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CID Faculty Working Paper No. 382
July 2020

Working Papers

Center for International Development
at Harvard University

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First Version: April 2020
This Version: July 2020

Abstract

In this paper, we develop a heterogeneous agent general equilibrium framework to analyze optimal joint policies of a lockdown and transfer payments in times of a pandemic. In our model, the effectiveness of a lockdown in mitigating the pandemic depends on endogenous compliance. A more stringent lockdown deepens the recession which implies that poorer parts of society find it harder to subsist. This reduces their compliance with the lockdown, and may cause deprivation of the very poor, giving rise to an excruciating trade-off between saving lives from the pandemic and from deprivation. Lump-sum transfers help mitigate this trade-off. We identify and discuss key trade-offs involved and provide comparative statics for optimal policy. We show that, ceteris paribus, the optimal lockdown is stricter for more severe pandemics and in richer countries. We then consider a government borrowing constraint and show that limited fiscal space lowers the optimal lockdown and welfare, and increases the aggregate death burden during the pandemic. We finally discuss distributional consequences and the political economy of fighting a pandemic.

Keywords: COVID-19, lockdown, fiscal policy, government borrowing constraint, political economy, inequality, developing countries

JEL: E62, F4, H12, I14, I18

*We would like to thank Eduardo Fernandez Arias, Andres Gomez, Frank Neffke, Maik Schneider, Federico Sturzenegger, Mariano Tommasi, Rodrigo Wagner, and seminar participants at the Harvard Growth Lab and the Universidad de San Andrés for valuable comments and suggestions.

1 Introduction

COVID-19 is the most severe global pandemic since the Spanish flu of 1918/1919, and it threatens millions of lives. To fight the pandemic and to limit its death burden, governments all over the world impose drastic measures. For lack of more targeted policies, they opt for partial and full lockdowns that bring significant parts of the economy to a halt. The economic consequences are dramatic with unemployment rising and GDP falling at unprecedented rates. A surging economic literature suggests that the enormous value at stake in terms of the pandemic may justify such drastic measures, even if they result in income losses in the order of 10, 20, 30% of GDP or higher.[1] Yet, these shocks need to be absorbed at the individual and at the country level, and some countries may be less able to do so than others.

To cushion the economic shocks, central banks in industrialized countries loosen monetary policy and governments announce enormous fiscal stimuli, financed via public debt. As of May 2020, total announced fiscal measures amount to more than 10% of GDP in countries like the US, South Korea, Switzerland, or Australia, for example.[2] Similar measures may not be feasible in large parts of the developing world. Many developing countries had limited fiscal space to begin with, only to see it collapse as a consequence of the global impact of COVID-19 on commodity prices, tourism, remittances, and capital flows (Hausmann, 2020; Hevia and Neumeyer, 2020). These global shocks are by themselves a major hit to the economy, increasing the need for supportive policies even further. While such policy measures are important in industrialized countries, they would be even more valuable in developing countries where a larger fraction of the population is in poverty. Hunger is projected to almost double in the wave of the COVID-19 pandemic (World Food Programme (WFP), 2020).

What does this imply for policy? How does the optimal lockdown depend on accompanying policy measures to alleviate the economic shock? How do such measures impact compliance with a lockdown and welfare? What are the cost of limited fiscal space in a pandemic and how does it affect policy? What if parts of the population are at or close to subsistence, already struggling with surviving the recession caused by the global shock? Are lockdowns a luxury good that is desirable only for countries rich enough to be able to bear the consequences?

In this paper, we provide first answers to these questions by presenting a tractable economic model that allows to *jointly* analyze optimal lockdowns and lump-sum transfers. Our main

[1] See e.g. Acemoglu et al. (2020); Eichenbaum et al. (2020a); Farboodi et al. (2020); Hall et al. (2020).

[2] Source: https://projects.iq.harvard.edu/covidpt/global-policy-tracker, accessed on 5/8/2020. When including loans, equity injections, and guarantees, the total volume of fiscal stimuli even exceeds 30% of GDP in the case of Germany and Italy (International Monetary Fund (IMF), 2020a).

focus is on developing countries, but the mechanisms we analyze matter more generally in countries where parts of society suffer from deprivation or are threatened to 'die of despair' (Case and Deaton, 2020). We outline our model in Section 2: There are two periods, the present and the future, the latter can be interpreted as an infinite horizon steady state. Also, there is a continuum of households that differ in their ability. Households inelastically supply 1 unit of labor and derive utility from consumption of a final good that is produced with constant returns to scale using efficiency units of labor as the only input. To survive, per-period consumption of a household needs to meet a subsistence level $\bar{c} > 0$. In the first period, the economy is unexpectedly hit by a pandemic. The pandemic causes deaths from the disease and a temporary loss in TFP—the latter can also be attributed to global economic shocks due to the global nature of the pandemic. The government can decide to fight the pandemic via a lockdown, which requires households to reduce their labor supply, and impacts TFP via e.g. distorted value chains, but also via a better control of the pandemic. In line with the idea that the trade-off between economic loss and loss of life need not be negative everywhere (see e.g. Acemoglu et al. 2020, Figure 1), we allow the TFP effect of a lockdown to be positive for small values of the lockdown. Eventually, however, it will be distortive, and aggregate TFP declines. The government can accompany the lockdown with transfer payments to be financed via international borrowing at a fixed rate, but subject to a borrowing constraint. Households comply with the lockdown as long as it allows subsistence.

In Section 3, we show that the lockdown and transfer payments have intricate effects on the economy. On the one hand, the lockdown reduces social interactions and thus mitigates the pandemic and its death burden. On the other hand, it deepens the recession, which lowers compliance with the lockdown and may imply that poorer parts of the population suffer from deprivation and are no longer able to subsist. Transfer payments can help these parts of the population through the recession and, more generally, allow for consumption smoothing between the present and the future. They also increase compliance with the lockdown and thus have an indirect beneficial effect on the pandemic. On the other hand, they lower future utility, and more so, the smaller the future population to service the debt.

These heterogeneous and interdependent effects notwithstanding, our set-up is tractable enough to analytically characterize optimal policy. We show that optimal policy always involves a trade-off between 'lives and livelihoods', i.e. between economic losses and loss of life. As excruciating as this trade-off may be, in developing countries a lockdown may involve an even more horrible trade-off: One between saving households from the pandemic or saving them from dying from deprivation. We show that the optimal lockdown always fights the pandemic at the margin, but this fight against the pandemic might imply deprivation of the poorest households, particularly in societies where part of the population is close to subsistence and if the future weighs large compared to the present. This being said, we provide numerical illustrations that consistently suggest that with optimal transfers in place—i.e. in an unconstrained optimum—a very small or even zero fraction of the population is threatened to die from deprivation. In other words, in the absence of borrowing constraints, subsistence of the poorer parts of society imposes limits on the optimal fight against a pandemic. Indeed, we show analytically that the optimal lockdown is more stringent for richer countries, i.e. poorer countries are more heavily concerned by deprivation vis-à-vis the pandemic. In that sense, a lockdown may be seen as a 'luxury good'. Importantly, lump-sum transfers help alleviate this trade-off, and we show that at the margin these policy tools are complements, i.e. ceteris paribus larger transfers rationalize a more stringent lockdown.

We next consider borrowing constraints, which have a major effect on optimal policy, deaths, and welfare. In particular, the complementarity between the lockdown and lump-sum transfers implies that—when confronted with a borrowing constraint—it is optimal to fight less the pandemic. Intuitively, if part of the population is close to subsistence and the government does not have the fiscal space to support the poor, fighting a pandemic via a lockdown that deepens the recession is very costly. As a consequence, the aggregate death burden is higher, welfare lower, and if the borrowing limit is small, it may no longer be true that the government can save the vast majority of its population from dying from deprivation, highlighting the dire need of developing countries to receive financial support in times of a pandemic.

We illustrate these effects and their impact on optimal policy by means of a numerical example in Section 4. This example delivers additional insights on the effects of borrowing constraints: It suggests that optimal transfers as a share of steady state GDP are lower in richer countries. Accordingly, the gap between constrained and unconstrained optimal policies is larger for poorer countries, i.e. borrowing constraints are particularly costly for the most vulnerable societies. It further shows that the distance between constrained and unconstrained optimal policies is larger, the more severe the disease.

The pandemic and the optimal policy response have important distributional consequences. On the one hand, the lockdown benefits the least—and may even hurt—the poorest households in a society. These households may not be able to afford full compliance with the lockdown and, hence, face a higher probability of dying from the disease. In the extreme, they may even not be able to live through the recession and die from deprivation. On the other hand, for the same reasons, these households are the ones that benefit the most from lump-sum transfers. We discuss these distributional consequences and the implied political economy of fighting a pandemic in Section 5, where we derive a single-crossing result for individual preferences over policies and discuss its implications. In this section, we also show that supporting vulnerable parts of society in times of a pandemic is in the self-interest of the rich if the externality of working on the pandemic is sufficiently large and the future sufficiently important vis-à-vis the present. We further discuss robustness of our main findings to changes in our simplifying assumptions.

Finally, Section 6 concludes.

Relation to literature

Our paper contributes to the rapidly growing literature that uses economic models to analyze the COVID-19 pandemic and policy options. A series of recent papers build on the so called SI(E)R model (Kermack et al., 1927) or variants thereof to analyze the evolution of the pandemic and optimal containment policy. Atkeson (2020), Bodenstein et al. (2020), and Rampini (2020) perform policy experiments. Acemoglu et al. (2020), Alvarez et al. (2020), and Piguillem and Shi (2020) solve planning problems that trade off the economic losses from a lockdown and the death burden of the disease. Eichenbaum et al. (2020a), Farboodi et al. (2020), Garibaldi et al. (2020), Jones et al. (2020), Krueger et al. (2020) endogenize agents' responses to the pandemic and emphasize externalities in the presence of a pandemic that relate to the transmission of the disease and the congestion in the healthcare system.[3] von Carnap et al. (2020) apply the Eichenbaum et al. (2020a) model to show that—due to lower income and differences in the demographic structure—optimal lockdowns are much smaller in developing countries when compared to the US.[4] None of these papers consider (complementary) policy tools and how they affect optimal lockdowns and key trade-offs involved.[5] Baqaee and Farhi (2020), Bigio et al. (2020), Caballero and Simsek (2020), Faria-e-Castro (2020), and Céspedes et al. (2020) on the other hand, study macroeconomic shocks during the COVID-19 pandemic and policy options to counteract these, but do not consider policies to fight the pandemic itself. Moreover, with the exception of Céspedes et al. (2020), these papers do not consider borrowing constraints.[6]

By contrast, we analyze optimal *joint* policies of lockdown and lump-sum transfers in set-ups with and without borrowing constraints. In that sense, our paper is closest to Guerrieri

[3]Eichenbaum et al. (2020b) analyze smart containment policies that involve intense testing and quarantining infected people to find that—if feasible—it is drastically more efficient than general lockdowns.

[4]von Carnap et al. (2020) also introduce a subsistence consumption level, which will play an important role in our analysis, but they consider representative agents and, hence, consumption is always above subsistence.

[5]Eichenbaum et al. (2020a) model containment measures as a tax on consumption whose revenues are rebated as lump-sum transfers. This tax, however, cannot be optimized independently from the lockdown.

[6]Céspedes et al. (2020) consider private borrowing constraints that may bind in times of a pandemic due to the endogenous value of collateral. They argue that by relaxing liquidity constraints, governments can focus the economy on the good equilibrium. This, however, requires ample fiscal space, i.e. they provide a channel for need of fiscal space in times of a pandemic that is complementary to the one we consider.

et al. (2020), Glover et al. (2020), and Alon et al. (2020). Guerrieri et al. (2020) present a theory of sector-specific 'Keynesian supply shocks', i.e. negative supply shocks that have negative demand spillovers to other sectors, in a two-sector model with nominal rigidities. Most relevant for our purposes, they also consider an extension where the sector-specific supply shock is explicitly modeled as a lockdown in response to a pandemic. They provide a sufficient condition for first-best efficiency of a complete lockdown of one sector in combination with stabilizing monetary policy intervention plus social insurance in case of a borrowing constraint for households. Glover et al. (2020) introduce heterogeneous agents into a macro-epidemiological model with two-sectors and costly transfers between working and non-working parts of the population, where households may not work due to age, health status, or a government-imposed closure of their sector. They use a calibrated version of their model to highlight distributional conflicts of lockdown policies and, quite intuitively, show that the optimal lockdown of the 'luxury goods' sector is smaller the costlier is redistribution. Our work differs along several dimensions: We consider international borrowing that allows for intertemporal consumption smoothing. More importantly, we allow for feedback effects from Macro-policies—debt-financed lump-sum transfers in our case—on the pandemic and the effectiveness of a lockdown in fighting the pandemic. Specifically, we consider an economy with a continuum of heterogeneous agents and analyze how lump-sum transfers affect compliance with a lock-down, the ability of households to live through the recession, and, hence, the optimal lockdown. Notably, this focus on countries where parts of society may suffer from deprivation also distinguishes our work from most of the aforementioned list of papers.

Alon et al. (2020) build on Glover et al. (2020) to perform a preliminary quantitative account of optimal policy in a developing country set-up with an informal sector. Lockdowns are effective only in the formal sector and households can choose in which sector to work. The government can support households via costly transfers, financed via taxes and emergency bonds. According to their analysis, welfare losses from the pandemic are smaller and optimal lockdowns tend to be less strict in developing countries. We complement their work along several dimensions. Most importantly, we introduce a subsistence consumption level and show how this may give rise to much larger welfare losses in the developing world, in particular if government borrowing is constrained. Moreover, we present a tractable model, which allows us to analytically identify and discuss main effects and, hence, to analyze key trade-offs in a transparent way and to derive robust comparative statics.

Chang and Velasco (2020) discuss feedback effects from economic policy on the pandemic in a stylized set-up that is different from ours along important dimensions. In their model, a subset of the population has been tested for the disease, and those who have been found to be susceptible then endogenously choose whether or not to work.[7] There may be multiple equilibria where either all or none of these susceptible individuals work. Lockdown and targeted transfers to unemployed workers may complement each other if it requires a 'big push' to shift the economy from a 'bad' to a 'good' equilibrium and neither of the policy measures is sufficient in itself. Our main mechanisms are different and we identify and discuss various convoluted effects of a lockdown and transfer payments on the pandemic and welfare. Moreover, we consider the effect of a borrowing constraint on optimal policy and welfare, and discuss political economy-effects of fighting a pandemic in developing countries.[8]

Finally, our paper also contributes to the broader economics literatures analyzing infectious diseases on the one hand (e.g. Bell et al. (2006), Bloom and Canning (2006), Greenwood et al. (2019)), and causes and consequences of government borrowing constraints on the other hand (e.g. Stiglitz and Weiss (1981), Bulow and Rogoff (1989), Gavin and Perotti (1997), Caballero and Krishnamurthy (2001), Kaminsky et al. (2004), Eichengreen et al. (2007), Chari et al. (2020)).[9] We add to these literatures in that we consider optimal joint policies of containment measures to combat a pandemic and transfer payments to support households in need, and how they are affected by a government borrowing constraint.

2 Model

2.1 Economic Environment

We consider an economy with a continuum of measure 1 of households who differ in their ability a. To simplify the exposition, we assume that there are two periods only: In the first period—the present—, the economy faces a pandemic. The second period—the future—is a post-pandemic period, which we can think of as a reduced form representation for the present value of an infinitely repeated post-pandemic steady-state, as further discussed below. The pandemic imposes two costs on the economy: It causes a fraction of the population to

[7]Agents who get tested positive are compelled to stay home. The decision to work of the susceptible therefore entails a positive externality as they are healthier than the average. Similarly, Eichenbaum et al. (2020b) argue that testing, if *not* combined with strict containment of infected, has a negative externality because a positive test increases the incentives to work for selfish agents.

[8]See e.g. Gourinchas (2020) and Loayza and Pennings (2020) for informal discussions of emerging policy issues in the COVID-19 pandemic, the latter with a focus on developing countries. Loayza (2020) discusses policy options to fight the pandemic in developing countries where poverty is rising and fiscal space is limited. Ray et al. (2020) consider the case of India and discuss horrible trade-offs in times of a pandemic in developing countries.

[9]While our focus is on a borrowing constraint, at a more general level we consider (short-run) constraints on the government to mobilize resources. In that sense our work is also, but less closely, related to the literature analyzing causes and consequences of limited fiscal capability, e.g. Aizenman et al. (2007), Besley and Persson (2009), and Gersbach et al. (2019).

die from the disease and involves a total factor productivity (TFP) loss during the time of the pandemic. Depending on the development stage of the economy, this recession may cause an additional death-toll if part of the population is at or close to subsistence. The government can decide to fight the pandemic by imposing a lockdown θ. The lockdown, however, deepens the recession as it decreases aggregate labor supply and has a negative effect on TFP. Households comply with the lockdown only if it allows subsistence. To lessen the burden of the recession, the government can cushion the lockdown with lump-sum transfers T which have to be financed via borrowing, subject to a borrowing constraint. In the second period, the government levies a lump-sum tax to finance its debt-payments.

2.1.1 Households

Households differ in their ability a which is distributed according to some atomless distribution with CDF $F(a)$ and support $\mathcal{A}$. In what follows, we will use $f(a)$ to denote the associated PDF, and identify households by their ability. Households inelastically supply 1 unit of labor. During the pandemic, the government can impose a lockdown θ and force its citizens to lower their labor supply. We assume that households comply with this lockdown up to the point where compliance would imply deprivation as further detailed below, and use $l(a)$ to denote the period-1 labor supply of household a.

Households receive instantaneous utility of consumption according to

$$u(c) = v(c) - \bar{c}, \quad v(c) := c^{\alpha}, \quad 0 < \alpha \leq 1,$$

where c denotes the level of consumption and $\bar{c}$ denotes the subsistence level.[10] If in any given period $c < \bar{c}$ the household dies. We assume that this death—and a potential death caused by the pandemic—occurs at the end of the period.

We choose the consumption good to be the numéraire. There are no private opportunities to lend or borrow,[11] implying that per-period consumption just equals per-period income. Let w denote the wage rate per efficiency unit of labor. Consumption of household a in period $s \in \{1, 2\}$ is then given by

$$\begin{aligned} c^1(a;\theta,T) &= w^1 \cdot a \cdot l(a) + T \qquad (1) \\ c^2(a;\tau) &= w^2 \cdot a - \tau, \end{aligned}$$

where T is a lump-sum transfer financed via government borrowing as detailed below, and τ the lump-sum tax that the government levies in the second period to pay for its debt.

[10]As an alternative, we could consider Stone-Geary-type preferences, $u(c) = (c - \bar{c})^{\alpha}$. This would not fundamentally change our main insights and we therefore opt for the slightly simpler specification in the main text. We briefly discuss the case of more than one goods in Section 5.1.

[11]See Section 5.1 for a discussion.

Consumption in the second period is conditional on survival. The expected lifetime utility of household a is then given by

$$U(a) = \left[w^1 \cdot a \cdot l(a) + T\right]^{\alpha} - \bar{c} + \pi(a) \cdot \beta \cdot \left[\left(w^2 \cdot a - \tau\right)^{\alpha} - \bar{c}\right].$$

In the above, β is the discount factor which—consistent with our interpretation of period 2 as the present value of an infinitely repeated steady state—we think of as being large, i.e. $\beta = \frac{\tilde{\beta}}{1-\tilde{\beta}}$ for some per-period discount factor $\tilde{\beta}$ close to 1. $\pi(a)$ is the probability that household a survives the first period. For most of what follows, we do not need to explicitly consider the utility of household a, and we therefore postpone the discussion of $\pi(a)$ to Section 5.2, where we consider the distributional consequences and political economics of combating a pandemic.

In what follows, we will not need the time-superscripts ^{s} and we therefore omit them throughout.

2.1.2 Pandemic

The pandemic hits the economy in the first period only.[12] Its severity depends on the amount of interactions between households in the economy, which can be summarized by the aggregate labor supply L in period 1.[13] Accordingly, we summarize the pandemic by a function $P(L)$, with the following properties:

Assumption 1
$P(L) = L^{\lambda}, \quad \lambda > 1$

L is equal to 1 in the steady-state, but may be lower during the pandemic as further detailed below. Hence, $P(L) \in [0, 1]$ and we can therefore think of P as summarizing how bad the pandemic is relative to the case of $L = 1$. The pandemic is the less severe the more effectively the government lowers L, and it is convex in L. We will get back to this point shortly.

The pandemic causes a recession as detailed in Section 2.1.4 below. In addition, the pandemic has a health burden. As far as pure health expenditures are concerned, these can be interpreted as being reflected in the TFP effect discussed below as such health expenditures

[12]That is, we implicitly assume that either recovered households are immune against the disease, in line with the widespread use of S(E)IR(D) models in the economics literature on COVID-19, such that herd-immunity can be reached by the end of period 1. Or, that a treatment or vaccine is available by the end of period 1. Hence, our set-up is flexible enough to accommodate situations where recovered households are not (permanently) immune.

[13]Note that there are no private savings in the economy, i.e. L can be seen as not only summarizing supply-side channels of disease transmission but, up to the lump-sum transfers of the government considered below, demand-side channels as well.

lower income available for other forms of consumption. The pandemic, however, also causes fatalities $d(P)$, where we assume:

Assumption 2
$d(P) = \delta \cdot P, \quad \delta > 0$

In the above, δ is the death burden of the pandemic with full employment ($L = 1$). The death burden is then linear in the size of the pandemic, i.e. we can think of P as being 'normalized' in terms of its death burden. $P(L)$ can therefore be seen as summarizing various forces: The effect of a lockdown on the rate of disease transmission, the effect of the rate of disease transmission on the overall extend of the pandemic and the size of its peak, and the effect of the latter on the death burden of the pandemic. We argue in Appendix B.1 with reference to a simple SIR model that the overall relationship between $d(P)$ and L is plausibly convex, in line with our assumptions.

2.1.3 Policy instruments

The government can decide to fight the pandemic by imposing a lockdown $\theta \in [0, 1]$, where a lockdown of size θ requires households to reduce their labor supply in period 1 from its steady state level 1 to a level $(1-\theta)$. Next to decreasing L, the lockdown lowers TFP, due to e.g. distorted supply chains as discussed below. The government can cushion the effects of the pandemic and the imposed lockdown via a lump-sum transfer t which is to be financed via foreign borrowing and which is expressed as a fraction of (pre-pandemic) steady state GDP, i.e. the per capita transfers are

$$T = t \cdot \bar{A} \cdot \mu_a,$$

where here and below we use $\mu_a := \int_{a \in \mathcal{A}} a \cdot f(a) da$ to denote the average ability in the economy, and where $\bar{A}$ denotes steady-state TFP as detailed below. Borrowing is subject to a constraint b as a percentage of steady-state income, that is

$$t \leq b. \tag{2}$$

Borrowing comes at an interest rate r. Consistent with our interpretation of period 2 as the present value of an infinitely repeated steady-state, we assume that the foreign debt is infinitely rolled over,[14] with interest payments financed via a lump-sum tax, for simplicity.

The lump-sum transfer—next to smoothing consumption—has the main effect of saving poorer households from deprivation and increasing compliance with the lockdown. We consider these effects in Section 2.2 and discuss production first.

[14] If $r > \tilde{\beta}$, this may not be optimal. Note, however, that this is not essential as we are imposing no restrictions on r and an optimal pay-back scheme given r would simply lower the effective cost of borrowing.

2.1.4 Production

Production is constant returns to scale with respect to its only input efficiency units of labor, $a \cdot l(a)$,

$$Y = A \cdot \int_{a \in \mathcal{A}} a \cdot l(a) \cdot f(a) da,$$

where A is a total factor productivity (TFP) term, which is equal to $\bar{A}$ in the steady state. TFP is, however, lower in period 1. For one thing, the pandemic causes a recession. For another, the government-induced lockdown will lower TFP further as it e.g. distorts supply chains in the economy. Taken together, TFP in the first period is given by

$$A := \bar{A} \cdot \gamma_P \cdot g(\theta), \tag{3}$$

where γ_P is a baseline effect in period 1 and $g(\theta)$ a lockdown-induced TFP effect. γ_P can be interpreted as capturing both a direct domestic effect of the pandemic and negative foreign economic shocks due to the global nature of the pandemic. We make the following assumption on these effects:

Assumption 3
(i) $g(0) > g(1)$ *(ii)* $g''(\cdot) < 0$ *(iii)* $\gamma_P \cdot g(\cdot) < 1$

Here and below, we use a superscript $'$ ($''$) to denote the first (second) derivative of a function.

Note that Assumption 3 does not rule out the possibility that TFP is increasing in θ for θ close to 0, i.e. $g'(0) > 0$. We allow for (but do not require) this possibility as a simple way of introducing a potential positive contemporaneous net effect of a lockdown on the economy as in e.g. Acemoglu et al. (2020, Figure 1). In either case, TFP is always lower than in the steady state (Assumption 3(iii)), TFP is concave, reflecting the idea that a lockdown is increasingly distortionary the stricter it is (Assumption 3(ii)), and TFP is lower with a full lockdown than with no lockdown (Assumption 3(i)).

Markets are perfectly competitive such that labor earns its marginal product and the wage per efficiency unit of labor is simply given by

$$w = A. \tag{4}$$

We assume that $\bar{A} \cdot a \geq \bar{c}$ for all $a \in \mathcal{A}$, i.e. that in normal times all households can subsist.

2.2 Labor supply and aggregate deaths for given policy

Due to the pandemic and the lockdown, the economy is in a recession in period 1. As a consequence, poor households may—given full compliance with the lockdown—see their income fall below the subsistence level. Using Equation (4) in Equation (1), and combining it with the fact that under full compliance $l(\cdot) = (1-\theta)$, we observe that this is the case for all households with ability $a \leq a_2$, where

$$a_2 := \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{(1-\theta) \cdot A}. \tag{5}$$

We assume that the government has the power to fully enforce the lockdown up to subsistence, that is households $a \leq a_2$ supply just enough labor to avoid dying from deprivation.[15] For sufficiently poor households, however, and if the recession is deep, the household income falls below the subsistence level even if they supply 1 unit of labor. This is the case for households $a \leq a_1$, where

$$a_1 := \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{A}. \tag{6}$$

Taken together, this implies for labor supply in period 1 of household a:

$$l(a) = \begin{cases} (1-\theta) & \text{if } a \geq a_2 \\ \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{a \cdot A} & \text{if } a_2 > a \geq a_1 \\ 1 & \text{otherwise} \end{cases}, \tag{7}$$

and for aggregate labor supply in period 1

$$L = F(a_1) + \int_{a_1}^{a_2} \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{a \cdot A} \cdot f(a) da + (1-\theta) \cdot [1 - F(a_2)]. \tag{8}$$

These discussions point to a horrible moral trade-off that policy makers may face in developing countries: If part of the population is at or close to subsistence and the recession is bad, a fraction $F(a_1)$ of the population may die from deprivation, whether or not they themselves are infected by the pandemic, implying that the total death burden in period 1 is given by:

$$D = \underbrace{F(a_1)}_{\text{economic}} + \underbrace{[1 - F(a_1)] \cdot \delta \cdot P(L)}_{\text{pandemic}} \tag{9}$$

In other words, in developing countries governments may not only be confronted with choosing between lives and livelihoods, but between lives and lives. The government can use the lockdown to trade off between these two sources of deaths, and accompany the lockdown with transfers to alleviate either of them. We discuss these issues and government policy more generally next.

[15] There may be sources of imperfect compliance other than a threat of deprivation. Incorporating these would reinforce the complementarity between a lockdown and transfers and would increase the need for fiscal space, but the various effects of the two policy instruments discussed below and our main insights would qualitatively be the same.

3 Policy

The government chooses the size of the lockdown θ and the lump-sum transfers t to maximize aggregate welfare of its citizens. As the previous discussions show, poorer households face a higher probability of dying. We will get back to analyzing the ensuing distributional consequences of a lockdown and political economy implications in Section 5.2. For now, we focus on aggregate welfare effects and assume that the government, in its optimization problem, assigns the same value to each life. In our model, this essentially boils down to assuming that the ability distribution in the surviving population is the same as in the entire population, irrespective of who exactly is dying.[16] Aggregate welfare is then given by:[17]

$$\begin{aligned} W = & \int_{a\in\mathcal{A}} \left[v(A\cdot a\cdot l(a) + t\cdot \bar{A}\cdot \mu_a) - \bar{c}\right]\cdot f(a)da \\ & + (1-D)\cdot\beta\cdot\int_{a\in\mathcal{A}}\left[v\left(\bar{A}\cdot a - r\cdot\frac{t\cdot\bar{A}\cdot\mu_a}{1-D}\right)-\bar{c}\right]\cdot f(a)da. \end{aligned} \tag{10}$$

The government chooses (θ, t) to maximize (10) subject to $\theta\in[0,1]$, (2), and taking into account the effect of its policy on A, $\{l(a)\}_{a\in\mathcal{A}}$, P, and D. Using (7), the government decision problem boils down to

$$\begin{aligned} \max_{\theta,t} \quad W = & \int_{a\in\mathcal{A}:a\leq a_1}\left[v(A\cdot a + t\cdot\bar{A}\cdot\mu_a)-\bar{c}\right]\cdot f(a)da + \int_{a_1}^{a_2}\left[v(\bar{c})-\bar{c}\right]\cdot f(a)da \\ & + \int_{a\in\mathcal{A}:a\geq a_2}\left[v((1-\theta)\cdot A\cdot a + t\cdot\bar{A}\cdot\mu_a)-\bar{c}\right]\cdot f(a)da \\ & + (1-D)\cdot\beta\cdot\int_{a\in\mathcal{A}}\left[v\left(\bar{A}\cdot a - r\cdot\frac{t\cdot\bar{A}\cdot\mu_a}{1-D}\right)-\bar{c}\right]\cdot f(a)da \\ \text{s.t.} \quad & \theta\in[0,1] \\ & t\leq b, \end{aligned} \tag{11}$$

where A, a_1, a_2, and D are as defined in Equations (3), (5), (6), and (9).

[16]Alternatively, this may be interpreted as a simple economy where households do not differ in their innate abilities, but in the productivity of their realized jobs, assuming that the distribution of jobs is scale invariant. While we would argue that attaching a lower value to the lives of poorer households is not desirable for the purpose of an aggregate-welfare analysis, it is nevertheless interesting to note that our main insights would qualitatively be the same in such case. The main difference would be that the cost of death from deprivation would be lower vis-à-vis the cost of death from the pandemic, which are arguably more equally distributed across the population. As a consequence, it would ceteris paribus be beneficial to fight more the pandemic and spend less on saving poorer parts of society from deprivation.

[17]Households may not be able to pay the lump-sum tax in future. In such case, the tax burden would need to be higher for richer households. To simplify the exposition, we ignore this possibility, which would not materially impact our results and, in particular, would not at all affect our analysis of Section 3.1. We get back to how debt-service costs are financed in Section 5.2, where we consider the distributional consequences of fighting a pandemic.

θ and t have various effects on the economy: They jointly impact consumption today and tomorrow—and differentially so for households with different abilities—, TFP, aggregate labor supply, the pandemic, and hence the death burden in period 1. Optimal policy therefore involves intricate and convoluted trade-offs, and we will study these next. We begin with considering high-level trade-offs between lives and livelihoods on the one hand, and economic and pandemic fatalities on the other hand, before zooming in on the details. Throughout, we will use a superscript * to denote variable values in the social optimum, and $\theta^*(t)$ to denote the optimal lockdown for given transfers, i.e. $\theta^*(t^*) = \theta^*$.

In light of the COVID-19 pandemic, it is debated whether or not fighting the pandemic involves a trade-off between saving lives and livelihoods. In our model, we allow for the possibility that fighting the pandemic may have a positive effect on the economy by not ruling out that $g'(0) > 0$, i.e. that fighting the pandemic initially has a positive *net* effect on TFP. Nevertheless, as the following proposition shows, at the margin fighting the pandemic always involves a trade-off between saving lives and livelihoods.

Proposition 1 (Lives vs livelihoods and pandemic vs economic fatalities)
Let $\theta^*(t)$ be the solution to decision problem (11) for a given t.

(i) $\frac{dD}{d\theta}\Big|_{\theta=\theta^*(t)} < 0$

(ii) $\frac{dL}{d\theta}\Big|_{\theta=\theta^*(t)} < 0$

The proof of Proposition 1 is given in Appendix A.1. In words, Proposition 1(i) implies that for any given t, the optimal choice of the lockdown is such that a marginal increase in the lockdown would decrease the aggregate death toll in period 1. The government nonetheless prefers not to increase θ further precisely because of the economic costs involved.

Underlying the aggregate death toll in period 1 are two sources of fatalities: Deaths caused by the disease ('pandemic deaths') and deaths caused by the recession ('economic deaths'). The lockdown affects both causes of deaths: Economic deaths because it impacts TFP in period 1 and pandemic deaths because it impacts aggregate labor supply, both directly and indirectly via TFP. The latter effect on L is actually *positive*, because a lower TFP makes compliance with the lockdown harder and, for sufficiently high θ, this indirect effect may dominate, implying that a lockdown *increases* L at the margin.[18] Nevertheless, as Proposition 1(ii) shows, this is never optimal: For any given t we have $\frac{dL}{d\theta}\Big|_{\theta=\theta^*(t)} < 0$, implying that, at the margin, the lockdown lowers the pandemic and alleviates the associated death burden. Governments may then face a trade-off not only between lives and livelihoods, but between

[18] This is the case if e.g. $\lim_{\theta\to 1} g(\theta) = 0$, in which case for $\theta \to 1$ no household will be able to comply and L approaches 1.

saving their people from dying from the disease or from deprivation. This trade-off may be particularly relevant in countries where part of the population is at or close to subsistence, and if the future is important vis-à-vis the period of the pandemic.[19] Importantly, however, the government can use transfer payments to mitigate this trade-off. We discuss these issues next.

3.1 Optimal policy

In this section, we analyze optimal combinations of lockdowns and transfer payments. Throughout, we consider the limiting case of $\alpha = 1$ (linear utility) and β large, which allows analyzing the main effects of interest in a transparent way.[20] We discuss the general optimization problem in Section 3.2 and argue that our main insights from this section are likely to apply to the general case as well. We corroborate these discussions with a numerical example in Section 4.

For $\alpha = 1$ and β large, decision problem (11) boils down to

$$\begin{aligned} \max_{\theta,t} \quad & \tilde{W} = (1-D)\cdot\left[\bar{A}\cdot\mu_a - \bar{c}\right] - r\cdot t\cdot\bar{A}\cdot\mu_a \\ \text{s.t.} \quad & \theta\in[0,1] \\ & t\leq b, \end{aligned} \tag{12}$$

and the first-order conditions for optimal policy are

$$\frac{d\tilde{W}}{d\theta} = -\frac{dD}{d\theta}\cdot\left[-\bar{c}+\bar{A}\cdot\mu_a\right] = 0 \tag{13}$$

$$\frac{d\tilde{W}}{dt} = -\frac{dD}{dt}\cdot\left[-\bar{c}+\bar{A}\cdot\mu_a\right] - r\cdot\bar{A}\cdot\mu_a \geq 0, \tag{14}$$

$$\left[-\frac{dD}{dt}\cdot\left[-\bar{c}+\bar{A}\cdot\mu_a\right] - r\cdot\bar{A}\cdot\mu_a\right]\cdot[t-b] = 0, \tag{15}$$

where Equation (15) is the complementary slackness condition for the borrowing constraint, and where we have simplified the exposition by ignoring the possibility that $\theta\in\{0,1\}$ or $t=0$ is optimal. With all the weight on the future, the optimal lockdown is trivially such that a

[19]There are two possible scenarios where this trade-off would not arise: First, if the death burden of the pandemic is small and $g'(\theta) > 0$ for a wide range of θ such that $\frac{dA}{d\theta}\big|_{\theta=\theta^*} \geq 0$. Note, however, that this will never be optimal if the death burden of the disease is sufficiently large and / or the future weighs sufficiently strongly relative to the pandemic period—see Section 3.1. Second there may be no trade-off between pandemic and economic deaths if the population is sufficiently rich and / or transfer payments are sufficiently large such that no-one has to suffer from deprivation. We will study the effects of t and $\bar{A}$ on the optimal lockdown in the following sections.

[20]It is also broadly consistent with a large value of life as assumed in e.g. Eichenbaum et al. (2020a), Farboodi et al. (2020), Glover et al. (2020), Jones et al. (2020).

marginal change of the lockdown would not affect the aggregate death toll, $\frac{dD}{d\theta}\big|_{\theta=\theta^*} = 0$. Note that by Proposition 1(ii), this necessarily implies that $\frac{dA}{d\theta}\big|_{\theta=\theta^*} < 0$, i.e. the optimal lockdown trades off a lower death burden from the pandemic (*'pandemic deaths effect'*) against a higher death burden from the recession (*'economic deaths effect'*):

$$\frac{dD}{d\theta} = \underbrace{-f(a_1) \cdot \frac{a_1}{A} \cdot \frac{dA}{d\theta} \cdot [1 - d(P)]}_{\text{economic deaths effect } (>0)} + \underbrace{[1 - F(a_1)] \cdot \delta \cdot P'(L) \cdot \frac{dL}{d\theta}}_{\text{pandemic deaths effect } (<0)}, \tag{16}$$

where here and below the (> 0 / < 0) next to the label of an effect indicates its sign at the optimal solution. Equation (16) highlights the horrible moral trade-off between saving its people from the pandemic and saving its people from deprivation that the government in a developing country may face.

The effect of a lockdown on the pandemic is transmitted via its effect on economic activities of households, which in our model are summarized by the aggregate labor supply, L,

$$\frac{dL}{d\theta} = \underbrace{-\int_{a_1}^{a_2} \frac{a_1}{a \cdot A} \cdot \frac{dA}{d\theta} \cdot f(a)da}_{\text{subsistence effect } (>0)} \quad \underbrace{- [1 - F(a_2)]}_{\text{compliance effect } (<0)} . \tag{17}$$

The lockdown impacts labor supply through two channels. It has a direct negative effect on the labor supply of all households that are rich enough to be able to fully comply with the lockdown ($a > a_2$, *'compliance effect'*). In addition, it indirectly affects L via the deeper recession, which forces households with intermediate abilities ($a \in (a_1, a_2]$) to increase their labor supply in order to be able to subsist (*'subsistence effect'*).

In the absence of borrowing constraints, the optimal transfers just balance the marginal costs of borrowing and the marginal benefits from a lower death burden (Equation (14)). Transfers unambiguously lower both sources of death: They directly save poor households from deprivation and indirectly help fight the pandemic as they enable households that are just at subsistence ($a \in [a_1, a_2)$) to decrease their labor supply

$$\frac{dD}{dt} = \underbrace{-f(a_1) \cdot \frac{\bar{A} \cdot \mu_a}{A} \cdot [1 - d(P)]}_{\text{economic deaths effect } (\leq 0)} + \underbrace{[1 - F(a_1)] \cdot \delta \cdot P'(L) \cdot \frac{dL}{dt}}_{\text{pandemic deaths effect } (<0)} \tag{18}$$

$$\frac{dL}{dt} = \underbrace{-\int_{a_1}^{a_2} \frac{\bar{A} \cdot \mu_a}{a \cdot A} \cdot f(a)da}_{\text{subsistence effect } (<0)}. \tag{19}$$

As these discussions suggest, the lockdown and transfer payments interact in non-trivial ways in shaping economic and pandemic outcomes. Consider, for example, the economic deaths effect of transfers. This effect depends on the exogenous ability-adjusted aggregate productivity ($\bar{A} \cdot \mu_a$) as well as on the endogenous death burden of the pandemic ($d(P)$), TFP in period 1 (A), and the deprivation cutoff (a_1). Each of these variables is going to be affected by the lockdown: A marginal increase in θ decreases the death burden of the pandemic, which, ceteris paribus, makes saving households from deprivation more desirable. The basic intuition is simple: With a high death burden from the pandemic households may be freed from dying from deprivation to then only find themselves dying from the disease.[21] The lockdown further lowers A, which, ceteris paribus, makes transfer payments more effective in terms of saving households from deprivation because it increases the marginal effect of t on the deprivation-cutoff a_1. Finally, the lockdown increases this cutoff itself. How this affects the effectiveness of transfers in terms of saving households from deprivation depends on the shape of $f(\cdot)$ at a_1. It will make transfers even more effective if $f(\cdot)$ is increasing.

Consider, on the other hand, the effect of transfers on the pandemic deaths effect of a lockdown: Transfers impact this effect via three channels: a_1, $P'(L)$, and $\frac{dL}{d\theta}$. Transfers decrease a_1 which, ceteris paribus, strengthens the pandemic deaths effect of a lockdown as it increases the share of the population that may die from the disease. It further lowers the labor supply and, hence, alleviates the pandemic, which ceteris paribus weakens the pandemic deaths effect of a lockdown due to decreasing returns in the effect of L on P (Assumption 1). Finally, its effect on $\frac{dL}{d\theta}$ is ambiguous in general, and it depends again on the shape of $f(\cdot)$. Transfers reinforce the effect of θ on L if $f(a_2) \geq f(a_1)$.[22]

In addition to these discussed effects, the lockdown impacts the pandemic deaths effect of transfers and transfers impact the economic deaths effect of the lockdown. The mutual dependency of θ and t is therefore highly complex with multiple effects possibly going in opposite directions. Nevertheless, for a broad range of parameter values, optimal policies are such that in the neighborhood of (θ^*, t^*), the lockdown and transfers complement each other. In particular, this is always the case if the following Assumption holds:

Assumption 4

$$\frac{(\lambda - 1) \cdot \int_{{a_1}^*}^{{a_2}^*} l^*(a) \cdot f(a)da}{L^*} \leq 1$$

While Assumption 4 is based on the endogenous ${a_1}^*$ and ${a_2}^*$, it is worth noting that it is

[21] This ignores feedback effects from an improved nutrition on the risk of dying from the pandemic. A careful analysis of such feedback effects would be an interesting endeavor for future research.

[22] Differentiating Equation (17) yields, after some straightforward rearrangements,

$$\frac{d^2L}{d\theta dt} = \frac{dA}{d\theta} \cdot \frac{\bar{A} \cdot \mu_a}{A^2} [f(a_2) - f(a_1)] - f(a_2) \cdot \frac{\bar{A} \cdot \mu_a}{(1-\theta) \cdot A} + \int_{a_1}^{a_2} \frac{\bar{A} \cdot \mu_a}{a \cdot A^2} \cdot \frac{dA}{d\theta} \cdot f(a)da.$$

always satisfied if e.g. $\lambda \leq 2$, i.e. if $P(L)$ is not too convex.[23] With this assumption at hands, we can show the following result:

Lemma 1

Let Assumption 4 be satisfied. Then

$$\left.\frac{d\theta^*(t)}{dt}\right|_{t=t^*} > 0.$$

The proof of Lemma 1 is given in Appendix A.2. Note that Assumption 4 is sufficient but not necessary for the result to hold.

With these considerations in mind, we now proceed with analyzing decision problem (12).

3.1.1 Unconstrained policies

The solution to decision problem (12) depends on country (e.g. $\bar{A}$ and $f(a)$), pandemic (e.g. δ and γ_P), and policy characteristics (e.g. b, r, $g(\theta)$). We consider, in turn, the arguably most important characteristic from each of these categories. Specifically, we first consider the unconstrained optimization problem (b large) and ask how the optimal lockdown is affected by the death burden of the pandemic (δ) and the income of the country ($\bar{A}$). We then turn to the constrained optimization problem in the next section, and ask how the optimal lockdown is affected by a borrowing constraint. We discuss the interactions between these parameters in our numerical illustration below.

Death burden of the disease (δ)

An important question concerning optimal policy is how it is going to be affected by the severity of the pandemic, and its death burden in particular. The latter is summarized by δ in our model. Intuitively, a higher death burden should render fighting the pandemic more important and, hence, increase the optimal lockdown. Indeed, for a given policy (θ, t), $\delta \cdot P'(L)$ increases with δ, which in turn reinforces the pandemic deaths effect of a lockdown—and of the transfers, for that matter. A higher δ, however, also increases $d(P)$, the total death burden of the pandemic. This weakens the economic deaths effect, i.e. it decreases the marginal benefits of saving households from deprivation as a higher fraction of these might then end up dying from the disease. Accordingly, a higher δ inevitably increases the marginal benefit of a lockdown, i.e.

$$\left.\frac{d^2 D}{d\theta d\delta}\right|_{\theta=\theta^*, t=t^*} < 0, \tag{20}$$

[23] This immediately follows from the fact that $\int_{a_1{}^*}^{a_2{}^*} l^*(a) \cdot f(a) da \leq L^*$.

while prima facie its effect on $\frac{dD}{dt}$ is ambiguous. As shown in Lemma 1, the optimal choices for θ and t are interdependent. Note that condition (20) is therefore not sufficient to conclude that the optimal lockdown increases with the death burden of the disease. Yet, this is nevertheless the case for a broad range of parameter values, as we now discuss.

Note first that the optimal policy response to a higher δ is always such that the pandemic death burden is smaller than it would be without policy adjustment, as shown in Lemma 2.

Lemma 2
Let θ^*, t^* $(\hat{\theta}, \hat{t})$ denote the optimal policy before (after) a marginal increase in δ and x^* $(\hat{x})$ the value of an endogenous variable given this policy. Then,

$$[1 - F({a_1}^*)] \cdot P(L^*) > [1 - F(\hat{a}_1)] \cdot P(\hat{L}).$$

The proof of Lemma 2 is given in Appendix A.3. There are in principle two ways of lowering the pandemic death burden for a given δ: via a higher economic death burden ($F(a_1)$) and via a mitigated pandemic ($P(L)$). Hence, in extreme cases, a 'fatalistic' policy response to a higher δ may be optimal, where more people die from deprivation and the pandemic is worse, i.e. $P(L)$ increases. This is because these two changes mutually reinforce each other. Such a policy response, however, would lower the pandemic death burden *only* via more deaths from deprivation. Put differently, such a policy response 'saves' households from the disease by forcing them into deprivation. Under plausible restrictions on parameter values, this cannot be optimal, in particular if the cost of saving people from deprivation are not too large relative to the value-of-live, in line with a—from a lifetime perspective—relatively short crisis period. In such case $P(L)$ optimally declines in response to a higher δ which, in turn, implies that the optimal θ is increasing.[24] In Proposition 2 we show that this is always the case for δ sufficiently small.

Proposition 2 (Lockdown and death burden of disease)

$$\frac{d\theta^*}{d\delta} > 0.$$

in a right neighborhood of $\delta = 0$.

The proof of Proposition 2 is given in Appendix A.4. While Proposition 2 is a local result for δ small, under weak parameter restrictions this result extends to a broad range of δ. We discuss this in Appendix B.2, where we also provide a sufficient—but not necessary—condition for $\frac{d\theta^*}{d\delta} > 0$ that is nevertheless naturally satisfied under reasonable assumptions. We corroborate this theoretical finding by our numerical illustration below.

[24] $P(L)$ also declines if t increases. Nevertheless, θ must necessarily increase in such a case due to Lemma 1 and Condition (20).

Aggregate TFP ($\bar{A}$)

We next consider the effect of steady state TFP, $\bar{A}$, on optimal policy. Ceteris paribus, a larger $\bar{A}$ (weakly) increases the income of every household in the economy, and it should therefore increase the policy space for fighting the pandemic via a lockdown. Indeed, a higher $\bar{A}$ frees parts of the population from deprivation, allows a broader range of households to fully comply with the lockdown, and therefore decreases aggregate labor supply, i.e. it decreases a_1, a_2, and L in our model.

To understand how a higher TFP affects optimal policy, however, we need to consider how it impacts the optimality conditions (13) and (14). $\bar{A}$ impacts these conditions through various, opposing channels. We focus our discussions on the lockdown, which is our main instrument of interest in this section. Consider the economic deaths effect of a lockdown first. On the one hand, a larger $\bar{A}$ increases this effect as it lowers L and, hence the pandemic and the death burden of the pandemic, which lowers the risk that household are saved from deprivation to then find themselves dying from the disease ($[1-d(P)]$ increases). On the other hand, with a larger $\bar{A}$, the subsistence cutoff a_1 responds less to a change in θ ($\left[\frac{dA}{d\theta}\cdot\frac{a_1}{A}\right]$ decreases), which provides additional space for a lockdown. Finally, the density of the population at the cutoff changes ($f(a_1)$), which alleviates the economic deaths effect of the lockdown if $f'(a_1)\geq 0$. This is typically the case, in particular if $f(\cdot)$ is single-peaked in line with empirical income distributions.

Consider the pandemic deaths effect next: A change in $\bar{A}$ has again three different effects here. On the one hand, it frees households from deprivation and therefore increases the share of the population that might die from the disease ($[1-F(a_1)]$ increases). On the other hand, it lowers L and therefore $P'(L)$, i.e. it decreases the marginal returns of fighting the pandemic. $\bar{A}$ finally affects $\frac{dL}{d\theta}$. This effect is ambiguous in general, but a higher TFP necessarily amplifies the effect of θ on L (i.e. it decreases $\frac{dL}{d\theta}$) if $f(a_2)\geq f(a_1)$.[25]

Taken together, the net effect of a change of $\bar{A}$ on the optimality condition for θ is not obvious. Moreover, this condition also depends on the optimal t. Yet, as the following proposition shows, the optimal lockdown is stricter in richer countries.

Proposition 3 (Lockdown as a luxury good)

$$\frac{d\theta^*}{d\bar{A}} > 0.$$

[25]Differentiating Equation (17) with respect to $\bar{A}$ yields, after some straightforward simplifications,

$$\frac{d^2L}{d\theta d\bar{A}} = -\frac{da_1}{d\bar{A}}\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot[f(a_2)-f(a_1)] + \frac{da_1}{d\bar{A}}\cdot\frac{f(a_2)}{1-\theta} - \int_{a_1}^{a_2}\frac{da_1}{d\bar{A}}\cdot\frac{dA}{d\theta}\cdot\frac{f(a)}{A\cdot a}da.$$

The proof of Proposition 3 is given in Appendix A.5. It is worth noting that richer countries also suffer from a smaller proportionate welfare loss of the pandemic when compared to poorer countries: If a country with a higher $\bar{A}$ would just mimic the policy of poorer countries, it would suffer from a smaller welfare loss due to a smaller death burden. A simple revealed preference argument then implies that with the optimal policy the welfare loss can only be even smaller.

Corollary 1 (Welfare loss from pandemic)
The proportionate welfare loss from the pandemic is decreasing in $\bar{A}$.

3.1.2 Constrained policy

So far, we have analyzed unconstrained policies. These policies require the mobilization of substantial fiscal resources in order to finance the lump-sum transfers—see also our numerical illustration below. Developing countries often have limited fiscal space in normal times, only to see this space further tightened up in the time of the pandemic as discussed in the introduction. They therefore may be forced to tailor their policy to the fiscal resources available. As we show next, this has important consequences for optimal policy, the aggregate death rate, and welfare, highlighting the dire need for increased fiscal space in the developing world during times of a pandemic.

Specifically, consider an economy for which $b = t^*$, i.e. initially the borrowing constraint is just non-binding, and suppose that b marginally declines. How does this affect policy, welfare, and the aggregate death burden in period 1? Intuitively, the borrowing constraint limits the ability of the government to cushion the economic consequences of the lockdown and, hence, worsens its economic deaths effect. As a consequence, the optimal lockdown and welfare should decline while the total death burden in period 1 should go up. Indeed, from Lemma 1 we know that, at the margin, θ and t are complements, i.e. the optimal response to a binding borrowing constraint is a smaller lockdown. A simple revealed preference argument immediately implies that, as a consequence of this policy change, welfare declines: Consider some $\hat{b} < t^*$ and let $(\hat{\theta}, \hat{t})$ denote the optimal policy with this binding borrowing constraint. Clearly, $(\hat{\theta}, \hat{t})$ is also an option in the case of $b = t^*$. The fact that the government nevertheless chooses (θ^*, t^*) therefore reveals that welfare must be higher in the unconstrained optimum. Moreover, the aggregate death burden must be higher with a binding borrowing constraint than without: The smaller transfers $\hat{t} < t^*$ imply that future debt service costs are smaller, i.e. the negative term in $\tilde{W}$—see (12)—is smaller. Welfare can therefore only be larger with (θ^*, t^*) than with $(\hat{\theta}, \hat{t})$ if the aggregate death burden is higher in the constrained optimum, i.e. the positive term in $\tilde{W}$ is smaller as well. In other words: limited fiscal space costs lives during the pandemic.

We summarize these insights in the following proposition:

Proposition 4 (Borrowing constraint and optimal policy)

(i) *Suppose the borrowing constraint is binding. In response to a marginal decline in* b*,* $\tilde{W}^*$ *declines and* D^* *increases.*

(ii) *Let Assumption 4 be satisfied and suppose that* $b = t^*$*. In response to a marginal decline in* b*,* θ^* *declines.*

It is worth noting that Proposition 4(i) is a global result, i.e. the welfare and death burden implications of borrowing constraints hold for any initial $b \leq t^*$. As opposed to that, part (ii) is a local result for $b = t^*$. Nevertheless, the arguments underlying Lemma 1 are not knife-edged, i.e. the result always holds for b in a left neighborhood of t^*. Moreover, for a broad range of parameter values the result also holds for any initial $b \leq t^*$. We discuss this further in Appendix B.3, where we provide a sufficient—but not necessary—condition for Proposition 4(ii) to hold for all $b \leq t^*$. This condition is based on a general functional form for $g(\cdot)$, a natural extension of Assumptions (4) and weak restrictions on the shape of $f(\cdot)$ at the cutoffs a_1 and a_2, which matters for how the lockdown and transfers interact in shaping economic and pandemic outcomes. We corroborate this theoretical finding by our numerical illustration below. Before turning to our numerical illustration, however, we briefly consider the general government decision problem and how it affects the key trade-offs involved in designing policy.

3.2 General case

In this section, we consider the general decision problem (11) and discuss the main effects of θ and t. Mathematical details on these effects are provided in Appendix B.4.

The general decision problem differs in two ways from the limiting case considered in our previous discussions: Utility is concave ($\alpha \leq 1$), and β is finite. The first thing to note when considering this generalized decision problem is that none of our previous arguments is knife-edged, i.e. all of our results immediately apply to economies where β is sufficiently large and α is in a left neighborhood of 1.

Nevertheless, it is insightful to consider the general decision problem and the main effects of policy in this context. Generally speaking, β finite implies that consumption in the current period also matters for welfare, while $\alpha < 1$ implies that the level of consumption in a given period impacts marginal utility. In our previous discussions, the recession in period 1 mattered because it impacts a_1, a_2, and L and, hence, the aggregate death burden in period 1. This is still the case in the general decision problem. With β finite, however, there is a direct negative *'recession effect'* of a lockdown, reflecting simply the fact that, ceteris paribus, a stricter lockdown lowers consumption in period 1.[26]

With $\alpha < 1$, transfer payments have a *'consumption smoothing effect'*, which replaces the cost-of-debt $(-r \cdot \bar{A} \cdot \mu_a)$ from the simplified decision problem: Higher transfers now increase period-1 consumption at the cost of lowering period-2 consumption, where both effects are evaluated at the respective (average) marginal utility.[27] For t low, this effect may even be positive, but an interesting insight that emerges from our previous discussions is that—in the unconstrained optimum—the transfers always 'overshoot' along this dimension. The reason is simply that higher transfers lower D, which in turn implies that the consumption smoothing effect must be negative.

Finally, the change in α also impacts the marginal benefit from a lower D. In the limiting case considered above this is given by the average (across households) steady-state utility gross of taxes, $\bar{A} \cdot \mu_a - \bar{c}$. Transfers do not matter for this marginal benefit because, for a given t, the total debt service costs in period 2 are independent of D and because utility is linear. With $\alpha < 1$, however, this is no longer the case, and two effects can be distinguished: On the one-hand a *'value-of-life effect'* which is simply the expected period 2 utility given t (and, hence, period 2 income). This effect is positive and smaller the larger t. On the other hand, there is a *'debt-burden effect'*, which simply reflects the fact that with a lower D there are more households to service the debt in period 2, which is beneficial due to concave utility. This effect is the larger the larger t.

While we cannot definitely pin down the implications for our previous results, an inspection of these effects nevertheless delivers valuable insights. Consider first the complementarity between the two policy instruments. Ceteris paribus, the value-of-life effect and the debt-burden effect tend to reinforce this complementarity. In particular, the marginal benefit of a lower D, which is reflected in the sum of these two effects, is now increasing in t, as shown in Appendix B.4. In the light of Proposition 1(i), this tends to increase the marginal benefit of a lockdown. When it comes to the recession effect of the lock-down, t has two opposing effects: On the one hand, it increases consumption of all households who are not at subsistence and,

[26]Strictly speaking, for low θ this effect can potentially be reversed due to the fact that we allow $g'(\theta) > 0$ for θ small. By Proposition 1, however, this effect must always be negative at the optimal solution.

[27]A pandemic might also lower the (marginal) utility of consumption (Sturzenegger, 2020). In such case consumption smoothing is less beneficial, in line with the limiting case of $\alpha = 1$ considered above, and a 'hibernation' of the economy might be optimal, where production and consumption get reduced simultaneously. Importantly, however, this does not reduce any need for transfers arising from a subsistence level of consumption as considered here. See Sturzenegger (2020) for a discussion.

hence, decreases their marginal utility and thus the utility burden of a stricter lockdown for them. On the other hand, it lowers the share of the population at subsistence and, hence, increases the share of the population who see their incomes decline in response to a lower θ. Nevertheless these discussions point to important factors that reinforce the complementarity between the policy tools, which then would immediately imply that Proposition 4(ii) applies to the general decision problem as well.

Consider a change in δ next. For a given policy, this impacts $\frac{dD}{d\theta}$ in the exact same manner as before. In case of the general decision problem, however, it increases the marginal benefits of the lockdown through a second channel: ceteris paribus, a higher D increases the joint value-of-life and debt-burden effect in essentially the same way as previously discussed for a change in t—see Appendix B.4. These discussions suggest that it is optimal to fight a more severe pandemic with a tighter lockdown in case of decision problem (11) as well.

Finally, consider an increase in $\bar{A}$. This impacts the value-of-life effect and the debt-burden effect via two channels: First, it impacts $\frac{dD}{d\theta}$. Ceteris paribus, this effect is the same as in the simplified decision problem. Second, this effect gets amplified because the value-of-life and the debt-burden are increasing in $\bar{A}$. In addition, a higher $\bar{A}$ ceteris paribus amplifies the recession effect through its effect on marginal utilities and through its effect on the cutoffs a_1 and a_2. While this latter effect lowers the net gains of a lockdown in richer countries, these discussions suggest that—for a moderate length of the pandemic—our comparative statics result with respect to $\bar{A}$ applies to the general decision problem as well as.

In summary, these discussions reveal important additional channels through which policy impacts welfare in decision problem (11). While we cannot definitely pin down comparative statics result for this problem, our discussions in this section—along with the fact that our prior results were not knife-edged—suggest that our main insights from the simplified decision problem prevail, i.e. that the optimal lockdown increases with δ and $\bar{A}$ and decreases with b. We next provide a numerical example to show that this is indeed robustly the case for our parameter choices.

4 Numerical illustration

In this section, we present a simple numerical illustration of our model and comparative statics results. We begin with briefly discussing our parameter choices. Further details are provided in Appendix C.1 and robustness checks in Appendix C.2.

4.1 Parameter values

In our model, the pandemic hits the economy in period 1 and we therefore choose a period length of 1 year, i.e. the (partial) lockdown of the economy applies to an entire year. Accordingly, we choose $\tilde{\beta} = 0.97$ as the 'per-period' discount factor, which implies $\beta = 32.33$ for the total weight of the future. Moreover, we choose $r = 0.06$, which corresponds to a 300 basis points spread over a risk-free interest rate of $\sim 3\%$ that mirrors $\tilde{\beta} = 0.97$ in a simple steady-state version of our model.

With regards to the ability distribution, we calibrate a shifted log-normal distribution such that the steady-state income distribution in our model is broadly consistent with the data. Specifically, we assume that

$$a \sim \underline{a} + z, \text{ where } z \sim \text{log-normal}(\mu, \sigma),$$

which leaves us with three parameters to calibrate: $\underline{a}$, μ, and σ. To calibrate these parameters, we first normalize the subsistence level $\bar{c}$ and steady state TFP $\bar{A}$ to be equal to 1, and then require that, in the steady state, the poorest households are just at subsistence, which yields $\underline{a} = 1$. We further assume that the subsistence income level is 40% of the median income, which yields $\mu \approx 0.41$. We finally choose σ to best—in a mean-squared error sense—match the decile income shares in low-income countries according to the World Income Inequality Database, which yields $\sigma \approx 1.1$. While the ratio of minimum to median income may seem high, it is important to bear in mind that in our list of low-income countries on average 40% of the population live on less than 1.90$ per day (2011 PPP). To put the parameterized ability distribution further into perspective: it implies that more than 99.3% of the population can handle a 10% recession during the pandemic with no transfers. Hence, our choice for the ability distribution may be seen as conservative when contrasting this number with the projection that the world population who suffers from acute hunger might increase by $130m$ during the COVID-19 pandemic (World Food Programme (WFP), 2020). We plot the implied income distribution in Appendix C.1, and provide a robustness check using an alternative specification in Appendix C.2.

With respect to the TFP effect of the pandemic and the lockdown, we choose the following functional form for $g(\cdot)$

$$g(\theta) = 1 - |\theta - \kappa_1|^{\kappa_2},$$

and then choose $\gamma_P = 0.85$, $\kappa_1 = 0$, and $\kappa_2 = 2$ in our baseline calibration. This implies that an entirely uncontrolled pandemic causes a 15% recession which, recall, may also be attributable to external shocks arising from the global nature of the pandemic.

In our model, policy impacts the pandemic via the aggregate labor supply L. We summarize the effect of L on the pandemic and, hence, the death burden of the pandemic, by a function $P(L) = L^{\lambda}$, and choose $\lambda = 3$ in our baseline calibration.

δ corresponds to the fatality rate of an uncontrolled pandemic. This may be different in developing countries than in industrialized countries due to e.g. differences in health care, demographics or health status. We choose $\delta = 0.04$ in our baseline calibration, which is in the same range but slightly higher as fatality rates assumed in the literature on industrialized countries (Alvarez et al., 2020; Eichenbaum et al., 2020a; Glover et al., 2020). One of our comparative statics exercises below studies the effects of a change in δ on optimal policy and welfare.

Finally, we choose $\alpha = 0.5$ in our baseline specifications, which implies a moderate consumption-smoothing motive, but the basic pattern is very similar for alternative choices of α. We show this in Appendix C.2 for the case of $\alpha = 1$, which is consistent with our theoretical derivations of Section 3.1.

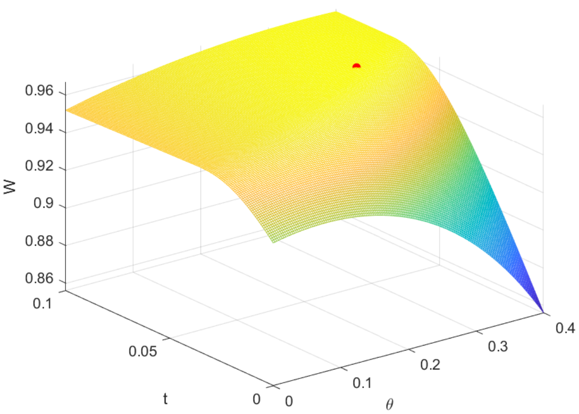
4.2 Numerical results

We now use our numerical example to illustrate our results of the previous sections and gain further insights on optimal policy.

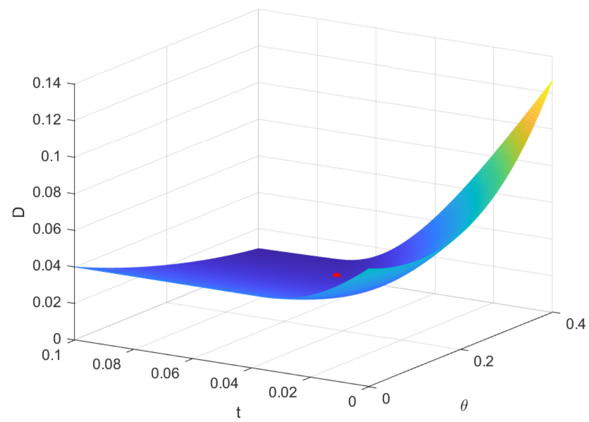
Figure 1 shows welfare W, the aggregate death burden D, and compliance with the lockdown ($(1 - L)/\theta$) in 3-D plots with θ on the x-axis and t on the y-axis. In each plot, the red dot indicates the (unconstrained) optimal policy. These plots provide some important and robust insights: Observe first from Figure 1a that the welfare function is single-peaked, albeit relatively flat around the optimal policy, lending support to our discussion of first-order conditions in the previous sections. For low levels of t, welfare is steeply increasing in t, mirroring a declining aggregate death burden (Figure 1b) that is driven by a smaller economic death burden on the one hand, but also a smaller pandemic death burden thanks to improved compliance with the lockdown (Figure 1c). This suggests large welfare losses from borrowing constraints, a point that we will return to below. Stricter lockdowns are costly for low levels of t, but beneficial for larger levels of t, reflecting the complementarity between these two policy variables. The optimal policy involves a sizeable lockdown of $\theta \sim .3$ and transfers that amount to almost 6% of steady state GDP. The welfare loss is $\sim 3.3\%$ and the aggregate death burden is $\sim 1.4\%$. Note that both W and D become relatively flat for high-enough t. This is when no or only very few households die from deprivation. Across a broad set of parameter specifications this is the case for (unconstrained) optimal policy,

Figure 1: Policy space and key outcomes

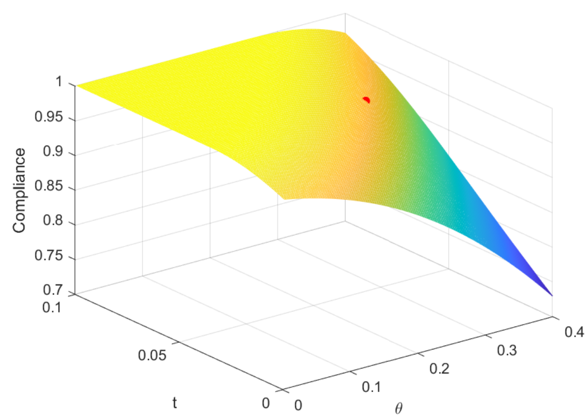
(a) Welfare



(b) Aggregate death burden



(c) Compliance

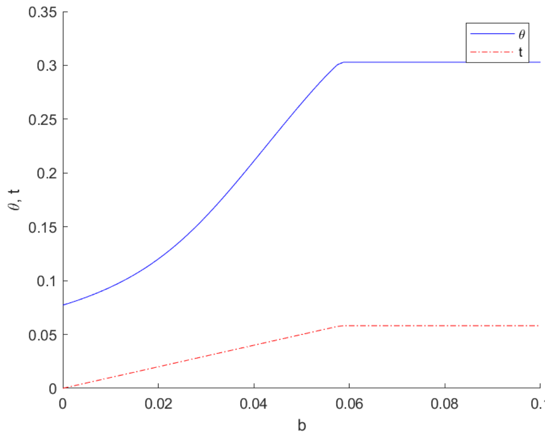


i.e. fighting the pandemic does not justify a sizeable economic death burden. Compliance is typically below 1 and in particular also for the optimal policy, reflecting the fact that some households cannot afford to fully comply with the lock-down.

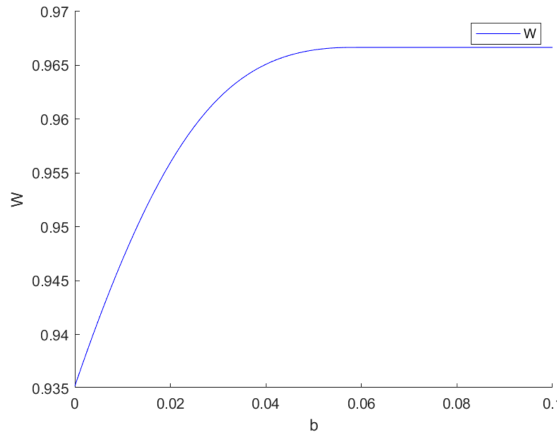
Figure 2 depicts the effect of a borrowing constraint on policy, welfare, and the death burden. In line with our theoretical predictions (Proposition 4), it shows that the optimal lockdown is smaller in the constrained optimum (Figure 2a) and that, as a consequence, welfare is lower (Figure 2b) and the aggregate death burden higher (Figure 2c). The aggregate death burden is higher because the government has less scope both to fight the disease and to financially support households. In fact, while in the unconstrained optimum aggregate deaths are almost entirely accounted for by the disease, this is no longer the case with a borrowing constraint, and Figure 2c reveals a horrible moral trade-off between saving lives from the pandemic and rescuing poorer households from deprivation.

Figure 2: Optimal policy with borrowing constraint

(a) Policy



(b) Welfare



(c) Aggregate death burden

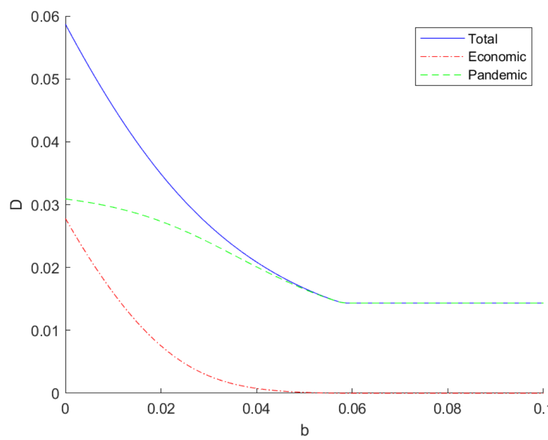


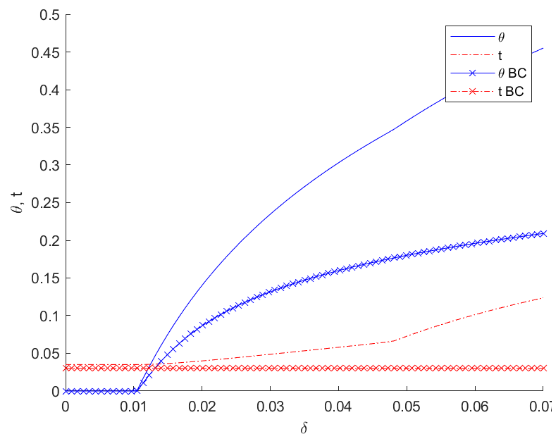
Figure 3 returns to our comparative statics exercises of Section 3.1.1, but considers both constrained and unconstrained policies. The left-hand-side panels focus on the death burden of the pandemic (δ), while the right-hand-side panels focus on steady state TFP ($\bar{A}$). In each figure, crossed out lines refer to the respective constrained optimum for $b = 0.03$, which corresponds to $\sim 50\%$ of the unconstrained optimal borrowing in our baseline specification.

Consider δ first. For δ sufficiently small, the optimum is a corner solution with $\theta = 0$.[28] For larger δ ($\sim \delta > 0.01$), however, the optimal θ is positive and increasing in δ, in line with Proposition 2. As Figure 3a suggests, this is true not only for the unconstrained optimum, but for the constrained optimum as well. Not surprisingly, welfare is smaller the larger δ both

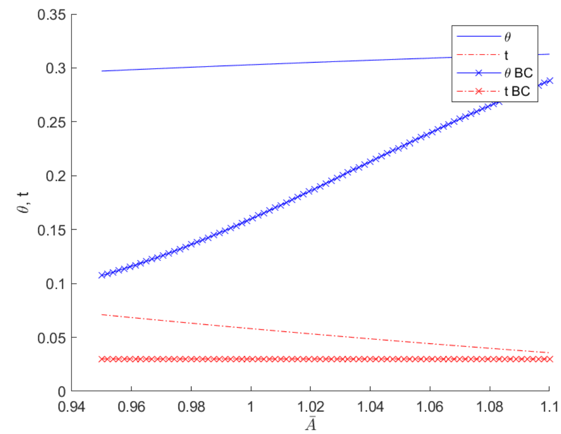
[28]While this is not our main point of interest, it is nevertheless interesting that this corner solution is consistent with the fact that e.g. governments do not fight the Influenza via lock-downs, despite the fact that it has an estimated global annual death burden of 290'000 to 650'000 (https://www.who.int/news-room/fact-sheets/detail/influenza-(seasonal)).

Figure 3: Comparative statics

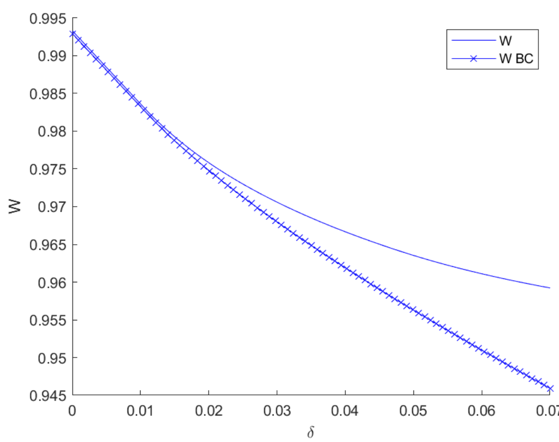
(a) δ: Policy



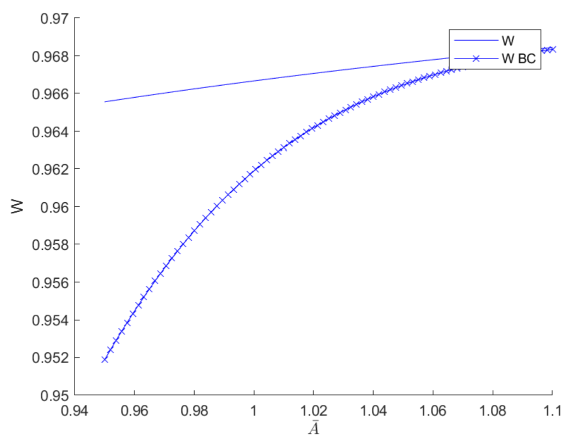
(b) $\bar{A}$: Policy



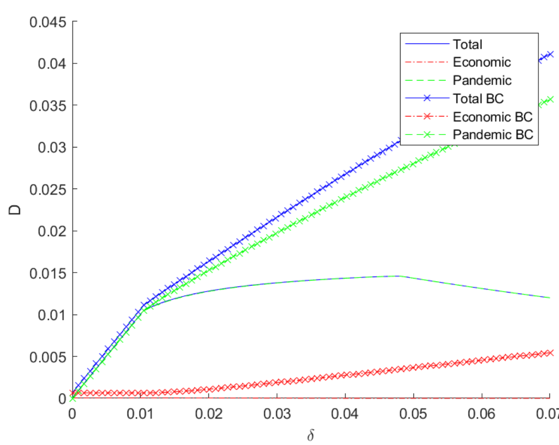
(c) δ: Welfare



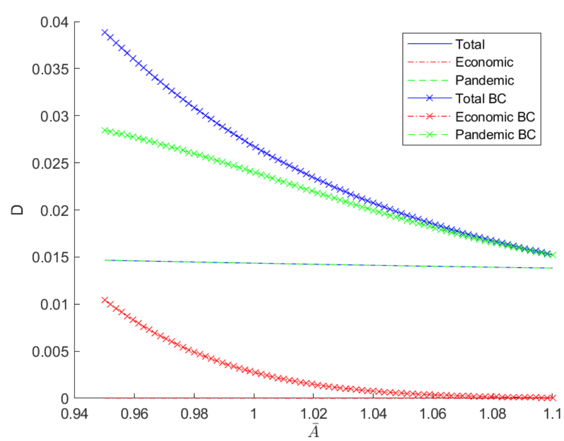
(d) $\bar{A}$: Welfare



(e) δ: Aggregate death burden



(f) $\bar{A}$: Aggregate death burden



in the constrained and the unconstrained optimum. Moreover, the welfare cost of a borrowing constraint are higher for more severe pandemics as shown in Figure 3c. This is a reflection of the fact that more severe pandemics justify stricter lockdowns, which are particularly costly in the presence of a borrowing constraint. As Figure 3e shows, the death burden is larger the higher δ and, not surprisingly, more so in the constrained than in the unconstrained optimum. As previously discussed, in the unconstrained optimum aggregate deaths are almost entirely accounted for by the disease, with economic deaths very small but positive. This is no longer true in the constrained optimum. In the latter case, the government is forced to trade off these two causes of fatalities, and this trade-off gets worse, the more severe the disease. Note that, in the unconstrained optimum, the aggregate death rate is hump-shaped in δ. For δ sufficiently high, economic deaths are 0, and the complementarity between θ and t implies that the death rate in the unconstrained optimum declines in δ.

The right-hand side of Figure 3 considers variations in $\bar{A}$. In line with Proposition 3, the optimal θ is larger in richer countries. This is true both in the constrained and the unconstrained optimum. In the latter case, optimal transfers are decreasing with $\bar{A}$, as fewer households are endangered by deprivation. As a consequence, the gap between the constrained and the unconstrained optimal transfers is larger in countries with lower $\bar{A}$, and governments in these countries see themselves forced to more heavily cut back on the lockdown in order to prevent deprivation. In other words, the gap between unconstrained and constrained optimal policies is smaller in richer countries, both in terms of policy choices and in terms of welfare implications, i.e. borrowing constraints are particularly costly in poor countries.

5 Robustness and Extensions

In this section, we provide further discussions. We begin with reconsidering some of our assumptions and discuss robustness of our main insights to alternative specifications. We then consider the distributional consequences and political economy of the pandemic.

5.1 Discussion

Our work is centered on the analysis of jointly optimal lockdowns and transfer payments and the policy implications and welfare cost of limited fiscal space. To keep our analysis tractable and thus build intuition of key effects at play, we make various simplifying assumptions. Yet, our main insights are likely to prevail in alternative set-ups and might even get reinforced as we now briefly discuss.

We take a simplistic view on fiscal policy. On the one hand, targeted transfers to support vulnerable parts of society would improve the efficiency of fiscal policy in our model and, hence, reduce the need for borrowing. Note, however, that in our model all households might potentially benefit from the consumption-smoothing effect of transfers—more on that later. Moreover, lump-sum transfers may be a feasible first response to an unexpected pandemic in the absence of pre-existing more sophisticated policy instruments.[29] On the other hand, several forces tend to increase the need for fiscal space: For one thing, the global economic shock causes a major recession and falling tax revenues. In the short-run, it may not be feasible to 1:1 reduce precommitted government expenditures, thus increasing the need for additional fiscal space via borrowing. For another, we do not include any direct costs of fighting the pandemic, such as investments in the healthcare system or testing capacities, which require additional fiscal resources. And finally, we do not consider longer-run adverse effects of the pandemic on the economy.[30] Counteracting policies such as temporary layoff assistance or financial assistance to businesses increase the need for fiscal space even further and tend to reinforce the complementarity between fiscal policy and a lockdown.

Similarly, a general lockdown is a blunt policy instrument, and smart testing and containment policies (Eichenbaum et al., 2020b; Piguillem and Shi, 2020; Romer, 2020) or alternating lockdowns (Meidan et al., 2020) may substantially improve efficiency. Nevertheless, they are unlikely to fundamentally change the trade-offs we consider and, in fact, our reduced form approach to modeling the pandemic as a function of aggregate labor supply may be interpreted as reflecting such alternative containment policies.

At a general level, the role of lump-sum transfers in our model is to enable intertemporal substitution of consumption by households. In principle, such intertemporal substitution can also be realized via private borrowing. Yet, private borrowing may be substantially hindered in times of uncertainty due to major health and economic shocks, and this is particularly true for the most vulnerable parts of a society, i.e. those that are most in need of intertemporal substitution. Moreover, note that all households would like to borrow in the first period. Who would they be borrowing from? The rest of the world. But international contract enforcement is difficult because the judge is in one country and the collateral (or the bailiff) in another. For this reason, international finance is constrained.[31] Indeed, the massive fiscal stimuli and transfer payments that are put in place in countries all across the world point to an important role of the state in facilitating intertemporal substitution.

[29] In either case, the possibility of targeted transfers would have qualitatively the same effect as a lower r in our model in the sense that they would lower the cost per dollar of transfers to vulnerable parts of society.

[30] See Barro et al. (2020) and Jordà et al. (2020) for analyses of the longer-run consequences of the Spanish flu.

[31] Of course, in times of a global pandemic all countries would seek to borrow, i.e. there must be some sort of borrowing from rich to poor. Nevertheless, in an ideal setting this should involve net borrowing of poor countries and, if such borrowing is limited due to e.g. a flight of capital to safety, this only reinforces borrowing constraints.

Finally, our mechanisms might also get reinforced if we consider more than one goods. Consider the case of two goods, a basic good with strictly positive subsistence consumption level and a luxury good. Then, if a sector-specific lockdown of the luxury good sector is infeasible or costly,[32] the lockdown might put inflationary pressure on basic goods. This is because households at or close to subsistence consumption of the basic good will increase their relative demand for the basic good during the recession, and if this increase in relative demand is not matched by an increase in relative supply, the relative price of the basic good has to increase to induce rich households to increase their relative demand for the luxury good.

5.2 Distributional consequences and political economy

So far, we have focused on aggregate welfare as well as distributional consequences of the pandemic *across* countries. Yet, the pandemic and aggregate-welfare optimal policy have important distributional consequences also *within* countries with possibly profound consequences for the political economics of lockdown policies. We briefly discuss these issues next.

For our analysis of optimal policy, we did not need to take a stand on the individual risk of dying during the pandemic. Yet, this risk is unequally distributed. Most importantly, under the weak assumption that—ceteris paribus—households are more likely to catch the disease the more they work,[33] the probability to die is monotonically decreasing in a household's ability, i.e. the poor have to disproportionally bear the health burden of the pandemic.

Households also differ in their individual preferences over policies, and this has interesting political economy implications. A detailed account of these implications is beyond the scope of our paper and depends, among others, on the household-level probability of dying from the disease and on how exactly the future debt-service costs are financed, which did not matter for our analysis of Section 3.1. Nevertheless, it is informative to briefly consider the case where $\alpha = 1$ and where future debt service costs are financed via a 'flat tax', i.e. there is no consumption smoothing motive and the future tax burden is proportionate to income. With these assumptions, we have the following result:

Proposition 5 (Single-crossing preferences)
Let $\alpha = 1$ and suppose that future debt service costs are financed via a flat tax τ on income

[32] This may be because it is not politically or administratively feasible to discriminate between sectors, because a sector-level lockdown increases the need for costly redistribution (Glover et al., 2020), or because there are input-output linkages between the sectors (Guerrieri et al., 2020).

[33] Note that this is consistent with our aggregate modeling of the pandemic— Assumptions 1 and 2—if

$$d(a) := \delta \cdot l(a) \cdot L^{\lambda-1}.$$

and let households with equal labor supply in period 1 *have the same probability of catching the disease. Moreover, let* $\succ_a$ *denote preferences of household* a *over policies. Let* $\mathcal{P}$ *denote the set of admissible policies* $p = (\theta, t)$*. Let* $\tilde{d}$ *denote the probability of dying from the disease of a household who fully complies with the lockdown and suppose that policies are ordered according to* $\tilde{d} + t \cdot \frac{\bar{A} \cdot \mu_a}{\beta \cdot \bar{c}}$*, i.e.* $p^h > p^l \Leftrightarrow \tilde{d}^h + t^h \cdot \frac{\bar{A} \cdot \mu_a}{\beta \cdot \bar{c}} > \tilde{d}^l + t^l \cdot \frac{\bar{A} \cdot \mu_a}{\beta \cdot \bar{c}}$*, where here and below we use a superscript* ^{h} $(^l)$ *to denote the value of an endogenous variable given policy* p^h (p^l)*. Then, for any pair of policies* $p^h > p^l$ *and every pair of households* $a^h > a^l \geq \max\{{a_2}^h, {a_2}^l\}$ *it holds*

$$p^l \succ_{a^l} p^h \;\Rightarrow\; p^l \succ_{a^h} p^h$$
$$p^h \succ_{a^h} p^l \;\Rightarrow\; p^h \succ_{a^l} p^l.$$

The proof of Proposition 5 is given in Appendix A.6. In words, Proposition 5 implies that the preferences of households that can fully comply with a lockdown satisfy the single-crossing property. It is important to note what this does—and does not—imply. It does not directly allow to apply a median voter theorem because depending on the policy a 'median voter' may not be able to fully comply with the lockdown and the above result does not apply. If, however, ex-ante we restrict the set of policies such that the median voter can always fully comply, the policy outcome in a Downsian two-party competition would be her preferred policy. Note that this is a plausible restriction on policy: If, for example, the median income was 250% of subsistence, it would imply that all policies were admitted that involve recessions of $\sim$ 60% or less, and even deeper recessions for positive transfers. More generally, the outcome would be the preferred policy of some decisive voter. Hence, Proposition 5 also suggests that, ceteris paribus, we should expect to observe more intense policy measures against the pandemic and less supporting transfers in societies where political power is in the hands of an elite, i.e. where the decisive household is richer, and vice versa in societies with populist governments.[34]

While Proposition 5 points to important conflicts of interest over policies, it is interesting to note that providing financial support to vulnerable parts of society in times of a pandemic is in the self-interest of households with highest ability if the negative externality of working on the pandemic is sufficiently large and the future is sufficiently important vis-à-vis the present. We show this in Proposition 6 where, for simplicity, we consider the case of $\alpha = 1$, $b = 0$, i.e. no government borrowing, and $d(a) = d(P)$, i.e. the risk of catching the disease only depends on aggregate behavior. We then show that for any given θ, all households $a \geq \mu_a$ are better off providing some support to the poor in times of a pandemic if β is sufficiently large.

[34] There may, however, be opposing forces not captured by our model, e.g., if the elite can capture a disproportionate share of domestic income and thus has strong economic interest in keeping the economy running.

Proposition 6 (Individual gains from supporting the poor)
Let $\alpha = 1$, $b = 0$, $d(a) = d(P)$, and consider lump-sum transfers $t \cdot \bar{A} \cdot \mu_a$ financed via a contemporaneous tax proportionate to steady-state income, i.e. net transfers to household a are $t \cdot \bar{A} \cdot [\mu_a - a]$. For every $\theta > 0$ such that $\mu_a \geq \frac{\bar{c}}{(1-\theta)\cdot A} > \inf\{\mathcal{A}\}$, there exists a $\bar{\beta}(\theta)$ such that for $\beta \geq \bar{\beta}(\theta)$ it holds that

$$\left.\frac{dU(a)}{dt}\right|_{\theta,t=0} > 0, \quad \text{for all } a \in \mathcal{A}.$$

The proof of Proposition 6 is given in Appendix A.7. Note that Condition $\mu_a \geq \frac{\bar{c}}{(1-\theta)\cdot A} > \inf\{\mathcal{A}\}$ in Proposition 6 simply guarantees that L is decreasing in t at $t = 0$ by ruling out that all households can fully comply with the lockdown and that household a_2 is a net contributor to the transfers.

6 Conclusion

In this paper, we have analyzed optimal joint policies of a lockdown and transfer payments in times of a pandemic. We have focused on a developing country set-up and shown that both policy instruments interact in non-trivial ways in combating the pandemic and supporting the poor through the recession. Our work points to important distributional consequences of the pandemic: Within countries, poorer households disproportionally die. Across countries, combating the pandemic is costlier for poorer countries, implying that these countries suffer from a higher death burden and a greater welfare loss. This is true with unlimited fiscal space, and may get much worse if government borrowing is constrained. In such cases, developing countries see themselves forced to fight less the pandemic in order to protect the poor. But still, this may not be sufficient to save all of them from deprivation due to the multiple global economic shocks in times of a pandemic. This is not just a theoretical possibility. Forecasts show that COVID-19 could almost double the number of people suffering from acute hunger (World Food Programme (WFP), 2020). In the words of Arif Husain, Chief Economist of the World Food Programme: '*COVID-19 is potentially catastrophic for millions who are already hanging by a thread. It is a hammer blow for millions more who can only eat if they earn a wage. Lockdowns and global economic recession have already decimated their nest eggs. It only takes one more shock – like COVID-19 – to push them over the edge*'.[35] Our work may help clarify some of the horrible trade-offs involved, and it corroborates the urgent calls for action to support developing countries in the COVID-19 crisis (Bolton et al., 2020;

[35] https://www.wfp.org/news/covid-19-will-double-number-people-facing-food-crises-unless-swift-action-taken, accessed on 5/14/2020.

International Monetary Fund (IMF), 2020b). This is the right thing to do. Considering the global nature of the pandemic, it is also the smart thing to do (Hausmann, 2020).

Appendix

A Proofs

A.1 Proof of Proposition 1

(i) We proceed by contradiction.

Let $\hat{\theta}(t)$ be a candidate solution and suppose that $\frac{dD}{d\theta}\big|_{\theta=\hat{\theta}(t)} \geq 0$. Note that this implies $\frac{dA}{d\theta}\big|_{\theta=\hat{\theta}(t)} < 0$. This is because if $\hat{\theta}(t)$ was indeed optimal, then a marginal increase in θ would strictly lower the aggregate labor supply and, hence, the pandemic by Proposition 1(ii). As a consequence, $\frac{dD}{d\theta}\big|_{\theta=\hat{\theta}(t)} \geq 0$ requires a larger economic death burden, which in turn requires a lower A. Now, a larger θ weakly lowers labor supply of all households but for households $a \in (a_1, a_2]$ who increase their labor supply, but remain at subsistence. In combination with $\frac{dA}{d\theta}\big|_{\theta=\hat{\theta}(t)} < 0$, this implies that

$$\frac{d}{d\theta}\left[\int_{a\in\mathcal{A}} \left[v(A\cdot a\cdot l(a) + t\cdot \bar{A}\cdot \mu_a) - \bar{c}\right]\cdot f(a)da\right]_{\theta=\hat{\theta}(t)} < 0.$$

Hence, a marginal decrease in θ would strictly increase the first summand in Equation (10). Moreover, as $\frac{dD}{d\theta}\big|_{\theta=\hat{\theta}(t)} \geq 0$ by assumption, a marginal decrease in θ would weakly increase the second summand, a contradiction to $\hat{\theta}(t)$ being optimal.

(ii) We proceed by contradiction.

Let $\hat{\theta}(t)$ be a candidate solution and suppose that $\frac{dL}{d\theta}\big|_{\theta=\hat{\theta}(t)} \geq 0$. Note that this implies $\frac{dA}{d\theta}\big|_{\theta=\hat{\theta}(t)} < 0$, for if not, an increase in θ causes every household to decrease their labor supply, some of them strictly. Hence, starting from $\hat{\theta}(t)$, a marginal decrease in θ would weakly decrease L and therefore P and at the same time strictly increase A. This would strictly increase period 1 consumption of every household $a \in \mathcal{A}\backslash[a_1, a_2)$ and not affect period 1 consumption of households $a \in [a_1, a_2)$, who remain at subsistence, i.e. they exactly offset the higher A by a lower labor supply. It follows that

$$\frac{d}{d\theta}\left[\int_{a\in\mathcal{A}} \left[v(A\cdot a\cdot l(a) + t\cdot \bar{A}\cdot \mu_a) - \bar{c}\right]\cdot f(a)da\right]_{\theta=\hat{\theta}(t)} < 0.$$

In addition, a marginal decrease in θ would decrease D as it weakly alleviates the pandemic ($\frac{dL}{d\theta}\big|_{\theta=\hat{\theta}(t)} \geq 0$), and the higher A lowers the economic death burden, a contradiction to $\hat{\theta}(t)$ being optimal.

□

A.2 Proof of Lemma 1

Using Equation (19) in Equation (17) yields

$$\frac{dL}{d\theta} = \frac{dA}{d\theta} \cdot \frac{a_1}{\bar{A} \cdot \mu_a} \cdot \frac{dL}{dt} - [1 - F(a_2)] .$$

Using the above in Equation (16) yields

$$\begin{aligned} \frac{dD}{d\theta} =& - f(a_1) \cdot \frac{a_1}{A} \cdot \frac{dA}{d\theta} \cdot [1 - d(P)] + [1 - F(a_1)] \cdot \delta \cdot P'(L) \cdot \frac{dA}{d\theta} \cdot \frac{a_1}{\bar{A} \cdot \mu_a} \cdot \frac{dL}{dt} \\ & - [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot \delta \cdot P'(L) \\ =& a_1 \cdot \left[\frac{dA}{d\theta} \cdot \frac{1}{\bar{A} \cdot \mu_a} \cdot \frac{dD}{dt} - \underbrace{\frac{1}{a_1} \cdot [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot \delta \cdot P'(L)}_{:=\mathscr{E}} \right], \end{aligned} \tag{A.1}$$

where the second equality follows from using (18). The optimal solution for θ satisfies $\frac{dD}{d\theta} = 0$, which is the case when the term in the big squared brackets is 0, i.e. the result follows from showing that this term is decreasing in t. Now, at (θ^*, t^*), $\frac{d\tilde{W}}{dt}$ crosses 0 from above, implying that

$$\left. \frac{d^2 D}{dtdt} \right|_{\theta=\theta^*, t=t^*} > 0$$

and, hence, that $\frac{dA}{d\theta} \cdot \frac{1}{\bar{A} \cdot \mu_a} \cdot \frac{dD}{dt}$ is decreasing in t at $t = t^*$. The desired result then follows from the fact that $\mathscr{E}$ is non-decreasing in t, which implies

$$\left. \frac{d^2 D}{d\theta dt} \right|_{\theta=\theta^*, t=t^*} < 0$$

and therefore

$$\left. \frac{d^2 W}{d\theta dt} \right|_{\theta=\theta^*, t=t^*} > 0.$$

To see that $\mathscr{E}$ is non-decreasing in t, note that $[1 - F(a_2)]$ and $[1 - F(a_1)]$ are both increasing in t. Moreover,

$$\frac{d}{dt} \left[\frac{P'(L)}{a_1} \right] = \frac{d}{dt} \left[\frac{\lambda \cdot L^{\lambda-1}}{a_1} \right] = \lambda \cdot (\lambda - 1) \cdot \frac{L^{\lambda-2}}{a_1} \cdot \frac{dL}{dt} - \lambda \cdot \frac{da_1}{dt} \cdot \frac{L^{\lambda-1}}{{a_1}^2}, \tag{A.2}$$

which is positive if

$$(\lambda - 1) \cdot \frac{dL}{dt} \cdot \frac{1}{L} - \frac{da_1}{dt} \cdot \frac{1}{a_1} \geq 0.$$

Using Equation (19) and the fact that $\frac{da_1}{dt} = -\frac{\bar{A} \cdot \mu_a}{A}$ yields

$$-\frac{(\lambda - 1) \cdot \int_{a_1}^{a_2} \frac{\mu_a \cdot \bar{A}}{a \cdot A} \cdot f(a) da}{L} + \frac{\mu_a \cdot \bar{A}}{a_1 \cdot A} \geq 0,$$

which, using Equation (7) can be rearranged to yield

$$\frac{(\lambda - 1) \cdot \int_{a_1}^{a_2} l(a) \cdot f(a) da}{L} \leq 1$$

and, hence, is true for (θ^*, t^*) by Assumption 4.

□

A.3 Proof of Lemma 2

To show the result, we proceed by contradiction. Throughout, we consider a marginal increase in δ to $\hat{\delta} > \delta$, and use (θ^*, t^*) to denote the initially optimal policy, $(\hat{\theta}, \hat{t})$ to denote the new optimal policy, and x^* $(\hat{x})$ the value of an endogenous variable x in the old (new) optimum.

Note first that δ does not have a direct effect on a_1, a_2 and, hence, L. Using Equation (9) in the objective in decision problem (12)

$$\tilde{W} = [1 - F(a_1) - (1 - F(a_1)) \cdot \delta \cdot P(L)] \cdot \left[\bar{A} \cdot \mu_a - \bar{c}\right] - r \cdot t \cdot \bar{A} \cdot \mu_a,$$

it therefore follows that for any given policy, a change in δ impacts welfare only via its direct effect on the pandemic death burden, $(1 - F(a_1)) \cdot \delta \cdot P(L)$. Hence, for any given policy (θ, t) it holds

$$\tilde{W}(\theta, t; \hat{\delta}) - \tilde{W}(\theta, t; \delta) = -\left[\bar{A} \cdot \mu_a - \bar{c}\right] \cdot (1 - F(a_1)) \cdot P(L) \cdot \left[\hat{\delta} - \delta\right], \tag{A.3}$$

where we used $\tilde{W}(\theta, t; \delta)$ to denote welfare given θ, t and δ. Now, consider a policy $(\hat{\theta}, \hat{t})$ and suppose by way of contradiction that

$$(1 - F(\hat{a}_1)) \cdot P(\hat{L}) \geq (1 - F({a_1}^*)) \cdot P(L^*). \tag{A.4}$$

Using Equation (A.3), we get

$$\begin{aligned}
\tilde{W}(\theta^*, t^*; \hat{\delta}) - \tilde{W}(\hat{\theta}, \hat{t}; \hat{\delta}) =& \tilde{W}(\theta^*, t^*; \hat{\delta}) - \tilde{W}(\theta^*, t^*; \delta) - \left[\tilde{W}(\hat{\theta}, \hat{t}; \hat{\delta}) - \tilde{W}(\hat{\theta}, \hat{t}; \delta)\right] \\
&+ \tilde{W}(\theta^*, t^*; \delta) - \tilde{W}(\hat{\theta}, \hat{t}; \delta) \\
=& -\left[\bar{A} \cdot \mu_a - \bar{c}\right] \cdot \left[\hat{\delta} - \delta\right] \cdot \left[(1 - F({a_1}^*)) \cdot P(L^*) - (1 - F(\hat{a}_1)) \cdot P(\hat{L})\right] \\
&+ \tilde{W}(\theta^*, t^*; \delta) - \tilde{W}(\hat{\theta}, \hat{t}; \delta) \\
&\geq 0,
\end{aligned} \tag{A.5}$$

where the inequality follows from Condition (A.4), $\hat{\delta} > \delta$, and the optimality of (θ^*, t^*) given δ. Inequality (A.5) is a contradiction to $(\hat{\theta}, \hat{t})$ being optimal because given $\hat{\delta}$ and starting from (θ^*, t^*), the government can increase welfare by marginally increasing θ while holding fixed t by Condition (20). Hence, given $\hat{\delta}$ the optimal policy must involve strictly larger welfare than when choosing (θ^*, t^*). The contradiction establishes the result.

□

A.4 Proof of Proposition 2

To show the result, we distinguish three cases.

(A) If given $\delta = 0$ the optimal transfer is $t^* = 0$, the result follows immediately from Condition (20) and Lemma 1.

(B) If $\frac{dD}{dtd\delta}\big|_{t=t^*,\theta=\theta^*,\delta=0} < 0$ and $t^* > 0$, the result follows again from Condition (20) and Lemma 1.

(C) Finally, for the case of $\frac{dD}{dtd\delta}\big|_{t=t^*,\theta=\theta^*,\delta=0} \geq 0$ and $t^* > 0$, we consider a marginal decrease in t such that the first-order condition for t is again satisfied. We then show that given the marginal changes in δ and t, it is beneficial to increase θ. The result then follows from Lemma 1.

Using the implicit function theorem and recalling that we hold constant θ, we get that the first-order condition for t continues to hold if[36]

$$\frac{dt}{d\delta} = -\frac{\frac{d^2D}{dtd\delta}}{\frac{d^2D}{dtdt}}, \tag{A.8}$$

i.e. given (A.8) we have

$$d\left[\frac{dD}{dt}\right] = \frac{d^2D}{dtd\delta} \cdot d\delta + \frac{d^2D}{dtdt} \cdot dt = 0. \tag{A.9}$$

The changes in δ and t both affect $\frac{dD}{d\theta}$. In particular, using Equation (A.1), we get

$$\frac{d^2D}{d\theta d\delta} = \frac{a_1 \cdot \frac{dA}{d\theta}}{\bar{A} \cdot \mu_a} \cdot \frac{d^2D}{dtd\delta} - [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot P'(L) \tag{A.10}$$

and

$$\frac{d^2D}{d\theta dt} = \frac{a_1 \cdot \frac{dA}{d\theta}}{\bar{A} \cdot \mu_a} \cdot \frac{d^2D}{dtdt} - \frac{\bar{A} \cdot \mu_a}{A} \cdot \left[\frac{\frac{dA}{d\theta}}{\bar{A} \cdot \mu_a} \cdot \frac{dD}{dt} + \frac{f(a_2)}{1-\theta} \cdot [1 - F(a_1)] \cdot \delta \cdot P'(L) + ... \right. \tag{A.11}$$

$$\left. + f(a_1) \cdot [1 - F(a_2)] \cdot \delta \cdot P'(L) - [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot \delta \cdot P''(L) \cdot \int_{a_1}^{a_2} \frac{1}{a} \cdot f(a)da, \right.$$

[36]Differentiating Equation (18) with respect to δ and t, respectively, yields, after some straightforward simplifications,

$$\frac{d^2D}{dtd\delta} = \frac{\bar{A} \cdot \mu_a}{A} \cdot \left[f(a_1) \cdot P(L) - [1 - F(a_1)] \cdot P'(L) \cdot \int_{a_1}^{a_2} \frac{1}{a} \cdot f(a)da\right] \tag{A.6}$$

$$\frac{d^2D}{dtdt} = \left(\frac{\bar{A} \cdot \mu_a}{A}\right)^2 \cdot \left[f'(a_1)\,[1 - d(P)] - 2 \cdot f(a_1) \cdot \delta \cdot P'(L) \cdot \int_{a_1}^{a_2} \frac{1}{a} \cdot f(a)da + ... \right. \tag{A.7}$$

$$\left. + [1 - F(a_1)] \cdot \delta \cdot P''(L) \left[\int_{a_1}^{a_2} \frac{1}{a} \cdot f(a)da\right]^2 + \frac{[1 - F(a_1)]}{a_1} \cdot \delta \cdot P'(L) \cdot [f(a_2) - f(a_1)]\right].$$

Hence, both $\frac{d^2D}{dtd\delta}$ and $\frac{d^2D}{dtdt}$ are finite. Moreover, $t^* > 0$ implies that $\frac{d^2D}{dtdt}\big|_{\theta=\theta^*,t=t^*} > 0$, which is the case for $\delta = 0$ if $f'(a_1{}^*) > 0$.

where by Equation (A.1) we have

$$\frac{dA}{d\theta} \cdot \frac{1}{\bar{A} \cdot \mu_a} \cdot \frac{dD}{dt} = \frac{1}{a_1} \cdot [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot \delta \cdot P'(L) \tag{A.12}$$

at the optimal solution. Totally differentiating $\frac{dD}{d\theta}$ and using $d\theta = 0$ along with Equations (A.8) to (A.12) yields

$$d\left[\frac{dD}{d\theta}\right] = -\left[1 - F(a_2)\right] \cdot \left[1 - F(a_1)\right] \cdot P'(L) \cdot d\delta + \frac{\frac{d^2D}{dtd\delta}}{\frac{d^2D}{dtdt}} \cdot \frac{\bar{A} \cdot \mu_a}{A} \cdot d\delta \cdot \delta \tag{A.13}$$

$$\cdot \left[\frac{1}{a_1} \cdot [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot P'(L) + \frac{f(a_2)}{1-\theta} \cdot [1 - F(a_1)] \cdot P'(L) + \ldots \right.$$

$$\left. + f(a_1) \cdot [1 - F(a_2)] \cdot P'(L) - [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot P''(L) \cdot \int_{a_1}^{a_2} \frac{1}{a} \cdot f(a)da\right].$$

Clearly,

$$d\left[\frac{dD}{d\theta}\right]_{\theta=\theta^*, t=t^*, \delta=0} < 0, \tag{A.14}$$

and the result follows from continuity of the above expressions in δ, t, θ.

□

A.5 Proof of Proposition 3

To show the result, we proceed in four steps. Throughout, we let (θ^*, t^*) denote the initially optimal policy and consider a marginal increase in $\bar{A}$ to $\hat{\bar{A}}$.

Step 1 The government can undo the effect of the change in $\bar{A}$ on a_1, a_2, and L by lowering t while holding constant θ. In particular, a_1 is constant when choosing $\tilde{t}$ such that

$$\frac{\bar{c} - \tilde{t} \cdot \hat{\bar{A}} \cdot \mu_a}{\hat{A}} = \frac{\bar{c} - t^* \cdot \bar{A} \cdot \mu_a}{A^*}.$$

Using the fact that θ is held constant and, hence, that $\frac{\hat{\bar{A}}}{\hat{A}} = \frac{\bar{A}}{A^*}$, this can be rearranged to

$$\tilde{t} = t^* - \frac{\bar{c}}{\mu_a}\left[\frac{1}{\bar{A}} - \frac{1}{\hat{\bar{A}}}\right].$$

Clearly, $\hat{a}_1 = {a_1}^*$ implies

$$\hat{a}_2 = \frac{\hat{a}_1}{1-\theta^*} = \frac{{a_1}^*}{1-\theta^*} = {a_2}^*$$

and, hence, $\hat{L} = L^*$.

Step 2 For a given θ, $\frac{dA}{d\theta} \cdot \frac{1}{A}$ does not depend on $\bar{A}$. Hence, step 1 implies that

$$\left.\frac{dL}{d\theta}\right|_{\theta=\theta^*,t=\tilde{t};\hat{\bar{A}}} = \left.\frac{dL}{d\theta}\right|_{\theta=\theta^*,t=t^*;\bar{A}}$$

and therefore

$$\left.\frac{dD}{d\theta}\right|_{\theta=\theta^*,t=\tilde{t};\hat{\bar{A}}} = \left.\frac{dD}{d\theta}\right|_{\theta=\theta^*,t=t^*;\bar{A}},$$

i.e. given $\tilde{t}$ it is just optimal to not adjust θ.

Step 3 For a given θ, $\frac{\bar{A}}{A}$ does not depend on $\bar{A}$. Hence, step 1 implies that

$$\left.\frac{dL}{dt}\right|_{\theta=\theta^*,t=\tilde{t};\hat{\bar{A}}} = \left.\frac{dL}{dt}\right|_{\theta=\theta^*,t=t^*;\bar{A}}$$

and therefore

$$\left.\frac{dD}{dt}\right|_{\theta=\theta^*,t=\tilde{t};\hat{\bar{A}}} = \left.\frac{dD}{dt}\right|_{\theta=\theta^*,t=t^*;\bar{A}}.$$

But then

$$-\left.\frac{dD}{dt}\right|_{\theta=\theta^*,t=\tilde{t};\hat{\bar{A}}} = \frac{r \cdot \bar{A} \cdot \mu_a}{-\bar{c} + \bar{A} \cdot \mu_a} > \frac{r \cdot \hat{\bar{A}} \cdot \mu_a}{-\bar{c} + \hat{\bar{A}} \cdot \mu_a},$$

i.e. given $\theta = \theta^*$, the optimal $\hat{t}$ is such that

$$\hat{t} > \tilde{t} = t^* - \frac{\bar{c}}{\mu_a}\left[\frac{1}{\bar{A}} - \frac{1}{\hat{\bar{A}}}\right].$$

Step 4 Steps 2, 3 and the complementarity between θ and t (Lemma 1) imply that $\hat{\theta} > \theta^*$.

□

A.6 Proof of Proposition 5

In period 2 the proportionate income tax is used to finance the debt service costs and, hence,[37]

$$\tau \cdot a \cdot \bar{A} = \frac{r \cdot t \cdot \bar{A} \cdot a}{1 - D}.$$

[37]Strictly speaking, this expression assumes that the future ability distribution is not affected by policy, in line with our aggregate welfare analysis—see Footnote 16 for a discussion. This is not essential for the proof. More generally, it holds that

$$\tau \cdot a \cdot \bar{A} = \frac{\mu_a}{\tilde{\mu}_a} \cdot \frac{r \cdot t \cdot \bar{A} \cdot a}{1 - D},$$

where $\tilde{\mu}_a$ denotes the average ability in period 2 that may depend on policy. Accounting for the term $\frac{\mu_a}{\tilde{\mu}_a}$ would not affect our subsequent arguments.

Given policy (θ, t), the expected lifetime utility of household $a \geq a_2$ is therefore given by

$$(1-\theta) \cdot A \cdot a + t \cdot \bar{A} \cdot \mu_a - \bar{c} + (1-\tilde{d}) \cdot \beta \cdot \left[\bar{A} \cdot a - r \cdot \frac{t \cdot \bar{A} \cdot a}{1-D} - \bar{c}\right]$$

where, as before, we use $\tilde{d}$ to denote the probability of dying from the disease of a household who fully complies with the lockdown. Now, consider a pair of policies $p^h > p^l$, and suppose that $p^l \succ_a p^h$ for some household $a \geq \max\{{a_2}^h, {a_2}^l\}$. This is the case if and only if

$$a \cdot \left[(1-\theta^l) \cdot A^l - (1-\theta^h) \cdot A^h\right] + \beta \cdot \bar{A} \cdot a \cdot \left[(1-\tilde{d}^l) \cdot \left(1 - \frac{r \cdot t^l}{1-D^l}\right) - (1-\tilde{d}^h) \cdot \left(1 - \frac{r \cdot t^h}{1-D^h}\right)\right]$$
$$> \left(\tilde{d}^h - \tilde{d}^l\right) \cdot \beta \cdot \bar{c} + \left(t^h - t^l\right) \cdot \bar{A} \cdot \mu_a.$$

Now, the right-hand side is positive and independent of a. It follows that the left-hand-side is positive as well and, hence, increasing in a, which proves that $p^l \succ_{a'} p^h$ for every $a' \geq a$. Similar arguments can be used to show the opposite case.

□

A.7 Proof of Proposition 6

To show the result, we derive a $\bar{\beta}(\theta)$ such that for $\beta \geq \bar{\beta}(\theta)$, the utility of all households is strictly increasing in t at $t = 0$.

Using the expression for net transfers we get for the full-compliance and the subsistence cutoffs

$$a_2 = \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{(1-\theta) \cdot A - t \cdot \bar{A}} \tag{A.15}$$

$$a_1 = \frac{\bar{c} - t \cdot \bar{A} \cdot \mu_a}{A - t \cdot \bar{A}}, \tag{A.16}$$

and for aggregate labor supply

$$L = F(a_1) + \int_{a_1}^{a_2} \frac{\bar{c} - t \cdot \bar{A} \cdot [\mu_a - a]}{A \cdot a} \cdot f(a)da + (1-\theta) \cdot [1 - F(a_2)]. \tag{A.17}$$

Differentiating with respect to t and simplifying terms yields

$$\frac{dL}{dt} = -\int_{a_1}^{a_2} \frac{\bar{A} \cdot [\mu_a - a]}{A \cdot a} \cdot f(a)da. \tag{A.18}$$

Now, $\mu_a \geq \frac{\bar{c}}{(1-\theta) \cdot A} > \inf\{\mathcal{A}\}$ in combination with $\theta > 0$ implies that for $t = 0$ it holds that $\frac{dL}{dt} < 0$ and $a_2 \leq \mu_a$. Hence, all households $a \leq a_2$ are net transfer recipients and they thus prefer any $t > 0$ over $t = 0$. Consider then households $a > a_2$. The expected lifetime utility of these households is given by

$$U(a) = (1-\theta) \cdot A \cdot a + t \cdot \bar{A} \cdot [\mu_a - a] - \bar{c} + \beta \cdot [1 - d(P)] \cdot \left[\bar{A} \cdot a - \bar{c}\right].$$

Differentiating with respect to t yields

$$\frac{dU(a)}{dt} = \bar{A} \cdot [\mu_a - a] - \beta \cdot \delta \cdot \lambda \cdot L^{\lambda-1} \cdot \frac{dL}{dt} \cdot \left[\bar{A} \cdot a - \bar{c}\right]. \tag{A.19}$$

As noted above, for $t = 0$ it holds that $\frac{dL}{dt} < 0$, implying that the second summand on the right-hand-side is positive while the first summand is negative for all $a > \mu_a$. Hence,

$$\left.\frac{dU(a)}{dt}\right|_{\theta, t=0} > 0, \quad \text{for all } a \in \mathcal{A}$$

if the right-hand-side of Equation (A.19) is non-decreasing in a.[38] Using Equations (A.15) to (A.18), we get that this is the case if

$$\beta \geq \left\{ \delta \cdot \lambda \cdot \left[F\left(\frac{\bar{c}}{A}\right) + \int_{\frac{\bar{c}}{A}}^{\frac{\bar{c}}{A \cdot (1-\theta)}} \frac{\bar{c}}{A \cdot a} \cdot f(a) da + (1-\theta) \cdot \left(1 - F\left(\frac{\bar{c}}{A \cdot (1-\theta)}\right)\right)\right]^{\lambda-1} \right.$$
$$\left. \cdot \int_{\frac{\bar{c}}{A}}^{\frac{\bar{c}}{A \cdot (1-\theta)}} \frac{\bar{A} \cdot [\mu_a - a]}{A \cdot a} \cdot f(a) da \right\}^{-1} := \bar{\beta}(\theta).$$

□

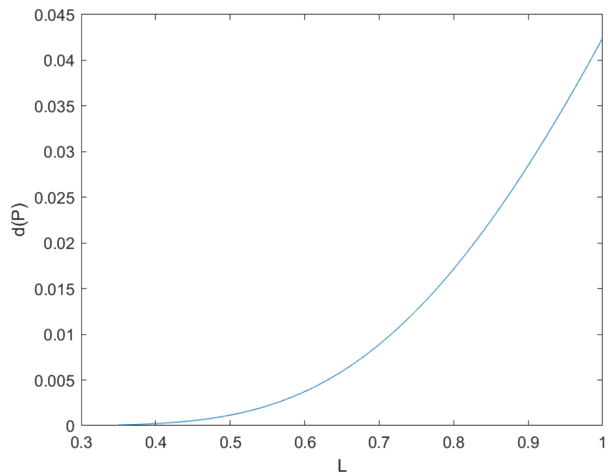
B Mathematical appendix

B.1 Motivation of $P(L)$ convex

In this part of the appendix, we briefly motivate our choice of a convex relationship between L and P in the context of a basic SIR model.

In our model, $P(\cdot)$ summarizes the effect of the pandemic on the death burden of the pandemic, $d(P) = \delta \cdot P$. Let I_{tot} denote the total share of the population that had the disease once the pandemic is over and I_{max} the share of the population that has the disease at the peak of the curve. Assuming that $S \approx 1$ initially, i.e. almost everyone is susceptible, the

[38] Note that with unbounded support of $F(\cdot)$, this is also necessary for *all* households to benefit from an increase in t in the right neighborhood of $t = 0$.



Notes: The figure depicts a stylized example of the death burden of the pandemic as a function of L. For a given L, R_0 has first been computed as $R_0 = 1 + 2 \cdot L^2$, which implies an R_0 of 3 for $L = 1$ and a full control of the pandemic for $L = 0$. Second, I_{tot} and I_{max} have been computed from Equations (B.1) and (B.2). Finally, these have been used to compute $d(P) = 0.5 \cdot {I_{max}}^2 \cdot I_{tot}$.

basic SIR model implies simple relationships between I_{tot}, I_{max} and the basic reproduction rate of infection R_0 (see, e.g. Kermack et al. (1927) and Hethcote (1989))

$$R_0 \cdot I_{tot} = -\ln(1 - I_{tot}) \tag{B.1}$$

$$I_{max} = 1 - \frac{1}{R_0} \cdot (1 + \ln(R_0)). \tag{B.2}$$

It further implies that the reproduction rate of the disease is quadratic in L (Alvarez et al., 2020).[39] Now, let us decompose the overall death burden into the death rate times the fraction of the population that got infected, and let the death rate be some function $h(I_{max})$ of the size of the peak. This yields

$$d(P) = h(I_{max}) \cdot I_{tot}.$$

The exact shape of $h(I_{max})$ is speculative, but it is plausibly convex with limited ICUs and a heterogeneous effect of intensive care on the death probability of infected persons.[40] The following figure takes into account these discussions and provides a stylized example of the total death burden of the disease as a function of L. The implied relationship is indeed convex. Further details are provided in the footer of the figure.

[39]This depends on the 'matching function' between susceptibles and infected—see Garibaldi et al. (2020) and Acemoglu et al. (2020) for a discussion.

[40]While not directly comparable, this is nevertheless broadly consistent with the assumption of a quadratic congestion effect in the healthcare system by Eichenbaum et al. (2020a).

B.2 Details on pandemic and optimal lockdown

In this part of the appendix, we argue why for a broad range of parameter values the optimal θ is increasing in δ. We begin with some general discussions and then provide a sufficient condition that is slack but nevertheless naturally satisfied under reasonable parameter restrictions. Throughout, we assume that $f(a)$ is single-peaked, consistent with empirical income distributions, and that it has support $[1, \bar{a}]$, where $\bar{a}$ may be infinite, i.e. we normalize ability in terms of the lowest ability.

For any given δ, the optimal policy solves first-order conditions (13) and (14). Using Equation (A.1), the former is satisfied iff

$$\frac{dA}{d\theta} \cdot \frac{1}{\bar{A} \cdot \mu_a} \cdot \frac{dD}{dt} = \frac{1}{a_1} \cdot [1 - F(a_2)] \cdot [1 - F(a_1)] \cdot \delta \cdot P'(L). \tag{B.3}$$

Hence, the optimal θ is increasing in δ if a marginal change in δ and t that leaves $\frac{dD}{dt}$ unchanged increases the right-hand-side of the above equation. If so, it would be beneficial to increase θ in response to this change, which implies that θ^* must increase by Lemma 1.

Now, the first-order condition for t implies that

$$f(a_1) \leq \frac{r \cdot A}{-\bar{c} + \bar{A} \cdot \mu_a} \cdot \frac{1}{1 - d(P)} \leq \frac{r \cdot A}{-\bar{c} + \bar{A} \cdot \mu_a} \cdot \frac{1}{1 - \delta} \tag{B.4}$$

which shows that a_1 and $f(a_1)$ are small for reasonable values of δ and r.[41] In economic terms this is to say that the cost of saving households from deprivation during the crisis are relatively low compared to the value of a life such that optimal policy saves the (vast) majority of households from deprivation. In this appendix, we limit attention to the case of a single-peaked $f(\cdot)$, in line with empirical income distributions. A small $f(a_1)$ then typically implies that $f'(a_1)$ is large relative to $f(a_1)$.[42] The key point is that in such case the change in t that is needed to rebalance first-order condition (14) is small such that the net effect of the changes in δ and t on the right-hand-side of Equation (B.3) is positive.

The following proposition makes this point more rigorously by providing a set of sufficient conditions for the optimal θ to increase in response to an increase in δ. As the proof shows, this sufficient condition is clearly not necessary and the result holds for a wide range of parameter values where this condition is not satisfied. Still, it is satisfied under reasonable assumptions as we briefly discuss below.

[41] Reasonable parameter choices imply $\frac{A}{-\bar{c}+\bar{A}\cdot\mu_a} < 1$. To see this note that if e.g. $A = 1$, i.e. there is no TFP loss in the first period, and the poorest household is just at subsistence in normal times, $\frac{A}{-\bar{c}+\bar{A}\cdot\mu_a} = 1$ would imply that the ratio of average to minimum income is 2.

[42] For example, in case of a standard log-normal distribution $f(x)$, this ratio is large for small x.

Proposition 7
Define $k_1 := \frac{f'(a_1{}^*)}{f(a_1{}^*)}$, $k_2 := \frac{f(a_1{}^*)}{[1-F(a_1{}^*)]}$, $k_3 := \frac{f(a_2{}^*)}{[1-\theta^*]\cdot[1-F(a_2{}^*)]}$. *Let Assumption 4 be satisfied and suppose that* $f(a_2{}^*) \geq f(a_1{}^*)$. *Then*

$$\frac{d\theta^*}{d\delta} > 0$$

if $\delta \leq \frac{k_1}{[5+k_1+k_2+k_3]}$

Proof We show that Condition (A.14) is satisfied. The result then follows from Lemma 1.

Consider $\frac{d^2D}{dtdt}$ first (see Equation (A.7)). Ignoring the last two summands on the right-hand-side, which are positive, using that $f'(a_1{}^*) = k_1 \cdot f(a_1{}^*)$ and noting that[43]

$$P'(L^*) \cdot \int_{a_1{}^*}^{a_2{}^*} \frac{1}{a} \cdot \; f(a) \; da \leq 2$$

yields

$$\left.\frac{d^2D}{dtdt}\right|_{\theta=\theta^*,t=t^*} \geq \left(\frac{\bar{A}\cdot\mu_a}{A}\right)^2 \cdot f(a_1{}^*) \cdot [k_1 - (k_1+4)\cdot\delta].$$

Combining this with $\left.\frac{d^2D}{dtd\delta}\right|_{\theta=\theta^*,t=t^*} \leq \frac{\bar{A}\cdot\mu_a}{A} \cdot f(a_1{}^*)$ (see Equation (A.6)) we get

$$\left.\frac{\frac{d^2D}{dtd\delta}}{\frac{d^2D}{dtdt}}\right|_{\theta=\theta^*,t=t^*} \cdot \frac{\bar{A}\cdot\mu_a}{A} \leq \frac{1}{k_1 - [k_1+4]\cdot\delta}.$$

Consider Equation (A.13) next. Ignoring the last term in the big squared brackets, which is negative, and using that $a_1 \geq 1$ along with the definitions of k_1 and k_2 yields that δ times the term in big squared brackets is less than or equal to

$$[1-F(a_2{}^*)] \cdot [1-F(a_1{}^*)] \cdot P'(L^*) \cdot \delta \cdot [1+k_2+k_3].$$

Taken together, this implies that

$$d\left[\frac{dD}{d\theta}\right] \leq [1-F(a_2{}^*)] \cdot [1-F(a_1{}^*)] \cdot P'(L^*) \cdot \left[-1 + \frac{\delta\cdot[1+k_2+k_3]}{k_1-[k_1+4]\cdot\delta}\right] \cdot d\delta,$$

i.e. it is negative if

$$\delta \leq \frac{k_1}{[5+k_1+k_2+k_3]}.$$

[43]Using that $P'(L) = \lambda \cdot \frac{P(L)}{L}$ and Equation (7) yields

$$P'(L^*) \cdot \int_{a_1{}^*}^{a_2{}^*} \frac{1}{a} \cdot f(a)da = \frac{1}{a_1{}^*} \cdot \frac{\lambda}{\lambda-1} \cdot \frac{(\lambda-1)\cdot\int_{a_1{}^*}^{a_2{}^*} l^*(a)\cdot f(a)da}{L^*} \cdot P(L^*) \leq 2,$$

where $\frac{1}{a_1{}^*} \leq 1$ and $P(L^*) \leq 1$. For $\lambda \geq 2$ the inequality thus follows from Assumption 4. For $\lambda < 2$ it follows from $\frac{\int_{a_1{}^*}^{a_2{}^*} l^*(a)\cdot f(a)da}{L^*} \leq 1$.

□

As shown in Condition (B.4), for reasonable assumptions on r and μ_a, $f(a_1)$ is small. In such cases, k_1 is typically large as noted above, and the result therefore holds for a broad range of parameter values.

B.3 Optimal lockdown with a borrowing constraint

In this part of the appendix, we provide a sufficient condition that allows to extend Proposition 4(ii) to all $b \leq t^*$. Our result is based on the following assumptions:

Assumption 3′

$(i)\ g(\theta) = 1 - |\theta - \kappa_1|^{\kappa_2}, \quad (ii)\ 0 \leq \kappa_1 < \frac{1}{2}; \quad (iii)\ \kappa_2 > 1$

Assumption 4′

$$\frac{(\lambda - 1) \cdot \int_{a_1{}^*(b)}^{a_2{}^*(b)} l^*(a) \cdot f(a) da}{L^*} \leq 1$$

Assumption 5

$(i)\ f'(a_1{}^*(b)) \geq 0; \quad (ii)\ f(a_2{}^*(b)) \geq f(a_1{}^*(b))$

In these assumptions, we use $a_1{}^*(b)$ ($a_2{}^*(b)$) to denote the value of a_1 (a_2) given the optimal policy with borrowing limit b. Assumption 3′ is a fairly general functional form satisfying the conditions in Assumption 3. Assumptions 4′ is a natural extension of Assumption 4. Assumption 5 is a weak restriction on the shape of $f(\cdot)$ at the cutoffs a_1 and a_2, which matters for how the lockdown and transfers interact in shaping economic and pandemic outcomes. It is always satisfied with a uniform distribution of abilities and, more generally, tends to be true in economies with single-peaked income distributions and where the majority of the population can afford to comply with the optimal θ. This is indeed the case for the parameter values underlying our numerical illustration in Section 4. With these assumptions, we can show the following result:

Proposition 4′

(ii) Let Assumptions 3′, 4′, and 5 be satisfied and suppose that b is binding. In response to a marginal decline in b, θ^* declines.

Proof For the purpose of this proof, let $\theta^*(b)$ and $t^*(b)$ denote the optimal θ and t, respectively, given b. The optimality of $\theta^*(b)$ implies that

$$\left.\frac{dD}{d\theta}\right|_{\theta=\theta^*(b)} = 0. \tag{B.5}$$

We therefore show that $\left.\frac{d^2D}{d\theta dt}\right|_{\theta=\theta^*(b)} < 0$, which proves the desired result.

Differentiating Equation (16) with respect to t yields:

$$\begin{aligned}\frac{d^2D}{d\theta dt} = &-\underbrace{f'(a_1)\cdot\frac{da_1}{dt}\cdot a_1\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot[1-d(P)]}_{:=\mathscr{K}}-\underbrace{f(a_1)\cdot\frac{da_1}{dt}\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot[1-d(P)]}_{:=\mathscr{L}}\\ &+\underbrace{f(a_1)\cdot a_1\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot\delta\cdot P'(L)\cdot\frac{dL}{dt}}_{:=\mathscr{M}}-\underbrace{f(a_1)\cdot\frac{da_1}{dt}\cdot\delta\cdot P'(L)\cdot\frac{dL}{d\theta}}_{:=\mathscr{N}}\\ &+\underbrace{[1-F(a_1)]\cdot\delta\cdot P''(L)\cdot\frac{dL}{d\theta}\cdot\frac{dL}{dt}}_{:=\mathscr{O}}+\underbrace{[1-F(a_1)]\cdot\delta\cdot P'(L)\cdot\frac{d^2L}{d\theta dt}}_{:=\mathscr{P}}.\end{aligned} \tag{B.6}$$

Recall that $\frac{da_1}{dt} < 0$, $\frac{dL}{dt} < 0$, $P'(L) > 0$ and $P''(L) > 0$. Moreover, $\left.\frac{dA}{d\theta}\right|_{\theta=\theta^*(b)} < 0$ as discussed in Section 3.1. At $\theta = \theta^*(b)$, $-\mathscr{K}$ is therefore negative by Assumption 5(i). $-\mathscr{N}$ is negative by Proposition 1(ii). Using Equations (16) and (B.5), we get for $\theta = \theta^*(b)$:

$$\mathscr{L} = [1-F(a_1)]\cdot\delta\cdot P'(L)\cdot\frac{dL}{d\theta}\cdot\frac{da_1}{dt}\cdot\frac{1}{a_1}$$

and therefore

$$-\mathscr{L}+\mathscr{O} = [1-F(a_1)]\cdot\delta\cdot\frac{dL}{d\theta}\cdot\left[-\frac{da_1}{dt}\cdot\lambda\cdot\frac{L^{\lambda-1}}{a_1}+\lambda\cdot(\lambda-1)\cdot L^{\lambda-2}\cdot\frac{dL}{dt}\right].$$

The term in brackets is positive as shown in the proof of Lemma 1. In combination with Proposition 1(ii), this implies that

$$-\mathscr{L}+\mathscr{O} < 0.$$

Finally, differentiating (17) with respect to t, using $\frac{da_1}{dt} = -\frac{\bar{A}\cdot\mu_a}{A}$, rearranging terms, and substituting the result in $\mathscr{P}$ yields

$$\mathscr{P} = [1-F(a_1)]\cdot\delta\cdot P'(L)\cdot\left[\frac{\bar{A}\cdot\mu_a}{A}\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot[f(a_2)-f(a_1)]-\frac{\bar{A}\cdot\mu_a}{A}\cdot\frac{f(a_2)}{1-\theta}-\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot\frac{dL}{dt}\right]$$

At $\theta = \theta^*(b)$, the first term in the big squared brackets is negative by Assumption 5(ii). Clearly, the last term is negative as well. Consider then the middle term, which is negative as well. Moreover,

$$\begin{aligned}&\mathscr{M}-[1-F(a_1)]\cdot\delta\cdot P'(L)\cdot\frac{\bar{A}\cdot\mu_a}{A}\cdot\frac{f(a_2)}{1-\theta}\\ =&\delta\cdot P'(L)\cdot\left[f(a_1)\cdot a_1\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot\frac{dL}{dt}-[1-F(a_1)]\cdot\frac{\bar{A}\cdot\mu_a}{A}\cdot\frac{f(a_2)}{1-\theta}\right]\\ =&\delta\cdot P'(L)\cdot\frac{\bar{A}\cdot\mu_a}{A}\cdot\left[-f(a_1)\cdot\frac{dA}{d\theta}\cdot\frac{1}{A}\cdot\int_{a_1}^{a_2}\frac{a_1}{a}\cdot f(a)da-[1-F(a_1)]\cdot\frac{f(a_2)}{1-\theta}\right]\\ <&0.\end{aligned}$$

The second equality follows from using Equation (19) and from rearranging terms. The inequality follows from noting that in the third row, the term outside the big squared brackets is positive, while the term inside these brackets is negative. To see this latter result, note first that the first term in the big squared brackets is positive, while the second term is negative. Second, that

$$[1 - F(a_1)] > \int_{a_1}^{a_2} \frac{a_1}{a} \cdot f(a)da.$$

Third, that

$$f(a_2) \geq f(a_1)$$

by Assumption 5(ii). And fourth that

$$\frac{1}{1-\theta} \geq -\frac{dA}{d\theta} \cdot \frac{1}{A}. \tag{B.7}$$

To see this last result note that

$$-\frac{dA}{d\theta} \cdot \frac{1}{A} = \kappa_2 \cdot \frac{(\theta - \kappa_1) \cdot |\theta - \kappa_1|^{\kappa_2 - 2}}{1 - |\theta - \kappa_1|^{\kappa_2}}$$

by Assumption $3'$, which can be verified to be decreasing in κ_1. Consider then the case of $\kappa_1 = 0$. In this special case, Inequality (B.7) can be rearranged to

$$\kappa_2 \cdot \theta^{\kappa_2 - 1} - (\kappa_2 - 1) \cdot \theta^{\kappa_2} \leq 1.$$

It is easy to verify that for $\theta \in [0, 1]$ the left-hand-side is increasing in θ. Moreover, for $\theta = 1$ the above condition holds with equality. This completes the proof.

□

B.4 Details on discussions of Section 3.2

Ignoring corner solutions for the sake of exposition, the first order conditions for government decision problem (11) are

$$\begin{aligned} \frac{dW}{d\theta} = & \int_{a \in \mathcal{A}: a \leq a_1} v'\left(A \cdot a + t \cdot \bar{A} \cdot \mu_a\right) \cdot \frac{dA}{d\theta} \cdot a \cdot f(a)da \\ & + \int_{a \in \mathcal{A}: a_2 \leq a} v'\left((1-\theta) \cdot A \cdot a + t \cdot \bar{A} \cdot \mu_a\right) \cdot \left[-A + \frac{dA}{d\theta} \cdot (1-\theta)\right] \cdot a \cdot f(a)da + \frac{dD}{d\theta} \cdot \beta \cdot \bar{c} \\ & - \frac{dD}{d\theta} \cdot \beta \cdot \int_{a \in \mathcal{A}} \left[v\left(\bar{A} \cdot a - r \cdot \frac{t \cdot \bar{A} \cdot \mu_a}{1-D}\right) + r \cdot \frac{t \cdot \bar{A} \cdot \mu_a}{1-D} \cdot v'\left(\bar{A} \cdot a - r \cdot \frac{t \cdot \bar{A} \cdot \mu_a}{1-D}\right)\right] \cdot f(a)da \\ = & 0 \end{aligned} \tag{B.8}$$

and

$$\frac{dW}{dt} = \int_{a\in\mathcal{A}:a\leq a_1} v'\left(A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \bar{A}\cdot \mu_a \cdot f(a)da \tag{B.9}$$
$$+ \int_{a\in\mathcal{A}:a_2\leq a} v'\left((1-\theta)\cdot A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \bar{A}\cdot \mu_a \cdot f(a)da$$
$$- \frac{dD}{dt}\cdot \beta \int_{a\in\mathcal{A}} \left[v\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right) + r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\cdot v'\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\right]\cdot f(a)da$$
$$+ \frac{dD}{dt}\cdot \beta \cdot \bar{c} - \beta \cdot \int_{a\in\mathcal{A}} v'\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\cdot r\cdot \bar{A}\cdot \mu_a \cdot f(a)da$$
$$=0,$$

where $\frac{dD}{d\theta}$, $\frac{dD}{dt}$, $\frac{dL}{d\theta}$, and $\frac{dL}{dt}$ are as given in Equations (16) to (19). These first-order conditions can be decomposed into the following effects:

Recession effect of a lockdown:

$$\int_{a\in\mathcal{A}:a\leq a_1} v'\left(A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \frac{dA}{d\theta}\cdot a\cdot f(a)da$$
$$+ \int_{a\in\mathcal{A}:a_2\leq a} v'\left((1-\theta)\cdot A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \left[-A + \frac{dA}{d\theta}\cdot (1-\theta)\right]\cdot a\cdot f(a)da$$

Value-of-life effect:

$$-\frac{dD}{d\theta}\cdot \left[-\beta\cdot \bar{c} + \beta\cdot \int_{a\in\mathcal{A}} v\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\cdot f(a)da\right]$$

Debt-burden effect:

$$-\frac{dD}{d\theta}\cdot \beta\cdot \int_{a\in\mathcal{A}} r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\cdot v'\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\cdot f(a)da$$

Consumption-smoothing effect of transfers:

$$\int_{a\in\mathcal{A}:a\leq a_1} v'\left(A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \bar{A}\cdot \mu_a\cdot f(a)da$$
$$+ \int_{a\in\mathcal{A}:a_2\leq a} v'\left((1-\theta)\cdot A\cdot a + t\cdot \bar{A}\cdot \mu_a\right)\cdot \bar{A}\cdot \mu_a\cdot f(a)da$$
$$- \beta\cdot \int_{a\in\mathcal{A}} v'\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\cdot r\cdot \bar{A}\cdot \mu_a\cdot f(a)da$$

We discuss these effects and their implications for our comparative statics results in Section 3.2. In these discussions, we make further use of the fact that the marginal benefit of a lower D as jointly captured by the value-of-life and the debt-burden effect

$$\beta\cdot \left[-\bar{c} + \int_{a\in\mathcal{A}} v\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right) + r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\cdot v'\left(\bar{A}\cdot a - r\cdot \frac{t\cdot \bar{A}\cdot \mu_a}{1-D}\right)\cdot f(a)da\right]$$

is increasing in t and D. This immediately follows from the fact that for any concave function $v(x)$

$$v(x-\epsilon)+\epsilon \cdot v'(x-\epsilon)$$

is increasing in ϵ.

C Details on the numerical example

In this part of the appendix, we provide further details on the numerical example of Section 4.

C.1 Details on the ability distribution

To calibrate the ability distribution, we assume that it follows a shifted log-normal distribution

$$a \sim \underline{a}+z, \quad z \sim \text{log-normal}(\mu, \sigma),$$

where $\underline{a}=1$ as explained in Section 4. We then use the ratio of median to subsistence income, ϕ. With a shifted log-normal distribution this is given by

$$\phi := \frac{\text{median}(a)}{\underline{a}} = \frac{\underline{a}+\exp(\mu)}{\underline{a}}$$

and, hence, allows to calibrate μ as

$$\mu = \ln(\phi-1)+\ln(\underline{a}),$$

which yields $\mu \approx 0.41$. Finally, we choose σ to minimize

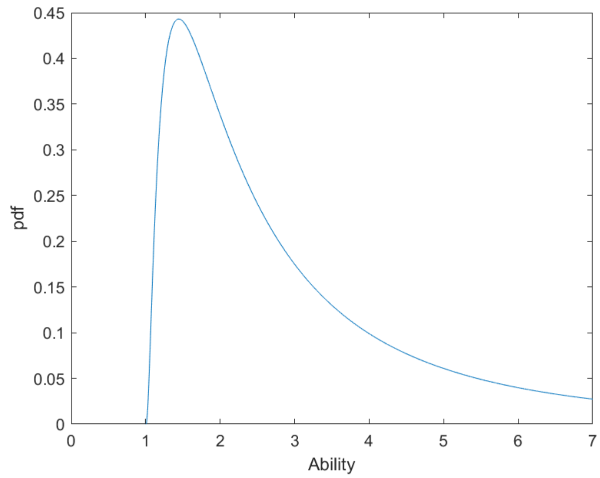
$$\min_{\sigma} \sum_{i=1}^{10} \left[\frac{s_i-\hat{s}_i}{s_i}\right]^2,$$

where s_i and $\hat{s}_i$ are the income shares of the i^{th} decile in the data and for our fitted distribution, respectively. To reduce measurement error in the income distribution for developing countries, we use the average decile shares across all low-income countries for which these shares are included in the World Income Inequality Database (UNU-WIDER, 2019).[44] This yields $\sigma \approx 1.1$.

Figure 4 depicts the implied ability distribution and Table 1 shows the actual and the fitted decile shares.

[44] Specifically, we include all countries that report these shares for at least one year from 2012 onwards and take for each country the latest available observation.

Figure 4: Fitted ability distribution



Notes: The figure depicts the fitted ability distribution used in the baseline calibration of our model:

$$a \sim 1 + z, \quad z \sim \text{log-normal}(0.41, 1.1)$$

Table 1: Actual and fitted decile shares

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
actual	0.026	0.039	0.049	0.058	0.068	0.080	0.095	0.116	0.152	0.318
fitted	0.033	0.039	0.046	0.053	0.061	0.073	0.088	0.111	0.154	0.342

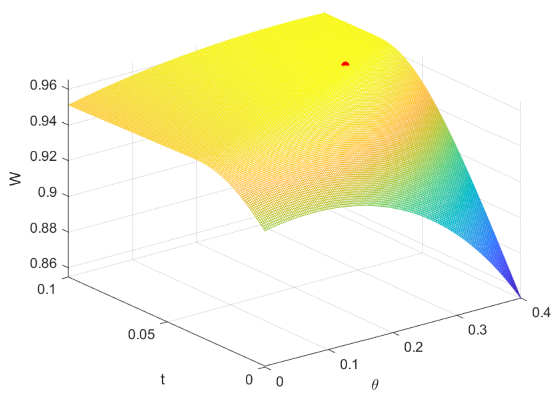
C.2 Alternative parameter specifications

In this part of the appendix, we provide robustness checks for our numerical illustration. Specifically, we show the counterparts of Figures 1 to 3, first for the case of $\alpha = 1$, which we considered in Section 3.1, and second for an alternative fitted ability distribution. In this alternative, the ratio of median to minimum income is 3.5, which implies $\mu = 0.92$, $\sigma = 0.97$. All other parameter values are as in our baseline calibration.

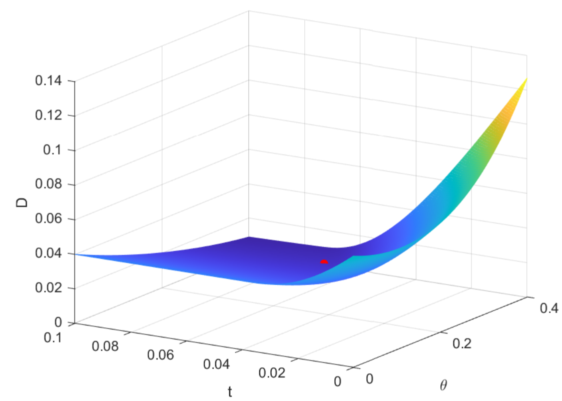
C.2.1 $\alpha = 1$

Figure 5: Policy space and key outcomes ($\alpha = 1$)

(a) Welfare



(b) Aggregate death burden



(c) Compliance

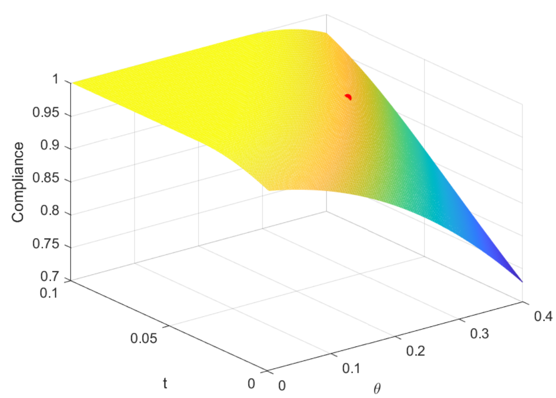
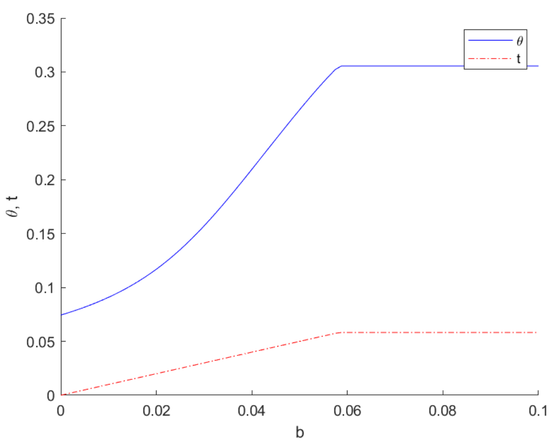
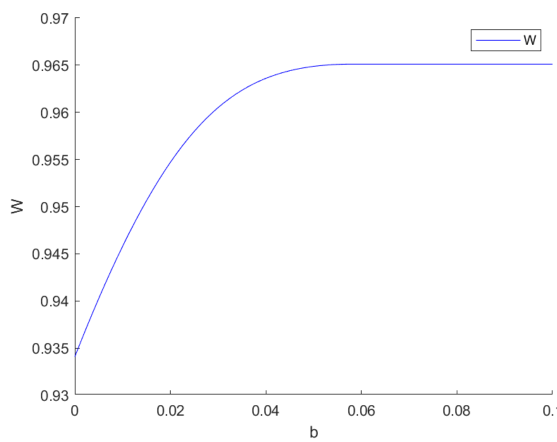


Figure 6: Optimal policy with borrowing constraint ($\alpha = 1$)

(a) Policy



(b) Welfare



(c) Aggregate death burden

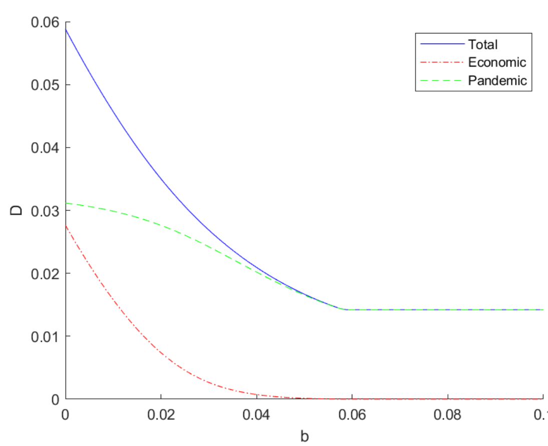
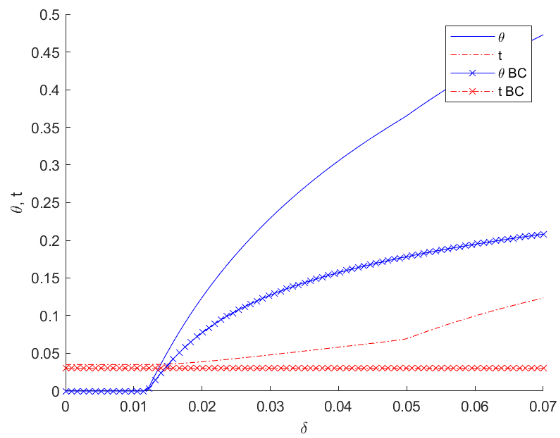
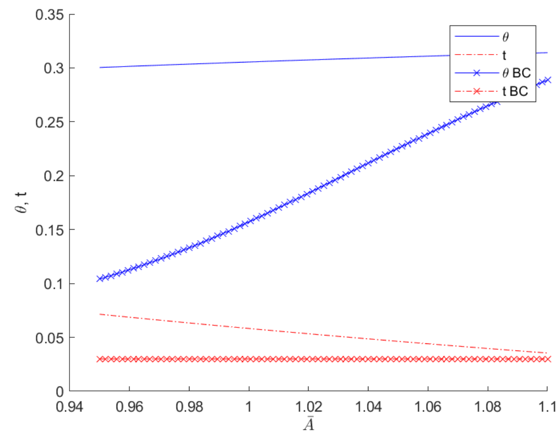


Figure 7: Comparative statics ($\alpha = 1$)

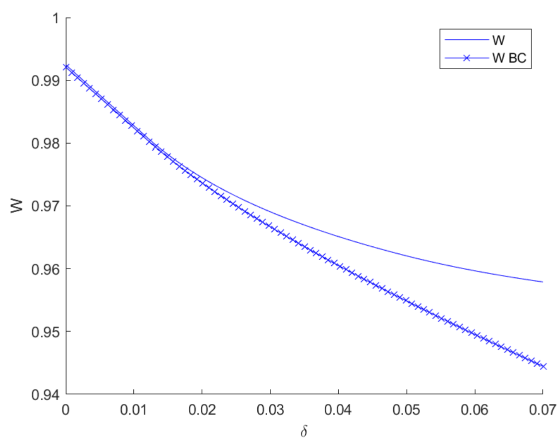
(a) δ: Policy



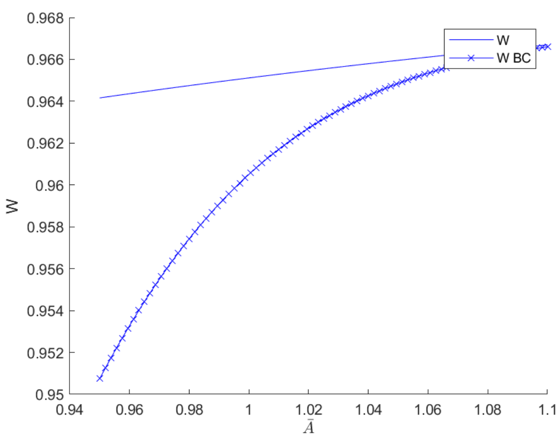
(b) $\bar{A}$: Policy



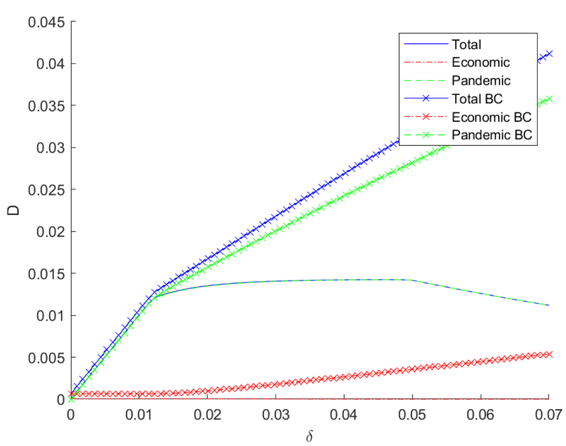
(c) δ: Welfare



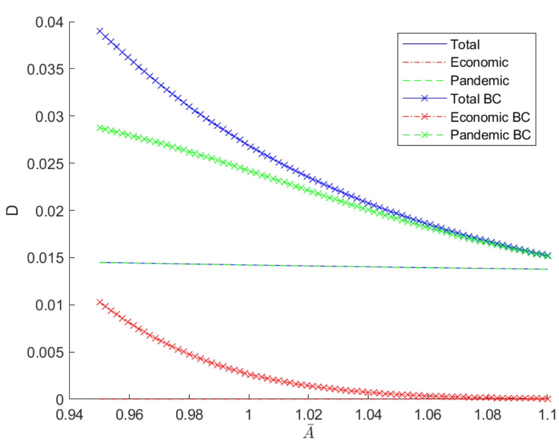
(d) $\bar{A}$: Welfare



(e) δ: Aggregate death burden



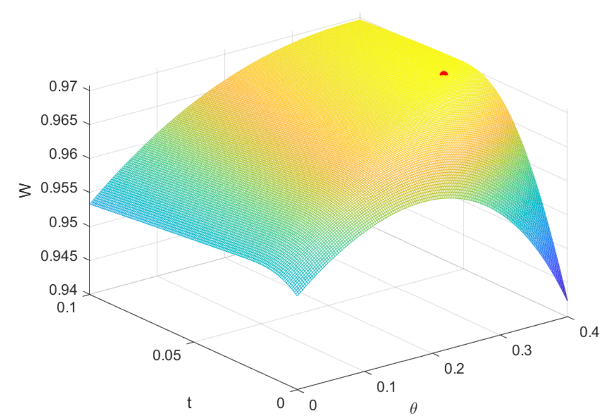
(f) $\bar{A}$: Aggregate death burden



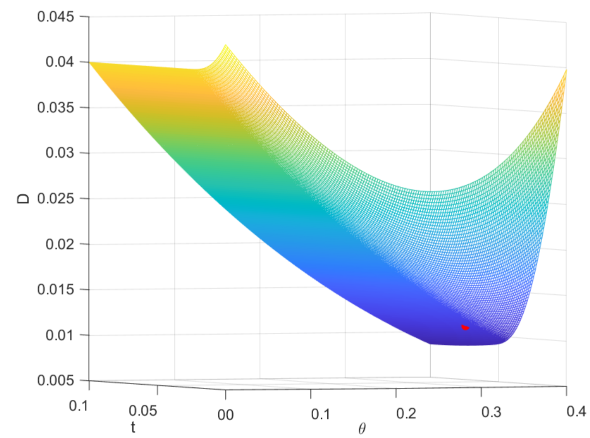
C.2.2 $\mu = 0.92$, $\sigma = 0.97$

Figure 8: Policy space and key outcomes ($\mu = 0.92$, $\sigma = 0.97$)

(a) Welfare



(b) Aggregate death burden



(c) Compliance

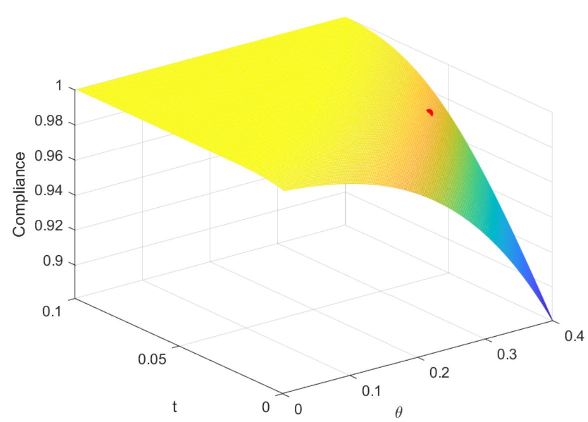
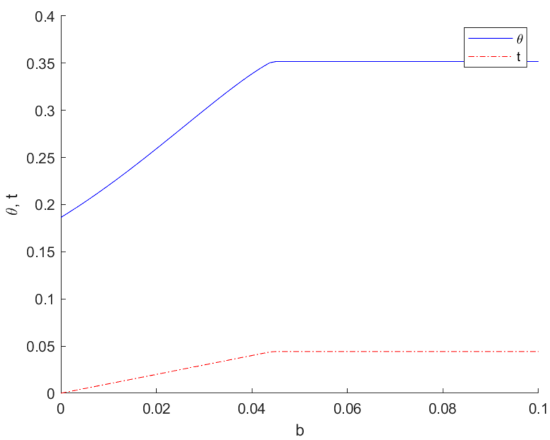
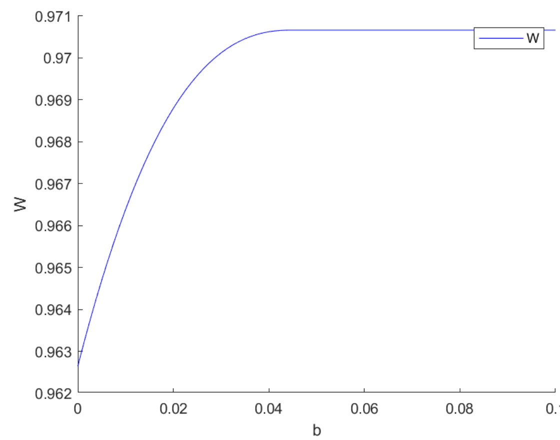


Figure 9: Optimal policy with borrowing constraint ($\mu = 0.92$, $\sigma = 0.97$)

(a) Policy



(b) Welfare



(c) Aggregate death burden

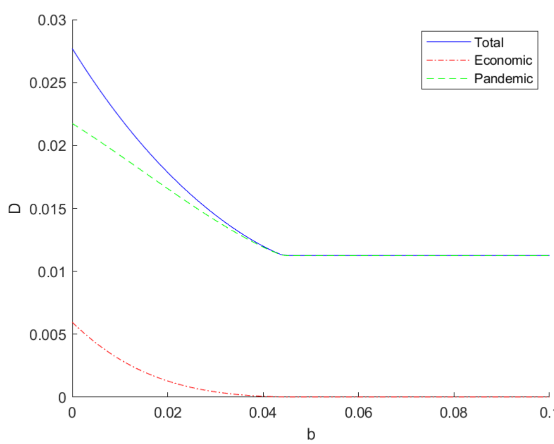
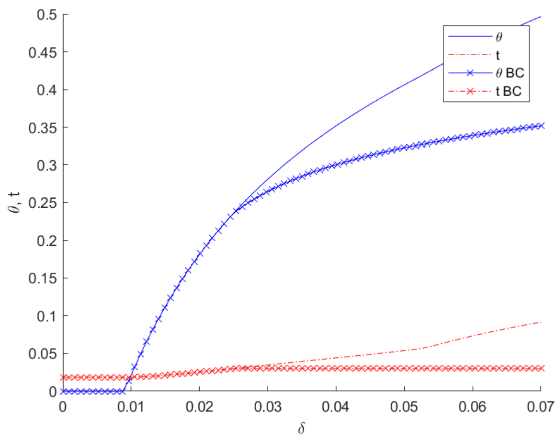
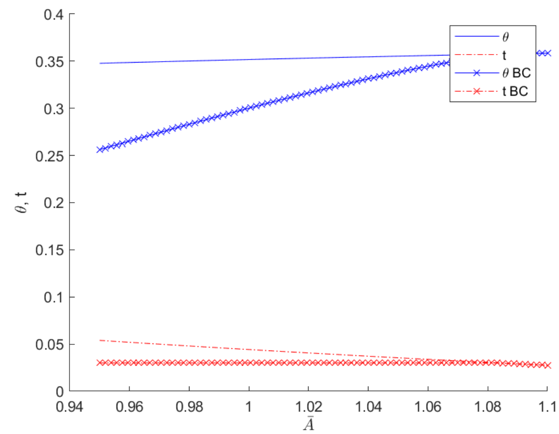


Figure 10: Comparative statics ($\mu = 0.92$, $\sigma = 0.97$)

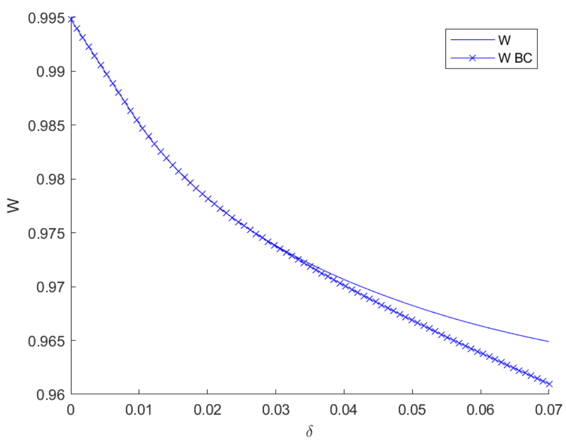
(a) δ: Policy



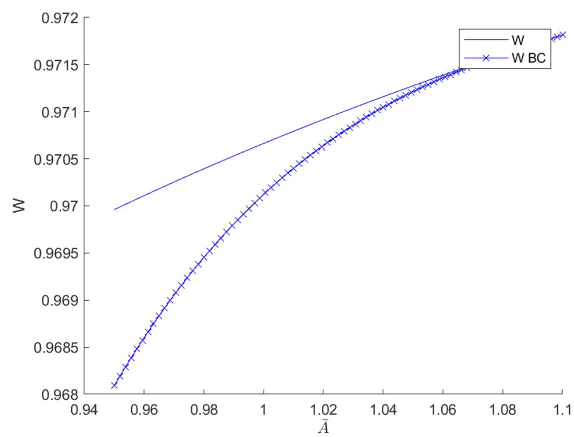
(b) $\bar{A}$: Policy



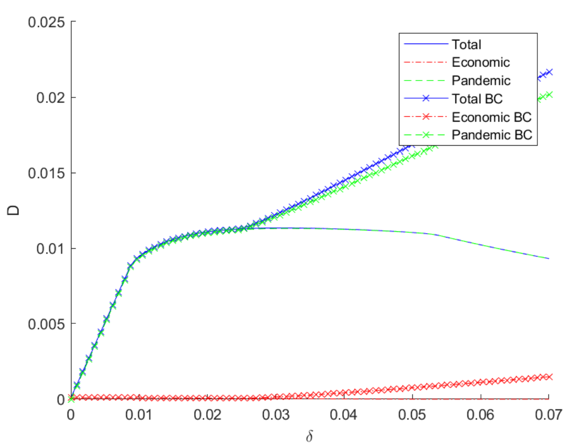
(c) δ: Welfare



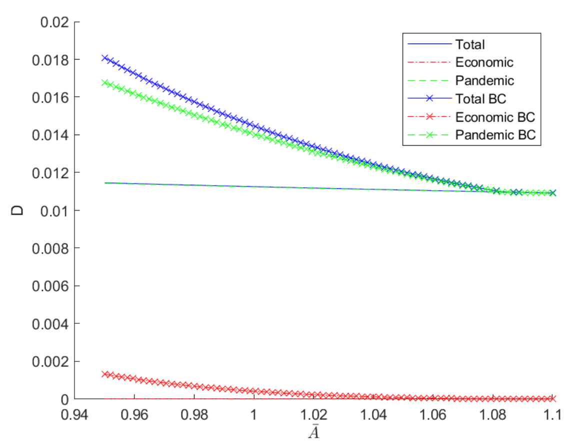
(d) $\bar{A}$: Welfare



(e) δ: Aggregate death burden



(f) $\bar{A}$: Aggregate death burden



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