Glacial Isostatic Adjustment in a region of complex Earth structure: The case of WAIS

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Glacial Isostatic Adjustment in a region of Complex Earth Structure: The Case of WAIS

A dissertation presented
by
Evelyn Powell
to
The Department of Earth and Planetary Sciences

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
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Harvard University
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Glacial Isostatic Adjustment in a region of Complex Earth Structure: The Case of WAIS

ABSTRACT

In this thesis, we consider surface loading effects associated with our dynamic cryosphere. Glacial Isostatic Adjustment (GIA) models have been used to constrain the extent of past ice sheets and viscoelastic Earth structure, and to correct geodetic and geological observations for ice age effects. These models, however, often only consider depth-dependent variations in Earth viscosity and lithospheric structure. Seismic, geological, and geodetic evidence indicates the Antarctic Ice Sheet is underlain by complex, high amplitude variability in 3-D viscoelastic structure. In contrast with West Antarctica’s low viscosity mantle, Canada, the location of the former Laurentide Ice Sheet, is underlain by a thick craton and mantle viscosities higher than the global average. GIA modeling with 3-D mantle structure requires greater model specificity and fidelity, but will also provide a deeper understanding of the past and future evolution of the cryosphere.

Our investigation is motivated by two questions: How does 3-D Earth structure impact observations of GIA-induced deformation, and how will 3-D Earth structure affect predictions of sea-level change? We compute gravitationally self-consistent uplift, gravity, and sea-level changes and show that 3-D Earth structure will have significant effects on sea-level changes associated with West Antarctic Ice Sheet melt during interglacial periods. Further, we show that Antarctica’s viscoelastic structure will impact geodetic observables even for timescales when the Earth is commonly treated as a purely elastic body. We demonstrate how the bias in crustal deformation induced by this 3-D structure will impact standard methods to use GPS observations to infer viscoelastic structure in West Antarctica. Finally, we use sea-level modeling to estimate the emergence of an island from Canada’s waters in order to corroborate an Indigenous people’s land claim.
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This thesis is dedicated to my Father & Mother. And to my great Aunt, whose name I share & example I follow.
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Truly, I stand on the shoulders of giants.
1

Introduction

1.1 Preface

Improving our understanding of sea-level change and the climatic response to human-induced global warming are issues of pressing societal importance. Critical to this endeavour is the ability to model how the Earth responds to evolving ice and ocean loads. Accurate modeling enables the interpretation of markers of paleogeography, the inference of Earth’s mantle properties, and the
projection of sea-level changes. The Antarctic Ice Sheet represents the largest present-day mass of ice and the largest potential contributor to future sea-level rise, and will be the primary focus of this thesis.

1.2 A Surface Loading Problem

The Earth’s cryosphere – from continental ice sheets to alpine glaciers – is a dynamic mass load that evolves on a variety of timescales. Today, we live in a time period of relative warmth, or an interglacial, with polar ice sheets extant only on Greenland and Antarctica. Approximately 25 thousand years ago, however, much of the northern hemisphere was blanketed by kilometers of ice during the Last Glacial Maximum (LGM). Cyclic variability in the ice age Earth system on timescales of thousands to hundreds of thousands of years is paced by changes in the Earth’s orbital parameters (or, Milankovitch cycles), which control the geographic distribution of solar insolation and thus affect global temperatures and climate (e.g., Huybers, 2006). Such climatic effects lead to a corresponding, cyclic expansion and retreat of continental ice sheets, as seen in ice volume proxies, such as oxygen isotope records (e.g., Lisiecki & Raymo, 2005). On shorter (seasonal to decadal) timescales, local changes to an ice sheet’s stress state can affect the large-scale dynamics of an ice sheet. For example, the breakup of Antarctica’s Larsen B Ice Shelf in 1999, and the resulting reduction of the ice shelf’s buttressing force, led to an observed speedup of local outlet glaciers. This increase in mass discharge was observed via satellite altimetry, but also by increased uplift rates from nearby GPS stations (Nield et al., 2014).

A time-dependent surface load - e.g., the cycles of ice accumulation, discharge, and melt described above - will drive deformation of the solid Earth. Specifically, it will result in a displacement field, \( \mathbf{u}(x, t) \), and a perturbed gravitational field, \( \varphi(x, t) \), where \( x \) denotes position and \( t \) is time. The evolution of these fields is governed by two equations. The first is Newton’s second law dictating
the conservation of momentum:

\[ F = m\dot{\mathbf{u}}, \quad (1.1) \]

where \( m \) is mass. The second is Poisson’s equation for gravity:

\[ \nabla^2 \varphi = 4\pi G\rho, \quad (1.2) \]

where \( G \) is the gravitational constant, and \( \rho \) is density. Note that this latter equation reduces to Laplace’s equation, \( \nabla^2 \varphi = 0 \), at points external to the Earth. These governing equations for the Earth are discussed in more detail in Section 1.3.

Crucially, to solve this system of equations, we require a constitutive equation that describes how Earth materials will strain, \( \varepsilon \), in response to an applied stress, \( \sigma \). That is to say, we need to know (or have an approximation for) the rheology of the Earth.

1.2.1 Rheology of Earth materials

Rheology is defined as the study of the macroscopic, deformational response of a material to an applied stress. Physical materials—such as iron, rock, or ice—respond to forcings in a manner that is dependent not only on their material properties, but also on the nature of the applied forcing (e.g., its duration and magnitude). We discuss three rheological classifications that apply to the Earth’s rocky material: elastic, viscous, and viscoelastic, below. For illustrative purposes, we will consider simple, 1-D models of each of these rheologies (and their associated constitutive equations) in turn.

Elastic Rheology

We can consider an elastic rheology using the simple example of a spring. In this case, an application of constant stress results in instantaneous displacement of the spring (strain), but no subsequent
changes. After the stress is removed, material with an elastic rheology will return to its original shape (potentially oscillating before reaching this state). This response is mathematically described by Hooke’s law,

\[ \sigma = k \varepsilon, \]  

(1.3)

which states that strain is proportional to stress. Here \( k \) is the spring constant, or more formally Young’s Modulus, with units Pa. An example of this type of behavior for the Earth would be the near-instantaneous propagation of seismic waves in cold mantle materials.

Microscopically, elastic strain is accommodated in Earth materials by extension and compression of the crystal lattice. Mantle silicates are thought to have \( k \approx 10^{11} \) Pa. Note, however, that \( k \) will not always be constant in the Earth even for material with the same mineral composition, but rather is a function of both pressure and temperature.

As an aside, should the forcing magnitude be sufficiently large, a material that might normally behave elastically can instead fail in a brittle manner. In this case, a forcing would result in initial fracture of the material, but no subsequent change in the shape of the fragments. An example of brittle behavior for the Earth would be rupture of the crust during an earthquake.

Viscous Rheology

Next, we consider viscous deformation, in which materials are said to exhibit fluid behavior. We can visualize a viscous, fluid-like response using a dashpot: a piston which is surrounded by a fluid in a small chamber. Here, we will consider the behavior of a Newtonian fluid, defined as having a strain rate that is linearly dependent on the stress. In this case, an application of a constant stress results in zero instantaneous displacement. However, over time, there will be a gradual change in shape (i.e., the dashpot opens). After the stress is removed, the displacement will cease, but the material will not recover any of its original shape.
The 1-D constitutive relation of a Newtonian fluid is given by:

\[ \sigma = \eta \varepsilon, \]  

(1.4)

where \( \eta \) is the viscosity. A fluid’s viscosity quantifies its resistance to flow, and has units Pa s.

An example of a viscous response of the Earth would be solid-state thermochemical convection in the Earth’s mantle which evolves on timescales of millions of years. In this case, mantle material is said to creep, a process whereby strain is accommodated microscopically by motion of crystal lattice defects and grain boundaries (Ranalli, 1991). These defects can include, for example, interstitial vacancies (missing atoms in the crystal lattice) and dislocations (essentially misalignments in the crystal structure). Overall, the mantle is characterized by a global average viscosity of approximately \( 10^{21} \) Pa s. However, this viscosity has both a pressure dependence as well as a strong, exponential dependence on temperature governed by an Arrhenius relationship. Small changes in temperature can produce exponentially large changes in the solid mantle’s viscosity.

**Viscoelastic Rheology**

There are many ways to define a viscoelastic rheology, but here we will discuss the simplest case, a Maxwell Body. In a Maxwell Body, the archetypal elastic spring and viscous dashpot are connected in series, and any applied forcing will act equally on both elements. An application of a constant stress will in this case produce an initial, elastic-like displacement (strain), followed by a subsequent gradual, viscous change in shape. If the stress is removed, only the initial, elastic strain is recovered. Microscopically, the Earth’s minerals accommodate viscoelastic strain using the full range of effects described above—both the irrecoverable, creeping motion of crystal lattice defects, and recoverable deformation of the crystal lattice.
The constitutive equation for a 1-D Maxwell viscoelastic body is:

\[
\dot{\epsilon} = \frac{1}{k} \dot{\sigma} + \frac{1}{\eta} \sigma.
\] (1.5)

The system has a characteristic timescale termed the Maxwell Time, \(\tau_M\), which is given by \(\tau_M = \eta/k\). The Maxwell time represents the timescale required for elastic and viscous strain to be approximately equal in response to an applied stress. For short timescale forcings (\(\tau \ll \tau_M\)), the material will exhibit elastic behavior, and for long timescale forcings (\(\tau \gg \tau_M\)), the response will be dominated by viscous behavior. Although the viscosity of Earth’s lithosphere and mantle vary globally and with depth, adopting a typical value of mantle viscosity of approximately \(\eta = 10^{21}\) Pa s and a Young’s modulus for mantle materials of \(k = 10^{11}\) Pa, the Maxwell Time is \(\sim 1000\) years. Thus, this simple viscoelastic body broadly captures the observed dependence of the Earth’s response on the time scale of the forcing. Rapid rupture of a fault (\(\tau \ll \tau_M\)) leads to the (largely) elastic propagation of seismic waves, while the temperature contrast between the core-mantle boundary and surface of the Earth, which has existed since the time of the proto-planet (\(\tau \gg \tau_M\)), is the driving force behind viscous mantle convective flow.

As a final point, the rheological response of a viscoelastic material is typically characterized by a strong temperature dependence. A cold material has a high \(\eta\), and in this case the above constitutive equation will reduce to \(\dot{\epsilon} \approx \frac{1}{k} \dot{\sigma}\), which describes a purely elastic response. A hot material, on the other hand, has a low \(\eta\), and so the constitutive equation reduces to \(\dot{\epsilon} = \frac{1}{\eta} \sigma\), which describes a purely viscous response). This strong dependence is responsible for the existence of the so-called lithosphere, or "plate" of plate tectonics. The lithosphere represents the region of the shallow mantle with a temperature low enough that the behaviour is elastic even over very long times scales.
1.2.2 Glacial Isostatic Adjustment (GIA)

Mass flux between the Earth’s cryosphere and ocean evolves on timescales comparable to this Maxwell time, and will drive deformations of the solid Earth and perturbations to the gravity field and rotational state. This ice-driven response process is termed “glacial isostatic adjustment” (GIA). The GIA process involves both a rapid (effectively instantaneous) elastic response to changes in the ice-plus-ocean load, and a time-evolving, viscous adjustment that is a function of the entire history of the loading (Peltier, 1982).

Understanding Earth’s deformational, gravitational, and rotational response to changes in ice and ocean loading is important for many reasons. By leveraging the coupled nature of the solid Earth and its cryosphere, observational data can be used in tandem with GIA modeling to constrain both the Earth’s interior structure and reconstruct the spatio-temporal history of past ice sheets (e.g., Lau et al., 2016; Peltier, 2004). Accurate models of the Earth’s GIA response are thus a basic requirement in the interpretation of ice-age related geological observables, such as relative sea-level markers. Applying this understanding to the geologic record of sea level during past warm periods (interglacials), we can gain insight into the stability of ice sheets in our progressively warming world.

Beyond paleoclimate applications, the Earth’s viscoelastic response also directly affects our ability to measure and monitor present-day ice loss. For example, one means of estimating ongoing ice mass loss is via the interpretation of satellite gravity measurements. Data from the Gravity Recovery And Climate Experiment (GRACE) satellites constrain integrated mass changes; that is, these data cannot discern a difference between ongoing mass changes due to post-glacial rebound associated with the last major deglaciation (starting 25 thousand years ago), versus mass flux associated with modern glacial melting. Thus, ongoing GIA due to the last deglaciation represents a contaminating signal in GRACE satellite gravity observations. Unfortunately, the evolution of Antarctic ice sheet since the LGM is not well known. Consequently, estimates of modern mass loss in the Antarctic have
significant uncertainties, which can be of the same order of magnitude as the estimate itself (e.g. Chen et al., 2009; Guo et al., 2012; Gunter et al., 2014; van der Wal et al., 2015).

1.2.3  Glacial Isostatic Adjustment on a 3-D Earth

Although most GIA calculations assume that Earth structure varies only with depth, plate tectonic and seismic tomography observations indicate that there are lateral variations in both mantle viscosity and lithospheric thickness (Ritsema et al., 1999). High viscosity subducted slabs descend into the mantle, and low viscosity regions of the mantle (e.g., plumes) ascend to create volcanic provinces. The Earth’s lithosphere—the upper, cold portion of the mantle and crust which does not viscously flow—also varies in thickness. The thickest areas are found below the centers of continental cratons, while the thinnest areas are found underlying the oceans, with a thickness that largely follows a square-root age dependence.

The importance of these lateral variations in Earth structure for GIA calculations is a current topic of investigation. Many studies have demonstrated that this spatially varying, 3-D Earth structure can significantly affect GIA processes. For example, Sella et al. (2007) found that GIA predictions from 1-D viscoelastic models cannot explain the horizontal displacements measured by GPS on the North American plate and suggested that 3-D modeling was required to improve fits between predictions and observations. In another study, Austermann et al. (2013) argued that the presence of a subducted slab with higher viscosity than the surrounding mantle beneath Barbados biased estimates of global mean sea level based on coral markers by muting crustal deformation associated with ocean loading. The bias is of magnitude ~7 meters, equivalent to the ice volume stored in the present-day Greenland Ice Sheet. Others have found that 3-D Earth structure could impact GIA observables in Antarctica, including present-day geodetic measurements (Kaufmann et al., 2005; A et al., 2013; van der Wal et al., 2015). Antarctica’s 3-D Earth structure is discussed in more detail below.
Antarctica’s complex Earth structure

The present-day lithospheric and mantle properties of Antarctica are heavily shaped by its protracted tectonic history. The Antarctic continent first became isolated from the other continents as part of its breakup from the ancient supercontinent Gondwana, via a rifting process that began approximately 180 Ma between Africa and East Antarctica (near the modern-day Weddell Sea; Figure 1.1a). This breakup then proceeded in a clockwise direction, and ended with the rifting of New Zealand from West Antarctica’s Marie Byrd Land around 84 Ma (Fitzgerald, 2002). Today, subduction of oceanic lithosphere beneath the Antarctic Peninsula associated with this continental reconfiguration continues.

In addition to moving away from the other continents, West Antarctica has also experienced phases of internal (failed) rifting that have resulted in a thinned rift zone and ongoing active volcanism (Figure 1.1a). In contrast, East Antarctica is characterized by an old, thick craton. Whilst reconstructions of plate tectonic motions treat the East Antarctic craton as a coherent block since the time of the original split from Gondwana, West Antarctica is instead made up of several distinct micro plates (including, at one time, a plate shared with New Zealand). For example, paleomagnetic data demonstrate that these microplates were quite mobile, experiencing differential displacement and rotation with respect to the East Antarctic craton (Cande et al., 2000). West Antarctica is thought to have experienced at least two phases of extension since the breakup of Gondwana, leading to the formation of the major continental rift system called the West Antarctic Rift System (WARS; Wörner, 1999). The main phase of crustal extension (and thus thinning) in the WARS occurred between 105 and 85 Ma (Fitzgerald, 2002).

Consistent with other continental rift systems (e.g., the East African Rift system), WARS is an active volcanic province. Volcanic rocks that protrude from the ice sheet have been sampled and dated, and found to be predominantly mafic (e.g., Hole & LeMasurier, 1994). Figure 1.1a shows
Figure 1.1: Antarctica. a) Map of Antarctica, with grounded ice in light gray and floating ice shelves in darker gray. b) Bed elevation (in km) from BEDMAP2 (Fretwell et al., 2013). c) Lithospheric thickness (in km) across the Antarctic Plate from An et al. (2015b). d) Inferred mantle temperature (°C) at 100 km depth from An et al. (2015b).
the location of volcanoes that have been active since the Holocene (red triangles). In addition to direct sampling, geophysical prospecting indicates the presence of further subglacial volcanoes. These methods include the analysis of aeromagnetic data (e.g., Behrendt, 1999), the identification of ash layers in glacial ice from airborne radar data (Corr & Vaughan, 2008), geomorphological identification of volcanic edifices (Vries et al., 2018; compare Figure 1.1a and Figure 1.1b), and detection of earthquakes caused by magmatic activity beneath active volcanoes (Lough et al., 2013). As would be expected for basalt-dominated rift volcanism, the 138 subglacial cones identified by Vries et al. (2018) are morphologically consistent with shield volcanoes.

West Antarctica’s widespread volcanism is indicative of an elevated geothermal heat flux. Ice core drilling efforts have further confirmed this high heat flux; for example, Fisher et al. (2015) measured a heat flux of $285 \pm 80$ mW/m$^2$, which they compare to a much lower continental average of $\sim 65$ mW/m$^2$ from a global heat flow database. This elevated heat flux is also supported by additional, indirect measurements, including global seismic data (Shapiro & Ritzwoller, 2004), satellite geomagnetic data (Maule et al., 2005), and models of ice dynamics associated with observed subglacial melt (Schroeder et al., 2014). Importantly, the geothermal heat flux sets the ice sheet’s boundary conditions. Elevated heat flux potentially leads to additional meltwater at the base of the ice sheet, which is corroborated by the existence of an active subglacial hydrologic system in West Antarctica (Fricker et al., 2007). This basal water acts to lubricate the base of the ice sheet, thereby influencing ice dynamics (e.g., Winsborrow et al., 2010) and the flux of ice mass that results in sea-level changes.

As one might expect for a region with a high heat flux and failed rift system, West Antarctica is believed to have a thin lithosphere and an upper mantle of low viscosity. As described in Section 1.2, the viscosity of the mantle determines a region’s characteristic timescale of deformation (i.e., Maxwell time), and any ongoing GIA response to changes in ice loading. West Antarctic GPS observations of anomalously rapid crustal uplift in response to recent mass loss are indicative of vis-
coelastic (as opposed to purely elastic) mantle deformation (Barletta et al., 2018). Using GIA modeling, the authors find a best fit to the GPS uplift rate data with a lithosphere of 50-60 km thickness (as compared to a global average thickness of approximately 100 km), and a viscosity in the shallow upper mantle that is two orders of magnitude lower than a global average upper mantle viscosity value of approximately $5 \times 10^{20}$ Pa s. We will discuss this issue in more detail in Chapter 4.

The inferred presence of a hot, low-viscosity mantle underlying West Antarctica is also supported by seismic observations. Seismic evidence indicates that the West Antarctic lithosphere is relatively thin (~65 km), in contrast with the stable East Antarctic craton, which has a thickness in excess of ~200 km (e.g., Morelli & Danesi, 2004; Heeszel et al., 2013; Hoggard et al., 2020). A map of lithospheric thickness from (An et al., 2015a) is shown in Figure 1.1c. Furthermore, anomalously slow seismic velocities characterize the sub-lithospheric mantle below West Antarctica (Ritzwoller et al., 2001; Morelli & Danesi, 2004; Schaeffer & Lebedev, 2013; Hansen et al., 2014; Heeszel et al., 2016; Lloyd et al., 2020). These seismic velocities were converted to temperature anomalies using standard scalings by An et al. (2015b); their estimate is shown in Figure 1.1d. This thermal interpretation of the anomalously slow seismic velocities (as opposed to a compositional interpretation) is supported by the presence of mafic rocks and is in agreement with geodynamic modeling of regional extension and dynamic uplift (Faccenna et al., 2008; Austrermann et al., 2015).

Antarctica’s crustal architecture, which is broadly defined by thicker crust in the East and thinner in the West, has been inferred through studies of seismic receiver functions (Winberry & Anandakrishnan, 2004) and seismic tomography (Lloyd et al., 2020). The base of the ice sheet has been extensively surveyed using airborne ice-penetrating radar, resulting in detailed maps of subglacial topography (e.g., Goff et al., 2014). BEDMAP2, a pan-Antarctic bed topography map, is shown in Figure 1.1b (Fretwell et al., 2013). These maps indicate that most of the West Antarctic Ice Sheet (WAIS) is grounded on topography that is 100s to 1000s of m below sea level, and hence WAIS is called a “marine-based” ice sheet.
Crucially, West Antarctica’s bed topography slopes inward. That is, the depth of the subglacial topography increases from the ice sheet margin toward the ice sheet interior. Because the stability of marine-based ice sheets is strongly sensitive to the underlying bed slope (Schoof, 2007), this particular bed configuration has led to what is called the Marine Ice Sheet Instability (MISI) hypothesis. The ice mass flux occurring at the grounding line increases nonlinearly with the bed depth at this point (approximately to the $5^{th}$ power), and so when a marine-based ice sheet retreats onto deeper bedrock, this positive flux feedback is thought to accelerate ice flux and potentially result in a runaway ice retreat scenario (Weertman, 1974). Gomez et al. (2010, 2015) demonstrate, however, that the uplift and local sea-level fall proximal to a melting ice sheet can act in opposition to this positive feedback, acting as a self-stabilizing mechanism for marine-based ice sheets. The magnitude of this negative feedback will depend on the local mantle viscosity structure that controls the GIA response. Broadly speaking, a higher-viscosity mantle would lead to a smaller stabilizing effect that occurs over a longer duration, while a lower-viscosity mantle would lead to a larger, shorter duration stabilizing effect. Thus, there is an active feedback loop between Antarctica’s crustal and mantle structure, ice sheet stability, and the accompanying complex deformation.

Taken together, the studies detailed above suggest significant, 3-D variations in Antarctica’s lithospheric thickness and upper mantle viscosity, with a particularly strong gradient between the shallow mantle beneath the warmer western sector to the colder mantle underlying the eastern sector of the continent. In this thesis, we will expand on how this complexity influences the region’s GIA response. This structure will specifically impact the magnitude and pattern of crustal deformation and sea-level changes associated with ice mass changes in Antarctica, and this will, in turn, affect the stability of the ice sheet and the spatial variation of sea-level rise.

In the next section, we explore the physics involved in modeling the response to surface loading in more detail. The effort is complicated by the fact that the meltwater also represents a surface load, and is itself a function of the ice mass distribution, the shape of the ocean basins, and the underlying
solid Earth structure.

1.3 Sea-Level Physics

The height of the ocean is not static: as continental ice sheets waxed and waned according to Earth’s past climatic conditions, a corresponding transfer of water flowed out of and into the ocean basins. Changes in sea level ultimately shape the exposure of land and will present an ongoing challenge for coastal communities as the ocean continues to rise. These changes also represents a second load (in addition to ice mass changes) that we must account for in GIA modeling. In this section, we will highlight the different ways to refer to sea level (from global to local); historical sea-level variability; as well as several important governing physical processes.

1.3.1 Global mean sea-level

Global mean sea level (GMSL) changes reflect variations in global ice volume, ocean heat content and salinity. Past changes in GMSL are generally estimated using geological markers of sea level (e.g., uplifted beach terraces) and proxy data (e.g., oxygen isotope records). Figure 1.2 shows three different GMSL curves extending from the Last Interglacial (LIG; \( \sim 120 \) ka) to to present day (Lambeck & Chappell, 2001; Peltier & Fairbanks, 2006; Pico et al., 2017). The three models agree well over the time period extending from the Last Glacial Maximum to present day because GMSL is well tightly constrained in the post-LGM period through sediment and coral records with associated radiocarbon or uranium-series dating (among other proxies; see Lambeck et al., 2014). In contrast, the pre-LGM component of the GMSL curves varies significantly between the three models, reflecting the sparsity and uncertainty of sea-level data and proxies across this time period. This discrepancy between GMSL models reflects - in a direct way - uncertainty in ice volumes stored in grounded ice sheets. For example, the present-day Antarctic ice sheet will contribute \( \sim 60 \) m to the ocean basins
Figure 1.2: GMSL Reconstructions. Global mean sea-level change relative to present day for three different models: black = LC01 (Lambeck & Chappell, 2001); orange = ICE-5G (Peltier & Fairbanks, 2006); blue = ICEPC2 (Pico et al., 2017), which is a recent refinement of ICE-5G. The dashed red line represents present-day sea level.

(if fully melted) (Fretwell et al., 2013), and a difference of approximately this size is seen between model ICEPC2 (in blue) and ICE5G (in orange) at \( \sim 45 \) ka.

Sea level does not rise and fall uniformly, as in a bathtub, and so a global mean perspective on sea level is not always useful or appropriate.

1.3.2 Beyond Global Mean Sea Level: Full Physical Representation

Beyond GMSL, sea level can be mathematically defined as the radial distance between the sea surface and solid surface. A perturbation to the elevation of either will result in a sea-level change, with uplift of the sea surface leading to a sea-level rise, and uplift of the solid Earth leading to sea-level fall (and vice versa). For our purposes, the sea surface is defined as the gravitational potential coinciding with mean sea surface height (that is, we neglect dynamic effects from ocean circulation and tidal activity).
Changes in regional sea-level can differ significantly from GMSL because of the gravitational and deformational effects of surface mass loading. For example, when an ice sheet grows, the additional surface mass exerts an additional, instantaneous gravitational pull on the surrounding ocean. This phenomena leads to a local sea-level rise extending ~ 2000 km from the growing ice sheet (the near-field of the ice sheet), and a corresponding sea-level fall beyond, in the far-field of the ice sheet (Figure 1.3A). This load self-gravitation will have the opposite effect on sea level for a melting ice sheet (Figure 1.3B), and in both cases, the magnitude of the sea-level changes can be significant. If an ice sheet melts by an amount that will raise GMSL by 1 m, the sea-level fall in the near-field will be of order 10 m, while the sea level rise in the far-field can reach up to 1.4 m (a 40% increase from the expected GMSL rise).

In the “near-field” of the ice sheets (or their former location), deformational processes dominate the time-evolving sea-level signal. As an ice sheet grows, the excess mass loads the underlying crust and mantle, causing it to viscoelastically subside and local sea level to therefore rise (Figure 1.4A). When an ice sheet melts, on the other hand, the area beneath the ice load viscoelastically rebounds, initiating a period of local sea-level fall (Figure 1.4A). Today, this behavior can be seen in, for example, Hudson Bay. Post-glacial rebound there is on the order of 1 cm/yr, leading to the observed raised ancient beach terraces.

Also in the near field, the ice loading causes mantle material to viscously creep outward from the surface load, creating what is termed a “peripheral bulge” (Figure 1.4A). The growth of these peripheral bulges leads to a local uplift of the solid surface and therefore a local sea-level fall. When the ice load is removed from the continent, the opposite effect occurs. The surrounding peripheral bulge subsides as mantle material viscously creeps back toward the zone of rebound (Figure 1.4B), leading to a local sea-level rise. In the United states, the continued subsidence of the peripheral bulge associated with LGM ice cover has led to a present-day sea-level rise along both coastlines of up to ~ 3 – 5 mm/yr.
Figure 1.3: Load self-gravitation. Load self-gravitation occurring during periods of active ice mass change for A) glaciations and B) deglaciation and interglacials. The yellow star represents a sea-level marker at the starting time. Figure modified from Borreggine et al., in review, illustration by Maximilian Werner.
Figure 1.4: Ocean Syphoning. Illustration of crustal deformation processes in the near field of ice cover (at left) and the contribution of peripheral bulge dynamics to ocean syphoning (at right) occurring during periods of active ice mass change for A) glaciations and B) deglaciation and interglacials. Mass is not conserved in these cartoons, that is, they do not include the expected rise/fall in GMSL due to ice melt/gain. The yellow star represents a sea-level marker at the starting time. Figure modified from Borreggine et al., in review, illustration by Maximilian Werner.
Deformation of the ocean basins, especially the uplift/subsidence of peripheral bulges within the ocean, leads to what is called “ocean syphoning,” where water moves toward/away from the far-field of the ice cover. During a period of ice sheet growth, accommodation space for water in the near field is reduced via the uplift of the bulge. Water is therefore pushed outward, and sea-level rises in the far field (Figure 1.4A). In contrast, during deglaciation periods and interglacial periods, the accommodation space in the near field increases as the peripheral bulge subsides, and water migrates toward the location of the former ice sheets, leading to a sea-level fall in the far-field (Figure 1.4B). Ocean syphoning associated with changes to the ocean basin accommodation space due to both ice and ocean load changes involves a signal of up to $\sim 0.5$ mm/yr. This is evident in coral records in the equatorial Pacific, which indicate that sea level has fallen $\sim 3$ m since the end of the last deglaciation phase at $\sim 5$ ka.

**Sea-Level Theory**

To calculate sea-level changes resulting from ice mass changes, mass conservation requires that we account for the influx of water into and out of the ocean basins in a gravitationally self-consistent manner. This is complicated by the fact that the redistribution of ocean mass perturbs the gravitational field (both by the direct effect of the mass flux and the deformation it induces) and is, in turn, governed by the gravitational field.

Farrell & Clark (1976) provide a theoretical framework for predicting gravitationally self-consistent sea-level changes driven by ice mass changes for the case of a non-rotating Earth with constant shoreline geometry (i.e., without transgressions or regressions). Under their theory, the change in the height of the ocean surface, $\Delta S$, can be found using the sea-level equation given by:

$$\Delta S(x, t) = C(x) \Delta S_L(x, t), \quad (1.6)$$
where \( x \) is a position denoted by colatitude and longitude, \( C \) is the so-called ocean function, which has a value of unity over the ocean and zero elsewhere, and \( \Delta SL \) is the change of the global sea-level field. Note that \( \Delta SL \) will itself be a function of \( \Delta S \) - which represents the surface mass load - and thus the equation is an integral equation.

The change in the global sea level is determined by how the solid surface and the gravitational equipotential corresponding to the sea surface will be perturbed by a load. This load is given by \( \rho_i \Delta I + \rho_w \Delta S \), where \( \Delta I \) is the change in the ice load, and \( \rho_i \) and \( \rho_w \) are the densities of ice and water, respectively. Perturbations in the two bounding surfaces of sea level are governed by the coupled equations of Laplace’s equation for gravity, and Newton’s second law describing the conservation of momentum. For the Earth, these two governing equations can be written as:

\[
- \nabla \cdot \mathbf{T} + \nabla (\rho \mathbf{u} \cdot \nabla \Phi) - \nabla \cdot (\rho \mathbf{u} \nabla \Phi) + \rho \nabla \varphi = 0, \tag{1.7}
\]

where \( \Phi \) is the Earth’s unperturbed gravitational potential, \( \mathbf{u} \) is the vector displacement and \( \mathbf{T} \) is the deviatoric stress tensor; and:

\[
(4\pi G)^{-1} \nabla^2 \varphi = \begin{cases} 
- \nabla \cdot (\rho \mathbf{u}), & \text{if } \mathbf{x} \text{ is within the solid Earth} \\
\rho^{-1} \varphi \partial_\rho \rho, & \text{if } \mathbf{x} \text{ is within the fluid Earth} \\
0, & \text{otherwise}
\end{cases} \tag{1.8}
\]

(Al-Attar & Tromp, 2014). In practice, the surface deformation and gravitational field perturbation are commonly found using the viscoelastic Love number number approach of Peltier (1974), in which semi-analytical solutions to the above equations may be found. The major drawback of this approach, however, is that it is only valid for 1-D models (i.e., models of Earth structure that vary with depth alone).
This sea-level theory has been modified via several improvements. The sea-level equation now includes time-varying coastal shoreline migration (Johnston, 1993; Milne & Mitrovica, 1998; Mitrovica & Milne, 2003), as well as the shoreline migration associated with marine based ice sheets (Mitrovica & Milne, 2003; Gomez et al., 2010). Further improvements have included the feedback into sea level of load-induced changes in the Earth’s rotation vector (Milne & Mitrovica, 1996).

1.4 Overview of Modeling Methods

The modeling performed in this thesis assumes an elastically compressible, Maxwell viscoelastic Earth model with (in general) 3-D variability in mantle viscosity structure (see description above), and explores the sea-level response to a prescribed ice mass loading history. The sea-level calculations here are based on a gravitationally self-consistent version of ice age sea-level theory that accurately accounts for all deformational, gravitational and rotational effects as well as time-varying shoreline migration due to local transgressions/regression and changes in the perimeter of grounded, marine-based ice sectors (Mitrovica & Milne, 2003; Kendall et al., 2005; Gomez et al., 2010), as described above.

We note that since we are primarily concerned with modern sea-level changes in Chapters 2-4 (or changes from a configuration we assume to closely resemble modern-day conditions), the initial (i.e. present-day) topography is assumed to be known. This allows us to simplify our calculations relative to typical ice age sea-level calculations, which require an iterative procedure that converges to the final, present day topography. The latter case is employed in Chapter 5 using the full theory described by Kendall et al. (2005).

Our sea-level calculations require two main inputs: a model for viscoelastic mantle structure and the space–time history of ice cover. While we will prescribe the relevant adopted ice histories in each respective chapter, we briefly discuss the viscoelastic Earth models adopted throughout the thesis in
the following discussion.

1.4.1 Construction of viscoelastic Earth models

We will consider three different Earth models, distinguished by their viscosity structure within the mantle. All three Earth models used here have elastic and density structure given by the 1-D seismic Preliminary Reference Earth Model (PREM; Dziewonski & Anderson, 1981). Realistic lateral variations in the elastic moduli have an insignificant effect on GIA (Mitrovica et al., 2011).

Elastic, 1-D Earth model

The first Earth model is a purely elastic model ($M_{EL}$). In this case, our computations depend only on the net change in ice volume between the beginning and end of the calculation and not on the duration of the simulation.

Viscoelastic, 1-D Earth model

The second Earth model ($MVE_{1D}$) assumes a depth-varying viscosity profile comprised of an elastic (effectively infinite viscosity) lithosphere of 96-km thickness, an upper mantle viscosity of $5 \times 10^{20}$ Pa s, and a lower mantle viscosity of $5 \times 10^{21}$ Pa s that extends from 670 km down to the core-mantle boundary. This model falls within the class of 1-D viscosity models favored in most GIA-based inferences of mantle viscosity (e.g., Lambeck et al., 1998; Mitrovica & Forte, 2004; Lambeck et al., 2014).

Viscoelastic, 3-D Earth model

Our third Earth model ($MVE_{3D}$) incorporates 3-D variability in both the thickness of the elastic lithosphere and mantle viscosity. Lateral variations in lithospheric thickness are taken from An
et al. (2015b) for the Antarctic plate, and Conrad & Lithgow-Bertelloni (2006) elsewhere. The lithosphere is scaled to produce a global mean lithospheric thickness of 96 km, which results in an average thickness of 65 km across the West Antarctic and 200 km across the East Antarctic (Figure 1.5A).

The 3-D viscosity variation below the lithosphere is estimated from seismic shear wave velocity ($v_s$) anomalies. To start, we constructed a global seismic velocity model by combining three different seismic tomography models. We use high resolution regional Antarctic tomography models in the West (Heeszel et al., 2016) and East (An et al., 2015a), and a global background model S40RTS (Ritsema et al., 2011). The first two models extend to 350 km depth, and S40RTS is adopted at greater depths across the entire mantle. To build our seismic velocity model, we begin by adopting the S40RTS model as the global model. The (An et al., 2015a) model was then patched into this global model and at each depth its velocities were shifted up or down so that the average value across the full Antarctic domain of the (An et al., 2015a) model matched the average of the S40RTS model over the same Antarctic domain. The Heeszel et al. (2016) model was then patched into the (An et al., 2015a) model so that at each depth the average across its smaller domain matched the average across the same domain of the (An et al., 2015a) model.

Variations in seismic wave speed are mapped into temperature perturbations following the depth-dependent scaling procedure described in Latychev et al. (2005). First, density variations are found using:

$$
\frac{\partial \ln \rho(r, \theta, \varphi)}{\partial \ln v_s(r)} = \frac{\partial \ln \rho}{\partial \ln v_s(r)} \frac{\partial \ln v_s(r, \theta, \varphi)}{\partial \ln v_s(r)} , \quad (1.9)
$$

where $r$, $\theta$ and $\varphi$ are the radius, co-latitude and east longitude, respectively, $v_s$ is the shear wave velocity, and $\rho$ the density. The velocity-to-density scaling factor, $\partial \ln \rho / \partial \ln v_s$, is assumed to be a function of depth alone. We adopt the profile of Forte & Woodward (1997), which was modified,
Figure 1.5: Viscoelastic 3-D Earth model. (A) Lithospheric thickness (km), and mantle viscosity variations at depths of (B) 100 km, (C) 125 km, (D) 150 km, (E) 200 km, and (F) 300 km in the 3-D viscoelastic Earth model $MVE_{3D}$. Panels (B)-(F) represent the logarithm of mantle viscosity variations relative to a background, spherically averaged (1-D) viscoelastic model $(\log(\nu_{3D}/\nu_{1D}))$. Areas in white in (B)-(F) lie within the lithosphere. The blue asterisk in (A) indicates the location of Marie Byrd Land. Figure is adapted from Hay et al. (2017).
on the basis of convection-related geodynamic constraints, from a profile of Karato (1993) based on constraints from mineral physics. Next, temperature variations are found using the following expression:

\[ \delta T(r, \theta, \varphi) = -\frac{1}{\alpha(r)} \delta \ln \rho(r, \theta, \varphi), \]

(1.10)

where \( T \) is temperature, and \( \alpha(r) \) is the depth-dependent coefficient of thermal expansion. We adopt the \( \alpha(r) \) derived by Chopelas & Boheler (1992).

Finally, temperature variations are mapped into estimates of 3-D mantle viscosity. We assume an exponential dependence of the viscosity field, \( \eta(r, \theta, \varphi) \), on the temperature variation. This dependence is described by:

\[ \eta(r, \theta, \varphi) = \eta_0(r) \exp \left( -\varepsilon \delta T(r, \theta, \varphi) \right), \]

(1.11)

where \( \eta_0(r) \) is the reference radial profile of mantle viscosity, and the parameter \( \varepsilon \) prescribes the magnitude of peak-to-peak lateral variations in viscosity. Note that when \( \varepsilon = 0 \), the system collapses to the spherically symmetric case. The spherically averaged depth profile of viscosity in the 3-D models will generally match \( MVE_{1D} \), but in each chapter we will specify if a different background model is adopted. For the scaling of temperature anomalies to viscosity perturbations about the global average, we used the scaling factors 0.037, 0.023, and 0.04 for the models by Heeszel et al. (2016), An et al. (2015a), and S40RTS, respectively. The scaling factor varies in order to account for the amplitude differences between the s-wave models, and thus to avoid significant viscosity discontinuities across the model domains.

The resulting viscosity model is shown in Figure 1,5B-F. Assuming a background viscosity profile given by \( MVE_{1D} \), viscosities in the 3-D model are as low as \( \sim 10^{18} \) Pa s below Marie Byrd Land (blue asterisk in Figure 1,5A).
1.4.2 Finite Volume Method

Unless otherwise noted, our results are generated using a finite volume numerical scheme and software described in Latychev et al. (2005) to solve the governing equations (Equations 1.7 and 1.8). In the calculations in this thesis, we utilize a global grid with a radial discretization that honors all first and second-order PREM interfaces (Dziewonski & Anderson, 1981). With 67 radial layers in the mantle and lithosphere and similar horizontal and spatial resolution at a given depth, the spatial resolution in the global domain is variable: 12-15 km from the surface to the base of the crust, ~25 km from this depth to 220 km, and ~50 km to the base of the mantle.

A major advantage of the finite-volume software is that it allows for higher regional grid refinement within a global grid. In this way, we can allocate our computational resources to the areas we are especially interested in, or require high spatial resolution calculations. In Chapters 2-4, we implement within our global grid an area of additional refinement in the Antarctic subdomain, which extends from the South Pole to approximately 60°S (the outer boundary is not axisymmetric), and vertically down from the Earth’s surface to a depth of 350 km. In this regionally refined grid we double the spatial (horizontal and vertical) resolution described above for the shallow mantle: 6 km to the base of the crust, 12 km to 220 km depth, and 25 km to 350 km depth. The final grid has a total of approximately 27 million grid nodes.

1.5 The Dissertation.

This thesis has 6 chapters, all of which explore how 3-D Earth structure affects both observations and our ability to infer Earth structure and processes.

• In Chapter 2, we present an assessment of the impact of 3-D Earth structure on interglacial sea-level analyses. We show that sea-level changes in the both near field and far field are af-
fected by 3-D Earth structure, and that inferences of West Antarctica’s possible contribution to GMSL have been underestimated by \( \sim 30\% \).

- In Chapter 3, we probe how 3-D Earth structure will affect Antarctic geodetic measurements on modern (i.e. short, annual-to-decadal) timescales. We show that while gravity measurements (as taken by the GRACE satellite mission) will be relatively insensitive to lateral Earth structure, GNSS measurements for crustal deformation rates, in contrast, are highly sensitive to the local structure.

- In Chapter 4, we consider the post-glacial inverse problem. Using synthetic data, we demonstrate that an inversion for 1-D mantle viscosity structure in West Antarctica will be biased by the existence of 3-D mantle structure.

- In Chapter 5, we use sea level retrodictions to help resolve land disputes regarding Canada’s Akimiski Island. Specifically, we timestamp important Cree oral history events in order to establish that the test for Aboriginal title are met.

In the final chapter, we present our conclusions and discuss some possible future applications of the results of this dissertation.
Effect of 3-D Earth structure on sea-level predictions from interglacial WAIS melt

2.1 Introduction

Interglacials are geological time periods characterized by warm global average temperatures that last for thousands of years. These intervals separate glacial periods within an ice age; for example, the current Holocene interglacial marked the end of the past deglaciation ~ 8 ka. It is often argued that understanding the response of ice sheets during these past periods of relative ice age warmth will improve constraints on ice sheet stability in our future, warming world. In this chapter, we will perform sea-level modeling of a hypothetical interglacial period in order to contribute to this aim.

Reconstructions of global mean sea level (GMSL) provide the primary constraint on the minimum volume of past ice sheets during these interglacial periods. For example, GMSL during Marine Isotope Stage (MIS) 5e, the most recent, or “Last Interglacial” (LIG; 130–116 ka), is thought to have reached 5.5–9.0 meters above present-day values (Kopp et al., 2009; Dutton & Lambeck, 2012; O’Leary et al., 2013). And although GMSL during MIS 11, the interglacial of longest duration in the past 500 kyr (424–395 ka), is debated (Hearty et al., 1999; McMurtry et al., 2007), recent work indicates a peak value close to 10 m above present-day sea level (Raymo & Mitrovica, 2012; Chen et al., 2014). Both examples indicate substantial melting of polar ice sheets (see below). But many out-
standing questions concerning interglacials remain: Which ice sheets contributed to the GMSL rise, and by how much? When and how quickly did these ice sheets collapse? Did GMSL oscillate, potentially indicating periodic ice sheet growth under warm conditions? Tackling these questions through careful interpretation of the geologic record is crucial to reducing uncertainties over ice sheet behavior during periods of sustained global warming. In this chapter, we will examine some of the physics that has previously been neglected in studies of these questions.

Let us consider the above estimate of peak GMSL during the Last Interglacial, which has been derived from several approaches. These include the interpretation of the diverse geological record of sea-level changes (Kopp et al., 2009; Thompson et al., 2011; Dutton & Lambeck, 2012; Grant et al., 2012; O’Leary et al., 2013; Dutton et al., 2015; Polyak et al., 2018; Barlow et al., 2018; Rohling et al., 2019), climate and ice-sheet modeling (Stone et al., 2013; Fogwill et al., 2014; DeConto & Pollard, 2016; Goelzer et al., 2016; Sutter et al., 2016; Plach et al., 2019; Clark et al., 2020), and ice core analyses (Dahl-Jensen et al., 2013; Yau et al., 2016). Current estimates of the contribution to LIG GMSL from thermal expansion and the complete melting of mountain glaciers are 0.7 ± 0.3 m (Shackleton et al., 2020) and 0.3 ± 0.1 m (Marzeion et al., 2020), respectively. Estimates of the potential contribution from the Greenland Ice Sheet vary (Otto-Bliesner et al., 2006; Helsen et al., 2013; Stone et al., 2013; Goelzer et al., 2016; Plach et al., 2019; Clark et al., 2020), but an upper bound of ~3.0 m is consistent with a number of these studies (Plach et al., 2019). A common assumption is that the maximum potential contribution of the West Antarctic Ice Sheet (WAIS) to the total GMSL rise during the LIG is 3.3 m (Joughin & Alley, 2011; Dutton & Lambeck, 2012; Steig et al., 2015; Hein et al., 2016; Thomas et al., 2020), and that higher values of total GMSL would indicate that additional melting from the East Antarctic Ice Sheet also occurred (Gilford et al., 2020; Turney et al., 2020). Specifically, the above estimates have led to the view that the lower bound of 5.5 m on peak GMSL during the LIG does not require East Antarctic Ice Sheet contribution, whilst the upper bound of 9.0 m does (Dutton & Lambeck, 2012; Helsen et al., 2013). Thus,
there is still substantial uncertainty regarding the fate of polar ice caps during interglacial warm periods.

The geographic pattern of sea-level change following the melting of ice sheets and glaciers is a strong function of the location and time scale of the ice mass change (Farrell & Clark, 1976). This behaviour occurs because melting of ice sheets results in deformational, gravitational and rotational effects in a process called glacial isostatic adjustment (GIA). Unfortunately, historical and geological sea-level records only sample this signal at specific points in space and time, which makes accounting for the sources of complexity in this signal non-trivial. One lens through which we may estimate the contribution of polar ice sheets to GMSL, however, is via forward modeling of the sea-level change arising from prescribed mass loss.

In the special case of rapid melt (i.e., melt events spanning decadal to century time scales), the resulting uniquely identifying geometries of sea-level change have come to be known as sea-level fingerprints. While this specific class of variability has been recognized since the late 1800s (Woodward, 1888), modern predictions of sea-level fingerprints date to the 1970s. Clark & Lingle (1977) applied the gravitationally self-consistent sea-level theory of Farrell & Clark (1976) to predict sea-level changes in response to future melting of WAIS. The same governing sea-level theory, which assumed a non-rotating Earth and fixed shorelines, was adopted in subsequent studies (e.g., Conrad & Hager, 1997). The assumption of fixed shorelines, however, neglects lateral ocean migration due to the onlap or offlap of water at coastlines experiencing local sea-level variations, as well as changes in the perimeter of grounded, marine-based ice sheets.

The so-called “fingerprinting” of sea-level change received renewed attention beginning at the turn of the current century. This included efforts to use sea-level observations such as tide gauges and U/Th age-dated coral samples to infer meltwater contributions of ice dating to both the modern (e.g., Mitrovica et al., 2001; Plag & Jüttner, 2001; Plag, 2006 and ice age (Clark et al., 2002) worlds. Mitrovica et al. (2001) and Tamisiea et al. (2001) introduced the feedback of rotational ef-
fects into their predictions of sea-level fingerprints by including a term in the theory for the response to a centrifugal driving potential associated with load-induced polar motion. Moreover, Mitrovica et al. (2009) and Gomez et al. (2010) adopted a generalized sea-level theory to incorporate shoreline migration into the calculations. The Gomez et al. (2015) study focused on WAIS collapse, and highlighted the importance of incorporating shoreline migration in predictions of sea-level fingerprints following the retreat (or advance) of grounded ice in marine settings. Finally, Mitrovica et al. (2011) summarized various technical issues associated with the calculation of sea-level fingerprints, investigated the relative accuracy of published predictions, and demonstrated that 3-D variability in the elastic structure of the mantle would have negligible impact on these predictions. We note that with the exceptions of Bamber et al. (2009) and Mitrovica et al. (2009), nearly all predictions of sea-level fingerprints assume that the melting events are sufficiently rapid that the Earth’s mantle may be treated as being purely elastic.

It is important to consider viscous effects on sea-level predictions, however, especially because WAIS is grounded below sea level. When ice sheets grounded in marine-based sectors collapse, in addition to an increase in GMSL, two unique phenomena occur. First, the area of the global ocean increases because the retreating ice sheet exposes marine sectors and creates new accommodation space for water. Second, viscoelastic post-glacial rebound will steadily reduce the height and thus the volume of this accommodation space and in doing so will increase the associated GMSL rise (Milne et al., 1999; Bamber et al., 2009; Mitrovica et al., 2009; Gomez et al., 2010). Note that this is a general physical effect, and is not limited to any particular interglacial. Bamber et al. (2009) calculated the magnitude of the water expelled by this GIA effect in the case WAIS collapse using 1-D mantle viscosity models and found that it would be small. Specifically, the authors simulated a collapse of marine-based sectors of WAIS over 500 years and tracked the GMSL changes over the following 10 kyr in which no changes to ice volume occurred. They found that the subsequent viscous deformation would increase the GMSL signal by ~0.3-0.4 m at the end of their 10 kyr simulation. In-
deed, the commonly adopted 3.3 m bound on the contribution of WAIS to Last Interglacial GMSL is based on Bamber et al. (2009)’s calculation. The mantle viscosity models used in the study did not, however, capture the complexity of the mantle’s viscoelastic structure beneath WAIS, including both the range and lateral variability of viscosity.

The use of unrealistic mantle viscosity models also occurs in the interpretation of far-field sea-level records. Because near-field sea-level records are highly sensitive to the exact geometry of local ice melting in addition to the local mantle viscosity structure, they are less useful as constraints on GMSL than observations from the far-field. Crawford et al. (2018) demonstrated, on the other hand, that the sea-level response at a far-field site is most impacted by mantle viscosity structure both beneath the site itself and within the region of the Earth beneath the ice melting zone, with additional sensitivity to structure along the path between these two locations. When predicting the GIA signal associated with ice mass changes during the interglacial of interest and the subsequent ice age cycle, many studies have assumed that any viscous response may be approximated using 1-D models of mantle viscosity (Raymo & Mitrovica, 2012; O’Leary et al., 2013; Chen et al., 2014; Hay et al., 2014).

In this chapter, we will study the effects of 3-D mantle viscosity structure on sea-level predictions for an example, “idealized” interglacial period, with particular attention to the complex viscoelastic structure beneath WAIS. We will use the sea-level projection modeling described in Chapter 1. That is, our sea-level projections are based on the gravitationally self-consistent sea-level theory described by (Gomez et al., 2010). All calculations in this chapter are computed using the finite volume software described in Latychev et al. (2005), using the global grid with regional refinement in Antarctica. The spatio-temporal histories of ice cover and the viscoelastic models of mantle structure we consider are described in their respective sections.
2.1.1 Outline

In the following sections, we will consider three types of sea-level predictions in turn: near-field sea-level fingerprints, global mean sea-level predictions, and far-field sea-level records. We perform an idealized case study and simulate GIA due to ice melt during a generic interglacial period. We will consider three case studies overall: (1) moderate mass loss (melt) over time scales of decades, characterized by thinning of ice cover; (2) rapid (instantaneous) and complete collapse of WAIS marine based sectors; and (3) more realistic mass loss over timescales of thousands of years, where major collapse of grounded ice in marine-based settings is predicted (e.g., Joughin et al., 2014; Levermann et al., 2014). Specifically, we will be implementing 3-D mantle viscosity models and comparing their predictions with the predictions based on purely 1-D models.

2.2 Sea-Level Fingerprints

In this section, we explore the impact of variable viscoelastic mantle structure on predictions of near-field sea-level fingerprints of relatively rapid WAIS mass loss. As described above, previous sea-level fingerprint studies of the Antarctic GIA response have assumed either an elastic rheology, or a stiff mantle viscosity of $\sim 10^{21}$ Pa s. The mantle viscosity structure beneath WAIS, however, has lateral variability that is inferred to vary over 3-5 orders of magnitude, reaching values as low as $\sim 10^{18}$ Pa s. This implies that the West Antarctic mantle could have a local Maxwell time as low as a decade. Thus, viscous effects may play a significant role in sea-level changes over this time scale. That is, published fingerprints of WAIS melt based on elastic or 1-D viscoelastic Earth modeling may be subject to significant error, particularly in the near field of the ice loss. For this part of our study, we will therefore explore the impact of 3-D Earth models on Antarctic GIA deformation, and reconsider mass loss (melt) over time scales of decades, characterized by thinning of ice cover. The goal here is to provide updated predictions of these fingerprints of melting in this region of complex,
3-D viscoelastic Earth structure.

The WAIS melt geometry we consider in this section, (WA1), follows most previous fingerprint analyses. We will remove a thin uniform layer of ice across WAIS. This choice allows us to perform comparisons with previous results from the literature. While this is not a realistic melt scenario, our goal is to focus on the geographic perturbation to the sea-level fingerprint driven by Earth model variability, rather than the impact of melt geometry. We will consider three different cases distinguished on the basis of the melt duration: 25 yr, 50 yr, and 100 yr.

We will also consider three different Earth models distinguished by their structure within the mantle, as described in Chapter 1. The first is a purely elastic model (MEL). In this case, the computed sea-level change depends only on the net change in ice volume between the beginning and end of the calculation and not on the duration of the simulation. As noted, the vast majority of fingerprint calculations have been performed using purely elastic Earth models. The second Earth model (MVE1D) assumes a depth-varying viscosity profile comprised of an elastic (effectively infinite viscosity) lithosphere of 96 km thickness and uniform upper and lower mantle viscosities of $5 \times 10^{20}$ Pa s and $5 \times 10^{21}$ Pa s, respectively. A third Earth model (MVE3D) incorporates 3-D variability in both the elastic thickness of the lithosphere and mantle viscosity. It has a mean lithospheric thickness of 96 km, with a mean thickness of 65 km across the West Antarctic and 200 km across the East Antarctic. The 3-D viscosity variation below the lithosphere is estimated from seismic velocity heterogeneity using the multi-step method discussed by Austermann et al. (2013) and described in Chapter 1. In the conversion of temperature variations to viscosity, the Earth model MVE3D is constrained to have a spherically averaged depth profile that matches the model MVE1D.

All of our sea-level predictions in this section will be normalized by the total eustatic (i.e., global mean) sea-level change associated with the melt scenario, and are therefore non-dimensional. As discussed by Mitrovica et al. (2011), normalized sea-level fingerprints can be scaled to consider melt
events of similar geometry but different net changes in ice mass.

2.2.1 Sea-level Fingerprint Simulations: Results & Discussion

We first calculate the (normalized) sea-level fingerprint using the elastic model, MEL (Figure 2.1). This elastic calculation shows the salient features characteristic of previously published (elastic) sea-level fingerprints of WAIS melting, including: a zone of sea-level fall in the vicinity of the ice melt that peaks at \(~9.4\) times the eustatic value for the WA1 melt event; and a far-field sea-level rise that peaks at \(~26\)% higher than the eustatic value in the North Atlantic, North Pacific, and Indian Oceans.

Figure 2.2 shows results obtained for viscoelastic Earth models MVE1D and MVE3D relative to the elastic calculation. For both viscoelastic models, we consider three simulations of the uniform ice melt geometry WA1 distinguished on the basis of the duration of the melt event: 25 yr, 50 yr, or 100 yr. In all cases, we plot the total sea-level change from the start to the end of the simulated deglaciation. We note that if we continued the simulation beyond the end of the deglaciation, the impact of viscous effects on the sea-level predictions would increase, and in this sense the results in Figure 2.2 represent lower bounds on the absolute magnitude of this impact.

The viscoelastic calculations based on the 1-D viscosity profile MVE1D are shown in the left column of Figure 2.2. As one would expect, the discrepancy from the elastic simulation (i.e., the excess signal from viscous effects) increases as the melt duration increases. Specifically, the peak difference from the elastic case (Figure 2.1A inset) over West Antarctica is 1.1\%, 2.1\% and 4.5\% for the 25 yr, 50 yr, and 100 yr simulations, respectively. As we have noted, the Earth model MVE1D is within the class of 1-D Earth models commonly inferred in GIA analyses.

Next, we consider analogous results based on the 3-D viscoelastic Earth model MVE3D (right column of Figure 2.2). In contrast to predictions based on model MVE1D, viscous effects become pronounced within just a few decades when MVE3D is adopted. In the near field, these viscous
Figure 2.1: Elastic sea-level fingerprint of WAIS melt. Normalized (dimensionless) sea-level change across a simulation based on the WA1 ice history assuming a purely elastic Earth model. The peak sea-level fall in the West Antarctic is -9.37 and the peak far-field sea-level rise is 1.26. The prediction is normalized relative to the effective eustatic value of the adopted ice history. Since these calculations adopt an elastic Earth model, the predictions are independent of the modeled duration of WAIS collapse. Note that the color bar is saturated at -0.8. The inset shows the normalized fingerprint centered over the South Pole. Figure is adapted from Hay et al. (2017).
Figure 2.2: Impact of viscous relaxation on computed sea-level fingerprints. Left column – difference between the normalized (and therefore dimensionless) sea-level change for simulations based on the 1-D viscoelastic Earth model MVE1D and the elastic Earth model MEL. The top, middle and bottom frames show results for WAIS melt events of duration 25 yr, 50 yr, and 100 yr, respectively. Right column – as in the left frames, except the viscoelastic calculations are based on the 3-D viscoelastic Earth model MVE3D. All calculations are based on the WA1 (uniform thinning) ice history and are normalized by the effective eustatic value of this history (1 mm). Figure is adapted from Hay et al. (2017).
effects are characterized by localized sea-level fall due to post-glacial uplift in West Antarctica and an offshore zone of localized sea-level rise due to the subsidence at the periphery, both of which are common features of ice age sea-level predictions (e.g., Milne & Mitrovica, 2008). The amplitude of these effects is non-negligible. For the 25 yr, 50 yr, and 100 yr melt simulations, the peak sea-level fall due to viscous effects occurs over the zone of minimum mantle viscosity and reaches amplitudes of 22.8%, 35.4%, and 54.1% of the peak elastic signal. Viscous effects clearly imprint a significant signature on the pattern and magnitude of the computed sea-level fingerprints throughout the near field. Predictions of the near field sea-level fingerprint of WAIS melting spanning the next few decades to century should be based on modeling that incorporates the complex mantle viscosity structure below the region.

2.2.2 Sea-level Fingerprint Simulations: Summary & Next Steps

Fingerprints of sea-level change following the rapid melting of an ice sheet are almost exclusively computed by assuming that the solid Earth response to melting is purely elastic (or in rare cases, adopt a simplified, 1-D viscosity model). Our results demonstrate that the predicted sea-level fingerprint of WAIS melt based on typical 1-D GIA models will exhibit only minor differences from the associated elastic fingerprint for time scales of ice melt of order a century. The assumption of a predominantly elastic response is questionable in the case of ice mass changes in West Antarctica, however, given that geological and geophysical evidence suggests that sub-lithospheric viscosity below the region is approximately two orders of magnitude lower than values typically adopted in ice age sea-level calculations (see the Introduction for more details).

We have demonstrated, using a realistic 3-D viscoelastic Earth model, that sea-level predictions in the near field of WAIS will depart significantly from previously published elastic fingerprints in as little as a few decades. This result has implications for studies of sea-level change due to both ongoing mass loss from WAIS over the next century and large scale collapse of WAIS on century-to-
millennial time scales - issues we will return to in the next section. We have also demonstrated that the amplitude of these viscous effects is not accurately modeled using standard 1-D viscosity profiles. As an example, both Bamber et al. (2009) and Mitrovica et al. (2009) considered the impact of viscous effects on the elastic fingerprints of WAIS collapse over time scales of centuries and longer by adopting 1-D Earth models very similar to MVE1D. We conclude that their calculations significantly underestimated the impact of viscous relaxation in the near field of the sea-level fingerprints. We explore the resulting effects of this viscous deformation on GMSL predictions in the following section.

2.3 Global Mean Sea Level Predictions of WAIS Collapse

On the basis of the viscous affects described above, the rate and magnitude of the GMSL rise due to post-glacial rebound of West Antarctica following WAIS collapse requires reappraisal. To compute GMSL, we calculate the total volume of meltwater leaving West Antarctica and divide it by the area of the ocean, excluding areas originally covered by WAIS. Based on the results from the previous section, one would expect an amplified GMSL signal relative to previous estimates of GMSL associated with WAIS collapse. This is due to the expected additional contribution to GMSL associated with meltwater being displaced by the uplift of exposed, marine-based sectors (Mitrovica et al., 2009; Gomez et al., 2010). How large will this signal be, and how will it affect our estimation of the potential contribution of WAIS collapse to GMSL?

In this section, we consider an Earth model (MVE3D*) that is identical to MVE3D with the exception that in the conversion of temperature variations to viscosity, the Earth model MVE3D* is constrained to have a spherically averaged depth profile of viscosity in the upper mantle of $10^{20}$ Pa s. We chose this value in order to satisfy additional constraints on West Antarctic mantle viscosity from GNSS observations (the robustness of these constraints is discussed in Chapter 4). With this
choice of background upper mantle viscosity, the radially averaged viscosity from the base of the
lithosphere to a depth of 400 km beneath West Antarctica varies from $6 \times 10^{18}$ Pa s to $2 \times 10^{20}$ Pa s
(Figure 2.3). The average viscosities in the vicinity of the Amundsen Sea Embayment ($\sim 10^{19}$ Pa s)
and in the southern sector of the Antarctic Peninsula ($10^{20}$ Pa s) are consistent with inferences based
on analyses of GNSS observations at sites within these regions (Zhao et al., 2017; Barletta et al.,
2018, Figure 2.3).

The spatio-temporal history of ice cover we consider in this section, (WA2), is the model de-
scribed by Bamber et al. (2009, see their Figure S2), where predominantly marine-based sectors of
the WAIS collapsed (Figure 2.4). Note that the grounded, marine-based sectors of WAIS coincide
with low viscosity areas in MVE3D* (compare Figures 2.4 and Figures 2.3). We will consider an in-
stantaneous deglaciation and track the sea-level changes for a total duration of 10 kyr. Although this
is not a realistic melt scenario, our goal is to quantify both the magnitude of the GMSL signal arising
due to viscous uplift of marine-based sectors of West Antarctica and the time scale over which
this uplift occurs after unloading has ceased.

### 2.3.1 WAIS collapse GMSL simulation: Results & Discussion

For reference, we first reproduce the results from the Bamber et al. (2009) study, who implemented
a linear collapse of WAIS (WA2) over 500 years, and tracked the subsequent GMSL changes over
the following 10 kyr. The GMSL rise computed assuming no uplift of the exposed marine sectors of
West Antarctica is $\sim 3.2$ m. The amplitude of rebound calculated using two 1-D models of mantle
viscosity structure (with a lithospheric thickness of 100 km, an upper mantle viscosity of either $5 \times$
$10^{20}$ Pa s or $10^{21}$ Pa s, and a lower mantle viscosity of $10^{22}$ Pa s) is small. An elastic contribution adds
$\sim 0.05$ m to GMSL and a viscous contribution increases slowly from $\sim 0.05$ m in 2 kyr to $\sim 0.3$–
$0.4$ m after 10 kyr (Bamber et al., 2009, Figure 2.5, shaded region).

As discussed above, we expect increased viscoelastic uplift of marine based sectors when the
Figure 2.3: 3-D viscoelastic Earth model used in Section 2.3. Mean viscosity from the base of the lithosphere to 400 km depth across the Antarctic and Southern Ocean. The symbols indicate the location of GPS sites in inferences of mantle viscosity by Zhao et al. (2017) in the southern Antarctic Peninsula (squares) and Barletta et al. (2018) in the Amundsen Sea Embayment (triangles).
Figure 2.4: WA2 melt geometry. The black line shows the perimeter of the melt zones as described by Bamber et al. (2009) relative to Antarctic bedrock topography from ETOPO2 with ice thickness subtracted. This perimeter does not include peripheral regions of land-based ice mass loss that are also included in the melt model.
Figure 2.5: Time series of GMSL after WAIS collapse. Evolution of GMSL for simulations of sea-level change driven by WAIS collapse. Black line – prediction based on the 3-D viscoelastic Earth model MVE3D* (Figure 2.3) and an instantaneous deglaciation, WA2. Shaded region is the range of GMSL contribution reproduced from Bamber et al. (2009) for the case of a 500 yr collapse of WAIS. Figure is adapted from Pan et al. (2021).
region’s complex 3-D mantle viscosity is considered. Figure 2.6 shows snapshots of the sea-level change in the West Antarctic computed by the MVE3D* model at 0, 500, 2000, and 4000 years in our simulation following an instantaneous deglaciation of WA12. The sea-level fall (and viscous uplift) occurs in regions of West Antarctica that correspond to the zones of ice melt (Figure 2.6b-d). Due to the mantle’s low viscosity and thin lithosphere, the magnitude of the predicted sea-level fall in West Antarctica increases rapidly to ∼600 m in the first 2 kyr (Figure 2.6c), and approximately plateaus at ∼650 m by 4 kyr (Figure 2.6d).

The resulting expulsion of water into the open ocean increases the GMSL rise associated with the collapse scenario, with a final value at 10 kyr of 4.2 m, which is ∼30% larger than the routinely cited upper bound of 3.3 m (Figure 2.5, black line). In accordance with the timing seen in Figure 2.6, this increase occurs rapidly, exceeding 4 m only ∼1 kyr after ice collapse, and achieving 90% of the viscous contribution to GMSL in ∼2 kyr. We note that while the total GMSL rise associated with WAIS collapse will be higher if additional melting also occurs from land-based ice located beyond the periphery of marine-based sectors that are included in our melt model, the water outflow mechanism described here is not relevant for land-based ice.

2.3.2 WAIS collapse GMSL simulation: Summary and Next Steps

Our results demonstrate that the GMSL rise following the full collapse of marine-based sectors of WAIS is 4.2 m, 30% higher than previously thought. The additional contribution of ∼1 m to this signal is from water flux out of West Antarctica due to post-glacial rebound of the seafloor, and will be largely established within 1–2 kyr of the end of the melt event. The water flux explored in this section will also impact site-specific relative sea-level histories, which are used to tackle questions about ice sheet stability in prior warm periods. Even if a full collapse of WAIS marine-based sectors did not occur, this mechanism will still be important for accurately establishing GMSL rise. In this case, the magnitude and rate of water expulsion will depend on the detailed geometry of marine
Figure 2.6: Post-glacial sea-level change after WAIS collapse. Snapshots of predicted sea-level changes (a-d) 0, 500, 2000 and 4000 yrs after an instantaneous collapse of marine-based sectors of WAIS. Calculations are performed using the 3-D viscoelastic Earth model MVE3D*. Figure is adapted from Pan et al. (2021).
exposure and local viscosity structure.

2.4 **Far-Field Sea Level**

In this section, we investigate the effect of lateral variations in mantle viscosity on predictions of far-field sea-level changes associated with protracted WAIS mass loss over the entire duration of a model interglacial. As discussed earlier, GMSL during prior interglacials is often estimated using the geological record of sea level in the far field. In order to interpret these sea-level records, one must make a correction for the GIA signal associated with ice age cycles. A common approach taken to make this correction is to use a calculation in which GMSL across the interglacial period is assumed to be equal to the present-day value (e.g., Raymo & Mitrovica, 2012; Dutton & Lambeck, 2012; O’Leary et al., 2013; Chen et al., 2014; Polyak et al., 2018). Any residual sea-level signal in the geological record is then attributed to “excess melt” (i.e., melting of ice sheets and glaciers beyond their present-day state) during the interglacial of interest.

The above approach, however, neglects any GIA effects arising due to the excess melt itself. For example, Dutton & Lambeck (2012) used MIS 5e coral records from Western Australia and the Seychelles, and assumed that the residual signals at the two sites define lower and upper bounds on peak GMSL, respectively. This analysis yielded a peak estimate of 5.5–9.0 m for the Last Interglacial. Hay et al. (2014) subsequently revised this value to 5.5–7.5 m by showing that GIA effects resulting from excess melting of either the Greenland or West Antarctic Ice Sheets would have produced a sea-level rise at the Seychelles 15% higher than the GMSL change. Although several studies estimating GMSL have recognized that the excess melting signal would introduce geographic sea-level variability, and that there can be substantial differences between predictions based on instantaneously melting this excess ice or a more physically realistic ice-sheet collapse, these studies generally assume that any viscous response may be estimated using 1-D models of mantle viscosity (Raymo & Mitrovica,
In this section, we test the accuracy of using a 1-D mantle viscosity model to correct sea-level signals in the far field generated by the excess melt from a time-varying model of ice sheet collapse (Gomez et al., 2015). We will compare the far-field residual sea-level signal (i.e., the far-field sea-level prediction minus the predicted GMSL component of sea-level rise) computed using this ice model in the presence of either a 1-D or 3-D mantle viscosity structure. We note that the impact of lateral variations in viscosity on the GIA signal during an interglacial associated with ice mass flux in the prior glacial cycle – which we are not considering – has recently been explored by Austermann et al. (2021). Our goal is to quantify the level of inaccuracy introduced by modeling the response to excess melting using 1-D Earth models.

Figure 2.7 summarizes the ice sheet history we adopt in this section (WA3). The history is adapted from a coupled ice sheet-sea level simulation of future WAIS melting described in Gomez et al. (2015). In order to simulate the sea-level response to a protracted collapse of WAIS, we have linearly scaled the timing of melt (the simulation was originally run for a model time of 600 years) so that the collapse takes place over 6000 years. The WA3 ice mass loss rate is muted for the first 1 kyr, but subsequently increases, with an approximately linear melt signal until $1.9 \times 10^6$ Gt of ice has melted by 6 kyr (see Figure 2.7a). Using present-day bedrock topography, this maps into a volume of ice above flotation of $\sim 3$ m. Snapshots of ice height at the beginning, middle, and end of the simulation are provided in Figures 2.7b-d. The simulation ends with a collapse of nearly all marine-based sectors of WAIS, and a marginal increase in ice volume within the EAIS. For our purposes, this melt model serves as a reasonable model for a protracted collapse of WAIS in which some (but not all) marine-based sectors are exposed by the retreating ice sheet during an interglacial.

We consider two 3-D Earth models. The first is based on the SL2013sv seismic tomography model in the upper mantle and SEMUCB-WM1 in the lower mantle, linearly blended together over the depth range 300-400 km (Schaeffer & Lebedev, 2013; French & Romanowicz, 2014).
Figure 2.7: Model of protracted WAIS collapse. (a) Ice mass lost (Gt) from the ice melting scenario WA3. (b-d) Snapshots of the thickness of the Antarctic Ice Sheet (km) at the start (b), middle (c), and end (d) of the 6 kyr ice melt history. The red contour in frames (c) and (d) shows the extent of the ice sheet at the start of the simulation.
conversion of upper mantle shear wave velocity anomalies into lateral variations in temperature and viscosity uses parameterizations derived from recent mineral physics experiments on the impact of anelasticity on seismic wave speed (Yamauchi & Takei, 2016). Anelasticity parameters are calibrated using constraints that include seismic attenuation measurements and the thermal structure of oceanic lithosphere (see Richards et al., 2020, for details). The thickness of the elastic lithosphere is taken to be the depth of the 1175°C isotherm, a value that accords with seismological observations in oceanic lithosphere and yields variations from 0 km along mid-ocean ridges up to 350 km in the thickest portions of cratons (Hoggard et al., 2020, Figure 2.8a). This calibration procedure yields a viscosity field which varies laterally by three orders of magnitude in the upper mantle (Figure 2.8b). We label this model as $M_{3D-A}$.

To illustrate the sensitivity of sea-level modeling results to the choice of Earth model, we also present calculations using a 3-D model constructed in earlier work focusing on the Antarctic near field (Hay et al., 2017, Section 2.2). The global lithospheric thickness variations are shown in Figure 2.8c. The viscosity field of this model, which varies laterally by 5 orders of magnitude in the upper mantle, is shown in Figure 2.8d. This model will be referred to as $M_{3D-B}$.

The lithospheric thickness of both models described above is scaled to globally averaged value of 96 km, and the lateral variations in structure are superimposed on 1-D (i.e., depth-varying) viscosity of $5 \times 10^{20}$ Pa s in the upper mantle (shallower than 670 km) and $5 \times 10^{21}$ Pa s in the lower mantle.

In addition, we consider results based on two 1-D Earth models. The first is identical to the spherically averaged profile of the 3-D Earth models (i.e., elastic lithosphere of thickness 96 km, and upper and lower mantle viscosity of $5 \times 10^{20}$ Pa s and $5 \times 10^{21}$ Pa s, respectively). This model will be termed $M_{1D}$. The second model is based on a regionally averaged version of $M_{3D-A}$, where the averaging encompasses the entire West Antarctic and extends over the entire upper mantle below the lithosphere and the lower mantle. This model (henceforth $M_{1D-ANT}$) has a lithospheric thickness of 60 km, and upper and lower mantle viscosities of $1 \times 10^{20}$ Pa s and $3 \times 10^{21}$ Pa s, respectively.

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Figure 2.8: 3-D Viscoelastic Earth Models Used in Section 2.4. (a) Lithospheric thickness (km), and average upper mantle viscosity variations in the 3-D viscoelastic Earth model $M_{3D-A}$ described in the text. Panel (b) depicts the logarithm of depth-averaged upper mantle viscosity variations relative to a background upper mantle viscosity of $5 \times 10^{20}$ Pa s, \( \log(\tau_{3D}/\tau_{ld}) \). (c-d) As in (a-b), but for Earth model $M_{3D-B}$. Note the difference in scale between (b) and (d).
These two Earth models allow us to compare predictions generated using either of the 3-D Earth models to calculations based on a 1-D model that is within the class of models inferred in independent analyses of GIA data from cratonic regions ($M_{1D}$; Lambeck et al., 1998; Mitrovica & Forte, 2004) or a 1-D model tuned to structure beneath the zone of excess ice melt, the West Antarctic ($M_{1D-ANT}$).

2.4.1 Protracted WAIS Collapse Simulations: Results & Discussion

Figure 2.9a shows a prediction of the sea-level change at the end of the 6000-year simulation based on the 3-D Earth model $M_{3D-A}$, while Figures 2.9b,c are maps of the difference between this prediction and the 1-D Earth models $M_{1D}$ and $M_{1D-ANT}$, respectively. Figures 2.9d-f are analogous predictions based on the 3-D Earth model $M_{3D-B}$.

The general features of the 3-D Earth model predictions (Figures 2.9a and d) are the same:

1. a major drawdown in sea level beneath WAIS (this signal is obscured by the continent mask) extending out to southern South America and New Zealand that is driven by long wavelength post-glacial elastic uplift and gravitational migration of water away from the collapsed ice sheet;

2. sea-level rise just offshore of the West Antarctic that punctuates zone (1) and reflects the development of a region of viscous crustal subsidence at the periphery of the ice sheet;

3. a so-called “quadrential” signature in the far-field that is driven by rotational effects (Milne & Mitrovica, 1998). Melting from WAIS acts to reorient the south pole toward West Antarctica and the north pole toward Eurasia (Gomez et al., 2010), and this drives sea-level rise in North America and the southern Indian Ocean, and sea-level fall in the southwest Pacific Ocean and Eurasia. The signal in the southwest Pacific is dominated by near-field effects,
Figure 2.9: Sea-Level Predictions of Protracted WAIS collapse. 

(a) Sea-level change (m) at the end of the 6000-year simulation predicted using the ice history WA3 and the viscoelastic Earth model \( M_{3D-A} \). (b,c) The difference in the sea level prediction (a) and predictions based on the 1-D Earth models \( M_{1D} \) and \( M_{1D-ANT} \), respectively (i.e., 3-D prediction minus 1-D prediction). (d-f) Analogous to (a-c) with the exception that the 3-D Earth model \( M_{3D-B} \) is adopted. The black triangles in frame (a) denote the locations of 6 sites considered in the sea level time series of Figure 2.10.
while in Eurasia it is largely masked by continents but is evident in the eastern Mediterranean and Black Sea; and

4. also in the far field, a crustal tilting signal near continental shorelines (downward towards oceans) due to ocean loading that is superimposed on the larger scale quadrential geometry.

The well-developed peripheral subsidence signal is a consequence of the low upper mantle viscosity in the vicinity of the West Antarctic of both 3-D Earth models; this region is more extensive in the case of the prediction based on Earth model $M_{3D-B}$ relative to $M_{3D-A}$, because the upper mantle viscosity is lower in the former (Figure 2.8). The peripheral subsidence signal is also predicted in the case of the model $M_{1D-ANT}$ but not $M_{1D}$ (these fields are not shown).

The remaining frames in Figure 2.9 indicate that the impact of introducing 3-D Earth structure is greater in the case of predictions generated with Earth model $M_{3D-B}$ than $M_{3D-A}$. Once again, this reflects the significantly higher amplitude lateral variability of mantle viscosity in the former relative to the latter (Figures 2.8b, d). In the near field, the discrepancy between these 3-D model simulations and the 1-D predictions is lower in the case of the 1-D model $M_{1D-ANT}$ that was tuned to viscosity structure beneath the West Antarctic (compare Figures 2.9b to 2.9c and 2.9e to 2.9f), which is as one would expect. In contrast, in the far field, the discrepancy is lower when the standard GIA-based inference of the mantle viscosity profile based on observations from cratons, $M_{1D}$ is adopted. Recall that the model $M_{1D}$ was adopted as the spherically averaged viscosity profile for both 3-D Earth models. In the case of the model $M_{3D-A}$, lateral variations in viscosity structure perturb the prediction of far field sea level change at the end of the simulation by an amount within the range $-0.2$ to $0.3$ m (Figure 2.9b). The analogous range in the case of model $M_{3D-B}$ is $-0.5$ to $0.3$ m (Figure 2.9e).

Next, in Figure 2.10, we show predicted times series of the difference in sea-level predicted using 1-D and 3-D Earth models across the 6 kyr simulations. The six sites have commonly been consid-
Figure 2.10: Local Sea-Level Predictions of Protracted WAIS collapse. Difference in sea-level change predicted using 3-D and 1-D Earth models across the 6 kyr simulation at six sites (as labelled at top of each frame; see Figure 2.9 for locations). Each line indicates a different 3-D/1-D Earth model pair, as specified in the top left frame of the figure.

In LIG analyses (e.g., Barlow et al., 2018): San Salvador Island, Bahamas; Bristol Channel, UK; Bab-el-Mandeb, Red Sea; La Digue Island, Seychelles; Cape Range, Western Australia; and Eyre Peninsula, Southern Australia. The location of these six sites is shown in Figure 2.9a. As one would expect on the basis of Figure 2.9, the curves involving predictions generated using the Earth model $M_{3D-B}$ tend to show larger magnitude differences with the 1-D model simulations than the predictions based on $M_{3D-A}$ (red versus blue lines). The primary exceptions are the predictions at the site Bab-el-Mandeb in the Red Sea.

However, the curves in Figure 2.10 allow us to consider a subtler question, namely: In modeling predictions of far-field sea level change in response to melting from the West Antarctic Ice Sheet using 1-D Earth models, is it more accurate to adopt a 1-D model that captures viscoelastic structure beneath the West Antarctic, as model $M_{1D-ANT}$ does (dotted lines), or a model that captures the
global-scale spherical average of the 3-D structure, as $M_{1D}$ does (solid lines)? Crawford et al. (2018) have shown that the sensitivity of sea level predictions in the far field of melt zones to Earth structure is broad and extends along a path from the site of the prediction to the site of the melt, with peak sensitivity close to the two end points. That is, the far-field sea level prediction will not only be sensitive to viscosity structure local to the site of interest, which may or may not be close to the global average viscosity structure, but also to the dynamics of the response close to the West Antarctic.

This complex sensitivity is reflected in Figure 2.10. In some cases, the ability (or inability) of the two 1-D Earth models to capture the prediction based on the 3-D Earth model is comparable (Figures 2.10c,f) and in other cases it depends on the specific 3-D and 1-D Earth models being considered (Figures 2.10b,e). In any event, we conclude from Figures 2.9 and 2.10 that the neglect of 3-D structure in modeling the far-field sea level response to collapse of the West Antarctic Ice Sheet is of order 0.3 m or less, which represents 10% of the GMSL rise associated with the collapse. However, we note that this signal will compound the error introduced by neglecting lateral variations in Earth structure in predictions of interglacial sea level change driven by the preceding glacial cycle (Austermann et al., 2021).

2.4.2 Protracted WAIS collapse far-field simulation: Summary

We have presented predictions of the sea-level signal associated with a model of protracted interglacial collapse of WAIS using 3-D viscoelastic Earth models inferred from seismic tomography fields. We have found that a prediction of the viscoelastic sea-level signal (i.e., the complete deformational, gravitational and rotational signal driven by the ice and ocean loading) based on 1-D Earth models can differ from the signal predicted with 3-D Earth models, even in the far field of the collapsing ice sheet – the region that has served as the focus of this section.
2.5 Conclusions

We have found that 3-D mantle viscosity structure will lead to significant differences in modeled sea-level changes arising due to WAIS melt. Our results demonstrate:

1. Sea-level fingerprints calculated using 1-D elastic models significantly underestimate the impact of near field viscous deformation;

2. WAIS’s potential contribution to GMSL given a full collapse is ~30% higher than previously thought;

3. While the contribution from viscous effects is smaller in the far field, it is a systematic effect rather than random, and therefore should be included in the uncertainty of estimates of GMSL during prior interglacials.

The strong, and geographically variable near-field signal of viscous effects associated with the 3-D mantle viscosity models (e.g., Figure 2.2, Figure 2.6) suggests that coupled ice sheet-sea level models of WAIS evolution (e.g., Gomez et al., 2015), should incorporate this complexity in viscoelastic Earth structure. The water expulsion mechanism explored in sections 2.3 and 2.4 will also be important for the collapse of marine-based sectors of the East Antarctic Ice Sheet if they sit atop low viscosity shallow mantle.

Our results also suggest that analyses of interglacial sea-level highstands should consider the impact of 3-D viscoelastic Earth structure, even at far-field sites. This structure introduces geographic variability in the sea-level response to WAIS of ~0.3 m, 10% of the total GMSL change of the modeled ice sheet collapse. Of course, the accuracy of predictions of sea-level change associated with melting of other ice sheets (e.g., Greenland) and glaciers during interglacial periods will also be impacted by lateral structure in lithospheric thickness and mantle viscosity. We therefore emphasize that any analyses of relative sea-level markers consider the signal discussed here in addition to other
sources of uncertainty. These include errors in the uranium-thorium dating of corals (Stirling & Andersen, 2009) and uncertainty in sea-level marker indicative range (Hibbert et al., 2016), as well as other geodynamical signals that affect sea level, including (for the case of the LIG) ongoing deformational effects associated with the largely unconstrained MIS6 ice sheets (e.g., Dusterhus et al., 2016), regional tectonic uplift, and dynamic topography (Austermann et al., 2017).

Our findings also have implications for other, non-interglacial, time periods. For example, our results suggest that inferences of Antarctic ice volumes since Last Glacial Maximum based, in part, on analyses of sea-level data sets from the region and 1-D Earth models akin to MVE1D (e.g., Whitehouse et al., 2012) may need to be re-evaluated. Finally, our results from the sea-level modeling performed in this chapter suggest that it is also important to include the solid Earth’s response to modern day melting of WAIS. We focus on this signal and how it might be biasing estimates of modern mass loss in geodetic measurements in the next chapter.
Effects of 3-D Earth structure on modern Antarctic geodetic measurements

3.1 Introduction

The Earth’s climate is changing, with warming that will lead to an increase in ice melting and a corresponding global mean sea-level rise. Projecting the variability of this sea-level rise requires an accurate estimate of the size and geometry of the melt water sources. One way to quantify this ice mass flux is to measure the associated deformation of the Earth. When ice melting occurs, the resulting mass redistribution (ice plus ocean) perturbs the Earth’s gravitational field and solid surface. These effects can be surveyed using a suite of geodetic methods, including, for example, satellite-based gravity observations and the Global Navigation Satellite System (GNSS). Properly analyzing this data, however, requires making assumptions about the timescale of Earth’s rheological response to the loading. On shorter time scales (decades-centuries) the Earth’s response is often assumed to be primarily elastic, whereas on longer (e.g., ice-age) time scales, the Earth’s response is clearly viscoelastic. In this chapter, we use the West Antarctic as a case study to investigate the transition between these two regimes. In particular, we adopt a series of ice-melting scenarios extending over the past 25 years and projecting into the next half century and incorporate 3-D viscoelastic Earth structure in Glacial Isostatic Adjustment (GIA) simulations to explore the timescale over which viscous forces become significant in driving gravity perturbations and 3-D crustal motions in the South Pole re-
Geodetic measurements have provided important constraints on ice melt. For example, data from the Gravity Recovery and Climate Experiment (GRACE)—including the primary satellite mission operational from March, 2002 to October, 2017 and the current follow on mission GRACE-FO (launched May, 2018)—is used to map geoid anomalies into surface mass changes (Wahr et al., 1998). A published analysis of GRACE data extending over the period April, 2002, to January, 2009, has argued that the West Antarctic lost an average of $132 \pm 26$ Gt of ice per year during this period (Chen et al., 2009). This ice melt signal, particularly in the Amundsen Sea sector, appears to have accelerated since 2002 (e.g., Velicogna et al., 2014; Shepherd et al., 2018). However, substantial uncertainty in these estimates comes from the contribution to mass changes from movement of the solid Earth associated with past loading, including over the last glacial cycle (i.e. GIA).

GNSS measurements in Antarctica are sensitive to changes in ice mass cover on both modern (annual to century) and GIA (thousands of years) timescales. Various approaches have been used to separate the modern signal from the older GIA component of the crustal deformation. Within these analyses, the crustal response to modern melting has been computed using either purely elastic Earth models (e.g., Bevis et al., 2009; Thomas et al., 2011; Argus et al., 2014; Martín-Español et al., 2016; Caron et al., 2018; Schumacher et al., 2018), or by augmenting these calculations to include viscous relaxation to explain anomalously rapid uplift rates in specific areas of the West Antarctic (e.g., Nield et al., 2014; Zhao et al., 2017; Barletta et al., 2018). With few exceptions (e.g., Argus et al., 2014; Zhao et al., 2017), only the vertical component of GNSS measurements has been considered in such analyses.

Although geodetic data have primarily been interpreted through the lens of elastic rheologies, Antarctica has a complex geologic setting that is not well described by purely elastic models. The East Antarctic is characterized by an old, cold craton with a thick lithosphere in excess of 200 km (Morelli & Danesi, 2004; Heesz et al., 2013), while the West Antarctic is dominated by a failed rift
system (Wörner, 1999) that has thinned the lithosphere to \(<100\) km (An et al., 2015a; Heeszel et al., 2016). Thermal interpretations of seismic tomographic images (Ritzwoller et al., 2001; Morelli & Danesi, 2004; Hansen et al., 2014; Lloyd et al., 2015; Heeszel et al., 2016) suggest that mantle viscosities below parts of West Antarctica are significantly less than both the regional Antarctic and global average, with viscosities as low as \(\sim 10^{18}\) Pa s under volcanic Marie Byrd Land (Kaufmann et al., 2005; Hay et al., 2017). While uncertainty remains in mapping seismic wave speed anomalies to viscosity structure, these estimates are consistent with inferences of low asthenospheric mantle viscosity based on analyses of GNSS-determined crustal uplift rates in the Antarctic Peninsula (Nield et al., 2014; Zhao et al., 2017) and Amundsen Sea Embayment region (Barletta et al., 2018). Analysis of xenoliths collected from Marie Byrd Land suggest that local viscosities in the shallow mantle below this area may be as low as \(10^{16}\) Pa s (Chatzaras et al., 2016, Krueckenber, pers. comm.).

Antarctica’s complicated 3-D Earth structure has already been studied in the context of GIA in response to the last ice age (Kaufmann et al., 2005; A et al., 2013; van der Wal et al., 2015; Gomez et al., 2015, 2018). However, such low viscosities indicate that the Maxwell time is \textit{less than a year} (See Chapter 1 for more information on Maxwell rheologies), suggesting that viscous effects will play a large role even in the response to modern melting over the West Antarctic. Previous studies have considered the impact of viscous relaxation on the response to modern melting at other sites characterized by shallow mantle viscosities of order \(10^{18}\) Pa s. These include examinations of the crustal response to melting in the Antarctic Peninsula (Nield et al., 2014; Zhao et al., 2017), the Amundsen Sea Embayment (Barletta et al., 2018), Patagonia (Richter et al., 2016), Iceland (Auriac et al., 2013), eastern Greenland (Khan et al., 2016), and Alaska (Taminiau et al., 2005; James et al., 2009). This issue also motivated the study of Hay et al. (2017), who explored the impact of 3-D low-viscosity structure beneath the West Antarctic on the sea-level fingerprints of modern melting and concluded that peak sea-level fall in the West Antarctic associated with a local melting event of
duration 25 years will increase by 25% relative to a purely elastic simulation.

We have two goals in the present study. First, we seek to estimate the contribution of 3-D variations in mantle viscosity beneath West Antarctica to predictions of the gravitational field and crustal deformation response to ice mass flux over the past 25 years, and projected forward over the next half-century. Second, given the significant technical requirements involved in treating 3-D viscoelastic Earth structure in such loading calculations, we explore whether 1-D viscosity models can be found that provide a reasonable approximation of these 3-D effects.

3.2 Methods

To predict the Earth’s response to a change in ice loading, one must account, in a gravitationally self-consistent manner, for the flux of water into and out of the ocean basins. In the present study we adopt the sea-level theory described by Gomez et al. (2010). This theory assumes the initial topography is known, and it incorporates effects associated with time varying shoreline geometry (Johnston, 1993; Milne & Mitrovica, 1998; Mitrovica & Milne, 2003) and load-induced Earth rotation variations (Mitrovica et al., 2005).

In studies adopting 1-D viscoelastic Earth models, loading calculations inherent to the sea-level theory are usually based on viscoelastic Love number theory (Peltier, 1974). The incorporation of 3-D Earth structure requires a more complex treatment of load-induced perturbations to the gravitational field and crust, and in this regard we adopt the finite volume treatment of Latychev et al. (2005). With recent improvements (e.g., Hay et al., 2017; Gomez et al., 2018), we extend the treatment to include a laterally varying resolution in the computational grid to accommodate available regional models of higher spatial resolution. The global model we adopt is characterized by an average spatial (horizontal and vertical) resolution of 12 km to the base of the crust, 25 km to a depth of 220 km, and 50 km to the core mantle boundary. The regional model, which asymmetrically covers
the Antarctic plate spatially, extends to depths of approximately 350 km, and it is characterized by an average spatial (horizontal and vertical) resolution of 5 km to the base of the crust, 12 km to a depth of 220 km, and 25 km to a depth of 350 km. We use the finite volume software and gridding scheme in all calculations presented in this Chapter, including those in which Earth structure varies only with depth.

Our calculations require two inputs: a model for the Earth’s viscoelastic structure, and the space-time history of ice cover. We describe these inputs below.

3.2.1 Earth Models

We consider a suite of Earth models in this study. All Earth models assume a Maxwell viscoelastic mantle rheology that is compressible in the elastic limit. The elastic and density structure of the models is provided by the 1-D seismic model PREM (Dziewonski and Anderson, 1981).

The first Earth model is purely elastic ($M_{EL}$). In this case, the computed perturbations to the solid surface and gravitational field do not depend on the duration of the simulation, only on the net change in ice volume between the beginning and end of the calculation. The second model, $M_{1D}$, has viscoelastic structure that varies with depth alone. In particular, a 96 km thick lithosphere overlies uniform upper and lower mantle viscosities of $5 \times 10^{20}$ Pa s and $5 \times 10^{21}$ Pa s, respectively. This viscosity profile is shown in Figure 3.1A (black curve).

The third model, $M_{3D}$, is defined by an elastic lithosphere of variable thickness (Figure 3.1B) and 3-D mantle viscosity structure (Figure 3.1C). Globally, we adopt the spatially varying lithospheric thickness model of Conrad & Lithgow-Bertelloni (2006), but within the Antarctic plate we use the higher resolution lithospheric model of An et al. (2015a). The full model is scaled to yield a global mean lithospheric thickness of 96 km. The 3-D mantle viscosity structure of $M_{3D}$ is built from three different seismic tomography studies that span global to regional (Antarctic) scale (Ritsema et al., 2011; Heeszel et al., 2016; An et al., 2015b). The model, which is described in full detail in
Figure 3.1: Earth Models. A) Two 1-D Earth models described in the text. The black line is the model $M_{1D}$ which is in the class of viscosity profiles favored in most analyses of GIA data. The red line is the regional, depth-varying viscosity structure beneath Marie Byrd Land, used to construct model $M_{MBL}$. The gray line marks the boundary between the upper and lower mantle and the dotted black and red lines mark the base of the elastic lithosphere in the $M_{1D}$ and $M_{MBL}$ models, respectively. B) Lithospheric thickness (km), and C) mantle viscosity variation at 125 km depth of the Earth model $M_{3D}$. The latter figure represents the logarithm of mantle viscosity variations relative to the global background, 1-D viscoelastic model (i.e., $\log(\nu_{3D}/\nu_{1D})$). Areas in white in (C) lie within the elastic lithosphere and the dashed circle over Marie Byrd Land represents the region over which the average of viscosity, with depth, is used to construct the 1-D model $M_{MBL}$. Frames (B) and (C) have been modified from Hay et al. (2017).
Hay et al. (2017) and Chapter 1, involves a free parameter that controls the level of lateral variability in mantle viscosity. In our standard run, this parameter is chosen such that the Earth model is characterized by a 5 order of magnitude (peak to peak) variation in viscosity in the asthenosphere beneath East and West Antarctica, where the latter region has a minimum viscosity of $\sim 10^{18}$ Pa s.

To test the sensitivity of the results to this choice, we consider two other values of the free parameter that yield minimum viscosities of $10^{17}$ Pa s and $10^{19}$ Pa s beneath West Antarctica (models $M_{3D-L}$ and $M_{3D-H}$, respectively). Globally, the 3-D Earth models are constrained to have a spherically averaged depth profile that matches the model $M_{1D}$. The results are relatively insensitive to this choice of spherically averaged viscosity given that we tune our model free parameter to yield specific lower bounds on viscosity ($10^{17}, 10^{18}$ or $10^{19}$ Pa s) beneath West Antarctica.

Finally, the fourth model, $M_{MRL}$, is a second 1-D viscoelastic model constructed from the regional mantle viscosity structure beneath Marie Byrd Land in model $M_{3D}$, the area overlying the asthenospheric low viscosity zone in the $M_{3D}$ model. The model has a 71 km thick lithosphere and a highly variable mantle viscosity profile (Figure 3.1A, red curve) constructed by taking the cylindrical average of viscosity with depth from the surface to the core mantle boundary, using a 555 km radius circle centered on 79°S, 124°W (Figure 3.1C).

3.2.2 Ice Models

The normalized, uniform melting or full collapse scenario generally used to calculate sea-level fingerprints, including in the study of Hay et al. (2017), does not capture the complex geometry or magnitude of recent mass loss in Antarctica. Ice mass redistribution occurs primarily via ice streams within the ice sheet and via calving and melting at the periphery of the ice sheet (Bennett, 2003; Shepherd et al., 2018). Antarctica contributed $0.27 \pm 0.11$ mm/yr equivalent sea-level rise over the period from 1993-2010 and this rate has increased considerably since 2010 (Harig & Simons, 2015; Martín-Español et al., 2016; Shepherd et al., 2018).
Figure 3.2: Modern ice thickness changes used to construct ice histories. A) Mean annual ice thickness change inferred from GRACE satellite gravity observations 2003-2014 (Harig & Simons, 2015). B) The mean annual ice height change inferred by Martín-Espiñol et al. (2016) for the period 2003-2013. C) GMSL times series for the ice histories described in the text. Main frame: I-MEG. Inset: I-GR (Orange), I-ME (black).
In this Chapter, we adopt a suite of ice melt histories over the Antarctic. The first two models extend over 25 years (1992-2017), consistent with the period over which the modern Antarctic Ice Sheet has been significantly out of mass balance, i.e., 1992 to present day (Shepherd et al., 2018). The first ice history, I-GR, is based on geographically variable melt rates inferred from GRACE satellite gravity data collected from 2003-2014 (Harig & Simons, 2015). Specifically, the geometry of the ice melt is defined by the mean annual change in ice thickness over that time period (Figure 3.2A), and we apply a constant melt volume change equivalent to 0.26 mm/yr of global mean sea-level rise over the entire 25-year simulation (Figure 3.2C, inset; Harig & Simons, 2015).

Our second ice history, I-ME adopts the full spatiotemporal evolution of the Martín-Español et al. (2016) reconstruction from 2003-2013. (Figure 3.2B shows the mean annual ice thickness change across the entire period.) For the period 1992-2002 we use the 2003 mass flux geometry in the Martín-Español et al. (2016) reconstruction and we follow the integrated, time-varying mass flux inferred by the IMBIE team for this 10-year period (Shepherd et al., 2018, Figure 3.2C, inset). For 2014-2017, the I-ME ice history adopts the 2013 mass flux geometry in the Martín-Español et al. (2016) reconstruction and follows the integrated mass flux inferred by Shepherd et al. (2018) for the same 4-year period (Figure 3.2C, inset).

Finally, our third ice history (I-MEG) is identical to I-ME over the period 1992-2017, but it extends this history for 50 additional years, to 2067. Over this latter period, the geometry of the mass flux is held fixed, and the integrated magnitude of the flux follows the trend predicted by Golledge et al. (2019) in their coupled ice sheet/ice shelf simulations of the Antarctic Ice Sheet over the 21st century (Figure 3.2C).
3.3 Results and Discussion

The Maxwell time associated with the model $M_{1D}$ is of the order of a millennium and, as a consequence, all of the 25-year simulations we performed yielded negligible differences between predictions based on the $M_{1D}$ and $M_{EL}$ Earth models. We therefore omit results generated using the $M_{1D}$ model in the figures discussed below. For the purposes of this study, the $M_{1D}$ model, a standard mantle viscosity profile inferred from GIA data sets, is essentially indistinguishable from a purely elastic Earth model.

In the following sections, we plot predictions based on the model $M_{3D}$ and the difference in predictions based on the pair of models $(M_{3D}, M_{EL})$ and $(M_{3D}, M_{MBL})$. The first of these pairs represents the viscous signal embedded within the 3-D Earth model simulation. The difference in the second pair quantifies the extent to which the regional, 1-D viscosity model $M_{MBL}$ captures the viscous signal within the $M_{3D}$-based simulation.

3.3.1 Geoid rate predictions

Figure 3.3A shows the rate of change in the height of the geoid at the end of the 25-year simulation computed using the $M_{3D}$ model and the I-GR ice history. This rate, which incorporates perturbations associated with both the ice mass flux and the associated adjustment of the solid Earth, has a peak negative value of $\sim-3.5$ mm/yr over the West Antarctic and a peak positive value of 0.6 mm/yr over the East Antarctic. Figures 3.3B and 3.3C show the difference in predictions of the geoid height rate change generated using the 3-D Earth model prediction and the two 1-D models $M_{EL}$ and $M_{MBL}$, respectively. The magnitude of the peak geoid rate over the West Antarctic in Figure 3.3A is 6% smaller ($\sim-0.2$ mm/yr) than the analogous predictions based on the model $M_{EL}$, reflecting an increased compensation of the geoid signal (due to ice mass loss) associated with uplift of the crust due to viscous mantle flow relative to the compensation computed using a purely elas-
Figure 3.3: Impact of viscous relaxation on geoid height due to modern ice mass flux. A) Predicted rate of change of geoid height after 25 years of loading calculated using the ice history I-GR and the Earth 3-D model $M_{3D}$. B-C) Differences between the geoid height rate after 25 years of loading predicted using the 3-D Earth model $M_{3D}$ and the 1-D Earth models B) $M_{EL}$ and C) $M_{MBL}$ (i.e., 3-D prediction minus 1-D prediction). D-F) As in (A-C) except that the calculations are based on the ice history I-ME.

The difference in the peak rate over the West Antarctic predicted using the $M_{MBL}$ and $M_{3D}$ models is less, $\sim 1\%$ (Figure 3.3C), indicating that the 1-D model tuned to the regional viscosity variation beneath the West Antarctic accurately captures the 3-D Earth model prediction.

The results in Figures 3.3D-F are analogous to the top row of the figure, but based on the more spatially resolved ice history I-ME. The peaks in the predicted signal based on the model $M_{3D}$ are more localized, reflecting the geometry of the underlying mass flux (e.g., Figure 3.2B), and the amplitudes are significantly higher. The viscous signal in the geoid rate peaks at 0.37 mm/yr (Fig-
ure 3.3E; 1.3% of the peak in frame D), and, as in the predictions based on the ice history I-GR, this signal is captured to within 0.3% accuracy with the 1-D viscoelastic model $M_{\text{MBL}}$ derived from regional structure below the West Antarctic (Figure 3.3F).

These results indicate that low viscosity structure beneath the West Antarctic has a relatively small impact on predictions of geoid rate, and that analyses of GIA-corrected GRACE measurements over the West Antarctic do not incur significant errors in assuming that modern ice mass loss drives a purely elastic solid Earth response.

### 3.3.2 Crustal deformation rate predictions

Next, we consider predictions of crustal deformation rates computed using ice history I-ME (Figures 3.4, 3.5 - top row, and 3.7-3.9). The vertical component of these rates has served as the primary data set in analyses of GNSS measurements across the Antarctic.

Crustal uplift rates predicted using the 3-D Earth model $M_{3D}$ (Figure 3.4A) are characterized by a peak value of 90 mm/yr at the location of greatest ice mass flux in the I-ME history, near the Amundsen Sea Embayment (Figure 3.2B). The viscous component of the uplift field peaks at 14 mm/yr (Figure 3.4B). The ratio of Figures 3.4B and 3.4A indicates that the viscous signal reaches $\sim 20\%$ of the full calculation in regions where significant uplift rates are predicted (Figure 3.5A). A significant component of this viscous signal is captured in the calculation based on the 1-D, regionally inferred viscoelastic Earth model $M_{\text{MBL}}$; within the zone of pronounced ice melting in the West Antarctic, the discrepancy between predictions based on models $M_{\text{MBL}}$ and $M_{3D}$ (Figure 3.4C) ranges from -5.6 mm/yr to 3.3 mm/yr (compare Figures 3.5A and 3.5B; we return to this point below).

These results demonstrate that adopting elastic Earth models to correct GNSS measurements of vertical crustal rates for the signal due to modern melting in the West Antarctic will underestimate the magnitude of the correction, and thus overestimate the residual contribution from the GIA pro-
Figure 3.4: Impact of viscous relaxation on crustal motion due to modern ice mass flux. A) Predicted vertical crustal rate after 25 years of loading calculated using the ice history I-ME and the Earth model $M_{3D}$. B-C) Differences between the crustal uplift rate after 25 years of loading predicted using the 3-D Earth model $M_{3D}$ and the 1-D Earth models (B) $M_{EL}$, and (C) $M_{MBL}$ (i.e., 3-D prediction minus 1-D prediction). D) Arrows show the horizontal crustal rate after 25 years of loading calculated using the ice history I-ME and the Earth model $M_{3D}$. Scale bar at bottom right. Color contours represent the magnitude of the horizontal rates. (Arrows relate to predictions at sites situated at the tail of the arrow.) E) Vector differences between the horizontal crustal rate after 25 years of loading predicted using the 3-D Earth model $M_{3D}$ and the 1-D Earth model $M_{EL}$ (i.e., 3-D prediction minus 1-D prediction). Contours represent the magnitude of the 3-D prediction minus the magnitude of the 1-D prediction. F) As in (E), except for the 1-D Earth model $M_{MBL}$. 
Figure 3.5: Comparisons between predictions of crustal uplift rates based on 3-D and 1-D Earth models. A) Percent difference between predictions of crustal uplift rates after 25 years based on ice history I-ME (or, equivalently, the I-MEG history) and the Earth models $M_{3D}$ and $M_{1D}$ (3-D – 1-D prediction divided by 3-D prediction; i.e., Figure 3.3B / Figure 3.3A). Results are only shown for sites in which predictions using $M_{3D}$ are greater than 10% of the peak prediction for this model. B) As in (A), except for the 1-D Earth model $M_{1D}$. (C-D) As in (A-B), except for predictions at year 50 of the projected ice history I-MEG. (E-F) As in (A-B), except for predictions at year 75 of the projected ice history I-MEG.
cess. Alternatively, using a GIA corrected field of measured crustal uplift to estimate modern mass flux would overestimate this flux if a purely elastic response is adopted to compute the response to recent melting.

Next, we turn to predictions of horizontal crustal motions predicted at the end of the 25-year I-ME simulations. Figure 3.4D shows the results based on model $M_{3D}$; Figure 3.4E shows the contributions to this field from viscous effects (i.e., the vector difference of predictions based on models $M_{EL}$ and $M_{3D}$), while Figure 3.4F is the difference in predictions based on models $M_{MBL}$ and $M_{3D}$.

Within the zone of crustal uplift (Figure 3.4A), the horizontal rate predictions based on the 3-D Earth model $M_{3D}$ emanate outward from the zone of peak uplift (Figure 3.4D), with peak rates that reach 15 mm/yr. A comparison of this prediction with the viscous signal (Figure 3.4E; note the different scale of the arrows in the two frames) indicates that the outward pattern in this region is dominated by elastic flexure (James & Morgan, 1990). Nevertheless, the viscous signal, which drives horizontal deformation inward toward the areas of melt (James & Morgan, 1990), exceeds 3 mm/yr within the zone of crustal uplift, and remains above 1 mm/yr well outside this region, particularly in oceanic crust to the north (Figure 3.4E). The prediction based on the 1-D regional viscosity model $M_{MBL}$ within the West Antarctic fails to capture the viscous signal in the 3-D simulation (that is, the residuals in Figure 3.4F are of similar magnitude to the viscous signal in Figure 3.4E) and is, in general, comparable in performance to the elastic model as an approximation to the 3-D viscoelastic simulation based on $M_{3D}$.

We next turn to time series of crustal deformation rates as would be measured by GNSS sites in West Antarctica (locations are detailed in Table 3.1; Figure 3.6). Figures 3.7-3.9 track predicted crustal rates at three representative GNSS sites. Rates are computed using a sliding window of 5 years. These sites lie on an east-west arc that spans the zone used to average viscosity in the construction of the 1-D regional model $M_{MBL}$ (compare the location of the three sites in Figure 3.6 and the dashed circle in Figure 3.1C). We show the prediction generated using the 3-D viscoelastic
Table 3.1: GNSS Stations. Station information of 10 West Antarctic GNSS sites (as labeled; see Figure 3.6 for location).

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Earth model $M_{3D}$ (left column; black lines) and the difference in the predictions (right column) based on model $M_{3D}$ and either model $M_{EL}$ (blue lines) or model $M_{MBL}$ (red lines). The figures provide a measure of the progression in the difference over the full 25-year time window between a prediction based on the 3-D viscoelastic model $M_{3D}$ and a prediction that either adopts a purely elastic response of the solid Earth or a viscous response based on the 1-D regionally-inferred viscosity profile.

GNSS site 5 (TOMO) lies closest to the region of highest ice mass flux and both the prediction of crustal uplift and the viscous component of this signal are the largest of the three sites. The viscous component reaches 8.8 mm/yr, 0.3 mm/yr and 0.5 mm/yr for crustal rates of uplift, eastward and northward horizontal deformation, respectively, at the end of the 25-year simulation. The prediction based on the 1-D model tuned to the regional viscosity profile beneath this region of the West Antarctic, $M_{MBL}$, performs well in recovering the uplift rate signal generated with the model $M_{3D}$, but the discrepancy between predictions of horizontal rates between these two models ($M_{MBL}$ and $M_{3D}$) is significantly larger than the viscous signal (i.e., $M_{3D} - M_{EL}$) after 25 years (1.1 mm/yr ver-
Figure 3.6: GNSS Site Locations. The locations of the ten GNSS sites considered here, described in Table 3.1, are superimposed on mean ice height changes from Figure 3.2B.
sus 0.3 mm/yr in the eastward direction, and 1.6 mm/yr versus 0.5 mm/yr in the northward direction). That is, one would incur a greater error using the regional 1-D viscosity model $M_{MBL}$ than a purely elastic model in predicting horizontal crustal rates at this site computed using the 3-D viscoelastic model $M_{3D}$.

The viscous signal at site 3 (CLRK) is of order $\sim$1 mm/yr or less, and simulations based on model $M_{MBL}$ have more success in capturing this component of the $M_{3D}$-based horizontal crustal response. For example, at the end of 25 years, the viscous signal in the $M_{3D}$ response for the three crustal deformation components is 1.1 mm/yr, 0.9 mm/yr and 0.3 mm/yr, respectively, while the analogous predictions for the $M_{MBL}$ simulation (i.e., $M_{3D} - M_{MBL}$) are lower: 0.1 mm/yr, 0.5 mm/yr and 0.2 mm/yr.

Finally, site 4 (SDLY) is closest to the mantle region of lowest viscosity in the Earth model $M_{3D}$ (see Figure 3.1C). As for site 3, the model $M_{MBL}$ is able capture nearly all of the viscous component of crustal uplift at site 4, and a substantial fraction of the component for the horizontal-east rate, associated with the prediction based on the 3-D Earth model $M_{3D}$; however, it does only marginally better than the model $M_{EL}$ in predicting the horizontal-north rate computed using the 3-D viscoelastic model $M_{3D}$.

Clearly, the magnitude of the viscous response in predictions of 3-D crustal rates, and the ability of the 1-D model $M_{MBL}$ to recover this response, will depend on the location of the site relative to both the geometry of the ice mass flux and the detailed variability in viscosity and lithospheric thickness that characterizes any 3-D model prediction. Tables 3.2 & 3.3 explore this issue further by showing predicted vertical and horizontal crustal rates, respectively, at all 10 GNSS sites in Figure 3.6 at the end of the 25-year simulation for model $M_{3D}$ and the differences $M_{3D} - M_{EL}$ and $M_{3D} - M_{MBL}$.
Figure 3.7: Solid surface deformation rates predicted at GNSS site #3 (CLRK) as a function of time using the ice history I-ME. Rows 1-3 show predicted vertical rates, eastward horizontal rates and northward rates as a function of time, respectively, across the 25-year simulation (1992-2017). The first column shows predictions based on the 3-D viscoelastic Earth model $M_{3D}$ (black lines). The second column shows the residual between the following pairs of predictions: Blue - $(M_{3D} \cdot M_{EL})$; Red - $(M_{3D} \cdot M_{MBL})$. All rates are computed with a running time window of 5 years.
Figure 3.8: Deformation rates predicted at GNSS site #4 (SDLY) using the ice history I-ME. As in Figure 3.7, for GNSS site #4 (SDLY).
Figure 3.9: Deformation rates predicted at GNSS site #5 (TOMO) using the ice history I-ME. As in Figure 3.7, for GNSS site #5 (TOMO).
Table 3.2: Modern Vertical Crustal Deformation Rates. Predictions of vertical crustal deformation rates at 10 GNSS sites (Table 3.1; see Figure 3.6 for location) at year 25 of the ice history I-ME. We show predictions based on the 3-D viscoelastic Earth model \( B \), the viscous component of this signal (prediction \( B \) minus prediction \( B = \)), and the difference in the predictions based on the Earth models \( B \) and \( B = \).

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Table 3.3: Modern Horizontal Crustal Deformation Rates. Predictions of horizontal crustal deformation rates at 10 GNSS sites (Table 3.1; see Figure 3.6 for location) at year 25 of the ice history I-ME. For each component, we show predictions based on the 3-D viscoelastic Earth model \( M_{3D} \), the viscous component of this signal (prediction \( M_{3D} \) minus prediction \( M_{EL} \)), and the difference in the predictions based on the Earth models \( M_{3D} \) and \( M_{MBL} \).

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3.3.3 Sensitivity Analysis: Varying the Minimum Viscosity

Next, we repeat the calculations based on the I-ME ice history, but vary the mapping from seismic velocity anomalies to viscosity to consider Earth models in which the minimum viscosity below the West Antarctic is reduced to $10^{17}$ Pa s and increased to $10^{19}$ Pa s (models $M_{3D-L}$ and $M_{3D-H}$, respectively). The viscous signal in predictions of the peak magnitude of geoid rate, crustal uplift rate and the two components of the horizontal crustal rate (i.e., the difference in the peak magnitude of these quantities computed using the set of 3-D Earth models and the $M_{EL}$ model) as a function of the minimum viscosity below West Antarctica is summarized on Figure 3.10. The viscous signal in the peak geoid rate is less than 1 mm/yr for all 3 cases. In contrast, the viscous signal in the crustal deformation rates ranges from 6 to 32 mm/yr for uplift and from 1 mm/yr to 9 mm/yr for horizontal deformation.

3.3.4 Sensitivity Analysis: Projections Across the Next 50 Years

As a final analysis, we perform a simulation that extends the calculation based on ice history I-ME for an additional 50 years using the GMSL trend predicted by Golledge et al. (2019) for the Antarctic Ice Sheet. The Golledge et al. (2019) projection of Antarctic ice mass flux was generated using a coupled ice sheet-ice shelf model forced with a climatology based on CMIP5 outputs, with the addition of ice sheet – climate feedbacks, and in the period 2017-2067 it projects a GMSL rise of ~60 mm (Figure 3.2C, main frame). To be consistent with our construction of the I-ME model, we assume that the ice melt geometry across this 50-year period is given by the mass flux in the final year of the Martín-Español et al. (2016) reconstruction, and, as noted earlier, we scale the total melt to follow the GMSL curve of Golledge et al. (2019). We denote the model as I-MEG, and emphasize that the first 25 years of the 75-year ice history are identical to model I-ME.

The bottom two rows of Figure 3.5 show results analogous to the top row - predictions of crustal
Figure 3.10: Sensitivity of peak geoid and crustal uplift predictions to variations in mantle viscosity. A) Peak difference in the rate of change of the geoid predicted using the 1-D Earth model $M_{EL}$ and the 3-D Earth model $M_{3D}$ (i.e., 1-D prediction minus 3-D prediction) as a function of the minimum viscosity in the sub-lithospheric mantle below the West Antarctica. B) As in (A), except for crustal uplift rate. C) As in (B), except for the peak difference in the horizontal crustal rate in the (blue) east-west and (red) north-south direction.
uplift rates at the 25-year mark of the I-ME simulation - at years 50 and 75 of the I-MEG simulations. Once again, the left frame on each row represents the contribution, in percent, of the viscous signal relative to the signal predicted using the 3-D viscoelastic model \( M_{3D} \) (i.e., \( M_{3D} \) prediction minus \( M_{EL} \) prediction, divided by the former). The right frame provides a measure of the ability of the 1-D, regionally tuned model, \( M_{MBL} \), to capture these viscous effects (i.e., \( M_{3D} \) prediction minus \( M_{MBL} \) prediction, divided by the former). In the case of the right column, one should focus on the region close to Marie Byrd Land since the 1-D viscosity profile was based on averaging the viscosity below this region (Figure 3.1C, dashed circle). However, the large discrepancies evident at other sites in the West Antarctic (Figures 3.5D,F) emphasizes that a 1-D viscosity model derived from mantle viscosity variations below one region cannot be considered an appropriate model for computing the response in the West Antarctic as a whole.

Moving down the left column of Figure 3.5 indicates that viscous effects peak at 20\%, 35\% and 55\% of the signal in the prediction of crustal uplift rates based on the 3-D viscoelastic model \( M_{3D} \) at years 25 (i.e., calendar year 2017, as discussed above), 50 and 75 of the simulation. A comparison of these values with the results in the right column indicates that using the 1-D viscosity model \( M_{MBL} \) in place of \( M_{EL} \) captures only about half of this viscous signal near the zone of major ice mass flux.

Finally, Figures 3.11-3.13 and Tables 3.4 & 3.5 are analogous to Figures 3.7-3.9 and Tables 3.2 & 3.3, except the figures track predictions of crustal rates at same three GNSS sites for the entire 75-year duration of the ice history I-MEG. The conclusions derived from Figures 3.7-3.9 for sites within the region of significant mass flux – namely, that the regional model \( M_{MBL} \) does a reasonable job at capturing the viscous effects in crustal uplift rates predicted using the 3-D model \( M_{3D} \), and that the same is not in general true for horizontal rates – continue to hold across the longer simulation. We note also that the viscous signals (blue lines) and the residuals between predictions based on models \( M_{3D} \) and \( M_{MBL} \) (red lines) increase monotonically over time for all three crustal rate components.
Figure 3.11: Solid surface deformation rates predicted at GNSS site #3 (CLRK) as a function of time using the ice history I-MEG. Rows 1-3 show predicted vertical rates, eastward horizontal rates and northward rates as a function of time, respectively, across the 75-year simulation (1992-2067). The first column shows predictions based on the 3-D viscoelastic Earth model $M_{3D}$ (black lines). The second column shows the residual between the following pairs of predictions: Blue - $(M_{3D} - M_{EL})$; Red - $(M_{3D} - M_{MBL})$. All rates are computed with a running time window of 5 years.
Figure 3.12: Deformation rates at GNSS site #4 (SDLY) using the ice history I-MEG. As in Figure figJCL, for GNSS site #4 (SDLY).
Figure 3.13: Deformation rates at GNSS site #5 (TOMO) using the ice history I-MEG. As in Figure fig:figJCLC, for GNSS site #5 (TOMO).
Table 3.4: Projection Vertical Crustal Deformation Rates. Predictions of vertical crustal deformation rates at 10 GNSS sites (Table 3.1; see Figure 3.6 for location) at year 75 of the ice history I-MEG. We show predictions based on the 3-D viscoelastic Earth model $M_{3D}$, the viscous component of this signal (prediction $M_{3D}$ minus prediction $M_{EL}$), and the difference in the predictions based on the Earth models $M_{3D}$ and $M_{MBL}$.

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Table 3.5: Projection Horizontal Crustal Deformation Rates. Predictions of horizontal crustal deformation rates at 10 GNSS sites (Table 3.1; see Figure 3.6 for location) at year 75 of the ice history I-MEG. For each component, we show predictions based on the 3-D viscoelastic Earth model $M_{3D}$, the viscous component of this signal (prediction $M_{3D}$ minus prediction $M_{EL}$), and the difference in the predictions based on the Earth models $M_{3D}$ and $M_{MBL}$.

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3.4 Conclusions

The Antarctic Ice Sheet is a central focus of studies investigating the impact of global climate change, and geodetic measurements, including GRACE satellite gravity data and surveying using GNSS, play a key role in many such studies. These studies follow two distinct approaches. First, the geodetic measurements are corrected for the ongoing influence of the ice age (i.e., GIA) and the residual signal is used to estimate modern ice mass flux. Second, an independent estimate of modern ice mass flux is used to correct the observational data, leaving a signal that is analyzed to constrain the GIA process. In both these approaches, a mapping is required between modern ice mass flux and perturbations to the Earth system associated with this flux.

The goal of the present study has been to use a 3-D model of mantle viscosity to quantify the impact of viscous relaxation of the solid Earth within the Antarctic region on predictions of geoid height changes and crustal deformation rates driven by modern melting, a component of the response that has sometimes been neglected in previous work. Our analysis has involved simulations of duration 25 and 75 years; the former is consistent with the period during the modern over which mass flux from the Antarctic is thought to have been significant (Shepherd et al., 2018), and the latter allows us to estimate the viscous signal associated with Antarctica’s projected melting (Schlegel et al., 2018; Golledge et al., 2019; Bulthuis et al., 2019) as the Earth moves further into a warming world. Moreover, we have considered a series of ice histories, and quantified the extent to which 1-D models of mantle viscosity can accurately account for viscous effects.

We have found that the viscous signal in predictions of peak geoid height changes in a laterally varying Earth model (i.e., $M_{3D}$) are at the level of 0.5 mm/yr at the end time of the 25-year simulations, and conclude that studies analyzing existing GRACE gravity data by assuming that modern mass flux drives a purely elastic response of the solid Earth will marginally overestimate the associated geoid signal at this level. This minor level of inaccuracy can be decreased by modeling the geoid
height changes using a 1-D Earth model with a radial viscosity profile tuned to the regional viscosity variation under the West Antarctic. The ability of a regionally tuned, 1-D viscosity profile to accurately reproduce the present-day geoid signal computed using an Earth model with lateral variations in Earth structure has also been demonstrated in studies of glacial isostatic adjustment in response to the last ice age (Paulson et al., 2005).

Our results demonstrate that viscous effects on crustal deformation rates due to modern ice mass flux are significantly larger than the corresponding effects on the geoid height. A et al. (2013) reached the same conclusion for signals associated with glacial isostatic adjustment. The viscous signal in peak uplift rates predicted using 3-D viscoelastic Earth models at sites near areas of significant ice melting ranges from 6 to 32 mm/yr at the 25-year mark in the simulations when the minimum viscosity beneath the region is varied from $10^{19}$-$10^{17}$ Pa s (Figure 3.10B). A portion of this viscous signal can be modeled by adopting a 1-D viscosity profile more characteristic of the low viscosity region beneath the West Antarctic ($M_{ML}$). Specifically, results from our suite of simulations extending 25 to 75 years indicate that the $M_{ML}$ model captures approximately 50% of the viscous signal predicted using the 3-D viscoelastic Earth model $M_{3D}$ (Figure 3.5).

The inadequacy of 1-D Earth models in reproducing horizontal crustal rates computed using a more complex Earth model with 3-D variations in viscoelastic structure is pronounced (Figures 3.4, 3.7-3.9, & 3.11-3.13), and the discrepancy between the two will depend on the details of the melt geometry and rheological variability. Indeed, at some GNSS sites within zones of high ice mass flux in the West Antarctic, we have found that the 1-D viscosity profile derived from a regional averaging of the viscosity field below the region ($M_{ML}$) yields horizontal crustal rate predictions that are more discrepant from predictions based on the 3-D viscoelastic Earth model $M_{3D}$ than predictions generated using an elastic model ($M_{EL}$) (Figure 3.9).

We conclude that horizontal crustal motions due to modern ice mass flux in the Antarctic cannot in general be accurately modeled using any 1-D Earth model, and thus the signal due to modern
melting in GNSS derived horizontal motions in the West Antarctic should be analyzed using models that incorporate the full complexity of viscoelastic Earth structure beneath the region. This conclusion has relevance for the results of Zhao et al. (2017), who were unable to find a 1-D viscoelastic model that satisfactorily fit their horizontal crustal rate observations. The sensitivity of these observations to lateral viscosity structure suggests that they have limited utility in constraining 1-D (i.e., depth varying) viscoelastic structure.

At the end of the 25-year simulation based on the ice history I-ME and the 3-D viscoelastic Earth model $M_{3D}$, the viscous signal within the zone of major ice mass flux reached 14 mm/yr for uplift rates and 2.5 mm/yr for horizontal rates. These values exceed average uncertainties in GNSS estimates of these motions (e.g., Barletta et al., 2018). This signal is systematic, not random (Figures 3.4-3.13), and this suggests that viscous effects should be incorporated in predictions of the response to the modern melt signal in regions where shallow mantle viscosities are less than $\sim 10^{21}$ Pa s now that the time scale of melting from the region has reached a quarter century (or more).

As a final point, the ice history I-ME and the Earth model $M_{3D}$ were adopted as representative of a growing suite of spatially high-resolution reconstructions of ice mass flux in the West Antarctic and 3-D mantle viscosity field below this region. Our goal was not to present accurate predictions of crustal deformation rates at specific sites within the West Antarctic, but rather to explore the range of viscous contributions to these predictions in a region of such complexity, and also to assess the ability of regionally tuned, 1-D viscosity profiles to capture the viscous signal. Any prediction of crustal rates or geoid anomalies in the West Antarctic will be sensitive to the choice of ice history (including the ice age component of this history) and the details of the Earth model, and efforts to improve constraints on either of these inputs based on geodetic observations must address their coupled non-uniqueness. As recent articles have demonstrated (e.g., Nield et al., 2014; Zhao et al., 2017; Barletta et al., 2018), and the present study emphasizes, in the West Antarctic this effort must contend with viscous effects in the response of the solid Earth to melting over the last few decades.
and century. This complication will become ever more relevant and important in the future as we continue to use space-based geodetic techniques to monitor the stability of the Antarctic Ice Sheet in a progressively warming world.
Using Antarctic GNSS observations to infer viscoelastic Earth structure

4.1 Introduction

The ability to precisely model the solid Earth’s viscoelastic response to ice melting is critical to understanding the impact of ongoing climate change in the West Antarctic. For example, ice streams are the largest sources of ice mass loss in Antarctica (Bennett, 2003; Shepherd et al., 2018). To constrain the discharge potential of these ice streams, accurate models of melt-induced solid Earth deformation are required to track the position of the grounding line, the point where ice is no longer in contact with the ground and becomes a floating ice shelf (Schoof, 2007; Gomez et al., 2010; Robel et al., 2018; Morlighem et al., 2020). On a broader scale, geodetic experiments that are used to estimate the rate of ice melting, such as GNSS measurements of crustal uplift or GRACE observations of gravity, are also sensitive to the rheology of the Earth’s mantle (Wahr et al., 1998; Bevis et al., 2009; Velicogna et al., 2014; Zhao et al., 2017; Barletta et al., 2018; Schumacher et al., 2018).

Seismic tomography studies have shown that mantle structure beneath the Antarctic may be complex and spatially varying, however, which poses a challenge for modeling the solid Earth’s response to changes in ice loading (Ritzwoller et al., 2001; Morelli & Danesi, 2004; Hansen et al., 2014). The conversion from seismic velocity anomalies to viscosity is uncertain (e.g., Ivins & Sam-
mis, 1995; Wu et al., 2013); however, under the assumption that thermal effects dominate the anomalies, standard mappings between the two indicate that the lithosphere in West Antarctica is thinner (<100 km thick), and the upper mantle is lower in viscosity (2-3 orders of magnitude), than both the mantle structure beneath the East Antarctic craton (Lloyd et al., 2015; Heeszel et al., 2016; Lloyd et al., 2020), and the global average mantle viscosity (Lambeck et al., 2014; Lau et al., 2016). The low inferred upper mantle viscosity beneath WAIS implies that the solid Earth will respond viscously on decadal timescale rather than multi-millennial timescales, and so this signal must be considered when modeling regional ice dynamics during the modern era (Hay et al., 2017; Barletta et al., 2018; Powell et al., 2020).

In addition to seismological techniques, GNSS observations of the solid Earth’s response to recent ice mass change have been used to further refine our knowledge of West Antarctica’s regional mantle structure. In these Antarctic GNSS studies, a best-fit viscosity profile is found by comparing observations of crustal uplift to modeled responses from known or inferred ice melting histories (Nield et al., 2014; Wolstencroft et al., 2015; Zhao et al., 2017; Barletta et al., 2018). But importantly, for computational feasibility, these GNSS-informed Earth models are chosen to be one-dimensional, i.e., depth varying, with an upper mantle viscosity profile discretized into 3-5 isoviscous layers.

The above discussion raises an important question, namely: What do the 1-D viscosity profiles inferred from GNSS observations of crustal uplift represent in an area with complex 3-D mantle structure? While 1-D viscoelastic Earth models have been shown to successfully fit GNSS observations of crustal uplift (e.g., Barletta et al., 2018), GNSS data are geographically sparse, and thus these models may not capture the regional variation in crustal uplift and may not have a simple relationship to the underlying 3-D Earth structure. To explore these issues, we will test the hypothesis that GNSS-derived 1-D viscosity profiles capture a characteristic lateral average of the Earth’s 3-D viscosity structure near the locations of the specific GNSS field sites. Specifically, we perform a
test by generating synthetic crustal deformation rate data for sites in the West Antarctic, and search for the best-fitting 1-D viscosity profiles using a standard forward modeling approach (Zhao et al., 2017; Barletta et al., 2018). The synthetic data will be based on the location of GNSS sites in West Antarctica and computed from a prescribed ice history and 3-D viscosity field, where the latter is constructed from seismic velocity anomalies obtained from recent tomographic modeling. Our 1-D forward models will adopt the same ice history used to compute the synthetics, and this the test represents the “best case scenario” in which the ice history is assumed to be known perfectly. Nevertheless, the performance of such 1-D models in a region of complex, 3-D mantle structure has important implications for ongoing efforts to use GNSS observations to constrain local viscosity and monitor ongoing ice melt in the West Antarctic.

4.2 Methods

4.2.1 Generating Synthetic Uplift Data

To create our synthetic dataset of uplift in the West Antarctic, we use the 3-D viscoelastic Earth model of Hay et al. (2017) and Powell et al. (2020), which was described in detail in Chapter 1. We then load our 3-D viscoelastic Earth model with a prescribed ice history, and the accompanying global redistribution of water mass. This spatiotemporal history of ice cover is the 25-year ice history (1992-2017) described in Powell et al. (2020), and in Chapter 3 of this thesis (model I-ME). Figure 4.1a shows the average ice height change over the 2003-2013 time period in the West Antarctic. In total, this modern ice melt history contributes ~5.4 mm of global mean sea level rise over the full 25-year time period (Figure 4.1b). The accompanying water mass redistribution is generated by solving the gravitationally self-consistent sea level equation (Farrell & Clark, 1976), as described in Chapter 1. We use an extended form of the sea-level equation that accurately accounts for migrating shorelines (including marine ice grounding lines) (Johnston, 1993; Mitrovica & Milne, 2003).
and a signal associated with perturbations in Earth’s rotation (Mitrovica et al., 2005), as described in Gomez et al. (2010b). Our numerical calculations are based on the finite volume methodology described in Latychev et al. (2005), with a regional grid refinement in the Antarctic, as described in Chapter 1.

We compute the total uplift rates at 10 West Antarctic GNSS sites in the Amundsen Sea Embayment and Marie Byrd Land (Figure 4.1a; Table 4.1). In order to isolate the viscous component of the uplift rate, we repeat the same calculation using a purely elastic Earth model, and then apply a linear regression to the last 5 years of the residual (3-D viscoelastic minus elastic) uplift time series. The choice of a 5-year window reflects the short observational timescale of many of the actual GNSS data sets. The name and location of the GNSS sites used in this study are given in Table 4.1.
The table also lists the viscous uplift rates computed as described above, as well as the published uncertainties, \( \sigma \), associated with the GNSS observations (Schumacher et al., 2018; Barletta et al., 2018). This procedure yields our final, synthetic ground-truth “observations” that form the basis of our case study.

<table>
<thead>
<tr>
<th>Station</th>
<th>Name</th>
<th>Lat (deg)</th>
<th>Lon (deg)</th>
<th>Viscous Uplift (mm/yr)</th>
<th>( \sigma ) (mm/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PATN</td>
<td>-78.03</td>
<td>204.98</td>
<td>0.21</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>MCAR</td>
<td>-76.32</td>
<td>215.70</td>
<td>0.75</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>CLRK</td>
<td>-77.34</td>
<td>218.13</td>
<td>1.06</td>
<td>0.8</td>
</tr>
<tr>
<td>4</td>
<td>SDLY</td>
<td>-77.14</td>
<td>234.03</td>
<td>1.18</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>TOMO</td>
<td>-75.80</td>
<td>245.34</td>
<td>8.83</td>
<td>3.0</td>
</tr>
<tr>
<td>6</td>
<td>BERP</td>
<td>-74.55</td>
<td>248.12</td>
<td>5.12</td>
<td>0.7</td>
</tr>
<tr>
<td>7</td>
<td>BACK</td>
<td>-74.43</td>
<td>257.52</td>
<td>3.16</td>
<td>1.5</td>
</tr>
<tr>
<td>8</td>
<td>INMN</td>
<td>-74.82</td>
<td>261.12</td>
<td>3.98</td>
<td>2.4</td>
</tr>
<tr>
<td>9</td>
<td>THUR</td>
<td>-72.53</td>
<td>262.56</td>
<td>1.32</td>
<td>0.8</td>
</tr>
<tr>
<td>10</td>
<td>LPLY</td>
<td>-73.11</td>
<td>269.70</td>
<td>1.26</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.1: Synthetic dataset. Predictions of synthetic viscous uplift rates at 10 GNSS sites (as labeled; see Figure 4.1a for location) at year 2017 of the model ice history. The values in the final column are uncertainties determined from actual GNSS-derived observations (Stations 1-4: Schumacher et al., 2018; Stations 5-10: Barletta et al., 2018).

### 4.2.2 Fitting Synthetic Uplift Data with 1-D Viscosity Models

Once we have generated our synthetic GNSS uplift observations, we then attempt to fit this data with 1-D mantle viscosity models. To do this, we first construct a suite of 1,960 different 1-D Earth models with upper mantle viscosity values that span a wide parameter space. The viscosity structure consists of an elastic lithosphere of thickness 60 km (consistent with the thickness from the 3-D model at the location of the GNSS sites), a lower mantle viscosity of \( 5 \times 10^{21} \) Pa s beginning at a depth of 670 km, and an upper mantle viscosity discretized into the 3 depth layers adopted by Barletta et al. (2018). These depth layers are: the shallow upper mantle (SUM), which extends from the base of the lithosphere to 200 km depth; the deep upper mantle (DUM), which extends from 200-
400 km depth; and the transition zone (TRZ), which extends from 400-670 km depth. Here, and in the remaining text, we will report viscosity values in units of log_{10}(Pa s). Our forward calculations sample viscosity values in the range 17.9-20.5 in the SUM layer, 19.3-21.9 in the DUM layer, and 19.3-21.1 in the TRZ layer.

After generating our suite of candidate forward models, we calculate the predicted uplift rates using the ice history specified above. While the creation of our synthetic dataset based on a complex 3-D mantle structure required the use of a computationally expensive finite volume method (Latychev et al., 2005), the 1-D approximation used here allows for significant simplifications. We adopt the spectral formulation of Mitrovica et al. (1994) and the pseudo-spectral sea level theory of Kendall et al. (2005), both based on viscoelastic Love number theory (Peltier, 1974). We truncate this pseudo-spectral calculation at spherical harmonic degree 512, which corresponds to a 21 arc-minute spatial resolution (approximately 35 km at the equator). The viscous component of the uplift rates is computed in the same manner described above for our synthetic dataset. That is, we fit a linear trend to the 1-D viscoelastic-minus-elastic crustal uplift time series spanning the last five years.

For each 1-D forward model simulation, we calculate the reduced chi-squared misfit, $\chi^2_{red}$, between the synthetic “ground truth observations” based on our initial 3-D Earth model, and the viscous uplift rates at the GNSS sites calculated using 1-D viscosity profiles:

$$\chi^2_{red} = \frac{1}{N - 1} \sum_{j=1}^{N} \left( \frac{O_{GPS,j} - C_{GPS,j}}{\sigma_{GPS,j}} \right)^2,$$  \hspace{1cm} (4.1)

where $O_{GPS,j}$ are the synthetic viscous uplift rates (“observations”) at the GNSS sites (Table 4.1), $C_{GPS,j}$ are the predicted uplift rates, $\sigma_{GPS,j}$ are the reported uncertainties in the uplift rates at each GNSS station (Table 4.1), and $N$ is the number of sites.
4.3 Results

How accurate are 1-D viscosity models that are inferred from West Antarctic GNSS data? Specifically, how well does our best-fitting 1-D model resemble the actual 3-D Earth that was used to compute the synthetics uplift rates? Figure 4.2 shows the regional variation in the 3-D viscosity field, which has been averaged over depth within each of the three upper mantle layers. If the uplift rate at each GNSS site were most sensitive to the local Earth structure in its vicinity, a logical presumption would be that each layer of the best-fitting 1-D viscosity model represented a lateral average of the Earth’s structure around the GNSS sites. To test this suggestion, we begin by calculating, as a function of depth, the average viscosity of the 3-D Earth model within a total volume defined by a lateral averaging radius (a cylinder of radius 300 km) around each of the 10 GNSS sites. We then average the viscosity in all ten cylinders together. The result, as a function of depth, is shown by the black profile in Figure 4.3. The minimum and maximum viscosity is shown by the gray box. Next, we averaged the viscosity profile (black line) in each of the three layers used in the forward modeling, yielding values of 19.7 (SUM), 20.3 (DUM), and 19.5 (TRZ) (purple line). This gives an overall “average 1-D viscosity” of the ground-truth 3-D Earth model based on the spatial distribution of GNSS sites, which can be compared to our 1-D forward models.

Figure 4.4 shows the viscosity in each of the three upper mantle layers laterally averaged as described above, but using radii ranging from 100–500 km around each GNSS site. A site-by-site breakdown is also shown. Overall, we find that the average viscosity structure is relatively insensitive to the choice of the lateral averaging radius. This insensitivity likely arises because the synthetic uplift rate at each GNSS site is sampling a distinct region within the 3-D viscosity field. This is due to both the complex structure in this viscosity field (Figure 4.2) and the varying spatial scale of ice loading in the vicinity of each site (Figure 4.1a).

Figure 4.3 shows a comparison of the locally-averaged viscosity profile for our 3-D ground-truth
Figure 4.2: Depth-averaged viscosity within three upper mantle layers of the 3-D Earth model. (a-c) Regional variation in average viscosity of the 3-D Earth model within the SUM, DUM and TRZ layers defined in the text.
Figure 4.3: Comparison of mantle viscosity between the 3-D Earth model and the best-fitting 1-D model. The black line depicts the depth-dependent mantle viscosity of our 3-D Earth model, which has been laterally averaged over a radius of 300 km and then averaged again across all 10 sites. The minimum and maximum viscosity in each layer are depicted by the shaded region. The average viscosity from the 3-D model when using the same three depth layers adopted in the forward modeling is shown in purple. The best-fitting 1-D viscosity profile is shown in orange.
Figure 4.4: Viscosity inference from 3-D model as a function of lateral averaging radius. The viscosity inference from the 3-D Earth model as a function of the lateral averaging radius applied to each GNSS station and to all 10 GNSS stations. Results are shown for all three layers adopted in the 1-D forward calculations (as labeled).
model to the 1-D forward model that best fits the synthetic observations (orange line). The latter has the following viscosity values: 18.5 (SUM), 20.1 (DUM), and 20.3 (TRZ). Agreement between the best-fit 1-D model and the 3-D model viscosity inference is best in the middle layer DUM, but the discrepancy reaches approximately an order of magnitude in the shallowest and deepest layers SUM (18.5 versus 19.7) and TRZ (20.3 versus 19.5), respectively. The size of the misfit in each layer appears to be related to the level of variability of viscosity in the 3-D model in the same layer. Consistent with the gray shading in Figure 4.3, it is clear from Figure 4.2 that the 3-D Earth model is characterized by significantly greater variability in the shallow sub-lithospheric mantle and in the transition zone than in the intermediate DUM layer.

Given the reported GNSS data uncertainty, however, our best-fitting 1-D viscosity profile does not represent a unique inference. In Figure 4.5, we show the reduced chi-squared misfit in the uplift rates between our broad suite of 1-D forward models and the laterally averaged 3-D model at the GNSS sites. Each frame shows the $\chi^2_{\text{red}}$ values as a function of viscosity for two of the three upper mantle regions, with the third region fixed to the viscosity value of the best-fit model (orange line, Figure 4.3). Of the 1,960 1-D viscosity models we considered, 300 yield a misfit to data at all GNSS stations that is not statistically significant to 95% confidence from the misfit of the best-fit profile. This non-uniqueness is reflected by the broad range of 1-D viscosity models that are characterized by relatively low misfit in Figure 4.5. Within the set of 300 models that satisfy this misfit criteria, all values for the viscosity of the DUM considered in the analysis (19.3-21.9) and a wide range of the values in the other upper mantle layers were nearly equally likely. Lower bounds were apparent for viscosities in the shallowest (SUM, 18.3) and deepest (TRZ, 19.9) of the three layers (Figure 4.6).

We next consider the regional spatial pattern of the viscous uplift rate between predictions based on the 3-D Earth model (Figure 4.7a) and the best-fit 1-D model (Figure 4.7b) relative to the position of the 10 GNSS sites. The viscous component of the uplift rate calculated using both models peaks at $\sim$10 mm/y. Notably, the sampling of the synthetic uplift field by the network of GNSS
Figure 4.5: Misfit between synthetic uplift data and the 1-D forward models. Misfit as a function of: a) viscosities in the DUM and SUM layers, with the TRZ layer fixed at 20.3; b) the TRZ and SUM layers, with the DUM layer fixed at 20.1; c) the TRZ viscosity and DUM layers, with the SUM layer fixed at 18.5. The best-fit model (orange line, Figure 4.3) is indicated by the orange x.
Figure 4.6: Histograms of 1-D models that fit synthetic observations. The spread in viscosity values of the subset of 1-D models which yield an acceptable misfit to the synthetic data at the 10 GPS sites (300 models out of 1,960; see text) for the (a) SUM, (b) DUM, and (c) TRZ layers. The corresponding average viscosity of the 3-D model (when using the same depth layers as in the forward modeling) is shown in purple, while the best-fitting 1-D viscosity profile is shown in orange (see also Figure 4.3).
sites misses the zone with a large viscous uplift component in the 3-D Earth model, in Marie Byrd Land. Figure 4.7c shows the regional misfit between the viscous uplift rate predicted by the best-fit 1-D Earth model and the synthetic field based on the 3-D Earth model (i.e., Figure 4.7a minus Figure 4.7b). The misfit is less than ~0.5 mm/y at all GNSS sites with the exception of site 5, where it exceeds 1 mm/y; we note that site 5 has the largest published uncertainty (3 mm/y) of all sites considered in our analysis (Table 4.1). This small level of misfit confirms the relatively good performance of the 1-D viscosity model, as measured by the weighted $\chi^2_{red}$ metric (Equation 4.1), in predicting the synthetic uplift rates at the specific locations of the GNSS sites. However, the misfit grows at distance from the sites, most notably in Marie Byrd Land, an area of relatively significant melt (Figure 4.1a), low viscosity (Figure 4.4) and thus a high rate of uplift computed using the 3-D Earth model (Figure 4.7a). The best-fit 1-D Earth model underestimates the viscous uplift rate by up to ~4 mm/y in this case because it fails to capture the localized zone of very low viscosity beneath the area.

4.4 Discussion

We have explored the robustness of inferences of 1-D mantle viscosity based on GNSS uplift data from a region of complex 3-D viscoelastic Earth structure. The non-uniqueness introduced when using 1-D modeling to infer West Antarctica’s viscosity structure from uplift observations can reach an order of magnitude in viscosity at all depths in the upper mantle (see Figures 4.5 and 4.6). The broad spread evident in the histograms of Figure 4.6 is due, in part, to correlations that exist between the viscosity values inferred for pairs of upper mantle layers (SUM,DUM) and (DUM,TRZ). Correlation coefficients for each pair, using the 300 acceptable solutions described above, are ~0.59 and ~0.20, respectively. While there may be more optimal choices for the layer depths in regard to the inherent resolving power of the GNSS data, we emphasize that all the 1-D models that con-
Figure 4.7: Predicted Uplift Rates. a-b) Viscous uplift rate at the end of the 25-year loading history generated using the a) 3-D Earth model described in the text and b) the best-fitting 1-D Earth model. c) Residual uplift rate found by removing the viscous uplift rate predicted by the best-fitting 1-D model in frame (b) from the field in frame (a).
tribute to the histogram values in Figure 4.6 provide an excellent fit to the synthetic uplift rates. Further, Figure 4.6 demonstrates that the sensitivity of the inference degrades in the deepest layer, TRZ. In particular, none of the 1-D models that provide an acceptable fit to the synthetic GNSS observations have a viscosity in the TRZ layer that matches the laterally averaged 3-D model results (Figure 4.6c).

As we have noted, the synthetic uplift rate at each GNSS site is sampling a distinct region within the 3-D viscosity field. While it is generally true that increasing the number of GNSS sites would improve our spatial resolution by sampling across multiple regions a complex 3-D viscosity field, this would not necessarily translate to improved results (i.e. reduced misfit) if such observations are only being fit with a 1-D viscosity profile.

This raises an interesting point. Published inferences of 1-D viscosity profiles beneath Antarctica have been based on large GNSS networks, which sample an area greater than the length-scale of lateral variability in the structure implied by seismic tomography imaging (Barletta et al., 2018). Would the performance of our 1-D models improve if they are compared to a more localized area of the 3-D Earth model? We have found that repeating the analysis shown in Figure 4.3 for groupings of proximal sites (e.g., 1·3 in in Figure 4.1a) does not consistently bring the best-fit 1-D model based on each of these groupings into closer accord with the local depth average of the 3-D Earth model (Figure 4.8).

The limited resolving power of uplift rates, whether considered in isolation or as part of a network, indicates that any analysis of GNSS uplift data based on 1-D Earth models will not capture extreme viscosity values beneath the region. The orange line and grey shading on Figure 4.3, which differ by orders of magnitude, represent an example of this limitation. We conclude that local minima and maxima in viscosity may be significantly lower/higher than the viscosities that have been inferred in previous 1-D analyses (Nield et al., 2014; Wolstencroft et al., 2015; Zhao et al., 2017; Barletta et al., 2018)
Figure 4.8: Comparison of mantle viscosity between the 3-D Earth model and the best-fitting 1-D model for three GNSS site groupings. (a-c) As in Figure 4.3, except the analysis is now performed for the following three groupings of proximal GNSS sites: a) [1,2,3], b) [4,5,6] and c) [7,8,9,10]. These tests do not show any consistent improvement in the discrepancy between the best-fit 1-D model (orange line) and the regionally averaged depth profile of the 3-D model (purple line) relative to the results generated for the full network (Figure 4.3).
Another result of relevance to previous analyses of GNSS data in West Antarctica is that while the best-fit 1-D model performs well in fitting the viscous uplift rates at the GNSS sites, it does not capture the regional pattern of uplift rates away from these sites (Figure 4.7c). In our analysis, this limitation is most evident in Marie Byrd Land, where a zone of significant melt and low viscosity is not sampled by the GNSS network. We conclude that uplift fields computed from 1-D modeling should not be used to make regional scale predictions.

Our analysis of uncertainties in viscosity inferences represents a best-case scenario since we have not considered the possibility of errors in the local ice history. A more realistic estimation would involve errors in the 25-year ice history considered by our calculations (Figure 4.1). Additionally, Antarctic GNSS data are thought to be contaminated by signals associated with past ice dynamics. For example, Nield et al. (2016) show that ice loading associated with periods of Siple Coast ice stream stagnation over the last 2000 years (in particular, Kamb Ice Stream variability over the last 200 years) lead to a present-day crustal response that is measurable in GNSS data. The authors conclude that the history of this stagnation must be included in ice loading models if the local upper mantle viscosity is less than $5 \times 10^{20}$ Pa s, which is likely the case for much of West Antarctica.

This argument points to a subtle issue: namely, the timescale of ice loading to which the uplift data will be sensitive is itself a function of the local mantle viscosity. Thus, if the Siple Coast ice streams had multiple stagnation and reactivation cycles (Catania et al., 2012), then the number of loading cycles needed to accurately reproduce present-day uplift rates would itself depend on the viscosity the GNSS uplift data is being used to constrain. This complexity will be exacerbated by transient mantle rheology effects, which may be relevant for this region (Lau et al., 2021).

Despite our conclusions regarding the high level of uncertainty and bias introduced by the use of 1-D viscosity modeling to interpret West Antarctic structure, such models can still be useful in specific applications. For example, Kingslake et al. (2018) adopt 1-D Earth models to argue that isostatic rebound is a plausible mechanism to explain the re-advance of the West Antarctic Ice Sheet in
the Weddell and Ross Seas during the Holocene to their present grounding line positions. Additionally, Gomez et al. (2015) and Konrad et al. (2015) adopt 1-D Earth models to show the impact of rapid uplift in low viscosity zones of the West Antarctica on future grounding line retreat in a warming climate. Larour et al. (2019), on the other hand, find that short-wavelength uplift (at scales of a kilometer) in the immediate vicinity of grounding lines, computed using 1-D elastic Earth models, impact grounding-line migration of WAIS. But while this component of deformation is important, full 3-D modeling of the Earth’s viscoelastic relaxation has been shown to be necessary to accurately capture the migration of the grounding line (Gomez et al., 2018).

4.5 Future Directions: Adjoint modeling

If a 1-D viscosity model is insufficient for the mantle beneath WAIS, the question arises: How can we rigorously invert for 3-D viscoelastic mantle structure using GNSS observations? That is, we wish to invert for a set of model parameters which define the viscosity field, $m$, that minimize the misfit between a given set of GNSS-related observables, $J$, and the output of the forward GIA solution, $f(m)$. For example, Equation 4.1 defines the $\chi^2$ misfit; we note, however, that there are other reasonable choices of misfit functions. There are several complications to such an inversion. First, the forward problem - i.e., computing crustal deformation rates using a 3-D viscoelastic Earth model - is numerically expensive. Second, our inverse problem is non-linear. Crustal deformation rates are non-linear functions of viscosity (or the logarithm of viscosity). The first of these rules out a brute force method of determining the “best” 3-D model given that the viscosity model space cannot be fully sampled through forward calculations. In this case, a better approach involves computing the gradients of $\hat{J}$. In practice, one would begin with a starting viscosity model $m$ and use a gradient based techniques, such as the steepest descent algorithm, to iteratively improve the model parameters until reaching a minimum misfit. Of course, the second complication, non-linearity, means that
one would have to establish whether a local minimum in misfit represents a global minimum.

However, this standard approach requires that one compute the gradient of the misfit $\bar{J}$ with respect to model parameters. This gradient, which we can express as $\delta \bar{J}$, is defined by the following equation:

$$
\delta \bar{J} = \int_{MS} K_\eta \delta \ln \eta \, dV,
$$

where $\eta$ denotes viscosity, and $\ln \eta$ represents our model parameters, i.e., the logarithm of viscosity within some chosen discretization of the mantle. $K_\eta$ is known as the ‘viscosity sensitivity kernel’ (or Fréchet kernel) defined using the same discretization as the model parameters. So, the above equation maps changes in the viscosity field to changes in the misfit function. Note that the non-linearity of the problem is reflected in the dependence of the Fréchet kernel on the viscosity.

The simplest way to calculate these Fréchet kernels in 4.2 is to use a finite difference approximation. That is, calculate $K_\eta$ by perturbing one model parameter (e.g., viscosity at a given location) at a time and use a forward calculation of the perturbed viscosity model to calculate a gradient. However, as we have noted, a single forward calculation using a 3-D Earth model is computationally expensive; thus any reasonable discretization of the mantle would require a large number of such calculations to construct $K_\eta$, making the computation of partial derivatives unfeasible.

Instead, so-called adjoint modeling offers a means to efficiently calculate the required partial derivatives. Adjoint modeling has the major advantage of requiring only two calculations - the so-called forward and adjoint problems - to generate the sensitivity kernels, and the computational expense of solving the adjoint problem is comparable to a single forward problem. The adjoint equations for the surface loading of a 3-D Maxwell viscoelastic Earth model were recently derived by Al-Attar & Tromp (2014). Crawford et al. (2018) extended this work to derive the governing adjoint equations for a loading that includes a gravitationally self-consistent sea-level component.

In its present state, the adjoint theory has two limitations. First, the theory assumes a non-rotating
Earth. Second, while the theory incorporates vertical crustal motions, since these play a role in the sea level treatment, an analogous theory that covers horizontal crustal motions is lacking. However, the required extensions to the theory are being developed (Prof. Al-Attar, personal communication). The numerical implementation of the existing theory is also characterized by limited spatial resolution as a consequence of memory requirements. A plausible future direction would be to adjust the finite volume code described in Latychev et al. (2003) to solve the forward and adjoint equations. Although this would be a non-trivial effort, it would take advantage of the software’s regional grid refinement.

4.5.1 Some Preliminary Results

Here, as an illustration of the power of the technique, we use adjoint modeling to compute sensitivity kernels of Antarctic GNSS uplift observations to ice loss from the Last Glacial Maximum. The spatio-temporal history of ice cover used was ICE-5G (Peltier, 2004). We consider two input mantle viscosity models. The first is one-dimensional and has an elastic lithosphere of 120 km thickness, an upper mantle of $5 \times 10^{20}$ Pa s, and a lower mantle of $5 \times 10^{21}$ Pa s. The second model has a 3-D variation in mantle viscosity. Due to the resolution limitations discussed above, we adopt a global seismic model of lower resolution than those treated in the rest of this thesis, S20RTS (Ritsema et al., 1999). The maximum spherical harmonic degree of the following calculations was 64 degrees, which corresponds to a spatial resolution of approximately 300 km.

Figure 4.9 presents a viscosity sensitivity kernel for GNSS station #6 (BERP). Positive values indicate that a viscosity increase in the region will lead to a higher uplift prediction at the GNSS station. Correspondingly, where the kernel is negative, a viscosity increase will lead to a lower uplift prediction at the GNSS station. The viscosity sensitivity is concentrated close to the measurement location and to where there has been significant changes in ice thickness (e.g., in the northern hemisphere beneath the location of the former Laurentide ice sheet). The kernel indicates increased sensitivity of
the surface observation to variations in the upper mantle viscosity. Physically, this can be attributed to the concentration of mantle flow in the upper mantle due to the viscosity contrast at the 660 km discontinuity.

The adjoint modeling allows us to directly answer the question: Over what spatial length scale are GNSS data sensitive? Figure 4.10 shows depth slices in the Antarctic region through the viscosity sensitivity kernel for both the 1-D background viscosity model and the 3-D background model. The location of the station is given by the black triangle outlined in yellow. For both the 3-D and 1-D viscosity kernels, the viscosity sensitivity is concentrated close to the measurement location. The spatial length scale of sensitivity increases with depth, but the amplitude of sensitivity decreases with depth. Specifically, the sensitivity in the 1-D case is concentrated below the ocean near the station, rather than on land. This sensitivity reflects the importance of the ocean loading signal, which is the main driver of the crustal deformation at this site. In the ICE-5G ice history, limited ice mass changes occur proximal to the GNSS station; mass loss of grounded ice in the Antarctic is
concentrated instead in the Weddell and Ross Sea sectors. Once again, a positive value here indicates that an increase in viscosity will lead to an increased uplift (or decreased subsidence). Physically, this makes sense: an increased mantle viscosity will result in reduced viscous flow in response to the ocean load, and thus a smaller subsidence signal associated with the ocean loading. Of course, it remains to be tested how this spatial scale of sensitivity will depend on both the spatial resolution of modeling as well as the spatial resolution of the surface loading. A highly concentrated melt scenario proximal to the GNSS station, for example, may result in more localized sensitivity.

With adjoint modeling, we are also able to explore the question: How does the sensitivity depend on the 3-D mantle structure? In comparing the left and right columns of Figure 4.10, we note clear differences between the sensitivity predicted using a 3-D viscosity and 1-D viscosity model. In the 3-D case, there is increased sensitivity on land. The increased (negative) sensitivity here is likely due to the lower shallow mantle viscosity of the 3-D model, which leads to increased viscous flow relative to the 1-D case.

This preliminary analysis will be improved by adopting adjoint methods that allow for the computationally efficient calculation of higher resolution Fréchet kernels (Al-Attar & Tromp, 2014; Crawford et al., 2018). In future work, I will also explore the unique sensitivity of crustal deformation motions, as measured by a GNSS receiver, to variations in the adopted ice history. Additionally, future analyses of the GNSS dataset should also consider horizontal crustal rates, which will have distinct sensitivities to Earth structure and ice history (e.g., Powell et al., 2020). With a suite of such kernels, I will be able to quantify the resolving power, or non-uniqueness associated with previous inferences of mantle viscosity based on GNSS observations. This effort will be an important, next step toward the inference of a 3-D viscosity model for the region using a gradient based inversion scheme. Overall, the goal is to build a rigorous inversion of the GNSS observations as a tool for monitoring the stability of Antarctica’s ice sheets.
Figure 4.10: Uplift Sensitivity Kernel of GNSS Station #6 for 1-D and 3-D models. Three depth slices at (top) 150 km, (middle) 300 km, and (bottom) 500 km through the viscosity sensitivity kernel corresponding to an uplift measurement at GNSS station #6 made at the present day. Positive kernel values indicate a viscosity perturbation at that location will lead to increased uplift (decreased subsidence) predicted at the GNSS site.
4.6 Conclusions

A variety of data sets, including seismic tomographic imaging, indicate that the viscoelastic Earth structure beneath the West Antarctic is complex and characterized by lateral viscosity variations that can exceed three orders of magnitude. We find that this 3-D structure drives a regional crustal uplift field that is poorly captured by 1-D viscosity models. Although there are significant practical limitations to establishing a denser GNSS network in West Antarctica, improved observations of crustal rates and an improved understanding of the resolving power of the observations will be vital to efforts to improve our understanding of the impact of climate change on WAIS and the role that Earth structure plays in controlling the evolution of this region.
5.1 Introduction

Present-day Canada was created through land acquisitions. When the Dominion of Canada was formed in 1867, only the present-day provinces of Ontario, Quebec, New Brunswick, and Nova Scotia were included in the boundaries of the new country, and the provinces of Quebec and Ontario were only a fraction of their present size (Figure 5.1A). In 1870, Rupert’s Land and the North-Western Territory would be acquired by the Dominion of Canada through an Imperial (British) Order-in-Council; these lands would be amalgamated to form the Northwest Territories (Figure 5.1A-B) (Cauchon & Cockburn, 1867; Rupert’s Land and North-Western Territory – Enactment No. 3, 1870). Furthermore, the Canadian government would have to compensate “Indians,” as it was recognized that they had claims to these lands (Cauchon & Cockburn, 1867; Rupert’s Land and North-Western Territory – Enactment No. 3, 1870). Previously, the British Crown had recognized indigenous rights to land in North America through the Royal Proclamation of 1763 (INAC, 2016). This document asserted that these lands had to be acquired through consent (i.e., ceded or purchased; INAC, 2016).

From 1870 to 1999, the Northwest Territories was partitioned into new provinces and territories.
Figure 5.1: 1800s Canada. Canada in A) 1867 and B) 1870, both from Tsuji et al. (2009).
Moreover, the boundaries of several of the existing provinces were extended (Figure 5.2; AANDC, 2019). In keeping with indigenous land rights in North America, treaties between the Government of Canada and indigenous groups had to be signed (AANDC, 2019).

Although the Ojibwa and James Bay Cree of northern Ontario signed Treaty No. 9 in 1905–06 and the adhesions to the treaty in 1929–30, there was no mention of the western James Bay marine islands (Treaty No. 9., 1905 – 06; Figure 5.3). The western James Bay Cree (or Omushkego Cree) of northern Ontario have realized the importance of the absence of marine islands from Treaty No. 9 and the adhesions, and they maintain that they have never relinquished their claim to Akimiski Island (and the other western James Bay marine islands) through treaty or any other means (Parliament of Canada, 1999). Nonetheless, on 1 April 1999, “the islands in Hudson Bay, James Bay [which includes Akimiski Island] and Ungava Bay that [were] not within Manitoba, Ontario or Quebec” were included in the newly established, Inuit-dominated territory of Nunavut, Canada.
Figure 5.3: Akimiski Island Location. The location of Akimiski Island in relation to the western James Bay First Nations of Moose Cree (Moose Factory), Fort Albany, Kashechewan, and Attawapiskat, from Tsuji et al. (2009).
(Nunavut Act, 1993; c.28, Part 1, 3(b)) even though the Inuit never asserted Aboriginal title (i.e., land rights to resources, such as, water, timber, and minerals) to the western James Bay islands, including Akimiski Island (Nunavut Land Claims Agreement, 1993).

The western James Bay region of northern Ontario, Canada, is part of the Mushkegowuk Territory and is inhabited by Omushkego Cree who live in four coastal First Nations (Moose Factory, Fort Albany, Kashechewan, and Attawapiskat) and one town, Moosonee (Figure 5.3). Place of residence is not static and movement of people between the communities is common. Akimiski Island is located ~16 km from the mouth of the Attawapiskat River and is the largest island in James Bay (NASA, 1994, 1997; Figure 5.3).

Herein lies the problem—two Canadian Indigenous groups lay claim to Akimiski Island: the Inuit through the Nunavut Act (1993) and the western James Bay Cree through Aboriginal title. However, this dispute can be settled, because a test of Aboriginal title exists in Canada (Denhez, 1982; Delgamuukw v. British Columbia, 1997; Hurley, 2000; also refer to Tsilhqot’in Nation v. British Columbia, 2014). The common-law test for proof of Aboriginal title is as follows:

1. The Aboriginal group is, and was, an organized society.

2. The organized society has occupied the specific territory over which it asserts Aboriginal title since time immemorial. The traditional use and occupancy of the territory must have been sufficient to be an established fact at the time of assertion of sovereignty by European nations.

3. The occupation of the territory by the Aboriginal group was largely to the exclusion of other organized societies.

4. The Aboriginal group can demonstrate some continuing current use and occupancy of the land for traditional purposes.
5. The group’s Aboriginal title and rights to resource use have not been dealt with by treaty.

6. Aboriginal title has not been eliminated by other lawful means (INAC, 1993: 5 – 6, 2003:8).

Pritchard et al. (2010) found no evidence (published or online) in the academic databases, grey literature, and published Inuit oral history that supports Inuit title to Akimiski Island, specifically, with reference to the 2nd and 4th criteria of the test of Aboriginal title. In fact, the Inuit Land Use and Occupancy Project (Milton Freeman Research Limited, 1976), a comprehensive record of Inuit land use in the Northwest Territories, Canada, did not refer to historical or present Inuit land use or occupation of Akimiski Island. This work is the authoritative Inuit land use and occupancy record and was the basis for the Inuit land claim that resulted in the formation of the territory of Nunavut in Canada. Indeed, in the Nunavut Land Claims Agreement (1993), beginning in Article 3.1.1 of this document, it states that the “Nunavut Settlement Area shall be composed of ‘Area A’... and ‘Area B’, being the Belcher Islands, associated islands and adjacent marine areas in Hudson Bay, described in Part 3 [Area B: section 3.3.1; p. 17]”. Marcopet, King George, Salliquit, and Belcher Islands were all mentioned (p. 19 – 20), but Akimiski Island was not named. Further, in Schedule 3-1, the Nunavut Settlement Area map (section 3.4.1) does not include Akimiski Island within Area B (the southernmost area of Nunavut; p. 21); however, a disclaimer appears that states “for general information purposes only” (p. 21). In Schedule 9-1, Existing Conservation Areas (Section 9.1.1, Part 1:83), migratory bird sanctuaries within the Nunavut Settlement Area were listed; eight bird sanctuaries were named, but the Akimiski Island Bird Sanctuary was not among them.

By contrast, General et al. (2017), using published and on-line evidence retrieved from the academic databases, grey literature and published Cree oral history, showed that all criteria of the common-law test of Aboriginal title were met to support Cree title to Akimiski Island; however, the written record only alluded to the Cree using Akimiski Island prior to European contact. Thus, traditional use and occupancy of Akimiski Island could only be definitively ascertained for post-
European contact, not pre-European contact. In other words, criterion 2 of the test for Aboriginal title was not fully addressed. To fully test criterion 2, Cree Elders share their oral history with respect to Akimiski Island, specific to the time period that corresponds to pre-European contact.

In this chapter, we employ state-of-the-art sea-level modeling to time stamp important Cree oral history events. It should be emphasized that Cree oral history highlights that the shorelines of the western James Bay region have been continuously evolving due to post-glacial isostatic adjustment (McDonald et al., 1997). Indeed, the Earth has gone through ice-age cycles and at the so-called last glacial maximum, the ancient Laurentide ice sheet covered Canada and the northeastern U.S. (Tsuji et al., 2009). Simplistically speaking, when the Laurentide ice sheet receded, the unloading associated with the melting of the ice sheet initiated rebound of the crust in the James Bay region that is locally evident as land emergence (or sea-level fall). Adding further, Martini & Glooschenko (1984, p.244) suggest that “Akimiski Island was totally submerged 7500 years ago by the early-postglacial Tyrrell sea [the forerunner of Hudson Bay and James Bay; Dean 1994] ... emersion may have been initiated approximately 3500 – 4000 yrs ago.” Taking into account that the Indigenous peoples’ archeological history in the western James Bay region goes back approximately 6000 years (Woodland Heritage Northwest, 2004), the ancestors of the Cree would have been in the area to witness the emergence of Akimiski Island, with this information becoming part of their oral history.

5.2 Cree Oral And Written History of Akimiski Island

In addition to Cree oral history from published sources, oral historical data were collected from 2007 to 2008, using the semi-directed interview format, which is culturally appropriate (Tsuji et al., 2007). Only Omushkego Cree Elders (≥ 60 years of age) were interviewed unless other knowledgeable community members were identified by personnel of First Nations organizations. Individual semi-directed interviews (n = 92; 71 males and 21 females) were conducted in person in either
English or Cree, at a location agreed upon by the participant. Oral consent for the interview was given by all participants, and some interviews were recorded (separate oral consent was obtained for this activity). It should be noted that “high” Cree (cf. conversational Cree that people of the Mushkegowuk Territory often employ) was used by some Elders in recounting their oral history; thus, members of the research team included people proficient in “high” Cree. In addition, interviews with some Elders required more than one session for various reasons (e.g., participants became tired).

During the semi-directed interview, participants were asked to recall any information related to Akimiski Island prior to the arrival of the Europeans (i.e., white man). It should be emphasized that the oral history is limited, as one Elder (interview identifier: sex and participant number, F4) suggested that her Elders would know, and another stated “50 years ago, [we] would have got a lot more information” (M6).

5.2.1 Oral History Relating to Usage of Akimiski Island

The oral history describes Akimiski Island as being bountiful with respect to food:

Before the white man arrived, the island was rich in food, geese, ducks, fish, rabbits.

(F10)

Families used to live on Akimiski because food was good, geese, ducks, fish, before the white man.

(M14)

Pre white man. Yes [Cree] hunted there. There was beaver, caribou, rabbit.

(M17)
These accounts of the bountiful resources of Akimiski Island pre-European arrival converge with the written record of the early post-contact years. For example, as reported by Father Albanel in 1671 – 72:

‘Three days’ journey into the depth of the [James] bay, toward the Northwest [northern Ontario], is a large river called by some Savages [east-coast Cree] Kichesipiou, and by others Mousousipiou, ‘Moose river,’ on which are many nations [west-coast Cree]; while on the left, as you advance, lies the well-known Island of Ouabaskou [Akimiski], forty leagues long by twenty wide, abounding in all kinds of animals...On the Island of Ouabaskou, if the Savages [east-coast Cree] are to be believed, they are so numerous that in one place, where the birds shed their feathers at molting time, any Savages or deer coming to the spot are buried in feathers over their heads, and are often unable to extricate themselves.

(Thwaites 1959: 203 – 205)

The written record also mentions the bountifulness of Akimiski Island with respect to caribou soon after first contact (Lytwyn & Lytwyn, 2002).

Perhaps the abundance of resources on Akimiski Island is the reason why several of the Elders describe Akimiski Island as being relatively highly populated prior to the arrival of Europeans:

Heard from the Elders [his Elders] that hundreds of Cree lived on Akimiski before the white man. Huge birch bark canoes were used to go across to the island. Hundreds lived on the island, just for survival as there was a lot of fishing, hunting, trapping, and berries. People never got sick.

(M6)

There is not much I could tell you about Akimiski Island before the white man had arrived here, all I could say that a lot of our people lived there.

(M9)

Before the paleskin arrived, the island was full of Native families. Guns were never used as there was no steel. Bow and arrows was used to kill game, rabbits, geese, ducks. Fishing was plenty. Beaver was also killed in those days. [There was] berry picking in summer time.
One Elder was particularly knowledgeable:

This knowledge has been passed down through the generations. Cree had always used the island as it was plentiful with wild game, game birds, caribou, rabbits, loons. People lived all around the island .... The Cree moved around the island as groups. They used caribou fences where they would herd caribou into a small opening where the hunters would be waiting to shoot [with bows and arrows] the caribou as they came through. [The caribou fence was] funnel-shaped [with] pointed sticks pointing inward so the animals had to follow the fence. Mennikamee [is the] name of fence and place [campsite or hunting ground] on Akimiski.

Cree oral history and the post-European record converge on the caribou fence issue, as Lytwyn & Lytwyn (2002, 84 - 85) writes that during “spring migration, caribou usually crossed frozen rivers, and the best method of hunting then was to build fences or hedges with snares set in them to trap the animals....did not require European technology, which suggests that caribou could be harvested easily during both spring and fall in the period before European contact.” Adding further, Lytwyn & Lytwyn (2002, 153) notes that, “In the vicinity of Albany Fort [Fort Albany], the caribou hunt was focused on Akimiski Island. The HBC [Hudson’s Bay Company] traders at Albany Fort tried on a number of occasions to open up a commercial trade with the lowland Cree hunters on the island ... [in the year] 1727.” It should be mentioned that Lytwyn & Lytwyn (2002)’s study of the Hudson Bay Lowland Cree (which included the western James Bay Cree) “delved into every corner of the Hudson’s Bay Company archives, from account books to miscellaneous files.”

Oral history also describes the first contact of Cree living on Akimiski Island with Europeans:

Ship was beached on the north side of the island. The ship stayed awhile because they came on high tide. The white men made a v-ditch in the beach to let the water come in. Four Cree came to investigate and one white man was left on guard, who was [a] cook. The others were making a ditch. [The] white man fired the gun into the air to warn his shipmates; Cree thought that they were shot at.
Evidently, Akimiski Island was occupied and used extensively by the Cree prior to European contact according to Cree oral history.

Similar to the Cree oral history documented in Section 5.2.2, Pritchard et al. (2010) report that published Cree oral history indicated that in the past the Inuit did use islands in the western James Bay region, including Akimiski Island, but the period of time was ambiguous:

Atwaywuk [the term], it is supposed to apply to the Inuit people. They came from the Bay[Hudson Bay], because the Ennui [Cree] people used to occupy the land on the West coast of James Bay[.] On the West Coast of Hudson Bay, a place at the junction they call Great Whale River, that’s up north and that’s occupied by the Inuit people on the shores, and one of the islands on the Belcher Islands in that small islands within James Bay, and the larger island we call Kamanski [sic; Akimiski]. They [Inuit] occupy that land a long time ago, and those people use to hunt seals, whales and polo [sic] bears. So, when they were a long time the Muskego [Cree] also hunt the seals, and that’s what the Inuit people hoped for they didn’t want the Muskego people to kill off the food, because the Muskego had plenty other kinds inland... The Inuit people that packed [sic, attacked] the Muskego after the European came they killed off some people.

(Bird 2002: 7)

The Cree historian Bird (1999, 15 – 16) also recounts a story that he has heard only once:

One time in the James Bay area, because the Inuit people used the in land [sic] which we call, ‘akaneski’ [Akimiski] in James Bay and also those small islands. So they used to attack a small group of families and then the whole tribe began to aware of that and they were very annoyed and they said ... ‘let us kill off if we can.’ And it happens after the European came, because the Omushkegowak [Cree] and also the Inuit did have a gun, not everybody. So when the west coast of James bay people, in a place called Ekwan and Attawapiskat and Kashachewan, they came together and they said, ‘lets go attack the Inuit people in the Akimiski Island.’ Akimiski Island, Inuit people used to live on the southeast end of the Akimiski Island and some of them to the north end .... Omushkegowak ... gathered the best 100 warriors ... So they said sail right into the end of the southeast coast of the Akimiski Island where the Inuit were camping. So they went there and they killed them off, they wanted to kill them off, all of them ... chase off into the waters these people, women and children and all and they killed them
.... Omushkegowack people failed to eliminate totally because of this shaman power [Inuit turned into seals when they entered the water] .... there is another story that says from within the west coast of James Bay ... after they [Cree] clear off the Inuit people [from Akimiski Island], they scare them off into far up north, Inuit people did not stop harassing the Omushkegowack people of the south west coast of Hudson Bay and the west coast of James Bay. They [Cree] usually attack the Inuit people from the Cape Henrietta Maria, where the ice always stuck during the month of June and July and part of August, before it’s melt.

(Bird 1999:15 – 16)

As Pritchard et al. (2010) note, the Bird (1999) story diverges from the Hudson’s Bay Company Archives, as no mention of Inuit occupation of Akimiski Island or a skirmish between Inuit and Cree over Akimiski Island, post-European contact, appears in the written record of the Hudson’s Bay Company Archives (Lytwyn & Lytwyn, 2002; HBCA, 1919 – 41, 1938 – 40). However, this time discrepancy has been resolved using Cree oral history collected here. Most Elders spoke in general terms of a Cree-Inuit conflict over the Government of Canada’s plan in the 1950s to relocate Inuit to Akimiski Island, with the Inuit ultimately rejecting the relocation plan because the environment was not to their liking (i.e., treed) (F16, M42, M58, M60, M69). However, one Elder was detailed in his account:

HBC [Hudson’s Bay Company] wanted Inuit on Akimiski [Island]. Something to do with the beaver, so that the Inuit could bring beaver and replenish the beaver as a harvest on the island .... Indian people in Attawapiskat heard that this was going to happen. [The Cree] had guns and would defend their land and kill the Inuit if they came. [Canadian] Indian Affairs heard and gave support to the Cree. HBC did go through with their [beaver] plan [but did not include the Inuit].

(M48)

There is convergence in the written record on this point, as Cummins (1992, 274) relates how an Inuit population was being considered for relocation to Akimiski Island, but in the end the Inuit were not relocated:

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A memorandum from V. M. Gran (Superintendent, James Bay Agency) to the Regional Supervisor, North Bay, dated July 5, 1962, states that “Mr. Jock Fyffe of Northern Affairs...was attempting to get possession of [Akimiski] for relocation of Eskimo people.” The point of his (Mr. Gran’s) letter was to inform the Regional Supervisor that the necessary action was being taken to insure “that [Akimiski would be] retained for Indian trapping.”

(Cummins 1992: 274)

5.2.2 Oral History Relating to the Evolution of Akimiski Island

Cree oral history also addresses the evolution of Akimiski Island:

No island [at first], just a sandbar.

(M28)

Skeleton of a whale when island just starting to form, [the island was made of] gravel [back then]. South side of the island, one of the old stories, named south side story, Whale Point.

(M35)

The evolution of Akimiski Island is of primary importance to the objective of this study, as evident from the following oral history:

Inuit at Akimiski Island first, but not know what year. Legend tells about how Akimiski Island started as a sandbar and Inuit would come to hunt seals. Once there were trees [on Akimiski Island], there were no more seals and the Inuit stopped coming. Cree also battled with the Inuit and drove them off. The Inuit never came back.

(M25)

Alright, it’s about Agamiski [sic], what my dad told me, two months ago [before he passed away]. Before white man came ... first there were Eskimos there he said, there were small trees, just a sandbar, then Eskimos ... were there because there was lots of
seals. They like them seals, them Eskimos. That’s what my dad said, and then after lots of years I guess there were trees, big ones. It’s about a seven, eight miles long now that I know. When there were trees there were no more seals. The Eskimos and Cree fought, the Eskimos left and went home.

(M71)

In Voices from the Bay (McDonald et al., 1997)—a compilation of Inuit and Cree cultural knowledge for the Hudson and James Bay regions, where 28 communities participated including the western Hudson Bay Cree community of Peawanuck and the western James Bay Cree communities of Attawapiskat, Kashechewan, Fort Albany, and Moose Factory, as well as the most southerly Inuit community in Hudson Bay, Sanikiluaq—similar stories were recorded, although not specific to Akimiski Island and not specific to seals:

Rocks are exposed on sandy beaches and shallow areas are now shoals. Shoals are forming new islands near Arviat, York Factory, Peawanuck, Lake River, Moose Factory, We-minjd, and in the Belcher Islands .... Emerging shorelines are very obvious in James Bay and along the southwestern coast of Hudson Bay where shoals have risen above sea level .... Large rocks and sandbars are now visible, and as an island in southwestern Hudson Bay slowly merges with the shorelines fewer walrus are visiting it .... A decline in local walrus numbers observed by James Bay and southwestern Hudson Bay Cree is associated with changing shorelines and habitat alteration. Walrus used to inhabit Cape Hope Island, but the depressions they made in the ground are now overgrown with willow. Lots of walrus also inhabited an island in the Winisk area until it began merging with the coastal shoreline in the early 1980s. Now they return only to visit, in groups of two or three.

(McDonald et al. 1997:37 – 42)

According to Cree oral history, the Inuit used Akimiski Island to hunt seals when the island was first emerging. However, the Inuit later abandoned the island because the number of seals at the island decreased, either because of the evolution of the island habitat, the Cree forcing them off the island, or both. What is unknown is the timing of the emergence of Akimiski Island. Of importance for the Cree to satisfy criterion 2 of the test for Aboriginal title, Cree traditional use and occupancy
of Akimiski Island needs to be established prior to the 1600s (General et al., 2017), the time of assertion of sovereignty by European nations (INAC, 1993, 2003).

5.3 Methods

5.3.1 The Emergence of Akimiski Island: Sea-Level Retrodiction

The evolution of topography and shorelines in the James Bay region is dominated by ongoing variations in sea level driven by the last ice age. Before we can retrodict paleo-topography or shoreline changes, we must know the sea level in the past relative to today.

We calculate geographically variable sea-level changes using a state-of-the-art ice age sea-level theory and numerical algorithm that accurately accounts for changes in shoreline geometry and perturbations in Earth rotation (Milne et al., 1999; Kendall et al., 2005; Mitrovica et al., 2005). The numerical algorithm (Kendall et al., 2005) outputs sea-level (and topography) changes from the start of the solution to the present day and involves an iteration that converges to present-day topography. The theory requires, on input, models for both the spatio-temporal history of ice cover since the Last Glacial Maximum (≈25,000 years before present) and Earth structure. We describe these inputs below.

For our first input, we adopt a slightly modified version of the ICE-6G ice history (Argus et al., 2014; Peltier et al., 2015). We scale the entire Laurentide ice history so that predictions of crustal uplift rates based on the model are consistent with the observed rate at the Moosonee site (9.3 ± 0.3 mm/yr; see Tsuji et al., 2016), which was determined by surveying using the Global Positioning System (NRC, 2003). The site is located at the southern tip of James Bay (Figure 5.3) and is the closest such site to Akimiski Island. The required scaling was 0.86.

For our second input, we adopt a 3-D viscoelastic Earth model preferred by Clark et al. (2019) in their study of the Cascadia region. Clark et al. (2019) demonstrated that sea-level predictions based
on this specific combination best fit observations of post-glacial decay times in Hudson Bay and James Bay, while simultaneously fitting relative sea-level histories along the Pacific Northwest coast of the United States. Aspects of the adopted Earth model are summarized in Figure 5.4. The model is characterized by an elastic lithosphere with a thickness that varies globally (Conrad & Lithgow-Bertelloni, 2006) and includes tectonic plate boundaries. The lithospheric thickness under the Hudson Bay and James Bay region (i.e., cratonic Canada) is in the range of 120 – 140 km (Figure 5.4A). The spherically averaged (i.e., depth dependent) component of the viscosity field matches the VM5a viscosity profile to which the ICE-6G model is coupled (Argus et al., 2014; Peltier et al., 2015). Beneath the lithosphere, the viscosity field varies by about an order of magnitude in the upper mantle (i.e., above 670 km depth in the mantle) beneath the Hudson Bay and James Bay region (Figure 5.4B – D). Finally, we note that the Earth model has a 1-D elastic and density structure given by the seismically inferred model PREM (Dziewonski & Anderson, 1981).

All sea-level calculations are performed using finite-volume software covering a volume that extends from the core-mantle boundary to the Earth’s surface (Latychev et al., 2005). The computational domain is discretized using tetrahedral elements with a spatial resolution that varies from 12 km at the surface to 50 km at the base of the mantle.

To retrodict past shoreline locations we superimpose the computed changes in sea level onto a present-day topography grid. We reconstruct the topography within James Bay over the past 2500 years using the sea-level methodology described above and a present-day topography grid for the region given by SRTM30 (http://www.webgis.com/srtm30. html), which has a spatial resolution of 900 m. Our focus in this application is in estimating the time of emergence of Akimiski Island. The island presently has an area of 5180 km² (NASA, 1997), and its western edge lies ~16 km (NASA, 1994) from the Province of Ontario (Figure 5.4 and Figure 5.5: bottom right).

Finally, in reconstructing past topography and shoreline changes, we also add a modern global change - induced sea-level rise (associated with recent ice mass flux and thermosteric and dynamic
Figure 5.4: Earth Model. Aspects of the 3-D viscoelastic Earth model adopted in the main simulation. (A) Lithospheric thickness variation (km) across the Hudson Bay and James Bay regions. (B-D) Viscosity variations at three depth slices (as labeled at bottom right). The values on the figure refer to the logarithm of the viscosity variation relative to the 1-D background model, VM5, in the upper mantle \((0.5 \times 10^{21} \text{ Pa s})\).
effects) of 1.2 mm/yr across the 20th century, following Tsuji et al. (2016), based on a probabilistic analysis of global sea-level changes since 1990 (Hay et al., 2015). We note that varying this rate from 0.0–2.4 mm/yr changes the estimated emergence time discussed below by only ± 10 years.

5.3.2 Sensitivity Tests

To test the sensitivity of our predictions to variations in the ice history and Earth model, we performed a series of additional simulations in which we varied both inputs. More specifically, we explore the sensitivity of the estimate of the emergence time of Akimiski Island by running nine additional simulations of GIA based on 3-D viscoelastic Earth models. We will refer to the simulation using the ice and Earth models described above as simulation number one (henceforth Sim1), and describe the nine additional simulations below.

Simulations 2 and 3 are identical to Sim1, with the exception that our scaling of lateral viscosity variations (e.g., Figure 5.4B–D) is increased by a factor of 2.7 and 7.4, respectively. These viscosity fields are all ultimately based on heterogeneity inferred from the seismic tomography model S40RTS (Ritsema et al., 2011).

Simulations 4 and 5 are identical to Sim 1 with the exception that the underlying seismic tomography fields are given by the Savani model of Auer et al. (2014) and the SEMUM2 model of French et al. (2015), respectively. Both models have lateral viscosity variations that are tuned to be comparable to the variations in Sim1 (Figure 5.4B–D).

Simulation 6 replaces the lithospheric thickness and mantle viscosity variations in Sim1 by the Earth model derived by Hoggard et al. (2020) and Richards et al. (2020).

Simulation 7 replaces the lithospheric thickness model used in Sim1 (Conrad & Lithgow-Bertelloni, 2006) with the model of Watts (2001).

Simulation 8 is identical to Sim3, with the exception that the spherically averaged viscosity model adopted in the latter (VM5) is replaced by a model with upper and lower mantle viscosities of
$5 \times 10^{20}$ Pa s and $5 \times 10^{21}$ Pa s, respectively.

All the above simulations use the ICE-6G history (Peltier et al., 2015). Simulation 9, in contrast, adopts the ICE-5G ice history and assumes the spherically averaged Earth structure of VM2 (Peltier, 2004). Simulation 10 adopts the ANU ice history, spherically averaged viscosity structure characterized by upper and lower mantle viscosities of $1.5 \times 10^{20}$ Pa s and $5 \times 10^{22}$ Pa s, respectively, and a global average lithospheric thickness of 48 km (Lambeck et al., 2014). Simulations 9 and 10 have lateral viscosity variations that are tuned to be comparable to the variations in Sim1 (Figure 5.4B–D).

In all 10 cases, the Laurentide ice history is scaled to match the GPS-derived uplift rate at Moosonee ($9.3 \pm 0.3$ mm/yr; NRC, 2003; Tsuji et al., 2016). For Sim1, the required scaling was 0.86, and for the remaining simulations, the scaling varied from 0.80 – 0.89.

5.4 Results

We estimate the emergence of Akimiski Island using our sea-level retrodiction. Figure 5.5 shows a simulation of changes in topography and shoreline location in James Bay for six time slices extending back to 2.5 ka. Areas in shades of blue on each frame are water covered. As previously noted, the region is currently experiencing a sea-level fall at a rate of $\sim$1 cm/yr and thus land has been progressively emerging since the area became ice-free during the Early Holocene. On the basis of this simulation, we estimate that Akimiski Island first emerged from James Bay at 2030 years ago (see Figure 5.5, top middle, southern coast) and reached 50% of its current areal extent 1000 years ago.

The sensitivity study yielded nine additional, but comparable, estimates of the emergence time based on simulations that varied both the ice history and the Earth model. Figure 5.6 provides estimates of the emergence time of Akimiski Island (in years before present) for all 10 simulations. These estimates show reasonable consistency and range from 1870 – 2083 yrs. Their mean value, with one standard deviation uncertainty, is $1985 \pm 78$ yrs. We therefore suggest an emergence time.
Figure 5.5: James Bay Paleotopography. Topography in the James Bay region for six time slices spanning the period from 2500 years ago (2.5 ka) to present-day (0.0 ka), as labeled on each frame. The snapshots of paleotopography are reconstructed as described in the text. Present-day (0.0 ka) shorelines are presented as dotted lines in the time slices. Prediction is based on the ice and 3-D Earth model of Simulation 1.
of Akimiski Island of 2000 ± 100 years.

We note that that relative sea level (RSL) histories predicted by all 10 3-D models are consistent with observations from the western James Bay region. Figure 5.7 shows predictions of relative sea level (RSL) change over the past 8 kyr in western James Bay for all 10 simulations superimposed on a composite RSL history compiled by Vacchi et al. (2018, Region 3). The predictions are consistent with the observations and they capture the extent of uncertainty explicit in the RSL record.

5.5 Discussion and Conclusions

Clearly, Cree traditional use and occupancy of Akimiski Island was “sufficient to be an established fact at the time of assertion of sovereignty by European nations” (INAC, 1993:5, 2003:8); thus, fully addressing criterion 2 of the test for Aboriginal title. Indeed, our sea-level retrodiction has time stamped the Cree oral history of the emergence of Akimiski Island to 2000 ± 100 years ago,
Figure 5.7: Relative Sea Level Predictions in James Bay. Predictions of RSL change in western James Bay over the past 8 kyr generated using all 10 GIA simulations described in the main text and appendix, superimposed on observational constraints compiled by Vacchi et al. (2018) in their (western James Bay) Region #3. The predictions are generated at a site located at the mean position of all sites in Region #3.
and the Inuit leaving Akimiski Island to the time period 2.0 to 1.5 ka. Five hundred years (i.e., 2.0 to 1.5 ka) would be sufficient time for an emergent sandbar (i.e., seal habitat) to evolve into boreal forest (see Martini & Glooischenko 1984, for a detailed description of emergent coasts of Akimiski Island). As the Cree have addressed all the criteria of the common-law test for proof of Aboriginal title with respect to Akimiski Island (General et al., 2017, and the present study) while, the Inuit have not (Pritchard et al., 2010), the Cree have sufficient basis to enter into the Recognition and Implementation of Indigenous Rights process. This new process replaces the Comprehensive Land Claims policy (Crown-Indigenous Relations and Northern Affairs Canada, 2018, 2019).

Akimiski Island is important for social and cultural reasons: the Cree continue to hunt, fish, trap, and gather (e.g., berries, medicinal plants, wood) on the island, and the island is dotted with graves, spiritual sites, and Cree seasonal camps (Tsuij et al., 2011; General et al., 2017). In addition, the mineral wealth of the Far North region of Ontario is well documented (Gamble, 2017; Ministry of Energy, Northern Development and Mines, 2019; Northern Ontario Business, 2020). Indeed, Akimiski Island has already been prospected for diamonds (Tsuij et al., 2009); thus, land rights have substantial economic implications for the First Nations people of this region (Tsuij et al., 2016).

It is unfortunate that the Government of Canada does not follow its own policy with respect to the common-law test for proof of Aboriginal title when settling comprehensive land claims, and puts the onus of responsibility of proving Aboriginal title on groups who have limited financial resources. As noted by Senator Lorna Milne of the Government of Canada:

> many of the complaints [boundary and Aboriginal title issues related to Akimiski Island] were originally with the Nunavut Act itself. That is when they should properly have been addressed. Unfortunately, they were not addressed at that time. You [First Nations representatives] are quite right: the [Canadian] government did not do its job.

(Parliament of Canada 1999: 33)

Lastly, the word Akamaski [Akimiski] is derived from Cree words—Aka (across) and Aski (land)—

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that is, “saying that there is land across here” (M48).
Conclusion

The cryosphere, solid Earth, and oceans are inherently coupled. A better understanding of the Glacial Isostatic Adjustment (GIA) process allows for improved monitoring of ice sheets and better understanding of past and future ice sheet stability. For the majority of this thesis, we have focussed on scenarios involving mass flux from the West Antarctic Ice Sheet (WAIS). The ice sheet sits above a hot (and therefore low viscosity) upper mantle and thin lithosphere. GIA-related changes to bed topography will impact local ice behavior and sea level on a global scale. Broadly, our investigation
was motivated by the following questions: How does 3-D Earth structure affect predictions of sea level change, geodetic observations, or our ability to infer Earth structure and processes? We have addressed elements of these questions in each of the chapters of this thesis.

In Chapter 2, we explored the sensitivity of sea-level predictions to 3-D mantle viscosity structure. We found that in the near field, viscous deformation will be a significant contributor to the sea-level signal, even in the case of unloading on “short” timescales of decades to centuries. Next, we showed that ongoing viscoelastic uplift of the marine based sectors in response to collapse of WAIS will act to push meltwater out to the global ocean. Our modeling indicates that previous estimates for WAIS’s contribution to global mean sea level during interglacial periods has been underestimated by as much as 30%. The exact timing and magnitude of this outflux is dependent on the 3D mantle viscosity structure and the spatio-temporal pattern of ice retreat. The expulsion mechanism will be active whenever the grounded portion of marine-based ice sheets retreats. The water expulsion mechanism is partially balanced by viscous subsidence of the nearby peripheral bulge, and thus the net sea-level signal will also depend on the local mantle viscosity of oceanic regions outside the glaciated sectors. Finally, we turned to a case study of protracted WAIS collapse. Our goal was to test how effective 1-D models may be in estimating the GIA sea level signal, given that 3-D Earth structure exists globally, not just in the Antarctic. In this case, we found that 1-D models can be reasonably accurate for predictions at far-field sea level sites, but, not surprisingly, the degree of misfit between 1-D and 3-D calculations will depend on the viscosity variability considered.

In the following two chapters, we explored the impact of 3-D viscoelastic structure beneath Antarctica on geodetic observations and inferences that have been based upon these observations. Specifically, in Chapter 3, we considered GPS measurements of crustal deformation and satellite gravity observations. We demonstrated that deformation arising from such structure will be imperfectly modeled by standard calculations that only consider radially-varying (1-D) Earth structure. Moreover, our results highlight the extreme sensitivity of horizontal crustal rates (rarely considered
in modeling analyses of the region) to both 3-D mantle structure and ice history. These findings motivated the study in Chapter 4, in which we addressed the following key question: How are inferences of 1-D mantle viscosity based on GPS network measurements related to the underlying 3-D viscosity field? We demonstrated that the inferences can differ by an order of magnitude from regional averages of 3-D structure, and that 1D modeling will not capture the extreme range of lateral viscosity variations below WAIS.

In Chapter 5, we presented a case study highlighting the importance of accurate GIA modeling on a societal questions that extend beyond modern climate. Specifically, we used sea-level modeling to investigate the timing of emergence of Akimiski Island from James Bay, Canada. In doing so, we timestamped human occupation of the island predating European contact. Our analysis is currently being used to support a land claim by the James Bay Cree based, in part, on their oral history.

Future efforts will focus on time periods in which we have relatively plentiful data. For example, estimates of individual ice sheet contributions to sea level from the Last Glacial Maximum to the present, including the sources of several so-called meltwater pulses during this period, remains an ongoing challenge. Better constraints in this regard are important for improving the accuracy of predictions of ongoing ice age effects in the modern world, which are routinely used to correct modern geodetic measurements of processes (e.g., present-day rates of global sea-level change) active in our progressively warming world.

The Antarctic Ice Sheet may in particular require re-appraisal for its budget and contribution to post-LGM sea level rise. Ice sheet and climate modeling efforts consistently indicate that excess ice volumes during the LGM were \( \sim 7 \) m in units of equivalent global average sea-level rise during the deglaciation (Whitehouse et al., 2012; Golledge et al., 2013). GIA efforts to budget out the deglaciation sea-level signal via analysis of far field sea level records, however, require the Antarctic Ice Sheet to contribute a much higher value of 15-30 m (e.g., Lambeck et al., 2014). This inconsistency is termed the “missing ice” problem (Simms et al., 2019).
Complicating this issue is Antarctica’s relative paucity of data compared to other glaciated or formerly glaciated regions. In North America, dated moraine deposits on land can be used to demarcate the extent of the ice sheet over time. In Antarctica, such markers are often difficult to access (they are sometimes submerged) and to date due to inherited radiocarbon. An additional complication, however, is that both the interpretation of far field sea level records and the construction of AIS ice models are typically based on predictions using 1-D models of Earth structure. We have shown that 3-D Earth structure will impact sea level and crustal deformation in both the near field and far field. We hypothesize, therefore, 1) that inconsistencies seen between far field sites may partially be due to lateral variations in the Earth’s 3-D viscosity structure and 2) that the Antarctic Ice Sheet may have been larger during the Last Glacial Maximum than climate models imply.

A first test of these hypotheses will involve exploring whether the incorporation of lateral viscosity variations can improve the fit of predictions to the full suite of far-field sea-level records. Another approach would be to generate new ice history models from coupled ice sheet/GIA simulations. One can build such reconstructions of the AIS evolution from the LGM to present using the methodology of Gomez et al. (2018), in which an ice sheet model is coupled to a high-resolution GIA model of sea-level change that incorporates 3-D variability in mantle viscoelastic structure (Latelychev et al., 2005). Given the significant data gap and the range of uncertainties associated with both the inference of 3-D mantle viscosity and the parameters adopted in the ice sheet simulations (e.g., the basal sliding coefficient), the best way forward may be to build a large suite of post-LGM AIS reconstructions. This effort would mirror the work of (Tarasov et al., 2012) for the North American ice complex, who used coupled GIA/ice sheet models and explored the parameter space to isolate a distribution of glaciologically-self-consistent deglacial histories (rather than just one model) that was calibrated against available data.

A rigorous analysis of data, as discussed in Chapter 4, requires that one quantify the intrinsic resolving power, or sensitivity, of these observations to GIA modeling inputs. These data include
GPS, gravity, or sea-level observations, and the inputs include both the 3-D mantle viscosity field and the ice mass history. Unfortunately, the large parameter space associated with GIA modeling inputs makes the usual computation of partial derivatives (Fréchet kernels) unfeasible for this purpose. However, adjoint modeling techniques (Crawford et al., 2018) may be used instead to calculate these kernels. Combining them with gradient-based optimization techniques suggests the possibility of inverting all available sea-level to construct a 3-D viscosity field beneath Antarctica (and its uncertainty).

In this thesis, we have demonstrated the importance of a sophisticated and nuanced understanding of solid Earth processes in analyzing GIA observations. A broad suite of relevant observations – including GPS measurements of crustal deformation rates, satellite and land-based gravity anomalies, and geological and geodetic records of sea-level change – can be brought to bear to test a wide range of hypotheses for critical events in modern and past climate of the Earth.
References


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Rupert’s Land and North-Western Territory – Enactment No. 3 (1870). Part 1. Order of Her Majesty in Council admitting Rupert’s Land and the North-Western Territory into the union, dated the 23rd day of June 1870.


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Woodland Heritage Northwest (2004). Stage one and two project report archeological/cultural heritage potential site assessment for southwest alternative winter road right of ways from Hearst northerly approximately 320 km to the Victor diamond project site. 134 College Street, Thunder Bay, Ontario: Woodland Heritage Northwest.
Woodward, R. S. (1888). On the form and position of the sea level with special references to its
dependence on superficial masses symmetrically disposed about a normal to the earth’s surface.
Report 48.

Wu, P., H. Wang, & H. Steffen (2013). The role of thermal effect on mantle seismic anomalies
under Laurentia and Fennoscandia from observations of Glacial Isostatic Adjustment. Geophysical

Wörner, G. (1999). Lithospheric dynamics and mantle sources of alkaline magmatism of the

Geophysical Research: Solid Earth, 121 (11), 7790–7820.

Yau, A. M., M. L. Bender, A. Robinson, & E. J. Brook (2016). Reconstructing the last interglacial

Zhao, C., M. A. King, C. S. Watson, V. R. Barletta, A. Bordoni, M. Dell, & P. L. Whitehouse
(2017). Rapid ice unloading in the Fleming Glacier region, southern Antarctic Peninsula, and its
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