I_{ij}$ in state $s$.

Proof. All proofs are in the Appendix.

The liquidation decision in equation (1.6) shows that because liquidations always result in resource losses, banks only liquidate assets when the collateral constraint binds, that is $\Lambda_1^i(s) > 0$. When forced to liquidate assets, banks prefers to liquidate assets with lower liquidation discounts, $(1 + r_{ij}) - \gamma_j(s)$, and higher collateral haircuts $h_j(s)$.

The investment decision in equation (1.5) reflects the motivations for foreign investment. “Specialization” reflects that banks invest in an asset when the marginal cost is low (e.g. activity specialization) or the return is high (e.g. lending in an under-serviced market). “Diversification” indicates the value of an asset that pays off when the collateral constraint binds and the marginal value of bank wealth is high. “Liquidity,” measures the value to the bank of being able to liquidate an asset when faced with binding collateral constraints. “Collateral,” measures the value of an asset for use as collateral, decreasing in both the haircut and the liquidation discount.

The diverse set of motivations for cross-border investment in our model allows for different countries to have different levels and compositions of foreign investment exposures, allowing us to capture a number of important potential applications of the model.

1.3.1 Relationship to Empirics

In our model, bank specialization in domestic activities ($r_{ii} > r_{ij}$), for example due to information advantages or lower costs, can generate two empirical regularities: home bias and retrenchment patterns.

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23For example, recent work by De Marco et al. (2019) emphasizes the role of information in determining European banks’ holdings of sovereign debt.
Home bias in investment is an empirical regularity whereby banks tend to overconcentrate in domestic assets compared to their world market weight. Nearly 70% of bank claims are on domestic counterparties.\textsuperscript{24} Equation (1.5) indicates that domestic specialization motivate banks to scale up domestic investment when $r_{ii} > r_{ij}$.

A second empirical regulatirty is that domestic bank retrenchment generally coincides with reductions in foreign investment into the domestic economy.\textsuperscript{25} When $r_{ii} > r_{ij}$, the forgone final return from liquidating domestic investment is larger than from liquidating foreign investment, motivating domestic banks to retrench and liquidate foreign assets. Foreign banks likewise retrench, coinciding with the empirical pattern.

Both of these empirical regularities are meaningful to the spillover and regulatory problem studied in the baseline model. Home bias implies that domestic banks maintain a substantial exposure to domestic risk and domestic fire sales. Foreign retrenchment implies that distressed banks liquidate assets in foreign countries, contributing to foreign stability risks.

1.3.2 Applications

We now map our baseline model into three important real world applications.

**EU Financial Integration:** Risk sharing and increased financial stability are two key motivations for both cross-border financial integration and organization under a common supervisory framework (the Single Supervisory Mechanism, or SSM) in the EU.\textsuperscript{26} Our model is able to capture the European experience with a block $I \subset [0, 1]$ of countries (“the EU”) have both partly uncorrelated and partly correlated returns. The uncorrelated component motivates cross-border investment for diversification (equation 1.5). However, a

\textsuperscript{24}BIS CBS, as cited in the introduction.

\textsuperscript{25}See e.g. Avdjiev et al. (2018), Broner et al. (2013), Davis and Van Wincoop (2018), and Forbes and Warnock (2012).

\textsuperscript{26}See ECB (2018a) and ECB (2018b).
correlated EU wide shock leads all member states to fire sell assets in tandem, fueling an EU wide crisis.

**Developed and Emerging Markets:** A large literature has emphasized that emerging markets may wish to use capital controls to manage domestic fire sales. Our model captures emerging market stability concerns through a set of developed economies \( D \subset [0, 1] \) and a set of emerging markets \( E \subset [0, 1] \). Emerging markets have high expected returns, making them a target for developed economy investment (equation (1.5)). Developed economies have lower collateral haircuts due to stronger legal protections and lower uncertainty, making them a target for emerging markets looking for safe and liquid assets (equation (1.5)). Due to larger emerging market haircuts, developed economies retrench during domestic crises (equation 1.6)), contributing to emerging market instability.

**“Systemically” Important Countries:** Some countries have larger foreign banking presences, and so may be important in the sense that they can generate larger spillovers. For example, the US has a large and growing presence in global investment banking, leading to concerns about European exposure to US shocks and retrenchment. Our model captures systemically important countries through a block of countries \( I \subset [0, 1] \) that have large endowments \( A_i \) and are efficient in foreign lending (e.g. low costs \( \Phi_{ij} \)). These countries maintain a large global banking presence (equation (1.5)), and their retrenchment may increase foreign financial instability.

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27See Erten et al. (2019) for an overview and IMF (2012) for a policy perspective.

28See the literature on global imbalances, for example Caballero et al. (2008), Maggiori (2017), and Mendoza et al. (2009).

1.4 Globally Optimal Policy

The presence of prices in constraints suggests existence of inefficient pecuniary externalities, warranting regulatory intervention. We begin by studying the optimal policy that would be chosen by a *global* planner looking to implement a globally Pareto efficient allocation. This global planning problem provides a benchmark for globally efficient policies.\(^{30}\)

The global planning problem is a constrained-efficient problem, where the global planner maximizes a weighted sum of bank welfare

\[
V^G = \int \omega_i \int c_i(s)f(s)dsdi
\]

subject to the same constraints (equations (1.1), (1.2), (1.3)) as faced by banks, but internalizing the equilibrium pricing equation (1.4).\(^{31}\)

We place welfare weights of zero on arbitrageurs for exposition, and show in the Appendix that qualitatively similar results apply with positive weights. We characterize the solution of the global planning problem by its decentralization: the complete set of date 0 wedges \(\tau = \{\tau^c_i, \tau^D_i, \tau^I_{ij}, \tau^L_{ij}(s)\}\) and date-0 lump sum transfers \(T_i\) that implement the globally optimal allocation. The complete set of wedges place on country \(i\) banks consist of: a wedge \(\tau^c_i(s)\) on consumption in state \(s\); a wedge \(\tau^D_i\) on date 0 debt; a wedge \(\tau^I_{ij}\) on investment in country \(j\); and a wedge \(\tau^L_{ij}(s)\) on liquidations of country \(j\) assets in state \(s\).\(^{32}\) Given welfare weights \(\omega_i\), the globally efficient allocation is characterized as follows.

**Proposition 3.** The globally efficient allocation can be decentralized using liquidation wedges

\[
\tau^L_{ij}(s) = -\underbrace{\Omega_{i,i}(s)}_{\text{Domestic Spillovers}} - \underbrace{\int \Omega_{i,j}(s)di'}_{\text{Foreign Spillovers}} \underbrace{\forall j}_{\text{Equal Treatment}}
\]

\(^{30}\)For expositional purposes, we present results in this section and in Section 1.5 for interior results. Results generalize to cases that include corner solutions. See Section 1.7 and Appendix A.5.

\(^{31}\)We do not give the global planner controls over arbitrageurs to prevent subsidizing asset purchases, which may be considered a form of bailout. We study fiscal backstops (“bailouts”) in Section 1.6.

\(^{32}\)As usual, the decentralizing wedge is set equal to the gap between the social first order condition and the private first order condition.
where $\Omega_{ij}(s) \leq 0$ is given by

$$
\Omega_{ij}(s) = \frac{\partial \gamma_i(s)}{\partial L^j_i(s)} \left[ \lambda^j_i(s) L_{ij}(s) - \frac{\lambda^0_i(s)}{\lambda^0_i} \left[ L_{ij}(s) + (1 - h^j_i(s)) [R^j_i(s) I_{ijij} + L_{ij}(s)] \right] \right]
$$

(1.8)

All other wedges are 0.

The globally efficient allocation corrects for a fire sale spillover problem: higher liquidations reduce liquidation prices and collateral values, tightening banks’ collateral constraints further and forcing more liquidations. When foreign banks also invest domestically, a domestic fire sale has spillovers onto both domestic and foreign banks. The globally optimal policy restricts liquidations in a manner that accounts for this total fire sale spillover onto domestic and foreign banks. Foreign spillovers are large when foreign banks are forced to liquidate domestic assets or face binding collateral constraints at the same time that the domestic liquidation price is particularly sensitive to additional liquidations.

Moreover, globally efficient policy applies equal treatment: the wedge on liquidations of the country $i$ asset does not depend on the domicile of the bank liquidating it. Because they generate the same total fire sale spillover by liquidating a domestic project, both domestic and foreign banks have the same wedge placed on doing so. Although foreign banks can contribute to domestic instability by retrenching, they are not treated different from domestic banks in terms of regulatory policy.

### 1.5 Non-Cooperative Policies

The globally efficient policy of Section 1.4 is predicated on a global planner setting policy. However, in practice individual countries have regulatory jurisdiction over banks within their borders. In this section, we provide a theory of how country-level governments design macroprudential regulation in the presence of fire sales, and ask whether they implement the globally efficient policy independently and non-cooperatively. We show that independent governments using *quantity regulation* are unable to achieve efficient policy, and show that
the departures from efficiency help understand existing cooperative regimes. By contrast, independent governments using *Pigouvian taxation* have the potential to achieve efficient policies.

### 1.5.1 Country Planners

Each country has a designated government, or “social planner,” who represents and acts in the interests of domestic agents. The social welfare function of country planner $i$ is equal to domestic bank welfare,

$$V_i^p = \int c_i(s) f(s) ds.$$

The social planner has a complete set of “macroprudential” wedges on both domestic banks and domestic allocations of foreign banks. The wedges of the country $i$ planner on country $i$ banks are $\tau_{ii} = (\tau_{ii}^c, \tau_{ii}^D, \tau_{ii}^L)$, and are fully contingent as in Section 1.4. The wedges of the country $i$ planner on country $j$ banks are $\tau_{ij} = (\tau_{ij}^l, \tau_{ij}^L)$, again fully contingent, reflecting that the country $i$ planner can only directly influence the domestic activities of foreign banks.

Wedges are taxes from the perspective of banks, meaning that revenue is collected from their use. We will interpret *revenue-neutral* wedges as quantity restrictions, appealing to duality results between quantity restrictions in revenue neutral wedges in problems with single regulators.\(^{33}\) We will refer to *revenue-generating* wedges as Pigouvian taxation. In this case of quantity regulation, regulators use wedges to control allocations, but not to generate revenues. In this case of Pigouvian taxation, there is also a revenue motive.\(^{34}\)

The total *date 0* wedge burden borne by country $i$ banks (excluding remissions) is

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\(^{33}\)See e.g. Erten et al. (2019). In Appendix A.4.7, we discuss a planning problem that uses explicit quantity restrictions.

\(^{34}\)Because wedges are the means of controlling allocations, we rule out explicit side payments.
\[ T_{i,i} + \int_j T_{i,ij} dj, \]
where
\[
T_{i,i} = \int_s \tau_{i,i}^c(s)c_i(s)f(s)ds + \tau_{i,i}^D D_i + \tau_{i,i}^I I_i + \int_j \tau_{i,ij}^I I_{ij} dj \\
+ \int_s \left[ \tau_{i,ii}^L(s)L_{ii}(s) + \int_j \tau_{i,ij}^L(s)I_{ij}(s) dj \right] f(s)ds
\]
is the burden to the domestic planner, and
\[
T_{j,ij} = \tau_{j,ij}^I I_{ij} + \int_s \tau_{j,ij}^L(s)L_{ij}(s)f(s)ds
\]
is the burden to foreign planner \( j \). To ease exposition, we adopt inner product notation
\[
T_i = \tau_i^c c_i + \tau_i^D D_i + \tau_i^I I_i + \tau_i L_i \quad \text{and} \quad T_{i,ij} = \tau_{i,ij}^I I_{ij} + \tau_{i,ij}^L L_{ij}.
\]

Let \( T_{i,i}^* \) denote the equilibrium tax revenue collected by country planner \( i \) from all domestic banks, where in equilibrium \( T_{i,i}^* = T_{i,i} \). The equilibrium tax revenue \( T_{i,i}^* \) is remitted lump-sum to domestic banks, under both quantity restrictions and Pigouvian taxation.

Under quantity regulation, the equilibrium tax revenue collected from foreign banks is remitted globally to foreign banks. As a result, the equilibrium date 0 tax burden of country \( i \) banks is
\[
T_{i,i}^{\text{Quantity}} = T_{i,i} - T_{i,i}^* + \int_j T_{j,ij} dj - T_i^G
\]
where \( T_i^G \) is global remitted revenues, which country \( i \) takes as given.\(^{35}\)

Under Pigouvian taxation, the equilibrium tax revenue collected from foreign banks is remitted to domestic banks. The equilibrium date 0 tax burden is
\[
T_{i,i}^{\text{Pigou}} = T_{i,i} - T_{i,i}^* + \int_j T_{j,ij} dj - \int_i T_{i,ij} dj.
\]

In contract to quantity regulation, the country \( i \) planner now accounts for how changes in policy affect the revenue \( \int_j T_{i,ij} dj \) collected from foreign banks.

\(^{35}\)In particular, there is the globally remitted revenue \( T_i^G = \int\int T_{i,ij} dijdj \) arising from the wedges, which corresponds to remitting revenue to foreigners. We assume this is remitted according to some allocation rule \( \int_i T_i^G dj = T_i^G \).
In both cases, taxes appear in the banks’ date 0 budget constraint, as

$$\Phi_{ii}(I_{ii}) + \int_j \Phi_{ij}(I_{ij}) dj \leq A_i + D_i - T_i.$$  \hspace{1cm} (1.9)

Banks optimally choose contracts as in Section 1.2, now taking into account the additional tax burden.

**Equilibrium Concept:** A non-cooperative equilibrium of the model is a Nash equilibrium between country planners, in which every country planner optimally chooses wedges $\tau_i = (\tau_{i,jr}, \{\tau_{ij}\})$, taking as given the wedges $\tau_{-i}$ set by all other country planners, in order to maximize domestic social welfare, accounting for how tax rates affect the demand functions of domestic and foreign banks.

**Implementability:** Banks are subject to wedges by planners in all countries they operate in. Because country planner $i$ has a complete set of wedges over country $i$ banks, we can adopt the standard approach whereby country planner $i$ directly chooses the allocations of country $i$ banks, and then back out the implementing wedges. Although country planner $i$ only has controls over part of foreign banks’ contracts, we derive an implementability result that shows that the same procedure can be applied for choosing the domestic allocations of foreign banks.

**Lemma 4 (Implementability).** Under both quantity regulation and Pigouvian taxation, country planner $i$ can directly choose the domestic allocations of foreign banks, with implementing wedges

$$\tau_{i,ji} = -\tau_{i,ji} - \frac{\partial \Phi_{ji}}{\partial I_{ji}} + E\left[\frac{\lambda_j^1}{\Lambda_j^0}(1 + r_{ji})R_i\right] + \frac{1}{\lambda_j^0}E\left[\Lambda_j^1(1 - h_i)\gamma_i R_i\right]$$  \hspace{1cm} (1.10)

$$\tau_{i,ji}(s) = -\tau_{i,ji}(s) + \frac{\lambda_j^1(s)}{\lambda_j^0} \left(\gamma_i(s) - (1 + r_{ji})\right) + \frac{1}{\lambda_j^0}\Lambda_j^1(s)h_i(s)\gamma_i(s)$$  \hspace{1cm} (1.11)

where the Lagrange multipliers and the foreign wedges $\tau_{j,ji}$ are constants from the perspective of country planner $i$.

As a result, the optimization problem of country planner $i$ can be written as maximizing social
welfare $V^P_i$, directly choosing allocations $(c_i, D_i, I_i, L_i, \{l_{ji}, L_{ji}\})$, subject to equations (1.9), (1.2), (1.3), and (1.4).

To choose a domestic allocation of foreign banks, country planner $i$ unwinds the wedges placed by the foreign country planner, and then sets the wedge equal to the marginal value to foreign banks of that activity in the domestic country.\(^{36}\) Because foreign banks have only a marginal investment presence in country $i$, allocations in country $i$ can be influenced without filtering to the entire foreign bank contract. As a result, we can define the optimization problem of country planner $i$ using a standard approach.

Lemma 4 allows for solving the problem using a standard approach. Importantly, notice that the implementing wedges on foreign banks do not appear in the objective function of country $i$ planner under quantity regulation, because revenues are remitted to foreign banks. By contrast, they do appear in the objective function under Pigouvian taxation, through the budget constraint due to revenue collections.

1.5.2 Non-Cooperative Quantity Regulation

We now characterize the solution to the problem under quantity regulation, where revenue from wedges on foreign banks is remitted to foreign banks.

Proposition 5. Under non-cooperative quantity regulation, the equilibrium has the following features.

1. The domestic liquidation wedges on domestic banks are

$$\tau^L_{i,ji}(s) = -\frac{\Omega_{i,ji}(s)}{\text{Domestic Spillovers}}$$

(1.12)

where $\Omega_{i,ji}$ is defined as in Proposition 3.

\(^{36}\)Notice that the non-tax terms in equations (1.10) and (1.11) are terms in the first order conditions from the competitive equilibrium. Intuitively, the Lagrange multipliers $\xi_{ij}$ do not appear because the planner can always ensure that the first order conditions hold with equality at corner solutions by setting tax rates appropriate. See Appendix A.5.2.
2. The domestic liquidation wedges on foreign banks generate an allocation rule

\[ L_{ji}(s) \Omega_{ji}(s)_{\text{Domestic Spillovers}} = 0. \]

In other words, if \( |\Omega_{ji}(s)| > 0 \) then \( \tau_{ji}^l(s) \) is set high enough that foreign banks do not liquidate domestic assets in state \( s \).

3. All other wedges on domestic and foreign banks are 0.

Proposition 5 reflects how country planners use regulation to manage spillovers from domestic fire sales. First, country planners place wedges on domestic liquidations by domestic banks that account for the fire sale spillover cost to domestic banks. Because planners do not care about the welfare of foreign banks, the domestic wedges do not account for spillovers to foreign banks.

Second, country planners place wedges on liquidations by foreign banks. Because planners again do not care about foreign bank welfare, their objective is to eliminate foreign banks’ contributions to domestic fire sales whenever there is an adverse domestic spillover. Country planners thus prevent foreign banks from contributing to domestic fire sales even while allowing domestic banks to take risks that lead to domestic fire sales. This effective ban on domestic liquidations by foreign banks (e.g. a ban on outflows) is too strong in practice, and arises because there is no domestic benefit to foreign investment in the baseline model. In Appendix A.4.1, we obtain less strong versions of this result that capture the same logic: country planners are not concerned with the benefits to foreign banks of activities in the domestic economy, and allow foreign activities and liquidations only to the extent they benefit the domestic economy.

Finally, the domestic planner does not tax foreign liquidations by domestic banks. This happens because the investment presence of domestic banks in any single foreign country is marginal, so that country planners do not internalize their fire sale impact in foreign countries.
Departures from Global Efficiency: Non-cooperative quantity regulation differs from globally efficient policy in two important ways.

The first departure is that unlike globally efficient policy, non-cooperative policy does not account for foreign spillovers. As a result, the globally efficient wedge $\tau^L_{ii}(s)$ is generally higher than the non-cooperative wedge $\tau^L_{i,ii}(s)$. Non-cooperative regulation features too little regulation of domestic banks due to the foreign spillovers from domestic fire sales. This is a multilateral problem, as the domestic fire sale potentially affects all foreign countries investing domestically.

The second departure is unequal treatment of foreign banks for domestic activities. Whereas globally efficient policy features equal treatment of domestic and foreign banks, under non-cooperative quantity regulation foreign banks are regulated more stringently than domestic banks. This regulatory gap $\tau^L_{i,ii}(s) - \tau^L_{i,ii}(s)$ reflects a bilateral problem: the marginal benefit to foreign banks of liquidating the domestic asset outweighs the marginal cost to the domestic economy. Nevertheless, foreign liquidations are banned because that positive surplus accrues to foreign banks, and not to the domestic economy.

These two departures provide a theory of optimal cooperation under quantity regulation. Optimal cooperation serves both to enforce equal regulatory treatment of foreign banks, and to increase regulation of domestic banks.

1.5.3 Non-Cooperative Pigouvian Taxation

We now characterize the non-cooperative equilibrium under Pigouvian taxation, where wedge revenues from foreign banks are remitted domestically. Recall that the change in the remission rule is the only change relative to quantity regulation.

Proposition 6. Under non-cooperative Pigouvian taxation, the equilibrium has the following features.
1. The domestic liquidation wedges on domestic and foreign banks are

\[
\tau_{Li,j}(s) = \tau_{Lj,i}(s) = - \Omega_{i,j}(s) - \int_{i}^{j} \Omega_{i,j}(s) di' \quad \forall j
\]

where \( \Omega_{i,j}(s) \) are as defined in Proposition 3.

2. The wedges on domestic investment by foreign banks are

\[
\tau_{Lj,i} = \frac{\partial^2 \Phi_{j,i}}{\partial I_{ji}^2} I_{ji} \geq 0
\]

Monopolist Motive

3. All other wedges are 0.

In contrast to quantity regulation, non-cooperative planners using Pigouvian taxation account for both the benefits to foreign banks of liquidating domestic assets, and for the spillover effects of domestic fire sales to foreign banks, leading to implement the efficient wedges on asset liquidations.

This difference arises from the motive to collect revenue from foreign banks. To build intuition, suppose that we start from the non-cooperative equilibrium of Proposition 5, in which foreign liquidations are banned, and perturb the equilibrium so that foreign banks are allowed to liquidate a small amount \( \epsilon \) of assets. There are two immediate effects. First, the domestic planner is able to collect \( \tau_{Li,j}(s)\epsilon \) in tax revenue from foreign banks, rather than nothing, which increases domestic welfare. The second effect is that the domestic fire sale is exacerbated, which has cost \( \tau_{Lj,i}(s)\epsilon \) to domestic banks. However, the bilateral tension of the non-cooperative regulatory equilibrium arises when the marginal benefit \( \tau_{Lj,i}(s) \) to foreign banks exceeds the marginal spillover cost \( \tau_{Li,j}(s) \), so that \( (\tau_{Lj,i}(s) - \tau_{Li,j}(s))\epsilon > 0 \). As a result, the domestic planner benefits from allowing some foreign bank liquidations, rather than none. In other words, taxation allows for the marginal surplus of foreign bank liquidations to accrue to the domestic planner, rather than foreign banks, leading the domestic planner to implement an optimal policy. This helps to understand why the wedge on liquidations by foreign banks is equal to that on domestic banks.
It is less immediate why the domestic planner also internalizes the fire sale spillovers to foreign banks when setting taxes. The reason is that larger fire sale spillovers reduce the marginal benefit foreign banks receive from domestic activities, which results in lower tax revenue collection for the domestic planner. Formally, the derivative of revenue collected from country $j$ banks in the total domestic liquidations $L_A^i(s)$ is given by

$$
\frac{1}{f(s)} \frac{\partial T^*_{ij}}{\partial L_A^i(s)} = \frac{1}{f(s)} \frac{\partial \gamma_i(s)}{\partial L_A^i(s)} \frac{\partial}{\partial \gamma_i(s)} \left[ \tau_{ij} I_{ij} + \int_{s'} \tau_{ij}(s') L_{ij}(s') f(s') ds' \right]
$$

$$
= \frac{\partial \gamma_i(s)}{\partial L_A^i(s)} \left[ \frac{\lambda_j^1(s)}{\lambda_j^0} L_{ij}(s) + \frac{1}{\lambda_j^0} \Lambda_j^1(s) h_i(s) I_{ij}^1(s) + \frac{1}{\lambda_j^0} \Lambda_j^1(s) (1 - h_i(s))(s) R_i(s) I_{ij} \right]
$$

$$
= \Omega_{ij}(s)
$$

which is precisely the fire sale spillover effect.

The revenue generation motive is critical to ensuring country planners account for effects on foreign banks when setting policy. Quantity regulation does not generate a revenue generation motive. Although the implementing wedges under quantity regulation are affected by changes in domestic liquidation prices, this results in a need for the country planner to alter the stringency of domestic regulation required to achieve a given allocation, but this higher stringency has no domestic welfare consequences. The country planner does not account for foreign banks’ welfare as a result.

The revenue generation motive generates one additional force: a monopolistic revenue extraction motive.\(^{37}\) This is similar to standard monopolistic tendencies: the country planner distorts allocations of foreign banks in order to increase their willingness to pay for domestic activities. In this model, this monopolistic motivation operates through a tax on foreign investment scale when there is curvature in the domestic investment cost function. If a country is sufficiently substitutable with other countries from an investment perspective, it will not have monopoly power and these latter terms will drop out. In practice, the high

\(^{37}\)Note that we have expressed this motive as a derivative of price (tax rate) in the quantity. This is equivalent to expressing it in a more familiar way of a derivative of quantity (demand) in price. In our model, it is simpler to solve for quantities, and then back out the implementing prices (taxes).
mobility of global banking assets suggests that substitutability is a plausible assumption.\footnote{See e.g. the work of Coppola et al. (2019) on global capital flows and tax havens.}

**Efficiency of Non-Cooperative Pigouvian Taxation:** When countries use Pigouvian taxation, the only difference between the non-cooperative wedges of Proposition 6 and cooperative policy is the monopolistic distortion that gives rise to (inefficient) positive taxes on investment. As a result, if the monopolistic distortion is zero, then non-cooperative taxation implements the cooperative outcome, and hence is efficient. We formalize this result below.

**Proposition 7.** Suppose that for all $i$ and $j \neq i$, \[ \frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0. \] Then, the non-cooperative equilibrium under taxation is globally efficient. There is no scope for cooperation.

Proposition 7 suggests an alternative to cooperative regulatory agreements exists in the model. If countries switch to Pigouvian taxation to manage fire sale spillovers, country planners can achieve the cooperative outcome in a non-cooperative manner. They do so even though each country maximizes domestic welfare only, even though domestic liquidation prices appear in foreign bank constraints, and even though domestic planners have market power over domestic liquidation prices.

The sufficient condition of Proposition 7 requires a notion of substitutability between countries. The condition \[ \frac{\partial^2 \Phi_{ij}}{\partial I_{ij}^2} = 0 \] implies that the (partial equilibrium) elasticity of investment with respect to the tax rate is infinite. The infinite elasticity is a limiting case in which countries have no monopoly power over foreign banks, and so implement an efficient outcome.

Proposition 6 provides an exact efficiency result in a limiting case of an infinite elasticity. In practice, countries may not always have close investment substitutes, and so may have some monopoly power. In these cases, Pigouvian taxation nevertheless provides an advantage by restricting the need for cooperation to foreign bank activities. Cooperation is no longer required over domestic activities of domestic banks. Moreover, it transforms the source of inefficiency from a multilateral spillover problem into a bilateral monopolist prob-
The problem of cooperation is to ensure that foreign banks, who in the non-cooperative equilibrium are taxed excessively, are taxed the same as domestic banks, who are taxed efficiently.

A final advantage is that Pigouvian taxation changes the information required to determine the need for and terms of a cooperative agreement. The multilateral financial stability spillovers that must be corrected under regulation are general equilibrium effects that arise in the future, and there may be substantial disagreements between countries as to their magnitudes and cross-country correlations.

By contrast, the information required to enforce a cooperative agreement under taxation in the baseline model is the elasticity of investment with respect to the tax rate (the cost of investment). Moreover, the elasticities needed are partial equilibrium elasticities, not general equilibrium ones. The tax formula of Proposition 6 implies that cooperation is required when the elasticity of investment in the tax rate (cost of investment) is low, and not required when it is high.

**Relation to Side Payments:** Our model rules out explicit side payments between planners and banks when designing quantity regulation or Pigouvian taxation. The revenue generation of Pigouvian taxation acts like an implicit side payment from banks to the government. In this sense, Pigouvian taxation allows foreign banks to conduct activities in exchange for an implicit side payment commensurate with the externality generated. Moreover because the fire sale reduces the side payment a foreign bank is willing to pay, the planner internalizes the impact of the domestic fire sale on foreign bank welfare through the side payment received.

**1.5.4 Discussion**

Our theory has both positive implications that help to understand the design of existing bank regulatory regimes and cooperative agreements, and normative implications that help to inform the optimal design of regulatory policy.
Real World Quantity Regulation and Cooperation

Home and Host Country Regulation: A common regulatory arrangement in practice is that the domestic planner regulates activities of the domestic subsidiary of a foreign bank holding company, while the foreign planner regulates the overall balance sheet of the bank holding company. The former is host country regulation, while the latter is home country regulation. Non-cooperative regulation in our model resembles this combination, with the domestic planner managing risk specific to the domestic economy, and the foreign planner managing how risks translate back into the foreign economy.

Basel III and the European Banking Union: The model suggests non-cooperative regulation of domestic banks is overly lax while at the same time there is unequal treatment of foreign banks. Both the Basel III accords and the European Banking Union aim to enhance bank regulatory standards to address cross-border stability risks, for example by strengthening bank capital and liquidity requirements. Moreover, equal treatment is recognized as an important aspect of cooperative regulatory policy in practice.

Ring Fencing, SPOE, and MPOE: Non-cooperative regulation in the model prohibits domestic liquidations by foreign banks. This resembles equity capital and liquidity ring fencing, where countries require foreign banks to adhere to domestic regulatory standards.

39For example, the US applies TLAC requirements to both systemically important US banks and to the US IHCs of systemically important foreign banks. The objective of these requirements is to “help to reduce risks to financial stability.” 82 FR 8266.

40It can also be interpreted as a quantity based capital control, such as a restriction on outflows.

41Basel III serves to “rais[e] the resilience of the banking sector by strengthening the regulatory capital framework,” with a particular mind to cross-border interconnectedness and global risks (BIS (2010)). The SSM of the European Banking Union seeks to ensure “the stability of the financial system by ensuring that banking supervision across the euro area is of a high standard” (ECB (2018b)).

42The ECB lists a goal of the Single Supervisory Mechanism as “ensuring a level playing field and equal treatment of all supervised institutions” (ECB (2018b)). In the capital control context, the IMF states that it “is generally preferable that [capital flow management measures] not discriminate between residents and non-residents” (IMF (2012)).
at the level of their domestic operations, potentially independent of their parent company.\footnote{In applying US TLAC requirements to US IHCs of foreign systemically important banks, the rules were “tailored to the potential risks presented by the U.S. operations of foreign GSIBs to the U.S. financial system.” Commenters on the rules suggested considering allowing guarantees from the parent to qualify towards TLAC requirements to reduce “ring fencing and ‘misallocation risk,’” the final TLAC rule did not allow parent guarantees to qualify towards TLAC requirements because they “would not be pre-positioned in the United States and available for use during a period of stress without additional actions by the foreign GSIB parent.” 82 FR 8266.}

For capital ring fencing, this connects to the debate between single-point-of-entry (SPOE), in which an international bank holds loss-absorbing capital at an international level and so shares losses and spillovers across jurisdictions, and multiple-point-of-entry (MPOE) resolution, in which an international bank holds loss-absorbing capital at a country level to ensure domestic stability.\footnote{See Bolton and Oehmke (2019) for a formal analysis of SPOE versus MPOE. See Financial Stability Board (2013) and Tucker (2014) for additional discussions.} Our model suggests that non-cooperative regulators prefer to adopt MPOE regimes, while cooperative policy more closely resembles SPOE.\footnote{This coincides with this notion that SPOE is an efficient strategy for banks that “operate in a highly integrated manner (through, for example, centralised liquidity, trading, hedging and risk management)” (Financial Stability Board (2013)).}

For liquidity ring fencing, this relates to a recent proposal by the US Federal Reserve Board to “impose standardized liquidity requirements on the U.S. branch and agency network of a foreign banking organization” (Federal Reserve System (2019b)). This has raised concerns that the policy would lead other countries to adopt similar proposals.\footnote{The Federal Reserve stated that it “intends to further evaluate” concerns from commenters that such a requirement “could lead other jurisdictions to implement similar requirements” and “lead to market fragmentation,” so that such rules should be “discussed...at the global level by international regulatory groups” (Federal Reserve System (2019a)).}

Consistent with these concerns, our model suggests that countries would engage in excessive liquidity ring fencing.

**Branches versus Subsidiaries and Reciprocity:** In the model, the host country regulator regulates domestic activities of foreign banks. In practice, this corresponds to cross-border bank expansion using subsidiaries, which are typically regulated by the host country,
rather than branches, which are typically regulated by the home country.\textsuperscript{47} This raises a natural question of whether cooperative policy could be implemented by bilateral treaties or reciprocity agreements that allow expansions using branches (or otherwise enforce equal treatment).\textsuperscript{48} The answer is no - although such agreements may alleviate unequal treatment, they do not fix the under-regulation of domestic banks.

**Design of Bank Regulation and Capital Controls**

The contrast between the inefficiency of non-cooperative quantity regulation and the potential efficiency of non-cooperative Pigouvian taxation has normative implications regarding the optimal design of macroprudential policies and capital control measures.

**Emerging Markets and Capital Controls:** Emerging markets in practice use both quantity- and priced-based capital control measures to manage capital flows.\textsuperscript{49} Provided emerging markets have low monopoly power, our model suggests that they set *price-based* capital controls in a globally efficient manner even when they act non-cooperatively. By contrast, quantity controls are not set efficiently and are overly restrictive. Our model provides an argument in favor of use of price-based capital controls, rather than quantity-based ones.

**Macroprudential Regulation:** Common macroprudential regulatory requirements are minimum equity capital and liquidity requirements, which are quantity restrictions. However, there have been proposals for and discussions of Pigouvian taxes, such as a tax on

\textsuperscript{47}This distinction is not always true in practice. First, host country regulators can require certain activities to occur in subsidiaries. For example, the US generally requires insured deposits to be held in subsidiaries (see Nolle (2012)). Second, host regulators in theory have the ability to regulate domestic branches of foreign banks, for example by imposing liquidity requirements (Federal Reserve System (2019a) and Federal Reserve System (2019b)).

\textsuperscript{48}Indeed, reciprocity is recognized as important in practice. For example, Federal Reserve Chair Jerome Powell stated that “U.S. regulators have a long-standing policy of treating foreign banks the same as we treat domestic banks...a level playing field at home helps to level the playing field for U.S. banks when they compete abroad.” Federal Reserve Press Release, “Federal Reserve Board finalizes rules that tailor its regulations for domestic and foreign banks to more closely match their risk profiles,” October 10, 2019.

\textsuperscript{49}See e.g. IMF (2012).
debt, as an alternative to quantity regulations. Our results suggest that the Pigouvian tax approach may lead to efficient outcomes if countries have low monopoly power, whereas quantity regulation does not.

**Relationship to the Quantity Regulation Paradigm:** In practice, macroprudential policies commonly take the form of quantity restrictions, rather than taxes. We highlight two complementary reasons this may have arisen. First, revenue-neutral Pigouvian taxation may be perceived as roughly equivalent to quantity regulation. Even in academic debates, duality is a common assumption. Moreover, quantity regulation can incorporate Pigouvian-like features, for example using risk weights and capital surcharges, which tie quantity requirements to the banks’ risk profile. In this context, it is not particularly surprising to see a perception of duality. Second, Pigouvian taxes may be politically more difficult to implement than quantity restrictions. Taxes can be politically unpopular, particularly when the impact is perceived as falling on the general public. In a world of perceived duality, quantity regulation may simply be politically easier to implement.

One of our core contributions is to argue that even when quantity and price regulation are equivalent in a purely domestic context, they are not equivalent in the international context. Quantity regulation does not generate revenue for country planners. As we have shown, this revenue generation is critical to ensuring non-cooperative efficiency. Our results provide a new argument in favor of the Pigouvian tax approach.

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50See for example Cochrane (2014), De Nicoló et al. (2014), Kocherlakota (2010), and Tucker (2016).

51See e.g. Erten et al. (2019). Indeed in our model, quantity regulation in Section 1.5 could also be considered a revenue-neutral Pigouvian tax.

52See e.g. Greenwood et al. (2017).

53Although largely outside the banking context, see for example Mankiw (2009) and Masur and Posner (2015). Relatedly, Baker III et al. (2017) argue that revenue neutrality is important to ensuring political support for a carbon tax.
Asymmetric Countries and Difficulties of Cooperation  

Cooperation is often perceived to be difficult when countries are sufficiently asymmetric.\textsuperscript{54} Asymmetric agreements may require explicit international transfers, which may be politically difficult to implement. Pigouvian taxation functions by implementing the required transfers in a decentralized manner, and may help facilitate efficient outcomes when countries are relatively asymmetric (e.g. developed economies and emerging markets).

1.6 Bailouts and Fiscal Backstops

In this section, we study the main extension of the banking model: fiscal backstops, or “bailouts.” Fiscal backstops – such as deposit insurance, lender of last resort (LOLR), asset purchases, and debt guarantees – are an important consideration for financial stability and may be complementary to an effective regulatory regime.\textsuperscript{55} International fire sale spillovers motivate studying cooperation over fiscal backstop measures. In practice, common fiscal backstops include the ECB as a EU-wide LOLR, and proposals for an EU Common Deposit Insurance scheme.\textsuperscript{56}

1.6.1 Incorporating Bailouts

Incorporating bailouts into the model requires a notion of how a bailout can be used to affect banks’ liquidations at the country level – for example, bailing out the local subsidiary of a foreign bank. We extend the baseline model by incorporating country-specific debt claims $D_{ij}$, which is the amount owed to (or from) investors from bank operations in a specific country, rather than an overall debt contract $D_i$. These claims reflect the ability of the bank to reallocate funds across operations. We model bailouts as ex ante lump

\textsuperscript{54}See for example Bolton and Oehmke (2019) and Dell’Ariccia and Marquez (2006).


\textsuperscript{56}See Acharya et al. (2016) for discussion and analysis of the ECB as a LOLR. See European Commision (2015) for a proposal for Common Deposit Insurance.
sum transfer commitments $T_{ij}(s) \geq 0$, which provides a tractable way of representing the various possible bailout instruments.\(^{57}\) The date 1 country-level liquid net worth of bank $i$ is $A_{ij}^{1}(s) = -D_{ij} + T_{ij}(s)$, which may be negative. The date 1 budget constraint accounts for the bailout transfers,

$$c_i(s) \leq A_i^1(s) + (\gamma_i(s) - 1) L_{ii}(s) + R_i(s) I_i + \int_j ((\gamma_j(s) - 1) L_{ij}(s) + R_j(s) I_{ij}) dj,$$  \hspace{1cm} (1.14)

where $A_i^1(s) = A_{ii}^1(s) + \int_j A_{ij}^1(s) dj$ is the total liquid net worth of the bank, and where for simplicity we set $r_{ii} = r_{ij} = 0$. The date 0 bank budget constraint is unchanged relative to before, except that total date 0 bank debt is $D_i = D_{ii} + \int_j D_{ij} dj$.

Banks now face country-level collateral constraints that force country-level liquidations, given by

$$L_{ij}(s) \geq -\frac{1}{h_j(s) \gamma_j(s)} A_{ij}^1(s) - \frac{(1 - h_j(s))}{h_j(s)} R_j(s) I_{ij} \quad \forall j.$$  \hspace{1cm} (1.15)

In addition, an incomplete markets constraint limits the ability of the bank to reallocate funds across operations

$$\Gamma_i \left( \phi_{ii}(D_{ii}) + \int_j \phi_{ij}(D_{ij}) dj \right) \geq 0,$$  \hspace{1cm} (1.16)

and so forces liquidations in specific countries. Applying collateral constraints at the country level allows us to consider how bailouts of operations in a specific country affect the riskiness of banks’ activities in that country.

We treat $D_{ij}$, rather than $L_{ij}(s)$, as the choice variable of banks for this section that determines liquidations. In doing so, we internalize the liquidation rule that arises from equation (1.15) into the date 1 budget constraint.\(^{58}\)

**Macroprudential Regulation versus Bailouts:** Country planners can reduce liquidations either with bailouts or macroprudential regulation. Because bailouts are state contingent, the two policies are not perfect substitutes and both may be desirable to mitigate fire sales.

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\(^{57}\)See Appendix A.3.6 for a discussion of time inconsistent bailouts. Although in theory fiscal backstops such as deposit insurance and LOLR rule out bad equilibria without being used on the equilibrium path, in practice these measures often are associated with undesirable transfers and moral hazard.

\(^{58}\)In absence of bailouts, the model of this section falls into the framework of Section 1.7 and Appendix A.5.
**Financing Bailouts:** Bailouts are financed by domestic taxpayers, with a utility cost $V_i^T(T_i)$ of tax revenue collections. Country planners trade state-contingent claims on taxpayer revenue, yielding a tax-bailout budget constraint

$$\int_s \left[ T_{i,ii}(s) + \int_j T_{i,jii}(s) + \int_j T_{i,jij}(s) \right] f(s) ds \leq G_i + T_i$$

(1.17)

where $T_{i,ii}(s) + \int_j T_{i,jii}(s) + \int_j T_{i,jij}(s)$ is required revenue for bailouts in state $s$. $G_i$ is an existing inter-country tax revenue claim, with $\int_i G_idi = 0$, which we use in decentralizing the cooperative outcome.

### 1.6.2 Globally Efficient and Non-Cooperative Policies

We now characterize the globally efficient bailout policies, and discuss the non-cooperative bailout rules. We leave formal characterizations of regulatory policies and of the non-cooperative results to Appendix A.3.

**Globally Efficient Bailouts:** We begin by considering the globally efficient allocation, which includes the choice of bailouts. The global planner places welfare weights $\omega_i^T$ on taxpayers, so that the global objective function is

$$V^G = \int_i \left[ \omega_i \left[ \int_s c_i(s) f(s) ds + \omega_i^T V_i^T(T_i) \right] \right] di.$$

The global planning problem includes a set of unrestricted lump-sum transfers $\int_i G_idi \leq 0$ that reallocate claims on tax revenues. The following proposition characterizes the globally efficient bailout rule.\(^{60}\)

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\(^{59}\)See Appendix A.3.1 for a foundation.

\(^{60}\)The Appendix also characterizes cooperative regulation, optimal tax collection, and an irrelevance result for bailout sharing rules.
Proposition 8. The globally efficient bailout rule for $T_{ij}^1(s)$ is

$$
\frac{\omega_i \omega_j^H}{\lambda_i^0} \left[ \frac{\partial V^T_i}{\partial t_f} \right] \geq B_{ij}^1(s) + \Omega_{ij}^B(s) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} + \int \Omega_{ij}^B(s) d\ell' \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)}
$$

(1.18)

where the terms $B_{ij}^1$, $\Omega_{ij}^B$, and $\Omega_{ij}^F$ are defined in the proof.

The globally efficient bailout rules trade off the marginal cost of the bailout to taxpayers against both the direct benefit to the bank receiving the bailout, and the spillover benefits from reduced liquidations and fire sales. As in the baseline regulatory problem, globally efficient policy considers the complete set of spillovers when designing bailouts.

Non-Cooperative Quantity Regulation and Bailouts: The non-cooperative bailout rules differ from efficient rules in manners similar to the regulatory model. For bailouts of the domestic activities of domestic banks, the domestic planner neglects foreign spillovers and provides too weak of a backstop. At the same time, when deciding whether to bail out the domestic activities of a foreign bank, the domestic planner additionally neglects the benefit that foreign bank receives from the bailout, resulting in unequal treatment. Optimal cooperation increases backstops of all banks, including domestic ones.

Non-Cooperative Pigouvian Taxation and Bailouts: Under Pigouvian taxation, bailout rules are still not efficient. The reason is that bailouts are chosen by governments, not by banks, so that there is not an equilibrium tax rate associated with them. This implies that this problem can be fixed if there is a mechanism in place to charge banks for the bailouts they expect to receive. Such a mechanism, which is effectively a Pigouvian tax on bailouts, restores efficiency, including over bailout rules. One example of such a mechanism would be a deposit insurance levy.\(^{61}\)

\(^{61}\)As we argue in the appendix, responsibility for bailouts would most naturally fall upon the host country.
1.6.3 Discussion

We now discuss the relationship between the results of the bailout model and bailout policy and cooperation in practice.

**Bailout Home Bias:** Our model predicts that country planners provide stronger backstops for domestic banks and domestic operations. There are several examples of home bias in deposit insurance, including: US deposit insurance not applying to foreign branches of US banks; Iceland’s decision not to honor deposit guarantee obligations to UK depositors after its deposit guarantee scheme was breached; and EU policies against deposit insurance discrimination by nationality.62

**Common LOLR and Common Deposit Insurance in the EU:** Our model predicts overly weak fiscal backstops. This coincides with the EU motivation for Common Deposit Insurance, whose purpose is to “increase the resilience of the Banking Union against future crises” (European Commission (2015)). It further coincides the ECB acting as a common LOLR to the European Union.

**Asymmetric Contributions to Backstops:** Concerns may arise about sharing a fiscal backstop if countries benefit asymmetrically from it, for example if some countries are net contributors while others are net recipients.63 Proposition 8 implies bailouts by foreign countries may be optimal if they mitigate the domestic fire sale and promote cross-border investment. In this sense, asymmetric bailout cooperation requires financial integration.


63 One manifestation of this is countries with large banking sectors may have difficulty providing their own backstop (e.g. the case of Iceland).
1.7 A General Framework

In this section, we study whether the insights of the baseline model extend to other externality problems, such as environmental externalities. In addition to generalizing the results of the baseline model, we characterize classes of externalities for which the results of the baseline model, in particular non-cooperative Pigouvian tax efficiency, apply.\(^{64}\)

1.7.1 Model

We frame the general problem as follows. In each country \(i\), there is a representative multinational agent. The representative multinational agent has a vector \(a_{ij} = \{a_{ij}(m)\}_{m \in M}\) of continuous and real-valued actions available in country \(j\), where \(M\) is an indexing set and where \(a_{ij}(m) \geq 0\).\(^{65}\) The action \(a_{ij}(m) = 0\) indicates not conducting activity \(m\) in country \(j\). Multinational agents are home biased, so that domestic actions are a mass while foreign actions are a density.

We use country-level aggregates to capture spillover effects in the model. In particular, define \(a^A_j(m) = a_{jj}(m) + \int_i a_{ij}(m)\,dj\) to be the aggregate action \(m\) in country \(j\), with \(a^A_j = \{a^A_j(m)\}\). In the baseline model, the relevant aggregate for spillovers was aggregate liquidations \(L^A_j(s)\), which affected multinational banks by determining the liquidation price. In the example of multinational firms generating local pollution, the relevant aggregate would be total local pollution (or the activity that generates it).

Country \(i\) multinational agents have a utility function \(U_i(u_i(a_i), u^A_i(a_i, a^{A}))\), where \(u_i(a_i) = u_{ii}(a_{ii}) + \int_j u_{ij}(a_{ij})\,dj\) and \(u^A_i(a_i, a^{A}) = u^A_{ii}(a_{ii}, a^A_i) + \int_j u^A_{ij}(a_{ij}, a^A_j)\,dj\).\(^{66}\) This pref-

\(^{64}\)To simplify exposition, we assume solutions are interior, omit world prices (e.g. of state-contingent securities), omit heterogeneous within-country agents, assume that aggregates are determined linearly, and omit non-regulatory government actions. Appendix A.5 extends results of this section along each of these dimensions.

\(^{65}\)An example of an indexing set is \(M = \{0\} \cup \{1\} \times S\), which denotes an action \(a_{ij}(0)\) at date 0 and an action \(a_{ij}(1, s)\) at date 1 in state \(s\). We can impose that there are only actions \(M' \subset M\) in country \(j\) by making actions \(m \notin M'\) valueless.

\(^{66}\)Note that \(u_i, u^A_i\) can be functions in a generalized sense - for example, a vector of real numbers or a vector of functions defined over an underlying state space.
herence structure provides a flexible way to add up the utility impact of activities in different countries - for example, a consumption good in each country - while ensuring sufficient continuity, that is a change in foreign activities generates a utility impact proportional to the measure of those activities. Multinational agents face constraint sets $\Gamma_i(W_i, \phi_i(a_i), \phi^A_i(a_i, a^A)) \geq 0$, where $W_i = A_i - T_i$ is the wealth of the multinational agents (accounting for taxes), and where $\phi_i, \phi^A_i$ are defined analogously to $u_i, u^A_i$ and have the same motivation. Taken together, the optimization problem of country $i$ multinational agents is

$$\max_{a_i} U_i(u_i(a_i), u_i^A(a_i, a^A)) \quad \text{s.t.} \quad \Gamma_i(W_i, \phi_i(a_i), \phi^A_i(a_i, a^A)) \geq 0,$$

(1.19)

where all multinational agents take the vector $a^A$ of aggregates as given.

### 1.7.2 Globally Efficient and Non-Cooperative Policies

We first characterize the globally efficient allocation, which is virtually identical to Proposition 3 except for the definition of spillovers.

**Proposition 9.** The globally efficient allocation can be decentralized by wedges

$$\tau_{ji}(m) = - \underbrace{\Omega_{ij}(m)}_{\text{Domestic Spillovers}} - \underbrace{\int_{m'} \Omega_{i',j}(m')\,dm'}_{\text{Foreign Spillovers}} + \underbrace{l_i^0 L_i}_{\text{Equal Treatment}}$$

where $\Omega_{ij}(m)$ is given by

$$\Omega_{ij}(m) = \frac{\omega_i}{\lambda_i^0} \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_i^A}{\partial a^A_j(m)} + \frac{1}{\lambda_i} \frac{\partial \Gamma_i}{\partial \phi^A_i} \frac{\partial \phi^A_i}{\partial a^A_j(m)}$$

where $\lambda_i$ is the Lagrange multiplier on the constraint set, and where $\lambda_i^0 \equiv \lambda_i \frac{\partial \Gamma_i}{\partial W_i}$ is the marginal value of wealth to country $i$ multinational agents.

Globally optimal policy in the general model features the same two core features as the baseline model. First, globally optimal policy enforces equal treatment of foreign agents, so that they are able to enjoy equally the benefits of cross-border activities. Second, globally
optimal policy accounts for both domestic and foreign spillovers when designing regulation. There are two forms of spillovers in the general model that are reflected in $\Omega_{ij}(m)$. The first set of spillovers are direct utility spillovers, a leading example of which is pollution externalities. The second second of spillovers are constraint set spillovers, a leading example of which is the fire sale externalities in the baseline model.

**Non-Cooperative Quantity Regulation:** We leave the formal characterization to the appendix, but the inefficiencies of quantity regulation are effectively identical to those highlighted in Proposition 5 for each activity $m$. Non-cooperative quantity regulation still neglects international spillovers and results in unequal treatment. Rather than necessarily being banned, foreign bank activities are allowed only to the extent they benefit the domestic economy. Indeed, we show that non-cooperative quantity regulation is generically inefficient in settings with externalities arising from cross-border activities.

**Pigouvian Taxation and the Form of Externality:** The efficiency of non-cooperative Pigouvian taxation highlighted in Proposition 7 applies in the general model under two conditions. The first is a similar notion of no monopoly rents, which carries the same intuition and is formalized in the Appendix. The second is the following on the manner in which foreign aggregates can affect a domestic agent.

**Assumption 10.** For all $i$ and $j \neq i$, $u_{ij}^A$ and $\phi_{ij}^A$ are homogeneous of degree 1 in $a_{ij}$, holding $a_j^A$ fixed. That is, $u_{ij}^A(\beta a_{ij}, a_j^A) = \beta u_{ij}^A(a_{ij}, a_j^A)$ and $\phi_{ij}^A(\beta a_{ij}, a_j^A) = \beta \phi_{ij}^A(a_{ij}, a_j^A)$.

Assumption 10 states that when a domestic agent is exposed to an aggregate in a foreign country, that exposure scales with the agent’s activities in that foreign country. For example, in the case where action $m$ has a local price $\gamma_j(a_j^A(m))$ attached to it, we obtain a linear form $\gamma_j(a_j^A(m)) a_{ij}(m)$, which satisfies Assumption 10. This is the case in the baseline model, where banks are affected by foreign aggregate liquidations through equilibrium prices.

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67 Note that the requirement of no monopoly rents implies that our theory does not provide a solution to terms of trade manipulation, given that the monopolist distortion is similar to terms of trade manipulation.
Assumption 10 therefore restricts the form that cross-border externalities can take. Notice, however, that Assumption 10 places no restrictions on the form that domestic externalities that affect domestic agents can take.

### 1.7.3 Local versus Global Externalities

The results of the general theory of this section, and in particular the efficiency of non-cooperative Pigouvian taxation, can be reflected by considering two classes of externalities: local externalities and global externalities. These broad classes of externality problems allow for the model to be translated into a number of alternative settings of potential interest.

**Local Externalities:** Local externalities are a class of purely domestic externalities, where the aggregates $a_i^A$ only appear in the utility functions and constraint sets of country $i$ multinational agents. For example, local externalities can include local pollution and economic costs of bank failures. Local externalities result in unequal treatment of foreign banks under quantity regulation, whereas Pigouvian taxation results in efficient policies. As a result, non-cooperative Pigouvian taxation continues to implement the global optimum.

**Global Externalities:** Global externalities are externalities that affect foreign agents, so that aggregates $a_i^A$ appear in the utility functions and constraint sets of foreign agents. In addition to the example of fire sales in the baseline model, other examples of interest include global pollution and climate change.

Assumption 10 indicates that the way in which a domestic aggregate (or “externality”) affects foreign agents is crucial to the efficiency of Pigouvian taxation. Assumption 10 requires that the domestic externality affect foreign agents proportional to their domestic activities. This is true in the case of local pecuniary externalities, such as fire sales, where an equilibrium local price is the source of the externality.

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68 Of course, the actions $a_j^A$ can still appear in the utility functions and constraint sets of country $j$ multinational agents.
However, Assumption 10 is not satisfied for externality problems arising from global pollution or climate change. Consider a simple model where there are no cross-border activities and a single domestic activity, “production” \( a_{ii} \). Production generates carbon emissions, leading to country welfare \( u_i(a_{ii}) - \int_j a_{jj}dj \). It is immediate that this does not satisfy Assumption 10, and so non-cooperative Pigouvian taxation does not generate the globally efficient outcome.

Intuitively, the difference arises from the revenue collection motive. When Assumption 10 holds, increases in the domestic externality affect the value foreign agents get from domestic activities, leading the domestic planner to account for the impact on foreign agents. By contrast when Assumption 10 does not hold, as under global warming, the lost surplus to foreign agents from the domestic externality is not fully captured in willingness to pay for domestic activities, and the resulting equilibrium is not efficient.

### 1.8 Extensions

Appendix A.4 provides extensions to the model of multinational banking, while in Appendix A.5 provides extensions to the general model of Section 1.7.

In Appendix A.4.1, we allow for positive welfare weights on arbitrageurs and for an extended set of stakeholders in bank activities, such as SMEs who benefit from bank lending. The qualitative results of the model are unaffected, except that planners become more accepting of foreign liquidations when they are associated with positive spillovers (either arbitrageur surplus or by allowing greater domestic activity by foreign banks). Fire sale spillovers to foreign banks are still not internalized under quantity restrictions, while Pigouvian taxes are efficient.

In Appendix A.4.2, we allow for domestic multinational banks to be partly foreign-owned. We show that foreign ownership leads non-cooperative planners to over-weight spillovers to the domestic economy. Neither quantity regulation nor Pigouvian taxation achieves the efficient outcome when there is foreign ownership, and when both fire sales are real economy spillovers are important to determination of optimal regulation.
In Appendix A.4.3, we study the possibility that a country maintains an large (mass) investment scale in a foreign country. Although the results on non-cooperative regulation are essentially as in the baseline model, non-cooperative Pigouvian taxation generates excessive taxation relative to the global optimum, as both the home and host countries tax the same fire sale spillover. This motivates delegating prudential authority to the host country.

In Appendix A.4.4, we study the possibility that country planners might use wedges in a protectionist manner to shield domestic banks from competition by introducing endogenous local capital goods and prices. Non-cooperative quantity regulation leads to protectionism, whereas Pigouvian taxation remains efficient. This suggests that price-based barriers to capital flows can provide a means of achieving stability goals while reducing perverse protectionist tendencies.

In Appendix A.4.5, we allow for the presence of both local and global arbitrageurs. Global arbitrageurs can generate an additional global spillover and channel of contagion. This channel is important to the extent that the marginal purchaser of additional bank liquidations is global, rather than local. We briefly discuss in the context of empirical retrenchment patterns, where declines in foreign inflows tend to accompany domestic retrenchment. This suggests that local arbitrageurs may be the marginal pricing agent, which would downplay this channel.

In Appendix A.4.6, we consider two possible types of regulatory arbitrage. We first consider arbitrage between a country planner and the cooperative agreement, which disrupts cooperation but does not affect the optimality of Pigouvian taxation, where full autonomy is retained. We second consider arbitrage in the form of an unregulated “shadow banking” sector. Wholly domestic shadow banks do not affect our qualitative results, whereas a partly international shadow banking sector generates uninternalized global spillovers that warrant cooperation even under Pigouvian taxation. However, if country planners are able to regulate shadow banks, the efficiency of non-cooperative Pigouvian taxation is restored.

In Appendix A.4.7, we consider a quantity regulation game with quantity ceilings rather than revenue-neutral wedges. This problem yields the same outcome as the revenue neutral
wedges.

In Appendix A.5 provides several extensions to the general model studied in Section 1.7 and show that the qualitative results still hold. Appendix A.5.1 extends the model to include global goods with endogenous global prices (e.g. Arrow securities). Appendix A.5.2 allows for the possibility of (linear) local constraints on foreign bank allocations, thus allowing for corner solutions. Appendix A.5.3 allows for the possibility of heterogeneous agents. Appendix A.5.4 considers the case where the activities of multinational banks aggregate non-linearly. Appendix A.5.5 allows for a general set of government actions, such as bailouts. Appendix A.5.6 considers the case where country planners have different preferences than agents, and shows that Pigouvian taxation is efficient provided spillovers in foreign countries are limited to constraint set spillovers such as fire sales.

1.9 Conclusion

We study a model of cross border banking, in which endogenous cross-border propagation of fire sales generates international financial stability spillovers. We characterize globally efficient banking activities, and compare it to the outcome achieved by non-cooperative national governments using quantity regulations. Absent cooperation, countries under-regulate domestic banks and over-regulate foreign banks, not accounting for welfare impacts of domestic regulation on foreign banks. This provides a theory of optimal cooperation, which both enforces equal treatment of foreign banks and increases regulation of domestic banks, and helps to understand the architecture of existing cooperative regimes.

Our most surprising normative contribution is to show that non-cooperative Pigouvian taxation can also implement the globally efficient allocation, eliminating the need for cooperation. From a policy perspective, this suggests that giving a more prominent role to Pigouvian policies in the macroprudential regime may be desirable. By doing so, policymakers may be able to reduce the need for cooperative regulatory agreements, and so avoid the inherent difficulties of cooperation.
Chapter 2

Bail-Ins, Optimal Regulation, and Crisis Resolution

2.1 Introduction

In the aftermath of the 2008 financial crisis, the question of orderly bank resolution has received significant attention on both sides of the Atlantic. In many advanced economies, governments employed bailouts to stem financial turbulence in late 2008 and early 2009. Bailouts were arguably very effective at stabilizing financial markets, but have been criticized for leading to moral hazard and perverse redistribution. As a result, the US (Title II of the Dodd-Frank Act) and the EU (Bank Recovery and Resolution Directive) have introduced “bail-ins,” which allow the government to impose haircuts on (long-term) debt holders. The Dodd-Frank Act lists ensuring that “creditors and shareholders will bear the losses of the financial company” as one of the primary purposes of bail-ins, and requires that “[n]o

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1 Co-authored with Andreas Schaab

2 Two examples in the US are the Troubled Asset Relief Program (TARP), which authorized the government to buy toxic bank assets, and the Temporary Liquidity Guarantee Program (TLGP), which provided for guarantees of bank debt.

3 The Dodd-Frank Wall Street Reform and Consumer Act (Dodd-Frank Act) lists “protect[ing] the American taxpayer by ending bailouts” as one of its main objectives, and lists “minimiz[ing] moral hazard” (Section 204) as one of the purposes of bail-ins.
taxpayer funds shall be used to prevent the liquidation of any financial company under [Title II].”\(^4\) Nevertheless, there is limited formal economic analysis on bail-ins.

Two important concerns arise from the introduction of bail-ins. The first is that if bank solvency can be improved by replacing standard debt with bail-in debt, then what prevents banks from efficiently doing so using private contracts?\(^5\) Moreover, why are bail-ins a preferable instrument to other liability instruments, such as equity, or to other recapitalization methods such as bailouts.

The second is that during a crisis, the possibility of haircuts may destabilize financial markets. In the words of former Treasury Secretary Timothy Geithner, “the consequence of the haircuts imposed on creditors in the case of Lehman and Washington Mutual was a dramatic escalation in the scope and intensity of the run” (Geithner (2016)). More recently in the resolutions of Veneto Banca and Banco Popolare di Vicenza in 2017, the Italian government spared senior debt holders from a bail-in in part due to concerns about investor confidence in other fragile Italian banks.\(^6\)

Understanding these issues requires a framework in which debt is part of an optimal liability structure, so as to understand the impacts of a regulator changing that liability structure. We provide such as framework in an optimal contracting model based on an incentive problem. Banks must monitor the quality of their loans both at the onset of the lending relationship and in its continuation. Because monitoring is not contractible at either stage, the optimal contract must incentivize monitoring, which involves the bank keeping a sufficient stake (“agency rent”) in its loan performance not only ex ante but also in continuation. Banks write optimal liability contracts in a complete markets setting, but must respect the underlying incentive problem.

\(^4\)Dodd-Frank Act Sections 204 and 214.

\(^5\)For example, banks could use contingent convertible (CoCo) securities that have gained traction in Europe, which are an internal recapitalization instrument with a trigger event (for example, the bank’s capital ratio falling below some threshold) for either a principal write-down or a conversion into equity.

\(^6\)For example, then Bank of Italy Deputy Governor Fabio Panetta stated “I think resolution would have been very costly not just in monetary terms but also in terms of confidence.” Political concerns and guarantee obligations were also important factors in the decision. Financial Times, “Why Italy’s €17bn bank rescue deal is making waves across Europe,” June 26, 2017.
In this framework, we show that the privately optimal optimal bank contract can be implemented with a combination of two debt instruments: non-bail-inable, or standard, debt and bail-in debt. Standard debt has a face value that does not depend on the bank’s return, and leads to insolvency and liquidation when bank returns are low. This provides strong monitoring incentives to the bank for initial monitoring, by eliminating the bank’s continuation agency rent. Bail-in debt provides weaker incentives, by holding the bank to the continuation agency rent, but does not require a costly liquidation of the bank. Bail-in debt is useful because although it cushions against liquidation, it still provides maximal cash flow transfer on the downside. Both instruments retain the upside for the bank. Banks privately find it optimal to issue only standard debt when liquidation values are not too low.

We then study the design of socially optimal bank regulation in the presence of a fire sale externality – more bank liquidations reduce the recovery value to any individual bank in liquidation. The social planner writing the bank’s contract internalizes the fire sale, but must account for the underlying incentive problem of the bank. As with the privately optimal contract, the socially optimal contract can be implemented with a combination of standard and bail-in debt. However, the social planner reduces the use of standard debt to mitigate the fire sale. A bail-in regime can implement this socially optimal contract by changing the contingency properties of bank debt ex post, and so is the optimal regulatory regime.

The model helps to understand the role of bail-ins in the regulatory regime, as opposed to other forms of liability regulation such as equity requirements. Bail-in debt is less efficient at addressing the bank’s incentive problem than standard debt, but avoids social costs of bank failures. By contrast, equity worsens the bank incentive problem even further by giving away the upside of the bank to investors. This provides the role for bail-in debt in the regulatory regime.

The model further sheds light on why bail-ins are particularly appropriate regulatory mechanism for banks, as opposed to non-financial corporates. Non-financials may have higher average liquidation discounts than banks, but are less exposed to fire sales. Our model predicts that non-financials adopt capital structures that are easier to resolve and
make socially efficient use existing debt restructuring mechanisms such as Chapter 11. By contrast, banks take on capital structures that are more difficult to resolve, making socially inefficient use of private restructuring mechanisms due to the fire sale externality.

We then introduce the possibility of time-inconsistent bailouts as a recapitalization method, and study the trade-off between bail-ins and bailouts. Whereas banks privately choose not to issue bail-in debt in order to capitalize on bailouts, optimal regulation ensures that there is sufficient bail-in debt to recapitalize the banks without engaging in bailouts. Bail-ins fully replace bailouts. Whereas bailouts are a socially costly resolution method since they have to be financed by distortionary taxes, the costs of bail-ins are efficiently priced into bank contracts ex ante. As a result, the planner prefers recapitalization via bail-ins over bailouts. This coincides with a core principle of post-crisis resolution, that the costs of bank resolution should be borne by bank investors and not by taxpayers.\footnote{See Sections 204 and 214 of the Dodd-Frank Act as cited above. The Dodd-Frank Act further states that “[t]axpayers shall bear no losses from the exercise of any authority under this title” (Section 214). See also e.g. French et al. (2010).}

The model sheds light on the difference between the pre- and post-crisis worlds, and the role of bail-ins. Prior to the crisis, private use of bail-in debt was limited because the private cost of bankruptcy was low relative to the social cost, due to fire sale externalities and moral hazard from bailouts. Bail-ins reduce the social cost of bank failures while respecting the underlying private incentive problem that gave rise to debt contracts in the first place, and so constitute optimal regulation in the model.

We next turn to the second key question of the paper, of whether bail-ins can destabilize bank refinancing efforts. In order to do so, we distinguish between short- and long-term debt, and model one of the important institutional features of bail-ins: they subordinate long-term debt to short-term debt in the event of bank failure. We argue that our optimal contracting model in fact naturally generates this property.

We demonstrate the existence of rollover crises for fundamentally solvent banks. A rollover crisis is a self-fulfilling prophecy in which long-term debt purchasers believe they are about to be bailed in and become unwilling to purchase newly issued long-term debt,
so that the bank cannot refinance itself and is forced into bankruptcy and liquidation. Since the outstanding stock of short-term debt is senior to long-term debt in resolution, long-term debt indeed receives no payoff at this point, justifying the equilibrium beliefs. Bail-ins generate refinancing instability in a bank that was fundamentally solvent, and can lead to inefficient bank failures. Moreover, we show that private covenants are unable to rule out the rollover crisis.

Finally, we study policy options to bolster market stability and prevent rollover crises, as a complement to an effective bail-in regime. First, we show that an expanded lender of last resort facility (LOLR), which promises to lend long-term debt to banks, can eliminate rollover crises. Second, we show that an extension of temporary guarantees to new long-term debt during crisis times can help stabilize the market and prevent rollover crises, even though these guarantees are not fulfilled in equilibrium. These proposals have precedent in programs used by the US government during the 2008 financial crisis; for example, the Temporary Liquidity Guarantee Program (TLGP), which extended debt guarantees to new issuances of long-term debt.

These results imply that even while bail-ins may be an optimal regulatory regime, they can result in instabilities for banks during times of market stress. As a result, “bailout” policies that protect new debt issuances are a desirable complement to a well-functioning bail-in regime, even while bailout policies that protect existing debt are not.

Related Literature. First, we relate to a growing literature on bail-ins. Dewatripont and Tirole (2018) explore how bail-ins can complement liquidity regulation. Keister and Mitkov (2017) show that banks may not write down their (deposit) creditors if they anticipate government bailouts. Chari and Kehoe (2016) use a costly state verification model and show that bail-ins are not required in the optimal regulatory regime. In their model, costly state verification implies that standard debt contracts are the only renegotiation-proof contracts.

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8In addition to the papers described below, see also e.g. Berger et al. (2020), Bolton and Oehmke (2019), and Colliard and Gromb (2018). See also a related literature on optimal derivatives protection, e.g. Biais et al. (2016) and Biais et al. (2019).
so that the possibility of bail-ins leads to a reduction in standard debt issuance but not the
use of bail-in debt. Mendicino et al. (2018) numerically explore the optimal composition
of bail-in debt and equity in the presence of both private benefit taking and risk shifting,
taking contracts as given and with a regulatory objective of protecting insured deposits.
Pandolfi (2018) studies a related incentive problem to ours, but takes standard debt contracts
as given. The paper argues that bailouts may be desirable in conjunction with bail-ins when
bail-ins limit investment scale by weakening bank incentives. Walther and White (2020)
show that precautionary bail-ins of long-term debt can signal adverse information about
a bank’s balance sheet and cause a bank run, leading to an overly weak bail-in regime.
Our contribution is to derive an optimal bank contract that combines standard and bail-in
debt, to rationalize bail-ins as optimal regulation, and to study rollover crises and their
implications for the crisis resolution toolkit.

Second, we relate to a literature on contingent convertible debt securities as a recapital-
ization instrument, including studying the possibility of multiple equilibria arising from
the conversion trigger. Multiplicity in our model arises from the relationship between
short-term and long-term debt, and results in a bank run. We further study the role of
lender of last resort and debt guarantees in preventing multiplicity.

Third, we relate to the literature on theories of debt in both the banking and corporate
finance contexts. Our model incorporates an incentive theory of debt and combines two
views of the role of debt: that debt is valuable both for cash flow transfer and for the explicit
liquidation. In our model, standard debt generates liquidations and bail-in debt generates
cash flow transfers.

Fourth, we relate to the literature on macroprudential regulation. This literature

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9See e.g. Flannery (2002), Raviv (2004), and Sundaresan and Wang (2015). See also Flannery (2014) for a
broader overview of the literature.

10For example, Calomiris and Kahn (1991), Dewatripont and Tirole (1994), Diamond and Dybvig (1983),
and Townsend (1979). We also connect in particular to the related literature that emphasizes the monitoring role

11For example, Bianchi (2011), Bianchi (2016), Bianchi and Mendoza (2010), Caballero and Krishnamurthy
features two common motivations for macroprudential regulation – pecuniary externalities (e.g. fire sales) and fiscal externalities (bailouts) – and studies optimal ex ante regulation, possibly in conjunction with ex post bailouts. Our focus is on ex post bail-ins as an optimal policy, and whether it is a complement or substitute to macroprudential regulation and bailouts.

Finally, we connect to the literature on debt dilution. A large finance literature has focused on different methods of debt dilution, including issuance of new senior debt and maturity shortening. Brunnermeier and Oehmke (2013) show that maturity shortening can result from a deliberate desire to dilute longer-maturity creditors. He and Milbradt (2016) show that multiple equilibria can arise if long-term creditors expect to be diluted by future short-term creditors, resulting in a gradual maturity shortening. In our model, multiplicity arises because bail-ins imply existing short-term debt dilutes new long-term debt and results in an immediate (fundamental) bank run.

2.2 Model

We develop a three-period model with three economic agents: banks, investors, and arbitrageurs. Banks sign contracts with investors to raise investment funds, while arbitrageurs purchase bank projects that are liquidated prior to maturity. We tailor the model to address the core trade-off of bail-ins: between standard debt and bail-in debt. Our baseline model will have no role for instruments such as equity, or for other trade-offs that affect the use of debt (e.g. tax benefits).\(^{12}\)

The three-period economy, \(t = 0, 1, 2\), has a unit continuum of banks, investors, and arbitrageurs. Banks invest in a project of variable scale \(Y_0 = A_0 + I_0 > 0\) by using their own funds, \(A_0 > 0\), and funds \(I_0 \geq 0\) from (date 0) investors. Investors are deep-pocketed at date

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\(^{12}\)In the appendix, we extend the baseline model to incorporate a role for equity in the capital structure and additional trade-offs such as tax benefits of debt.
0 and can finance any investment scale.

Banks and investors are risk-neutral and do not discount the future. We denote bank consumption by \((c_0, c_1, c_2)\), so that bank expected utility is given by \(E_0 [c_0 + c_1 + c_2]\). We denote the payments to investors by \((x_1, x_2)\). \(x_t\) is the actual amount received by investors, and is distinct from the face value of liabilities. Investor expected utility from the bank contract is \(E_0 [-I_0 + x_1 + x_2]\). Contracts are subject to limited liability constraints, given by

\[
c_0, c_1, c_2, x_1, x_2 \geq 0 \tag{2.1}
\]

Banks need to refinance any liabilities that mature at date 1. They raise these funds from a set of (date 1) risk-neutral, no-discounting, deep-pocketed investors. Any projects they are forced to liquidate at this point are purchased by arbitrageurs, who generate an equilibrium liquidation price.

The economy features idiosyncratic uncertainty, but no aggregate uncertainty.\(^{14}\)

### 2.2.1 Bank Projects

Banks extend financing to firms, thereby establishing a lending and monitoring relationship with those firms. When first extending funds to firms, banks monitor their borrowers, ensuring that the projects undertaken are of good quality. In doing so, banks develop specialized knowledge of that firm, and are uniquely able to monitor and collect from the firm in continuation. This relationship is the foundation of banking in our model. We refer to these relationships as bank projects.\(^{15}\)

Our model proceeds similarly to a multi-period version of Innes (1990). The bank project experiences a stochastic quality shock \(R\) at date 1, adjusting its scale to \(Y_1 = RY_0\), at which point uncertainty is resolved. The shock \(R\) is idiosyncratic with a density \(f_e(R)\) that has support over \([R, \bar{R}]\). The state \(R\) is contractible, but the distribution of \(R\) depends on the

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\(^{13}\)In our model, these are equivalent to the limited liability constraints \(c_1 + c_2 \geq 0\) and \(x_1 + x_2 \geq 0\).

\(^{14}\)See Appendix B.2.3 for aggregate uncertainty.

\(^{15}\)For simplicity, firms in our model earn zero surplus from the lending relationship.
bank’s non-contractible monitoring effort \( e \in \{H, L\} \), where \( e = H \) is high monitoring effort and \( e = L \) is low monitoring effort. \( f_e(R) \) satisfies the monotone likelihood ratio property (MLRP), that is \( \frac{\partial}{\partial R} \left[ \frac{f_L(R)}{f_H(R)} \right] < 0 \). MLRP is a standard assumption in generating debt contracts, and implies that high (low) returns are a signal that the bank exerted high (low) monitoring effort. We assume throughout the paper that optimal contracts induce high monitoring, so that \( e = H \).

Because monitoring effort is non-contractible, the bank chooses \( e \) to maximize bank utility after contracts have been signed. Given the consumption profile under the contract signed, the bank exerts high monitoring effort if

\[
E [c_1(R) + c_2(R) | e = H] \geq E [c_1(R) + c_2(R) | e = L] + BY_0
\]

where \( B > 0 \) is a private benefit of exerting low monitoring effort (“shirking”). We rearrange this incentive compatibility constraint to obtain the representation

\[
E \left[ (c_1(R) + c_2(R)) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \middle| e = H \right] \geq BY_0. \tag{2.2}
\]

Higher payoffs \( c_1(R) + c_2(R, s) \) in states where the likelihood ratio \( \frac{f_L(R)}{f_H(R)} \) is low relax incentive compatibility because these states signal that monitoring effort was high.

Although the quality shock \( R \) is realized at date 1, the project does not mature until date 2 and yields no dividend at date 1. If the project survives to date 2, it generates 1 unit of the consumption good per unit of final scale, \( Y_2 = Y_1 \). Only a portion \( (1 - b)Y_2 \) is pledgeable to investors, while the remaining portion \( bY_2 \) is retained by the bank. This non-pledgeable portion \( bY_2 \) is an agency rent in the continuation monitoring or collection problem of the bank.\(^{16}\) Holding projects to maturity implies a maximum pledgeability constraint \( c_2 \geq bY_2 \).

Projects can be liquidated prematurely at date 1, in which case they yield \( \gamma Y_1 < Y_1 \) units of the consumption good at date 1 and nothing at date 2, with the proceeds accruing entirely

\(^{16}\)See e.g. Holmstrom and Tirole (1997). We assume that the continuation payoff under shirking is sufficiently low that shirking in continuation is never optimal.
to investors.\footnote{We think of the liquidation discount as arising from selling projects to second-best users, who have not developed the knowledge of the firm lending relationship that the bank has. In this sense, we also assume the banker is not severable from the bank.} We assume that $\gamma < 1 - b$, so that liquidating the project is not desirable from an investor repayment perspective and so are not desirable ex post. Liquidations may be ex ante efficient for incentive reasons, since the bank can be paid 0 when liquidated but must be paid the agency rent $bY_2$ when not liquidated.

Finally, because banks are risk-neutral and do not discount the future, banks are indifferent to whether they consume at date 1 or date 2. We set $c_1(R) = 0$ to ease the exposition going forward.

### 2.2.2 Bank Liabilities

In order to raise investment $I_0 \geq 0$, banks pledge state-contingent liabilities to investors at date 0, which promise a face value of $L_1(R) \geq 0$ to be paid in period $t$. Banks may pledge a face value of liabilities in excess of pledgeable income. In such states, the bank is then unable to repay its liabilities in full. It enters bankruptcy and liquidates its assets.

Without loss of generality, we assume that banks pledge one-period liabilities contracts, so that $L_2(R) = 0$. This is without loss of generality in our model since investors are indifferent to the period in which repayment occurs. Given a liability structure $L_1(R)$, the resulting payoff profiles of banks and investors are

$$
(c_2(R), x_1(R)) = \begin{cases} 
(RY_0 - L_1(R), L_1(R)), & L_1(R) \leq (1 - b)RY_0 \\
(0, \gammaRY_0), & L_1(R) > (1 - b)RY_0 
\end{cases}
$$

(2.3)

where $c_1(R) = x_2(R) = L_2(R) = 0$. To understand this payoff profile, if $L_1(R) \leq (1 - b)RY_0$, the bank can roll over its face value of liabilities by raising money from date 1 investors, who break even at the same face value $L_1(R)$. Date 0 investors receive $x_1(R) = L_1(R)$, while the bank receives $c_2(R) = Y_2 - L_1(R)$. If instead $L_1(R) > (1 - b)RY_0$, the face value of liabilities exceeds pledgeable income and the bank is liquidated, yielding payoffs $x_1(R) = \gamma(s)RY_0$ and $c_2(R) = 0$. 

\footnote{We think of the liquidation discount as arising from selling projects to second-best users, who have not developed the knowledge of the firm lending relationship that the bank has. In this sense, we also assume the banker is not severable from the bank.}
The voluntary investor participation constraint states that investors must at least break even in expectation on the contract they signed, and it is given by

\[ Y_0 - A_0 \leq E \left[ x_1(R) \mid e = H \right]. \tag{2.4} \]

where \( I_0 = Y_0 - A_0 \) is the amount financed by investors.

Finally, we assume that liabilities \( L_1(R) \) must be monotone, that is

\[ R \geq R' \Rightarrow L_1(R) \geq L_1(R'). \tag{2.5} \]

Monotonicity is a common assumption in many settings of optimal contracts or security design. It generates the flat face value of liabilities in high-return states.\(^{18}\)

### 2.2.3 Arbitrageurs and Liquidation Values

At date 1, arbitrageurs can purchase bank projects and convert them into the consumption good using a production technology \( F(\Omega)Y_0 \), where \( \Omega \) is the economy-wide fraction of bank projects purchased relative to the initial scale.\(^{19}\) Arbitrageur surplus at date 1 from purchasing projects is 

\[ \gamma = \gamma(\Omega) = \frac{\partial F}{\partial \Omega}, \quad \Omega = \int_R \alpha(R)Rf_H(R)dR \tag{2.6} \]

where \( \alpha(R) \in \{0, 1\} \) indicates whether or not a bank liquidates in state \( R \), with \( \alpha(R) = 1 \) denoting liquidation. When \( \frac{\partial \gamma}{\partial \Omega} = \frac{\partial^2 F}{\partial \Omega^2} < 0 \), there is a fire sale spillover from bank liquidations: more liquidations reduce the liquidation value.

\(^{18}\)For example, one justification offered is that banks would be incentivized to pad their returns, for example by secretly borrowing from a third party. Without monotonicity, optimal contracts in our model have the “live-or-die” feature (see Innes (1990)) above the thresholds \( R_u(s) \) defined in Proposition 11, so that \( L_1(R, s) = 0 \) for \( R > R_u(s) \). The form of banks’ liability structure below \( R_u(s) \) would be the same.

\(^{19}\)Second best users via production is a common foundation for arbitrageurs.
2.2.4 Bank Optimal Contracting and Equilibrium

Every bank takes the equilibrium liquidation values $\gamma$ as given, and signs a contract $(L_1, Y_0)$ with investors. Banks maximize their own expected utility

$$\max_{L_1, Y_0} E \left[ c_2(R) | e = H \right],$$

subject to incentive compatibility (2.2), investor participation (2.4), monotonicity (2.5), and limited liability (2.1), where $c_2$ and $x_1$ are given by equation (2.3). Figure 2.1 presents a simple timeline underlying this contracting problem.

![Timeline](image)

Since all banks are identical ex ante, all banks sign the same equilibrium contract. Therefore, $a(R) = 1_{L_1(R) > (1-\delta)RY_0}$, where $1_{\cdot}$ is the indicator function. An equilibrium of the economy is a set of liquidation values $\gamma$ such that the contract $(L_1, Y_0)$ is optimal, given $\gamma$, and such that liquidation values are determined by equation (2.6), given $(L_1, Y_0)$.

2.3 Privately and Socially Optimal Contracts

In this section, we characterize the privately optimal contract written by banks that take as given the liquidation value $\gamma$. We compare the privately optimal contract to the contract written by the social planner who internalizes the fire sale spillover. In both cases, the optimal contract can be implemented by a combination of two debt instruments. The first, which we call standard debt, has a fixed face value that does not depend on $R$, and liquidates the bank in low-return states. The second, which we call bail-in debt, has a face value that
can be written down based on $R$, and restores bank solvency when total debt exceeds the pledgeable income. Although the bank and planner both agree that the optimal liability structure combines standard and bail-in debt, they disagree on the relative use of the two instruments. The planner wishes to use less standard debt than the bank, internalizing the fire sale spillover.

### 2.3.1 Privately Optimal Contract

We begin by characterizing the privately optimal bank contract in terms of two thresholds, $R_l$ and $R_u$. We then associate these two thresholds with the two debt instruments. These thresholds are sufficient statistics for the privately optimal liability structure of the bank.

**Proposition 11.** A privately optimal bank contract has a liability structure

$$L_1(R, s) = \begin{cases} 
(1 - b)R_lY_0, & R \leq R_l \\
(1 - b)RY_0, & R_l \leq R \leq R_u \\
(1 - b)R_uY_0, & R_u \leq R 
\end{cases}$$

where $0 \leq R_l \leq R_u \leq \overline{R}$. The bank is liquidated if and only if $R \leq R_l$. These thresholds, when interior and not equal, are given by

$$\frac{\mu b \left( \frac{f_L(R_l)}{f_H(R_l)} - 1 \right)}{\text{Incentive Provision}} = b + \lambda (1 - b - \gamma) \tag{2.7}$$

$$0 = E \left[ \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \right] \quad | \quad R \geq R_u, e = H \tag{2.8}$$

where $\mu > 0$ is the Lagrange multiplier on incentive compatibility (2.2) and $\lambda > 1$ is the Lagrange

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20For the remainder of the paper, we assume that the thresholds are interior and not equal, except when explicitly stated otherwise. Although $R \in [\underline{R}, \overline{R}]$ with $\underline{R} \geq 0$, it is possible that when $\overline{R} > 0$, the bank finds it optimal to set $R_l, R_u < \overline{R}$ in which case there is only standard debt but the debt level is low enough that the bank is always solvent. Generally speaking, $R_l$ will be interior when the likelihood ratio $\frac{f_L(R)}{f_H(R)}$ is sufficiently large, that is when $\overline{R}$ is a sufficiently good signal of low effort. $R_u$ will be interior when $\frac{f_L(R)}{f_H(R)}$ is sufficiently small and $\mu > \lambda - 1$, that is when $\overline{R}$ is a sufficiently good signal of high effort.
Proof. All proofs are contained in the appendix.\footnote{In the proof of this proposition, see Appendix B.1.1 for a comment on non-uniqueness of face value of liabilities $L_1(R)$ below the threshold $R_l$. Non-uniqueness arises in this region because any face value of liabilities above $(1 - b)R_0$ results in bank liquidation. We have chosen the face value of liabilities that correspond to standard debt, which seems most natural in the context of banks and bail-ins. Moreover, uniqueness is restored if there is an $e \to 0$ premium for standard debt, for example due to tax benefits of debt. The face value of liabilities is unique above $R_l$.}\\

Before discussing the properties of the optimal contract, we associate these two thresholds $R_l$ and $R_u$ with the two debt instruments that we discussed before. We associate $R_l$ with standard debt and $R_u$ with bail-in debt.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Illustration of Privately Optimal Contract.}
\end{figure}

To understand this terminology consider Figure 2.2, which illustrates the optimal liability contract. There are three regions of the liability structure. The first region, where $R \leq R_l$, is one where the total face value of liabilities is constant, but exceeds the pledgeable income of

\[ (1 - b)R_0 \]
the bank. In this region, the bank is liquidated, and investors only receive partial repayment on the face value of their debt contracts. This is a standard debt contract.

In the third region, above $R_u$, investors receive a constant payoff equal to the total face value of liabilities, so that $(R_u - R_i)Y_0$ corresponds to the total level of bail-in debt. What distinguishes bail-in debt from standard debt is that in the second region, where $R_l \leq R \leq R_u$, the face value of bail-in debt is written down to $((1 - b)R - R_l)Y_0$. This recapitalizes the bank and allows it to continue operating, rather than being liquidated. We refer to this as bail-in debt because its face value can be written down (“bailed in”) based on the idiosyncratic state.

**Corollary 12.** The optimal contract can be implemented with a combination of standard debt with face value $(1 - b)R_lY_0$, which cannot be written down contingent on the idiosyncratic state $R$, and bail-in debt with face value $(1 - b)(R_u - R_l)$, which can be written down contingent on the idiosyncratic state.

For the remainder of the paper, we associate standard and bail-in debt with the thresholds $R_l$ and $R_u$, respectively, rather than writing out their associated (face value) liabilities.

The core property of standard debt is that it forces liquidations in low-return states. Equation (2.7) describes the marginal trade-off the banker faces in replacing a unit of bail-in debt with a unit of standard debt. On the one hand, liquidating the bank results in a total resource loss $b + \lambda(1 - b - \gamma)$ to the bank and investors. On the other hand, pledging to liquidate the bank provides higher-powered monitoring incentives at date 0, reflected in the term $\mu b \left(\frac{f_l(R_l)}{f_H(R_l)} - 1\right)$, by depriving the bank of the non-pledgeable income $bRY_0$. In particular, the liquidation threshold features $\frac{f_l(R_l)}{f_H(R_l)} > 1$. That is, at $R_l$ the likelihood ratio is greater than 1, implying that the state provides a stronger signal that low effort may have been exerted.

By contrast, for a given level of standard debt, an additional unit of bail-in debt does not change liquidations but does transfer value from banks to investors. The marginal trade-off is summarized in equation (2.8). On the one hand, the binding investor participation
constraint implies this transfer is valuable \((\lambda - 1 > 0)\), as it allows the bank to increase project scale. On the other hand, increasing the total debt level reduces bank consumption in high-return states, where the likelihood ratio \(\frac{f_1(R)}{f_H(R)}\) is low and the signal of high effort is stronger. This weakens bank monitoring incentives and tightens the incentive compatibility constraint (2.2). The optimal level of bail-in debt equalizes these two effects on the margin.

Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of standard and bail-in debt: the ex ante incentive problem \((B > 0)\), limited pledgeability \((b > 0)\), and costly liquidations \((\gamma < 1)\). In the absence of any one of these elements, contracts in our model would not combine standard and contingent debt.\(^{22}\) If we had \(B = 0\), there would be no initial incentive problem and no reason for costly liquidations to occur, and hence no motivation for standard debt. If \(b = 0\) and there were no limited pledgeability, the bank could leave itself with no continuation agency rent with bail-in debt, and there would be no motivation for standard debt. If \(\gamma = 1\), then liquidations would not be costly, and the bank would not require bail-in debt.

**Why Didn’t Banks Issue Bail-in Debt before 2008?:** Although Proposition 11 states that privately optimal bank contracts combine standard and bail-in debt, bail-in debt is largely a post-crisis innovation that was “introduced” by bail-in regulation: it places contingencies into debt contracts where few (if any) had previously existed. This leads to the question: Under what conditions would the privately optimal contract feature no bail-in debt?\(^{23}\)

The case where banks do not issue bail-in debt, that is \(R_u = R_f\), is a corner solution of the model. This case is summarized in the following corollary to Proposition 11.

\(^{22}\)See Appendix B.2.1.

\(^{23}\)Unfavorable tax or regulatory treatment may also have contributed to a lack of bail-in debt issuance. Prior to the crisis, regulatory requirements were generally equity requirements, which bail-in debt would not count towards. We see some support for this force mattering in the fact that CoCos have grown in use post-crisis in the EU, where they often count towards regulatory capital requirements, but not in the US, where they do not count towards regulatory capital requirements.
Corollary 13. Suppose that the solution $R_l$ to equation (2.7) satisfies

$$E \left[ \lambda - 1 - \mu \left(1 - \frac{f_L(R_l)}{f_H(R)}\right) \right] \bigg| R \geq R_l, e = H \leq 0$$

(2.9)

Then, banks do not issue bail-in debt, that is $R_l = R_u$.

To understand Corollary 13, the left-hand side of equation (2.9) is the marginal value of increasing $R_u$ above $R_l$, as in equation (2.8) from Proposition 11. When this marginal value is negative, the incentive costs of increasing the total debt level outweigh the investor repayment benefits, and the bank chooses $R_l = R_u$.

A key determinant of the trade-off between standard and bail-in debt is the cost of liquidations, $\gamma$. When liquidation values are not too low, the resource loss from liquidations is not too large and $R_l$ increases,\(^{24}\) pushing the bank towards $R_l = R_u$. Banks are less likely to issue bail-in debt when liquidation values are high.

2.3.2 Socially Optimal Contract

The social planner internalizes the fire sale spillover and writes bank contracts to maximize social welfare. We assume that the social planner places a welfare weight of zero on arbitrageurs, an assumption which we relax in the appendix. Since all investors in our model receive no surplus in expectation, social welfare is equal to bank utility.\(^{25}\) The only difference between the private bank contracting problem and the social planning problem is that the planner internalizes the fire sale spillover. We assume away the possibility of bailouts until Section 2.4.

The social planner writes contracts subject to the same conditions as banks, namely incentive compatibility (2.2), investor participation (2.4), monotonicity (2.5), and limited liability (2.1), with $c_2$ and $x_1$ given by equation (2.3). In other words, any socially optimal contract must also be privately feasible.

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\(^{24}\)To see this, combine equation 2.7 with MLRP.

\(^{25}\)The planner in our model could generate surplus for investors by promising them liabilities with expected value greater than $I_0$. We assume the planner places a low enough welfare weight on investors that this is not desirable.
Proposition 14. The socially optimal contract can be implemented by a combination of standard and bail-in debt. These debt levels are associated with endogenous thresholds $R_l$ and $R_u$. Whenever $R_l < R_u$ is interior, $R_l$ is given by

$$
\mu b \left( \frac{f_l(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda \left((1 - b) - \gamma\right) + \lambda \left[ \frac{\partial \gamma(\Omega)}{\partial \Omega} \right] f_R \int_{R_l}^{R_u} R f_H(R) dR
$$

while $R_u$ is given as before by equation (2.8).

Even though the planner uses the same debt instruments as the bank, the term $\frac{\partial \gamma}{\partial \Omega} < 0$ generates an additional social cost of liquidation in the planner’s optimality condition for $R_l$. This liquidation cost term represents the only difference between the private and social optimality conditions in equations (2.7) and (2.10), respectively. Fire sale spillovers generate an additional social cost: the project liquidations of one bank increase the resource loss to all other banks that liquidate projects at the same depressed prices. By contrast, there is no additional wedge in the determination of $R_u$, since a greater total debt level arising from more bail-in debt does not change total liquidations. Relative to private banks, the planner prefers relatively less use of standard debt in favor of more use of bail-in debt. However, because the planner respects the underlying incentive problem, the planner prefers to replace standard debt with bail-in debt, which generates the cash flow transfer without costly liquidations.

In the case of Corollary 13, where banks had not found it optimal to issue bail-in debt, the additional social cost of bankruptcy implies optimal regulation can introduce bail-in debt into the bank’s capital structure. The planner’s motivation to do so does not result from an incentive to complete markets by writing contracts that banks could not write on their own. Rather, private contracts feature too little or no bail-in debt because banks do not internalize the fire sale spillover.

Bail-ins as Optimal Regulation: Proposition 14 provides the contract that the planner writes for banks, under ex-ante regulation which contractually specifies bail-ins. Bail-ins
are often associated with an ex post resolution authority: the planner takes a debt contract with a fixed face value, and writes down that face value ex post. Both forms of authority are used in practice, with the US emphasizing ex post resolution and the EU being more accommodating of contractual recapitalization.26

In our model, there is a straightforward equivalence between the two regulatory methods. Under the ex ante (contractual) implementation, the planner caps bank issuance of standard debt at \( R_l \) and requires all remaining debt \( R_u - R_l \) to have contractual write-down provisions that restore solvency to the bank. Under the ex post (resolution authority) implementation, the planner caps the amount of non-bail-inable debt at \( R_l \) and designates the remaining \( R_u - R_l \) to be bail-inable. The resolution authority can write down bail-inable debt ex post to recapitalize the bank, resulting in the same outcome. Since the model predicts that these two methods are equivalent, it is not necessarily surprising that we see both methods used in practice.27

2.3.3 Bail-in Debt or Equity?

Proposition 11 highlights why standard debt can be a valuable loss-absorbing instrument for banks, relative to equity. Bail-in debt combines the incentive properties of standard debt with the loss-absorbing properties of equity. It generates a cash flow transfer below \( R_u(s) \) and a flat investor payoff above \( R_u(s) \), similar to standard debt, but does so without liquidating the bank. By contrast, equity transfers the upside of the bank to investors, which worsens incentives. Bail-in debt therefore achieves a capital structure that standard debt and equity combined cannot. Under the incentive problem of the baseline model, banks

---

26In the US, banks are required to maintain a certain level of total loss-absorbing capital (TLAC), principally long-term debt and equity, to safeguard the bank against poor returns. Debt used to satisfy TLAC requirements must be plain-vanilla, implying a fixed face value, while debt with contractual contingencies cannot generally be used to satisfy TLAC requirements. In particular, “eligible external LTD [is] prohibited from including contractual triggers for conversion into or exchange for equity.” 82 FR 8266. See Avdjiev et al. (2017) for background on the European case.

27It is not necessarily surprising to see some preference for ex post implementation, given there may be the potential for regulatory arbitrage around contractual triggers. See Appendix B.2.9 for some suggestion of this nature.
prefer bail-in debt to equity as a loss-absorbing instrument.

As in the privately optimal contract, bail-in debt, rather than equity, is used as the loss absorbing instrument. Although the social planner and the bank disagree about the costs of bankruptcy, they agree about the underlying incentive problem. As a result, the planner replaces standard debt with bail-in debt, which addresses the underlying incentive problem that standard debt was designed to solve. This provides a role for bail-in debt rather than equity.28

Although the baseline model does not feature equity, in Appendix B.2.6 we add a role for equity in the model by incorporating risk aversion and risk shifting. We show that the core trade-off between standard debt and bail-in debt exists as in the baseline model.

### 2.3.4 Relationship to Debt Renegotiation and Restructuring

In practice, bail-in debt is generally associated with banks. However, the core (private) optimal contracting model of the paper could also be applied to non-financial corporates, some of whom may have high liquidation discounts even in the absence of fire sales. In principle, this suggests that non-financial corporates might also wish to use bail-in debt.

One interpretation in this spirit can be provided in the context of debt renegotiation and restructuring. Chapter 11 of the US Bankruptcy Code provides a reorganization and debt restructuring process for non-financials, allowing them to avoid liquidation under Chapter 7.29 Our model predicts that if failures of non-financial firms are not associated with externalities such as fire sales or bailouts, they will use Chapter 11 efficiently.

In comparison to non-financials, banks take on sufficiently more short-term debt and do not make use of a Chapter 11 type process. In our model, one way to understand this difference is as follows. Banks and other financials may be relatively easier to liquidate

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28 In Appendix B.2.6, we add risk shifting and risk aversion, which generates a role for both bail-in debt and equity. The intuition for the role of bail-in debt is unchanged.

29 To the extent non-financials can control bankruptcy decisions ex ante, for example with their capital structure choices, a process like Chapter 11 provides an ex post alternative to ex ante contractual write-down arrangements. This is in the spirit of the equivalence between ex ante contractual provisions and ex post bail-ins in our model.
during normal times, but are also associated with externalities and bailouts in crises.\textsuperscript{30} Our model suggests that both of these forces encourage banks to adopt capital structures that leave them with larger quantities of short-term, or non-resolvable, debt, and therefore do not make use of Chapter 11. By contrast, non-financials may have greater liquidation discounts on average, but may not be as vulnerable to fire sales or as likely to receive bailouts.\textsuperscript{31} Our model suggests that they adopt capital structures that make them easier to resolve ex post, and make use of Chapter 11.\textsuperscript{32}

One important consideration is that the design of Chapter 11 may simply not be appropriate for banks due to the financial nature of their activities.\textsuperscript{33} Reflecting this, the US Treasury Department has adopted a proposal for a Chapter 14 bankruptcy process, with the aim of creating a process in the spirit of Chapter 11 that is tailored to banks.\textsuperscript{34} Our model predicts that banks would privately under-utilize Chapter 14 relative to the social optimum, leaving a role for a bail-in regime.\textsuperscript{35}

2.3.5 Discussion

In this subsection, we provide additional discussion, including connecting our results to real-world regimes.

\textsuperscript{30}See Appendix B.2.3 for the case of aggregate risk.

\textsuperscript{31}In this sense, we might think of financial companies as having a high average liquidation value $\gamma$, but also having a high sensitivity $\frac{\partial \gamma}{\partial W}$. By contrast, non-financials may have a lower average liquidation value, but also a much lower sensitivity $\frac{\partial \gamma}{\partial W}$.

\textsuperscript{32}For example, the percent of nonfinancial corporate debt that is short-term is approximately 32%, while the ratio of nonfinancial corporate debt to the market value of equities is approximately 34%. (US Flow of Funds)

\textsuperscript{33}See French \textit{et al.} (2010).


\textsuperscript{35}For example, by over-issuing difficult-to-resolve short-term debt.
Short and Long-Term Debt: In practice, bail-in regimes focus on write-downs of long-term debt.\textsuperscript{36} Short-term debt is generally given priority over long-term debt in resolution as part of bail-in regimes. The optimal contract can naturally be implemented by issuing $R_l$ in non-bail-inable short-term debt along with $R_u - R_l$ in bail-inable long-term debt. Moreover, our model predicts that short-term debt should enjoy absolute priority over long-term debt in bankruptcy and liquidation, given this implementation.\textsuperscript{37} In the threshold state $R_l$, long-term debt is fully bailed in while short-term debt is fully repaid. If on the other hand the bank underwent normal bankruptcy and short- and long-term debt were equal claimants, long-term debt would receive positive repayment. This would be inconsistent with a No Creditor Worse Off principle of bank resolution, which requires that resolution be ex post \textit{Pareto efficient} relative to insolvency.\textsuperscript{38}

CoCos: Bail-in debt in our model can be interpreted as a form of \textit{contingent convertible} (CoCo) debt, a form of contractual bail-in instrument that has gained prominence in Europe.\textsuperscript{39} The most natural interpretation in this context is that bail-in debt in our model is a principal write-down CoCo debt security that applies at the point of non-viability.\textsuperscript{40}

Firing the Banker and Control Rights: We assumed that the banker was inseverable from the bank, and so could not be fired. In this sense, short-term debt can be viewed as enforcing a transfer of control rights to investors, whose best option is to liquidate the bank. The

\textsuperscript{36}For example, in the US the top-tier bank holding company is subject to a “clean holding company” requirement, which bars it from issuing short-term debt to external investors. See 12 CFR §252.64.

\textsuperscript{37}In practice, short-term debt priority has three implementations. The first is contractual: bail-in debt is junior to short-term debt. The second is organizational: short-term debt is issued at the operating subsidiary, whereas long-term debt is issued at the top-tier holding company. The third is legal: national bankruptcy law confers priority to short-term debt in the case of banks. The three are equivalent in our model, which may help understand why the method of guaranteeing short-term debt priority varies across countries.

\textsuperscript{38}See e.g. Article 73 of BRRD.

\textsuperscript{39}See Avdjiev \textit{et al.} (2017) and Flannery (2014) for more background on CoCos.

\textsuperscript{40}Although bail-in debt is most naturally expressed as a principal write-down in our model, it can also be expressed as a debt-equity conversion by converting it to an equivalent valued equity stake. As a result, our model does not speak to the optimal form of CoCos (principal write-down and debt-equity conversion).
model could be extended to include costly firings of the banker, where the best option after the control rights transfer might not be liquidation. If there were a trade-off between liquidation and change of management (without liquidation), banks would over-use costly liquidations relative to firing the banker, not internalizing the fire sale. The planner might prefer to fire the banker rather than liquidate, to avoid the fire sale.41

**Dynamic Agency Problems:** We simplified the continuation agency problem for expositional simplicity. Although a formal dynamic model is beyond the scope of this paper, it is worth discussing briefly the forces that would arise with multiple periods of effort choice. In this setting, liquidations ("standard debt") and maximal cash flow transfers ("bail-ins") would still be valuable tools for incentive provision after low returns, while greater payoffs to the bank after high returns would likewise be valuable. However, incentive provision would be linked over periods, likely leading to history dependence - future punishments enhance current incentives when they follow after current low returns, but exacerbate current incentives when they follow after current high returns. Moreover, if future shirking (low effort) reduced agency rents, then an incentive scheme that induced future shirking might constitute an optimal form of money burning, similar to liquidation. This might lead to an additional conflict between a social planner, who wishes to reduce overhang to increase effort and reduce the probability of failures, and the bank, who wants to use overhang as a method of punishment.

**2.4 Bailouts and Time Consistency**

We now introduce bailouts to the model and study their role in the optimal regime. Bailouts may be welfare-enhancing in our model, even if they are chosen ex post without commitment,

41 Of course, this assumes no diminishing marginal value in changing management. For example, a planner might face an issue similar to a fire sale if forced to change the management at many banks simultaneously, where the marginal ability of the new banker hired declines as the planner is forced to search for a greater number of new bankers.
because they can mitigate the fire sale spillover. However, a core principle of post-crisis regulation is that banks and bank investors, not taxpayers, should bear the costs of bank resolution.

We first study the private bank contracting problem and show that bailouts can completely eliminate banks’ incentives to issue bail-in debt. This offers another explanation for why banks wrote few contingencies into their debt contracts prior to the crisis. We then show that optimal regulation limits the use of standard debt so that no ex post bailouts occur: bail-ins fully replace bailouts. A planner who could tie her hands and never engage in bailouts would always prefer to do so.

The contracting problem is the same as before, except that banks understand they may be bailed out when insolvent.

2.4.1 Ex Post Bailout Authority

The planner can bail out insolvent banks in order to prevent liquidations and fire sales. Bailouts are chosen ex post according to a utilitarian welfare function, and are financed by taxpayers.

The bailout required to recapitalize an insolvent bank is $T_1(R) = L_1(R) - (1 - b)RY_0$. Bailouts are associated with two costs. First, bailouts have an ex post (date 1) political or administrative cost $\kappa Y_1$, which we think of as corresponding to a political backlash against bailouts. Second, bailouts have an ex ante (date 0) cost $(\tau - 1)T_1(R)$ to taxpayers, which we think of as corresponding to distortionary costs of taxation. The distortionary cost $\tau - 1$ is sufficiently large that transfers from taxpayers to banks are not welfare-enhancing for redistributive reasons alone. The timing of the two costs in our model generates simple

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42 Another view (Chari and Kehoe (2016)) is that a planner may be tempted ex post to bail out banks to prevent resource losses from liquidation, even in the absence of fire sale spillovers. The results of this section also hold in this case.

43 The political cost scales with bank size in order to prevent banks from “outgrowing” the cost.

44 See the proof of Proposition 16 for a formal condition. Even if redistribution were desirable, they could be done with ex ante lump-sum rather than ex post bailouts. We can interpret bank resources $A$ as including any desirable redistribution.
We conjecture a threshold rule for bailouts, so that insolvent banks with \( R \geq R^{BO} \) are bailed out, and then verify the rule is optimal. The optimal threshold \( R^{BO} \) is the solution to the ex post bailout problem:

\[
\max_{R^{BO} \leq R_i} \left( \int_{R^{BO}}^{R_i} (\gamma(\Omega) - 1) RY_0 f_H(R) dR - \int_{R^{BO}}^{R_i} \kappa RY_0 f_H(R) dR \right) = \kappa
\]

where the transfer \( T_1(R) \) does not appear in the objective function because the planner is utilitarian and the distortionary cost arises ex ante. The ex post bailout decision trades off losses from bank liquidation against the political cost of bailing out banks. The optimal bailout rule, when interior\(^{45}\), is given by

\[
1 - \gamma(\Omega) - \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_{R^{BO}}^{R_i} Rf_H(R) dR = \kappa
\]

Equation (2.11) implies a threshold bailout rule, as conjectured, with a unique solution \( R^{BO} \).\(^{46}\) If the planner continued bailing out banks beyond \( R^{BO} \), concavity of \( \gamma \) implies that total loss falls below the political cost of bailouts. As a result, we have a threshold rule.

Even though \( R^{BO} \) is unique, equation (2.11) implies strategic complementarities in bank risk taking: the planner only has an incentive to engage in bailouts ex post if the equilibrium contract features \( R_i > R^{BO} \).\(^{47}\) If the equilibrium contract sets \( R_i < R^{BO} \), but a single bank instead writes an alternative contract \( R'_i > R^{BO} \), that bank will not be bailed out. As a result, there may be multiple equilibria of the date 0 private bank contracting problem. Our main result on the private contracting problem provides a generic result, and is agnostic to the

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\(^{45}\) The optimal bailout rule may be a corner solution at \( R \) if the left-hand side of equation (2.11) is lower than \( \kappa \) at \( R \), or a corner solution at \( R_i \) if the right-hand side of equation (2.11) is greater than \( \kappa \) at \( R \).

\(^{46}\) Uniqueness follows since since \( \gamma \) is non-decreasing and weakly concave in \( \Omega \). To ensure strict (rather than weak) optimality of the threshold rule, we can add a distortionary cost \( t_1 \rightarrow 0 \) of transfers at date 1.

\(^{47}\) See e.g. Farhi and Tirole (2012). To see the complementarity in our model, the left hand side of equation 2.11 is increasing in the marginal bank that is liquidated in equilibrium. When there are bailouts, the marginal bank that is liquidated has return \( R = R^{BO} \). When \( R'_i < R^{BO} \), the marginal bank that is liquidated has \( R = R'_i \), and so the value of rescuing any failing bank is below the cost \( \kappa \).
equilibrium that is actually selected.

2.4.2 Privately Optimal Contract

As before, the privately optimal bank contract combines standard and bail-in debt. The only difference is that insolvent banks are now bailed out when the equilibrium contract sets $R_l \geq R^{BO}$.\(^{48}\) We show that whenever the equilibrium contract sets $R_l > R^{BO}$, then $R_u = R_l$ and banks issue no bail-in debt. Bailouts crowd out bail-ins.

**Proposition 15.** Suppose there are time-inconsistent bailouts. If the equilibrium private contract sets $R_l \geq R^{BO}$, then it also sets $R_u = R_l$. There is bail-in debt only if there are no bailouts.

To understand Proposition 15, suppose that the equilibrium contract features $R_u > R_l \geq R^{BO}$. In states $R_l \leq R \leq R_u$, bail-in debt is written down and the bank consumes $c_2 = bRY_0$. Suppose that a single bank deviated by writing a contract that set $R'_l = R_u$, and otherwise left contract terms unchanged. The bank now receives bailouts over the range $R_l \leq R \leq R_u = R'_l$, meaning that standard debt holders are fully repaid, while the bank consumes $c_2 = bRY_0$. The investor participation constraint is relaxed, so that this contract strictly dominates the equilibrium contract, contradicting that $R_u > R_l \geq R^{BO}$ was an equilibrium optimal contract.\(^{49}\)

Proposition 15 provides a moral hazard view of limits to the private use of bail-in debt: banks do not use bail-in debt when they expect to be bailed out. The moral hazard view is particularly strong in the presence of fire sale spillovers, where resource losses are larger and bailout incentives stronger. The moral hazard perspective is complementary to the high liquidation values and fire sale views in Section 2.3.

\(^{48}\)Bank contract incentives do not change in the region of the contract above $R_l$. Below $R_l$, banks want to maximize on the bailout subsidy whenever possible. Due to liability monotonicity (2.5), this implies standard debt.

\(^{49}\)This result does not rely on fire sale spillovers. If $1 - \gamma > \kappa$ but $\gamma$ does not depend on $\Omega$, then $R^{BO} = R$ and every insolvent bank is bailed out. Proposition 15 implies that no bail-in debt is issued.
2.4.3 Socially Optimal Contract With Bailouts

The moral hazard problem of Proposition 15 generates an additional role for bank regulation. We study the socially optimal contract in the presence of ex post bailouts, and show that it sets \( R_I \leq R^{BO} \) so that no bailouts occur. This is true whether or not there are fire sale spillovers.

**Proposition 16.** Suppose there are time-inconsistent bailouts, and suppose that \( \tau \) is sufficiently large that ex ante transfers from taxpayers to banks are not welfare enhancing. The socially optimal contract sets \( R_I \leq R^{BO} \), and there are no bailouts. Bank welfare is non-decreasing in \( R^{BO} \).

When banks fail ex post, the planner is tempted to bail them out to mitigate fire sales and prevent resource losses. However, bailouts are costly to taxpayers in a socially undesirable way. Although bail-ins are costly to investors ex post, this cost is priced into bank contracts ex ante. The planner prefers to recapitalize failing banks using bail-ins rather than bailouts, avoiding perverse redistribution from taxpayers to banks and bank investors. A planner would therefore prefer to commit never to bail out failing banks and instead use bail-ins to mitigate fire sales.

In the absence of commitment power, bailouts are chosen in a time-inconsistent manner. A planner that prefers ex ante to liquidate a bank for incentive reasons may prefer ex post to bail out the bank to mitigate fire sales. Since liquidation is not time consistent, the planner must choose between bail-in debt and bailouts to recapitalize the bank. Since bailouts are socially costly, the planner chooses bail-ins. Bail-ins are an imperfect but time-consistent substitute for a commitment against bailouts. Nevertheless, since the planner is forced to increase the use of bail-in debt to prevent bailouts, the optimal bank contract is distorted relative to the optimal commitment contract. Bank welfare increases in the strength of the commitment against bailouts.\(^{50}\)

Proposition 16 reflects the principle that banks and bank investors, not taxpayers, should bear the cost of bank recapitalization. Optimal regulation fully replaces bailouts with

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\(^{50}\)We can think of the cost \( x \) as reflecting the strength of commitment against bailouts.
bail-ins.

2.5 Rollover Crises

We now turn to the second question of the paper, which is whether the prospect of bail-ins is a source of bank instability. We characterize rollover crisis equilibria, which are self-fulfilling prophecies where bail-in debt holders believe they are about to be bailed in, and refuse to refinance an otherwise healthy bank at date 1. The bank becomes too fragile to recapitalize itself and suffers a bank run, justifying the equilibrium beliefs.

To generate rollover crises, we introduce a notion of fragility in the date 1 economy. We extend the model to four periods, and incorporate uncertainty at date 2 via a second quality shock. We incorporate standard short-term debt and bail-inable long-term debt, recalling that the optimal contract in the previous sections could be implemented with this combination.\(^{51}\) Given the uncertainty in continuation, banks will try to refinance the maturing short-term debt by replacing it with long-term debt in order to avoid future liquidations.

This extended model will feature a best equilibrium with successful refinancing, under which the model collapses to the representation of the baseline model. As a result, optimal contracts are the same as in Section 2.3. However, we will also show the existence of the rollover crisis equilibrium at date 1.

2.5.1 Model

There are now four periods, \( t = 0, 1, 2, 3 \). Figure 2.3 provides an illustration of the extended timeline. Banks sign complete markets contracts at date 0 and experience a quality shock \( R \) at date 1, with the same incentive problem at date 0. Banks also experience a quality shock \( R_2 \) at date 2, with \( R_2 \sim F_2 \) on \([R_2, \overline{R}]\) and \( E[R_2] = 1 \), after which uncertainty resolves. Projects mature at date 3 and pay off \( Y_3 = Y_2 = R_2 Y_1 \) units of the consumption good, but

\(^{51}\)In practice, bail-in debt is typically long-term debt.
may be liquidated prior to maturity.

**Figure 2.3: Timeline of Extended Model.**

\[ t=0 \quad \text{Monitoring} \quad t=1 \quad R \text{ realized} \quad t=2 \quad R_2 \text{ realized} \quad t=3 \quad \text{Final Payoff} \]

The date 1 limited pledgeability constraint takes the form of a maximum date 1 debt level \( R^b Y_1 \), where \( R^b \in [R, \overline{R}] \), so that we have

\[ \int_{R \leq R^b} R f_2(R_2) dR_2 + \int_{R > R^b} R^b f_2(R_2) dR_2 \equiv 1 - b. \quad (2.12) \]

As before, \( (1 - b)RY_0 \) is the maximum pledgeable (expected) repayment to investors. Liquidations are not necessary for incentive provision in the continuation problem, meaning that all liquidations on the equilibrium path will occur at date 1. However, the liquidation value \( \gamma(\Omega) \) at date 1 is persistent and applies also at date 2. We think of dates 1 and 2 as subsequent stages of a crisis, where fire sale prices remain depressed throughout.

So far, this model is identical to the baseline model up to the additional period, assuming that we trivialize the rollover problem. The optimal contract can therefore be implemented by the same combination of standard and bail-in debt as in Section 2.3. In particular, appealing to the implementation with short-term and long-term debt, it can be implemented with \( R_l \) of short-term debt, used to liquidate the bank at date 1, and with \( R_u - R_l \) of long-term debt that can be written down to restore solvency.

Under this approach, at date 1 the bank rolls over its short-term debt. Without loss of generality a solvent bank will always choose to use bail-in debt at date 1, because it maximizes stability at date 2. As discussed in Section 2.3, short-term debt is senior to bail-in debt under the optimal contract, provided that a no creditor worse off condition applies. This is also an institutional feature of bail-in regimes.
2.5.2 Refinancing Problem

We now focus on the refinancing problem at date 1. To simplify exposition, we focus on the limiting case where \( R_l = R_u \), so that there is no initial long-term debt.\(^{52}\) Let \( D_1 \) denote the initial level of standard (short-term debt) and \( Y_0 \) the project scale. The bank at date 1 refines itself using a combination of short-term debt \( D_2 \) and long-term debt \( L_3 \).\(^{53}\)

Throughout this section, we study the refinancing problem of a fundamentally solvent bank, with \((1 - b)R Y_0 < D_1 < (1 - b)R Y_0\). Fundamentally insolvent banks are always liquidated. We will also rule out all equilibria except two: the best equilibrium with successful refinancing (as in the baseline model), and the rollover crisis equilibrium.

At date 1, the bank operates in a Walrasian market, where \( q_1^D \) is the price of a new unit of short-term debt, and \( q_1^L \) is the price of a new unit of long-term (bail-in) debt. We assume all contracts are fully visible to all creditors to rule out conventional rat race dynamics.\(^{54}\) This assumption is reflected in the following pair of constraints

\[
q_1^D D_2 \leq \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} D_2 f_2(R_2) dR_2 \tag{2.13}
\]

\[
q_1^L L_3 \leq \int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2) dR_2 \tag{2.14}
\]

which state that the expected payoff to debt holders must be at least as high as the amount they pay, given market prices.\(^{55}\) They imply the bank cannot deliberately dilute new bank creditors.

**Best Equilibrium:** Under these constraints, there is a unique best equilibrium under which the bank successfully refines itself and never liquidates after date 1.

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\(^{52}\)This assumption is not required to generate rollover crises.

\(^{53}\)Our results generalize immediately to the case where banks can also issue other liabilities that are junior to short-term debt under the same logic as will follow.

\(^{54}\)We could also interpret these as bond covenants.

\(^{55}\)These conditions are stronger than needed, in order to guarantee the debt issuances of the best equilibrium are unique. Rollover crises are not affected by using these stronger conditions. See Appendix B.2.10.
Lemma 17. There is a unique best equilibrium with no liquidations after date 1. It has prices $\bar{q}_1^D = 1$ and $\bar{q}_1^L < 1$. It has short-term debt issuance $\bar{D}_2 = R Y_0$ and long-term debt issuance $\bar{T}_3 = \frac{D_1 - D_2}{\bar{q}_1^L}$.

The best equilibrium of Lemma 17 is the refinancing problem outcome associated with the optimal contracts considered in the first part of the paper. Under this equilibrium, the problem behaves as-if periods 2 and 3 were combined into a single period. In other words, our baseline model can be viewed as a representation of this best equilibrium.

2.5.3 Rollover Crisis Equilibrium

We now study the existence of the worst rollover crisis equilibrium, where the bank fails to refinance itself and is immediately liquidated, despite being fundamentally solvent. To be clear, we are not looking for traditional bank run equilibria, and we assume that a costless lender of last resort stands ready to stop a sunspot bank run. Rollover crises will result in a bank run, however, because short-term debt alone cannot refinance the bank.

We assume that the maximum amount pledgeable to investors is higher in continuation than in liquidation even with short-term debt,

$$\sup_{d_2 \leq R_b} \int_{d_2}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{R} d_2 f_2(R_2) dR_2 > \gamma$$

(2.15)

where we define $d_2 = D_2 / Y_1$ to be the continuation debt-to-asset ratio. Repayment to short-term debt holders therefore cannot be increased by liquidating the bank. As a result, if the bank cannot refinance itself, there is sufficient short-term debt to liquidate the bank.

We now show that a rollover crisis equilibrium generically exists in a region above the short-term debt level $R_l$.

Proposition 18. Let $D_1 = (1 - b) R_l Y_0$ be the short-term debt level, with $R_l > R$. A rollover crisis equilibrium exists for all date-1 returns $R \in [R_l, R^*)$, where $R^*$ is given by

$$\frac{D_1}{R^* Y_0} = \sup_{d_2 \leq R_b} \int_{d_2}^{d_2} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{R} d_2 f_2(R_2) dR_2$$

(2.16)
In the rollover crisis equilibrium, the bank is unable to refinance itself at date 1 and is liquidated. The equilibrium price of new long-term debt is $q_1^L = 0$.

To understand the intuition behind the rollover crisis equilibrium, consider the threshold bank for which $D_1 = (1 - b)Y_1$. This bank is just able to recapitalize itself in the best equilibrium. It can survive only by issuing the securities packages with $D_2 \leq RY_1$ and $L_3 = RbY_1 - D_2$, which pledges all pledgeable income to investors without any liquidations in continuation. In particular, there are no securities packages $(D_2, 0)$ and price $q_1$ that would refinance the bank in a Walrasian equilibrium. As a result, this threshold bank, if faced with the only option of refinancing itself with short-term debt, would suffer a run by its short-term creditors.\(^56\) This generates the rollover crisis equilibrium: holders of long-term debt expect to be bailed in and quote a price of 0, leading to a bank run and liquidation. Since new long-term debt is subordinated to short-term debt, it would receive no recovery value in the liquidation, justifying the equilibrium price of 0 and completing the equilibrium.

Rollover crisis equilibria result from sunspots in long-term debt markets, unlike conventional bank runs which are the result of sunspots in short-term debt markets. They are justified by the bail-in regime, which promises to subordinate long-term debt to short-term debt in resolution, and relies on the fundamental fragility of short-term debt relative to long-term debt. A bank on the brink of non-viability has an excessively high short-term-debt-to-assets ratio which it needs to unwind in order to remain viable. If it cannot, it becomes non-viable and subject to a run.

Rollover crises exist because the bail-in regime confers explicit priority to short-term debt over long-term debt. The conferred priority applies not only under bail-in resolution, but also under liquidation, leading to the instability. This further implies that a rollover crisis exists for any other liabilities that a bank could issue, such as equity, provided those

\(^{56}\) A lender of last resort also cannot break even on any securities package $(D_2, 0)$ below $R^*$. Above $R^*$, there exists a price $q_1^D$ and debt issuance $D_2$ such that $q_1^D D_2 = D_1$, in other words the bank can refinance itself from a short-term lender of last resort.
instruments are subordinated to short-term debt. Although “deposit priority” may help alleviate traditional bank runs, it has the potential to generate instability in long-term debt markets.

2.5.4 Propagation of Rollover Crises

The existence of rollover crises relies only on the presence of liquidation discounts, and not on fire sale spillovers. However, we show that fire sale spillovers contribute to the propagation of rollover crises, leading to more frequent crises and larger fire sales. The rollover crisis problem is particularly severe in times of market stress.

Because we have multiple equilibria, we must adopt an equilibrium selection rule. We adopt a simple selection rule: banks experience a rollover crisis with probability $0 < p < 1$ and experience the best equilibrium with probability $1 - p$ when in the rollover crisis region. By the law of large numbers, equilibrium liquidations and the liquidation value are a solution to

$$\gamma^* = \gamma(\Omega^*), \quad \Omega^* = \epsilon + \int_{\mathbb{R}} R f_H(R) dR + p \int_{R^*}^\infty R f_H(R) dR \quad (2.17)$$

where $\epsilon$ is an exogenous liquidation shock that we use to illustrate the feedback loop. The rollover crisis threshold $R^*$ depends on the equilibrium liquidation value $\gamma^*$, as illustrated in Proposition 18, so that we have a fixed point problem. We characterize the feedback loop by starting from $\epsilon = 0$, and then study the equilibrium response of total liquidations $\Omega^*$ with respect to $\epsilon$. If there is no feedback loop, then $\frac{\partial \Omega^*}{\partial \epsilon} = 1$.

**Proposition 19.** Starting from an equilibrium of the date-1 economy with $\epsilon = 0$, an exogenous increase in liquidations $\epsilon$ generates a total increase in equilibrium liquidations

$$\frac{\partial \Omega^*}{\partial \epsilon} \bigg|_{\epsilon=0} = 1 + \frac{p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \left| \begin{array}{c} R^* f_H(R^*) \\ \epsilon=0 \end{array} \right|}{1 - p \left| \frac{\partial \gamma(\Omega^*)}{\partial \Omega^*} \right| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \left| \begin{array}{c} R^* f_H(R^*) \\ \epsilon=0 \end{array} \right|} \quad (2.18)$$

The core of the feedback loop in Proposition 19 is the sensitivity of the liquidation discount to liquidations, $\frac{\partial \gamma}{\partial \Omega^*}$. When this sensitivity is higher, $\frac{\partial \Omega^*}{\partial \epsilon} \bigg|_{\epsilon=0}$ increases, and the
exogenous shock to liquidations is amplified by the propagation of rollover crises. The increase in liquidations lowers the liquidation value, expanding the region of crises. More banks become subject to rollover crises, pushing down liquidation values further. By contrast, if liquidation values are not sensitive to liquidations \( \frac{\partial W}{\partial l} = 0 \) then there is no feedback loop and no propagation.

In normal times, when fire sale spillovers are limited, rollover crises may therefore be relatively contained. By contrast, during financial crises, when spillovers may be more severe, they can propagate, increasing bank failures and exacerbating fire sales. This propagation effect reflects the concern that even though bail-ins may have beneficial properties for bank resolution during relatively normal times or for the resolution of individual banks, they may generate adverse effects during times of systemic crises.\(^57\)

### 2.5.5 Impact of Covenants

A natural conjecture is that bond covenants, which are a common tool for addressing typical debt dilution or rat race incentives, would be effective in preventing rollover crises. We show that bond covenants do not prevent rollover crises.

A bond covenant is an arbitrary set of restrictions on the refinancing structures available to the bank at date 1,

\[
C_1(D_2, L_3|D_1, Y_1, q_1) \leq 0,
\]

where \( C_1 \) is some vector-valued function.\(^58\) Define the vacuous covenant by \( C_1 = 0 \), which places no restrictions on the bank. All previous results have assumed the vacuous covenant.

**Proposition 20.** If a rollover crisis equilibrium exists under the vacuous covenant \( C_1 = 0 \), then it also exists under any other covenant \( C_1(D_2, L_3|D_1, Y_1, q_1) \).

Proposition 20 shows that covenants are not a solution to rollover crises. To understand

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\(^57\)In Appendix B.2.11, we show that rollover crises can generate multiple aggregate equilibria of the date 1 economy, even for a fixed equilibrium selection rule \( p \), due to the propagation effect.

\(^58\)For example, the covenant \( C_1(D_2, L_3|D_1, Y_1, q_1) = D_2 - R Y_1 \) restricts short-term debt issuance at date 1.
why, rollover crises are justified in equilibrium by the outstanding stock of short-term debt, not by new short-term debt. As a result, covenants cannot rule out rollover crises.

### 2.6 Policy Responses to Rollover Crises

In Section 2.5, we showed that the socially optimal contracts written by the planner in Section 2.3.2 were susceptible to rollover crises at date 1. We now consider ex post policies the planner could adopt to prevent rollover crises.\(^59\) We consider two policies that have precedent in policies employed during the 2008 financial crisis. The first is an expanded lender of last resort that extends both short- and long-term loans to banks. The second is temporary guarantees of new issuances of long-term debt.

#### 2.6.1 Expanded Lender of Last Resort

A common solution to sunspot bank runs by short-term debt holders (i.e. depositors) is a lender of last resort (LOLR), which extends short-term loans to fundamentally solvent banks faced with a sunspot bank run.\(^60\) Although rollover crisis equilibria in our model are a form of coordination failure, the coordination failure arises from long-term debt, not short-term debt. A conventional LOLR extending short-term loans is therefore not a sufficient policy in this case,\(^61\) but a LOLR facility might be successful if it provided long-term debt loans to distressed banks.\(^62\) A facility of this form is not without precedent. For example, the Capital Purchase Program (CPP) implemented under TARP in 2008, made $250 billion available for

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\(^{59}\) The planner could also limit rollover crises by adjusting ex ante contract terms, but this would not rule out rollover crises unless the planner set \(R_l \leq R\) and would distort the optimal contract.

\(^{60}\) For example, the Federal Reserve set up the Term Securities Lending Facility (TSLF) in March 2008 to provide liquidity to financial institutions and restore credit market functioning, allowing them to swap illiquid collateral for liquid Treasury securities, which could then act as collateral in new funding agreements in credit markets.

\(^{61}\) A conventional LOLR facility could provide a temporary stopgap while the government attempted to orchestrate a public or private sector rescue program.

\(^{62}\) These loans would be subordinated to short-term debt, and would be potentially bail-inable and uncollateralized. The LOLR would break even in expectation but not on every realized path. This LOLR facility differs starkly from the usual facility and violates Bagehot best practice principles.
the purchase of preferred stock in banks, with the goal of making funding available in a
fragile market. The nature and goals of CPP were similar to those of our proposal.63

We model the extended LOLR facility as follows. At the same time as the bank accesses
private markets, it can also access the LOLR which is willing to make available to the bank
any package \((D_2^{LOLR}, L_3^{LOLR})\) at prices \(q_1^{LOLR}\) so that it breaks even in expectation. LOLR
claims rank \textit{pari passu} with private sector claims of the same instrument.64 Provided that the
bank successfully refines with issuances \((D_2, L_3, D_2^{LOLR}, L_3^{LOLR})\), the break-even prices of
the LOLR are given by

\[
q_1^{D,LOLR} = 1 - \int_{D_2 + D_2^{LOLR} \geq R_2 Y_1} \left[1 - \frac{\gamma R_2 Y_1}{D_2 + D_2^{LOLR}} \right] f_2(R_2) dR_2 \quad (2.19)
\]

\[
q_1^{L,LOLR} = \int_{D_2 + D_2^{LOLR} \leq R_2 Y_1} \min \left\{ \left( R_2 Y_1 - D_2 - D_2^{LOLR} \right) \left( \frac{1}{L_3 + L_3^{LOLR}} \right), 1 \right\} f_2(R_2) dR_2 \quad (2.20)
\]

Given market prices \(q_1\) for private sector debt, the bank can refinance itself if there are a
securities package \((D_2, L_3, D_2^{LOLR}, L_3^{LOLR})\) and corresponding LOLR prices \(q_1^{LOLR}\) that allow
the bank to raise \(D_1\) in revenue, while satisfying the private sector no-rat-race conditions.65

Because the bank can always refinance itself successfully by borrowing best equilibrium
quantities from the LOLR at best equilibrium prices, rollover crises are eliminated. Since
LOLR debt and private sector debt rank \textit{pari passu}, private sector prices must be the best
equilibrium prices. The bank can then refinance itself entirely from the private sector.

\textbf{Proposition 21.} \textit{Suppose there is an extended LOLR facility. Suppose private sector and LOLR
claims rank \textit{pari passu}. Rollover crises are eliminated. The bank can refinance itself entirely in private
markets at best equilibrium prices.}

\footnote{63The US Treasury Department, in describing CPP, states that it “helped bolster the capital posi-
tion of viable institutions of all sizes and built confidence in these institutions and the financial system
as a whole.” https://www.treasury.gov/initiatives/financial-stability/TARP-Programs/bank-investment-
programs/cap/Pages/default.aspx.}

\footnote{64In practice, the government tends to give itself high priority on claims in resolution. See 12 CFR §380.21.}

\footnote{65The proof of Proposition 21 restates these conditions when LOLR debt is included.}
Proposition 21 is similar in spirit to the standard LOLR solution to sunspot bank runs: by making loans available to the bank, the LOLR prevents the onset of the crisis. Although these loans only happen off equilibrium in the model, in practice such a facility would likely be used to some degree by banks during crises. This leads to two natural practical concerns. The first is that because the LOLR extends uncollateralized, long-term loans and would lose money on some realized paths (but not in expectation), the operating principles are different from those of a standard LOLR. The second is that the facility would be subject to familiar moral hazard concerns: it may unintentionally lend to insolvent banks, or lend at prices lower than break-even prices.\textsuperscript{66} This would lead the LOLR to subsidize distressed banks, amounting to a partial bailout.

2.6.2 Debt Guarantees

Deposit insurance is a second common solution to sunspot bank runs. In its idealized form, deposit insurance prevents the run and is not filled on the equilibrium path. In a rollover crisis, although insurance would allow the bank to roll over its short-term debt and avoid liquidation, insurance would be filled on the equilibrium path.

We consider instead an extension of temporary guarantees to new issuances of long-term debt, and show that these guarantees can successfully rule out rollover crises without being filled on the equilibrium path. This has precedent in the Temporary Liquidity Guarantee Program (TLGP) instituted in 2008, under which the US government provided guarantees to new issuances of senior unsecured debt with the goal of “preserving confidence in the banking system and encouraging liquidity.”\textsuperscript{67} These guarantees were “temporary” in that they expired no later than June 2012.

We model guarantees as follows: at the beginning of date 1, the government extends a temporary guarantee to all new issuances of long-term debt. Guarantees oblige the govern-

\textsuperscript{66}One particular manifestation of this is that the break-even prices of long-term debt are bank-dependent, because it is not guaranteed to be repaid in full. A LOLR operating during stress times may not wish to price discriminate between banks to avoid reputational damage to distressed banks.

ment to cover any losses relative to face value during the guarantee period. Guarantees expire at the end of date 1, after which the debt is once again subject to the bail-in regime. In other words, if at date 1 the bank enters bankruptcy and liquidates, the government is obligated to pay \( 1 - q_1^{L,B} \) to a holder of new long-term debt, where \( q_1^{L,B} \leq 1 \) is the recovery value of new long-term debt in liquidation. This guarantee extension eliminates rollover crises.

**Proposition 22.** *A temporary guarantee of new long-term debt that expires at the end of period 1 eliminates rollover crises. Guarantees are not filled on the equilibrium path.*

Proposition 22 is closely related to the rationale for deposit insurance. Since debt is guaranteed, investors cannot expect a price of 0, ruling out the rollover crisis and allowing the bank to refinance itself. Guarantees are never filled on the equilibrium path because they expire the end of date 1. This “temporary” aspect is consistent with the principle of TLGP, where the guarantees expired after a certain time frame.

It is too strong in practice to assume that guarantees can be timed perfectly to never be filled. Some guarantees would be filled on the equilibrium path, leading to moral hazard concerns. Proposition 22 is an idealized result that helps explain why debt guarantee programs such as TLGP may be a valuable part of a crisis resolution toolkit.

### 2.6.3 Bailouts versus Debt Guarantees

Sections 2.6.1 and 2.6.2 show that expanded LOLR facilities and long-term debt guarantees can be valuable components of a planner’s crisis resolution toolkit. However, post-crisis

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68. We have assumed away existing long-term debt. Guarantees would not need to be extended to existing long-term debt. This is consistent with TLGP.

69. The fact that long-term debt does not mature until the final period increases its value relative to short-term debt even in the absence of future bail-ins, as it cannot trigger early liquidations. The results that follow would apply, possibly in a weaker form, even if the government cannot bail-in this debt in the future either.

70. The Debt Guarantee Program (DGP) portion of TLGP, which involved guarantees of new loans, at its peak guaranteed approximately $345 billion in debt. Approximately $153 million in guarantees were filled. A second component of TLGP, the Transaction Account Guarantee Program, provided guarantees for noninterest-bearing transaction accounts and resulted in estimated losses of $2.5 billion. These numbers are reported by the FDIC as of February 2019 (https://www.fdic.gov/regulations/resources/tlgp/index.html).
regulation has limited the ability of the US government to engage in such policies as part of a commitment against future bailouts.\footnote{See Geithner (2016).} Our model suggests an important distinction between bailouts of existing debt and protection of new debt issued during stress times, even when such protection is “bailout-like.” Optimal regulation in Section 2.4 replaces bailouts of existing debt with bail-ins. Guarantees serve a different function and can be a valuable stabilization tool during a crisis. A commitment against bailouts of existing debt need not preclude protection of new debt during a crisis.

### 2.7 Extensions

In Appendix B.2, we provide a set of extensions.

In Appendix B.2.1, we discuss the role of the agency problems in generating optimal contracts that combine standard and bail-in debt. We show that in the absence of both the ex ante and interim agency problems \((B, b > 0)\), contracts would not combine standard and bail-in debt.

In Appendix B.2.2, we account for arbitrageur welfare and characterize sufficient conditions under which the socially optimal contract in Section 2.3.2 is Pareto efficient relative to the privately optimal contract.

In Appendix B.2.3, we allow for the possibility of aggregate risk. The optimal contract takes the same form, but both instruments are contingent on the aggregate state. Banks inefficiently limit contingencies on aggregate risk when there are fire sales or bailouts.

In Appendix B.2.4, we consider the interaction between macroprudential (asset-side) and liability-side regulation by introducing multiple investment projects. While bail-ins still constitute optimal liability regulation, asset-side regulation is also required to implement the social optimum.

In Appendix B.2.5, we extend the model to allow for heterogeneous investors with different risk tolerances and different exposures to the banking sector, and discuss the
allocation of bail-in securities. We show that retail investors, who maintain greater exposures to individual banks, and institutional investors who experience spillovers from fire sales should hold safer (non-bail-inable) claims.

In Appendix B.2.6, we extend the model to incorporate equity-like claims into the bank’s capital structure by incorporate bank risk aversion and risk shifting. The disagreement between the bank and planner is over the use of standard debt versus loss-absorbing capital (bail-in debt + equity), and not over the composition of loss-absorbing capital.

In Appendix B.2.7, we allow for standard debt to command a premium over other instruments, including bail-in debt. This increases use of standard debt and helps to explain why, in practice, the level of standard debt banks employ is so high. The marginal trade-off for banks is still influenced by the incentive problem and leads to use of bail-in debt. In absence of the incentive problem, there would be no reason to use bail-in debt over equity.

In Appendix B.2.8, we allow for insured deposits at banks. The planner faces a trade-off between greater deposit insurance (i.e. taxpayer) losses when liquidating the bank, and worse bank incentives when bailing out the bank. Bailouts may be desirable to lessen the taxpayer burden of deposit insurance. All non-deposit investors are fully bailed in whenever the planner bails out the bank. This motivates the possibility of having a deposit guarantee scheme, even in the absence of other bailouts.

In Appendix B.2.9, we characterize the (Pigouvian) tax wedges needed to decentralize the socially optimal contract. Tax wedges are needed only on standard debt, and not on bail-in debt, but are also needed on any other liability that can generate liquidations. This suggests why regulators may wish bail-in debt to be simple and subject to a regulatory trigger is to avoid regulatory arbitrage.

In Appendix B.2.10, we characterize the weaker form no-rat-race conditions used to rule out deliberate dilution motives.

In Appendix B.2.11, we characterize conditions under which rollover crises can generate multiple equilibria of the date 1 economy.

In Appendix B.2.12, we study rollover crises, early triggers, and the relative roles of
de jure and de facto seniority in rollover crises. First, we argue that the existence region of rollover crises defined in Proposition 18 is invariant to early triggers. Second, we argue that affording de jure pari passu status to a set of non-bail-inable long-term debt claims during a crisis can potentially alleviate rollover crises, but may be problematic.

2.8 Conclusion

We characterize optimal bank contracts under a monitoring incentive problem. Privately optimal bank contracts combine standard and bail-in debt. Banks’ private use of bail-in debt is limited when there are fire sales or bailouts. A bail-in regime, which increases use of bail-in debt relative to standard regime, is the optimal regulatory policy. Optimal regulation replaces bailouts with bail-ins. This helps to understand the design of bail-in regimes in the US and EU.

We also show that the prospect of bail-ins can have destabilizing effects on bank refinancing during times of market stress, leading to rollover crises and bank failures. Rollover crises can be addressed using policies such as extended lender of last resort facilities and debt guarantees. These crisis resolution tools complement an effective bail-in regime.
Chapter 3

A Theory of Dynamic Inflation Targets

3.1 Introduction

The policy discussion has increased attention on the question of whether central bank inflation targets should be perpetually fixed, as in the US, or whether they should be adjusted. For example, a central bank may wish to adjust its target upward during normal times to bring itself away from the zero lower bound and allow for a larger monetary policy response during a recession.\(^2\) Given that inflation targets are designed to anchor inflation expectations, this raises the natural concern that allowing a flexible target would undermine its purpose. Moreover, there are different methods by which adjustment could occur, with possibilities including allowing adjustment at fixed points in time or for flexible adjustment within a restricted band.

This paper provides a framework to study whether inflation targets should be adjusted over time, and if so what the process governing target adjustment should be. To do so, we study a mechanism design problem of a government designing a transferable utility

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\(^1\)Co-authored with Andreas Schaab

\(^2\)See e.g. Ball et al. (2016).
mechanism to control how a central bank sets monetary policy (inflation). The central bank has private information over persistent structural economic shocks which determine the optimal rate of inflation, meaning that optimal inflation can vary systematically over time. However, the central bank also has a time consistency problem in inflation setting that arises from a Phillips Curve relationship. This results in a trade-off between flexibility to allow monetary policy to respond to structural shocks, and commitment to manage the time consistency problem. Additionally, information persistence implies that when the central bank reveals its information, it affects the beliefs of both the government and of the firms that determine the inflation-output relationship.

The main result of this paper is that a dynamic inflation target implements the constrained efficient level of inflation, and moreover is the optimal mechanism when the social cost of the transfer (enforcement) mechanism governing central bank behavior is negligible relative to the inflation-output trade-off. Under this mechanism, the central bank’s inflation setting in each period is managed by the target, with linear incentives for deviations of inflation from the target. Moreover, central bank can update the target one period in advance. As a result, in every period the central bank both sets inflation based on the existing target, and also decides whether to update the target for the next period. In equilibrium under this mechanism, the target is always equal to next-period expected inflation, giving rise to the interpretation as a dynamic inflation target.

The intuition behind the optimality of the dynamic inflation target is as follows. First, the target itself manages the time consistency problem in inflation setting by penalizing inflation in excess of the target. Second, requiring the target to be adjusted one period in advance manages the time consistency problem in target adjustment. If the central bank were allowed to contemporaneously adjust its inflation target, it would adjust its target to its desired inflation level, de-anchoring inflation expectations and rendering the target meaningless. By adjusting in advance, the central bank internalizes its future time consistency problem and sets the target efficiently. This allows inflation expectations to remain anchored to the target, even though it is adjusted over time. Moreover, the information effects of changing
the target on the beliefs of the government and firms offset each other at the constrained efficient allocation due to the fact that both firm behavior and government transfers are based on inflation expectations.

The optimality of a dynamic inflation target suggests that controlled target adjustment may be preferable to a perpetually fixed target, and moreover provides a means of controlling target adjustment. It suggests that adopting an approach similar to that of the Bank of Canada, where the target can be adjusted only at fixed five year intervals, may be an effective way of allowing for target adjustment without de-anchoring inflation expectations.

We explore the robustness of our results to costly enforcement (transfers) - that is, the mechanism used to control central bank behavior has first order welfare costs. Although the dynamic inflation target still implements the constrained efficient allocation, the government no longer finds it exactly optimal to do so due to the enforcement costs. Nevertheless, we show that the properties of the optimal mechanism still bear features resembling a dynamic inflation target, in particular a time-varying slope term that governs the time consistency problem. We further show that there are three sufficient statistics that capture the full history dependence of the optimal mechanism: one reflecting the time consistency problem (the slope), and two reflecting information persistence. Finally, we show that the mechanism reverts to the dynamic inflation target at both extremes of the distribution.

**Related Literature:** We related to the literature on time consistency in monetary policy, as well as to the broader mechanism design literature on the trade-off between commitment and flexibility. Walsh (1995) shows that an inflation target is an optimal mechanism in a static context with transferable utility. Halac and Yared (2019) considers the trade-off between instrument-based rules and target-based rules in a delegation framework. Several papers study the commitment-flexibility trade-off in a delegation framework (Amador et al. (2006), Athey et al. (2005)), including with persistent information (Halac and Yared (2014)). Beshears et al. (2019) considers the trade-off in a population with present bias and

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3See e.g. Barro and Gordon (1983), Canzoneri (1985), Kydland and Prescott (1977), Persson and Tabellini (1993), and Rogoff (1985) for the former.
inter-agent transfers. Our contribution is to study the problem of inflation setting in a transferable utility framework with both persistent information and outside agents who derive information from reports under the mechanism, and to show its implications for inflation target adjustment. In our model, there are two commitment-flexibility trade-offs: first in allowing for present inflation to respond to shocks (“the target”), and second in allowing for adjustment of future targets to shocks (“changing the target”).

3.2 Model

Our economy has a government, a monetary authority (central bank), and a continuum of small firms. Time is infinite and discrete, indexed by $t = 0, 1, \ldots$ Allocations in this economy are summarized by two scalar variables, inflation $\pi_t$ and output $y_t$. We introduce these labels from the start to allude to the standard New Keynesian model, but everything that follows extends to the general case where $\pi_t$ and $y_t$ are vectors. Both inflation and output lie in compact sets, $\pi_t \in [\underline{\pi}, \overline{\pi}] \subset \mathbb{R}$ and $y_t \in [\underline{y}, \overline{y}] \subset \mathbb{R}$. The relationship between inflation and output is determined by price-setting firms.

The government and central bank interact in a principal-agent framework. The central bank learns about the state of the economy, and uses this information to determine monetary policy. However, the central bank is subject to a time-consistency problem. As a result, the government will design a mechanism to determine how the bank sets monetary policy, subject to truthful reporting conditions. Firms are not directly under the control of the government, so their actions will be taken as a constraint on the problem.

**Government.** The social preferences of the government are given by

$$E \sum_{t=0}^{\infty} \beta^t U_t(\pi_t, y_t, \theta_t),$$

(3.1)

where $\beta$ is the discount factor, and $\theta_t \in \Theta = [\underline{\theta}, \overline{\theta}]$ denotes an economic shock with conditional density $f(\theta_t | \theta_{t-1}).$ The government does not directly observe the shock $\theta_t$, which will be observed and reported by the central bank.
Interpretation of \( \theta_i \). We think of \( \theta_i \) as corresponding to the true economic shock to the economy, which the central bank learns information about. In this sense, the government welfare function reflects a notion of social welfare that incorporates the true shock - for example, a true supply side shock or shock to the loss function for inflation and output deviations. However, both the firms and the government rely on the central bank to collect and report information about that true type. As such, both the government and firms form decision rules based on the reported type, not the true type. The government’s beliefs matter because it designs the mechanism that maps the reported type (beliefs) into an allocation rule. The firm’s beliefs also matter, because firms form a decision rule based on those beliefs. In both cases, that decision rule responds to beliefs, while the “true” welfare function is related to the true type.

**Central Bank.** The central bank has preferences over both social welfare, and over transfers \( T_t \) from the government, so that its preferences are

\[
\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ u_t(\pi_t, y_t, \theta_t) + T_t \right].
\]

(3.2)

The central bank observes the shock realization \( \theta_i \) in every period, and so is tasked with choosing inflation and output in each period.

In writing preferences, we have assumed that \( T_t \) is welfare neutral from the perspective of the government, and is only used as a control mechanism. This can be seen as a limiting case where the costs of controlling central bank behavior are negligible relative to the underlying social welfare problem. In Section 3.4, we study the case where transfers are not neutral from the perspective of the government.

**Interpretation of Transfers.** Rather than monetary transfers, the practical analogs of the control mechanism \( T_t \) may be closer to policies such as Congressional scrutiny, reputational risk, or
firing the central banker. For example, a central bank being awarded high $T_t$ may face a low degree of Congressional scrutiny in its policy determination. So far, $T_t$ could be a utility transfer, or it could be a form of money burning.

**Firms and Phillips Curve.** Firms, whose actions are not directly controlled by the government or central bank, create a “Phillips Curve” relationship between inflation and output, given by

$$y_t = F_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\mu_t]). \quad (3.3)$$

The object $\mu_t$ is the belief of firms about the current state $\theta_t$, which are formed from the report of the central bank. Because $\theta_t$ will feature persistence, beliefs about the current state of $\theta_t$ translate into beliefs about the next period state $\theta_{t+1}$ and hence the next-period inflation $\pi_{t+1}$. Because firms cannot be directly controlled, equation (3.3) is an implementability condition from the perspective of the government and central bank.

To simplify exposition, we internalize the Phillips Curve relationship of equation (3.3) into preferences, and write

$$u_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\mu_t], \theta_t) = U_t(\pi_t, \pi_{t+1}, \theta_t, \mu_t) = U_t(\pi_t, F_t(\pi_t, \mathbb{E}_t[\pi_{t+1}|\mu_t]), \theta_t)$$

where we use $U_t$ and $u_t$ for more compact notation.

**Lucas Critique.** A key concern of this Phillips Curve relationship is a Lucas critique - firm price setting behavior may change in response to inflation policy changes, such as target changes. Our Phillips Curve relationship is robust to a Lucas critique provided that expected future (next period) inflation is sufficient for determining how changes in future policies affect firm behavior. For example, higher expected inflation may lead firms to

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4In the U.S., for example, this process is multifaceted. The central bank Chairwoman is directly held accountable to Congress in the form of bi-annual, as well as extraordinary, Congressional testimonies. The institutions adopted by modern central banks to allow for active monitoring by stakeholders and maintain accountability vary across countries but most are highly complex.

5See e.g. L’Huillier and Schoenle (2019).
increase the frequency with which they update prices, altering the slope of the Phillips Curve.

3.2.1 Constrained Efficient Allocation

Before characterizing the mechanism structure, we characterize the constrained efficient allocation that could be achieved if the government and firms observed $\theta_t$, and the government mandated with commitment the inflation decisions of the central bank. This provides an efficiency benchmark which respects the Phillips Curve relationship between inflation and output determined by firms.

The constrained efficient planning problem of the government is given by the optimization problem

$$\max_{\{\pi_t\}} \sum_{t=0}^{\infty} \beta^t E_0 \left[ u_t(\pi_t, E_t(\pi_{t+1} | \theta_t), \theta_t) \right],$$

where $\pi_t = \pi_t(\theta^t)$ is adapted to the full history of realized shocks, and where $\theta_{-1}$ is an initial condition of the model. Taking the first order conditions, we obtain the necessary conditions for optimality,

$$\frac{\partial u_t}{\partial \pi_t} = -\frac{1}{\beta} \frac{\partial u_{t-1}}{\partial E_{t-1}(\pi_t | \theta_{t-1})} \quad \forall t \geq 1 \quad (3.5)$$

$$0 = \frac{\partial u_0}{\partial \pi_0} \quad (3.6)$$

Because there is no Phillips curve constraint for $\pi_0$ (it would occur at date $-1$), the optimality condition for $\pi_0$ simply sets the marginal value of increasing inflation to 0, yielding a standard first-order condition that would also be chosen by the central bank. For $t \geq 1$, the first-order condition reflects the Phillips curve relationship, and internalizes that inflation at date $t$ affects output at date $t - 1$ through the constraint set. The LHS of equation (3.5) is date $t$-adapted, whereas the RHS is date $t - 1$-adapted. Therefore, the RHS is constant from the perspective of time $t$, implying that the marginal (flow) utility from inflation is constant at date $t$ in histories $\theta^t$ proceeding from the same history $\theta^{t-1}$. 

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Discretionary monetary policy. Suppose that the central bank were left to its own devices to set inflation \((T_t = 0)\) with discretion. At period \(t\) for any \(t\), the central bank finds it optimal to set \(\partial u_t / \partial \pi_t = 0\) state by state. In particular at date \(t\), the central bank neglects the impact of inflation on the previous period’s Phillips Curve, which no longer serves as a constraint of the problem. This results in inflationary bias and reflects a standard Barro and Gordon (1983) time consistency problem. This motivates studying a mechanism to control central bank inflation setting.

A Sufficient Statistic. Suppose that we evaluate the constrained efficient allocation, and define the wedge

\[
\nu_{t-1} = -\frac{1}{\beta} \frac{\partial u_{t-1}}{\partial \mathbb{E}_{t-1}(\pi_t | \theta_{t-1})}.
\]

This wedge \(\nu_{t-1}\) is a date \(t - 1\) adapted constant that is a sufficient statistic for the shock history \(\theta^{t-1}\) in determining the allocation rule \(\pi_t, \pi_{t+1}, \ldots\) for inflation. \(\nu_{t-1}\) is the wedge between the constrained efficient allocation rule for \(\pi_t\) and the central bank’s allocation rule under discretion. In other words, this wedge reflects the inflationary bias that arises from the central bank’s time consistency problem.

3.2.2 Mechanism Structure

We study direct and full-transparency mechanisms, under which the central bank makes a report of its type each period.\(^6\) By full transparency, we mean that there is no pooling of central bank types in reporting. We denote this report by \(\tilde{\theta}_t\). Along the equilibrium path, agents’ posterior will therefore be the degenerate distribution at the reported type, or \(\mu_t = \tilde{\theta}_t\). Note that we abuse notation here because \(\mu_t\) is a full distribution in general.

Restricting attention to full transparency mechanisms is not without loss of generality. In principle, the government could want to pool central bank types to shroud the type and manipulate firm beliefs. In assuming mechanisms under which the central bank truthfully

\(^6\)Note that under an indirect mechanism, e.g. a report \(\sigma(\theta)\) for some rule \(\sigma\), the true type can be uncovered by unwinding the indirect report in the usual manner.
reveals its type, we assume away such motivations. Given that central bank transparency has received increasing emphasis since the financial crisis, we impose the restriction to full transparency.\(^7\)

A mechanism in our model is a mapping from the history of *reported* types into a transfer and allocation rule, given by \((\pi_t, T_t) : \Theta^t \rightarrow \mathbb{R}^2\). Although the date \(t\) allocations depend on the entire history of reported types, we will characterize state space reduction results which will allow us to characterize sufficient statistics for information histories.

### 3.2.3 Incentive Compatibility, Time Consistency, and Information

At every date \(t\), the central bank makes a report \(\tilde{\theta}_t\) of its true type \(\theta_t\), subject to incentive compatibility. We characterize the local incentive compatibility constraints used in solving the relaxed problem. We will apply a *first order approach* to solving the problem.\(^8\) We use incentive compatibility to identify three driving forces of the model: a time consistency problem from the Phillips Curve, and two informational problems related to shock persistence.

Suppose that we define the value function of the central bank by \(W_t(\theta^t)\), where \(\theta^{t-1}\) is the history of *reported* types whereas \(\theta_t\) is always the current true type. The central bank’s reporting problem at date \(t\) is given by

\[
W_t(\theta^t) = \max_{\tilde{\theta}_t} \mathbb{E}_t \left[ \sum_{s=t}^{\infty} \beta^s \left( U_s(\pi_s, \pi_{s+1}, \theta_s, \tilde{\theta}_s) + T_s \right) \right] | \theta_t
\]

where the history-dependence is implicit in the policies \((\pi_s, T_s)\). The current report \(\tilde{\theta}_t\) affects not only the current allocations \((\pi_t, T_t)\), but through history dependence also affects all future allocations. In addition, it also affects firm beliefs \(\tilde{\theta}_t\). Formally, we can write the local

\(^7\)See e.g. Powell (2019).

\(^8\)We make use of techniques developed in Pavan et al. (2014) for a first order approach with persistent shocks. We do not provide or verify a monotonicity condition that can be used to verify that the first order approach yields a globally incentive compatible mechanism.
incentive compatibility condition as
\[
\mathbb{E}_t \sum_{s=t}^{\infty} \left[ \beta^{s-t} \frac{\partial \pi_t}{\partial \theta_t} \right] = -\mathbb{E}_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{\partial U_t}{\partial \pi_t} \frac{\partial \pi_s}{\partial \theta_t} + \frac{\partial U_t}{\partial \pi_{s+1}} \frac{\partial \pi_{s+1}}{\partial \theta_t} \right) + \frac{\partial U_t}{\partial \theta_t} \right]
\]

Local incentive compatibility gives the change in total transfers required to implement an allocation. The change in transfers depends on changes in the allocation rule, which reflect both the contemporaneous utility impact and the impact through the Phillips Curve relationship.

There are three key forces that appear in this decision rule: one reflecting time consistency, and two reflecting information considerations related to persistent shocks.

The first, reflecting time consistency, is the absence of the impact on the previous period’s Phillips Curve. This reflects the Barro and Gordon (1983) time consistency logic, that the central bank neglects the previous period Phillips Curve when determining current inflation. This is the first force that the government must account for when designing the mechanism.

The second force, related to persistent shocks, is that the central bank evaluates expectations with respect to the true type \( \theta_t \), whereas the government designs policy and forms expectations based on the reported types. This creates an informational friction where the central bank has an incentive to manipulate the beliefs of the government, while its own beliefs are held constant.

The third and related force is that an analogous information manipulation incentive exists for firm beliefs. Firms form beliefs based on the reported type, meaning that the central bank can manipulate their actions with the information it reports. This appears through the appearance of inflation expectations of firms in the Phillips Curve relationship, which are formed based on their beliefs. In the conventional New Keynesian framework, the central bank will want to bias firm inflation expectations downward in order to improve the contemporaneous inflation-output trade-off.

These three forces affect not only the central bank’s inflation decision, but also its reporting decision. When designing a mechanism, the government must account for all three of these effects.
3.2.4 Naive Target Adjustment

It is well known that an inflation target is a method of controlling inflationary bias. This proposal arises in static mechanism design problems as a way to enforce efficient inflation policies. Relative to the static setting, however, we feature a dynamic problem of persistent information, where the optimal inflation rate moves substantially over time, and where there are additional informational incentives over the beliefs of firms and the government. As a result, a static target would not be able to achieve the constrained efficient level of inflation, and is unlikely to constitute an optimal mechanism.

However, motivated by the previous literature, a benchmark proposal would be to simply grant the central bank flexibility to adjust its target. This would allow the central bank to accommodate structural shocks that change the inflation-output trade-off in a fundamental way. It is clear, however, that this type of mechanism would simply reintroduce the Barro and Gordon (1983) time consistency problem and undermine the target. The central bank would have an incentive in every period to evaluate its optimal policy \( \frac{\partial u}{\partial \pi} = 0 \), and then to set its inflation target to coincide with that inflation level. Rather than only resetting its target in response to structural shocks, the central bank also resets its target to accommodate its inflationary bias.

However, recognizing that the central bank would set its inflation target each period to coincide with its discretionary policy, firms in the previous period would form inflation expectations that coincided with expected inflation under discretion. In other words, firm inflation expectations are no longer anchored to the current target because they know it will simply be adjusted in the next period to accommodate the inflationary bias. This results in a de-anchoring of inflation expectations. Naive adjustment undermines the target by allowing the central bank to set its target to its desired inflation, rather than the intended goal of forcing inflation to match the target.
3.3 Dynamic Inflation Target

We show that a “dynamic inflation target” mechanism can implement the constrained efficient allocation when the target is set one period in advance.

In particular, equations (3.5) and (3.7), along with their sufficient statistic implications, suggest the following mechanism: use the transfer rule $T_t$ to penalize deviations from a target. In other words, we look over a class of mechanisms defined by the rule

$$-b_{t-1}(\pi_t - \tau_{t-1}),$$

where $\tau_{t-1}$ is the target, and $b_{t-1}$ is the slope of the punishment for inflation in excess of the target. A target of this form seeks to correct the time consistency problem in central bank inflation setting by incentivizing the central bank to set inflation to its target.

However, in the presence of structural shocks, the target might need to be adjusted over time to accommodate a changing efficient level of inflation. A persistent structural shock may drift the optimal inflation rate far from the present target in a persistent manner, resulting in larger gains from letting the target adjust and suggesting a structural change in the commitment-flexibility trade-off.

As a result, in addition to choosing inflation, the central bank must decide each period whether and how to adjust the target. Under our proposed mechanism, the central bank adjusts its target for the following period. It does so by reporting its type $\tilde{\theta}_t = \theta_t$, which generates a mapping $\tilde{\theta}_t \rightarrow (b_t, \tau_t)$ from the history of reported types into the next period target. In practice, we can also think of the central bank as directly choosing its own target, represented by $(b, \tau)$, from among a set of pairs.

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In sum, we propose the following mechanism. In each period, the central bank chooses inflation $\pi_t$ and reports its type to determine the next period target. We show that this “dynamic inflation target” implements the constrained efficient level of inflation in a locally incentive compatible mechanism, which moreover admits a key state space reduction property.
Proposition 23 (Dynamic Inflation Targeting). A dynamic inflation target implements the constrained efficient allocation in a locally incentive compatible mechanism. The mechanism is represented by the Bellman equation

\[
\mathcal{W}(v_{-1}, E_{-1}(\pi), \theta) = \sup_{\pi, \tilde{q}} \left\{ \begin{array}{c}
-\nu_{-1}[\pi - E_{-1}(\pi)] + U\left(\pi, E(\pi_{+1} | v_{-1}, \tilde{\theta}), \theta\right) \\
+ \beta E\left[ \mathcal{W}\left(v(v_{-1}, \tilde{\theta}), E(\pi_{+1} | v_{-1}, \tilde{\theta}), \theta_{+1} \right) | \theta \right] \end{array} \right\}, 
\]

(3.9)

where \(v\) and \(E(\pi_{+1})\) are given by their constrained efficient values. \(v_{-1}\) is a sufficient statistic for the history \(\tilde{\theta}^{-1}\) of reported types.

Proposition 23 shows that in a setting with persistent structural shocks, the optimal mechanism collapses to a simple dynamic inflation target where inflation always meets the target in expectation, that is \(\tau_{-1} = E_{-1}\pi\). The slope of the target mechanism is always given by \(b_{-1} = v_{-1}\).

Our mechanism features a time-varying inflation target, where each period the time consistency in inflation setting is corrected by the target. As in the static model, the marginal welfare cost of future inflation is always through expectations in the Phillips curve, and so is constant on the margin across all future states. As a result, a linear inflation penalty for exceeding the target corrects the time consistency problem. This component of the mechanism is analogous to standard inflation targeting logic (e.g. Walsh (1995)) from the static setting.

However, the inflation target also needs to be adjusted in response to structural shocks in order to allow inflation to coincide with its constrained efficient rule. The key insight of Proposition 23 is that the central bank optimally resets this target one period in advance. In essence, if the central bank observes a persistent shift in the constrained efficient inflation rate from 2% to 3%, the correct response of the central bank is to adjust the inflation target for the next period from 2% to 3%.

Moreover, the current target \((v_{-1}, E_{-1}\pi)\) is a sufficient statistic for the history \(\tilde{\theta}^{-1}\) of
reported types. This sufficient statistic property follows precisely as in the constrained efficient allocation, as the slope of the target \( v_{-1} \) reflects the previous period time consistency problem. This implies that all history dependence of the mechanism is captured in the inflation target, and greatly reduces the knowledge required to adjust the target.

Target adjustment is subject to all three frictions identified above: time consistency, government belief manipulation, and firm belief manipulation. A naive inflation target adjusted contemporaneously resulted in an un-anchoring of inflation expectations, as the central bank could simply set its target to its desired inflation. By requiring instead the target to be set one period in advance, the central bank overcomes this time consistency problem. When setting the target at date \( t - 1 \), the central bank accounts for the current Phillips Curve relationship, overcoming the time consistency problem.\(^9\)

If shocks were not persistent, time consistency would be the only force in the model, and Proposition 23 would still characterize optimal policy. This reflects the logic of the static model, with the additional insight that by setting the target in advance, the time consistency problem in target adjustment is overcome. In this sense, the optimality of the dynamic inflation target directly reflects the logic of the standard time consistency problem. However, if there is no shock persistence and shocks are small, there may be little value to adjustment the target over time, that is little commitment versus flexibility trade-off.

By contrast when shocks are large and persistent, there may be a strong incentive to adjust the target. At the same time, however, two additional incentive problems emerge: government and firm belief manipulation. It is therefore surprising that this simple target and adjustment mechanism remains relevant. In the constrained efficient allocation, these two informational effects exactly offset one another. In particular, when the central bank considers on the margin changing its report \( \tilde{q}_t \), this generates a welfare impact\(^{10}\)

\(^9\)This is similar to the static setting, where the central bank is willing “ex ante” to set up a targeting mechanism for itself. It is also closely related to the literature on optimal mechanisms to control present bias (e.g. Amador et al. (2006)), where agents are willing to set up mechanisms to control their own time consistency problems.

\(^{10}\)See the proof of Proposition 23 for a detailed derivation.
\[
\frac{\partial u}{\partial E(\pi_{t+1} | \tilde{\theta})} dE[\pi_{t+1} | \tilde{\theta}] + \beta \nu \frac{dE[\pi_{t+1} | \tilde{\theta}]}{d\tilde{\theta}}
\]

Under the constrained efficient allocation, we have \(\frac{\partial u}{\partial E(\pi_{t+1} | \tilde{\theta})} + \beta \nu = 0\). In economic terms, on the one hand the central bank wishes to bias inflation expectations of the government upward, which increases the target and reduces penalties for exceeding its target. On the other hand, the central bank also wishes to bias the inflation expectations of firms downward, in order to economize on the Phillips Curve relationship and improve the contemporaneous inflation-output trade-off. At the constrained efficient allocation where the slope of the target is exactly equal to the Phillips Curve impact, the marginal reduction in penalties for exceeding the target is exactly equal to the Phillips Curve impact, and these two forces exactly offset one another.

It is worth remarking two forces exactly cancel each other because central bank and government align at date \(t - 1\) over the future inflation-output trade-off. In fact, cancellation occurs by construction. The central bank at date \(t - 1\) agrees with the government about optimal inflation from dates \(t\) and onward, internalizing the Phillips Curve relationship, but also wishes to manipulate firm beliefs. As a result, the government designs its transfer rule to offset firm belief manipulation. This results in these two informational effects exactly canceling one another. In a more general setup with preference disagreement, the government would also have to account for differences in preferences in controlling the reported type, leading to additional forces in the transfer rule. Moreover, when considering the optimal mechanism when transfers are costly (Section 3.4), counteracting firm belief manipulation will be costly to the government, leading to distortions in the allocation rule relative to constrained efficiency.

**Dynamic Inflation Target as an Optimal Mechanism.** Proposition 23 makes a statement about implementability: a dynamic inflation target can implement the constrained efficient allocation of inflation. It does not state that it is an optimal mechanism. It is immediate that
if we assume the control mechanism has no direct welfare consequence to the government – that is, $T_t$ does not appear in the government objective function – then the dynamic inflation target is also an optimal mechanism, because it maximizes the government’s objective function. This implies that if the government’s primary concern is with the inflation-output trade-off, and not with the costs of implementing the mechanism, the dynamic inflation target is optimal.

### 3.3.1 On the Nature of Inflation Targets

The dynamic inflation target of Proposition 23 sheds light on the important features of the target in anchoring inflation expectations. In particular, the target’s efficacy does not result from it being constant and unchanging, but rather from it being committed to sufficiently in advance. De-anchoring of expectations arises in the naive adjustment process because the central bank is able to match its target to its desired inflation, rather than forcing inflation to match the target. By ensuring target setting in advance, the central bank is forced to set its inflation to its target instead.

### 3.3.2 Target Adjustment in Practice

Proposition 23 provides a mechanism to implement the constrained efficient allocation, which takes the form of a dynamic inflation target. The key property of the target adjustment process is that it occurs one period in advance.

The inflation target adjustment in our model happens at the frequency of the underlying structural shock, in the sense that large movements in the target will generally coincide with large and persistent shock realizations. This suggests that, in practice, target adjustment would either require the new target to either be announced sufficiently far in advance of it taking effect, or the target being adjusted contemporaneously but for a long enough interval of time. This latter method minimizes the time consistency problem by forcing the target to remain in force for a duration of time, so that the central bank is guided primarily by the long-term consequences of the target and not by short-term inflationary bias.
What does a period in advance mean in practice? In the model, the key property of a period is the expectations appearing through the Phillips Curve, which generates the time consistency problem. This relationship arises traditionally because firms adjust prices only infrequently. One notion of a period in advance, then, would be a notion of the duration of firm prices resetting. The target adjustment should happen sufficiently far in the future that a sufficient number of firms will have had the chance to reset prices, in order to minimize time inconsistency. In other words, as the frequency of price adjustment increases, the interval of time constituting a “period” falls.

A related force is the lag between central bank policy setting and its impact on output. As this lag increases, the effective frequency of price resetting also increases from the perspective of time consistency. A longer lag would reduce the interval of time constituting a period.

On the other hand, it is also important to know the duration of structural shifts in the underlying shock. When the frequency of structural shocks is low relative to the frequency of price adjustment, a central bank observing a structural shock does not expect to observe another one until well after firm prices have adjusted. We would therefore expect lower frequency structural shocks to increase the notion of period length.

The notion of the “correct” length of a period must consider all three of these forces. Fundamentally, the trade-off reflects a form of commitment versus flexibility trade-off in target adjustment. On the one hand, granting greater flexibility (more frequent target adjustments) allows responses to structural shocks. On the other hand, greater flexibility also exacerbates the time consistency problem.

Comparing our mechanism to real-world inflation target regimes. A close analogue to our mechanism in practice is the adjustment process for the Bank of Canada.\textsuperscript{11} The Bank of Canada revisits its inflation target every five years at fixed intervals, with the possibility of adjusting its target for the next five-year interval based on new information. Our results

\textsuperscript{11}For background, see https://www.bankofcanada.ca/core-functions/monetary-policy/inflation/.
suggest that this type of adjustment process can implement the efficient level of inflation by providing a means of responding to structural economic shocks without generating substantial time consistency problems in target setting.

### 3.3.3 Relation to Conservative Central Banker

Although inflation targets are one solution to time consistency problems, appointment of a conservative central banker is another possible solution. The linear penalty for inflation around expectations could be understood as a time-varying central banker, where rather than increasing (decreasing) the target, control of the central bank is handed over to a more dovish (hawkish) central banker. Under this interpretation, higher $n_{-1}$ reflects more hawkish central bankers.

This interpretation requires a clearer interpretation of $v^E \pi$. Under this interpretation, $v^E \pi$ is constant except when transitioning between central bankers types $\theta_t$, when it jumps. This suggests that implementing the mechanism in this form requires some notion of scrutiny of a central bank when it decides to transition power.

### 3.4 Second-Best Mechanisms

In Section 3.2, we showed that a dynamic inflation target can implement the constrained efficient level of inflation. It is clear that this mechanism is optimal when the control mechanism is costless to the government. In practice, the control measures may not in fact be costless. If the implementation or enforcement of the mechanism is costly, it is no longer costless for the government to achieve the constrained efficient outcome. In this section, we study the optimal mechanism when enforcement is not costless.

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12See Rogoff (1985).
3.4.1 Model

We capture the social cost of implementing and enforcing a monetary policy mechanism by assuming that transfers are now costly to the government. Social preferences are now

$$\max \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( U_t(\pi_t, \pi_{t+1}, \theta_t, \hat{\theta}_t) - \kappa T_t \right) \right],$$

(3.10)

where $\kappa \geq 0$ captures the importance of this social cost.\(^{13}\)

In previous sections, the costless nature of the mechanism implied participation constraints were irrelevant. We now introduce a date 0 participation constraint, given by

$$\mathcal{W}_0 \geq 0.$$  \hspace{1cm} (3.11)

Without loss of generality, we have normalized the outside option in the participation constraint to 0.\(^{14}\) Recall that a mechanism is a mapping $(\pi_t, T_t) : \Omega^t \to \mathbb{R}^2$.

We now characterize local incentive compatibility from the Envelope Theorem, in order to solve the problem using a first-order approach that relaxes the required monotonicity constraint.\(^{15}\) The Bellman representation of the central bank’s problem is

$$\mathcal{W}_t(\theta_t) = \sup_{\hat{\theta}_t} \left\{ U_t(\pi_t, \pi_{t+1}, \theta_t, \hat{\theta}_t) + T_t + \beta \mathbb{E}_t [\mathcal{W}_{t+1} | \theta_t] \right\}$$

(3.12)

As before, $\pi_t, T_t$ are functions of the history of reported types, not true types, while $\mathcal{W}_t$ is a function of both past reported types and the current true type.

We adopt the notational convention that $\frac{\partial U_t}{\partial \theta_t}$ is the derivative of $U_t$ in the true type $\theta_t$, while $\frac{\partial U_t}{\partial \pi_t}$ is the derivative in the reported type. In equilibrium, the reported type and the true type coincide, but the derivatives are distinct. Applying the Envelope Theorem and

\(^{13}\)This corresponds to a standard (quasilinear) transferable utility model. As usual, $T_t$ may also correspond to non-quasilinear utilities, provided they are transferable in this form.

\(^{14}\)If instead the outside option where $\mathcal{W}_0$, the mechanism we derive still applies, except that the period-0 transfer would be adjusted by $\mathcal{W}_0$ for all types.

\(^{15}\)See Pavan et al. (2014). We do not provide results on when the first order approach yields a globally incentive compatible mechanism.
integrating over types, we obtain an integral incentive compatibility condition

$$W_t(\theta^t) = \int_{\mathbb{E}} \partial U_t(\theta^{t-1}, s_t) ds_t + \beta \int_{\mathbb{E}} \mathbb{E}_t \left[ W_{t+1} \frac{\partial f(\theta_{t+1}|s_t)/\partial s_t}{f(\theta_{t+1}|s_t)} | s_t \right] ds_t$$  \hspace{1cm} (3.13)$$

Integral incentive compatibility relates the total date-\(t\) utility to the central bank to two information rents. Note that due to shock persistence, the central bank earns information rents not only due to the effect on current flow utility, but also on the conditional probability distribution.  \(^{16}\)

Integral incentive compatibility (3.13) gives a Bellman representation to the value function, in terms of only the allocation rule. We can re-express this Bellman equation in sequence form by iterating the Bellman equation forward. Doing so, we obtain the following result characterizing this sequence representation.

**Lemma 24.** The value function \(W_t\) can be represented as

$$W_t(\theta^t) = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s B_t^s(\theta^{t+s}) \mid \theta_t \right] \quad \forall t,$$

where \(B_t^s\) is given by

$$B_t^s(\theta^{t+s}) = \int_{s_t \leq \theta_t, \ldots, s_{t+s} \leq \theta_{t+s}} \frac{\partial U_t(\theta^{t-1}, s_t, \ldots, s_{t+s})}{\partial s_{t+s}} \prod_{k=0}^{s-1} \frac{\partial f(\theta_{t+k+1}|s_{t+k})/\partial s_{t+k}}{f(\theta_{t+k+1}|\theta_{t+k})} ds_{t+s} \ldots ds_t.$$

Lemma 24 allows us to represent the principal’s optimization problem in a tractable way. Given an allocation rule for inflation, we use the characterization of the value function in Lemma 24 as well as the Bellman equation to characterize the transfer rule which implements the allocation,

$$T_t = W_t - U_t - \beta \mathbb{E}_t[W_{t+1}|\theta_t].$$

We can then substitute the implementing taxes into the government’s utility function, and obtain the following result characterizing the relaxed social planning problem.

\(^{16}\)Without loss of generality, we have set the constant of integration \(W_t(\theta^{t-1}, \emptyset) = 0\) and have used the outside option normalization \(W_0(\emptyset) = 0\).
Lemma 25. The relaxed social planning problem can be written as
\[
\max_{\pi_t} \mathbb{E}_{-1} \left[ \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\kappa}{1+\kappa} B_0^t + U_t \right] \right],
\]
where $B_0^t$ is given as in Lemma 24. The implementing transfer rule is given by
\[
T_t = W_t - U_t - \beta \mathbb{E}_t [W_{t+1} | \theta_t],
\]
where $W_t$ is given as a function of the allocation rule as in Lemma 24.

3.4.2 The Second-Best Allocation Rule

Lemma 25 provides a characterization of the relaxed social planning problem, subject to integral incentive compatibility. From this, we can characterize the optimal allocation.\(^{17}\)

Proposition 26. The solution to the optimal allocation rule is given by the first-order conditions
\[
\beta \frac{\partial U_t}{\partial \pi_t} f(\theta_t | \theta_{t-1}) = -\frac{\partial U_{t-1}}{\partial \pi_t} \left[ \text{Time Consistency} \right]
\]
\[
+ \frac{\kappa}{1+\kappa} \left[ \Gamma_{t-1}(\theta^{t-1}) \frac{\partial^2 U_{t-1}}{\partial \theta_{t-1} \partial \pi_t} + \beta \Gamma_t(\theta^t) \frac{\partial^2 U_t}{\partial \theta_t \partial \pi_t} f(\theta_t | \theta_{t-1}) \right] \quad \forall t \geq 1
\]
\[
\frac{\partial U_0}{\partial \pi_0} = \frac{\kappa}{1+\kappa} \Gamma_0(\theta^0) \frac{\partial^2 U_0}{\partial \theta_0 \partial \pi_0}
\]
where $\Gamma_t(\theta^t)$ is given by the recursive sequence
\[
\Gamma_t(\theta^t) = \Gamma_{t-1}(\theta^{t-1}) \frac{1-F(\theta^t | \theta_{t-1})}{f(\theta_t | \theta_{t-1})} \mathbb{E}_{t-1} \left[ \left. \frac{\partial f(s_t | \theta_{t-1})}{f(s_t | \theta_{t-1})} \right| s_t \geq \theta_t \right] \quad \text{(3.14)}
\]
with initial condition $\Gamma_0(\theta^0) = \frac{1-F(\theta_0)}{f(\theta_0)}$.

There are three effects that cause the allocation rule to deviate from simple optimization of

\(^{17}\)We characterize the optimal allocation assuming that $\pi_t$ is interior.
flow utility, \( \frac{\partial U_t}{\partial \pi_t} = 0 \), which would be the decision of the central bank if left uncontrolled by the government.

The first effect is a pure time consistency effect, reflected by the term \( -\frac{\partial U_{t-1}}{\partial \pi_t} \). This effect arises because period-\( t \) inflation appears in period \( t - 1 \) flow utility, due to the Phillips curve relationship. This effect would exist even in the absence of persistent shocks and costly transfers. If transfers are costless or information is not persistent, then it is the only effect, and we return to the characterization in Section 3.2.

If transfers were not costly (\( k = 0 \)), the optimal allocation would be the constrained efficient allocation, and the dynamic inflation target would be the optimal mechanism. However, there are two effects arising from the fact that transfers are now costly. These effects reflect both the persistence of information and the time consistency problem. Because the central bank earns an information rent, implementing a given allocation rule is now costly. Suppose that on the margin, the government changes the allocation \( \pi_t(\theta^t) \), holding all else fixed. This affects the required transfer at two points. First, it affects the required transfer in period \( t \), which is related to the cross partial \( \frac{\partial^2 U_t}{\partial q_t \partial \pi_t} \) owing to the Envelope Theorem characterization of transfers. Supposing that \( \frac{\partial^2 U_t}{\partial q_t \partial \pi_t} > 0 \) and that \( \Gamma_t > 0 \), then \( \frac{\partial^2 U_t}{\partial q_t \partial \pi_t} > 0 \) implies that an increase in \( \pi_t \) increases \( \frac{\partial U_t}{\partial \pi_t} > 0 \), which in turn increases the required transfer, generating a cost for the government. This is a standard effect that exists even without a time consistency problem.

However, changes in \( \pi_t \) also affect the required transfers at date \( t - 1 \), due to the Phillips curve relationship. Because \( \pi_t \) affects \( U_{t-1} \), we have the analogous property applying. In particular if \( \Gamma_{t-1} > 0 \) and \( \frac{\partial^2 U_{t-1}}{\partial q_{t-1} \partial \pi_{t-1}} \), then the effect is analogous. An increase in \( \pi_t \) causes an increase in \( \frac{\partial U_{t-1}}{\partial \pi_{t-1}} \), which increases the required transfer. In other words, this second effect is an interaction between the time consistency problem and the information persistence problem.

In either case, note that when the RHS is positive, we have \( \frac{\partial U_{t-1}}{\partial \pi_t} + \rho \frac{\partial U_t}{\partial \pi_t} f(\theta_t|\theta_{t-1}) > 0 \). This implies that from the social perspective (accounting for time consistency), it would be desirable to increase \( \pi_t \) further to reach the constrained efficient allocation. However, it is
costly to provide the correct incentive scheme to do so, and as a result the time consistency problem is not fully corrected. Costly enforcement reduces the ability to combat the time consistency problem. Notice that this implies that the inflation target is strict than the constrained efficient allocation.

Lastly, the initial condition \( \Gamma_0(\theta^0) = \frac{1-F(\theta_0)}{f(\theta_0)} \) implies a conventional first-order condition for period-0 inflation

\[
\frac{\partial U_0}{\partial \pi_0} - \frac{\kappa}{1+\kappa} \Gamma_0(\theta^0) \frac{\partial^2 U_0}{\partial \theta_0 \partial \pi_0} = 0
\]

This reflects a standard virtual value relationship. In our model, the component \((1 - F(\theta_0))/f(\theta_0) > 0\) of virtual value is relevant in both period 1 and in period 0, due to the time consistency problem. Moreover, this value \((1 - F(\theta_0))/f(\theta_0) > 0\) is persistent over time through the evolution of \( \Gamma_1 \). In other words, if it is large initially, then \( \Gamma_1 \) is also larger in magnitude along every history.

It should be noted that the constrained efficient allocation is still implementable, but is no longer optimal. In particular, at the constrained efficient allocation, the informational effects still exactly offset one another, and a dynamic inflation target still implements that allocation. However, at the constrained efficient allocation the marginal benefit of higher inflation is zero, while the marginal cost of the enforcement mechanism is not. As a result, the government no longer finds it optimal to implement the constrained efficient allocation that would have arisen under zero costs. This helps to understand why the optimal mechanism here deviates from a precise dynamic inflation target: once moved away from the constrained efficient allocation, the informational effect on firms no longer exactly offsets the informational effect on government transfers.

Nevertheless, the properties of the optimal mechanism still bear similarities to the dynamic inflation target. In particular, the time consistency term is still a constant given the Phillips curve relationship, generating a component of the optimal mechanism that resembles the dynamic target.
3.4.3 Sufficient Statistics for the Optimal Mechanism

Although in principle we are now solving a dynamic mechanism design problem with persistence and a time consistency problem, the allocation rule derived in Proposition 26 implies a sufficient statistics result for characterizing the optimal allocation rule. In particular, at date $t$, we need to know two sufficient statistics for past information in order to derive the allocation rule.

The first sufficient statistic is the function $\Gamma_t(\theta^t)$, which itself has two sufficient statistics - the constants $\Gamma_{t-1}$ and $\theta_{t-1}$. This first pair of sufficient statistic summarizes the information relevant from the fact that shocks are persistent over time. It is separate from the time consistency problem, and would exist even if there were no time consistency problem. This sufficient statistic is irrelevant, however, in the absence of persistent information. In other words, if there is time consistency problems but idiosyncratic shocks, we do not need to keep track of this information.

The second sufficient statistic is a “wedge” $v_{t-1}$, which is a date $t-1$ adapted constant given by

$$v_{t-1} = \frac{1}{\beta} \left[ \frac{\partial U_{t-1}}{\partial \pi_t} - \kappa \Gamma_{t-1} \frac{\partial^2 U_{t-1}}{\partial \theta_{t-1} \partial \pi_t} \right]$$

$v_{t-1}$ is a sufficient statistic for the time consistency problem, including the interaction between the time consistency problem and the information persistence problem. $v_{t-1}$ would equal zero in absence of the time consistency problem, whether or not there were persistent shocks.

Using this triple $(\Gamma_{t-1}, \theta_{t-1}, v_{t-1})$, we can solve for the optimal allocation from date $t$ onward. From there, we can use the central bank value function to back out the required transfer rule. As a result, this triple are sufficient statistics for the shock history $\theta^{t-1}$ under the optimal mechanism. This fact is summarized in the following corollary to Proposition 26.

**Corollary 27.** The triple $(v_{t-1}, \theta_{t-1}, \Gamma_{t-1})$ of date $t-1$ adapted constants is a sufficient statistic.
for the history of shocks $\theta^{t-1}$ at date $t$. The allocation rule can be written as

$$
\frac{\partial u_t}{\partial \pi_t} = v_{t-1} + \frac{\kappa}{1 + \kappa} \Gamma_t \frac{\partial^2 u_t}{\partial \theta_t \partial \pi_t} f(\theta_t | \theta_{t-1}) \quad \forall t \geq 1
$$

(3.16)

Corollary 27 further helps to understand how this mechanism resembles a dynamic inflation target. Over time, the time consistency term $v_{t-1}$, which is a constant from a date $t$ perspective, evolves in response to shocks. Smaller slopes imply lower inflation penalties and higher average inflation, whereas larger slopes imply larger penalties and lower inflation. In this sense, the slope $v_{t-1}$ now tells us information about the nature of the target.

3.4.4 Reversion to Dynamic Inflation Target

Proposition 26 tells us that the optimal mechanism reverts to a dynamic inflation target at both extremes of the shock distribution. In other words, there is both a no top distortion and a no bottom distortion result: if at period $t$ we have $\theta_t = \overline{\theta}$ or $\theta_t = \underline{\theta}$, then $\Gamma_t = 0$, and hence $\Gamma_{t+s} = 0 \quad \forall t \geq s$. This implies that the entire costly transfer term disappears, at which point the optimal mechanism reverts to the dynamic inflation target.

**Corollary 28.** If $\theta_t \in \{\overline{\theta}, \underline{\theta}\}$, then $\Gamma_{t+s} = 0 \quad \forall s \geq 0$. $v_{t+s-1}$ is a sufficient statistic at date $t + s \quad \forall s \geq 0$. The optimal allocation at dates $t + s$ ($s \geq 1$) is implemented by a dynamic inflation target (as in Proposition 23).

The no top distortion result closely resembles normal top distortion results in the absence of time consistency. The normal result states that after $\theta_t = \overline{\theta}$, we revert to the optimal allocation rule $\frac{\partial u_{t+s}}{\partial \pi_{t+s}} = 0 \quad \forall s \geq 0$. In our model, the optimal allocation rule instead satisfies $\frac{\partial u_{t+s-1}}{\partial \pi_{t+s}} + \kappa \frac{\partial u_{t+s}}{\partial \pi_{t+s}} f(\theta_{t+s} | \theta_{t+s-1}) = 0$, due to the Phillips curve. No top distortion reverts the economy to this efficient relationship. However, this efficient relationship is still distorted by the time consistency problem, when the central bank is left to set inflation in an uncontrolled manner. As a result, the wedge $v_{t+s}$ are still relevant, even after reaching a state $\theta_t = \overline{\theta}$. 

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Moreover due to information persistence, we also have a no distortion at the bottom result.\textsuperscript{18}

As a result, not only does the optimal mechanism have a component that resembles the dynamic inflation target throughout the shock distribution, but it reverts fully to the dynamic inflation target at the limits of the distribution.

### 3.4.5 Second best with Average Transfers

In the baseline model, we impose the assumption that the outside option takes the form $W_0(\theta^0) \geq 0$. We might alternatively have expressed this in the form

$$\int_{\theta_0} W_0(\theta^0) f(\theta_0|\theta_{-1}) d\theta_0 \geq 0$$

The core difference between these two assumptions from a modeling perspective is on the timing of information arrival versus the participation decision. Under the baseline assumption, either $\theta_0$ is already known to the central bank, or the central bank has the opportunity to revert to the outside option after learning $\theta_0$. Under the second assumption, $\theta_0$ is not known to the central bank, and the central bank does not have the option to revert to the outside option after learning it.

Under this alternative structure, the optimality of the dynamic inflation target returns. In particular, implementable allocations are still defined as in Lemma 24, while the transfer rule is $T_t(\theta^t) = W_t - U_t - \beta \mathbb{E}_t [W_{t+1} | \theta_t]$. The average participation constraint implies that we have

$$0 = \mathbb{E}_{-1} W_0 = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t (U_t + T_t),$$

which is markedly different from the baseline model. In particular, substituting this expression into social welfare, we obtain the social optimization problem

$$\max_{\{\pi_t\}} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t (1 + \kappa) U_t$$

implying that the optimal allocation rule is constrained efficient. From here, we obtain the optimality of the dynamic inflation target.

\textsuperscript{18}See Pavan et al. (2014) for related results.
**Proposition 29.** Suppose that the participation constraint takes the form

\[
\int_{\theta_0} W_0(\theta^0) f(\theta_0|\theta_{-1}) d\theta_0 \geq 0
\]

Then, the optimal mechanism is a dynamic inflation target, and yields the constrained efficient allocation.

The intuition behind Proposition 26 is straight-forward: under the average constraint, the government can capture the full social surplus and simply reduce the average transfer to the central bank at date 0 to satisfy the participation constraint. This implies that the government chooses the mechanism and allocation that maximize social surplus, which is the dynamic inflation target.

### 3.4.6 Discussion

The results of this suggestion imply that even under costly enforcement (transfers), the optimal mechanism has properties that resemble a dynamic inflation target, in particular the time-varying slope \( v_{t-1} \) associated with the time consistency problem. Moreover, the optimal mechanism reverts to the dynamic inflation target at the extremes of the shock distribution.

Taken together, if policymakers believe that the costs of enforcement are second-order next to the social welfare inflation-output trade-off, or believe that the target adjustment to (e.g.) 4% is a movement close to the upper limit of the distribution, then adopting a simple dynamic inflation target may be a good approximation to the true optimal mechanism. Moreover, the dynamic inflation target can always be used to implement the constrained efficient allocation, even if these statements are not true.

### 3.5 Conclusion

We provide a theory of dynamic inflation targets. We show that a dynamic inflation targeting mechanism can implement the constrained efficient level of inflation, and that
when enforcement costs of using the mechanism are negligible the dynamic inflation target is the optimal mechanism. The key property of the dynamic inflation target is that it is updated *one period in advance*, which mitigates the time consistency problem in target setting. This suggests that the key property of target efficacy is advanced commitment. The results provide guidance on the mechanism by which to adjust an inflation target without exacerbating the underlying time consistency problem, suggesting that a mechanism of adjustment at restricted points in time, for example every five years as with the Bank of Canada, would be the desirable adjustment method.
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Appendix A

Appendix to Chapter 1

A.1 General Framework: Non-Cooperative

This Appendix presents the results of the non-cooperative problem in the general framework of Section 1.7.

A.1.1 Non-Cooperative Setup

The setup of the non-cooperative problem is analogous to the setup in the baseline model. Country planner $i$ maximizes the welfare of domestic agents using a complete set of wedges $\tau_{i,i} = \{\tau_{i,j}(m)\}_{j,m}$ on the actions of domestic agents, and wedges $\tau_{i,j} = \{\tau_{i,j}(m)\}_m$ on domestic actions of foreign agents. The total tax burden faced by country $i$ agents from the domestic planner (excluding remissions) is therefore $T_{i,i} = \tau_{i,i}a_{i,i} + \int_j \tau_{i,j}a_{i,j}dj$, while the total tax burden from foreign planner $j$ is given by $T_{j,ij} = \tau_{j,ij}a_{i,ij}$. These taxes appear in the wealth of the multinational agent.

As in the baseline model, under quantity regulation wedges are revenue-neutral, while under Pigouvian taxation wedges generate revenues from foreign banks.

Implementability: As in the baseline model, the approach to implementability is standard for domestic agents. Moreover, an implementability result analogous to Lemma 4 holds in
the general environment, allowing us to apply the standard approach for domestic actions of foreign agents.

**Lemma 30.** The domestic actions of foreign agents can be chosen by the domestic planner, with implementing wedges

\[ t_{ij}(m) = \tau_{ij}(m) + \frac{1}{\lambda_j^0} \left[ \omega_j^i \frac{\partial U_j}{\partial a_{ji}(m)} \frac{\partial u_{ji}}{\partial a_{ji}(m)} + \omega_j^i \frac{\partial U_j^A}{\partial a_{ji}(m)} \frac{\partial u_{ji}^A}{\partial a_{ji}(m)} + \Lambda_j \frac{\partial \Gamma_j}{\partial \phi_j} \frac{\partial \phi_{ji}}{\partial a_{ji}(m)} + \Lambda_j^i \frac{\partial \Gamma_j^i}{\partial \phi_j^i} \frac{\partial \phi_{ji}^i}{\partial a_{ji}(m)} \right] \]

where \( t_{ij}, \lambda_j^0, \omega_j^i, \frac{\partial U_j}{\partial a_{ji}(m)}, \frac{\partial u_{ji}}{\partial a_{ji}(m)}, \frac{\partial U_j^A}{\partial a_{ji}(m)}, \frac{\partial u_{ji}^A}{\partial a_{ji}(m)}, \frac{\partial \Gamma_j}{\partial \phi_j}, \) and \( \frac{\partial \Gamma_j^i}{\partial \phi_j^i} \) are constants from the perspective of country planner \( i \).

The intuition behind these implementability conditions is analogous to the baseline model: the planner first unwinds the wedge placed by the foreign planner, and then sets the residual wedge equal to the benefit to foreign agents of conducting that activity.

### A.1.2 Non-Cooperative Quantity Regulation

We now characterize the non-cooperative equilibrium under quantity regulation, where wedges are revenue neutral. We obtain the following characterization of the non-cooperative equilibrium.

**Proposition 31.** Under non-cooperative quantity regulation, the equilibrium has the following features.

1. The domestic wedges on domestic activities of domestic agents are

\[ \tau_{ii}(m) = -\Omega_{ii}(m) \]

while the domestic wedges on foreign activities of domestic agents are 0.

2. The domestic wedges on foreign banks generate an allocation rule

\[ \Omega_{ij}(m)a_{ji}(m) = 0 \]
Proposition 31 reflects logic closely related to the baseline model. On the one hand, regulatory policies applied to domestic agents account for spillovers to domestic agents, but not to foreign agents. On the other hand, regulatory policies applied to foreign agents’ domestic activities do not account for benefits to foreign agents of domestic activities. Foreign agents are allowed to conduct domestic activities only to the extent the domestic benefits of those activities outweigh domestic costs.

This characterization leads to a generic inefficiency result in the presence of cross-border activities. We say that there are cross border activities if \( \exists M' \subset M \) and \( I, J \subset [0,1] \) such that \( a_{ii}(m), a_{ji}(m) > 0 \forall m \in M', i \in I, j \in J. \)

**Proposition 32.** Suppose that a globally efficient allocation features cross border activities over a triple \((M', I, J)\). The non-cooperative equilibrium under quantity regulation generates this globally efficient allocation only if the globally efficient allocation features \( \Omega_{i,i}(m) = \int_{i'} \Omega_{i,i'}(m)di' = 0 \forall m \in M', i \in I. \)

Proposition 32 provides a strong and generic result that quantity regulation does not generate an efficient allocation when there are regulated cross-border activities. In particular, cross-border activities must generate no net domestic externality to avoid the problem of unequal treatment, and cross-border activities must generate no net foreign externalities to avoid the problem of uninternalized foreign spillovers. Notice that efficient under Proposition 32 requires no regulation of cross-border activities in the globally efficient policy.

### A.1.3 Non-Cooperative Pigouvian Taxation

Finally, we characterize non-cooperative Pigouvian taxation and its optimality.

**Proposition 33.** Suppose Assumption 10 holds. The equilibrium under non-cooperative Pigouvian taxation has the following features.
1. The domestic wedges on domestic activities of domestic agents are

\[ \tau_{i,ii}(m) = -\Omega_{i,i}(m) - \int_j \Omega_{j,ii}(m) dj \]

while domestic wedges on foreign activities of domestic agents are 0.

2. The domestic wedges on domestic activities of foreign agents are

\[ \tau_{i,ji}(m) = \tau_{i,ji}(m) - \frac{\partial \tau_{i,ji}}{\partial a_{ji}(m)} a_{ji} \]

As in the baseline model, the derivatives of foreign tax revenue in domestic liquidation prices yield the foreign spillovers, so that planners account for these effects in designing policy. However, revenue collection generates a monopolistic distortion. The generalized problem therefore reflects the same logic as Proposition 6, with the only difference being the nature of the spillovers and of the monopolistic distortion. As in the baseline model, when this monopolist distortion is zero, non-cooperative Pigouvian taxation results in a globally efficient allocation.

**Proposition 34.** Suppose Assumption 10 holds, and suppose that \( u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A \) are linear in \( a_{ij} \) (given \( a_{ij}^A \)) for all \( i \) and \( j \neq i \). Then the non-cooperative equilibrium under taxation is globally efficient, and there is no scope for cooperation.

**Proof.** Observe that when \( u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A \) are linear in \( a_{ij} \) (given \( a_{ij}^A \)) for all \( i \) and \( j \neq i \), then the non-cooperative tax rates align with the cooperative ones, resulting in an efficient allocation.

Non-cooperative taxation is globally efficient if Assumption 10 holds, and if the monopolistic distortions are zero. The assumption of linearity on \( u_{ij}, \phi_{ij}, u_{ij}^A, \phi_{ij}^A \) ensures that (partial equilibrium) elasticities of foreign activities with respect to tax rates are infinite, so that monopolistic distortions are zero. This reflects the same notion of sufficient substitutability as in the baseline model, and generalizes Proposition 7 to a broader class of problems.
As in the baseline model, Proposition 34 provides a limiting case of exactly efficiency. Comparing Propositions 33 and 9 reveals that even without exact efficiency, there are three appealing properties of Pigouvian taxation. The first is that the need for cooperation is restricted to foreign activities of multinational agents. The second is that cooperation is needed only to correct bilateral monopolist problems. The third is that the information needed to determine the magnitude of these problems is a set of partial equilibrium elasticities. This provides a potential method to evaluate the need for cooperation in practice.

A.2 Proofs

Proofs from the baseline model (Sections 1.3-1.5) and Section 1.7 are contained in this appendix. Proofs from the bailouts model (Section 1.6) are contained in Appendix A.3.

A.2.1 Proofs of Baseline Model

Propositions 3, 5, 6, and 7 as well as Lemma 4, are applications of the corresponding propositions in Section 1.7.\(^1\)

The proof of Lemma 2 is given below.

\(^1\)For the proof of Proposition 6, one might be concerned that the planner’s objective function becomes linear in foreign investment when \(\partial^2 \Phi_{ji} / \partial I_{ji} = 0\). This problem is easily addressed by introducing curvature elsewhere into the problem, for example by incorporating a local capital price (Appendix A.4.4) or by incorporating a small utility spillover to the domestic economy from total investment scale. Such spillovers would satisfy Assumption 10.
Proof of Lemma 2

The bank Lagrangian is

\[ L_i = \int_s c_i(s) f(s) ds + \lambda_i^0 \left[ A_i + D_i - \Phi_{ii}(I_{ii}) - \int_j \Phi_{ij}(I_{ij}) dj \right] + \int_s \lambda_i^1(s) \left[ \gamma_i(s) L_{ii}(s) + (1 + r_{ii})(R_i(s) I_{ii} - L_{ii}(s)) \right. \\
+ \int_j \left[ \gamma_{ij}(s) L_{ij}(s) + (1 + r_{ij})(R_j(s) I_{ij} - L_{ij}(s)) \right] dj - c_i(s) - D_i \right] f(s) ds \\
+ \int_s \Lambda_i^1(s) \left[ - D_i + \gamma_i(s) L_{ii}(s) + \int_j \gamma_{ij}(s) L_{ij}(s) dj + (1 - h_i(s)) C_{ii}(s) \right. \\
+ \int_j (1 - h_j(s)) C_{ij}(s) dj \right] f(s) ds \\
+ \int_s \left[ \xi_{ij}(s) L_{ij}(s) + \xi_{ii}(s) (R_i I_{ii} - L_{ii}(s)) \right. \\
+ \int_j \left( \xi_{ij}(s) L_{ij}(s) + \xi_{ij}(s) (R_j I_{ij} - L_{ij}(s)) \right) \right] f(s) ds \\
\]

where we recall that \( C_{ij}(s) = \gamma_j(s) \left[ R_j(s) I_{ij} - L_{ij}(s) \right]. \)

**FOC for \( I_{ij} \)**

Taking the first order condition in \( I_{ij} \), we obtain

\[ 0 \geq -\lambda_i^0 \frac{\partial \Phi_{ij}}{\partial I_{ij}} + E \left[ \lambda_i^1(s)(1 + r_{ij}) R_j \right] + E \left[ \Lambda_i^1(s)(1 - h_j) \gamma_j R_j \right] + E \left[ \xi_{ij}(s) R_j \right]. \]

Expanding the first expectation, we obtain the result.

**FOC for \( L_{ij}(s) \)**

Taking the first order condition in \( L_{ij}(s) \), we obtain

\[ 0 = \left[ \lambda_i^1(s)(\gamma_j(s) - (1 + r_{ij}))(\gamma_j(s) - (1 + r_{ij})) + \Lambda_i^1(s)(\gamma_j(s) - (1 - h_j(s)) \gamma_j(s)) + \xi_{ij}(s) - \xi_{ij}(s) \right] f(s) \]

which yields the result.
A.2.2 Section 1.7 Proofs

Proof of Proposition 9

The Lagrangian of the global planner is

\[ \mathcal{L}^G = \int \left[ \omega_i U_i \left( u_i(a_i), u_i^A(a_i, a^A) \right) + \Lambda_i \Gamma_i \left( A_i + T_i, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \right] di - \lambda^0 \int \mathcal{T}_i di. \]

From here, we have

\[ \frac{d\mathcal{L}^G}{d\alpha_{ij}(m)} = \frac{\partial \mathcal{L}_i}{\partial \alpha_{ij}(m)} + \frac{\partial \mathcal{L}_j}{\partial a^A_j(m)} + \int \frac{\partial \mathcal{L}_{ij}'}{\partial a^A_{ij}(m)} d\alpha_{ij}'. \]

so that we obtain the required wedge

\[ \tau_{ij}(m) = -\frac{1}{\lambda^0} \left[ \frac{\partial \mathcal{L}_j}{\partial a^A_j(m)} + \int \frac{\partial \mathcal{L}_{ij}'}{\partial a^A_{ij}(m)} d\alpha_{ij}' \right] \]

where we define \( \lambda^0_i \equiv \Lambda_i \frac{\partial \mathcal{L}_i}{\partial\mathcal{W}_i}. \) Next, by the Envelope Theorem we can characterize the derivative

\[ \frac{\partial \mathcal{L}_i}{\partial a^A_i(m)} = \omega_i \frac{\partial U_i}{\partial a^A_i(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_i^A}{\partial a^A_i(m)}. \]

Finally, defining \( \Omega_{ij}(m) = \frac{1}{\lambda^0} \frac{\partial \mathcal{L}_i}{\partial a^A_i(m)} \) and using that \( \lambda^0 = \lambda^0_i \) (from the FOC for \( \mathcal{T}_i \), we obtain

\[ \tau_{ij}(m) = -\Omega_{ij}(m) - \int \Omega_{ij}'(m) d\alpha_{ij}'. \]

giving the result.

Proof of Lemma 30

Taking the Lagrangian of bank \( i \)

\[ \mathcal{L}_i = \omega_i U_i \left( u_i(a_i), u_i^A(a_i, a^A) \right) + \Lambda_i \Gamma_i \left( A_i - T_i, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \]
and taking the first order condition in \( a_{ij}(m) \), we obtain

\[
0 = \omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_i}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \phi_i^A}{\partial a_{ij}(m)}.
\]

Defining \( \lambda_i^0 = \Lambda_i \frac{\partial \phi_i}{\partial a_{ij}(m)} \) and rearranging, we obtain

\[
\tau_{i,j}(m) = -\tau_{i,j}(m) + \frac{1}{\lambda_i} \left[ \omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i}{\partial u_i^A} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_i}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \phi_i^A}{\partial a_{ij}(m)} \right]
\]

giving the relevant equation. From here, notice that the allocations \( a_{ij} \) and aggregates \( a_i^A \) appear to first order only in the tax rate equations in country \( j \). As a result, considering any candidate equilibrium, the first order conditions for optimality for allocations by country \( i \) banks outside of country \( j \) are not affected (to first order) by policies in country \( j \), and so continue to hold independent of \( a_{ij} \) and \( a_i^A \). As a result, any allocation \( a_{ij} \) can be implemented with the above tax rates. The implementability result follows.

**Proof of Proposition 31**

Substituting in the equilibrium tax revenue, the optimization problem of the country \( i \) social planner is

\[
\max_{a_i, \{a_j\}} \omega_i U_i \left( u_i(a_i), u_i^A(a_i, a^A) \right) \quad \text{s.t.} \quad \Gamma_i \left( A_i - \int_j \tau_{i,j} a_{ij} d_j, \phi_i(a_i), \phi_i^A(a_i, a^A) \right) \geq 0
\]

and subject to the implementability conditions of Lemma 30. Note that the wedges rates \( \tau_{i,j} \) do not appear except in the implementability conditions, meaning that they are set to clear implementability but do not contribute to welfare. As a result, the Lagrange multipliers on implementability are 0, and the Lagrangian of planner \( i \) is given by

\[
\mathcal{L}_{SP}^i = \omega_i U_i \left( u_i(a_i), u_i^A(a_i, a^A) \right) + \Lambda_i \Gamma_i \left( A_i - \int_j \tau_{i,j} a_{ij} d_j, \phi_i(a_i), \phi_i^A(a_i, a^A) \right).
\]

First of all, note that the social planner does not internalize impacts on foreign aggregates. As a result, \( \frac{d\mathcal{L}_{SP}^i}{da_{ij}(m)} = \frac{\partial \mathcal{L}_{SP}^i}{da_{ij}(m)} \). Social and private preferences align, and therefore we have
\( \tau_{i,ij}(m) = 0. \)

Next, consider a domestic policy \( a_{ii}(m) \). Here, we have \( \frac{\partial L^{SP}_{ii}}{\partial a_{ii}(m)} = \frac{\partial L^{SP}_{ii}}{\partial a_{ii}^A(m)} + \frac{\partial L^{SP}_{ii}}{\partial a_{ii}^A(m)} \). To align preferences, the domestic planner therefore sets

\[
\tau_{i,ii}(m) = -\frac{1}{\lambda_i^0} \frac{\partial L^P_i}{\partial a^A_i(m)} = -\Omega_{i,i}(m)
\]

where the final equality follows as in the proof of Proposition 9.

Finally, consider \( a_{ji}(m) \). Here, we have

\[
\frac{\partial L^{SP}_{ji}}{\partial a_{ji}(m)} = \frac{\partial L^{SP}_{ji}}{\partial a_{ji}^A(m)} = \lambda_i^0 \Omega_{i,j}(m),
\]
giving the allocation rule.

**Proof of Proposition 32**

Given a globally efficient allocation with cross border activities over \((M', I, J)\), suppose that the non-cooperative equilibrium under quantity regulation generates this allocation. From Proposition 31, \( a_{ji}(m) > 0 \) implies that \( \Omega_{ji}(m) = 0 \) over \((M, I, J)\). Using Propositions 9 and 31, \( \tau_{i,ii}(m) = \tau_{ii}(m) \) and \( \Omega_{i,j}(m) = 0 \) implies that \( \int_I \Omega_{i',j}(m) dt' = 0 \) over \((M, I, J)\), completing the proof.

**Proof of Proposition 33**

It is helpful to begin by characterizing the derivative of revenue from foreign agents in the domestic aggregate. Using the implementability conditions of Lemma 30, the revenue collected by planner \( j \) from country \( i \) agents is

\[
T_{j,ij}^s = -\tau_{i,ij} a_{ij} + \frac{1}{\lambda_i^0} \left[ \omega_i \frac{\partial U_i}{\partial a_{ij}} u^A_{ij} + \omega_i \frac{\partial U_i}{\partial a_{ij}^A} u_{ij}^A + \lambda_i \frac{\partial \phi_{ij}}{\partial a_{ij}} a_{ij} + \lambda_i \frac{\partial \phi_{ij}^A}{\partial a_{ij}^A} \right].
\]

Applying Assumption 10, \( \frac{\partial u_i^A}{\partial a_{ij}} a_{ij} = u^A_{ij} \) and \( \frac{\partial \phi_{ij}^A}{\partial a_{ij}} a_{ij} = \phi^A_{ij} \), so that we obtain

\[
T_{j,ij}^s = -\tau_{i,ij} a_{ij} + \frac{1}{\lambda_i^0} \left[ \omega_i \frac{\partial U_i}{\partial a_{ij}} u_{ij}^A + \omega_i \frac{\partial U_i}{\partial a_{ij}^A} u^A_{ij} + \lambda_i \frac{\partial \phi_{ij}}{\partial a_{ij}} a_{ij} + \lambda_i \frac{\partial \phi_{ij}^A}{\partial a_{ij}^A} \right].
\]

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Finally, differentiating in $a_{ij}^A(m)$, we obtain
\[
\frac{\partial T_{ij}^s}{\partial a_{ij}^A(m)} = \frac{1}{\lambda_i^0} \left[ \omega_i \frac{\partial U_i}{\partial u_{ij}} \frac{\partial u_{ij}^A}{\partial a_{ij}^A(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_{ij}^A} \frac{\partial \phi_{ij}^A}{\partial a_{ij}^A(m)} \right] = \Omega_{ij}(m)
\]
which is the spillover effect. From here, the country $i$ social planner’s Lagrangian is given by
\[
L_{SP}^i = \omega_i U_i \left( u_i(a_i), u_i^A(a_i, a^A) \right) + \Lambda_i \Gamma_i \left( A_i - \int_j \tau_{ij} a_{ij} dj + \int_j T_{ij}^* dj, \phi_i(a_i), \phi_i^A(a_i, a^A) \right).
\]
From here, derivation follows as in the proof of Proposition 31, except for the additional derivative in revenue. For $a_{ij}(m)$, there is no additional revenue derivative, and so $\tau_{ij}(m) = 0$ as before. For $a_{ii}(m)$, we have following the steps of Proposition 31
\[
\tau_{ii}(m) = -\frac{1}{\lambda_i^0} \frac{\partial L_{SP}^i}{\partial a_{ii}^A(m)} - \frac{1}{\lambda_i^0} \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_{ii}^A} \int_j \frac{\partial T_{ii}^s}{\partial a_{ii}^A(m)} dj = -\Omega_{ii}(m) - \int_j \Omega_{jj}(m) dj
\]
giving the first result.

Finally, considering a foreign allocation $a_{ji}$, we have
\[
0 = \frac{d L_{SP}^i}{da_{ji}(m)} = \frac{\partial L_{SP}^i}{\partial a_{ji}^A(m)} + \lambda_i^0 \left[ \frac{d T_{ij}^s}{da_{ji}^A(m)} + \int_j \frac{\partial T_{ij}^s}{\partial a_{ji}^A(m)} dj \right].
\]
From here, noting that we have $\frac{d T_{ij}^s}{da_{ji}^A(m)} = \tau_{ij}(m) + \frac{\partial \tau_{ij}}{\partial a_{ji}^A(m)} a_{ji}$, we obtain
\[
0 = -\tau_{ii}(m) + \tau_{ij}(m) + \frac{\partial \tau_{ij}}{\partial a_{ji}^A(m)} a_{ji}
\]
which rearranges to the result.

### A.3 Bailouts Model: Additional Results and Derivations

In this appendix, we provide the additional results and derivations from the bailouts section, including the characterization of taxpayers, the relevant implementability conditions, characterization of regulatory policy, and characterization of non-cooperative taxation.
A.3.1 Taxpayers

We provide a foundation for the reduced-form indirect utility function \( V_i(T_i) \) of tax revenue collections from taxpayers, and show a tax smoothing result.

A unit continuum of domestic taxpayers are born at date 1 with an endowment \( T_i^1(s) \) of the consumption good. Given tax collections \( T_i^1(s) \leq T_i^1(s) \), taxpayers enjoy consumption utility \( u_i^T(T_i^1(s) - T_i^1(s),s) \).\(^2\) These tax collections generate total bailout revenue \( T_i = \int_s q(s)T_i^1(s)f(s)ds \).\(^3\) We characterize the optimal tax collection problem of country planner \( i \), who has decided to collect a total \( T_i \) in tax revenue for use in bailouts.

**Lemma 35.** Taxpayer utility can be represented by the indirect utility function

\[
V_i^T(T_i) = \int_s u_i \left(T_i^1(s) - T_i^1(T_i,s),s\right) f(s)ds
\]  
(A.1)

where \( T_i^1(T_i,s) \) is given by the tax smoothing condition

\[
\frac{1}{q(s)}u_i' \left(T_i^1(s) - T_i^1(s),s\right) = \frac{1}{q(s')}u_i' \left(T_i^1(s') - T_i^1(s'),s'\right) \quad \forall s,s'.
\]

Lemma 35 allows us to directly incorporate the indirect utility function \( V_i^T(T_i) \) into planner \( i \) preferences, and to use total revenue collected \( T_i \) as the choice variable. It implies that countries engage in tax smoothing without cooperation, but does not guarantee that they engage in the globally efficient level of bailouts.

**Proof of Lemma 35**

The optimization problem is

\[
\max_{\omega_i^T} \int_s u_i \left(T_i^1(s) - T_i^1(s),s\right) f(s)ds \quad \text{s.t.} \quad \int_s q(s)T_i^1(s)f(s)ds \geq T_i
\]

\(^2\)We impose \( u_i^T(0,s) = +\infty \). We think of \( u_i^T \) as incorporating both consumption preferences and distortionary effects of taxation.

\(^3\)We could assume that the government pays a different price vector \( \bar{q} \) for bailout claims, with derivations largely unchanged.
The FOCs are
\[ \omega_i^T \frac{\partial u_i}{\partial c_i^T(s)} (T_i^1(s) - T_i^1(s)) f(s) - \mu q(s) f(s) = 0 \]

Combining the FOCs across states, we obtain the result.

### A.3.2 Implementability Conditions

We characterize the implementability conditions for domestic allocations of foreign banks, in a manner analogous to the characterization in Lemma 4. Note that now, the domestic choice variables of foreign banks are \((D_{ij}, I_{ij})\).

**Lemma 36.** *Country planner \(j\) can directly choose all domestic allocations of foreign banks, with implementing wedges*

\[
\tau^l_{j,ij} = -\tau^l_{i,ij} - \frac{\partial \Phi^l_{ij}}{\partial I_{ij}} + \frac{1}{\lambda^0_i} E \left[ \lambda^1_i(s) \left( (\gamma_j(s) - 1) \frac{\partial I_{ij}(s)}{\partial I_{ij}} + R_{ij}(s) \right) \right] \quad (A.2)
\]

\[
\tau^D_{j,ij} = -\tau^D_{i,ij} - E \left[ \frac{\lambda^1_i(s)}{\lambda^0_i} \left( 1 + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial A^l_{ij}(s)} \right) \right] + \frac{\Lambda^0_i \partial \Gamma_i \partial \phi_{ij}}{\lambda^0_i \partial \phi_i \partial D_{ij}} \quad (A.3)
\]

Using Lemma 36, we can characterize the non-cooperative equilibrium in the same manner as the baseline model. In particular, we isolate the decision problem of the country \(i\) planner, who optimizes domestic bank welfare choosing domestic and foreign allocations, subject to domestic bank constraints and to the implementability conditions of Lemma 36, taking as given foreign planner wedges and foreign bailouts.

### A.3.3 Non-Cooperative and Cooperative Regulation

We now characterize optimal non-cooperative regulation.

In the non-cooperative equilibrium, country planners choose both the wedges and the bailouts \(T^1_{i,ij}(s)\), taking as given the wedges and bailouts of other countries, to maximize domestic social welfare

\[
V^P_i = \omega_1 \left[ \int_s c_i(s) f(s) ds + \omega_i^T V^T_i (T_i) \right].
\]
The following proposition describes optimal bailout policy under regulation.

**Proposition 37.** The bailout rules in the non-cooperative equilibrium under quantity regulation are as follows.

1. The optimal bailout rule for the domestic operations of a domestic bank is

   \[
   \frac{\omega_i \omega_i^T}{\lambda_i^0} \frac{\partial V_i^T}{\partial T_i} \geq B_{ii}^1(s) + \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)} \Omega_{ii}^B(s) \quad \text{(A.4)}
   \]

   Three factors govern the non-cooperative bailout rules: the social cost of taxes, the direct benefit of bailouts to banks, and the domestic fire sale spillover. When choosing bailouts of domestic activities of domestic banks, the domestic planner considers all three factors, but neglects spillover costs to foreign banks. Moreover, the domestic planner neglects the benefits of alleviating foreign fire sales when choosing bailouts of foreign activities of domestic banks, and neglects the benefits of the bailout transfer when choosing bailouts of domestic activities of foreign banks. Country planners are *home biased* in their bailout decisions, generally preferring to bail out domestic activities of domestic banks.

   Relative to the globally efficient bailout rule, non-cooperative planners under-value all bailout activities, including bailouts of domestic activities of domestic banks, not accounting for either benefits or spillovers to foreign banks. The cooperative agreement increases
bailouts of both domestic and foreign banks. Multilateral fire sale spillovers imply the need for multilateral bailout cooperation.

**Proposition 38.** Optimal non-cooperative regulation is given as follows.

1. Domestic taxes on domestic banks’ domestic activities are given by

\[
\tau_{i,ii}^D = E \left[ \Omega_{ii}(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^1(s)} \right]
\quad (A.7)
\]

\[
\tau_{i,ii}^I = -E \left[ \Omega_{ii}(s) \frac{\partial L_{ii}(s)}{\partial I_{ii}} \right]
\quad (A.8)
\]

while other domestic taxes on domestic banks are zero. \( \Omega_{ii}(s) \) is defined in the proof.

2. If there is an adverse price spillover \(-\Omega_{ii}(s) > 0\), then regulation of foreign banks is equivalent to a ban on foreign liquidations.

To understand Proposition 38, the fact that liquidations are now determined indirectly, rather than directly, implies that the spillovers \( \Omega_{ii}(s) \) now form a basis to price the cost of policies that increase liquidations. This is reflected in the optimal tax rates.

At the same time, the domestic planner prefers sufficiently stringent regulation to prevent foreign banks from contributing to domestic fire sales. This is equivalent to requiring foreign banks to maintain domestic allocations that set \( L_{ji} = 0 \).

Next, we can characterize regulatory policy under the optimal cooperative agreement (global planning).

**Proposition 39.** Optimal cooperative policy consists of taxes on investment scale and debt, given by

\[
\tau_{ij}^D = E \left[ \Omega_{ij}(s) \left[ \Omega_{ij}(s) + \int_p \Omega_{ij}(s) d\hat{i} \right] \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right]
\quad (A.9)
\]

\[
\tau_{ij}^I = -E \left[ \Omega_{ij}(s) \left[ \Omega_{ij}(s) + \int_p \Omega_{ij}(s) \right] \frac{\partial L_{ij}(s)}{\partial I_{ij}} \right]
\quad (A.10)
\]

The intuition of Proposition 39 is analogous to the intuition of Proposition 3. Globally optimal policy accounts for the full set of spillovers. Note that cooperative policy no longer
features equal treatment in tax rates, to the extent that the responses of different banks’ liquidation rules are different on the margin. There is equal treatment in the sense that the basis of spillover effects $\Omega_{ij}(s)$ are the same, independent of which country generates the spillover.

Finally, we can characterize the optimal tax collection and bailout sharing rules of the cooperative agreement.

**Proposition 40.** Globally optimal tax collection and bailout sharing are as follows.

1. Optimal cross-country bailout sharing is given by

$$\omega_i \omega_j \frac{\partial V^T_i(T_i)}{\partial T_i} = \omega_j \omega_i \frac{\partial V^T_j(T_j)}{\partial T_j} \quad \forall i, j \quad (A.11)$$

2. Any bailout sharing rule $(T^1_{i,ij}(s), T^1_{j,ij}(s))$ satisfying $T^1_{ij}(s) = T^1_{i,ij}(s) + T^1_{j,ij}(s)$ can be used to implement the globally optimal allocation. Different bailout sharing rules differ in the initial distribution of tax revenue claims $G_i$. We can set $T^1_{i,ij}(s) = 0$ whenever $i \notin \{i', j\}$ without loss of generality.

The bailout sharing rule (A.11) implies that tax burdens of bailouts are smoothed across countries in an average sense but not state-by-state at date 1, so that some countries may be net contributors or recepients of bailouts in any given state $s$.\textsuperscript{4} Bailout sharing rule irrelevance describes equivalent set of bailout sharing rules, and implies that in principle bailout obligations can be delegated entirely to one country (or to one international organization).\textsuperscript{5}

### A.3.4 Non-Cooperative Taxation

We next consider non-cooperative taxation in the model.

\textsuperscript{4}For example, if countries have the same indirect utility functions and are equally weighted globally, then expected tax burdens are the same across countries.

\textsuperscript{5}For example, the responsibility for deposit insurance can be entirely vested in a single entity (the host country, the home country, or an international deposit guarantee scheme). Once the bailout authority has been delegated to a single entity, the goal of the global planner will be to ensure that that entity chooses bailouts optimally. In practice, imperfectly controllable political economy distortions may lead to bailout funds being misused. See Foarta (2018).
Proposition 41. Suppose that the monopolist distortion is 0. Then, non-cooperative optimal taxation is as follows.

1. Domestic taxes on domestic banks’ domestic activities are

\[
\tau_{i,ii}^D = E \left[ \left( \Omega_{i,i}^B(s) + \int_{i'} \Omega_{i',i}^B(s)di' + \int_{i'} \Delta_{i,ii}^T(s)T_{i'i}(s)di' \right) \frac{\partial L_{i,ii}(s)}{\partial A_{ii}^1(s)} \right] \tag{A.12}
\]

\[
\tau_{i,ii}^I = -E \left[ \left( \Omega_{i,i}^B(s) + \int_{i'} \Omega_{i',i}^B(s)di' + \int_{i'} \Delta_{i,ii}^T(s)T_{i'i}(s)di' \right) \frac{\partial L_{i,ii}(s)}{I_{ii}} \right] \tag{A.13}
\]

where \( \Delta_{i,ii}^T(s) \) is defined in the proof.

2. Domestic taxes on foreign banks’ domestic activities are

\[
\tau_{i,ji}^D = E \left[ \left( \Omega_{i,ji}^B(s) + \int_{i'} \Omega_{i',ji}^B(s)di' + \int_{i'} \Delta_{i,ji}^T(s)T_{i'i}(s)di' \right) \frac{\partial L_{i,ji}(s)}{\partial A_{ji}^1(s)} \right] \tag{A.14}
\]

\[
\tau_{i,ji}^I = -E \left[ \left( \Omega_{i,ji}^B(s) + \int_{i'} \Omega_{i',ji}^B(s)di' + \int_{i'} \Delta_{i,ji}^T(s)T_{i'i}(s)di' \right) \frac{\partial L_{i,ji}(s)}{I_{ji}} \right] \tag{A.15}
\]

Although the result here appears largely as in the baseline model, there is one substantive difference: the additional terms \( \Delta_{i,ii}^T(s)T_{i'i}(s) \) that arise in the revenue derivatives. These terms arise whenever there are bailouts by some country (not necessarily \( i \)) of domestic activities of foreign banks. This effect arises because bailout revenue is not a choice variable of private agents, but rather is an untaxed and unpriced action of governments. In absence of bailouts \( (T_{ji}'(s) = 0) \), this term disappears and we revert to the effective characterizations in the first half of this paper. In other words, bailouts lead to a violation of Assumption 10.\(^6\)

Finally, we could consider the bailout rule for banks. The bailout rule for domestic banks is of the same form as in Proposition 37, except for the change in the spillover. Importantly, however, it is immediate to observe that the bailout rule for foreign banks does not consider the direct revenue benefit to banks from bailout revenue, because there is no tax on bailouts (i.e. it is not a private choice variable). However, there are effects on tax revenue.

\(^6\)Note that the bailouts model features the nonlinear aggregates property of Appendix A.5.4, but that Assumption 10 is still the relevant assumption in that section.
Proposition 42. The optimal bailout rule for domestic activities of a foreign bank is as in Proposition 37, but with the spillover effect in Proposition 41.

Proposition 42 indicates that in addition to imperfect internalization of spillovers, planners are not able to account for direct benefits to foreign banks of receiving bailouts. As a result, cooperation is likely to be required over bailouts of cross-border banks even if non-cooperative Pigouvian taxation is able to achieve close-to-optimal internalization of spillovers. However, it is worthwhile to note that if the terms $\Delta_{ij}^T$ are close to zero, then Pigouvian taxation transforms the bailout problem to a bilateral problem, where the domestic planner simply neglects the benefit to foreign banks of receiving a bailout. Transforming the problem into a bilateral surplus problem, rather than a multilateral problem, may simplify cooperation over bailouts. For example, it may allow for simple agreements such as reciprocity on provision of deposit insurance and access to LOLR facilities.

A.3.5 Restoring Non-Cooperative Optimality With Bailout Levies

The above results imply that the existence of bailouts limits the ability for non-cooperative Pigouvian taxation to generate efficient policies. This failure arises because bailouts are not priced or otherwise optimally chosen by private banks. This implies that if bailouts were chosen by private banks, either explicitly or implicitly, we could restore efficiency.

Suppose that banks can in fact purchase bailout claims from the government, or alternatively that banks are charged ex ante for the bailout claims they will receive. In particular, banks can purchase claims $T_{ij}^1(s) \geq 0$ at date 0, at a cost $q > 1$, so that bailout claims are more expensive than state-contingent securities. The first-order condition for bailout claim purchases in state $s$ is

$$
\tau_{ij}^T(s) = -\tau_{ij}^T(s) - \bar{q} + \frac{\lambda^1_i(s)}{\lambda^0_i} \left( 1 + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right).
$$

(A.16)

Following the logic of previous sections, we have $\tau_{ij}^T = 0$, since banks now purchase bailout claims and since country $i$ does not internalize impacts on foreign fire sales. As a result,
domestic planners never force banks to increase their backstop for foreign activities. On the other hand, the revenue that country planner $j$ raises at date 0 from taxing the bailout purchases is of country $i$ banks is $\tau_{ij}^T(s)T_{ij}(s)$. From here, it is easy to see that the efficiency results of the baseline model are restored. Planner $j$ accounts for the direct benefits of bailouts, and also for the spillover costs.

The results of this section imply that bailout cooperation is also not necessary if it can be given a “market mechanism” and taxed. In practice, we could think about these taxes as corresponding to levies on banks for deposit insurance or access to lender of last resort, with the levies calibrated based on how much the bank expects to receive from them. Such levies are consistent with the fact that the Single Resolution Fund in the EU is funded by bank levies, and the Orderly Liquidity Fund in the US is designed to recoup expenditures from either the resolved bank or from other large financial institutions.\footnote{See https://srb.europa.eu/en/content/single-resolution-fund for the former, and US Department of Treasury (2018) for the latter.}

The framework suggests that bailout policies are most naturally delegated to the host country, who can internalize the benefits and spillovers to foreign banks when using Pigouvian regulation combined with a market mechanism for bailouts. For example, this could correspond to a host country insuring the deposits of the local subsidiary of a foreign bank. This synergizes with other possible considerations, such as benefits to domestic depositors of deposit insurance, that might help to ensure that bailout policies are time consistent.

**Time Consistency and Bailout Sharing:** The results of this section assume that bailouts and bailout sharing rules are chosen ex ante with commitment. In practice, a key concern may be time consistency problems, where countries that ex post are obliged to send bailout funds to foreign countries renege on their international claims. If there are time consistency problems that prevent non-cooperative sharing of taxpayer funds, there may be a role for cooperation to enforce risk sharing agreements. However, the results of this section imply that the role of cooperation would be limited to enforcement of risk sharing, and would not...
need to specify the level of risk sharing.

### A.3.6 Time Inconsistent Bailouts

Suppose that we had time inconsistent bailouts. As a result, the liquidation rule becomes

\[ L_{ij}(s) \geq -\frac{1}{h_j(s)\gamma_j(s)} A_{ij}^1(s) - \frac{(1 - h_j(s))}{h_j(s)} R_j(s) I_{ij} \]

where the bailout transfer is now chosen ex post, and for simplicity we assume it is targeted. Suppose for simplicity that the government is utilitarian ex post and the cost of taxpayer funds is linear with marginal cost of 1, and suppose that there is no fire sale. An ex post utilitarian government always fully bails out its own banks, regardless of investment location. As a result, the bailout rule satisfies

\[ 0 \geq -\frac{1}{h_j(s)\gamma_j(s)} A_{ij}^1(s) - \frac{(1 - h_j(s))}{h_j(s)} R_j(s) I_{ij}. \]

Given that banks know they will be bailed out, they internalize the effects of debt and investment on bailout transfers, given by the above rule. At the same time, banks are always bailed out by their domestic planner.

The result is a moral hazard problem: higher debt issuance, for example, increases the bailout subsidy. The bailout cost is not internalized by banks, creating a role for regulation. However, domestic regulators only regulate domestic banks, because they are not tempted to bail out foreign banks. Nevertheless, notice that there is equal treatment in the sense that if both domestic and foreign banks expect to be bailed out, simply by different planners, then both are regulated for the externality cost of that bailout. As a result, the equilibrium achieved is in fact efficient.

Now, suppose instead that bailouts have a cost greater than the resource loss, so that planners never bail out banks to save resources, but that bank liquidations also have an externality cost $\kappa$. When $\kappa$ is large, domestic planners bail out any failing bank, regardless of domicile, ex post. This reintroduces a motivation to regulate foreign banks, and results in the bans on foreign liquidations to avoid the cost of taxpayer bailouts. By contrast, domestic
banks are not banned but are restricted to account for the bailout cost. This restores unequal treatment and inefficiency.

This suggests that the moral hazard view of bailouts may affect who regulates and bails out what banks, but does not fundamentally alter the intuitions of the regulatory model. When moral hazard is concentrated in domestic banks, the regulatory equilibrium features efficient outcomes even though domestic regulators only regulate domestic banks. When moral hazard also arises from foreign banks, the regulatory equilibrium features inefficient outcomes and over-regulation of foreign banks, for the same reason as in the baseline model.

A.3.7 Bailout Proofs

Proof of Lemma 36

The Lagrangian of the country $i$ bank problem is given by

$$
\mathcal{L}_i = \int_s c_i(s)f(s)ds + \lambda_i^0 \left[ A_i + D_i - T_i - \Phi_i(I_{ii}) - \int_j \Phi_{ij}(I_{ij})dj \right]
+ \int_s \lambda_i^1(s) \left[ A_i^1(s) + (\gamma_i(s) - 1) L_{ii}(s) + R_i I_i \right.
+ \left. \int_j ((\gamma_i(s) - 1) L_{ij}(s) + R_i I_i) dj - c_i(s) \right] f(s)ds
+ \Lambda_i^0 \Gamma_i \left( \phi_i(D_{ii}) + \int_j \phi_{ij}(D_{ij})dj \right)
$$

where we have implicitly internalized the demand liquidation function

$$
L_{ij}(s) = \max\{0, -\frac{1}{h_j(s)\gamma_j(s)} A_{ij}^1(s) - \frac{(1 - h_j(s))}{h_j(s)} R_j I_{ij}(s)\}.
$$

Taking the FOC is $I_{ij}$ and rearranging, we obtain

$$
\tau_{j,ij} = -\tau_{i,ij} - \frac{\partial \Phi_{ij}}{\partial I_{ij}} + \frac{1}{\lambda_i^0} E \left[ \lambda_i^1(s) \left( (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial I_{ij}} + R_{ij}(s) \right) \right].
$$

Similarly, taking the FOC for $x_{ij}(s)$ and rearranging, we obtain

$$
\tau_{j,ij} = -\tau_{i,ij} - E \left[ \frac{1}{\lambda_i^0} \lambda_i^1(s) \left( 1 + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} \right) \right] + \frac{1}{\lambda_i^0} \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial D_{ij}}.
$$
Proof of Propositions 37 and 38

As in the baseline model, the implementing tax rates of Lemma 36 do not otherwise appear in the country $i$ planning problem. These constraints simply determine these tax rates, for the chosen allocation.

Now, consider the decision problem of the country $i$ planner. The only twist is that the liquidation discount is now given by the equation

$$
\gamma_i(s) = \gamma_i \left( L_{ii}(s) + \int_j L_{ij}(s) dj, s \right),
$$

where we have adopted the shorthand $\gamma_i = \frac{\partial F_i}{\partial I_A}$. From here, we characterize the response of the liquidation price to an increase $e$ in total liquidations. Totally differentiating the above equation in total liquidations, we have

$$
\frac{\partial \gamma_i(s)}{\partial e} = \frac{\partial \gamma_i(s)}{\partial L_A^i(s)} \left[ 1 + \frac{\partial [L_{ii}(s) + \int_j L_{ij}(s) dj]}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial e} \right],
$$

where $L_{ii}(s)$ and $L_{ij}(s)$ depend on $\gamma_i(s)$ due to the collateral constraint. Rearranging from here, we obtain the equilibrium country $i$ price response

$$
\frac{\partial \gamma_i(s)}{\partial e} = \frac{1}{1 - \frac{\partial \gamma_i(s)}{\partial L_A^i(s)} \frac{\partial \gamma_i(s)}{\partial \gamma_i(s)}}.
$$

This characterization is useful, since externalities in this proof arise from changes in total liquidations.

Now, consider the Lagrangian of the country $i$ planner. The Lagrangian of the planner can be written as

$$
\mathcal{L}_i^{SP} = \mathcal{L}_i + \omega_i^T V_i^T (T_i) + \lambda_i^T \left[ G_i + T_i - \int_s \left[ T_{i,ii}^1(s) + \int_j T_{i,ij}^1(s) + \int_j T_{i,ii}^1(s) \right] f(s)ds \right],
$$

where $\mathcal{L}_i$ internalizes the liquidation response and liquidation price relationships.

We first characterize the regulatory policies (Proposition 38), and then characterize the bailout policies (Proposition 37).
**Regulatory Policies**: Consider first the domestic allocations of domestic banks. For foreign allocations and consumption of the bank, the planner and bank derivatives coincide, and no wedges are applied, that is \( \tau_{i,ij}^D = \tau_{i,ij}^I = 0 \) for all \( j \neq i \).

For domestic investment, the planner’s derivative is
\[
\frac{\partial L_{i}^S}{\partial I_{ii}} = \frac{\partial L_{i}}{\partial I_{ii}} + \int_s \frac{\partial L_{i}}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \epsilon} \frac{\partial L_i(s)}{\partial I_{ii}} ds \equiv \lambda_i^D \Omega_i^B(s) f(s)
\]
so that the domestic tax on domestic investment scale is given by
\[
\tau_{i,ii}^I = -E \left[ \Omega_i^B(s) \frac{\partial L_{ii}(s)}{\partial I_{ii}} \right],
\]
which is simply the expected spillover effect. Next, we can apply the same argument to taxes on domestic state-contingent securities \( D_{ii} \). We have
\[
\frac{\partial L_{i}^S}{\partial D_{ii}} = \frac{\partial L_{i}}{\partial D_{ii}} + \int_s \frac{\partial L_{i}}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial \epsilon} \frac{\partial L_i(s)}{\partial D_{ii}}
\]
so that the required tax rate is
\[
\tau_{i,ii}^D = E \left[ \Omega_i^B(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^I(s)} \right].
\]

Finally, considering domestic allocations of foreign banks, we only have the price spillover effect. This implies that there is a liquidation ban whenever there is an adverse price spillover, \(-\Omega_{i,ij}^B(s) > 0\).

Note that we can formally characterize the spillover effect \( \Omega_{i,ij}^B(s) \) by evaluating
\[
\frac{\partial L_{i}}{\partial \gamma_j(s)} = \lambda_i^1(s) \left[ L_{ij}(s) + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial \gamma_j(s)} \right] f(s)
\]
so that we have
\[
\Omega_{i,ij}^B(s) = \frac{\frac{\partial L_{i}}{\partial \gamma_j(s)} \frac{\partial \gamma_j(s)}{\partial \epsilon}}{\lambda_i^0 f(s)} = \frac{\lambda_i^1(s)}{\lambda_i^0} \left[ L_{ij}(s) + (\gamma_j(s) - 1) \frac{\partial L_{ij}(s)}{\partial \gamma_j(s)} \right] \frac{\partial \gamma_j(s)}{\partial \epsilon}.
\]
**Bailout Policies:** We next characterize the optimal bailout policies. Consider first the bailout rule for domestic activities of domestic banks, where we have

$$\frac{\partial L_{SP}^i}{\partial T_{i,ii}^i(s)} = \frac{\partial L_i}{\partial T_{i,ii}^i(s)} + \frac{\partial \gamma_i(s)}{\partial e} \frac{\partial L_{ii}(s)}{\partial A_{ii}^i(s)} - \lambda_i^T f(s).$$

Now, the FOC for tax collection tells us that $\lambda_i^T = -\omega_i^T \frac{\partial V_T^i}{\partial T_{i,ii}^i}$. Noting that $\frac{\partial V_T^i}{\partial T_{i,ii}^i} < 0$, we rearrange and obtain the bailout rule

$$\frac{\omega_i^T}{\lambda_i^0} \left| \frac{\partial V_T^i}{\partial T_{i,ii}^i} \right| \geq B_{ii}^i(s) + \Omega_{ii}^i(s) \frac{\partial L_{ii}(s)}{\partial A_{ii}^i(s)}.$$

The remaining two equations follow simply by noting that the spillover term does not appear in the FOC for bailouts of foreign activities of domestic banks, while the bank benefit term does not appear in the FOC for bailouts of domestic activities of foreign banks.

**Proof of Propositions 39 and 40**

The Lagrangian of the global planner is given by

$$L_G^i = \int \left[ L_i + \omega_i^T V_T^i(T_i) + \lambda_i^T \left( G_i + T_i - \int T_{i,ii}^i(s) ds \right) \right] ds + \int \left[ \lambda_0^T T_i + \lambda_i^T G_i \right] di$$

where the last terms reflect the set of lump sum transfers. The FOC for $G_i$ implies $\lambda_T = \lambda_i^T$ while the FOC for $T_i$ implies $\lambda_0 = \lambda_0^T$. From here, the regulation and bailout rules follow by the same steps as in the non-cooperative equilibrium, except that now the full set of spillovers appear, and the benefits to banks of bailouts are always accounted for.

Next, the relationship $\lambda_T = \lambda_i^T$ gives the tax sharing rule. Bailout irrelevance arises by setting $G_i = \int \left[ T_{i,ii}^i(s) + \int T_{i,ii}^j(s) dj + \int T_{i,ii}^j(s) dj \right] dj - T_i$, for the desired bailout rule.

**Proof of Propositions 41 and 42**

The country planner Lagrangian is the same as under regulation, except that there is now also tax revenue collected from foreign banks.
The tax revenue collected by country $j$ from country $i$ banks is given by

$$T_{ij}^* = \tau_{ij} + \tau_{ij}^D D_{ij}$$

so that differentiating in total liquidations in state $s$, we have

$$\frac{\partial T_{ij}^*}{\partial e} = \frac{\partial}{\partial \gamma_j(s)} \left[ \frac{\lambda_i^1(s)}{\lambda_i^0(s)} (\gamma_j(s) - 1) \left[ \frac{\partial L_{ij}(s)}{\partial I_{ij}} I_{ij} - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} D_{ij} \right] \right] \frac{\partial \gamma_j(s)}{\partial e} f(s)$$

from here, we note that $L_{ij}(s)$ is homogeneous of degree 1 in $(I_{ij}, A_{ij}^1(s))$, given $\gamma_j$, so that we can write

$$\frac{\partial T_{ij}^*}{\partial \gamma_j(s)} = \frac{\partial}{\partial \gamma_j(s)} \left[ \frac{\lambda_i^1(s)}{\lambda_i^0(s)} (\gamma_j(s) - 1) \left[ L_{ij}(s) - \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} T_{ij}^1(s) \right] \right] \frac{\partial \gamma_j(s)}{\partial e} f(s)$$

$$= \Omega_{ij}^T(s) f(s) + \Delta_{ij}^T(s) T_{ij}^1(s) f(s)$$

where we have defined $\Delta_{ij}^T(s) = - \frac{\partial}{\partial \gamma_j(s)} \left[ \frac{\lambda_i^1(s)}{\lambda_i^0(s)} (\gamma_j(s) - 1) \left[ \frac{\partial L_{ij}(s)}{\partial A_{ij}^1(s)} T_{ij}^1(s) \right] \right] \frac{\partial \gamma_j(s)}{\partial e}$.

From here, results on regulation follow by the usual steps. Moreover, results on bailouts also follow the usual steps, noting the bailout has indirect effects on tax rates through the liquidation price, but does not have direct effects due to the linear nature of $L_{ij}(s)$.

### A.4 Extensions of the Banking Model

In this Appendix, we present a number of extensions to and discussions of the model, as applied to the banking context. To ease exposition, we express all results in this appendix for interior solutions, except for foreign allocations under non-cooperative regulation.

#### A.4.1 Extended Stakeholders, Real Economy Spillovers, and Arbitrageurs

Bank activities and failures can affect a wide range of economic agents, leading to spillovers from the financial sector to the real sector of the economy. In the baseline model, there were no benefits from foreign banking, and fire sales affected only the consumption profiles of agents in the “financial sector” (banks and arbitrageurs); moreover, arbitrageurs were given zero welfare weight. In practice, the spillover to the real economy’s “extended stakeholders”
in the financial sector are an important consideration for macroprudential policy. Examples of extended stakeholders include: (1) SMEs, who rely on banks for financing; (2) bank/SME employees, whose employment changes based on bank activities; (3) retail customers, who benefit from retail banking (e.g. deposit) activities; and (4) other local banks, who hold related assets and are affected by fire sales.

For simplicity, we model these spillovers as a reduced form utility spillover onto “extended stakeholders.” The indirect utility function of extended stakeholders \( V^E_i(\bar{I}^A_i, \bar{L}^A_i) \), where \( I^A_i = I_{ii} + \int_j I_{ij}dj \) is total investment in country \( i \) at date 0. This additional indirect utility function appears in social welfare, so that country \( i \) social welfare is now

\[
V^P_i = \int_s c_i(s)f(s)ds + \omega^E_i V^E_i(\bar{I}^A_i, \bar{L}^A_i)
\] (A.17)

where \( \omega^E_i \) is the aggregate welfare weight on extended stakeholders. The model setup is otherwise the same. Notice that positive welfare weights on arbitrageurs is a special case of this setup, where the indirect utility function \( V^E_i \) is the expected consumption surplus of arbitrageurs.

This model is a straight-forward application of Section 1.7 when we interpret \( V^P_i \) as representative agent welfare with total utility spillovers \( \omega^E_i V^E_i \). Note that it is irrelevant that \( V^E_i \) does not satisfy homogeneity of degree 1, since it is for country \( i \) allocations. As a result, all of the core properties of Section 1.7 go through: quantity restrictions are inefficient and features unequal treatment + uninternalized foreign spillovers, whereas Pigouvian taxation is efficient in absence of monopoly rents.

Nevertheles, in contrast to the baseline model, there are “real economy” domestic spillover from banking activities \( \Omega^{L,E}_{i,i} = \omega^E_i \frac{\partial V^E_i}{\partial L^A_i} \) and \( \Omega^{L,E}_{i,j} = \omega^E_i \frac{\partial V^E_i}{\partial L^A_j(s)} \) which may make the domestic government more accepting of international banking activities.

In particular, suppose that there is a positive domestic liquidation spillover perfectly

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8“...The objective of the [Basel III] reforms is to improve the banking sector’s ability to absorb shocks... thus reducing the risk of spillover from the financial sector to the real economy... “B]anks provide critical services to customers, small and medium-sized enterprised, large corporate firms and governments who rely on them to conduct their daily business.” BIS (2010)
offsets the domestic fire sale cost at the global optimum. Section 1.7 implies that the domestic planner would on the margin regulate foreign banks correctly. However, the domestic planner would nevertheless still under-regulate domestic banks. A cooperative agreement would still be required to achieve global efficiency, and would increase regulation of domestic banks.

**Positive Arbitrageur Welfare Weights:** With positive welfare weights on arbitrageurs, we have \( V_i^E = E \left[ F_i^A(L_i^A(s), s) - \gamma_i^A(s)L_i^A(s) \right] \). This delivers a positive utility spillover from liquidations, 
\[
\frac{\partial c_i^A(s)}{\partial L_i^A(s)} = -\frac{\partial \gamma_i^A(s)}{\partial L_i^A(s)}L_i^A(s) \geq 0
\]
and so positive welfare weights on arbitrageurs tend to make domestic planners more accepting of liquidations, both by domestic and foreign banks.

**Extended Stakeholders Without Fire Sales:** What if instead, there were no fire sales but there were spillovers to extended stakeholders? Optimal regulation would account for the domestic spillover, but there is no longer any international spillover. As a result, optimal cooperation only is needed to enforce equal treatment, but does not require an increase in regulation of domestic banks.

**A.4.2 Dispersed Bank Ownership**

Banks in practice are multinational not only in their activities, but also in ownership: even though a bank is headquartered in one country, part of its equity can be owned by foreigners.

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9Although positive spillovers from bank liquidations may seem counter intuitive, they can be understood in two ways. The first way to understand why there might be a positive liquidation spillover is that it is a reduced-form representation of the idea that banks may not be able to provide valuable services if they are subject to too strict regulatory stringency. For example, they may choose not to lend to certain types of borrowers entirely. This may lead domestic regulators to be more accepting of foreign bank liquidations, to avoid crowding out these valuable lending services. The second way to understand a positive spillover is that optimal regulation in the model may imply investment subsidies, despite the fact that we are using “regulation.” In this sense, it may be natural to impose an instrument restriction \( \tau_{i,ji}^I \geq 0 \), which we could also reflect as a utility cost for choosing negative wedges under regulation. When banning liquidations reduces the value of domestic investment, this may make the value of domestic investment negative, making the allocation unfeasible. As a result, domestic regulators need to allow some foreign liquidations in order to motivate them to invest domestically.
This invites a natural question: do regulatory incentives change when part of the value of banks accrues to non-domestic agents, and if so does it cause inefficiencies?

Formally, we extend the model as follows. \( c_i(s) \) is the total payoff to the (domestic and foreign) equity holders of country \( i \) banks, whose objective is as in the baseline model.\(^{10}\) Country \( i \) equity holders are risk-neutral agents who own a portfolio of bank equity shares, with exogenous portfolio shares \( \{a_i, 1 - a_i\} \). \( a_i \) is the level of domestic ownership of country \( i \) banks. For simplicity, we assume that foreign equity holdings of domestic banks is equally dispersed across all countries. The consumption profile of country \( i \) equity holders is \( c_i^{\text{Eq}}(s) = a_i c_i(s) + \int_{s'} (1 - a_i') c_i'(s) ds' \).\(^{11}\)

As we will show, non-cooperative and cooperative policy are invariant to foreign ownership if there are only spillovers to multinational banks. To this end, we allow for a second set of spillovers onto purely domestic agents, which we reflect by a reduced-form social cost of bank liquidations. For example, this may correspond to fire sale spillovers to wholly domestic local banks, or to real economy spillovers (as in Appendix A.4.1). Incorporating the value of equity holders and the reduced-form spillover cost, country \( i \) social welfare is now

\[
V_i^P = a_i \int c_i(s)f(s)ds + \int_{s'} (1 - a_i') c_i'(s) ds' ds' - \omega_i^E \int L_i^A(s)f(s)ds,
\]

where \( \omega_i^E \) is the welfare weight on the additional spillover cost.

Note that this falls into the framework of Section 1.7, except that the “aggregate” \( c_i'(s) \) now appears in the utility of foreign multinational banks. It does not satisfy the form of Assumption 10, and so will not generate efficient results under either quantity restrictions or taxes.

Expressing the inefficiency in the form of consumption payout is unintuitive. Instead, let us represent it in form of more intuitive arguments. Suppose for simplicity that the global

\(^{10}\)Even if banks only care about domestic equity holders, \( a_i \) is a constant scaling of the objective function that does not affect optimization.

\(^{11}\)We treat the ownership structure of banks as given, implying that regulation (or taxation) is either chosen after ownership is determined or is chosen simultaneously but is not easily verifiable or contractible. This is related to the time consistency problem of Farhi and Tirole (2018), which revolves around debt holdings by foreigners.
planner is utilitarian, so that the global welfare function is

\[ V^G_i = \int \left[ \int_{s} c_i(s) f(s) ds - \omega^E_i \int_{s} L^{A}_i(s) f(s) ds \right] ds. \]

Consider instead the optimization function of country planner \( i \). Their objective function is a monotone transformation of the optimization problem

\[ V^{P,s}_i = \int_{s} c_i(s) f(s) ds - \frac{1}{\kappa_i} L^{A}_i(s) f(s) ds. \]

From here, we see that the spillover is equivalent to an upweighting of domestic real economy spillovers. As a result, even under Pigouvian taxation, the efficient outcome is not achieved because of foreign ownership of domestic banks. Foreign ownership leads to an inefficient upweighting of other domestic interests.

In this case, the “global spillover” that must be corrected is equivalent to a positive utility spillover \((\frac{1}{\kappa_i} - 1) L^{A}_i(s)\). It is positive in this case because the spillover to extended stakeholders is negative, leading to over-regulation of domestic banks.

As a result, if there are no net spillovers to domestic non-bank agents (or those spillovers are not valued), the equilibrium is efficient even under foreign ownership. This implies that the goal of macroprudential regulation matters substantially for determining the efficiency properties under foreign ownership. When policy orients principally around mitigating fire sale impacts on multinational banks \( (\omega^E_i = 0) \), there is no (or only a small) distortion, as foreign ownership only results in a constant scaling of the social objective function.

A second implication is that the composition of the banking sector matters. When the banking system consists only of multinational banks (no other spillovers), foreign ownership is irrelevant to optimal policy design. By contrast when there is a mixture of multinational and local banks, increasing foreign ownership of multinational banks up-weights the relative weight that domestic planners place on spillovers to local banks, leading to excessively stringent regulation of multinational banks.
A.4.3 Large Presences in Other Countries

In the baseline model, banks only maintain a small investment presence in any single foreign country. We extend the model to allow banks to hold investments in a country that are large relative to the host country size. A natural example is a developed economy investing into an emerging market.

We extend the model to feature two types of countries. A “large” country is a country \( i \in [0, 1] \), as in the baseline model. A block of small countries is a point \( i \in [0, 1] \), which represents a unit continuum of small countries indexed by \( i_k \), with \( k \in [0, 1] \). The combined measure of small countries is therefore the same as a large country. We use the terminology “large” and “small” to indicate the relative sizes of the two countries. Both types of countries are small from a global perspective.

When a large country \( i \) makes investments in a small country \( j_k \) that are proportional to the measure of country \( j_k \), the results from before are the same. We consider instead a large country that makes an investment in a small country \( j_k \) that disproportionate to its measure. We focus on a case where there is a single block \( j_0 \) of small countries, and a single large country \( i_0 \) investing into that block. In other words, all other countries are “large,” but make investments comparable to the measure of the country they are investing in. To ensure tractability, \( i_0 \) is the only (foreign) country that invests into the block \( j_0 \).

Although \( i_0 \) maintains a large presence from its perspective in the block \( j_0 \), it maintains a small presence from its perspective (relative to its total investment portfolio) in any individual country \( j_{0,k} \) in the block. By contrast, both \( i_0 \) and \( j_{0,k} \) recognize that \( i_0 \) is large in \( j_{0,k} \). The assumption that \( i_0 \) has a large presence in \( j_{0,k} \), but that this presence is small relative to the entire contract of \( i_0 \), helps to maintain tractability.\(^{12}\)

To the same end, we simplify the problem of countries \( j_{0,k} \) in the block \( j_0 \). In every country \( j_{0,k} \), the domestic investment scale is a fixed amount \( I_{j_{0,k}|j_{0,k}} > 0 \), with a fixed

\(^{12}\)If \( i_0 \)’s presence in \( j_{0,k} \) were also large from the perspective of \( i_0 \), then changes in liquidation prices or contract terms in country \( j_{0,k} \) would filter back to the entire contract of country \( i_0 \) in a measurable way. In other words, the country \( j_{0,k} \) would internalize that she was affecting the entire contract of \( i_0 \).
liquidation rule \( L_{j_{0,k}}(s) > 0 \). Any net worth of country \( j_{0,k} \) is saved in state-contingent securities and consumed at date 2, so that the utility of country \( j_{0,k} \) is given simply by

\[
V_{j_{0,k}}^P = E[R_{j_{0,k}}]L_{j_{0,k}} + E[\gamma_{j_{0,k}}(s) - 1]L_{j_{0,k}}(s) + A_{j_{0,k}}
\]

where we have the liquidation price determined by total liquidations

\[
L_{A,j_{0,k}}(s) = L_{j_{0,k}}(s) + L_{i_{0,j_{0,k}}}(s).
\]

For simplicity, country \( j_{0,k} \) arbitrageurs are given zero welfare weight, as in the baseline model.

**Globally Optimal Policy**

Globally optimal policy takes the same form as before. The spillover to country \( j_{0,k} \) from a change in the liquidation price is simply

\[
\Omega_{i_{0},j_{0,k}} = L_{j_{0,k}}(s)
\]

since the marginal value of date 0 wealth is always 1, and the marginal value of date 1 wealth is also 1. In other words, there is only a price spillover. Globally optimal regulation therefore sets

\[
\tau_{i_{0},j_{0,k}} = -\frac{\partial \gamma_{j_{0,k}}(s)}{\partial L^A_{j_{0,k}}(s)} \left[ \Omega_{i_{0},j_{0,k}}(s) + \Omega_{j_{0,k},j_{0,k}}(s) \right].
\]

**Non-Cooperative Implementability**

We begin by studying the optimal policy of country \( i_0 \). Its policy towards its domestic banks is the same as before, except that country \( i_0 \) now has a large presence in the block \( j_0 \). As a result, the country \( i_0 \) planner internalizes the effects of its liquidations on the liquidation

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13If other countries invested in block \( j_0 \), there would be additional effects on how changes in the liquidation price affected the demand functions of those countries. If \( j_0 \) had a variable investment scale, then country \( i_0 \) would internalize how its policy affected the entire contract of the country \( j_0 \), which would affect the liquidation policies and hence the fire sale price. If countries in \( j_0 \) invested in \( i_0 \) (and there were many such countries), then \( i_0 \) would internalize how liquidations in \( j_0 \) affected the contracts in \( j_0 \), which would in turn affect the decisions of \( j_0 \) in country \( i_0 \).
price in country $j_{0,k}$.

**Lemma 43.** Optimal taxes by country $i_0$ are the same as the baseline model, except for the taxes on liquidations in countries $j_{0,k}$ is the block $j_0$, which are given by

$$
\tau_{i_0,j_{0,k}}^L(s) = - \frac{\partial \gamma_{j_{0,k}}(s)}{\partial L^A_{j_{0,k}}(s)} \Omega_{i_0,j_{0,k}}(s)
$$

(A.18)

*Proof.* The uninternalized spillover from an increase in liquidations in country $j_{0,k}$ onto country $i_0$ banks is $\frac{\partial \gamma_{j_{0,k}}(s)}{\partial L^A_{j_{0,k}}(s)} \Omega_{i_0,j_{0,k}}(s)$. The country $i_0$ planner internalizes this effect due to the large investment scale. The problem is otherwise the same as the baseline model. ■

Equation (A.18) of Lemma 43 gives us the tax rate set by planner $i_0$ on liquidations in $j_{0,k}$. In contrast to the baseline model, this tax is non-zero because the country $i_0$ planner internalizes the fire sale spillover onto $i_0$ banks in country $j_{0,k}$, where it has a large presence relative to the size of the $j_{0,k}$ banking sector.

From here, we can replicate the implementability conditions of Lemma 4 in country $j_{0,k}$. Because the investment of country $i_0$ in $j_{0,k}$ is small from the perspective of country $i_0$’s entire investment portfolio (even though it is large from the perspective of country $j_{0,k}$), the implementability characterization is exactly as in Lemma 4.

**Non-Cooperative Regulation**

As in the baseline model, liquidations by country $i_0$ are banned in country $j_{0,k}$ provided that they reduce the liquidation price. The fact that $i_0$ has a large presence, rather than a small one, from the perspective of country $j_{0,k}$ does not alter this result.

**Non-Cooperative Taxation**

Consider next non-cooperative taxation. We obtain the following characterization.

**Proposition 44.** Suppose the monopolist distortion is 0. Then:
1. The optimal liquidation tax set by country \( j_{0,k} \) is
\[
\tau^L_{j_{0,k},i_{0,j_{0,k}}} (s) = - \frac{\partial \gamma_{j_{0,k}}(s)}{\partial L^A_{j_{0,k}} (s)} \left[ \Omega_{j_{0,k},i_{0,k}}(s) + \Omega_{i_{0,k},j_{0,k}}(s) \right]
\]

2. There is excessive taxation relative to the global optimum, that is
\[
\tau^L_{i_{0,i_{0,j_{0,k}}}} (s) + \tau^L_{j_{0,k},i_{0,j_{0,k}}} (s) > \tau^L_{j_{0,k}} (s)
\]

Proof. Given the problem structure, the optimization problem of country planner \( j_{0,k} \) is
\[
\max_{\lambda_{j_{0,k}}^{i_{0,j_{0,k}}}, \tau_{j_{0,k}}{i_{0,j_{0,k}}}} E[R_{j_{0,k},i_{0,j_{0,k}}} J_{j_{0,k},i_{0,j_{0,k}}} + E[(\gamma_{j_{0,k}}(s) - 1)L_{j_{0,k},i_{0,j_{0,k}}}(s)] + \left[ A_{j_{0,k}} + T^*_{j_{0,k},i_{0,k}} \right].
\]
Suppose that the monopolist distortion is 0. The optimal taxes follow as in the proof of Proposition 6. Noting from here that \( \tau^L_{i_{0,i_{0,j_{0,k}}}} (s) > 0 \) while \( \tau^L_{j_{0,k},i_{0,j_{0,k}}} (s) = \tau^L_{i_{0,j_{0,k}}}(s) \), there is excessive taxation.

Proposition 44 illustrates that although the optimal liquidation tax is set correctly by the host country, there is excessive taxation in equilibrium, because the home country also sets a positive tax on liquidations to account for the fire sale spillover.

Proposition 44 suggests a motivation for delegating taxation to the host country, that is to say requiring \( \tau^L_{i_{0,j_{0,k}}}(s) = 0 \). Whereas the home country is able to internalize the spillovers onto itself, it does not internalize the spillovers to other banks in country \( j_{0,k} \). By contrast, country \( j_{0,k} \) does, when using taxation.14

A.4.4 Local Capital Goods and Protectionism

Although financial stability and fire sales have been highlighted as justifications for post-crisis regulation, cooperative agreements predate the crisis, including the previous Basel accords. In this context, regulators may care about additional considerations such as domestic spillovers (Appendix A.4.1). Additionally, regulators may care about controlling...

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14Note that delegating regulation to the host country is not a perfect solution, because \( \tau^L_{i_{0,j_{0,k}}}(s) = 0 \) then acts as restriction. However, the home country \( i_{0} \) will internalize that its other regulations affect liquidations in country \( j_{0,k} \), and so may distort other regulations with the intent of indirectly affecting liquidations in \( j_{0,k} \).
local costs of investment, for example wishing to ensure that (less strictly regulated) foreign banks are not at a competitive advantage over domestic banks.\footnote{The mechanism in this extension is closely related to Dell’Ariccia and Marquez (2006), who use a similar model to motivate a coordinated increase in domestic regulation. Our main contribution relative to their paper is to allow for common agency, to study the impacts of Pigouvian taxation, and to relate this mechanism to fire sales.}

We consider such a motivation by extending the model to include a common domestic investment price. In particular, we augment the model with local capital goods, which are used to produce domestic projects. For simplicity, we rule out all spillovers besides capital good prices in this sections. As a result, there are no fire sales and no extended stakeholder spillovers.

Banks can produce projects using both initial endowment, and with units of a local capital good. Bank $i$ purchases a vector $k_i$ of local capital goods, with $k_{ij}$ being the capital good of country $j$, at prices $p_j$. When using $k_i$ of the capital good, it costs an additional $\Phi_{ii}(I_{ii},k_{ii}) + \int_j \Phi_{ij}(I_{ij},k_{ij})dj$ to produce the vector $I_i$ of projects. The date 0 budget constraint of bank $i$ is

$$p_i k_{ii} + \int_j p_j k_{ij}dj + \Phi_{ii}(I_{ii},k_{ii}) + \int_j \Phi_{ij}(I_{ij},k_{ij})dj \leq A_i + D_i.$$

The optimization problem of banks is otherwise unchanged, except that $k_i$ is now a choice variable of banks.

In each country, there is a representative capital producing firm. The capital producing firm produces the capital good out of the consumption good with an increasing and weakly convex cost function $K_i(K_i)$, and so has an optimization problem

$$\max_{k_i} p_i K_i - K_i(K_i).$$

The resulting equilibrium capital good price in country $i$ is

$$p_i = \frac{\partial K_i(K_i)}{\partial K_i}, \quad K_i = k_{ii} + \int j k_{ij}dj.$$  \hspace{1cm} (A.19)

The local capital producing firm cannot be controlled by country planners, so that equation
(A.19) is an implementability condition of the model. Note that $\frac{\partial p_i}{\partial K_i} \geq 0$.\footnote{In order to ensure that firm profits are bounded above, we will assume that $\frac{\partial p_i}{\partial K_i} = 0$ above some point $K^*$, which amounts to assuming that $K_i(K_i)$ becomes linear on the margin above $K^*$.}

Finally, the social planner places a welfare weight $\omega^K_i$ on the capital producing firm, so that the social welfare function is

$$V^P_i = \int c_i(s)f(s)ds + \omega^K_i\left[p_i(K_i)K_i - K_i(K_i)\right].$$

From here, note that the model is in the form of Section 1.7 when we interpret profits of the capital producing firm as a utility spillover to the domestic representative bank.

From here, we see that there are spillovers to both domestic and foreign agents from changes in capital purchases, given by

$$\Omega^K_{i,i} = -\frac{\partial p_i}{\partial K_i}K_{ii} + \omega^K_i \frac{\partial p_i}{\lambda^K_i}K_i$$

$$\Omega^K_{j,i} = -\frac{\partial p_i}{\partial K_i}K_{ji}$$

The spillover from the capital price increase is the additional resource cost to the bank of purchasing their existing level of the capital good. This is closely related to the direct price spillover under fire sales.

Let us suppose that we are in an environment where the domestic planner wishes to subsidize domestic banks by keeping capital cheap. We represent this by the limiting case $\omega^K_i = 0$. In this case, there is a negative spillover from increases in the capital price to both domestic and foreign banks, which make capital more expensive.

The globally efficient policy subsidizes capital by limiting capital purchases of all banks. By contrast, non-cooperative quantity regulation is protectionist and bans foreign banks from purchasing the domestic capital. In effect, it shields domestic banks from foreign competition.

Nevertheless, the “pecuniary externality” here falls within the class of problems under Assumption 10. As a result, assuming no monopoly power, the non-cooperative equilibrium under Pigouvian taxation is globally efficient.
**Relationship to the Pre-Crisis World:** In addition to understanding the Basel accords, this result also helps contextualize the historical aversion to capital control measures or other barriers to capital flows. In a purely non-cooperative environment, countries are tempted to engage in inefficient protectionism to shield domestic banks from foreign competition. Protectionism is inefficient because all countries do so, and so countries benefit from agreements against protectionist policies. For example, agreements might allow expansion via branches, rather than subsidiaries, in addition to lifting other barriers to capital flows. Our results suggest that although quantity-based measures lead to inefficient protectionist policies, priced-based measures (taxes) do not. This provides another advantage of tax-based policies in the international context.

**Differences from Fire Sales:** Although the general characterizations in this extension are closely related in a general sense to the characterizations of the main paper under fire sales, there are two important differences.

The first important difference is the form of restrictions on foreign banks. Under fire sales, non-cooperative policies were meant to restrict premature liquidations. This corresponded most naturally to either ring fencing type policies, or to restrictions on capital outflows. By contrast, with local capital prices, non-cooperative policies are meant to restrict investment in the first place, and so more closely resemble either greater regulation on domestic activities of foreign banks, or bans on capital inflows. The motivation under the former is to enhance domestic financial stability, while the motivation under the latter is more protectionist in nature.

The second important distinction is in the implications for cooperation. Under fire sales, cooperation was required among countries who invest across borders and who share common crisis states. By contrast under local capital goods, cross border investment alone determines the need for cooperation.
A.4.5 Global Resale Markets

In the baseline model, investment resale markets are local: local arbitrageurs always buy liquidated projects. We extend the model to allow for global arbitrageurs, so that resale markets are partly global. For simplicity, we will assume that local and global arbitrageurs are not valued by any country planner. At the end of the section, we provide a brief discussion of relation to the literature on empirical capital flows.

In addition to local arbitrageurs, there is also a representative global arbitrageur, who has a production technology $F^G \left( \int F^G_i \left( L^G_i(s), s \right) di, s \right)$, where $F^G_i(s) : \mathbb{R} \to \mathbb{R}_+^M$ and $F^G : \mathbb{R}_+^M \to \mathbb{R}_+$. Global markets may be fully integrated, may be partially segmented, or may be fully segmented.\(^\text{17}\)

The optimality conditions of the global arbitrageur are

$$\frac{\partial F^G(s)}{\partial g_i(s)} \frac{\partial F^G_i(s)}{\partial L^G_i(s)} = \gamma_i(s)$$

while the optimality conditions of local arbitrageurs are $\frac{\partial F_i(s)}{\partial L^A_i(s)} = \gamma_i(s)$, as before. Inverting these equilibrium conditions, we obtain the demand functions of global and local arbitrageurs, respectively, as $L^G_i(\gamma_i(s), F^G(s), s)$ and $L^A_i(\gamma_i(s), s)$. Note that the demand functions of global arbitrageurs also depend on $F^G(s)$ through the derivative $\frac{\partial F^G_i(s)}{\partial g_i(s)}$. As a result, we also have $\gamma_i(s)$ being a function of $F^G(s)$.

From the perspective of individual country planners, this problem is no different from that in the baseline model, except for the change in the equilibrium price determination. In particular, the sensitivity of the equilibrium price to total country $i$ liquidations, from the perspective of country planner $i$, is given by

$$1 = \frac{\partial L^A_i(s)}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial L^L_i(s)} + \frac{\partial L^G_i(s)}{\partial \gamma_i(s)} \frac{\partial \gamma_i(s)}{\partial L^L_i(s)}$$

which gives us the equilibrium movement of the country $i$ price from the perspective of

\(^{17}\)For example, partial segmentation could be two regions, $I_1 \cup I_2 = [0, 1]$, with production $F^G = F^G_1(\int_{i \in I_1} F^G_i di) + F^G_2(\int_{i \in I_2} F^G_i di)$. Full segmentation would be $F^G = \int_{i \in I} F^G_i di$.  

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country planner \(i\). Defining \(\alpha^A_i(s) = L^A_i(s)/L^{\text{Tot}}_i(s)\), we have

\[
\tilde{\epsilon}_{\tau_i, L^{\text{Tot}}_i}(s) = \frac{1}{\alpha^A_i(s)\tilde{\epsilon}_{L^A_i, \tau_i}(s) + (1 - \alpha^A_i(s))\tilde{\epsilon}_{L^G_i, \tau_i}}
\]

which relates the elasticity of the liquidation price in total liquidations to a weighted sum of the demand elasticities of local and global arbitrageurs.

Suppose first that \(\partial F^G(s)/\partial F^G(s)\) is constant, so that \(I^G_i\) is not a function of \(F^G\). Then, this problem proceeds as in the baseline model, with the domestic liquidation price being a function only of domestic liquidations. All results are unchanged.

By contrast, suppose that \(\partial F^G(s)/\partial F^G(s)\) is not constant. In this case, suppose that an increase in foreign liquidations (in countries outside of \(i\)) induces an increase in a single direction \(F^G_0(s)\). From here, we can characterize the resulting equilibrium price change.

**Lemma 45.** Holding country \(i\) allocations fixed, the equilibrium country \(i\) price change from an increase in foreign liquidations \(F^G_0(s)\) is

\[
\frac{\partial \gamma_i}{\partial F^G_0} = \frac{(1 - \alpha^A_i(s))\tilde{\epsilon}_{L^G_i, \tau_i}(s)}{(1 - \alpha^A_i(s))\tilde{\epsilon}_{L^G_i, \tau_i}(s) + \alpha^A_i(s)\tilde{\epsilon}_{L^A_i, \tau_i}(s)} \frac{\partial^2 F^G}{\partial F^G_0} \frac{\partial F^G_i}{\partial F^G_0} \frac{\partial L^G_i}{\partial F^G_0}
\]

where \(\alpha^A_i(s)\) is the share of liquidations purchased by local arbitrageurs, and \(\tilde{\epsilon}_{L^G_i, \tau_i}\) (\(\tilde{\epsilon}_{L^A_i, \tau_i}\)) is the demand elasticity of global (local) arbitrageurs in the local liquidation price.

Lemma 45 illustrates the change in equilibrium liquidation price in country \(i\) as a direct result of changes in foreign country liquidations filtering through global resale markets. This change is not internalized by any individual planner, and so reflects an uninternalized global spillover.

This uninternalized global resale market is governed by two forces. The first is a measure of the marginal importance of global arbitrageurs in the country \(i\) resale market. When either global arbitrageur demand (on the margin) for local liquidated assets is relatively inelastic, or when global arbitrageurs account for a relatively small share of domestic purchases, spillovers from global resale markets are muted. Similarly, if global resale markets have relatively low impacts on each other, then likewise global resale market spillovers are muted.
From here, we obtain the following result.

**Proposition 46.** Suppose that the global resale market spillover is 0. Then, the non-cooperative equilibrium under Pigouvian taxation is globally efficient under the conditions of Proposition 9.

**Proof.** When $\partial \gamma_i / \partial F_0^G = 0$, additional global spillover is 0, and so the proof follows as before.

There are two cases under which global resale market spillovers are 0. The first is that local arbitrageurs are the relevant marginal pricing agent, that is $(1 - \alpha_i^A(s))\zeta_{L_i^G, h_i}(s) = 0$. The second that that global resale markets are not interconnected, that is $\partial^2 F^G / \partial F^G \partial F_0^G = 0$.

The derivative $\frac{\partial^2 F^G(s)}{\partial F^G(s) \partial F_0^G}$ indicates that the global asset market spillover depends on a notion of asset similarity between countries. In particular, suppose that there are two blocks of countries, $I$ and $I' = [0, 1] \setminus I$, so that asset resale markets are integrated within each block, but not across blocks. This can be represented by a global technology $F^G = F_I^G \left( \int_{i \in I} F_i^G(L_i^G(s), s) + F_{I'}^G \left( \int_{i \in I'} F_i^G(L_i^G(s), s) \right) \right)$. Here, we have $\frac{\partial^2 F^G(s)}{\partial F_I^G(s) \partial F_{I'}^G(s)} = 0$, and there are no spillovers on global resale markets between the two blocks.

**Relation to Empirics**

In practice, the empirical literature has highlighted that international capital flows tend to co-move, with declines in foreign inflows at the same time as declines in domestic outflows.\textsuperscript{18} If in the model global arbitrageurs were the principal pricing agents, this would suggest that there should be foreign inflows at the same time as retrenchment. This is suggestive of a role for local arbitrageurs in determining prices.\textsuperscript{19}

\textsuperscript{18}See e.g. Broner et al. (2013).

\textsuperscript{19}Of course, these foreign inflows might be offset by outflows by foreign banks, generating the empirical patterns.
Proof of Lemma 45

For exposition, we suppress the $s$ notation on equilibrium objects. The domestic liquidation price is a function $g_i(L_{\text{Tot}}^i, F^G, s)$ of domestic total liquidations and of foreign liquidations $F^G$, while the equilibrium demand functions are $L_A^i(\gamma_i(s))$ and $L_G^i(\gamma_i(F^G, s))$. Market clearing implies that

$$L_{\text{Tot}}^i = L_A^i \left( \gamma_i(L_{\text{Tot}}^i, F^G, s) \right) + L_G^i \left( \gamma_i(L_{\text{Tot}}^i, F^G, s) \right)$$

so that differentiating in a single direction $F^G_0$, holding $L_{\text{Tot}}^i$ fixed, we have

$$0 = \frac{\partial L_A^i}{\partial \gamma_i} + \frac{\partial L_G^i}{\partial \gamma_i} + \frac{\partial L_G^i}{\partial F^G_0}.$$

Next, differentiating the optimality condition of the global arbitrageur in $F^G_0$, we have

$$\frac{\partial^2 F^G}{\partial F^G \partial F^G_0} \frac{\partial F^G_i}{\partial F^G_0} + \frac{\partial^2 F^G}{\partial F^G_0} \frac{\partial F^G_i}{\partial F^G_0} \frac{\partial (L_G^i)^2}{\partial L_G^i \partial F^G_0} = \frac{\partial \gamma_i}{\partial F^G_0}.$$

Finally, note that from the optimality condition of the global arbitrageur, we also obtain the price response $\frac{\partial F^G_i}{\partial s} \frac{\partial^2 F^G_i}{\partial (L_G^i)^2} = 1$. Substituting into the above equation and rearranging, we obtain

$$\frac{\partial \gamma_i}{\partial F^G_0} = \frac{(1 - \alpha_i^A(s)) \xi_{L_G^i, \gamma_i}(s)}{(1 - \alpha_i^A(s)) \xi_{L_G^i, \gamma_i}(s) + \alpha_i^A(s) \xi_{L_G^i, \gamma_i}(s)} \frac{\partial^2 F^G}{\partial F^G_0} \frac{\partial F^G_i}{\partial L_G^i}$$

where $\xi$ are the elasticities, and $\alpha_i^A$ is the share of liquidations purchased by local arbitrageurs.

A.4.6 Regulatory Arbitrage

An important consideration in macroprudential regulation is regulatory arbitrage. In our model, there are two potential sources of regulatory arbitrage. First, country planners may be unwilling or unable to form cooperative agreements around all activities, creating possibilities for arbitrage around the cooperative agreement. Second, there may be a group of agents (“shadow banks”) beyond the regulatory control of country planners.
Arbitrage Around Cooperative Agreements

The possibility for arbitrage around cooperative agreements may arise if planners retain autonomy over a set of instruments, which the global planner cannot control. In the baseline model, this could be captured by restricting the set of wedges the global planner could place on country planners.

It is obvious that such arbitrage has the potential to disrupt cooperative agreements. However, this form of arbitrage has no bearing on the efficiency of non-cooperative Pigouvian taxation, where all authority is vested in country planners and efficiency is nevertheless obtained.

Regulatory arbitrage of this form can therefore be seen as an additional difficulty of cooperation. It provides another argument for advantages of adopting Pigouvian taxation.

Unregulated Sectors (“Shadow Banks”)

A second form of regulatory arbitrage arises if there is a “shadow banking” sector, which conducts similar activities to banks but cannot be regulated.

Suppose for simplicity that country planners are not concerned with the welfare of the shadow banking sector. Given this, the welfare-relevant aspect of the shadow banking sector is their contribution to financial stability. The only welfare-relevant aspect of the shadow banking sector, therefore, is their asset liquidations $L_{i}^{SB}(s)$, which lead total domestic liquidations in country $i$ to be $L_{i}^{A}(s) = L_{ii}(s) + \int_{j} L_{ji}(s) dj + L_{i}^{SB}(s)$.

Suppose first that the shadow banking sector is wholly domestic. In this case, we represent the liquidations of the shadow banking sector as a function $L_{i}^{SB}(\gamma_i(s), s)$ of the domestic liquidation price in state $s$. This function is taken as exogenous by country planners. The equilibrium liquidation price is given as before by arbitrageur optimization, that is

$$\gamma_i(s) = \frac{\partial F_i(L_i^A(s), s)}{\partial L_i^A(s)}, \quad L_i^A(s) = L_{ii}(s) + \int_{j} L_{ji}(s) dj + L_i^{SB}(\gamma_i(s), s)$$

where we now have a fixed-point relationship. Inverting this fixed point relationship and
rearranging, we obtain

$$(\mathcal{F}_i')^{-1}(\gamma_i(s), s) - L_i^{SB}(\gamma_i(s), s) = L_{ii}(s) + \int L_{ji}(s) dj$$

so that the equilibrium liquidation price determination takes the same generic form as the baseline model - the equilibrium response of the shadow banking sector simply feeds into the ability of the arbitrageur sector to absorb losses. Under natural assumptions, $L_i^{SB}(\gamma_i(s), s)$ increases in $\gamma_i(s)$, so that the effects of reducing bank liquidations are partially offset by increasing risks in the shadow banking sector. However, because the problem remains of the same form as the baseline model, all results of the baseline model continue to apply, including the efficiency of non-cooperative Pigouvian taxation.

Consider next the case where the shadow banking sector is international, but home-biased. In this case, we can instead represent shadow banking sector liquidations by a function $L_i^{SB}(\gamma_i(s), \gamma_{-i}(s), s)$. The arbitrageur pricing relationship takes the same form

$$(\mathcal{F}_i')^{-1}(\gamma_i(s), s) - L_i^{SB}(\gamma_i(s), \gamma_{-i}(s), s) = L_{ii}(s) + \int L_{ji}(s) dj$$

except that now, the liquidation price in country $i$ depends on liquidation prices abroad. This generates an uninternalized spillover of the same generic nature as the model of global arbitrageurs in Appendix A.4.5. This implies that global shadow banking activities may provide an alternate need for regulatory cooperation. Moreover, as in the case of global resale markets, this implies that the “liquidation demand elasticity” of the shadow banking sector is relevant for determining the need for cooperation. When the shadow banking sector’s response is highly elastic, the need for cooperation will be greater. When the shadow banking sector’s response is relatively inelastic, the need for cooperation is smaller.

Finally, notice that the inefficiency arises because shadow banks are not regulated. Were shadow banks regulated, they would fall under the framework of Section 1.7 and the efficiency of Pigouvian taxation would be restored. Surprisingly, this implies that by solving the problem of unregulated shadow banks, country planners can also solve the problem of international cooperation due to resale market spillovers.
A.4.7 Quantity Restrictions in the Form of Ceilings

In the baseline model, we adopt a revenue-neutral wedge approach to quantity restrictions. Here, we consider a variant of the problem where planners are allowed to place ceiling restrictions, and make use of the fact that all wedges in the non-cooperative equilibrium were non-negative in the baseline model. For expositional simplicity, we will restrict attention to quantity ceilings in liquidations, as no other ceilings would be used in equilibrium.\(^\text{20}\)

Suppose that each planner can place quantity ceilings \(L_{ij}(s)\) and \(L_{ji}(s)\) on liquidations by banks. Banks facing multiple quantity ceilings must respect the more stringent ceiling, that is \(L_{ij}(s) \leq L_{ji}(s) = \min\{L_{i,ij}(s), L_{j,ij}(s)\}\). From here, the demand function of banks for liquidations can be written as

\[
L_{ji}(s) = \begin{cases} 
0, & \frac{\lambda_i^1(s)}{\lambda_j^1(s)} (\gamma_i(s) - (1 + r_{ji})) + \frac{1}{\lambda_j^1(s)} \Lambda_i^1(s) h_i(s) \gamma_i(s) < 0 \\
\in [0, L_{ji}(s)], & \frac{\lambda_i^1(s)}{\lambda_j^1(s)} (\gamma_i(s) - (1 + r_{ji})) + \frac{1}{\lambda_j^1(s)} \Lambda_i^1(s) h_i(s) \gamma_i(s) = 0 \\
L_{ji}(s), & \frac{\lambda_i^1(s)}{\lambda_j^1(s)} (\gamma_i(s) - (1 + r_{ji})) + \frac{1}{\lambda_j^1(s)} \Lambda_i^1(s) h_i(s) \gamma_i(s) > 0
\end{cases}
\]

Consider first the global planning problem. We implement the global constrained efficient allocation with quantity ceilings \(L_{ij}(s) = L_{ij}^*(s)\), where \(L_{ij}^*(s)\) is the constrained efficient liquidation rule. Notice that the Lagrange multipliers \(\tau_{ij}^*(s)\) on the quantity restriction constraints are given by the wedge formulas in Proposition 3.

From here, let us consider the regulatory game between country planners. Conjecture an equilibrium where all foreign planners impose quantity ceilings that correspond with the optimal foreign liquidations in the equilibrium under Proposition 5, and consider the optimal policy of country \(i\). Suppose that country planner \(i\) wishes to impose the equilibrium of Proposition 5. It does so by banning foreign liquidations in the associated states of Proposition 5, and by restricting domestic liquidations to their equilibrium level under Proposition 5. This enforces the equilibrium under Proposition 5. But note that the equilibrium under Proposition 5 was an optimal response for country \(i\) when country \(i\) had unrestricted control over allocations. Finally, note that the Lagrange multipliers on the

\(^{20}\)For this section, we assume the bank optimization problem is convex.
A.5 Extensions of the General Model

In this Appendix, we provide extensions of the general model presented in Section 1.7.

A.5.1 World Prices

We now extend the model to incorporate world prices, for example allowing for state contingent securities prices at date 0 to be endogenous. We show that provided that global prices only enter constraints through the wealth level, the problem is unaffected. This result is in line with Korinek (2017) and follows similarly.

Let \( x_i = \{x_i(n)\}_{n \in \mathbb{N}} \) be a vector of global goods held by country \( i \), so that market clearing implies \( \int x_i(n)di = 0 \). Global goods trade at prices \( q \), so that the wealth level of country \( i \) multinational agents is

\[
W_i = A_i - T_i - \sum_n q(n)x_i(n).
\]

Global goods enter into \( u_{ii}, u_{ii}^A, \phi_{ii}, \phi_{ii}^A \), but prices do not enter except through the wealth level. Note that because global goods enter into domestic functions, they do not influence Assumption 10. From here, we obtain the following result.

**Proposition 47.** The optimal cooperative wedges are of the same form as Proposition 9, with no wedges on \( x_i \). Pigouvian taxation is efficient under the same conditions as Proposition 34.

**Proof.** Consider the global planning problem, which yields a Lagrangian of similar form to Proposition 9

\[
L^G = \int_i \left[ \omega_i U_i \left( u_i(a_i, x_i), u_i^A(a_i, x_i, a_i^A) \right) \right] + \Lambda_i \Gamma_i \left( A_i + \mathcal{T}_i \phi_i(a_i, x_i), \phi_i^A(a_i, x_i, a_i^A) \right) di
\]

\[
- \lambda^0 \int_i \mathcal{T}_i di - Q \int_i x_i di
\]

where we have suggestively denoted \( Q(n) \) to be the Lagrange multiplier on the global goods.
market clearing for good \( n \). Differentiating in \( x_i(n) \), we obtain
\[
0 = \frac{\partial L_i}{\partial x_i(n)} - Q(n)
\]
so that world prices \( q(n) = Q(n)/\lambda^0 \) form an equilibrium (recall that \( \lambda_i^0 = \lambda^0 \)). In other words, cooperative policy is as in Proposition 9, with no wedges placed on \( x_i \).

Proposition 47 may apply, for example, to a global market for liabilities at date 0.

A.5.2 Local Constraints on Allocations

We extend Section 1.7 to incorporate local constraints on allocations. Note that such constraints are already available through \( \Gamma_i \) for domestic allocations, but that such constraints are not available in countries \( j \neq i \). The extension captures, for example, the constraints \( 0 \leq L_{ij}(s) \leq R_i(s)I_{ij} \) and \( I_{ij} \geq 0 \) imposed in the main paper.

Suppose that in country \( j \), there is a vector of linear constraints \( \chi_{ij}(a_i^A) a_{ij} \leq b_{ij} \) on allocations, where \( \chi_{ij}(a_i^A) \) potentially depends on aggregates in country \( j \) and where \( b_{ij} \geq 0 \).21 We impose linearity in the spirit of the required conditions for optimality of Pigouvian taxation in Proposition 34. We obtain the following revised implementability result for foreign allocations, which mirrors Lemma 30

**Lemma 48.** Any domestic allocation of foreign banks satisfying constraints \( \chi_{ij}(a_i^A) a_{ij} \leq b_{ij} \) is optimally implemented with the wedges in Lemma 30.

**Proof.** For expositional ease, we suppress the notation \( \chi_{ij}(a_i^A) \) and simply write \( \chi_{ij} \). Let \( v_{ij} \geq 0 \) be the Lagrange multipliers on the local feasibility constraints \( b_{ij} - \chi_{ij} a_{ij} \geq 0 \). The first order condition for an action \( m \) is
\[
0 = \omega_i \frac{\partial U_i}{\partial u_i} \frac{\partial u_{ij}}{\partial a_{ij}(m)} + \omega_i \frac{\partial U_i^A}{\partial u_i} \frac{\partial u_{ij}^A}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial W_i} \left( -\tau_{ij}(m) - \tau_{ij}(m) \right) \\
+ \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_{ij}}{\partial a_{ij}(m)} + \Lambda_i \frac{\partial \Gamma_i^A}{\partial \phi_i} \frac{\partial \phi_{ij}^A}{\partial a_{ij}(m)} - v_{ij} \chi_{ij}(m)
\]

21 We impose \( b_{ij} \geq 0 \) to ensure that non-participation \( (a_{ij} = 0) \) is always feasible.
So that rearranging, we obtain
\[ \lambda^0_i \tau_{i,j}(m) + v_{ij} \chi_{ij}(m) = -\lambda^0_i \tau_{i,j}(m) + \frac{\partial U_i}{\partial u_{ij}} \frac{\partial u_{ij}}{\partial a_{i,j}(m)} + \frac{\partial U_i}{\partial u_{ij}^A} \frac{\partial u_{ij}^A}{\partial a_{i,j}(m)} 
+ \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i} \frac{\partial \phi_i}{\partial a_{i,j}(m)} + \Lambda_i \frac{\partial \Gamma_i}{\partial \phi_i^A} \frac{\partial \phi_i^A}{\partial a_{i,j}(m)}. \]

Notice that the left-hand side is constant for a given allocation, and is the same formula as in Lemma 30. Denote it to be \( \lambda^0_i \tau_{i,j}^*(m) \), so that we have \( \tau_{i,j}(m) = \tau_{i,j}^*(m) \) if \( v_{ij} = 0 \). Given corner solutions, there may be multiple vectors of tax rates that implement this allocation.

We can express the problem of country planner \( j \) therefore maximizing tax revenue collected while implementing the same allocation, that is
\[
\max_{v_{ij}, \tau_{i,j}} \tau_{i,j}^* a_{ij}^* \quad \text{s.t.} \quad \lambda^0_i \tau_{i,j}(m) + v_{ij} \chi_{ij}(m) = \lambda^0_i \tau_{i,j}^*(m), \quad v_{ij} \left( b_{ij} - \chi_{ij} a_{ij}^* \right) = 0
\]
where the second constraint is complementary slackness. Substituting in for \( \tau_{i,j} \) and substituting in the complementary slackness condition, we obtain
\[
\max_{v_{ij} \geq 0} \tau_{i,j}^* a_{ij}^* - \frac{1}{\lambda^0_i} v_{ij} b_{ij}
\]
Because \( b_{ij} \geq 0 \), revenue collection is maximized at \( v_{ij} = 0 \) so that we have \( \tau_{i,j} = \tau_{i,j}^* \). As a result, the implementability conditions of Lemma 30 hold. \[\blacksquare\]

Lemma 48 implies that implementability constraints are the same as in Section 1.7. The only difference is that now the constraint set on local allocations is a constraint of the local planner. Note that this implies that the local planner directly internalizes spillovers of domestic aggregates onto the constraint set \( \chi_{ij}(a_j^A) a_{ij} \leq b_{ij} \), so that such spillovers are not an issue.

From here, all results proceed as in Section 1.7. Intuitively, the only adjustment we need to make is that \( \chi_{ij}(a_j^A) a_{ij} \leq b_{ij} \) is now a constraint set of planner \( j \). Without loss of generality, scale the Lagrange multiplier \( v_{ij} \) by \( \lambda^0_i \), and define the “local constraint spillover”
of a change in aggregates by

$$\Omega_{ij}^{LC}(m) = -v_{ij} \frac{\partial \chi_{ij}}{\partial a_{ij}^j(m)} a_{ij}$$

so that we can define the total domestic local constraint set spillover as

$$\Omega_j^{LC}(m) = -\int_i \Omega_{ij}^{LC}(m) di$$

From here, it follows that the results of Section 1.7 apply, treating the total domestic spillover as $$\Omega_{ij}(m) + \Omega_j^{LC}(m)$$.

Note that if the local constraints were non-linear, this would not generally hold, as we would not be able to recover the complementary slackness condition precisely in the above proof. As a result, the domestic planner may have an incentive to manipulate the tax rates that implement corner solutions in order to increase revenue. This would amount to another form of "monopolistic" revenue distortion in the model.

**A.5.3 Heterogeneous Agents**

We extend the model of Section 1.7 by allowing for agents banks within a country. Suppose that in each country, there are $$K = \{1, ..., K\}$$ agents, who differ in their utility functions and constraint sets, whom we index $$i_k$$. Some agents may not be able to conduct cross-border activities, in which case foreign actions would not appear in their utility function or constraint sets. Agents of type $$i_k$$ have relative mass $$\mu_{i_k}$$ and are assigned a social welfare weight $$\omega_{i_k}$$.

It is easy to see that we can treat the problem as if there were a single representative agent in country $$i$$. In particular, define $$a_i = \{a_{i_k}\}_{k \in K}$$, $$U_i = \sum_k \mu_{i_k} \omega_{i_k} U_{i_k}$$, and $$\Gamma_i = (\Gamma_{i_1}, ..., \Gamma_{i_k})$$. The

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22To see that $$\tau_{ij}(m) = 0$$ constitutes an equilibrium policy for $$j \neq i$$, suppose that $$\tau_{ij}(m)$$ is set to clear the first-order condition. Then, the first order condition of country planner $$i$$ for $$a_{ij}(m)$$ is satisfied with equality, and so we must have $$v_{ij} = 0$$, so that there is no value to country planner $$i$$ of relaxing the local constraints in country $$j$$ at the equilibrium. As a result, the preferences of country planner $$i$$ align with country $$i$$ banks over actions in country $$j$$, and we have $$\tau_{ij}(m) = 0$$. 

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problem is as-if we have a single representative agent who solves

$$\max_{a_i} U_i \quad \text{s.t.} \quad \Gamma_i \geq 0,$$

since this decision problem is fully separable in $a_{ik}$ and yields the optimality conditions of each agent type. The only difference relative to Section 1.7 is that there are $K$ different measures of wealth, $W_{ik}$. Domestic lump sum transfers imply that $\lambda^0_{ik} = \lambda^0_i$ is independent of $k$, and the characterization of optimal policy follows as in Section 1.7.

### A.5.4 Nonlinear Aggregates

In Section 1.7, we assumed that aggregates are linear, that is $a_i^A(m) = a_{ii}(m) + \int_j a_{ij}(m) dj$. The welfare-relevant aggregates may not necessarily be linear. We can represent this by $a_i^A(m) = z_{ii}(a_{ii}, m) + \int_j z_{ij}(a_{ij}, m) dj$ for some functions $z$. The key change in the model is that we now have spillover effects that depend on the identity of the country investing, as in the bailouts model. The optimality of non-cooperative Pigouvian taxation follows from the same steps and logic as the baseline model. This clarifies once again that the homogeneity property of Assumption 10 applies to allocations, not to aggregates.

The possibility for non-linear aggregation helps to generalize the results to settings where regulation is set at an initial date, but the economy is not regulated thereafter. This is the case in the bailouts model when we assume away bailouts, where planners set policies at date 0 but then do not intervene at date 1.

### A.5.5 General Government Actions

We extend the model to feature more general government actions, for example bailouts as in Section 1.6. In particular, country planner $i$ can take actions $g_{ijk}(m) \geq 0$ (for either $i = j$ or $i = k$), which affect country $j$ agents in the same way as action $m$ in country $k$. As such, we can define the total domestic action of agent $i$ as

$$\pi_{ii}(m) = a_{ii}(m) + g_{ii}(m)$$
and the total foreign action of agent \(i\) as
\[
\bar{a}_{ii}(m) = a_{ii}(m) + g_{i,ij}(m) + g_{ij}(m).
\]

This classification allows for a rich set of both agent and government actions. For example, a domestic action \(m\) that can only be taken by the government, such as government debt issuance or a bailout, could feature a feasibility constraint \(a_{ii}(m) = 0\). From here, the domestic aggregates are given by
\[
a_{i}^A(m) = \bar{a}_{ii}(m) + \int f_{ji}(a_{ii}, g_{ii}, a_{ij}) dj.
\]

The flow utility of the country \(i\) representative agent is now given by
\[
\max_{a_{ii}, g_{ii}, a_{ij}, a_{i}^A} U_i(a_{ii}, g_{ii}, a_{ij}, a_{i}^A) \quad \text{s.t.} \quad G_i W_i, \quad \text{subject to Lagrange multipliers of 0 by the representative agent, but not by the social planner.}
\]

From here, we begin by characterizing the globally efficient allocation. Observe first that the optimal wedges for private actions are still given by the equations in Proposition 9.

**Proposition 49.** The globally efficient allocation can be decentralized by the wedges of Proposition 9. The globally efficient government actions \(g_{i,jk}\) (for either \(i = j\) or \(i = k\))
\[
- \frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} \geq \frac{\partial \mathcal{L}_j}{\partial a_{j,jk}(m)} + \frac{\partial \mathcal{L}_k}{\partial a_{k,jk}(m)} + \int \frac{\partial \mathcal{L}_{i'}}{\partial a_{k,jk}(m)} di'
\]
where \(\frac{\partial \mathcal{L}_i}{\partial g_{i,jk}(m)} = \omega_i \frac{\partial U_i}{\partial g_{i,jk}(m)} + \Lambda_i \frac{\partial r_i}{\partial g_{i,jk}(m)} \) and so on.

**Proof.** The proof of the decentralizing wedges follows as in the proof of Proposition 9. The government action rules follow directly from the derivatives of the global Lagrangian.

The globally efficient allocation of government actions is a generalization of the optimal
bailout rule of Proposition 8, with analogous intuition. Note that for \( j \neq i \), we have an action smoothing result: \( \frac{\partial L_i}{\partial g_{ji}(m)} = \frac{\partial L_j}{\partial g_{ji}(m)} \), that is the marginal cost of providing the action is smoothed across countries. For example, this corresponds to bailout sharing.

From here, the non-cooperative results on quantity regulation follow as in the baseline model and bailouts section. Taking either \( i = j \) or \( i = k \), the neglected terms are always the terms that affect other countries, namely the foreign spillovers and either the spillover \((i = j)\) or the benefit \((i = k)\). For domestic actions, there are neglected foreign spillovers, while for domestic actions on foreign banks there is unequal treatment when the cost of providing the action is held fixed.

On the other hand, suppose that choices of foreign government actions \( g_{ij} \) and \( g_{ji} \) are delegated to agents, but can be taxed.\(^{23}\) Once this is imposed and governments use Pigouvian taxation, these foreign government actions are no different from regular actions from a technical perspective,\(^ {24}\) and the efficiency of Pigouvian taxation is restored.

### A.5.6 Preference Misalignment

We now suppose that there is a difference in preferences between country planners and multinational agents, that is country planners have a utility function \( V_i(v_i(a_i), v_A(a_i, a_A)) \). Preference differences may arise due to paternalism, political economy, or simple corruption. For simplicity, we incorporate the welfare weights into the planner utility function.

We will define efficient policies with respect to those of country planners. This is a natural efficiency benchmark, as country planners agree to cooperative agreements.\(^ {25}\) Under this definition, globally efficient policy can be characterized as follows.

**Proposition 50.** The globally efficient wedges are given by

\[
\tau_{ji}(m) = -\Delta_{ji}(m) - \Omega^p_{ji}(m) - \int_{i'} \Omega^p_{i',j}(m) di'
\]

\(^ {23}\)Notice that \( g_{ij} \) is delegated to country \( i \) agents and \( g_{ji} \) to country \( j \) agents.

\(^ {24}\)Excepting that there is a non-linear aggregate arising from \( u^p_{ij} \), which is covered above.

\(^ {25}\)See Korinek (2017) for the same argument.
where we have

\[
\Delta_{ji}(m) = \frac{1}{\lambda_j} \left[ \frac{\partial V_j}{\partial v_j} \frac{\partial v_{ji}(m)}{\partial a_{ji}(m)} - \frac{\partial U_i}{\partial u_j} \frac{\partial u_{ji}(m)}{\partial a_{ji}(m)} \right]
\]

and where \( \Omega_{i,j}^v \) are defined analogously to \( \Omega_{i,j} \), but with the planner utility functions.

Proof. The proof follows as usual by writing country social welfare as \( U_i + (V_i - U_i) \) and comparing the planner and agent first order conditions.

Globally efficient policy accounts for spillovers onto the welfare of country planners in a standard way. However, it also must correct for the difference in preferences, yielding the first term \( \Delta_{ji}(m) \).

From here, characterization of optimal quantity regulation follows as in Section 1.7, except with the spillovers defined above. Regulation of domestic agents accounts for both the preference difference and spillovers to country planner welfare, but does not account for spillovers to foreign planners. Regulation of foreign agents allows them to conduct activities only to the point that it increases domestic planner welfare. The result is uninternalized spillovers and unequal treatment.

The result for Pigouvian taxation is more subtle. Considering tax revenue collections with no monopolist distortion, we have the tax revenue collection \( \tau_{ji}(m)a_{ji}(m) \). Note first that differentiating in \( a_{ji}(m) \), we obtain the total revenue impact (assuming no monopoly rents)

\[
\tau_{ji}(m) + \int \frac{\partial \tau_{ji}}{\partial a_{ji}'}(m) a_{ji}'(m) = \tau_{ji}(m) + \int \Omega_{i,j}(m) di'
\]

where we note that \( \tau_{i,ji}(m) \) is now the benefit to the foreign agent net of the wedge placed by the foreign planner, which unwinds the preference difference. This results in the difference \( \Delta_{ji}(m) \) being correctly accounted for. However, the spillovers defined above are the spillovers to the agent, not the planner. This implies setting correct policy requires \( \Omega_{j,ji}^v = \Omega_{j,ji} \) when \( j \neq i \). The simplest way for this requirement to hold is if spillovers onto foreign agents are limited to constraint set spillovers, for example the fire sales of the baseline model.

Finally, it should be noted that these results imply that country planners can achieve the cooperative outcome using Pigouvian taxation. However, this section does not address
whether the cooperative outcome is superior to the non-cooperative outcome. This latter claim requires a normative stand on whether the preferences of the planner or the agent are the normatively legitimate preferences, which depends on the source of preference difference. Although interesting for future work, such analysis is beyond the scope of this paper.
Appendix B

Appendix to Chapter 2

B.1 Proofs

B.1.1 Proof of Proposition 11

Consider the program

\[
\max_{L_1,Y_0} E \left[ c_2(R) \mid e = H \right]
\]

Subject to

\[
E \left[ c_2(R) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] \geq BY_0
\]

\[
Y_0 - A = \int_{R} \gamma(s)RY_0 f_H(R) dR + \int_{R=0} L_1(R)f_H(R) dR
\]

\[
R \geq R' \Rightarrow L_1(R) \geq L_1(R')
\]

\[
c_2(R) \geq (1 - a(R))bRY_0
\]

\[
L_1(R) \geq 0
\]

and recall the second to last constraint is limited pledgeability. It is helpful to redefine the problem in the investor payoff space, and then to define the implementing liability structure \(L_1(R)\). Total investor payoff \(x(R)\) is given by

\[
x(R) = a(R)\gamma RY_0 + (1 - a(R))RY_0 - c_2(R)
\]
where \( \alpha(R) \in \{0, 1\} \) is the liquidation rule. We treat \( \alpha(R) \) as a choice variable, and then back out the liability structure that implements it. Note that because banks are repaid 0 when \( \alpha(R) = 1 \), it is irrelevant whether we multiply \( c_2(R) \) by \( 1 - \alpha(R) \). Given this characterization, investor voluntary participation can be rewritten as

\[
Y_0 - A = E [\alpha \gamma R Y_0 + (1 - \alpha) R Y_0 - c_2 | e = H].
\]

We begin by studying the optimization problem not subject to liability monotonicity, and show that it generates a non-monotone contract. The Lagrangian of this relaxed problem is

\[
\mathcal{L} = E[c_2 | e = H] + \mu \left[ E \left[ c_2 \left(1 - \frac{f_L(R)}{f_H(R)}\right) | e = H\right] - BY_0 \right] + \lambda \left[ E [\alpha(R) \gamma(s) R Y_0 + (1 - \alpha) R Y_0 - c_2 | e = H] + A - Y_0 \right] + E [\chi(c_2 - (1 - \alpha)b RY_0) | e = H] + E [\zeta((\alpha \gamma R Y_0 + (1 - \alpha) R Y_0 - c_2)) | e = H]
\]

From here, first order condition for bank consumption as

\[
0 = f_H(R) + \mu \left(1 - \frac{f_L(R)}{f_H(R)}\right) f_H(R) - \lambda f_H(R) + \chi(R)f_H(R) - \zeta(R)f_H(R)
\]

\[
= \left[1 - \lambda + \mu \left(1 - \frac{f_L(R)}{f_H(R)}\right)\right] f_H(R) + \chi(R) - \zeta(R)
\]

By MLRP, there is a threshold \( R^* \) such that \( \chi(R) > 0 \) for \( R \leq R^* \) and \( \zeta(R) > 0 \) for \( R \geq R^* \), implying that \( x(R) = L_1(R) = 0 \) for all \( R \geq R^* \). This threshold is given by

\[
1 - \lambda + \mu \left(1 - \frac{f_L(R^*)}{f_H(R^*)}\right) = 0.
\]

However, this contract violates liability monotonicity unless \( L_1(R) = 0 \) for all \( R \). Therefore, we have an upper pooling region in the optimal contract, where liabilities and investor repayment are constant.\(^1\)

It is worth remarking that the contract not subject to monotonicity is of the live-or-die

\(^1\)If \( L_1(R) = 0 \), then the entire contract is pooled. If \( R^* = \mathcal{R} \), then the results that follow apply setting \( R_u = \mathcal{R} \) to be the pooling threshold.
form.\(^2\) It implies that banks will be either liquidated or held to the agency rent when \(R < R^*\), with all remaining repayment going to investors. By contrast, the bank receives the full resources of the bank when \(R > R^*\). This contract is optimal because it provides strong incentives to the bank. Because all agents are risk-neutral, they are willing to accept this extreme payoff structure. However, this payoff structure violates liability monotonicity, and so is not implementable.

We now characterize the optimal contract using the following strategy. First, we conjecture pooling thresholds \(R_u\) with corresponding liabilities \(x_u = x(R_u) = L_1(R_u)\), so that \(x(R) = x_u\) for all \(R \geq R_u\). The live-or-die result of the contract not subject to monotonicity implies such a pooling threshold exists.\(^3\) We then solve for the optimal contract below \(R_u\), taking as given \(R_u\) and \(x_u\), subject to a relaxed monotonicity constraint \(x(R) \leq x_u \forall R \leq R_u\), and verify that the resulting contracting is monotone. In doing so, we characterize the space of implementable contracts (that satisfy monotonicity). Finally, we optimize over the choice of \(R_u\) and \(x_u\).

Conjecture pooling thresholds \(R_u\) with liabilities \(x_u\). The associated Lagrangian is given by

\[
\mathcal{L} = E[c_2|e = H] + \mu \left[ E \left[ c_2 \left(1 - \frac{f_L(R)}{f_H(R)}\right) |e = H\right] - BY_0 \right] \\
+ \lambda \left[ E \left[ a\gamma RY_0 + (1 - a)R Y_0 - c_2(R) |e = H\right] + A - Y_0 \right] \\
+ E \left[ \chi \left(c_2 - (1 - a) bRY_0 \right) |e = H\right] \\
+ E \left[ v \left(x_u - (a\gamma RY_0 + (1 - a)R Y_0 - c_2) \right) |e = H\right]
\]

where the final line is the relaxed monotonicity constraint, and where we have anticipated that limited liability \(x(R) \geq 0\) does not bind below \(R_u\) for feasible contracts. Taking the derivative in consumption \(c_2(R)\) for \(R \leq R_u\), we obtain

\[
0 = 1 + \mu \left(1 - \frac{f_L(R)}{f_H(R)}\right) - \lambda + \chi(R) + v(R).
\]


\(^3\)Note that this is without loss, since the pooling threshold could be \(R_u = \mathbb{R}\) if \(R^* = \mathbb{R}\).
Observe that the resulting contract is non-monotone if \( R_u > R^* \) (we would have \( \zeta(R) > 0 \) so that \( x(R) = 0 \)), by the same logic as above. Therefore, we can discard candidate contracts with \( R_u > R^* \). This implies that \( 1 + \mu \left( 1 - \frac{f_L(R_u)}{f_H(R_u)} \right) - \lambda < 0 \) among the set of viable contracts.

Now, consider the derivative in liquidations \( a(R) \), given by

\[
\frac{\partial L}{\partial a(R)} \propto \lambda (\gamma - 1) + \chi(R)b + v(R)(1 - \gamma)
\]

When \( a(R) = 1, v(R) = 1 \) is possible at at most a single point, in particular at \( \gamma R Y_0 = x_u \).

\( a(R) = 1 \) therefore generically implies \( \chi(R) > 0 \) and \( v(R) = 0 \). From the FOC for \( c_2(R, \mu) \), we have (almost everywhere) that when \( a(R) = 1 \)

\[
\chi(R) = \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right)
\]

which, combined with the liquidation rule, yields

\[
\frac{\partial L}{\partial a(R)} \propto \lambda (\gamma - 1) + \left( \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \right) b.
\]

By MLRP, there is a threshold rule \( R \leq R_l \) for liquidations.

Finally, in the region (if non-empty) between \( R_l \) and \( R_u \), by MLRP we have

\[
1 + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda < 1 + \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) - \lambda < 0
\]

so that we have either \( \chi(R) > 0 \) or \( v(R) > 0 \). This implies that \( x(R) = \min \{ (1 - b) R Y_0, x_u \} \) for all \( R_l \leq R \leq R_u \).

As a result, the optimal contract is a three-part liability structure. First, there is a threshold \( R_l \) such that \( a(R) = 1 \) and \( x(R) = \gamma R Y_0 \) for \( R \leq R_l \), and \( a(R) = 0 \) for \( R \geq R_l \). Second, there is a threshold \( R_u \geq R_l \) such that \( x(R) = \min \{ (1 - b) R Y_0, x_u \} \) for \( R \leq R_u \) and \( x(R) = x_u \) for \( R \geq R_u \). Note finally that there cannot be a discontinuity in liabilities at \( R_u \).

\[\text{Implicitly, we are treating } a(R) \text{ as a continuous variable in performing the differentiation. To do so, we implicitly incorporate the constraint } a(R)(1 - a(R)) = 0, \text{ which ensures that implementable contracts must set } a(R) \in \{0, 1\}. \text{ The logic below is unaffected.}\]
If there were a discontinuity, we would have

$$x_u > \lim_{R \uparrow R_u} x(R) = (1 - b)R_uY_0$$

and liabilities would exceed pledgeable income at $R_u$. The capital structure is therefore continuous at $R_u$.

Finally, the above capital structure can be implemented by a liabilities contract $L_1(R) = (1 - b)R_lY_0$ for $R \leq R_l$ and $L_1(R) = x(R)$ for $R > R_l$. This liability structure is monotone, and so we have implementable contracts.

In sum, the optimal contract lies within a class of contracts characterized by thresholds $R_l$ and $R_u$ and corresponding liability structure above. This proves the first part of the proposition.

Now, we characterize the optimal thresholds $R_l$ and $R_u$. Considering the case where these thresholds are interior, $\underline{R} < R_l \leq R_u \leq \overline{R}$ we have the optimization problem

$$\max_{L_1,Y_0} \int_{R_l}^{R_u} bRY_0f_H(R)dR + \int_{R_u}^{\overline{R}} [R - (1 - b)R_u]Y_0f_H(R)dR$$

subject to

$$\int_{R_l}^{R_u} bRY_0(f_H(R) - f_L(R))dR + \int_{R_u}^{\overline{R}} [R - (1 - b)R_u]Y_0(f_H(R) - f_L(R))dR \geq BY_0$$

$$Y_0 - A = \int_{\underline{R}}^{R_l} \gamma RY_0f_H(R)dR + \int_{R_l}^{R_u} (1 - b)RY_0f_H(R)dR + \int_{R_u}^{\overline{R}} (1 - b)R_uY_0f_H(R)dR$$

Under the same multiplier convention, the optimality condition for $R_l$ is

$$0 = -bR_lY_0 - \mu bR_lY_0 \left(1 - \frac{f_L(R_l)}{f_H(R_l)}\right) + \lambda (\gamma - (1 - b))R_lY_0$$

which reduces to

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1\right) = b + \lambda (1 - b - \gamma).$$

Similarly, the optimality condition for $R_u$ is

$$0 = \int_{R_u}^{\overline{R}} [-(1 - b)Y_0f_H(R) - \mu (1 - b)Y_0(f_H(R) - f_L(R)) + \lambda (1 - b)Y_0f_H(R)]dR,$$
which reduces to

\[ 0 = E \left[ \lambda - 1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \right] \bigg| R \geq R_u, e = H. \]

This completes the proof.

A Remark on Contract Uniqueness

The optimal contract is not generally unique in the following sense. In the region \( R \leq R_l \), the bank only needs a liability face value that is sufficient to liquidate the bank, and so any contract with monotone face value \( L_1(R) > (1 - b)Y_0 \) in this region is optimal. We selected the contract with a flat face value below \( R_l \) due to its correspondence to standard debt. The face value of liabilities above \( R_l \) is uniquely determined. Moreover, in the presence of an \( e \to 0 \) premium for standard debt (e.g. as in Appendix B.2.7), the implementation using standard debt becomes uniquely optimal.

B.1.2 Proof of Corollary 12

Consider the proposed liability structure. The amount \((1 - b)R_lY_0\) of standard debt liquidates the bank when \( R \leq R_l \), generating the lower region. \((1 - b)(R_u - R_l)\) is written down in the region \( R_l \leq R \leq R_u \), so that the bank is always held to the agency rent over this region. The full debt level \((1 - b)R_uY_0\) is repaid above \( R_u \). Therefore, we replicate the contract in Proposition 11.

B.1.3 Proof of Corollary 13

Characterizing the value of \( R_l \) from equation (2.7), we then take its solution and plug it into the RHS of equation (2.8). The RHS of equation (2.8) corresponds to the value of increasing \( R_u \) on the margin. Supposing that it is \( \leq 0 \) at \( R_l \), then the value of increasing \( R_u \) above \( R_l \) on the margin is non-positive. Moreover, by MLRP the RHS of equation (2.8) is non-positive for all \( R > R_l \). As a result, we move to a corner solution where \( R_u = R_l \). Note that \( R_l \) may fall below the value implied by equation (2.7), but over the region below this value, the bank
prefers to issue standard debt, per the optimality condition of equation (2.7).

**B.1.4 Proof of Proposition 14**

Consider the program of the social planner

$$\max_{L_1, Y_0} E [c_2 | e = H]$$

subject to

$$E \left[ c_2 \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \bigg| e = H \right] \geq BY_0$$

$$Y_0 - A = \int_{R_{xu}} \gamma(\Omega) RY_0 f_H(R) dR + \int_{R_{xu}} L_1(R) f_H(R) dR$$

$$R \geq R' \Rightarrow L_1(R) \geq L_1(R')$$

$$c_2 \geq (1 - a(R)) bY_0$$

$$\Omega = \int a(R) Rf_H(R) dR$$

The proof follows as in the proof of Proposition 11. Redefine the payoff space over \(x(R)\) and solve for the optimal contract without imposing monotonicity. The first order condition for \(c_2(R)\) is the same as in the proof of Proposition 11, since \(c_2(R)\) does not directly affect \(\Omega\). This implies as before that we obtain a pooling region at the top.

As before, take \(R_u\) and \(x_u\) as given, and solve for the optimal contract for \(R \leq R_u\). The same steps imply that implementable contracts must satisfy \(R_u < R^*\). The FOC for optimal liquidations \(a(R)\) is now

$$\frac{\partial L}{\partial a(R)} \propto \lambda \left( \gamma(\Omega) - 1 \right) RY_0 f_H(R) + \chi(R)bRY_0 f_H(R) + \nu(R) (1 - \gamma) RY_0 f_H(R)$$

$$+ \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial a(R)} \lambda \int_{R^*} a(R') R' Y_0 f_H(R') dR'$$

Substituting in the derivative \(\frac{\partial \Omega}{\partial a(R)} = R f_H(R)\), we obtain

$$\frac{\partial L}{\partial a(R)} \propto \lambda \left( \gamma - 1 \right) + \chi(R)b + \nu(R) (1 - \gamma) + \frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R^*} a(R') R' f_H(R') dR'$$

The additional wedge \(\frac{\partial \gamma(\Omega)}{\partial \Omega} \lambda \int_{R^*} a(R') R' f_H(R') dR'\) is negative and independent of \(R\). The
same steps apply as in the proof of Proposition 11, yielding a liquidation threshold rule \( R_l \). Because as before \( R_u < R^* \), we have \( x(R) = \min \{ (1 - b)RY_0, x_u \} \) in the region \( R_l \leq R \leq R_u \). Thus, the set of candidate optimal contracts is the same as in the private equilibrium, and the implementation of Corollary 12 holds.

Lastly, we characterize the optimal choices of \( R_l \) and \( R_u \) for interior solutions. The optimality condition for \( R_u \) is identical to the private optimality condition, since it does not affect the liquidation value. By contrast, the social optimality condition for \( R_l \) satisfies

\[
b + \lambda (1 - b) - \gamma = \mu b \left( \frac{f_l(R_l)}{f_H(R_l)} - 1 \right) + \frac{\lambda}{R_l Y_0 f_H(R_l)} \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} RY_0 f_H(R) dR.
\]

Substituting in \( \frac{\partial \Omega}{\partial R_l} = R_l f_H(R_l) \) and rearranging, we obtain

\[
\mu b \left( \frac{f_l(R_l)}{f_H(R_l)} - 1 \right) = b + \lambda (1 - b) - \gamma - \lambda \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} Rf_H(R) dR.
\]

This completes the proof.

**B.1.5 Proof of Proposition 15**

The proof proceeds by contradiction. Suppose that the equilibrium optimal contract sets \( R_u > R_l \geq R^{BO} \). Because \( R_l \geq R^{BO} \), banks are bailed out when insolvent in any state \( R \geq R^{BO} \). Consider any single bank writing a contract, and suppose that bank offers an alternate contract that sets \( R'_l = R_u \), but otherwise leaves the liability contract otherwise unchanged. Since the bank is now bailed out in states \( R_l \leq R \leq R_u \) rather than bailed in, investor repayment increases, the participation constraint is relaxed, and investors increase date 0 payment to the bank. Suppose the bank immediately consumes that additional payment, so that project scale is unaffected. The consumption profile of the bank after date 0 under the revised contract is identical to the original, so that the contract remains incentive compatible. Moreover, bank welfare is strictly higher under the revised contract than the original, because the bank consumes the additional date 0 payment from investors. But then this contract yields higher utility than the equilibrium contract, contradicting that it the
equilibrium contract was optimal. As a result, if in equilibrium \( R_l \geq R^{BO} \), then \( R_u = R_I \).

**B.1.6 Proof of Proposition 16**

We adopt the following proof strategy. We will consider a contract that results in bailouts, and show that it is equivalent to a contract that: (1) features bail-ins (rather than bailouts) ex post; and, (2) implements an ex ante lump sum transfer from taxpayers to the bank. Because ex ante transfers from taxpayers to banks are assumed undesirable, it follows immediately that the same (bail in) contract without the ex ante lump sum transfer dominates the bailout contract. Finally, we will derive the required condition on \( \tau \) so that ex ante transfers from taxpayers to banks are indeed not optimal.

Define contracts over a space \( \{R_l, R_u, T\} \), where \( T \) is an ex ante lump-sum transfer from the government (via taxpayers) to banks, above and beyond any bailouts. We use these transfers as an accounting tool to compare contracts.

Consider an implementable contract \( \Gamma = \{R_l, R_u, 0\} \) with \( R_l > R^{BO} \). Let \( T (R_l, R_u) \) be the (ex ante) value of the bailout transfer under this contract, and define an alternative contract \( \Gamma' \) by \( R'_l = \min \{R_l, R^{BO}\} \), \( R'_u = R_u \), and \( T' = T (R_l, R_u) \). The contract \( \Gamma' \) is implementable: bank consumption is identical, and investor repayment is satisfied by ex ante transfers rather than ex post (bailout) transfers. As a result, welfare under \( \Gamma' \) is at least as high as under the contract \( \Gamma \).

Now, consider an alternate contract \( \Gamma'' = \{R'_l, R'_u, 0\} \). This contract is implementable because it generates the same consumption-to-asset ratio \( c_2(R)/Y_0 \) as contract \( \Gamma' \), with only the project scale \( Y_0 \) being different. In order to compare the welfare of these contracts, note that for any incentive compatible bank contract with thresholds \( \{R_l, R_u\} \) and transfers \( T \),

\[5\text{ Note that it is possible that the bank may, in the presence of bailouts, no longer find it optimal to offer a contract that enforces high effort, due to the bailout guarantee. The above argument is unaffected. We can generally rule out the possibility that bailouts induce low effort by introducing aggregate risk and assuming that the probability of bailout states is not too high.}

\[6\text{ We will not have to consider political costs of bailouts for the proof, but we state “at least as high” for formality.} \]
equilibrium bank welfare is given by

\[ V(R_l, R_u)Y_0(R_l, R_u, T|R_l^*, R_u^*) \]

where we have defined

\[ V(R_l, R_u) = \int_{R_l}^{R_u} b R f_H(R) dR + \int_R^{R_u} (R - (1 - b) R_u) f_H(R) dR. \]

Total investor repayment under the contract \( \Pi \) per unit of scale is given by

\[ \Pi = \int_{R_l}^{R_l} \gamma R f_H(R) dR + \int_{R_l}^{R_u} (1 - b) R f_H(R) dR + \int_{R_u}^{R_l} (1 - b) R_u f_H(R) dR \]

so that the project scale is given by

\[ Y_0(R_l, R_u | R_l^*, R_u^*) = \frac{A + T}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)}. \]

Substituting in, we obtain equilibrium bank welfare under the contract as

\[ \frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)} (A + T). \]

Social welfare at date 0, given a lump sum tax cost \( \tau > 1 \), is given by

\[ \frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)} (A + T) - \tau T \]

Now, we can compare contracts \( \Gamma' \) and \( \Gamma'' \). Because these two contracts differ only in the date 0 lump sum transfer but feature the same thresholds, a sufficient condition for the welfare gain from switching to contract \( \Gamma'' \) from contract \( \Gamma' \) is

\[ \tau T' - \frac{V(R_l', R_u')}{1 - \Pi(R_l', R_u' | R_l^*, R_u^*)} T' \geq 0 \]

or in other words

\[ \tau \geq \max_{R_l, R_u} \frac{V(R_l, R_u)}{1 - \Pi(R_l, R_u | R_l^*, R_u^*)}. \]

Under this condition, contract \( \Gamma'' \) is preferable to the contract \( \Gamma' \). Because contract \( \Gamma' \) yields at least as high welfare as contract \( \Gamma \), then contract \( \Gamma'' \) is preferable to contract \( \Gamma \). In

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7 We can ignore the political costs of bailouts.
other words, there are no bailouts.

Note that because \( R_l \leq R^{BO} \) acts as a constraint on the socially optimal contract, it immediately follows that bank welfare is non-decreasing in \( R^{BO} \). In particular, if any contract \( \Gamma = \{ R_l, R_u, 0 \} \) that is implementable given \( R^{BO} \) is also implementable for any \( R^{BO'} \geq R^{BO} \), implying that social welfare cannot be lower. Lastly, recall that social welfare and bank welfare are the same in the absence of bailouts.

### B.1.7 Proof of Lemma 17

If the best equilibrium is to have no liquidations, it must set \( D_2 \leq RY_1 \). Short-term debt is then always repaid in full, and \( \bar{q}_1^D = 1 \), since it is senior. Because \( RY_1 < D_1 \), the bank cannot refinance itself with short-term debt \( D_2 \leq RY_1 \) alone, and therefore must raise some long-term debt. Long-term debt will not be fully repaid, and has price \( q_1^L < 1 \). Given the lower long-term debt price, the “no rat race” condition of equation 2.13 binds and we have \( D_2 = RY_1 \). The total funds raised from long-term debt must satisfy the repayment condition

\[
D_1 - RY_1 = \int_{R} \min \{ L_3, R_2 Y_1 - RY_1 \} f_2(R_2) dR_2.
\]

Noting that the RHS is increasing in \( L_3 \), there is a unique solution \( L_3 \) to this equation. From here, \( q_1^L \) is given by the definition \( q_1^L = \frac{D_1 - RY_1}{L_3} \). The prices and quantities satisfy the no rat race conditions by construction. As a result, the best equilibrium has unique prices and quantities, and does not feature liquidations.

### B.1.8 Proof of Proposition 18

Suppose that markets quote \( q_1^L = 0 \). The bank can only raise funds by issuing short-term debt, with maximum raisable funds

\[
\sup_{D_2 \leq R_b Y_1} \int_{R}^{D_2 / Y_1 \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 / Y_1}^R D_2 f_2(R_2) dR_2
\]
Define \( d_2 = D_2/Y_1 \), then the bank cannot refinance itself if
\[
\frac{D_1}{Y_1} > \sup_{d_2 \leq R_0} \int_{[d_2, R]} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{R} d_2 f_2(R_2) dR_2.
\]

The right-hand side is a constant that does not depend on \( D_1, Y_1 \), or \( R \). As a result, we obtain a threshold rule in \( Y_1 \) (given \( D_1 \)), which implies a threshold rule in \( R^* \), so that rollover crises may exist when \( R \leq R^* \). Lastly, note that equation (2.15) guarantees that the existing stock \( D_1 \) is sufficient to liquidate the bank whenever there is a rollover crisis. As a result, a hypothetical unit of new long-term debt receives no recovery value in liquidation, completing the equilibrium.

Finally, at \( R = R_l \) we have
\[
\frac{D_1}{R_l Y_0} = (1 - b) > \sup_{d_2 \leq R_0} \int_{[d_2, R]} \gamma R_2 f_2(R_2) dR_2 + \int_{d_2}^{R} d_2 f_2(R_2) dR_2
\]
so that a rollover crisis equilibrium always exists at \( R = R_l \). Hence, there is always an interval of existence \( R \in [R_l, R^*] \).

\textbf{B.1.9 Proof of Proposition 19}

Consider the fixed point problem
\[
\gamma^* = \gamma \left( \epsilon + \int_{R_l}^{R} R f_H(R) dR + p \int_{R_l}^{R^*} R f_H(R) dR \right).
\]

Totally differentiating in \( \epsilon \) and evaluating at \( \epsilon = 0 \), we obtain
\[
\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\partial \gamma}{\partial \Omega^*} \left( 1 + p \frac{\partial R^*}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*) \right)
\]
which rearranges to
\[
\frac{\partial \gamma^*}{\partial \epsilon} = \frac{\frac{\partial \gamma}{\partial \Omega^*}}{1 - \frac{\partial \gamma}{\partial \Omega^*} p \frac{\partial R^*}{\partial \gamma^*} R^* f_H(R^*)}.
\]
Next, evaluating the derivative in $\frac{\partial \Omega^*}{\partial \epsilon}$ and substituting in, we obtain

$$\frac{\partial \Omega^*}{\partial \epsilon} = 1 + p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \epsilon} R^* f_H(R^*)$$

$$= 1 + \frac{p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}{1 - p \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} R^* f_H(R^*)}.$$

Finally, differentiating equation (2.16) in $\gamma^*$

$$\frac{\partial R^*}{\partial \gamma^*} = -\frac{(R^*)^2 Y_0}{D_1} \int_{d_1}^{d_2} R_2 f_2(R_2) dR_2$$

and substituting in, we obtain the final result

$$\frac{\partial \Omega^*}{\partial \epsilon} = 1 + \frac{p \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}{1 - p \left| \frac{\partial R^*(\gamma^*)}{\partial \gamma^*} \right| R^* f_H(R^*)}.$$

Noting that $\frac{\partial R^*}{\partial \gamma^*}$ and $\frac{\partial \gamma^*}{\partial \epsilon}$ are negative, we have adopted the absolute value notation for clarity.

**B.1.10 Proof of Proposition 20**

Suppose that markets quote $q_1^L = 0$. Let $C^0$ be the space of tuples $C = (q_1, D_2, L_3)$ with $q_1^L = 0$. A rollover crisis equilibrium exists under the vacuous covenant if there does not exist any tuple $C \in C^0$ such that $q_1^L D_2 \geq D_1$ and that is consistent with the no rat race conditions (2.13) and (2.14).

Now, consider an alternate covenant $C_1$. When markets quote $q_1^L = 0$, the set of tuples that satisfy covenant $C_1$ is $C^1 \subset C^0$. But since no tuple $C \in C^0$ refinance the bank, no tuple $C \in C^1$ refinance the bank either. As a result, existence of rollover crises under the vacuous covenant implies existence of rollover crises under any other covenant.

**B.1.11 Proof of Proposition 21**

The modified private sector no rat race conditions, given pari passu claims, are

$$q_1^D D_2 \leq \int_{D_2 + D_2^{LOLR} \geq R_2 Y_1} \gamma R_2 Y_1 \frac{D_2}{D_2 + D_2^{LOLR}} f_2(R_2) dR_2 + \int_{D_2 + D_2^{LOLR} \leq R_2 Y_1} D_2 f_2(R_2) dR_2$$

(B.2)
Conjecture a rollover crisis equilibrium with \( q_1^L = 0 \). Suppose the bank borrows best equilibrium quantities from the LOLR, and does not borrow from the private sector. The break-even LOLR debt prices are given by

\[
q_{1,LOLR}^D = \frac{1}{D_2} \int_{D_2 \geq R_2 Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2 Y_1} \overline{D}_2 f_2(R_2) dR_2 = \overline{q}_1^D
\]

\[
q_{1,LOLR}^L = \frac{1}{L_3} \int_{L_3 \leq R_2 Y_1} \min \{ (R_2 Y_1 - D_2) , L_3 \} f_2(R_2) dR_2 = \overline{q}_1^L
\]

which are the best equilibrium prices, given that \( \overline{D}_2 = R Y_1 \). Therefore, the bank raises total funds \( \overline{q}_1^D \overline{D}_2 + \overline{q}_1^L L_3 = D_1 \) from the LOLR, and successfully refines itself. The rollover crisis equilibrium is eliminated.

Finally, because the LOLR and private sector rank pari passu, in equilibrium \( q_{1,LOLR}^D = q_1^D \) and \( q_{1,LOLR}^L = q_1^L \). Equilibrium market prices are therefore best equilibrium prices, and the bank can refinance itself entirely from the private sector using best equilibrium issuances. The best equilibrium is restored.

**B.1.12 Proof of Proposition 22**

Suppose that a rollover crisis equilibrium occurs, and consider the value of a hypothetical unit of new private sector long-term debt. The government guarantee implies the unit is repaid its full face value of 1, so that \( q_1^L = 1 \). But then the bank can refinance itself with long-term debt, and a rollover crisis equilibrium does not exist.\(^8\)

Consider now instead the best equilibrium prices \( \overline{q}_1 \). The bank can always refinance itself with the best equilibrium quantities \( (\overline{D}_2, \overline{L}_3) \), and therefore survives until period 2.

\(^8\)If \( q_1^L = 1 \), then the stronger no rat race conditions we have applied in the main paper will be violated. Instead, we need to use the weaker form no rat race conditions in Appendix B.2.10. Under these conditions, we have the quoted price \( \overline{q}_1^L = 1 \). The bank chooses issuance \( (\overline{D}_2, \overline{L}_3) = (0, \overline{D}_2 + \overline{L}_3) \), which lowers the price to the level in equation (B.11) and gives us an equilibrium.
Guarantees expire at the end of period 1, and so are never filled, regardless of the refinancing package chosen by the bank. As a result, the bank indeed chooses the best equilibrium quantities \((D_2, L_3)\), and the best equilibrium is restored.

B.2 Extensions

B.2.1 Role of Agency Problems and Liquidation Costs

Our model features three ingredients that are jointly necessary to generate contracts that consist of combinations of contingent and standard debt. First, there is an ex ante incentive problem, that is \(B > 0\), which implies a conventional incentive-based deviation from Modigliani-Miller. Second, there is a limited pledgeability problem (e.g. a continuation incentive problem), that is \(b > 0\). Finally, there are costly liquidations, that is \(\gamma < 1\). In the absence of any one of these elements, contracts in our model would not combine contingent and standard debt.

Corollary 51. Optimal contracts do not consist of both contingent and standard debt if \(B = 0\), \(b = 0\), or \(\gamma = 1\). In particular, considering each deviation:

1. If \(B = 0\), then optimal contracts feature only equity (without loss of generality).
2. If \(b = 0\), then optimal contracts feature only bail-in debt.
3. If \(\gamma = 1\), then optimal contracts feature only feature standard debt.

When \(B = 0\), a standard Modigliani-Miller logic applies. The bank can ensure incentive compatibility with any monotone consumption policy \(c_2(R)\), and in particular has no need of or desire for liquidations. As a result, without loss of generality the bank uses entirely equity financing. However, the second and third cases show that \(B > 0\) is not sufficient to generate contracts that combine contingent and standard debt. When \(B > 0\), optimal contracts employ some debt instrument for ex ante incentive reasons. If \(b = 0\), then all income is pledgeable to investors, and the bank can set \(c_2(R) = 0\) without liquidating. Banks use only bail-in debt. If \(\gamma = 1\) but \(b > 0\), there is a limit to pledgeable income,
but no bankruptcy costs from liquidation. Banks can repay any amount \( x_1(R) \leq RY_0 \) by liquidating bank projects, and the pledgeability constraint ceases to be relevant. Banks use only standard debt.

In both the second and third cases, the key property of debt is the full transfer of value of the bank from the bank to investors in low-return states. This corresponds to a common understanding of debt in the optimal contracting literature: the core property of debt is its payoff profile \( x_1(R) = \min\{RY_0, R_uY_0\} \),\(^9\) which corresponds to a full value transfer in low-return states. In the absence of pledgeability limitations, this value transfer is achieved with bail-in debt. In the absence of bankruptcy costs, this value transfer is achieved with standard debt.

If there are incentive problems \((B > 0)\), limited pledgeable income \((b > 0)\), and bankruptcy costs \((\gamma < 1 - b)\), then bail-in debt cannot enact full value transfer, whereas standard debt can enact full value transfer, but comes at a resource cost. A role emerges for both forms of debt in the optimal contract.

This suggests why banks are a natural candidate for this hybrid capital structure. Bank liquidations tend to be costly from an investor recovery perspective. Banks are also likely to face incentive problems in their lending that limit ability to pledge full returns to investors.

**Proof of Corollary 51**

We split the proof into the different cases.

**Case 1:** Suppose first that \( B = 0 \), but \( b > 0 \) and \( \gamma(s) < 1 - b \). We impose \((1 - b)E[R] < 1\) to obtain a finite solution.

The result is a Modigliani-Miller type result. Incentive compatibility is now

\[
E \left[ c_2(R) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) | e = H \right] \geq 0.
\]

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\(^9\)For example, see Hébert (2018).
Let \( c_2(R) \) be some monotone consumption rule. We have

\[
E \left[ c_2(R) \left( 1 - \frac{f_L(R)}{f_H(R)} \right) \mid e = H \right] = \text{cov} \left( c_2(R), 1 - \frac{f_L(R)}{f_H(R)} \right) \geq 0
\]

where the inequality follows from MLRP. As a result, any monotone consumption rule is implementable. We can span the frontier of expected repayment splits between banks and investors, with \( \Pi \in [0, (1-b)E[R]] \) to investors and \((1-b)E[R] - \Pi\) to bankers, with monotone consumption rules (e.g. equity). Because all agents are risk-neutral, all that matters is the expected revenue division, and there is no need to liquidate the bank. Equity allocation rules \( E \in [0,1-b] \), with investors receiving shares \( E \) and banks retaining equity \( 1-E \), generate monotone consumption profiles and so are incentive compatible. They also span the range of possible surplus divisions. As a result, pure equity constitutes an optimal contract.

**Case 2**: Consider next \( b = 0 \). The RHS of (2.7) then collapses to \( \lambda(1-\gamma) \) while the LHS collapses to 0, and so banks never choose to liquidate. Optimal contracts use only bail-in debt.

**Case 3**: Consider finally \( \gamma = 1 \). Any face value \( L_1(R) \leq \text{RY}_0 \) can then be repaid by liquidating assets, so that bank consumption is \( c_2(R) = \text{RY}_0 - L_1(R) \) for any \( L_1(R) \leq \text{RY}_0 \). Therefore for any liability structure \( L_1(R) \), we can define

\[
(c_2(R), x(R)) = \begin{cases} 
(\text{RY}_0 - L_1(R), L_1(R)), & L_1(R) \leq \text{RY}_0 \\
(0, \text{RY}_0), & L_1(R) \geq \text{RY}_0 
\end{cases}
\]

where the relevant liquidation function \( \alpha(R) \in [0,1] \) is defined from the liability structure. For example, without loss of generality we could define \( \alpha(R) = \frac{x(R)}{\text{RY}_0} \). As a result, minimum pledgeability never binds.

Defining the problem in the repayment space, we then have

\[
\max \int_{R} [\text{RY}_0 - x(R)] f_H(R) dR,
\]
subject to
\[
\int_R [RY_0 - x(R)] (f_H(R) - f_L(R)) dR \geq BY_0
\]
\[Y_0 - A = \int_R x(R)f_H(R)dR\]
\[R \geq R' \Rightarrow x(R) \geq x(R')\]
with \(0 \leq x(R) \leq RY_0\). Relaxing monotonicity, the FOC for \(x(R)\) is given by
\[
\frac{\partial L}{\partial x(R)} = \left[ -1 - \mu \left( 1 - \frac{f_L(R)}{f_H(R)} \right) + \lambda \right] f_H(R)
\]
yielding a threshold rule \(R^*\) such that \(x(R) = RY_0\) for \(R \leq R^*\) and \(x(R) = 0\) for \(R \geq R^*\). This results in an upper pooling region \(R_u\) with liabilities \(x_u\). Because \(R_u < R^*\) as in the proof of Proposition 11, we have \(x(R) = RY_0\) for all \(R \leq R_u\). Continuity implies \(L_1(R) = R_uY_0\) for all \(R\), and so the contract is standard debt.

### B.2.2 Pareto Efficiency

We now study whether the socially optimal contract in Section 2.3 is Pareto efficient relative to the privately optimal contract. Because investors receive zero surplus from bank contracts, contracts are always (weakly) Pareto efficient from their perspective. We only need to consider arbitrageurs, who purchase liquidated bank project.

To show a Pareto improvement can be obtained, we will require transfers at date 0 to arbitrageurs. To develop this, we assume arbitrageurs can consume at date 0 at utility \(\frac{1}{\beta}c^A\) for \(\beta < 1\). Arbitrageurs are borrowing constrained at date 1 and so cannot borrow against future income. As a result, arbitrageur welfare is given by
\[
\frac{1}{\beta}c^A + (\mathcal{F}(\Omega) - \gamma(\Omega))\Omega Y_0.
\]
From here, we obtain the following result.

**Proposition 52.** Let \(\frac{\partial \gamma}{\partial \Omega} < 0\). Then, there exists a \(\beta > 0\) such that if \(\beta < \beta\), the movement from the privately optimal contract to the socially optimal contract can be made Pareto efficient with an ex ante lump sum transfer from banks to arbitrageurs.
Pareto efficient improvements arise because arbitrageurs are borrowing constrained. Efficiency is achieved by transferring resources to arbitrageurs at date 0 in order to compensate them for resource losses from lower surplus from bank liquidations.

**Proof of Proposition 52**

Using incentive compatibility, we associate a level of $R_l$ with a level of $R_u$:

$$
\int_{R_l}^{R_u(R_l)} b R (f_H(R) - f_L(R)) dR + \int_{R_u(R_l)}^{R} [R - (1 - b)R_u(R_l)] (f_H(R) - f_L(R)) = B.
$$

Define investor repayment per unit of investment scale by:

$$
\Pi(R_l) = \int_{R_l}^{R} \gamma(R_l) R f_H(R) dR + \int_{R_l}^{R_u(R_l)} (1 - b) R f_H(R) dR + \int_{R_u(R_l)}^{R} (1 - b) R_u(R_l) f_H(R) dR.
$$

Let $T$ be a lump sum tax on banks. The scale of the bank is then given by:

$$
Y_0 = A - T + \Pi(R_l) Y_0 \Rightarrow Y_0 = \frac{A - T}{1 - \Pi(R_l)}.
$$

From here, we can obtain bank utility

$$
V(R_l, T) = V(R_l) \frac{A - T}{1 - \Pi(R_l)},
$$

where $V$ is given by

$$
V(R_l) = \int_{R_l}^{R_u(R_l)} b R f_H(R) dR + \int_{R_u(R_l)}^{R} [R - (1 - b)R_u(R_l)] f_H(R) dR.
$$

Now, let $R^S_l$ be the socially optimal contract and let $R^P_l$ be the privately optimal contract. Define $T \left( R^P_l, R^S_l \right)$ by

$$
V \left( R^S_l, T \left( R^P_l, R^S_l \right) \right) = V(R^P_l, 0).
$$

which is the maximum transfer that can be taken from banks, while moving from the private to social contract, that leaves them indifferent to the contract change. Solving for $T \left( R^P_l, R^S_l \right)$, we obtain

$$
T \left( R^P_l, R^S_l \right) = A \left[ 1 - \frac{V \left( R^P_l, 0 \right)}{V \left( R^S_l, 0 \right)} \right] = A \left[ 1 - \frac{V^P}{V^S} \right].
$$

Now, let $L^A \left( R^P_l, R^S_l, T \left( R^P_l, R^S_l \right) \right)$ be the ex post losses to arbitrageurs from moving from
the contract \((R_p^T, 0)\) to the contract \((R_i^S, T (R_i^P, R_i^S))\). These losses can be bounded above by the surplus they receive from purchasing liquidated projects in the private equilibrium, that is

\[ |L^A| \leq Y^P (\mathcal{F}(\Omega^P) - \gamma(\Omega^P)\Omega^P) \]

Suppose that we transfer \(T (R_i^P, R_i^S)\) to arbitrageurs to compensate them for the losses \(L^A\). The total change in arbitrageur welfare can be bounded below as follows

\[
\Delta^A = \frac{1}{\beta} T \left( R_i^P, R_i^S \right) - |L|
\geq \frac{1}{\beta} T \left( R_i^P, R_i^S \right) - Y^P (\mathcal{F}(\Omega^P) - \gamma(\Omega^P)\Omega^P)
= \frac{1}{\beta} A \left[ 1 - \frac{V^P}{V^S} \right] - Y^P (\mathcal{F}(\Omega^P) - \gamma(\Omega^P)\Omega^P)
\]

This gives us the sufficient condition

\[
\beta \leq \frac{A \left[ 1 - \frac{V^P}{V^S} \right]}{Y^P (\mathcal{F}(\Omega^P) - \gamma(\Omega^P)\Omega^P)} \equiv \bar{\beta}
\]

for achieving Pareto efficiency when moving from the privately optimal contract to the socially optimal contract.

**B.2.3 Aggregate Risk**

To incorporate aggregate risk into the model, we add an aggregate state \(s \in S\) of the economy at date 1. For expositional simplicity, we assume that \(S\) is a finite set, with probability measure \(\pi(s)\).

The aggregate state \(s\) affects the return distribution, so that we have \(f_c(R|s)\). All contracts can be written on the aggregate state. MLRP now applies contingent on the aggregate state, and liability monotonicity is also contingent on the aggregate state.

From here, the characterization of privately optimal contracts follows almost identically to before.
Proposition 53. A privately optimal bank contract has a liability structure

\[
L_1(R, s) = \begin{cases} 
(1 - b)R_l(s)Y_0, & R \leq R_l(s) \\
(1 - b)RY_0, & R_l < R \leq R_u(s) \\
(1 - b)R_u(s)Y_0, & R_u(s) \leq R 
\end{cases}
\]

where \(0 \leq R_l(s) \leq R_u(s) \leq \bar{R}\) are aggregate-state-contingent thresholds. The bank is liquidated if and only if \(R \leq R_l(s)\). These thresholds, when interior and not equal, are given by

\[
\mu b \left( \frac{f_l(R_l(s)|s)}{f_H(R_l(s)|s)} - 1 \right) = b + \lambda (1 - b - \gamma(s)) \tag{B.4}
\]

\[
0 = E \left[ \lambda - 1 - \mu \left( 1 - \frac{f_l(R|s)}{f_H(R|s)} \right) \bigg| R \geq R_u(s), s, e = H \right] \tag{B.5}
\]

where \(\mu > 0\) is the Lagrange multiplier on incentive compatibility (2.2) and \(\lambda > 1\) is the Lagrange multiplier on investor participation (2.4).

Proof. The proof follows the same steps as the proof of Proposition 11. ■

In contrast to the baseline model, both instruments are contingent on the aggregate state, reflecting that the terms of bank contracts adjust to verifiable events that are beyond a bank’s control. For example, if all else equal a state \(s\) has lower returns due to an aggregate (TFP) shock, equation (2.7) implies it should have a lower liquidation threshold.\(^{10}\)

In the context of CoCos, conditioning the level of bail-in debt on both the idiosyncratic state (i.e. individual bank health) and aggregate state (i.e. banking sector health) resembles a dual price trigger.\(^{11}\)

From here, the results on the socially optimal contract proceed identically, with the state contingency. Similarly, the bailout results can also be derived, where the result is that no bail-in debt is issued for state \(s\) whenever there are bailouts in state \(s\).

\(^{10}\)See Dewatripont and Tirole (2012) for a related argument.

\(^{11}\)See e.g. Allen and Tang (2015) and McDonald (2013).
This helps to understand the limits of bank contingencies on verifiable aggregate risk. Although aggregate risk is verifiable and not a result of bank shirking, banks neglect fire sales and expect to receive bailouts in bad aggregate states. This limits the extent to which they write contingencies on aggregate risk.

**Bail-in Equivalence with Aggregate Risk**

When there is aggregate risk ($|S| > 1$), the planner can still implement the optimal contract with bail-ins, but the implementation is more complicated because $R_l(s)$ depends on the aggregate state. Without loss of generality, order the states such that $R_l(s_1) \leq \ldots \leq R_l(s_{|S|})$. For exposition, let us assume that $R_u(s) = R_u$ is constant.\(^{12}\) Under a bail-in implementation, the bail-inability of debt depends on the aggregate state. To implement the optimal contract, take $R_u$ to be a bank’s total debt. In state $s$, a portion $R_u - R_l(s)$ of debt is bail-inable, while $R_l(s)$ is non-bail-inable. If we interpret $s_1$ as a crisis and $s_{|S|}$ as a boom, decentralizing the optimal contract with ex post bail-ins implies that a larger fraction $(R_u - R_l(s_1))$ of debt is eligible to be bailed in by the planner during a crisis, while a smaller fraction $(R_u - R_l(s_{|S|}))$ is eligible to be bailed in by the planner during a boom. As a result, implementing the socially optimal contract requires a set of rules defining whether debt is bail-inable, depending on the aggregate state. Such rules either must be contractually pre-written into debt contracts, or must be written into the rules governing the operations of the bail-in authority.

In the US, such rules could be implemented using the organizational structure of the bank. Bank holding companies are required to maintain an amount of loss-absorbing debt at the level of the top-level holding company. The goal is to resolve the top-level holding company while allowing operating subsidiaries to continue operations without being affected by the resolution of the holding company. In principle, however, if a full write-down of the liabilities at the holding company level is not sufficient to recapitalize the bank, recapitalization would require bail-ins of debt at the operating subsidiaries. One

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\(^{12}\)For example, this can arise if the return distribution $f(R|s)$ does not depend on $s$ but the liquidation function $\gamma(s)$ does.
could structure the governing rules of the bail-in authority to condition the ability of that authority to resolve operating subsidiaries based on the state of the economy. Operating subsidiaries could be resolved by the bail-in authority in crises, but not in normal times.

It is not clear whether aggregate state contingent rules governing the bail-in authority could credibly be implemented and followed. A bail-in authority is likely to be tempted to recapitalize a bank if there is enough long-term debt available to do so, suggesting the potential for time inconsistency in bail-ins.

B.2.4 Macroprudential Regulation and Bail-ins

In the baseline model, the fact that banks have a single investment project means that liability-side regulation is sufficient. In practice, banks asset allocations also affect their risk profiles. We now show that macroprudential (asset-side) regulation is a necessary complement to bail-ins when banks can affect risks using both sides of their balance sheet.

We augment the model as follows. Banks choose a contractible vector \( \theta = (\theta_1, ..., \theta_N) \) of asset allocations. The total return \( R \) on bank scale \( Y_0 \) follows a density \( f_e(R|\theta) \), which depends on the allocation \( \theta \). \( f_e(R|\theta) \) satisfies MLRP (conditional on \( \theta \)) over the relevant range of allocations \( \theta \). To simplify exposition, the support of \( R \) is an interval \( [R, \bar{R}] \) that does not depend on \( \theta \). Otherwise, the setup is the same as before.\(^{13}\)

As before, optimal liability contracts combine contingent and standard debt, and the trade-off between standard and bail-in debt reflects the same forces as before.\(^{14}\) We now characterize the optimal asset allocation rule under the socially optimal contract.

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\(^{13}\) In Appendix B.2.4, we show how a standard asset allocation problem generates a density function of this form. If the shirking benefit \( B(\theta) \) depended on the allocation, e.g. because riskier assets are more difficult to monitor, the planner and banker would agree on how \( \theta \) affects \( B \). Assets in our model all sell at the same discount and generate the same fire sale spillover. If they differed in terms of liquidation discounts and fire sale spillovers, there would be an additional regulatory incentive on this margin.

\(^{14}\) Given that \( \theta \) is contractible, the proof follows the same steps as Proposition 11.
Proposition 54. The socially optimal contract has FOC for $\theta_n$

$$0 = E \left[ \lambda x(R) + c_2(R) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|\theta)/\partial \theta_n}{\partial f_H(R|\theta)/\partial \theta_n} \right) \right) \frac{\partial f_H(R|\theta)/\partial \theta_n}{f_H(R\theta)} \right]$$

$$+ \lambda E \left[ \frac{\partial \gamma}{\partial \Omega} \Omega Y_0 \cdot \left( \int_{R_l}^{R_i} R \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR \right) \right]$$

The first line of Proposition 54 reflects the private trade-off to banks of a change in asset composition, corresponding to changes in the return distribution. These changes are weighted by the (weighted) sum of payoff to investors in those states, and to banks in those states, where the weighting reflects both the direct value of payoffs, and the incentive value of payoffs. The second line of Proposition 54 reflects the social cost of changes in asset composition. The social cost arises when changes in the return distribution affect the magnitude of the fire sale spillover, by altering the measure $\Omega$ of bank liquidations. When an asset increases the probability that the banks’ total return is lower than $R_l$, larger allocations to that asset result in more severe fire sale spillovers. The social cost term penalizes investment in such assets. The social cost term exists whenever $R_l > R$, that is whenever liability-side regulation has not completely eliminated bank failures.

Proposition 54 illustrates that macroprudential (asset) regulation is a necessary complement to bail-ins (liability regulation). Macroprudential regulation and bail-ins co-exist in the regulatory regime because they control fire sales in different manners. For a given level of asset risk, bail-ins mitigate fire sales by reducing the liquidation threshold. For a given liquidation threshold, macroprudential regulation mitigates fire sales by reducing the probability that a bank will fall below that threshold. These two aspects of regulation are not generally perfect substitutes, so they co-exist under the optimal regulatory regime.

Even though macroprudential regulation and bail-ins are not perfect substitutes, Proposition 54 suggests that bail-ins are a partial substitute for macroprudential regulation. Stronger liability regulation pushes the magnitude of the additional wedge in the asset allocation

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15Macroprudential regulation in our model closely risk weights on loss-absorbing capital.
decision towards zero, by reducing the size of the liquidation region.

Multiple Assets Density Function

Suppose that there are \( N + 1 \) assets between which the bank allocates its funds. Denote \( \omega \in [\omega, \overline{\omega}] \) to be the underlying idiosyncratic state of the bank, with associated density \( f_\omega^e(\omega) \), where \( e \in \{ H, L \} \). Suppose that \( f_\omega^e(\omega) \) satisfies MLRP, so that \( \frac{\partial}{\partial \omega} \left( \frac{f_\omega^H(\omega)}{f_\omega^L(\omega)} \right) > 0 \).

Asset \( n \in \{ 1, ..., N + 1 \} \) generates a return \( R_n(\omega) \) per unit. Let \( \theta = (\theta_1, ..., \theta_{N+1}) \) be a vector that determines the asset allocations \( \theta_1 Y_0, ..., \theta_{N+1} Y_0 \). Allocations \( \theta \) satisfy a technological restriction \( \mathcal{F}(\theta) = 0 \), for example there may be a concave technology. Note that to coincide with the previous parts, we assume the technology is linear in the scale \( Y_0 \), and only (potentially) concave in the asset weights. If \( \mathcal{F}(\theta) = \sum_{n=1}^{N+1} \theta_n - 1 \), we have a simple linear technology with equal cost of investment across assets.

We invert \( \theta_{N+1} \) from \((\theta_1, ..., \theta_N)\) via \( \mathcal{F} \), so that we can internalize the constraint. We denote the total return to the bank, given an asset allocation vector \( \theta \), by

\[
R(\omega \theta) = \sum_{n=1}^{N+1} \theta_n R_n(\omega) Y_0
\]

where \( \theta_{N+1} \) is derived from the technology \( \mathcal{F}(\theta) = 0 \), given \( \theta_1, ..., \theta_N \).

Suppose that conditional on \( \theta \), there is an injective mapping between \( \omega \) and \( R \). In this case, \( R \) identifies \( \omega \), given \( \theta \), and we can write contracts on \( R \). We assume that the mapping is injective over the relevant range of asset allocations \( \theta \). For example, this will be the case if asset allocations are non-negative \( (\theta_n \geq 0) \) and individual asset returns are monotone in \( \omega \). Without loss of generality, we assume the injective mapping is monotone increasing: high states \( \omega \) identify high returns \( R \), consistent with the interpretation of \( e = H \) as “high effort.”

Denote \( R^{-1}(R|\theta) \) to be the inverse function mapping the total return \( R \) into the idiosyncratic state \( \omega \). The inverse function does not depend directly on \( e \), but rather the density will depend on \( e \). We now derive the density of \( R \), conditional on \( \theta \). We have

\[
F_e(R|\theta) = \Pr(R(\omega|\theta) \leq R|e) = \Pr(\omega \leq R^{-1}(R|\theta) \mid e) = F_e^\omega \left( R^{-1}(R|\theta) \right).
\]
Differentiating in $R$, we obtain the density function:

$$f_e(R|\theta) = f_e^{\omega} \left( R^{-1}(R|\theta) \right) \frac{\partial R^{-1}(R|\theta)}{\partial R}$$

We impose the simplifying assumption that the support $[R, R]$ of the density is invariant to the allocation $\theta$. If the support depended on the portfolio allocation, we would have boundary terms in derivatives. The principal term of relevance would be how the lower boundary of the support moves in the asset allocation, which reflects changes in the measure of the liquidation region. These effects are qualitatively the same as the direct effects of changing the measure from changes in the density. For simplicity, we keep the support fixed.

Finally, we can show that this function satisfies monotone likelihood. Differentiating the likelihood ratio in $R$, we obtain

$$\frac{d}{dR} \left( \frac{f_H(R|\theta)}{f_L(R|\theta)} \right) = \frac{d}{dR} \left( \frac{f_H^{\omega}(R^{-1}(R|\theta))}{f_L^{\omega}(R^{-1}(R|\theta))} \right)$$

$$= \frac{\partial f_H^{\omega}}{\partial \omega} \frac{\partial R^{-1}(R|\theta)}{\partial R} f_L^{\omega} - \frac{\partial f_L^{\omega}}{\partial \omega} \frac{\partial R^{-1}(R|\theta)}{\partial R} f_H^{\omega}$$

$$= \frac{\partial}{\partial \omega} \left( \frac{f_H^{\omega}}{f_L^{\omega}} \right) \frac{\partial R^{-1}(R|\theta)}{\partial R}$$

$$> 0$$

where in the last line, we have used MLRP on $f_e^{\omega}$ combined with monotonicity of $R^{-1}$.

As a result, we obtain a representation of the problem as a density $f_e(R|\theta)$. Implicitly, we differentiate in $(\theta_1, ..., \theta_N)$, where we have internalized $\theta_{N+1}$ as arising from the technology.
Proof of Proposition 54

Consider the optimal contract of the social planner. Holding fixed the debt levels \( R_i \) and \( R_u \), the derivative of the planner’s Lagrangian in \( \theta_n \) is given by

\[
0 = E \left[ c_2 \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right] + \mu E \left[ c_2 \left( \frac{\partial f_L(R|\theta)}{f_H(R|\theta)} - \frac{\partial f_L(R|\theta)}{f_H(R|\theta)} \right) \right]
+ \lambda E \left[ \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right]
+ \lambda E \left[ \int_R^{R_i} \frac{\partial \gamma(\Omega)}{\partial \Omega} \frac{\partial \Omega}{\partial \theta_n} \frac{\partial \gamma(Y_0 f_H(R|\theta))}{\partial \theta_n} dR \right]
\]

where the first two lines reflect the private bank trade-off, and the last line reflects the social trade-off. Liquidations are given by

\[
\Omega = \int_R^{R_i} f_H(R|\theta) dR
\]

so that we have

\[
\frac{\partial \Omega}{\partial \theta_n} = \int_R^{R_i} \frac{f_H(R|\theta)}{\partial \theta_n} dR.
\]

Substituting in above, we obtain

\[
0 = E \left[ \lambda x(R) + c_2(R) \left( 1 + \mu \left( 1 - \frac{\partial f_L(R|\theta)}{\partial \theta_n} \right) \right) \frac{\partial f_H(R|\theta)}{f_H(R|\theta)} \right]
+ \lambda E \left[ \frac{\partial \gamma(\Omega)}{\partial \Omega} \gamma(Y_0) \left( \int_R^{R_i} \frac{\partial f_H(R|\theta)}{\partial \theta_n} dR \right) \right]
\]

giving the result.

B.2.5 Heterogeneous Investors and the Allocation of Securities

In the baseline model, investors are homogeneous and risk neutral, so that the distribution of standard and bail-in debt among investors is irrelevant. A key practical concern is what investors should hold what form of debt, since bail-in debt holders will experience losses when it is written down. Particular concern has been expressed about protecting retail
investors from losses that are large relative to their wealth\textsuperscript{16}, and to preventing institutional investors who are potentially exposed to fire sales from bearing losses from bail-ins.\textsuperscript{17}

To capture these elements, we extend the model to include two classes of bank investors, “institutional” and “retail.” To make the problem interesting, we include aggregate risk. Institutional investors are able to invest across all banks, but still retain exposure to the aggregate state and have preferences that may depend on bank liquidation discounts. Retail investors are only able to invest in a single bank and retain exposure to the idiosyncratic return of that bank. For simplicity, we abstract away from other potential components of these investors’ portfolio choice problems, instead allowing for state dependent preferences. All investors are price takers, and purchase state-contingent payoffs from the banks they invest in. Nevertheless, we show that in equilibrium all investors purchase a combination of the standard and bail-in debt contracts issued by banks.

Denote \( q(R, s) \) the (endogenous) probability-normalized price of a unit of payoff from a bank that realizes state \((R, s)\).\textsuperscript{18} Institutional investors are indexed by \( i \in I \), have initial wealth \( w^i_0 \), and preferences \( u^i_0(c^i_0) + E[u^i_1(c^i_1|s, \gamma(s))] \). Retail investors are indexed by \( j \in J \), have initial wealth \( w^j_0 \), and preferences \( u^j_0(c^j_1) + E[u^j_1(c^j_1|s)] \). Both \( I \) and \( J \) are finite sets, and we interpret each investor type as corresponding to a continuum of (atomistic) agents of that type. Both types of agents have period-0 budget constraints given by

\[
c^k_0 + \sum_s \pi(s) \int_R q(R, s)x^k(R, s)f_H(R|s)dR = w^k_0, \quad k \in I \cup J.
\]

However, they differ in their choice of \( c_1 \). Institutional investors are able to diversify across banks, so that \( c^i_1(s) = \int_R x^i(R, s)f_H(R|s)dR \). Retail investors are not able to diversify across banks, and so have \( c^j_1(R, s) = x^j(R, s) \). Given the contract payoff \( x(R, s) \) from the bank,

\textsuperscript{16}The resolution of four Italian banks in 2015 sparked a political backlash due to losses to retail investors. Financial Times, “Italy bank rescues spark bail-in debate as anger at Renzi grows,” December 22, 2015.

\textsuperscript{17}Article 44 of BRRD states that “[m]ember states shall ensure that in order to provide for the resolvability of institutions and groups, resolution authorities limit...the extent to which other institutions hold liabilities eligible for a bail-in tool.”

\textsuperscript{18}Note that the bank will go bankrupt in some states, implying not all liabilities are repaid at full face value. For simplicity, we price units of payout directly, rather than face value.
market clearing for liabilities is given by
\[ \sum_{k \in I \cup J} \mu_k x_k^I(R, s) = x(R, s) \]
where \( \mu_k \) is the mass of investors of type \( k \in I \cup J \).

We focus on the case where the mass of retail investors is sufficiently small that it does not exhaust the returns of the bank in any state \((R, s)\). That is, \( \sum_j \mu_j x_j^I(R, s) < x(R, s) \). As a result, both retail and institutional investors price bank liabilities on the margin. We now characterize the equilibrium of the private economy without government intervention.

**Proposition 55.** Suppose that in equilibrium \( \sum_j \mu_j x_j^I(R, s) < x(R, s) \). In the private equilibrium:

1. The price \( q(R, s) = q(s) \) depends only on the aggregate state \( s \).
2. Optimal bank contracts combine standard and bail-in debt.
3. Retail investors only purchase standard debt, and their consumption profile \( c_1^I(R, s) = c_1^I(s) \) only depends on the aggregate state \( s \). Consumption profiles of retail investors are given by
   \[ \frac{\partial u_1^I(c_1^I(s)|s)}{\partial c_1^I(s)} = q(s) \frac{\partial u_0^I(c_0^I)}{\partial c_0^I} \]
4. Institutional investors purchase both standard and bail-in debt. Consumption profiles of institutional investors are given by
   \[ \frac{\partial u_1^I(c_1^I(s)|s, \gamma(s))}{\partial c_1^I(s)} = q(s) \frac{\partial u_0^I(c_0^I)}{\partial c_0^I} \]

Even though retail investors are tied to a specific bank, their equilibrium consumption profile does not depend on the idiosyncratic state. This implies not only that retail investors exclusively purchase standard debt, but also that retail investors are first in line for repayment in the event of bank liquidation. In other words, in equilibrium they purchase claims that have the highest priority for repayment. Since retail investors are often depositors, one natural interpretation of this result is that of deposit priority.\(^{19}\) However, it extends beyond

\(^{19}\)These deposits are not insured in this section, but are repaid due to their priority. In Appendix B.2.8, we
deposits, and furthermore suggests that retail bondholders may also benefit from priority. This suggests a role for non-bail-inable long-term debt, as a way to codify protection for retail investors.

Institutional investors are not exposed to the idiosyncratic state due to their ability to diversify, but are exposed to the aggregate state. Institutional investors face greater losses on the aggregate state when either they are more risk tolerant, or less exposed to bank fire sales. This suggests that the ideal holders of bail-in debt will be institutional investors with limited risk aversion (or ability to diversify using other securities) and limited commonality with the banking sector, so that they are not affected by fire sales.

Finally, consider what would happen if we relaxed the assumption $\sum_j \mu^j x_1^j(R,s) < x(R,s)$. Consider an aggregate state $s$ where $\sum_j \mu^j x_1^j(R,s) = x(R,s)$ for a range of returns $R \leq R^*$. For $R > R^*$, institutional investors are the marginal pricing agent, and $q(R,s) = q(s)$ is a constant. For $R < R^*$, retail investors are the marginal pricing agents, and $q(R,s) \geq q(s)$. Given monotone liabilities contracts, $q(R,s)$ will be falling in $R$. Contracts will still be debt, but the optimal thresholds are affected by the fact that retail investors suffer larger losses in liquidation, pushing $q(R,s)$ higher above $q(s)$. This generates an additional trade-off for the bank in deciding the optimal composition of standard and bail-in debt.

Proof of Proposition 55

Suppose that there is a state-contingent Arrow price $q(R,s) = q(s)$ that depends only on the aggregate state. Contracts still take the form of standard and bail-in debt, following the same steps as in the proof of Proposition 11.

Now, consider the investor side. Begin first with institutional investors, whose Lagrangian
is given by

\[ \mathcal{L}^i = u_0^i \left( c_0^i \right) + \sum_s \pi(s) u_1^i \left( c_1^i | s, \gamma(s) \right) + \lambda^i \left[ w_0^i - c_0^i - \sum_s \pi(s) \int_R q(R, s) x^i(R, s) f_H(R | s) dR \right] \\
+ \sum_s \pi(s) \mu^i(s) \left[ \int_R x^i(R, s) f_H(R | s) dR - c_1^i(s) \right]. \]

Given the non-negativity constraint \( x^i(R, s) \geq 0 \), we have

\[ \frac{\partial \mathcal{L}^i}{\partial x^i(R, s)} = - \left[ \lambda^i q(R, s) - \mu^i(s) \right] \pi(s) f_H(R | s) \leq 0. \]

This equation holds with equality only at the lowest value of \( q(R, s) \) in state \( s \). In other words, investors only purchase \( x^i(R, s) > 0 \) if \( q(R, s) = q(s) \), where \( q(s) \) is defined to be the lowest price of a state-contingent security for some return state \( R \) in state \( s \).

Suppose then that in equilibrium \( \sum_j \mu^j x^j(R, s) < x(R, s) \). Then, at least one institutional investor \( i \) is purchasing \( x^i(R, s) > 0 \). As a result, we have \( q(R, s) = q(s) \) for all \( R \) in state \( s \), that is the price is constant in aggregate state \( s \). Moreover, \( q(s) \lambda^i = \mu^i(s) \).

From here, we can obtain \( \lambda^i \) from the FOC for \( c_0^i \) and \( \mu^i(s) \) from the FOC for \( c_1^i \).

Substituting in, we obtain

\[ \frac{\partial u_1^i \left( c_1^i(s) | s, \gamma(s) \right)}{\partial c_1^i(s)} = \frac{\partial u_0^i \left( c_0^i \right)}{\partial c_0^i} q(s). \]

giving us the characterization of the consumption rules of institutional investors.

Finally, consider type-\( j \) (retail) investors. Given the constant price \( q(s) \), their Lagrangian is

\[ \mathcal{L}^j = u_0^j \left( c_0^j \right) + E \left[ u_1^j \left( c_1^j(R, s) | s \right) \right] + \lambda^j \left( w_0^j - c_0^j - \sum_s \pi(s) \int_R q(s) c_1^j(R, s) f_H(R | s) dR \right), \]

so that we have optimality condition for \( c_1^j(R, s) \)

\[ \frac{\partial u_1^j \left( c_1^j(R, s) | s \right)}{\partial c_1^j(R, s)} = \lambda^j q(s). \]

As a result, \( c_1^j(R, s) = c_1^j(s) \) is constant within state \( s \). The indifference condition follows immediately by combining with the FOC for \( c_0^j \). This concludes the proof.
B.2.6 Risk Aversion and Risk Shifting

The baseline model featured no role for equity-like instruments in the bank’s capital structure. We extend the model to incorporate risk aversion and risk shifting, ingredients known to generate a role for equity-like claims. Optimal contracts still feature a region of liquidations and a region of “bail-ins,” where the bank is held to its continuation agency rent. Above the bail-in region, the contract involves equity-like claims.\(^{20}\)

Banks are risk averse and have utility \(u(c_1 + c_2)\) from consumption, while investors are risk averse and have utility \(v(x_1 + x_2)\). Bank utility and marginal utility are finite at 0, and we normalize \(u(0) = 0\). We incorporate risk shifting by extending the bank’s monitoring decision to \(e \in \{L, H, RS\}\), where \(e = RS\) is “risk shifting” and \(e \in \{L, H\}\) are the high and low monitoring choices from before. Risk shifting does not generate a private benefit but affects the return density, \(f_{RS}(R)\).\(^{21}\) Define the likelihood ratios \(\lambda_{L,H}(R) = \frac{f_L(R)}{f_H(R)}\) and \(\lambda_{RS,H}(R) = \frac{f_{RS}(R)}{f_H(R)}\). Risk shifting inefficiently pushes mass towards the extremes of the distribution, which we formalize by defining a point \(R_{RS} \in [\underline{R}, \overline{R}]\) such that \(\frac{\partial \lambda_{RS,H}(R)}{\partial R} < 0\) for \(R < R_{RS}\) and \(\frac{\partial \lambda_{RS,H}(R)}{\partial R} \geq 0\) for \(R \geq R_{RS}\).

As before, we assume optimal contracts enforce \(e = H\). The no-risk-shifting constraint is

\[
\int_R u(c(R)) (f_H(R) - f_{RS}(R)) dR \geq 0 \tag{B.6}
\]

while the incentive constraint is the same as before, except with \(u(c(R))\). Investor participation is given by

\[
Y_0 - A = \int_R v(x(R)) f_H(R) dR.
\]

Define \(\overline{\lambda}_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R)\) and \(\mu = \mu_L + \mu_{RS}\).

To simplify exposition, we will assume that the characterization that follows satisfies both consumption monotonicity for the bank and liability monotonicity for investors.\(^{22}\)

\(^{20}\)See e.g. Hilscher and Raviv (2014) for analysis of CoCo design on risk shifting.

\(^{21}\)We could incorporate a private benefit or cost of risk shifting without qualitatively changing results.

\(^{22}\)Note that because both agents are risk averse, there is less scope for live-or-die contracts.
Characterization of contracts in settings that do not satisfy monotonicity is beyond the scope of this paper. Moreover, we assume that the region \(1 + \mu(1 - \lambda_H(R)) < 0\) is a connected set. This simplifies exposition.

**Proposition 56.** Let \(|S| = 1\). Suppose that the region \(1 + \mu(1 - \lambda_H(R)) < 0\) is a connected set. The privately optimal contract is as follows.

1. In the region where \(1 + \mu(1 - \lambda_H(R)) < 0\), there are liquidations and bail-ins.

2. In the region where \(1 + \mu(1 - \lambda_H(R)) \geq 0\), there are bail-ins and “equity.” The equity sharing rule is

\[
u'(c(R)) \left(1 + \mu(1 - \lambda_H(R))\right) = \lambda v' \left(RY_0 - c(R)\right)
\]

The motivations behind the liquidation region and the bail-in region are as in the baseline model. Consider next the “equity” region. First, bank risk aversion moderates payouts to the bank, smoothing the bank consumption profile on the upside and so giving away some of the equity value to investors. Second, bank consumption decreases with the average likelihood \(\lambda_H(R)\). In the region \(R \leq R_{RS}, \lambda_H(R)\) is decreasing in \(R\) and so banker consumption is increasing. However, when \(R \geq R_{RS}, \lambda_{LS,H}\) is falling while \(\lambda_{RS,H}\) is rising. This second effect, which comes from the risk shifting motivation, moderates payoffs to banks in high return states, which signal a higher likelihood that the bank engaged in risk shifting.

We could also derive the socially optimal contract, which would internalize the fire sale spillover cost of liquidations. However, conditional on not liquidating, bank and planner incentives are aligned, suggesting that the planner needs only to control the trade-off between liquidations and non-liquidations, and not the trade-off between bail-ins and “equity.”

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23If effort were a continuous choice variable that affected bank returns, there would be an incentive to govern this margin. See Mendicino et al. (2018) for a numerical study of this problem.
Proof of Proposition 56

Given the assumption of consumption monotonicity, if there is a liquidation region, it satisfies a threshold rule \( R \leq R_i \). We define the optimal contract in terms of this threshold rule and in terms of liaiblities \( x(R) \) above this threshold. The bank’s Lagrangian is given by

\[
\mathcal{L} = \int_{R \geq R_i} u(c(R)) f_H(R) dR \\
+ \mu_L \left[ \int_{R \geq R_i} u(c(R)) (f_H(R) - f_L(R)) dR - BY_0 \right] \\
+ \mu_{RS} \left[ \int_R u(c(R)) (f_H(R) - f_{RS}(R)) dR \right] \\
+ \lambda \left[ A + \int_{R \leq R_i} v(\gamma RY_0) f_H(R) dR + \int_{R \geq R_i} v(RY_0 - c(R)) f_H(R) dR - Y_0 \right] \\
+ \int_{R \geq R_i} \chi(R) [c(R) - bRY_0] f_H(R) dR
\]

Define \( \overline{\lambda}_H(R) = \frac{\mu_L}{\mu} \lambda_{L,H}(R) + \frac{\mu_{RS}}{\mu} \lambda_{RS,H}(R) \) and \( \mu = \mu_L + \mu_{RS} \). We can combine the second line and obtain

\[
\mathcal{L} = \int_{R \geq R_i} u(c(R)) f_H(R) dR \\
+ \mu \left[ \int_{R \geq R_i} u(c(R)) \left[ 1 - \overline{\lambda}_H(R) \right] f_H(R) dR - \frac{H_L}{\mu} BY_0 \right] \\
+ \lambda \left[ A + \int_{R \leq R_i} v(\gamma RY_0) f_H(R) dR + \int_{R \geq R_i} v(RY_0 - c(R)) f_H(R) dR - Y_0 \right] \\
+ \int_{R \geq R_i} \chi(R) [c(R) - bRY_0] f_H(R) dR
\]

The derivative in \( R_l \) is given by

\[
\frac{1}{f_H(R_l)} \frac{\partial \mathcal{L}}{\partial R_l} = -u(c(R_l)) \left[ 1 + \mu \left( 1 - \overline{\lambda}_H(R_l) \right) \right] + \lambda \left[ v(\gamma RY_0) - v(RY_0 - c(R_l)) \right]
\]

so that liquidations may be optimal when \( 1 + \mu \left( 1 - \overline{\lambda}_H(R_l) \right) < 0 \), that is when the average likelihood ratio is high. At low values of \( R_l \), both the risk shifting and shirking problems have high likelihoods, so that \( \overline{\lambda}_H \) is large. As a result, bank consumption contributes negatively to welfare. Provided that this negative contribution outweighs the resource cost to investors, we have \( R_l > R \).
Next, consider the region above $R_l$. The FOC for consumption $c(R)$ is

$$0 = u'(c(R)) (1 + \mu(1 - \overline{\lambda}_H(R))) - \lambda v' (RY_0 - c(R)) + \chi(R)$$

so that we have $\chi(R) > 0$ when $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$. As a result, for all values $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$, we either have liquidation or bail-in.

Finally, for $1 + \mu(1 - \overline{\lambda}_H(R)) > 0$, we either have bail-in or an interior consumption value. When consumption is interior, it satisfies a risk sharing rule

$$u'(c(R)) (1 + \mu(1 - \overline{\lambda}_H(R))) = \lambda v' (RY_0 - c(R))$$

giving us an “equity” sharing rule.

Finally, the only role of assuming $1 + \mu(1 - \overline{\lambda}_H(R)) < 0$ is a connected set in the proof is to ensure that it there are no points with $1 + \mu(1 - \overline{\lambda}_H(R)) \geq 0$ below $R_l$.

**B.2.7 Premium for standard debt**

In the baseline model, the incentive problem is the only motivation for issuance of standard debt. In practice, standard debt can enjoy a premium relative to all other instruments, meaning it can pay a lower rate of return to investors. There are two natural stories for such a premium. The first is that standard debt takes the form of demand deposits, which enjoy a liquidity premium and require a lower rate of return. The second is that standard debt enjoys preferential tax treatment. We show that contracts still feature standard and bail-in debt, and that the trade-off is largely the same up to the consideration of the return premium. We then discuss potential issues with a pure premium story for standard debt.

Suppose that standard debt has required return $1 + r$, where $r > 0$. We obtain the following result.

**Proposition 57.** Suppose the model is extended to include a premium for standard debt. Optimal contracts combine standard and bail-in debt. The private optimality condition for standard debt is

$$\mu b \left( \frac{f_i(R_i)}{f_H(R_i)} - 1 \right) = b + \lambda \left[ (1 - b) - \gamma \right] + r \left[ \lambda \left[ (1 - b) - \gamma \right] - \lambda \frac{1 - F_H(R_i)}{R_i f_H(R_i)} \right].$$

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while the optimality condition for bail-in debt is the same as in Proposition 11. The tax on R_l that decentralizes the socially optimal contract is

\[ \tau_l = -(1 + r)R_l f_H(R_l) \frac{\partial \gamma(\Omega)}{\partial \Omega} \int_R^{R_l} R Y_0 f_H(R) dR \]

while the tax on bail-in debt is \( \tau_u = 0 \).

Relative to the baseline case where \( r = 0 \), when \( r > 0 \) we have the term

\[ r \left[ \lambda [(1 - b) - \gamma] - \lambda \frac{1 - F_H(R_l)}{R_l f_H(R_l)} \right] \]

in the private optimality condition, reflecting an additional cost/benefit trade-off of increasing use of standard debt. This term contains two additional effects of the presence of the liquidity premium. On the one hand, the higher liquidity premium implies that the costs of liquidation go up, because the resources lost would have been repaid to investors who have a high willingness to pay. On the other hand, replacing bail-in debt with standard debt increases payoff to investors with high willingness to pay in non-liquidation states. The bank privately trades off these two forces in choosing the optimal standard debt level, in addition to the incentive forces.

**Premium versus Incentive Problems**

If \( r > 0 \), then the bank is willing to issue standard debt even in the absence of an incentive problem, that is if \( B = b = 0 \) and hence \( \mu = 0 \). The premium story alone can generate use of standard debt in the bank’s capital structure. However, in the absence of the incentive problem the logic of Corollary 51 applies. The bank (without loss of generality) uses equity as its other instrument.\(^{24}\) The planner can implement optimal regulation with an equity requirement. By including the incentive problem, our model provides a role for bail-in debt in optimal contracts.

\(^{24}\) As a technical aside, of course a bank with no incentive problem and an expected return greater than 1 would, given linear technology, scale up to infinity. This issue is fixed simply by assuming that banks operate a concave technology \( Y_0 = f(l_0) \) to produce projects.
What if instead $B > 0$, $b = 0$, and $r > 0$, so that standard debt has value from a premium perspective, but not from an incentive perspective (relative to bail-in debt). In this case, the optimal contract would combine standard and bail-in debt. However, this story on its own is problematic for two reasons.

The first is that because bail-ins typically apply to long-term debt, which were also non-contingent prior to the crisis, the premium story has to revolve around premiums on long-term debt, which is likely due to tax incentives. But if the government is subsidizing (non-contingent) long-term debt, this suggests it must provide some fundamental economic benefit. Our model provides a fundamental economic benefit of non-contingent long-term debt.

A second and closely related way to understand this issue is that in the event that $b = 0$, banks have strong incentives to protect themselves against liquidations by backing their non-contingent claims with liquid assets such as treasuries. This relates to a fundamental question in the banking literature: why are illiquid assets paired with fragile (often deposit) financing? Our model endogenously pairs illiquid assets with fragile (non-contingent) financing, rather than exogenously imposing it. Optimal regulation in our model respects the fundamental activity of banks: backing illiquid assets with fragile funding. A model that relies exclusively on a standard debt premium naturally lends itself to a “narrow bank” type result, where not only the planner but also banks prefer to use safe treasuries to keep the bank from ever failing. Indeed in Appendix B.2.7, we show a narrow banking result where the bank fully backs its non-contingent debt with liquid treasuries, eliminating the link between illiquid assets and fragile bank financing.

We could nevertheless adopt this view. The main result that would change is the non-optimality of bailouts (Proposition 16), which would no longer generically hold. We would be back into an incomplete markets world, in which bailouts may be desirable to mitigate fire sales, in a standard way. Moreover in the case of deposit insurance, the planner would always prefer to bail out the bank, rather than liquidating and repaying depositors. Bailing out the bank would save resources without distorting bank incentives, and so would be
strictly preferred to liquidation.

A Narrow Banking Result

We show a “narrow banking” result as a simple extension of this section. Suppose that the bank can purchase safe assets (“treasuries”), which yield a deterministic return of 1. Treasuries do not need to be monitored. To avoid earning infinite profits, suppose the bank can issue a maximum of $\bar{D}$ in non-contingent “deposits” that pay the premium $r > 0$.

First, note that if $r = 0$ but $b > 0$, we are back in the baseline model. The bank has no incentive to purchase treasures because liquidations are optimal for incentive provision.

Consider next the case where $r > 0$ but $b = 0$, so there is no incentive benefit for liquidations. The bank always issues $\bar{D}$ in deposits, since it can always back an additional unit of deposit with $\frac{1}{1+r}$ treasuries and immediately consume the surplus without otherwise affecting its contract. Conjecture an implementable contract with treasury purchases $T < \bar{D}$, bail-in debt $L$, and project scale $Y_0$. The threshold bankruptcy state of the bank is $R_{Y_0} + T = \bar{D}$, while the payoff profile to the bank is $c(R) = \max\{RY_0 + T - \bar{D} - L, 0\}$. Suppose the bank increases $T$ and $L$ both by $\epsilon$. The bank consumption profile does not change, so the contract is still incentive compatible. However, the bankruptcy threshold falls and investor repayment increases, meaning the bank can do strictly better by implementing this change and immediately consuming the surplus at date 0. Hence, $T = \bar{D}$ under the privately optimal contract.

The condition $T = \bar{D}$ tells us that banks privately find it optimal to back their deposits entirely with safe treasuries, so that there is no risk to their depositors. This resembles a “narrow banking” proposal in that non-contingent debt holders are shielded completely from risks of illiquid lending. This applies not only to insured deposits, but to any deposit-like activity (e.g. wholesale deposits). Illiquid lending is no longer associated with standard debt.
Proof of Proposition 57

Relative to the baseline model, the only change is that the participation constraint becomes

$$Y_0 - A = \int_{R_l}^{R_u} (1 + r)(1 + r)bR_lY_0 + x_1(R)\frac{dR}{Y_0f_H(R)dR}$$

where $x_1(R)$ is repayment pledged to other investors. Note that it is immediate that standard debt enjoys priority over other liabilities, since it has the lower required rate of return. The proof that optimal contracts combine standard and bail-in debt follows as in the proof of Proposition 11. As a result, the optimization problem that determines $R_l$ and $R_u$ is the same as before, except that the participation constraint is now

$$Y_0 - A = \int_{R_l}^{R_u} (1 + r)(1 + r)bR_lY_0 + (1 - b)(R - R_l)\frac{dR}{Y_0f_H(R)dR}$$

This yields the private optimality condition for $R_l$

$$0 = -bR_lY_0f_H(R_l) - \mu bR_lY_0 \left(1 - \frac{f_L(R_l)}{f_H(R_l)}\right)f_H(R_l)$$

$$+ \lambda [(1 + r)\gamma R_lY_0f_H(R_l) - (1 + r)(1 - b)R_lY_0f_H(R_l)] + \lambda \int_{R_l}^{R_u} r(1 - b)Y_0f_H(R)dR$$

which rearranges to

$$\mu b \left(\frac{f_L(R_l)}{f_H(R_l)} - 1\right) = b + \lambda [(1 - b) - \gamma] + r \left[\lambda [(1 - b) - \gamma] - \lambda \frac{1 - f_H(R_l)}{R_l f_H(R_l)}\right]$$

Because $R_u$ is not directly impacted by the liquidity premium, the optimality condition for $R_u$ is as before, assuming that $R_u > R_l$.

The planning problem features a wedge of the same form as before. The only difference is that the wedge is now weighted by $1 + r$, reflecting the higher liquidation losses. In other words, the planning problem is decentralized by the tax

$$\tau_l = -(1 + r)R_l f_H(R_l) \frac{d\gamma(\Omega)}{d\Omega} \int_{R_l}^{R_u} RY_0f_H(R)dR.$$

As before, $R_u$ does not contribute to liquidations, and therefore $\tau_u = 0$. 213
B.2.8 Insured Deposits and Bailouts

In addition to fire sale spillovers and moral hazard, another goal of bail-ins is to reduce the costs of protecting insured deposits. We consider the addition of a group of insured deposits, and explore how the planner chooses to protect depositors.

For simplicity, we assume that \( \gamma < 1 - b \) does not depend on liquidations (no fire sale spillover). We further allow for the planner to commit ex ante to the desired combination of bailouts and insurance, so that the planner can always tie their hands and commit to no bailouts if desired. As a result, bailouts in this section will only occur if they are ex ante optimal.

The bank is constrained to issue insured deposits as a fixed fraction of its total assets, that is it issues \((1 - b)R_d Y_0\) in insured deposits for some fixed threshold \(R_d > R\). We abstract away from the socially optimal determination of \(R_d\), instead focusing on how the planner chooses to protect a given set of depositors.\(^{25}\) The bank is always insolvent if \(R < R_d\), absent intervention, regardless of its other liabilities. Because deposits are insured, the planner is liable for any shortfall relative to the face value \((1 - b)R_d Y_0\). Insured deposits are always at the top of the creditor hierarchy in liquidation.\(^{26}\)

Because the bank chooses bailouts with commitment, we set the political cost \(\kappa = 0\). When the planner bails out the bank, the bailout cost is

\[
\text{Cost}_{\text{No Liquidation}} = \tau \left( (1 - b)R_d Y_0 + x(R) - (1 - b)RY_0 \right)
\]

where \(x(R)\) is any liabilities in excess of \((1 - b)R_d Y_0\) that the planner does not write down.

When the planner instead allows the bank to fail, the creditor hierarchy implies the cost to deposit insurance is

\[
\text{Cost}_{\text{Liquidation}} = \min \{ \tau \left( (1 - b)R_d Y_0 - \gamma Y_0 \right), 0 \}
\]

When \(x(R) = 0\), the cost of rescuing the bank with a bailout is lower than the cost of

\(^{25}\)For example, the planner may use deposit insurance to backstop risk averse depositors.

\(^{26}\)In practice, banks may issue wholesale funding which is not insured but runs prior to resolution.
rescuing the bank under liquidation, due to the loss of pledgeable income in liquidation.

The planner solves for the optimal contract, which includes the rescue decision (either via bailout or via liquidation and repayment by insurance). We constrain bank consumption to be monotonic, that is $c(R)$ must be nondecreasing in $R$, which was satisfied by optimal contracts in the baseline model. This implies that bailouts must be monotonic: if a an insolvent bank $R$ is bailed out, then all insolvent banks $R' \geq R$ must also be bailed out. This rules out the possibility that the planner bails out a bank with $R < R_d$ to protect depositors but liquidates a bank with $R > \frac{1-b}{\gamma} R_d$ for incentive reasons.

**Proposition 58.** Suppose that $c(R)$ must be monotonic, there are no fire sales, and there are insured deposits. The socially optimal contract consists of insured deposits $R_d$, standard debt $R_l \geq R_d$, and bail-in debt $R_u \geq R_l$. The following are true regarding the use of deposit insurance and bailouts.

1. If $R_l > R_d$, there is deposit insurance but no bailouts. The bank is liquidated when $R \leq R_l$.
2. If $R_l = R_d$, there is a threshold $R_L \leq R_d$ such that the bank is liquidated when $R \leq R_L$ and bailed out when $R_L \leq R \leq R_d$. The indifference condition is for bailouts (when interior) is

$$b + \tau (1 - b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right).$$

Proposition 58 illustrates the trade-off between two mechanisms for protecting insured deposits. Bailing out the bank reduces the taxpayer cost of deposit insurance, but provides worse incentives for the bank. Whenever the planner allows use of standard debt in excess of insured deposits, that is $R_l > R_d$, then necessarily the planner will commit to rescue depositors but not the bank. In this case, there is deposit insurance but no bailouts.

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27 A technical aside is that it is possible that the planner does not find it optimal to allow the bank to scale up as much as possible due to the cost of insuring deposits. We assume this is not the case, for example if $R_d$ is close to $R$.

28 If $c(R) > c(R')$ but $R < R'$, the bank could increase its payoff ex post by destroying assets to bring its return down to $R$. We look for contracts where value destruction is not ex post optimal.

29 If bailouts are chosen in a time-inconsistent manner and if $R_d > R^{BO}$, there will have a mixture of bailouts and insurance independent of whether or not it is desirable. The planner will optimally set $R_l = R_d$. 

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If $R_l = R_d$ and $R_L < R_d$, the planner uses bailouts ex post in order to reduce the cost of protecting depositors. This may or may not imply that the planner wishes to restrict use of insured deposits ex ante to avoid bailouts, depending on the motivation for deposit insurance. If deposit insurance is a way to provide a backstop to risk-averse depositors that the bank cannot provide itself, or if it is a way to stop sunspot runs, the planner may wish to allow enough insured deposits that it sometimes engages in bailouts.

**Proof of Proposition 58**

Due to consumption monotonicity, there is a threshold $R_L \geq R$ for bank liquidation, with $R_L = R$ corresponding to no liquidations. As in the proof of Proposition 16, there are no bailouts above $R_d$, due to the taxpayer burden. We can thus split the problem into two parts.

First, suppose that the liquidation threshold satisfies $R_L > R_d$, and suppose that the planner finds it optimal to engage in bailouts in a state $R < R_d$. By consumption monotonicity, there are also bailouts for $R_d \leq R \leq R_L$. But then because transfers to regular investors are wasteful, it is optimal to set $R_L = R_d$, as in the proof of Proposition 16. The optimal contract does not feature both $R_L > R_d$ and bailouts.

Consider then the form of the optimal contract when $R_L > R_d$. Because there are no bailouts, the social objective function is

$$\int c_2(R)f_H(R)dR - \int_{R \leq R_L} \tau \max\{(1 - b)R_d - \gamma R, 0\} Y_0 f_H(R)dR$$

while the corresponding investor participation constraint is

$$Y_0 - A = \int_{R \leq R_L} \max\{(1 - b)R_d, \gamma R\} Y_0 f_H(R)dR + \int_{R \geq R_L} ((1 - b)R_d + x(R)) f_H(R)dR$$

and where incentive compatibility is the same as in the baseline model. From here, note that the trade-off above $R_L$ is the same as in the baseline model. The model again combines standard and bail-in debt, as in the baseline model.

Consider next the optimal contract when $R_L < R_d$. $R_L$ then also corresponds to the bailout threshold, such that there are bailouts when $R_L \leq R \leq R_d$, and where $R_L = R_d$.
corresponds to no bailouts. The resulting social objective function is

\[ \int c_2(R) f_H(R) dR - \int_R^{R_L} \tau [(1 - b) R_d - \gamma R] Y_0 f_H(R) dR - \int_{R_L}^{R_d} \tau (1 - b) (R_d - R) Y_0 f_H(R) dR \]

while investor repayment is given by

\[ Y_0 - A = (1 - b) R_d Y_0 + \int_{R_d}^{R} x(R) f_H(R) dR \]

reflecting that depositors are always repaid. Finally, incentive compatibility is as in the baseline model. Optimal contracts again combine standard and bail-in debt.

Consider the choice of the liquidation threshold \( R_L \). The trade-off is the same as in the baseline model, except that an increase in the liquidation threshold leads to a tax burden on taxpayers rather than a cost to investors. That is, the FOC for the liquidation threshold is

\[ 0 = -b R_L - \mu b R_L \left( 1 - \frac{f_L(R_L)}{f_H(R_L)} \right) - \tau \left[ (1 - b) R_d - \gamma R_L - (1 - b)(R_d - R_L) \right] \]

which simplifies to

\[ b + \tau (1 - b - \gamma) = \mu b \left( \frac{f_L(R_L)}{f_H(R_L)} - 1 \right) . \]

The only change is that the effective costs of liquidations has risen, due to the greater burden on taxpayers (\( \tau > \lambda \)). If the solution to this equation features \( R_L < R_d \), then there are bailouts in states \( R_L \leq R \leq R_d \).

**B.2.9 Decentralizing the Socially Optimal Contract**

Proposition 14 describes the structure and optimality conditions of the socially optimal contract. We decentralize the optimal contract with wedges \( \tau_l(s) \) and \( \tau_u(s) \) on debt issuance, and show that tax wedges \( \tau_l(s) \) on issuance of standard debt alone are sufficient to decentralize the socially optimal contract. In other words, the planner sets \( \tau_u(s) = 0 \) for all \( s \). We do this for the case with aggregate risk.
The total tax burden $T$ of the bank is

$$T(R_l, R_u|R_l^*, R_u^*) = \sum_s \pi(s) [\tau_l(s) (R_l(s) - R_l^*(s)) + \tau_u(s) (R_u(s) - R_u^*(s))]$$

where $\{R_l, R_u\}$ are the contracts of an individual bank, and $\{R_l^*, R_u^*\}$ are the terms of the socially optimal contract. Equilibrium taxes are remitted lump-sum to banks ex ante. The modified investor participation constraint is

$$Y_0 = A - T(R_l, R_u|R_l^*, R_u^*) + E [x_1(R, s)|e = H].$$

Recall that $(1 - b)R_l(s)Y_0$ is the level of standard debt. $R_l(s)$ is therefore a measure of the equilibrium standard-debt-to-asset ratio of the bank. Similarly, $R_u(s)$ is a measure of the total-debt-to-asset ratio. Tax wedges are thus defined against debt-to-asset ratios. Indeed in practice, many regulatory requirements are expressed as ratios rather than as levels.

From here, we characterize the tax wedges that align private and social incentives on the margin and so decentralize the socially optimal contract.

**Proposition 59.** The marginal tax wedges on contingent and standard debt that decentralize the social optimum are given by

$$0 \leq \tau_l(s) = -R_l(s)f_H(R_l(s), s) \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l(s)}^{R_l^*(s)} RY_0f_H(R|s)dR$$

$$\tau_u(s) = 0$$

**Taxes on Other Liabilities**

Proposition 59 characterizes taxes for standard and bail-in debt, but admits a Lucas critique: banks may find it optimal to include a liability other than standard or bail-in debt in its capital structure as a result of the taxes. Issuance of other (non-debt) liabilities would amount to a form of regulatory arbitrage. We now characterize these wedges, which are only required for alternate liabilities that generate additional liquidations.

Starting from the socially optimal contract $\{R_l, R_u\}$, consider some alternate liability with a continuous and monotone (in $R$) face value $L_1^{alt}(R, s) \geq 0$. The privately and socially
optimal contracts feature no issuance of this liability. Its issuance cannot be negative because it is a liability.\footnote{To guarantee sufficient smoothness, we assume that bail-in debt is (on the margin) written down to restore solvency in response to a marginal increase in issuance of liability $a$. As a result, issuance of $a$ has the effect of increasing the threshold bankruptcy state, rather than causing bankruptcies over the entire region $(R_t(s), R_u(s))$. This is consistent with the notion that bail-in debt is written down to restore solvency whenever possible. As a result, the total value of liabilities is constant in the write-down region, with only the distribution of payoffs between holders of liability $a$ and bail-in debt holders changing. Because all these investors are risk neutral, these changes are efficiently priced into contracts.}

The optimal tax on $L_1^{\text{alt}}$ is a simple combination of the tax rates on standard debt for the states where $a$ causes the threshold bankruptcy state to increase. In particular, we define the tax burden resulting from $\epsilon > 0$ units of issuance of liability $L_1^{\text{alt}}$ by $\tau^{\text{alt}} \epsilon$. We obtain the following result.

**Proposition 60.** Let $\{R_l, R_u\}$ be the socially optimal contract, and $\{\tau_t\}$ the implementing tax wedges. Consider a liability with a face value $L_1^{\text{alt}}(R, s) \geq 0$ that is continuous and monotone nondecreasing in $R$. Then, the required tax rate on issuance of $L_1^{\text{alt}}$ is

$$
\tau^{\text{alt}} = \sum_{s \in S} \pi(s) \tau_t(s) \frac{L_1^{\text{alt}}(R_t(s), s)}{(1 - b) Y_0}
$$

Proposition 60 tell us that the tax on any other liability $a$ is related to the extent to which that liability moves the liquidation thresholds across states. Any alternate instrument for which $L_l^{\text{alt}}(R_t(s), s) = 0 \forall s \in S$, that is which does not increase the liquidation threshold, does not require any regulatory tax. As a result, if standard debt is the only instrument the bank issues that generates bankruptcies, then it is the only instrument that needs to be taxed.

This result may help to understand why the US requires loss-absorbing debt to be plain-vanilla and not have trigger events. Although in principle regulators could allow a richer set of CoCo instruments to qualify as regulatory capital, these instruments may be harder to evaluate and require wedges if they do not result in the appropriate set of write-downs. The regulator may simply find it easier to require that loss-absorbing debt be simple and subject to a regulatory trigger in order to prevent regulatory arbitrage.
Proof of Proposition 59

Because there is no disagreement between the bank and planner on $R_u(s)$, we can set $\tau_u(s) = 0$. By contrast for $R_l(s)$, the additional wedge in the planner’s FOC relative to the bank’s is the additional term is

$$\frac{\lambda \partial \Omega(s)}{\partial R_l(s)} \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l}^{R(s)} RY_0 f_H(R|s) dR$$

as derived in Proposition 14. Substituting $\frac{\partial \Omega(s)}{\partial R_l(s)} = R_l(s) f_H(R_l(s)|s)$ and setting equal to the bank tax burden $-\lambda \tau_l(s) \pi(s)$, we obtain

$$-\lambda \tau_l(s) \pi(s) = \lambda \pi(s) R_l(s) f_H(R_l(s)|s) \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l}^{R(s)} RY_0 f_H(R|s) dR$$

which rearranges to

$$\tau_l(s) = -R_l(s) f_H(R_l(s)|s) \frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \int_{R_l}^{R(s)} RY_0 f_H(R|s) dR$$

giving us our result. Non-negativity follows since $\frac{\partial \gamma(s, \Omega(s))}{\partial \Omega(s)} \leq 0$.

Proof of Proposition 60

Consider an alternate liability $L^{alt}_1(R, s)$ satisfying the conditions of the proposition. From Proposition 14, any such liability is not used under the optimal contract, that is $L^{alt}_1(R, s) = 0$. As a result, the first order condition for optimal use of $L^{alt}_1(R, s)$ is satisfied with potential inequality.

Since $L^{alt}_1(R, s) \geq 0$ satisfies monotonicity conditional on $s$, a marginal increase in its issuance generates a monotone liability structure. Suppose that the bank issues an amount $\epsilon$ of $L^{alt}_1(R, s)$. The marginal increase in the bankruptcy threshold for increasing $\epsilon$ marginally above 0 is given by

$$\frac{\partial R_l(s)}{\partial \epsilon} = \frac{1}{(1 - b) Y_0} L^{alt}_1(R_l(s), s).$$

As a result, the planner’s FOC is given by

$$\sum_s \frac{\partial L^{sp}_s}{\partial R_l(s)} \frac{\partial R_l(s)}{\partial \epsilon} + \sum_s \pi(s) \int_{R \geq R_0(s)} L^p_1(R, s) \left[ \lambda - 1 - \left( 1 - \frac{f_l(R|s)}{f_H(R|s)} \right) \right] f_H(R|s) dR \leq 0$$
where we have $\frac{\partial L_{SP}}{\partial R_1(s)}$ given as before. Consider instead the FOC of the bank, which is given by

$$0 \geq - \lambda \tau + \sum_s \frac{\partial L}{\partial R_1(s)} \frac{\partial R_1(s)}{\partial e} + \sum_s \pi(s) \int_{R \geq R_u(s)} L^a(R, s) \left[ \lambda - 1 - \left( 1 - \frac{f_L(R|s)}{f_H(R|s)} \right) \right] f_H(R|s) dR$$

The required tax rate that aligns the private and social incentives is

$$-\tau = \sum_s \pi(s) \frac{\partial L}{\partial R_1(s)} \frac{\partial R_1(s)}{\partial e}.$$

By construction, $-\lambda \tau \pi(s) = \frac{\partial L_{SP}}{\partial R_1(s)} - \frac{\partial L^B}{\partial R_1(s)}$, giving

$$\tau = \sum_s \pi(s) \tau \frac{\partial R_1(s)}{\partial e}.$$

Finally, substituting in $\frac{\partial R_1(s)}{\partial e} = \frac{1}{(1-b)Y_0} L^a(R_1(s), s)$, we obtain in equilibrium

$$\tau = \sum_s \pi(s) \tau \frac{L^a(R_1(s), s)}{(1-b)Y_0}$$

giving us the required tax.

**B.2.10 No Rat Race Conditions**

In the main paper, we have used equations (2.13) and (2.14) to rule out rat race dynamics. These conditions are stronger than is necessary as we discuss in this section. Suppose instead that the market quotes debt prices $\bar{q}^L_1$ and $\bar{q}^D_1$. We define the equilibrium prices associated with an issuance $(D_2, L_3)$ by

$$q^D_1 = \min\{\bar{q}^D_1, \hat{q}^D_1\} \quad (B.9)$$

$$q^L_1 = \min\{\bar{q}^L_1, \hat{q}^L_1\} \quad (B.10)$$

where $\hat{q}^D_1$ and $\hat{q}^L_1$ are the payoff-neutral prices, given by

$$\hat{q}^D_1 = \frac{1}{D_2} \left[ \int_{D_2 \geq R_2Y_1} \gamma R_2 Y_1 f_2(R_2) dR_2 + \int_{D_2 \leq R_2Y_1} D_2 f_2(R_2) dR_2 \right]$$
\[ q_1^L = \frac{1}{L_3} \left[ \int_{D_2 \leq R_2 Y_1} \min\{R_2 Y_1 - D_2, L_3\} f_2(R_2) dR_2 \right] \]

Under these conditions, equilibrium prices are the maximum between the payoff-neutral prices, and the prices quoted by the market. In other words, the bank internalizes that certain issuances may lower market prices for its debt, but can never increase them.

This distinction is not relevant for the majority of the analysis, and so we present the simpler conditions of equations (2.13) and (2.14). The simpler conditions have the benefit that the best equilibrium is unique, whereas it is not unique under the extended conditions. To illustrate why it is not unique, suppose that \((\bar{D}_2, \bar{L}_3)\) are the best equilibrium quantities. Then, there is also a best equilibrium associated with quantities \((0, \bar{D}_2 + \bar{L}_3)\). The prices are \(q_D^1 = 1\) and

\[ q_L^1 = \frac{1}{\bar{D}_2 + \bar{L}_3} \left[ \int_{R} \min\{R_2 Y_1, \bar{D}_2 + \bar{L}_3\} f_2(R_2) dR_2 \right] \quad (B.11) \]

However, this is not an equilibrium under the no rat race conditions (2.13) and (2.14), since the bank always prefers to issue \(D_2 = R Y_1\) to capitalize on the higher price of short-term debt.

Rollover crisis equilibria are not affected by this definition, because given a market price \(\bar{q}_L^1 = 0\), we have \(q_L^1 = 0\) regardless of the payoff-neutral price \(\hat{q}_L^1\).

### B.2.11 Rollover Crises and Aggregate Multiplicity

Rollover crises can generate multiple aggregate equilibria of the economy for a fixed equilibrium selection rule \(p\) for any individual bank, provided that \(\gamma\) is sufficiently sensitive to additional liquidations. This reflects another form of fragility during crisis times, similar to the feedback loop.

**Proposition 61.** Fix \(p \in (0,1)\) and suppose that \(\frac{\partial \gamma}{\partial \Omega} < 0\ \forall \Omega \geq 0\). For any aggregate equilibrium \(\gamma^*\), \(\exists c > 0\) such that if at \(\gamma^*\) we have

\[ \left| \frac{\partial \gamma}{\partial \Omega} \right| > c, \]

then there also exist at least two additional equilibria: one with lower \(\gamma\) and one with higher \(\gamma\).
When liquidation values fall rapidly in response to sell-offs, our economy becomes more fragile and subject to multiple equilibrium. Crisis times (where $\gamma$ may be highly responsive) are likely characterized by such heightened sensitivity. A bail-in regulatory regime is therefore more likely to contribute to fragility in long-term debt markets during crises. Finally, we note that Proposition 61 is generic: there is always a sufficiently high sensitivity that produces multiple aggregate equilibria. The only question is how high that elasticity is.

**Proof of Proposition 61**

Consider equation (2.17)

$$\gamma^* = \gamma \left( \int_R^{R_l} Rf_H(R) + p \int_{R_l}^{R^*(\gamma^*)} Rf_H(R)dR \right)$$

and define the gap

$$\Delta(\gamma^*) = \gamma^* - \gamma \left( \int_R^{R_l} Rf_H(R) + p \int_{R_l}^{R^*(\gamma^*)} Rf_H(R)dR \right)$$

which is the gap between the equilibrium price, and the implied equilibrium price from the liquidation function. Given an inherited contract with $R_l > R$, take the range of values $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ that can be obtained (where $\underline{\gamma} = \gamma(E[R])$ and $\overline{\gamma} = \gamma(0)$. Since $R_l > R$, we have $\Delta(\overline{\gamma}) > 0$. Since $p < 1$, we have $\Delta(\gamma) < 0$.

Every zero of $\Delta$ is an equilibrium of the economy at date 1. Suppose that we have an equilibrium at $\gamma^*$, that is $\Delta(\gamma^*) = 0$, and suppose that $\Delta'(\gamma^*) < 0$. Then, for sufficiently small $\epsilon > 0$ we have $\Delta(\gamma^* + \epsilon) < 0 < \Delta(\gamma^* - \epsilon)$. Given that $\Delta(\overline{\gamma}) < 0$ and $\Delta(\underline{\gamma}) > 0$, then by continuity we must have two additional zeros, one at a point above $\gamma^*$, and one at a point below $\gamma^*$. Given these points are zeros of $\Delta$, they are also equilibria.

Finally, we characterize when we have $\Delta'(\gamma^*) < 0$. Differentiating, we obtain the required condition

$$1 < \frac{\partial \gamma}{\partial \Omega} \frac{\partial \Omega}{\partial R^*} \frac{\partial R^*}{\partial \gamma}.$$
We have $\frac{\partial \Omega}{\partial R^*} = pR^* f_H(R^*)$. Differentiating equation (2.16), we obtain

$$-\frac{D_1}{(R^*)^2 Y_0} \frac{\partial R^*}{\partial \gamma^*} = \int_{R^*}^{d^*_2} R_2 f_2(R_2) dR_2$$

where $d^*_2$ is the solution to the supremum problem in equation (2.16). Observe that both $\frac{\partial \Omega}{\partial R^*}$ and $\frac{\partial R^*}{\partial \gamma^*}$ depend on $\gamma^*$, but not on $\frac{\partial \gamma}{\partial \Omega}$. Note that both of these depend on $\gamma$, but not on $\frac{\partial \gamma}{\partial \Omega}$. As a result, we have multiple aggregate equilibria if at $\gamma^*$ we have

$$\left| \frac{\partial \gamma}{\partial \Omega} \right| > c$$

where $c$ is given by

$$\frac{1}{c} = \left| \frac{\partial \Omega}{\partial R^*} \frac{\partial R^*}{\partial \gamma^*} \right| = pR^* f_H(R^*) \frac{(R^*)^2 Y_0}{D_1} \int_{R^*}^{d^*_2} R_2 f_2(R_2) dR_2$$

concluding the proof.

**B.2.12 Early Triggers and De Jure vs De Facto Seniority**

In our model, we abstracted away from existing long-term debt, and considered bail-ins as granting short-term debt de jure seniority over long-term debt. We now discuss the impact of early triggers, which write down existing stocks of long-term debt, and discuss the differences between de jure and de facto seniority of short-term debt over long term debt. Our discussion is high-level and preliminary, and leaves open a set of considerations for further research.

**Early Triggers**

First we ask whether early triggers - writing down existing long-term debt prior to resolution - can help recapitalize the bank and prevent rollover crises. An early trigger can be likened to a precautionary bail-in, and writes down existing long-term debt at the beginning of period 1, prior to accessing markets. Early triggers are a property of Contingent Convertible (CoCo) instruments, where the trigger (for example, based on a sufficient drop in the bank’s stock price) results in an automatic write-down or conversion from debt to equity with the
goal of alleviating debt overhang in a distressed bank.\textsuperscript{31}

Equation 2.16 in Proposition 18 is a condition that defines existence of rollover crises when there is no existing long-term debt, in other words early triggers have in effect already been applied. Once the bank is in the rollover crisis region defined in Proposition 18, it is already too late for early triggers. This is not to say early triggers have no benefits in mitigating rollover crises, as they may be able to help prevent the bank from drifting into a rollover crisis region. For example, early triggers may help a bank with weak covenants. If there is a long-term debt overhang, the bank will be tempted to roll over short-term debt rather than refinance using new long-term debt, pushing itself towards the region of rollover crises. In this case, early triggers can help by alleviating debt overhang and reducing the debt dilution incentive.

\textit{De Jure vs De Facto Seniority}

Second, we consider the difference between \textit{de jure} and \textit{de facto} seniority. Since rollover crises result from the fact that short-term debt is \textit{de jure} senior to long-term debt under bail-ins, one possible solution would be to allow for issuance of non-bail-inable long-term debt. Non-bail-inable long-term debt would rank \textit{pari passu} short-term debt in insolvency proceedings, but could not be used as a recapitalization tool. Non-bail-inability could be contractually designated, or could be implemented (e.g. in the US) by differentiating between top-tier BHC debt that can be bailed in (and is subordinated), and operating subsidiary debt which cannot be bailed in and is \textit{de jure pari passu} with subsidiary short-term debt.

Because \textit{de jure pari passu} status increases recovery values to long-term debt in resolution, hypothetical liquidation values of long-term debt will be positive, which may allow the bank to refinance itself even at the low prices. This relies fundamentally on the stability of

\textsuperscript{31}A second possible implementation of early triggers leverages the institutional structure of bail-in resolution as currently practiced in the U.S. under Title II. Write-downs of debt during a bail-in are applied at the level of the top-tier bank holding company (BHC). Long-term debt of operating subsidiaries (and intermediate BHCs) in the company structure are only eligible for write-downs once the debt of the top-tier BHC has been fully bailed in. If a BHC issues new long-term out of operating subsidiaries instead of out of the top-tier BHC, that debt will be senior to the top-tier BHCs liabilities in a bail-in resolution. This effect is similar to an early trigger.
long-term debt relative to short-term debt: even if the two instruments carried the same price, the bank may be able to refinance itself with long-term debt even when it cannot with short-term debt, because long-term debt does not induce future liquidations. To reflect this, we assume that long-term debt does not liquidate the bank at date 2, even if the bank is insolvent then. Banks refinanced at date 1 with non-bail-inable long-term debt always survive to the final period, and there are no liquidation discounts.

Formally, suppose that the liquidation value of new long-term debt is $q_{L,B}^{L,B} > 0$. The bank can issue at most $L_3 \leq R_b Y_1$ in new long-term debt while maintaining its agency rent, so that the most the bank can raise using long-term debt alone is $q_{L,B}^{L,B} R_b Y_1$. This implies that the bank can refinance itself provided that $q_{L,B}^{L,B} \geq \frac{D_1}{K_b Y_1}$.

**Proposition 62.** Rollover crises are avoided if the bankruptcy value of new long-term debt satisfies $q_{L,B}^{L,B} \geq \frac{D_1}{K_b Y_1}$.32

Suppose that new, non-bail-inable long-term debt ranks pari passu with short-term debt. In this case, its price in a hypothetical date-1 bankruptcy is $q_{L,B}^{L,B} = \frac{\gamma Y_1}{D_1}$, so that the bank is guaranteed to avoid a rollover crisis if $\frac{1}{\gamma} \left( \frac{D_1}{Y_1} \right)^2 \leq R_b$. Higher liquidation values $\gamma$ and lower short-term-debt-to-asset ratios $D_1/Y_1$ are associated with greater ability of the bank to refinance itself using long-term debt. Banks that are closer to insolvency or face large liquidation discounts (e.g. due to a crisis) are less likely to be able to avoid rollover crises this way.

A practical issue with this approach is that even if the planner opens up issuance of non-bail-inable long-term debt during stress times, short-term debt still enjoys an extent of de facto seniority over legally pari passu long-term debt, due to its shorter maturity.33 Even if long-term debt is de jure pari passu with short-term debt, the maturity subordination may push the effective value of $q_{L,B}^{L,B}$ below the threshold for viability.

Finally, we have assumed that non-bail-inable long-term debt does not contribute to future solvency problems by studying a case where the bank refinances entirely with long-

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32 The proof of Proposition 62 uses the weaker (true form of the) no rat race conditions of Appendix B.2.10.

33 See e.g. Brunnermeier and Oehmke (2013) and He and Milbradt (2016).
term debt. In practice, the bank will also roll over a potentially substantial amount of short-term debt. In this case, non-bail-inable long-term debt contributes to future solvency problems, reducing the future value to the bank. This may make this solution non-viable, or leave the bank with a trade-off between trying to avoid a current rollover crisis by issuing non-bail-inable debt, and trying to avoid future solvency issues.

In sum, non-bail-inable debt may help to alleviate the rollover crisis problem, but faces a set of issues and trade-offs. Future research exploring these trade-offs may be valuable.

**Proof of Proposition 62**

The proposition is immediate, since the bank can issue \( L_3 = R_b Y_1 \) and obtain at least \( D_1 \), except that we have to verify that the no-rat-race conditions hold. In particular, suppose there is a hypothetical price \( q_1^{L,B} > 0 \), and suppose that the bank issues long-term debt \( L_3 = R_b Y_1 \). For long-term debt to be fairly priced under this issuance, we must have

\[
q_1^L R_b Y_1 \leq \int_{\mathbb{R}} \min\{R_2 Y_1, R_b Y_1\} f_2(R_2) dR_2 = (1 - b) Y_1
\]

or in other words, we must have \( q_1^L \leq \frac{1-b}{R_b} \).

The fact that the no rat race conditions are violated when \( q_1^L > \frac{1-b}{R_b} \) owes to the fact that we defined stronger no rat race conditions than are necessary, since they were easier to work with in the main part of the paper. Instead, apply the weaker conditions of Appendix B.2.10. Now, suppose that \( q_1^{L,B} > \frac{1-b}{R_b} \). The payoff neutral price associated with the issuance \((D_2, L_3) = (0, R_b Y_1)\) is given by \( q_1^L = \frac{1-b}{R_b} \). As a result, under the true no rat race conditions, banks issuing \((D_2, L_3) = (0, R_b Y_1)\) results in a price

\[
q_1^L = \min\{q_1^{L,B}, \frac{1-b}{R_b}\} = \frac{1-b}{R_b}
\]

and so the bank can successfully refinance itself for any \( q_1^{L,B} \geq \frac{D_1}{R_b Y_1} \).
Appendix C

Appendix to Chapter 3

C.1 Proofs

C.1.1 Proof of Proposition 23

To verify given our conjectured mechanism, we verify the mechanism is locally incentive compatible at the constrained efficient allocation by studying one shot deviations. Adopting the sequence form of the problem, we have the (WLOG) date 0 value function

\[ W = \sup_{\pi_0, \tilde{\theta}_0} E_0 \sum_{t=0}^{\infty} \beta^t \left[ -v_{t-1} \left( \pi_t - E_{t-1} \left[ \pi_t|v_{t-2}, \tilde{\theta}_{t-1} \right] \right) + U_t \left( \pi_t, E_t \left[ \pi_{t+1}|v_{t-1}, \tilde{\theta}_t \right], \theta_t \right) \right]. \]

The first order condition for \( \pi_0 \) is

\[ 0 = -v_{-1} + \frac{\partial U_0}{\partial \pi_0} \]

giving the first order condition for inflation. Next, consider a change in the report \( \tilde{\theta}_0 \). By the Envelope Theorem, we have

\[
\frac{\partial W_0}{\partial \tilde{\theta}_0} = -E_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{\partial U_t}{\partial \tilde{\theta}_0} \left( \pi_{t+1} - E_t \left[ \pi_{t+1}|v_{t-1}, \tilde{\theta}_t \right] \right) \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{dU_t}{dE_t} \left( \pi_t, E_t \left[ \pi_{t+1}|v_{t-1}, \tilde{\theta}_t \right], \theta_t \right) + \beta v_{t} \right] dE_t \left[ \pi_{t+1}|v_{t-1}, \tilde{\theta}_t \right] \frac{d}{d\tilde{\theta}_0} \left[ \pi_{t+1}|v_{t-1}, \tilde{\theta}_t \right].
\]
By Law of Iterated Expectations, we obtain

$$\frac{\partial W_t}{\partial \theta_0} = -E_0 \sum_{t=0}^{\infty} \beta^{t+1} \frac{\partial u_t}{\partial \theta_0} E_t \left[ \pi_{t+1} - E_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right] \left| \theta^t \right. \right]$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{dU_t \left( \pi_{t+1} E_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right], \theta_t \right)}{dE_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right]} + \beta v_t \right] dE_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right] \left| \theta^t \right. \right]$$

Now, substituting in at the constrained efficient allocation along a truthful equilibrium, we have

$$\frac{dU_t \left( \pi_{t+1} E_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right], \theta_t \right)}{dE_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right]} + \beta v_t = 0$$

and

$$E_t \left[ \pi_{t+1} - E_t \left[ \pi_{t+1} | v_{t-1}, \hat{\theta}_t \right] \left| \theta^t \right. \right] = 0.$$ Therefore, we have the first order condition in the report \( \hat{\theta} \) also holding.

As a result, this mechanism implements the constrained efficient allocation.

### C.1.2 Proof of Lemma 24

Suppose that we take the Bellman equation:

$$W_t(\theta^t) = \int_{\theta}^{\theta_t} \frac{\partial U_t(\theta^{t-1}, s_t)}{\partial s_t} ds_t + \beta \int_{\theta}^{\theta_t} E_t \left[ W_{t+1} \frac{\partial f(\theta_{t+1} | s_t)}{\partial s_t} | s_t \right]$$

and iterate it forward once. Iterating forward once, we obtain

$$W_t(\theta^t) = \int_{\theta}^{\theta_t} E_t \left[ \frac{\partial U_t(\theta^{t-1}, s_t)}{\partial s_t} ds_t + \frac{\partial f(\theta_{t+1} | s_t)}{\partial s_t} | s_t \right]$$

$$+ \beta \left[ \int_{\theta}^{\theta_t+1} \frac{\partial U_t(\theta^{t-1}, s_t, s_{t+1})}{\partial s_{t+1}} \right]$$

$$+ E_{t+1} W_{t+2} \frac{f(\theta_{t+2} | s_{t+1})}{f(\theta_{t+2} | s_{t+1})} | s_{t+1}$$

Iterating forward, suppose that we define the following recursive operator. In particular, we define

$$B_t^0(g, \theta) = \int_{\theta}^{\theta_t} g ds_t.$$

Note that for the function \( \delta_t^0 = \frac{\partial U_t(\theta^{t-1}, s_t)}{\partial s_t} \), we have that \( B_t^0 \) is the first term in the infinite series defining \( W_t \).

Next, we define

$$B_t^1(g, \theta) = \int_{\theta}^{\theta_t} E_t \left[ \frac{\partial f(\theta_{t+1} | s_t)}{\partial s_t} | s_t \right] ds_t$$

Consider the function \( \delta_t^1 = \int_{\theta}^{\theta_t+1} \frac{\partial U_{t+1}(\theta^{t-1}, s_{t+1})}{\partial s_{t+1}} ds_{t+1} \). Taking the function \( B_t^1(g^1, \theta_t) \) and
multiplying by \( \beta \), we obtain the second term in the infinite series for \( \mathcal{W}_t \).

From here, we define a recursive operator. Consider a function \( g_t^s \) that is a date \( t + s \)
adapted function. We define the operator

\[
B^2_t \left( g_t^2, \theta_t \right) = B^1_t \left( B^1_{t+1} \left( g^2_{t+1}, \theta_{t+1} \right), \theta_t \right),
\]

so that we have

\[
B^2_t = \int^\theta_t \left[ \frac{\partial f(\theta_t | s_t)}{\partial s_t} \int^\theta_{t+1} \left[ \frac{\partial f(\theta_{t+1} | s_{t+1})}{\partial s_{t+1}} g^2_{t+1} (s_{t+1}, \theta_{t+1}) \right] ds_{t+1} \right] ds_t d\theta_{t+1}
\]

which, when \( g^2_{t+1} (s_t, s_{t+1}, \theta_{t+1}) = \int^\theta_{t+1} \frac{\partial h_{t+1} (\theta^{-1}, s_t, s_{t+1})}{\partial s_{t+1}} ds_{t+1} \) and multiplied by \( \beta^2 \), gives us the next term in the infinite series defining \( \mathcal{W}_t \). Continuously defining these recursive operators as such, and defining functions

\[
g_t^s (s_t, ..., s_{t+s-1}, \theta_{t+s}) = \int^\theta_{t+s} \frac{\partial h_{t+s} (\theta^{-1}, s_t, ..., s_{t+s})}{\partial s_{t+s}} ds_{t+s},
\]

we obtain the infinite series that characterizes \( \mathcal{W}_t \).

In other words, we can construct such recursive operators. From here, we look to simplify these operators. Let us start from the operator \( B^1_t \left( g, \theta_t \right) \). In particular, we have

\[
B^1_t \left( g, \theta_t \right) = \int^\theta_t \left[ \frac{\partial f(\theta_t | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) \right] ds_t d\theta_t = \int^\theta_t g (s_t, \theta_{t+1}) ds_t d\theta_{t+1}
\]

\[
= \int^\theta_t \left[ \frac{\partial f(\theta_t | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) \right] ds_{t+1} d\theta_{t+1}
\]

\[
= \int^\theta_t \left[ \frac{\partial f(\theta_t | s_t)}{\partial s_t} g (s_t, \theta_{t+1}) ds_t \right] d\theta_{t+1}
\]

\[
= \int^\theta_t \left[ \frac{\partial f(\theta_t | s_t)}{\partial s_t} \right] f(\theta_{t+1} | \theta_t) d\theta_{t+1}
\]

\[
= \int^\theta_t \left[ \frac{1}{f(\theta_{t+1} | \theta_t)} \left[ \int^\theta_{t+1} \frac{\partial f(\theta_{t+1} | s_{t+1})}{\partial s_{t+1}} g (s_{t+1}, \theta_{t+1}) ds_{t+1} \right] \right] d\theta_{t+1}
\]

Applying this operator to the function \( g^1_{t+1} = \int^\theta_{t+1} \frac{\partial h_{t+1} (\theta^{-1}, s_t, s_{t+1})}{\partial s_{t+1}} ds_{t+1} \), we obtain

\[
B^1_t \left( g, \theta_t \right) = \int^\theta_t \left[ \frac{1}{f(\theta_{t+1} | \theta_t)} \left[ \int^\theta_{t+1} \frac{\partial h_{t+1} (\theta^{-1}, s_t, s_{t+1})}{\partial s_{t+1}} f(\theta_{t+1} | \theta_t) ds_{t+1} \right] \right] d\theta_{t+1}
\]

which is of the form in the Lemma.
Now, let us consider the second operator. We have

\[ B_t^2 (g, \theta_t) = B_t^1 \left( B_{t+1}^1 (g, \theta_{t+1}), \theta_t \right) \]

Recall that the simplified operator above expresses

\[ B_t^1 (g, \theta_t) = E_t \left[ \frac{1}{f(\theta_{t+1}|\theta_t)} \left[ \int_\mathcal{G} \frac{\partial f(\theta_t + | s_t) \partial f(s_t)}{\partial s_t} g(s_t, \theta_{t+1})ds_t \right] \theta_t \right] \]

In other words, we have along history \((\theta_{t-1}, s_t)\)

\[ B_{t+1}^1 (g, \theta_{t+1}) = E_{t+1} \left[ \frac{1}{f(\theta_{t+2}|\theta_{t+1})} \left[ \int_\mathcal{G} \frac{\partial f(\theta_{t+2}|s_{t+1}) \partial f(s_{t+1})}{\partial s_{t+1}} g(s_{t+1}, \theta_{t+2})ds_{t+1} \right] \theta_{t+1} \right] . \]

Applying this into the operator defining \(B_t^2\), we obtain

\[ B_t^2 (g, \theta_t) = E_t \left[ \frac{1}{f(\theta_{t+1}|\theta_t)} \left[ \int_\mathcal{G} \frac{\partial f(\theta_{t+1}|s_t) \partial f(s_t)}{\partial s_t} B_{t+1}^1 (g, \theta_{t+1}) ds_t \right] \theta_t \right] \]

\[ = E_t \left[ \int_\mathcal{G} E_{t+1} \left[ \int_\mathcal{G} \frac{\partial f(\theta_{t+1}|s_t) \partial f(s_t)}{\partial s_t} \frac{\partial f(s_{t+1}) \partial f(\theta_{t+2}|s_{t+1})}{\partial s_{t+1}} g(s_t, s_{t+1}, \theta_{t+2}) ds_{t+1} \right] | \theta_{t+1} \right] ds_t | \theta_t \right] \]

\[ \overset{\text{LIE}}{=} E_t \left[ \int_\mathcal{G} \left[ \int_\mathcal{G} \frac{\partial f(\theta_{t+1}|s_t) \partial f(s_t)}{\partial s_t} \frac{\partial f(s_{t+1}) \partial f(\theta_{t+2}|s_{t+1})}{\partial s_{t+1}} g(s_t, s_{t+1}, \theta_{t+2}) ds_{t+1} \right] | \theta_{t+1} \right] ds_t | \theta_t \right] . \]

Finally, substituting in \(s_t^2 = \int_\mathcal{G} \frac{\partial f(\theta_{t+2}|s_{t+1}) \partial f(s_{t+1})}{\partial s_{t+1}} ds_{t+2}\), we get the next expression from the Lemma. From here, the result follows from repeated iteration.

**C.1.3 Proof of Lemma 25**

For any allocation rule, \(T_t\) provides the implementation. Recall that the government’s welfare is given by

\[ \max E_{-1} \left[ \sum_{t=0}^{\infty} \beta^t U_t - \kappa T_t \right] , \]

while the central bank’s welfare function is

\[ \mathcal{W}_0 = E_0 \sum_{t=0}^{\infty} \left[ \beta^t U_t + T_t \right] . \]
In other words, we always have

\[-E_0 \sum_{t=0}^{\infty} T_t = E_0 \sum_{t=0}^{\infty} \beta^t U_t - W_0.\]

Substituting in above, by Law of Iterated Expectations we obtain the planning problem

\[
\max E_{-1} \left[ -\kappa W_0 + \sum_{t=0}^{\infty} \beta^t (1 + \kappa) U_t \right],
\]

and where lastly, we use Lemma 4 substitute in for \( W_0 \) to obtain the result.

C.1.4 Proof of Proposition 26

Recall that our objective function for the second-best optimization problem was given by

\[
\max \int_{\theta_0}^{\infty} \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\kappa}{1 + \kappa} B_0^t(s_0, \theta_0) + U_t(\pi_t, \pi_{t+1}, \theta, \theta_t) \right] dF_0(\theta_0)
\]

Note that given the optimal mechanism implements truthful reporting, we may substitute in \( \tilde{\theta}_t = \theta_t \). Recalling the form of the operators, we denote the realized value of the operator \( B_0^t \) by

\[
B_0^t(\theta_t) = \frac{1}{\prod_{k=0}^{t-1} f(\theta_{k+1}|\theta_k)} \int_{s_0 \leq \theta_0 \leq \theta_t} \frac{\partial U_t(s_0, \ldots, s_t)}{\partial s_t} \prod_{k=0}^{t-1} \frac{\partial f(\theta_{k+1}|s_k)}{\partial s_k} ds_t \ldots ds_0
\]

so that \( B_0^t(\theta_t) \) is a random variable derived from the history \( \theta_t \) of shocks. Given the definition of this random variable, denote \( E_{-1} \) to be the beginning-of-period-0 expectation, not conditional on the information \( \theta_0 \). From here, we can rewrite the objective function of the government as

\[
\max E_{-1} \left[ \sum_{t=0}^{\infty} \beta^t \left[ -\frac{\kappa}{1 + \kappa} B_0^t(\pi_t, \pi_{t+1}, \theta_t|\theta_t^{-1}) + (1 + \kappa) U_t(\pi_t, \pi_{t+1}, \theta_t) \right] \right]
\]

From here, consider the optimal choice of inflation \( \pi_t(z_t) \), for a realized history \( \theta_t = z_t \) of shocks. Note that the solution can be written in the form (for \( t \geq 1 \))

\[
\frac{\partial U_t}{\partial \pi_t(z_t)} f(z_t^{t-1}) + \beta \frac{\partial U_t}{\partial \pi_t(z_t)} f(z_t) = \frac{\kappa}{1 + \kappa} E_{-1} \sum_{s=t-1}^{t} \beta^{s-t-1} \frac{d}{d \pi_t(z_t)} B_0^s(\pi_s, \pi_{s+1}, \theta_s|\theta_s)
\]

so that all that remains is to characterize the derivatives of \( B_0^s \) with respect to \( \pi_t(z_t) \). When
where here, we applied the fact that we have chosen a specific history \( z \)

As a result, the right-hand side of the first-order condition becomes

It is worth remembering then, when we substitute into the expectation, that

Note the subtlety that the \( q \)'s, as the path under the integrals that leads to the history \( z' \) under the integrals. It is worth remembering then, when we substitute into the expectation, that \( \theta_t \) is a random variable, and \( z' \) is (fixed) the history being differentiated along, and so is not a random variable.

By exactly the same logic, we obtain \( \forall t \geq 2 \)

As a result, the right-hand side of the first-order condition becomes \( \forall t \geq 2 \)

where here, we applied the fact that we have chosen a specific history \( z' \), so that the cross-
partials above are *not* random variables, but rather are specific realizations of those random variables. By contrast, the part inside the expectation corresponds to histories which contain these specific histories, and so are random variables.

Now, consider these two expectations. Now, we define \( \Omega_t(z^t) \) by

\[
\Omega_t(z^t) = E_{-1}\left[ \prod_{k=0}^{t-1} \frac{1}{f(\theta_{k+1}|\theta_k)} \prod_{k=0}^{t-1} \frac{\partial f(\theta_{k+1}|z_k)}{\partial z_k} \right]
\]

\[
= \int_{z_{t-1}}^{z_t} \cdots \int_{z_0}^{z_t} \prod_{k=0}^{t-1} \frac{\partial f(\theta_{k+1}|z_k)}{\partial z_k} f(\theta_0) d\theta_1 \cdots d\theta_0
\]

\[
= \int_{z_{t-1}}^{z_t} \cdots \int_{z_0}^{z_t} \prod_{k=0}^{t-1} \frac{\partial f(\theta_{k+1}|z_k)}{\partial z_k} f(\theta_0) d\theta_1 \cdots d\theta_0 d\theta_t
\]

\[
= \Omega_{t-1}(z^{t-1}) \int_{z_t}^{z_{t-1}} \frac{\partial f(\theta_t|z_{t-1})}{\partial z_{t-1}} d\theta_t
\]

which is well-defined for all \( t \geq 1 \). However, it requires an initial condition \( \Omega_0(z^0) \). It is helpful to define this initial condition in the date 1 FOC. Note that at date 1, we have

\[
B_{t=1}(\theta^{t-1}) = B_0^0(\theta^0) = \int_{\theta^0} \frac{\partial U_0}{\partial \theta_0} d\theta_0
\]

so that we have \( \frac{d}{d \theta_t(z^t)} B_{t=1}^{t-1}(\theta^{t-1}) = 1_{z_0 \leq \theta_0} \frac{\partial U_0}{\partial \theta_t(z^t)} \). In particular then, the expectation is simply

\[
E_{-1}[1_{z_0 \leq \theta_0}] = \int_{z_0} \frac{\partial f(\theta_0)}{\partial \theta_0} d\theta_0 = 1 - F(z_0)
\]

so that we have initial condition \( \Omega_0(z^0) = 1 - F(z_0) \).

This gives us a state space reduction property, where we can fully determine \( \Omega_t \) from \( \Omega_{t-1} \) and \( z_{t-1} \) by a recursive sequence, where the initial value is \( \Omega_0(z^0) = 1 - F(z_0) \).

From here, we can substitute back into the FOCs and dividing through by \( (1 + \kappa) f(z^{t-1}) \)

\[
\frac{\partial U_{t-1}}{\partial \tau_t} + \beta \frac{\partial U_t}{\partial \tau_t} f(z_t|z_{t-1}) = \frac{\kappa}{1 + \kappa} \left[ \frac{\Omega_{t-1}(z^{t-1})}{f(z^{t-1})} \frac{\partial^2 U_{t-1}}{\partial z_{t-1} \partial \tau_t} + \beta \frac{\Omega_t(z^t)}{f(z^t)} \frac{\partial^2 U_t}{\partial z_t \partial \tau_t} f(z_t|z_{t-1}) \right]
\]

From here, we define \( \Gamma_t(z^t) = \frac{\Omega_t(z^t)}{f(z^t)} \). Note that we have

\[
\Gamma_t(z^t) = \frac{\Omega_t(z^t)}{f(z^t)} = \frac{\Omega_{t-1}(z^{t-1})}{f(z^{t-1})} \int_{z_t}^{z_{t-1}} \frac{\partial f(\theta_t|z_{t-1})}{\partial z_k} d\theta_t = \Gamma_{t-1}(z^{t-1}) \int_{z_t}^{z_{t-1}} \frac{\partial f(\theta_t|z_{t-1})}{\partial z_k} d\theta_t
\]
giving us our key result for \( t \geq 1 \).

Note that the relevant initial condition is \( \Gamma_0 = \frac{1-F(z_0)}{f(z_0)} \). This is the standard term in evaluating the virtual value in static mechanism design problems, and it is not surprising that it appears here. What it notable is that this term appears in the date 1 optimality condition, in addition (as we will see) to the date-0 one. This is because of the time consistency problem.

Lastly, we can evaluate the FOC for \( \pi_0 \). In \( \pi_0 \), there is no time consistency element, and we are left with the simple tradeoff between current \( \pi \) and transfers. Repeating the steps from above, we obtain the simple condition

\[
\frac{\partial U_0}{\partial \pi_0} = \frac{\kappa}{1+\kappa} \Gamma_0(z_0) \frac{\partial^2 U_0}{\partial z_0 \partial \pi_0}
\]

which is a standard virtual value condition. This gives the full result.

### C.1.5 Proof of Corollary 27

The proof follows immediately from Proposition 26, noting that the triple can be used to evaluate the allocation rule from date \( t \) and onward, and that the transfer rule can be defined from the value function.

### C.1.6 Proof of Corollary 28

The proof follows immediately from the definition of \( \Gamma_t \), which is equal to zero if \( \theta_t \in \{\theta, \overline{\theta}\} \). When \( \Gamma_t = 0 \), the allocation rule is constrained efficient for all \( \Gamma_{t+k}, k \geq 1 \), so the optimal mechanism reverts to constrained efficiency, which is implemented by the dynamic inflation target.

### C.1.7 Proof of Proposition 29

The optimization problem

\[
\max_{\{\pi_t\}} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t (1 + \kappa) U_t
\]
yields the constrained efficient allocation rule. The result of Proposition 23 gives the mechanism that implements this allocation rule, giving the result.