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## Citation

Zhang, Mian, Brandon Buscaino, Cheng Wang, Amirhassan Shams-Ansari, Christian Reimer, Rongrong Zhu, Joseph Kahn, and Marko Loncar. 2018. "Broadband Electro-optic Frequency Comb Generation in an Integrated Microring Resonator." *Nature*, 11 March 2019.

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# 1 **Broadband electro-optic frequency comb generation in** 2 **an integrated microring resonator**

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1       **Optical frequency combs consist of equally spaced discrete optical frequency**  
2 **components and are essential tools for optical communication, precision metrology,**  
3 **timing and spectroscopy<sup>1-9</sup>. To date, wide-spanning combs are most often generated**  
4 **by mode-locked lasers<sup>10</sup> or dispersion-engineered resonators with third-order Kerr**  
5 **nonlinearity<sup>11</sup>. An alternative comb generation method uses electro-optic (EO)**  
6 **phase modulation in a resonator with strong second-order nonlinearity, resulting in**  
7 **combs with excellent stability and controllability<sup>12-14</sup>. Previous EO combs, however,**  
8 **have been limited to narrow widths by a weak EO interaction strength and a lack of**  
9 **dispersion engineering in free-space systems. In this work, we overcome these**  
10 **limitations by realizing an integrated EO comb generator in a thin-film lithium**  
11 **niobate (LN) photonic platform that features a large EO response, ultra-low optical**  
12 **loss and highly co-localized microwave and optical fields<sup>15</sup>, while enabling dispersion**  
13 **engineering. Our measured EO frequency comb spans more than the entire**  
14 **telecommunications L-band (over 900 comb lines spaced at ~ 10 GHz), and we show**  
15 **that future dispersion engineering can enable octave-spanning combs. Furthermore,**  
16 **we demonstrate the high tolerance of our comb generator to modulation frequency**  
17 **detuning, with frequency spacing finely controllable over seven orders of magnitude**  
18 **(10 Hz to 100 MHz), and utilize this feature to generate dual frequency combs in a**  
19 **single resonator. Our results show that integrated EO comb generators, capable of**  
20 **generating wide and stable comb spectra, are a powerful complement to integrated**  
21 **Kerr combs, enabling applications ranging from spectroscopy<sup>16</sup> to optical**  
22 **communications<sup>8</sup>.**

23       The migration of optical frequency comb generators to integrated devices is motivated  
24 by a desire for efficient, compact, robust, and high repetition-rate combs<sup>11</sup>. At present,

1 almost all on-chip frequency comb generators rely on the Kerr (third-order,  $\chi^{(3)}$ ) nonlinear  
2 optical process, where a continuous wave (CW) laser source excites a low-loss optical  
3 microresonator having a large Kerr nonlinear coefficient. This approach has enabled  
4 demonstration of wide-spanning Kerr frequency combs from the near- to mid-infrared in  
5 many material platforms such as silicon, silicon dioxide, silicon nitride and magnesium  
6 fluoride<sup>17-22</sup>. Owing to the complex nature of the parametric oscillation process, however,  
7 the formation dynamics and noise properties of the Kerr combs are not yet fully  
8 understood and are still under active investigation<sup>23,24</sup>. Sophisticated control protocols are  
9 typically required to keep Kerr combs stabilized.

10 An alternative frequency comb-generation method uses the electro-optic (EO)  
11 effect in materials with second-order ( $\chi^{(2)}$ ) nonlinearity. Conventionally, EO frequency  
12 comb generators pass a CW laser through a sequence of discrete phase and amplitude  
13 modulators<sup>25</sup>. Such EO comb generators can feature remarkable comb power and flat  
14 spectra, and can support flexible frequency spacing. They usually have narrow frequency  
15 span, however, comprising only tens of lines and spanning only a few nanometers<sup>25-27</sup>.  
16 Therefore, highly nonlinear fiber is typically required to further broaden the comb  
17 spectrum, increasing the system complexity and size<sup>13</sup>. Broader EO combs can be  
18 generated using an optical resonator to increase the nonlinear interaction strength<sup>12,28</sup>. In  
19 a canonical resonator-based EO comb generator, a CW laser is coupled to a bulk nonlinear  
20 crystal resonator containing an EO phase modulator (Fig. 1a), and comb lines are  
21 generated solely through the  $\chi^{(2)}$  process. When the modulation frequency matches a  
22 harmonic of the resonator free spectral range (FSR), the optical sidebands generated by  
23 the phase modulator are resonant. In a low-loss resonator, the light passes through the  
24 modulator many times before being dissipated or coupled out, efficiently generating many

1 comb lines spaced at the modulation frequency (Fig. 1b). The output frequency comb can  
 2 be predicted accurately by closed-form solutions<sup>28</sup> with spacings equal to the modulation  
 3 frequency. The overall flatness of the comb strongly depends on the round-trip  
 4 modulation strength and the optical resonator loss. In particular, at frequencies away from  
 5 the pump frequency, the comb line power decreases exponentially: the optical power in  
 6 the  $q$ th comb line is  $P_q \propto e^{-\frac{|q|l}{\beta}}$ , where  $\beta = V_p/V_\pi$  is the phase modulation index,  $V_p$  is the  
 7 microwave drive peak amplitude,  $V_\pi$  is the half-wave voltage of the phase modulator,  $l =$   
 8  $\frac{\kappa}{FSR}\pi$  is the round-trip electric-field loss coefficient of a resonator with damping rate  $\kappa =$   
 9  $\frac{\omega_0}{Q}$ ,  $Q$  is the resonator quality factor, and  $\omega_0$  is the optical frequency. It is therefore clear  
 10 that strong phase modulation (large  $\beta$ ) and a high- $Q$  optical resonator (small  $l$ ) are crucial  
 11 for generating flat and broad EO combs. Furthermore, dispersion sets a fundamental limit  
 12 on the total comb bandwidth by introducing frequency-dependent phase shifts that cause  
 13 comb lines far from the pump frequency to fall out of resonance (see Methods). Although  
 14 EO frequency combs generated by free-space or fiber-based optical cavities have been  
 15 designed and extensively studied for over 25 years<sup>12,28,29</sup>, practical comb widths are still  
 16 limited to a few tens of nanometers by a combination of weak modulation and limited  
 17 dispersion engineering<sup>29</sup>.

18 Here we overcome these limitations of traditional discrete-component-based  
 19 implementations by monolithically integrating an EO comb generator on a thin-film  
 20 lithium niobate (LN) nanophotonic platform. Leveraging the large  $\chi^{(2)}$  nonlinearity, strong  
 21 microwave and optical field overlap, and ultra-low loss optical waveguides enabled by  
 22 this platform, we demonstrate integrated EO combs with performance superior to bulk  
 23 EO comb generators. Our devices feature nearly two orders of magnitudes increase in

1 comb width compared to previous integrated EO combs<sup>30,31</sup> based on InP and Si  
2 platforms, where the effective EO modulation processes, created either by doping (Si) or  
3 operating near the material's absorption band edge (InP), induce high optical losses.

4 We demonstrate an EO frequency comb with over 900 unique frequencies spaced  
5 by 10.453 GHz, spanning 80 nm over part of the telecommunication C-band, the entire  
6 L-band and part of the U-band (Fig. 2). Our comb generator uses a low-loss LN microring  
7 resonator with loaded  $Q \sim 1.5$  million, which is integrated with microwave electrodes for  
8 efficient phase modulation<sup>15</sup> via the strong second-order nonlinearity of LN ( $r_{33} = 30$   
9 pm/V) (Fig. 2a). Importantly, the tight confinement of the light (waveguide width = 1.4  
10  $\mu\text{m}$ ) allows for gold electrodes to be placed only 3.3  $\mu\text{m}$  away from the edge of the  
11 resonator, resulting in efficient microwave delivery to achieve strong phase modulation  
12 while not affecting the resonator  $Q$  factor. The two microwave electrodes are driven so  
13 the top and bottom sections of the resonator experience opposite phase shifts enabling  
14 phase matching between the microwave and circulating optical field. The microresonator  
15 is modulated by an external microwave synthesizer with peak voltage  $V_p = 10$  V ( $\beta =$   
16  $1.2\pi$ ) at a frequency near the resonator FSR, and the generated comb spectrum (Fig. 2b)  
17 is well predicted by theory (see Methods). The comb spectrum has a slope of  $\sim 1$  dB/nm  
18 (Fig. 2b left inset), corresponding to less than 0.1 dB power variation between adjacent  
19 comb lines. The comb lines have greater than 40 dB signal-to-noise ratio (SNR) near the  
20 pump frequency, where the measurement is limited by the noise floor and 20 pm  
21 resolution bandwidth of the optical spectrum analyser (OSA).

22 We develop a theoretical model to quantify the fundamental limits of the wide  
23 spanning EO combs generated on our integrated platform. Traditional EO comb span is  
24 limited to a narrow width by a combination of weak microwave modulation strength and

1 native material dispersion, which hinders the constructive interference needed for  
 2 cascaded frequency conversion to generate comb lines far from the pump frequency<sup>29</sup>. In  
 3 contrast, the integrated EO comb generators feature large modulation strength and the  
 4 ability to engineer dispersion, which enables broader EO comb generation. To understand  
 5 the limitations of EO comb generation process, we look at the resonance condition for a  
 6 comb line at optical frequency  $\omega_q$ . In a traditional resonator, the round-trip constructive  
 7 interference condition is  $|\Delta\phi_q| < 2l$ , where  $\Delta\phi_q = \omega_q T - 2\pi N$  is the accumulated  
 8 round-trip phase,  $T$  is the round-trip time and  $N$  is the number of optical cycles per round-  
 9 trip (chosen to minimize  $|\Delta\phi_q|$ ). For optical frequencies that satisfy this condition, the  
 10 optical field interferes constructively within the resonator. When the resonator length is  
 11 modulated, as in an EO comb generator, the resonance condition is modified into a  
 12 dynamic one, where constructive interference occurs periodically at the microwave  
 13 modulation frequency  $\omega_m$  inside the resonator (i.e.,  $|\Delta\phi_q + \beta \sin \omega_m t| < 2l$ ). Any  
 14 frequency that does not satisfy this dynamic resonance condition will halt the frequency  
 15 conversion process, thus limiting the comb width. This condition is reflected in the  
 16 measured transmission spectrum of a microring resonator under microwave modulation  
 17 (Figure 2b right inset). With no microwave modulation ( $\beta \sim 0$ ), the transmission spectrum  
 18 exhibits a Lorentzian shape. By contrast, when the electrodes are strongly modulated  
 19 (large  $\beta$ ), the half-width at half-maximum of the transmission spectrum broadened by a  
 20 factor of approximately  $\beta$ , confirming that the tolerable absolute accumulated phase  
 21  $|\Delta\phi_q|$  is increased to  $\beta$ . It is therefore clear that it is the strong phase modulation achieved  
 22 in our EO comb generator allowed for the continued cascade of phase modulation even  
 23 in the presence of dispersion.

1           To verify the round-trip phase model experimentally, we detune the optical and  
2 microwave frequencies to generate different comb shapes and widths. By increasing the  
3 microwave detuning up to 30 MHz (Fig. 3a), we observe significant reduction in the comb  
4 frequency span, which is predicted well by the round-trip phase model (Fig. 3b). Any  
5 frequency components having total accumulated phases larger than  $\beta$  cannot resonate,  
6 thus limiting the comb bandwidth. Taking advantage of this well understood dynamic  
7 resonance condition, we can generate asymmetric combs by appropriately choosing the  
8 optical and microwave detuning (Fig. 3c, d). EO combs driven off resonance, such as this  
9 one, could be used as low-noise sources for optical communications due to the noise-  
10 filtering properties of the optical resonator<sup>32</sup> (see Methods).

11           The ability to engineer the dispersion of integrated EO comb generators could  
12 allow achievable comb bandwidth over a full octave. Traditionally, the span of EO comb  
13 generators is restricted by the dispersion of bulk materials, whereas our EO comb  
14 generators tightly confine light in optical waveguides enabling fine tuning of dispersion.  
15 Our simulation shows that with a higher microwave modulation frequency of 50 GHz, a  
16 higher optical pump power (currently only 2 mW in our experiment), and a dispersion  
17 engineered LN rib waveguide resonator that minimizes variation in FSR, it is possible to  
18 generate an EO comb spanning over an octave (Fig. 3e and Methods).

19           Perhaps the most attractive properties of EO comb generators are their excellent  
20 configurability and stability. Leveraging the high tolerance to the detuning of the  
21 modulation frequency from the resonator FSR, we drive the microresonator electrodes  
22 with two phase-locked microwave sources at various frequency offsets from 10.453 GHz,  
23 spanning over seven orders of magnitude, ranging from 10 Hz to over 100 MHz. The  
24 comb generator is optically pumped close to zero detuning at a resonance near 1600 nm

1 and the output of the comb generator is then connected to a high-speed photodetector,  
2 allowing observation of coherent beating between comb lines (Fig. 4). Due to the strong  
3 phase modulation, this dual-driven EO comb contains frequency components far beyond  
4 the ring resonator linewidth without modulation (120 MHz). The ability to vary the  
5 frequency spacing of resonator-based EO combs over seven orders of magnitude is in  
6 stark contrast with Kerr-based combs, whose frequency offset is predetermined by the  
7 fabricated resonator dimensions<sup>5</sup>. This flexibility in comb drive frequencies may enable  
8 applications requiring reconfigurable dynamic range, such as dual-comb-based optical  
9 ranging<sup>7,9</sup> and spectroscopy<sup>3-6</sup>. Two independent microresonators can be integrated onto  
10 the same LN chip with high fabrication tolerance to avoid potential aliasing of the comb  
11 lines.

12 Our work using high- $Q$  microring resonators and highly confined optical  
13 waveguides for EO comb generation is the first step towards a new generation of  
14 integrated EO comb sources. Based on our demonstration of an EO comb that is almost  
15 two orders of magnitude larger than prior integrated EO combs, dispersion engineering  
16 and high frequency modulation can soon enable efficient octave-spanning EO comb  
17 generators. Importantly, the approach demonstrated here can be used to realize EO combs  
18 all over the LN transparency window, including visible and near-IR, *simultaneously*. With  
19 the added ability to integrate filters and resonators adjacent or inside EO comb generators  
20 on the same chip, comb line power and hence SNR can be further increased by nearly 20  
21 dB<sup>33</sup>. Our approach allows for complex EO circuits to be integrated on the same chip and  
22 thus has the potential to transform microresonator frequency comb applications. For  
23 example, high-performance EO combs featuring high power and flat combs could enable  
24 Tb/s optical communications links that rely on stable, low-noise combs as sources for

1 high capacity wavelength-division multiplexed systems on a single chip<sup>8</sup>. Furthermore,  
2 the EO comb generator demonstrated in this work provides many stable coherent optical  
3 frequencies with electrically adjustable frequency spacing, paving the way for efficient  
4 dual-comb spectroscopy<sup>3-6</sup> on a chip or highly-reconfigurable comb-based ranging<sup>7,9</sup>.

## 5 **References**

- 6 1. Ye, J., Schnatz, H. & Hollberg, L. W. Optical frequency combs: from frequency  
7 metrology to optical phase control. *IEEE J. Sel. Top. Quantum Electron.* **9**, 1041–  
8 1058 (2003).
- 9 2. Papp, S. B. *et al.* Microresonator frequency comb optical clock. *Optica* **1**, 10–14  
10 (2014).
- 11 3. Coddington, I., Newbury, N. & Swann, W. Dual-comb spectroscopy. *Optica* **3**,  
12 414–426 (2016).
- 13 4. Bernhardt, B. *et al.* Cavity-enhanced dual-comb spectroscopy. *Nat. Photonics* **4**,  
14 55–57 (2010).
- 15 5. Dutt, A. *et al.* On-chip dual-comb source for spectroscopy. *Sci. Adv.* **4**, e1701858  
16 (2018).
- 17 6. Suh, M.-G., Yang, Q.-F., Yang, K. Y., Yi, X. & Vahala, K. J. Microresonator  
18 soliton dual-comb spectroscopy. *Science* **354**, 600–603 (2016).
- 19 7. Trocha, P. *et al.* Ultrafast optical ranging using microresonator soliton frequency  
20 combs. *Science* **359**, 887–891 (2018).
- 21 8. Pfeifle, J. *et al.* Coherent terabit communications with microresonator Kerr  
22 frequency combs. *Nat. Photonics* **8**, 375–380 (2014).
- 23 9. Suh, M.-G. & Vahala, K. J. Soliton microcomb range measurement. *Science* **359**,  
24 884–887 (2018).

- 1 10. Cundiff, S. T., Ye, J. & Hall, J. L. Optical frequency synthesis based on mode-  
2 locked lasers. *Rev. Sci. Instrum.* **72**, 3749–3771 (2001).
- 3 11. Kippenberg, T. J., Gaeta, A. L., Lipson, M. & Gorodetsky, M. L. Dissipative Kerr  
4 solitons in optical microresonators. *Science* **361**, eaan8083 (2018).
- 5 12. Kourogi, M., Nakagawa, K. & Ohtsu, M. Wide-span optical frequency comb  
6 generator for accurate optical frequency difference measurement. *IEEE J. Quantum*  
7 *Electron.* **29**, 2693–2701 (1993).
- 8 13. Beha, K. *et al.* Electronic synthesis of light. *Optica* **4**, 406–411 (2017).
- 9 14. Xiao, S., Hollberg, L., Newbury, N. R. & Diddams, S. A. Toward a low-jitter 10  
10 GHz pulsed source with an optical frequency comb generator. *Opt. Express* **16**,  
11 8498–8508 (2008).
- 12 15. Wang, C. *et al.* Integrated lithium niobate electro-optic modulators operating at  
13 CMOS-compatible voltages. *Nature* **562**, 101 (2018).
- 14 16. Millot, G. *et al.* Frequency-agile dual-comb spectroscopy. *Nat. Photonics* **10**, 27–30  
15 (2016).
- 16 17. Del’Haye, P. *et al.* Octave Spanning Tunable Frequency Comb from a  
17 Microresonator. *Phys. Rev. Lett.* **107**, 063901 (2011).
- 18 18. Okawachi, Y. *et al.* Octave-spanning frequency comb generation in a silicon nitride  
19 chip. *Opt. Lett.* **36**, 3398–3400 (2011).
- 20 19. Griffith, A. G. *et al.* Silicon-chip mid-infrared frequency comb generation. *Nat.*  
21 *Commun.* **6**, 6299 (2015).
- 22 20. Savchenkov, A. A. *et al.* Tunable Optical Frequency Comb with a Crystalline  
23 Whispering Gallery Mode Resonator. *Phys. Rev. Lett.* **101**, 093902 (2008).

- 1 21. Liang, W. *et al.* Generation of near-infrared frequency combs from a MgF<sub>2</sub>  
2 whispering gallery mode resonator. *Opt. Lett.* **36**, 2290–2292 (2011).
- 3 22. Gräupner, P., Pommier, J. C., Cachard, A. & Coutaz, J. L. Electro-optical effect in  
4 aluminum nitride waveguides. *J. Appl. Phys.* **71**, 4136–4139 (1992).
- 5 23. Herr, T. *et al.* Universal formation dynamics and noise of Kerr-frequency combs in  
6 microresonators. *Nat. Photonics* **6**, 480–487 (2012).
- 7 24. Yi, X. *et al.* Single-mode dispersive waves and soliton microcomb dynamics. *Nat.*  
8 *Commun.* **8**, 14869 (2017).
- 9 25. Metcalf, A. J., Torres-Company, V., Leaird, D. E. & Weiner, A. M. High-Power  
10 Broadly Tunable Electrooptic Frequency Comb Generator. *IEEE J. Sel. Top.*  
11 *Quantum Electron.* **19**, 231–236 (2013).
- 12 26. Sakamoto, T., Kawanishi, T. & Izutsu, M. Asymptotic formalism for ultraflat  
13 optical frequency comb generation using a Mach-Zehnder modulator. *Opt. Lett.* **32**,  
14 1515–1517 (2007).
- 15 27. Ozharar, S., Quinlan, F., Ozdur, I., Gee, S. & Delfyett, P. J. Ultraflat Optical Comb  
16 Generation by Phase-Only Modulation of Continuous-Wave Light. *IEEE Photonics*  
17 *Technol. Lett.* **20**, 36–38 (2008).
- 18 28. Ho, K. P. & Kahn, J. M. Optical frequency comb generator using phase modulation  
19 in amplified circulating loop. *IEEE Photonics Technol. Lett.* **5**, 721–725 (1993).
- 20 29. Kourogi, M., Widiyatomo, B., Takeuchi, Y. & Ohtsu, M. Limit of optical-  
21 frequency comb generation due to material dispersion. *IEEE J. Quantum Electron.*  
22 **31**, 2120–2126 (1995).
- 23 30. Dupuis, N. *et al.* InP-Based Comb Generator for Optical OFDM. *J. Light. Technol.*  
24 **30**, 466–472 (2012).

- 1 31. Demirtzioglou, I. *et al.* Frequency comb generation in a silicon ring resonator  
2 modulator. *Opt. Express* **26**, 790–796 (2018).
- 3 32. Kim, J., Richardson, D. J. & Slavík, R. Cavity-induced phase noise suppression in a  
4 Fabry–Perot modulator-based optical frequency comb. *Opt. Lett.* **42**, 1536–1539  
5 (2017).
- 6 33. Kouroggi, M., Enami, T. & Ohtsu, M. A Coupled-Cavity Monolithic Optical  
7 Frequency Comb Generator. *IEEE Photonics Technol. Lett.* **8**, (1996).

## 8 **Acknowledgements**

10 This work is supported by: National Science Foundation award numbers ECCS-1609549,  
11 ECCS-1740291 E2CDA, ECCS-1740296 E2CDA, DMR-1231319; the Harvard  
12 University Office of Technology Development, Physical Sciences and Engineering  
13 Accelerator Award; and Facebook, Inc. Device fabrication is performed at the Harvard  
14 University Center for Nanoscale Systems, a member of the National Nanotechnology  
15 Coordinated Infrastructure Network, which is supported by the National Science  
16 Foundation under award number ECCS-1541959.

## 17 **Author contributions**

18 M.Z., B.B, C.W., J.K. and M.L. conceived of the experiment. M.Z., C.W. and A.S.  
19 designed and fabricated the devices. B.B performed theoretical modelling and numerical  
20 simulations of the integrated EO comb. M.Z. and R.Z. designed waveguide dispersion.  
21 M.Z., C.W., A.S. and C.R. carried out the device characterization. M.Z. and B.B.  
22 performed the data analysis. M.Z. and B.B. wrote the manuscript with contributions from  
23 all authors. J.K. and M.L. supervised the project.

## 24 **Author information**

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2           **Competing interests:** M. Z., C. W., and M. L. are involved in developing lithium-  
3 niobate technologies at HyperLight Corporation.

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## 1 **Figures legends**

2 **Figure 1 | Resonator-enhanced electro-optic comb generator.** **a**, Schematic of a  
3 canonical electro-optic (EO) comb generator comprising an EO ( $\chi^{(2)}$ ) phase modulator  
4 inside a Fabry-Pérot (FP) resonator. A continuous-wave (CW) laser is coupled into the  
5 resonator and an optical frequency comb is generated at the output **b**, EO comb generation  
6 principle. A microwave signal, with modulation frequency equal to the free spectral range  
7 (FSR) of the optical resonator, couples light between different resonator modes. As a  
8 result, the input-coupled CW light is modulated, giving rise to sidebands at the  
9 modulation frequency, which are then recirculated to be modulated again. The modulation  
10 index determines the strength of coupling between nearby frequency components after  
11 passing through the modulator. **c**, Integrated microring EO comb generator. The FP  
12 resonator can be replaced by a microring resonator that is EO modulated at a frequency  
13 matching the FSR of the ring. Similar to the FP resonator, a CW laser coupled into the  
14 ring resonator will be converted to a frequency comb in the output optical waveguide.

1 **Figure 2 | Integrated electro-optic comb generator. a**, Micrograph of a fabricated  
2 lithium niobate microring resonator (a shorter device is shown here for illustration  
3 purpose, see Methods for details). The black lines are etched optical waveguides and the  
4 yellow regions are gold microelectrodes. The gold electrodes are driven such that the  
5 phase shifts in the two sides of the microresonator are opposite, which is required to break  
6 the symmetry of different azimuthal order optical modes, enabling efficient frequency  
7 conversion. **b**, Measured output spectrum of the electro-optic comb generated from the  
8 microring resonator, demonstrating  $> 80$  nm bandwidth and more than 900 comb lines  
9 with a slope of 1 dB/nm. The input optical power is 2 mW and the microwave peak driving  
10 amplitude is  $V_p = 10$  V. Note that the signal-to-noise-ratio of the comb lines exceeds 40  
11 dB but is limited by the noise floor and resolution of the optical spectrum analyzer. Insets:  
12 left, magnified view of several comb lines showing a line-to-line power variation of  $\sim 0.1$   
13 dB. Right, measured transmission spectrum for several different modulation indices ( $\beta$ ).  
14 When the modulation is turned on, the optical resonance is broadened by twice the  
15 modulation index. This behaviour is predicted well by the round-trip phase model (see  
16 Methods). NT: normalized transmission. NOD: normalized optical detuning  
17

1 **Figure 3 | Controllability of the electro-optic comb spectrum.** **a**, Measured electro-  
2 optic (EO) comb output spectrum (light, shaded) for various values of modulation  
3 frequency detuning from the resonator free spectral range. Numerical simulation of the  
4 comb envelopes (dark, lines, see Methods) match the measured spectra. **b**, Calculated  
5 round-trip phase versus wavelength for the modulation frequency detuning values in (a).  
6 The light grey shaded region highlights the constructive interference condition region  
7 beyond which EO comb generation is suppressed. This region is bounded by  $\pm\beta$ , the  
8 round-trip modulation index. Insets show a zoomed-out view of the round-trip phase vs.  
9 wavelength. The calculated cut-off frequency matches well with experimental data, as  
10 shown by the dashed lines extending to (a). **c, d**, Measured and simulated comb spectrum  
11 and round-trip phase versus wavelength in presence of both optical and microwave  
12 detuning. Different comb shapes, such as a single-sided EO comb can be generated. **e**,  
13 Simulated round-trip phase versus wavelength for traditional bulk devices (black), the  
14 measured integrated device (blue), and dispersion-engineered integrated devices  
15 (orange). The simulations demonstrate that integrated EO combs can achieve larger  
16 dispersion-limited bandwidths than devices based on bulk crystals and dispersion  
17 engineering can enable octave-spanning EO combs.

1 **Figure 4 | Dual-tone electro-optic comb generation** **a**, Demonstration of coherent  
2 beating of the electro-optic (EO) comb. The measured beat-note power spectral density  
3 (PSD) is shown on a logarithmic scale to highlight the flexibility in control of the EO  
4 comb spacing over seven orders of magnitude, from 10 Hz to 100 MHz. **b**, Experimental  
5 setup. The EO comb generator is driven by a superposition of two phase-locked  
6 microwave signals with various values of frequency offset  $\Delta$ . The optical output is  
7 detected by a fast photodiode (PD), and the beat notes are detected by a radio-frequency  
8 spectrum analyzer (RSA). **c-f**, Magnification of the individual beat notes for various comb  
9 spacings on a linear frequency scale. This measurement, which demonstrates frequency  
10 components well beyond the resonator bandwidth in the absence of modulation, confirms  
11 that phase modulation changes the resonance condition to tolerate large microwave  
12 detuning. Additionally, this measurement demonstrates the extreme flexibility in comb  
13 frequency spacing for practical applications such as dual-comb spectroscopy or comb-  
14 based ranging.  
15

## 1 **Methods**

### 2 **Fabrication details**

3 All devices are fabricated on x-cut single crystalline thin-film lithium niobate (LN) wafers  
4 (NANOLN). The wafer stack consists of a 600 nm thin-film LN layer, a 2  $\mu\text{m}$  thermally  
5 grown  $\text{SiO}_2$  layer and a 500  $\mu\text{m}$  silicon handle layer. Standard electron-beam (e-beam)  
6 lithography is used to pattern optical waveguide and micro-racetrack resonators. The  
7 patterns are then transferred into the LN layer using argon ( $\text{Ar}^+$ ) plasma etching in an  
8 inductively coupled plasma reactive ion etching (ICP-RIE) tool<sup>34</sup>. The etch depth is 350  
9 nm, leaving a 250 nm thick LN slab behind, which enables efficient electric field  
10 penetration into the waveguide core. Gold contact patterns are then created using aligned  
11 e-beam lithography, and the metal is transferred using e-beam evaporation methods and  
12 lift-off processes. The chip is then diced and the facets are polished for end-fire optical  
13 coupling. A 10 GHz FSR micro-racetrack measures 200  $\mu\text{m}$  by 6.2 mm. For illustration  
14 purposes, a 25 GHz FSR ring with otherwise the same design measuring 200  $\mu\text{m}$  by 2.7  
15 mm is displayed in Fig. 2a, where the straight section has a reduced length.

16

### 17 **Microwave driving circuitry**

18 The 10 GHz microwave drive signal is generated by a radio-frequency (RF) synthesizer  
19 and amplified by an electrical power amplifier. The amplified electrical signal is passed  
20 through a microwave circulator and delivered to the microelectrodes. As the  
21 microelectrodes represent a capacitive load, most of the electrical driving signal is  
22 reflected back to the circulator and terminated at the circulator output by a 50- $\Omega$  load.

23

1 In the dual-drive EO comb generation experiment, two RF synthesizers are phase-locked  
2 via a common 10 MHz clock and are free to operate at different frequencies. The two  
3 sinusoidal microwave signals are power balanced and combined using an RF power  
4 splitter and passed through the amplifier-circulator circuitry described previously.

5

## 6 **Optical characterization and detection**

7 Light from a tunable laser (SANTEC TS510) is launched into, and the comb output is  
8 collected from, the LN waveguides by a pair of lensed optical fibers. The output comb is  
9 passed to an optical spectrum analyser (OSA) having a minimum resolution of 20 pm.  
10 This finite resolution accounts for the limited signal-to-noise ratio observed in Fig. 2b (~  
11 20 dB). The shot-noise-limited signal-to-noise ratio is much higher, as the comb shot  
12 noise lies below the OSA noise floor. Although the measurement in the paper is chosen  
13 to center at 1600 nm, the frequency comb center wavelength can be flexibly chosen  
14 between 1500 nm to 1680 nm of the tunable laser's range without affecting much of the  
15 generated comb width.

16

17 In the dual-drive EO comb measurements, the modulated light is passed to a fast  
18 photodetector (New Focus 1544A) and the resulting electrical signal is sent to a RF  
19 spectrum analyzer to record the beating in the RF domain.

20

## 21 **Measurement and calculation of resonator parameters**

22 As demonstrated by Equation (4) below, there are four resonator parameters that fully  
23 characterize the EO comb spectrum: the internal round-trip transmission coefficient  $\alpha$ ,  
24 the power coupling coefficient  $k$ , the coupler insertion loss of the coupler  $\gamma$ , and the phase

1 modulation index  $\beta$ . Finding each of these four parameters by fitting to the comb  
 2 spectrum of Equation (4) is difficult because the output comb can be fully determined by  
 3 a subset of these independent parameters (e.g., increasing the modulation index has the  
 4 same effect as decreasing the loss in the resonator). Instead, each of the parameters must  
 5 be measured separately.

6  
 7 We find  $\alpha$  and  $k$  by measuring the total transmitted power without phase modulation  
 8 (Figure 2b right inset). By fitting to the expected transmission of an all-pass ring  
 9 resonator, we find  $Q = 1.5 \times 10^6$ ,  $\alpha = 0.95$  and  $k = 0.027$ . Then we perform a grid  
 10 search optimization for  $\gamma$  and  $\beta$  comparing the measured output spectrum (Fig 2b) with  
 11 the spectrum determined from the output time-domain electric field of Equation (3)  
 12 below. We find a best fit for  $\gamma = -0.004$  dB and  $\beta = 1.2 \pi$ , where the average difference  
 13 between experimental and theoretical comb line power is 0.6 dB. The relative uncertainty  
 14 in the measurement of  $\beta$  in this case is  $\pm 4\%$ , calculated by finding the furthest fit within  
 15 a 95% confidence interval and calculating the resulting  $\beta$ .

16  
 17 The output power transmission for nonzero modulation indices (Figure 2b right inset) is  
 18 calculated by sampling the output electric field with Equation (3) and averaging the power  
 19 over more than 100 modulation periods.

## 21 **Dispersion engineering in thin-film LN waveguides**

22 To achieve wide-spanning EO combs, the waveguide dispersion should be engineered  
 23 such that the group velocity (or the FSR) of the ring is roughly a constant across the entire  
 24 frequency range. We simulate the dispersion of the waveguide using finite element

1 methods (LUMERICAL Mode Solutions). The simulation accounts for the LN material  
 2 anisotropy and the finite waveguide etching angle (around  $70^\circ$  from horizontal). The  
 3 round-trip phase of the light inside the resonator is calculated by integrating the simulated  
 4 group velocity dispersion twice to determine the total frequency-dependent phase-shift.  
 5 For the device we demonstrate here, with a waveguide ridge height of 350 nm, waveguide  
 6 width of 1.4  $\mu\text{m}$ , slab thickness of 250 nm, and  $\text{SiO}_2$  top cladding of 1.5  $\mu\text{m}$  the dispersion  
 7 of the waveguide is weakly normal and supports an EO comb cut-off bandwidth of  $\sim 250$   
 8 nm. We find that for an air-cladded waveguide with a 600 nm thin-film LN layer, 550 nm  
 9 etch depth and 1.8  $\mu\text{m}$  waveguide width, a comb spanning  $\sim 1.3$  octave can be generated  
 10 with a round-trip modulation frequency of 50 GHz and strength of  $1.2 V_\pi$ , as shown in  
 11 Fig 3e. The waveguide dispersion can be tailored for low microwave drive powers at the  
 12 expense of a smaller comb span. For an air-cladded waveguide with a 650 nm thin-film  
 13 LN layer, etch depth of 620 nm and width 2400 nm, an octave spanning comb can be  
 14 generated with a phase modulation strength of only  $0.3 V_\pi$ . These results are presented in  
 15 the accompanying extended data figure.

16

### 17 **Microwave driver power consumption**

18 The current EO comb generator features a direct capacitive drive electrode design, where  
 19 the electrical power consumption  $P_E$  can be estimated as

$$20 \quad P_E = \frac{1}{2} C V_p^2 \omega_M \quad (1)$$

21 Where  $C \sim 200$  fF is the estimated capacitance<sup>35</sup>,  $V_p$  is the peak voltage and  $\omega_M$  is the  
 22 microwave frequency. For the broad comb shown in Fig 2., the calculated electrical power  
 23 consumption is  $\sim 630$  mW.

24

1 There are several ways to reduce the electrical power consumption. Presently the  
 2 electrode gaps are not optimized and can be reduced to directly increase the electro-optic  
 3 efficiency. A microwave resonator with a quality factor of  $Q_M$  can be used to dramatically  
 4 enhance the driving voltage, as only a narrow band microwave source is required. A  
 5 microwave resonator has an enhanced voltage  $V_{p,eff}$  of

$$6 \quad V_{p,eff} = \sqrt{\frac{2P_E Q_M}{\omega_M C}} \quad (2)$$

7 Comparing (2) with (1), the effective pumping power is increased by a factor of  $Q_M$ . This  
 8 means for a moderate  $Q_M = 20$  at 10 GHz, the power consumption can be reduced to  $\sim 30$   
 9 mW.

10

11 To estimate the minimum electrical power required to generate an octave spanning EO  
 12 comb, we consider the first case where the resonator is driven to  $1.2 V_\pi$  at 50 GHz FSR.  
 13 Here the capacitance of the device is reduced by a factor of 5 as the ring resonator  
 14 becomes smaller to achieve a 50 GHz FSR. At the same time, the  $V_\pi$  also increases by a  
 15 factor of 5 due to the shorter electrodes. For  $Q_M = 20$ , the calculated power consumption  
 16 is  $\sim 750$  mW. Through dispersion engineering and higher optical  $Q$  microresonators, it is  
 17 possible to achieve an octave spanning EO comb even at low drive voltages of  $V_p = 0.3$   
 18  $V_\pi$ . In this case, the electrical power consumption is further reduced to only  $\sim 45$  mW.

19

## 20 **Canonical EO comb generator design**

21 The concept of a comb generator using a resonator to enhance frequency generation by  
 22 an EO phase modulator dates to 1972<sup>36</sup>. Theoretical and experimental work<sup>12,28</sup> on these  
 23 comb generators continued in the 1990s. Recent advances in low-loss integrated LN

1 photonic platform<sup>34</sup> has motivated re-examination of these comb generators. This section  
 2 provides details on important characteristics of these EO comb generators.

3 A canonical waveguide-based comb generator is shown in Fig. 1c of the main text. A  
 4 single-frequency input with electric field  $E_{in}(t) = \hat{E}_{in} e^{i\omega_0 t}$  is coupled, with power  
 5 coupling coefficient  $k$  and insertion loss  $\gamma$ , to a resonator having round trip time  $T$  at  
 6 center frequency  $\omega_0$  and round trip power loss  $\alpha$ . The resonator contains a phase  
 7 modulator driven with modulation index  $\beta$  and frequency  $\omega_m$ . The output electric field  
 8 is<sup>28</sup>

9  
 10  $E_{out}(t)$

$$11 \quad = \sqrt{(1-\gamma)(1-k)} E_{in}(t) - k \sqrt{\frac{1-\gamma}{1-k}} \sum_{n=1}^{\infty} r^n e^{-i\beta F_n(\omega_m t)} E_{in}(t - nT), \quad (1)$$

12

13 where  $r = \sqrt{(1-\gamma)(1-k)\alpha}$  is the round trip electric field transmission and  $F_n(\omega_m t) =$   
 14  $\sum_{i=1}^n \sin \omega_m(t - iT)$  is the modulator coherence function. The parameter  $l = 1 - r$ ,  
 15 corresponding to the round-trip electric field loss, is used in the main text for simplicity.

16 When the optical carrier is resonant in the resonator ( $\omega_0 T = 2\pi m_1$ ) and the microwave  
 17 drive signal is resonant ( $\omega_m T = 2\pi m_2$ ), the modulator coherence function becomes  
 18  $F_n(\omega_m t) = n \sin \omega_m(t - iT)$  and the output electric field can be simplified to

19

$$20 \quad E_{out}(t) = \left[ \sqrt{(1-\gamma)(1-k)} - k \sqrt{\frac{1-\gamma}{1-k}} \frac{r e^{-i\beta \sin \omega_m t}}{1 - r e^{-i\beta \sin \omega_m t}} \right] E_{in}(t). \quad (2)$$

21

1 This output electric field corresponds to an optical frequency comb spaced at the  
 2 modulation frequency. The power in the  $q$ th comb line away from the center frequency  
 3 can be found by rewriting Equation (1) as

4  $E_{out}(t)$

$$5 \quad = \sqrt{(1-\gamma)(1-k)} \hat{E}_{in} e^{i\omega_0 t} - k \sqrt{\frac{1-\gamma}{1-k}} \sum_{n=1}^{\infty} r^n e^{-i\beta n \sin \omega_m t} \hat{E}_{in} e^{i\omega_0 t} \quad (3)$$

$$6 \quad = \sqrt{(1-\gamma)(1-k)} \hat{E}_{in} e^{i\omega_0 t} - k \sqrt{\frac{1-\gamma}{1-k}} \sum_{q=-\infty}^{\infty} \hat{E}_{in} e^{i(\omega_0 + q \omega_m) t} \sum_{n=1}^{\infty} r^n J_q(\beta n),$$

7  
 8 where  $J_q$  is the  $q$ th order Bessel function of the first kind. The power of the  $q$ th (nonzero)  
 9 comb line is then

$$10 \quad P_q = k^2 \frac{1-\gamma}{1-k} P_{in} \left| \sum_{n=1}^{\infty} r^n J_q(\beta n) \right|^2. \quad (4)$$

11 Kourogı et. al.<sup>12</sup> found an approximation for the power of the  $q$ th comb as  $P_q \propto e^{-\frac{|q|(1-r)}{\beta}}$ .

12

13 In the presence of optical and microwave detuning from resonance, the comb spectrum  
 14 can still be calculated. When the optical carrier is off resonance, the total round-trip phase  
 15 is  $\omega_0 T = 2\pi m_1 + \phi_{opt}$ . Similarly, when the microwave carrier is off resonance the  
 16 total round-trip phase is  $\omega_m T = 2\pi m_2 + \phi_{micro}$ . Using these expressions in Equation  
 17 (1), we can find the following expression for the power in the  $q$ th comb line:

18  $P_q$

$$19 \quad = k^2 \frac{1-\gamma}{1-k} P_{in} \left| \sum_{p=-\infty}^{\infty} \sum_{n=1}^{\infty} (r e^{i\phi_{opt}})^n e^{i p \frac{\pi}{2}} J_{q-p}(\beta_o(\phi_{micro}, n)) J_p(\beta_e(\phi_{micro}, n)) \right|^2. \quad (5)$$

1 The modified even and odd modulation indices ( $\beta_e$  and  $\beta_o$ , respectively) are

$$3 \quad \beta_e(\phi_{micro}, n) = \beta \left[ \frac{1}{2} \cot \phi_{micro}/2 - \frac{\cos \left( n + \frac{1}{2} \right) \phi_{micro}}{2 \sin \phi_{micro}/2} \right] \quad (6)$$

$$4 \quad \beta_o(\phi_{micro}, n) = \beta \left[ -\frac{1}{2} + \frac{\sin \left( n + \frac{1}{2} \right) \phi_{micro}}{2 \sin \phi_{micro}/2} \right]. \quad (7)$$

5  
6 It is clear here that in the regime of low optical detuning, the slope of the comb decreases  
7 by a factor of  $\cos(\phi_{opt})$ . This effect has been studied and reported in<sup>37</sup>. The effect of  
8 microwave detuning is harder to visualize, but results in a destructive interference  
9 condition for large values of  $q$  in Equation (5). This effect is demonstrated experimentally  
10 and theoretically in Fig. 3a and 3b of the main text.

## 12 Noise Properties

13 The optical phase noise of the comb lines is important in applications that require high  
14 optical signal-to-noise ratios, such as high-capacity optical communications. It is well  
15 known that the optical phase noise contribution from the pump laser does not increase  
16 with increasing comb line index<sup>28</sup>. By contrast, the phase noise contribution from the  
17 microwave modulation signal increases in power with comb line quadratically with  $q$ .  
18 This can be shown by modifying the modulator coherence function to include the effects  
19 of microwave modulation phase noise  $\theta(t)$ :

$$21 \quad F_n(\omega_m t) = \sum_{i=1}^n \sin \omega_m (t - iT + \theta(t - iT)). \quad (8)$$

23 The output optical field can then be written as:

1

2  $E_{out}(t)$ 

3

$$= \sqrt{(1-\gamma)(1-k)} \hat{E}_{in} e^{i\omega_0 t}$$

4

$$- k \sqrt{\frac{1-\gamma}{1-k}} \hat{E}_{in} \sum_{q=-\infty}^{\infty} \sum_{n=1}^{\infty} r^n J_q(\beta n) e^{i(\omega_0 + q\omega_m)t + iq\theta(t)}. \quad (9)$$

5

6 The phase noise amplitude increases linearly with increasing comb line index  $q$ ,

7 corresponding to a quadratic increase in phase noise power.

8

9 For applications that require few comb lines, this increase in microwave phase noise is

10 often negligible because quartz crystal oscillators have very low phase noise. For

11 applications requiring many comb lines, however, the effect of microwave phase noise

12 may be noticeable. Recently, there has been experimental evidence of microwave phase

13 noise suppression in EO comb generators<sup>32,38</sup>. In these studies, the phase noise of free-

14 space resonator-enhanced EO combs is measured and simulated. When the optical and

15 microwave frequencies are resonant, higher order comb lines do not experience a

16 quadratic increase in phase noise power. Instead, high frequency phase noise components

17 are attenuated such that the high frequency phase noise is comparable for all comb lines.

18 Furthermore, detuning the optical and microwave frequencies from the resonator FSR can

19 further reduce the phase noise power. These experiments suggest that EO comb

20 generators can generate low-noise comb lines over their entire dispersion-limited

21 bandwidth. Additionally, integrated platforms, such as the one presented in the main text,

22 enable additional filtering cavities and structures to be readily included in the resonator

23 structure.

1

## 2 **Round-Trip Phase Model**

3 To include the effect of dispersion, we introduce a round-trip phase model. In particular,  
 4 we consider the destructive interference that occurs due to the microwave detuning  
 5 motivates a phase-based resonance approximation for the viable comb bandwidth.  
 6 Previous analytical work<sup>29</sup> provided a mathematical treatment of the dispersion limits of  
 7 resonator-based EO comb generators. Here, as a complement to that work, we clarify the  
 8 physical interpretation of the round-trip phase model and demonstrate its application to  
 9 combs of arbitrary bandwidth within a given dispersion-limited window.

10

11 The resonance condition of an optical frequency  $\omega_q$  in a microresonator without EO  
 12 modulation is  $|\omega_q T - 2\pi N| < 2l$ , where the total round-trip phase offset  $\Delta\phi_q = \omega_q T -$   
 13  $2\pi N$ ,  $T = 1/FSR$  is the round-trip time and  $N$  is the number of optical cycles per round-  
 14 trip that ensures that  $|\Delta\phi_q| < 2\pi$ . Frequency components outside of the resonance are  
 15 attenuated by destructive interference, and thus do not resonate. When the resonance  
 16 condition is satisfied, the optical fields constructively interfere inside the resonator at  
 17 every time and spatial location.

18

19 In a resonator containing an EO phase modulator, the (now time-dependent) resonance  
 20 condition becomes  $|\Delta\phi_q + \beta \sin 2\pi f_m t| < 2l$ , where  $\beta$  is the modulation index and  $f_m$   
 21 is the modulation frequency. Here, it is clear that the resonance condition can be satisfied  
 22 for much larger round-trip phase offsets  $\Delta\phi_q$  because within the round-trip resonator  
 23 propagation time, the modulation term oscillates between negative and positive  $\beta$  (i.e.  
 24  $-\beta < \beta \sin 2\pi f_m t < \beta$ ).

1

2 This effect may be understood by plotting the total transmission of the EO comb generator  
 3 for various  $\beta$ , as shown in Fig, 2b, right inset, of the main text. The transmission is  
 4 calculated by averaging the output power of a time-domain representation of the electric  
 5 field given in Equation (1). The optical power output depends primarily on the  
 6 interference between the input optical field and the optical field inside the resonator. As  
 7 in a microresonator without EO modulation, the dips in the transmission spectrum  
 8 correspond to a large built-up field inside the resonator. For various values of  $\beta$ , the width  
 9 of the resonance increases, indicating that for large modulation indices, the resonance  
 10 condition can be satisfied for various detuning values. As shown in Fig. 2b, the amount  
 11 of detuning is approximately equal to the modulation index  $\beta$ , as is predicted by the phase  
 12 model when  $\Delta\phi_q = \phi_{opt}$ .

13

14 We can now determine the contributions to the optical phase offset  $\Delta\phi_q$  as a function of  
 15 frequency. The optical phase offset, as discussed in the previous section, does not induce  
 16 frequency-dependent phase shifts. However, microwave signal detuning and dispersion  
 17 effects are frequency dependent.

18

19 Once the resonator has reached steady state, the output field is an EO comb spaced at the  
 20 modulation frequency  $f_m$ , such that the  $q$ th comb line frequency is  $f_q = f_0 + qf_m$ . A  
 21 mismatch between the microwave frequency and the resonator free spectral range,  $\Delta f_m$ ,  
 22 results in a frequency-dependent phase offset  $\phi_{micro}(q) = 2\pi q \Delta f_m T$ .

23

1 For an arbitrary dispersion profile, it is possible to find the frequency-dependent phase  
 2 offset by integrating the group velocity dispersion profile of the waveguide. However, if  
 3 the dispersion is approximately linear with frequency, the dispersion-related phase offset  
 4 is  $\Delta\phi_{disp}(q) = 2\pi(q f_m)^2 \beta_2 L$  where  $\beta_2 L$  is the round-trip group velocity dispersion in  
 5  $\text{fs}^2/\text{mm}$ .

6

7 To first order we then have a model for the total phase offset as a function of frequency,  
 8  $\Delta\phi_q = \Delta\phi_{opt} + \Delta\phi_{micro}(q) + \Delta\phi_{disp}(q)$ . In fact, this model agrees exactly with the  
 9 analytical model for the output comb shape developed in<sup>12</sup>. In the case of maximum comb  
 10 bandwidth, corresponding to zero microwave detuning and optical detuning satisfying

11  $\phi_{opt} + \beta = 0$ , the maximum dispersion-limited bandwidth is  $\Delta f_{comb} = \frac{1}{\pi} \sqrt{\frac{2\beta}{\beta_2 L}}$ , agreeing

12 with up to a factor of  $\sqrt{2}$  due to the difference in FSR of a Fabry-Pérot resonator and ring  
 13 resonator of identical length<sup>29</sup>.

14

15 Using this model, it is a straightforward optimization problem to start with the frequency-  
 16 dependent round-trip resonance condition and alter the optical and microwave detuning  
 17 so that the resonance condition is satisfied only for a desired frequency region, as is done  
 18 to demonstrate the one-sided comb in Fig. 3c of the main text.

## 1 **Methods references**

2

- 3 34. Zhang, M., Wang, C., Cheng, R., Shams-Ansari, A. & Lončar, M. Monolithic ultra-  
4 high-Q lithium niobate microring resonator. *Optica* **4**, 1536–1537 (2017).
- 5 35. Wang, C., Zhang, M., Stern, B., Lipson, M. & Lončar, M. Nanophotonic lithium  
6 niobate electro-optic modulators. *Opt. Express* **26**, 1547–1555 (2018).
- 7 36. Kobayashi, T., Sueta, T., Cho, Y. & Matsuo, Y. High-repetition-rate optical pulse  
8 generator using a Fabry-Perot electro-optic modulator. *Appl. Phys. Lett.* **21**, 341–  
9 343 (1972).
- 10 37. Saitoh, T. *et al.* Modulation characteristic of waveguide-type optical frequency  
11 comb generator. *J. Light. Technol.* **16**, 824–832 (1998).
- 12 38. Kim, J., Richardson, D. J. & Slavík, R. Cavity effect on phase noise of Fabry-Perot  
13 modulator-based optical frequency comb. in *2016 IEEE Photonics Conference*  
14 *(IPC)* 503–504 (2016). doi:10.1109/IPCon.2016.7831201

## Data availability

The data sets generated and/or analysed during the current study are available from the corresponding authors on reasonable request.

1

### Extended data figure legends

2 **Extended Data Figure 1 | Dispersion engineering and tailored EO comb phase**  
3 **matching condition. a**, Simulated dispersion for an air-cladded lithium niobate (LN)  
4 ridge waveguide with top width  $w = 1800$  nm, film thickness  $t = 600$  nm and etch depth  
5  $h = 550$  nm. **b**, The phase matching condition for generating EO comb sidebands. The  
6 grey area shows the region of phase matching, with round-trip modulation strength,  $\beta$ .  
7 With a 50 GHz microwave drive and  $\beta = 1.2\pi$ , an EO comb spanning over 1.3 octave can  
8 be generated. **c**, Simulated dispersion for an air-cladded LN waveguide with a different  
9 geometry optimized for octave spanning comb with small microwave driving amplitude.  
10 **d**, An octave-spanning EO comb is shown with  $\beta = 0.3\pi$ . **e**, Another example of an air-  
11 cladded LN waveguide with dispersion engineered for broad comb generation. **f**, Such  
12 EO comb generation features a flat response over 600 nm. A broad comb spanning less  
13 than an octave can be generated in devices with small microwave modulation amplitudes  
14 and high- $Q$  factor optical resonators.