The Traditional Ordering of College Preparatory Math Courses, and an Evaluation of a Non-Traditional Ordering

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This work is dedicated to my parents, Morris and Grace Orzech,
and to my lovely wife, Doctor Sharon Antonucci.

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Abstract

A math course sequence consisting of a course in algebra and a course in geometry, followed by another course in algebra, has been an enduring feature of U.S. high school curriculum since the later 19th century. In recent decades, there has been interest in alternative math content, and curricular arrangements of math content, but a traditional sequence of courses, commonly titled Algebra I, Geometry, and Algebra II, has endured in most locales. Interestingly, with many reforms of the high school math curriculum having been proposed over the past century, almost nothing has been written on the possibility of changing the order of the traditional sequence while maintaining the three-course structure. In Chapter One of this dissertation, I recount the history of the three-course sequence, of alternatives that have been proposed, and of an ongoing trend towards encouraging more students to complete more of the sequence. In Chapters Two and Three I undertake an empirical analysis of the experience of a large urban district that mandated that all students follow a non-traditional sequence of consecutive Algebra I and II, followed by Geometry. I find no evidence of positive effects from this mandate, and substantial evidence of harmful effects, both direct and indirect. In Chapter Two I study Algebra II outcomes, finding that, conditional on Algebra I performance, students earned lower grades in Algebra II when they enrolled in the two courses consecutively. In Chapter Three I study Geometry outcomes. I find that students were less likely to enroll in Geometry when they delayed enrollment by an additional year, and that when the difficulty of the Algebra II course increased, it had a negative impact on student test scores in Geometry in the next year.

Chapter One

Introduction

What is the origin of the Algebra I – Geometry – Algebra II course sequence that is an enduring feature of the American high school curriculum and of the expectation, in the early twenty-first century, that every student will complete some or all of this sequence? The 1893 report of the Committee of Ten offers a temptingly easy answer: this committee, chaired by Harvard President Charles W. Eliot, proposed such a sequence as part of its attempt to codify a uniform curriculum for the nation's high schools. The report further stated that this sequence (or at least the first part of it) would be appropriate for all students, regardless of their future goals. The current situation, therefore, could be seen as the inevitable fulfillment of the vision of an august group of educators. Upon further examination, however, the history is more complicated: Rates of high school math enrollment dropped sharply in the decades after the Report, and rival curricular visions emerged regularly throughout the next hundred years. Nonetheless, 125 years after Charles Eliot's Committee released its report, its vision is closer than ever to realization.

In this chapter I argue that three things largely explain the current success of the traditional sequence. The first relates to the circumstances of the sequence's birth: The Committee of Ten justified its prescriptions based on the theory of mental discipline which, although eventually set aside by professional psychologists, seems to inform popular views of math education's purpose through the present day. This established the standard curriculum as a gold-standard of intellectual rigor, and offered a measure of protection from demands that the curriculum perpetually justify and reform itself in relation to a developing body of knowledge and practice, as did fields such as history or biology.[1] The second relates to its content, specifically its prescribing a secondary curriculum dominated by algebra: Whatever the original rationale for this, heavy emphasis on algebra would become indispensable in the second half of the twentieth century when calculus emerged as a principal goal of secondary math education. These things established the conditions that any serious candidate to replace the standard sequence would have to meet: it would have to prove itself at least as "rigorous" as the standard sequence, and provide at least as strong a preparation for calculus, or else offer something that people considered more important. No proposed alternative has succeeded at meeting these conditions, and although one can imagine alternatives that would, I will consider possible reasons why they have not emerged.

The third factor in the sequence's ultimate (or at least current) success is different in type, in that it is largely indifferent to the particularities of the sequence itself: In the mid-1980s there was a confluence of political, social, and economic forces that mitigated strongly in favor of curricular uniformity, not only across classrooms and schools, but also across districts and states. Largely due to the two factors mentioned above, the traditional sequence was the strongest candidate at that moment in history to be the consensus choice.

[1] Of course, the idea that the objects of mathematical knowledge differ in type from the objects of other types of knowledge did not originate with the Committee of Ten. What the Committee did was enshrine the idea that working with these objects developed mental faculties that were transferable to other fields of endeavor.

I begin this chapter with a prologue describing the background to, and work of, the Committee of Ten, which largely articulated the standard sequence, and framed its place in the broader curriculum. I then briefly describe the decades from 1893 to 1958, to illustrate some of the pressures that worked against math education as an integral part of secondary curriculum. I follow this with three sections on the reasons for the traditional sequence's survival and success, and a conclusion in which I consider future prospects for reform of the traditional sequence, particularly the ordering of the courses.

In the first of these sections, I consider why curricula that have aimed to emphasize abstract ideas and the drawing of conceptual linkages, and to subordinate computational practice and drills, have not had more success. I principally discuss the New Math of the 1960s and, in less detail, the Math Wars of the 1990s, to illustrate how such attempts to bring school math in line with the academic discipline of mathematics often fail: An archetypal criticism of such reform curricula is that they lack rigor.[2] Although I render a mixed judgment on the fairness of this criticism, it reflects the longstanding cultural status of the traditional sequence, and does have some basis in fact.[3]

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[2] The theory of mental discipline seems to have gone from being self-evident to being discredited without ever providing a truly rigorous definition of "rigor". The popular conception of a rigorous math curriculum probably means primarily a curriculum in which there are clear and present standards of "success" and "failure", "right" and "wrong", and probably also a curriculum that students find at least moderately challenging.

[3]They also reflect a tension within the modern university between research mathematics (or at least a subpart thereof), and other disciplines that depend on math departments to train their students. It is somewhat difficult, and beyond the scope of this chapter, to address the ways in which this does or does not matter for the fate of curricula in districts and states, although it has multiple points of tangency to the section on Calculus.

In the second section, I consider why the traditional sequence's emphasis on algebra has made it, if not indispensable, at least advantaged relative to alternative curricula that have been proposed in recent decades. This discussion centers on the dominant place of calculus in math education, which reinforces demand for a high school curriculum emphasizing algebra. Training in calculus requires strong intuition for the behavior of a wide variety of functions (principally continuous ones), and the ability to fluently and accurately manipulate their algebraic representations. A principal avenue to supplanting the traditional curriculum with something less focused on algebra would be to replace calculus as the end-goal of the curriculum with material that makes less intensive use of algebra.[4] I describe the two principal alternatives that have been proposed, discrete math and statistics, and consider why they have not (at least so far) enjoyed more success.

In the third section, I describe the emergence of socio-political forces since the early 1980s that have created demand for a national consensus around what math students should study, and for curricula that would be amenable to standardized testing - demand that I argue the traditional sequence was uniquely positioned to meet. In this section, I describe the period from the 1983 publication of *A Nation at Risk* to the present, in which the standard sequence not only maintained its place as the predominant college-preparatory curriculum, but also came to be followed by the overwhelming majority of the nation's high school students.

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[4] I use "algebra" here in its common, rather than its contemporary technical, sense, to mean generalized arithmetic: lines of expressions and equations that must be manipulated and re-expressed..

I conclude by posing an additional question: Of all of the reforms proposed to the standard sequence, why does the literature contain barely a passing reference to the possibility of changing the course ordering? I offer some thoughts on this and, in the second and third chapters of this dissertation, offer the first (to my knowledge) empirical study of such a reform.

Historical Background

The Committee of Ten

In late nineteenth century America, there existed a "wide gap" between elementary and college education, as characterized by Theodore Sizer (Sizer, 1961). Although the large majority of students completed their formal education by fourteen years of age (eighth grade, for those in graded schools) there was broad variation in the character of the institutions attended by those who continued on, generally with a view to attending college, or entering a profession. Some attended "grammar schools" (Sizer, 1961), which provided a core curriculum of Latin, Greek, and mathematics, intended to train and discipline students' minds in preparation for college study and, ultimately, religious and secular leadership roles. Others attended "academies", which took a more utilitarian perspective on subject matter, offering training in modern languages such as French and German, and contemporary science and technology (Sizer, 1961). Still other students attended "common schools", which were publicly funded and administered by local authorities, and offered curriculum of highly variable content and rigor, determined by local needs and capacities (Sizer, 1961).

In the decades after the Civil War, the fragmented nature of secondary education grew increasingly unsatisfactory as colleges began to evolve from being finishing schools for largely parochial elites, to being training institutes for an increasingly national, and ultimately global, technocracy. Previously, most students planning to attend college would have planned to attend a specific college, and would have attended a nearby secondary school[5] with a curriculum specifically tailored to that college's entrance requirements: Famous examples include the Boston, Cambridge, and Roxborough Latin Schools, which specialized in preparing students to study at Harvard College. In the latter third of the nineteenth century, several forces were rapidly transforming American society, and educational demands: First, rapid urbanization, and immigration flows from southern and central Europe, expanded school enrollments and raised demands that schools serve as a force for civic cohesion. Second, science and technology took on new salience due not only to rapid industrialization and mechanization of the economy, but also the prestige attaching to the work of figures such as Charles Darwin and Herbert Spencer (Sizer, 1961). This created demands on colleges to provide relatively specialized technical training in new and diverse areas.

The Committee of Ten was created by the National Education Association in the early 1890s in response to the need for a common understanding across cities and regions of what the nation's colleges could expect of students who had completed a course of secondary schooling. The committee was chaired by Harvard University President Charles W. Eliot, and the majorities of both the main committee and its sub-committees

[5] This could have been an institution of one of the types mentioned above, or a pre-college institution operated by a college (Sizer, 1961).

were comprised of university presidents and faculty, with school principals and teachers in the minorities. Although my principal interest is the Committee's legacy for math education, it is necessary to understand the place of mathematics in the broader context of the committee's work.

The Committee appointed a number of subject area "conferences", which issued individual reports in addition to the Committee's overall report. The final report indicates that the work of the Latin, Greek, and Mathematics conferences had primacy: It states that the members of other conferences (Physics, Astronomy, and Chemistry; Natural History; History, Civil Government, and Political Economy; Geography; English; and Other Modern Languages) "ardently desired to have their respective subjects made equal to Latin, Greek, and Mathematics, but they knew that educational tradition was averse to this desire, and that many teachers and directors of education felt no confidence in these subjects as disciplinary materials."[6] The principal justification for teaching Latin, Greek, and mathematics, then, was their ability to "discipline" the mind; the transmission of content was (at least ostensibly) a distinctly secondary goal.[7]

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[6] To underline what a different world from our own this report comes from, note that Latin and Greek were each assigned to their own sub-committee, while "History, Civil Government, and Political Economy" which in a modern university might occupy three or four departments and two professional schools, were considered a single area of concern.

[7] Although not stated in the report, it is difficult for an early twenty-first century reader not to perceive, as well, a desire to transmit a cultural patrimony. The study of Latin and Greek would have the (less than) incidental effect of grounding students in what we now call the Western canon. It seems likely that, for educated men of Charles Eliot's generation, the idea of an educated person not familiar with the major texts of the ancient world would have been oxymoronic – one might therefore be tempted to read the invocation of mental discipline as providing these children of the Enlightenment with cover for fundamentally atavistic urges.

The math sub-committee recommended that that the study of "systematic algebra" (NEA, 1894, p.23) begin in ninth grade with five hours of weekly study, and that in tenth and eleventh grade math courses be equally divided between geometry and algebra.[8] In twelfth grade, "trigonometry and higher algebra"[9] was to be offered to "candidates for scientific schools" (NEA, 1894, p.35). This curriculum was justified by reference to the habits and dispositions of mind it would inculcate in the learner, rather than the applicability of the knowledge gained to any particular area of endeavor:

Training in geometry, which had its basis in the work of Euclid, was historically largely of a piece with training in Latin and Greek. From this perspective, it might make more sense to ask why math education includes anything other than geometry, than to demand a rationale for geometry itself. Nonetheless, the Committee justified it by saying that "whatever [this] training may accomplish for [a student] geometrically, there is no student whom it will not brighten and strengthen intellectually as few other exercises can." (NEA, 1894, p.116).

The more interesting case is algebra which, as a distinct type of mathematics, was of more recent provenance.[10] The report again offered no specific field of endeavor for

[8] One anomaly is that while the Committee proposed that sophomore and junior year both be divided equally between algebra and geometry, this seems to have been implemented as (or evolved into) sophomore year being taken up entirely with geometry, and junior year with a more advanced course in algebra.
[9] This is a forerunner of the course now commonly known as "Precalculus". Determining the origin of this course title is beyond the scope of the current project, but anecdotal evidence suggests that as late as the 1980s it was not yet in widespread use.
[10] The establishment of algebraic knowledge as (more or less) fully distinct from geometric knowledge can be dated to Euler's 1765 *Elements of Algebra.*

which the study of algebra was to serve as preparation but, as with geometry, explained how it develops distinct virtues of the mind:

> Oral exercises in algebra, similar to those in what is called "mental arithmetic," are recommended. Such exercises are particularly helpful in conducting brief and rapid reviews. *Quickness and accuracy* in both oral and written work should be rigidly enforced. (NEA, 1894, p.112, italics added)

The invocation of "quickness and accuracy" as its own justification might be understood against the background of rapid industrialization, and particularly the spread of assembly-line processes, in the post-Civil War era.[11] Speculation about the conscious or unconscious motives of the Committee of Ten aside, however, the fact is that the study of geometry and algebra was intended to build minds that were "disciplined", "quick", "accurate", "bright", and "strong" in activity and expression.

These animating values of mental discipline would continue to hold cultural sway long after they were discarded by the field of psychology: The idea that the brain possessed such faculties in way that would transfer freely across different activities was discredited by psychologists by the early twentieth century (Garrett & Davis, 2003). Nonetheless, as late as the 1990s, math instruction that did not reward students for being quick, and accurate (i.e. disciplined) in executing computations would fall under popular suspicion of making students mentally dull, and weak.

[11] Historically, there is a pattern of the brain being understood in terms of the most current technology of the day. Gary Marcus discusses this in, among other places: https://www.nytimes.com/2015/06/28/opinion/sunday/face-it-your-brain-is-a-computer.html Retrieved 11/30/2017.

Math Education in the Unites States from 1893 to 1958

Although the available evidence suggests that the Committee's prescriptions were not implemented on any large scale in the period following the report (e.g. Dexter, 1906), it is evident that they survived the subsequent decades, presumably as the standard preparatory course for the relatively small subset of students who went on to postsecondary education. The first serious attempt to replace the standard sequence did not arise until the 1950s, when a post-World War II wave of émigré European mathematicians had settled in American universities, and postwar technological competition with the Soviet Union had taken hold.

While the intervening six decades defy easy characterization, a key development that separates the postwar period from the late nineteenth century is an enormous increase in the rate of high school enrollment: Even with the rapid expansion after the Civil War it was barely above 10% in 1893; by 1960, high school enrollment was well on its way to being universal (Snyder, 1993). High schools' mission had expanded from preparing students to join a middle-class, white-collar milieu, to serving the full spectrum of needs of the entire American adolescent population. As the high school population grew, many students simply opted not to study math (Progressive Education Association, 1940, p.10), and those that did often enrolled in locally designed courses focused on the application of specific computational techniques to specific technical problems (Kliebard & Franklin, 2003).

Reform and Rigor

The New Math

Largely led by university research mathematicians, the New Math aimed to grow the nation's pipeline of mathematical talent by creating continuity from K-12 study to undergraduate and graduate-level study, a major goal of the broader education reform movement of that period (e.g. Beberman, 1958; Bruner, 1960; Dow, 1991).[12] The emergence of the New Math in the 1950s (and not earlier) was due to two factors. First, the influx of research mathematicians holding Ph.D.'s to American universities following the Second World War (Tucker, 2013): the division of mathematical knowledge into algebraic and geometric species was an artifact of the late Enlightenment that was on its way to obsolescence by the time of the Committee of Ten Report, but newer developments were confined to England and the European Continent until the late 1940s.[13] Second, although the first New Math-type project was launched in 1951 (by the University of Illinois Committee on School Mathematics (UICSM) under the leadership of Max Beberman), the 1958 launch of Sputnik sparked a broader set of (federally funded) initiatives aimed at increasing the flow of high school graduates into scientific and technical college majors.

[12] A more complete account of the New Math movement would also note the influence of the latest work of child development theorists, particularly Harvard psychologist Jerome Bruner. Bruner is perhaps most famous for his hypothesis that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (Bruner, 1960, p.33), and he took leave from Harvard during this period to lead development of an innovative middle grades social studies curriculum. Titled *Man, a Course of Study* (MACOS), it attempted to inculcate the habits of mind of professional anthropologists and archaeologists in adolescent learners (see Dow, 1991 for a complete account of this project).

[13] Important figures in this vanguard were David Hilbert in Germany, Russell and Whitehead in England, and "N. Bourbaki" (actually the pseudonymous name used by a group of mathematicians) in France. I offer a brief account of the historical bifurcation and reunification of mathematical knowledge in the appendix.

The hallmark of most, if not all, New Math curricula, was integrating content around a small number of unifying concepts with an emphasis on making connections and seeing unity, rather than the serial presentation of algebraic and geometric topics with an emphasis on a defined set of proofs and algorithms.[14] Much development in mathematics consists of abstracting common properties of familiar objects, and then describing those objects as instantiations of a more general class. The degree of intellectual ambition embodied in such curricula makes them likely to be very difficult for teachers to implement faithfully, especially those who lack deep mathematical training.

Although initially welcomed with enthusiasm by both the general public (Mueller, 1966) and the academy (Duren, 1967), the New Math came under criticism beginning in the mid-1960s due to a widespread perception that students studying from these curricula were being denied foundational technical skills. This criticism arose not only in the popular media (Mueller, 1966), but also from respected scientists and mathematicians (e.g. Feynman, 1965; Kline, 1973). Concerns were compounded by a decline in SAT Math scores that began in the late 1960s (Usiskin, 1985).[15]

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[14] This reflects that fact that in the post-war period in math departments in leading American universities were dominated by the Bourbaki movement (founded in Paris in 1934 and publishing under the pseudonym Nicolas Bourbaki). Bourbaki's goal was to provide a new foundation for mathematics, based on unifying structures that could bring together apparently diverse sub-disciplines (Atiyah, 2007). One of the most common structures that New Math textbooks used to unify mathematical concepts was the "set" (Usiskin, 2003): An important example is offered by introductory texts of the School Mathematics Study Group (SMSG) (a major National Science Foundation (NSF) funded reform effort undertaken in the late 1950s), Introduction to Algebra and First Course in Algebra, which both begin with chapters on sets: Set theory was invented by German mathematician Georg Cantor between 1874 and 1884 (Johnson, 1972), placing it in the era of mathematical development that the proponents of the New Math wanted to draw into the school curriculum.

[15] Powell and Steelman (1984) consider this trend, which continued through the early 1980s, and argued that the decline in scores was at least partially attributable to changes in the demographic composition of the test-taking population.

It is a low bar for the historiographer to demonstrate that New Math curricula were widely perceived as lacking rigor, in the sense of not training computational proficiency. One might observe that the most famous and influential critique was a book titled "Why Johnny Can't Add"[16] (Kline, 1973). One might look to Tom Lehrer's satirical song "New Math", which notes that taking away seven from thirteen leaves five, "well, six actually; but the idea is the important thing!"[17] One might also note that the New Math was supplanted by something called the "Back to Basics" movement. The harder work is looking beyond this simplification, and finding some room for nuance: It is implausible that professional mathematicians would have thought it unnecessary for students to be able to perform arithmetic accurately and fluently. The evidence suggests, rather, that they underestimated the difficulty of imparting this to students, let alone of doing so while grappling with strange and unfamiliar teaching materials.

Max Beberman criticized as a "national scandal" the "undue haste" with which the New Math was introduced in schools,[18] and Kline (1973) criticized its fundamental diagnosis that curriculum was the key problem to be addressed (e.g. p.12) suggesting, on the contrary, that the problem was a lack of qualified teachers (p.17). While it is beyond the scope of this chapter to evaluate this claim in any depth, two pieces of evidence lend

[16] Morris Kline's 1973 book, not to be confused with Ruth Dunbar's 1956 Saturday Review article of the same name.
[17] See, for example, https://www.youtube.com/watch?v=DfCJgC2zezw, Retrieved 11/30/2017
[18] http://www.nytimes.com/1971/01/26/archives/dr-max-bebermanls-dead-at-45-a-creator-of-new-mathematics.html, Retrieved 11/24/2017

support: First, the New Math classroom vignettes with which Kline begins his book. For example:

> [The teacher asks] "Is 7 a number?" The students, taken aback by the simplicity of the question, hardly deem it necessary to answer; but the sheer habit of obedience causes them to reply affirmatively. The teacher is aghast. "If I asked you who you are, what would you say?"
>
> The students are now wary of replying, but one more courageous youngster does do so: "I am Robert Smith."
>
> The teacher looks incredulous and says chidingly, "You mean that you are the name Robert Smith? Of course not. You are a person and your name is Robert Smith. Now let us get back to my original question: Is 7 a number? Of course not! It is the name of a number. 5 + 2, 6 + 1, and 8 - 1 are names for the same number. The symbol 7 is a numeral for the number.
>
> The teacher sees that the students do not appreciate the distinction and so she tries another tack. "Is the number 3 half of the number 8?" she asks. Then she answers her own question: "Of course not! But the numeral 3 is half of the numeral 8, the right half." (Kline, 1973, p.2)

While Kline's example tends toward caricature, it suggests that many lessons were derailed because teachers lacked a sufficiently broad grasp of mathematics to understand (for example) when and why it was useful to distinguish between a quantity and its representation. Furthermore, in reading the architects of New Math curricula it is clear that they were perfectly cognizant of the issues that Kline raises. Again, to offer one example, in his remarks on SMSG's First Course in Algebra, Henry O. Pollak (1965) wrote:

> We maintain the distinction between numbers and numerals for some time and in fact find it very useful in connection with consideration of simplification. … [eventually] we drop the fine distinction and admit that the student will be able to tell the difference. There is no point tangling yourself up in this language for very long after the student has seen what you are driving at. (p.16)

It seems plausible, therefore, that the architects of the New Math put too much emphasis on curriculum design and too little on implementation.[19] It does not appear that they were self-indulgently oblivious to the potential for the difficulties observed by their critics and caricaturists.

It would be unfair to say that the New Math curricula were inherently lacking in rigor, but the public had little patience for, or confidence in, math instruction that did not prioritize disciplined computational fluency and accuracy. Teachers apparently struggled to find the necessary balance between training students to be computationally fluent, and imparting a deeper and more abstract understanding of mathematical structures and, fairly or unfairly, the novel aspects of the New Math were named as the culprit.[20]

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[19] This criticism would also be raised about later rounds of reform that drew from the same well of inspiration as the New Math, for example Ralston (2003), and Wu (1997), writing about the Math Wars (see below), concur that an important factor in the failure of math education reform is often the inability of teachers to faithfully implement reform curricula.

[20] More technical criticisms of the New Math were also raised, although it seems unlikely that these were decisive. Kline (1973), for example, was highly critical of the New Math curricula's tendency to emphasize the abstract and formal at the expense of the concrete and intuitive. He argued that doing so deprived students of opportunities to gain familiarity with concrete instances, which were necessary to developing their own grasp of abstract ideas. He also criticized a tendency to emphasize formal proof and deductive logic at the expense of intuition, quoting Henri Lebesgue, a late nineteenth century French mathematician as follows:

> No discovery has been made in mathematics, or anywhere else for that matter, by an effort of deductive logic; it results from the work of creative imagination which builds what seems to be truth, guided sometimes by analogies, sometimes by an esthetic ideal, but which does not hold at all on solid logical bases. Once a discovery is made, logic intervenes to act as a control; it is logic that ultimately decides whether the discovery is really true or is illusory; its role therefore, though considerable, is only secondary. (Lebesgue, H., quoted in Kline, M., 1973, p.59)

The New Math also came under criticism from scientists such as Richard Feynman, who argued that "mathematics which is used in engineering and science … is all really old mathematics, developed to a large extent before 1920" (Feynman, 1965, p.10). In his view, the New Mathematics reflected the increasing self-referentiality of the discipline of pure mathematics, while neglecting not only the needs of educated laymen, but also those of other academic disciplines that use mathematics.

The Math Wars

Although the New Math, as such, did not survive the 1970s, it has found continuance by other means, with the most large-scale example being the "Math Wars", that followed from the implementation of the National Council of Teachers of Mathematics' (NCTM) 1989 *Curriculum and Evaluation Standards* in California.

In 1992, California introduced a new math curriculum framework, which reflected pro-reform, or "progressive" views (Ralston, 2003). This framework was heavily influenced by a set of model standards published by the National Council of Teachers of Mathematics (NCTM) in 1989, the creation of which involved many of the same people who contributed to the 1989 NCTM Standards (Kilpatrick, 2004). In 1993, shortly after the framework was published, a "traditionalist" reaction against the document broke out: The ensuing controversy has been widely termed "the Math Wars".[21]

In their overarching vision for school math, the NCTM Standards were in many ways the heir to the New Math's vision of a modern math curriculum that would supersede more traditional approaches:

> The *Standards* [embody] a vision of school mathematics in which [the historical] purposes [of secondary school mathematics] are embedded in a context that is broader and more consistent with accelerating changes in today's society. High school graduates during the remainder of this century can expect to have four or more career changes. To develop the requisite adaptability, high school mathematics instruction … must provide experiences that encourage and enable students to value mathematics, gain confidence in their own mathematical ability,

[21] Suzanne Wilson's *California Dreaming: Reforming Mathematics Education* (2003) offers a thorough and remarkably even-handed account of this complicated and protracted phenomenon.

> become mathematical problem solvers, communicate mathematically, and reason mathematically. (NCTM, 1989, p.123).

This contains more than an echo of the rationale for the New Math offered thirty years earlier by Edward G. Begle, when he argued that “[no one can] foretell which mathematical skills will be required in the future by a given profession” and that school math must therefore impart a deep understanding of the “basic concepts and structure of mathematics that would enable the student to learn new (perhaps yet undiscovered) mathematics in the future” (Begle, 1968, p.239).

While it is beyond the scope of this chapter to describe the many ways in which the reforms around the Math Wars differed from the New Math period, there is a strong commonality in the concerns that were raised about the alleged dulling effect of the proposed reforms. The emphasis on student use of technology, which Ralston (2003) argues was an innovation in the Math Wars period, gave rise to concerns that the new curriculum was, in the words of one dissenter, “creating a new learning disability: Computer-Assisted Mathematical Incompetence” (Escobales, 1997, p.542). Indeed, while some saw the use of technology as offering the possibility of “focus[ing] on networks of mathematical ideas rather than solely on the nodes of the network in isolation” (NCTM, 1989, p.149), others voiced concern about neglecting students’ computational acumen. Klein and Milgram, for example, were sharply critical of “the view that the four standard arithmetic algorithms are obsolete [and] superfluous” (Klein & Milgram, 2000, p.2).

Beyond the Math Wars

The 2010 *Common Core State Standard for Mathematics*, the successor to the NCTM *Standards* documents, offers a framework for making "the mathematics curriculum in the United States … more focused and coherent", echoing the rhetoric of the earlier reform movements described above. The authors show remarkable reticence, however, about what students' exposure to the material should actually look like:

> These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B. (National Governors' Association, 2010, p.5)

In light of the foregoing, this sounds less like a call to let a hundred flowers bloom, than like a resigned acceptance that if any deep reform is to occur, it will have to be incremental, on a classroom-by-classroom basis.

Nonetheless, the influence of the New Math persists at the high school level, even if it is manifested within a more traditional sequential structure based around a separation of algebraic and geometric content. One recent example of this is the Center for Mathematics Education (CME) series. Although this consists of Algebra 1, Geometry, and Algebra 2 texts, the CME curriculum:

> makes a conscious choice not to think of each course as a list of topics to cover, but rather as an opportunity to develop mathematical themes in different areas of mathematics. These themes provide students with insight about what it means to 'think like a mathematician' and can be applied to many different (even non-mathematical) situations.

Considering the legacy of the New Math more broadly, researchers of math education still argue that drawing connections between ideas will help students develop stronger understanding of those ideas (e.g. Hiebert & Carpenter, 1992), and the search for "a theme that transcends content and grade level" (Usiskin, 2003, p.17) has continued. Clement and Sowder (2003) go further, proposing that unifying concepts can be provided not only by content, but also by processes: Examples of such organizing principles in recent decades have included problem solving (Krulik, 1980), algorithmic thinking (Pollak, 1983), sense-making (Kaput, 1993), measurement (Clement & Sowder, 2003), and algebraic thinking (National Governors Association, 2010).

Summary

One important reason why the traditional sequence has persisted, therefore, is that it embodies a recognizable (i.e. traditional) body of knowledge and skills with unambiguous (i.e. rigorous) standards of success and failure, and does so more effectively than at least one major family of alternatives that has been offered. The alternative curricula discussed in this section regarded traditional knowledge and skills as subservient to abstract ideas. These ideas, and the entire mode of thought associated with them, were unfamiliar not only to parents, but also to many teachers. The proponents of such curricula may have hoped that using basic knowledge and skills to provide entry into abstract mathematics would provide motivation to students. In practice, however, it appears to have led teachers to neglect the more basic content, or to present it in ways that students and parents found confusing. In turn, parents were alarmed to see that their children were failing to develop the expected skills in their math classes, and that nothing of (to them, at least) clear value was being taught in its place.

The traditional sequence was therefore not to be transcended by a push to teach the more abstract foundations of its content. Other alternatives, which I now turn to discussing, accept the idea that a math curriculum essentially consists of a body of knowledge of skills, but aim to supplant the traditional sequence's particular body of knowledge and skills.

The Dominance of Calculus

A second reason for the persistence of the traditional sequence is that it ensures students receive a heavy dose of algebra. The "college preparatory" sequence of Algebra I, Geometry, and Algebra II could, with fairness, also be called a "calculus preparatory" sequence: Algebraic topics not only make up two-thirds of the content, but also are generally selected and presented so as to provide a seamless transition into differential calculus,[22,23] a course which stands as the gatekeeper to many professions. Although the original rationale for granting algebra a central place in the high school curriculum had to do with the mental habits it was assumed to engender, after the launch of Sputnik and the NDEA it became important because it provided the requisite skills to study calculus, and thereby engineering. This presents an additional criterion for any proposed replacement for the standard sequence: it must either provide superior preparation for calculus, or prepare students for something generally recognized as more valuable.

[22] Balomenos, Ferrini-Mundy & Dick (1987) argue that although many teachers view Geometry as an isolated course it is, in fact, also crucially important for grasping calculus.
[23] A more complete discussion of the extent to which computational and manipulative skill is truly a requirement for the competent use of calculus would have to deal with the calculus reform movement, and with the spread of affordable and portable computer algebra systems (e.g. TI-Nspire™ CAS Handheld, Wolfram Alpha) that can calculate complex derivatives and integrals at the push of a button. Here, I simply assume that, for the foreseeable future, studying calculus will mean learning to apply a wide variety of differentiation and integration techniques manually.

Reforms have generally failed to meet these criteria: Although the UICSM Project, the original New Math curriculum, was initially developed to improve freshman readiness for calculus,[24] and may have had some success with the most able students (e.g. Duren, 1967; Rossman & Hayden, 2016), more recent reform efforts have been criticized for providing inadequate preparation for calculus (e.g. Klein, et al., 1999; Wu, 1997).

The other approach, taken by reformers in recent decades has been to argue that the large majority of students would be better served by a course of study culminating in computer science or statistics, than one culminating in calculus. This, in turn, would provide leeway for replacing algebra with other content in earlier courses. In this section, I consider the two major alternatives that have been proposed to either supplant, or stand co-equal with calculus: discrete (or "finite") math, and statistics. I consider the arguments offered for and against these alternatives, and the reasons for the durability of calculus, one of the pillars of demand for the traditional sequence.

The Place of Calculus

Bressoud (1992) argues that mathematics' taking shape as an academic discipline was inextricably linked with the birth of calculus. He points out that Oxford and Cambridge only established their first chairs of mathematics in the mid-17th century, around the time that Newton's *Philosophiae Naturalis Principia Mathematics* (1687) was published. Newton's work revolutionized people's view of the natural world, and of the

[24] At the behest of the University of Illinois engineering department

possibility of understanding it through mathematics (Douglas, 1986). This means that, in addition to being central to how mathematicians see their discipline, calculus is foundational to the modern form of other disciplines, including physics, astronomy, and (more recently) economics – one can plausibly argue that math would have a much less central place (or at least a much different place) in education in a world without calculus.[25]

In the final decade of the twentieth century the place of calculus in the school curriculum began to be seriously challenged. An important example of such a challenge is the 1990 report *Reshaping School Mathematics* by the Mathematical Sciences Education Board of the National Research Council (NRC), which offered a remarkably bold and comprehensive broadside against the centrality of calculus. The 1990 report set out to deal with "changing perspectives on the need for mathematics, the nature of mathematics, and the learning of mathematics" (NRC, 1990, p.xi). These issues were selected from others raised by the 1989 report *Everybody Counts* and the 1989 NCTM Standards for "their compelling and inevitable impact on the organization of the mathematics curriculum" (NRC, 1990, p.xi).

Reshaping School Mathematics could not have been clearer about its antipathy toward tradition, and especially calculus. From the outset, it declared that the prevailing curriculum was "controlled" by an "outdated" assumption that "mathematics is a fixed and unchanging body of facts and procedures" (NRC, 1990, p.4). While echoing the

[25] It also means that there is a humanist argument for the place of calculus in a liberal education, independent of the subject's widespread utility (e.g. Mayor & Brown, 1964).

rhetoric of the New Math era, the report went on to note that the ancient root of the word "curriculum" is the word for a "deeply rutted" path for chariots (NRC, 1990, p.4), and (not to put too fine a point on it) out that "even calculus … is three centuries old" (NRC, 1990, p.4). [26] Among its concluding principles was a demand for a "zero-based" curriculum process (NRC, 1990, p.38).[27] The report went on to recommend (re)building the secondary math curriculum around five overarching topics: algebra, geometry, discrete math, data analysis, and optimization (NRC, 1990, p.46). The inclusion of algebra, and of trigonometry under the rubric of geometry, indicated that the goal was not to eradicate calculus from the curriculum (nor would that have been possible), but two other topics, data analysis and discrete math, were emphatically not related to calculus, and optimization was defined (perhaps pointedly) so that it neither required nor implied the study of calculus: "Optimization include[s] mathematical modeling, `what if` analysis, systems thinking, and network flows" (NRC, 1990, p.46). At present, these sub-topics would typically be divided between courses in discrete math and statistics. In the next two sub-sections I discuss the history of proposals to include discrete math in the school curriculum, and then of proposals to include statistics and data analysis.[28]

[26]This hostility has antecedents in earlier periods – even during the New Math period one of its proponents, G. Baley Price, wrote under the heading *The Revolution in Mathematics*, "the general public … seem[s] to feel that mathematics was completed by Newton, and that … there is no opportunity, need, or occasion for [math courses] to change" (Price, 1961, p.1).

[27] "Zero-based" is an accounting term, denoting a budgeting process in which each item's place and status in the budget is decided without reference to its place and status in previous years' budgets.

[28] I have been using the terms "data analysis" and "statistics" somewhat interchangeably up to this point. A third term "probability" arises less in the literature. Unless quoting directly, I will use the term "statistics" to refer to all three of these, but here I provide a brief discussion of the distinction: Joe Blitztein (Statistics 110: Probability, Harvard University, retrieved 4/11/2017 from http://itunes.apple.com) describes probability as the science of making predictions about the data that will be generated by a known process, and statistics as the science of making inferences about data-generating processes from known data. Through early high school, "statistics" is likely to refer to visualizing data through charts, interpreting data displays, calculating simple descriptive statistics such as means, medians, and quartiles, and calculating probabilities arising from simple games. Many of the questions dealt with by professional statisticians arise naturally from extending these simple settings, and some of them provide content for high-school level

Discrete mathematics

"Discrete math" (also called "finite math") is shorthand for "math other than calculus": Calculus is concerned with evaluating the behavior and properties of continuous functions over infinitesimally small intervals of their domain. Therefore, defining mathematical topics as "discrete" or "finite" signifies that they deal with objects that cannot be treated using calculus. Because computers deal with processes and objects that are countable, discrete, digital, and recursive, interest in these grew in the post-war period with the spread of computing technology (Hart, 1985). Although discrete math includes (but is not limited to) topics applicable to computer science, such as graph theory, combinatorics, Boolean algebra, and logic, Meyer (2007) points out that discrete math also includes sub-fields of mathematics with strong roots in the social sciences of the 1940s and 1950s, such as social choice theory and game theory.[29]

In the mid-1980s calls to supplant calculus, and calculus preparatory topics, with discrete math grew stronger (e.g. Ralston, 1984). Such calls met with skepticism about both their wisdom (Douglas, 1986; MacLane, 1984), and their feasibility (Hart, 1985). MacLane (1984) admitted that "some discrete mathematics has real substance" (MacLane, 1984, p.373), but derided other parts as "creatures of fleeting fashion" and the

courses in statistics (e.g. http://www.collegeboard.com/html/apcourseaudit/courses/statistics.html). Two additional points on terminology: First, authors frequently (and incorrectly) use "data" and "statistics" interchangeably. Second, statistics and mathematics are generally considered distinct, if closely related, disciplines, and "data analysis" may be used partly to avoid the term "statistics" in discussing math curriculum.

[29] Examples of foundational results in these fields from this period are, for social choice theory, Kenneth Arrow's impossibility theorem: (Arrow, K. J. (1950). A difficulty in the concept of social welfare. *Journal of political economy*, *58*(4), 328-346.), and, for game theory, John Nash's Nash equilibrium: (Nash, J. (1951). Non-cooperative games. *Annals of mathematics*, 286-295.)

overall concept as constituting a “grab bag of all sorts of things” whose introduction into schools would promote a “new captivity by computers” (MacLane, 1984, p.373).

While it is possible that MacLane (1984) evinced the prejudices of a research mathematician,[30] Ronald Douglas (1986) met the arguments of computer scientists such as Ralston on their own terms. Douglas (1986) acknowledged that computers, by their very nature, deal with discrete processes and finite differences, and that areas of discrete math such as graph theory and Boolean algebra are applicable to computer programming. Nonetheless, he argued, calculus is foundational to modern practice of the natural sciences, and the solution to problems from “differential calculus … using finite differences [which is required if they are to be handled by computers] is next to meaningless without an understanding of calculus” (Douglas, 1986, p.251).

Hart (1985) expressed further skepticism, asking whether *Discrete Mathematics [is] the New Math of the Eighties?* He noted the large difference in point of view between the movements - the New Math was a structuralist enterprise more concerned with general principles, while discrete math consists of very particular content, dictated by specific circumstances – but he also noted that the implementation of New Math involved substantial investment in teaching materials and in teacher training, and that it failed in spite of these. Given that such supports for the teaching of discrete math were much scarcer in the eighties, it seemed even less likely to precipitate revolutionary change.

[30] And also those of the founder of Category Theory, a mathematical sub-discipline that is called “abstract nonsense” even by its practitioners. See, for example, Marquis, J. P. (2010). Category theory. In What is category theory? (pp. 221-255).

Although some discrete math topics, such as truth tables and matrices, have appeared in high school math curricula, they generally appear as isolated curiosities, a small minority of the material covered. According to the College Board, almost ten times as many students sat the AP® Calculus AB or BC exams in 2015 as sat the AP® Computer Science exam (420,000 vs. 49,000). There is no official AP® course in Linear Algebra, the other college-level class that would follow logically from a primer in discrete math. Taken together, this evidence suggests that a high school curriculum that emphasizes the algebra of continuous functions is unlikely to be superseded by one that substantially replaces that algebraic content with discrete math topics.

Statistics

In the late twentieth century, there was also a surge of enthusiasm for giving statistics greater prominence in the undergraduate curriculum (Moore, 1988), which spilled over into the high school curriculum with the 1997 introduction of the AP® Statistics exam (Gould & Peck, 2004).[31] Although fewer than half as many students sat the AP® Statistics exam in 2015 as sat the AP® Calculus AB or BC exams, there are stronger reasons than for discrete math to expect that statistics may gradually erode calculus's place at the pinnacle of the curriculum, and gradually transform what students are required to study in earlier grades.

[31] It had been nearly eighty years since the Kilpatrick report of 1920 had first called for including data analysis in the curriculum, and some of the statistical topics that the 1959 CEEB report recommended for advanced seniors, such as arithmetic mean, median, and frequency histograms were standard fare in ninth grade, if not earlier.

Calls for expanding statistics education, beginning in the late 1980s, were primarily driven by increased demand both from industry and from diverse academic disciplines, including economics, sociology, psychology, and public health (Moore, 1988; Moore, 1992), although more civic-minded arguments were also raised, especially in the early twenty-first century. Industrial and academic demand for statistical training continued into the late 2000s and 2010s, when "big data" seemed to be revolutionizing every field of human endeavor (e.g. Economist, 2010); and the U.S. Bureau of Labor Statistics projected extraordinary labor market demand for statisticians.[32] More civic-minded arguments for expanding students' exposure to statistics also emerged from various quarters: Rothstein (2001) argued that it is often difficult for juries to deliberate competently without statistical literacy, and that media literacy increasingly requires statistical literacy, noting at least a twenty-fold increase in the number of data displays appearing in the New York times since the 1970s. Arthur Benjamin of Harvey Mudd College, in an early 2009 TED talk that has been viewed over two million times, averred that "if all the American citizens knew about probability and statistics, we wouldn't be in the economic mess that we're in today."[33] Another notable phenomenon in this regard has been the rise of statistician and pollster Nate Silver to quasi-celebrity status, and the increasing insinuation of not only his polling results, but also his profession, into the very narrative of U.S. elections. Some commentators (e.g. Goyal, 2012) have suggested that this phenomenon could serve as an engine for revitalizing math education.

[32] https://www.bls.gov/ooh/math/statisticians.htm, retrieved May 12, 2017
[33] Benjamin makes this remark at 1:46 in the video, retrieved on 4/11/2017 from https://www.youtube.com/watch?v=BhMKmovNjvc

The increasing prevalence of statistics education may have better long-term prospects of fomenting change in the school math curriculum than other innovations and reforms discussed in this chapter. One reason for this is that a primary driver is real demand, not only from industry, but also a broad swath of academia. Another is that raising the mathematical acumen of the nation's high school students is proving to be an ongoing challenge (which I describe in more detail in the subsequent sections and chapters). Data analysis-based statistics courses offer relevant and engaging learning activities while using only relatively basic mathematics (e.g. Moore, 1992), and offer a way to keep students engaged in meaningful quantitative work through their high school years.

Nonetheless, statistics faces some of the same challenges as other reforms in gaining greater curricular prominence. Although it can engage students, and a wide variety of instructional materials are available, many teachers are likely not to have appropriate training to teach the statistical skills, knowledge, and intuition that fill the gap between the most elementary topics and the highly theoretical (and calculus based) content that characterizes the courses taken by many math majors (e.g. Froelich, Kliemann, & Thompson, 2008; Hayden & Kianifard, 1992). A further challenge is that, while labor market demand for statisticians might be expected to draw more students to statistics classes, it is also likely to draw individuals with statistical training away from teaching positions (e.g. Moore & Cobb, 2000). Finally, because of the role that uncertainty plays in statistical reasoning, it may struggle with a public that expects math classes to traffic in right and wrong, rather than better and worse, answers.

Other Obstacles to Supplanting Calculus

An additional hurdle faced by any attempt to supplant calculus is the cultural prestige that attaches to an algebra-based curriculum and its culmination in a calculus course. This is reflected in both the upper and lower echelons of academia, both in the calculus requirement that most medical schools set for admission, and in the algebra (but not geometry) proficiency requirement that most postsecondary institutions set for even a two-year degree (Blum, 2007).[34] Although these requirements have been called into question (e.g. Hacker, 2016; Muller & Kase, 2010), there has not yet been any widespread movement to lift them.

The historical situation has led to a self-reinforcing dynamic in which relatively stronger demand for calculus courses leads to a relatively stronger supply of teachers qualified to teach them. As noted above, far more students sit the AP® Calculus AB or BC exams than sit the AP® Statistics and AP® Computer Science exams combined. It is beyond the scope of this study to determine the extent to which this disparity is demand-driven, and to what extent it is supply-driven. That is, to what extent parents and students see calculus as sending a stronger signal of ability on college applications (or genuinely find the courses more interesting), and to what extent the past dominance of calculus has

[34]Another remarkable example of the prestige that attaches to the traditional algebra-based curriculum was offered by Robert Moses. Moses and Cobb (2001), argued that there is, in fact, no necessity to algebra driving science and technology education. Nonetheless, they claimed that because prevailing conditions in the United States make algebra proficiency a necessary condition for academic and professional advancement, algebra education is a civil right. The moral authority with which Moses speaks among many progressive educators adds to the burden that one must carry in arguing to de-emphasize calculus and calculus-preparatory material.

led to a relatively stronger supply of teachers willing and able to teach it. In any case, there is a strong constituency (or set of constituencies) that will resist attempts to introduce curricula that do not give centrality to the algebra of continuous functions.

Summary

A second important reason why the traditional sequence has persisted, therefore, is that its emphasis on algebra aligns it with a widespread demand for training in calculus. Although not all extant proposed alternatives to the traditional sequence de-emphasize algebra, many (as described above) do. Critics often dismiss the demand for calculus as reflecting outdated assumptions or even a kind of intellectual chauvinism (in the case of medical school requirements). Although these criticisms are not baseless, they often fail to recognize the epoch-making nature of the development of calculus in terms of how both the natural and social worlds are understood. A diverse and powerful set of constituents will oppose any attempt to introduce curricula that provide weaker preparation for calculus. To the extent that this opposition is answerable, the indispensability of calculus, and concerns about equitable access to calculus, will have to be taken seriously, and addressed directly.

The Push for Uniformity: 1983 and Beyond

The current state of affairs, in which virtually all students are expected to complete a substantial portion of the traditional college-preparatory curriculum, came about in response to three types of concerns, with a substantial assist from federal policymakers. First, national security concerns, articulated in *A Nation at Risk*, along with other reports released in 1983, provided an impetus for increasing high school math credit requirements. Later in the decade, a complementary set of social justice and economic concerns were raised, which sharpened curricular prescriptions to include not only raising math credit requirements, but also eventually prescribing the same curriculum and assessment standards for all students. The enactment of this prescription was incentivized and promoted by federal legislation, especially the reauthorizations of the Elementary and Secondary Education Act (ESEA) that took place in 1994 (IASA) and 2000 (NCLB).

National Security Concerns: A Nation at Risk

Prompted by a sense of crisis that had been building throughout the previous decade, several reports on education were issued in 1983, the most famous of which was *A Nation at Risk*, published by Ronald Reagan's National Commission on Excellence in Education: In the late 1960s a decline in SAT Math scores had begun, which continued through the 1970s (Usiskin, 1985).[35] At the same time Japan and West Germany had emerged as industrial competitors to the United States even as the Soviet Union continued to stand as a military and ideological competitor.[36] Against this backdrop, *A Nation at Risk* famously likened America's "mediocre educational performance" to something that might have been imposed by an "unfriendly foreign power [as an] … act of war" (Bell, 1983, p.5).

[35] Powell and Steelman (1984) consider this trend, which continued through the early 1980s, and argued that the decline in scores was at least partially attributable to changes in the demographic composition of the test-taking population.

[36] In addition to the best known of the reports, *A Nation at Risk,* which repeatedly compares the U.S. with Japan and West Germany, other reports issued in 1983 include: *Educating Americans for the 21st Century* released by the National Science Board (NSB); *High School: A Report on Secondary Education in America* released by the Carnegie Foundation for the Advancement of Teaching; *The Mathematical Sciences Curriculum K-12: What is Still Fundamental and What is Not"*, NSB; *Academic Preparation for College: What Students Need to Know and Be Able to Do*, College Board. The first NSB report and the Carnegie Foundation report both reference Japan and Germany in their introductions.

A Nation at Risk recommended expanding the amount of secondary math coursework required of students to three yearlong courses, although it did not specify the content of those courses. In 1982, most states required zero or one math courses for high school graduation (Goodman, 2017). By 1990, high school students in 40 of the 50 states faced increased math credit requirements: In 31 states the requirement had increased to two credits, and in 9 states to three credits (Goodman, 2017). The exact nature of the math courses to be taken for those credits, however, was largely left to local discretion.

Although *A Nation at Risk* had antecedents - it marked the third time since 1940 that anxieties had been raised about the nation's schools as a liability on the international stage[37] – this time would be different, due to additional concerns that would be raised later in the 1980s, and an unprecedented federal push for reform beginning in the 1990s.

Social Justice Concerns

In the mid-1980s, researchers grew concerned that allowing high school students substantial latitude in the particulars of their coursework, historically a key practice in accommodating the diversity of the American high school population, was not a benign

[37] This type of concern points to a second development that separated the Committee of Ten's time period from the mid-20th century: the emergence of the United States as a pre-eminent global power after the Second World War, along with an ongoing set of practical concerns (and neuroses) about maintaining that status. It is notable that education reforms have repeatedly been argued for based on their necessity for supporting the nation's military, technological, and economic strength. Other notable examples of this theme include: figures such as Admiral Nimitz (Nimitz, 1942) and General Somervell (Somervell, 1942) raising concerns during World War II about the lack of mathematical and technological aptitude among young military recruits, which influenced math education during the war, and contributed to the 1950 establishment of the National Science Foundation. Another was the 1958 launch of Sputnik, the fallout from which is described in detail, above. A later example was 2012's *U.S. Education Reform and National Security* (Council on Foreign Relations Press).

practice, Rather, they argued, it was perpetuating, and even exacerbating, historical inequalities and injustices.

Federal education policy in the decades after Sputnik gradually evolved from a focus on identifying and developing top talent in math and science to identifying and closing achievement gaps based on race and income (Lappan & Wanko, 2003). This manifested in the Supreme Court's 1954 Brown v. Board of Education of Topeka ruling,[38] followed by the ESEA of 1965, the education component of Lyndon Johnson's Great Society anti-poverty initiative.[39] Although *de jure* equalization of access to educational opportunity spread in subsequent years, by the late 1970s it was becoming apparent that ESEA's effect on income-based inequality had been minimal (Lappan & Wanko, 2003).

As states and districts began to demand more math credits for high school graduation, the reigning ethos in the comprehensive American high school was to continue allowing students as much choice as possible in the level of rigor and often the general outline of their program. This practice became a target of criticism, with commentators using consumerist metaphors to describe and criticize it: Powell, Farrar, and Cohen did so in their 1985 work titled *The Shopping Mall High School: Winners and Losers in the Educational Marketplace,*[40] while *A Nation at Risk* decried a "cafeteria-

[38]which found that "separate educational facilities [for white and black students] [were] inherently unequal" (Brown v. Board of Educ., 1954, p.483).
[39] Although this history coincides with that of the New Math period described above, the two phenomena targeted two largely distinct groups of students: those most likely to become members of the scientific elite, and the poorest and most historically disadvantaged students in the highest poverty schools.
[40] Powell, Farrar, and Cohen characterized the classrooms in the schools they studied as functioning under "treaties" (Powell, Farrar & Cohen, 1985, Chapter 2), whereby teachers and students more or less tacitly

style curriculum in which the appetizers and desserts can easily be mistaken for the main courses" (Bell, 1983, p.61).

In the mid-1980s, researchers also raised the possibility that students following diverse curricular tracks, even if nominally voluntary, had a negative impact on student outcomes. Criticizing the view that tracking simply met demand for different courses of study from students with different goals, and different levels of ability, these researchers wanted to demonstrate that tracking reinforced and even created disparities in outcomes by offering some students less rigorous material and lower quality instruction, and thus restricted their opportunities to learn (e.g. Gamoran, 1987; Oakes, 1985; Oakes 1990).[41]

The indictments of Oakes, Gamoran, and others, were augmented by other research findings about variation in student course-taking patterns, indicating that they were mirroring other historical inequalities. First, a 1985 analysis of 1981-82 student transcript data from the National Center for Education Statistics' (NCES) High School and Beyond study indicated that higher socio-economic status (SES) students, white students, and male students exhibited more intensive participation in math and science courses than their lower SES, black, Hispanic, and female peers (West, Miller & Diodato, 1985). These concerns were compounded by concurrent findings in labor economics, to which I now turn.

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negotiated within individual classrooms over the level of rigor that teachers would demand in exchange for orderly student conduct.

[41] McDonnell (1995) provides an interesting history of the phrase "opportunity to learn", from its origins as a technical concept in international comparisons conducted in the 1960s, to its adoption in the 1990s as what she calls a "hortatory policy instrument" (McDonnell, 1995, p.313).

Labor Market Concerns

In the 1990s and early 2000s, concern was also growing that a high school diploma did not, on its own, signal readiness either for college study or the modern workplace (e.g. Quality Counts 1997; Reality Check 2002). Because population projections suggested that historically underperforming groups were going to form an increasingly important segment of the workforce through the twenty-first century, this dovetailed with ongoing concerns about demographic inequalities in education outcomes.

The Department of Labor's 1987 Employment and Training Administration report, *Workforce 2000: Work and Workers for the 21st Century*, found that while white males made up 47 percent of the current labor force, they would make up only 15 percent of new labor market entrants over the next 13 years. Further, the report found that "very few new jobs will be created for those who cannot … use mathematics" (Johnston & Packer, 1987, p.xiii). In 1989, the Congressional Task Force on Women, Minorities, and the Handicapped in Science and Technology issued its final report, noting that white men also made up a majority of those pursuing college majors in science and engineering, and lamenting that, "our pool of talent for new scientists and engineers is predominantly female or minority or disabled – the very segments of our population we have not attracted to science and engineering careers in the past" (Oaxaca & Reynolds, 1989, p.21). These findings meant that expanding access to math courses was an issue not only of justice, but also of economic survival. This alignment of interests between progressive social activists and business concerns would lead to a broad policy consensus in support of a substantially larger and more activist federal role in education policy.

Policy Response

Although *A Nation at Risk* declared that the "Federal Government has *the primary responsibility* to identify the national interest in education" (Bell, 1983, p.79, italics in original), the Reagan White House resisted proposals for federal intervention in education (McLeod, 2003), and it was only under Reagan's successors that such a responsibility was taken up. The federal policy response to these concerns can be understood as having two parts, which are collectively known as the Standards and Accountability Movement (e.g. Foote, 2007).

The initial policy response to *A Nation at Risk* resembled that undertaken in the post-Sputnik era: the government used grant funding to support the development of new curricular materials, and in some cases to incentivize their adoption. This launched the era of national standards. In 1994, however, the federal government began to promote its policy preferences using not only the incentive of grants, but also the threat to withhold Title I funds – anti-poverty grants established in connection with the 1965 ESEA - from states that did not comply with their requirements. This launched the accountability portion of the movement under which schools were required to provide evidence of whether students were, in fact, meeting the standards that states had established.

The standards movement

In the same year that *A Nation at Risk* was published, a National Science Board commission chaired by prominent Republican William T. Coleman proposed that professional organizations, rather than the federal government, take the lead in directing change in their fields (Coleman & Selby, 1983). An important result of this recommendation was the process that led to NCTM's *Curriculum and Evaluation Standards for School Mathematics*, which were commissioned in 1986, and published in 1989 (McLeod, 2003).

Published during the administration of George H.W. Bush, which was less hostile to federal involvement in education, the NCTM Standards received regulatory and material support from the National Science Foundation (NSF). In terms of regulatory support, the NSF required that education research grant proposals make reference to "generally accepted standards" (McLeod, 2003), with math education proposals having few clear options aside from the NCTM Standards. In terms of material support, the NSF funded the development of thirteen "standards based" math curriculum projects, including five at the high school level – interestingly, all of which offered alternatives to the traditional sequential curriculum. Paradoxically, as I will argue in the next section, the success of the accountability portion of the Standards and Accountability movement probably helped to undermine adoption of these curricula, by putting pressure on states and districts to have all students study the same material in approximately the same arrangement.

The 1989 Standards have influenced the design of many state standards frameworks (Howe, 1998) (California's, for example, as discussed above in connection with the Math Wars), and a revised and updated version was published in 2000 under the title *Principles and Standards for School Mathematics.* The writers of the *Common Core State Standards for Mathematics*, in turn, adopted elements of the *Principles and Standards* in their work (e.g. National Governors Association, p.6).

The accountability movement and the resulting push for uniformity

The more activist portion of the federal policy response to these concerns was embodied in the 1994 reauthorization of the ESEA, titled the *Improving America's Schools Act* (IASA), and its 2001 successor, *No Child Left Behind* (NCLB). The IASA attempted to address the apparent failure of the nation's schools to meet the nation's workforce needs by mandating that states create not only academic content standards, but also aligned testing programs, to monitor whether those standards were being met (Lappan & Wanko, 2003). NCLB went even further, imposing accountability for raising students to the standards that states established: The law required detailed reporting of test scores to the federal government, and sanctions for schools that failed to meet benchmarks in a timely fashion.[42]

Although the NSF-funded curricula of the 1990s offered alternatives to the standard sequential curriculum it seems likely that, whatever other resistance they faced,

[42]A notable feature of NCLB was that each state was allowed to determine its own definition of "proficient", and it gradually became apparent that the definition of proficiency varied widely from state to state (Hamilton, Stecher & Yuan, 2008). Concern about this was an important source of pressure for the project that led to the Common Core State Standards.

they posed a challenge to those trying to design standardized assessments for all of the students in a given state. In some cases, the reasons for this are clear: It will always be more difficult to reliably assess how deeply students grasp abstract ideas and their relationships than to assess their accuracy in selecting and executing well-defined algorithms and chains of reasoning. This makes New Math-style curricula, and curricula built around open-ended applied problems, less-than-ideal candidates for preparing students for standardized tests.[43]

A less ambitious type of non-sequential curriculum, curriculum integrated "by strands" (Usiskin, 2003, p.20), allows for a serial presentation of discrete skills while still offering certain advantages over the traditional sequence.[44] In this model, the traditional distinctions between algebra and geometry (and any other areas of math included, such as combinatorics or logic) are maintained, but students are exposed to all areas in each year.[45] At the high school level, however, it is much less common than a sequential curriculum, although the reasons require more explanation.

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[43] See the appendix for a description and discussion of the COMAP curriculum, an NSF-funded *Standards*-based curriculum based on applications of mathematics to rich real-world problems.

[44] There are sound arguments in favor of even this relatively modest reform. First, students do not experience year-long gaps in their exposure to algebra and geometry which may lead to learning loss (House, 2003). Second, these curricula may be more equitable because they level the playing field among learners whose learning style may be favored by one area of mathematics or another (House, 2003). Third, such curricula allow "informal development of intuition along the multiple roots of mathematics" (Moore & Steen, 1990, p.4). Finally, when courses are not constrained by their titles to include (and exclude) certain topics, it is much easier to add and delete topics, especially from discrete math and statistics. This final argument is relevant to my discussion in the third section of this chapter concerning the ongoing dominance of calculus in influencing the curriculum.

[45] In this sense, virtually all elementary and middle school math courses in the United States are integrated.

A case study in adoption (and eventual rejection) of an integrated-by-strands math curriculum is provided by New York State, and suggests one reason why such curricula may not be more prevalent. From 1988 until 2002 New York offered ninth through eleventh grade courses simply titled Course I, Course II, and Course III (Paul & Richbart, 1985). In 2002 these were replaced with a two-exam sequence titled Math A and Math B, which both contained a mixture of algebra and geometry (with a smattering of logic, combinatorics, and data analysis),[46] and which allowed for flexible pacing. In 2004 the Mathematics Standards Committee of the New York State Board of Regents recommended changing to a three-course (and three-exam) sequence with the titles "Algebra", "Geometry", and "Algebra II and Trigonometry" – in June of 2008 the phasing in of these new exams began.[47] In making this recommendation the committee cited an undesirable amount of variation in course "titles and content" (Blais, et al., 2005, p.1) from district to district, and the fact that the new course titles were "commonly understood in the field of mathematics" (Blais, et al., 2005, p.2). In short, the flexibility offered by the highly general course titles was seen as a liability.

This case points to a second issue: in addition to New York, the other three largest states (California, Texas, and Florida) also follow the traditional sequence in their assessment regimes.[48] These four states comprise a full third of the U.S. population, and

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[46] http://www.nysedregents.org/archive-regents.html

[47] Although the second and third of these exams retained their titles, the first was ultimately called "Integrated Algebra", presumably to reflect the presence of a small number of basic geometry concepts (such as the Pythagorean Theorem and basic area formulas) and data analysis topics (such as measures of central tendency and histograms).

[48] The 2015 reauthorization of the ESEA, the "Every Student Succeeds Act" (ESSA) allows states to use college-entry exams, such as the SAT and ACT as their official state assessments. Although this could mitigate the effect of the testing regimes on curricula, the content of those tests is very traditional, and very highly standardized, so this may be a limited spur to innovation.

three of them (California, Texas, and New York) effectively mandate which textbooks schools may purchase with public money (Schoenfeld, 2004). This creates strong disincentives for textbook publishers to invest in developing materials that do not align with these states' curricular programs.

California does offer an alternative set of assessments titled Math I, Math II, and Math III, to support classrooms using integrated curricula. These tests, however, use the same items as the sequential tests, simply in a different arrangement, which means that preparation for these tests does not require a greater degree of conceptual linkage across strands than does preparation for the sequential tests, nor can these courses stray far from standard algebra and geometry content. Under such a regime, all versions of the curriculum will to some degree be hostage to the most conservative version in common use.

The Consequences of Expanded Enrollment in the Traditional Sequence

The combined effect of IASA and NCLB, and of the various documents that formed their background, has been that states and districts have increasingly chosen to foreclose any option other than working to develop proficiency in a traditional algebra-heavy curriculum. In the next chapter, I review research on specific policies aimed at improving achievement and attainment in this curriculum. Overall, however, evidence suggests that the effect of this has been to water down traditional courses, with little or no evidence of increased rigor in the math instruction that students receive.

According to the 2012 NCES report *The Nation's Report Card: Trends in Academic Progress 2012* (U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, 2012, cited as "NAEP 2012", hereafter), there has been a large increase in attainment rates in the college preparatory course sequence since the mid-1980s: In 1978, only 37% of seventeen-year-olds reported "second-year algebra or trigonometry" as their highest-level math course, with an additional 6% reporting "pre-calculus or calculus" (a total of 43%). In 2012, 54% of students reported second-year algebra or trigonometry as their highest course, and 23% reported pre-calculus or calculus (a total of 77%). In spite of these enrollment increases, the average level of tested achievement has remained flat over this 34-year period, at a scale score of 305 – this was also the average score in 2012 for students who had completed second-year algebra or trigonometry. The NAEP report explains a score of 300 (very close to 305 on the score scale) as follows:

> Students at this level are developing an understanding of number systems. They can compute with decimals, simple fractions, and commonly encountered percents. They can identify geometric figures, measure lengths and angles, and calculate areas of rectangles. These students are also able to interpret simple inequalities, evaluate formulas, and solve simple linear equations. They can find averages, make decisions based on information drawn from graphs, and use logical reasoning to solve problems. They are developing the skills to operate with signed numbers, exponents, and square roots. (NAEP, 2012, p.36)

Given that (ostensibly) at most one year of further study stands between these students and a course in differential calculus, one is inclined to question whether their Algebra II classes matched the Committee of Ten's vision of a second algebra course targeted specifically at "candidates for scientific schools" (NEA, 1894, p.35). Further, the traditional syllabus for an Algebra II course - training facility in the algebraic manipulation of polynomial, exponential, rational, and trigonometric functions, and building intuition for their behavior and properties – seems to have been aimed principally at preparing students for the study of calculus. As taught in the early twenty-first century, the course appears to be falling far short of that goal.

Summary

The third reason why the traditional sequence has persisted to the present day (and come to be the default option for a large majority of the nation's students), therefore, is the emergence of demand for a (relatively) uniform national curriculum for all students that would allow large-scale standardized evaluation. Given the absence of viable (or at least popular) alternatives, as described in the previous two sections, the traditional sequence was a strong candidate for consensus. Being composed of a well-worn body of knowledge and skills that could be evaluated against clear and unambiguous standards of correctness (i.e. "rigorously"), it was a strong candidate to support standardized testing regimes. Furthermore, the fact that the traditional sequence had generally been the default option for college-bound students made its expansion an obvious way to address equity concerns. Nonetheless, if the increased prevalence of the traditional sequence leads to its being watered down, this may yet lead to renewed demand for alternatives that will have more enduring success than has been seen in the past.

Conclusion: On the Ordering of the Sequence

If the foregoing has not proven that the current circumstances regarding the traditional sequence were precisely inevitable, it has at least shown that the particular (and probably most obvious) alternatives that have emerged probably never had strong chances of success. Although the standard sequence seems to hold the advantage in the competition between sequential and integrated curricula, one might ask if the sequencing itself might be subject to revision and, specifically, if it might be preferable for students to complete two consecutive years of algebra before geometry. Especially in light of evidence that contemporary Algebra II courses may cover similar content to Algebra I courses of thirty years ago, placing the two classes back-to-back could allow the second year to include less review, and therefore cover more new material. Such a reform could be cheap and easy to implement, and it is not unheard of - Usiskin (2003), and former NCTM president Michael Shaughnessy (quoted in Postal, 2013) both indicate awareness of districts having used this non-traditional course ordering[49] – but I am not aware of any theoretical literature arguing for or against it, or empirical literature studying its implementation.

The structure of the traditional sequence was based on (among other things) an assumption that most students would not complete more than two years of high school math, and that as many students as possible should gain some exposure to both Algebra and Geometry. With increasing expectations that most students will complete the entire sequence, this rationale holds less sway.

Given the absence of discussion, scholarly or otherwise, on the issue of course ordering, one is left to speculate, and I offer the following thoughts: First, there may be a blind spot in the research community around tinkering with the traditional arrangement of courses. Those who wish to retain the status quo are, by nature, conservative. Those who

[49] The reference to Shaughnessy is actually more ambiguous. Writing about a district that was considering reversing the course ordering, a journalist wrote: “Nationally, [the district] won't be alone, though the more common course progression is geometry between the two algebra courses, said Michael Shaughnessy, the immediate past president of National Council of Teachers of Mathematics.” (citation available on request)

are inclined to question the status quo tend to raise more fundamental questions about what is taught and why. Second, the infrastructure around education research is geared more to elementary than to secondary instruction: Colleges of education face much more demand to train teachers of elementary math simply because there are more such positions, thus creating a bias toward a focus on elementary education in these research communities.[50] Further, the field of secondary math research is more contested between a relatively small number of education researchers, and research mathematicians who have a stake in what high schools turn out. It is plausible that this creates bias in favor of more revolutionary (or simply more mathematically interesting) interventions at the high school level than the one that I study in Chapters 2 and 3.

A third possibility is that teachers and administrators still harbor a traditional sense of Algebra II as a more elite course than Algebra I or Geometry. Although recent research suggests that the typical student who has completed Algebra II knows no more math than the typical student who had completed Algebra I in 1978, there may be resistance among teachers and administrators to fully acknowledging this fact. So long as Algebra II is regarded as the gateway course to high math and STEM fields, it is likely to remain at the end of sequence.

[50] I am indebted to Jon Star for this insight.

Chapter Two

Introduction

Since the early 1980's, states and school districts have been trying to increase both the math attainment (number of years of math enrollment) and achievement (measured performance in math courses) of their students. In general, this has meant trying to promote successful completion of some or all of the standard college preparatory sequence of Algebra I, followed by Geometry, followed by Algebra II, laying the groundwork for college level coursework, especially Calculus.

Policies to promote participation in the college preparatory sequence have largely focused on inducing more students to take Algebra I, and inducing students to take it earlier, in seventh or eighth, rather than ninth grade. Results from such policies have generally been disappointing. Low achieving students tend to experience a higher risk of poor grades and course failure under such policies, and to be no more likely to advance through the pre-college sequence. High achieving students tend to have lower test scores under such policies due to reduced peer ability level in their math classes. Nonetheless, policy makers face continued pressure to see more students succeed not only in Algebra I, but in the entire pre-college sequence: The question confronting states and districts, therefore, is not whether to promote the standard pre-college math sequence to the vast majority of students, but how to do so.

In this dissertation, I study an unusual policy, which mandated that students enroll in Algebra I and II consecutively, and in Geometry in the following year. This is the first piece of research I am aware of to study such an arrangement. In this, the second chapter of my dissertation, I investigate whether and how Algebra II course grades differed for students who enrolled in Algebra I and II consecutively, rather than taking Geometry in between. Briefly, I find that the probability of earning an A or a B in Algebra II was slightly lower for students who enrolled in Algebra I and II consecutively, and the probability of earning a D or an F slightly higher. Negative effects were stronger for students with weaker prior achievement. Although I find only negative effects of the policy, I cannot rule out that they were due to the abruptness with which the policy was implemented, and that future study of a more carefully implemented version of the policy could be worthwhile.

This chapter has five sections: In the first I review the empirical literature on high-school math policies targeting student outcomes in high-school math, and describe the specific context for my study. In the second I describe the data that I use and the measures that I construct. In the third section I describe my methods, in the fourth I describe my results, and in the fifth I discuss their policy significance and implications.

Background and Policy Context

Prior Literature

Observational research has generally found that students who proceed further through the pre-college math sequence have better educational outcomes (e.g. Adelman, 2006; Gamoran & Hannigan, 2000; Long, Conger & Iatarola, 2012), but causal evidence on the effect of math coursework is much less consistent in documenting positive effects of policies that aimed to increase math course-taking beyond the middle school level. On one hand, studies of policies that raised math credit requirements without mandating specific courses have found that such policies were beneficial for students who were induced to take more courses (Goodman, 2017). On the other hand, causal evidence on policies aimed specifically at promoting attainment in the standard pre-college sequence, which were motivated by earlier observational findings, has indicated negative effects of such policies (Stein, Kaufman, Sherman & Hillen, 2011). These studies found that such policies caused harm throughout the distribution of prior achievement, both to low performing students who were induced to take more challenging coursework (Allensworth, Nomi, Montgomery & Lee, 2009; Clotfelter, Ladd & Vigdor, 2015), and to high performing students who had lower performers introduced into their classes (Nomi, 2012; Penner, Domina, Penner & Conley, 2015).

Policies aimed at promoting participation in the standard pre-college sequence have generally focused on accelerating enrollment in Algebra I, expanding enrollment in Algebra I, or both. Acceleration policies have aimed for large-scale or universal shifts of Algebra I enrollment from its traditional place in high school to the middle school grades. Such policies have been motivated by two types of findings. First, observational research found that students who completed Algebra I before starting high school completed more advanced coursework before graduating from high school, had stronger outcomes in high school math courses, and were more likely to graduate and to attend college (Schneider, Swanson & Riegle-Crumb, 1998; Smith, 1996). Taken together, these findings suggested that Algebra I serves as a gateway to later success. Second, studies have found that, conditional on prior achievement, low income, black, and Hispanic students have been less likely to enroll in Algebra I in middle school than their white and higher income counterparts (Bozick & Ingels, 2008; Dougherty, Goodman, Hill, Litke & Page, 2015; Stein et al., 2011). These inequalities led to policies mandating that all students, or all students meeting a certain test score threshold, enroll in Algebra I in middle school. Causal evidence on the effects of such policies has shown few positive effects. Clotfelter, Ladd, and Vigdor (2015) found that students induced to take Algebra I in middle school generally experienced more course failures than they would have otherwise, and were not more likely to persist or succeed in the standard pre-college sequence. Dougherty, Goodman, Hill, Litke, and Page (2017) found that targeted acceleration into Algebra I in seventh grade did increase students' college readiness, but that only a small minority of students remained in an accelerated math track through junior year, and most accelerated students earned Cs and Ds in their more rigorous math classes, passing, but not excelling.

A second type of policy is a universal mandate requiring students to take Algebra I, but leaving the course in its standard place in ninth grade, or allowing flexibility as to its timing. The best evidence on these policies suggests that while they lead some students to complete higher level math courses than they would have otherwise, they also lead to larger numbers and higher proportions of course failures (Allensworth et al., 2009; Stein et al., 2011). Furthermore, Nomi (2012) found harm to higher ability students from a universal mandate in Chicago, for whom the policy's primary consequence was to lower the peer ability level in their math classes.

The extant research presents policy-makers with inadequate guidance: On one hand, most policies that drastically increase, or universalize, enrollment in specific courses and/or that accelerate their timing have been shown to have harmful effects. On the other hand, many students who would likely benefit from taking more, and more rigorous, math courses do not do so when left to their own devices, and mandates are arguably the most equitable tool for expanding opportunity. Furthermore, policies that mandate participation in all or part of the pre-college math sequence seem likely to remain popular, for at least two reasons. First, research has shown that in the absence of such mandates the social distribution of access to college preparatory math courses is highly uneven, and mandates are seen as a tool for making such access fair and equal. Second, federal education policy is increasingly aimed at broadening postsecondary enrollment, and several states are planning to take advantage of the flexibility provided by the 2015 Elementary and Secondary Education Act reauthorization (Every Student Succeeds Act of 2015) to use college-entrance exams for high school accountability purposes (Gewertz, 2016). Given this situation, there is a need for new kinds of policies and practices to promote student success in the standard pre-college math sequence: the policy that I consider below is not only novel, but holds out the possibility of being relatively simple and inexpensive to implement.

Policy Context

In spring of 2013, a large urban district in Florida mandated that every student who enrolled in Algebra I in the 2012-13 school year or later take Algebra II in the next school year, with a mandatory Geometry course to follow (Algebra II was optional under state rules). This policy reversed the standard ordering of Geometry followed by Algebra II, which is almost universal in the U.S.[51] According to the district, this policy was terminated after two years, and students who enrolled in Algebra I in 2014-15 (whom I do not observe) reverted to taking Geometry and Algebra II in the traditional order.

Three rationales for the policy were offered in conversations with district leaders. First, it was suggested that students might be more successful in Algebra II if they enrolled immediately after Algebra I. Second, there was hope that because Geometry was required, enrolling students in Algebra II prior to Geometry would increase the total number of college preparatory math courses that students completed. A third rationale, which the available data did not allow me to evaluate, was that enrolling students in Geometry closer to the time that they sat the SAT would improve their performance on the Geometry sub-section of that test, which the district had identified as an area of weakness.

In the district, which had approximately 190,000 students in the 2014-15 school year,[52] students enrolled in Algebra I in seventh, eighth, or ninth grade, with higher achieving students generally enrolling earlier. One complication for this study is that in the same year that the course reordering took effect, the district also began enrolling more students in Algebra I in seventh or eighth grade meaning that some students who enrolled

[51] It also technically mandated Algebra II for all affected students (the state of Florida mandates that students take Algebra I and Geometry, but not Algebra II, for high school graduation), but this was of relatively little consequence: The district's rate of Algebra II enrollment had been high, and increasing steadily, in the years leading up to the policy, meaning that Algebra II enrollment was already extremely common, if not quite universal.
[52] National Center for Education Statistics, Common Core of Data (CCD), 2014-15

in Algebra I and II consecutively also enrolled in Algebra I earlier than they would have otherwise. I address this in my analysis below.

I also exclude seventh grade Algebra I enrollees from my analysis, because the policy context for those students differs substantially than for other students: In addition to reversing the order in which they enrolled in courses, it made Algebra II a middle school class, composed entirely of relatively high-performing peers.[53] It is therefore impossible to disentangle the effect of the course reordering on these students from the effect of a large shift in peer characteristics. Results for seventh grade Algebra I enrollees are presented in the appendix.

My principal research question in this chapter is how Algebra II course grades changed when students enrolled in Algebra I and II consecutively. I also investigate how rates of Algebra II enrollment changed, not only because this is an interesting question in its own right, but also because it is important to understand whether and how the sample of Algebra II enrollees differed between the pre- and post-policy period.

Data and Measures

Data

I use data on students in a large urban school district in Florida. Seventh and eighth-grade students in the district attended 35 middle schools (grades six to eight) and three K-8 schools, while ninth-grade students attended 19 high schools. There were also a

[53] Results (available on request) indicate that there was little overlap between the prior-achievement distributions in these students' Algebra II classes before and after the policy was implemented.

number of charter schools serving the district, as well as schools serving special populations, such as schools for the hospitalized or homebound, schools located in juvenile detention or addiction treatment facilities, and schools for students who were over-age. The principal restriction that I impose on my analytic sample is to exclude students who, at any point, were enrolled in a school other than one of the 57 district-operated comprehensive K-8, middle, or high schools, and may therefore have been exempted from some or all district policies.[54]

My final analytic dataset for this chapter contains 49,834 students who enrolled in Algebra I between 2009-10 and 2012-13. 12% of those students (5,855) first took Algebra I in seventh grade (results for whom are presented separately, in the appendix), 25% (12,353) in eighth grade, and 63% (31,626) in ninth grade. 40,870 of those students enrolled in Algebra II within two years of completing Algebra I and have grades reported in both Algebra I and Algebra II. I exclude 12,555 students in the second policy-exposed cohort who enrolled in Algebra I in 2013-14 from this analysis because their Algebra II course grades are not comparable to those from earlier cohorts. This is due to the introduction of a new set of Algebra II standards, and an Algebra II EOC test, the first in the state's history – both of these substantially increased the difficulty of the Algebra II course. I deal with this issue in more detail in the next chapter. More detail on the data preparation process is provided in the appendix.

[54] I exclude 4,963 students because they were enrolled in a charter school in one or more years, and an additional 4,625 students who spent some or all of their time in a school serving a special population. I do account for these students in constructing measures of classroom characteristics.

The district provided a second dataset containing information about classroom and teacher assignments. Because I was only able to associate 75.6% of Algebra II enrollees with a section of that course, I use the information provided by this data only in ancillary analyses that I use to strengthen my interpretation my main results. I describe these data, and the results based on them, in the appendix.[55]

Measures

Treatment variable

The question predictor in this analysis, *Post-policy*, is an indicator variable equal to 1 for students who enrolled in Algebra I in the 2012-13 school year, and 0 for those who enrolled prior.

Achievement measures

Grades: Student course grades were reported as A, B, C, D, or F with no plus/minus grades. If a student enrolled in a course in multiple years, I use the grade from the first year that they enrolled. In general, fewer than 1% of students are missing grades for a year in which they enrolled in Algebra I or II or Geometry. When students enrolled in multiple sections of the same class in a given year I assign the average grade across those sections, using a standard five-point scale (with A=4 and F=0), and round the result down if it falls exactly between two grades (e.g. 3.5 to a B, not an A).

[55] See appendix Tables A10 and A11

Test scores: In the interest of maximizing the size, diversity, and timespan coverage of my sample I do not use standardized test results in my main analysis. I provide a description of the available test results, and the limitations associated with them, in the appendix. Results from models controlling for available test scores are presented in the appendix, and do not differ substantively from those presented below.

Demographic variables: I use binary indicators to control for a standard set of student demographic characteristics: Race/ethnicity (Hispanic, black, Asian, white, and "other", which includes Native American, multi-racial, and unspecified), eligibility for free-or-reduced price lunch (FRL), classification for special education (SPED), and classification as limited-English proficient (LEP).

Methods

My central question in this chapter is how consecutive enrollment in Algebra I and II affected students' course grades in Algebra II, as compared with enrollment in Algebra I and Geometry prior to Algebra II. Answering this question requires not only describing the difference in Algebra II course grades between students who followed the two course orderings, but also ruling out alternative explanations for any differences observed. My strategy for doing this has three parts: First, I establish that there is an exogenous source of variation in the order in which students enrolled in their courses. Second, I describe how outcomes differed between students who followed the two different orderings (conditional on student characteristics and school fixed effects). Third, having eliminated selection, observable differences between students, and fixed characteristics of schools, as explanations for observed differences in outcomes, I conduct a series of supplemental analyses to rule out other explanations.

I describe most of these supplemental analyses at the end of the results section, with detailed results in the appendix, but present the three most important in detail: First, I conduct a difference-in-differences analysis to isolate the effect of the reordering from the effect of the Algebra I enrollment acceleration. Second, I conduct another difference-in-differences analysis to isolate the effect of the reordering from the effect of increased Algebra II enrollment under the policy – enrollment by students who presumably would not have enrolled under the standard ordering. Third, I compare not Algebra II grades across the two groups, but grades in students' first post-Algebra I class. I do this to rule out the possibility that outcomes were affected by students simply being younger when they enrolled in Algebra II immediately after Algebra I rather than waiting an additional year.

Establishing Exogenous Variation

The first step in my analysis is to establish the extent to which the policy actually effected a change in enrollment patterns. Prior to the 2012-13 school year the district's policy was for students to follow the standard course ordering, while in 2012-13 the policy was for students to enroll in Algebra I and II consecutively, with Geometry to follow. The policy was announced near the end of the 2011-12 school year, meaning that students had limited opportunity to select in or out of the reordered course sequence.[56] In

[56] It does appear, as I describe in more detail in Chapter 3, that a relatively small number of 2012-13 eighth grade Algebra I enrollees did defy the policy by enrolling in Geometry immediately after Algebra I.

order to formally describe the effect of the policy on actual course enrollment behavior, I fit the following model:

$$(1)\ Consecutive\ Alg\ I\ and\ II_{is} = \varphi_s + \beta_1 * PostPolicy_{is} + \beta_2 *\ Grd\ 8\ Alg\ I\ _{is}$$
$$+\beta_3 *\ Post - policy\ x\ Grd\ 8\ Alg\ I_{is} + AlgI\ Course\ Grade_{is} * \Theta$$
$$+Demographics_{is} * \Psi + \varepsilon_{is}$$

With *Consecutive Alg I and II* a 0/1 indicator for student *i* enrolling in Algebra II in high school *s* immediately after Algebra I, and φ_s a vector of school fixed effects. β_1 gives the change in the percentage of ninth-grade Algebra I enrollees enrolling in Algebra I and II consecutively after the policy came into effect, and the sum of β_1 and β_3 gives the change in the percentage of eighth-grade enrollees. Ideally, these estimates would both be close (or equal) to 100%.

Quantifying Policy Effects

Having established the relationship between my treatment variable, *Post-policy*, and actual changes in course enrollment patterns, I turn to studying the relationship between the treatment variable and Algebra II outcomes, specifically course grades. The basic model that I use is the linear probability model:[57]

[57] The ease of interpreting results from a linear probability model outweigh its shortcomings, principally that the OLS assumption of normally distributed errors is necessarily violated when using a dichotomous outcome, and that it is theoretically possible to predict nonsensical probabilities, outside of the [0,1] range. I corroborate point estimates and statistical significance levels from this linear probability model using a set of logistic regression analyses that do not suffer from these shortcomings – results from these are available from the author on request.

$$(2)\ A\ \ in\ Algebra\ II_{is} = \varphi_s + \beta * PostPolicy_{is} + AlgI\ Course\ Grade_{is} * \Theta + \pi * Grade\ 8\ Alg\ I\ enrollment_{is} + Demographics_{is} * \Psi + \varepsilon_{is}$$

With β the primary coefficient of interest, providing an estimate of the difference in probability of earning a given grade (an "A" in the example above) in Algebra II before and after the policy was enacted, net of a vector of school fixed effects (φ_s), and other control variables, as described above.

I also fit a version of this model adding a vector of interactions between the treatment variable, *Post-policy*, and the Algebra I course grade indicators, in order to examine how the probability of receiving a given grade in Algebra II changed for students at each level of Algebra I achievement.[58]

Supplemental Analyses

The analyses above rule out the possibility that any difference in Algebra II outcomes is due to selection, to observable differences between students, or to fixed characteristics of schools. Nonetheless, there remain other possible explanations for observed differences in outcomes that need to be addressed. I describe my approach to three of these issues below. At the conclusion of my presentation of results I discuss my approach to other possible explanations that I consider and rule out.

[58]In the appendix I also present results from a version controlling for Algebra I EOC test scores (Table A8), middle school test scores (Table A9), and classroom fixed effects (Table A11).

Difference-in-difference analyses

As mentioned earlier, there was a sharp acceleration in middle school Algebra I enrollment that changed the composition of the groups enrolling in Algebra I in a given grade just as the course re-ordering was being implemented, and this complicates my effort to make comparisons between pre- and post-policy cohorts.

The middle-school Algebra I acceleration induced a substantial deterioration in the mean prior achievement of eighth-grade enrollees, by approximately 0.2 standard deviations.[59] Because the impact of the enrollment acceleration varied substantially across the district's 38 middle and K-8 schools, however, it is possible to conduct a difference-in-difference analysis for eighth-grade Algebra I enrollees to isolate the impact of the course reordering from that of the acceleration.

In some middle schools, the number of Algebra I enrollees increased by less than 10% in the 2012-13 school year, while in others it increased by over 250%. It stands to reason that post-acceleration students from middle schools with small increases in Algebra I enrollments would be relatively similar to pre-acceleration students, in not only observable, but also unobservable ways. In contrast, post-acceleration students from middle schools that had a large increase in Algebra I enrollments are likely to be quite different, on average, from pre-acceleration students. Therefore, any shift in outcomes

[59] Based on sixth-grade FCAT scores.

that is equally strong for eighth-grade enrollees from middle schools with small, and with large, increases in enrollments, is more likely to be attributable to the course reordering.[60]

To investigate whether the difference in conditional Algebra II outcomes before and after the reordering (the first difference) differed between students who graduated from middle schools that accelerated smaller and larger percentages of their eighth-grade students into Algebra I (the second difference) in 2012-13, I use the following OLS model:

$$(3)\ Algebra\ II\ Grade_{is} = \beta_0 + \beta_1 * PostPolicy$$
$$+\beta_2 * \log(Increase\ in\ G8\ AlgI\ Enrollment_s)$$
$$+\beta_3 * \log(Increase_s\ )x\ Post - Policy + AlgI\ Course\ Grade_{is} * \Theta$$
$$+ Demographics_{is} * \Psi + \varepsilon_{is}$$

With β_3 the primary quantity of interest, providing an estimate of how any post-policy change in Algebra II grade-point average varies with the magnitude of the Algebra I enrollment acceleration that occurred in a student's middle school. I use the natural logarithm of the enrollment increase because the untransformed distribution of enrollment increases has a very long right tail. I also fit expanded versions of Model (2)

[60] I note that any effect observed at higher levels in schools with large post-acceleration increases cannot be interpreted as a pure effect of the acceleration, but rather as the effect of an acceleration combined with consecutive Algebra I and Algebra II – something that is likely to be of fairly narrow interest.

allowing the post-policy coefficient to vary in the same way, and provide results from this model in the appendix.[61]

I use a very similar strategy to investigate whether changes in Algebra II course grades were due to an increase in Algebra II enrollment rates that occurred after the policy was implemented. The enrollment increase (which I document below) was largely restricted to ninth-grade Algebra I enrollees, and I approach this question by fitting a version of Model (3) only for ninth-grade Algebra I enrollees, and substituting post-policy change in high-school Algebra II enrollment rates for the logarithm of the change in middle school Algebra I enrollment rates.

Alternative outcome

Another plausible explanation for differences between pre- and post-policy students' Algebra II grades is that policy-exposed students were simply younger when they enrolled in Algebra II. Were this the case, one would expect the effect of the policy to be weaker when the outcome is student grades in their grade in their first post-Algebra I course, rather than specifically in Algebra II.

To test this hypothesis, I re-fit Model (2), substituting students' grades in their first post-Algebra I math course (irrespective of content) for grades in Algebra II (irrespective of timing). Were differences in outcome due to the maturity that students

[61] I provide results from this model because they are directly comparable with those from Model (2), but do not provide them as main results because the very small size of some cells combined with the difference-in-differences design creates a lack of statistical power that makes some estimates unstable.

were bringing to their math classes I would expect to observe no effect, or at least a weaker effect, of the policy on this outcome.

Results

Means of key variables are provided in Table 1: The analytic sample is approximately one-quarter black, and one-third each white and Hispanic. Approximately 60% of students were eligible for Free-or-Reduced-Price Lunch (FRL), just under 10% were classified for special education services, and around 15% as limited English proficient. Approximately 60% earned a B or a C in Algebra I. These percentages are generally within one percentage point of the corresponding figures for the unrestricted sample, and are generally very similar for the pre- and post-policy groups.[62]

Policy Implementation

In the three years prior to the 2012-13 school year, the overwhelming majority of students enrolled in Geometry immediately after Algebra I, while in the 2012-13 school year the overwhelming majority enrolled in Algebra II immediately after Algebra I. This is starkly illustrated by Figure 1, and regression results indicate that, even net of controls, fewer than 5% of students enrolled in Algebra I and II consecutively before the policy was implemented, while over 90% did afterwards (Table 2).[63] This result is robust to the inclusion (or exclusion) of student controls and school fixed effects.

[62] Correlations between variables are generally moderate to weak – a correlation matrix is provided in the appendix.
[63] Table 1 indicates that the effect of the policy was somewhat weaker for eighth-grade Algebra I enrollees, with 87% enrolling in Algebra I and II consecutively after the policy (0.048-0.027+0.901-0.057), and 95% of ninth-grade Algebra I enrollees (0.048+0.901).

This indicates that, at least to the extent that enrollment in Algebra I in 2012-13 (and not earlier) was assigned by birth timing (i.e. randomly), there should be little or no selection bias in evaluating the effect of the policy. More formally, it indicates that the treatment predictor, *Post-policy*, is a very strong instrument for enrolling in Algebra I and II consecutively. Consequently, although I do not conduct a formal two-stage least-squares analysis, the treatment effects estimated below are likely to be somewhat conservative – roughly 10% smaller than would be estimated by an IV analysis.

Treatment Effects

The percentage of students enrolling in Algebra II within two years of Algebra I increased after the policy came into effect, and the increases were larger for students with lower Algebra I grades. Overall, the rate of Algebra II enrollment increased by 7.4 percentage points ($p<0.001$) (Table 3). There was no statistically significant increase for students who earned A's in Algebra I, and only a 4.4 percentage point increase for those who earned B's ($p<0.001$), but increases of over 16 percentage points for students who earned D's or F's ($p<0.001$ in both cases). These increases were almost entirely among the subset of students who enrolled in Algebra I in ninth grade. Later in this section I consider whether changes in the distribution of Algebra II grades (as described immediately below) were likely due to these enrollment increases.

Students who were exposed to the policy were less likely to earn A's and B's in Algebra II, and more likely to earn D's and F's, than were those who faced a policy mandating the standard course ordering. This is true both for students who earned A's and B’s in Algebra I, and those who earned C’s, D’s, or F’s (Figure 2, left and right panels, respectively). Regression results indicate that there is statistically significant pattern of post-policy students having lower conditional performance in Algebra II, and that this pattern is robust to the inclusion of controls for prior achievement, student demographics, and school fixed effects.

After controlling for prior achievement and student demographics, the probability of earning an A or a B in Algebra II declined by, respectively, 2.7 percentage points ($p<0.001$) and 1.5 percentage points ($p<0.05$), and the probability of earning a C or a D increased by 2.3 and 1.5 percentage points ($p<0.001$ in both cases) (Table 4). A more fine-grained analysis, by Algebra I course grade, offers two additional findings: First, there were no statistically significant differences in Algebra II grades for students who earned A’s in Algebra I, and effects were generally larger for students who earned lower grades in Algebra I. Second, the decline in performance largely reflects post-policy students being less likely to earn the same grade (or a higher grade) in Algebra II as in Algebra I, and being more likely to earn the next lower grade (Table 4, lower panel): For example, policy-exposed students who earned C’s in Algebra I were 3.5 percentage points less likely to B’s in Algebra II ($p<0.01$) and 4.2 percentage points more likely to earn D’s ($p<0.01$).

Difference-in-difference Results

Eighth-grade Algebra I enrollees

After controlling for student demographics and prior achievement, difference-in-difference results indicate that the decline in post-policy Algebra II grades was not associated with the magnitude of the increase in eighth-grade Algebra I enrollments in students' middle schools. After controlling for student background and school fixed effects, the difference-in-difference coefficient, *Increase x Post-policy*, is substantively small and at best marginally significant (-0.0628, p=0.108), as compared with the magnitude of the main effect of the treatment coefficient (-0.187, p<0.001) (Table 5, Column 2).[64]

The fact that the observed differences in Algebra II outcomes did not vary with the size of the eighth-grade enrollment increase in students' schools suggests that they are not due to unobserved compositional differences induced by the enrollment acceleration.

Ninth-grade Algebra I enrollees

Across the nineteen high schools in the district, the post-policy increase in Algebra II enrollment rates among ninth-grade Algebra I enrollees ranges from 4.2 to 17.1 percentages points, with a median of 10.8 percentage points. I find no evidence that the post-policy decline in Algebra II course grades is associated with the size of this increase. The difference-in-difference estimate from this analysis is substantively and

[64] In Appendix Table A7 I present the set of difference-in-difference estimators associated with the estimates presented in Table 4. These corroborate the main results presented here, although some of the cell-sizes are too small to allow robust estimation.

statistically zero (Table 6, Column 2), indicating that my results are not due to unobserved differences between the types of ninth-grade Algebra I enrollees enrolling in Algebra II before and after the policy.

Alternative Outcome: Post-Algebra I Grades

Another plausible explanation for the decline in Algebra II grades is that post-policy students were simply younger when they enrolled in Algebra II, and that their lower grades were attributable to this. Were this the case, one would expect the effect of the policy to be weaker when the outcome is student grades in their grade in their first post-Algebra I course, rather than specifically in Algebra II. In fact, the exact opposite is the case.

The probability of earning an A or a B in one's first post-Algebra I class was much lower when the prescribed course was Algebra II than when it was Geometry, by 3.5 and 7.4 percentage points, respectively ($p<0.001$ in both cases) (Table 7). The probability of earning a D increased by 5.9 percentage points ($p<0.001$). These results reflect a tendency for students (at least those earning A's, B's, or C's) to have a substantially greater probability of scoring *two* full grade points lower in their first post-Algebra I math course than in Algebra I (as opposed to one grade point lower in the results presented above) (Table 7, lower panel). This contradicts the hypothesis that the decline in Algebra II grades observed above was due to pre-policy students simply having an extra year of maturity, irrespective of course content.

Additional Robustness and Validity Checks

I conduct four additional analyses to increase confidence in my interpretation of my results as providing evidence for a negative effect on Algebra II performance of taking the course immediately after Algebra I. I describe these results briefly below, and present more detailed results in the appendix.

First, I seek to validate my use of course grades as a measure of achievement by testing for the presence of grade inflation after the policy was enacted.[65] A major threat to the validity of this analysis would be for the meaning of course grades to change after the policy was implemented: Specifically, I am addressing the concern that students earning a given grade in Algebra I in the 2011-12 school year had lower grades in Algebra II than students who earned the same grade in 2012-13 because Algebra I grading standards were lower in 2012-13, and the decline in conditional Algebra II performance were simply an artefact of this. The first step in addressing this concern is to document the degree (if any) to which Algebra I grading standards differed between 2011-12 and 2012-13. I check for this possibility in two ways: First, I regress pre- and post-policy Algebra I course grades on Algebra I EOC test scores to check for differences in the level of achievement associated with each grade point, as measured by the test, before and after the policy. This offers no evidence of grade inflation.[66] Second, I fit a version of Model (2) using only the 2011-12 and 2012-13 Algebra I cohorts (for which I have EOC test

[65] That is, inflation in Algebra I grades. Another concern is deflation of Algebra II grades – that is, higher grading standards in Algebra II after the policy. Because there was no Algebra II EOC test in the sampled period, it is impossible to test for this.
[66] See Appendix Table A13

scores), including those scores. Results from this model do not differ substantively from those presented above and, in fact, find moderately larger effects.[67]

Second, I investigate the extent to which negative effects of the policy might have been due to disruption caused by the new policy, including the double cohort of Algebra II enrollees that it created. Although it is not possible to examine this in depth, one channel by which such a disruption would likely have operated would have been through less experienced teachers being pushed into teaching Algebra II sections. To assess this, I create an indicator variable for teachers who were observed teaching Algebra II for the first time in 2013-14, and an interaction of this indicator with the treatment variable, and include these in an expanded version of Model (2). This is effectively a difference-in-differences model that allows the effect of the policy to vary by whether a student is being taught by an experienced or inexperienced Algebra II teacher. The interaction coefficient from this model (the difference-in-differences estimator) is statistically and substantively zero, indicating that at least one potentially important mechanism of disruption in the first year of the policy did not have an effect on student grades.[68]

Third, I fit a version of Model (2) using only the 2011-12 and 2012-13 Algebra I cohorts and including classroom fixed effects. These cohorts enrolled in Algebra II in the same year, and were mostly programmed into the same classrooms. This controls for all

[67] See Appendix Table A8
[68] See Appendix Table A10

classroom characteristics that were invariant across all students in a given classroom. Results from this model did not differ substantively from those reported above.[69]

Fourth, I conduct a placebo test in which I re-fit two versions of Model (2) defining (falsely) 2011-12 and 2010-11, in turn, as the "post-policy" year, and excluding any subsequent cohorts. Based on the data visualization in Figure 2, I select this, rather than controlling for a time trend in Model (2), as a more appropriate strategy for testing whether the first post-policy year is actually exceptional in the broader context of this time period under consideration. I do not find a statistically significant difference in Algebra II grade conditional on Algebra I grades between any other pair of consecutive cohorts.[70]

Discussion

Students who enrolled in Algebra I and II consecutively had lower grades in Algebra II, conditional on their prior math achievement, than students who enrolled in Geometry before Algebra II. The evidence presented here strongly suggests that enrolling in Algebra II immediately after Algebra I was harmful to all but the strongest Algebra I students, at least in terms of its impact on their Algebra II course grades. For the strongest students in the sample, those who earned A's in Algebra I, it appears that the policy had no effect.[71] It also appears that the probability of a negative effect increased as students'

[69] See Appendix Table A11
[70] See Appendix Table A14
[71]For students who enrolled in Algebra I in seventh grade (see the appendix) the evidence is more ambiguous, but is less consistent with the reordering being beneficial than with it being neutral. Further, to the extent that the reordering had a salutary effect on these students it is at least as plausible that it was due to the incidental fact that it pushed the course into middle school, thus enhancing the overall level of the class for these students.

Algebra I performance declined. I see no persuasive evidence for any other explanation of the decline in Algebra II grades after the course reordering other than the order of the courses.

A further consideration is how student performance in the year following Algebra I changed after the reordering. As described above, these results give even greater cause for pessimism about reordering Algebra II and Geometry: when Geometry grades from the immediate post-Algebra I grade are compared with Algebra II grades from the same grade there was an increased tendency for comparable students to score not one, but two grade points lower, and the patterns of statistical significance are even stronger than in the analyses described above. This is particularly concerning if poor performance in one year's math course leads to lower performance in future years' math courses through, for example, a discouragement effect; I consider this possibility in the next chapter, and do find evidence for such a phenomenon.

Does this mean that it is simply inadvisable to enroll students in Algebra I and Algebra II consecutively? I would caution against drawing such a strong conclusion on the basis of these findings. One must take into account the particular circumstances of this intervention: By all accounts it was hastily implemented, being announced late in preceding school year and without any systematic plan for providing additional support to the students who were most likely to struggle. There is relatively little basis for speculating about specific mechanisms. One possibility is suggested by the fact that some teachers complained that the trigonometry portion of the Algebra II curriculum relied on content from the Geometry curriculum.[72] Although this does not seem like an insurmountable obstacle - the geometry and algebra of similar triangles that form the basis of trigonometry are often covered in middle school or in Algebra I courses – it does indicate that Geometry and Algebra II cannot necessarily be treated as entirely independent courses. Another possibility, noted above, is that teachers raised grading standards in Algebra II in anticipation of the new standards and EOC test that were to be released the following year. I am skeptical that this occurred, principally because teachers were already very busy in this period accommodating present changes in policy and standards, without trying to anticipate future changes.

Given a sunnier picture of student outcomes after the reordering, this could be a particularly attractive reform: It would be easy and inexpensive (if not free) to implement on a large scale, and boosting success in Algebra II could increase student readiness for Calculus and other college-level math courses. These results suggest that any district wanting to experiment with consecutive Algebra I and II enrollment would be well advised to plan carefully, and be prepared to provide substantial support to students who did not excel in Algebra I.

[72] E.g. Martin & Postal, 2016, "Orange students struggle with algebra 2 exam", retrieved from http://www.orlandosentinel.com/features/education/os-orange-algebra-two-20160630-story.html on 9/11/2017.

Chapter Two Tables and Figures

Table 1

Means of student characteristics by pre- and post-policy

	Pre	Post
Black	26.3%	25.3%
Hispanic	34.5%	36.3%
White	31.9%	31.6%
Asian	4.5%	4.1%
Other	2.8%	2.7%
Special Ed.	9.9%	8.1%
FRL-eligible	57.9%	63.3%
Limited English Proficient	16.9%	13.1%
Grade 8 Alg. I	23.6%	39.2%
Enrolled in Alg. II	78.0%	86.8%
A in Alg. I	11.4%	15.8%
B in Alg. I	28.1%	31.9%
C in Alg. I	30.6%	28.5%
D in Alg. I	20.2%	15.9%
F in Alg. I	9.7%	8.0%
N	31,391	12,588

Table 2

Regression results predicting change in consecutive Algebra I and II enrollment rates before and after the introduction of a course-reordering policy

	(1)	(2)	(3)
Post-policy	0.874***	0.901***	0.901***
	(0.000)	(0.000)	(0.000)
Grade 8 Alg. I enrollment		-0.0271***	-0.0502***
		(0.000)	(0.000)
Post-policy x Grd. 8 Alg. I		-0.0570***	-0.0412***
		(0.000)	(0.000)
A in Algebra I			0.151***
			(0.000)
B in Algebra I			0.139***
			(0.000)
C in Algebra I			0.118***
			(0.000)
D in Algebra I			0.0853***
			(0.000)
Constant	0.0408***	0.0482***	-
	(0.000)	(0.000)	-
High School Fixed Effects	N	N	Y
Demographic controls	N	N	Y
N	35,400	35,400	35,193
R-sq	0.762	0.765	0.778

p-values in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Table 3

Change in probability of enrolling in Algebra II within two years of Algebra I after policy implementation

		Probability of Alg. II enrollment
	All students	0.074***
		(0.006)
Algebra I grade	A	0.019
		(0.015)
	B	0.044***
		(0.007)
	C	0.078***
		(0.007)
	D	0.164***
		(0.012)
	F	0.169***
		(0.016)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Table 4

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background and high school fixed effects

		Algebra II grade				
		A	B	C	D	F
	All students	-0.027*** (0.005)	-0.015* (0.007)	0.004 (0.007)	0.023** (0.007)	0.015*** (0.004)
Algebra I grade	A	-0.016 (0.017)	0.021 (0.019)	0.001 (0.017)	0 (0.010)	-0.005 (0.003)
	B	-0.024* (0.009)	-0.026* (0.012)	0.026 (0.016)	0.023 (0.012)	0.001 (0.003)
	C	-0.008 (0.004)	-0.035** (0.011)	-0.008 (0.012)	0.042** (0.015)	0.009 (0.005)
	D	0 (0.005)	-0.024* (0.012)	-0.039*** (0.011)	0.023 (0.015)	0.039** (0.014)
	F	-0.005 (0.010)	-0.009 (0.019)	-0.032 (0.023)	-0.065*** (0.018)	0.111*** (0.032)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Table 5

Difference-in-differences results predicting Algebra II grade (0-4 scale) for eighth-grade Algebra I enrollees as a function of policy-exposure, size of middle school Algebra I acceleration, and their interaction

	(1)	(2)	(3)
Post-policy	-0.160***	-0.193***	-0.187***
	(0.000)	(0.000)	(0.000)
Algebra I grade (0-4 scale)	0.509***	0.513***	0.508***
	(0.000)	(0.000)	(0.000)
Log increase in 8th grade Alg. I enr.		0.219***	0.126**
		(0.000)	(0.005)
Post-policy by log increase		-0.0647	-0.0628
		(0.099)	(0.108)
High school fixed effects	Y	Y	Y
Demographic controls	Y	N	Y
N	9,756	9,756	9,756
R-sq	0.256	0.256	0.258

p-values in parentheses

* p<0.05 ** p<0.01 *** p<0.001

Table 6

Difference-in-differences results predicting Algebra II grade (0-4 scale) for ninth-grade Algebra I enrollees as a function of policy-exposure, size of post-policy high school Algebra II enrollment increase, and their interaction

	(1)	(2)
Post-policy	-0.0881*	-0.135
	(0.01)	(0.060)
Algebra I grade (0-4 scale)	0.392***	0.388***
	(0.000)	(0.000)
Increase in Alg. II enrollment		0.792
		(0.219)
Post-policy by increase		0.510
		(0.478)
High school fixed effects	Y	Y
Demographic controls	Y	Y
N	24,230	24,230
R-sq	0.173	0.174

p-values in parentheses

* p<0.05 ** p<0.01 *** p<0.001

Table 7

Change in probability of earning a given grade in one's first post-Algebra I math class after introduction of policy, overall and by Algebra I course grade, net of student background and high school fixed effects

		Grade in first post-Algebra I math class				
		A	B	C	D	F
	All students	-0.035***	-0.074***	0.006	0.059***	0.044***
		(0.010)	(0.012)	(0.011)	(0.010)	(0.009)
Algebra I grade	A	-0.105***	0.016	0.065***	0.018**	0.007*
		(0.022)	(0.017)	(0.016)	(0.006)	(0.003)
	B	-0.039**	-0.131***	0.063***	0.080***	0.027***
		(0.012)	(0.022)	(0.019)	(0.015)	(0.006)
	C	-0.016	-0.096***	-0.046**	0.095***	0.062***
		(0.009)	(0.019)	(0.017)	(0.019)	(0.016)
	D	-0.006	-0.024	-0.052**	0.01	0.072***
		(0.007)	(0.014)	(0.016)	(0.013)	(0.017)
	F	-0.001	-0.017	-0.044	0	0.062
		(0.008)	(0.016)	(0.025)	(0.021)	(0.038)

standard errors in parentheses

* p<0.05 ** p<0.01 *** p<0.001

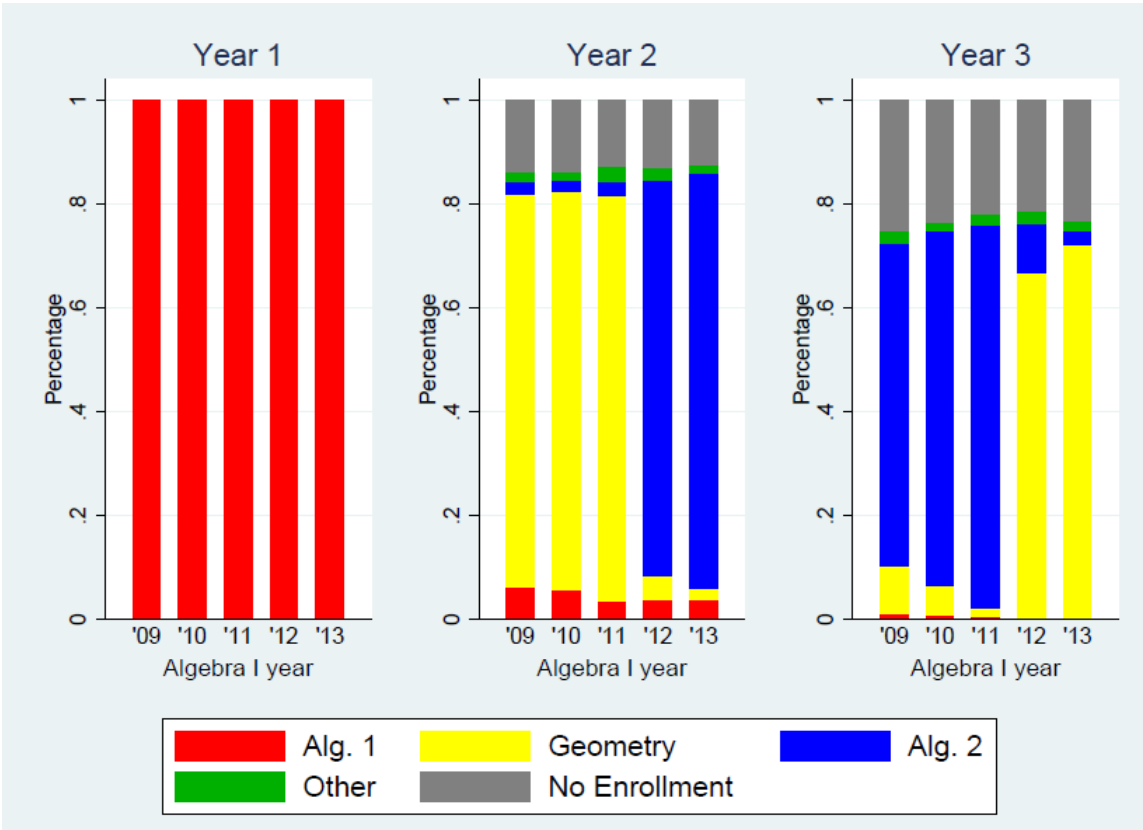

Figure 1: *Course enrollments in first, second, and third year of enrollment in the standard sequence, by year of initial Algebra I enrollment.*

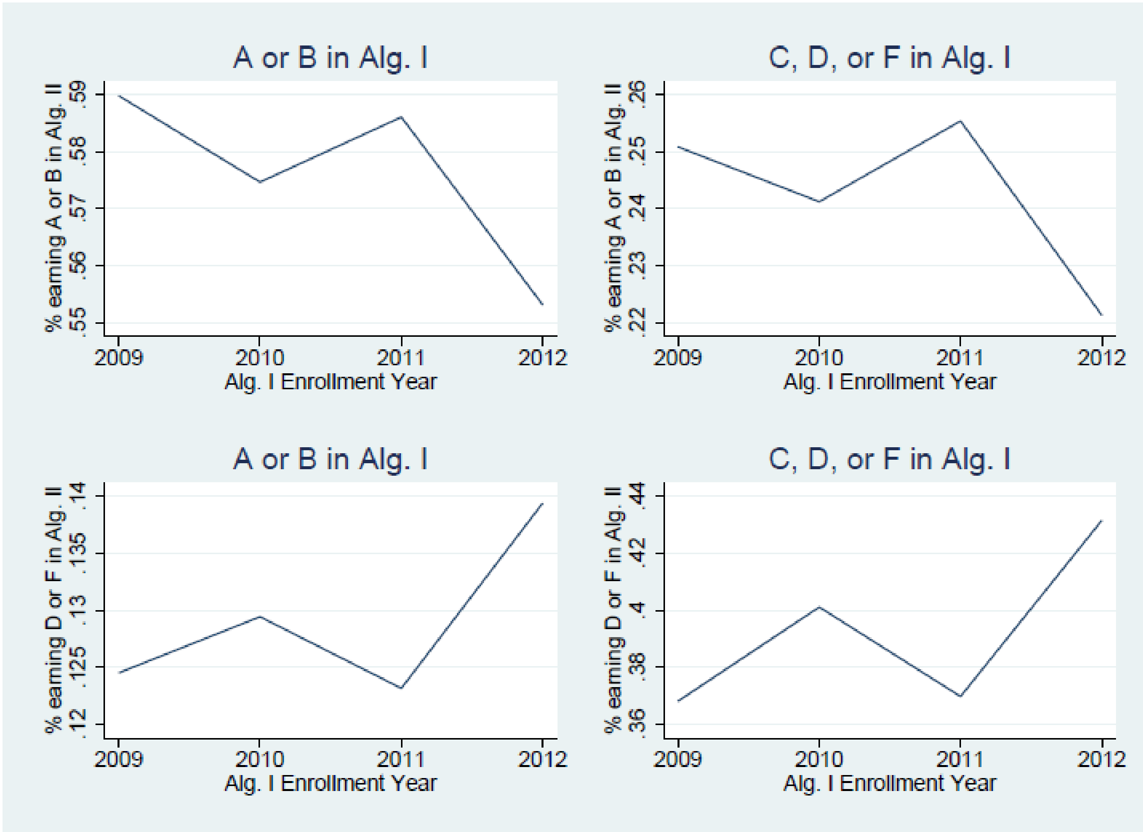

Figure 2: *Percentage of students earning A's or B's (top panel) and D's or F's (bottom panel) in Algebra II by year of Algebra I enrollment and Algebra I course grade.*

Chapter Three

Introduction

In this, the third chapter of my dissertation, I investigate how students' Geometry outcomes changed in a large urban district in Florida when they were mandated to enroll in Algebra I and II consecutively, prior to Geometry, rather than following the traditional ordering of Algebra I, Geometry, and then Algebra II.

I study two outcomes related to Geometry: First, I investigate whether rates of Geometry enrollment changed when students were mandated to enroll in Algebra II before having the opportunity to enroll in Geometry – a mandate with which they generally complied. Second, I investigate how a sharp increase in Algebra II standards during the period in which the reordering was in effect, which led to a deterioration in Algebra II results, impacted Geometry performance. I find that students who enrolled in Algebra I and II consecutively were less likely to enroll in Geometry, and that persistence into a third year of math course-taking declined – this result was not simply due to Geometry being postponed. I also find that student achievement in post-Algebra II Geometry courses declined when the difficulty of the Algebra II course increased, and that a discouragement effect was the most likely cause.

This chapter has five sections. In the first section, I provide the background and context and state my research questions. In the second section I describe additional features of my data and measures that were not relevant to the analysis in chapter two. In the third section I describe my methods, and in the fourth my results. I conclude with a discussion of those results.

Background and Policy Context

The general background and context to this chapter is the same as that of the previous chapter: A large urban school district in Florida mandated that all students who enrolled in Algebra I in the 2012-13 or 2013-14 school years enroll in Algebra II in the next year, and Geometry in the following year. Students in the district enroll in Algebra I in seventh, eighth, or ninth grade, based roughly on their level of math achievement in primary and middle school. In this section I describe additional features of the policy context, not mentioned in the second chapter, that are relevant to the questions that I ask in this portion of the study.

My first question, how rates of Geometry enrollment changed under the policy, is important because, unlike Algebra II, Geometry is a required course in Florida, and failure to complete it creates a serious obstacle to graduation and, for dropouts, to returning to school.

Although a convincing comparison of Geometry test scores, or of course grades, before and after the course reordering is not possible[73], it is possible, and useful, to study

[73]The available data do not permit a before and after comparison of Geometry grades or test scores: there is no set of Geometry outcome measures other than enrollment, and possibly credit attainment, that supports comparisons between the pre- and post-policy periods. For students who enrolled in Algebra I in the 2009-10 and 2010-11 school years (and Geometry in the 2010-11 and 2011-12 school years) there was no Geometry EOC test. Students who enrolled in Algebra I in the 2011-12 school year, and Geometry in the 2012-13 school year (the final pre-policy cohort), sat a Geometry EOC test indexed to Florida's Next Generation Sunshine State Standards (NGSSS) (this exam was also administered in the 2013-14 school year, although very few students in the district enrolled in Geometry that year, due to the course

the consequences of a change that occurred in the second year that the reordering was in effect: In the 2014-15 school year Florida introduced a new set of Algebra II standards, along with its first-ever Algebra II EOC test. As I will document below, this test was substantially harder than any standardized math test students had previously taken, with most students who had posted strong results on earlier assessments earning mediocre scores, and students who had posted mediocre results on earlier assessments generally earning abysmal scores. To be clear, a change in Geometry performance due to a change in Algebra II standards would not be a direct consequence of the course re-ordering policy. Rather, it would be a consequence of an unrelated policy – an increase in the difficulty of Algebra II – but one that would not have occurred in the absence of the course re-ordering.

Without entertaining discussion of whether the difficulty of the Algebra II EOC was excessive or appropriate, it appears that, prior to 2014-15, the district's Algebra II course was not very rigorous: As I argue in chapter one, a student who has mastered the material in a traditional Algebra II course should be on track to enroll in an introductory course in differential calculus with, at most, an additional year of study, generally a subsequent course explicitly titled "Pre-Calculus". It seems significant, therefore, that virtually no student in the district enrolled in Calculus immediately after Algebra II and,

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reordering). Students who enrolled in Algebra I in the 2012-13 and 2013-14 school year, most of whom enrolled in Geometry in the 2014-15 and 2015-16 school years, sat a Geometry EOC test indexed to the Florida Standards Assessment (FSA) system. Although it is not clear that FSA standards differed greatly from the NGSSS standards, the tests did not produce comparable scores, and because students in a given graduation cohort do not all take a given course in the same year, standardizing scores will not address the problem. Although it is not clear that course grades were impacted by the changes in the tests being used, Florida's education statute 1008.22 mandates that EOC test scores comprise 30% of course grades, making it difficult to be confident that course grades are comparable across years.

moreover, that only a minority enrolled in Calculus even after taking Pre-Calculus[74]. Most students either did not persist in math, or enrolled in courses to prepare them to test out of remedial math courses at state colleges and universities or, in a few cases, enrolled in courses in discrete math or statistics.

Data and Measures

The data and measures used in the analyses for this chapter are generally the same as those used in chapter two. The principal difference is that the analyses in this chapter make use of an additional 12,555 students who enrolled in Algebra I in the 2013-14 school year. These students do not differ substantively from those who enrolled in previous years in terms of their demographic and background characteristics. As in the previous chapter, I restrict my main analysis to students who enrolled in Algebra I in eighth or ninth grade, and provide results for seventh-grade enrollees in the appendix. Here I describe additional features of the data and measures that are unique to these analyses.

The only additional restriction I impose is that, in answering my second question as to how Geometry outcomes changed after the introduction of the Algebra II EOC, I only include students who complied with the policy. I do this out of necessity, because the change of Geometry tests means that usable measures of Geometry outcomes are only available for compliers: students who enrolled in Geometry in the 2013-14 school year immediately after completing Algebra I (the vast majority of non-compliers) sat the NGSSS Geometry EOC. In the results section below, I examine the differences between

[74] See Appendix Figures A1 and A2 for course-taking beyond the second year after Algebra I.

compliers and non-compliers below to assess the possibility of bias resulting from this restriction[75]. As well, I introduce one additional measure that does not appear in the second chapter:

Enrollment in Geometry within two years of Algebra I: This is a 0/1 indicator for being observed to enroll in Geometry for the first time in either the first or second year after initial enrollment in Algebra I - both before and after the course reordering, most students who are observed to enroll in Algebra I do so within two years after Algebra I. Although a small number of students enroll in the third or fourth year, I am unable to observe students in the 2012-13 Algebra I cohort in their fourth post-Algebra I year, and 2013-14 Algebra in their third or fourth post-Algebra I years. This censoring makes it infeasible to extend the window beyond two years. Exploratory results suggest that, after the reordering, slightly more students delayed Geometry enrollment beyond their second post-Algebra I year, but that taking account of this would not appreciably change my results.

Methods

Research Question 1: Geometry Enrollment

I approach my first research question, of how the probability of enrolling in Geometry within two years of Algebra I changed under the policy, using the following linear probability model:

Model (1):

[75] To preview this result, I find that any bias is almost certain to be in the opposite direction from the observed effect.

$$
\begin{aligned}
GeomWithin2Years_{is} \\
&= \varphi_s + \beta * Post - policy_{is} + AlgI\ Score\ Level_{is} * \Theta \\
&+ AlgI\ Score\ Level_{is} x\ Post - policy_{is} * \Pi \\
&+ \tau * Grade\ 8\ AlgI_{is} + Demographics_{is} * \Psi + \varepsilon_{is}
\end{aligned}
$$

With φ_s a vector of fixed effects for the first high school that student *i* attended[76] and standard errors clustered at the school level. The inclusion of indicators for Algebra I EOC score levels and their interactions with the *Post-policy* indicator allows me to estimate how changes in rates of Geometry completion varied by student prior achievement.

I also fit a version of Model (1) with the outcome being enrollment in both Geometry and Algebra II within two years of Algebra I. Comparing results from this model with those from the main version of Model (1) indicates the extent to which any decline in Geometry enrollment is due simply to patterns of non-persistence in math courses that exist irrespective of the ordering of courses[77], and to what extent there was an additional decline in persistence after the policy came into effect.

Research Question 2: Geometry EOC Performance

I approach my second question, of how Geometry performance was affected by the increase in the difficulty of Algebra II, in three steps. In the first step, I establish

[76] Last middle school for seventh grade Algebra I enrollees.
[77] This is likely due to movement out of district schools, but the available data offer no way to be sure.

whether and how Geometry performance, conditional on Algebra I performance, changed in the second year of the course reordering. In the second step I investigate whether discouragement from a poor Algebra II EOC test result could have negatively influenced Geometry EOC test performance. I do this using a regression discontinuity, comparing Geometry scores for students who scored just above and below the cut scores for different Algebra II performance levels. In the third step, I consider possible explanations for discontinuous changes (to the extent that I observe them) in Geometry outcomes for students just above and below Algebra II cut scores to understand whether discouragement is the most plausible explanation. I now explain each step in more detail.

In the first step, I fit an OLS regression of Geometry EOC scores on Algebra I EOC scores, including 2012-13 and 2013-14 Algebra I enrollees who sat the FSA Geometry EOC test. I include an indicator for 2013-14 enrollees, and test an interaction of this indicator with Algebra I EOC scores in order to allow both the average level and the slope of the relationship to vary from year to year. The model is:

Model (2):

$$
\begin{aligned}
&Geometry\ EOC\ Score_{is} \\
&\qquad = \varphi_s + \beta_1 AlgI\ EOC\ Score_{is} + \beta_2 2013\ AlgI\ Enrollee_{is} \\
&\qquad + \beta_3 2013 Enrollee_{is}\ x\ AlgI\ EOC\ Score_{is} + \ \tau * Grade\ 8\ AlgI_{is} \\
&\qquad + Demographics_{is} * \Psi + \varepsilon_{is}
\end{aligned}
$$

with β_2 the primary coefficient of interest, and standard errors clustered at the school level. β_2 provides an estimate of the average difference in Geometry EOC score between

2012-13 and 2013-14 enrollees conditional on Algebra I EOC score, and net of student demographic controls and fixed effects for school of Geometry enrollment.

A statistically significant value of β_2 on its own, however, would not establish a causal link between the difficulty of the Algebra II course and Geometry outcomes. It would (and does) strongly suggest that something occurred (or failed to occur) in the second year that negatively impacted Geometry performance, and the most notable difference across the two years observed was the introduction of an extremely difficult Algebra II EOC test. Based on the results of this model, I hypothesize that students' poor performance on the Algebra II EOC test was an important factor in weakening performance on the Geometry EOC test, and conduct a second analysis to investigate this hypothesis.

For this second analysis, I employ a regression discontinuity, in which I compare students who scored just above and just below the cut score to qualify as Level 3 on the Algebra II EOC test. This is an important cut, because Level 3 carries the label "Satisfactory" while Level 2 carries the label "Below Satisfactory" (Level 1 is "Inadequate", while levels 4 and 5 are "Proficient" and "Mastery"). The distinction between Levels 2 and 3 is therefore the distinction between receiving a positive and a negative evaluation. Further, as a practical matter, so few students scored above Level 3 on the Algebra II EOC as to make the analysis infeasible for higher cut scores. The basic model is:

Model (3):

$$
\begin{aligned}
&Geometry\ EOC\ Score_{is} \\
&\qquad = \varphi_s + \beta_1 AlgII\ EOC\ Score_{is} + \beta_2 AboveCut_{is} \\
&\qquad + \beta_3 AlgII\ EOC\ Score_{is} x AboveCut_{is} + \tau * Grade\ 8\ AlgI_{is} \\
&\qquad + Demographics_{is} * \Psi + \varepsilon_{is}
\end{aligned}
$$

with β_2 the primary coefficient of interest, providing an estimate of any incremental benefit (above that predicted by the local trend, β_1), to scoring just above the cut. A positive, statistically significant estimate of β_2 would strongly suggest that Geometry performance for 2013-14 Algebra I enrollees was weakened by a mechanism operating through the pathway of Algebra II scores. This strategy assumes that students who just failed to score at Level 3 and students who just succeeded in scoring at Level 3 are effectively equivalent in expectation, net of any linear relationship between Geometry and Algebra II scores in the vicinity of the cut score[78]. While this helps to establish whether Algebra II scores were a mechanism affecting Geometry scores, it is not informative about the nature of this mechanism, and additional steps are required.

I further hypothesize that Algebra II EOC scores acted on Geometry EOC scores through a discouragement effect, resulting from students receiving especially disappointing feedback about their aptitude in math. I use variants on Model 3, above, to consider possible mechanisms by which the difference between scoring at Level 2 or 3 could, in itself, have influenced outcomes: First, I consider the possibility that the

[78] Another assumption required for the validity of the regression discontinuity design is that individuals cannot manipulate which side of the cut score they fall on. Because the EOC tests were scored off-site by non-district personnel, and largely by machines, I see no reason to doubt that this assumption is satisfied.

probability of having a valid Geometry score varied around the cut, and specifically that higher ability students who scored just below the cut score were more likely to select out of enrolling in Geometry in a district school. I investigate this hypothesis by using the right-hand-side of Model 3 to predict the probability of having a valid Geometry EOC score. Second, I want to rule out the possibility that schools programmed students into different Geometry sections based on which side of the cut score they fell on. I investigate this by fitting Model 3 with the mean Algebra II test score in students' Geometry classes as the outcome. If neither of these outcomes differs between students who score just below and just above the cut score between Levels 2 and 3, it would rule out two important institutional channels through which Algebra II scores could have acted on Geometry scores, suggesting that they acted through a psychological channel.

Results

Research Question 1: Geometry Enrollment

In general, students were less likely to enroll in Geometry within two years of completing Algebra I after the policy was implemented[79] (Figure 1). Regression results indicate that the observed decline of roughly ten percentage points in Figure 1 is highly significant, and robust to the inclusion of controls for student demographics and prior achievement (-0.098, $p<0.001$) (Table 1, Column 1). More fine-grained results indicate that there was a decline in Geometry enrollment rates at all levels of prior achievement, although it was approximately twice as large for students who scored at levels 1 or 2 on

[79] The available data suggest that there was not a substantial offsetting increase in the number of students enrolling in Geometry in their third post-Algebra I year.

their Algebra I EOC tests (-0.139, p<0.001) compared with students who scored at levels 3, 4 or 5 (Table 1, Column 1, lower panel).

It appears that the decline in Geometry enrollment was primarily a function of the course being postponed, rather than to an effect specifically of having enrolled in Algebra I and II consecutively: The probability of students enrolling in both Geometry and Algebra II within two years of Algebra I exhibited much smaller, but still highly significant declines after the policy came into effect. Overall, the probability of enrolling in both courses declined by 3.2 percentage points (p<0.001) (Table 1, Column 2), and this did not vary systematically with students' prior achievement.

Research Question 2: Geometry EOC Performance

I now turn to comparing the Geometry EOC test performance of the two cohorts of students who enrolled in Algebra I and II consecutively, and investigating the effect of the newly introduced Algebra II EOC test on the second group. The second group (those who enrolled in Algebra I in the 2013-14 school year) differs from the first group (those who enrolled in Algebra I in the 2012-13 school year) primarily in that their Algebra II course was indexed to a new set of standards (the FSA standards) and, for the first time in Florida's history, concluded with a mandatory EOC test. Especially for eighth and ninth grade Algebra I enrollees, this test was likely to have been extremely frustrating and disappointing: The majority of these students scored at Levels 1 and 2, and (for those who scored at Level 3 or above on their Algebra I EOC test), the majority scored at least two levels lower than they had on their Algebra I EOC test[80]. Even for seventh grade enrollees, who fared somewhat better, only a minority scored at the same level (or a higher level) on their Algebra II EOC test as on the Algebra I EOC test.

Because it is only possible to include students who complied with the policy in this analysis, I begin by comparing compliers and non-compliers for each cohort and, where compliance was substantially below 100%, evaluating the possibility of bias resulting from selective non-compliance. I then compare Geometry scores (conditional on Algebra I scores) between the 2012-13 and 2013-14 cohorts and, in cases where I observe a substantial decline from the first cohort to the next (I never observe an increase), evaluate the hypothesis that the Algebra II EOC test had a role in causing that decline.

Compliance: checking for bias

In the first year that students were mandated to take Algebra I and II consecutively, approximately ten percent of eighth grade enrollees did not comply with the policy, enrolling in Geometry in ninth grade, and Algebra II in tenth grade (virtually all ninth grade Algebra I enrollees did comply, as did virtually all students in the policy's second year). This non-compliance is observed in all high schools in the sample, and it appears that, district-wide, students were offered the option of following the standard ordering in the first year of the policy. The group that availed itself of the option was, on average of higher ability than the group that complied (even within the subset of eighth grade enrollees). Although this creates a near certainty of selection bias in my observed

[80] A matrix comparing Algebra I and Algebra II EOC test levels is provided in Appendix Table A15.

results, it is in the opposite direction from the observed effect: I find that the 2012-13 cohort would have stronger Geometry outcomes than the 2013-14 cohort, which is less likely to be observed if higher ability students select out of the 2012-13 sample, but not the 2013-14 sample.

Geometry EOC test results

Conditional on Algebra I performance, Algebra I enrollees from the 2013-14 cohort who sat the FSA Geometry test earned scores that, conditional on Algebra I EOC performance, were consistently lower than those earned by the 2012-13 cohort (Figure 2). This is corroborated by regression results[81], which indicate that, after controlling for Algebra I EOC scores and demographic controls, Geometry EOC scores were 0.18 standard deviations lower ($p<0.001$) for the 2013-14 cohort than for the 2012-13 cohort (Table 2, Column 2).

Regression discontinuity: Geometry vs. Algebra II EOC test results

Turning to my regression discontinuity analysis, Figure 3 shows a noticeable discontinuous increase in conditional Geometry scores for students just above the cut score between Levels 2 and 3. Regression estimates using bandwidths of between +/-8 and +/-16 points around the cut score find an increase of between 0.117 and 0.163 standard deviations (with an average of 0.139) associated with scoring just above the cut

[81] I restrict the sample for the regression results to the 87.4% of the sample with Algebra I EOC scores between 375 and 450. Outside of this range there is some evident non-linearity (Figure 2), and it also appears that there are too few test-takers to permit confident estimation of any relationship that does exist.

score, net of the local trend of Geometry on Algebra II scores[82] (p<0.01 for all bandwidths) (Table 3, Column 1). At bandwidths smaller than +/-8 the estimated difference is in fact much larger, but the trend line cannot be estimated. At bandwidths above +/- 16 points the estimate begins to shrink and lose statistical significance, although the estimated trend is including students who are increasingly different from those near the cut score[83].

My hypothesis, however, is not only that the difficulty of the Algebra II test was the mechanism by which Geometry performance fell in 2013-14, but also that it acted through a discouragement effect. A subsidiary set of regression discontinuity results indicate that, if there was a discouragement effect, it did not operate through students below the level 3 cut score being less likely to enroll in Geometry after Algebra II. This is illustrated in Table 3, Column 2, where the cut-score coefficient from a version of the regression discontinuity model predicting probability of Geometry enrollment is statistically and substantively zero for all bandwidths.

I also want to rule out another possible explanation for the discontinuous increase in Geometry scores for students scoring just above the level 3 cut on the Algebra II test: that students enrolled in different sections of Geometry based on their Algebra II score level. Fitting the regression discontinuity model with the mean Algebra II test score in students' Geometry classes as the outcome, indicates that in neither case is there a

[82] The version of the model presented allows the trend to differ on either side of the cut score. A restricted version not allowing this provides a substantively identical estimate of the discontinuity.
[83] An illustration of this is provided in Appendix Figure A3.

substantively or statistically significant difference across the cut score at any bandwidth[84].

Another limitation of the regression discontinuity results is that they only provide an estimate of the local effect of Algebra II score levels on students near the cut between Levels 2 and 3, and do not necessarily support claims about students further from that score point. This limitation is not entirely remediable, although the function of the regression discontinuity analysis in this argument is not simply to provide a local estimate that is interesting in itself. Rather, it provides an existence proof for the proposition that discouragement and frustration in Algebra II can impact performance in a later Geometry class. Given the results presented above, this seems very likely (if not strictly proven). Furthermore, combined with the fact that the vast majority of students, even those scoring at level three, scored at a level much lower than that to which they had been accustomed, these results are consistent with the Geometry performance of students in the 2013-14 cohort having been negatively impacted by poor results on the Algebra II EOC test.

Discussion

I find evidence consistent with students having been harmed (both directly and indirectly) by the course reordering policy, and no evidence of any benefits, the same general conclusion that I drew in the previous chapter. The harm took two forms in the case of Geometry: First, when Geometry enrollment was postponed, fewer students were observed to enroll in this required course. This could be largely, but not entirely,

[84] Results are available on request

explained entirely by the postponement. Second, when the difficulty of the Algebra II course increased, it caused students to have lower achievement in their subsequent Geometry course. While this is technically not a consequence of the re-ordering itself, it would not have occurred in the absence of the reordering, and it has consequences for any future proposals to enroll students in Algebra II earlier in their schooling. I discuss each of these findings in turn.

The clearest reason to consider reduced probability of Geometry enrollment to be harmful is that the course is required for graduation. Because the reduced probability of Geometry enrollment was equally prevalent at high and low levels of prior achievement, it seems plausible that at least some students simply left the district and enrolled in Geometry elsewhere. Particularly among ninth-grade enrollees who earned lower scores in Algebra I, however, failure to appear in Geometry classes in eleventh grade probably reflect a substantial number of students who failed to clear a hurdle to high school graduation.

The second result, the negative impact on Geometry achievement of the Algebra II EOC test should also be of interest to districts that are considering a switch to a consecutive Algebra I and II curriculum, or that have implemented such a curriculum and are facing (or considering) an increase in the rigor of their Algebra II course. It is beyond the scope of this paper to quantify levels of rigor, but I would propose that Algebra II courses are set between two poles: courses that cover little more than would have been covered by a typical Algebra I class of the late 1970s, and courses that aim to prepare as many students as possible to enroll in a Calculus course. These results suggest that the closer a district's Algebra II course is to the latter pole, the stronger the argument for postponing that course until students have satisfied their graduation requirements in math and face less risk of harm from being confronted with negative information about the objective level of their math aptitude.

In summary, the results presented in chapters two and three provide no endorsement for a policy of consecutive Algebra I and II. Given, however, that the negative effects observed were generally small, and in at least some cases probably related to specific features of the policy context, schools or districts may still wish to experiment with this intervention, and this study may provide helpful guidance.

Chapter Three Tables and Figures

Table 1

Change in enrollment probability within two years of Algebra I for post-policy Algebra I enrollees, overall and by Algebra I EOC level, compared with average for prior three years. Regression adjusted including controls for student prior achievement and demographics, and school fixed effects.

		(1)	(2)
		Geometry enrollment	Geom. & Alg. II enrollment
	All students	-0.098***	-0.032***
		(0.005)	(0.005)
Algebra I EOC level	5	-0.061***	-0.043*
		(0.017)	(0.018)
	4	-0.072***	-0.052***
		(0.013)	(0.014)
	3	-0.076***	-0.025***
		(0.007)	(0.007)
	2	-0.139***	-0.047***
		(0.009)	(0.01)
	1	-0.139***	-0.004
		(0.013)	(0.013)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Table 2

Regression results predicting Geometry EOC test score as a function of Algebra I EOC test score, year of Algebra I enrollment, student demographics, and school fixed effects

	(1)	(2)
Alg. I EOC test score	0.877***	0.758***
	(0.000)	(0.000)
2013-14 Alg. I enrollee	-0.151***	-0.175***
	(0.000)	(0.000)
Black		-0.153***
		(0.000)
White		0.118***
		(0.000)
Asian		0.0928***
		(0.000)
Other		0.0471
		(0.128)
SPED		-0.134***
		(0.000)
FRL-eligible		-0.0688***
		(0.000)
LEP		-0.148***
		(0.000)
Constant	-0.0672**	0.0986***
	(0.003)	(0.000)
N	14108	14108
R-sq	0.486	0.531

p-values in parentheses

* p<0.05 ** p<0.01 *** p<0.001

Table 3
Regression discontinuity results predicting Geometry EOC score and probability of Geometry enrollment as a function of Algebra II EOC score, and being above the Level 2/3 cut score on the Algebra II test

	(1)	(2)
Bandwidth	Discontinuity in Geometry EOC test scores	Discontinuity in probability of Geometry enrollment
+/-8	0.163**	0.022
	'(0.056)	'(0.022)
+/-9	0.152**	0.028
	'(0.052)	'(0.021)
+/-10	0.148**	0.021
	'(0.049)	'(0.019)
+/-11	0.149**	0.022
	'(0.046)	'(0.019)
+/-12	0.136**	0.017
	'(0.045)	'(0.018)
+/-13	0.132**	0.016
	'(0.043)	'(0.018)
+/-14	0.131**	0.008
	'(0.042)	'(0.018)
+/-15	0.117**	0.006
	'(0.041)	'(0.017)
+/-16	0.123**	0.009
	'(0.040)	'(0.017)

standard errors in parentheses
* p<0.05   ** p<0.01   *** p<0.001

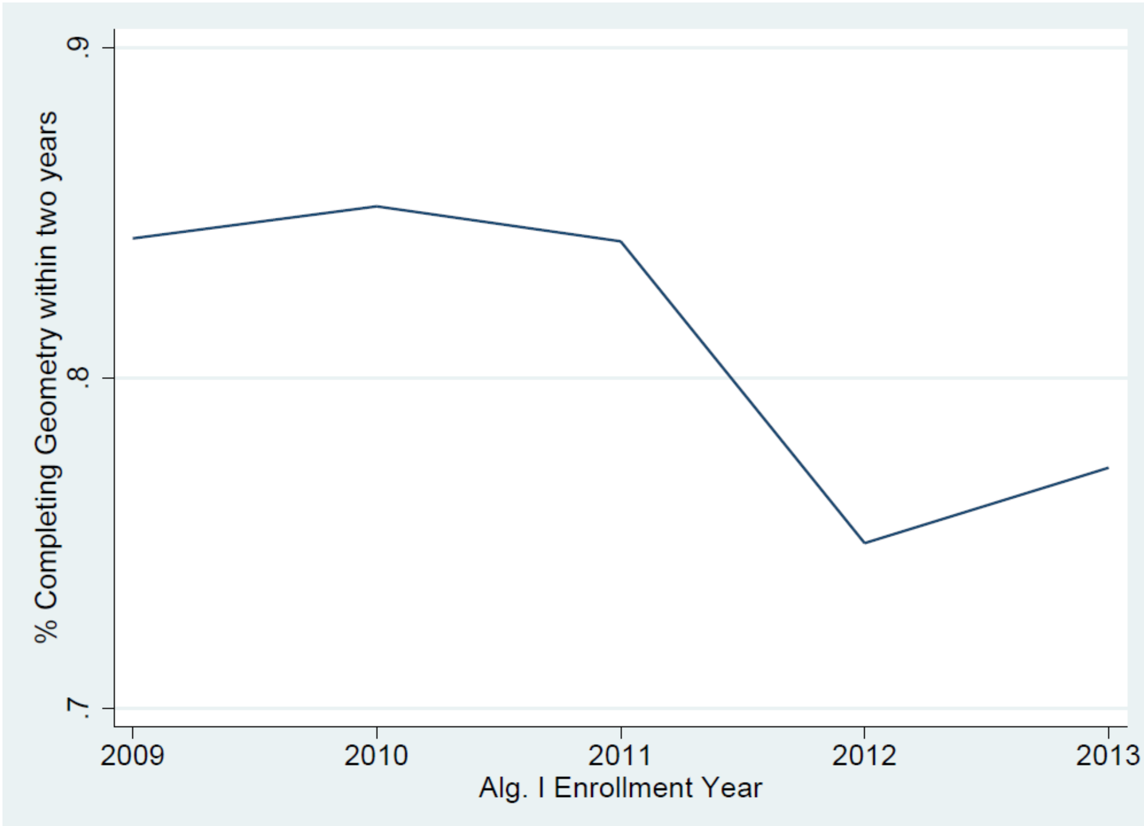

Figure 1: *Probability of completing Geometry within two years of Algebra I by year of Algebra I enrollment.*

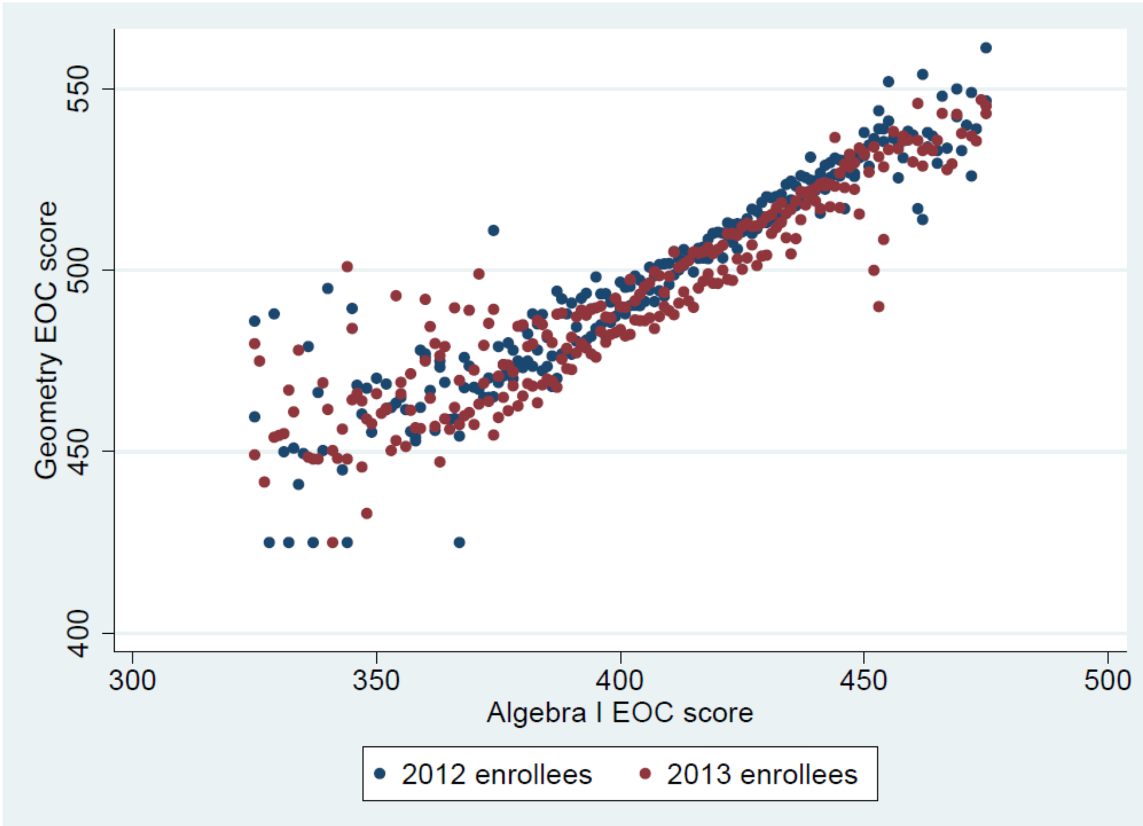

Figure 2: *Geometry EOC test score as a function of Algebra I EOC score for first and second post-policy cohorts (2012-13 and 2013-14 Algebra I enrollees).*

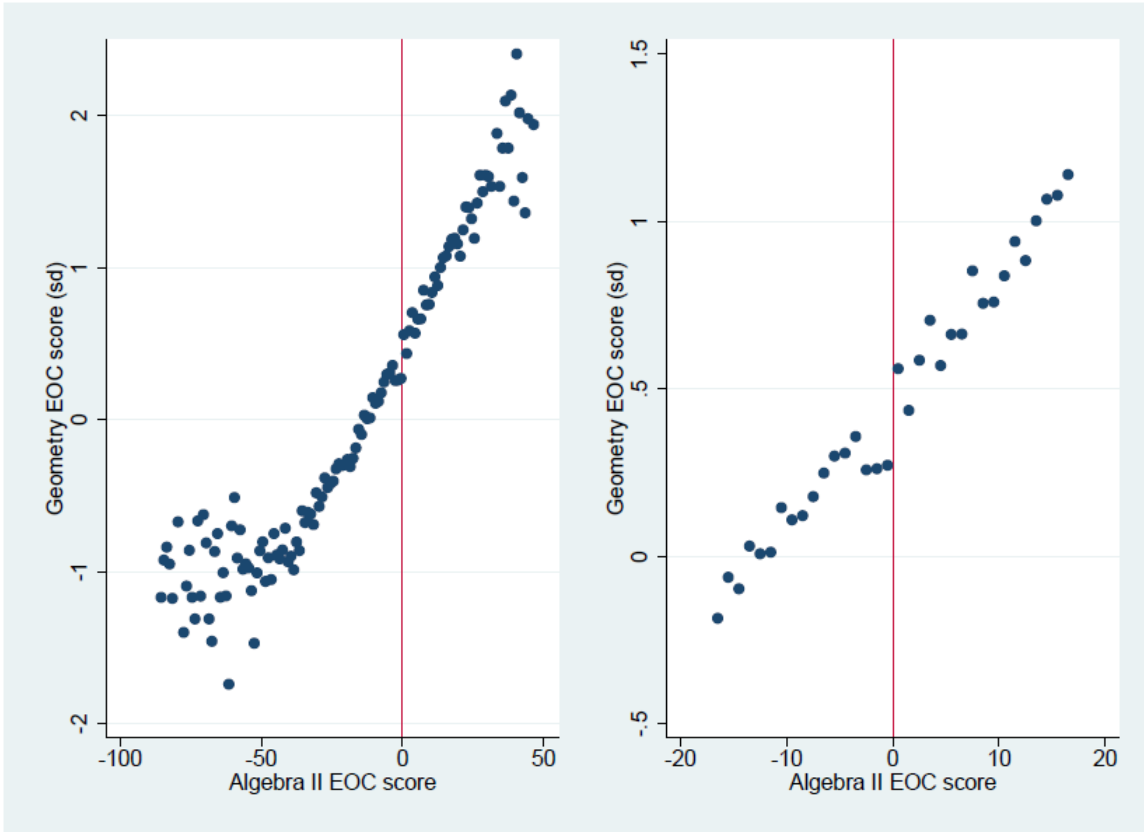

Figure 3: *Geometry EOC test score as a function of Algebra II EOC test score (centered on zero at cut score between level 2 - below satisfactory - and 3 - satisfactory). Full sample (left panel) and +/-16 points (right panel).*

Appendix One: Appendix to Chapter One

A Brief History of Mathematics[85]

Until the middle of the second millennium A.D., geometry constituted the whole of Western mathematics (Usiskin, 2003). Euclid's Elements was the foundational text (Heath, 1956), and while it contained material that is now handled algebraically, such as a proof of the infinitude of primes (IX, 20) or a method for finding square roots (X, 9), these were treated geometrically, with magnitudes being inextricably linked to physical representations, such as lengths or areas.[86]

Although algebra (in some form) dates at least to the late first millennium A.D. – the word itself is from the title of a ninth-century Persian text – it was introduced into European mathematics by Fermat (1601-1665) and Descartes (1595-1650), the latter stating in his 1637 essay *Géometrie* that "it is possible to construct all the problems of ordinary geometry by doing no more than [solving equations]" (1954 translation, quoted from Usiskin, 2003). At that point in history, however, algebra was not a separate discipline (or sub-discipline), but merely a tool for handling problems that were conceived of physically – even Newton's development of calculus in the late seventeenth century relied on geometric arguments (Usiskin, 2003).

Euler's 1770 Elements of Algebra marked the emergence of algebra as a topic of study in its own right (Usiskin, 2003), and algebra proceeded to develop along a separate path from geometry through most of the nineteenth century, until two events in the late nineteenth and early twentieth century opened the possibility of reunifying the two areas. First, in 1899 (five years after the Committee of Ten Report), "the German mathematician David Hilbert showed that geometry was logically consistent if we assumed that arithmetic was logically consistent." Second, "within ten years Bertrand Russell and Alfred North Whitehead supplied details showing that arithmetic, algebra, geometry, and analysis could be viewed as emanating deductively from a common origin in logic." (Usiskin, 2003, p.16).

Although there have been efforts to rebuild the curriculum around a unifying structure, most notably in the New Math movement of the 1960s, a radical separation of algebra and geometry prevails in most current American curricula. As Usiskin shows, this separation is an artefact of the late Enlightenment, and reflects neither the ancient

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[85] This history draws largely on a history offered by Usiskin (2003). I have attempted to locate and review the sources that he cites, and cite them myself when they were both physically and intellectually accessible. There are also a few points at which I reference sources not mentioned in Usiskin (2003), or add my own interpretive gloss. Any and all errors are my own.

[86] An online edition of Euclid's Elements is available at: http://aleph0.clarku.edu/~djoyce/elements/

structure of mathematical thought, nor its contemporary divisions.[87] Over the course of the twentieth and early twenty-first centuries there have been attempts to think beyond the traditional algebra/geometry division, and to find other bases for organizing the high school math curriculum, which I now turn to describing.

Models of Integration: Integration through applications

Another species of integrated curriculum attempts to develop mathematical content through the study of non-routine applied problems that provide opportunities to develop mathematics and defend one's reasoning. The history of math offers many examples of new mathematics coming about this way: two of the most famous are the origin of graph theory in Euler's Bridges of Königsberg problem (Shields, 2012), and the development of calculus to describe motion (Boyer, 1959).

A curriculum that exemplifies this approach was developed by the Consortium on Mathematics and its Applications (COMAP), one of five NSF-funded high school curricula developed to embody the 1989 NCTM Standards. COMAP's high school series titled *Mathematics: Modeling Our World* (MMOW), consists of courses 1, 2, and 3, and a pre-calculus textbook (COMAP also created a college-level text titled *For All Practical Purposes*). In the MMOW overview the authors explain:

> Mathematical modeling is the process of looking at a situation, formulating a problem, finding a mathematical core, working within that core, and coming back to see what mathematics tells us about the original problem. We do not know in advance what mathematics to apply. The mathematics we settle on may be a mix of geometry, algebra, trigonometry, data analysis, and probability. … Because *Mathematics: Modeling Our World* brings to bear so many different mathematical ideas ... *this approach is truly integrated*. (Garfunkel, Godbold & Pollak, 1998, p.6, my italics)

The MMOW units are based around real-world settings and the chapters combine radically divergent areas of mathematics: Unit 1 in Course 2 titled *Decision Making in a Democracy* includes chapters on percentages, graph theory, "paradox", and matrices. Unit 2, on *Secret Codes* includes chapters on representing functions, matrix operations, modular arithmetic (also known as "clock arithmetic"), and frequency distributions.

Such curricula encounter several objections: First, there are concerns that the lack of focus on specific areas of math makes it easy for students to miss crucial concepts and skills and become distracted by content that, while interesting and worthy, is peripheral

[87] The Mathematics Subject Classification, a 47-page document containing an exhaustive list of mathematical subdisciplines for publication and cataloguing purposes, is available at: http://www.ams.org/msc/pdfs/classifications2010.pdf

(e.g. MacLane, 1984). Second, there are concerns that giving credit to students for explaining their reasoning can come at the expense of incentives for precision (e.g. Wu, 1996). Third, in spite of many areas of mathematics being rooted in real world problems, many pure mathematicians look askance at such curricula, seeing an inadequate substitute for a curriculum animated by the "motivation, attitude, [and] technique" (Halmos, 1981, p.14) that often drive pure mathematics (e.g. Halmos, 1981, *Applied Mathematics is Bad Mathematics*). Finally, such texts may be poorly suited to preparing students for the typical state math assessment, which restricts itself to the routine application of traditional content; although technical proficiency is a likely by-product of setting up and solving COMAP problems, the time required for their investigation may limit coverage of mandated content.

In fairness, MMOW offers considerable structure to guide students. Although the MMOW texts use open-ended problems more extensively than do traditional textbooks, they are accompanied by a solutions manual featuring "answers to all of the activities, individual works, assessment problems, and supplementary materials" (Garfunkel, Godbold & Pollak, 1998, p.3). Nonetheless, although numbers are not available, the COMAP texts seem unlikely to have seen widespread adoption as primary classroom texts, for the reasons mentioned above. They seem more likely to be attractive for enrichment purposes.

Appendix Two: Appendix to Chapters Two and Three

Seventh Grade Algebra I Enrollees: Description of Results

The subset of seventh grade Algebra I enrollees has a higher percentage of white and Asian students, and a lower percentage of black and Hispanic students, than the subset of eighth and ninth grade enrollees (Appendix Table A1). It also has fewer FRL-eligible students, although they still make up a substantial minority of the group. Almost half of seventh grade enrollees earned A's in Algebra I, and almost 90% earned A's or B's.

For this group the post-policy indicator is an almost perfect instrument for enrolling in Algebra I and II consecutively, with fewer than 1% enrolling consecutively in the pre-policy period, and around 98% enrolling consecutively in the post-policy period (Appendix Table A2). The effect of the post-policy indicator, however, was different for this subset of students:

Regression results controlling for student background and middle school fixed effects indicate that there was little or no significant change in the overall distribution of Algebra II course grades after the policy came into effect (Appendix Table A2, top row). A more fine-grained analysis indicates, however, that students who took Algebra I and II consecutively were more likely to earn the same grade in Algebra II as in Algebra I, and less likely to earn a lower grade (Appendix Table A2, bottom panel).

Turning to the questions addressed in Chapter 3, it appears that while the percentage of seventh-grade Algebra I enrollees completing Geometry (or observed completing Geometry) fell after the policy came into effect, this was entirely due to the course being postponed. Overall, the percentage of students completing Geometry within two years of Algebra I fell by 6.1 percentage points, but the percentage completing both Geometry and Algebra II within two years of Algebra I was unchanged (Appendix Table A4). This indicates that the decline in Geometry completion for these students was not due to prior exposure to Algebra II, but simply to patterns of attrition that were equally present before and after the policy took effect.

Geometry EOC test performance conditional on Algebra I EOC test performance underwent a slight decline in the second year of the policy, after the Algebra II EOC test was introduced. This decline was statistically significant, but much smaller than for eighth and ninth grade enrollees (-0.06sd, $p<0.001$ vs. -0.18sd, $p<0.001$). This may be due to seventh-grade enrollees being stronger students, and therefore more resilient, or to the transition to high school after Algebra II providing a psychological "reset". In any case, the decline of one-twentieth of a standard deviation is of barely substantive significance, and I do not undertake further analysis.

Although the evidence suggests that the policy had a mildly beneficial effect for seventh-grade Algebra I enrollees, this is less interesting than the result for eighth and ninth-grade enrollees for two reasons. First, for these students the policy moved Algebra II into middle school, and greatly strengthened the peer-ability composition of their classes: there is almost no overlap between the prior-achievement distributions of their Algebra II classes between the pre- and post-policy periods. This is at least as plausible an explanation of their improved performance as is the ordering of Algebra I and II. Second, this is a very high achieving group of students, generally having high levels of success under existing arrangements. Finding ways to effect marginal improvements in their already strong performance is therefore not a central policy objective, especially if it comes at the expense of lower-performing students.

Appendix Table A.1

Means of student characteristics by pre- and post-policy (seventh grade Alg. I enrollees)

	Pre	Post
Black	13.6%	14.4%
Hispanic	22.4%	24.9%
White	48.1%	45.1%
Asian	12.2%	11.4%
Other	3.7%	4.2%
Special Ed.	0.9%	1.2%
FRL-eligible	36.5%	44.1%
Limited English Proficient	4.7%	3.1%
Grade 8 Alg. I	0.0%	0.0%
Enrolled in Alg. II	91.7%	96.5%
A in Alg. I	45.0%	45.3%
B in Alg. I	37.9%	38.2%
C in Alg. I	12.9%	14.2%
D in Alg. I	3.9%	2.0%
F in Alg. I	0.3%	0.3%
N	3,812	2,043

Appendix Table A2

Regression results predicting change in consecutive Algebra I and II enrollment rates before and after the introduction of a course-reordering policy (seventh grade Alg. I enrollees)

	(1)	(2)	(3)
Post-policy	0.976***	0.976***	0.975***
	(0.000)	(0.000)	(0.000)
A in Algebra I			0.143***
			(0.000)
C in Algebra I			0.123***
			(0.000)
D in Algebra I			0.0989***
			(0.000)
Black			0.00680
			(0.173)
White			0.000864
			(0.815)
Asian			0.00924
			(0.052)
Other			0.00218
			(0.762)
SPED			-0.00556
			(0.676)
FRL-eligible			-0.00123
			(0.707)
Limited English Proficient			-0.0144*
			(0.030)
Constant	0.00285	0.00285	-
	(0.078)	(0.078)	-
High School Fixed Effects	N	N	Y
N	5487	5487	5486
R-sq	0.960	0.960	0.961

p-values in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Appendix Table A3

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background and middle school fixed effects (seventh grade Alg. I enrollees)

		Algebra II grade				
		A	B	C	D	F
	All students	0.04 (0.029)	-0.009 (0.021)	-0.024 (0.015)	0.002 (0.011)	-0.009** (0.003)
Algebra I grade	A	0.117** (0.040)	-0.062* (0.029)	-0.042** (0.015)	-0.010*** (0.003)	-0.003* (0.001)
	B	-0.007 (0.032)	0.078** (0.030)	-0.053* (0.021)	-0.007 (0.018)	-0.010* (0.004)
	C	-0.065** (0.025)	-0.062 (0.046)	0.110*** (0.033)	0.045 (0.036)	-0.027* (0.011)
	D	-0.130* (0.059)	-0.083 (0.060)	0.092 (0.108)	0.117 (0.097)	0.005 (0.035)
	F	-0.031 (0.080)	0.146 (0.285)	-0.482*** (0.112)	0.463 (0.296)	-0.096** (0.029)

standard errors in parentheses

* $p<0.05$ ** $p<0.01$ *** $p<0.001$

Appendix Table A4

Change in enrollment probability within two years of Algebra I for post-policy Algebra I enrollees, overall and by Algebra I EOC level, compared with average for prior three years. Regression adjusted including controls for student prior achievement and demographics, and school fixed effects (seventh grades Alg. I enrollees).

		(1)	(2)
		Geometry enrollment	Geom. & Alg. II enrollment
	All students	-0.061***	-0.007
		(0.009)	(0.01)
Algebra I EOC level	5	-0.053***	0.002
		(0.015)	(0.016)
	4	-0.029	0.016
		(0.017)	(0.019)
	3	-0.089***	-0.033*
		(0.015)	(0.017)
	2	-0.100**	-0.012
		(0.046)	(0.05)
	1	-0.236	0.12
		(0.176)	(0.192)

standard errors in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Appendix Table A5

Regression results predicting Geometry EOC test score as a function of Algebra I EOC test score, student demographics, and school fixed effects (seventh grade Alg. I enrollees)

	(1)	(2)
Alg. I EOC test score	0.616***	0.586***
	(0.000)	(0.000)
2013-14 Alg. I enrollee	-0.0551***	-0.0587***
	(0.000)	(0.000)
Black		-0.0553
		(0.053)
White		0.101***
		(0.000)
Asian		0.145***
		(0.000)
Other		0.0732
		(0.074)
SPED		-0.0426
		(0.589)
FRL-eligible		-0.0580**
		(0.002)
LEP		-0.183***
		(0.000)
Constant	0.444***	0.417***
	(0.000)	(0.000)
N	3491	3491
R-sq	0.524	0.538

p-values in parentheses

* p<0.05 ** p<0.01 *** p<0.001

Appendix on Data Preparation

The initial student dataset contained 293,511 non-duplicate rows, each representing a math course enrollment for one of 72,053 students. Students enter the dataset in seventh, eighth, or ninth grade, with their first Algebra I enrollment - only a negligible number of the district's students first enrolled in Algebra I before seventh grade or after ninth grade[88]. The principal restriction that I impose is to exclude students who, at any point, were enrolled in a school other than one of the 57 district-operated comprehensive K-8, middle, or high schools[89]. I also add rows for students who were not enrolled in math in every year through twelfth grade (or the highest grade that my data would permit me to observe), so that each student would have a record for each grade from their first year of Algebra I enrollment through twelfth grade. I code the course for these rows as "no enrollment" – there is no way to know how many of these null rows represent students who dropped out of school or left the district, and how many simply stopped taking math[90].

In a small but non-trivial minority of cases (around nine percent), students were recorded as having multiple enrollments in the same course in a single year. Although in some instances students appear to have enrolled in multiple sections of the same course in a given year, or in two half-credit sections, it is not possible to determine which semester a given half-credit course belonged to and, in some cases, it is implausible that the data are accurate (e.g. students taking six full credits of Algebra in ninth grade). For cases with multiple sections of the same class in a given year, therefore, I assign each section the average grade (on a four-point scale) for all sections in that year, and weight the binary course-taking indicator variables (see below) so that no student can contribute more than one full instance of enrollment in a given course in a given year.

Constructing classroom sections

The district provided a dataset containing 278,142 non-duplicate rows, with each row containing a student identifier (corresponding to the identifiers in the course-

[88] As a check on this proposition, the size of the graduating cohorts for which complete data is available (i.e. students who should have started ninth grade together in 2011, 2012, 2013) was compared with official NCES counts. Before imposing restrictions on my sample, my counts are within 1% of the NCES counts, suggesting that a negligible number of students enrolled in Algebra I outside of the seventh-to-ninth grade window.

[89] I exclude 4,963 students because they were enrolled in a charter school in one or more years, and may therefore have been exempted from some district policies. I exclude an additional 4,625 students because they spent some or all of their time in a school serving a special population. I do account for these students in constructing measures of classroom characteristics.

[90] I also make only limited use of the 12,555 students who enrolled in Algebra I in 2013-14 and Algebra II in 2014-15, as these students were the first to sit the newly introduced Algebra II EOC exam which, by all appearances, led to a sharp decline in measured course outcomes.

enrollments dataset), a teacher identifier, a course code, a school year, a school code, and a code for the period during which the class met. There was nothing to specify whether a course was a full- or half-credit course and, therefore, in which semester half-credit courses convened. In general, I assume that a unique combination of teacher, period, school year, and course code represent a section or classroom. Grouping students into sections depends on being able to match a unique instance of enrollment from the course-enrollments dataset to a unique section in the dataset containing teacher and class period data.

The groupings so formed contain between 1 and 63 students, 16.7%, or 2,899 of these groupings, contain three or fewer students. Excluding these, the modal group size is 21. 73.5% of the sections, containing 92.5% of the non-duplicate rows, contained between 10 and 32 students (inclusive). Given that 10 and 32 seem like bounds outside of which course sections are likely to represent either special cases or errors, I assign "missing" section ids to such sections. Through this process, I am able to associate 81.25% of the students in my analytic sample with a section of Algebra I, including 90.1% of seventh grade enrollees, 87.9% of eighth grade enrollees, and 76.4% of ninth grade enrollees. I am able to associate 75.6% of students who enrolled in Algebra II with a section of that course.

Classroom characteristics

I use classroom level data to construct measures of classroom characteristics that I then control for in my analyses, and also fit an alternate model using classroom fixed effects. In creating these measures, I include students who are excluded as individual cases in my final analysis due to spending time in charter schools or schools for special populations.

Class size: The number of students enrolled in a section of Algebra II. I allow this variable to range from 10 to 32, and treat the small number of cases in which sections appear to contain larger or smaller numbers of students as missing classroom data.

Mean fifth grade FCAT score: Fifth grade is the most recent score that is available for every year and grade-level of Algebra I enrollee, with the exception of seventh grade enrollees in 2013-14. The Algebra II sections of the final cohort of seventh grade enrollees are of the least interest of all the Algebra II sections that I study, and I can tolerate the penalty of lacking a measure of mean prior achievement for those sections.

Algebra II teacher teaching Algebra II for the first time in 2013-14: In the 2013-14 school year, there was a double cohort of students enrolled in Algebra II – those who had enrolled in Algebra I in the 2011-12 school year and followed the standard ordering, and those who had enrolled in Algebra I in the 2012-13 school year and taken Algebra II consecutively. This meant that many teachers were assigned to teach Algebra II who had not previously taught the course. The district has generally high teacher turnover, and a large number of inexperienced teachers teaching in any given year, giving reason to expect that the impact of this increase in Algebra II assignments would make little difference. Nonetheless, I include this variable to account for the possibility that student performance in Algebra II may have been harmed by teacher inexperience in the year of the reordering.

Appendix to Main Results

Appendix Table A6
Correlations of student demographic characteristics, grades, and sixth-grade FCAT performance levels

	Algebra 1 grade	Algebra 2 grade	G6 FCAT Score	FRL eligible	SPED classified	LEP classified
Algebra 1 grade	1					
Algebra 2 grade	0.5121*	1				
	(0.000)					
G6 FCAT Score	0.5014*	0.3801*	1			
	(0.000)	(0.000)				
FRL eligible	-0.2262*	-0.1923*	-0.3162*	1		
	(0.000)	(0.000)	(0.000)			
SPED classified	-0.1626*	-0.1133*	-0.2870*	0.0602*	1	
	(0.000)	(0.000)	(0.000)	(0.000)		
LEP classified	-0.1111*	-0.0649*	-0.3218*	0.1989*	0.0343*	1
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Hispanic	-0.0960*	-0.0957*	-0.1314*	0.2553*	0.0291*	0.2939*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Black	-0.1289*	-0.0959*	-0.2308*	0.2205*	-0.0024	-0.0583*
	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.000)
White	0.1473*	0.1271*	0.2654*	-0.4295*	-0.0062	-0.2361*
	(0.000)	(0.000)	(0.000)	(0.000)	(1.000)	(0.000)
Asian	0.1359*	0.1144*	0.1370*	-0.0595*	-0.0414*	0.0274*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Other	0.0173	0.0067	0.0307*	-0.0188*	-0.0048	-0.0608*
	(0.069)	(1.000)	(0.000)	(0.023)	(1.000)	(0.000)

Notes:
1: *p*-values, in parentheses are Bonferroni corrected to account for the large number of comparisons being made.
2: Algebra 1 and 2 grades are on a 0-4 scale (0=F, 4=A)

Appendix Table A7

Difference-in-difference estimates for association between increase in middle-school Algebra I enrollments and the change in Algebra II grades for eighth-grade Algebra I enrollees from those schools

		Algebra II grade				
		A	B	C	D	F
	All students	-0.013 (0.017)	-0.019 (0.017)	0.02 (0.011)	0.014 (0.011)	-0.002 (0.003)
Algebra I grade	A	0.004 (0.050)	-0.046 (0.046)	0.044 (0.030)	-0.002 (0.016)	0.001 (0.006)
Algebra I grade	B	-0.037 (0.065)	-0.043 (0.098)	0.113 (0.076)	-0.016 (0.064)	-0.017 (0.052)
Algebra I grade	C	-0.062 (0.073)	-0.069 (0.107)	0.064 (0.110)	0.074 (0.147)	-0.007 (0.044)
Algebra I grade	D	-0.063 (0.075)	0.082 (0.099)	0.284* (0.142)	-0.135 (0.139)	-0.168 (0.134)
Algebra I grade	F	0.004 (0.050)	-0.046 (0.046)	0.044 (0.030)	-0.002 (0.016)	0.001 (0.006)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Versions of main model with additional controls

Appendix Table A8

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background, high school fixed effects, and Algebra I EOC scores (2011-12 and 2012-13 cohorts only)

		Algebra II grade				
		A	B	C	D	F
	All students	-0.020***	-0.011	-0.003	0.022**	0.012***
		'(0.005)	'(0.007)	'(0.009)	'(0.008)	'(0.004)
Algebra I grade	A	-0.027*	0.024	0.001	0.008	-0.006*
		'(0.011)	'(0.014)	'(0.011)	'(0.007)	'(0.003)
	B	-0.039***	-0.008	0.030*	0.016	0.001
		'(0.009)	'(0.011)	'(0.012)	'(0.011)	'(0.003)
	C	-0.013*	-0.034*	-0.016	0.056***	0.007
		'(0.007)	'(0.016)	'(0.014)	'(0.015)	'(0.007)
	D	0.002	-0.019	-0.047**	0.018	0.047***
		'(0.007)	'(0.015)	'(0.018)	'(0.020)	'(0.013)
	F	0.008	0.018	-0.019	-0.077*	0.071
		'(0.012)	'(0.019)	'(0.026)	'(0.034)	'(0.037)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Appendix Table A9

Change in probability of earning a fiven Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background, high school fixed effects, and sixth grade FCAT scores

		Algebra II grade				
		A	B	C	D	F
	All students	-0.029*** '(0.006)	-0.015* '(0.007)	0.005 '(0.008)	0.025*** '(0.007)	0.013*** '(0.003)
Algebra I grade	A	-0.051* '(0.021)	0.041* '(0.019)	0.016 '(0.017)	-0.001 '(0.009)	-0.006** '(0.002)
Algebra I grade	B	-0.038*** '(0.009)	-0.026* '(0.012)	0.035* '(0.014)	0.029** '(0.009)	-0.001 '(0.002)
Algebra I grade	C	-0.020*** '(0.004)	-0.030* '(0.013)	-0.005 '(0.011)	0.045** '(0.015)	0.010* '(0.005)
Algebra I grade	D	-0.01 '(0.006)	-0.017 '(0.014)	-0.039** '(0.014)	0.028 '(0.015)	0.038** '(0.012)
Algebra I grade	F	-0.009 '(0.011)	-0.001 '(0.019)	-0.031 '(0.028)	-0.070** '(0.025)	0.110* '(0.043)

standard errors in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Appendix Table A10

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background, high school fixed effects, and an indicator for having a teacher teaching Algebra II for the first time

		Algebra II grade				
		A	B	C	D	F
	All students	-0.031***	-0.016*	-0.001	0.030***	0.017***
		'(0.006)	'(0.008)	'(0.009)	'(0.009)	'(0.004)
Algebra I grade	A	-0.032**	0.021	0.003	0.012	-0.005
		'(0.012)	'(0.013)	'(0.011)	'(0.007)	'(0.003)
	B	-0.048***	-0.012	0.031*	0.023	0.006*
		'(0.010)	'(0.012)	'(0.012)	'(0.012)	'(0.003)
	C	-0.024***	-0.039*	-0.015	0.067***	0.012
		'(0.006)	'(0.016)	'(0.014)	'(0.015)	'(0.007)
	D	-0.011	-0.027	-0.042*	0.028	0.052***
		'(0.007)	'(0.015)	'(0.017)	'(0.019)	'(0.013)
	F	-0.008	0.014	-0.009	-0.070*	0.073
		'(0.012)	'(0.018)	'(0.025)	'(0.032)	'(0.038)

standard errors in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Appendix Table A11

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background, high school fixed effects, and an indicator for having a teacher teaching Algebra II for the first time

		Algebra II grade				
		A	B	C	D	F
	All students	-0.031***	-0.016*	-0.001	0.030***	0.017***
		'(0.006)	'(0.008)	'(0.009)	'(0.009)	'(0.004)
Algebra I grade	A	-0.032**	0.021	0.003	0.012	-0.005
		'(0.012)	'(0.013)	'(0.011)	'(0.007)	'(0.003)
	B	-0.048***	-0.012	0.031*	0.023	0.006*
		'(0.010)	'(0.012)	'(0.012)	'(0.012)	'(0.003)
	C	-0.024***	-0.039*	-0.015	0.067***	0.012
		'(0.006)	'(0.016)	'(0.014)	'(0.015)	'(0.007)
	D	-0.011	-0.027	-0.042*	0.028	0.052***
		'(0.007)	'(0.015)	'(0.017)	'(0.019)	'(0.013)
	F	-0.008	0.014	-0.009	-0.070*	0.073
		'(0.012)	'(0.018)	'(0.025)	'(0.032)	'(0.038)

standard errors in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Additional results for robustness and validity checks

Appendix Table A12

Difference-in-differences results predicting Algebra II grade (0-4 scale) for ninth-grade Algebra I enrollees as a function of policy-exposure, size of post-policy high school Algebra II enrollment increase, and their interaction

	(1)
Post-policy	-0.135
	(0.060)
Increase in Alg. II enrollment	0.792
	(0.219)
Post-policy by increase	0.510
	(0.478)
Algebra I grade (0-4 scale)	0.388***
	(0.000)
High school fixed effects	Y
Demographic controls	Y
N	24,230
R-sq	0.174

p-values in parentheses
* p<0.05 ** p<0.01 *** p<0.001

Appendix Table A13

Difference (in standard deviations) between mean Algebra I EOC score of students earning a given grade in Algebra I in 2011-12 and 2012-13, by grade of Algebra I enrollment, net of school fixed effects and demographic controls

		Algebra I enrollment grade	
		8	9
Algebra I grade	A	0.126***	0.069
	B	0.077	-0.03
	C	0	-0.093
	D	0.005	-0.091
	F	0.508***	0.007

Notes: The general lack of statistical significance indicates that the underlying level of achievement that grades signified remained largely unchanged before and after the policy came into effect.

Appendix Table A14

Change in probability of earning a given Algebra II grade after introduction of policy, overall and by Algebra I course grade, net of student background and high school fixed effects in a given non-policy year, as compared with prior years (placebo test).

		Algebra II grade				
		A	B	C	D	F
Placebo "policy" year	2011-12	0.002	-0.012	-0.013	0.01	0.013**
		(0.009)	(0.012)	(0.008)	(0.014)	(0.006)
	2010-11	0.006	0.002	0.006	-0.012	-0.002
		(0.009)	(0.008)	(0.008)	(0.008)	(0.004)

standard errors in parentheses

* p<0.05   ** p<0.01   *** p<0.001

Appendix Table A15

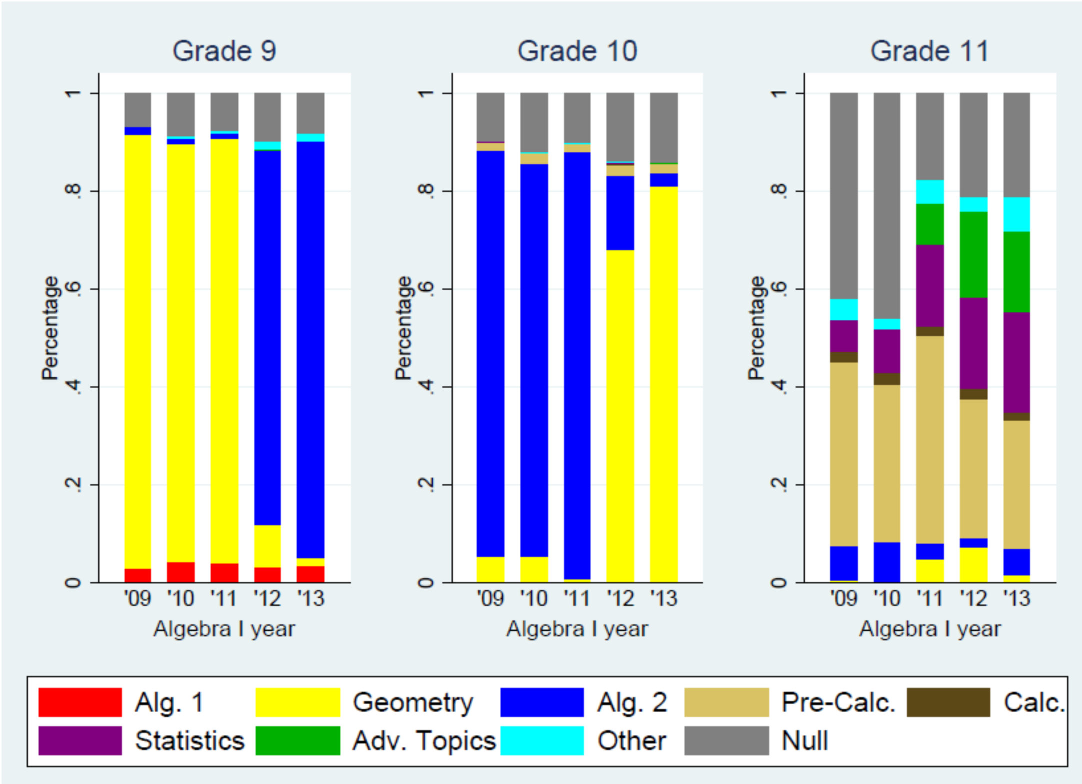
Geometry and Algebra II EOC test score levels by Algebra I EOC test score levels, for each grade of Algebra I enrollment

Grade 8 enrollees

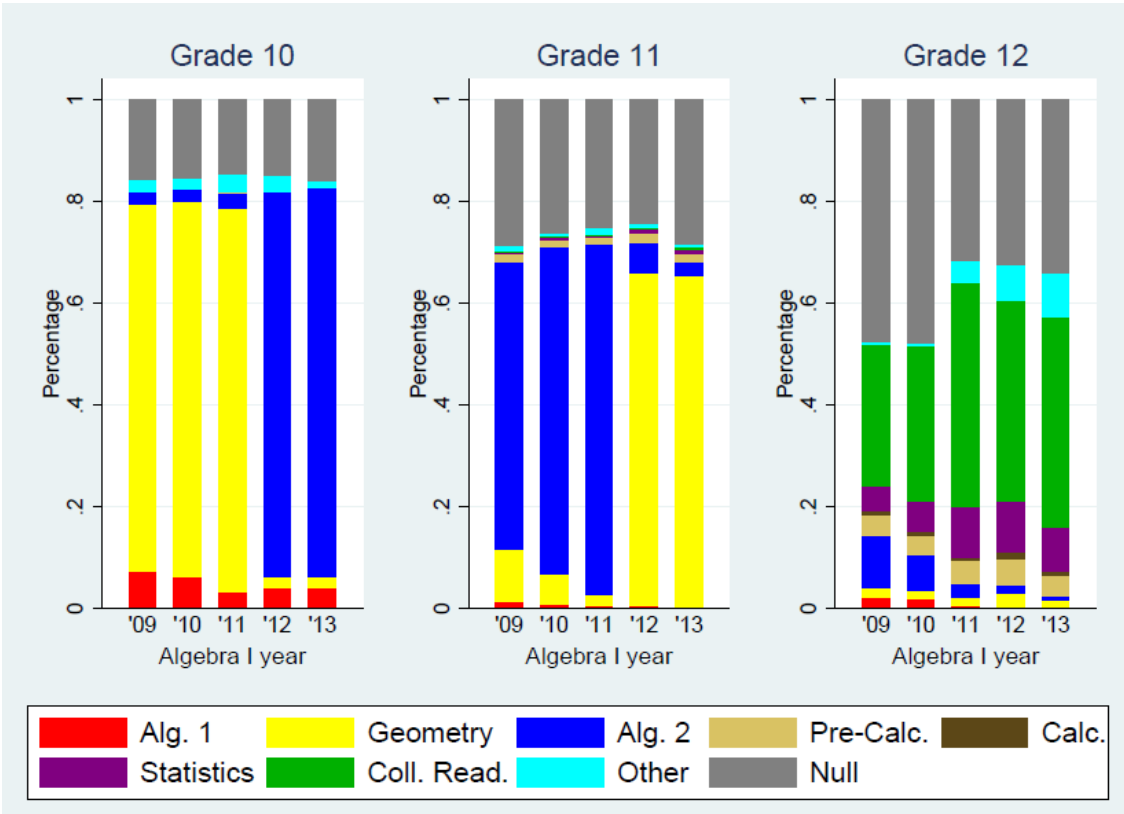
Algebra I level		Algebra II level					
		1	2	3	4	5	Total
	1	55	2	0	0	0	57
		96.5%	3.5%	0.0%	0.0%	0.0%	100.0%
	2	496	54	11	0	0	561
		88.4%	9.6%	2.0%	0.0%	0.0%	100.0%
	3	1,345	727	284	12	2	2,370
		56.8%	30.7%	12.0%	0.5%	0.1%	100.0%
	4	161	329	380	37	11	918
		17.5%	35.8%	41.4%	4.0%	1.2%	100.0%
	5	21	85	404	135	105	750
		2.8%	11.3%	53.9%	18.0%	14.0%	100.0%
	Total	2,078	1,197	1,079	184	118	4,656
		44.6%	25.7%	23.2%	4.0%	2.5%	100.0%

Grade 9 enrollees

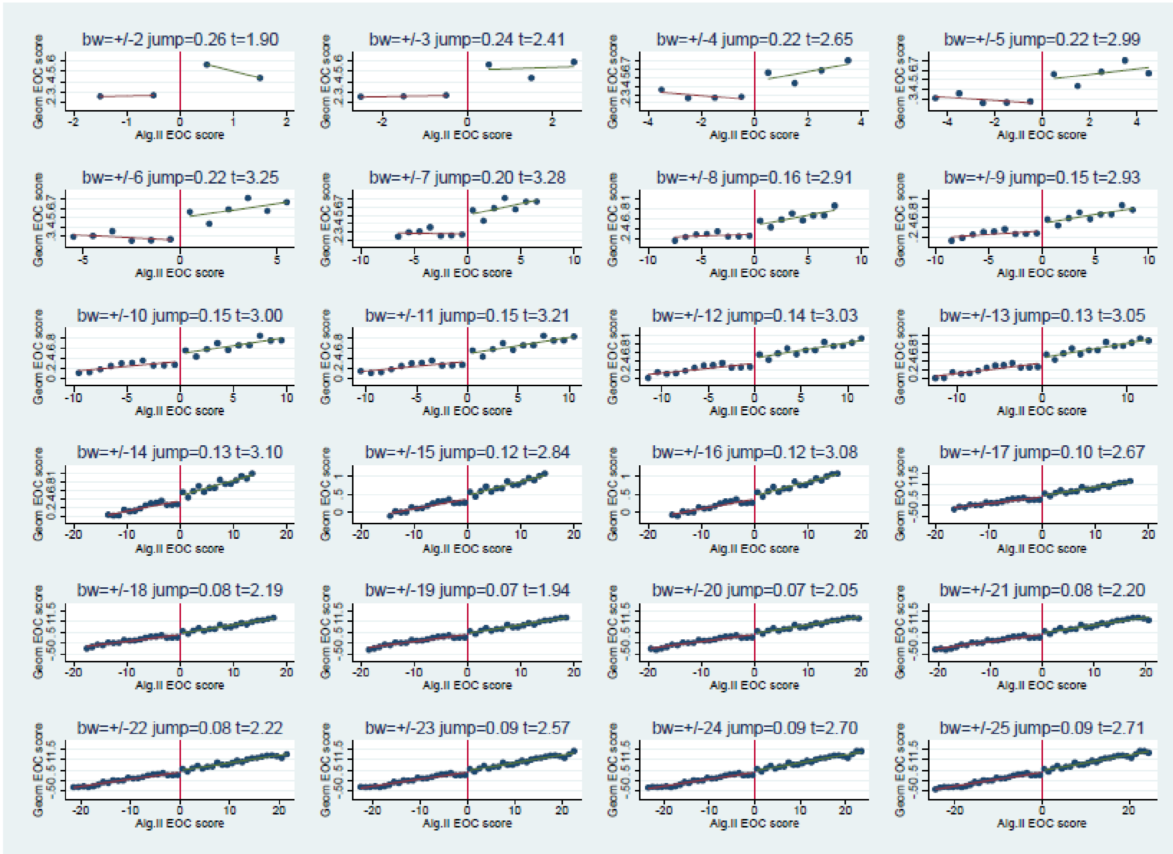
Algebra I level		Algebra II level					
		1	2	3	4	5	Total
	1	723	8	0	0	0	731
		98.9%	1.1%	0.0%	0.0%	0.0%	100.0%
	2	1,632	60	13	0	0	1,705
		95.7%	3.5%	0.8%	0.0%	0.0%	100.0%
	3	1,657	341	72	5	1	2,076
		79.8%	16.4%	3.5%	0.2%	0.1%	100.0%
	4	95	83	91	7	1	277
		34.3%	30.0%	32.9%	2.5%	0.4%	100.0%
	5	13	23	68	21	13	138
		9.4%	16.7%	49.3%	15.2%	9.4%	100.0%
	Total	4,120	515	244	33	15	4,927
		83.6%	10.5%	5.0%	0.7%	0.3%	100.0%



Appendix Figure A1: *Course enrollments for eighth-grade Algebra I enrollees in their first, second, and third post-Algebra I year, by year of first Algebra I enrollment.*



Appendix Figure A2: *Course enrollments for ninth-grade Algebra I enrollees in their first, second, and third post-Algebra I year, by year of first Algebra I enrollment.*



Appendix Figure A3: *Illustrated regression discontinuity results (coefficients are adjusted for school fixed effects and student controls) for all bandwidths from +/-2 to +/-25.*

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