Grants Vs. Investment Subsidies

Ashok S. Rai and Tomas Sjöström

CID Working Paper No. 84
December 2001

© Copyright 2001 Ashok S. Rai and Tomas Sjostrom and the President and Fellows of Harvard College

Working Papers
Center for International Development at Harvard University
Grants Vs. Investment Subsidies

Ashok S. Rai*   Tomas Sjöström†

December 2001

We thank Beatriz Armendariz, Phil Bond, Mark Gersovitz, Magnus Hattlebakk, Dilip Mookherjee, Robert Townsend and seminar participants at Boston College, IFPRI, University of Virginia, Yale, NEUDC and the Cornell/ISPE conference for useful comments. The usual disclaimer applies.

*Corresponding author. Center for International Development, Harvard University, 79 JFK St, Cambridge MA 02138. ashok_rai@harvard.edu
†Department of Economics, Pennsylvania State University, University Park, PA 16802. jts10@psu.edu
Abstract

How should a government intervene to help the credit constrained poor? We study an economy where productivity and wealth are unobserved, and loans must be collateralized. We show that the efficient policy typically consists of offering both a grant and an investment subsidy. Everybody will take the grant and only the relatively productive will take the subsidy. This policy reduces but does not eliminate investment distortions.

**JEL Codes:** D82, H20, I38

**Keywords:** Credit constraints, collateral, intervention, loan subsidy, grant.
1 Introduction

Credit market imperfections can prevent the poor from making profitable investments, such as sending their children to school or starting businesses.\footnote{Many empirical studies have shown that personal wealth is positively associated with entrepreneurship (see Paulson and Townsend \cite{9} for Thailand or the Hubbard \cite{6} for the U.S). Banerjee \cite{1} surveys the literature on credit constraints and development.} In response, governments often subsidize investment. For instance, enrollment subsidies to induce schooling are a common intervention in developing countries. Examples include Bangladesh’s Food for Education, Brazil’s Bolsa Escola and Mexico’s Progresa, all programs which give transfers conditional on school attendance. Suppose there are no externalities to schooling however, so that the private return to investment equals the social return. Then subsidizing investment in schooling distorts incentives and leads to a deadweight loss. An investment subsidy encourages families with low returns from investment (or high opportunity costs) to invest, even though it is socially inefficient for them to do so. If these families were instead given a lump-sum grant, they would invest in education only if their return exceeded the cost of investment, which is socially efficient. Should education subsidy programs therefore eliminate the deadweight loss by replacing the subsidy with lump-sum grants? This article investigates the optimal mix of lump-sum grants and investment subsidies in an efficient government intervention.

We study a model where productivity and wealth are unobserved, but investment is observable. Only agents who post sufficient collateral with a bank can borrow. Without government intervention, banks will not lend to the productive poor since they have insufficient collateral. The government has the same information and enforcement constraints as banks. Unlike banks, however, the government wishes to maximize social welfare and has raised public funds at a cost to do so. Since the social cost of raising a dollar is greater than a dollar, the government wants to minimize its expenditures. How should the government spend its money? A lump-sum grant to all agents is expensive, since grants will be given to rich agents who do not need it, as well as to unproductive poor agents who will not (and should not) invest. On the other hand, investment subsidies typically induce unproductive borrowers to invest even though they should not. We derive the constrained efficient intervention, and we show that it typically consists of offering both a grant and an investment subsidy. All agents will take the grant, but only relatively productive agents will choose to invest and receive the subsidy.
The optimal policy screens productive and unproductive borrowers in a way which involves minimal information and bureaucratic co-ordination on the part of the government. The government can implement the efficient scheme by operating two separate departments: one that gives a grant to everybody, and another that gives an investment subsidy only to those who invest. It is crucial that agents cannot costlessly pretend to invest, claim the investment subsidy, and divert the money to consumption. If they could do this then every agent would claim the investment subsidy and there would be no screening of types. Our model applies to situations where investment cannot easily be falsified. A leading example is investment in human capital (education) when school attendance can be monitored. Investment in physical capital may be more difficult to monitor, however. There is considerable policy debate about whether the public funds used to subsidize microcredit programs such as the Grameen Bank should be given as lump-sum grants instead (Morduch [8]). If subsidized loans are given only to those agents who will invest, then they are equivalent to the investment subsidies in our model. So we suggest that subsidized lending may be part of an optimal intervention, as long as the government can ensure that the loans are actually used for investment. A similar argument is relevant to recent discussions on whether the World Bank should give grants instead of subsidized loans (Meltzer Commission Report [7]).

The optimal mix of grants and investment subsidies depends on the relative weights the government puts on equity and efficiency. Consider two extreme cases for illustration. First suppose that the government cares only about efficiency, i.e., it wishes to induce the productive poor to invest at the lowest possible cost. Such a government will want to subsidize investment, but since this subsidy encourages unproductive agents to invest it could lead to a large deadweight loss from excessive investment. By offering a lump-sum grant to everybody, the investment subsidy and hence the deadweight loss can be reduced, while still making investment possible for the productive poor (who can use both the grant and the subsidy to invest). Suppose instead the government cares only about equity, i.e., it wants to help the poorest regardless of their productivity. Such a government will not want to use investment subsidies because they go only to agents who invest. These agents are by necessity disproportionately rich since the poorest will still be credit constrained. Such a government will eliminate investment subsidies completely and give only lump-sum grants. This argument, albeit derived in a highly stylized model, suggests that governments should supplement enrollment subsidies with lump-sum grants if they care mainly about efficiency, but it should replace enrollment subsidies by lump-sum grants if they
care mainly about equity.

Several papers including Hoff and Lyon [5] and de Meza and Webb[4] have studied how governments should intervene to correct for credit market failures. While information asymmetries are the underlying friction in these papers, enforcement difficulties are the root of the credit market imperfection in our model. Hoff and Lyon compare grants and loan subsidies and argue that grants dominate loan subsidies because they improve the quality of borrowers. This is similar to the distortional effect of subsidies in our model. We do not restrict the government’s choice of intervention to one of these two instruments, however. Instead we take a mechanism design perspective, and solve for the constrained efficient intervention which is typically a mix of the two instruments. Moreover, we show how the optimal mix depends on the government’s trade-off between equity and efficiency. In a credit rationing model with excessive lending, de Meza and Webb[4] argue for subsidies to inactivity to improve the pool of borrowers. Such policies are of no help in our model.

In order to focus on the distributional role of grants and investment subsidies, we assume that rich agents can costlessly hide their wealth and claim to be poor. We therefore abstract from commonly used techniques such as the work requirements or in-kind transfers to sort between rich and poor (Besley and Coate [2], Blackorby and Donaldson [3]). Even though the rich and poor have the same preferences, and productivity and wealth is not necessarily correlated in our model, grants and investment subsidies have different distributional consequences. Typically the optimal subsidy will be strictly less than the full cost of the investment, and the remaining cost will have to be financed by loans. But then only those agents who can post sufficient collateral will invest and claim the investment subsidy. So investment subsidies will not reach the poorest of the poor. Since grants are universal, they are preferred by a government which has a strong desire for equity.

The paper is organized as follows. In section 2 we describe the economy. In section 3 we show that a simple policy of giving a grant and a subsidy is welfare maximizing. In section 4 we analyze the mix between grants and investment subsidies, first in a discrete example and then in the more general case. And in section 5 we conclude.
2 The Economy

Consider an economy with risk neutral agents and a competitive credit market. Each agent has a project which requires one dollar of investment. An agent’s type is \((\rho, w)\) where \(\rho\) is the return on the agent’s project and \(w\) is the agent’s wealth. The wealth is not liquid and so cannot be invested directly.\(^2\) It is an asset such as land which can be posted as collateral. Agents who want to invest can borrow on the credit market if they can post sufficient collateral. We assume \(\rho\) and \(w\) are independent random variables, where \(\rho \in [\rho_L, \rho_H]\) with distribution \(F\) and \(w \in [w_L, w_H]\) with distribution \(G\). The agent’s type is his private information. In particular, if the agent is asked to reveal his wealth, he can hide some (or all) of it. However, he cannot exaggerate his wealth: an agent whose true wealth is \(w\) cannot reveal more than \(w\) (this assumption is needed for collateral to make sense, since if wealth could be exaggerated, any agent could post unlimited amounts of “fake” collateral). Similarly, the existence of a project can be revealed by the agent. Thus a contract where an agent receives a loan conditional on investing is enforceable: the agent will simply be asked to show that he is indeed operating a project. The project may imply investment in fixed capital, renting factory space etc., and the agent who invests must prove that he indeed has invested in these variables. Alternatively, the investment may be in human capital, in which case the agent must prove that he is attending school in order to demonstrate investment. As discussed in the introduction, the assumption that investment can be monitored is important for us, for otherwise the investment subsidy could be claim by all agents whether or not they invest. In many cases, this assumption is plausible. On the other hand, it is also plausible that an agent who invests but does not want to reveal it can hide his project. We do allow this.\(^3\) So if an agent does not invest he cannot prove that he has not invested.

For simplicity and without loss of generality, there is no discounting and the interest rates on loans are zero. Thus the repayment on a loan of size \(x\) obtained from a bank is just \(x\). We assume \(\rho_L < 1\) and \(w_L < 1\) to make the problem non-trivial. For any \(r \leq \rho_H\), define \(\phi(r)\) as the average return for

\(^2\)Letting some fraction of the agents’ wealth be in liquid form would not change our results. Liquid wealth would allow some partial self-financing of projects, which would complicate the model without adding any new insight.

\(^3\)This assumption is not crucial. Our results would be go through if agents were unable to hide their investments.
agents with productivity $\rho \geq r$:

$$\phi(r) \equiv E[(\rho - r)|\rho \geq r] = \frac{1}{1 - F(r)} \int_r^{\rho_H} (\rho - r)dF(\rho)$$

Assume that all banks face an enforcement constraint: without collateral, the bank cannot prevent the agent from simply absconding without repaying. So banks must secure a loan of size $x$ with collateral which is worth at least $x$ to the borrower, in order to ensure repayment.

Without loss of generality, we assume that agents consume at the end of the period. So the timeline for an agent who invests is:

| Borrow and invest | Output $\rho$ is realized | Repay and consume |

If there is no government intervention, then agents with $\rho < 1$ would not invest even if they could borrow a dollar, since the repayment of the dollar would exceed the project return. Agents with $\rho \geq 1$ and $w < 1$ would like to borrow but would have insufficient collateral to do so. They are credit constrained. Only agents with $\rho \geq 1$ and $w \geq 1$ are both willing and able to invest in a project. This is illustrated in figure 1. The total surplus in this laissez-faire economy is

$$\int_1^{\rho_H} \int_1^{\rho_H} (\rho - 1)dF(\rho)dG(w) = (1 - G(1))(1 - F(1))\phi(1),$$

(1)

where $(1 - G(1))(1 - F(1))$ is the fraction of agents who invest in the absence of any intervention and $\phi(1)$ is the expected surplus for a typical investor.

### 3 Economic Policy

Now we introduce a principal (donor or government) with shadow cost of public funds $\lambda > 1$. The principal has no information or enforcement advantages over the credit market. Just like the agents, the principal can borrow

---

4 Other justifications of collateral could be considered, and they would not significantly change our analysis. For example, if project returns are stochastic and unobserved ex post, then an agent can always claim that the project yielded no surplus so that he cannot repay. In this case, collateral is used to make him report his returns truthfully.

5 Notice that the credit constraints in this model arise from the difficulty of enforcing repayments and not from informational asymmetries. Even if type were observed, agents with $\rho \geq 1$ and $w < 1$ would still be credit constrained. Notice also that as long as the loan is secured by collateral, banks do not need to observe whether or not agents invest in order to make loans.
on the financial markets at zero interest rate. Like the banks, the principal cannot observe an agent’s type or his realized project return, nor can she enforce repayment on loans unless they are collateralized. Moreover, the principal cannot force an agent to participate in her scheme. Thus, the principal’s mechanism must respect incentive compatibility, individual rationality, and enforcement constraints. In this section, we show that the outcome of any mechanism that satisfies these constraints can be replicated by a very simple policy consisting of a grant and an investment subsidy. This result will be used to characterize optimal policies in the next section.

Using the revelation principle, we can restrict attention to the following type of mechanism. Each agent reports his type \((\rho, w)\) to the principal. The principal then tells the agent whether or not he should invest, and gives him a transfer \(x(\rho, w)\). Let \(i(\rho, w) = 1\) if the agent of type \((\rho, w)\) is told to invest, and \(i(\rho, w) = 0\) otherwise. (Since agents are risk neutral, it will not be useful assign probabilities of investment strictly between zero and one.) This mechanism is feasible because an agent who is told to invest \((i(\rho, w) = 1)\) can be required to verify that she actually is operating a project. An agent with \(i(\rho, w) = 0\) is free to do what she wants. However, it is without loss of
generality to assume that all agents with \( i(\rho, w) = 0 \) prefer not to invest.\(^6\)

Agents who are told to invest but have insufficient funds to finance the investment will take a loan from a bank.\(^7\) Whether or not the principal observes the agent taking a loan from the bank is immaterial. Instead, what is important is that the principal can verify if agents with \( i(\rho, w) = 1 \) are actually operating projects. Notice also that if an agent reveals his type to the principal, banks will not benefit from this information. The reason is that banks will lend to agents (regardless of their type) if and only if they post sufficient collateral, and so additional information does not change behavior of the banks.

The principal’s mechanism must respect individual rationality, enforcement and incentive compatibility constraints. Individual rationality requires that transfers are non-negative (agents will not show up voluntarily to “pay a tax”). Thus, for each \((\rho, w)\),

\[ x(\rho, w) \geq 0 \tag{2} \]

The enforcement constraint implies that if an agent is told to invest, then he must have sufficient collateral. If \( x(\rho, w) < 1 \) then an agent of type \((\rho, w)\) will only need a loan of size \( 1 - x(\rho, w) \) in order to invest, for the transfer \( x(\rho, w) \) can be used as partial financing. (If \( x(\rho, w) \geq 1 \) then of course the agent does not need a loan at all). Thus, for each \((\rho, w)\), the enforcement constraint is

\[ w + x(\rho, w) \geq 1 \text{ if } i(\rho, w) = 1 \tag{3} \]

Incentive-compatibility requires that each agent prefers to announce his type truthfully. Thus, we need to check that type \( t^* \) cannot benefit from pretending to be type \( t \neq t^* \). Let \( c(t^*, t) \) denote the consumption of an agent whose true type is \( t^* = (\rho^*, w^*) \) but who claims to be type \( t = (\rho, w) \), where \( w \leq w^* \). Notice that we need only consider the case \( w \leq w^* \), because wealth cannot be exaggerated so an agent of type \((\rho^*, w^*)\) cannot claim to be a type \((\rho, w)\) with \( w > w^* \). Notice also that \( c(t^*, t^*) \) denotes the consumption for type \( t^* \) if he behaves truthfully.

What does type \( t^* \) obtain by behaving like type \( t \)? Suppose first that \( i(\rho, w) = 1 \). If \( x(\rho, w) < 1 \), then the agent will need a loan of size \( 1 - x(\rho, w) \),

\(^6\)Suppose there is a mechanism where some type \((\rho, w)\) is not told to invest, but he prefers to invest and therefore does so. That mechanism can be replaced by an equivalent mechanism where type \((\rho, w)\) is told to invest.

\(^7\)This is without loss of generality. We could allow the principal himself to give loans, but since the principal can obtain funds at the interest rate of zero (which is also the bank lending rate) there is no advantage in doing this.
which can be obtained at a zero interest rate. Moreover, since type \( t = (\rho, w) \)
must have enough collateral to borrow this amount, if type \( t^* \) pretends to be type \( t \) then type \( t^* \) must also have enough. The type \( t^* \) agent produces \( \rho^* \) and his final consumption will be

\[
c(t^*, t) = \rho^* - (1 - x(\rho, w)) = \rho^* + x(\rho, w) - 1
\]

If instead \( x(\rho, w) \geq 1 \), then the agent needs no loan. He will invest one dollar and consume \( x(\rho, w) - 1 \). His final consumption is again \( c(t^*, t) = \rho^* + x(\rho, w) - 1 \). If \( i(\rho, w) = 0 \) then the agent will simply consume his transfer, \( c(t^*, t) = x(\rho, w) \). Thus, in general, for any \( (\rho, w) \) we will have

\[
c(t^*, t) = x(\rho, w) + i(\rho, w) (\rho^* - 1)
\]

The incentive compatibility condition is: for all \( t^* = (\rho^*, w^*) \) and all \( t = (\rho, w) \) with \( w \leq w^* \),

\[
c(t^*, t^*) \geq c(t^*, t) \tag{4}
\]

Our first proposition shows that any intervention constrained by (2),(3) and (4) can be implemented by a simple policy of grants and investment subsidies that are non-decreasing in wealth. The intuition is the following. For a given wealth level and investment decision, the transfers cannot depend on the return that agents claim to have, because all agents could claim the return with the highest associated transfer. Moreover, transfers cannot be decreasing in wealth, for then rich agents would hide their wealth. Transfers cannot be used to penalize agents who invest, for then agents who invest would hide their projects. However, a feasible intervention can favor those who invest relative to those who do not invest (since projects are verifiable), and it can also be biased in favor of the rich (because poor agents cannot pretend to be rich, by assumption).

**Proposition 1** Any individually rational, enforcement constrained and incentive compatible intervention can be implemented by the following policy. Each agent receive a wealth-dependent grant \( g(w) \geq 0 \). Those agents who invest also receive (in addition to the grant) a wealth-dependent subsidy of \( s(w) \geq 0 \). The functions \( g(w) \) and \( g(w) + s(w) \) must be non-decreasing in \( w \).

**Proof.** The proof has two steps. First, we derive certain conditions that any individually rational, enforcement constrained and incentive compatible intervention must satisfy. Second, we show that these conditions imply that the intervention can be mimicked by a simple grant/subsidy scheme.

---

8
Step 1. If \( i(\rho, w) = i(\rho^*, w^*) \), then the incentive-compatibility condition (4) reduces to

\[
x(\rho^*, w^*) \geq x(\rho, w).
\]

Thus, for any two agents who take the same action (invest or not), the transfer must be non-decreasing in the agent’s wealth. Now, suppose \( i(\rho, w) = i(\rho^*, w^*) \) and \( w = w^* \). Then, (4) says that

\[
x(\rho^*, w) = x(\rho, w).
\]

That is, two types with the same wealth, but different productivity levels, who take the same action (invest or not), must get the same transfer. Thus, the transfer depends only on your wealth and your action. This allows us to write the transfer as

\[
x(\rho, w) = \hat{x}(w, i(\rho, w)).
\]

(5)

Moreover, suppose \( w = w^* \) and \( \rho^* \neq \rho \). Then, (4) implies both

\[
c(t^*, t^*) \geq c(t^*, t)
\]

(6)

and, reversing the roles of \( t \) and \( t^* \),

\[
c(t, t) \geq c(t, t^*).
\]

(7)

Using (5), the two conditions (6) and (7) can be written as

\[
\hat{x}(w^*, i(\rho^*, w^*)) + i(\rho^*, w^*) (\rho^* - 1) \geq \hat{x}(w^*, i(\rho, w^*)) + i(\rho, w^*) (\rho^* - 1)
\]

and

\[
\hat{x}(w^*, i(\rho, w^*)) + i(\rho, w^*) (\rho - 1) \geq \hat{x}(w^*, i(\rho^*, w^*)) + i(\rho^*, w^*) (\rho - 1)
\]

If we add the two conditions, we get:

\[
(i(\rho^*, w^*) - i(\rho, w^*)) (\rho^* - \rho) \geq 0
\]

Thus, \( \rho^* > \rho \) implies \( i(\rho^*, w^*) \geq i(\rho, w^*) \). So, if two agents \( t \) and \( t^* \) have the same wealth but type \( t^* \) is more productive, then either they take the same action, or agent \( t^* \) invests while agent \( t \) does not. Therefore for each wealth level \( w \), there is a cut-off level \( \hat{\rho}(w) \) such that type \( (\rho, w) \) invests if
and only if \( \rho \geq \hat{\rho}(w) \).\(^8\) Now, consider types \((\hat{\rho}(w), w)\) and \((\hat{\rho}(w) - \delta, w)\). The IC conditions are

\[
x(\hat{\rho}(w), w) + \hat{\rho}(w) - 1 \geq x(\hat{\rho}(w) - \delta, w)
\]

\[
x(\hat{\rho}(w) - \delta, w) \geq x(\hat{\rho}(w), w) + \hat{\rho}(w) - \delta - 1
\]

Thus,

\[
\hat{\rho}(w) - 1 \geq x(\hat{\rho}(w) - \delta, w) - x(\hat{\rho}(w), w) \geq \hat{\rho}(w) - \delta - 1
\]

so that as \( \delta \to 0 \),

\[
x(\hat{\rho}(w) - \delta, w) - x(\hat{\rho}(w), w) \to \hat{\rho}(w) - 1
\]

But

\[
x(\hat{\rho}(w) - \delta, w) = \hat{x}(w, 0)
\]

since \( \hat{\rho}(w) \) is the cut-off value, and

\[
x(\hat{\rho}(w), w) = \hat{x}(w, 1)
\]

Thus,

\[
\hat{x}(w, 0) - \hat{x}(w, 1) = \hat{\rho}(w) - 1
\]

so that

\[
\hat{x}(w, 1) + \hat{\rho}(w) - 1 = \hat{x}(w, 0)
\]

Now, define \( g(w) \equiv \hat{x}(w, 0) \) and \( s(w) \equiv \hat{x}(w, 1) - \hat{x}(w, 0) \). Since wealth can be hidden, incentive compatibility implies that the two functions \( \hat{x}(w, 0) = g(w) \) and \( \hat{x}(w, 1) = g(w) + s(w) \) are non-decreasing in \( w \). Individual rationality implies \( g(w) \geq 0 \). Also, the fact that it cannot be verified that an agent does not invest forces \( \hat{x}(w, 1) \geq \hat{x}(w, 0) \), that is \( s(w) \geq 0 \).

Step 2. The conditions found in step 1 imply that the intervention can be replaced by the following simple policy. Each agent reveals his wealth \( w \) to the principal. The principal immediately gives him a grant \( g(w) \) and asks the agent if he wants to invest. If the answer is no, the agent consumes

\(^8\)There must be a cut-off level such that type \( t \) invests if \( \rho > \hat{\rho}(w) \) and does not invest if \( \rho < \hat{\rho}(w) \). However, strictly speaking, an agent who is the “cut-off type” \( \rho = \hat{\rho}(w) \) may or may not invest. Without loss of generality we assume that he does invest if \( \rho = \hat{\rho}(w) \).
his grant. If the answer is yes, the agent receives an additional “investment subsidy” of $s(w)$, goes to the bank, obtains a loan of $1 - g(w) - s(w)$, and starts his project. We claim that this policy satisfies all constraints and gives exactly the same outcome as the initial mechanism (in step 1).

First of all, notice that no agent wants to hide his wealth since $g(w)$ and $g(w) + s(w)$ are both non-decreasing in $w$. Second, an agent will prefer to invest in the project if and only if

$$\rho + g(w) + s(w) - 1 \geq g(w)$$

which is equivalent to

$$\rho \geq 1 - s(w) = 1 - \hat{x}(w, 1) + \hat{x}(w, 0) \equiv \hat{\rho}(w)$$

But that means he invests if and only if he invests in the initial mechanism. Finally, agents who invest consume

$$\rho + g(w) + s(w) - 1 = \rho + \hat{x}(w, 1) - 1$$

and those who do not invest consume $g(w) = \hat{x}(w, 0)$ as in the initial mechanism. ■

Let $V[c, \rho, w]$ denote the value the principal assigns to giving $c$ units of consumption to type $t = (\rho, w)$. The social welfare function is

$$W = \int_{w_L}^{w_H} \int_{\rho_L}^{\rho_H} V[c(\rho, w), \rho, w] dF(\rho)dG(w) - \lambda \int_{w_L}^{w_H} \int_{\rho_L}^{\rho_H} x(\rho, w)dF(\rho)dG(w)$$

(8)

A policy that maximizes this expression subject to individually rationality, enforcement, and incentive compatibility constraints, will be called a socially optimal policy.

The fact that wealth cannot be exaggerated makes it possible for the principal to systematically favor the rich over the poor. For example, anyone who can prove that he already has a dollar could be given another dollar without violating incentive compatibility. But such policies are ruled out by a minimum sense of equity. Assume $V$ is non-increasing in $w$, so the principal does not favor inequity. Then, it is clear that the principal will never benefit from letting $g(w)$ and $g(w) + s(w)$ be strictly increasing in $w$. Policies that give higher transfers to rich agents who take the same actions as poor agent are dominated by policies that give no more to the rich than to the poor. Therefore, we may restrict attention to very simple policies: the government gives each agent a lump-sum transfer or grant $g \geq 0$, and agents who invest
get an investment subsidy \( s \geq 0 \) (in addition to the grant). Neither the grant nor the investment subsidy depends on wealth. For convenience, this result is stated formally in the next proposition.

**Proposition 2** Suppose the social welfare weight given to an individual is non-increasing in his wealth. Then, any socially optimal policy can be replicated by a simple policy where each agent gets a grant \( g \geq 0 \), and those agents who invest also receive (in addition to the grant) a subsidy of \( s \geq 0 \). The grants and the subsidies do not depend on wealth levels.

Notice that this policy of offering a grant \( g \), as well as the choice of whether or not to take an additional subsidy \( s \), is just one of several policies that can implement a socially optimal outcome. For instance, the principal could offer agents a choice between a grant of \( g \) and a subsidized loan of size \( 1 - g \) with repayment \( 1 - s - g \) that is conditional on investment. The outcome would be the same. Alternatively, the principal could offer a choice between an unconditional transfer \( g \) and a transfer \( g + s \) that is conditional on investment.

## 4 Optimal Intervention

### 4.1 Discrete Case

The intuition for the trade-off between grants and investment subsidies can be brought out most clearly in a discrete example. Although the propositions in the previous section were stated for the continuous case, they go through equally well for the discrete case. Suppose there is an agent who needs a dollar to invest. The productivity (the return on a dollar invested) of the agent is either \( \rho_L \) or \( \rho_H \), where \( \rho_L < \rho_H \). The wealth of the agent is either 0 or \( \bar{w} \), where \( \bar{w} < 1 \). Wealth and productivity levels are independently drawn and private information for each agent. Since \( \bar{w} < 1 \), no agent can collateralize a loan of one dollar. Thus, we may call agents with wealth \( \bar{w} \) “fairly poor” and those with wealth 0 “very poor.” Assume \( 0 < \rho_L < 1 < \rho_H \), so only the high-productivity types can generate a social surplus from their projects. Finally, to simplify the calculations we assume \( \bar{w} < \rho_L \).

The probability that the agent’s productivity is high is \( \pi \), the probability his wealth is high is \( p \). We will assume \( V[c, \rho, w] \) is of the form \( V[c, \rho, w] = ch(w) \), where \( h(0) \) is the social weight given to very poor agents, and \( h(\bar{w}) \) the weight given to fairly poor agents is \( h(\bar{w}) \). The average weight is 1:

\[
p h(\bar{w}) + (1 - p) h(0) = 1 \tag{9}
\]
The social welfare function is

\[ W = ph(\bar{w}) [\pi c(\rho_H, \bar{w}) + (1 - \pi)c(\rho_L, \bar{w})] + (1 - p)h(0) [\pi c(\rho_H, 0) + (1 - \pi)c(\rho_L, 0)] - \lambda \times (\text{total transfer}) \]

Notice that \( \lambda > 1 \) and (9) guarantees that if there were no investment opportunities, then the government would not want to give a lump-sum transfer to everybody, because the average value of a dollar consumption is only a dollar. Thus, the only justification for an intervention is to encourage investment by the productive poor. The question is whether such an intervention should use grants, subsidies, or both.

By proposition 2, we can focus on policies of the following type: the government gives a lump-sum transfer \( g \geq 0 \) to all agents, and an investment subsidy \( s \geq 0 \) to all agents who invest. Now, suppose the government wants the productive rich to invest. They need \( 1 - \bar{w} \) from the government in order to do so. We shall compare two policies to induce the productive rich to invest. The first will involve only a grant, while the second will involve both a grant and a subsidy.

**Policy 1** consists simply of a grant \( g = 1 - \bar{w} \), and no subsidy, \( s = 0 \). The type \((\rho_H, \bar{w})\) agents will take the grant \( g = 1 - \bar{w} \), borrow \( 1 - g = \bar{w} \) (which they can collateralize with their wealth), invest a dollar, and consume \( \rho_H + g - 1 = \rho_H - \bar{w} \). All others will consume the grant \( g = 1 - \bar{w} \) without investing. The total welfare with this policy is

\[ W_1 = p\pi h(\bar{w}) (\rho_H - \bar{w}) + p(1 - \pi)h(\bar{w}) (1 - \bar{w}) + (1 - p)h(0) (1 - \bar{w}) - \lambda (1 - \bar{w}) \]

\[ = p\pi h(\bar{w}) (\rho_H - 1) - (\lambda - 1) (1 - \bar{w}) \]

Notice that the grant has to be given to all agents, which is costly. Alternatively, the government could give an investment subsidy. To discourage unproductive agents from investing, the subsidy cannot be greater than \( 1 - \frac{1}{2}L \). But, since \( 1 - \rho_L + \bar{w} < 1 \), the subsidy must be complemented by a grant \( g = \rho_L - \bar{w} \) in order to make it possible for productive fairly poor agents to invest.

**Policy 2** consists of a grant \( g = \rho_L - \bar{w} \) and a subsidy \( s = 1 - \rho_L \). The type \((\rho_H, \bar{w})\) agents will take the grant \( g = \rho_L - \bar{w} \), borrow \( 1 - g - s = \bar{w} \), invest one dollar, and consume \( \rho_H + g + s - 1 = \rho_H - \bar{w} \). All others will consume the grant \( g = \rho_L - \bar{w} \). Total welfare is

\[ W_2 = p\pi h(\bar{w}) (\rho_H - \bar{w}) + p(1 - \pi)h(\bar{w}) (\rho_L - \bar{w}) + (1 - p)h(0) (\rho_L - \bar{w}) - \lambda p\pi (1 - \rho_L) \]

\[ = p\pi h(\bar{w}) (\rho_H - \rho_L) - (\lambda - 1) (\rho_L - \bar{w}) - \lambda p\pi (1 - \rho_L) \]
Clearly, policy 2 is cheaper for the government than policy 1. Indeed, with policy 1 the government gives $1 - \bar{w}$ to everybody, but with policy 2 they give $1 - \bar{w}$ only to those who invest, and only $\rho_L - \bar{w}$ to those who do not invest. In general, combining a grant and a subsidy allows the government to screen the agents and provide investment opportunities at a lower cost than by using only grants. The problem is that policy 2 reduces cost by reducing the transfers to those who do not invest, but those agents are disproportionately very poor. Now, if $\lambda < h(0)$, the government may well want to provide extra money to the very poor. The government cannot observe wealth directly, but it knows who does not invest, and so by reducing the investment subsidy and increasing the grant, the government transfers money from investors (none of which are very poor) to non-investors (who are disproportionately very poor). Thus, a government which is concerned with equity may prefer to eliminate the investment subsidy and give lump-sum grants to everybody instead, even though that raises their expenditures. Formally, it can be checked that $W_2 > W_1$ if and only if
\[
p\pi h(\bar{w}) + \lambda - 1 > \lambda p \pi
\]
or equivalently
\[
\frac{\lambda - 1}{\lambda - h(\bar{w})} > p \pi
\]

If the principal cares only about efficiency, then $h(\bar{w}) = 1$ and policy 2 is strictly preferred because it is cheaper. If the principal cares about equity then $h(\bar{w}) < 1 < h(0)$ and the left hand side of (10) is strictly between 0 and 1. The inequality (10) can be re-written as:
\[
h(\bar{w}) > \lambda - \frac{\lambda - 1}{p \pi} = 1 - \frac{(1 - p \pi) \lambda}{p \pi}
\]

If
\[
\lambda < \frac{1}{1 - p \pi}
\]
then the right hand side of (11) is positive. Thus, if $h(\bar{w})$ is sufficiently small (11) is violated and policy 1 dominates. We conclude that if the cost of public funds is not too great, $\lambda < 1/(1 - p \pi)$, and the principal cares enough about the poor, so that $h(\bar{w}) < (1 - (1 - p \pi) \lambda)/p \pi$, then she prefers giving only grants rather than combining grants and subsidies. The explanation is that a grant-subsidy mix gives a lower consumption to those agents who do not invest (a lower grant). But the agents who do not invest are disproportionately poorer than the agents who do invest. So the principal prefers the grant which is higher and the same for everybody.
4.2 Continuous Case

In this section, we show that the effects identified in the discrete example are also present in a more general model. In particular, as explained in the previous section, subsidies induce investment at a lower cost than grants. However, this has to be traded off against another effect: investment subsidies distort incentives and lead to “over investment” by less productive agents, and do not reach the poorest. The first order conditions for the principal’s problem represents the trade off between these two effects.

Let \( h(w) \) be the “welfare weight” given to the consumption of agents with wealth \( w \), where

\[
\int_{w_L}^{w_H} h(w) dG(w) = 1
\]

and \( h'(w) \leq 0 \) for all \( w \). Again we will assume \( V[c, \rho, w] \) is of the form

\[
V[c, \rho, w] = ch(w)
\]

Thus, the social welfare function is

\[
W = \int_{w_L}^{w_H} \int_{\rho_L}^{\rho_H} c(\rho, w) h(w) dF(\rho) dG(w) - \lambda \int_{w_L}^{w_H} \int_{\rho_L}^{\rho_H} x(\rho, w) dF(\rho) dG(w)
\]

(13)

By proposition 2, the optimal policy is of the form \((g, s)\). Let the agents who take the investment subsidy be called investors. Let the agents who do not take the investment subsidy be called consumers. Type \((\rho, w)\) can invest if he has enough collateral to finance his loan:

\[
w \geq 1 - s - g
\]

He prefers to invest only if his consumption from investing exceeds his consumption from not investing (and consuming the grant \( g \)):

\[
\rho - (1 - s - g) \geq g
\]

or equivalently, if

\[
\rho \geq 1 - s
\]

Therefore, a type \((\rho, w)\) will invest if and only if \( \rho \geq 1 - s \) and \( w \geq 1 - s - g \). This is illustrated in figure 2, which shows how an intervention
that involves $g > 0$ and $s > 0$ partitions types into those who invest and those who do not. Such an intervention induces some agents who were previously credit constrained to invest. But it also allows investment by relatively unproductive agents with $1 - s < \rho < 1$ for whom the social cost of the subsidy, $\lambda s$, exceeds the net return of their project.

From equation (13) social welfare is

$$W = \int_{1-s-g}^{w_H} \int_{1-s}^{p_H} (h(w)(\rho + g + s - 1) - \lambda s) dF(\rho) dG(w) + \int_{w_L}^{1-s-g} h(w) dG(w)$$

$$+ F(1-s) \int_{1-s-g}^{w_H} h(w) dG(w) - \lambda g$$

which can be rewritten as

$$\int_{1-s-g}^{w_H} h(w) \int_{1-s}^{p_H} (\rho - (1-s)) dF(\rho) dG(w)$$

$$- \lambda s(1 - G(1 - s - g))(1 - F(1-s)) + g \int_{w_L}^{w_H} h(w) dG(w) - \lambda g$$

$$= I(s, g) [\phi(1-s) \Psi(1-s-g) - \lambda s] - (\lambda - 1) g$$

where $\rho - (1-s)$ as the net return on the project, $\phi(1-s)$ is the expected
net return of investors and
\[ I(s, g) \equiv (1 - G(1 - s - g))(1 - F(1 - s)) \]
is the fraction of investors. We define \( \Psi(z) \) as the average welfare weight on agents with \( w \geq z \)
\[ \Psi(z) \equiv E\{h(w) \mid w \geq z\} = \frac{1}{1 - G(z)} \int_{z}^{w_H} h(w) dG(w) \]
The first order conditions for the maximization of \( W \) are:
\[ \frac{\partial W}{\partial g} = 0 = \frac{\partial I(s, g)}{\partial g} \left[ \phi(1 - s) \Psi(1 - s - g) - \lambda s \right] - I(s, g) \phi(1 - s) \Psi'(1 - s - g) + 1 - \lambda \quad (14) \]
and
\[ \frac{\partial W}{\partial s} = 0 = -I(s, g) \left[ \phi'(1 - s) \Psi(1 - s - g) + \phi(1 - s) \Psi'(1 - s - g) + \lambda \right] \]
\[ + \frac{\partial I(s, g)}{\partial s} \left[ \phi(1 - s) \Psi(1 - s - g) - \lambda s \right] \quad (15) \]
Interpreting these first-order conditions clarifies the trade-off between spending an additional dollar on grants or on subsidies. The condition (14) can be rewritten as
\[ \frac{\partial I(s, g)}{\partial g} \left[ \phi(1 - s) \Psi(1 - s - g) - \lambda s \right] - I(s, g) \phi(1 - s) \Psi'(1 - s - g) = \lambda - 1 \quad (16) \]
Recall that type \((\rho, w)\) invests in his project if and only if \( w \geq 1 - s - g \) and \( \rho \geq 1 - s \). If \( g \) is increased, a number \( \frac{\partial I(s, g)}{\partial g} \) of agents can now invest, and the expected return on their projects is \( \phi'(1 - s) \). A dollar to agents with wealth greater than \( 1 - s - g \) has welfare weight \( \Psi(1 - s - g) \). Moreover, all investors receive the subsidy \( s \) at social cost \( \lambda \). This explains the first term in (16). The second term is due to the fact that lowering the wealth levels of investors raises the average social value of each dollar going to the investors, since \( \Psi' \leq 0 \). Thus, this term is non negative. The right hand side is the net social cost of giving 1 dollar to all agents.
The condition (15) can be rewritten as

$$\frac{\partial I(s, g)}{\partial s} [\phi(1 - s)\Psi (1 - s - g) - \lambda s] = I(s, g) [\phi'(1 - s)\Psi (1 - s - g) + \phi(1 - s)\Psi'(1 - s - g) + \lambda]$$

(17)

If $s$ is increased the number of investors increases by $\frac{\partial I(s, g)}{\partial s}$. As before, each additional investor raises social welfare by

$$\phi(1 - s)\Psi (1 - s - g) - \lambda s$$

This explains the left hand side of (17). On the other hand, each of the $I(s, g)$ investors will benefit from the increased subsidy. Now, the average surplus of investors falls because less productive investors are attracted by the subsidy: the social cost of this is $\phi'(1 - s)\Psi (1 - s - g)$. On the other hand, the lower wealth levels of investors raises the average value of the surplus earned by the investors: this is the term $\phi(1 - s)\Psi'(1 - s - g) < 0$. Finally, each dollar has a cost $\lambda$. This explains the right hand side of (17).

5 Conclusions

In this paper we study the problem faced by a government that cannot observe which households are credit constrained, and which households are not. Subsidized loans and grants when offered by themselves are too blunt to distinguish between the two. But by offering both, the government can induce unproductive agents to reveal their type. This raises welfare by reducing inefficient investments (where the return is lower than the social cost), but does not eliminate such investments completely.

We make the extreme assumption that the government cannot target the poor and exclude the rich. If wealth was observed in our model, then the poor would still be credit constrained in the absence of any intervention. The efficient intervention would be wealth dependent and targeted only to those who cannot access credit market. It would take the form of a grant $g(w)$ and an investment subsidy $s(w)$, where $g(w) = s(w) = 0$ for those agents with $w \geq 1$ (access to the credit market). It will be incentive compatible for both the grant and the investment subsidy to be decreasing in $w$. For a given wealth level, though, by offering both a positive grant and a positive investment subsidy the principal would typically be able to sort between productive and unproductive borrowers. If the cost of public funds were high, the principal may choose to give only grants to the very poor.
References


