Improving Ontario Mathematics Performance: A Comparative Study of Underlying Factors for Mathematics Achievements in Ontario-Canada, Singapore and Shanghai-China

Improving Ontario Mathematics Performance: A Comparative Study of Underlying Factors for Mathematics Achievements in Ontario-Canada, Singapore and Shanghai-China

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Abstract

The downtrend of Ontario high school mathematics performance in international assessments since 2003 has drawn the attention of many education stakeholders seeking solutions for improvement. This comprehensive study specifically looks for elements that can improve international mathematics achievement of Ontario high school students. By comparing different attributes in the areas of sociocultural context, teacher qualification and compensation, curriculum, assessments, classroom activities, and students' attitudes and motivation about learning mathematics among Ontario, Singapore and Shanghai, it was found that the curriculum could benefit from incorporating more basic mathematics skills either by whole-class or flipped-class instruction to achieve resource (cost and time) effectiveness. Evidence showed that the qualification and ongoing training for mathematics teachers are crucial to students' achievement. It was further revealed that after-school math class, tutoring, appropriate amount of homework with suitable type of assessment can help reinforce students' knowledge of mathematics as well. In 2016, Ontario spent 60 million on a program entitled the Renewed Math Strategy mainly concentrating on teachers' training and resources for support. However, the program did not renew a focus on fundamental mathematics. If a pivotal part is missing in the design of mathematics education, the effectiveness of the pedagogy can be diminished. Each culture is different; those elements that work well in some countries might not function as effectively if they are incorporated in other countries, but there should be a combination of elements that best fits each culture. The results of this thesis make it clear that Ontario

policy makers need to do more to balance these elements to find the right recipe for success in achieving optimal mathematics education.

Dedication

I dedicate this thesis to my father, who was severely sick while I was working on this research. He is the role model whom I have learned to be diligent and persistent, the most important virtues that lead to my every success.

Equally, I also dedicate this work to my mother, my sister and my husband who have been caring and loving me unconditionally. I thank them for being there whenever I was in need of any help.

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I would also like to thank my mentor, Dr. Yongqian Wang, who has helped me organize my thoughts logically to construct the framework prior to my thesis proposal.

Last but not least, I am deeply grateful that my son has inspired me to pursue my timeless dream. Everything can be possible if one tries hard to achieve.

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Chapter I

Introduction

This chapter will present an overview of this study, along with information about the international assessments referenced throughout the thesis. In addition, data will be given about the recent high school mathematics performance of Ontario, Singapore and Shanghai, trends in their assessments, the two significant math approaches prevalent across the world, and the reaction from Ontario.

Overview

This section will explain the relevance of mathematics in society, along with the background, hypothesis, and purpose of this thesis.

Relevance of Mathematics in Society

Mathematics has a long history and is related to many aspects of our daily lives. Numbers were found recorded as early as 35,000 BCE on the Lebombo bone (Pegg, n.d.). In ancient times, various cultures such as the Babylonians, Egyptians, Greek, Chinese, and so forth had invented their own number systems to facilitate their daily needs. Today mathematics can be defined as "the science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects" ("Mathematics," 2006). Mathematics exists everywhere around us and its importance can be compared to that of language in terms of human progress. Most school systems have aimed at educating students with mathematical skills and knowledge to meet the challenges in the dynamic context of modern society. The Trends in International Mathematics and Science Study (TIMSS) and The Programme for International Student Assessment (PISA) are two reputable international assessments that provide feedback to participating countries to help them assess their mathematics achievement and work towards potential improvements in their education systems.

Background

The overall international math assessment results have revealed that over the past several decades a number of Asian countries have outperformed most Western countries in high school mathematics achievement as measured on these international assessments. This leads naturally to the question as to why Singapore and Shanghai-China are consistently achieving higher results as compared to Ontario-Canada in the international mathematics assessments. From studying the attributes of the educational environment of these two countries, it is theorized that there might be particular attributes of these other regions' educational systems that could be used to help Ontario students close the gap in math performance on these assessments.

Hypothesis and Purpose

The working hypothesis for this thesis is that Ontario can achieve better international mathematics performance, and that it can do so based on suggestions made through findings for the above research questions.

The purpose of this thesis is to compare math education comprehensively among the three regions by probing a wide range of various factors including sociocultural context, teacher qualification and compensation, curriculum, assessments, classroom activities, and student's attitudes and motivations about learning mathematics to reveal the underlying mechanisms for these High Achieving Countries to perform as well as they do. Accordingly, seeking possibilities and ways of applying the outstanding attributes to Ontario's education system will be discussed and recommended as ways to lead to potential improvement in math performance for high school students.

Though this is not a comprehensive study of all possible variations between regions in the world, this deep analysis of Shanghai and Singapore versus Ontario should be more informative than the results of a less detailed comparison made across many countries.

International Assessments

The performance of the three regions in this study is mainly based on the results of the two respectable international assessments, the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA).

The Trends in International Mathematics and Science Study (TIMSS)

TIMSS is an international assessment administrated by the International Association for the Evaluation of Educational Achievement (IEA) every four years since 1995 in the subjects of mathematics and science for fourth grade and eighth grade. "It provides education policy-makers, administrators, teachers, and researchers with powerful insights into how education systems are functioning as well as critical intelligence about the possibilities for education reform and improvement" (CMEC, 2017, para. 1).

The eighth grade mathematics content domains of TIMSS are Number, Algebra, Geometry, and Data and Chance; whereas the cognitive domains are Knowing, Applying and Reasoning (EQAO, 2012). Students' results are categorized into four benchmarks, advanced (scores of 625 or above), high (scores between 550 to 625), intermediate (scores between 475 and 550) and low (scores below 475) (EQAO, 2012). In 2011, 56 jurisdictions participated with results ranging from 331 to 613 average scores (EQAO, 2012).

The Programme for International Student Assessment (PISA)

PISA measures how well 15-year-old students among the Organisation for Economic Co-operation and Development (OECD) countries, together with partner countries and economies, apply their skills and knowledge to "meet the challenges of the future, rather than how well they master particular curricula" (OECD, 2014c, p. 3). Data is collected every three years to compare strengths and weaknesses in mathematics, reading and science, which are essential to students, educators, policy makers, parents and researchers for future educational planning (OECD, 2014c). The assessment started in 2003 and there were 65 countries and economies who participated in PISA 2012, with average scores ranging from 368 to 613. The overall mathematics results were based on process subscales and content subscales. Process subscales included Formulating, Employing and Interpreting; whereas content subscales were Change and Relationship, Space and Shape, Quantity and Uncertainty and Data (Brochu, Deussing, Houme, & Chuy, 2013). Participants were categorized into six mathematics proficiency levels, in which level 1 represented the weakest participants with 357.77 as a minimum score, and level 6 represented the strongest students with a minimum score of 669.30. See Appendix A for summary descriptions of proficiency levels.

Mathematics Performances in TIMSS 2011 and PISA 2012

The results of TIMSS 2011 and PISA 2012 for Ontario, Singapore and Shanghai will be examined in this section to compare across the three regions.

Performance of Students from Ontario

Being the largest province of Canada with a population of 13.79 million in 2015, Ontario has an independent educational system and is one of the Canadian Benchmarks in education ("Ontario Fact Sheet January 2016," 2016). Ontario has been participating in the two reputable international math assessments during the last two decades.

In TIMSS 2011 (eighth grade students, approximately 14 years old) and PISA 2012 (15 year old students), Ontario achieved 512 and 514 points respectively (see Table 1 and Table 2), in which 500 points was the scale center point. However, the performance of students from Ontario was approximately 16% behind students from the best performing countries achieving 613 score in both studies.

In TIMSS 2011, Ontario students performed best in Data and Chance with an average score of 531 and lowest in Algebra with an average score of 497 in the content domains. As far as the cognitive domains, Ontario did better in Reasoning questions with an average of 524 than in Knowing and Applying questions with scores of 503 and 510 respectively (EQAO, 2012). The results indicated that 4% of the students had achieved an advanced benchmark, and 31% of student attained a high benchmark (EQAO, 2012). Furthermore, 15% of students from Ontario achieved level 5 and above, followed by

21%, 27%, 23% and 14% achieving level 4, 3, 2, and below level 2 respectively in PISA 2012 (Brochu et al., 2013).

Table 1.
TIMSS 2011 Average Scores for Eighth Grade Mathematics Performance

Country / Benchmarking Participants	Average Score
Korea, Rep. of	613
Singapore	611
Chinese Taipei	609
Hong Kong SAR	586
Japan	570
Russian Federation	539
Quebec, Canada	532
Israel	516
Finland	514
Ontario, Canada	512
United States	509
England	507
Alberta, Canada	505
Hungary	505
Australia	505
Slovenia	505
Lithuania	502
TIMSS Scale Center Point	500
Italy	498
New Zealand	488
Kazakhstan	487
Sweden	484
Ukraine	479

Note. Data for average scores from Mullis et al. (2012); [p. 42-43].

Table 2.
PISA 2012 Estimated Average Scores for 15-year-old Mathematics Performance

Country / Benchmarking Participants	**Average Score**
Shanghai - China	613
Singapore	573
Hong Kong - China	561
Chinese Taipei	560
Korea	554
Macao - China	538
Japan	536
Quebec, Canada	536
Liechtenstein	535
Switzerland	531
The Netherlands	523
British Columbia, Canada	522
Estonia	521
Finland	519
Canada	518
Poland	518
Alberta, Canada	517
Belgium	515
Ontario, Canada	514
Germany	514
Vietnam	511
Saskatchewan	506
Austria	506
Australia	504
New Brunswick	502

Note. Data for averages scores from Brochu et al. (2013); [p. 19].

Performance of Students from Singapore

Singapore is a small country with a population of 5.49 million in 2015, and it has been well known for its remarkable education system for a considerable amount of time. It has been participating in both international assessments as well ("Singapore Population," n.d.).

Singapore was ranked second in both TIMSS 2011 and PISA 2012 mathematics assessments with an average score of 611 and 573 respectively (see Table 1 and Table 2). In TIMSS 2011, Singapore was only two points below the highest country score (that of Korea with 613), and 99 points above Ontario-Canada. In PISA 2012, Singaporean students performed an average of 59 points higher than those of Ontario-Canada.

In TIMSS 2011, students from Singapore did well in content domains ranging from scores of 607, for Data and Chance, to a score of 614 in Algebra. The small range implies that these students are generally strong in all four content domains (U.S. Department of Education, 2012). As far as cognitive domains, Singaporean students did best in Knowing with an average score of 617, followed by Applying (613) and Reasoning (604). In terms of benchmarks, 48% and 78% of eighth grade students in Singapore achieved advanced and high benchmark respectively (Mullis et al., 2012). These percentages of top performers were considerably higher than those of eighth grade students from Ontario. See Figure 1 for an overview of students' achievement in TIMSS 2011.

As far as results of the PISA 2012, 40% of Singaporean students achieved level 5 and above, followed by 22%, 17.5%, 12.2% and 8.3% achieving level 4, 3, 2, and below level 2 respectively (NCES, 2012).

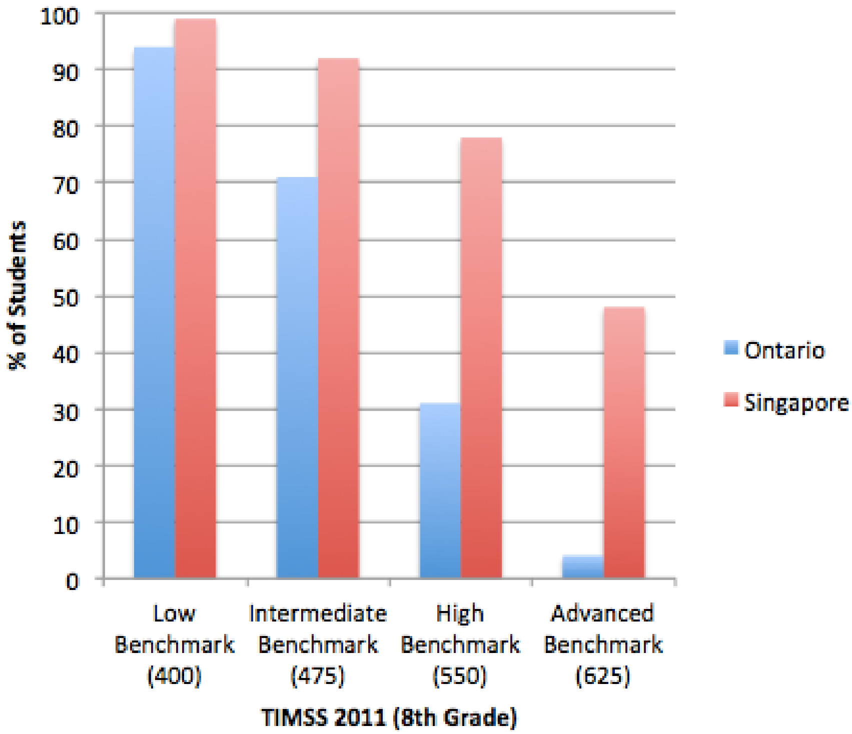


Figure 1. Overview of Students' Achievement in TIMSS 2011 (in percentage). Data for TIMSS 2011 from Mullis et al. (2012); [p. 114-115].

Performance of Students from Shanghai

Shanghai is the largest and most populous city in China; its population was over 24 million in 2013 (Wu, 2014). Though Shanghai-China did not participate in TIMSS 2011, it surprised many people with a very high score of 613 in the PISA 2012 results, which was 40 points higher than the results of the second highest scoring country, Singapore, and 99 points higher than Ontario-Canada. This high score implied that their mathematical standard was three school years above the OECD average (OECD, 2014a).

Moreover, Shanghai had 55.4% of its students achieving level 5 and above, along with 20.2%, 13.1%, 7.5% and 3.8% of students achieving level 4, 3, 2, and below level 2 respectively (NCES, 2012). In short, Shanghai had approximately 15% and 40% more high achievers than Singapore and Ontario in PISA 2012. See Figure 2 for an overview of students' achievement in PISA 2012.

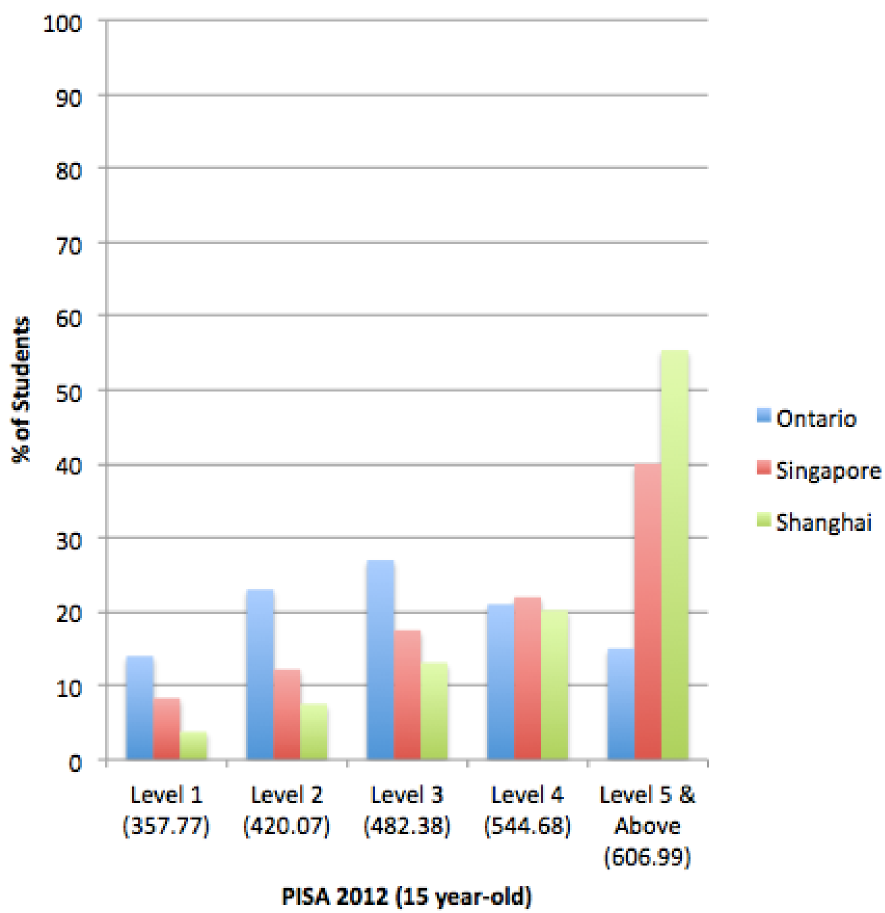


Figure 2. Overview of Students' Achievement in PISA 2012 (in percentage). Data for PISA 2012 from NCES (2012).

Trends of Assessments

The mathematics results of TIMSS 1995 – 2011 is employed to project the trends of the eighth graders for Ontario, Singapore and Shanghai; whereas mathematics performance of PISA 2003 – 2012 for these three regions is used to study the trends of the 15-year-old students.

Trends in TIMSS (1995 – 2011)

Figure 3 displays the trends in eighth grade TIMSS mathematics performance of significant regions between 1995 and 2011. The average age of participants was about fourteen years old.

The top five eighth grade mathematics performers of TIMSS 2011 were Republic of Korea, Singapore, Chinese Taipei, Hong Kong SAR and Japan respectively. These regions have occupied the top five positions from 1995 through 2011. Four of them, Republic of Korea, Singapore, Chinese Taipei and Hong Kong SAR, have shown improvement when compared to their first scores from the 1995 test.

In 2011, most western industrialized nations had average scores between 500 and 550. Ontario had an average score of 512, three points above ninth place United States, with an overall improvement of eleven points between 1995 and 2011. However, the graph of Ontario performance was concave down; it reached its peak score in 2003 and then started trending down slightly. Furthermore, the other two Canadian benchmarks, Quebec and Alberta, were decreasing in their mathematics performance over the period as well.

On the other hand, Tunisia, Morocco and Ghana were performing below the average throughout those years. Yet Tunisia and Ghana showed an upward trend starting in 2003, and scores for Moroccan students were decreasing since then.

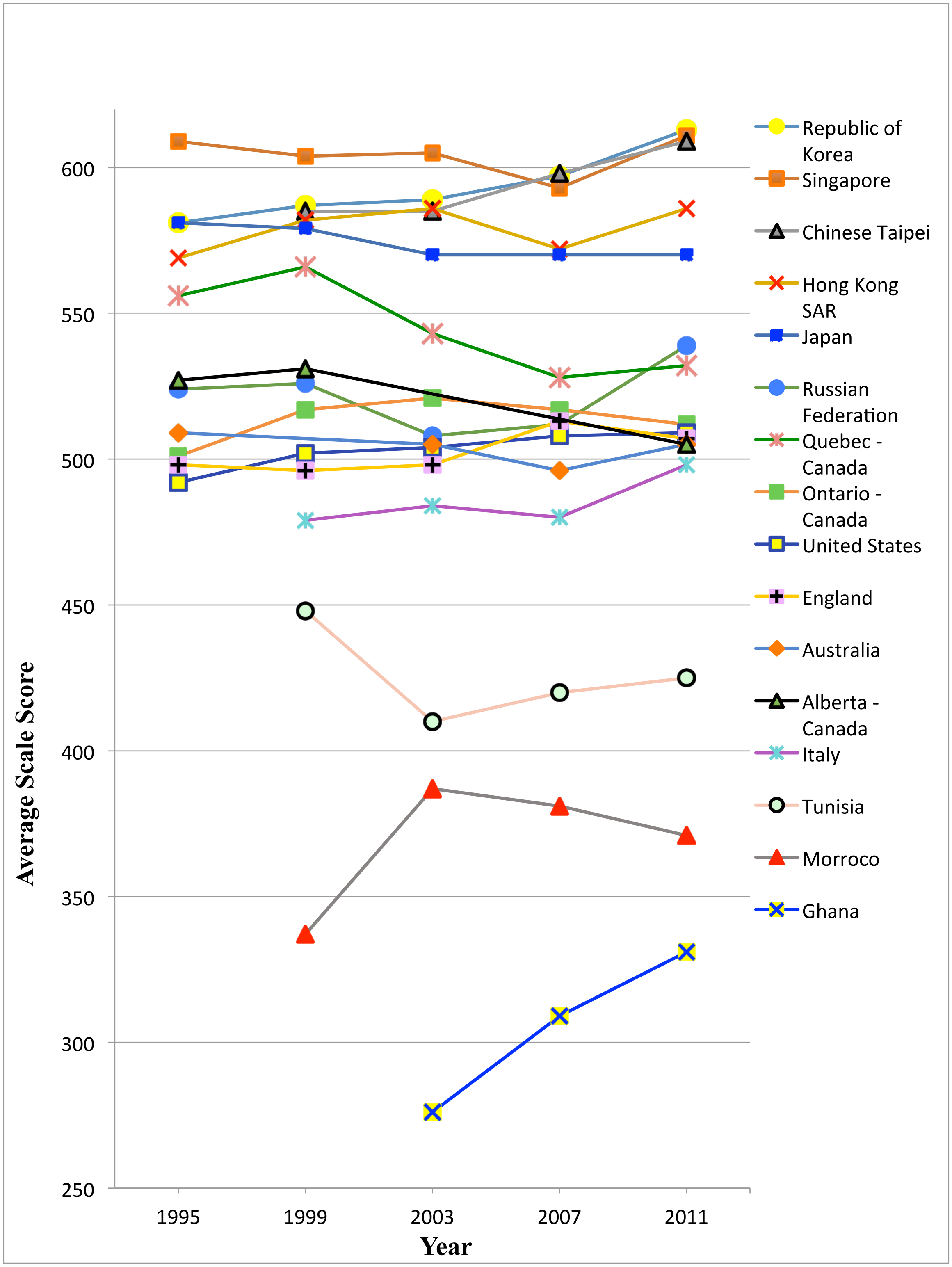


Figure 3. Trends in eighth grade TIMSS mathematics performance of significant regions between 1995 and 2011. Data for all regions from Mullis et al. (2012); [p. 56-59].

Trends in PISA (2003 – 2012)

As Shanghai did not participate in TIMSS, another international assessment will be used to study Shanghai, namely the Program for International Assessment, or PISA. Figure 4 displays trends in PISA mathematics performance of particular provinces and countries between 2003 and 2012.

Reading, science and mathematics are the three core domains being assessed in PISA; each subject rotates to be the major domain in which it will be tested in more depth (*Beyond PISA 2015*, n.d.). When mathematics became the major domain for PISA in 2012, Shanghai-China, Singapore, Hong Kong SAR, Chinese Taipei and Republic of Korea made up the top five scoring countries respectively in the PISA rankings. These Asian regions had shown overall improvements and were always among the top five between 2003 and 2012. However, it is conspicuous that Shanghai-China had a very high average score in 2012, ending up 40 points above that of Singapore, the second highest performing country.

In 2012, most western industrialized countries had average scores between 480 and 536. Quebec performed highest among all provinces of Canada with an average of 536, the same score as from the 2003 results. Ontario had an average score of 514, which showed a decline of 3% in average over the nine years; it had the same score as Germany which ranked 15th among the 65 participating countries, and was four points below the Canadian average of 518 (Brochu et al., 2013). The downward trend of Ontario PISA mathematics performance over the period 2003 – 2012 is clear to see. In contrast, Shanghai and Singapore were both improving between 2009 and 2012 in the top tier.

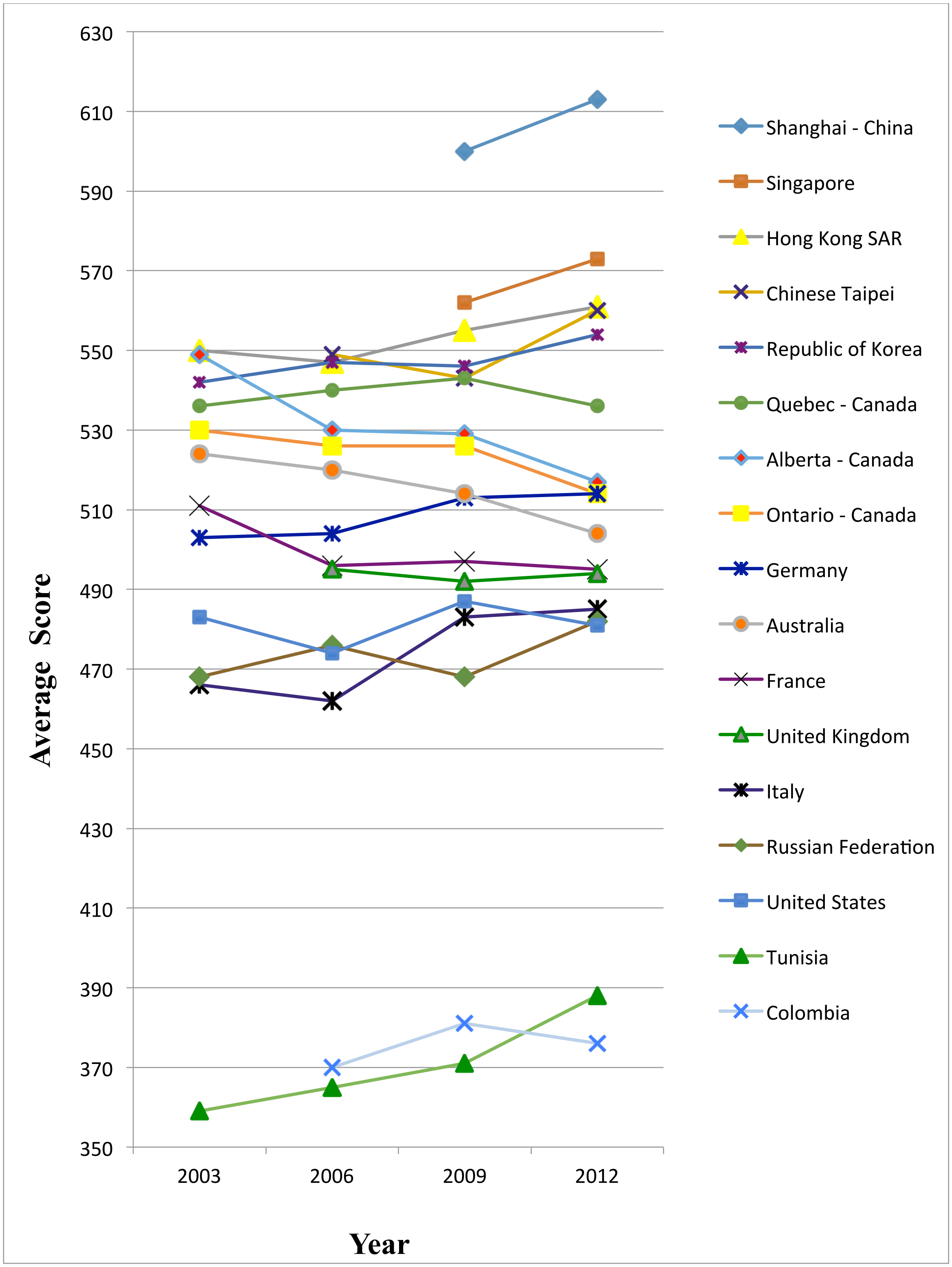


Figure 4. Trends in PISA mathematics performance of particular provinces and countries between 2003 and 2012. Data for Canada from Brochu et al. (2013); data for all other regions from NCES (2014).

Similar to the TIMSS results, the developing or third world countries were still in the bottom of the PISA performance chart. It is likely the case that those countries did not have sufficient resources to provide effective educational systems, which led to their unfavorable results. However, most western industrialized countries with ample governmental support were still exhibiting scores below those of the highest performing Asian countries. Why was it the case that Singapore and Shanghai continuously performed better than Ontario in mathematics assessments over these years?

The Two Significant Math Approaches

To compete in today's fast paced world, it is important for students to be enriched with relevant mathematical skills. During the time in which international testing highlighted variations in individual country performance, a number of countries have taken action to try to improve their results in comparison with those of other countries. The Singapore and Shanghai math approaches have become two of the most popular models for math education. For instance, schools in Kentucky and schools in Australia have adopted the Singapore math approach, and schools in the United Kingdom has taken up the Shanghai math approach.

Singapore Math Approach

Since Singapore has been proving the effectiveness of their mathematics education approach by posting the leading scores, some other participating regions like Kentucky, and Australia have taken up the Singapore Math approach hoping to be able to match the scores of other top countries.

Thirteen elementary schools in Kentucky, in the US had adopted a Singapore math approach in 2013 to meet the Common Core State Standards for teaching math in elementary schools, as Holliday stated “Singapore was one of the benchmarks used in the development of the Common Core math standards” (as quoted in Konz, 2013). According to Konz (2013), this Singapore teaching method “focuses on problem solving and in-depth understanding rather than rote memorization” (para. 1). Yet improvement has not been revealed as the result of 2015 National Assessment of Educational Progress at Grade 8 (see Figure 5) has indicated that the 2015 math eighth grade achievement is slightly lower (2%) than that in 2013 (DeCandia, 2015). Is it the case that the Singapore math approach is not being used as effectively in the U.S.? Or does it take a longer time to become effective?

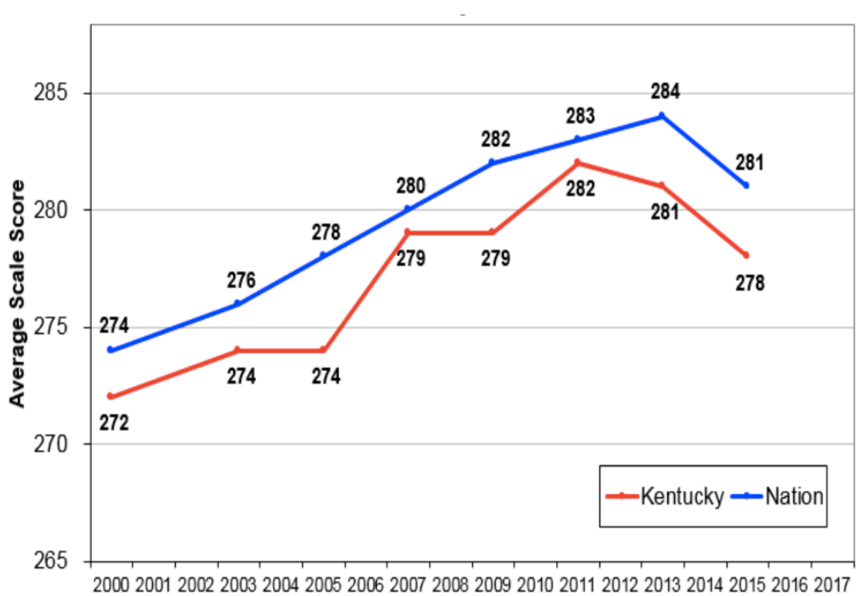


Figure 5. Math eighth grade Kentucky vs Nation NAEP trends between 2000 and 2015. Reprinted from *2015 NAEP Mathematics National Assessment Of Educational Progress At Grades 4 And 8*; [p. 34], by DeCandia, M., 2015, Copyright 2015 by Kentucky Department of Education.

Similarly, after the Australian Curriculum Assessment and Reporting Authority had compared the Australian and Singapore math curriculum, Scholastic Australia decided to publish new primary textbooks (Years 1 to 6) in May 2014 following the Singapore math approach; the textbooks include the teaching strategies that has prepared students of Singapore to achieve to the top in international assessments (Ferrari, 2014). Norm Hart, the President of the national executive of the Australian Primary Principals Association, reviewed the textbooks and said, "the system would enable kids to gain a much deeper and better understanding of mathematical content and ways of working" (Ferrari, 2014, para.12). However, Australia scored 494 points in PISA 2015, which indicated a drop of 10 points from PISA 2012 (OECD, 2016b; Brochu et al., 2013). It will probably to take a few years to see if this change will be effective in Australia indeed.

Shanghai Math Approach

Since students from Shanghai achieved such a high result in the PISA 2012, many educators and researchers worldwide have been studying elements of their educational system. The UK Education Minister, Elizabeth Truss, believes that the U.K. students can achieve the same high testing results as the students of Singapore, Japan, and China (Department for Education, 2014). In February 2014, she went to Shanghai to learn more about some of the possible factors behind their high performance (Howse, 2014). In July 2014, she announced a £11m funding which included bringing in 50 Shanghai math teachers to teach students and deliver Chinese-style teaching to teachers in 32 hub lead schools (Howse, 2014). Also, two English math teachers from each hub would go to China to learn the teaching approach in schools. Each hub would then support local schools in England to form a network focused on raising standards in math education (Department for Education, 2014). The newest PISA 2015 results (492 points) show that the U.K. has dropped two points from the PISA 2012 result (494 points) (OECD, 2016b; Brochu et al., 2013). In July 2016, another £41 million was funded to further support eight thousand primary schools, that is about 50% of the total in U.K., in this "Asian-style mastery approach" (Department for Education, 2016, para. 3). The effect of this reform to uplift math standards in the U.K. might not be revealed until the next international assessment in 2018.

Ontario Reaction

The dip in scores for students from Ontario on international mathematics assessments has aroused attention from various groups. Moreover, 57 percent of sixth grade students and 67 per cent of third grade students did not meet the provincial math standard in 2013 according to Ontario's Education and Accountability Office in 2013 (Logan, 2014). As reported in the Globe and Mail, "Parents petitioned for curriculum reform; business leaders sounded the alarm about the danger that poor math knowledge poses for economic growth; even math teachers are agitating for change" ("Canadian Education," 2014, para. 4). Parents, educators, business leaders, researchers, and members of the media had expressed their concerns and asked for education policy makers to make changes to curriculum and teaching practices in order to try to improve mathematical literacy.

In January 2014, Ontario's Education Minister, Liz Sandals, promised a four million dollar funding for mathematics teacher training to work towards an improvement in the region's mathematical performance. She believed curriculum was not the main cause of the decline and stated, “In jurisdictions similar to Ontario, where the results are better, we often find there’s more extensive teacher training, and in particular more extensive teacher training in how to teach math” (Rushowy, 2014a, para. 7).

While other countries are learning about new mathematics curricula and pedagogical approaches from Singapore and Shanghai, is it the case that improved teacher training can, by itself reverse the trend of lower international test scores in Ontario?

One possible approach that Ontario can take to try to achieve better international mathematics performance is by trying to determine what are the most effective attributes that can be implemented in its educational system. It is hoped that it might be possible to reveal the underlying mechanisms of educational success by comprehensively comparing mathematics education among Ontario, Singapore and Shanghai. One of the main goals for this thesis project is to identify a series of recommendations for improving Ontario’s high school mathematics performance by probing more deeply into a wide variety of factors related to potential differences in social context, teacher training, qualification, and compensation, differences in curriculum, variety in classroom activities, and assessments, as well as studying differences in students’ attitudes towards mathematics and education in general.

Summary

After reviewing the background and problems of this thesis, Chapter II will present information comparing societal differences between these various regions regarding their demography, social welfare systems, university entrance systems, as well as the educational policy, parental influence and students’ attitude towards mathematics among these three regions. In Chapter III, differences in teacher qualification, training, compensation and societal attitudes towards the teaching profession will be discussed. Chapter IV will analyze the curriculum by inspecting the curriculum focus and pedagogy approach in each region to seek real factors for improving. In Chapter V, the importance of assessments will be investigated to find out any difference in preparation for assessments and assessments for university/college entrance among these regions. Chapter VI will study the different classroom activities in terms of pedagogical focus, teaching method, student's participation, class size and instructional time. Chapter VII will focus on the intrinsic value, utility value, ability belief index, attitude towards schools and students' happiness index to identify any attributes that will be able to contribute to mathematical achievement. The final chapter will identify and analyze the key differences of the attributes among the three regions to outline possible suggestions for improving math education, as well as to compare these suggestions with the 2016 Ontario Renewed Math Strategy to seek outstanding elements for further potential advancement.

Chapter II

Sociocultural Context

Different nations express different values and beliefs related to their societies' predominant attitudes towards education. These embedded values and beliefs are often greatly influenced directly or indirectly by tradition and social context over a considerable amount of time (Leung, Graf & Lopez-Real, 2006). By analyzing a nation's population density, demographics, social welfare system, education policy, college and university admissions procedures, parental influence on students, and students' attitudes towards mathematics, one can gauge the potential impact of societal values and beliefs on mathematical learning. In this chapter, we investigate these areas as they relate to Ontario, Singapore and Shanghai respectively. After preliminary findings are listed, we then determine possible connections between these elements and mathematics performance as measured on standardized international test assessments.

Sociocultural Context of Ontario

According to Discover Canada (2012), Canada had become "a self-governing Dominion" ("Confederation," para. 3) after the British North America Act passed in 1867, and is currently populated with diverse ethnic groups. Its largest province Ontario is considered as "the most multicultural city in the world" ("About Ontario," 2016, "Languages," para. 2). According to the Canada 2011 Census, 71.7% of the Ontario population is White; 7.6% is South Asian; 5% is Chinese, 4.3% is Black, 2.4% is

Aboriginals, and 9% is comprised of other ethnic origins. ("Demographics of Ontario," 2016). The population density was 14.1 people per square kilometer as of 2011 ("Focus on Geography Series, 2011," 2015).

The official language is English; however, French is also used in certain regions and government publications. More than 100 other languages and dialects exist and the most prevalent are Chinese, Italian, German, Polish, Spanish, Punjabi, Ukrainian and Portuguese ("About Ontario," 2016).

Canada has a strong social welfare and health care system that particularly protects the younger generation as well as minorities; as a result of this, the overall financial stress is less intense in Canada for these groups as compared to that felt in many other countries. The total net social spending in 2009 was 21.6% of GDP (OECD, 2016c). It is also important to note that the social welfare system has been characterized as one that "values education and teachers, prizes the public good, and doesn't abandon the weak in its efforts to become economically stronger" (Hargreaves, 2011).

The Ministry of Education (2009) has articulated in the *Ontario's Equity and Inclusive Education Strategy* that "Canadians embrace multiculturalism, human rights, and diversity as fundamental values" (p. 7). In the strategy, Kathleen Wynne, the former Minister of Education, stressed the importance of providing safe, respectful and accepting environments to multi-cultural students with various strengths, needs and interests to learn (Ministry of Education, 2009). Ontario does not have standardized provincial tests for university entrance; students are required to obtain a High School Diploma after completing 18 compulsory credits, 12 optional credits and 40 hours of community involvement activities (Ontario Ministry of Education, [n.d.]). The enrollment of Grade

12 students in 2008-2009 was 218,712 students (“Quick Facts,” 2012), and there were 199,593 Ontario post-secondary graduates (University and Colleges) in 2013 (Statistics Canada, 2016). With about 20 public universities and 24 colleges of applied arts and technology in Ontario (“About Ontario”, 2016), the competition to get into university or college is not as challenging as in some other nations, yet it is believed that graduates will be well prepared to compete worldwide. Besides the economic factor for education, Ontario is aiming for a cohesive society built on principles of safety, acceptance and respect through education (Ministry of Education, 2009).

After Ontario had encountered a downward trend in mathematics performance both provincially and internationally in 2014, people from many parts of society showed deep concern as they knew that this implied the presence of a number of issues with the education system that needed to be addressed. In 2014, Education Minister Liz Sandals stated that improving student mathematics achievement was one of the top current priorities of the Ministry of Education (Rushowy, 2014b). Sandals suggested that Ontario families should employ tutoring and homework support websites whenever extra help was required (Rushowy, 2014a). The 2007 Survey of Canadian attitudes toward learning (2007) has indicated that “72% of Canadian parents, with children aged 5-24, report that homework is often a source of household stress” (para. 3); at that point around one third of parents had hired tutors for their kids in Canada.

In 2011, 81% of Ontario English-speaking parents expected their Grade 4 children to complete their education through at least an undergraduate level, and that 43% of parents expected their Grade 4 children to complete postgraduate study (“How important are educational expectations,” 2013). A 2015 report exploring parental influences on teens' science education has shown that 75% of Canadian parents believed that their children require basic knowledge of mathematics and science for their future job, and 46% of the polled parents said that they would spend more money to improve these two subjects if they could afford it ("Canadian Parents have Influence," 2015). This implies that a significant proportion of parents had high expectations and they wanted to work to improve their children's academic performance. In fact, "80% of Canada's top 25 jobs required a university degree" in 2016 ("Back to School 2016 Quick Facts," 2016). In 2012, 73.4% of Canadian students agreed or strongly agreed with the statement that "Mathematics is an important subject for me because I need it for what I want to study later on" (OECD, 2013a, p. 287).

Sociocultural Context of Singapore

As an independent country, Singapore has had a much briefer history; it became independent from the Malaysian Federation just over 50 years ago in August 1965 ("Singapore," 2016). It is quite a small country geographically, with only 660 square kilometers ("Singapore Country Profile," 2016), along with very humid and hot weather and few natural resources. More than 75% of the total population has Chinese ancestry; followed by Malays and Indians subpopulations. The official languages are English, Mandarin Chinese, Malay and Tamils. However, English is the main language used in government and education. The population density of Singapore is relatively high, with 7,126 people per square kilometer in 2010 ("Demographics of Singapore," 2017).

The main governmental party, the People's Action Party, led by Lee Kuan Yew, diligently transformed Singapore into a global financial hub and wealthy nation within its first five decades. With a low tax rate, Singapore has attracted more than 7,000 international companies that currently operate there. Today, it ranks third in income per-capita worldwide, and it is the only Asian country that has a AAA credit rating from the three major credit rating agencies, S & P, Moody's, and Fitch (Larkin, 2012). It can be argued that one key to this economic success is Singapore's focus on education and the government's insistence on continuously improving its school system effectively and efficiently.

Singapore's welfare system is designed in such a way that each generation, individual and family is responsible for paying for themselves on their own before seeking help from the government (Fund, 2015). In 2009, the social development expenditure was about 6.4% of GDP (Lim, 2016). An associate professor at Lee Kuan Yew School of Public Policy has indicated that the Singapore pension fund, the Central Provident Fund, can only provide sufficient spending money for the bottom 30% of the work force after their retirement; the fund is inadequate for middle and higher income groups to retire on their expected expenditures due to inflation (Shih, 2015).

The majority of the younger generation thus realizes that acquiring a college degree is a basic necessity to pursue success in the future; a 21-year-old College student, Sherie Melek, said, "I've known since I was a young child that a college degree is something I would need to be successful" (Larkin, 2012, para. 19). At home, parents have been instilling the value of education to their children since they are young. In a survey done for the Friends Provident International poll in 2013, 61% of Singaporean parents planned to send their child to a local university ("New Survey," 2013). A 13-year-old student, Sie Yu Chuah, told the reporter that his parents had high expectation of him, "I have to do well. Excel at my studies. That's what they expect from me" (Vasagar, 2016).

Furthermore, University graduates have higher overall salaries than Polytechnic and ITE graduates (Ministry of Manpower, 2017). Children are motivated to study hard as they know there is a need. The *Financial Times* printed a speech given by Prime Minister Lee Hsien Loong to Singaporeans that "To survive, you have to be exceptional" (Vasagar, 2016). Usually teachers dominate in classroom talk and students are normally listening in an orderly manner; "parents, students, teachers and policy makers share a highly positive but rigorously instrumentalist view of the value of education at the individual level" (Hogan, 2014).

Due to the country's ethnic diversity, Singapore's government has developed an education system with different pathways which enhance the holistic growth of each student to attain his/her full potential. "Students are placed in classes and schools tailored to their abilities" (Larkin, 2012, para. 17). University applicants are required to complete the Singapore – Cambridge GCE A Level. Besides academic subjects, schools also develop "physical, aesthetic, moral and socio-emotional aspects" (Ministry of Education, Singapore, 2015, p. vi) according to students' interests and talents. The education system prepares students to develop a "core of competencies, values and character, to ensure that they have the capabilities and dispositions to thrive in the 21st century" (Ministry of Education, Singapore, 2015, p. vi). Students therefore learn to be caring, respective, responsible and honest to propel a harmonious multicultural country.

In addition, the Ministry of Education has required mathematics as being fundamental to develop 21st century competencies in Singapore (Ministry of Education, Singapore, 2013). Hence, mathematics is essential for both personal lives and professional pursuits; it is regarded as an important subject being taught throughout the school years to prepare students for further academic and occupational pursuits so as to promote economic growth and innovative development (Ministry of Education, Singapore, 2013). Without requiring this level of mathematics skills students are unlikely "to proceed to higher levels of recognition" (Nickles, 2008, para. 9), even nursing school also expects students with good mathematics skills. In 2012, 87.4% of Singapore students agreed or strongly agreed the statement that "Mathematics is an important subject for me because I need it for what I want to study later on" (OECD, 2013a, p. 287).

Sociocultural Context of Shanghai - China

According to the Shanghai Statistics Bureau, more than 39% of the Shanghai population consists of long-term migrants, mostly coming from Anhui, Jiangsu, Henan, and Sichuan provinces ranging from 300 km to 2,000 km away from Shanghai ("Demographics of Shanghai", 2017). The Shanghai 2010 Census reflected that 98.8% of the residents were of the Han Chinese ethnicity; the 1.2% minority were mainly Japanese, America, and Korean ("Demographics of Shanghai", 2017). The population density was 3,631 people per square kilometer in 2010 ("Shanghai Now Densest City in China," 2011).

Though China is often still considered a developing country, it has a history of more than 5000 years. Traditionally, education is valued as "the major path to climb the social ladder" (Hsieh, 2013, p. 51) and considered as the only way to become successful in life. In China, there is a strong belief that it is the nurturing of children that determines academic performance; hence, Shanghai officials "launched a reform effort hinged on making the curriculum much more rigorous, and they invested heavily to do so" ("How

Shanghai's Students Stunned the World," 2011) in the previous decades. There is an old Chinese proverb which says that "a book holds a house of gold"; this saying exhibits the significance placed on the value of education in pursuing prosperity and an affluent future in Chinese society.

Today, Chinese authorities are determined to raise national school standards so as to prepare the next generation of students to succeed in the global economy. The Chinese government has made it a goal to adopt a meritocratic education system intended to develop "key schools" as models for all of the other schools. These key schools are supposed to cultivate top students to then get admitted into top universities. High-performing teachers and administrators are then transferred from these key schools to help improve the quality of weaker schools ("How Shanghai's Students Stunned the World," 2011). With high expectation from family and school, students typically strive very hard in school to meet or exceed requirements.

China does not have an especially secure social welfare system at this point. Spending on public social welfare was about 7% of GDP in 2009 (OECD, 2014d). The developing welfare system has created a number of unequal circumstances for people, such as discrepancies in medical insurance, pension plans, minimum wage schemes, and so forth (Branigan, 2013). Though the official one-child policy ended in 2015, it caused significant stress among students since its inception in 1980, for an only child has to fulfill the expectations and hopes of parents on their own (Ryan, 2011). The National Higher Education Entrance Examination, also known as Gaokao, is a prerequisite for all senior high school students to enter undergraduate programs in China. In 2012, 9.15 million students took Gaokao countrywide and 75% of them got admitted to universities

(“The Gaokao,” 2012). The percentage might seem high but the competition to get into top-tier universities with very limited space was exceedingly high. Though the situation is similar to other nations, the two prestigious universities, Peking University and Tsinghua University, only admit the top 0.04% to 0.3% students in the National Higher Education Entrance Examination (“How hard is it,” 2015); whereas the 2016 Ivy League Admissions Statistics has indicated the overall acceptance rate for the Ivy League Universities in the United States ranged from 5.9% to 16.2%. In 2008, two philanthropists set up a 30 million yuen, approximately 4.9 million US dollars, initiative to award students in Enping, a city in Guangdong, who were able to get admitted into either of the two prestigious universities, Peking University and Tsinghua University; each student was to be awarded one million yuen, approximately US$163,000, and a house (Chen & Si, 2014). Unfortunately, no student received the award as of September, 2014.

Mathematics has been taught in China for more than 2000 years (Ryan, 2011). It was viewed as an important subject in the imperial examinations (400 – 900 AD). Thus, for a very long time, mathematics achievement has been highly respected as a critical element for attaining a “family’s success, prestige, power” (Ryan, 2011, p. 166) and quality of life. The significance placed on the value of mathematics persists to this day and is reflected in the current education system. In 2012, 79% of Shanghai students agreed or strongly agreed with the statement that “Mathematics is an important subject for me because I need it for what I want to study later on” (OECD, 2013a, p.287).

Mathematics is a crucial subject not only because it is a compulsory requirement for many professional majors, such as Engineering, Computer Science, Actuarial Studies, Astrophysics and so forth, but scores on mathematics tests are also factored in to count for at least one-quarter of the university entrance exam. As a result, students spend a large amount of time studying math, and this has been estimated by some to be almost double the amount of time that American students spend (NCEE, 2017). In general, Chinese students work especially hard on this core subject.

Summary of the Findings

A summary of the differences and similarities in the findings described in previous sections is presented in Table 3.

Table 3.
Comparison of Elements of Sociocultural Context

	Ontario - Canada	Singapore	Shanghai - China
Population Density (persons per square kilometer)	14.1	7,126	3,631
Demographic Majority	White (72%)	Chinese (75%)	Chinese (99%)
Social Welfare Spending of GDP (in 2009)	21.6%	6.4%	7%
Major Focus of Education Policy	Equity and inclusiveness	Holistic growth for attaining a student's full potential	Academic success
University / College Prerequisite	Ontario High School Diploma	Singapore – Cambridge A - Level	National Higher Education Entrance Examination
Parental Expectation on Education	High	High	High
Students Regard Mathematics as Important for Further Study	73.4%	87.4%	79%

To compare the sociocultural elements among these three regions with respect to their international mathematics performance, it appears that those places with a higher population density and higher student regard of mathematics education performed better, at least in the comparison of these three areas. In addition, for the past several decades, students from a number of Asian countries have continued to record higher test results than most countries from other continents. For students to excel in a subject, it requires both a recognized need to perform well, in addition to requiring a consistent drive as momentum. One could make the argument that those places with less emphasis on a widely supportive social welfare system can lead to there being a sense of a more competitive environment, one that could potentially then lead to students competing harder to get higher scores. However, clearly a weaker social welfare system by itself does not foster mathematical excellence in itself, in the case of Shanghai, it happens to be combined with other factors as well

The education system in Shanghai has been developed to place as much emphasis on academic success as possible, through a belief that this can elevate economic productivity. This focus can act as a catalyst for increasing mathematical performance in society. In Singapore, the prominent education system has extended its academic focus to students' holistic growth with the goal of helping students to achieve their full potential. In comparison, the Ontario education system appears to be more concerned with issues involving equity and inclusiveness. This can provide a more stable environment for students to learn in, but it might also result in students having less impetus to strive past their comfort zone.

Parental expectation is generally high in all regions. However, Li (2004) has stated in his study that "high parental expectations and children's striving for excellence are rooted in Chinese cultural heritage and are situationally motivated and historically transformed in different ways in response to the demands of the Canadian sociocultural context" (p. 167). Research has shown that Chinese and Japanese mothers ranked "effort" as the main factor contributing to school success of their children; whereas American mothers believe "ability" was the major reason (Linh, 1997). Merseth (1993) commented that "the belief in innate ability not only minimizes personal responsibility but also fosters the view that poor performance in mathematics is socially acceptable" (p.549). Merely depending on ability without making effort as well makes it challenging to reach one's full potential in most instances.

All of these regions value mathematics education as being essential to society. However, it is possible that students in Ontario are more likely to engage in mathematics more as a fulfillment of basic requirements, whereas students in Singapore and Shanghai might view their success in mathematics as an important asset of their post-secondary education and prospective career, which could potentially drive them to devote more time to learning the subject.

In general, these three regions have various amounts of societal support for mathematics learning. One can note that some of the elements that contribute to better mathematics test performance are associated with the Chinese cultural heritage, which emphasizes preparation of students to do well in assessments (Lee, 2000). Chinese students, whose parents typically have high academic expectations for their children tend to focus on mathematics and academic success, and in a highly competitive and less social welfare context might be driven in such a way that ends up with them achieving higher scores on international mathematics assessments.

Chapter III

Teacher Qualification and Compensation

Teachers play an essential role in education systems as they have the most amount of direct educational contact with students. As a result, the qualification and experience of teachers, as well as considerations of their overall compensation can have a significant influence on students' performance. Teachers also play a critical role in engaging students to learn positively and to inspire them to achieve their full potential. Ideally, students should be able to enjoy the learning process and teachers should be satisfied with their job. If these are both present, then this produces a self-supporting cycle in the educational system. In this chapter we seek to investigate possible correlations between students' mathematics performance and teacher qualifications, ongoing training and support, teacher compensation and the level of societal support and respect given to teachers.

Teacher Qualification and Compensation in Ontario – Canada

All Ontario teachers must be certified by the Ontario College of Teachers in order to teach in any Ontario public schools. A minimum of a three-year post secondary degree and a one-year four-semester education program must be completed before applying for such certification. Beginning in September 2015, the education program developed into a two-year training including 80 days of practicum, an increase in the amount of time devoted to studying special education, as well as use of technology and a focus on diversity training (Ontario College of Teachers, 2017).

In 2005, the Working Table on Teacher Development was established to recommend and propose training programs for teachers to enrich their skills and knowledge throughout their careers (Ontario Ministry of Education, 2009). In addition, the New Teacher Induction Program provides another year of professional support for new teachers to help promote effective teaching, learning and assessment practices (Ontario Ministry of Education, 2010).

However, the 2011 TIMSS report has indicated that only 18% of Ontario eighth grade mathematics teachers possessed a mathematics major and/or a major in mathematics education (Mullis et al., 2012). This percentage was relatively low as compared to those of Alberta, Quebec and compared to an international average, which were 45%, 67% and 85% respectively (Mullis et al., 2012). Furthermore, many Ontario elementary teachers did not take any mathematics courses after high school (Rushowy, 2014a); Professor Mary Reid of the University of Toronto stated that some teacher candidates did not appear to know basic mathematics algorithms; for instance, she gave as an example the fact that some teacher candidates thought "a remainder of 3" and "decimal 3" were the same thing (Brown, 2016). There was also no subsidy for teachers to take specialty math courses or summer learning until 2014 when four million dollars was granted for math professional training (Rushowy, 2014b).

Although many consider teaching to be a highly rewarding career, it is also often a stressful and challenging career. The Ontario Ministry of Education has been working to provide sufficient incentives to attract the best teachers in Ontario and as a result, salaries for Ontario teachers are among the highest in the world ("Is the Party," 2012). According to the research done by British Columbia Teachers' Federation (2014) for Canadian teacher salary rankings between 2013 and 2015, the average salary for teachers in Ontario ranged from US$48,800 to US$88,100 (as converted from Canadian dollars at a December 2013 Bank of Canada exchange rate of 0.9402). In addition, the teachers' retirement pension fund did extraordinary well in 2011 with approximately $11.7 billion, and approximately 10% in annual return, which is much higher than the largest Canadian bank RBC's net income of $4.85 billion in the same year ("Is the Party," 2012).

There is no doubt that most teachers work hard. Besides financial incentives, it is useful to consider if teachers are rewarded in terms of societal respect as well. According to the survey, "Public Attitudes Toward Education in Ontario 2012", done by Hart (2012) at the University of Toronto, 70% of the public and 76% of parents are satisfied with the teachers' job. Nonetheless, the status of teachers has been disparaged at times; Justin Trudeau was a former teacher before he began his political career and was questioned by his opponent Jason Kennedy, who was the Immigration Minister in 2013, as to whether a teacher was capable of being the Liberal leader (Maharaj, 2013). In 2015, Trudeau became the 23rd Prime Minister of Canada. At that point, some people in the media asked, "Why is being a teacher seen as not good enough to lead our country?" and "Why is the idea of a teacher as prime minister regarded as outlandish?" (Maharaj, 2013, para. 1, 2).

In terms of societal support, the Ontario teachers' unions are visibly powerful as they are able to negotiate with the government to renew teachers' contracts. When the unions are not able to achieve satisfactory contract results, they have sometimes directed teachers to go on strikes or they have specified work-to-rule plans. Students and parents are the ones who suffer during such strikes, and many in the public have become dissatisfied with such threatening actions. Unions have been criticized that "while good teachers get lots of respect, their union tends to erode it faster than it can be accumulated" (Worthington, 2012).

Teacher Qualification and Compensation in Singapore

The teaching system of Singapore is based on a "strong culture of dedication, collaborative learning, and professional excellence" (Ministry of Education, Singapore, 2015). Teachers receive training and support focusing on "pedagogy and connections between educational subjects" (NCEE, 2016, para. 6). All teachers have to complete an education program at the National Institute of Education for either a bachelor or post-graduate degree ranging from two to four years of study. In 2011, 83% of eighth grade Singapore students were taught by teachers with either a major in mathematics and/or in mathematics education (Mullis et al., 2012). Teacher training is free and those in teacher education programs receive 60% of a teacher's starting salary during this time, but they have to commit to teaching for three years after their training is complete (NCEE, 2016).

Moreover, in-service teachers can continue their teacher education through courses offered by National Institution of Education. Also, the Ministry of Education has established a Teachers' Network, which provides regular conferences, forums and seminars. In addition, study leave and scholarship are available for further advancement. In general, teachers are entitled to have approximately 100 hours of developmental training annually (Stewart, n.d.).

Singapore also provides reasonable levels of incentives to compensate teachers (Larkin, 2012). The average teachers salary was US$45,755 in 2013 (Dolton, Marcenaro-

Gutierrez, Pota, Boxser & Pajpani, 2013). Besides their basic salary, teachers may receive bonuses based on their annual evaluation results (NCEE, 2016). Also, the Ministry of Education established something referred to as the CONNECT plan (**CON**ti**N**uity, **E**xperience and **C**ommitment in **T**eaching) that offers S$3,200 to S$8,320 annually for each teacher and passes along this payout every three to five years until retirement. This is done with the goal of encouraging teachers to stay in the teaching field (Lee & Tan, 2010).

The Singapore education system has been influenced by a Confucian culture that places significant value on educators. At the same time, teachers play an important role in fostering high-level achievers in Singapore. Given that it is quite demanding to become a teacher in Singapore, the career is highly respected (NCEE, 2016). The 2013 Global Teacher Status Index is the first comprehensive study to compare the status of teachers in depth among 21 countries across the world; Singapore came in seventh with an index ranking of 46.3 out of 100, which reflected a relatively high ranking as stated in the study (Dolton et al., 2013).

Teacher Qualification and Compensation in Shanghai – China

In China, teachers are trained in special upper secondary schools, colleges and universities to teach in primary, junior secondary and senior secondary schools, respectively. Primary school teachers are further required to possess post-secondary and sub-degree diplomas. Note that a "sub-degree" is a pathway to a Bachelor degree which includes Associate Degree, Higher Diploma and Professional Diploma ("China (Mainland)", n.d.; "Introduction to sub-degree," 2016); it usually takes at least two years full-time to complete (UniSA, 2014). Secondary school teachers must have bachelor degrees and a professional certificate (NCEE, 2015c). Despite that, Shanghai has been raising the standard of teachers' qualifications over the past two decades. All Shanghai teachers are certified; most teachers have degrees and many teachers possess a Master's degree at this point (NCEE, 2017).

After beginning their teaching career, a mentor is assigned to each new teacher for about three years to assist in all aspects of teaching (NCEE, 2015c). All teachers are required to take continuous developmental training throughout their careers (NCEE, 2015c). New teachers are required to spend 120 hours in training during the first year, and thereafter spend a total of 360 hours every five years. Senior level teachers are required to spend a minimum of 540 hours of developmental training every five years in addition to the school-based teachers' research, which "evaluate and modify their own pedagogy in relation to student outcomes" (Liang, 2016, p. xix). Moreover, teachers and principals are switched between high and low performing schools for a period of time to learn and share experiences for increasing students' academic proficiency with the support of Teachers Professional Development Institutes (NCEE, 2017). In 2008, a web platform was created to share teaching and curriculum information, along with research papers and related resources (NCEE, 2015c).

Despite the amount of teacher training and support, teacher salaries are considerably lower in China as compared to teachers in the rest of the world. The average teacher salary in China was US$17,730 in 2013 with "purchasing power parity adjusted on currency exchange rate between countries that allows the exchange to be equal to the purchasing power of each country's currency" (Investopedia, 2016), which meant that this ranked 20 out of the 21 countries in the Varkey Gems Foundation's survey (Dolton et al., 2013). However, teachers are on a public payroll and their compensation is quite stable (NCEE, 2015c). 70% of a teacher's base salary depends on their teaching level, whereas 30% is a bonus paid according to their performance. Although teachers' salaries in China are relatively lower than those of other countries, the salaries of Shanghai teachers are among the highest of all the teachers in China (“How Shanghai's students,” 2011). In addition to their teaching salaries, teachers often make extra money through tutoring, which is popular in Chinese society.

Due to the prestigious value of education in Chinese tradition, teachers in Shanghai are highly respected (NCEE, 2017). There is an old saying “teacher for one day, father forever”, which underscores the perception that students have great respect for teachers, even if they have only taught for a short while. The 2013 Global Teacher Status Index indicated that China came in first with an index ranking of 100 out of 100 (Dolton et al., 2013). Moreover, the same study also indicated that China ranked teachers' social status the same as doctors'. Teaching is thus considered as a “respected” and “stable professional” in Shanghai (NCEE, 2015c).

Analysis of Findings

To compare the three regions in teaching qualification and compensation, Table 4 summarizes the information given in the previous sections:

Table 4.
Summary of teacher qualification and compensations

	Ontario - Canada	**Singapore**	**Shanghai - China**
Mathematics teachers with a Major in Mathematics and/or Mathematics Education (2011)	18%	83%	N/A
Developmental Training Requirement for Mathematics Teachers	Not specifically required; was not subsidized until summer of 2014 for specialty math courses and summer workshops to strengthen teaching skills	100 hours annually	120 hours for first year and 360 hours for every five years
Average Annual Salary in 2013 (in US$)	48,800 – 88,100	45,744 with extra bonus	17,730 with bonus and private tutoring income
2013 Global Teacher Status Index	N/A	46.3/100 (ranked seventh)	100/100 (ranked first)

Though teachers in these regions all possessed post-secondary educations in addition to teaching certificates, many Ontario mathematics teachers were neither mathematics majors nor mathematics education majors. It is possible that a lower level of mathematics knowledge and knowledge of mathematics teaching techniques might hinder teachers' effectiveness. It appears evident that it is less likely for a teacher to be able to help develop a highly accomplished mathematics student if the teacher was not particularly strong in the subject him or herself.

Shanghai and Singapore both have significant amounts of training and support for teachers' ongoing development; teachers have been spending 100 hours or more annually to continue to improve their skills by pursuing more in-depth fundamental knowledge as well as their pedagogical knowledge of mathematics (Li & Huang, 2013). In Shanghai, the interchanging system of teachers and principals might have a positive effect on overall educational standards. In contrast, Ontario has had comparatively low requirements for ongoing teacher training and support; the new program for teachers' training did not start until 2014.

Students' achievement reflects teachers' performance to a certain extent. It is typically suggested that high incentives can help to retain better teachers. However, while teacher salaries in Ontario are still among the highest worldwide, Ontario is decreasing in terms of students' overall mathematics performance. This might be due to the result of strong teachers' unions which have been fighting for higher benefits that in themselves are not necessarily connected to bringing in the best teachers. In contrast, Shanghai's teachers are the lowest paid among the three regions, but it is still the case that students in Shanghai perform the best in mathematics assessments. As a result, it is not possible to make a direct association between test outcomes and teacher salaries, and it might be the case that the emotional incentive of teachers' passion and their beliefs about their work also contribute to student success. Figure 6 shows some correlation between teachers' salaries and mathematics performance in the results from PISA 2012:

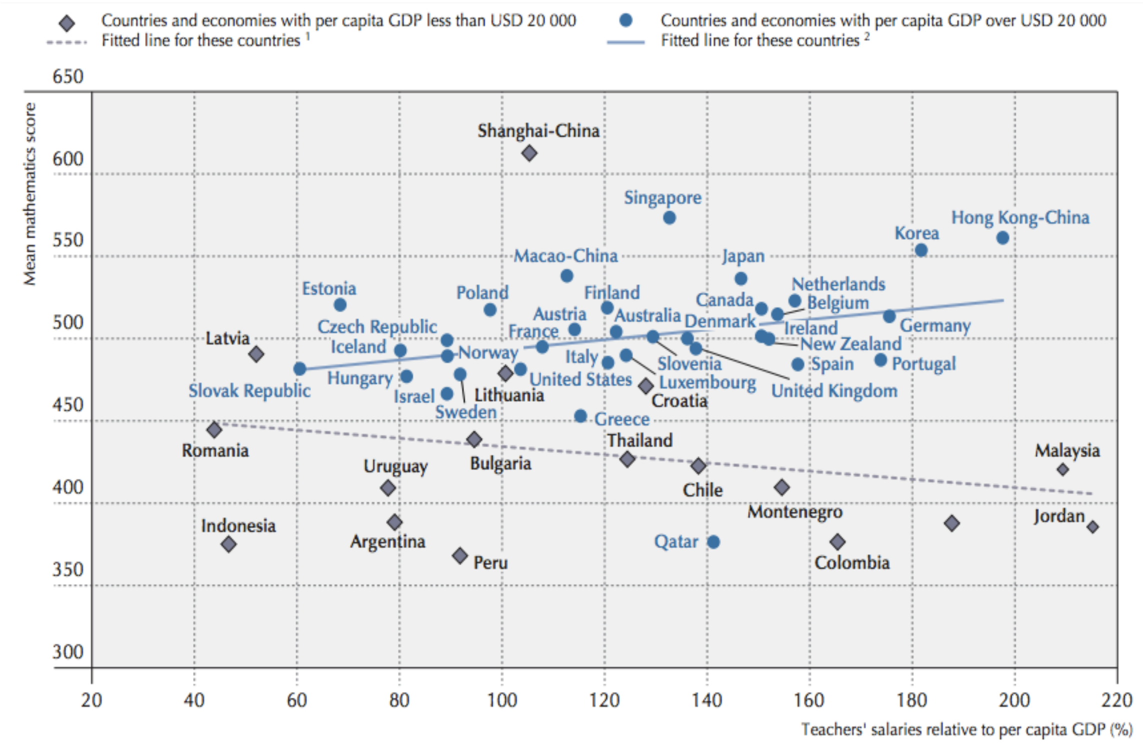


Figure 6. Teachers' salaries and mathematics performance in 2012 for 15-year-old students. Reprinted from *PISA 2012 Results in focus*; [p. 27], by OECD, 2014, Copyright 2014 by OECD.

On the other hand, good teachers should be highly respected as they play a vital role in developing knowledgeable students who can become successful professionals and top achievers. The data suggest that teachers in Shanghai enjoy higher social respect and social status than teachers in Singapore, although there is no comparable data about teachers in Ontario to compare to from the same teachers study. Despite the fact that Chinese culture deeply respects teachers, strikes are not prevalent among teachers in Singapore and Shanghai. The negative effects of strikes can arouse public adversity and can have an impact on students' educational experiences.

In summary, one can make a case that teacher training and pedagogical knowledge might be factors contributing to the decreasing math performance of students in Ontario, and the high performance of Shanghai might be related to the increased amount of time given to developmental training as well as to the relatively higher level of teacher status in society in China. After the evident decline in international mathematics performance, Ontario Education Minister Liz Sandals announced a $4 million funding campaign in January 2014, to support all of Ontario's mathematics teachers to undergo mathematics training including summer courses and extra professional development courses starting in the summer of 2014, with the goal of strengthening their mathematics teaching skills ("Province spends $4M," 2014). Another $60 million fund was released in April 2016 for more extensive training in mathematics education. What the effect of this money being spent in this way is still an open question.

Chapter IV

Curriculum

Although there are many elements to an effective educational system, the choice of curriculum can have a major impact as it has a direct influence on what goes on in the classroom on a day-to-day basis. In some sense a curriculum provides the blueprint for an education system. The type of curriculum that is chosen can reflect certain features of a nation's values and needs, as well as incorporating elements of its culture as well. The PISA 2012 result suggested that the mathematics performance of Shanghai is "three years of schooling" (OECD, 2014a, p. 4) more advanced than the average OECD jurisdictions. Thus, as part of the comparison between the three areas being studied in this thesis, it is important to identify the elements of curriculum that contribute to mathematics advancement by studying the educational focus and syllabi choices of Ontario, Singapore and Shanghai.

Curriculum of Ontario

Canada has undergone a series of major curriculum changes during the last decade. In succession, almost all Canadian provinces changed their curricula in response to an educational theory known as constructivism, which was advocated by Catherine Fosnot at City University in New York, among others. "From a constructivist perspective, meaning is understood to be the result of humans setting up relationships, reflecting on their actions, and modeling and constructing explanations" (Fosnot, 2005, p.

5). According to constructivist principles, students are encouraged to figure out mathematical algorithms themselves instead of being told through direct instruction. Another advocate John Van de Walle suggested that students should “develop their own understanding of math and invent their own ways of answering math questions” (Zwaagstra, 2011, p. 7). These influences created a new shift of educational philosophy from traditional mathematics to discovery-based learning, which was believed to better develop “problem solvers” and “creative thinkers” (Stokke, 2016).

In 2008, Ontario had also followed a path of changing its mathematics curriculum from traditional direct instruction to a discovery-based approach. The innovative approach was focused on ideas of creative thinking and problem solving, and placed a decreased emphasis on learning standard mathematics algorithms, practicing basic mathematics skills and rote memorization (Stokke, 2016). An eighth grade Ontario student, Zach Bedard, said, ”There are kids in my class who don’t know how to multiply six times seven” (Burke, 2012). As reported in *The Globe and Mail* by Morrow & Alphonso (2014), “Ontario in its school curriculum requires students to know the multiplication tables and solve problems using a variety of strategies, but does not specifically state that they must memorize them.” The new inductive thinking approach was comparatively more time consuming as it took up more students’ time in class to discover their own understanding of mathematics. Students eventually had less class time to learn and practice basic mathematics skills.

Ann Stokke (2016), an associate professor of mathematics at the University of Winnipeg, stated that “both the curriculum and commonly used texts place too much emphasis on open-ended problems, multiple strategies and hands-on materials” (para. 9), and their contents were sometimes difficult to follow, with an increased chance for confusion. Furthermore, the curriculum adopted a spiral approach in which mathematics concepts would be repeated subsequently over time (The University of Chicago, n.d.).

Curriculum of Singapore

When the People's Action Party (PAP) came to power in 1959, the education plan was chosen to place an emphasis on mathematics and science subjects to provide support for a rapid industrialization of Singapore. This was then followed by a revised education system providing various pathways to students with different academic abilities to achieve each student's "greatest potential" that was developed in the 1980s (Kaur, 2013). Since 1990, the Singapore curriculum has been based on a mathematical framework that focuses on mathematical problem solving by mastering basic concepts and skills, along with studying applications related to other subjects so as to develop autonomy and higher level thinking skills (Ministry of Education, Singapore, 2012).

This framework has been used as a guideline for a primary and secondary spiral curriculum which switched from being based just on a deductive approach to being based on both deductive and inductive approaches with respect to students' needs in a dynamic context. Typically a deductive approach is a more teacher-centered approach in which concepts are explained to students followed by additional practice; whereas an inductive approach typically involves a student-centered focus in which examples are presented to students so that they can observe concepts on their own (Bilash, 2009). The five key components of the Singapore framework are concepts, skills, processes, attitudes and metacognition. See Figure 7.

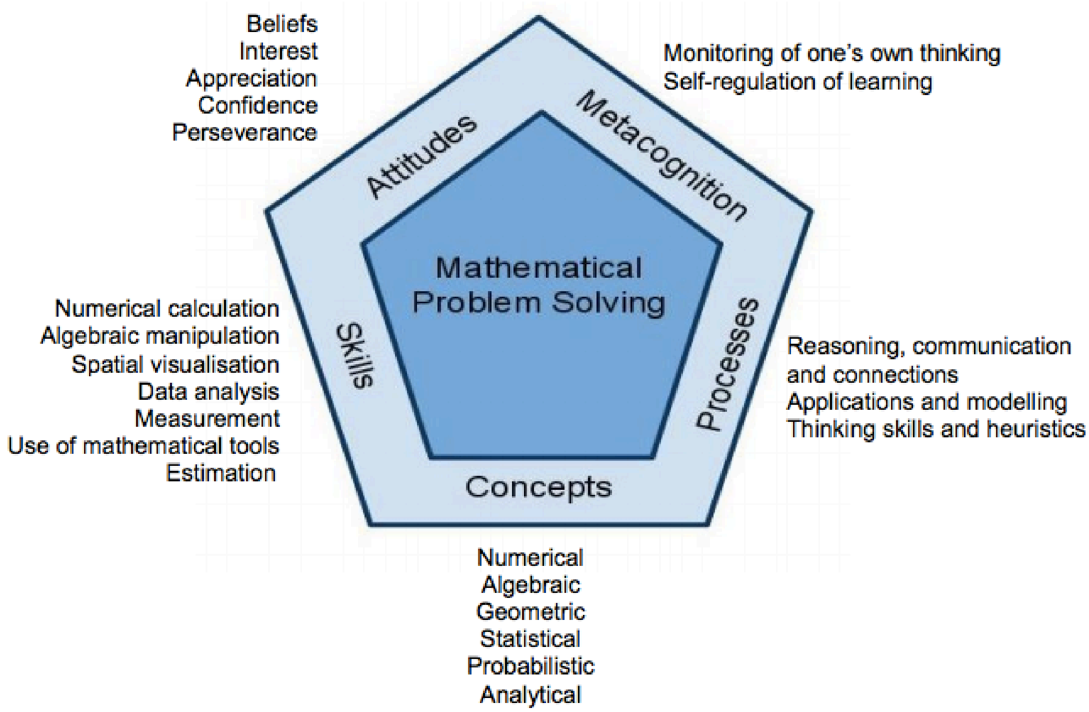


Figure 7. Mathematical framework of Singapore curriculum which reflects the 21st century competencies of the five inter-related components. Reprinted from *Primary mathematics Teaching and Learning Syllabus*; [p. 14], by Ministry of Education, Singapore, 2012, Copyright 2012 by Ministry of Education, Singapore.

Concepts are categorized into numerical, algebraic, geometric, statistical, probabilistic and analytical. Students are obligated to understand these interdependent concepts thoroughly in order to use them for further mathematical applications (Ministry of Education, Singapore, 2012).

Skills include numerical calculation, algebraic manipulation, spatial visualization, data analysis, measurement, use of mathematical tools, and estimation. Students have to learn and practice the mathematics principles and procedures of these skills to ensure proficiency via different processes (Ministry of Education, Singapore, 2012).

Processes include reasoning, communication and connections, applications and modeling, and thinking skills and heuristics. The main purpose of this component is to acquire mathematical knowledge and apply it to solve different real world problems (Ministry of Education, Singapore, 2012).

Metacongnition is the development of controlling one's thinking by solving "non-routine" and "open-ended problems" (Ministry of Education, Singapore, 2012, p. 17). Students learn and discuss how and when to apply strategies in various mathematical problems. This development varies for each individual student so that each student expresses himself differently in creating his own mathematical solutions.

Attitudes of students depend on their beliefs, interests, appreciation, confidence and perseverance in mathematics learning (Ministry of Education, Singapore, 2012). A critical aspect of this is the belief that a positive attitude is required for life-long success and should be developed as early as possible to enhance the overall learning process.

The structured curriculum based on this framework is intended to provide a strong mathematics foundation for students to achieve competence in real life situations. Also, the framework employs both innovative and inquiry-based learning theories to promote creativity and problem solving skills among students.

Curriculum of Shanghai

Shanghai has been pioneering education in China by reforming its curriculum over the past few decades as well. Prior to 1988, students concentrated only on core subjects; however, Shanghai underwent a major reform since that time, and established a "three-block curriculum" in 1988, which allowed students to take core subjects (the basic curriculum), elective subjects (adopting an enriched curriculum) and extra-curricular activities (using an inquiry-based curriculum) for a more well-rounded development (NCEE, 2015b). Since that time, Shanghai has integrated more subjects into its curriculum to include educational, physical and social development as well.

Shanghai's structured mathematics curriculum has been refined regularly to comply with its goal of developing conceptual and procedural skills. Policy makers strive to maintain a high quality of up-to-date teaching resources. "An enormous amount of thought and care has gone into the construction of the Shanghai mathematics curriculum, and it is anything but a straitjacket for teachers," explained by Nick Gibb (2015), an U.K. Minister of State for School. All schools in Shanghai use the same textbook published by the Shanghai education commission to align with the curriculum (Gibb, 2015). The Shanghai mathematics syllabus is linear with few topics taught in great depth. This linear approach ensures the education system "works as a whole" (Carroll, 2007). The deductive approach enables students learning mathematics concepts and skills to develop logic, mathematical reasoning, thinking and application (Li & Lappan, 2014). Since 2001, Shanghai has changed the content of its mathematics examinations to focus on assessing students' understanding instead of rote memorization. The emphasis of the curriculum on knowledge of "number and algebra", "space and graph", "statistics and probability" and "practice and applications" (Wei, 2014, para. 7) helps prepare students to solve real world problems.

Summary of Findings

A summary of the difference and similarities in elements of the various approaches to curricula among these three areas as described in this chapter is presented in Table 5.

Table 5.
Differences and Similarities in Curriculum Approaches

Curriculum	Ontario - Canada	Singapore	Shanghai - China
Main Focus	Problem solving and mathematical concepts exploration	Mathematical problem solving with strong mathematics foundation	Real world problem solving with developed mathematical concepts and skills
Structured vs. Innovative Approach	Innovative	Structured and innovative	Structured
Linear vs. Spiral Approach	Spiral	Spiral	Linear
Deductive vs. Inductive Approach	Inductive	Deductive and inductive	Deductive

Problem solving has become a main focus of the education philosophy in all three of these regions. Their curricula have been modified differently to equip their students with better problem solving skills. The curriculum of Ontario is innovative and inductive; students are allowed to explore mathematics concepts through various activities. In contrast, Shanghai has structured and deductive approaches in which teachers explain mathematics concepts before students do any practice. In comparison, Singapore follows a mixed model employing both approaches (Li & Lappan, 2014). Furthermore, both Ontario and Singapore have a spiral curriculum in which "material is revisited repeatedly over months and across grades" (The University of Chicago, n.d., para. 1), whereas Shanghai has a linear curriculum which "suggests an orderly procedure aimed at a more thoughtfully planned and a more dynamically conceived curriculum" ("Linear models," 2016).

Although the innovative Ontario curriculum was supposed to raise students' mathematics abilities to a higher level, teachers, students and parents have complained that the discovery-based approach to mathematics has not helped students to develop a firm arithmetic foundation to form a basis for exploring more complex mathematics (Urback, 2014). It is possible that the innovative Ontario curriculum has over-emphasized conceptual skills while at the same time under-emphasizing traditional math skills; this change in focus might have had an impact on the decrease in international mathematics performance over the past decade. Without a thorough grounding in basic mathematics skills, students might not be able to fully understand higher level concepts as well, and might not be able to solve mathematics problems as quickly as their counterparts during math assessments. For instance, the PISA 2012 results reflected that 80.7% of the Shanghai and 56.1% of Singapore students understood concepts related to quadratic functions and knew them well; however, only 30.9% of Canadian students were shown to possess the same knowledge (OECD, 2014b).

Shanghai, which uses a linear curriculum, achieved better mathematics results in international assessments. Merseth (1993) has commented that "the implementation of the spiral curriculum has been less than ideal" (p. 550) as concepts were not conveyed in great depth and repeated materials "deadens the mind and breeds low expectation" (p. 550). Nevertheless, the PISA 2012 results indicated that Canada and Singapore, which both adopted a spiral curriculum approach, as well as inductive approaches performed better on interactive tasks; whereas Shanghai and Singapore which adopted structured approaches performed better on knowledge-acquisition tasks. Singapore students who were exposed to both approaches did well on both tasks. See Figure 8.

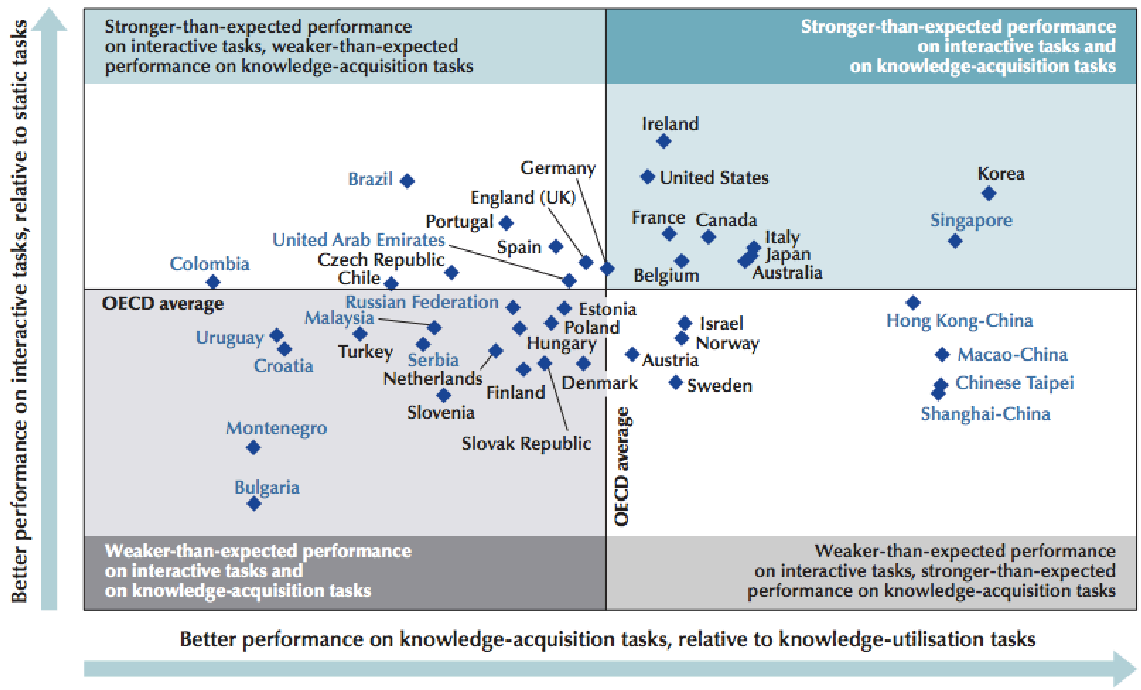


Figure 8. Students' strengths and weaknesses in problem solving. Reprinted from *PISA 2012 Results in Focus*; [p. 34], by OECD, 2014. Copyright 2014 by OECD.

All children are unique and it is critical for them to learn fundamental mathematical skills in order to provide them with the basis to become creative thinkers and prominent problem solvers. Having an understanding of basic skills is an important tool for unlocking the door of creativity. It can be argued that it would have been better for Ontario policy makers to have held back on implementing a discovery-based curriculum until teachers could have been properly instructed in the appropriate skills and techniques to support such a curriculum, and textbooks could have been developed to comply with the new curriculum. It might also have been helpful to have ensured that the new curriculum incorporated a "balanced approach" that "includes instruction in math concepts, math problems and math skills . . ." as stated by Sandals (Rushowy, 2014a, para. 17). As a result of issues relating to the incorporation of this curriculum reform, there is a possibility that the change in curriculum might have contributed to the decline in mathematics performance in both international and provincial assessments. At the same time, the curriculum in Shanghai also has its shortcomings as well. Content-oriented curricula require students to memorize mathematical facts instead of fully understanding the concepts (Leung, 2006). This can potentially hinder students' creativity and exploration in mathematics. Each curriculum approach has its advantages and disadvantages; a successful curriculum should balance different approaches in order to meet as many essential needs for students as possible.

Chapter V

Assessments

In our modern world the growth in the global economy has led to a sense of competition existing everywhere. Adults compete for jobs, students compete for acceptance to universities, people compete to win contests, and so forth. In each of these cases, some type of assessment is required to select who will be successful. An increasing number of corporations require some type of job test before an interview is granted. Different education systems, by and large, have different perspectives and approaches for assessing students. In many instances, examinations have been used to differentiate students' abilities. To determine if taking more tests and examinations leads to an increase in mathematics achievement, in this chapter we will focus on an examination of the existing assessments, students' preparation for assessments, as well as assessments required for entering colleges and universities, and the importance of assessments in these three regions.

Assessments in Ontario

In Canada there is no national examination for university entrance, and each province, such as Ontario, has an independent education system. There is, however, an Education Quality and Accountability Office (EQAO) in Ontario that administers provincial assessments and coordinates students' participation in international assessments (EQAO, 2015). The EQAO arranges provincial mathematics tests for students in third, sixth and ninth grade. Assessments are based directly on the curriculum; therefore, teachers can simply follow the guidelines of the curriculum to help get their students prepared for the test (Durham District School Board, 2013). The EQAO analyzes the test results to provide feedback to the provincial education systems in order to help improve students' learning as stated by Ministry of Education and Training (1999).

The declining performance of provincial mathematics performance has caught public attention in recent years. Figure 9 shows the trend of Ontario students in provincial mathematics performance from 2009 through 2016.

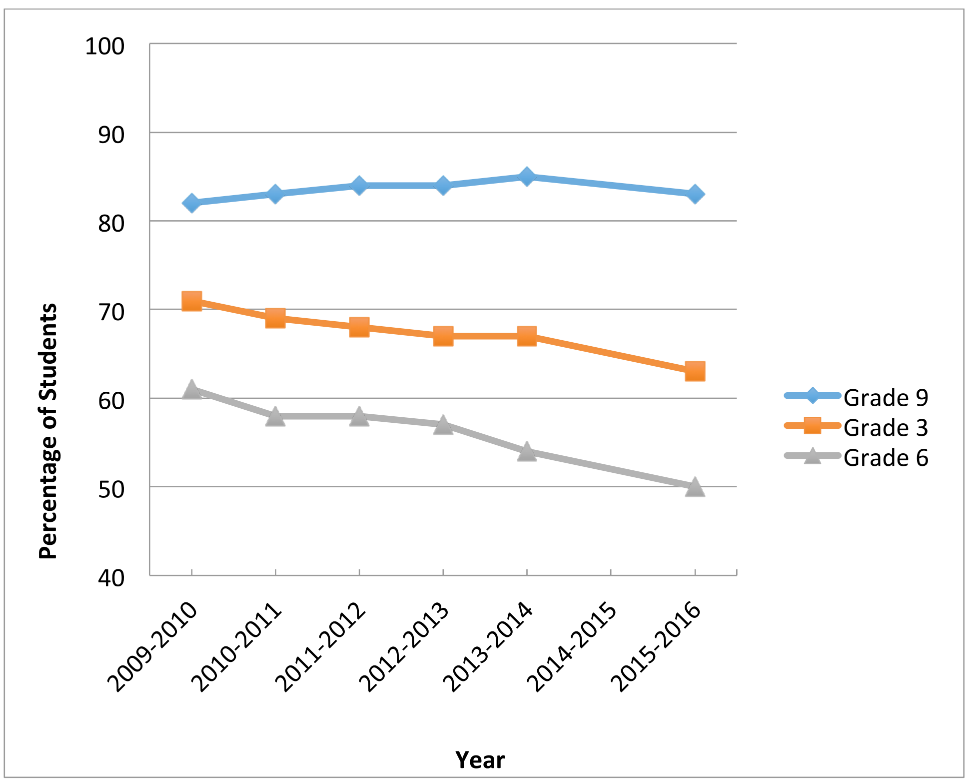


Figure 9. Trends in Ontario students performing at or above provincial standards from 2009 to 2016 for third, sixth and ninth grades. Data for 2015 – 2016 from EQAO (2016), Highlights of the provincial results (2015-2016). Data for 2009 – 2014 from EQAO (2014), Highlights of the provincial results (2013-2014). Data for 2014-2015 are unavailable due to teacher federation labor disruptions.

The results for third grade and sixth grade students meeting the standard have declined 11% and 18% respectively over the seven years; the percentage of ninth grade students meeting the standard was stable throughout the period ranging from 82% to 85% (EQAO, 2014a; EQAO, 2016b). These trends have indicated that some learning problems exist for the third and sixth grade cohort. 28% of the 114,872 students in the same cohort were unable to meet the standard in third grade (2013) and sixth grade (2016) according to EQAO (2016a), as stated as an "early identification of students who are not meeting the standard in Grade 3 is the key for their success in Grade 6" (p. 2). Thus, an early mathematics foundation is crucial in mathematics education.

In addition, there is a national assessment, "The Pan-Canadian Assessment Program" which is administered every three years across Canada to evaluate eighth grade science, mathematics and reading skills (EQAO, 2014b, p. 1). Students do not usually spend time on preparing for these national and provincial tests, as the results have no effect on their final school reports, and there is no particular influence on their schools in respect to the results of the test.

Without any pressure resulting from having to prepare for national examinations or provincial tests, the significance of national assessments is fairly low for students, and their concerns mainly involve the need to fulfill requirements for applying to colleges and universities. Credit is granted for every course in which the student's grade is 50 percent or higher in school (Ministry of Education and Training, 1999). There are University Preparation courses, University/College Preparations courses, College Preparation courses, Workplace Preparation courses, Transfer courses, and Open courses offered in grades 11 and 12 which prepare students with different abilities and interests for different educational paths (Ministry of Education and Training, 1999).

The average of high school entering grades for Ontario University in 2012 ranged from 77.8 (University of Ontario Institute of Technology) to 88.7 (Queen's University) (Dehaas, 2013). The acceptance rate varies from 64% for the University of Toronto to 42% for Queen's University (The Princeton Review, 2017; Google, 2014). From Statistics Canada (2015), the number of graduates from Ontario's public secondary schools in 2011/2012 was 149,139. There were 79,652 full-time fall entry students registered in 2012, in which 65,046 applicants were secondary school applicants according to Council of Ontario Universities (2015). Besides 23 public universities, there are many other tertiary educational opportunities including 17 private universities, 24 colleges, 3 Institutes of Technology and Advanced Learning, and more than 500 career colleges for students to attend (Higher Education in Ontario, 2017).

Assessments in Singapore

A significant emphasis on meritocracy in the Singapore education system has intensified the importance of assessments in selecting the best students for educational training ("Education in Singapore", 2017). The Singapore Examinations and Assessment Board (SEAB) administers national examinations and other assessments in Singapore. The major examinations include the Primary School Leaving Examination (PSLE), GCE 'N' Level, GCE 'O' Level and GCE 'A' Level ("Singapore Examinations and Assessment Board (SEAB)", 2017).

The PSLE is for students completing Primary 6 (11-12 year old); students are assigned to various streams, Express, Normal (Academic) and Normal (Technical), according to the results of the examination (“Education in Singapore,” 2017). GCE ‘N’ Level is for students completing Secondary 4 (equivalent to Grade 10) in Normal (Academic) and Normal (Technical). According to a students’ performance, he or she can then continue their education in either a technical or a vocational institute, or in a Secondary 5 Normal (Academic). GCE ‘O’ Level is for students completing Secondary 4 in Express or Secondary 5 in Normal (Academic). Qualified students can then enter either a Junior college (lasting 2 years) or a Millennia Institute (lasting 3 years) before taking the GCE ‘A’ Level, or students can enter a Polytechnics institute if their results are acceptable. The GCE ‘A’ Level is for students completing their post secondary education who want to pursue tertiary education including universities and colleges (“SEAB”, 2017).

In early school years, students are assigned to various streams based on high-stakes examination results to identify and help train top students (“Education in Singapore,” 2017). Teachers are accountable for students’ academic development as part of their annual performance appraisal for extra bonuses (Center for Global Education, 2017). Therefore, assessments are regarded as quite important throughout the school journey. In 2014, enrollment for Secondary 4 and Secondary 5 was 45,183 and 6,915 students respectively (Ministry of Education, Singapore, 2015); whereas the size of the incoming class at the university level was 17,870 (Ministry of Education, Singapore, 2015). Besides attending the six publicly funded universities, students have the choice to attend five Polytechnics, Institute of Technical Education and ARTS Institutions (Ministry of Education, Singapore, 2015). Since the education environment is relatively competitive (as compared to Ontario) from the earliest grades, teachers take care to make sure to cover the required curriculum thoroughly, and to help train students to perform well in these critical examinations (Hogan, 2014). Students usually practice many past examination papers and "assessment books" to prepare for future examinations "inside and outside the classrooms" ("Ten year series", 2017, para.2).

Assessment in Shanghai

The Chinese education system is based on "universal education and the use of merit as a criterion to determine students' access to schools" (Lee, 2000, p. 12); the system is comprehensive and examination-oriented. It is compulsory for all Shanghai students to take the National Higher Education Entrance Examination, also known as Gaokao, in their final year of senior high school if they wish to pursue further study. Gaokao is a nine hour high-stakes examination taken over the course of two days conducted by the provincial government for students who want to pursue tertiary education ("The Gaokao", 2012). Students' Gaokao scores determine whether they can continue their education in any of the approximately 2,000 universities and colleges in China. Although the number of students taking Gaokao can vary, in 2014 there were over nine million students taking Gaokao even though there were spots for just six million students in colleges and universities nationwide that year ("Higher education in China,", 2017; Chibber, 2014).

In Shanghai, there are approximately thirty national and municipal universities and colleges. It was estimated that there were 51,000 students in Shanghai attending Gaokao in 2016 (Gaokao, 2016). The competition is intense, especially for candidates who want to enter the top universities, such as Peking University and Tsinghua University. Students have to begin preparing years ahead taking practice exams and doing a considerable number of exercises. A Gaokao candidate Yang Wei made a bitter joke, “If you connected all of the practice tests I’ve taken over the past three years, they would wrap all the way around the world” (Larmer, 2014, para. 3).

The hyper-competitive environment in Shanghai has intensified the importance of assessment in propelling the “examination-driven education” that has been prevalent since the 7th Century for civil service examinations (Lingard, Martino, Rezai-Rashti, & Sellar, 2016, p. 47). Though there is only one national examination, Chinese students and teachers have been striving to do well in this particular system.

Overview of Findings

This chapter has been considering the question of how well students in Ontario are prepared to compete on mathematical assessments in contrast to students from the other regions. The comparison of this overview of assessments in the three regions is summarized in Table 6.

Table 6.
Comparison of Assessments Overview

	Ontario - Canada	**Singapore**	**Shanghai - China**
Assessments	EQAO provincial tests for third, sixth and ninth grades, and the national Pan-Canadian Assessment Program for eighth grade	PSLE (completed Primary 6), GCE ‘N’ (completed Secondary 4), GCE ‘O’ (completed Secondary 5) and GCE ‘A’ (completed post secondary education	National Higher Education Entrance Examination (at last year of senior high school)
Amount of Preparation for National Assessments	Low	High	High
Entrance Assessments for Universities / Colleges	No	Yes	Yes
Importance of National / Provincial Assessments	Not Important	Important	Important

In general, the impact of national assessments appears to be quite significant in many countries in Asia, such as Japan, Korea, Singapore, Vietnam and China (Leung, 2006). Curricula in China and Singapore are examination-driven for the most part. However, Shing-Tung Yau, the Fields Medalist of 1982, commented that the Chinese test-oriented math education system had produced less than twenty outstanding mathematicians as compared to hundreds in the U.S. over the same time period; the system did not seem as focused on nurturing the highest levels of talent (Zhao, 2016). Teachers allocate a considerable amount of time to cover examination materials to ensure that their students are able to do well in the national examination. One possible concern is that the education systems in these cultures place too much emphasis on rote memorization in order for students to be able to succeed on these assessments (Lingard et al., 2016). After this emphasis on short-term memorization, it is not clear whether or not students are able to retain their mathematics knowledge as well.

Some recent research studies have indicated that East Asian students are good at basic skills of computation, algebraic manipulation, algorithm application and routine problem solving; while other research has shown that students from European and American countries performed better in visual and graphical representation and open-ended problem solving (Leung, 2006). The studies indicated that in general students educated in China are good at memorization and taking tests, but they are weaker in terms of creativity and critical thinking. As stated in Chapter IV, referring to Figure 7 on p. 53, students in Shanghai were comparatively weak at problem solving when compared to the results for students from Singapore and Ontario in the PISA 2012. Students from Shanghai were good at solving static and analytical problems that they had seen or learned in textbook. However, they were not as good at solving problems when the use of more information was required to make sense of the problem situation (OECD, 2014a). In contrast, students in Canada and Singapore exhibited better problem solving skills involving processing abstract information.

Across the world, different cultures exhibit markedly different attitudes towards assessments. People in Shanghai typically regard assessments (examinations and tests) as the most important focus of education; this relates to the idea that better performance on assessments will result in a more successful future in the culture. However, people in Ontario often show a different viewpoint, one that places more emphasis on the quality of learning (Leung, 2006). As for Singapore, their education system embraces both perspectives though covering knowledge for examinations is still a dominant element. With the stress on performance on assessments, students from Shanghai and Singapore have performed better in PISA and TIMSS, whereas students from Ontario, which places more emphasis on creativity in education, have done better on the problem solving sections of these assessments.

Chapter VI

Classroom Activities

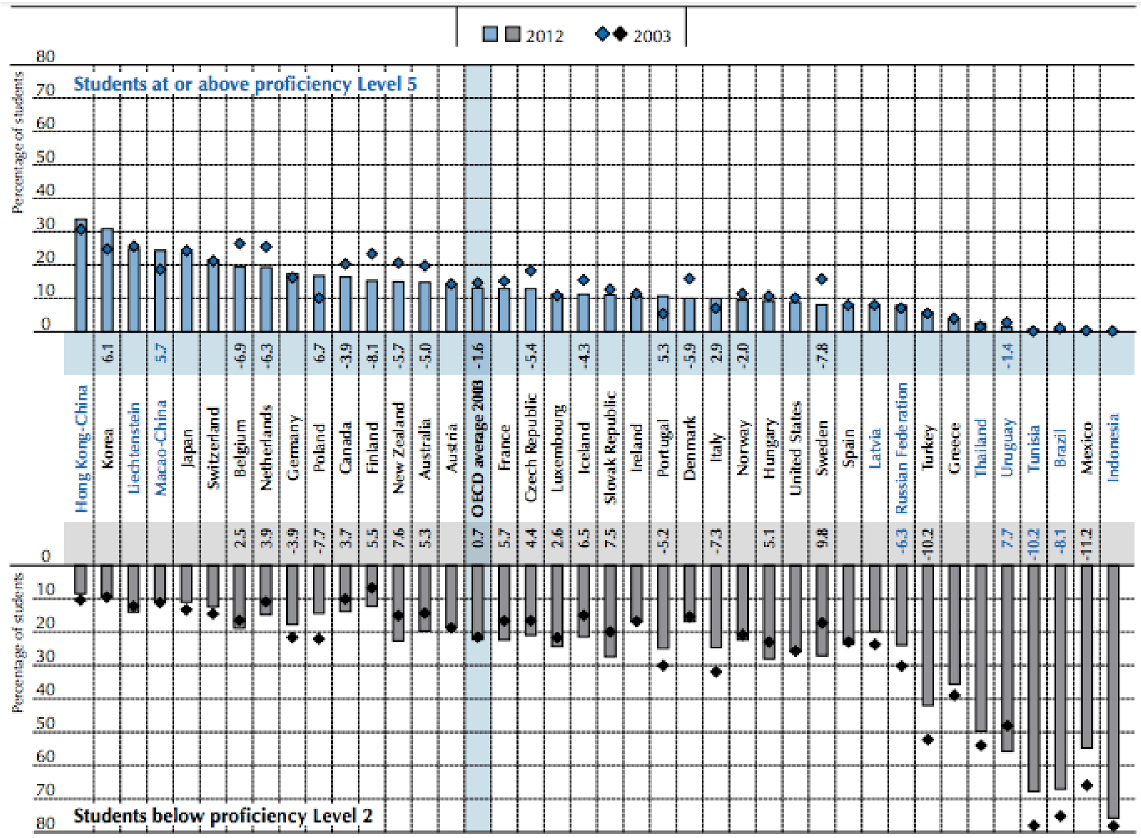
Due to different curricula and assessment approaches in Ontario, Singapore and Shanghai, classroom activities in these three regions vary somewhat so as to best meet their particular educational philosophies and needs. As part of this study of the differences in mathematical performance of these three different regions, the focus in this chapter is to identify attributes that might lead to improved mathematics performance by comparing different classroom activities including a focus on pedagogical approach, teaching method, student participation, class size and instructional time.

Classroom Activities in Ontario

After several educational reforms over the past few decades, the main instructional approach in Ontario schools has changed from "rote learning" to "child-centered learning" (NCEE, 2015a, para. 6). The pedagogy is focused on heuristics allowing students to become more active and open-minded during math class. They are permitted to have more freedom to explore mathematics concepts independently. Hence, the teaching style emphasizes individual work and the class itself tends to be more student-led.

It is worth considering whether class size might be an attribute affecting mathematics performance. Research done by Dee and West (2011) showed that smaller class size resulted in better non-cognitive skills, such as student engagement, for eighth grade students; whereas another research study done by Ehrenberg, Brewer, Gamoran & Willms, (2001) indicated that smaller classes was only one of the options that could improve learning for students. According to an OECD 2013 survey, the average Canadian middle school class size was 25.8 students, slightly more than the OECD average of 24.1 students (as cited in "Class size," 2016).

The London School of Economics has published a study on Israeli students (10 to 13-year-olds) stating that "instructional time has a positive and significant effect on achievement" (Mullis, 2012, p. 341). Thus, one can argue that sufficient instruction time is required for students to develop their mathematical skills. According to OECD (2013b), the PISA 2012 report indicated that Ontario students spent 325.4 minutes per week learning mathematics in class, which was approximately 50% more than the OECD average of 217.8 minutes per week, and slightly more than the overall Canadian average of 313.8 minutes per week (OECD, 2013b). In contrast, Canadian students only spent 222.8 minutes per week learning mathematics in class in 2003 (OECD, 2013b), substantially less than they did in 2012. However, over this time period there was a decrease of 3.9% of Canadian students achieving proficiency Level 5 or above, and an increase of 3.7% students performing below proficiency Level 2 in 2012 as compared to the results from 2003. It seems surprising that an increase of 91 minutes instructional time per week over the nine years resulted in declining performance. Figure 10 shows the percentage of low-performing students and top performers in mathematics in PISA 2003 and PISA 2012.



Notes: The chart shows only countries/economies that participated in both PISA 2003 and PISA 2012 assessments.
The change between PISA 2003 and PISA 2012 in the share of students performing below Level 2 in mathematics is shown below the country/economy name. The change between PISA 2003 and PISA 2012 in the share of students performing at or above Level 5 in mathematics is shown above the country/economy name. Only statistically significant changes are shown (see Annex A3).
OECD average 2003 compares only OECD countries with comparable mathematics scores since 2003.
Countries and economies are ranked in descending order of the percentage of students at or above proficiency Level 5 in mathematics in 2012.
Source: OECD, PISA 2012 Database, Table I.2.1b.

Figure 10. Percentage of low-performing students and top performers in mathematics in PISA 2003 and PISA 2012. Reprinted from *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and science (Volume I)*; [p. 70], by OECD, 2014, Copyright 2014 by OECD.

Classroom Activities in Singapore

After the People's Action Party came into power in 1959, the government placed great emphasis on educating the population. Pedagogy embraced both eastern and western traditions (Hogan, 2014), which promoted "academic excellence, creative thinking and collaborative learning" (Larkin, 2012, para. 17). The Singapore education system is considered as effective, for students are well prepared for national and international assessments in a teacher-oriented setting. Originally it was believed that "teaching is talking and learning is listening", "assessment is summative", and "knowledge is factual and procedural" (Hogan, 2014, para. 16). However, classroom activities in Singapore have been changed accordingly to better prepare students to the dynamic context of working in the global economy. The pedagogical framework advocated "Teach Less, Learn More" (Hogan, 2014, para. 21) in 2004, which stressed the quality of teaching over the quantity of learning. Nevertheless, the teaching style still incorporated whole class instruction as the norm to ensure "coverage of the curriculum, knowledge transmission, and teaching to the test" (Hogan, 2014, para. 23), then followed with individual work after whole class instruction to address the quality of learning. It was found that "traditional and direct instructional techniques are much better at predicting student achievement than high leverage instructional practices" (Hogan, 2014, para. 24). The major classroom activities include taking notes, organizing class materials, drilling mathematical skills, reviewing assessment feedback, solving and discussing new problems, and working on project presentations (Ministry of Education, Singapore, 2012). The top priority is to develop excellent conceptual understanding and to employ technology and hands-on activities to demonstrate abstract concepts whenever possible (Ministry of Education, Singapore, 2012). Thus the Singapore pedagogy combines "activity-based learning, teacher-directed inquiry and direct instruction" (Ministry of Education, Singapore, 2012, p. 23-24).

According to an OECD 2013 survey, the average middle school class size in Singapore is 35.5 students, which was about ten more students per class than Ontario schools on average ("Class size," 2016). The PISA 2012 report indicated that Singapore students spent 287.8 minutes per week learning mathematics in school, which was 37.6 minutes per week less than the amount in Ontario classrooms (OECD, 2013b). This then leads one to question the impact of classroom size and length of mathematics instruction on performance on mathematical assessments.

Classroom Activities in Shanghai

Shanghai teachers are highly respected, and they tend to be more authoritative in school where students are typically quite well disciplined. As a result of this discipline, Shanghai teachers can employ whole-class teaching more effectively than if they had to spend more time monitoring discipline issues. The typical Chinese education system belief is that routine practice is the most efficient way to learn mathematics (Wei, 2014). Traditionally, mathematics education is focused on conceptual and procedural development through constant practice. In PISA 2012, "Shanghai – China has the highest scores in mathematics, with a mean score of 613 points – 119 points, or the equivalent of nearly three years of schooling, above the OECD average" (OECD, 2014a, p. 17). It has been estimated that the mathematics skills of students in Shanghai are up to three years ahead of students in the UK (Wollaston, 2015).

Shanghai mathematics classrooms are characterized by teaching with small steps and variations, and topics are taught from basic to advanced levels through systematic exercises (Li & Huang, 2013). Research done by Professor David Reynolds of Southampton University has found that Shanghai teachers spent 72% of class time on whole-class teaching (Gibb, 2015). They break down difficult topics into small steps, coupling this with exercises that vary in difficulty, giving students practice until they understand the concepts and can explain these in front of the class. Recently, the teaching method has been expanded to encourage students to have more than one answer to every question (NCEE, 2017). Yet whole-class instruction is still the dominant way to teach in the examination-oriented system (Wei, 2014). “No pupil understanding is left to chance or accident: every step of a lesson is deliberate, purposeful and precise” (Gibb, 2015, para. 12).

In order to cover the required mathematics skills and processes with limited resources, most of the time students are listening in class. Teachers also use “logical reasoning” to influence and develop their thinking skills; however, Chinese teachers are typically not as good at integrating concepts across the curriculum (Wei, 2014). In addition, there is just a small number of teachers who encourage students to actively participate.

Due to the nature of the classroom atmosphere, the normal class size in China can be as large as 50 students (OECD, 2011). According to the PISA 2012 report, Shanghai students spent 269.5 minutes per week learning mathematics in class, which is the least time spent among the three regions being studied (OECD, 2013b). This leads one to wonder how it is that Shanghai students could spend the least amount of time studying mathematics in very large classrooms and yet still perform so well on international assessments.

Analysis of Findings

A comparison of classroom activities is illustrated in Table 7 after studying the pedagogical focus, teaching method, student participation, class size and instructional time of each region.

Table 7.
Comparison of Predominant Classroom Activities in Mathematics

	Ontario - Canada	**Singapore**	**Shanghai - China**
Pedagogical Focus - Direct instruction vs. Student led	Student led	Both	Direct instruction
Teaching Method - Whole-class teaching vs. individual work	Individual work	Both	Whole-class teaching
Student Participation - Active vs. passive	Active	Both	Passive
Class size (number of students in average as of 2012)	25.8	35.5	50
Instructional time per week (in minutes as of 2012)	325.4	287.8	269.5

In Shanghai the teaching emphasis is on whole-class instruction dominated by teachers; students tend to be more passive as teachers mostly talk. Nevertheless, Shanghai students performed significantly better than Ontario students whose teachers mainly incorporated innovative learning focusing on individual work and students' participation. As far as the Singapore education system which has been an inspiration for thoughtful pedagogy, teachers employed both methods to facilitate direct instruction traditional and student led pedagogy new mathematics teaching. However, inquiry-based learning was limited in class to give time for direct instruction of factual and procedural skills to ensure that the full curriculum was covered for examinations (Hogan, 2014). This then seems to imply that a whole-class teaching method is more effective for an examination-oriented context.

However, the findings have indicated that a bigger class size (number of students on average) does not necessarily negatively impact math performance. Ontario math class size is only half of that in Shanghai but the small class size itself does not guarantee better results on assessments. Still a smaller class might make it possible to enhance students' creativity and critical thinking skills, which were not particularly assessed in PISA and TIMSS. In addition, it has been found that class size is more influential for younger students than for 15-year-olds from various sources (OCED, 2013b).

In terms of length of class time, a study done by University of Chicago economists found that students performed slightly better when they spent more time in class (McMahon, 2014). In contrast, though, Ontario students spent 20% more instructional time than Shanghai students and yet performed 16% worse in the PISA 2012. Singapore students spent slightly more instructional time than the Shanghai students but also performed 40 points worse. This raises an important issue of what is an appropriate amount of instructional time that still needs to be investigated. The TIMSS 2011 report has stated "the relationship between instructional time and student achievement is highly dependent on the effectiveness of the educational system. If an education system essentially is ineffective, increasing the amount of instruction time will have diminishing returns" (Mullis et al., 2012, p. 341). As a result, this leads one to ask about the overall effectiveness of the Ontario educational system.

On top of class instructional time, attendance in after-school math lessons might be another factor affecting assessment results. The PISA 2012 report indicated that 70.7% of Shanghai and 67.5% of Singapore students participated in after-school math lessons (OECD, 2013b). However, only 27.9% of Ontario students took after-school math lessons (OECD, 2013b). More than 55% of Shanghai and 40% of Singapore students were top performers who are able to achieve Level 5 or 6 in PISA 2012 (OECD, 2014a). In contrast, only 15% of students from Ontario were top performers on the same test

(OECD, 2014a). Perhaps one important consideration, then, is attendance in after-school math classes for achieving higher mathematics performance.

In summary, employing a whole-class instructional approach to cover required mathematics skills and procedures can be beneficial for achieving high performance in international math assessments. This approach can potentially help a greater number of students acquire the necessary knowledge in order to be successful at a top level on such tests. The small-step teaching of the Shanghai linear approach can indeed fortify the mathematics basic skills which facilitate further advanced studies as well. In addition, after-school math lessons might enhance students' mathematics performance. At the same time, focusing on smaller class sizes and emphasizing active and individual work, along with prolonged instructional class time does not in itself promise better assessment results. One final caveat about these findings is that the international tests being considered do not properly assess students' mathematics creativity.

Chapter VII

Students' Attitudes and Motivation About Learning Math

The psychological profile of students consisting of their attitudes and motivations about learning mathematics can have a significant impact on their academic performance. The International Association for the Evaluation of Educational Achievement (IEA) has reported that "each successive TIMSS assessment has shown a strong positive relationship within countries between student attitudes towards mathematics and their mathematic achievement" (Mullis, 2012, p. 326). Students' motivation greatly depends on the value they place on learning math, along with beliefs about their ability (Mullis, 2012). These can be broken into an intrinsic value which reflects how much students like to learn according to their interests and enjoyment level and an external utility value that measures students' ideas about the importance and usefulness of mathematics, such as getting high grades, having a good career, making more money, and so forth (Mullis, 2012). Along with these, ability beliefs indicate how confident students are in their capability for learning mathematics (Mullis, 2012).

Investigating students' intrinsic and extrinsic motivation, ability beliefs, attitude towards school, and level of happiness at school in the three regions is useful to do to help identify whether any of these elements might have an influence upon the results of their mathematics assessments.

Students' Attitudes and Motivations in Ontario

The TIMSS 2011 report stated that there is a strong relationship between students' attitudes towards mathematics and their performance; simply put, students who like learning mathematics tend to perform better on mathematics assessment. See Figure 11 below. It is not clear, however, whether this is a causal relationship, or simply a reflection of a negative association that students might have with math due to their getting lower test results.

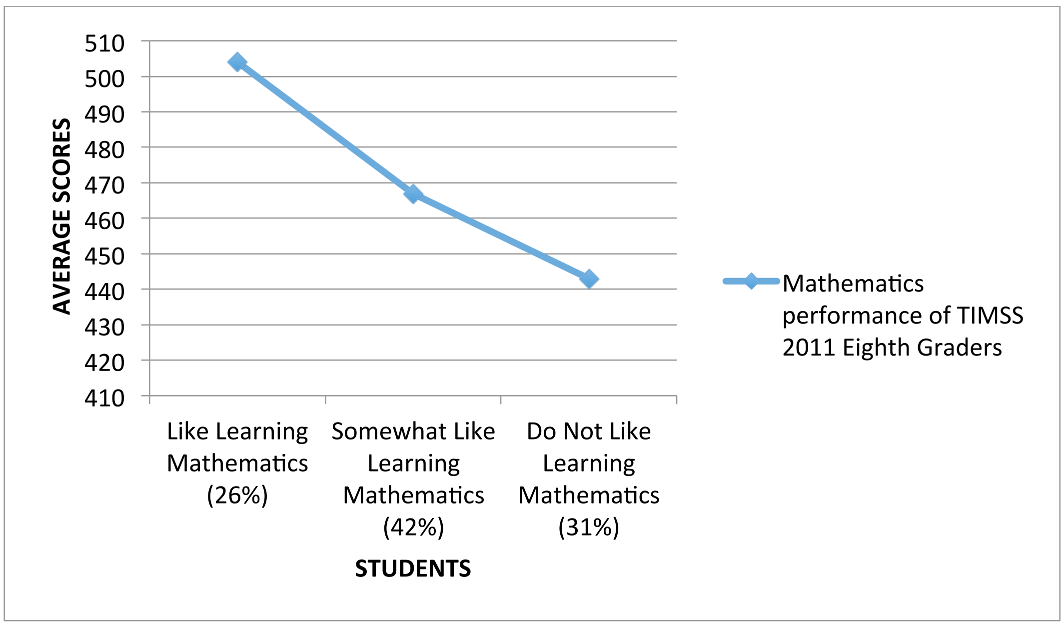


Figure 11. Relationship between attitude of eighth grade students towards mathematics and international achievement in TIMSS 2011.
SOURCE: *TIMSS 2011 International Results in Mathematics*; [p. 19], by Mullis, Martin, Foy & Arora, 2012, Copyright 2012 by International Association for the Evaluation of Educational Achievement (IEA).

The TIMSS 2011 report has also shown that 67% of eighth grade students in Ontario report liking or somewhat liking learning mathematics. Whereas, it showed that 76% of Singapore eighth grade students reported liking or somewhat liking learning mathematics as seen in Appendix B (Mullis et al., 2012). Since Shanghai did not participate in TIMSS 2011, the data of PISA 2012 will be used for more comprehensive analysis later in this chapter.

The PISA 2012 report (OECD, 2013a) has indicated that 53.9% of 15-year old Canadian students were interested in the things they learned in mathematics, and 85.7% of Canadian students agreed that learning mathematics would improve their career prospects and chances with respect to utility value. As for ability beliefs, 94.2% of 15-year old Canadian students believed that if they put in enough effort, then they could succeed in mathematics. In the matter of students' attitudes towards school, 95% of Canadian students agreed trying hard at school is important. Also, the report reflected that 80.9% of students agreed with the statement that "I feel happy at school" (OECD, 2013a, p. 251).

Students' Attitudes and Motivations in Singapore

It is well known that parental involvement plays a vital role in students' school success. Motivations of Singapore students to succeed are predominantly driven by their family; their desire to study hard is instilled by parents at a very young age (Larkin, 2012). Students with supportive home backgrounds are more likely to have "positive attitudes towards learning" (Mullis et al., 2012, p. 206).

In 2012, 77.1% of 15-year old Singapore students were interested in the things they were learning in mathematics as reported by OECD (2013a); 88.2% of Singapore students agreed that learning mathematics would improve their career prospects and chances. Furthermore, 98.1% of Singapore students believe that if they put in enough effort, they can succeed in mathematics.

Concerning students' attitudes towards school, 97.1% of Singapore students agreed that trying hard at school is important; also, 87.9% of the students agreed with the statement that "I feel happy at school" (OECD, 2013a, p. 251).

Students' Attitudes and Motivations in Shanghai

Shanghai is a particularly competitive region in many aspects due to its leading position in fast growing China. As mentioned before, Shanghai's students are passionate about mathematics as they are aware of the need to do well in their highly competitive society ("How Shanghai's Students," 2011). Motivation is an imperatively critical factor pushing students to achieve their full potential. Dr. Jerrim at the University of London stated that "the attitudes and beliefs East Asian parents instill in their children make an important contribution to their high levels of academic achievement" (as quoted in Weale, 2014, para. 10). Parents who have high expectations and who are supportive in the learning process can provide a more nurturing environment and can help to motivate their children to do well in academics (OECD, 2014a).

In 2012, 60.6% of 15-year old Shanghai students were interested in the things that they were learning in mathematics, and 72.7% of Shanghai students agreed that learning mathematics improves their career prospects (OECD, 2013a). Along with this, 92.1% of Chinese students believed that if they put in enough effort, they could succeed in mathematics.

As teachers are highly respected in Shanghai, students are typically quite attentive in class following what teachers say; sometimes they will ask questions of their teachers, but they will seldom challenge them (Wang, 2011). Less student independence leads to the possibility of more efficient whole-class teaching. It was reported in 2012 that 94.9%

of Shanghai students agreed that trying hard at school was important, and 80.9% of Shanghai students reported being happy in school (OECD, 2013a).

Summary of Findings

Students' attitudes and motivations appear to be closely related to their mathematics performance. Data related to these studies are summarized in Table 8 and Figure 12.

Table 8.
Summary of Students' Attitudes and Motivations (in Percentage)

	Ontario - Canada	**Singapore**	**Shanghai - China**
Intrinsic Value - Students interested in the things they learn in mathematics	53.9	77.1	60.6
Utility Value - Students agree that learning mathematics will improve their career prospects and chances	85.7	88.2	72.7
Ability Beliefs Index - Students believe they can succeed if they put in enough effort	94.2	98.1	92.1
Attitude towards school - Students agree trying hard at school is important	95	97.1	94.9
Student's Happiness Index Students feel happy at school	80.9	87.9	84.6

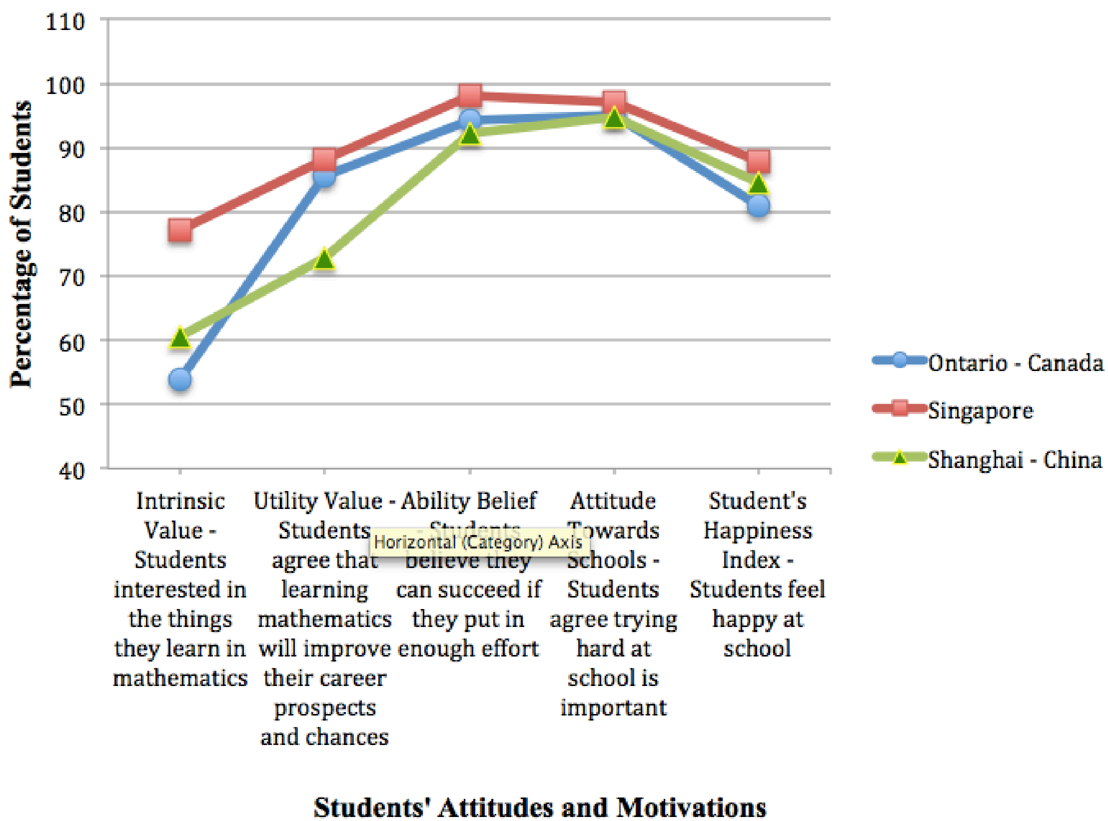


Figure 12. Summary of 2012 students' attitudes and motivations (in percentage). Data for intrinsic value, utility value, ability belief, attitude towards schools and student's happiness index from PISA 2012 Results: Ready to Learn: Students' Engagement, Drive and Self-Beliefs (Volume III); [p. 282], by OECD, 2013a. Copyright 2013 by OECD.

Table 8 and Figure 12 have shown that students in Singapore exhibited the greatest amount of positive attitudes and motivations towards learning mathematics, although Singapore was second to Shanghai in mathematics performance of the PISA 2012.

In terms of intrinsic value, this was the quality that was ranked lowest among all three regions. In particular, only 53.9% of Ontario students found mathematics interesting to learn, as compared to 60.6% and 77.1% of Shanghai and Singapore respectively. Though it was found that attitudes toward mathematics and science had correlations with mathematics and science achievement across 288 studies (Hattie, 2009), even with their high mathematics achievement, Shanghai students do not have the most positive attitudes toward mathematics in this study.

Besides, only 72.7% of Shanghai students agreed that learning mathematics would improve their career prospects and chances. Shanghai had the lowest percentage in this type of utility value as compared to Singapore with 88.2% and Ontario with 85.7% respectively. This raises questions regarding the underlying motivations causing Chinese students to strive hard in mathematics.

In addition, the results show that students were generally happy in the study; more than 80% of students felt happy at school. Though Canadian students have lower scores and fewer report feeling happy in school than students in Shanghai and Singapore, there is no strong relationship between the top test scorers and the happiest students overall. Singapore is comparatively high across all elements; whereas Shanghai is higher in mathematics performance but fewer of its students report being happy on average than those in Singapore. Regardless, the happiness index of students in all three regions was high in general.

Overall, students from all three regions placed high value on ability beliefs and attitude towards school. More than 90 % of students believed that they could succeed if they put in enough effort and they strongly agreed that trying hard at school is important. This lays a solid grounding for Ontario students to achieve if other essential factors are incorporated as well.

Chapter VIII

Conclusion

This study was driven by concerns about the declining performance of students from Ontario on international mathematics tests along with concerns about the state of provincial math standards. The goal of this research has been to determine what changes could be made to potentially help improve Ontario high school students' performance on international and provincial mathematics assessments. After comparing each of the various attributes in the previous six chapters, it has been shown that variations in sociocultural context, teacher qualifications and compensations, school curricula, preparation for assessments, classroom activities, along with students' attitudes and motivations are all contributing to high school mathematics performance to various degrees; this final chapter will be used to address a number of concluding elements.

First, we will begin by identifying the key differences in each of the areas that were investigated. By analyzing each of these differences, we can also identify possible changes that could be implemented in Ontario so as to improve the declining high school mathematics performance.

After doing this, we will be in position to make suggestions in the domains of curriculum, classroom activities, learning material, teacher qualification and training, assessment, homework, and supportive resources in the following section of the chapter.

At the same time that the work for this thesis was being done, the Ontario Ministry of Education began to make suggestions for change as well, unveiling the

Ontario Renewed Math Strategy in April 2016 supported with CAD $60 million in funding. The hope was that this program would help to improve students' performance (Ontario Ministry of Education, 2016). Therefore, it is worth reviewing the Ontario Renewed Math Strategy and then comparing our suggestions with it. After the comparison, we will discuss any outstanding elements that might also be helpful for improving mathematics performance in Ontario.

In the latter part of this chapter, we will address limitations of this study and present a summary of the conclusions for the entire thesis.

Summary of Key Differences

In the following sections, we will summarize the major differences that were found along each of the attributes that were studied in the previous chapters in the regions of Ontario, Singapore and Shanghai.

Distinctions in Sociocultural Context

It was considered that the high performance in mathematics assessments in regions in certain Asian countries might be related to a cultural heritage in which parents place a greater general emphasis on high academic expectations. Also, research showed that American parents chose "ability" over "effort" in greater numbers to describe math success, a belief that might also hinder their children from striving for mathematics excellence. In addition, densely populated regions such as Singapore and Shanghai might generate a more competitive environment for students to work harder, especially when these regions have less social welfare expenditure to protect their future.

Dissimilarity in Teacher Qualification and Compensation

Ideally, mathematics teachers should possess a solid mathematics background and have good pedagogical knowledge as well. However, Ontario was comparatively less stringent in these requirements prior to 2014, which could relate to students' weaker mathematics performance in that region. Despite teachers in Shanghai having the least average salary among the three regions, they have been compensated by a higher social status; thus, it is possible that this form of emotional incentive might attract better teachers in this instance.

Contrast in Curriculum

Ontario placed a higher emphasis on innovative, spiral and inductive curriculum, along with a relatively lower emphasis on basic skills in mathematics. This is in comparison to the structured, linear and deductive curriculum of Shanghai. Meanwhile, the carefully crafted spiral curriculum of Singapore, as demonstrated with what is commonly referred to as "Singapore Math", is more comprehensive than that of Ontario and Shanghai in comparison.

Variations in Assessments

In Singapore and Shanghai, results of assessments are regarded as an important index in both their education systems and in their broader culture as well; thus, their curricula are predominantly examination-driven. Moreover, official university and college entrance examinations are required in both regions. In contrast, Ontario students are not typically required to make any special effort to prepare for provincial, national and international assessments as compared to the preparation of students in Shanghai and

Singapore, as Canadian universities and colleges accept Ontario students mainly based on their school reports. In Ontario, the provincial and national assessments have no significant effect on students' school performance and further study. Teachers and students in Singapore and Shanghai are spending much more time on average preparing for various assessments beginning in their early years of school as compared to those in Ontario.

Differences in Classroom Activities

The whole-class teaching model of direct instruction by teachers is commonly practiced in classrooms in Shanghai. In contrast, classrooms in Ontario are usually student led with a focus on active individual work. Classrooms in Singapore manage to exhibit both modes. The average class size in Ontario is about half of that of Shanghai, and students in Ontario spent about 20% more time in mathematics classes than students in Shanghai.

Dissimilarity in Students' Attitudes and Motivation

Ontario students were very close to the high performing regions in terms of utility value, ability beliefs index, attitude towards school and student's happiness index. However, measurements of intrinsic value associated with mathematics are particularly low for students in Ontario; only 53.9% of students in Ontario were interested in the things that they learned in mathematics as compared to 77.1% of Singapore.

Potential for Change

After having specified these key differences among the three regions, we next consider an analysis of these differences to identify as many possible levers for affecting change as possible. First, we consider which elements present opportunities for change, and then follow that with identifying which elements will be substantially difficult to alter.

Possible Changes in Ontario

In the present day, communication between countries can be much closer due to the aide of technology; modern economic competition is essentially globalized at this point. The Canadian young generation needs to be well prepared to meet different challenges worldwide. A quote by the motivational speaker Denis Waitley (n.d.) expresses that, "Success is almost totally dependent upon drive and persistence. The extra energy required to make another effort or try another approach is the secret of winning." Seeking changes in requirements for teacher qualification, along with modifications in curriculum, assessment, and classroom activities combined with shifts in students' attitudes toward mathematics might help to improve the status of Ontario's mathematics performance.

In Canada, whether or not to be among the best in the international assessments does not appear to be a main focus at this moment as it is not noticeable that students are being encouraged to spend the majority of their time doing schoolwork. As an example of this, in the past decade various school boards in Ontario have made proposals for "homework-free holidays", "family-friendly homework policies" (CBC News, 2008), and so forth to limit time spent on homework. It is not easy to change popular beliefs but students should be aware that being successful in terms of mathematical achievement requires effort in addition to ability. If Canadian students do not spend as much time and put in as much effort as students in Shanghai and Singapore, their chance to match the assessment outcomes of these top performers will be diminished.

Teachers should be well trained and possess the necessary mathematical knowledge and pedagogical training to teach effective lessons. Stringent training for teacher candidates and teachers is needed. Though Ontario budgeted to spend four million dollars in 2014 on teacher training, the effectiveness of its new training plan is still not known. Regardless, as has been argued throughout this thesis, to achieve high mathematics performance does not depend on just one element but a combination of interrelated attributes.

Despite Liz Sandals', Ontario's Education Minister, belief that the Ontario curriculum was not the main cause of the declining mathematics performance (Rushowy, 2014b), Ontario's innovative curriculum might be an element causing students confusion if it is the case that teachers, parents and textbooks are not all fully supporting an effective learning process. Though there have always been debates about the emphasis placed on traditional versus discovery mathematics approaches, they are in fact deeply interrelated. For instance, students can learn about multiplication by counting arrays, and realize that basic multiplication simply relates to the concept of multiple additions. Once students learn this concept, they should then memorize the multiplication table to facilitate further advanced mathematics. Watkins & Biggs have suggested that "the strategy of memorization used by Chinese students is not simply for rote learning but an important method to achieve a deep understanding in which subject content is internalized and actively reflected upon" (as cited in Grant et al., 2014, p. 6).

Using discovery or other creative approaches should focus on finding alternatives to achieve a particular mathematics task. These alternatives might or might not be better than following traditional methods of calculation. Nevertheless, the crucial part of discovery learning is the process. Going through the thinking process, and becoming engaged with the mathematics concepts, students recall knowledge from memory to seek new methods to solve math problems. If students do not memorize basic skills, such as multiplication tables, then they will be more likely to encounter time-consuming processes which can hinder their mathematical journeys of discovery later on. As John Mighton, the founder of the JUMP Math program, said, "Kids need to know basic number facts, so they can work conceptually" (as quoted in Bennett, 2013). Each curriculum approach has its advantages and disadvantages; the Ontario curriculum can be adjusted to develop discovery skills at the same time as strengthening basic mathematics skills that might have been missed during discovery processes done at younger ages.

Even though requiring assessments can add stress to students, parents, teachers and schools, it is indispensable in modern society because it can assist in helping to identify qualified candidates for different needs. Students are more likely to review their mathematics knowledge if a given assessment has an impact on their future study. Hence, it might be the case that a provincial university and college entrance examination would lead to more students' paying attention to preparation for assessments. However, such a requirement might be difficult to introduce given possible reactions in Ontario due to the additional stress. It is possible that the result of using a provincial entrance examination might partially contribute to school results as a whole in that it could create a need for Ontario students to review mathematics in a more regular and sustainable manner.

Considering the nature of classroom activities, classrooms in Shanghai can have up to 50 students in whole-class instruction, whereas classrooms in Ontario have only about 26 students on average for student led classes. Since students in Shanghai are still outperforming other countries in mathematics assessments, it might be worth questioning whether Ontario should consider re-allocating educational resources. Certain mathematics class could be taught with whole-class instruction that would accommodate more students; performance would not necessarily decline as long as the mathematics content is delivered appropriately.

If mathematics courses could be made more interesting, then it seems likely that students would be able to achieve better results as long as other essential factors are also being incorporated. Making more effort or trying another approach are important ingredients to succeed; it seems evident that Ontario needs to make alterations in some of these educational elements to improve mathematics performance of high school students.

Elements Difficult to Alter in Ontario

Cultural heritage is developed over decades and centuries, so it is difficult for any region to alter its sociocultural context within a short period of time. Elements that work in one culture might not necessarily work in another culture, and so it is the case when we consider sociocultural comparisons between Ontario and other regions.

Population density is another particular element that is not going to change in the short term. Also, decreasing the amount that a region spends on social welfare will likely cause public resentment, as it is part of government policy to protect the minorities, and so this is another element that is unlikely to change in Ontario.

At the same time, it is hard to change the framework of teacher unions in Ontario; the influence of such unions is not likely to diminish in the near future. Hence, to raise the social status of Ontario teachers to that of Shanghai teachers is also unlikely to happen within a short period of time.

Suggestions for Improving Mathematics Performance

Every country has limited resources to spend per student. Finding ways to distribute resources efficiently and effectively to achieve the greatest amount possible is crucial for leading to success of any kind. In the previous sections, we have provided an overview of the significant differences among these regions along with possibilities as to what can be done in Ontario by considering elements that are more entrenched than others. Next we put forth our suggestions as to the type of improvements that might make a difference in the performance of Ontario students in mathematics. These include changes in curriculum, instruction and size of class, learning materials, teacher qualification and training, assessment, homework, and supportive resources.

Curriculum

In this fast-paced world, people always want to improve themselves so as to be perceived as pioneers. We are easily influenced by ideas involving discovery, innovation, creativity, critical thinking and so forth. The great emphasis on discovery-based mathematics curricula in Ontario might have contributed to its relatively lower mathematics performance in international assessments. Although this approach emphasizes thinking skills, at the same time it can provide less support for acquiring traditional mathematical calculation skills. The result is that students trained with discovery-based methods might not be able to do mathematics problems as quickly as others. Given this, the Ontario curriculum might need to be reviewed to see if it would be possible to incorporate both traditional and discovery-based mathematics approaches to benefit students for further educational and career success.

In a balanced curriculum, standard arithmetic algorithms and basic mathematics skills should ideally be introduced through a structured and deductive approach at appropriate times to facilitate students in advancing their thinking skills. The role of educators is to provide these mathematical tools to students to help give them opportunities for their own invention.

Instruction and Size of Class

Without basic skills, students might find it difficult to progress in a discovery process, and that might lead to confusion and frustration. This can also lead to disadvantages when it comes to considering more complex problem-solving tasks. Urback (2016), a journalist at *National Post*, explained, “If a student is left counting her fingers and toes to figure out 7 x 8 in Grade 3, there’s no way she’ll be able to multiply fractions in Grade 6” (para. 5). If seventh grade students do not know the distributive law of multiplication, it is likely that they will then have difficulties in learning algebra and calculus later on. It is thus recommended that specific class time should be designated to whole-class instruction to help deliver fundamental mathematics knowledge. In doing this, class sizes can be larger with whole-class instruction as the most important element is the effectiveness of the content being delivered. The more information that students are able to remember and the more that students are able to use their time to think, the more efficient students will be in finding solutions to problems at a higher level.

Flipped classroom. One approach for doing this type of whole-class instruction would be to use videotapes for pre-class preparation or after-class review. This would provide a cost effective and time efficient way to teach as it gives students and parents a chance to re-visit topics being covered as many times as they need. “Most importantly, all aspects of instruction can be rethought to best maximize the scarcest learning resource - time” (Tucker, 2012, para. 2). This so-called “flipped classroom” arrangement can free teachers’ instruction time to focus on solving students’ problems, as well as to focus class time on implementing more interesting mathematics activities. Some teachers in North America have claimed that the “flipped classroom” has a positive effect on attendance and academic achievement; a 10^{th} Grade Ontario math teacher, Donna Green, stated that students were excited as “it was something different than just going to math class and listening to a lecture for an hour” (as quoted in Deschamps, 2015). In a survey done by Bishop & Verleger (2013), it is revealed that “anecdotal evidence suggests that student learning is improved for the flipped compared to traditional classroom.” Nevertheless, whether this approach leads to effective results depends largely on the quality of the lessons being videotaped. Khan Academy is a non-profit organization that has created a large collection of education videos for more than 5000 topics by 2015 (“Khan Academy,” 2017). This existing resource could be used as part of the approach for achieving more effective learning. Moreover, funding for after-school mathematics classes could also reinforce mathematics knowledge and assist weaker students with their mathematics as well.

Learning Materials

To help provide a more comprehensive curriculum for all schools, it would be helpful for Ontario to incorporate uniform textbooks that explicitly include all the necessary topics and activities. Resources could be consolidated to ensure the required knowledge to be delivered effectively instead of having an array of potentially confusing textbooks. It would also be helpful to have all textbooks and class notes posted online for easy access to facilitate the learning process.

Teacher Qualification and Training

The effectiveness of teaching is imperative to student's success. Teachers should be given the necessary training to fully understand how to teach the materials effectively. Ontario might want to improve mathematical and pedagogical standards for teachers in their study of education prior to their beginning teaching. Along with continuous on-the-job developmental training, this could help teachers to be more qualified to teach mathematics in a dynamic context.

The Ontario Association for Mathematics Education (2017) has strived to promote "Excellence in Mathematics Education" by organizing annual conferences and providing information on other conferences, publication and classroom resources. Furthermore, the website EduGAINS.ca has provided teacher resources since 2012 to support Ontario teaching and learning programs. Educators can access resources that provide support for effective instruction and new technology learning tools (EduGAINS, 2017).

Assessment

Though as a country, Canada does not emphasize assessments as much as some other nations, appropriate assessment can be useful to alert educators as to how well all the elements of an educational system are working together. Assessment can also help to identify particular students who might need extra help. To drill students to perform well on examinations is not the primary goal of learning; it is important that the content of any assessment should align with appropriate learning goals so as to serve the purpose of testing. Various assessment samples can be posted on official websites to give students a chance to practice and review for tests, but this should not be made to be compulsory.

Homework

In addition to the previous suggestions, another factor to consider is the role of homework. The use and amount of time dedicated to homework might have important impact on mathematics performance but it has not been included in this study. According to the PISA 2012 report, 15-year-old students in Shanghai and Singapore had spent considerably more time in doing homework than students in Ontario. Students in Shanghai and Singapore reported 13.8 and 9.4 hours spent on homework on all subjects combined per week respectively; whereas students in Ontario reported spending 5.5 hours (OECD, 2013b). If one were to make a connection between homework and test scores, then this could mean that "students saw an increase of 17 score points or more per extra hour of homework" (Kohli, 2014, para. 4). Therefore, it is worth considering that an increase in mathematics assignments outside of class might be helpful to increase students' mathematical abilities, but more would need to be studied on what types of homework produce the best results.

Supportive Resources

Another informative website mathies.ca provides learning tools, games and activities, Gap Closing materials, Homework Help, and parental guides for students and parents in learning mathematics (Mathies, 2017). In particular, Homework Help sponsored by the Ontario government has provided real-time online free one-on-one tutoring to seventh and tenth grade students in public schools, which has been very helpful for students who might have been frustrated with their math work. With the aid of technology, students have been encouraged to be self-learners as the learning journey is boundless for them to explore.

Ontario Renewed Math Strategy (ORMS)

Although it has been the intention of this thesis to put forth a series of suggestions for possible improvements based on a close study of the state of math education in Ontario, Singapore and Shanghai, it is also the case that the Ontario Ministry of Education has been similarly putting forth suggestions for change as well. In this next section we discuss the potential overlap between findings raised in this thesis, and the suggestions that have been put forth by the Ontario Ministry of Education in a recent program titled "Ontario Renewed Math Strategy". This program involves extensive support for educators, students and parents according to their different needs starting in 2016. Key elements of this strategy include the following (Ontario Ministry of Education, 2016):

- Support for teachers and principals to deepen their knowledge and teaching skills by providing dedicated facilitators to help schools with more needs, one

Professional Activity (PA) Day for math educators to attain improvement, and Additional Qualifications (AQ) courses in math.

- For students with learning disabilities, educators are to be trained for special education with more professional support.
- Access for parents to resources helping their children with the math curriculum. Online math resources, such as Homework Help and SOS Devoirs, have been made available to support students outside classroom.
- At the elementary level (first through eighth grades), a 6o-minutes daily math class requirement has been set to ensure more thorough instruction and assessment through resources and evidence-based practices.
- Lead teachers with professional math training will be assigned in each elementary school to provide quality math class and share knowledge with other educators in school.
- For sixth through ninth grades, remedial supports will be provided through tutoring and summer courses.

Mitzie Hunter, the new Ontario Minister of Education, stated, “ We have to have balance and our curriculum focuses on that. It focuses on problem solving. It also focuses on the practice. It focuses on direct instruction and the assessment” (as quoted in Staff, 2016, para. 3).

Comparison of Thesis Suggestions and the Ontario Renewed Math Strategy

Having reviewed the key elements of the Ontario Renewed Math Strategy, we provide a comparison between suggestions made in this thesis and the elements of the ORMS in Table 9 to highlight the outstanding elements.

Table 9.
A Comparison Between Thesis Suggestions and the Renewed Math Strategy

Thesis Suggestions	**Ontario Renewed Math Strategy (Proposed in April 2016)**
Integrated curriculum incorporating both basic and discovery mathematics	No specific change in curriculum to include basic math facts learning; curriculum still focus on problem solving.
Whole class instruction for basic math skills	Effective math instruction is stressed
Videotaped lessons / Flipped classroom	Not mentioned
Uniform textbooks	Not mentioned
Online access for textbooks and class notes	Not mentioned
More training for teachers to teach effectively	Yes
Support teams for regular students, teachers and parents	Yes
Support for students with extra needs	Yes
More homework	No specific indication for more homework but there are online supports for homework assistance
Appropriate Assessment	Yes

The curriculum of the renewed strategy is still focused on problem solving, only with extra class time for more practice for students from first to sixth grade. However, an acknowledgement of the role of basic math skills in a comprehensive math education is missing in the approach. Whether students practice in the right way to achieve math success is uncertain. It has been criticized that “the solution here is to double down and devote more time to a flawed, ineffective, needlessly complex math program” (Urback, 2016, para. 6).

Attention is being paid to effective instruction, teacher training, support for teachers, students and parents, and assessment. Yet suggestions made in this thesis to employ more technology in education, such as videotaped lessons, using flipped classroom approaches, online textbooks, and so forth, are not included in the ORMS.

Limitations of this Study

This comparative study has focused on the PISA and TIMSS results of two regions, Singapore and Shanghai, that have outperformed Ontario. The same approach can apply to other regions as well, such as Finland, Germany, England, as well as other countries, if one wanted to undergo an even more comprehensive study.

One issue of PISA and TIMSS that should be pointed out is that these assess a particular set of mathematical skills using timed tests. This approach, however, does not assess all thinking mathematical skills. Time limits may undervalue Canadian performance if there is a stronger stress placed on thinking skills in the Canadian math curriculum. Using creative thinking processes can sometimes take a longer time to demonstrate than other types of math skills. Thus, the international test results might not reflect all of the mathematical abilities of Ontario's students.

Another limitation is that this study is based more on test data and published results but less on related empirical research comparisons.

Furthermore, the PISA 2015 and TIMSS 2015 results were not been released until the time when this thesis was almost completed at the end of 2016. As a result, this study has been mainly based on the data up to 2012. The newly released OECD (2016a) report has indicated Ontario continued a slight decline by slipping five points to 509 for its score in the PISA 2015 for 15-year-old students. Whereas the TIMSS & PIRLS (2016) result shows that scores for eighth grade students in Ontario improved 10 points in the TIMSS 2015, however the scores for fourth graders declined four points to 512. In any event, Ontario should continue to look to other regions over time for inspiration and to seek effective and efficient ways for improvements in mathematics performance.

Conclusion

In conclusion, one of the main goals for this thesis has been to use comparisons with two other top performers, Singapore and Shanghai, to suggest elements that might lead to improvements in Ontario high school students' mathematics achievement on international assessments. The same comparison framework that was presented in this thesis can also be applied to other regions for further study. To implement each of these suggested elements effectively will undoubtedly be quite challenging. However, if Ontario does not undertake effective changes in mathematics education, its students might continue to slip in international, national and provincial math assessments.

After analyzing all of the attributes studied in this thesis, it has been suggested that there are certain elements that can be changed that could lead to improvements in Ontario's mathematical performance in international assessments. Policy makers first have to identify the best blueprint for educators to teach and students to learn. It is suggested that the math curriculum needs to be balanced between discovery math approaches and an emphasis on basic math skills. Basic math skills can be taught in whole-class instruction; further, videotaped lessons or flipped classroom can be used as well to achieve the purpose of more effective classroom instruction. Teachers can be more thoroughly trained to make sure that they have solid grounding in the mathematics that they are teaching, and in appropriate mathematical pedagogy before and during their teaching career. An appropriate amount of homework can be assigned for students to practice and reinforce their math skills. Resources to help students outside classroom, such as after-school math classes, online tutoring, and extra learning materials, can be provided for assistance. Utilizing technology to provide cost effective resources, including online textbooks, learning materials, and videotaped lessons, is a current educational trend. After all, assessment is inevitably required to articulate potential further adjustments that could be made among the various educational elements.

Though the renewed Ontario math strategy has provided more resources and support for students, parents and educators, a focus on fundamental math skills has not been emphasized. Whether or not the new Ontario math strategy will successfully improve students' previous international performance will not be known for another few years. Yet Dr. A. Bonato at Ryerson University commented, "If you don't have the tools to do math, it will turn into an exercise of frustration. Our students need to learn the fundamentals of mathematics before they can progress to the more advanced topics so prevalent in STEM disciplines" (Bonato, 2016, para. 22). It is the major responsibility of policymakers and educators to prepare Ontario students for academic success, and to equip them with skills to become independent learners for life-long achievement. Ontario should continue to implement improvements by continued study of proven approaches by other regions; it should never be the case that a region gets complacent that it has done enough in mathematics education.

Appendix A Summary description of PISA 2012 mathematics proficiency levels

PISA 2012 mathematics proficiency levels – summary description*

Level	Lower score limit	Percentage of students able to perform tasks at this level or above	Characteristics of tasks
6	669.30	3.3% of students across the OECD and 4.3% in Canada	Students at Level 6 of the PISA mathematics assessment are able to successfully complete the most difficult PISA items. At Level 6, students can: • conceptualize, generalize and use information based on their investigations and modeling of complex problem situations, use their knowledge in relatively non-standard contexts. • link different information sources and representations and move flexibly among them. • demonstrate advanced mathematical thinking and reasoning and apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations. • reflect on their actions, and formulate and precisely communicate their actions and reflections regarding their findings, interpretations and arguments, as well as explain why they were applied to the original situation.
5	606.99	12.6% of students across the OECD and 16.4% in Canada	At Level 5, students can: • develop and work with models for complex situations, identifying constraints and specifying assumptions. • select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. • work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insights pertaining to these situations. • begin to reflect on their work and formulate and communicate their interpretations and reasoning.
4	544.68	30.8% of students across the OECD and 38.8% in Canada	At Level 4, students can: • work effectively with explicit models on complex, concrete situations that may involve constraints or call for making assumptions. • select and integrate different representations, including symbolic representations, linking them directly to aspects of real-world situations. • use their limited range of skills and reason with some insight, in straightforward contexts. • construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
3	482.38	54.6% of students across the OECD and 65.2% in Canada	At Level 3, students can: • execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be the basis for building a simple model or for selecting and applying simple problem-solving strategies. • interpret and use representations based on different information sources and reason directly from them. • demonstrate some ability to handle percentages, fractions, and decimal numbers, and to work with proportional relationships. • provide solutions reflecting that they have engaged in basic interpretation and reasoning.
2	420.07	77.1% of students across the OECD and 86.2% in Canada	At Level 2, students can: • interpret and recognize situations in contexts that require no more than direct inference. • extract relevant information from a single source and make use of a single representational mode. • employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers. • make literal interpretations of the results. **Level 2 is considered the baseline level of mathematical proficiency that is required to participate fully in modern society.**
1	357.77	92.0% of student across the OECD and 96.4% in Canada	At Level 1, students can: • answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. • identify information and carry out routine procedures according to direct instructions in explicit situations. • perform actions that are almost always obvious and follow immediately from the given stimuli.

*Adapted from OECD. (2013). *What students know and can do: Student performance in mathematics, reading and science. Volume I. Paris: Author.*

Note. Reprinted from *Measuring up: Canadian Results of the OECD PISA Study*; [p. 24], by Brochu, Deussing, Houme, & Chuy, 2013, Copyright 2013 by Council of Ministers of Education, Canada (CMEC).

Appendix B

Eighth grade students' attitudes towards learning mathematics in TIMSS 2011

Reported by Students

Students were scored according to their degree of agreement with five statements on the *Students Like Learning Mathematics* scale. Students who **Like Learning Mathematics** had a score on the scale of at least 11.3, which corresponds to their "agreeing a lot" with three of the five statements and "agreeing a little" with the other two, on average. Students who **Do Not Like Learning Mathematics** had a score no higher than 9.0, which corresponds to their "disagreeing a little" with three of the five statements and "agreeing a little" with the other two, on average. All other students **Somewhat Like Learning Mathematics.**

Country	Like Learning Mathematics		Somewhat Like Learning Mathematics		Do Not Like Learning Mathematics		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Morocco	48 (0.7)	398 (2.4)	40 (0.7)	353 (2.2)	12 (0.5)	340 (4.6)	11.2 (0.03)
Armenia	43 (1.0)	499 (3.1)	39 (0.8)	451 (3.4)	18 (1.0)	437 (4.8)	10.9 (0.05)
Jordan	42 (1.5)	442 (3.7)	39 (1.0)	388 (4.2)	19 (0.9)	376 (4.8)	10.9 (0.06)
Georgia	42 (1.3)	463 (5.0)	40 (1.0)	423 (4.1)	18 (1.0)	405 (6.2)	10.8 (0.06)
Malaysia	39 (1.3)	463 (5.0)	46 (0.9)	430 (5.6)	15 (0.9)	413 (8.1)	10.8 (0.05)
Iran, Islamic Rep. of	39 (1.1)	450 (5.4)	40 (0.8)	396 (4.2)	22 (0.9)	388 (4.5)	10.6 (0.05)
Ghana	38 (1.4)	370 (4.8)	51 (1.2)	314 (4.0)	10 (0.5)	299 (6.7)	10.9 (0.05)
Oman	38 (0.8)	420 (3.0)	45 (0.8)	342 (3.6)	17 (0.7)	324 (4.4)	10.8 (0.04)
Kazakhstan	38 (1.5)	506 (4.4)	52 (1.3)	478 (4.4)	10 (0.7)	475 (7.4)	10.9 (0.05)
Tunisia	38 (1.0)	448 (3.4)	40 (0.8)	415 (3.2)	23 (0.9)	405 (3.3)	10.6 (0.05)
Syrian Arab Republic	37 (1.1)	408 (5.2)	44 (1.0)	373 (4.8)	19 (0.9)	353 (6.3)	10.7 (0.05)
Ukraine	36 (1.7)	502 (4.9)	43 (1.2)	477 (4.1)	20 (1.2)	450 (4.9)	10.6 (0.07)
Lebanon	35 (1.2)	475 (4.6)	43 (1.0)	441 (4.2)	21 (1.1)	425 (5.6)	10.6 (0.06)
Singapore	32 (0.7)	637 (3.9)	44 (0.7)	610 (4.1)	23 (0.7)	578 (4.4)	10.4 (0.03)
Turkey	31 (1.0)	504 (6.0)	42 (0.7)	436 (3.9)	26 (1.0)	420 (3.5)	10.3 (0.05)
United Arab Emirates	31 (0.7)	488 (2.3)	42 (0.6)	448 (2.5)	27 (0.8)	432 (2.5)	10.2 (0.04)
Palestinian Nat'l Auth.	31 (1.1)	447 (5.0)	43 (1.0)	394 (4.1)	26 (1.1)	375 (5.1)	10.3 (0.05)
Russian Federation	29 (1.1)	567 (4.7)	49 (0.9)	537 (3.6)	22 (1.0)	509 (4.1)	10.3 (0.04)
Benchmarking Participants							
Abu Dhabi, UAE	32 (1.2)	485 (4.4)	42 (1.0)	441 (3.6)	26 (1.4)	420 (4.9)	10.3 (0.06)
Dubai, UAE	29 (1.0)	508 (3.5)	41 (0.9)	473 (3.1)	30 (1.0)	456 (3.1)	10.1 (0.05)
Ontario, Canada	26 (1.1)	546 (3.5)	41 (1.0)	513 (3.4)	34 (1.4)	481 (3.0)	9.9 (0.06)
North Carolina, US	24 (1.8)	556 (7.6)	44 (1.1)	542 (7.8)	31 (2.3)	516 (7.0)	9.9 (0.11)
Connecticut, US	22 (1.5)	552 (6.0)	40 (1.2)	526 (5.2)	38 (1.8)	495 (5.4)	9.7 (0.08)
Colorado, US	20 (1.6)	548 (5.9)	38 (1.7)	528 (4.8)	42 (2.1)	495 (5.8)	9.4 (0.10)
Massachusetts, US	19 (1.3)	585 (6.1)	40 (1.0)	568 (5.4)	41 (1.7)	543 (5.4)	9.4 (0.09)
Minnesota, US	18 (1.5)	578 (6.8)	41 (0.9)	555 (4.7)	41 (1.6)	521 (4.6)	9.5 (0.08)
Alabama, US	18 (1.9)	475 (10.7)	37 (0.9)	471 (6.7)	45 (1.7)	460 (5.3)	9.3 (0.11)
Florida, US	17 (1.1)	552 (9.7)	38 (1.4)	525 (6.9)	45 (1.7)	493 (6.2)	9.4 (0.08)
California, US	17 (0.9)	519 (6.4)	42 (1.3)	496 (6.1)	41 (1.8)	480 (5.0)	9.4 (0.07)
Alberta, Canada	16 (0.9)	531 (4.7)	44 (1.0)	514 (2.5)	40 (1.4)	486 (3.1)	9.4 (0.06)
Indiana, US	16 (1.4)	547 (6.2)	39 (1.3)	529 (5.3)	45 (2.0)	507 (5.0)	9.3 (0.10)
Quebec, Canada	12 (0.7)	557 (3.9)	43 (0.9)	540 (2.4)	44 (1.2)	517 (2.6)	9.3 (0.05)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS 2011

Note: Reprinted sections from *TIMSS 2011 international results in mathematics*; [p. 332-333], by Mullis, Martin, Foy, & Arora, 2012, Copyright 2012 by International Association for the Evaluation of Educational Achievement (IEA).

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