Particle Acceleration and Heating in Low Mach Number Collisionless Shocks

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Particle Acceleration and Heating in Low Mach Number Collisionless Shocks

A DISSERTATION PRESENTED
BY
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TO
THE DEPARTMENT OF ASTRONOMY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE SUBJECT OF ASTRONOMY AND ASTROPHYSICS

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Particle Acceleration and Heating in Low Mach Number Collisionless Shocks

Abstract

Low Mach number collisionless shocks occur during the mergers of galaxy clusters. Radio and X-ray observations reveal that particles are accelerated and heated in these shocks. However, exactly how the particles, especially the electrons whose radiative signature we directly observe, are actually energized has been poorly understood. In this thesis, I use multi-dimensional first-principles numerical simulations to elucidate the microphysics of particle acceleration and heating in low Mach number collisionless shocks.

The first part of the thesis focuses on particle acceleration. I show via simulations that electrons are efficiently accelerated in low Mach number quasi-perpendicular shocks, while protons are not. I identify a Fermi-like electron acceleration mechanism in which particle injection operates via shock drift acceleration, and long term evolution is sustained by electron self-generated waves driven by the oblique electron firehose instability. I then explore how the efficiency of electron acceleration depends on pre-shock conditions of the plasma. I find that the mechanism I have identified works for shocks with high plasma beta, at nearly all magnetic field obliquities, and for electron temperatures in the range relevant for galaxy clusters. My findings offer a natural explanation for the bright radio synchrotron emission observed in the outskirts of galaxy clusters. Previously, this radiation was considered problematic since electron acceleration was believed to be inefficient in low Mach number shocks.

In the second part of the thesis I focus on particle heating in low Mach number shocks.
I show that protons are heated to higher temperatures than electrons in perpendicular shocks. However, electrons still experience a non-trivial amount of irreversible heating, i.e., their entropy increases as they travel through the shock. I develop a model for particle irreversible heating which requires the presence of two elements: (i) temperature anisotropy between field-parallel and field-perpendicular directions, and (ii) a mechanism to break the adiabatic dynamics of particles. For electrons in shocks, the temperature anisotropy is induced by field amplification via compression, while adiabaticity is broken by the action of electron whistler waves which are triggered by the same anisotropy. I successfully validate the model through detailed comparisons with first-principles numerical simulations. I then explore how the efficiency of electron heating depends on pre-shock conditions of the plasma. I provide an empirical fitting formula for the irreversible heating of electrons in low Mach number shocks and use it to predict the post-shock electron-to-proton temperature ratio as a function of two key parameters, the sonic Mach number of the shock and the plasma beta of the upstream gas. The model predictions compare favorably with observational data.
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FOR MY FAMILY.
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Chapter 1

Introduction

1.1 Collisionless Plasma Astrophysics

Most of the baryonic matter in the observable Universe is in the form of ionized gas, i.e., it is a plasma. In this state, the free electrons and ions interact with one another through Coulomb collisions as well as their collective electromagnetic fields. Though, on the largest astrophysical scales, gas dynamics in the Universe is dominated by gravity, plasma physics regulates the physics on almost all other measurable scales.

Most of the plasma in the Universe is dilute and hot, which means that the particle mean free-path due to Coulomb collisions is much larger than the system size. This is the so-called collisionless limit, where particles only interact through their collective electromagnetic fields.

The absence of Coulomb collisions can lead to many interesting consequences. For instance, the different particle species in the plasma can maintain separate temperatures. In fact, even within a single species, the particles can be out of thermal equilibrium. Thus, the particle distribution may be anisotropic, with particles having different temperatures parallel and perpendicular to the local magnetic field. This can lead to a variety of plasma in-
stabilities that will ultimately regulate the degree of anisotropy. In addition, once a certain fraction of a given particle species gain more energy than the rest of the population, they can retain the additional energy and develop into a distinct non-thermal sub-population, often with a power-law energy distribution.

Collisionless shocks encompass all of the processes mentioned above. The study of these shocks is therefore a rich and fascinating subject, with a wide range of applications in astrophysics.

1.2 Collisionless Shocks

Shocks are formed when supersonic flows encounter an obstacle, e.g., when fast flows interact in head-on collisions, or over-run each other, or are stopped by an ambient medium. While collisional shocks are mediated by physical collisions among particles, collisionless shocks are mediated purely by electromagnetic processes, since physical collisions are extremely rare.

Collisionless shocks are ubiquitous in the Universe, and occur on a variety of scales. In the heliosphere, collisionless planetary bow shocks form when the supersonic solar wind hits the magnetosphere of a planet. On galactic scales, when a star’s death results in a supernova, the explosion ejects a large amount of stellar material at highly supersonic speed into the interstellar medium and forms collisionless shocks. Additionally, gamma-ray bursts, pulsar winds, jets from active galactic nuclei (AGN), etc., are all known to drive collisionless shocks. On yet larger scales, mergers of galaxy clusters generate some of the largest collisionless shocks in the Universe.

The astrophysical shocks listed above span a huge range of not only length scales, but also
of other parameters relevant to shock physics. Based on the flow speed, we can distinguish between non-relativistic and relativistic shocks. Shocks in gamma-ray bursts, pulsar winds and AGN jets fall into the category of relativistic shocks, while shocks in the heliosphere, supernova remnants and galaxy clusters are non-relativistic shocks.

Among non-relativistic shocks, one can further characterize shocks by their Mach number. This thesis is concerned with non-relativistic collisionless shocks with low sonic Mach number, $M_s \lesssim 5$ (in contrast to high Mach number shocks in supernova remnants\(^1\)), where the sonic Mach number,

$$M_s \equiv \frac{V_1}{c_s} = \frac{V_1}{\sqrt{\Gamma k(T_{1i} + T_{1e})/(m_i + m_e)}},$$

is defined as the ratio of the upstream flow velocity $V_1$ to the sound speed of the medium $c_s$, with the latter estimated via the upstream ion temperature $T_{1i}$ and electron temperature $T_{1e}$, the adiabatic index of the plasma $\Gamma$, and the masses of the two particle species, $m_i$ and $m_e$.

Since almost all astrophysical shocks are magnetized to some extent, shocks are also characterized by their magnetization through the plasma beta parameter,

$$\beta_p \equiv \frac{8\pi n_1 k(T_{1i} + T_{1e})}{B_1^2},$$

which is defined as the ratio of the thermal pressure to the magnetic pressure in the upstream, where $n_1$ is the upstream number density\(^2\), and $B_1$ is the strength of the upstream magnetic field. In this thesis, we primarily focus on collisionless shocks with high $\beta_p$ ($\beta_p \gg 1$).

\(^1\)Cosmological simulations also show that high Mach number accretion shocks shall exist beyond the virial radius of galaxy clusters, but they have not yet been observed.

\(^2\)Note that we assume here $n_{1i} = n_{1e}$ as most astrophysical plasma is charge neutral.
In the literature, collisionless shocks are often characterized by their Alfvénic Mach number,

\[ M_A \equiv \frac{V_1}{v_A}, \]  

defined as the ratio of the upstream flow velocity to the upstream Alfvén velocity,

\[ v_A \equiv \sqrt{\frac{B_0^2}{4\pi n_1 (m_i + m_e)}}. \]  

The sonic and Alfvénic Mach numbers are related by

\[ M_A = M_s \sqrt{\frac{\Gamma \beta_p}{2}}. \]  

Finally, another parameter that is used to classify shocks is the magnetic obliquity angle \( \theta_B \), which is the angle between upstream magnetic field lines and the shock propagation direction. Shocks with \( \theta_B = 90^\circ \) are called perpendicular shocks, those with \( 45^\circ \lesssim \theta_B < 90^\circ \) are called quasi-perpendicular shocks, those with \( 0^\circ < \theta_B \lesssim 45^\circ \) are called quasi-parallel shocks, and those with \( \theta_B = 0^\circ \) are called parallel shocks. This thesis investigates shocks with a variety of obliquity angles, with an emphasis on perpendicular and quasi-perpendicular shocks.

### 1.2.1 Astrophysical Low Mach Number Collisionless Shocks

As mentioned earlier, in this thesis I focus on collisionless shocks with low Mach number: \( M_s \lesssim 5 \). In the astrophysical context, these shocks occur mostly during mergers of galaxy clusters, though some planetary and heliospheric shocks also have low Mach numbers. The main difference between cluster merger shocks and planetary shocks is the magnetization.
The former usually have high beta $\beta_p \gg 1$, while the latter typically have $\beta_p \lesssim 1$. The major effect of plasma beta is that it often determines what kind of plasma instabilities occur in the shock to regulate particle kinematics. Since this thesis is for a degree in Astronomy and Astrophysics, I focus on high beta collisionless shocks in order to facilitate future applications of the results to astronomical observations of galaxy cluster merger shocks. However, as we will see throughout this thesis, much wisdom on collisionless processes in low Mach number shocks has been gained through previous studies of heliospheric shocks in the geophysics/space physics community, and we frequently draw on lessons from that work.

Galaxy clusters are the largest gravitationally bound structures in the Universe, with typical masses ranging from $10^{14} - 10^{15} M_\odot$ and typical radii from $2 - 10$ Mpc. According to the current paradigm of structure formation, galaxy clusters grow through mergers of subclusters; these mergers are the most energetic events since the Big Bang (Sarazin, 2002). As the subclusters collide with relative velocities $\sim 2000$ km/s, shocks are driven into the intracluster medium (ICM). These shocks are of low Mach number and dissipate a large fraction of the infall kinetic energy of the merging clusters (on the order of $10^{63} - 10^{64}$ ergs). This is the major heating source of the hot X-ray emitting plasma in the ICM, whose temperature is on the order of $10^7 - 10^8$ K (Ryu et al., 2003; Skillman et al., 2008; Brüggen et al., 2012). As with most astrophysical shocks, many cluster merger shocks also accelerate particles to relativistic energies.

The key signature of a shock is that it causes a density and temperature jump between the unshocked and shocked plasma. Cluster merger shocks are routinely detected through X-ray observations of the density and/or temperature jump of the shocked electrons (e.g. Markevitch et al., 2002; Finoguenov et al., 2010; Russell et al., 2010; Ogrean et al., 2013b;
Since the X-ray surface brightness profile drops quickly with redshift \( \propto (1 + z)^{-3} \), the X-ray samples of merger shocks are usually limited to the local Universe. An alternative probe of the shock discontinuity is the thermal Sunayev-Zeldovich (SZ, Sunyaev & Zeldovich (1972)) effect, which has the key advantage of being redshift-independent. Recently, Basu et al. (2016) used radio observation of the thermal SZ effect to reveal the pressure jump associated with a merger shock at relatively high redshift.

Apart from heating, non-thermal particle acceleration is also often observed in cluster merger shocks. Synchrotron radiation from non-thermal electrons accelerated to relativistic energies have been routinely observed at shock locations identified through X-ray observations (e.g., van Weeren et al., 2010; Lindner et al., 2014; Trasatti et al., 2015; Kale et al., 2017). The diffuse synchrotron emission from these relativistic electrons can extend up to several Mpc in size and are typically found near cluster virial radii. The radio spectra are characterized by a steep slope \( \alpha > 1 \), where \( S_\nu \propto \nu^{-\alpha} \), and a high degree of polarization (20-30 per cent at 1.4 GHz) (e.g. Feretti et al., 2012, for a review). The observational signature of proton acceleration can in principle be observed through collisional interactions of the non-thermal protons with thermal ions, leading to pion decays that end in gamma-ray emission (Pfrommer, 2008). However, current observation with the Fermi satellite are only able to place upper limits on the gamma-ray emission from shock accelerated protons (Ackermann et al., 2014).

A common theme here is that our observational knowledge of the thermal and non-thermal physics in low Mach number shocks in cluster mergers comes from the radiative signatures of the electrons, while the properties of the protons are essentially unconstrained by observations. Prior to this thesis, our understanding of how electrons are accelerated
and/or heated in low Mach number, high beta shocks was very poor, and this was a major bottleneck in the study of cluster merger shocks.

A collisionless shock is intrinsically a non-linear phenomenon. The complex interplay between plasma instabilities self-generated by different species of the plasma can only be self-consistently captured by first-principles kinetic simulations. One way to simulate these kinetic processes is via the particle-in-cell (PIC) method. This approach is followed throughout this thesis. The method is reviewed in Section 1.3.

PIC simulations are extremely expensive because collisionless shocks involve multi-scale physics due to the large separation in mass scale between electrons and protons. On the one hand, the dynamics of the shocks is governed by the proton gyro-frequency. On the other hand, the electron plasma frequency needs to be resolved in order to properly capture the electron kinetics. The separation between these two scales is proportional to $\sqrt{\beta_p m_i/m_e}$. Even after artificially reducing the mass ratio to $m_i/m_e \ll 1836$, most previous PIC studies of low Mach number collisionless shocks have only been able to study the low beta regime $\beta_p \ll 1$, where the separation of scales is less severe. Thanks to the rapid advance of computing power, I have been able to tackle the problem of particle acceleration and heating in low Mach number shocks in the numerically more challenging high beta regime, via PIC simulations. Below I briefly review previous physical models of particle acceleration and heating, and thereby motivate the studies carried out in this thesis.

1.2.2 Shock Acceleration

First-order Fermi acceleration, also called diffusive shock acceleration (DSA), has been widely invoked to explain the production of non-thermal particles in various collisionless shocks in astrophysics (see Blandford & Eichler, 1987, for a review). The theory of first-
order Fermi acceleration is based on an idea first put forward by Fermi (Fermi, 1949, 1954), and developed in the late 1970s almost simultaneously by a number of independent groups (Krymskii, 1977; Axford et al., 1977; Bell, 1978a,b; Blandford & Ostriker, 1978). In the first-order Fermi process, particles are repeatedly scattered by magnetic fluctuations present on the two sides of the shock, and as a result move back and forth across the shock front. Due to the velocity jump across the shock, the scattered particles always see a converging flow and the scattering always leads to an energy increase. The fractional energy gain from each cycle of shock crossing is a constant. Thus the particles can reach very high energies, provided they are confined to the shock vicinity and undergo many cycles of acceleration.

One remarkable feature of Fermi acceleration is that, regardless of the details of the particle scattering process, the spectrum of the accelerated particles naturally forms a power law, whose spectral index only depends on the density compression across the shock,

$$f(p) \propto p^{-3r/(r-1)}, \quad (1.6)$$

where $p$ is the magnitude of the particle momentum and $r$ is the density compression ratio across the shock. In the limit of a non-relativistic strong shock (high Mach number), $r \to 4$, and the momentum spectrum of the non-thermal particles approaches $f(p) \propto p^{-4}$.

The corresponding energy spectrum $f(E)$ can be calculated via

$$4\pi p^2 f(p) dp = f(E) dE \Rightarrow f(E) = 4\pi p^2 f(p) \frac{dp}{dE}. \quad (1.7)$$

In the non-relativistic particle energy regime, $E = p^2/2m$ and $dp/dE \propto p^{-1} \propto E^{-1/2}$, so

$$f(E)_{\text{non-rel}} \propto E^{-(2r+1)/2(r-1)}, \quad (1.8)$$
which reduces to \( f(E) \propto E^{-1.5} \) in the strong shock limit. In the relativistic particle energy regime, \( E \propto p \), so

\[
    f(E)_{\text{rel}} \propto E^{-(r+2)/(r-1)},
\]

which reduces to \( f(E) \propto E^{-2} \) in the strong shock limit.

The universal power-law feature of Fermi acceleration has been utilized to deduce the Mach number of shocks from the spectral index \( \alpha \) of the synchrotron radiation emitted by the accelerated particles, defined as

\[
    S_\nu \propto \nu^{-\alpha}.
\]

The synchrotron spectrum of a power-law energy distribution is itself a power law, and is related to the slope of the particle energy distribution \( \delta \) by

\[
    \alpha = \frac{\delta - 1}{2}.
\]

The compression ratio of a nonrelativistic hydrodynamic shock with Mach number \( M_s \) and adiabatic index \( 5/3 \) is

\[
    r = \frac{4M_s^2}{M_s^2 + 3}.
\]

Thus, using Equations (1.9),(1.11),(1.12), one derives

\[
    M_{s,\text{DSA}} = \sqrt{\frac{2\alpha + 3}{2\alpha - 1}}.
\]

However, one needs to be cautious when applying Equation (1.13), especially when deducing the Mach number of a shock from the synchrotron emission of shocked electrons. This is
because it has for long been an open problem whether electrons can actually participate in long-term Fermi acceleration. While Fermi acceleration provides an attractive framework to explain the power law distribution of accelerated non-thermal particles, it glosses over the injection problem. Namely, how do some of the particles distinguish themselves from the thermal pool and get injected into the Fermi process in the first place? What is the efficiency of shock acceleration, i.e. the number density of non-thermal particles? What is the nature of the magnetic fluctuations that are necessary for sustaining Fermi acceleration?

The injection problem for ions has been recently studied in a series of papers (Caprioli & Spitkovsky, 2013, 2014a,c,b; Caprioli et al., 2014) using hybrid kinetic simulations. It has been shown that quasi-parallel shocks can efficiently accelerate protons and that the particle spectrum agrees with the slope predicted by DSA. The protons are injected after having been specularly reflected at the shock potential barrier and energized via shock drift acceleration (SDA) up to an injection momentum $p_{\text{inj}} \approx 2.5 m_i v_{\text{sh}}$, where $m_i$ is the proton mass and $v_{\text{sh}}$ is the shock velocity. The injected protons then begin to diffuse around the shock by scattering off self-generated resonant streaming instability (in shocks with $M_s \lesssim 30$) and Bell’s non-resonant hybrid instability (in shocks with $M_s \gtrsim 30$).

Achieving the same injection momentum is significantly harder for electrons, as their initial momentum is a factor of $m_i/m_e = 1836$ smaller than that of the protons. Since hybrid kinetic simulations do not resolve the kinetics of electrons, fully kinetic particle-in-cell simulations are needed to study the electron injection problem. In the high Mach number regime, many authors have found from PIC simulations that the shock surfing acceleration (SSA) can inject high energy electrons into DSA (Dieckmann et al., 2000; McClements et al., 2001; Hoshino & Shimada, 2002; Schmitz et al., 2002; Amano & Hoshino, 2007; Matsumoto et al., 2012). In the SSA, large-amplitude electrostatic waves are excited at the leading
edge of the shock transition region by the Buneman instability, as a result of the interaction between the reflected ions and the incoming electrons. These electrostatic waves trap the incoming electrons in their electrostatic potential. As a consequence, the trapped electrons can be effectively accelerated by the shock motional electric field (the electric field resulting from the motion of the magnetized upstream region toward the shock). However, most of the previous work was based on simulations with a modest proton-to-electron mass ratio. Recently, Riquelme & Spitkovsky (2011) argued that the importance of SSA decreases as the ion-to-electron mass ratio increases toward the realistic value. In this limit, rather than SSA, they found that it is the growth of oblique whistler waves near the front of quasi-perpendicular shocks that pre-accelerate the electrons for long-term DSA.

The above discussion pertains to high Mach number shocks. Meanwhile, the electron injection problem in the low Mach number regime remains poorly understood. The SSA mechanism is unlikely to operate because the Buneman instability, essential for trapping the electrons near the shock for the SSA process, cannot be triggered in low Mach number shocks (Matsumoto et al., 2012). Previous semi-analytic injection models, such as the "thermal leakage" model (Malkov & Völk, 1998; Gieseler et al., 2000; Kang et al., 2002), predict extremely low acceleration efficiency for electrons in low Mach number shocks (Kang & Ryu, 2010). This is clearly contradicted by the bright synchrotron radiation observed in radio relics in galaxy clusters, which indicates the presence of a large number of accelerated electrons. Several workaround solutions have been proposed to alleviate the tension by adding ad hoc modifications to the energy distribution of the ambient unshocked thermal electrons. Kang et al. (2014) proposed that the electron acceleration efficiency would be increased if the pre-existing electrons follow a $\kappa$-distribution, which has a super-thermal power-law tail. Pinzke et al. (2013) postulated that there could be pre-existing accelerated
electrons from past AGN activity in the cluster core.

In the first part of this thesis, Chapters 2-3, we use multi-dimensional fully kinetic PIC simulations to study particle acceleration in low Mach number shocks, with an emphasis on the electron injection problem. We find that electrons can indeed be efficiently accelerated in low Mach number shocks, consistent with observations, and contrary to previous semi-analytical models. The electrons are injected into a Fermi-like process by experiencing multiple cycles of shock drift acceleration (SDA). The process is sustained by the oblique electron firehose instability, which is self-generated by the accelerated electrons.

1.2.3 Shock Heating

While the problem of acceleration of non-thermal particles in shocks has received considerable attention, the related problem of shock heating of (quasi)-thermal particles has been relatively ignored.

In collisionless shocks, the downstream state of the plasma, especially the electron and proton temperatures, is not uniquely determined by the fluid-type Rankine-Hugoniot shock jump condition, which only predicts the mean temperature jump, not the individual temperatures of the electrons and protons. Naively, one would expect electrons to be heated to a lower temperature than protons since, ahead of the shock, the bulk kinetic energy of protons is a factor of $m_i/m_e$ larger than for electrons. In the absence of a channel for efficient proton-to-electron energy transfer, a comparable ratio should persist between the post-shock temperatures of the two species.

Observationally, in situ spacecraft measurements of both electron and proton distribution functions in heliospheric shocks have revealed that electrons are indeed heated to lower temperatures than protons (Bame et al., 1979; Schwartz et al., 1988; Masters et al., 2011).
Multi-wavelength spectroscopy of SNR shocks (Ghavamian et al., 2001, 2002, 2003, 2007; Laming et al., 1996; Rakowski et al., 2008) lead to a similar conclusion.

In the case of galaxy cluster mergers, current observational diagnostics are all based on radiation emitted by electrons, so the proton properties (in particular, their temperature) are basically unconstrained. One usually makes the simplifying assumption that the electron temperature equals the mean plasma temperature (and so, the proton temperature). While Coulomb collisions will eventually drive electrons and protons to equal temperatures, the collisional equilibration timescale (Spitzer, 1962) for typical ICM conditions is as long as $10^8 - 10^9$ yrs. In fact, X-ray observations by Russell et al. (2012) have shown that the electron temperature just behind a merger shock in Abell 2146 is lower than the mean gas temperature expected from the Rankine-Hugoniot jump conditions, and thus lower than the proton temperature. In another study, Akamatsu et al. (2017b) compiled a list of merger shocks, estimating their Mach number from both X-ray ($M_{s,X\text{-ray}}$) and radio observations ($M_{s,\text{radio}}$), and noticed a slight bias of $M_{s,\text{radio}} \gtrsim M_{s,X\text{-ray}}$. Here, $M_{s,\text{radio}}$ is derived by measuring the power-law slope of the synchrotron emission, which is related — via the theory of diffusive shock acceleration — to the density compression at the shock (and so, to the Mach number, see Equation 1.13). On the other hand, $M_{s,X\text{-ray}}$ is obtained from the electron free-free emission by measuring the jumps in density and temperature across the shock. It follows that, if electrons have a lower temperature than protons behind the shock, $M_{s,X\text{-ray}}$ would have been underestimated.

In fact, it has long been thought that collisionless shocks can lead to a two-temperature structure at the outskirts of galaxy clusters (Fox & Loeb, 1997; Ettori & Fabian, 1998; Takizawa, 1999). Detailed cosmological hydrodynamic simulations have shown that this can significantly bias the X-ray and thermal SZ signatures (Wong & Sarazin, 2009; Rudd
In the absence of a physical model for electron heating in low Mach number shocks, these studies usually employ an *ad-hoc* subgrid approach to prescribe the electron heating efficiency in shocks. Either electrons are heated adiabatically, or the non-adiabatic (or “irreversible”) heating efficiency is quantified by a phenomenological (and often, arbitrary) parameter.

An accurate understanding of electron heating in collisionless shocks requires fully kinetic simulations to self-consistently capture the non-linear structure of the shock and the role of electron and proton plasma instabilities in particle heating. So far, PIC studies of electron heating in shocks have focused on the regime of high sonic Mach number ($M_s \gtrsim 10$) and low plasma beta ($\beta_p \lesssim 1$) appropriate for supernova remnant shocks. At very high Mach numbers ($M_s \gtrsim 30$), the Buneman instability can trap electrons in the shock transition region and heat them (Dieckmann et al., 2012). For lower Mach numbers ($10 \lesssim M_s \lesssim 30$), resonant wave-particle scattering induced by the modified two-stream instability (MTSI) can lead to significant electron heating at the shock front (Matsukiyo & Scholer, 2003; Matsukiyo, 2010).

Electron heating in the regime of low Mach number shocks remains poorly explored. In the second part of this thesis, Chapters 4-7, we address the problem of particle heating in low Mach number shocks using fully kinetic PIC simulations, with a focus on the irreversible heating of electrons. We find that electrons are indeed heated to temperatures lower than protons in low Mach number shocks. However, they do experience some level of irreversible heating. We develop a heating model for electrons, which requires the existence of electron temperature anisotropy and a mechanism that breaks the electron adiabatic behavior. The electron temperature anisotropy is induced by the field amplifications in the shock downstream and the electron whistler instability provides the means to break the electron
adiabatic behavior. I find the heating model is in excellent agreement with simulations.

1.3 Numerical Method

As we have repeatedly stated in this introductory chapter, an accurate understanding of particle acceleration and heating in collisionless shocks requires fully kinetic simulations. Here, we review the numerical method and code that is used throughout this thesis to simulate collisionless shocks from first principles.

1.3.1 Vlasov-Maxwell Equations

The fundamental equations that govern the evolution of a collisionless plasma are the so-called Vlasov-Maxwell equations. A collisionless plasma is completely described by its distribution functions \( f_\alpha (\vec{r}, \vec{p}, t) \). Here, \( f_\alpha (\vec{r}, \vec{p}, t) \) specifies the number of particles of the species \( \alpha \) having approximately the momentum \( \vec{p} \) near the position \( \vec{r} \) at time \( t \). The time evolution of the distribution function of a plasma with particle mass \( m_\alpha \) and electric charge \( q_\alpha \) is governed by the Vlasov equation

\[
\frac{\partial f_\alpha}{\partial t} + \frac{\vec{p}}{\gamma m_\alpha} \cdot \frac{\partial f_\alpha}{\partial \vec{r}} + q_\alpha \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) \cdot \frac{\partial f_\alpha}{\partial \vec{p}} = 0,
\]  

(1.14)
where \( \gamma \equiv 1/\sqrt{1 - v^2/c^2} \), \( \vec{p} = \gamma m_v \vec{v} \), and \( \vec{E}(\vec{r}, t) \) and \( \vec{B}(\vec{r}, r) \) are the electric and magnetic fields self-consistently generated by the plasma. The latter satisfy the Maxwell equations,

\[
\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t},
\]

(1.15)

\[
\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t},
\]

(1.16)

\[
\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_c,
\]

(1.17)

\[
\nabla \cdot \vec{B} = 0,
\]

(1.18)

where the charge and current density are obtained by taking appropriate moments of the distribution functions,

\[
\rho_c = \sum_\alpha \int q_\alpha f_\alpha d^3p,
\]

(1.19)

\[
\vec{J} = \sum_\alpha \int q_\alpha \vec{v}_\alpha f_\alpha d^3p.
\]

(1.20)

Together, the closed set of Equations (1.14)-(1.20) is referred to as the Vlasov-Maxwell equations.

### 1.3.2 Particle-in-cell Method

Analytic solutions of the Vlasov-Maxwell equations are known for only a few idealized systems. In most cases, one needs to solve the equations numerically. One approach is the so called particle-in-cell (PIC) method (Hockney & Eastwood, 1981; Birdsall & Langdon, 1991), which is used throughout this thesis.

The PIC method tracks the phase space of a large ensemble of particles under the influence of their self-generated electromagnetic fields. Once the electromagnetic field and particle
distribution are initialized, the PIC method evolves the system self-consistently as follows: (i) move the particles according to the Lorentz force; (ii) interpolate the charge and current source terms to the mesh; (iii) solve Maxwell’s equations to update the electromagnetic field; (iv) interpolate the field to the updated positions of the particles; (v) go back to (i) and iterate. Due to the collisionless nature of the plasma, there is no explicit particle-particle interactions. Rather the particles interact via collective effects through the electromagnetic field.

1.3.3 Basics of the TRISTAN-MP code

All PIC simulations in this thesis have been carried out with the TRISTAN-MP code, whose name stands for TRIdimensional ST ANford - Massively Parallel code. It is the parallel version (Spitkovsky, 2005) of the TRISTAN code originally developed by Buneman (1993). The code is written in a modular format in Fortran 95, and uses the MPI and HDF5 libraries to support parallelism and standardized parallel output files.

TRISTAN-MP scales quantities such that \( \epsilon_0 = 1 \) and hence \( \mu_0 = 1/c^2 \). The magnetic field in code units is scaled up by a factor of \( c \) to make the electric and magnetic field symmetric in Maxwell equations. With this scaling, the basic equations solved by TRISTAN-MP are

\[
\frac{\partial \vec{B}}{\partial t} = -c\nabla \times \vec{E}, \quad (1.21) \\
\frac{\partial \vec{E}}{\partial t} = c\nabla \times \vec{B} - \vec{J}, \quad (1.22)
\]

along with the Lorentz force equation, and the relativistic dynamic equations for the elec-
trons and ions,

\[
\frac{\partial \vec{p}}{\partial t} = q \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right), \tag{1.23}
\]

\[
\frac{\partial \vec{x}}{\partial t} = \vec{p}/m. \tag{1.24}
\]

The above set of equations is self-consistently evolved as follows. First, the electromagnetic fields and the particle positions and velocities are initialized in the simulation box. The initial electric and magnetic fields are made divergence-free (Equations (1.17),(1.18)), by initializing electrons and protons in the same spatial positions. The momenta of the particles are initialized from a drifting Maxwellian-Jüttner distribution. It is important to note that merely drawing particle momenta from a stationary Maxwellian-Jüttner distribution and then Lorentz boosting them with the desired drift velocity results in a wrong distribution for the particles (as pointed out by Melzani et al., 2013; Zenitani, 2015). I have updated the code to correctly load the drifting Maxwellian distribution according to the "flipping method" described in Zenitani (2015).

TRISTAN-MP uses space- and time-centered finite difference schemes to calculate spatial and temporal derivatives, so that the code is second-order accurate in both space and time. Electric and magnetic fields are discretized and stored in a staggered grid mesh system, known as the Yee lattice (Yee, 1966). The special arrangement of electric and magnetic fields of the Yee lattice inherently preserves a divergence-free magnetic field, provided that \( \nabla \cdot B = 0 \) at \( t = 0 \). This allows the code to avoid explicitly solving Equation (1.18). In addition, Equation (1.17) is ensured through a charge-conserving current deposition scheme (Villasenor & Buneman, 1992).

In order to obtain the Lorentz force experienced by the particles (Equation (1.23)), a tri-
linear interpolation function is used to interpolate the electromagnetic fields from the grid to particle positions. The momenta and positions of the particles are updated according to Equations (1.23) and (1.24) in a leap-frog fashion. Two choices of integrators are available in the code for the particle update: the Boris algorithm (Boris, 1971) and the Vay pusher (Vay, 2008), with the Vay pusher having some advantages in the relativistic regime. In this thesis, we have always used the more traditional Boris algorithm.

Once the positions and velocities of the particles are updated, they are used to calculate the current (with a three-point digital filter to significantly reduce numerical noise), which is then used as the source term in Equation (1.22).

Originally, TRISTAN-MP was only parallelized in directions transverse to the shock propagation direction. In these directions, where periodic boundary conditions are applied, the computational load is approximately uniform and the load balancing is fairly trivial. On the other hand, the computational load in the direction parallel to shock propagation varies in a time-dependent and position-dependent fashion due to the density compression and the propagation of the shock. To fully utilize numerical resources, I have developed a dynamic load balancing scheme for the code so that it can use multiple cores along the direction parallel to the shock as well.

1.4 Chapter Summary

In this thesis, I study via multi-dimensional PIC simulations the process of particle acceleration and heating in low Mach number collisionless shocks.

The first part of the thesis, Chapters 2-3, investigates particle acceleration in low Mach number shocks. In Chapter 2, I focus on the particle energy spectra and the accelera-
tion mechanism in a reference PIC simulation with Mach number $M_s = 3$ and a quasi-perpendicular magnetic field. I find that protons are not efficiently accelerated. In contrast, about 15% of electrons are efficiently accelerated, forming a non-thermal power-law tail in the energy spectrum with a slope of $\simeq 2.4$. I study in detail the electron acceleration mechanism and identify it to be as follows. A fraction of the incoming electrons are energized at the shock front via shock drift acceleration (SDA). The accelerated electrons are reflected back upstream by the mirror force of the shock-compressed magnetic field. In the upstream region, the interaction of these electrons with the incoming flow generates magnetic waves. In turn, the waves scatter some of the electrons propagating upstream back toward the shock, for further energization via SDA. Thus the self-generated waves allow for repeated cycles of SDA, similar to a sustained Fermi-like process.

In Chapter 3, I elucidate the nature of the upstream waves that are essential for sustaining the Fermi-like acceleration process. I demonstrate that the waves are triggered by the shock-reflected electrons propagating upstream via the oblique electron firehose instability, which is driven by the temperature anisotropy in the reflected electron distribution. I then systematically explore how the efficiency of wave generation and of electron acceleration depend on the magnetic field obliquity, the plasma beta, and the upstream electron temperature. I find that the mechanism works for shocks with high plasma beta ($\gtrsim 20$) at nearly all magnetic field obliquities, and for electron temperatures in the range relevant for galaxy clusters. My findings offer a natural solution to the conflict between the bright radio synchrotron emission observed from the outskirts of galaxy clusters and the low electron acceleration efficiency usually expected in low Mach number shocks.

The second part of the thesis, Chapters 4 - 7, explores particle heating in low Mach number perpendicular shocks. In Chapter 4 and 5, I focus on a representative shock with
$M_s = 3$ and $\beta_p = 16$. In this reference simulation, the post-shock electron temperature exceeds the adiabatic expectation by $\simeq 15\%$. That is, the electrons gain some entropy added as they cross the shock. The resulting downstream electron-to-proton temperature ratio is $\simeq 0.45$. I find that two basic ingredients are needed for electron entropy production: (i) an electron temperature anisotropy, induced by field amplification coupled to adiabatic invariance; and (ii) a mechanism to break the electron adiabatic invariance itself. In shocks, field amplification occurs at two major sites: at the shock ramp, where density compression leads to an increase in the frozen-in field; and farther downstream, where the shock-driven proton temperature anisotropy generates strong proton cyclotron and mirror modes. The electron temperature anisotropy induced by field amplification exceeds the threshold of the electron whistler instability. The growth of whistler waves breaks the electron adiabatic invariance, and allows for efficient entropy production. I develop an analytical model for electron irreversible heating and show that it is in excellent agreement with the simulation results.

In Chapter 6 and 7, I explore the dependence of the electron heating mechanism on the two principal shock parameters, the sonic Mach number $M_s$ and the plasma beta $\beta_p$. In Chapter 6, I first utilize periodic box simulations, designed to mimic the two field amplification sites identified in the previous paragraph, to understand in detail the effect of $M_s$ and $\beta_p$ under idealized conditions. Then in Chapter 7, I apply the lessons learned in Chapter 6 to interpret the parameter dependence of the electron heating mechanism in full shock simulations covering a wide range of Mach number and plasma beta. I find that electron entropy production is increasingly efficient with increasing Mach number, but is insensitive to plasma beta in the high beta regime I focus on. I provide a fitting formula for irreversible heating in electrons and use it to predict the electron-to-proton temperature ratio in the
shock downstream. I show that the predictions are in good agreement with observational data.
Chapter 2

Non-Thermal Electron Acceleration in Low Mach Number Collisionless Shocks. I. Particle Energy Spectra and Acceleration Mechanism

This thesis chapter originally appeared in the literature as


Abstract

Electron acceleration to non-thermal energies in low Mach number \( M_s \lesssim 5 \) shocks is revealed by radio and X-ray observations of galaxy clusters and solar flares, but the electron acceleration mechanism remains poorly understood. Diffusive shock acceleration, also
known as first-order Fermi acceleration, cannot be directly invoked to explain the acceleration of electrons. Rather, an additional mechanism is required to pre-accelerate the electrons from thermal to supra-thermal energies, so they can then participate in the Fermi process. In this work, we use two- and three-dimensional particle-in-cell plasma simulations to study electron acceleration in low Mach number shocks. We focus on the particle energy spectra and the acceleration mechanism in a reference run with $M_s = 3$ and a quasi-perpendicular pre-shock magnetic field. We find that about 15% of the electrons can be efficiently accelerated, forming a non-thermal power-law tail in the energy spectrum with a slope of $p \simeq 2.4$. Initially, thermal electrons are energized at the shock front via shock drift acceleration. The accelerated electrons are then reflected back upstream, where their interaction with the incoming flow generates magnetic waves. In turn, the waves scatter the electrons propagating upstream back toward the shock, for further energization via shock drift acceleration. In summary, the self-generated waves allow for repeated cycles of shock drift acceleration, similarly to a sustained Fermi-like process. This mechanism offers a natural solution to the conflict between the bright radio synchrotron emission observed from the outskirts of galaxy clusters and the low electron acceleration efficiency usually expected in low Mach number shocks.

2.1 Introduction

Collisionless shocks occur in a wide variety of astrophysical settings: examples include Earth’s bow shock, and the solar wind termination shock, supernova remnant (SNR) shocks in the interstellar medium, and structure formation shocks in the intracluster medium (ICM).
Particle acceleration is often associated with collisionless shocks. For instance, it is widely believed that Galactic cosmic rays with energies up to $\sim 10^{15}$ eV are ions accelerated by SNR shocks (e.g. Gaisser, 1990). The most successful mechanism for explaining ion acceleration is diffusive shock acceleration (DSA; Blandford & Ostriker, 1978; Bell, 1978; Drury, 1983; Blandford & Eichler, 1987), also known as first-order Fermi acceleration (Fermi acceleration for short, hereafter). In DSA, charged particles cross the shock back and forth as they scatter off plasma/magneto-hydrodynamic (MHD) waves existing ahead and behind the shock (in the upstream and downstream regions, respectively). Since the turbulence is convected roughly at the local flow speed, waves on the two sides of the shock effectively move towards each other due to the velocity jump at the shock. Hence the charged particles gain energy at each shock crossing.

While the DSA mechanism has been successful in explaining ion acceleration in various settings (Blandford & Eichler, 1987), e.g. in the Earth’s bow shock (see Burgess et al., 2012, for review) and SNR shocks (see Reynolds, 2008, for review). However, DSA cannot be straightforwardly invoked for the acceleration of electrons. To participate in the DSA process, thermal electrons need to cross the shock front multiple times. However, due to their small mass, electron gyro radii are very small compared to the shock thickness, which is controlled by the ion gyro radius. Thus, without undergoing some pre-acceleration, thermal electrons are expected to be tied closely to magnetic field lines and to be convected downstream without undergoing any significant DSA. This is known as the electron injection problem.

To understand electron acceleration in shocks, fully kinetic numerical simulations are essential to self-consistently capture the non-linear loop that links the accelerated particles – that generate turbulent magnetic fields – to the turbulence itself – which in turn governs
the particle acceleration. In recent years, particle-in-cell (PIC) methods (e.g. Birdsall & Langdon, 1991) have been used to simulate these kinetic processes.

So far, most of the work has focused on high Mach number shocks, where the Mach number $M_s$ is defined as the ratio of the shock speed to the sound speed of the ambient medium. Many authors have found that, in high Mach number shock, the shock surfing acceleration (SSA) mechanism can inject high-energy electrons into DSA (Dieckmann et al., 2000; Mcclements et al., 2001; Hoshino & Shimada, 2002; Schmitz et al., 2002; Amano & Hoshino, 2007; Matsumoto et al., 2012). In the SSA, large-amplitude electrostatic waves are excited at the leading edge of the shock transition region by the Buneman instability, as a result of the interaction between the reflected ions and the incoming electrons. These electrostatic waves trap the incoming electrons in their electrostatic potential. As a consequence, the trapped electrons can be effectively accelerated by the shock motional electric field (i.e., the electric field resulting from the motion of the magnetized upstream region toward the shock). However, most of the previous work was based on simulations with a modest ion-to-electron mass ratio. Riquelme & Spitkovsky (2011) argued that the importance of SSA decreases as the ion-to-electron mass ratio increases toward the realistic value. In this limit, rather than SSA, they found that it is the growth of oblique whistler waves near the front of quasi-perpendicular shocks that can pre-accelerate the electrons for long-term DSA.

Electron acceleration in low Mach number ($M_s \lesssim 5$) shocks has been poorly understood so far. The injection mechanism is expected to be different because the Buneman instability, essential for trapping the electrons near the shock for the SSA process, cannot be triggered at low Mach numbers (Matsumoto et al., 2012).

On the other hand, low Mach number shocks are of great astrophysical interest. Lin et al. (2003) observed electron acceleration above solar flare tops and footpoints using X-
ray data from Yohkoh and RHESSI. In galaxy clusters, low Mach number shocks have been identified in X-ray images (e.g. Markevitch et al., 2002; Russell et al., 2010; Akamatsu et al., 2012) and through observations of radio synchrotron emission by relativistic electrons accelerated at the shock (e.g. Willson, 1970; Fujita & Sarazin, 2001; Govoni & Feretti, 2004; van Weeren et al., 2010; Lindner et al., 2014). In addition, low Mach number shocks have been hypothesized to be present ahead of the G2 cloud (Narayan et al., 2012; Sadowski et al., 2013) and of the S2 star (Giannios & Sironi, 2013) when they interact with the hot accretion flow at the Galactic Center (Yuan & Narayan, 2014).

A few recent studies have explored electron acceleration in low Mach number shocks. Using one-dimensional (1D) PIC simulations, Matsukiyo et al. (2011) found efficient shock drift acceleration (SDA, e.g. Wu et al. (1984), Krauss-Varban & Wu (1989), Ball & Melrose (2001), Mann et al. (2006)) in low Mach number shocks. In the SDA process, particles gain energy from the shock motional electric field while drifting along the shock surface due to the gradient of the magnetic field at the shock front. However, the 1D nature of the simulations of Matsukiyo et al. (2011) prohibits a self-consistent study of self-generated waves in the upstream, since the wave-vector is confined to be perpendicular to the shock plane. Their results certainly suggest that SDA could be a potential injection mechanism, but they have no direct evidence of electron Fermi acceleration. Similarly, Park et al. (2012) and Park et al. (2013) studied perpendicular and quasi-perpendicular low Mach number shocks and found efficient SDA. However they did not see any evidence for sustained Fermi acceleration.

In this paper, we study electron acceleration in low Mach number shocks using fully kinetic 2D and 3D PIC simulations. We focus on results from a reference run where the upstream magnetic field is quasi-perpendicular to the shock normal. Yet, in a forthcoming paper (Guo et al., 2014b) we show that the results presented here can be generalized to
a wide range of obliquity angles. We find a self-consistent mechanism for electron Fermi acceleration in which electrons are injected by pre-heating via SDA. These pre-accelerated electrons self-generate magnetic waves in the upstream region, and the waves in turn facilitate Fermi acceleration. The organization of this paper is as follows. In Section 2.2, the simulation setup with the various physical and numerical parameters is described. In Section 2.3, the shock structure of the reference run is discussed. In Section 2.4, we present particle energy spectra and study in detail the electron acceleration mechanism of the reference run. We conclude with a discussion in Section 2.5. In a forthcoming paper (Guo et al., 2014b) we will study in detail the nature of the upstream waves and explore the parameter dependence of the electron energy spectrum and acceleration mechanism.

### 2.2 Simulation Setup

We perform numerical simulations using the 3D electromagnetic PIC code TRISTAN-MP (Spitkovsky, 2005), which is a parallel version of the publicly available code TRISTAN (Buneman 1993, p.67) that was optimized for studying collisionless shocks.

The computational setup and numerical scheme are described in detail in Spitkovsky (2008); Sironi & Spitkovsky (2009, 2011); Sironi et al. (2013). In brief, the shock is set up by reflecting an upstream electron-ion plasma, which follows a Maxwell-Jüttner distribution with the electron temperature $T_e$ equal to the ion temperature $T_i$, and bulk velocity $\vec{u}_0 = -u_0\hat{x}$, off a conducting wall at the leftmost boundary ($x = 0$) of the computational box (Figure 2.1). The interplay between the reflected stream and incoming plasma causes a shock to form, which propagates along $+\hat{x}$ at the speed $u_{sh}$. In the simulation frame, the downstream plasma is at rest.
The relation between the upstream bulk flow velocity and the plasma temperature is parametrized by the simulation-frame Mach number

$$M \equiv \frac{u_0}{c_s} = \frac{u_0}{\sqrt{2\Gamma k_B T_i/m_i}},$$

(2.1)

where $c_s$ is the sound speed in the upstream, $k_B$ is the Boltzmann constant, and $\Gamma = 5/3$ is the adiabatic index of the plasma, and $m_i$ is the mass of the ion. The incoming plasma carries a uniform magnetic field $\vec{B}_0$, whose strength is parameterized by the magnetization parameter

$$\sigma \equiv \frac{B_0^2/4\pi}{(\gamma_0 - 1) nm_i c^2},$$

(2.2)

where $\gamma_0 \equiv (1 - u_0^2/c^2)^{-1/2}$ and $n = n_i = n_e$ is the number density of the incoming plasma. The magnetic field orientation with respect to the shock normal (along $+\hat{x}$) is parameterized by the polar angle $\theta_B$ and azimuthal angle $\varphi_B$ (Figure 2.1). The incoming plasma is initialized with zero electric field in its rest frame. Due to its bulk motion in the simulation frame, the upstream plasma carries a motional electric field $\vec{E}_0 = -\left(\vec{u}_0/c\right) \times \vec{B}_0$.

In the literature, the Mach number $M_s$ is often defined as the ratio between the upstream flow velocity and the upstream sound speed in the shock rest frame (rather than in the downstream frame, as in Equation 1). In the limit of weakly magnetized shocks, the Mach number $M_s$ is related to our simulation-frame Mach number $M$ through the implicit relation

$$M_s = M \frac{u_{\text{up}}^{\text{sh}}}{u_0} = M \left(1 + \frac{1}{r(M_s) - 1}\right),$$

(2.3)

where $u_{\text{up}}^{\text{sh}}$ is the shock velocity in the upstream rest frame, equal to the upstream flow
velocity in the shock rest frame, and

$$r(M_s) = \frac{\Gamma + 1}{\Gamma - 1 + 2/M_s^2}$$  \hspace{1cm} (2.4)$$

is the Rankine-Hugoniot relation for the density jump from upstream to downstream.

For comparison with earlier work, where the magnetization is sometimes parametrized by the Alfvénic Mach number $M_A \equiv u_0/v_A$, where $v_A \equiv B_0/\sqrt{4\pi nm_i}$ is the Alfvén velocity, we remark that the relation between the magnetization and the Alfvénic Mach number is simply $M_A = \sqrt{2/\sigma}$. Alternatively, one could parameterize the magnetic field strength by the plasma beta $\beta_p \equiv 8\pi n k_B (T_e + T_i)/B_0^2$, which is given by $\beta_p = 4/(\sigma \Gamma M^2)$ under the assumption of $T_e = T_i$. We stress that in our simulations the upstream particles are initialized with the physically-grounded Maxwell-Jüttner distribution, instead of the so-called “κ-distribution” that was employed by, e.g., Park et al. (2013). The latter distribution artificially boosts the high energy component of the particle spectrum, thus artificially enhancing the acceleration efficiency in most astrophysical settings (with the possible exception of shocks in solar flares, where the κ-distribution might be a realistic choice).

We perform simulations in both 2D and 3D computational domains. In our 2D simulations, we use a rectangular simulation box in the $xy$ plane, with periodic boundary conditions in the $y$ direction (Figure 2.1). In 3D, we employ periodic boundary conditions both in $y$ and in $z$. For both 2D and 3D domains, all three components of particle velocities and electromagnetic fields are tracked. As a result, the appropriate adiabatic index $\Gamma$ takes the value of $5/3$, as expected from a plasma with velocity in all three components. We find that most of the shock physics is well captured by 2D simulations, when the field is lying in the simulation plane, i.e. $\varphi_B = 0^\circ$. Therefore, to follow the shock evolution for longer times
Figure 2.1: Simulation setup.
with fixed computational resources, we mainly utilize 2D runs, but we explicitly show in Appendix 2.A that our 2D results with an in-plane field configuration are in good agreement with full 3D simulations, while 2D results with an out-plane field configuration ($\varphi_B = 90^\circ$) do not agree with the 3D physics.

For accuracy and stability, PIC codes have to resolve the plasma oscillation frequency of the electrons

$$\omega_{pe} = \sqrt{\frac{4\pi e^2 n}{m_e}}, \quad (2.5)$$

and the corresponding plasma skin depth $c/\omega_{pe}$, where $e$ and $m_e$ are the electron charge and mass. On the other hand, the shock structure is controlled by the ion Larmor radius

$$r_{L,i} = \sqrt{\frac{2}{\sigma}} \sqrt{\frac{m_i}{m_e}} \frac{c}{\omega_{pe}} \gg \frac{c}{\omega_{pe}}, \quad (2.6)$$

and the evolution of the shock occurs on a time scale given by the ion Larmor gyration period $\Omega_{ci}^{-1} = r_{L,i} u_0^{-1} \gg \omega_{pe}^{-1}$. The need to resolve the electron scales, and at the same time to capture the shock evolution for many $\Omega_{ci}^{-1}$, is an enormous computational challenge, for the realistic mass ratio $m_i/m_e = 1836$. Therefore, we decide to employ a reduced mass ratio $m_i/m_e = 100$ for most of our runs. In Appendix 2.B, we discuss in detail the dependence of our results on the mass ratio. We find that the results are in perfect agreement between $m_i/m_e = 100$ and $m_i/m_e = 400$. Thus our results can be generalized to the realistic mass ratio of $m_i/m_e = 1836$, with the scalings that we discuss in Appendix 2.B.

To further optimize our use of computational resources, the incoming particles are initialized at a “moving injector”, which recedes from the wall in the $+\hat{x}$ direction at the speed of light. When the injector approaches the right boundary of the computational domain, we expand the box in the $+\hat{x}$ direction. This way both memory and computing time are
saved, while following at all times the evolution of the upstream regions that are causally connected with the shock. Further numerical optimization can be achieved by allowing the moving injector to periodically jump backward (i.e. in the $-\hat{x}$ direction), resetting the fields to its right (see Sironi & Spitkovsky (2009)). Since we expect the acceleration to happen close to the shock, we choose to jump the injector in the $-\hat{x}$ direction such that to keep a distance of at least a few tens of ion Larmor radii ahead of the shock. This suffices to properly capture the acceleration physics. We have checked that, albeit at relatively early times, simulations with and without the jumping injector show consistent results.

In the main body of this paper, we present the results from a reference run simulated on a 2D domain. The upstream plasma in this reference run is initialized with $T_i = T_e = 10^9 K = 86 \text{ keV} / k_B$ and $u_0 = 0.15 c$, which results in a simulation-frame Mach number $M = 2$. Using Equation 2.3, this corresponds to $M_s = 3$. The strength of the magnetic field is set so that the magnetization is $\sigma = 0.03$ and the field lies in the simulation plane at an oblique angle with respect to the shock normal, such that $\theta_B = 63^\circ$ and $\varphi_B = 0^\circ$. The parameters are chosen to resemble closely the simulation presented in Narayan et al. (2012), which studied a low Mach number shock ($M_s = 2$) that might be formed during the passage of the G2 cloud through the accretion disk at the Galactic Center. In this work, our choice of the Mach number ($M_s = 3$) and of the magnetization ($\sigma = 0.03$) is relevant also for shocks in galaxy clusters, where $M_s \sim 1.5 - 5$, $B_0 \sim 1 \mu G$, $n \sim 10^{-4} - 10^{-5} \text{ cm}^{-3}$ and $u_0 \sim 1000 \text{ km/s}$ (Matsukiyo et al., 2011). We remark that although the chosen value for the plasma temperature ($T_i = T_e = 10^9 K = 86 \text{ keV} / k_B$) is fairly high in the context of the plasma in ICM, where $k_B T \sim 10 \text{ keV}$ for rich clusters, the acceleration physics does not change much with temperature. We will explicitly show the dependence of our results on the flow temperature in a forthcoming paper (Guo et al., 2014b).
We employ a spatial resolution of 10 cells per electron skin depth \( c/\omega_{pe} \), have 32 particles per cell (16 per species) and use a time resolution of \( dt = 0.045 \omega_{pe}^{-1} \). The transverse box size is fixed at 76 \( c/\omega_{pe} \) (corresponding to 7.6 \( c/\omega_{pi} \), or about one ion Larmor radius). We have performed convergence tests which show that we can properly resolve the acceleration physics with 5 cells per \( c/\omega_{pe} \), and we have confirmed that simulations with a number of particles per cell up to 64 and a transverse box size up to 256 \( c/\omega_{pe} \) give essentially the same results.

### 2.3 Shock Structure

In this section we present the shock structure of our reference run. Figure 2.2 shows quantities related to ions in the left column and those related to electrons in the right column. Each column shows the longitudinal profiles of number density \( n \), transverse magnetic field \( B_y \) and total magnetic field strength \( B \), the momentum spaces \( p_x - x \), \( p_y - x \), \( p_z - x \), the average velocity profiles \( \langle v_x \rangle, \langle v_y \rangle, \langle v_z \rangle \), the temperature parallel to the magnetic field \( T_{||} \) and perpendicular to the field \( T_{\perp} \). Figure 2.3(a)-(c) shows 2D plots of the magnetic field components in units of \( B_0 \), after subtracting the background field \( \vec{B}_0 \) (i.e., we show \( (B_x - B_{x,0})/B_0 \), \( (B_y - B_{y,0})/B_0 \) and \( B_z/B_0 \), respectively). These quantities provide a good characterization of the waves seen in the magnetic field. Figure 2.3(d) shows the \( y \)-averaged value of the electric potential \( \Phi(x) = -\int_{\infty}^{x} \langle E_x(x') \rangle dx' \) normalized by the electron rest mass energy \([e\Phi/(m_e c^2)]\), vertical scale on the left\] or by the ion bulk kinetic energy \([e\Phi/(m_i u_0^2/2)]\), vertical scale on the right\].\(^1\) Figure 2.3(e)-(g) presents the \( y \)-averaged components of the electric field \( \langle E_x \rangle, \langle E_y \rangle, \langle E_z \rangle \). All quantities are measured in the simulation.

\(^1\)Strictly speaking, the quantity \( \Phi \) we define is not the electric potential, but rather the electromotive force along the \( x \) direction. Yet, for the sake of simplicity, in the following we refer to \( \Phi \) as the electric potential.
Figure 2.2: Shock structure of our reference run at time $\omega_{pe} t = 14625$ ($\Omega_{ci} t = 26.9$). The shock is at $\simeq 1115 \, c/\omega_{pe}$, indicated by the vertical dot-dashed lines, and moves towards the right. The downstream is to the left of the shock, and the upstream to the right. The left column shows quantities related to the ions and the right column quantities related to the electrons. From top to bottom, we present the ratios $n/n_0$, $B_y/B_{y0}$, $B/B_0$, the momentum phase space plots $p_x - x$, $p_y - x$, $p_z - x$, the $y$-averaged velocity profiles $\langle v_x \rangle$, $\langle v_y \rangle$, $\langle v_z \rangle$, and the temperatures parallel ($T_\parallel$) and perpendicular ($T_\perp$) to the magnetic field. The yellow box encloses $0 - 80 \, c/\omega_{pe}$ ($0 - 1 \, r_{Li}$) ahead of the shock. The cyan box encloses $80 - 160 \, c/\omega_{pe}$ ($1 - 2 \, r_{Li}$) behind the shock. The time evolution of the ion energy spectra in these two regions is shown in Figure 2.4(a) and(b). The green and the magenta boxes enclose regions at $80 - 160 \, c/\omega_{pe}$ behind the shock and $80 - 160 \, c/\omega_{pe}$ ahead of the shock, respectively. The time evolution of the electron energy spectra in these two regions is shown in Figure 2.4(c) and (d).
Figure 2.3: Electromagnetic fields in the reference run at time $\omega_{pe}t = 14625$ ($\Omega_{ci}t = 26.9$). Panels (a)-(c) show 2D plots of the magnetic field components in units of $B_0$, after subtracting the background field $\vec{B}_0$ (i.e., we show $(B_x - B_{x,0})/B_0$, $(B_y - B_{y,0})/B_0$ and $B_z/B_0$, respectively). The black arrows indicate the orientation of the upstream background magnetic field $\vec{B}_0$. Note that there are waves in all three components of the magnetic field. Panel (d) shows the $y$-averaged electric potential normalized by the electron rest mass energy, i.e. $e\Phi/(m_e c^2)$ (as indicated by the vertical axis on the left) or by the ion bulk kinetic energy, i.e. $e\Phi/(m_i u_0^2/2)$ (as indicated by the vertical axis on the right). Panels (e)-(g) show the $y$-averaged electric field components normalized by the upstream magnetic field strength, $\langle E_x \rangle/B_0$, $\langle E_y \rangle/B_0$, $\langle E_z \rangle/B_0$. Since the upstream plasma is initialized with zero electric field in its rest frame, the expected motional electric field $\vec{E}_0 = -(u_0/c) \times \vec{B}_0$ in the simulation frame in the upstream region is $\langle E_z \rangle/B_0 = u_0/c \sin \theta_B = 0.134$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_3}
\caption{Electromagnetic fields in the reference run at time $\omega_{pe}t = 14625$ ($\Omega_{ci}t = 26.9$). Panels (a)-(c) show 2D plots of the magnetic field components in units of $B_0$, after subtracting the background field $\vec{B}_0$ (i.e., we show $(B_x - B_{x,0})/B_0$, $(B_y - B_{y,0})/B_0$ and $B_z/B_0$, respectively). The black arrows indicate the orientation of the upstream background magnetic field $\vec{B}_0$. Note that there are waves in all three components of the magnetic field. Panel (d) shows the $y$-averaged electric potential normalized by the electron rest mass energy, i.e. $e\Phi/(m_e c^2)$ (as indicated by the vertical axis on the left) or by the ion bulk kinetic energy, i.e. $e\Phi/(m_i u_0^2/2)$ (as indicated by the vertical axis on the right). Panels (e)-(g) show the $y$-averaged electric field components normalized by the upstream magnetic field strength, $\langle E_x \rangle/B_0$, $\langle E_y \rangle/B_0$, $\langle E_z \rangle/B_0$. Since the upstream plasma is initialized with zero electric field in its rest frame, the expected motional electric field $\vec{E}_0 = -(u_0/c) \times \vec{B}_0$ in the simulation frame in the upstream region is $\langle E_z \rangle/B_0 = u_0/c \sin \theta_B = 0.134$.}
\end{figure}
frame at $\omega_{pe}t = 14625$ ($\Omega_{ci}t = 26.9$). The shock front is at $x \simeq 1115 \, c/\omega_{pe}$ (indicated by the vertical dot-dashed lines in all the panels) and moves towards the right with a velocity of $u_{sh} \simeq 0.076 \, c$. This velocity yields $M_s = M \left( u_{sh}^{up} / u_0 \right) \sim M \left( u_{sh} + u_0 \right) / u_0 \simeq 3$, in agreement with Equation (2.3), that applies to a weakly magnetized medium.

In Figure 2.2(a) and (g) we show the ion and electron density normalized by the upstream value (red curves). The plasma density is compressed in the downstream, as expected. The influence of the magnetic field on the shock jump conditions can be neglected when the upstream plasma beta is $\beta_p \gg 1$ (Tidman & Krall, 1971). For our reference run, we have $\beta_p = 20 \gg 1$, so the expected compression ratio, $r = n_2 / n_1$, where $n_2$ and $n_1$ are the downstream and upstream plasma density respectively, can be safely estimated from the Rankine-Hugoniot conditions for an unmagnetized shock (Equation (2.4)). With $M_s = 3$ and $\Gamma = 5/3$, the jump condition yields $r = 3$.

In the simulation, the compression ratio is $r \sim 4$ near the shock front and relaxes to $r \sim 3$ farther downstream, as expected. The compression of the magnetic field strength is closely related to the density compression, by the freezing of magnetic flux. Under the $\beta_p \gg 1$ condition, the compression of the magnetic field transverse to the shock normal ($B_y$ in this case) can be approximated by $r$. Indeed, in the simulation, we observe that the compression of $B_y$ (blue curve in Figure 2.2(a)) is almost identical to the density compression (red curve in Figure 2.2(a)). Since the gradient of the total magnetic field strength drives the drift motion of electrons in the process of SDA (Section 2.4.2.1), we also plot the profile of the total field $B$ (black curve). As the upstream magnetic field is quasi-perpendicular ($\theta_B = 63^\circ$), the compression of total magnetic field is only slightly less than the compression of the transverse component.

The overshoot of the compression ratio at the shock has been well studied, see, e.g., Leroy
et al. (1981); Wu et al. (1984). It is ascribed to a population of gyrating ions that stay close to the shock front. These ions are reflected by the ambipolar electric field (Figure 2.3(e)), induced by the different inertia of the ions and electrons entering the shock. The potential energy $e\Phi$ of the ions near the shock front is comparable to their bulk kinetic energy $m_i u_0^2/2$, as indicated by the vertical axis on the right side of Figure 2.3(d).\footnote{Both observations and numerical simulations of shocks show that $e\Phi \sim m_i u_0^2/2$ holds generally (Amano & Hoshino, 2007).} It follows that a fraction of the ions having $m_i v_x^2/2 < e\Phi$ will be reflected at the shock front, to form a stream of gyrating particles ahead of the shock. Upon reflection, they acquire energy as they drift towards the $+\hat{z}$ direction along the shock motional electric field (Figure 2.3(g)). The resulting energy gain allows the gyrating ions to overcome the potential barrier and thus advect downstream at their second encounter with the shock. The gyromotion during their first encounter affects the magnetic field structure, and contributes to the formation of the magnetic overshoot seen in Figure 2.2(a).

The existence of the reflected ions can be inferred from the ion phase space plots in Figure 2.2(b)-(d)). The reflected ions are located just ahead of the shock and they have large and positive values of $p_x$ and $p_z$. We note that the gyrostream formed by the reflected ions is confined within $0 - 1 r_{L,i}$ ahead of the shock (as delimited by the yellow box in Figure 2.2). Beyond a few Larmor radii ahead of the shock, the ions follow the distribution at initialization, in terms of average velocities (Figure 2.2(e)) and temperature (Figure 2.2(f)).

In the downstream, the ions are heated anisotropically, with the momentum dispersion along the field (i.e., along the $y$ direction) being smaller.\footnote{In the absence of efficient pitch angle scattering, it is harder for the ions to isotropize along the direction of the mean field.} The resulting ion temperature anisotropy in the downstream is shown in Figure 2.2(f).
potential barrier at the shock. Rather, the potential tends to pull them into the down-
stream region. Yet, we observe a significant fraction of electrons reflecting back upstream,
propagating far ahead of the shock. These are the electrons with very large momenta
\( p_{x,y,z} \gtrsim 3 m_e c \) ahead of the shock in the electron phase spce plots (Figure 2.2(h)-(j)). The
reflection happens, according to the theory of SDA (Section 2.4.2.1), due to the jump of
the magnetic field at the shock, which effectively acts as a magnetic mirror. These reflected
electrons have preferentially large momenta parallel to the upstream magnetic field, as seen
from the excess of electrons with large positive \( p_y \) in the upstream (Figure 2.2(i)). Since
the returning electrons preferentially move along the upstream magnetic field, the electron
temperature parallel to the magnetic field \( T_{e||} \) is larger than the perpendicular component
\( T_{e\perp} \) in the upstream (Figure 2.2(l)).

The electron temperature anisotropy is closely related to the waves in the magnetic field
shown ahead of the shock in Figure 2.3(a)-(c). The waves in \( B_z \) (Figure 2.3(c)) show two
oblique modes symmetric around the direction of the background magnetic field (indicated
by the black arrow in Figure 2.3(a)-(c)). The waves in \( B_x \) and \( B_y \) tend to prefer one of
the two oblique modes present in \( B_z \) (Figure 2.3(a) and (b), respectively). The oscillatory
pattern in \( \langle E_y \rangle / B_0 \) (Figure 2.3(f)) is associated with the upstream magnetic waves in \( B_z \),
which are advected roughly at the upstream fluid velocity \( \vec{u}_0 = -0.15 c \hat{x} \). More precisely,
by analyzing high time resolution data from the simulation, we measure that the average
phase velocity of the waves is indeed \( \vec{v}_w \approx -0.15 c \hat{x} \) in the simulation frame. The fact
that the waves are moving toward the shock suggests that the particles triggering the waves
must exist beyond the region where the waves are present. The waves clearly exist beyond a
distance of a few hundred \( c/\omega_{pe} \) ahead of the shock. So do the returning electrons that cause
the upstream electron temperature anisotropy (Figure 2.2(l)). On the other hand, there are
no returning ions beyond 80 $c/\omega_{pe}$ ahead of the shock, as evident from the ion phase spaces, the average velocity profile and the temperature plots (Figure 2.2(b)-(f)). This is a strong evidence that the upstream waves are driven by the returning electrons, not by the ions.

### 2.4 Particle Energy Spectra and Acceleration Mechanism

In this section, we first present the evolution of the energy spectra of ions and electrons in the upstream and downstream regions of the reference run. We show that the electron energy spectrum in the upstream develops a clear non-thermal component and the high energy tail stretches in time to higher and higher energies, indicating efficient acceleration persisting over time. Then in subsection 2.4.2, we describe the electron acceleration mechanism by identifying SDA as the injection mechanism and showing how it enables sustained Fermi acceleration.

#### 2.4.1 Spectral Evolution

Figure 2.4 follows the time evolution of the ion and electron energy spectra at fixed distances from the shock in the upstream and downstream regions, as marked by the colored boxes in Figure 2.2. At each instant in time, we define the maximum energy of ions or electrons as the Lorentz factor $\gamma_{i,\text{max}}$ and $\gamma_{e,\text{max}}$ at which the particle number density drops below $10^{-4.5}$, the lowest level shown in our spectrum plots. (The value of $10^{-4.5}$ is arbitrary, but our results are not sensitive to this choice.) We show in the subpanels of Figure 2.4 how $\gamma_{i,\text{max}}$ and $\gamma_{e,\text{max}}$ evolve over time.
Figure 2.4: Time evolution of the ion (left) and electron (right) energy spectra from $\Omega_{ci} t = 4.1$ up to 29.8, measured in the upstream (top) or downstream (bottom) regions. (a) ion energy spectra taken $0 - 80 \, c/\omega_{pe}$ ($0 - 1 \, r_L$) ahead of the shock (yellow box in Figure 2.2). (b) ion energy spectra taken $80 - 160 \, c/\omega_{pe}$ ($1 - 2 \, r_L$) behind the shock (blue box in Figure 2.2). (c) electron energy spectra taken $80 - 160 \, c/\omega_{pe}$ ahead of the shock (magenta box in Figure 2.2). (d) electron energy spectra taken $80 - 160 \, c/\omega_{pe}$ behind the shock (green box in Figure 2.2). Color indicates time, from blue to red as the simulation evolves from early to late times. The subplot of each panel traces the maximum particle energy over time for the particles in the same slab where the spectrum is computed. Dot-dashed lines in panels (c) and (d) represent the initial electron energy spectrum, namely, a Maxwellian distribution with $T = 10^9 K$ drifting at the bulk velocity $\vec{u}_0 = -0.15 \, c \hat{x}$. The dashed line in panel (c) shows the best-fit power law of the non-thermal component in the late-time upstream electron energy spectrum. Panels (a) and (b) show that the ion energy spectra barely evolve over time. In contrast, panels (c) and (d) show that the electrons continue to be accelerated to higher and higher energies over time.
From the ion energy spectrum ahead of the shock in Figure 2.4(a), we see that about 20% of the incoming ions are reflected at the shock and form a high energy component in the upstream spectrum (at $\gamma_i - 1 \gtrsim 0.03$). The high-energy end of the ion spectrum shows little temporal evolution, as revealed also by the fact that the maximum Lorentz factor $\gamma_{i,max}$ is nearly constant over time. Similarly, the downstream ion energy spectrum in Figure 2.4(b) shows little time evolution. In addition, we have confirmed that the ion energy spectra taken further upstream ($x - x_{\text{shock}} \gtrsim r_{L,i}$, where $x_{\text{shock}}$ is the shock position) strictly follow the initial drifting Maxwellian distribution at all times, which demonstrates that no reflected ions can reach this region. This agrees with the ion phase space plots in Figure 2.2(b)-(d). Based on the lack of evolution in the ion spectra, we conclude that the acceleration of ions will not proceed to higher energies. We point out that this conclusion is valid only in the quasi-perpendicular regime with $\theta_B \gtrsim 45^\circ$. For quasi-parallel field configurations ($\theta_B \lesssim 45^\circ$), ions can be efficiently accelerated to higher and higher energies by DSA (Caprioli & Spitkovsky, 2014a). But an investigation of the physics of ion acceleration is beyond the scope of this paper.

In contrast to the ion energy spectrum, the upstream electron energy spectrum (Figure 2.4(c)) clearly develops a non-thermal tail over time. The fractional number density of non-thermal electrons is roughly 15% and the spectral index $p$, defined as $dN/d\gamma \propto (\gamma - 1)^{-p}$, has a best-fit value $p = 2.4$. The steady growth of the maximum electron energy $\gamma_{e,max}$ clearly shows that electron acceleration is efficient and persistent over time.

The downstream electron spectrum (Figure 2.4(d)) initially follows a Maxwellian distribution, whose temperature is higher than in the upstream. After $\sim 20 \Omega_{ci}^{-1}$, a non-thermal component begins to develop and the maximum energy increases. This happens because some of the electrons that get accelerated in the upstream are eventually advected down-
stream after being reflected back toward the shock by the upstream waves (Section 2.4.2.3). However, the non-thermal tail in the downstream electron spectrum evolves more slowly than in the upstream spectrum, and it is hard to disentangle from the Maxwellian distribution. Because of this, we will only study the upstream electron energy spectrum in the following sections.

2.4.2 The Acceleration Process

In general, the electron acceleration process operating in the reference run can be summarized as follows. First, a fraction of the incoming electrons, whose pitch angle and energy satisfy the reflection criteria of the shock-drift acceleration (SDA) process, which we shall describe in detail in Section 2.4.2.1, will drift along the shock surface and gain energy from the motional electric field. After the energization at the shock front, they are reflected upstream with increased energy. Their momentum is preferentially oriented along the upstream magnetic field, causing an electron temperature anisotropy (with $T_{e\parallel} > T_{e\perp}$) in the upstream region. The induced anisotropy drives self-consistently the generation of upstream waves, which can scatter the reflected electrons propagating upstream (after the SDA process) back toward the shock, for further energization through SDA. We identify this self-sustained process as a form of Fermi acceleration.

We first study the details of SDA in Section 2.4.2.1. We then illustrate the electron Fermi acceleration process by showcasing the evolution of a typical non-thermal electron in Section 2.4.2.3.
2.4.2.1 Shock Drift Acceleration Theory

The non-relativistic theory of SDA including non-zero cross-shock electric potential has been reviewed by Ball & Melrose (2001). Mann et al. (2006) have developed the relativistic theory of SDA with vanishing cross-shock potential and Park et al. (2013) have studied the effect of non-zero cross-shock potential in the non-relativistic limit. In our reference run, the upstream electrons are hot and marginally relativistic, and the cross-shock potential is also significant compared to the electron rest mass energy. In this section, we summarize findings from previous work and generalize the fully relativistic SDA theory to properly treat arbitrary value of cross-shock potential.

We begin by addressing the relation between the drift motion of electrons at the shock and their energy gain by the SDA mechanism. The physics can be intuitively understood in the downstream frame. The gradient of the magnetic field (along $-\hat{z}$) near the shock front (see Figure 2.2(a)) causes an incoming electron to drift with $\vec{v}_\nabla B = (-p_\perp^2 / 2m_e c^2 \gamma B^3)(\vec{B} \times \nabla B)$ along the $-\hat{z}$ direction. Here, $p_\perp$ denotes the component of the particle momentum perpendicular to the magnetic field. The electron is then accelerated by the motional electric field $E_z$ (Figure 2.3(g)) in the downstream rest frame. The contribution to the electron energy gain by $E_z$ takes the form

$$\Delta \gamma_{\text{SDA}} = -\frac{e}{m_e c^2} \int E_z \, dz ,$$

(2.7)

where the integral is over the drift path. Assuming that the electric field near the shock is constant and is purely due to the upstream motional electric field, $E_z(x) \approx E_0 \equiv$
$u_0/cB_0 \sin \theta_B$, then the energy gain via SDA in the downstream frame can be estimated as

$$\Delta \gamma_{\text{SDA}} \simeq -\frac{e}{m_ec^2} E_0 \Delta z = -\sqrt{\frac{\sigma}{2} \frac{m_i}{m_e} \frac{u_0^2}{c^2} \sin \theta_B} \frac{\Delta z}{c/\omega_{pe}}. \quad (2.8)$$

The time scale of the SDA drift cycle is estimated by Krauss-Varban & Wu (1989); Krauss-Varban et al. (1989) to be $\sim \Omega_{ci}^{-1}$, which we have verified in our simulations.

Figure 2.5 shows the trajectory of an electron undergoing one cycle of SDA, at an early stage of the shock evolution. The SDA process operates from $\Omega_{ci} t \sim 3.5$ up to $\Omega_{ci} t \sim 6.5$, during which the particle stays within $\sim 50 \ c/\omega_{pe}$ ahead of the shock. The electron drifts along $-\hat{z}$ for a distance $\Delta z \simeq -200 \ c/\omega_{pe}$, and its energy gain is $\Delta \gamma \simeq 6$. Using the relevant parameters $\sigma = 0.03, u_0 = 0.15 c, \theta_B = 63^\circ$, we find that the energy gain indeed comes almost exclusively from the drift motion along $-\hat{z}$, in agreement with Equation (2.8).

In other words, $\Delta \gamma \sim \Delta \gamma_{\text{SDA}}$. After being reflected by the shock at $\Omega_{ci} t \sim 6.5$, the electron propagates back into the upstream.

In order to understand the efficiency of SDA and the conditions under which an incoming electron can participate in SDA, it is more convenient to switch to two other frames: the de Hoffman-Teller (HT) frame (de Hoffmann & Teller, 1950) or the upstream rest frame. The upstream rest frame is a convenient choice to analyze the properties of the incoming plasma.

From the downstream rest frame, it is obtained by simply boosting with velocity $u_0$ along the shock normal. The HT frame has the advantage that the motional electric field vanishes on both sides of the shock, since the flow velocity is parallel to the background magnetic field both ahead and behind the shock. The HT frame can be obtained, starting from the upstream rest frame, by boosting in the direction opposite to the upstream magnetic field.
Figure 2.5: The evolution of a typical electron undergoing shock drift acceleration (SDA). Color indicates time, from blue to red as time evolves from early to late. Panels (a) and (b) show the evolution of the electron Lorentz factor and z-coordinate as a function of its distance from the shock. Panel (c) shows the x-location of the electron on top of the spatio-temporal evolution of the y-averaged value of $B_z/B_0$. All quantities are measured in the simulation (downstream) frame.
with a velocity

\[ u_t = u_{sh} \sec \theta_B = \frac{u_{sh} + u_0}{1 + u_{sh}u_0/c^2} \sec \theta_B . \]  

Note that the HT frame can be defined only for shocks with \( u_t \leq c \). Shocks with \( u_t > c \) are characterized as superluminal, and the SDA physics explained below does not apply. For the parameters of our reference run, magnetic field configurations with \( \theta_B \gtrsim 77^\circ \) are superluminal. Our choice of the obliquity angle \( \theta_B = 63^\circ \) is thus well below the superluminality threshold.

Given that the motional electric field vanishes on both sides of the shock in the HT frame, we can assume that the total electron energy and the magnetic moment \( \mu \) are conserved, i.e.,

\[ \gamma^{HT}(x)m_e c^2 - e\Phi^{HT}(x) = \text{const} , \]  

\[ \mu^{HT}(x) \equiv \frac{\left[p^{HT}_\perp(x)\right]^2}{2m_e B^{HT}(x)} = \text{const} , \]

where the superscript HT refers to quantities in the HT frame and \( p_\perp \) denotes the momentum perpendicular to the magnetic field. Combining Equations (2.10) and (2.11), we can solve for the velocity parallel to the magnetic field in the HT frame \( v^{HT}_\parallel \) as a function of the magnetic field amplification at the shock, the electric potential energy change, the initial (denoted by subscript \( i \)) velocity perpendicular to the magnetic field \( v^{HT}_{i\perp} \) and the initial Lorentz factor \( \gamma^{HT}_i \)

\[ v^{HT}_\parallel(x) = c \sqrt{1 - \frac{1 + \left(\gamma^{HT}_i\right)^2 \left(v^{HT}_{i\perp}/c\right)^2 B^{HT}(x)}{\left[\Delta \phi(x) + \gamma^{HT}_i\right]^2}} , \]  

(2.12)
where we have defined the dimensionless parameter \( \Delta \phi = e \left[ \Phi^{HT}(x) - \Phi^{HT}_0 \right] / m_e c^2 \) for notational convenience. For a particle to be reflected at the shock in the HT frame, it needs to move towards the shock in the first place, which requires the initial velocity along the magnetic field line \( v_{i||}^{HT} \) to be negative. Secondly, the reflection occurs when the parallel velocity in Equation (2.12) vanishes. Combining these two conditions, we obtain the reflection conditions in the HT frame

\[
v_{i||}^{HT} < 0 ,
\]  
\[
v_{i\perp}^{HT} \geq c \sqrt{ \frac{B_0^{HT}}{B^{HT}(x)} \left[ \frac{\gamma_i^{HT} + \Delta \phi(x)}{\gamma_i^{HT}} \right]^2 - 1 } .
\]

Alternatively, we can rewrite the second equation as a condition on the pitch angle in the HT frame, defined as \( \alpha_i^{HT} \equiv \cos^{-1} \left( v_{i||}^{HT} / v^{HT} \right) \),

\[
\alpha_i^{HT} \geq \sin^{-1} \left[ \sqrt{ \frac{B_0^{HT}}{B^{HT}(x)} \left[ \frac{\gamma_i^{HT} + \Delta \phi(x)}{\gamma_i^{HT}} \right]^2 - 1 } \right] .
\]

This states that only particles with pitch angle larger than \( \alpha_i^{HT} \) can be reflected back toward the upstream. The effect of a positive potential jump \( \Delta \phi > 0 \) is to increase the minimum pitch angle needed for reflection, and thus reduce the number of particles satisfying the reflection condition. Physically, this stems from the fact that, if \( \Delta \phi > 0 \), the electric force tends to attract the electrons into the shock, so it is harder for them to reflect upstream.

The velocities in the HT frame are related to those in the upstream rest frame by the
relativistic velocity addition formulae

\[ v_{\parallel}^{\text{HT}} = \frac{v_{\parallel}^{\text{up}} - u_t}{1 - v_{\parallel}^{\text{up}}u_t/c^2}, \quad (2.16) \]

\[ v_{\perp}^{\text{HT}} = \frac{v_{\perp}^{\text{up}}}{\gamma_t \left(1 - v_{\parallel}^{\text{up}}u_t/c^2\right)}, \quad (2.17) \]

which imply that

\[ \gamma_{\text{HT}} = \gamma_{\text{up}}\gamma_t \left(1 - v_{\parallel}^{\text{up}}u_t/c^2\right), \quad (2.18) \]

where \( u_t \) is the relative velocity between the HT frame and the upstream rest frame defined in Equation (2.9) and \( \gamma_t = 1/\sqrt{1 - u_t^2/c^2} \) is the corresponding Lorentz factor. Applying the above Lorentz transformations to Equations (2.13)-(2.15), one can obtain the reflection conditions in the upstream rest frame.

The condition on \( v_{i\parallel}^{\text{up}} \) is simply

\[ v_{i\parallel}^{\text{up}} < u_t, \quad (2.19) \]

The condition on \( v_{i\perp}^{\text{up}} \) is straightforward to derive but lengthy. However, we can gain some insight by considering the limit \( \Delta \phi \to 0 \). In this case, the particles would be reflected back upstream if

\[ v_{i\perp}^{\text{up}} \geq \gamma_t \left(u_t - v_{i\parallel}^{\text{up}}\right) \tan \alpha_0, \quad (2.20) \]

where we have defined \( \alpha_0 = \sin^{-1}\left[B_0^{\text{HT}}/B^{\text{HT}(x)}\right]^{1/2} \). In the limit \( \Delta \phi \to 0 \), one can obtain a lower bound on the energy needed for reflection. Taking the equality in Equation (2.20), we find that the lower boundary of the allowed region for SDA reflection (at \( v_{i\parallel}^{\text{up}} < u_t \)) is
described by

\[
(v_{\text{up}}^i)^2 = (v_{\parallel i}^i)^2 + (v_{\perp i}^i)^2 \\
= (1 + \gamma_i^2 \tan^2 \alpha_0) (v_{\parallel i}^\text{up})^2 - 2 \gamma_i^2 u_t v_{\parallel i}^\text{up} \tan \alpha_0 \\
+ \gamma_i^2 u_t^2 \tan^2 \alpha_0 .
\]  

(2.21)

The minimum of this quadratic equation with respect to \( v_{\parallel i}^\text{up} \) gives a lower bound on the minimum velocity in the upstream rest frame for SDA reflection

\[
v_{i,\text{min}}^{\text{up}} = u_t \sqrt{\frac{\tan^2 \alpha_0}{\tan^2 \alpha_0 + 1/\gamma_i^2}} ,
\]  

(2.22)

which grows monotonically with \( u_t \). It follows that, with increasing \( u_t \), i.e., for higher shock velocity or magnetic obliquity, the minimum energy required to participate in the SDA process will increase. Thus, the fraction of particles that can participate in SDA decreases with increasing \( u_t \), and the reflection fraction drops further with increasing \( \Delta \phi > 0 \), as we have discussed before.

After reflection (which we shall indicate with the subscript \( r \)), the parallel velocities of the particles are reversed in the HT frame:

\[
v_{r\parallel}^\text{HT} = -v_{\parallel i}^\text{HT} , \quad v_{r\perp}^\text{HT} = v_{\perp i}^\text{HT} .
\]  

(2.23)

Transforming back to the upstream rest frame, we just need to switch the superscripts and change the sign of \( u_t \) in Equations (2.16)-(2.18). The post-reflection Lorentz factor and
velocity are related to the pre-reflection values by

\[ \gamma_r^{up} = \gamma_i^{up} \left[ 1 + \frac{2u_t (u_t - v_{i\|}^{up})}{c^2 - u_t^2} \right] \equiv \gamma_i^{up} (1 + \Delta_i \rightarrow r) , \quad (2.24) \]

\[ v_{r\|}^{up} = \frac{\gamma_i^{2} 2u_t - v_{i\|}^{up} (1 + u_t^2 / c^2)}{1 + \Delta_i \rightarrow r} , \quad (2.25) \]

\[ v_{r\perp}^{up} = \frac{v_{i\perp}^{up}}{1 + \Delta_i \rightarrow r} . \quad (2.26) \]

Since \( v_{i\|}^{up} < u_t \) is required for reflection (see Equation (2.19)), the term in the square brackets in Equation (2.24) is always larger than unity (i.e., \( \Delta_i \rightarrow r > 0 \)). Thus, all the reflected particles will gain energy. For the same reason, all the reflected particles will suffer a reduction in their perpendicular velocity (see Equation (2.26)). Given the net increase in energy, we conclude that, after reflection, all the particles will have a smaller perpendicular momentum and a larger parallel momentum, and thus move preferentially along the magnetic field. In addition, for a given \( v_{i\|}^{up} \), the fractional energy gain \( \Delta_i \rightarrow r \) increases with increasing \( u_t \).

### 2.4.2.2 Verification of SDA in the Simulation

To confirm that the SDA process indeed operates in our simulation, we trace the evolution of 12800 electrons injected into the reference run at \( \omega_{pe} t = 450 \) (\( \Omega_{ci} t = 0.83 \)) and check whether the properties of the reflected electrons agree with the predictions of SDA.

The predictions of SDA are indicated in the plots of velocity space \( v_{\|}^{up} - v_{\perp}^{up} \) in Figure 2.6. The white dashed half-circle indicates the limit \( v = c \), so only the regions within the half-circle have physical meaning. The vertical white dashed line marks the condition
Figure 2.6: Velocity space $v_{\parallel}^u - v_{\perp}^u$ of the sample of selected electrons at different stages of their evolution. The electrons are initialized in the reference run at $\omega_{pe}t = 450$ ($\Omega_{ci}t = 0.83$). In all the plots, the dashed half-circle indicates the speed of light. Panel (a) shows the velocity space of all the electrons that reach the shock, at the time when they first approach within $50 \, c/\omega_{pe}$ ahead of the shock. In panels (b) and (c) we plot, at two different times, the phase space of particles that are identified as eventually being reflected upstream from the shock. Panel (b) refers to the time when the identified electrons first approach within $50 \, c/\omega_{pe}$ ahead of the shock, coming from the upstream side. Panel (c) refers to the time when they leave the shock region, i.e., at the time when they first move beyond $50 \, c/\omega_{pe}$ away from the shock after the interaction. The 2D histograms of the velocity space plots are to be compared with the over-plotted predictions of the SDA theory (green and pink solid lines), computed assuming a magnetic compression ratio of $b = 4$ and a cross-shock potential of $\Delta \phi = 0.5$. The effect of different values of the dimensionless cross-shock potential $\Delta \phi$ is illustrated with the dash-dotted color lines ($\Delta \phi = 0.2$) and the dashed color lines ($\Delta \phi = 0$). The vertical white dashed line tracks the location where $v_{\parallel}^u = u_t$. The nearly-horizontal green curve to its left is the minimum requirement on $v_{\perp}^u$ for SDA reflection (Equations (2.14) and (2.17)). The nearly-horizontal pink curve to the right of $v_{\parallel}^u = u_t$ is the post-reflection mapping of the green curve (Equation (2.23)). The region enclosed by the green solid lines indicates the allowed region for SDA reflection. The region enclosed by the pink lines indicates the predicted region where the electrons will lie after reflection.
The nearly-horizontal green solid curve to its left indicates the limit in Equation (2.14), once transformed to the upstream rest frame using Equation (2.17). Only particles with velocity to the left of the white dashed line are approaching the shock, and only those with velocity within the region enclosed by the green solid lines are allowed for SDA reflection. The pink solid curve to the right of the white dashed line is the post-reflection mapping of the green curve (following Equations (2.24)-(2.26)). Thus, the SDA theory predicts that after reflection, the velocity of the particles should occupy the region enclosed by the pink solid lines.

We remark that $B_{HT}^0 / B_{HT}(x)$ and $\Delta \phi(x)$ in Equation (2.14) are both functions of position, and are likely to differ for each individual particle, depending on its location at the time of reflection. But to first order, we shall assume a constant value of the magnetic compression ratio $b \equiv B_{HT}(x) / B_{HT}^0$ and of the cross-shock potential $\Delta \phi$ in our SDA theory. For our reference run, we use $b = 4$, which is roughly the value of magnetic field compression in the HT frame at the shock overshoot. The dimensionless cross-shock potential is chosen to be $\Delta \phi = 0.5$, which we estimate from Lorentz transformations of the electromagnetic fields measured in the simulation frame. As we show below, a value of $\Delta \phi = 0.5$ also yields a reflection fraction – defined as the fraction of particles in the initialized Maxwellian distribution that satisfy the SDA reflection conditions – which matches the simulation results. To show the effect of different values of $\Delta \phi$, we also plot in Figure 2.6 the SDA predictions corresponding to $\Delta \phi = 0$ and $\Delta \phi = 0.2$ with colored dashed lines and dot-dashed lines, respectively. As we have discussed in the previous subsection, the allowed region for reflection shrinks with increasing $\Delta \phi$.

While tracing the selected sample of particles, we identify those electrons that have approached the shock within $50 \ c/\omega_{pe}$. After interacting with the shock, a subset of these
electrons will be reflected upstream. Our results are plotted as 2D histograms in Figure 2.6. Panel (a) shows the velocity space of all the electrons that have approached the shock, at the time when they are located at $\sim 50 c/\omega_{pe}$ ahead of the shock. We see that no electron with $v_{up}^\parallel > u_t$ has reached the vicinity of the shock, just a consequence of the fact that particles initialized with $v_{up}^\parallel > u_t$ would have propagated away from the shock. Despite the uncertainties in the values of the magnetic compression ratio and of the cross-shock potential jump, Figure 2.6(b) clearly demonstrates that only electrons satisfying the SDA reflection criteria (namely, the region enclosed by the green solid lines) will eventually be reflected back upstream. Also, the velocity distribution of the particles after reflection (Figure 2.6(c)) lies well within the region enclosed by the pink solid lines, in agreement with the SDA predictions.

As an additional test, we have used the SDA theory to compute a synthetic electron energy spectrum after one cycle of SDA, and we have compared the synthetic spectrum to the energy spectrum measured in the simulation just upstream from the shock. In the upstream rest frame, the initial electron velocity distribution is a Maxwellian with $kT_e/m_e c^2 = 0.17$, which is shown in Figure 2.7(a) as a 2D histogram. We identify the electrons that satisfy the reflection conditions (i.e., inside the region delimited by the green lines in Figure 2.7(a)), compute their post-reflection energy using Equation (2.24), and combine them with the sub-population of the initialized electrons that satisfies $v_{up}^\parallel < u_t$, so that the initialized electrons can approach the shock. Finally, we transform the energy of each electron from the upstream frame to the simulation (downstream) frame. The resulting synthetic spectrum is shown with the solid blue line in Figure 2.7(b). It agrees very well with the actual energy spectrum measured from the simulation at a relatively early time, $\Omega_{ci} t = 3.7$ (blue dashed line).
Figure 2.7: Panel (a) shows the velocity space \( v_{\parallel}^{up} - v_{\perp}^{up} \) of the injected electrons in the reference run. The SDA predictions are indicated with the green and pink solid lines as in Figure 2.6, assuming \( b = 4 \) and \( \Delta \phi = 0.5 \). The synthetic spectrum is constructed by combining two populations. The first population consists of electrons with \( v_{\parallel}^{up} < u_t \) (i.e., in the region to the left of the vertical white dashed line in panel (a)), which is required so that the electrons can interact with the shock. The second population consists of electrons that were originally within the region allowed for SDA reflection, as delimited by the green solid line. After reflection, they move to the region delimited by the pink solid line, and their energy increases according to Equation (2.24). Panel (b): The blue solid line shows the synthetic energy spectrum computed in the way described above, following the SDA predictions. The blue dashed line shows the electron energy spectrum measured in the simulation at a relatively early time (\( \Omega_{ci}t = 3.7 \)) at a distance of \( 50 - 100 \ c/\omega_{pe} \) ahead of the shock. This is in very good agreement with the synthetic spectrum. The red dashed line shows the spectrum measured at a later time (\( \Omega_{ci}t = 28.9 \)), which differs significantly from the synthetic SDA spectrum. The late-time spectrum shows a pronounced non-thermal component extending to much larger energies, suggesting the presence of a long-term Fermi-like acceleration mechanism.
It should be stressed that the maximum energy of the predicted SDA spectrum is time-independent, for a given set of shock parameters. However, at late times, the electron energy spectrum in our simulation keeps evolving, with the non-thermal tail stretching in time to higher and higher energies. The red dashed line in Figure 2.7(b) shows the electron energy spectrum measured at a later time, \( \Omega_{ci}t = 28.9 \). The maximum energy here is \( \gamma_{e,max} \gtrsim 20 \), three times larger than what is predicted for one-cycle of SDA, \( \gamma_{e,max} \sim 6 \) (compare the red dashed line with the blue solid line). Such long-term sustained acceleration can only be achieved by an additional stage of energization, akin to the Fermi mechanism.

2.4.2.3 Fermi Acceleration

In Figure 2.8, we show the evolution of a typical non-thermal electron that undergoes multiple SDA cycles, in a way resembling the well-known Fermi process. The energy and z-location of the particle (Figure 2.8(a) and (b)) show that the electron first gains energy at \( \Omega_{ci}t \sim 2.5 - 5 \) via SDA and is then reflected upstream. While the particle propagates upstream at \( \Omega_{ci}t \sim 5 - 9 \), it interacts with the upstream waves (Figure 2.8(c)), generated by previous populations of returning electrons streaming ahead of the shock. As a result of the interaction with the waves, the electron momentum parallel to the magnetic field is reduced, and eventually the electron is scattered back toward the shock at \( \Omega_{ci}t \sim 9 \). The interaction with the waves themselves does not yield a significant energy gain, yet it allows the particle to approach the shock for a second time. When the particle reaches \( 50 c/\omega_{pe} \) ahead of the shock at \( \Omega_{ci}t \sim 10.5 \), the gradient of the magnetic field confines the electron at the shock, and the particle gains energy again through a second SDA cycle. This appears clearly from both the drift motion along \(-\hat{z}\) and the corresponding increase in energy (Figure 2.8(a) and (b)).
Figure 2.8: The evolution of a typical non-thermal electron undergoing multiple SDA cycles, in a way that resembles the standard Fermi mechanism. Color indicates time, from blue to red as time evolves from early to late. Panels (a) and (b) show the evolution of the electron Lorentz factor and $z$-coordinate as a function of its distance from the shock. Panel (c) shows the $x$-location of the electron on top of the spatio-temporal evolution of the $y$-averaged value of $B_z/B_0$. All quantities are measured in the simulation (downstream) frame. The particle is scattered by the upstream waves plotted in panel (c) back toward the shock at $\Omega_{ci}t \sim 9$, which allows it to gain additional energy via a second cycle of SDA.
We remark that the conventional Fermi acceleration relies on scattering by waves both upstream and downstream of the shock. Although we do observe downstream waves in Figure 2.3(a)-(c), they are not necessary for the Fermi-like acceleration we are discussing here. As shown in the typical particle orbit in Figure 2.8, the particles are reflected upstream by the magnetic field gradient at the shock front via the magnetic mirror effect, without penetrating the downstream region. The upstream waves then scatter these reflected electrons back towards the shock for additional cycles of SDA. We note that the particles propagate into the upstream for a distance larger than a few Larmor radii, as expected in the standard Fermi acceleration mechanism. It is worth contrasting this behavior with that of the particle in Figure 2.5, which is not scattered back from the upstream region towards the shock. The main difference is that after the first cycle of SDA, the particle in Figure 2.5 returned upstream having a large component of the momentum parallel to the magnetic field. It propagated away from the shock, with little deflection by the upstream waves (which were still weak, at such early times). As a result, the interaction with the waves was not sufficient to scatter the electron back towards the shock for further SDA energization.

In general, particles emerging from the SDA acceleration with a small parallel momentum are more favorable for being scattered back toward the shock by the upstream waves. After being scattered, their parallel momentum is still small, and since their energy has increased due to the previous SDA cycle, they lie in the region of velocity space favorable for additional SDA reflection and acceleration. This makes this Fermi-like acceleration process – composed of multiple SDA cycles – extremely efficient. We also note that particles emerging from the SDA acceleration with a large parallel momentum, such as the one in Figure 2.5, though not likely to undergo Fermi acceleration, are still very important for the overall acceleration process. In fact, their large parallel momentum contributes to the
electron temperature anisotropy in the upstream region (see Figure 2.2(l)), which governs the growth of the upstream waves that scatter later generations of reflected electrons. The nature of the electron self-generated waves will be discussed in a forthcoming paper (Guo et al., 2014b). We stress that the electron self-generated waves exist and mediate efficient electron acceleration in low Mach number shocks for a wide range of physical parameters. As we will show in a forthcoming paper (Guo et al., 2014b), we reach similar conclusions across nearly all the magnetic obliquity angles, in the temperature range $T_e = 10^7 - 10^9 K$ and for magnetizations $\sigma = 0.01 - 0.03$. The excitation of the waves requires the parallel electron thermal pressure, $P_e^\parallel \equiv n_e k_B T_e^\parallel$, to be larger than the magnetic pressure. In addition, it relies on the free energy provided by the electron temperature anisotropy ($T_e^\parallel > T_e^\perp$) introduced by the returning electrons, which further depends on the SDA process. The detailed dependence on various physical parameters shall be presented in the forthcoming paper (Guo et al., 2014b).

### 2.5 Summary and Discussion

In this work, we study from first principles the physics of electron acceleration in a low Mach number ($M_s = 3$) shock, by means of fully kinetic PIC plasma simulations. In our simulation, the upstream plasma follows a Maxwell-Jüttner distribution with temperature $T_e = T_i = 10^9 K$ and a low magnetization parameter $\sigma = 0.03$ (equivalent to a plasma beta $\beta_p = 20$). The upstream magnetic field is oriented at an angle of $\theta_B = 63^\circ$ with respect to the shock normal. The physical parameters we choose are applicable to the bow shocks expected to form ahead of the G2 cloud (Narayan et al., 2012; Sadowski et al., 2013) and of the S2 star (Giannios & Sironi, 2013) upon interaction with the hot accretion
flow at the Galactic Center. Our parameters are also relevant to merger shocks in galaxy clusters, aside from the lower temperature of the intra-cluster plasma \((T \sim 10^7 \, K)\). A complete investigation of the parameter space will be presented in a forthcoming paper \((\text{Guo et al.}, \, 2014b)\), where we will show explicitly that our results can be generalized to lower temperatures. We emphasize that, in the upstream frame, the plasma is initialized according to the physically-grounded Maxwellian distribution, instead of the so-called “\(\kappa\)-distribution” that was employed by, e.g., \(\text{Park et al.} (2013)\). The latter distribution contains an additional supra-thermal tail that can artificially enhance the injection of electrons into the acceleration process.

We find that ions are not efficiently accelerated. In contrast, about 15% of the incoming thermal electrons are accelerated up to non-thermal energies. The upstream electron energy spectrum develops a non-thermal power-law tail with slope \(p \equiv -d\log N/d\log(\gamma - 1) \simeq 2.4\). The energy density carried by the high-energy electrons is \(\simeq 10\%\) of the bulk kinetic energy density of the incoming ions. The spectral cut-off energy of the upstream electron spectrum steadily grows with time, indicating that the acceleration process persists to late times. The radio synchrotron spectral index expected for a slope \(p \simeq 2.4\) of the electron energy distribution is \(\alpha \equiv d\log F_\nu/d\log \nu = (1 - p)/2 \simeq -0.7\) \((\text{Rybicki & Lightman}, \, 1979)\), which agrees with the radio spectral index \(\alpha = -0.6\pm0.05\) observed at the shock front of the radio relic in the galaxy cluster CIZA J2242.8+5301 \((\text{van Weeren et al.}, \, 2010)\). Incidentally, the polarization analysis of the radio relic shows that the magnetic field is quasi-perpendicular, which is consistent with our setup.

We study in detail the electron acceleration mechanism. We find that shock drift acceleration (SDA) governs the injection of electrons into a Fermi-like acceleration process, that self-consistently persists in the long-term evolution of the shock. We develop a fully-
relativistic theory of the SDA process and we compare it to the results of our simulation, finding excellent agreement. During the SDA process, a fraction of the incoming electrons gain energy from the shock motional electric field while drifting along the shock surface, and they are reflected back upstream. By tracing electrons from the simulation, we demonstrate that our SDA theory properly predicts the conditions required for participating in the SDA process (and so, for the subsequent reflection toward the upstream). We also show that the electron energy spectrum predicted by our SDA formalism, assuming one cycle of SDA acceleration, agrees well with the spectrum measured from the simulation at early times. However, the spectrum from the simulation at late times clearly indicates the existence of additional energization, beyond a single cycle of SDA. The additional energy gain is mediated by the upstream waves self-generated by the electrons streaming ahead of the shock after the SDA phase. The upstream waves are not primarily driving the electron energy gain. Rather, they scatter the electrons propagating upstream back toward the shock for multiple cycles of SDA, thus sustaining a long-term acceleration process akin to the Fermi mechanism.

Our study offers a possible solution to the electron injection problem in the low Mach number shocks present in galaxy clusters. The bright radio luminosity that is observed from radio relics in the outskirts of galaxy clusters seems to be in contradiction with the poor electron acceleration efficiency expected on theoretical grounds (e.g. Brunetti & Jones, 2014, for a review). Most theoretical models assume that the particles are injected via the so-called “thermal leakage” process (Malkov & Völk, 1998; Gieseler et al., 2000; Kang et al., 2002), i.e., supra-thermal particles can propagate from the downstream back into the upstream and get injected into the Fermi process. This model requires that the electrons should have a momentum at least a few times larger than the characteristic post-shock
ion thermal momentum, in order to be injected into the Fermi process. The minimum momentum for injection increases with decreasing Mach number and is at least a factor of $\sqrt{m_i/m_e}$ larger than the expected post-shock electron momentum (Kang & Ryu, 2010). As a result, the fraction of electrons that can participate in the Fermi process at low Mach number shocks is expected to be extremely small (Kang & Ryu, 2010). In contrast, the observed bright radio emission from radio relics requires a large number of accelerated electrons. Kang et al. (2014) propose to resolve this conflict by assuming the existence of electrons following a $\kappa$-distribution, which has an ad hoc supra-thermal tail with a power-law shape. However, the existence of such supra-thermal electrons at the outskirts of galaxy clusters has never been demonstrated (Pinzke et al., 2013).

The key issue of the thermal-leakage model summarized above is that it assumes that the electrons, in order to be injected into the Fermi process, need to be scattered by the MHD waves in the downstream region to propagate back into the upstream, and thus they have to possess very large momenta in the first place (so that their Larmor radius is larger than the scale of the MHD turbulence). In contrast, our mechanism, based on first-principle PIC simulations, does not involve any scattering in the downstream turbulence. Rather, the shock itself acts as a magnetic mirror, reflecting a fraction of the incoming electrons back upstream after the SDA stage. The minimum electron momentum required for reflection via the SDA mechanism is much lower (by a factor of $\sim m_e/m_i$) than that required in the thermal-leakage model. We emphasize that, while the electron injection efficiency in the thermal-leakage model should significantly decrease for higher mass ratios, our results are

\footnote{While the thermal leakage model was originally invoked for quasi-parallel shocks and has not been well developed for quasi-perpendicular shocks, it has been shown that the minimum momentum required for injection in quasi-perpendicular shocks is no less than in quasi-parallel shocks (see e.g. Zank et al., 2006, in the context of interplanetary shocks), so the injection efficiency in quasi-perpendicular shocks is expected to be even smaller.}
insensitive to the choice of $m_i/m_e$, as shown in Appendix 2.B. In addition, in our study we show that the electrons propagating upstream can self-generate magnetic waves, without any need for the MHD turbulence that is usually invoked in the thermal-leakage models described above.

We also remark that the SDA-mediated injection that we describe in this work is complementary to the injection mechanisms invoked in high Mach number and low plasma beta shocks, which have been extensively studied in application to Supernova Remnants. The often invoked electron shock surfing acceleration cannot operate in low Mach number shocks, since the Buneman instability is suppressed in hot plasmas. On the other hand, SDA cannot operate efficiently in high Mach number shocks, because the fraction of velocity space that allows injection via SDA is shrinking with increasing Mach number, resulting in poor acceleration efficiencies. The injection by whistler waves proposed by Riquelme & Spitkovsky (2011) in low beta flows is not playing any important role in high plasma beta shocks, where energization via SDA is dominating. On the other hand, the magnetic waves that allow for multiple SDA cycles in our mechanism are suppressed in low plasma beta flows, as will be demonstrated in a forthcoming paper (Guo et al., 2014b).

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2.A Dependence on the dimensionality

Our reference run is performed on a 2D spatial domain, although we solve for all the 3D components of the particle momenta and of the electromagnetic fields. To test the validity of our choice of a reduced dimensionality, we have run simulations with the same physical parameters as in our reference run, but in 1D and 3D computational domains. Similarly, to validate our choice for the orientation of the magnetic field (lying in the simulation plane, i.e., $\varphi_B = 0^\circ$ in our reference run), we have performed a 2D simulation with $\varphi_B = 90^\circ$ (referred to as 2D out-plane). For the 1D run, the box size along the $y$-dimension is only $1 \, c/\omega_{pe}$. For the 3D run, the box size along the $z$-dimension is equal to that along the $y$-dimension, which is $76 \, c/\omega_{pe}$ (corresponding to $7.6 \, c/\omega_{pi}$, or almost one ion Larmor radius).

In our 3D run, we choose to resolve the electron skin depth with 5 cells, and we initialize the upstream plasma with 2 computational particles per cell (one per species), as opposed to the larger value (32) that we can employ in 2D simulations.

In Figure 2.9(a), we compare the electron energy spectra from the reference run (2D in-plane) and the 3D run at $\Omega_{ci}t = 7.4$, the latest stage of evolution of the 3D simulation. We find that the energy spectra agree very well in terms of both the normalization and the power-law slope of the non-thermal tail. However the spectrum cuts off at a lower energy in the 3D run, and the maximum energy also grows slower at late times. This is likely an artifact of the lower number of computational particles per cell (2 in the 3D run versus 32 in the reference run). We have checked that the upstream waves in 3D show a similar pattern.
as in our reference run. Thus, Fermi acceleration is expected to operate in the same way in 2D and 3D.

The comparison of the electron energy spectra between the reference run, the 1D run and the 2D out-plane run (Figure 2.9(b)) shows that the spectra from both the 2D out-plane run and the 1D simulation present an artificially higher normalization than those from the 3D and 2D in-plane runs. The growth of the maximum energy is also slower than in the reference run, as shown in the subpanel. This indicates that the 2D out-plane configuration misses some of the important physics. Our conclusion is that the 2D in-plane configuration is a good choice to capture the acceleration physics of the full 3D problem.

2.B Dependence on the mass ratio

Since our simulations employ a reduced mass-ratio $m_i/m_e = 100$, it is natural to wonder how the results will change when a realistic mass-ratio of $m_i/m_e = 1836$ is used. We note that $u_t$ and the dimensionless electric potential jump $\Delta \phi$ are the key parameters that determine the minimum energy required for participating in the SDA process (Equation (2.22) and discussion thereafter). Their value controls the fraction of reflected electrons and the maximum energy gain per cycle (Equation (2.24)). To understand the effect of the mass ratio, we rewrite $u_t$ and $\Delta \phi \sim m_iu_0^2/(2m_c^2)$ in terms of the simulation parameters as

$$u_t = u_{up} \sec \theta_B = \sqrt{\frac{2\Gamma k_B T_i}{m_i}} M_s \sec \theta_B = \sqrt{\frac{2\Gamma k_B T_e}{m_e}} \sqrt{\frac{m_e}{m_i}} M_s \sec \theta_B ,$$

(2.27)

$$\Delta \phi \sim \frac{m_iu_0^2}{2m_c^2} = M^2 \Gamma \frac{k_B T_e}{m_c^2}.$$

(2.28)

This suggests that, for fixed $T_e = T_i$, $M$ and $M_s$, if we scale the obliquity angle $\theta_B$ such that $\sec \theta_B \sqrt{m_e/m_i} = \text{const}$, the acceleration efficiency should be unchanged. Since the
Figure 2.9: Panel (a) compares the electron energy spectra from the reference run (2D in-plane) and the 3D run measured at $\Omega_{ci} t = 7.4$ at a distance of $60 - 160 \, c/\omega_{pe}$ ahead of the shock. Panel (b) compares the electron energy spectra from the 1D run, the reference run (2D in-plane) and the 2D out-plane run measured at $\Omega_{ci} t = 15$ at a distance of $60 - 160 \, c/\omega_{pe}$ ahead of the shock.
relevant time scale for SDA and the shock evolution is $\Omega_{ci}^{-1}$, we should expect the spectra to be comparable at the same time in units of $\Omega_{ci}^{-1}$.

To test this prediction, we have run a simulation with the same physical parameters as in our reference run but with a larger mass ratio, $m_i/m_e = 400$. Correspondingly, we have changed the field obliquity to $\theta_B = 77^\circ$, which ensures that the value of $u_t$ is unchanged. Figure 2.10 shows the comparison at $\Omega_{ci} t = 11$ of the upstream electron energy spectra, between $m_i/m_e = 100$ (red) and $m_i/m_e = 400$ (blue). We see excellent agreement, as expected. This confirms that the efficiency of both the SDA mechanism and of the Fermi acceleration process is independent of the mass ratio, once the magnetic obliquity is properly rescaled. In addition, we observe that the upstream waves for $m_i/m_e = 400$ show a similar pattern as in our reference run.

We then conclude that our results, that employ a reduced mass ratio, can be generalized to the realistic mass ratio. For $m_i/m_e = 1836$, the corresponding field obliquity is $\theta_B = 84^\circ$. For this value of $\theta_B$, electron acceleration should proceed exactly in the same way as we have presented here for our reference run with $m_i/m_e = 100$. The independence of our mechanism on the mass ratio is to be contrasted with the conclusions by Riquelme & Spitkovsky (2011), finding that electron injection by oblique whistler waves in low plasma beta shocks depends sensitively on the mass ratio. In our high plasma beta shocks, the strength of the magnetic waves mediating electron acceleration does not depend on mass ratio, as we will show in the forthcoming paper (Guo et al., 2014b). In addition, our mechanism does not rely on direct acceleration by the electric field of the waves, as opposed to the work by Riquelme & Spitkovsky (2011).
Figure 2.10: Electron energy spectra and the evolution of the maximum Lorentz factor from the run with $m_i/m_e = 400$ (blue) and the reference run (red) measured at $\Omega_{ci} t = 11$ at a distance of $60 - 160 \, c/\omega_{pe}$ ahead of the shock. The spectra agree very well, as expected when we change the value of the mass ratio but keeping $u_t$ fixed (Equation (2.27)).
Chapter 3

Non-Thermal Electron Acceleration in Low Mach Number Collisionless Shocks. II.

Firehose-Mediated Fermi Acceleration and its Dependence on Pre-Shock Conditions

This thesis chapter originally appeared in the literature as

Abstract

Electron acceleration to non-thermal energies is known to occur in low Mach number ($M_s \lesssim 5$) shocks in galaxy clusters and solar flares, but the electron acceleration mechanism remains poorly understood. Using two-dimensional (2D) particle-in-cell (PIC) plasma simulations, we showed in Paper I that electrons are efficiently accelerated in low Mach number ($M_s = 3$) quasi-perpendicular shocks via a Fermi-like process. The electrons bounce between the upstream region and the shock front, with each reflection at the shock resulting in energy gain via shock drift acceleration. The upstream scattering is provided by oblique magnetic waves, that are self-generated by the electrons escaping ahead of the shock. In the present work, we employ additional 2D PIC simulations to address the nature of the upstream oblique waves. We find that the waves are generated by the shock-reflected electrons via the firehose instability, which is driven by an anisotropy in the electron velocity distribution. We systematically explore how the efficiency of wave generation and of electron acceleration depend on the magnetic field obliquity, the flow magnetization (or equivalently, the plasma beta), and the upstream electron temperature. We find that the mechanism works for shocks with high plasma beta ($\gtrsim 20$) at nearly all magnetic field obliquities, and for electron temperatures in the range relevant for galaxy clusters. Our findings offer a natural solution to the conflict between the bright radio synchrotron emission observed from the outskirts of galaxy clusters and the low electron acceleration efficiency usually expected in low Mach number shocks.
3.1 Introduction

There is considerable observational evidence that electrons are efficiently accelerated in low Mach number collisionless shocks in astrophysical sources. In particular, at the outskirts of galaxy clusters, where X-ray telescopes have unambiguously detected the existence of low Mach number shocks based on density and/or temperature jumps, radio observations reveal synchrotron emission from relativistic electrons, presumably accelerated at the shock fronts (e.g. Markevitch et al., 2007; Finoguenov et al., 2010; van Weeren et al., 2010; Akamatsu et al., 2012; Brüggen et al., 2012; Feretti et al., 2012; Brunetti & Jones, 2014). However, the physics of the electron acceleration mechanism remains poorly understood. This paper is the second in a series, focusing on the study of electron acceleration in low Mach number shocks by means of self-consistent PIC simulations.

In the first paper of this series (Guo et al., 2014a, Paper I hereafter), we focused on the particle energy spectra and the acceleration mechanism in a reference PIC run with Mach number \(M_s = 3\) and a quasi-perpendicular magnetic field. We found that about 15% of electrons are efficiently accelerated, forming a non-thermal power-law tail in the energy spectrum with a slope of \(p \simeq 2.4\). We identify the acceleration mechanism to be as follows. A fraction of the incoming electrons are energized at the shock front via shock drift acceleration (SDA). The accelerated electrons are reflected back upstream by the mirror force of the shock-compressed magnetic field. In the upstream region, the interaction of these electrons with the incoming flow generates magnetic waves. In turn, the waves scatter some of the electrons propagating upstream back toward the shock, for further energization via SDA. Thus the self-generated waves allow for repeated cycles of shock drift acceleration, similar to a sustained Fermi-like process.
In Paper I, we did not investigate the nature of the upstream waves, which are essential for sustaining the Fermi-like process. We show in this work that the waves are triggered by the shock-reflected electrons propagating upstream, via the electron firehose instability. In addition to clarifying the nature of the upstream waves, another goal of this paper is to explore the dependence of the efficiency of firehose-mediated electron acceleration on pre-shock conditions.

The reference run in Paper I was set up to capture the physical environment at the Galactic Center along the trajectory of the G2 cloud \cite{Narayan:2012, Sadowski:2013}, where the plasma temperature $T$ is very high, reaching $k_B T \sim 100$ keV, where $k_B$ is the Boltzmann constant. In the intracluster medium (ICM), where low Mach number shocks frequently occur due to mergers, the plasma temperature is lower, $k_B T \sim 1 - 10$ keV. The magnetic field pressure in the reference run was chosen to be a fraction $\sim 5\%$ of the plasma thermal pressure, and the field was quasi-perpendicular to the shock direction of propagation (with obliquity $\theta_B = 63^\circ$). Observationally, the magnetic field strength and obliquity cannot be easily constrained, though we expect a range of strengths and obliquities. With this as motivation, we explore here the dependence of the electron acceleration mechanism on various pre-shock parameters.

The paper is organized as follows. In Section 3.2, we describe the simulation setup and our choice of physical parameters. In Section 3.3, we summarize the shock structure and the electron acceleration mechanism described for the reference run in Paper I. In Section 3.4, we investigate the dependence of the SDA injection process on the pre-shock conditions. In Section 3.5, we study in detail the nature of the upstream waves, which are essential for sustaining long-term acceleration of electrons. In Section 3.6, we explore the dependence of the acceleration mechanism on the upstream magnetic obliquity, the flow magnetization (or
equivalently, the plasma beta) and the electron temperature. We conclude with a summary and discussion of our findings in Section 3.7.

3.2 Simulation Setup and Parameter Choice

We perform numerical simulations using the three-dimensional (3D) electromagnetic PIC code TRISTAN-MP (Spitkovsky, 2005), which is a parallel version of the publicly available code TRISTAN (Buneman, 1993) that has been optimized for studying collisionless shocks.

The computational setup and numerical scheme are described in detail in Paper I. In brief, the shock is set up by reflecting an upstream electron-ion plasma off a conducting wall at the left boundary \((x = 0)\) of the computational box. The upstream plasma is initialized as a Maxwell-Jüttner distribution with the electron temperature \(T_e\) equal to the ion temperature \(T_i\), drifting with a bulk velocity \(\vec{u}_0 = -u_0 \hat{x}\). The interaction between the reflected stream and the incoming plasma causes a shock to form, which propagates along \(+\hat{x}\) at a speed \(u_{\text{sh}}\). The relation between the upstream bulk flow velocity and the plasma temperature is parametrized by the simulation-frame Mach number

\[
M \equiv \frac{u_0}{c_s} = \frac{u_0}{\sqrt{2\Gamma k_B T_i/m_i}},
\]

where \(c_s\) is the sound speed in the upstream, \(k_B\) is the Boltzmann constant, \(\Gamma = 5/3\) is the adiabatic index of the plasma, and \(m_i\) is the ion mass. The incoming plasma carries a uniform magnetic field \(\vec{B}_0\), whose strength is parametrized by the magnetization

\[
\sigma \equiv \frac{B_0^2/4\pi}{(\gamma_0 - 1) n_0 m_i c^2},
\]

(3.2)
where $\gamma_0 \equiv (1 - u_0^2/c^2)^{-1/2}$ is the upstream bulk Lorentz factor and $n_0 = n_i = n_e$ is the number density of the incoming plasma. The magnetic field orientation with respect to the shock normal (aligned with $+\hat{x}$) is parameterized by the polar angle $\theta_B$ and the azimuthal angle $\varphi_B$. If the magnetic field lies in the $xy$ plane of the simulations, $\varphi_B = 0^\circ$. The incoming plasma is initialized with zero electric field in its rest frame. Due to its bulk motion, the upstream plasma carries a motional electric field $\vec{E}_0 = -(\vec{u}_0/c) \times \vec{B}_0$ in the simulation frame.

In the literature, the Mach number $M_s$ is often defined as the ratio between the upstream flow velocity and the upstream sound speed in the shock rest frame (rather than in the downstream frame, as in Equation (3.1)). In the limit of weakly magnetized shocks, the shock-frame Mach number $M_s$ is related to our simulation-frame Mach number $M$ through the implicit relation

$$M_s = M \frac{u_{\text{sh}}^{\text{up}}}{u_0} = M \left(1 + \frac{1}{r(M_s) - 1}\right),$$

(3.3)

where $u_{\text{sh}}^{\text{up}}$ is the shock velocity in the upstream rest frame, equal to the upstream flow velocity in the shock rest frame, and

$$r(M_s) = \frac{\Gamma + 1}{\Gamma - 1 + 2/M_s^2},$$

(3.4)

is the Rankine-Hugoniot relation for the density jump from upstream to downstream.

For comparison with earlier work, where the magnetic field strength is sometimes parametrized by the Alfvénic Mach number $M_A \equiv u_0/v_A$, where $v_A \equiv B_0/\sqrt{4\pi n m_i}$ is the Alfvén velocity, we remark that the relation between the magnetization and the Alfvénic Mach number is simply $M_A = \sqrt{2/\sigma}$. Alternatively, one could employ the plasma beta $\beta_p \equiv 8\pi n k_B (T_e + T_i) / B_0^2$, which is related to the magnetization as $\beta_p = 4/(\sigma \Gamma M^2)$, under
the assumption of temperature equilibrium $T_e = T_i$. We stress that in our simulations
the upstream particles are initialized with the physically-grounded Maxwell-Jüttner
distribution, instead of the so-called “$\kappa$-distribution” that was employed by, e.g., Park
et al. (2013). The $\kappa$-distribution might be a realistic choice for shocks in solar flares.
However, in most other astrophysical settings one expects the upstream particles to
populate a Maxwellian distribution. By using a $\kappa$-distribution, which artificially boosts the
high-energy component of the particle spectrum, one would unphysically overestimate the
electron acceleration efficiency.

In Paper I, we performed simulations in both 2D and 3D computational domains. We
found that most of the shock physics is well captured by 2D simulations in the $xy$ plane, if
the magnetic field lies in the simulation plane, i.e., $\phi_B = 0^\circ$. Therefore, to explore a wide
range of parameter space with fixed computational resources, in this paper we only utilize
2D runs with in-plane fields. We stress that all three components of particle velocities and
electromagnetic fields are tracked. As a result, the adiabatic index is $\Gamma = 5/3$.

For accuracy and stability, PIC codes have to resolve the plasma oscillation frequency of
the electrons

$$\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e}, \quad (3.5)$$

and the electron plasma skin depth $c/\omega_{pe}$, where $e$ and $m_e$ are the electron charge and mass.

On the other hand, the shock structure is controlled by the ion Larmor radius

$$r_{L,i} = \frac{2}{\sigma} \sqrt{m_i / m_e} \frac{c}{\omega_{pe}} \gg \frac{c}{\omega_{pe}}, \quad (3.6)$$

and the evolution of the shock occurs on a time scale given by the ion Larmor gyration
period $\Omega_{ci}^{-1} = r_{L,i} u_0^{-1} \gg \omega_{pe}^{-1}$. The need to resolve the electron scales, and at the same
time to capture the shock evolution for many $\Omega_{ci}^{-1}$, is an enormous computational challenge, especially for the realistic mass ratio $m_i/m_e = 1836$. We found in Paper I that simulations with two choices of the mass ratio, $m_i/m_e = 100$ and $m_i/m_e = 400$, show consistent results, so that the shock physics can be confidently extrapolated to the realistic mass ratio $m_i/m_e = 1836$, using the scalings presented in Appendix B of that paper. We therefore employ a reduced mass ratio $m_i/m_e = 100$ for all the runs presented in this paper.

We adopt a spatial resolution of 10 computational cells per electron skin depth $c/\omega_{pe}$ and we use a time resolution of $dt = 0.045 \omega_{pe}^{-1}$. Each cell is initialized with 32 particles (16 per species). The transverse box size is fixed at 76 $c/\omega_{pe}$. We have performed convergence tests which show that 5 cells per electron skin depth can resolve the acceleration physics reasonably well, and we have confirmed that simulations with a number of particles per cell up to 64 and a transverse box size up to 256 $c/\omega_{pe}$ show essentially the same results.

We carry out several runs with various values of $T_e$, $\sigma$ and $\theta_B$, while keeping $M = 2$ (corresponding to $M_s = 3$) fixed. The choice of $M_s = 3$ is representative of the Mach numbers of merger shocks in galaxy clusters ($M_s \sim 1.5 - 5$) in cosmological simulations (e.g. Ryu et al., 2003) and also inferred from X-ray and radio observations (e.g. Akamatsu et al., 2012). The effect of varying $M_s$ has been explored by means of PIC simulations in Narayan et al. (2012), and we shall briefly comment on the dependence on Mach number in Section 3.6. The upstream parameters of our runs are summarized in Table 3.1. We vary $T_e$ from $10^{7.5}K$ to $10^9K$, which overlaps the typical temperature range of the ICM ($T \sim 10^7 - 10^8K$). The magnetization $\sigma$ varies from 0.003 to 0.1, corresponding to a plasma beta ranging from $\beta_p = 200$ to $\beta_p = 6$, which is well motivated, based on the typical number density in the ICM ($10^{-4} - 10^{-2} \text{cm}^{-3}$) and on the magnetic field strength (a few $\mu$G, see e.g. Brunetti & Jones, 2014). We vary the obliquity angle $\theta_B$ across a wide range (from 13° up to 80°,
Table 3.1: Upstream Parameters Used for the Shock Simulations

<table>
<thead>
<tr>
<th>Run</th>
<th>( T_e = T_i ) [K(keV)]</th>
<th>( u_0/c )</th>
<th>( \theta_B )</th>
<th>( \sigma )</th>
<th>( \beta_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>63°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta13</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>13°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta23</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>23°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta33</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>33°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta43</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>43°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta53</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>53°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta68</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>68°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta73</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>73°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>theta80</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>80°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>sigle-1_43</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>43°</td>
<td>0.1</td>
<td>6</td>
</tr>
<tr>
<td>sigle-1_53</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>53°</td>
<td>0.1</td>
<td>6</td>
</tr>
<tr>
<td>sigle-1_63</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>63°</td>
<td>0.1</td>
<td>6</td>
</tr>
<tr>
<td>sigle-1_73</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>73°</td>
<td>0.1</td>
<td>6</td>
</tr>
<tr>
<td>sigle-2_43</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>43°</td>
<td>0.01</td>
<td>60</td>
</tr>
<tr>
<td>sigle-2_53</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>53°</td>
<td>0.01</td>
<td>60</td>
</tr>
<tr>
<td>sigle-2_63</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>63°</td>
<td>0.01</td>
<td>60</td>
</tr>
<tr>
<td>sigle-2_73</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>73°</td>
<td>0.01</td>
<td>60</td>
</tr>
<tr>
<td>sigle-3_63</td>
<td>( 10^9(86) )</td>
<td>0.15</td>
<td>63°</td>
<td>0.003</td>
<td>200</td>
</tr>
<tr>
<td>Te1e7.5</td>
<td>( 10^{7.5}(2.7) )</td>
<td>0.027</td>
<td>63°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>Te1e8.0</td>
<td>( 10^8(8.6) )</td>
<td>0.047</td>
<td>63°</td>
<td>0.03</td>
<td>20</td>
</tr>
<tr>
<td>Te1e8.5</td>
<td>( 10^{8.5}(27) )</td>
<td>0.084</td>
<td>63°</td>
<td>0.03</td>
<td>20</td>
</tr>
</tbody>
</table>
i.e., from quasi-parallel to quasi-perpendicular shocks), as it is usually not constrained by observations.

### 3.3 Shock Structure and Particle Acceleration

In Paper I, we analyzed the reference run (see Table 3.1) and showed that electrons are efficiently accelerated, with the upstream electron energy spectrum developing a clear non-thermal component over time. Before exploring the parameter dependence of the electron acceleration efficiency, we first summarize the shock structure and the electron acceleration mechanism as inferred from the reference run in Paper I.

Electrons are initially energized via shock drift acceleration (SDA) at the shock front. At the shock front (located at \( x \approx 1115 c/\omega_{pe} \) in Fig. 3.1), a fraction of the upstream thermal electrons get reflected by the mirror force of the shock-compressed magnetic field. The change in the total magnetic field strength \( B/B_0 \) (black curve in Fig. 3.1(a)) is dominated by the compression of the perpendicular component \( B_y \) (blue curve in Fig. 3.1(a)), which is related via flux freezing to the density compression (red curve in Fig. 3.1(a)). The plasma density increases at the shock by a factor of \( \sim 4 \), which is higher than the prediction (\( \sim 3 \)) of the Rankine-Hugoniot jump condition (see Equation (3.4)). This density overshoot is a common feature of quasi-perpendicular shocks (\( \theta_B \gtrsim 45^\circ \)), both in the non-relativistic (see e.g. Hoshino et al., 2001; Treumann, 2009; Umeda et al., 2009; Matsumoto et al., 2012) and in the ultra-relativistic regime (e.g., Sironi & Spitkovsky, 2011; Sironi et al., 2013). It is due to Larmor gyration of the incoming ions in the compressed shock fields, which decelerates the upstream flow and therefore increases its density.

While the electrons are confined at the shock front by the mirror force of the shock-
Figure 3.1: Shock structure of the reference run at time $\omega_{pe} t = 14625$ ($\Omega_{ci} t = 26.9$). The shock is at $x \simeq 1115 \ c/\omega_{pe}$, as indicated by the vertical dot-dashed lines, and moves to the right. Downstream is to the left of the shock, and upstream to the right. Panel (a) shows the ratios $n_e/n_0$ (red line), $B_y/B_{y,0}$ (blue line) and $B/B_0$ (black line). Panels (b)-(d) show the electron momentum phase spaces $p_x$-$x$, $p_y$-$x$, $p_z$-$x$, as a function of the longitudinal coordinate $x$. Panel (e) shows the electron temperatures parallel ($T_{e,\parallel}$) and perpendicular ($T_{e,\perp}$) to the magnetic field. Panels (f)-(h) show 2D plots of the magnetic field components in units of $B_0$, after subtracting the background field $\vec{B}_0$ (i.e., we show $(B_x - B_{x,0})/B_0$, $(B_y - B_{y,0})/B_0$ and $B_z/B_0$, respectively). The white arrows indicate the orientation of the upstream background magnetic field $\vec{B}_0$. Note that there are upstream waves in all three components of the magnetic field.
compressed field, they drift along the shock surface along the $-\hat{z}$ direction and are energized via SDA by the motional electric field $\vec{E}_0 \parallel \hat{z}$. As a result of the SDA process, the reflected electrons increase their momentum along the direction of the upstream background field, as revealed by the electron phase diagrams in Fig. 3.1(b)-(d). In the reference run, the upstream background magnetic field is nearly along the $+\hat{y}$ axis. Correspondingly, we notice a number of electrons having large momenta along the $+\hat{y}$ direction ($p_y \gtrsim 3m_e c$) far ahead of the shock front. Since the electrons accelerated by SDA gain momentum preferentially along the direction of the magnetic field, they induce an electron temperature anisotropy $T_{e\parallel} > T_{e\perp}$ in the upstream, over an extended region ahead of the shock (Fig. 3.1(e)). Here $T_{e\parallel}$ ($\perp$) is the electron temperature parallel (perpendicular) to the upstream magnetic field.

In quasi-perpendicular shocks, electrons are the only species that can propagate upstream after being reflected by the shock front. The ions either advect downstream or are confined within a distance of $\sim r_{L,i} \simeq 80 c/\omega_{pe}$ ahead of the shock. Beyond this distance, no shock-reflected ion is present, and the ion distribution is isotropic (as expected for the upstream medium at initialization). This suggests that it is the electron temperature anisotropy that triggers magnetic waves in the upstream, since the waves extend well beyond a few ion Larmor radii ahead of the shock (Fig. 3.1(f)-(h)). Their wave vector is oblique with respect to the background upstream field (indicated by the white arrows in Fig. 3.1(f)-(h)).

The magnetic field fluctuations, $\delta \vec{B} \equiv \vec{B} - \vec{B}_0$, grow preferentially perpendicular to the plane defined by the wave vector and the background field, since $\delta B_z$ is stronger than $\delta B_x$ or $\delta B_y$. We find that the waves have phase velocity equal to the upstream flow velocity $\vec{u}_0 = -0.15c \hat{x}$ in the simulation frame, which implies that they are purely growing modes in the upstream comoving frame. In Section 3.5, we confirm that the waves are indeed generated by the electrons, and that they are due to the electron firehose instability. The
self-generated waves mediate the second stage of electron acceleration (beyond the initial SDA phase), in which the reflected electrons are scattered back towards the shock by the upstream waves and undergo multiple cycles of SDA, in a process similar to the Fermi mechanism. The energy gain of the accelerated electrons is dominated by multiple cycles of SDA, whereas the direct contribution from the interaction with the upstream waves is marginal. The trajectory and energy evolution of a typical electron undergoing Fermi-like acceleration is shown in Fig. 8 of Paper I.

3.4 Injection via Shock Drift Acceleration

Since SDA plays a major role in the electron Fermi-like acceleration process summarized above, it is important to understand the main properties of SDA: (i) its efficiency, i.e., the fraction of electrons from a thermal distribution that will be reflected upstream by SDA (and thereby injected into the acceleration process); and (ii) the energy gain from each cycle of SDA. Our fully relativistic theory of SDA has been presented in detail in Section 4.2.1 of Paper I. In this section, we briefly summarize our previous findings and focus on the dependence of the SDA injection efficiency on the magnetic field obliquity angle $\theta_B$ and the electron temperature $T_e$.

SDA is only viable in subluminal shocks. In contrast, at superluminal shocks, the velocity required to boost from the upstream rest frame to the de-Hoffman-Teller (HT) frame ([de Hoffmann & Teller, 1950])

$$u_t = u_{\text{up}} \sec \theta_B = \sqrt{\frac{2k_B T_e}{m_e} \sqrt{\frac{m_e}{m_i}}} M_s \sec \theta_B$$

(3.7)

exceeds the speed of light. In superluminal shocks, no particle travelling along the magnetic
Figure 3.2: Dependence of the SDA injection efficiency on the obliquity angle $\theta_B$ (on the left) and the electron temperature $T_e$ (on the right), for a low Mach number shock with $M_s = 3$. Panel (a) shows the region in velocity space ($v_{\parallel}/c - v_{\perp}/c$) where electrons populating a thermal distribution with $T_e = 10^9$ K are allowed to participate in SDA, as a function of the magnetic obliquity $\theta_B$. The solid black semi-circle indicates the speed of light. The vertical colored lines indicate the values of $u_t$ corresponding to different choices of $\theta_B$, as indicated in the legend. For a given $\theta_B$, the region allowed for SDA reflection is to the left of the colored vertical line and above the semi-horizontal line of the same color (within the limit of the speed of light). The region to the right of the vertical colored line is the area in velocity space that the SDA-reflected particles will occupy. The dashed black semi-circle indicates the electron thermal velocity $v_{th,e} = \sqrt{2k_BT_e/m_e}$ for $T_e = 10^9$ K. The overlap between the region near the thermal velocity semi-circle and the allowed region for reflection indicates the SDA efficiency: the more they overlap, the higher the number of electrons participating in SDA, thus the higher the SDA efficiency. In the examples shown here the efficiency increases with decreasing $\theta_B$. Panel (b) shows a similar diagram investigating the dependence of the SDA efficiency on the electron temperature $T_e$. We fix $\theta_B = 63^\circ$ and $b = 4$ and scale the cross-shock potential as $\Delta \phi = 0.5 (T_e/10^9 K)$. Here the dashed colored semi-circles indicate the electron thermal velocity at different temperatures (color coding in the legend). For different $T_e$, the solid colored lines mark the boundaries of the region allowed for SDA reflection (to the left of the vertical line) and of the region occupied by the SDA-reflected electrons (to the right of the vertical line). Panel (c) shows the reflection fraction (left axis, in black) and average energy gain (right axis, in red) as a function of $\theta_B$, for three representative choices of $b$ and $\Delta \phi$. Panel (d) shows the reflection fraction (tick marks on the left axis) and average energy gain (tick marks on the right axis) as a function of $T_e$, with $\theta_B = 63^\circ$, $b = 4$ and $\Delta \phi = 0.5 (T_e/10^9 K)$.  

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field towards the upstream can outrun the shock, so the injection efficiency is expected to vanish. Therefore, in this work, we focus only on electron acceleration in subluminal shocks.

We confirm in Section 3.6.1 that in superluminal shocks the SDA process cannot operate, and the electron acceleration efficiency vanishes.

The efficiency of SDA decreases with increasing $u_t$, as the minimal electron energy required to participate in the SDA process increases with $u_t$ (see Section 4.2.1 in Paper I). In addition, the efficiency of SDA depends on the cross-shock electrostatic potential and the magnetic field compression at the shock. Inheriting the notation used in Paper I, we use $\Delta \phi$ to denote the change of potential energy of an electron as it crosses the shock from upstream to downstream, in units of its rest mass energy. We indicate with $b$ the compression of the magnetic field strength $B/B_0$ at the shock front. A decrease in $b$ or an increase in $\Delta \phi$ have the effect of increasing the minimum pitch angle required for SDA reflection in the HT frame. This leads to a lower fraction of incoming electrons that can participate in the SDA process.

The fractional energy gain from a single cycle of SDA increases monotonically with $u_t$ and is independent of $\Delta \phi$ or $b$ (see Equation (24) of Paper I)

$$\gamma_{i(r)}^{\text{up}} \equiv \gamma_i^{\text{up}} - \gamma_i^{\text{up}} = \frac{2 u_t (u_t - v_{i\parallel}^{\text{up}})}{c^2 - u_t^2}.$$  

Here $\gamma_i^{\text{up(r)}}$ is the electron Lorentz factor before (after, respectively) the SDA cycle, and $v_{i\parallel}^{\text{up}}$ is the particle velocity parallel to the magnetic field before SDA. All the quantities are

---

1 We remark that $b$ is typically larger than the value predicted by the Rankine-Hugoniot relations. As seen from the shock structure of the reference run, for quasi-perpendicular shocks the magnetic field compression is higher than in the far downstream (both the density and the transverse magnetic field show an overshoot at the shock front), thus $b$ is larger than predicted by the Rankine-Hugoniot relations. On the other hand, in quasi-parallel shocks $b$ approaches the Rankine-Hugoniot relation for the compression of the magnetic field strength, as the overshoot at the shock front is less prominent.
measured in the upstream rest frame.

To understand the dependence of the SDA process on the upstream conditions, it is useful to illustrate how the key parameters of SDA, namely $u_t$, $\Delta \phi$ and $b$, depend on the pre-shock properties. Equation (3.7) shows that, for fixed $m_i/m_e$ and $M$, the HT velocity $u_t$ increases with $T_e$ and $\theta_B$. In addition, the cross shock potential $\Delta \phi$ scales roughly as (Amano & Hoshino, 2007)

$$\Delta \phi \sim \frac{m_i u_0^2}{2 m_e c^2} = M^2 \Gamma \frac{k_B T_e}{m_e c^2}.$$  \hspace{1cm} (3.9)

The value of $\Delta \phi$ also depends on $\theta_B$ for fixed $M$ and $T_e$, which is not accounted for by the equation above. Intuitively, this can be understood as follows. The cross-shock potential develops as a result of the excess of ions in the overshoot formed at the shock front (see e.g. Gedalin & Balikhin, 2004). At smaller obliquities, the magnetic barrier of the shock-compressed field is weaker, so the upstream ions can advect downstream more easily along the magnetic field. It follows that the magnetic overshoot at the shock front is smaller in quasi-parallel shocks (i.e., $b$ decreases with decreasing $\theta_B$). Since the cross-shock potential is related to the overshoot in ion density, $\Delta \phi$ is expected to decrease at lower $\theta_B$, for fixed $M$ and $T_e$. An analytical theory of the exact dependence of $\Delta \phi$ and $b$ on the magnetic obliquity is beyond the scope of this paper. In the upcoming sections, we shall use the values of $\Delta \phi$ and $b$ measured from our simulations.

To illustrate the effect of various pre-shock conditions, we show in Fig. 3.2 how the SDA reflection fraction (i.e., the injection efficiency) and the average energy gain vary as a function of $\theta_B$ and $T_e$. The results in Fig. 3.2 are based on the analytical model of SDA presented in Paper I. Fig. 3.2(a) and (b) identify the region allowed for SDA reflection in
the velocity space $v_{\text{up}}^\parallel - v_{\perp}$, where $v_{\text{up}}^\parallel$ is the velocity component parallel (perpendicular, respectively) to the ambient upstream magnetic field. The solid black circle indicates the limit of the speed of light. The electron thermal velocity $v_{\text{th, } e} \equiv \sqrt{2k_B T_e/m_e}$ is indicated by the dashed semi-circles. The colored vertical lines denote $v_{\text{up}}^\parallel = u_t$, where $u_t$ is determined by Equation (3.7). To be reflected at the shock via SDA, an incoming electron has to move towards the shock, i.e., $v_{\text{up}}^\parallel < u_t$, and its transverse velocity $v_{\text{up}}^\perp$ should be larger than a critical value that depends on $v_{\text{up}}^\parallel$, $b$ and $\Delta \phi$. This critical threshold is indicated by the colored solid curve to the left of the vertical line having the same color. The area bounded by these two limits, together with the speed of light (i.e., within the black solid circle), indicates the region in velocity space allowed for SDA reflection. The overlap between the region near the thermal velocity semi-circle (dashed line) and the allowed region for reflection provides an estimate of the SDA efficiency: the more they overlap, the larger the number of electrons participating in SDA, so the higher the SDA efficiency. The region bounded by the colored curve to the right of the colored vertical line indicates the area in velocity space that the electrons occupy after SDA reflection.

Fig. 3.2(a) shows the effect of varying $\theta_B$ at fixed Mach number ($M_s = 3$) and electron temperature ($T_e = 10^9 K$). We use the values of $b$ and $\Delta \phi$ measured from our simulations, for different choices of $\theta_B$. At higher $\theta_B$, the HT velocity $u_t$ increases (see Equation (3.7)), and in fact the vertical colored lines in Fig. 3.2(a) — corresponding to $v_{\text{up}}^\parallel = u_t$ — shift to the right. With larger $\theta_B$, we find from our simulations that both $\Delta \phi$ and $b$ increase (see the legend in Fig. 3.2(a)). The increase in $\Delta \phi$ raises the minimum energy required for reflection and thus decreases the SDA efficiency. On the other hand, the increase in $b$ allows particles with a wider range of pitch angles to participate in SDA (see Section 4.2.1 in Paper I), thus balancing the effect of $\Delta \phi$ to some extent.
The combined effect of $\Delta \phi$ and $b$ is illustrated in Fig. 3.2(c), where we show the SDA injection efficiency as a function of $\theta_B$. The three black curves (dotted, dashed and solid, with tick marks on the left axis) illustrate the dependence of the injection efficiency on $\theta_B$, for three representative combinations of $b$ and $\Delta \phi$, as indicated in the legend. In our simulations, we find that at lower obliquities, the values of both $\Delta \phi$ and $b$ tend to decrease, so one should shift from the solid to the dashed and then to the dotted curve, for lower $\theta_B$. We find that the injection efficiency drops when $\theta_B \gtrsim 60^\circ$, i.e., when the shock becomes quasi-perpendicular, and it vanishes near $\theta_B \simeq 78^\circ$. This is because the shock becomes superluminal, i.e., $u_t > c$ in Equation (3.7), for our choice of $m_i/m_e = 100$, $M_s = 2$ and $T_e = 10^9 K$.

As discussed earlier, superluminal shocks are poor particle accelerators, since particles streaming along the field toward the upstream cannot outrun the shock, and the Fermi process is suppressed. However, in the context of the SDA theory, we find that electron acceleration becomes inefficient already for $u_t \gtrsim v_{th,e}$, which is more stringent than the superluminality criterion $u_t > c$. When $u_t \gtrsim v_{th,e}$, most of the thermal electrons cannot participate in the SDA process, so the injection efficiency becomes negligible. For the case of hot electrons ($T_e = 10^9 K$) considered in Fig. 3.2(a) and (c), the two criteria are similar, since the thermal velocity $v_{th,e} \sim 0.6c$ is quasi-relativistic. Yet, when studying colder electrons whose thermal velocity is non-relativistic, it is important to consider that the electron acceleration efficiency is already expected to decrease when $u_t \gtrsim v_{th,e}$.

The red curves in Fig. 3.2(c) illustrate the dependence of the average electron energy gain $\langle \Delta \gamma \rangle m_e c^2 / k_B T_e$ on the field obliquity (tick marks on the right axis). The mean energy gain increases monotonically with $\theta_B$, because the SDA fractional energy gain increases with $u_t$ (see Equation 3.8), and $u_t$ grows monotonically with $\theta_B$ (see Equation 3.7). We
point out that the average is measured over a population of electrons following a thermal distribution with $T_e = 10^9 K$ (as expected for the upstream electrons at initialization), which is appropriate for the first SDA cycle. When the electrons undergo further SDA cycles, the exact value of the mean energy gain will change, since the reflected electrons no longer follow a thermal distribution. However, the trend of higher energy gains for larger $\theta_B$ should remain the same.

Similarly, Fig. 3.2(b) and (d) illustrate the effect of varying $T_e$ on the SDA efficiency and the average energy gain at quasi-perpendicular ($\theta_B = 63^\circ$) low Mach number ($M_s = 3$) shocks. For the calculations presented in these two panels, we use a fixed value of the magnetic compression ratio ($b = 4$) and we scale the cross-shock potential with the electron temperature such that $\Delta \phi = 0.5 (T_e/10^9 K)$, as suggested by Equation (3.9) and verified in our simulations. In the low temperature regime ($T_e \lesssim 10^8 K$) most relevant for the ICM, the SDA efficiency and the electron energy gain (in units of $k_B T_e$) depend weakly on the electron temperature. The injection efficiency stays around 23%, while the mean electron energy gain is $(\Delta \gamma)m_e c^2/k_B T_e \sim 3$. At higher temperatures ($T_e \gtrsim 10^8 K$), the efficiency drops slowly with increasing $T_e$, while the electron energy gain (normalized to the thermal energy $k_B T_e$) increases with $T_e$.

In summary, at fixed electron temperature, the SDA efficiency decreases at higher magnetic obliquities, $\theta_B$, while the average energy gain increases with $\theta_B$. At fixed $\theta_B$, both the SDA efficiency and the average energy gain depend weakly on $T_e$ in the low temperature regime ($T_e \lesssim 10^8 K$) most relevant for galaxy clusters. When the temperature rises beyond $T_e \gtrsim 10^8 K$, the SDA efficiency decreases and the average energy gain moderately increases at higher electron temperatures.
3.5 The Upstream Waves

In this section, we investigate in detail the properties of the upstream waves that mediate the Fermi-like acceleration process. We first confirm that the waves are triggered by the electrons returning upstream after the SDA process and show that the waves are associated with the oblique mode of the electron firehose instability. We then investigate how the generation of the upstream waves depends on the plasma conditions.

3.5.1 Setup for Periodic-Box Simulations

To explore the physics of the upstream waves, we have performed 2D PIC simulations on a square computational domain in the upstream rest frame, with periodic boundary conditions along the $x$ and $y$ directions (periodic box simulations, hereafter). The simulations are targeted to capture the evolution of the upstream medium far ahead of the shock, in the upstream fluid frame. The plasma in the box consists of a background electron-ion plasma and an electron beam. The background plasma follows the thermal distribution initialized in our shock simulations. The electron beam mimics the properties of the SDA-reflected electrons, based on the SDA injection model discussed in Section 3.4. In this computational setup, the free energy in the electron beam is the only available source of instability.

According to the prediction of SDA, a certain fraction of the thermal electrons propagating toward the shock are reflected back upstream. As result of SDA, the reflected electrons have higher energy and smaller pitch angles. To mimic the properties of the SDA-reflected electrons, we extract four parameters from the SDA injection model described in Section 3.4: the fraction of electrons from the upstream thermal distribution that satisfy the reflection condition, $n_{\text{ref}}/n_e$ (normalized to the number density of the background electrons); the
maximum pitch angle of the reflected electrons, $\alpha_{\text{max}}$; the minimum and maximum kinetic energy of the reflected electrons, $\gamma_{b,\text{min}} - 1$ and $\gamma_{b,\text{max}} - 1$.

We set up our periodic box simulations as follows. For a given shock simulation listed in Table 3.1, the corresponding periodic box experiment contains a background electron-ion plasma with the same temperature and magnetic field strength as in the shock simulation. Since there is no shock in the periodic box simulations, the orientation of the magnetic field is arbitrary (in our shock simulations, the magnetic field direction was defined with respect to the shock normal, aligned with $x$). Yet, we decide to orient the ambient magnetic field with respect to the $x$ axis at the same angle $\theta_B$ as in the shock simulations, for easier comparison.\(^2\)

In addition to the background plasma, we initialize an electron beam whose number density is a fraction $n_{\text{ref}}/n_e$ of the electron density in the background. The beam electrons follow a power-law distribution in kinetic energy in the range $\gamma_{b,\text{min}} - 1 \leq \gamma_b - 1 \leq \gamma_{b,\text{max}} - 1$, with a slope of $-4$.\(^3\) The beam electrons are distributed isotropically in solid angle within a cone whose axis is aligned with the ambient magnetic field. The opening angle of the cone (or equivalently, the maximum electron pitch angle) is chosen to be $\alpha_{\text{max}}$.\(^4\) To ensure charge and current compensation in our periodic simulations, we balance the negative charge of the beam electrons with a corresponding excess of background ions. The beam current

\(^2\)Since all the shocks in Table 3.1 are non-relativistic, the change of magnetic obliquity when transforming from the simulation frame to the upstream rest frame is negligible.

\(^3\)We point out that the power-law distribution with a slope of $-4$ chosen for the beam electrons is just a convenient way to represent the energy spread of the SDA-reflected electrons. It should be distinguished from the power-law fit of the electron energy spectra measured in the shock simulations, which gives a spectral index $p \approx 2.4$ below the exponential cutoff (see Paper I). Also, we remark that for a slope as steep as $-4$, our results are insensitive to the exact value of the high-energy cutoff $\gamma_{b,\text{max}} - 1$.

\(^4\)As shown in Fig. 3.2(a)-(b), particles having small pitch angles (or equivalently, $v_{\parallel}^{\text{up}}/v_{\perp}^{\text{up}} \ll 1$) are outside the region occupied by SDA-reflected electrons (delimited by the colored curves to the right of the colored vertical lines). Yet, since the solid angle is small near $\alpha = 0^\circ$, we still choose $[0^\circ, \alpha_{\text{max}}]$ as a convenient proxy for the range of pitch angles of the reflected electrons.
Figure 3.3: Momentum space $p_x-p_y$ of the electron beam in our model, with $\theta_B = 63^\circ$, $\alpha_{\text{max}} = 57^\circ$, $\gamma_{\text{min}} - 1 = 0.4$, $\gamma_{\text{max}} - 1 = 6.7$. The momentum of the beam is centered around the direction of the background magnetic field, indicated by the white solid arrow. The beam electrons are distributed uniformly in solid angle, with pitch angle ranging from 0 to $\alpha_{\text{max}}$, as bounded by the dashed lines.
is neutralized by initializing the background electrons with a small bulk velocity. In our
shock simulations, the charge and current imbalance introduced by the beam of returning
electrons in the upstream is compensated self-consistently on short time scales (a few ω_{pe}^{-1}).

For our reference run, using b = 4 and Δφ = 0.5 as input parameters to our SDA injection
model, we obtain n_{ref}/n_e = 0.18, α_{max} = 57°, γ_{b, min} − 1 = 0.4 and γ_{b, max} − 1 = 6.7. Fig. 3.3
shows the p_x-p_y momentum space of the electron beam for our reference periodic box run.

We note that the beam electrons have a large momentum component along the direction
parallel to the magnetic field (indicated by the white solid arrow). When combined with
the background isotropic electrons, this will introduce an electron temperature anisotropy
T_e∥ > T_e⊥ in the beam-plasma system, which is essential for triggering the upstream waves
that we discuss below.

As regards to numerical parameters, we employ 5 cells per electron skin depth on a square
domain of 768 × 768 cells. Since the upstream waves we observe in the shock simulations
have small amplitudes |B_z|/B_0 ∼ 0.1 (Fig. 3.1), the noise level needs to be very low, in
order to clearly resolve their exponential growth and measure the growth rate. To achieve
such a high accuracy, we employ a large number of particles per cell (512 per species, for
the background electrons and ions).

### 3.5.2 Electron Oblique Firehose Instability

The setup of our periodic box simulations, with the electron temperature anisotropy T_e∥ >
T_e⊥ induced by the beam of SDA-reflected electrons, is unstable to the electron firehose
instability. Hollweg & Völk (1970) first discovered that high-beta plasmas with T_e∥ > T_e⊥
are unstable to waves propagating along the background field (the so-called parallel firehose
instability). Paesold & Benz (1999) demonstrated that the maximum growth rate of modes
Figure 3.4: 2D plot of the magnetic waves in the reference *periodic box simulation*. Note their similarity to the upstream waves in the reference *shock simulation* (Fig. 3.1(f)-(h)), in terms of wavelength and orientation of the wavevector. The white arrows indicate the orientation of the background magnetic field.
associated with electron anisotropies $T_{e\parallel} > T_{e\perp}$ in high-beta plasmas is attained at oblique angles, i.e. $\vec{k} \times \vec{B}_0 \neq 0$, where $\vec{k}$ is the wave vector. Later, it was discovered that the oblique mode is purely growing (i.e., with zero real frequency), and its growth rate $\Gamma_{\text{max}}$ lies in the range $\Omega_{ci} \ll \Gamma_{\text{max}} \lesssim \Omega_{ce}$, where $\Omega_{ci}$ and $\Omega_{ce}$ are the ion and electron cyclotron frequencies, respectively (Li & Habbal, 2000; Gary & Nishimura, 2003; Camporeale & Burgess, 2008).

Thus, the oblique mode grows faster than the parallel mode, whose growth rate is $\Gamma_{\text{max}} \lesssim \Omega_{ci}$ (e.g. Davidson, 1984; Yoon, 1990, 1995; Kunz et al., 2014). Also, the threshold of the oblique mode is lower than that of the parallel mode. Due to its faster growth rate and lower threshold, the oblique mode of the electron firehose instability is usually the dominant mode for anisotropic ($T_{e\parallel} > T_{e\perp}$) moderately magnetized plasmas, unless the wavevector is forced to align with the magnetic field (Gary & Nishimura, 2003).

Given the above expectations, there are two major aspects we wish to investigate using our 2D periodic box simulation with parameters appropriate to the reference shock run in Table 3.1. First, we want to verify that our periodic box simulation can reproduce the waves we found in the shock simulation. Then, we want to check if the waves are indeed associated with the electron oblique firehose instability.

The waves generated in our periodic box simulation are shown in Fig. 3.4. We find that their pattern closely resembles what we observed in the corresponding shock simulation (Fig. 3.1(f)-(h)). In particular, in both the shock and the periodic box simulations, the magnetic fluctuations $\delta \vec{B}$ are stronger along the $\hat{z}$ direction than in $\hat{x}$ or $\hat{y}$. This is expected for the oblique firehose instability, where the largest contribution to $\delta \vec{B}$ is predicted to be perpendicular to the plane formed by $\vec{B}_0$ and $\vec{k}$ (Gary & Nishimura, 2003). In both the shock and the periodic box simulations, the waves show two dominant wave-vectors, symmetric with respect to the ambient field $\vec{B}_0$. Their wavelength is $\sim 10 - 20c/\omega_{pe}$, much smaller
than the ion gyration radius \( r_{L,i} \sim 80 \, c/\omega_{pe} \) from Equation (3.6)), which confirms that the waves are governed by the electron physics. The waves in the periodic box simulation have zero real frequency (i.e., they are non-propagating, as expected for the oblique firehose instability), which agrees with the fact that the upstream waves in our shock simulation move together with the upstream flow, as discussed in Section 3.3.

Given the close similarity between the waves in our periodic box and in the shock simulations, and the fact that the periodic box experiment excludes other sources of instability — except for the beam of SDA-reflected electrons — we conclude that the upstream waves in the shock simulation are generated by the electrons returning upstream after the SDA process.

We now demonstrate that the properties of the waves in our periodic box simulation are fully consistent with the expectations of the electron oblique firehose instability. At initialization, the beam-plasma system in our reference periodic box is above the threshold for the oblique mode of the electron firehose instability (compare the red curve with the horizontal black dashed line in Fig. 3.5(b)),

\[
1 - \frac{T_{e\perp}}{T_{e\parallel}} - \frac{1.27}{\beta_{e\parallel}^{0.95}} > 0
\]  

(3.10)

where \( \beta_{e\parallel} \equiv 8\pi n_e k_B T_{e\parallel}/B_0^2 \) is the electron beta parallel to the background magnetic field, and we adopt the instability threshold derived by Gary & Nishimura (2003). The growth rate of the instability, as measured from the exponential phase \( (10 \lesssim \Omega_{ce} t \lesssim 50) \) in Fig. 3.5(c) (red curve), is \( \sim 0.05 \, \Omega_{ce} \). The predicted growth rate for a system with \( \beta_{e\parallel} = 10 \) and \( T_{e\perp}/T_{e\parallel} = 0.7 \), similar to our setup, is \( \sim 0.08 \, \Omega_{ce} \) (Camporeale & Burgess, 2008), which compares favorably with our result. Camporeale & Burgess (2008) further predict that the
wavelength of the fastest growing mode is \( \sim 15 c/\omega_{pe} \) and the wave vector is oriented at \( \sim 70^\circ \) with respect to \( \vec{B}_0 \). This is not very different from the wavelength (\( \sim 29 c/\omega_{pe} \)) and the wave vector orientation (at an angle of \( \sim 57^\circ \) with respect to \( \vec{B}_0 \)) measured in our periodic box simulation (wave pattern shown in Fig. 3.4). The agreement is reasonable, considering that our system does not match the particle distribution in Camporeale & Burgess (2008) exactly. In addition, by running simulations with \( m_i/m_e = 100 \) (our standard choice), \( m_i/m_e = 400 \) and the realistic case \( m_i/m_e = 1836 \), we have verified that the growth rate and dominant wavelength of the instability do not depend on the ion-to-electron mass ratio \( m_i/m_e \), as long as \( m_i/m_e \gg 1 \). This is indeed expected for the oblique firehose instability (Gary & Nishimura, 2003; Camporeale & Burgess, 2008), whereas for the parallel firehose instability \( \Gamma_{\text{max}} \propto m_e/m_i \).\(^5\) The fact that the oblique mode is insensitive to the mass ratio explains why our choice of a reduced mass ratio \( m_i/m_e = 100 \) is sufficient to capture the electron acceleration physics, as we have demonstrated in Appendix B of Paper I. Over time, the temperature anisotropy decreases (so, \( T_{e\perp}/T_{e\parallel} \) increases, as shown by the red line in Fig. 3.5(a)) and the wave energy \( \delta B^2/B_0^2 \) grows (red curve in Fig. 3.5(c)). This is consistent with the fact that the instability is triggered by the free energy of the temperature anisotropy and that the waves scatter the electrons towards isotropization (e.g. Hellinger & Trávníček, 2014).

Apart from the oblique firehose mode, other instabilities can be triggered by an electron temperature anisotropy \( T_{e\parallel} > T_{e\perp} \): the parallel electron firehose instability and the ordinary-mode instability (akin to the Weibel instability, Ibscher et al., 2012), whose properties are summarized in Table 3 of Lazar et al. (2014). We can confidently identify our instability as the oblique firehose mode, for the following reasons. The fact that the dominant mode

\(^5\)We have directly tested the scaling \( \Gamma_{\text{max}} \propto m_e/m_i \) expected for the parallel mode by performing a suite of 1D simulations (with different \( m_i/m_e \)) where the computational box is aligned with \( \vec{B}_0 \).
of our instability is purely growing rules out the parallel firehose instability, which has a non-zero real frequency. Also, the growth rate of the parallel firehose mode should depend on the ion-to-electron mass ratio, which is not the case in our simulations. In addition, the growth rate of our instability ($\lesssim \Omega_{ce}$) is incompatible with the expectations from the ordinary-mode instability, whose growth rate is $> \Omega_{ce}$. The fact that we do not observe strong fluctuations in the electric field component parallel to $\vec{B}_0$ further argues against the ordinary-mode instability.\footnote{We point out that the whistler waves discussed in Riquelme & Spitkovsky (2011) are triggered when $T_{e\parallel} < T_{e\perp}$, whereas the opposite temperature anisotropy (i.e., $T_{e\parallel} > T_{e\perp}$) is present in the upstream region of our shocks.}

In summary, the excellent agreement between our results and the theory of the oblique firehose mode suggests that the upstream waves ahead of low Mach number shocks are triggered by the returning electrons via the oblique firehose instability. We remark that, since the dominant mode is oblique with respect to both the shock normal and the upstream magnetic field, multi-dimensional shock simulations are of paramount importance to characterize the electron acceleration physics.

### 3.5.3 Dependence of Wave Generation on Plasma Conditions

To understand the conditions under which the electrons reflected by SDA can trigger the electron oblique firehose instability in the upstream region of low Mach number shocks, we perform five additional periodic box simulations. These are called theta43b, theta73b, sigle-1b, sigle-2b, Te1e8.0b, and they correspond to the shock simulations theta43, theta73, sigle-1_63, sigle-2_63, Te1e8.0. The beam parameters are listed in Table 3.2. Note that the values of $n_{\text{ref}}/n_e$, $(\gamma_{b,\text{min}} - 1)/k_B T_e$, $(\gamma_{b,\text{max}} - 1)/k_B T_e$ in Table 3.2 reflect the trends discussed in Section 3.4. In particular, the reflection fraction from SDA
Table 3.2: Parameters Used for the Periodic Box Simulations

<table>
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<tr>
<th>Run</th>
<th>$\frac{n_{ref}}{n_e}$</th>
<th>$\alpha_{max}$</th>
<th>$\gamma_{b,\text{min}}^{-1}$</th>
<th>$\gamma_{b,\text{max}}^{-1}$</th>
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<td>refb</td>
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<td>57°</td>
<td>2.4</td>
<td>40</td>
</tr>
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<td>theta43b</td>
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<td>0.02</td>
<td>31°</td>
<td>7.7</td>
<td>82</td>
</tr>
<tr>
<td>sigle-1b</td>
<td>0.18</td>
<td>57°</td>
<td>2.4</td>
<td>40</td>
</tr>
<tr>
<td>sigle-2b</td>
<td>0.18</td>
<td>57°</td>
<td>2.4</td>
<td>40</td>
</tr>
<tr>
<td>Te1e8.0b</td>
<td>0.25</td>
<td>74°</td>
<td>1.8</td>
<td>31</td>
</tr>
</tbody>
</table>
Figure 3.5: Time evolution of various quantities measured in periodic box simulations with parameters listed in Table 3.2 and indicated in the legend of panel (a). Panel (a) traces the electron temperature ratio $T_{e\perp}/T_{e\parallel}$. Panel (b) follows the quantity $1 - T_{e\perp}/T_{e\parallel} - 1.27/\beta_{e\parallel}^{0.95}$. The horizontal dashed line indicates the marginal stability threshold for a growth rate $\Gamma_{\text{max}} = 0.001 \Omega_{ce}$, where $\Omega_{ce}$ is the electron cyclotron frequency. Panel (c) traces the energy in the magnetic waves, normalized to the energy density of the background field.
increases at lower $\theta_B$, while the energy gain of the reflected electrons decreases. The SDA reflection fraction increases slightly at lower temperatures, while the energy gain normalized to $k_B T_e$ moderately decreases for smaller $T_e$ (see Fig. 3.2).

For the six periodic box simulations listed in Table 3.2, Fig. 3.5 shows the evolution of the electron temperature anisotropy $T_{e\perp}/T_{e\parallel}$ (panel (a)), the quantity $1 - T_{e\perp}/T_{e\parallel} - 1.27/\beta_{e\parallel}^{0.95}$ (panel (b)) that characterizes the departure from the instability threshold, and the wave energy $\delta B^2/B_0^2$ (panel (c)).

As apparent in Fig. 3.5, there is a clear dichotomy, depending on whether the electron anisotropy starts above or below the instability threshold (indicated with the horizontal black dashed line in panel (b)). For the two runs that start below the instability threshold, theta73b and sigle-1b, the instability never grows (the green and gray curves in Fig. 3.5(c) stay at the noise level) and the temperature anisotropy remains constant (Fig. 3.5(a)). Runs theta73b and sigle-1b fail to exceed the instability threshold for different reasons. In run theta73b, the temperature anisotropy is very weak ($T_{e\perp}/T_{e\parallel} \sim 1$), because the fraction of SDA-reflected electrons is very small. Even though SDA results in a substantial energy gain (third row in Table 3.2), the reflected electrons have a negligible effect on the overall plasma anisotropy. In contrast, for run sigle-1b the temperature anisotropy starts at the same level as in the reference run refb, where the instability does develop, as discussed in the previous subsection. However, due to the strong magnetization ($\sigma = 0.1$, giving $\beta_p = 6$), the electron parallel beta $\beta_{e\parallel} \sim 3$ is too small and the instability does not grow.

For the simulations meeting the instability threshold at initialization, viz., theta43b, Tel8.0b, sigle-2b, the waves do grow (blue, cyan and orange curves in Fig. 3.5(c)) and they isotropize the electron distribution (Fig. 3.5(a)), in analogy to the reference periodic box run refb. The saturation time of the wave energy (i.e., the end of the phase
of exponential growth) is roughly coincident with the time when the electron anisotropy falls below the instability threshold. These three runs meet the instability threshold, for the reasons that we now explain. In run theta43b, although the beam electrons are less energetic and populate a wider cone than in the run refb ($\alpha_{\text{max}} = 72^\circ$, as compared to $\alpha_{\text{max}} = 57^\circ$ in refb), the relative density of the electron beam is larger ($n_{\text{ref}}/n_e = 0.4$, as compared to 0.18 in refb). The combined effect is still to induce a strong temperature anisotropy $T_{e\perp}/T_{e\parallel} \simeq 0.75$, similar to the refb run. With comparable $\beta_{e\parallel}$ as in the refb run, the threshold is met. A similar argument applies to the case Te1e8.0b. In the run sigle-2b, the beam has the same properties as in run refb, and thus $T_{e\perp}/T_{e\parallel}$ starts exactly at the level. Since the magnetization is lower ($\sigma = 0.01$, as compared to 0.03 in refb), the electron parallel beta is even higher than in the run refb, making it easier to exceed the threshold.

In summary, the results of our periodic box simulations confirm the role of the threshold in Equation (3.10) for the excitation of the electron oblique firehose instability. In short, the pressure induced by the electron anisotropy should be stronger than the upstream magnetic pressure. This condition cannot be met when either the number density of returning electrons is too low or the upstream plasma is too strongly magnetized.

Finally, we point out that the upstream environment simulated in our periodic box runs differs in some respect from that in the shock simulations. In the shock simulations, a steady flow of electrons reflected via SDA is produced at the shock and propagates into the upstream, constantly driving the instability. In contrast, in the periodic box simulations, the electron anisotropy is set up at the initial time, and then the system relaxes towards isotropy as the instability grows. To investigate from first principles the dependence of the electron acceleration process on the pre-shock conditions, we still need to rely on fully-
consistent shock simulations, as we describe in the next section.

3.6 Dependence on the Pre-Shock Conditions

In this section, we explore the dependence of electron acceleration in low-Mach number shocks on pre-shock conditions. To do this, we vary the magnetic obliquity angle $\theta_B$, the magnetization $\sigma$ and the electron temperature $T_e$, as listed in Table 3.1. For completeness, we also comment briefly on the effect of varying the Mach number $M_s$, based on simulations presented in an earlier work (Narayan et al., 2012). All the electron energy spectra presented in this section are measured at the same time in units of $\Omega_{ci}^{-1}$ between $60c/\omega_{pe}$ and $160c/\omega_{pe}$ ahead of the shock.

3.6.1 Dependence on the Field Obliquity Angle $\theta_B$

We study the effect of the obliquity angle of the upstream magnetic field, $\theta_B$, by comparing the results from simulations with the same $T_e = 10^9, M_s = 3, \sigma = 0.03$ but different $\theta_B$, in the range from $\theta_B = 13^\circ$ to $80^\circ$. In this section, we focus on runs having $\sigma = 0.03$. In Section 3.6.2, we will briefly comment on the dependence on magnetic obliquity in shocks with $\sigma = 0.1$ and $\sigma = 0.01$.

The electron energy spectra measured at $\Omega_{ci}t = 29.8$ from the simulations theta33, theta43, theta53, theta68, theta73, theta80, along with the reference run (having $\theta_B = 63^\circ$), are presented in Fig. 3.6(a). As in Paper I, we define the maximum electron energy as the Lorentz factor $\gamma_{e,\text{max}}$ at which the particle number density drops below $10^{-4.5}$, the lowest level shown in our spectra. The subpanel of Fig. 3.6(a) traces the temporal evolution of the maximum energy of the upstream electrons. In Fig. 3.6(b), we show syn-
thetic electron energy spectra obtained by adding a component of SDA-reflected electrons to the thermal electrons initialized in the upstream (see Section 4.2.2 of Paper I). The spectra in Fig. 3.6(b) are based on our theory of SDA, considering a single acceleration cycle (hereafter, we shall call them SDA-synthetic spectra). So, they cannot account for the sustained Fermi-like acceleration process that is mediated by the oblique firehose waves. By comparing the left and right panels in Fig. 3.6, we can quantify the importance of the firehose-mediated Fermi-like acceleration.

As discussed in Section 3.4, no electron is expected to be accelerated via SDA in superluminal shocks, i.e., shocks with $u_t > c$ (Equation (3.7)). For $T_e = 10^9 K, m_i/m_e = 100, M_s = 3$, shocks with $\theta_B \gtrsim 78^\circ$ are superluminal. For this reason, the SDA-synthetic spectrum for $\theta_B = 80^\circ$ (purple curve in Fig. 3.6(b)) overlaps with the electron energy spectrum at initialization (dot-dashed curve in Fig. 3.6(b)), with no evidence for non-thermal electrons. The energy spectrum measured from our simulation theta80 also shows no sign of accelerated electrons (purple curve in Fig. 3.6(a)). This indicates that, without injection via SDA, no electrons can be accelerated.

Below the superluminal limit, the efficiency of SDA injection is expected to increase with decreasing $\theta_B$, see Fig. 3.2(c). As discussed in Section 3.4, the injection efficiency is still small if $u_t \gtrsim v_{th,e}$, since few of the thermal electrons will be fast enough to propagate back upstream, thus participating in the Fermi process. For $T_e = 10^9 K, m_i/m_e = 100, M_s = 3$, this limit correspond to an angle of $\theta_B \simeq 67^\circ$. Indeed, the electron non-thermal tails in shocks with $\theta_B$ below or close to this critical threshold contain a moderate fraction of electrons ($10\% - 20\%$), whereas the normalization of the non-thermal tail in the run theta73 is very low ($\sim 2\%$).

\footnote{The normalization of the non-thermal tail, a proxy for the electron injection efficiency, can be estimated from the point where the energy spectrum starts to deviate from a thermal distribution.}
Figure 3.6: Panel (a): upstream electron energy spectra measured at $\Omega_{ci}t = 29.8$ in the runs theta33, theta43, theta53, reference, theta68, theta73 and theta80, as indicated in the legend. The subplot shows the temporal evolution of the maximum energy of the upstream electrons. Panel (b): upstream electron energy spectra predicted by our SDA theory. In both panels, the black dot-dashed line corresponds to the drifting Maxwellian distribution at initialization, having $u_0 = 0.15c$ and $T_e = 10^9K$. 
The average energy gain resulting from one cycle of SDA increases with \( \theta_B \), as shown in Fig. 3.2(c) and suggested by the trend in the high-energy cutoffs of the SDA-synthetic spectra in Fig. 3.6(b). However, by comparing panel (a) and (b), we find that the maximum energy \( \gamma_{e,\text{max}} \) in our shock simulations theta53, reference, theta68 has evolved to a value that is much larger than expected from a single cycle of SDA (compare the high-energy cutoffs between the two panels). This suggests that the Fermi-like acceleration mechanism is operating in those runs, and this explains the steady growth in the maximum electron energy shown in the subpanel of Fig. 3.6(a). In such shocks, the fraction of SDA-reflected electrons is large enough that the resulting electron temperature anisotropy in the upstream region can trigger the oblique firehose waves that mediate the Fermi-like process at late times (e.g., see the waves in runs theta53, reference shown in Fig. 3.7(f)-(g)).

In contrast, the maximum energy of the electron spectrum from run theta73 saturates soon after \( \Omega_{ci} t \sim 15 \) to a value almost identical to that of the SDA-synthetic spectrum (\( \gamma_{e,\text{max}} \sim 18 \)), indicating that the Fermi-like acceleration mechanism does not operate in this case. This is consistent with the periodic box simulations in Section 3.5, where we have shown that with only \( \sim 2\% \) of the incoming electrons being reflected via SDA — a value appropriate for the run theta73 — the electron temperature anisotropy induced in the upstream is too weak to trigger the electron firehose instability. In Fig. 3.7(h), we explicitly show that no upstream waves are present in the upstream region for run theta73. In the absence of upstream waves, the electrons cannot undergo multiple cycles of SDA, and the maximum energy saturates to the value predicted by one cycle of SDA.

In the quasi-parallel regime (\( \theta_B \lesssim 45^\circ \)), the reflection fraction is expected to be relatively high, while the average energy gain from each SDA cycle is only a few times the electron thermal energy (Fig. 3.2(c)). Thus, we expect that after one cycle of SDA, the electron
energy spectra will not differ significantly from the drifting Maxwellian at initialization (see the green and blue curves in Fig. 3.6(b), for SDA-synthetic spectra). The measured spectra from runs theta33 and theta43 (green and blue curves in Fig. 3.6(a)) are also similar to a drifting Maxwellian. We find that the maximum electron energy $\gamma_{e,\text{max}}$ in runs theta33 and theta43 is steadily growing over time, indicating sustained Fermi-like acceleration, yet the acceleration rate is much slower than in quasi-perpendicular shocks. The same is true for runs theta13, theta23, whose results are not shown here. The increase in acceleration rate with magnetic obliquity is primarily driven by the fact that the energy gain per SDA cycle grows monotonically with $\theta_B$ (see Fig. 3.2(c)).

Fermi-like acceleration is expected to be efficient in low-obliquity shocks, albeit with a slow acceleration rate. As demonstrated in Section 3.5, due to the large SDA injection efficiency of quasi-parallel shocks (see Fig. 3.2(c)), the reflected electrons cause a sufficient temperature anisotropy in the upstream to enable the oblique firehose instability to grow. As Fig. 3.7(e) (corresponding to $\theta_B = 43^\circ$) shows, waves associated with the oblique firehose instability are triggered even with quasi-parallel shocks. By tracing a sample of selected electrons in runs theta23 and theta43, we have confirmed that a significant fraction of the reflected electrons are undergoing multiple cycles of SDA, by scattering off the upstream waves back toward the shock. This is similar to the behavior described in Fig. 8 of Paper I for a quasi-perpendicular shock with $\theta_B = 63^\circ$. Thus, we conclude that Fermi-like acceleration process consisting of multiple cycles of SDA operates efficiently in quasi-parallel shocks, albeit at a slower rate than in quasi-perpendicular shocks.

In quasi-parallel shocks, we observe that the electron spectra in our simulations peak at a slightly higher kinetic energy ($\gamma_e - 1 \sim 0.5$) than expected for the drifting Maxwellian at initialization, which peaks at $\gamma_e - 1 \sim 0.3$. In such shocks, we find that a significant
fraction of ions propagate upstream, in agreement with the results of hybrid simulations (e.g., Caprioli & Spitkovsky, 2013, 2014a). We speculate that the overall heating in the upstream electron spectrum might result from the interaction with long-wavelength modes driven by the reflected ions. On top of the ion waves, the electron firehose instability, growing on electron scales, will mediate efficient Fermi-like electron acceleration, as described above.

In summary, the injection process mediated by SDA cannot operate in superluminal shocks. In subluminal quasi-perpendicular shocks, the reflection fraction is moderate (∼10 – 20%) at angles such that $u_t \lesssim v_{th,e}$. Here, the beam of returning electrons induces a sufficient temperature anisotropy in the upstream such that oblique firehose waves can be generated, mediating long-term Fermi-like electron acceleration via multiple SDA cycles. Since the average energy gain from each SDA cycle is much larger than the electron thermal energy, the acceleration rate is fast. At obliquity angles such that $u_t \gtrsim v_{th,e}$, the SDA injection efficiency is poor, resulting in weak temperature anisotropies that do not meet the critical threshold for the excitation of the electron firehose instability. In the absence of firehose-driven waves, the electron acceleration process terminates after one cycle of SDA. In subluminal quasi-parallel shocks, the SDA efficiency is large, so the upstream waves can be promptly triggered and Fermi-like electron acceleration is very efficient. However, due to the small energy gain resulting from each cycle of SDA, the acceleration rate is slow.

### 3.6.2 Dependence on the Magnetization $\sigma$

To explore the effect of the flow magnetization $\sigma$, we have run simulations with $\sigma = 0.1$, 0.01 and 0.003, to be compared with our reference case $\sigma = 0.03$. We fix $T_e = 10^9 K$ and $u_0 = 0.15 c$, so that the Mach number stays fixed at $M_s = 3$. The range from $\sigma = 0.003$ to $\sigma = 0.1$ corresponds to a plasma beta varying from $\beta_p = 200$ to $\beta_p = 6$. At fixed
Figure 3.7: 2D plots of $B_z/B_0$ at $\omega_{pe} t = 4275$ from runs with different magnetizations $\sigma$ and magnetic obliquities $\theta_B$. Panels (a) - (d): from runs with $\sigma = 0.01$ and different obliquities: sig1e-2_43, sig1e-2_53, sig1e-2_63, sig1e-2_73 from top to bottom; Panels (e) - (h): from runs with $\sigma = 0.03$ and different obliquities: theta43, theta53, reference, theta73 from top to bottom; Panels (i) - (l): from runs with $\sigma = 0.1$ and different obliquities: sig1e-1_43, sig1e-1_53, sig1e-1_63, sig1e-1_73 from top to bottom. In all the panels, the background field orientation in the upstream region is indicated with the white arrows.
Figure 3.8: Electron upstream energy spectra at $\Omega_{ci}t = 16.2$ from the runs sigle-1_63, reference and sigle-2_63, as indicated in the legend. The black dashed curve shows the SDA-synthetic spectrum for the corresponding pre-shock parameters and the black dot-dashed curve shows the drifting Maxwellian distribution at initialization, having $u_0 = 0.15c$ and $T_e = 10^9$ K. In the run sigle-1_63, the upstream magnetic pressure is too strong for the electron firehose instability to be triggered, so no upstream waves are generated (Fig. 3.7) to sustain long term Fermi acceleration, and the maximum energy (red line in the subpanel) stops growing. The reference run and sigle-2_63 show similar results, because the SDA injection is similar, and their low magnetization allows upstream waves to grow (Fig. 3.7), to mediate long-term Fermi acceleration.
magnetization, we study a few magnetic obliquity angles.

We find that the electron acceleration in runs with $\sigma = 0.01$ \textit{(sig1e-2_43, sig1e-2_53, sig1e-2_63, sig1e-2_73)} shows strong similarities with our reference runs having $\sigma = 0.03$ \textit{(theta43, theta53, reference, theta73, respectively)}. On the other hand, electron acceleration via the Fermi-like process is suppressed at higher magnetizations, in the runs with $\sigma = 0.1$ \textit{(sig1e-1_43, sig1e-1_53, sig1e-1_63, sig1e-1_73)}. To illustrate the dependence on $\sigma$, we present in Fig. 3.8 the upstream electron energy spectra measured at $\Omega_{ci}t = 16.2$ from the runs \textit{sig1e-2_63, reference, sig1e-1_63}, having the same quasi-perpendicular obliquity angle $\theta_B = 63^\circ$. The subplot in Fig. 3.8 traces the evolution of the maximum energy of the upstream electrons. The SDA-synthetic spectrum for the corresponding pre-shock parameters is shown as a black dashed curve in Fig. 3.8. We remark that, based on our SDA theory, the SDA-synthetic spectrum has no explicit dependence on magnetization. In fact, in the weakly magnetized shocks considered here ($\sigma \ll 1$), the magnetic field energy does not significantly affect the shock structure, so our assumed values for the magnetic compression ratio $b = 4$ and the cross shock potential $\Delta \phi = 0.5$ still apply, regardless of $\sigma$.

As mentioned above, the simulations \textit{sig1e-2_63 and reference} yield similar electron acceleration efficiencies and a comparable acceleration rate (blue and red curves in Fig. 3.8). In both cases, $\sim 20\%$ of electrons populate a non-thermal tail in the energy spectrum, and the maximum energy evolves steadily to higher and higher values, well beyond the SDA-synthetic spectrum. This suggests that long-term Fermi-like acceleration is operating fast and efficiently. On the other hand, the electron acceleration in run \textit{sig1e-1_63} does not go beyond one cycle of SDA. The maximum energy saturates after $\Omega_{ci}t \sim 7$ to a value comparable to the high-energy cutoff in the SDA-synthetic spectrum ($\gamma_e - 1 \simeq 7$).
shape of the measured electron spectrum (green curve in Fig. 3.8) resembles closely the SDA-synthetic spectrum.

The similarity between the runs sigle-2_63 and reference stems from the fact that both the injection efficiency and the average energy gain via SDA are nearly identical in the two runs, so the returning electrons induce a similar temperature anisotropy in the upstream, irrespective of $\sigma$. We have demonstrated in Section 3.5 that for $\sigma = 0.03$, the oblique firehose instability is excited across a wide range of magnetic obliquities (see Fig. 3.7(a)-(d)). Having comparable electron anisotropy and lower magnetization (so, higher $\beta_e$), the instability threshold is easily met at shocks having $\sigma = 0.01$. The resulting waves mediate efficient Fermi-like acceleration (the wave patterns of runs sigle-2_43,sigle-2_53,sigle-2_63 are shown in Fig. 3.7(a)-(c), respectively). Generalizing this argument to different obliquities, we see why the electron acceleration in runs having $\sigma = 0.01$ (sigle-2_43, sigle-2_53, sigle-2_63, sigle-2_73) shows a similar efficiency as in our reference runs with $\sigma = 0.03$, (theta43, theta53, reference, theta73, respectively). In contrast, in shocks with $\sigma = 0.1$ (so, lower $\beta_e$), the electron anisotropy is not sufficient to satisfy the threshold criterion in Equation (3.10), so the growth of oblique firehose waves is inhibited (see Fig. 3.7(i)-(l)). In the absence of upstream waves, the electron acceleration process stops after one cycle of SDA.

In summary, the dependence of the electron acceleration physics on magnetization in weakly to moderately magnetized shocks ($\beta_p \gtrsim \text{a few}$) is largely determined by whether the upstream magnetic pressure is weak enough to allow the growth of the electron firehose instability, thereby allowing the electrons to participate in long-term Fermi-like acceleration process. The instability is suppressed in shocks having $\sigma = 0.1$ ($\beta_p = 6$), the strongest magnetization we have explored. Here, the electron acceleration process stops after one
cycle of SDA. On the other hand, lower magnetizations ($\sigma = 0.01 - 0.03$, corresponding to $\beta_p = 20 - 60$) allow firehose-driven waves to grow and support the Fermi-like process. We argue that this mechanism can operate down to at least $\sigma = 0.003$ ($\beta_p = 200$), since we find that in run sig3e-3_63 the same type of waves are generated in the upstream region.

At lower magnetizations (or equivalently, higher $\beta_p$), the ordinary-mode instability is likely to dominate (Lazar & Poedts, 2009; Lazar et al., 2010, 2014), in analogy to unmagnetized shocks. A discussion of the electron injection and acceleration mechanism at extremely low $\sigma$ (high $\beta_p$), is likely to differ from the scenario presented here, and is beyond the scope of this paper.

### 3.6.3 Dependence on the Electron Temperature $T_e$

To investigate the dependence of electron acceleration on the upstream electron temperature $T_e$, we perform simulations with lower temperatures than the reference run (which has $T_e = 10^9K$). We vary the temperature from $10^{7.5}K$ to $10^{8.5}K$ at fixed $\theta_B = 63^\circ$, $\sigma = 0.03$ ($\beta_p = 20$) and $M = 2$ ($M_s = 3$). In order to keep the Mach number $M$ fixed, we scale the upstream bulk flow velocity as $u_0 \propto \sqrt{T_e}$ (Table 3.1).

Fig. 3.9(a) shows the upstream electron energy spectra at $\Omega_{ci}t = 10.3$ from runs Te1e7.5, Te1e8, Te1e8,5, reference, while Fig. 3.9(b) shows the corresponding SDA-synthetic spectra. Unlike the other electron spectra presented in this section, the horizontal axis of these spectra measures $(\gamma_e - 1)m_e c^2/k_B T_e$, instead of $\gamma_e - 1$. This choice is motivated by the fact that the electron spectra from runs with different $T_e$ peak at $\gamma_e - 1 \sim k_B T_e$, so comparisons are easier after rescaling with $k_B T_e$. In addition, we have shown in Section 3.4 that the average energy gain in one SDA cycle also scales with $T_e$ (Fig. 3.2(d)), which further motivates our rescaling.
Figure 3.9: Panel (a): upstream electron energy spectra measured at $\Omega_c t = 10.3$ from the runs $T_e=10^{7.5}$, $T_e=10^8$, $T_e=10^8.5$, reference, as indicated in the legend. The subplot shows the temporal evolution of the maximum energy of the upstream electrons, in units of $k_B T_e$. Panel (b): upstream electron energy spectra predicted by SDA theory. In both panels, the black dot-dashed line corresponds to the drifting Maxwellian distribution at initialization, having $u_0 = 0.15 c$ and $T_e = 10^9 K$. 
We find that in all the runs considered here (Te1e7.5, Te1e8, Te1e8.5, reference), the maximum electron energy $\gamma_{e,\text{max}}$ evolves well beyond the prediction of a single cycle of SDA (see subpanel of Fig. 3.9(a)), indicating that long-term Fermi acceleration process operates efficiently in all the runs. This stems from the fact that the electron temperature anisotropy is only weakly dependent on $T_e$ (see Fig. 3.5), so the threshold for the oblique firehose instability is still met, and the upstream waves can grow and mediate long-term Fermi-like acceleration. Indeed, we observe similar wave patterns in all the runs mentioned above, regardless of $T_e$ (see Fig. 3.10).\(^8\)

As regards spectra, we find that, when the kinetic energy $(\gamma e-1)m_e c^2$ is measured in units of $k_B T_e$, the electron energy spectra from runs Te1e7.5 and Te1e8 nearly overlap. Both the normalization and the maximum energy of the non-thermal tail are almost identical. The agreement between these two runs can be understood from the SDA theory, since both the reflection fraction and the average energy gain (in units of $k_B T_e$) stay almost constant in the regime $T_e \lesssim 10^8 K$ (see Fig. 3.2(d)). With increasing temperature towards the relativistic regime (runs Te1e8.5 and reference), the normalization of the non-thermal tail tends to decrease, but the spectrum extends to higher energies (see Fig. 3.9(a)). Once again, SDA theory predicts both the lower injection efficiency and the higher maximum energy, as the temperature increases beyond $T_e \gtrsim 10^8 K$ (see Fig. 3.2(d)).

To summarize, Fermi-like electron acceleration operates efficiently over the whole temperature range we have explored, $T_e = 10^7.5 K - 10^9 K$, with fixed $M_s = 3$, $\sigma = 0.03$ and $\theta_B = 63^\circ$. The number density of non-thermal electrons stays roughly constant in the regime $T_e \lesssim 10^8 K$, but decreases slightly with increasing $T_e$ for $T_e \gtrsim 10^8 K$, as suggested\(^8\) The waves in the run Te1e7.5 look weaker, due to limited numerical accuracy at low temperatures. The strength of electromagnetic fields is weaker in lower temperature runs (e.g., $B_0 \propto u_0 \propto \sqrt{T_e}$), resulting in a lower signal to noise ratio.
Figure 3.10: 2D plots of $B_z/B_0$ at $\Omega_{ci}t = 10$ from the runs $Te = 10^{7.5}$, $Te = 10^{8}$, $Te = 10^{8.5}$, $Te = 10^{9}$. The orientation of the upstream magnetic field is indicated by the white arrows.
by Fig. 3.2(d). The acceleration rate is faster in the higher temperature regime $T_e \gtrsim 10^8 K$, since the average energy gain per SDA cycle increases with $T_e$. The acceleration rate at lower temperatures is slower, but it saturates at a constant value for $T_e \lesssim 10^8 K$. Based on the SDA theory presented in Section 3.4 and on the mechanism of wave generation described in Section 3.5, we expect that our results can be extrapolated to even lower temperatures (at fixed Mach number and magnetic obliquity) since both the SDA injection efficiency and the threshold for excitation of the firehose instability are independent of $T_e$.

### 3.6.4 Dependence on the Mach number $M_s$

We comment on the dependence of the electron acceleration physics on the Mach number $M_s$, based on simulations presented in Narayan et al. (2012).

From the discussion above, we know that the electron acceleration efficiency is ultimately related to the efficiency of SDA injection. In turn, the number of SDA-reflected electrons and their anisotropy determine whether the oblique firehose instability can grow in the upstream, governing the long-term Fermi acceleration. A key parameter that regulates the SDA injection efficiency is the HT velocity $u_t$, which scales linearly with $M_s$ (Equation (3.7)). When $u_t \gtrsim v_{th,e}$, or equivalently $M_s \sec \theta_B \sqrt{m_e/m_i} \gtrsim 1$, SDA injection is expected to be inefficient. For $\theta_B \gtrsim 45^\circ$ and mass ratio $m_i/m_e = 100$ (as employed in our reference runs), the requirement $u_t \lesssim v_{th,e}$ for efficient SDA injection is satisfied for Mach numbers $M_s \lesssim 5.5$, while for the realistic mass ratio $m_i/m_e = 1836$ the requirement is $M_s \lesssim 23$. As the Mach number increases towards this limit, the acceleration efficiency is expected to decrease. In contrast, since the energy gain per SDA cycle increases with $u_t \propto M_s$, the acceleration rate will be faster for higher $M_s$.

The results of 2D PIC simulations presented in Fig.2(b) of Narayan et al. (2012) illustrate
the dependence on $M_s$. There, the Alfvénic Mach number was fixed at $M_A = 8$, which corresponds to $\sigma = 0.03$, as in our reference run. The electron temperature is changed but the upstream flow speed is fixed, which effectively results in varying the Mach number. For the temperature range $T_e = 5 \times 10^7 K - 10^9 K$ explored in Narayan et al. (2012), the Mach number varies between $M_s \simeq 2$ and 9 ($M_s \propto T_e^{-1/2}$). The electron spectra from runs of different $M_s$ show that the normalization of the non-thermal tail decreases monotonically with increasing $M_s$. In particular, the acceleration efficiency drops from $\sim 10\%$ in the run with $M_s = 2$ ($T_e = 10^9 K$) to $\sim 4\%$ in the run with $M_s \simeq 4.5$ ($T_e = 2 \times 10^8 K$), and it is negligible ($\ll 1\%$) in the runs having $M_s \gtrsim 6$ ($T_e \lesssim 10^8 K$). These results are in agreement with our arguments above.

For higher Mach number shocks, a regime relevant for supernova remnants, injection via SDA becomes extremely inefficient and other pre-acceleration mechanisms, such as shock surfing acceleration or injection via whistler waves, will dominate (see, e.g., Dieckmann et al., 2000; Hoshino & Shimada, 2002; Schmitz et al., 2002; Amano & Hoshino, 2007; Riquelme & Spitkovsky, 2011; Matsumoto et al., 2012).

3.7 Summary and Discussion

In this paper, the second of a series, we complete our investigation of electron acceleration in low Mach number shocks ($M_s = 3$) by performing a suite of self-consistent 2D PIC simulations. In Paper I, we studied a reference shock that propagates in a high-temperature plasma ($T_e = 10^9 K$) carrying a quasi-perpendicular magnetic field (with magnetization $\sigma = 0.03$ and obliquity $\theta_B = 63^\circ$). We identified a Fermi-like electron acceleration mechanism whose injection is governed by shock drift acceleration (SDA). A fraction of the incoming
thermal electrons are reflected at the shock front by the mirror force of the shock-compressed field, and they are energized by the motional electric field while drifting along the shock surface. The reflected electrons propagate ahead of the shock, where their interaction with the upstream flow generates oblique magnetic waves in the upstream region. The waves scatter the reflected electrons back towards the shock for multiple cycles of SDA, in a process resembling the Fermi mechanism.

In the present work, we address the nature of the upstream waves, which are essential for maintaining the long-term Fermi-like acceleration. Using 2D periodic box simulations in the upstream frame, we study the interaction between the beam of SDA-reflected electrons and the pre-shock plasma. We confirm that the upstream waves are triggered by the electrons reflected at the shock during the SDA process. The distribution of reflected electrons is anisotropic, such that the temperature parallel to the field is larger than perpendicular \( (T_{e\parallel} > T_{e\perp}) \). We demonstrate that the waves are associated with the oblique mode of the electron firehose instability, which is driven by the electron temperature anisotropy and requires the pressure associated to the electron anisotropy to be stronger than the plasma magnetic pressure. It follows that the waves cannot be generated if the fraction of SDA-reflected electrons is too small or if the upstream magnetic field is too strong (i.e., low beta plasmas). In the absence of upstream magnetic waves, the Fermi-like acceleration process will be inhibited.

By means of fully-consistent 2D shock simulations, we systematically explore the dependence of the electron acceleration efficiency on the pre-shock conditions for low Mach number shocks \( (M_s = 3) \). We investigate the effect of the upstream magnetic field obliquity \( \theta_B \), of the magnetization \( \sigma \) and of the electron temperature \( T_e \).

We find that at superluminal shocks (i.e., where the de Hoffman-Teller velocity exceeds
the speed of light), the SDA process does not operate and no electron is accelerated. In subluminal shocks, the efficiency of electron acceleration depends on the magnetic obliquity $\theta_B$. Injection via SDA into the acceleration process is inefficient if $u_t \gtrsim v_{th,e}$, where $v_{th,e}$ is the electron thermal speed and the de Hoffman-Teller velocity $u_t$ can be written as $u_t \sim v_{th,e}M_s\sec \theta_B\sqrt{m_e/m_i}$. Here, few electrons are able to propagate back into the upstream, so the resulting temperature anisotropy induced in the pre-shock region is too weak to trigger the firehose instability. In the absence of upstream waves, the process of electron acceleration stops after one cycle of SDA. In contrast, for $u_t \lesssim v_{th,e}$ (still, for quasi-perpendicular shocks with $\theta_B \gtrsim 45^\circ$), the electron acceleration process is efficient and fast. In fact, the fraction of SDA-reflected electrons is large enough to generate firehose-driven waves in the upstream. These waves help the Fermi-like process by enabling multiple SDA cycles. Also, the acceleration rate is fast because each SDA cycle provides a significant energy gain. The Fermi-like process operates efficiently also in quasi-parallel shocks ($\theta_B \lesssim 45^\circ$), but the electron acceleration rate is slower. In fact, the SDA reflection efficiency increases at lower $\theta_B$, but the energy gain per SDA cycle is smaller. In addition, in quasi-parallel shocks, we find that a fraction of the incoming ions are reflected back into the upstream. The ion acceleration physics operates on timescales longer than the timespan of our simulations, but it is not expected to modify significantly the process of firehose-mediated electron acceleration described above.

When varying the magnetization $\sigma$ at fixed $T_e = 10^9$ K, we find that the electron acceleration physics does not depend on the flow magnetization so long as $\sigma \lesssim 0.03$. Neither the injection efficiency of SDA nor the energy gain per SDA cycle depends explicitly on $\sigma$, and the threshold condition for the excitation of the oblique firehose mode is satisfied at all $\sigma \lesssim 0.03$, as long as the magnetic obliquity is such that $u_t \lesssim v_{th,e}$. Oblique firehose waves
are still present at $\sigma = 0.003$, but we expect that the injection and acceleration physics at yet lower magnetizations could be different, as the shock transitions to the Weibel-mediated regime relevant for unmagnetized flows. In contrast, at high magnetizations ($\sigma = 0.1$), even though injection via SDA is efficient, the Fermi process cannot operate because the strong magnetic pressure in the upstream suppresses the growth of the firehose instability.

When varying the electron temperature $T_e$ at fixed $\sigma = 0.03$ and $\theta_B = 63^\circ$, we find efficient long-term electron acceleration across the whole temperature range $T_e = 10^{7.5} K - 10^9 K$. Both the SDA injection efficiency and the acceleration rate are insensitive to the electron temperature in the regime of non-relativistic electrons ($T_e \lesssim 10^8 K$). No major change is observed for trans-relativistic temperatures ($T_e \gtrsim 10^{8.5} K$), except that there is a slight tendency for a higher SDA injection efficiency and a larger energy gain per SDA cycle, as $T_e$ increases.

In summary, our study finds that efficient Fermi-like electron acceleration, whose injection is controlled by SDA, operates in low Mach number shocks for a variety of pre-shock conditions. The Fermi acceleration is mediated by oblique upstream waves generated by the electron firehose instability, which can only be excited if the plasma beta in the upstream region is sufficiently large. As the criterion in Equation (3.10) suggests, the electron firehose instability would be completely suppressed for $\beta_{e\parallel} \lesssim 1.3$, which corresponds to $\beta_p \lesssim 2.6$. For $\beta_p = 20 - 200$, we have demonstrated that the growth of the upstream firehose waves is allowed, given a sufficient electron anisotropy. In this respect, our work is complementary to earlier PIC studies of shocks propagating in low-beta plasmas (so, with high Mach number), as appropriate for supernova remnants. There, firehose-driven waves cannot grow, since $\beta_p \lesssim 1$. Also, a different injection mechanism, other than SDA, is required for efficient electron acceleration. For instance, Riquelme & Spitkovsky (2011) found that in shocks with
higher sonic Mach number ($M_s \simeq 7$, as opposed to our choice $M_s = 3$) and lower plasma beta ($\beta_p \lesssim 1$), electron injection is regulated by the interaction with oblique whistler waves near the shock front. Alternatively, Matsumoto et al. (2012) found that the shock surfing mechanism serves to inject electrons into Fermi acceleration at shocks with low plasma beta ($\beta_p \ll 1$) and high Alfvénic Mach number ($M_A \sim 30$) (see also e.g. McClements et al., 2001; Hoshino & Shimada, 2002; Amano & Hoshino, 2007).

The generality of our Fermi-like electron acceleration mechanism in low Mach number shocks offers a possible solution to the problem of electron injection in merger shocks of galaxy clusters. The bright radio emission that is observed from radio relics cannot be reconciled with the poor efficiency of the commonly-invoked “thermal leakage” model for electron injection (Malkov & Völk, 1998; Gieseler et al., 2000; Kang et al., 2002). As we discussed in Paper I, the thermal leakage model assumes that the electrons are scattered by downstream magnetic waves back into the upstream. This requires that the electrons have large momentum, a few times larger than the characteristic post-shock ion momentum, so that their Larmor radius is larger than the scale of the magnetic turbulence. The number of incoming thermal electrons that satisfy this stringent criterion is extremely small. In contrast, our mechanism, based on first-principles PIC simulations, does not involve any scattering by the downstream turbulence. Rather, the shock itself acts as a magnetic mirror, reflecting a fraction of the incoming electrons back upstream via SDA. The minimum electron momentum required for reflection via SDA is much lower (by a factor of $\sim m_e/m_i$) than that required in the thermal leakage model. For this reason, the electron injection fraction in our low Mach number shocks is as large as $\sim 10\% - 20\%$, which can explain the bright radio emission of galaxy cluster shocks.\(^9\)

\(^9\)Incidentally, electrons accelerated in radio relics are also invoked as a seed population of relativistic electrons for particle reacceleration by turbulence at radio halos (Brunetti & Lazarian, 2011).
In the thermal leakage model, due to the stringent constraint on the minimum momentum for electron injection, the number of accelerated ions is expected to exceed that of accelerated electrons by a large factor. The high-energy ions will interact with the thermal gas in the ICM and produce gamma-ray emission. Assuming the large ratio of ion-to-electron acceleration efficiencies predicted by the thermal leakage model, Vazza et al. (2014) found that, given the current observations of radio relics, which are powered by synchrotron emission of the shock-accelerated electrons, the predicted gamma-ray luminosity of nearby galaxy clusters, resulting from the accelerated ions, should be above the detection limit of the Fermi telescope. Yet, Fermi has not detected any gamma-ray signature from these systems. This apparent tension can be alleviated if the electron acceleration efficiency is much higher than expected from the thermal leakage model, as indeed predicted by our mechanism.\textsuperscript{10} In particular, we find that a fraction as large as $\sim 10\% - 20\%$ of electrons can be accelerated in quasi-perpendicular shocks, where ion acceleration is known to be extremely inefficient (e.g., Caprioli & Spitkovsky, 2013, 2014a). The ratio of electron-to-ion acceleration efficiency should then be higher than expected from the thermal leakage model, suggesting that the gamma-ray brightness of galaxy cluster shocks is likely to be significantly lower than estimated by Vazza et al. (2014). This could explain the lack of Fermi detections of galaxy clusters.

Our study might also help to clarify why some low Mach number shocks are not efficient electron accelerators. Using Chandra X-ray images, Russell et al. (2011) have unambiguously identified two merger shocks with $M_s \simeq 2.1$ and $M_s \simeq 1.6$ in the galaxy cluster Abell 2146. However, no radio emission is detected there. Currently, no convincing explanation has been proposed. We point out that the merger shocks in Abell 2146 are not located at the

\textsuperscript{10}The possibility of a higher ratio of electron-to-ion acceleration efficiencies – as a solution to the lack of Fermi detections of galaxy clusters – has already been invoked by Brunetti & Jones (2014).
outskirts of the galaxy cluster, where most radio relics have been detected. Near the cluster center, the magnetic field should be stronger than in the outskirts, though no measurement of magnetic field strength is available at the location of these two shocks. The pre-shock temperature and density inferred from X-ray observations (Russell et al., 2010) suggest that $T_e \sim 6 \times 10^7$ K and $n_e \sim 10^{-3}$ at both shocks. If $B_0 \sim 8 \mu$G (slightly stronger than typically field strength inferred in cluster outskirts), the plasma beta could be as low as $\beta_p \sim 2.5$, which would prevent the growth of the electron firehose instability. Without upstream waves, the process of electron acceleration would stop after one cycle of SDA, and electrons would not be accelerated to relativistic energies. This argument may also offer a generic explanation for the rarity of radio relics in the central regions of galaxy clusters, as discussed by Vazza et al. (2012).

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Chapter 4

Electron heating in low Mach number perpendicular collisionless shocks. I. Reference Run

The following two thesis chapters are based on the following publication


4.1 Introduction

Galaxy clusters grow via mergers of subclusters. A large fraction of the kinetic energy of the infalling subclusters is dissipated at low Mach number shocks ($M_s \lesssim 5$, where $M_s$ is the ratio of shock speed and pre-shock sound speed), which heat the intracluster medium (ICM) and sometimes accelerate particles to relativistic energies (Sarazin, 2002; Ryu et al., 2003; Brüggen et al., 2012). Merger shocks in clusters are collisionless. Due to the high ICM temperatures ($\sim 10^7 - 10^8$ K) and low densities ($10^{-2} - 10^{-4}$ cm$^{-3}$), the collisional mean free path ($\sim 10^{21} - 10^{23}$ cm) is as large as the size of the cluster.

Galaxy cluster shocks are routinely observed in the radio and X-ray bands. X-ray mea-
surements can quantify the density and temperature jumps between the unshocked (up-
stream) and the shocked (downstream) plasma (e.g., Markevitch et al., 2002; Finoguenov
et al., 2010; Russell et al., 2010; Ogrean et al., 2013a; Eckert et al., 2016; Akamatsu et al.,
2017b). The existence of shock-accelerated electrons is revealed by radio observations of
synchrotron radiation (e.g., van Weeren et al., 2010; Lindner et al., 2014; Trasatti et al.,
2015; Kale et al., 2017). Recently, the pressure jump associated with a merger shock
at relatively high redshift has been measured through radio observations of the thermal
Sunyaev-Zel’dovich (SZ) effect (Basu et al., 2016).

Since all of our observational diagnostics are based on radiation emitted by electrons,
the proton properties (in particular, their temperature) are basically unconstrained. One
usually makes the simplifying assumption that the electron temperature equals the mean
gas temperature (and so, the proton temperature). This assumption is unlikely to hold in
the vicinity of merger shocks. Ahead of the shock, the bulk kinetic energy of protons is
a factor of \( m_i/m_e \) larger than for electrons (here, \( m_i \) and \( m_e \) are the proton and electron
masses, respectively). In the absence of a channel for efficient proton-to-electron energy
transfer, a comparable ratio should persist between the post-shock temperatures of the two
species.

While Coulomb collisions will eventually drive electrons and protons to equal tempera-
tures, the collisional equilibration timescale (Spitzer, 1962) for typical ICM conditions is
as long as \( 10^8 \) – \( 10^9 \) yrs. In fact, X-ray observations by Russell et al. (2012) have shown
that the electron temperature just behind a merger shock in Abell 2146 is lower than the
mean gas temperature expected from the Rankine-Hugoniot jump conditions, and thus
lower than the proton temperature. In another study, Akamatsu et al. (2017b) compiled a
list of merger shocks, estimating their Mach number from both X-ray \( (M_{s,X-ray}) \) and radio
observations \((M_{s,\text{radio}})\), and noticed a slight bias of \(M_{s,\text{radio}} \gtrsim M_{s,\text{X-ray}}\). Here, \(M_{s,\text{radio}}\) is derived by measuring the power-law slope of the synchrotron emission, which is related — via the theory of diffusive shock acceleration — to the density compression at the shock (and so, to the Mach number). On the other hand, \(M_{s,\text{X-ray}}\) is obtained from the electron free-free emission by measuring the jumps in density and temperature across the shock. It follows that, if electrons have a lower temperature than protons behind the shock, \(M_{s,\text{X-ray}}\) would have been underestimated.

It has long been thought that collisionless shocks can lead to a two-temperature structure at the outskirts of galaxy clusters (Fox & Loeb, 1997; Ettori & Fabian, 1998; Takizawa, 1999). Detailed cosmological hydrodynamic simulations have shown that this can significantly bias the X-ray and thermal SZ signatures (Wong & Sarazin, 2009; Rudd & Nagai, 2009). In the absence of a physical model for electron heating in low Mach number shocks, these studies usually employ an \textit{ad-hoc} subgrid approach to prescribe the electron heating efficiency in shocks. Either electrons are assumed to be heated adiabatically, or the non-adiabatic (or “irreversible”) heating efficiency is quantified by a phenomenological, often arbitrary, parameter. While observations from heliospheric low Mach number shocks have shown that electrons do not get heated much beyond adiabatic compression (Bame et al., 1979; Ghavamian et al., 2013), there has also been direct evidence of electron entropy production, i.e., non-adiabatic heating, at low Mach number shock fronts (Parks et al., 2012).

What is the mechanism responsible for electron heating at collisionless shocks? This is a fundamental question of plasma physics, as the fluid-type Rankine-Hugoniot relations only predict the jump in the mean plasma temperature across the shock, without specifying how the shock-generated heat is distributed between the two species. To understand electron heating in collisionless shocks, fully-kinetic simulations with the particle-in-cell
(PIC) method (Birdsall & Langdon, 1991; Hockney & Eastwood, 1981) are essential to self-consistently capture the non-linear structure of the shock and the role of electron and proton plasma instabilities in particle heating.

So far, PIC studies of electron heating in shocks have focused on the regime of high sonic Mach number ($M_s \gtrsim 10$) and low plasma beta ($\beta p_0 \lesssim 1$) appropriate for supernova remnants. At very high Mach numbers, the Buneman instability can trap electrons in the shock transition region and heat them (Dieckmann et al., 2012). For lower Mach numbers, resonant wave-particle scattering induced by the modified two-stream instability (MTSI) can lead to significant electron heating at the shock front (Matsukiyo & Scholer, 2003; Matsukiyo, 2010).

The regime of low sonic Mach number and high beta most relevant for cluster merger shocks is still unexplored. In the series of studies starting from this chapter, we study electron heating in low Mach number perpendicular shocks by means of two-dimensional (2D) PIC simulations. In this chapter, we focus on the results from a reference shock simulation with $M_s = 3$ and $\beta p_0 = 16$. In upcoming chapters we will explore the dependence of our conclusions on sonic Mach number and plasma beta. The choice of a perpendicular magnetic field geometry is meant to minimize the role of non-thermal electrons that are self-consistently accelerated in oblique configurations, as we have shown in Chapters 2 and 3 (Guo et al., 2014a,b, respectively). Because there are no shock-accelerated electrons returning upstream in a perpendicular shock, the shock can settle to a steady state on a shorter time, thus allowing us to study the steady-state electron heating physics. However, we emphasize that we have verified with a suite of PIC simulations of quasi-perpendicular shocks (not shown here) that the physics of electron heating presented in this paper also applies to quasi-perpendicular configurations, as long as the non-thermal electrons are energetically
sub-dominant.

The rest of the paper is organized as follows. In Section 4.2 we lay out the theoretical framework for electron heating. Section 4.3 describes the setup of the reference shock simulation. Section 4.4 shows the shock structure of the reference simulation, where we emphasize that efficient electron irreversible heating occurs at two main locations. In the next chapter, we will use periodic box experiments to reproduce the shock conditions at these two major sites of entropy production and we will validate our heating model in the case of the reference shock simulation.

4.2 The Physics of Electron Heating

As electrons pass through the shock, they experience a density compression, which results in adiabatic heating. In addition, irreversible processes might operate, which will further increase the electron temperature. The purpose of this section is to present a general formalism for the physics of irreversible heating. Even though we will be primarily interested in electron heating, the model can be applied to any particle species. It relies on the presence of two basic ingredients: (i) a temperature anisotropy; and (ii) a mechanism to break the adiabatic invariance. We first describe the change in internal energy of an anisotropic fluid, and then consider the resulting change in entropy.¹

¹We point out that the model that we propose is reminiscent of the so-called “magnetic pumping” mechanism, where a periodically-varying external magnetic field is used in the laboratory to drive proton anisotropy and subsequent plasma heating (Spitzer & Witten, 1953; Berger et al., 1958; Borovsky, 1986).
4.2.1 The Change in Internal Energy

The work done on an isotropic gas with pressure $P$ and volume $V$ is simply $dW = -PdV$.
We shall generalize this expression to the case of an anisotropic gas having pressure perpendicular (parallel, respectively) to the magnetic field lines equal to $P_\perp$ ($P_\parallel$, respectively).
Consider a magnetic flux tube with length $L$, cross-sectional area $A$, volume $V = LA$, and field strength $B$. The magnetic flux through the area $A$ is $\Phi = BA$. In response to a compression (or expansion) perpendicular to the magnetic field, the volume will change as

$$dV_\perp = L \, dA = L \, d\left(\frac{\Phi}{B}\right) = -L \Phi \frac{dB}{B^2} = -V \ln B,$$

where we have used the fact that, because of flux freezing, $\Phi$ is a constant. In contrast, for compression (or expansion) along the field, the volume will change as

$$dV_\parallel = A \, dL = A \, d\left(\frac{V}{A}\right) = \frac{AN}{\Phi} \, d\left(\frac{B}{n}\right) = -V \ln \left(\frac{n}{B}\right),$$

where $N$ is the total number of particles in the volume element, with number density $n = N/V$. It follows that the work done on an anisotropic gas can be written as

$$dW = -P_\perp dV_\perp - P_\parallel dV_\parallel = P_\perp V d\ln B + P_\parallel V d\ln \left(\frac{n}{B}\right).$$
Defining the work done per particle as $dw = dW/N$, we find that it can be separated into a “perpendicular” component $dw_\perp$ and a “parallel” component $dw_\parallel$ as

$$dw = k_B T_\perp d\ln B + k_B T_\parallel d\ln \left(\frac{n}{B}\right).$$

(4.4)

It follows that $dw_\perp$ will change the internal energy per particle $u_\perp$ associated with motions perpendicular to the field, while $dw_\parallel$ will affect the energy per particle $u_\parallel$ associated with parallel motions.

In writing the energy equation for the perpendicular and parallel components, we need to take into account two additional processes: (i) In the presence of pitch angle scattering, heat can be transferred between the two components (as we show below, this will give rise to entropy increase). We denote the differential amount of transferred heat as $dq_{\perp\rightarrow\parallel}$, with the convention that $dq_{\perp\rightarrow\parallel} > 0$ if heat flows from the perpendicular to the parallel component. (ii) Pitch angle scattering may be caused by self-generated waves (e.g., sourced by the plasma anisotropy), whose energy needs to be provided by the plasma itself. The energy balance relations then read

$$du_\perp = dw_\perp - dq_{\perp\rightarrow\parallel} - de_{w,\perp},$$

(4.5)

$$du_\parallel = dw_\parallel + dq_{\perp\rightarrow\parallel} - de_{w,\parallel},$$

(4.6)

where we denote the wave energy per particle coming from the perpendicular (parallel, respectively) plasma energy as $de_{w,\perp}$ ($de_{w,\parallel}$, respectively). By summing the above two equations, we obtain the expected result that the net change of internal energy per particle
is equal to the external work minus the energy given to waves

\[ du \equiv du_{\perp} + du_{\parallel} = dw - de_{w\text{tot}} , \]  

(4.7)

where we denote the total energy per particle transferred to waves as \( de_{w\text{tot}} \equiv de_{w,\perp} + de_{w,\parallel} \), including magnetic, electric and bulk kinetic contributions (in practice, the magnetic term always dominates).

While the total wave energy per particle \( de_{w\text{tot}} \) is easy to extract from our simulations, the two contributions \( de_{w,\perp} \) and \( de_{w,\parallel} \) are hard to separate. Fortunately, as we show below, for the entropy calculation it suffices to measure the total energy per particle transferred to waves \( de_{w\text{tot}} \). We also remark that \( de_{w\text{tot}} \) accounts for the differential energy per particle transferred to waves, which might not necessarily equal the differential change in the energy residing in waves, which we shall call \( de_w \). More specifically, while for electron-driven waves \( de_{w\text{tot}} = de_w \), we will show in the next chapter that proton-generated waves will lose energy by performing work on the electron plasma, so the change in the energy residing in proton waves \( de_w \) will be smaller than the differential energy \( de_{w\text{tot}} \) transferred from protons to waves.

### 4.2.2 The Change in Entropy

For a non-relativistic bi-Maxwellian plasma with perpendicular temperature \( T_\perp \) and parallel temperature \( T_\parallel \), the entropy per particle (or specific entropy) is

\[ s \equiv -\frac{\int d^3p f \ln f}{\int d^3p f} = \ln \left( \frac{T_\perp T_\parallel^{1/2}}{n} \right) + C , \]  

(4.8)
where \( f(p) \) is the phase space distribution and \( C \) is a normalization constant. By differentiating,

\[
ds = \frac{dT_\perp}{T_\perp} + \frac{1}{2} \frac{dT_\parallel}{T_\parallel} - \frac{dn}{n}.
\] (4.9)

The temperature \( T_{\perp,\parallel} \) can be related to the internal energy per particle \( u_{\perp,\parallel} \) via the respective adiabatic index \( \Gamma_{\perp,\parallel} \) as

\[
u_{\perp,\parallel} = \frac{k_B T_{\perp,\parallel}}{\Gamma_{\perp,\parallel} - 1}.
\] (4.10)

For a non-relativistic gas, \( \Gamma_\perp = 2 \) (two degrees of freedom are available in the perpendicular direction), whereas \( \Gamma_\parallel = 3 \) (one degree of freedom). The equation above then becomes

\[
ds = \frac{du_\perp}{T_\perp} + \frac{du_\parallel}{T_\parallel} - \frac{dn}{n}.
\] (4.11)

Using Equations (4.5) and (4.6), we have

\[
ds = dq_{\perp\rightarrow\parallel} \cdot \left[ \frac{1}{T_\parallel} - \frac{1}{T_\perp} \right] - \frac{de_{w,\perp}}{T_\perp} - \frac{de_{w,\parallel}}{T_\parallel},
\] (4.12)

which shows that the entropy of the gas can change as a result of heat flowing internally between the parallel and perpendicular components (first term on the right hand side) or when generating the waves (second term). This can be rewritten in two equivalent forms:

\[
ds = \left[ \frac{1}{2} d\ln \left( \frac{T_\parallel}{(n/B)^2} \right) \right] \cdot \left[ 1 - \frac{T_\parallel}{T_\perp} \right] - \frac{de_{w,\perp}}{T_\perp},
\] (4.13)

\[
ds = - \left[ d\ln \left( \frac{T_\perp}{B} \right) \right] \cdot \left[ \frac{T_\perp}{T_\parallel} - 1 \right] - \frac{de_{w,\parallel}}{T_\parallel}.
\] (4.14)

As anticipated above, the two separate components \( de_{w,\parallel} \) and \( de_{w,\perp} \) of the wave energy per particle do not explicitly enter the entropy equation.
In Equations (4.13) and (4.14), the first term on the right hand side typically dominates. This clearly demonstrates that two ingredients are required for entropy generation: (i) the presence of a temperature anisotropy; and (ii) a mechanism to break the adiabatic invariance. Note that the CGL double adiabatic theory of Chew et al. (1956) predicts that, for adiabatic perturbations, \( T_\perp \propto B \) and \( T_\parallel \propto (n/B)^2 \), which follow from the conservation of the first and second adiabatic invariants. The form of Equations (4.13) and (4.14) is thus easy to understand. In most cases, it is the temperature anisotropy that provides the free energy for generating the waves responsible for breaking the adiabatic invariance.

We conclude with an important remark on the interpretation of the magnetic field \( B \) which appears in the above equations. This should be viewed as a large-scale field, so the particle response to its variation is properly modeled by the CGL approximation. In particular, the field that we have denoted as \( B \) must not include the magnetic contribution of the waves that break the particle adiabatic invariance. In practice, \( B \) will take into account all the magnetic contributions at scales much larger than the particle Larmor radius (for the species in question) and at frequencies much lower than the relevant gyration frequency. It follows that proton-generated waves that break the proton adiabatic invariance can still serve as large-scale \( B \) fields for the electron energy and entropy equations, as we further discuss in the next chapter.

### 4.3 Setup of the Shock Simulations

We perform numerical simulations using the three-dimensional (3D) electromagnetic PIC code TRISTAN-MP (Spitkovsky, 2005), which is a parallel version of the code TRISTAN (Buneman, 1993) that was optimized for studying collisionless shocks. In this section, we
describe the setup of our shock simulations, which parallels closely what we used in Guo et al. (2014a,b).

For shock simulations, we use a 2D simulation box in the $x - y$ plane, with periodic boundary conditions in the $y$ direction. Even though the simulations are two-dimensional in space, all three components of particle velocities and electromagnetic fields are tracked. The shock is set up by reflecting an upstream electron-proton plasma moving along the $-\hat{x}$ direction off a conducting wall at the leftmost boundary of the computational box ($x = 0$). The interaction between the reflected stream and the incoming plasma causes a shock to form, which propagates along $+\hat{x}$. In the simulation frame, the downstream plasma is at rest.

The upstream electron-proton plasma is initialized following the procedure described by Zenitani (2015), as a drifting Maxwell-Jüttner distribution with electron temperature $T_{e0}$ equal to the proton temperature $T_{i0}$ (i.e. $T_{e0} = T_{i0} = T_0$), and bulk velocity $V_0 = -V_0\hat{x}$. This gives a simulation-frame Mach number

$$M_{s,0} = \frac{V_0}{c_s} = \frac{V_0}{\sqrt{2\Gamma k_B T_0/m_i}},$$

(4.15)

where $c_s$ is the sound speed in the upstream, $k_B$ is the Boltzmann constant, $\Gamma = 5/3$ is the adiabatic index for an isotropic non-relativistic gas, and $m_i$ is the proton mass. Below, we will adopt the usual definition of Mach number, as the ratio between the upstream flow velocity and the upstream sound speed in the shock rest frame (rather than in the downstream frame of the simulations, as in Equation (4.15)), where the upstream moves
into the shock with speed $V_1$. We will then parameterize our results with the Mach number

$$M_s = \frac{V_1}{c_s}. \quad (4.16)$$

The shock-frame Mach number $M_s$ is related to the downstream-frame Mach number $M_{s,0}$ via

$$M_s = M_{s,0} \left[1 + \frac{1}{r(M_s) - 1}\right], \quad (4.17)$$

where the density jump $r(M_s)$ across the shock, in the limit of weakly magnetized flows, is equal to

$$r(M_s) = \frac{\Gamma + 1}{\Gamma - 1 + 2/M_s^2}. \quad (4.18)$$

In writing these relations we have assumed an isotropic gas, which is valid upstream of the shock by our initial conditions, and is also valid sufficiently downstream of the shock, as we will see in the discussion that follows.

The incoming plasma carries a uniform magnetic field $B_0$, and its associated motional electric field $E_0 = -\mathbf{V}_0/c \times \mathbf{B}_0$. The magnetic field strength is parametrized by the plasma beta

$$\beta_{p0} = \frac{8\pi n_0 k_B (T_{i0} + T_{e0})}{B_0^2} = \frac{16\pi n_0 k_B T_0}{B_0^2}, \quad (4.19)$$

where $n_{i0} = n_{e0} = n_0$ is the number density of the incoming protons and electrons. Alternatively, one could quantify the magnetic field strength via the Alfvénic Mach number

$$M_A = M_s \sqrt{\Gamma \beta_{p0}/2}. \quad (4.20)$$
The magnetic field is initialized in the simulation plane along the \( y \) direction, i.e., perpendicular to the shock normal. We find that the shock physics is properly captured by 2D simulations only if the field is lying in the simulation plane. A posteriori, this will be motivated by the fact that the plasma instabilities excited in the downstream region have wavevectors preferentially parallel or quasi-parallel to the background magnetic field. We have explicitly verified with an additional simulation having magnetic field initialized along \( z \) (so, perpendicular to both the shock normal and the simulation plane), that the electron heating efficiency is completely suppressed, just as in 1D simulation results (Appendix 5.A). Our choice of an in-plane magnetic field configuration will be justified again in the following sections.

For accuracy and stability, PIC codes have to resolve the plasma oscillation frequency of the electrons

\[
\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e}, \tag{4.21}
\]

and the electron plasma skin depth \( c / \omega_{pe} \), where \( e \) and \( m_e \) are the electron charge and mass, respectively. On the other hand, the shock structure is controlled by the proton Larmor radius

\[
r_{Li} = M_{A,0} \sqrt{m_i / m_e} \frac{c}{\omega_{pe}} \gg \frac{c}{\omega_{pe}}, \tag{4.22}
\]

where the shock-frame Alfvénic Mach number is \( M_{A,0} = M_{s,0} \sqrt{\Gamma \beta_{p0}/2} \). Similarly, the evolution of the shock occurs on a time scale given by the proton Larmor gyration period \( \Omega_{ci}^{-1} = r_{Li} V_0^{-1} \gg \omega_{pe}^{-1} \). The need to resolve the electron scales, and at the same time to capture the shock evolution for many \( \Omega_{ci}^{-1} \), is an enormous computational challenge for the realistic mass ratio \( m_i / m_e = 1836 \). Thus we adopt a reduced mass ratio \( m_i / m_e = 49 \) for our reference run, which is sufficient to properly separate the electron and proton scales. This
allows us to follow the system for long times, until the shock reaches a steady state. We have explicitly verified that the electron heating physics in our shock simulations is nearly the same for higher mass ratios (see Section 5.3, where we test up to \(m_i/m_e = 200\)). In addition, in Section 5.1 and Section 5.2 we demonstrate via analytical arguments and PIC simulations in periodic domains that the electron heating efficiency is nearly independent of \(m_i/m_e\) over the range from \(m_i/m_e = 49\) up to the realistic mass ratio.

As in Guo et al. (2014a,b), the upstream plasma is initialized at a “moving injector”, which recedes from the wall in the \(+\hat{x}\) direction at the speed of light. When the injector approaches the right boundary of the computational domain, we expand the box in the \(+\hat{x}\) direction. This way both memory and computing time are saved, while following at all times the evolution of the upstream regions that are causally connected with the shock. Further numerical optimization can be achieved by allowing the moving injector to periodically jump backward (i.e. in the \(-\hat{x}\) direction), resetting the fields to its right (see Sironi & Spitkovsky 2009). For a perpendicular shock (i.e., with magnetic field perpendicular to the shock direction of propagation), no particles are expected to escape ahead of the shock, so we choose to jump the injector in the \(-\hat{x}\) direction so as to keep a distance of a few proton Larmor radii ahead of the shock. This suffices to properly capture the heating physics of electrons and protons. We have checked, though only for relatively early times, that simulations with and without the jumping injector give identical results.

In the main body of this paper, we present the results from a reference run with \(M_s = 3\) and \(\beta_{p0} = 16\), as motivated by galaxy cluster shocks. The upstream plasma is initialized with \(T_{i0} = T_{e0} = 10^{-2}m_ec^2\) and \(V_0 = 0.05c\). We remark that even though our values for the plasma temperature and bulk speed are motivated by galaxy cluster shocks, the results can be readily applied to other systems (e.g., the solar wind), as long as the dimensionless ratios
$M_s$ and $\beta_{p0}$ are the same and all the velocities remain non-relativistic. We will investigate the dependence of the results on the Mach number and the plasma beta in Chapters 6 and 7.

We employ a spatial resolution of 10 computational cells per electron skin depth $c/\omega_{pe}$, which is sufficient to resolve the Debye length of the upstream electrons for our chosen temperature of $k_B T_{e0} = 10^{-2} m_e c^2$. We have tested that a spatial resolution of 7 cells per electron skin depth can still capture the electron heating physics. We use a time resolution of $dt = 0.045 \omega_{pe}^{-1}$. Each cell is initialized with 32 computational particles (16 per species), but we have tested that a larger number of particles per cell (up to 64 per species) does not change our results (Appendix 5.B). For the reference run, the transverse size of the computational box is $151 c/\omega_{pe}$, corresponding to $\sim 3 r_{Li}$, but we have tested that simulations with a transverse box size up to $256 c/\omega_{pe} \sim 5 r_{Li}$ show essentially the same results.

### 4.4 Shock Structure

In this section, we describe the structure of our reference shock run, with $M_s = 3$, $\beta_{p0} = 16$ and $m_i/m_e = 49$. We first discuss the proton dynamics and the generation of magnetic fluctuations sourced by the proton temperature anisotropy. Then, we present the electron dynamics and focus on the profile of electron irreversible heating. We will identify two main locations where the electron entropy increases: the shock ramp and the downstream region where proton-driven waves grow. The electron heating physics in these two regions will be investigated in the next chapter.
Figure 4.1: Shock structure and proton dynamics at $t = 25.6 \Omega_i^{-1}$. The $x$ coordinate is measured relative to the shock location $x_{sh}$, and it is normalized to the proton Larmor radius $r_{Li}$. From top to bottom, we plot: (a) the $y$-averaged 1D profiles of proton density (black, in units of the upstream value), magnetic field $B_y$ (green, in units of the upstream field $B_0$) and total magnetic field strength $B$ (red, in units of the upstream field $B_0$); (b) the cross-shock electrostatic potential energy $e\Phi$, in units of the proton upstream bulk energy $m_i V_i^2/2$; (c)-(e) the proton phase spaces $f(x - x_{sh}, p_{i,x})$, $f(x - x_{sh}, p_{i,z})$, and $f(x - x_{sh}, p_{i,y})$, where the proton momentum $p_{i,\alpha}$ is in units of $m_i v_{i,th0}$ and the proton thermal velocity is defined as $v_{i,th0} = \sqrt{k_B T_{i0}/m_i}$; (f) the proton temperature perpendicular ($T_{i,\perp}$, blue line) and parallel ($T_{i,\parallel}$, orange line) to the magnetic field, and the mean proton temperature $T_i \equiv (2T_{i,\perp} + T_{i,\parallel})/3$ (green line); (g) the proton anisotropy $T_{i,\perp}/T_{i,\parallel} - 1$ (blue line) and the anisotropy upper bound in Equation (4.24) (red dashed line).
Figure 4.2: 1D and 2D structure of magnetic fluctuations in our reference shock run at $t = 25.6 \Omega_{ci}^{-1}$. In panel (a), we plot the energy of magnetic field fluctuations in the $x$, $z$ and $y$ directions (blue, orange and green lines, respectively) normalized to the magnetic energy of the frozen-in field, which is defined as $B_{\parallel} = B_{\parallel} \hat{y} \equiv B_0 (n/n_0) \hat{y}$. Panels (b)-(d) show the 2D structure of the field fluctuations $\delta B_x = B_x / B_{\parallel}$, $\delta B_z = B_z / B_{\parallel}$ and $\delta B_y = (B_y - B_{\parallel}) / B_{\parallel}$, respectively. The $x$ coordinate is measured relative to the shock location $x_{sh}$, and it is normalized to the proton Larmor radius $r_{Li}$. In panels (b)-(d), the $y$ coordinate is in units of the proton Larmor radius $r_{Li}$. 
4.4.1 Proton Dynamics and Proton-Driven Instabilities

In this subsection, we describe the proton dynamics, with a focus on proton isotropization and thermalization downstream of the shock. Figure 4.1 shows the profiles of various quantities in the shock at time $t = 25.6 \Omega_{ci}^{-1}$, as a function of the $x$ coordinate relative to the shock location $x_{sh}$, in units of the proton Larmor radius $r_{Li}$ defined in Equation (3.6).

Panel (a) shows the $y$-averaged profile of the proton number density $n_i$ in units of the proton density in the upstream $n_{i0}$ (black line). The density compression at the shock reaches $n_i/n_{i0} \sim 3.5$ over a distance of $\sim r_{Li}$, consistent with the expectation that the thickness of a perpendicular shock should be of the order of the proton Larmor radius (Bale et al., 2003; Scholer & Burgess, 2006). The density oscillates on a typical length scale of $\sim r_{Li}$ after the overshoot and then relaxes to the Rankine-Hugoniot value of $\sim 2.8$ beyond a distance of $\sim 5 r_{Li}$ behind the shock.

The density pile-up at the shock is related to the electrostatic potential $\Phi$ that develops in the shock transition region. This phenomenon has been well studied via hybrid simulations of collisionless shocks (e.g. Leroy et al., 1981, 1982; Leroy, 1983). As shown in Figure 4.1(b), the potential energy $e\Phi$ reaches $\sim 60\%$ of the incoming proton energy $m_i V_1^2/2$. As a result, a significant fraction of the incoming protons are reflected back toward the upstream, leading to a pile-up of particles just in front of the shock. The reflected protons can be identified as the ones with positive $p_{i,x}$ and $p_{i,z}$ ahead of the shock in the phase spaces of Figure 4.1(c) and (d), respectively. As the reflected protons gyrate in the shock-compressed magnetic field, they gain energy from the upstream motional electric field. Upon their second encounter with the cross-shock potential, the reflected protons now have sufficient energy to penetrate the shock. In the downstream region just behind the shock, the protons keep gyrating...
in the $xz$ plane perpendicular to the shock-compressed magnetic field (compare the phase spaces in Figure 4.1(c) and (d), at $-4 \lesssim (x-x_{sh})/r_{Li} \lesssim 0$). The peaks in density seen in Figure 4.1(a) are then correlated with the locations where the proton gyro-phase is such that most protons have small $p_{i,x}$ (e.g., at $x-x_{sh} \sim -0.25 r_{Li}, -1.25 r_{Li}$ and $-2.75 r_{Li}$). The amplitude of the density oscillations gets smaller as the gyrating reflected protons become more and more phase-mixed with the directly transmitted protons, at $x-x_{sh} \lesssim -5 r_{Li}$.

Since the post-shock protons gyrate in the $xz$ plane perpendicular to the field, the momentum dispersion along the $y$ direction of the field is expected to be nearly the same on the two sides of the shock (see the $p_{i,y}$ phase space in Figure 4.1(e) near the shock). Further behind the shock, the dispersion in $p_{i,y}$ increases. This can be also quantified with the $y$-averaged profiles of the proton temperature perpendicular ($T_{i,\perp}$) and parallel ($T_{i,\parallel}$) to the background magnetic field, as in Figure 4.1(f). Here, the $jk$ component of the temperature tensor is defined as $k_{B} T_{jk}/m_{i}c^{2} \equiv \langle \gamma' v_{j}' v_{k}' \rangle/c^{2}$, where $v_{j}', v_{k}'$ are the particle velocities in the fluid comoving frame, $\gamma'$ is the comoving particle Lorentz factor, and the average is performed over the particle distribution at a given spatial location. In the case of our shock simulation where the magnetic field is along the $y$-direction, $T_{i,\perp} \equiv (T_{xx}+T_{zz})/2$ and $T_{i,\parallel} \equiv T_{yy}$. As Figure 4.1(f) shows, the mean proton temperature $T_{i}$, defined as

$$T_{i} = \frac{2 T_{i,\perp} + T_{i,\parallel}}{3}, \quad (4.23)$$

is nearly uniform in the downstream region (green line), but the parallel temperature (orange line) — which is continuous across the shock — increases with distance behind the shock, while the perpendicular temperature (blue line) shoots up at the shock and then experiences

\footnote{The factor of two that multiplies $T_{i,\perp}$ in the definition of $T_{i}$ comes from the fact that the perpendicular motion has two degrees of freedom.}
a modest decline. This is the same trend shown by the phase spaces in Figure 4.1(c)-(e).

The decrease in perpendicular temperature, and the resulting increase in parallel temperature, suggests that protons are being scattered in pitch angle. In fact, in the region $-4 \lesssim (x - x_{sh})/r_{Li} \lesssim -1$ where the variation in $T_{i,\perp}$ and $T_{i,\parallel}$ is most pronounced, strong magnetic waves are observed in Figure 4.2. Their wavelength is comparable to the proton skin depth, indicating that they are driven by protons (as opposed to electrons). In Figure 4.2(a), we compare the 1D profiles (averaged over the $y$ direction) of the magnetic fluctuations $\delta B^2_z$, $\delta B^2_y$ and $\delta B^2_z$, normalized to $B_{ff}^2$, where $B_{ff}$ is defined as the magnitude of the flux-frozen magnetic field (i.e., $B_{ff} \equiv B_0 (n/n_0) \hat{y}$, where $n$ is the $y$-averaged particle density). The energy of proton-driven waves peaks at $x - x_{sh} \sim -2.5 r_{Li}$. In Figure 4.1(a), they are responsible for the excess of magnetic field strength (red curve) above the flux-freezing prediction (which would correspond to the density profile, in black).

The dominant mode at $-4 \lesssim (x - x_{sh})/r_{Li} \lesssim -1$ in the $x$ and $z$ direction has a wavevector nearly parallel to the background field (Figure 4.2(b) and (c)), consistent with the proton cyclotron instability (Kennel, 1966; Davidson & Ogden, 1975). The waves in $\delta B_y$ are slightly weaker (compare the green line with the blue and orange curves in Figure 4.2(a)) and have oblique wavevectors (Figure 4.2(d)), as expected for the mirror mode (Chandrasekhar et al., 1958; Barnes, 1966; Hasegawa, 1975; McKean et al., 1993). The presence of mirror modes breaks the flux freezing condition, as shown by the fact that in Figure 4.1(a) the $y$-averaged transverse magnetic field profile $B_y/B_0$ deviates at $-5 \lesssim (x - x_{sh})/r_{Li} \lesssim -2$ from the density profile (in black, which tracks the flux freezing prediction).

Both the proton cyclotron instability and the mirror instability are sourced by proton temperature anisotropy. In fact, since the motion of downstream protons right behind

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3The frozen-in magnetic field is also used in the definition of $\delta B_y \equiv B_y - B_{ff}$. 

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the shock is mostly confined in the $xz$ plane, a large temperature anisotropy arises, with $T_{i,\perp} \gg T_{i,\parallel}$ (Figure 4.1(g)). The anisotropy provides free energy for the growth of proton cyclotron waves and mirror modes, which scatter the protons in pitch angle and reduce their anisotropy back to the upper bound corresponding to marginal stability Gary et al. (1997) (see the red dashed line in Figure 4.1(g)), which is

$$\frac{T_{i,\perp}}{T_{i,\parallel}} - 1 \simeq \frac{1.1}{\beta_{i,\parallel}^{0.55}}.$$  

(4.24)

Here, $\beta_{i,\parallel} = 8\pi n_i k_B T_{i,\parallel}/B^2$ is the local value of the proton plasma beta, computed with the parallel proton temperature.

4.4.2 Electron Dynamics and Heating

In this subsection, we describe the electron dynamics, with a focus on electron heating in the shock layer and in the downstream region. Due to their opposite charge and much smaller Larmor radius, the dynamics of electrons is drastically different from that of the protons.

Figure 4.3(a) shows the electron density profile (black line), which strongly resembles that of the protons (black line in Figure 4.1(a)), and thus ensures approximate charge neutrality. While a small degree of charge separation at the shock is responsible for establishing the electric potential $\Phi$ shown in Figure 4.1(b), the fact that $\Phi$ is nearly uniform at $x - x_{sh} \lesssim -5 r_{Li}$ suggests that charge neutrality is satisfied very well in the far downstream.

Figure 4.3(b)-(d) shows the electron phase space. Since electrons have opposite charge than protons, they are not reflected back upstream by the cross-shock potential. In fact, unlike for protons, there is no reflected electron population with $p_{e,x} \gtrsim 0$ just ahead of the
Figure 4.3: Shock structure and electron dynamics at $t = 25.6 \Omega_{ci}^{-1}$. From top to bottom, we plot: (a) the $y$-averaged profiles of electron density (black) and total magnetic field strength $B$ (red); (b)-(d) the electron phase spaces $f(x - x_{sh}, p_{e;x})$, $f(x - x_{sh}, p_{e;z})$, and $f(x - x_{sh}, p_{e;y})$, where the electron momentum $p_{e;\alpha}$ is in units of $m_e v_{e;th0}$ and the electron thermal velocity is $v_{e;th0} = \sqrt{k_B T_e/\mu_e}$; (e) the electron temperature perpendicular ($T_{e;\perp}$, blue) and parallel ($T_{e;\parallel}$, orange) to the magnetic field, and the mean electron temperature $T_e \equiv (2T_{e;\perp} + T_{e;\parallel})/3$ (green); (f) the electron anisotropy $T_{e;\perp}/T_{e;\parallel} - 1$; (g) the excess electron temperature $T_e$ over the adiabatic expectation $T_{e;ad} = (n_e/n_{e0})^{2/3} T_{e0}$ for an isotropic gas; (h) the electron entropy profile, measured as in Equation (4.27).
shock (compare Figure 4.3(b) with Figure 4.1(c)).

Figure 4.3(e) shows the temperature profile of electrons, for the perpendicular component $T_{e,\perp}$ (blue), the parallel component $T_{e,\parallel}$ (orange) and the mean temperature $T_e$ (green), which is defined as

$$T_e = \frac{2T_{e,\perp} + T_{e,\parallel}}{3}. \quad (4.25)$$

The profile of perpendicular temperature (blue line) follows closely the density compression (compare with the black line in Figure 4.3(a)) and starts to rise just ahead of the shock at $x - x_{sh} \sim 0.5r_{Li}$. This is consistent with the double adiabatic theory Chew et al. (1956), which predicts $T_{\perp} \propto B$ (and in flux freezing, $B \propto n$). The double adiabatic theory applies to electrons, since the density and magnetic field compression occurs on scales much larger than the electron Larmor radius. This is not true for protons, since the shock thickness and the scale length of the downstream oscillations seen in Figure 4.1(a) are set by the proton Larmor radius $r_{Li}$.

The parallel electron temperature (orange line in Figure 4.3(e)) initially remains unchanged, as the CGL theory predicts $T_{\parallel} \propto (n/B)^2$ and $B \propto n$ as a result of flux freezing (compare the green and black lines in Figure 4.1(a) in the vicinity of the shock). In the region between $x - x_{sh} \sim 0.5r_{Li}$ and 0, the perpendicular temperature of the electrons increases as a result of compression, while the parallel temperature stays the same as in the upstream. As a result, a strong electron anisotropy develops, growing up to $T_{e,\perp}/T_{e,\parallel} - 1 \sim 0.6$ at $x - x_{sh} \sim 0$ (Figure 4.3(f)). This excites the electron whistler instability, which creates the small-wavelength transverse magnetic waves in $\delta B_x$ and $\delta B_z$ seen in

\footnote{We remark, as we have already pointed out at the end of Section 4.2, that the field strength $B$ should include all the magnetic contributions at scales much larger than the electron Larmor radius and at frequencies much lower than the electron gyration frequency.}
the region \( x - x_{\text{sh}} \sim -0.25 r_{\text{Li}} \) of Figure 4.2(b) and (c) (see also the magnetic energy in \( \delta B_x^2 \) and \( \delta B_z^2 \) in Figure 4.2(a), at the same location). The electron whistler instability provides a mechanism for electron pitch angle scattering and thus reduces the electron temperature anisotropy, as shown in the downstream region of Figure 4.3(f).

As we have already discussed, if the electron fluid were to follow the double adiabatic predictions, we expect \( T_{e,\perp} \propto n \) and \( T_{e,\parallel} \propto \text{const} \). The fact that the perpendicular temperature profile in Figure 4.3 (f) (blue line) resembles the density profile (black line in Figure 4.3(a)), and the fact that \( T_e \sim T_{e,\perp} \) (compare green and blue curves in Figure 4.3(e)), suggests that most of the increase in electron temperature comes from adiabatic compression. However, the fact that \( T_{e,\parallel} \) is not constant across the shock implies that there must be some non-adiabatic processes as well. In order to quantify the degree of non-adiabatic, or “irreversible,” electron heating, we compare in Figure 4.3(g) the mean electron temperature \( T_e \) with the adiabatic prediction

\[
\frac{T_{e,\text{ad}}}{T_{e0}} = \left( \frac{n_e}{n_{e0}} \right)^{2/3}.
\]

(4.26)

This estimate of the adiabatic temperature assumes an isotropic gas, which is valid, given the small degree of electron anisotropy far downstream of the shock (see Figure 4.3(f) at \( x - x_{\text{sh}} \lesssim -1 r_{\text{Li}} \)). Figure 4.3(g) shows the excess of \( T_e \) above \( T_{e,\text{ad}} \) in units of the upstream electron temperature. Most of the irreversible heating occurs at two locations: \( x - x_{\text{sh}} \sim 0 \), i.e., in the shock transition region; and \( x - x_{\text{sh}} \sim -2.5 r_{\text{Li}} \), where the density suffers another compression, and strong proton-driven waves are generated (see Figure 4.2). These two locations are marked by the vertical dotted lines in Figure 4.2 and Figure 4.3, and the particle and wave properties there will be further studied below. In the far downstream, the temperature excess over the adiabatic estimate saturates at \( T_e - T_{e,\text{ad}} \sim 0.3 T_{e0} \) (Fig-
An alternative (and possibly, more rigorous) estimate of the degree of irreversible electron heating is given by the specific entropy $s_e$ (i.e., the entropy per particle), measured with the electron distribution function $f_e$ as

$$s_e \equiv -\frac{\int d^3p f_e \ln f_e}{\int d^3p f_e},$$

where the normalization is such that $\int d^3p f_e = n_e/n_{e0}$. To construct the spatial profile of $s_e(x)$, we first bin the particles by their $x$ position with a width of $\Delta x = 100$ cells. In each spatial bin, we compute $f_e(p)$ by constructing a three-dimensional histogram of the particle momenta. In each direction (i.e., $p_{e,x}$, $p_{e,y}$ and $p_{e,z}$), the central bin of the histogram lies at the mean momentum, and the histogram spans four standard deviations above and below the mean. Each standard deviation is resolved with 10 momentum bins.

Figure 4.3(h) shows the change of electron entropy with respect to the upstream value. In analogy to Figure 4.3(g), the increase in electron entropy is localized around $x - x_{sh} \sim 0$ and $x - x_{sh} \sim -2.5 r_{Li}$ (indicated by the gray and pink vertical dotted lines, respectively). The increase in electron specific entropy saturates at $\Delta s_e \sim 0.28$ in the far downstream.

### 4.4.2.1 Electron Whistler Waves

The physics of particle irreversible heating that we have described in Section 4.2 relies on two ingredients: a certain level of particle anisotropy, and a mechanism to break the adiabatic invariance. As we have shown above, a large-scale magnetic field amplification (e.g., resulting from shock compression of the upstream field) will lead to electron anisotropy with $T_{e,\perp} > T_{e,\|}$. In turn, this triggers the growth of whistler waves, which scatter the electrons
Figure 4.4: Space-time diagrams and power spectra at a distance of \( x - x_{sh} = 4 \frac{c}{\omega_{pe}} \) ahead of the shock (as indicated by the vertical dotted grey lines in Figure 4.2 and Figure 4.3), during the time interval \( 24.0 \leq \Omega_{ce} t \leq 27.4 \). For this plot, the unit of time is the electron cyclotron time \( \Omega_{ce}^{-1} \) (the corresponding unit of frequency is \( \Omega_{ce} \)) and the unit of distance along \( y \) is the electron skin depth \( \frac{c}{\omega_{pe}} \) (the corresponding unit for the wavevector \( k_y \) is \( \omega_{pe} / c \)). Panels (a)-(d) are the space-time diagrams of: (a) total magnetic field strength \( |B| \), in units of the upstream field \( B_0 \); (b)-(c) transverse magnetic field fluctuations \( \delta B_x / B_0 \) and \( \delta B_z / B_0 \); (d) electron anisotropy \( T_{e,\perp} / T_{e,\parallel} - 1 \). Panels (e) and (f) show the \((\omega, k_y)\) power spectra of the field fluctuations presented in panels (b) and (c), respectively. The solid black line is the predicted real part of the frequency of electron whistler modes, whereas the dashed black line is the predicted imaginary part (i.e., the growth rate). The agreement between the prediction and our measurement confirms that the fluctuations in panels (a)-(c) are produced by whistler waves.
Figure 4.5: Space-time diagrams and power spectra at $x - x_{\text{sh}} = -122 \, c/\omega_{pe} \sim -2.5 \, r_{Li}$ behind the shock (as indicated by the vertical dotted pink lines in Figure 4.2 and Figure 4.3), during the same time interval $24.0 \leq \Omega_{ci} t \leq 27.4$ as in Figure 4.4. Panels (a)-(f) are similar to the equivalent panels in Figure 4.4, the only difference being that the predictions in panels (e) and (f) here (solid black line for the whistler wave frequency, dashed black line for the growth rate) are computed considering the plasma properties only in regions where the electron anisotropy is well above the whistler threshold, more specifically $T_{e,\perp}/T_{e,\parallel} - 1 - 0.21/\beta_{e,\perp}^0 \geq 0.3$. In panel (g), where the quantity $T_{e,\perp}/T_{e,\parallel} - 1 - 0.21/\beta_{e,\perp}^0$ is shown, this would correspond to the dark green areas. Since panels (a)-(c) are dominated by long-wavelength slowly-propagating proton modes, we isolate electron waves via a high-pass filter in the power spectra of panels (e) and (f), keeping only the high-$\omega$ high-$k_y$ region delimited by the red dashed lines. The resulting space-time wave pattern is shown in panels (h) and (i), which reveal the presence of electron whistler waves.
in pitch angle, providing a mechanism to break the adiabatic invariance and generate irreversible heat. Below, we show that the two ingredients needed for entropy increase are indeed present in the two locations where the entropy profile shows the fastest increase (vertical dotted lines in Figure 4.2 and Figure 4.3).

At the shock (grey dotted line in Figure 4.2 and Figure 4.3), the electron temperatures are driven to $T_{e,\perp} > T_{e,\parallel}$ by shock-compression of the upstream field, via conservation of the first and second adiabatic invariants. In Figure 4.4, we show the space-time diagram of various quantities, in the time interval $20.0 \leq \Omega_{ci} t \leq 27.4$ and along the $y$ extent of the box. The $x$ location is fixed at the shock ramp (more precisely, $x - x_{sh} = 4 c/\omega_{pe}$). Shock-compression of the upstream field (see Figure 4.4(a), where the mean $|B|/B_0 \sim 2.2$) leads to a temperature anisotropy $T_{e,\perp}/T_{e,\parallel} - 1 \simeq 0.6$ (Figure 4.4(d)).

As a result of the strong temperature anisotropy, magnetic waves are excited throughout the $y$ range consistently over time. Panels (b) and (c) show the space-time diagrams of the magnetic fluctuations $\delta B_x$ and $\delta B_z$, revealing the presence of high-frequency and short-wavelength modes (as also seen in Figure 4.2(b) and (c) near the shock). Figure 4.4(e) and (f) show the corresponding power spectra, i.e. the modulus square of the 2D Fourier transforms of panels (b) and (c), as a function of frequency $\omega$ (horizontal axis) and wavenumber $k_y$ (vertical axis). The power spectrum displays a pronounced peak at frequency $\omega \simeq 0.5 \Omega_{ce}$ (here $\Omega_{ce} = (m_i/m_e)\Omega_{ci}$ is the electron gyrofrequency) and wavevector $k_y \simeq 0.5 \omega_{pe}/c$. We have compared this with linear theory of the electron whistler instability (e.g. Gary & Madland, 1985; Gary & Wang, 1996; Gary & Karimabadi, 2006) by solving the dispersion
relation

\[ 0 = D^\pm(k_y, \Omega) \]

\[ = \Omega^2 - c^2k_y^2 + \frac{\Omega^2}{k_z v_i} Z(\zeta_e^\pm) \]

\[ + \frac{\Omega}{k_z v_e^\parallel} Z(\zeta_e^\pm) \]

\[ + \omega_{pe} \left( \frac{T_{e\perp}}{T_{e\parallel}} - 1 \right) \left[ 1 + \zeta_e^\pm Z(\zeta_e^\pm) \right] , \] \hspace{1cm} (4.28)

where \( \zeta_e^\pm = (\Omega \pm \omega_{ce})/k_y v_e^\parallel, \) \( v_e^\parallel = (2k_B T_{e\parallel}/m_e)^{1/2}, \) \( \zeta_i^\pm = (\Omega \pm \omega_{ci})/k_y v_i, \)

\( v_i = (2k_B T_i/m_i)^{1/2}, \) and \( Z(\zeta) \) is the plasma dispersion function

\[ Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{\exp(-x^2)}{x - \zeta}. \] \hspace{1cm} (4.29)

The input values of \( v_i, v_e^\parallel, T_{e\perp}/T_{e\parallel} - 1 \) for the dispersion relation are taken from the time- and space-averages of the corresponding quantities over the same time period and spatial extent as the space-time diagram in Figure 4.4. The resulting theoretical prediction for the real part of the frequency is shown with a black solid line in panels (e) and (f), and it matches extremely well the contours of the power spectrum. The imaginary part of the frequency, i.e., the growth rate of the mode, is plotted with a dashed black curve. The value of \( k_y \) giving the fastest growth agrees well with the location of the peak of the power spectrum \( (k_y \simeq 0.5 \omega_{pe}/c) \). The excellent agreement between the simulation data and the electron whistler dispersion relation confirms that the waves in the shock ramp are produced by the electron whistler instability.

Figure 4.5 shows similar plots at the location indicated in Figure 4.2 and Figure 4.3 with a vertical dotted pink line: \( x - x_{sh} \simeq -2.5 r_{Li} \simeq -122 c/\omega_{pe} \). Here, field amplification is driven by a combination of two effects: the density (and so, the frozen-in magnetic field)
experiences another large-scale compression; in addition, the proton-driven waves shown in Figure 4.2 further increase the local magnetic field intensity.

As compared to Figure 4.4, the space-time diagrams show now a higher degree of inhomogeneity, imprinted by the anisotropy-driven long-wavelength proton modes. These fluctuations co-exist with weaker small-wavelength high-frequency modes, which only appear in localized patches in Figure 4.5(b) and (c). The high-frequency waves are generated in regions where field amplification (see Figure 4.5(a) at \( y \sim 80 c/\omega_{pe} \) and \( t \sim 100 \Omega_{ce}^{-1} \)) causes the electron anisotropy (Figure 4.5(d)) to exceed the threshold for whistler growth (Figure 4.5(g)), which is given by

\[
\frac{T_{e, \perp}}{T_{e, \parallel}} - 1 \approx \frac{0.21}{\beta_{e, \parallel}^{0.6}},
\]

where \( \beta_{e, \parallel} = 8\pi n_e k_B T_{e, \parallel}/B^2 \) is the local value of the parallel electron beta (Gary, 2005).

Figure 4.5(e) and (f) show the power spectra of \( \delta B_x \) and \( \delta B_z \). Most of the power is concentrated in low-frequency long-wavelength modes, generated by the proton cyclotron or mirror instabilities. However, there is still an appreciable amount of power in high-frequency short-wavelength modes peaking at \( \omega \sim 0.7 \Omega_{ce} \) and \( k_y \sim 0.7 \omega_{pe}/c \). We apply a high-frequency short-wavelength filter, in order to isolate the top right region in Figure 4.5(e) and (f) (the cutoff frequency and wavenumber of our filter are shown with dashed red lines). This allows us to extract (via an inverse Fourier transform) the space-time wave patterns of high-frequency short-wavelength modes, which are shown in panels (h) and (i). The two panels confirm that short-wavelength modes exist only in regions where the electron temperature anisotropy exceeds the electron whistler threshold (compare with the dark green regions in Figure 4.5(g)). We have measured the average electron and proton temperatures and
densities in the region where the whistler threshold is appreciably exceeded $(T_{e,\perp}/T_{e,\parallel} - 1 - 0.21/\beta_{e,\parallel}^0 \geq 0.3)$, in order to obtain linear theory predictions. The real part and imaginary part of the resulting dispersion relation are plotted in Figure 4.5(e) and (f) with solid and dashed black lines, respectively. The good agreement with the power spectra extracted from our shock simulation confirms the presence of patches of whistler waves in the second ramp (at $x - x_{sh} \sim -2.5 r_{Li}$) of the electron entropy profile.

To summarize, we have identified two major sites of electron entropy production in the shock downstream. One is at the shock ramp, and the other is at a distance of $\sim 2.5 r_{Li}$ behind the shock, where density compression and proton-driven waves both contribute to magnetic field amplification. Both sites show the presence of electron whistler waves triggered by electron temperature anisotropy. Whistler waves provide the pitch-angle scattering required to break electron adiabatic invariance and to generate entropy. In the next chapter, we further elucidate the physics of entropy production in these two sites, by means of periodic box simulations.
In the previous chapter, we formulated the general theory for electron heating. Two basic ingredients are needed for electron irreversible heating: (i) the presence of a temperature anisotropy, induced by field amplification coupled to adiabatic invariance; and (ii) a mechanism to break the adiabatic invariance itself. We then used a reference shock run with sonic Mach number $M_s = 3$ and plasma beta $\beta_{p0} = 16$ to show that efficient electron entropy production occurs at two major sites: at the shock ramp, where density compression coupled to flux freezing leads to field amplification and a high degree of electron anisotropy; and farther downstream, where density compression and long-wavelength magnetic waves induced by the proton temperature anisotropy both contribute to magnetic field growth. Regardless of the origin of field amplification, electrons are driven to a large degree of temperature anisotropy, exceeding the threshold of the electron whistler instability. The resulting growth of electron whistler waves — whose presence is one of the common denom-
inators of the two sites mentioned above — causes the violation of the electron adiabatic invariance, and allows for efficient entropy production.

In this chapter, we study in more detail the two scenarios that can lead to field amplification that naturally take place in the downstream of the shocks by using controlled experiments. The scenarios are motivated by the properties of locations where most of the entropy is produced in the shock downstream shown in the last chapter. The first scenario is uniform density compression, where background $B$ changes at the same rate as $n$ by flux-freezing condition. This closely mimics the first ramp, where the ion waves have not grown yet. The second scenario is uniform background density throughout time, but the change of $B$ and $n$ are induced by ion temperature anisotropy instability. This isolates the density oscillations in the downstream of the shock, but allows us to focus on the net effect of ion waves on electron heating. The key advantage of controlled experiments is that we can employ a much larger mass ratio to assess how the heating mechanism scales with mass ratio. In addition, we can afford much larger number of particles per cell to minimize numerical heating effects. We use the controlled experiments to explicitly show that the heating mechanism depends very weakly on the mass ratio, once there is reasonable scale separation between the electrons and protons. We then return to the reference shock simulation and show that there is excellent agreement between the theory and measurement. Finally, we demonstrate the weak dependence of the shock simulation on the mass ratio, which follows from the weak dependence found in the controlled experiments.
5.1 Electron Heating in the Shock Ramp

The first increase in electron entropy happens in the shock ramp. As a result of the shock-compression of the upstream field \( B \propto n \) by flux freezing, electrons become anisotropic and they trigger whistler waves. Below, we model the shock compression in a periodic box using a novel form of the PIC equations introduced in Sironi & Narayan (2015) and Sironi (2015), which incorporates the effect of a large-scale compression of the system. We briefly describe the simulation setup in Section 5.1.1, we discuss periodic box simulations applicable to our reference shock run in Section 5.1.2, and we describe the dependence on mass ratio in Section 5.1.3.

5.1.1 Simulation Setup

To emulate the conditions for electrons in the shock ramp, we set up a suite of compressing box experiments, using the method introduced in Sironi & Narayan (2015) and Sironi (2015). Here, we report only its main properties. We solve Maxwell’s equations and the Lorentz force in the fluid comoving frame, which is related to the laboratory frame by a Lorentz boost. In the comoving frame, we define two sets of spatial coordinates, with the same time coordinate. The unprimed coordinate system has a basis of unit vectors, so it is the appropriate coordinate set to measure all physical quantities. Yet, it is convenient to redefine the unit length of the spatial axes in the comoving frame such that a particle subject only to compression stays at fixed coordinates. This will be our primed coordinate system.
Figure 5.1: As a function of the comoving time of the electron fluid defined in Equation (5.2), we present the density profile experienced by electrons as they propagate from upstream to downstream (solid blue line). The time axis is shifted such that $\tau = 0$ just ahead of the shock. The shock-compression felt by incoming electrons can be approximated as $n_e/n_{e0} = (1 + 2.5\tau)$ with compression rate $q = 2.5 \Omega_{ci}$ (orange dashed line).
Then, compression with rate $q$ is accounted for by the diagonal matrix

$$L = \frac{\partial x}{\partial x'} = \begin{pmatrix}
(1 + qt)^{-1} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (5.1)$$

which has been tailored for compression along the $x$ axis, as expected in our shock.

A uniform ordered magnetic field $B_0$ is initialized along the $y$ direction (in analogy to the shock setup). We define $\Omega_{ci}$ as the proton Larmor frequency in the initial field $B_0$. Maxwell’s equations in the primed coordinate system (Sironi & Narayan, 2015) prescribe that the field grows in time as $B_0(1 + qt)$, which is consistent with flux freezing (the particle density in the box increases at the same rate). From the Lorentz force in the compressing box (Sironi & Narayan, 2015), the component of particle momentum aligned with the field does not change during compression, whereas the perpendicular momentum increases as $\propto \sqrt{1 + qt}$. This is consistent with the conservation of the first and second adiabatic invariants.

This method is implemented for 1D, 2D and 3D computational domains, with periodic boundary conditions in all directions. In the previous chapter, we have shown that the whistler instability is the dominant mode in the shock ramp. Its wavevector is nearly aligned with the field direction (i.e., along $\hat{y}$). It follows that the evolution of the dominant mode can be conveniently captured by means of 1D simulations with the computational box oriented along $y$, which we employ in this section. Yet, all three components of electromagnetic fields and particle velocities are tracked. In 1D simulations, we can employ a large number of particles per cell (typically, $10^4$ per cell) so we have adequate statistics for the calculation of the electron specific entropy from the phase space distribution function. In addition, in 1D
simulations we can readily extend our results up to the realistic mass ratio. Even though we only show results from 1D runs, we have checked that the main conclusions hold in 2D.

As a result of the large-scale compression encoded in Equation (5.1), both electrons and protons will develop a temperature anisotropy, and we should witness the development of both electron and proton anisotropy-driven modes. However, in our reference shock, no proton modes grow in the shock ramp (they only develop a few Larmor radii behind the shock). For this reason, in our compressing box runs, we artificially inhibit the update of the proton momentum. Effectively, this corresponds to the case of infinitely massive protons, which only serve as a charge-neutralizing fluid. The mass ratio in this section then only enters through the time scale, i.e. the compression rate is in units of $\Omega_{ci}$, which is a factor of $m_i/m_e$ smaller than $\Omega_{ce}$.

The compression rate $q$ is measured directly from our reference shock simulation. There, we can quantify the profile of electron density as a function of the co-moving time of the electron fluid, which is estimated via

$$
\tau \equiv \int \frac{dx'}{V_{xe}(x')},
$$

where $V_{xe}$ is the electron fluid velocity in the shock frame, and the integral goes from the upstream to the downstream region. Figure 5.1 shows the density profile as a function of $\tau$ from our reference run. The density oscillates on a timescale comparable to the proton gyration time $\Omega_{ci}^{-1}$, which is expected given that the proton dynamics controls the shock structure. At the ramp starting near $\tau = 0$, the electron density increases by a factor of $\sim 3.5$ within $\sim 1\Omega_{ci}^{-1}$. Even though the density increase is not perfectly linear, we find that a linear approximation with $q = 2.5\Omega_{ci}$ provides a reasonable fit (see the orange dashed
line in Figure 5.1). Note that our electron heating model (4.2) is agnostic with regard to
the exact profile of density compression, as long as the compression rate and the resulting
field amplification rate are much slower than the electron gyration frequency.

Below, we fix $q = 2.5 \Omega_{ci}$, and we run the simulations from time $t = 0$ to $t = 1\Omega_{ci}^{-1}$.
With increasing mass ratio, the separation between $q$ and the electron cyclotron frequency
$\Omega_{ce}$ will increase as $m_i/m_e$. As in our reference shock run, electrons are initialized to have
$k_B T_e 0 = 10^{-2} m_e c^2$, and the strength of the background magnetic field is set so that $\beta_{p0} = 16$.
We resolve the electron skin depth with 10 cells, so the Debye length is marginally resolved.
The box extent along the $y$ direction is fixed at $43 c/\omega_{pe}$, which is sufficient to capture
several wavelengths of the electron whistler instability. The box size does not need to scale
with the mass ratio, since we are artificially excluding the proton physics.

### 5.1.2 Application to the Reference Shock

As in our reference shock run, we employ a reduced mass ratio $m_i/m_e = 49$. In the periodic
compressing box, this means that our choice of $q = 2.5 \Omega_{ci}$ leads to a compression rate that
is a factor of $\sim 20$ lower than the electron gyration frequency.

To highlight the importance of the electron whistler instability in facilitating electron
entropy production, in Figure 5.2 we compare two simulations, one with the background
field $B_0$ in the $z$ direction, the other one with $B_0$ along the $y$ direction. Since our simulation
box is oriented along $y$ and the dominant wavevector of the electron whistler instability is
parallel to the background field, if the field lies along $z$ (which we call the “out-of-plane”
case, and indicate with dotted lines) we artificially suppress the growth of electron whistlers.
By comparing it with the “in-plane” simulation where the field is along $y$ (solid lines) and
whistler wave growth is allowed, we can demonstrate the importance of the electron whistler
Figure 5.2: Time evolution of various space-averaged quantities in a 1D periodic box whose compression rate $q = 2.5 \Omega_{ci}$ is chosen to mimic the effect of the shock ramp in the reference shock. We compare two field geometries, with background field lying either along the $y$ axis of the simulation box (“in-plane” configuration, solid lines) or along the $z$ direction perpendicular to the box (“out-of-plane” configuration, dotted lines): (a) energy in magnetic field fluctuations, normalized to the energy of the compressed magnetic field (the legend is appropriate for the in-plane configuration, whereas for the out-of-plane case the orange line refers to $\delta B_y^2$); (b) electron temperature perpendicular ($T_{e, \perp}$, blue lines) and parallel ($T_{e, \parallel}$, orange lines) to the background field; (c) electron temperature anisotropy (blue lines), and comparison with the threshold of the electron whistler instability, as in Equation (4.30) (dashed red line); (d) electron entropy change, measured from the electron distribution function as in Equation (4.27) (blue lines) or predicted from Equation (5.4) (red dashed); (e) electron energy increase in units of $k_B T_{e0}$, measured directly (blue solid) or predicted using Equation (5.3) (red dashed).
instability for entropy production.

Because of the absence of any electron-scale instabilities that could break the adiabatic invariance, the out-of-plane simulation is expected to follow adiabatic scalings. In fact, in the out-of-plane simulation, we see that $T_{e,\perp} \propto B \propto (1 + qt)$ (blue dotted line in Figure 5.2(b)), while $T_{e,\parallel} \propto (n/B)^2 \propto \text{const}$ (orange dotted line in Figure 5.2(b)), as expected from the double adiabatic theory. Since no whistler waves grow (notice that the fields stay at the noise level, see the dotted lines of Figure 5.2(a)), no mechanism exists that can transfer heat from the perpendicular to the parallel temperature. Therefore, the electron entropy remains constant (dotted line in Figure 5.2(d)).

The in-plane simulation shows a completely different behavior. Initially, $T_{e,\perp}$ and $T_{e,\parallel}$ follow the double adiabatic trends (solid lines in Figure 5.2(b)). At $\Omega_{ci} t \sim 0.3$, the increasing temperature anisotropy (blue solid line in panel (c)) leads to the exponential growth of electron whistler waves (solid lines in Figure 5.2(a)). At $\Omega_{ci} t \sim 0.4$, the wave energy is strong enough to pitch-angle scatter the electrons. As a result, heat is transferred from the perpendicular to the parallel direction. Both $T_{e,\perp}$ and $T_{e,\parallel}$ deviate from the adiabatic scalings and the temperature anisotropy is reduced.

At $t \sim 0.4 \Omega_{ci}^{-1}$, with the onset of pitch-angle scattering and the consequent breaking of adiabatic invariance, the electron specific entropy starts to rise (solid blue line in Figure 5.2(d)). The most rapid entropy increase happens near the end of the exponential whistler growth, at $t \sim 0.4 - 0.5 \Omega_{ci}^{-1}$. Here, the electron anisotropy is still large, and at the same time whistler waves are sufficiently powerful to provide effective pitch-angle scattering. In other words, both terms in the square brackets of either Equation (4.13) or Equation (4.14) are large. After the exponential growth, the electron whistler waves enter a secular phase where the wave energy (normalized to the compressed background field energy) stays almost
constant (solid green line in Figure 5.2(a)). In this phase, whistler waves are continuously generated as the large-scale compression steadily pushes the electron anisotropy slightly above the threshold of marginal stability (indicated by the red dashed line in Figure 5.2(c)). Both the ingredients needed for entropy increase (i.e., nonzero electron anisotropy and efficient pitch angle scattering mediated by whistler waves) persist during the secular phase, leading to further increase in the electron entropy, though at a more moderate rate compared to the exponential phase.

In Figure 5.2, we also explicitly validate the heating model described in Section 4.2. Following Equation (4.7), we expect the electron energy per particle to change as

\[
\frac{du_e}{dt} = k_B T_{e,\perp} d\ln B + k_B T_{e,\parallel} d\ln \left(\frac{n}{B}\right) - dE_{w,e}
\]

\[
= k_B T_{e,\perp} d\ln B - dE_{w,e},
\]

(5.3)

where \(e_{w,e}\) is the energy per particle in whistler waves (as we have discussed in Section 4.2, the electron energy transferred to electron modes stays entirely in the waves, so \(dE_{w,e} = dE_{w,\text{tot},e}\)), and we have used the fact that \(n/B \propto \text{const}\). In Figure 5.2(e), the blue solid line shows the measured change of electron internal energy from the in-plane run, while the red dashed line is obtained by integrating Equation (5.3). We find excellent agreement between the simulation results and the electron heating model.

The validation can also be extended to the entropy measurement, as we do in Figure 5.2(d). Again, the blue solid line shows the measured change in electron specific entropy (computed from the distribution function as in Equation (4.27)), while the red dashed line
is obtained by integrating

\[ ds_e = \left( \frac{1}{2} d \ln T_{e,\parallel} \right) \left( 1 - \frac{T_{e,\parallel}}{T_{e,\perp}} \right) - \frac{d e_{w,e}}{T_{e,\perp}}, \quad (5.4) \]

which follows from Equation (4.13) (an equivalent form can be obtained from Equation (4.14)). Once again, the model matches the simulation results extremely well.

While the increase of entropy is already an indication of irreversible heating, we go one step further and extend the simulations presented in Figure 5.2 to show intuitively how pitch-angle scattering induced by electron whistler can lead to net increase of electron temperature after cycle(s) of compression and expansion.

In Figure 5.3 we show an extension of the simulations shown in Figure 5.2. After compressing the box at \( q = 2.5 \) for \( 1\Omega_{ci}^{-1} \), we switch to expanding the box, so that the density and background magnetic field scale like \( 1/(1 + 2.5t) \) from \( t = 1\Omega_{ci}^{-1} \) to \( 2\Omega_{ci}^{-1} \).

In contrast, the in-plane simulation (solid lines) experience pitch-angle scattering during the compression phase and transfer energy from the perpendicular component to the parallel component, which results in entropy increase. Interestingly, the temperature increase in the in-plane simulation is less than in the out-of-plane simulation at the end of the compression.
phase. This is because $dw_\perp = T_{e,\perp} dB$ is consistently less in the in-plane simulation after the onset of pitch angle scattering compared to the out-of-plane simulation. In the expansion part of the simulation ($t > 1\Omega_{ci}$), the wave energy decays very quickly (solid line in Figure 5.3(a)). This is $T_{e,\perp}$ quickly falls below $T_{e,\parallel}$ and the electron whistler wave is no longer unstable. There is another instability when $T_{e,\parallel} > T_{e,\perp}$, electron firehose instability, but it has a larger threshold and also our 1D simulation restricts its maximally growing mode, which is usually oblique. During the expansion phase, $T_{e,\perp}$ roughly follows the decrease of external $B$ field, and $T_{e,\parallel}$ is roughly constant by adiabatically following $(n/B)^2$. However, due to the net increase of entropy (indicator of irreversible heating) during the compression phase, after a full cycle of compression and expansion, the electron temperature shows a net increase of $\sim 20\%$ (solid line in Figure 5.3(d)).

5.1.3 Dependence on the Mass Ratio

We now extend our compressing box experiments up to the realistic mass ratio and show that the electron entropy increase is nearly insensitive to $m_i/m_e$ (as long as the mass ratio is larger than a few tens). Figure 5.4 compares the evolution of the whistler wave energy (panel (a)), the electron temperature anisotropy (panel (b)), the rate $-d\ln(T_{e,\perp}/B)$ of breaking adiabatic invariance (panel (c)) and the electron entropy increase (panel (d)) when varying the mass ratio from $m_i/m_e = 49$ up to $m_i/m_e = 1600$ (from purple to red, see the legend in the second panel). Since we fix the compression rate to be $q = 2.5\Omega_{ci}$, a larger mass ratio corresponds to a lower compression rate in units of the electron gyration frequency $\Omega_{ce} = (m_i/m_e)\Omega_{ci}$.

Initially, the electron temperature anisotropy grows linearly in time as $T_{e,\perp}/T_{e,\parallel} - 1 = qt$, as a result of the large-scale compression. This proceeds until the energy in whistler waves
Figure 5.3: Comparison between in-plane and out-of-plane setups in 1D periodic box simulations in which the box is compressed from time $t = 0$ to $1\Omega_{ci}$ and expanded from $t = 1\Omega_{ci}$ to $2\Omega_{ci}$. The in-plane configuration (solid lines) which allows entropy production through electron whistler wave, ends up with a net energy gain at the end of the compression-expansion cycle, while the out-of-plane setup (dotted lines) comes back to the same energy it started with.
Figure 5.4: Dependence on mass ratio (up to $m_i/m_e = 1600$) of various space-averaged quantities in a 1D periodic box with compression rate $q = 2.5 \Omega_{ci}$ (the legend is in panel (b)). The background field is aligned with the box (in-plane configuration). We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the compressed field; (b) electron temperature anisotropy (solid lines) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (c) rate of violation of adiabatic invariance $-d\ln(T_{e,\perp}/B)$; (d) electron entropy change, measured from the electron distribution function as in Equation (4.27). After $\Omega_{ci} t \sim 1$ (vertical dotted black line in panel (d)), which corresponds to the end of the compression phase in the shock ramp, the entropy change is nearly independent of the mass ratio.
reaches a fraction $\sim 3 \times 10^{-2}$ of the compressed background field energy (Figure 5.4(a)).

At this point, whistler waves are sufficiently strong to scatter the electrons in pitch angle, breaking their adiabatic invariance and reducing the electron anisotropy by transferring energy from the perpendicular to the parallel component. In fact, notice that the peak in panel (c), i.e., the time when the electron adiabatic invariance is most violently broken, always corresponds to the time when the electron anisotropy in panel (b) shows the sharpest decrease.

The onset of efficient pitch-angle scattering (and so, the peak time of electron anisotropy) occurs earlier at higher mass ratio, at a time that decreases from $t \sim 0.35 \Omega_{ci}^{-1}$ at $m_i/m_e = 49$ down to $t \sim 0.1 \Omega_{ci}^{-1}$ at $m_i/m_e = 1600$. This can be understood from the competition between the large-scale compression rate (which increases the electron anisotropy) and the growth rate of whistler waves (which try to reduce the anisotropy via pitch angle scattering).

The compression rate in units of the electron cyclotron frequency is $q = 2.5 (m_e/m_i) \Omega_{ce}$, while the whistler growth rate (also in units of $\Omega_{ce}$) depends on how much the anisotropy exceeds the whistler threshold in Equation (4.30). In order to balance the two rates, a higher anisotropy is needed for larger $m_e/m_i$, i.e., for lower mass ratios. This has two consequences: first, the growth rate of the whistler instability (normalized to $\Omega_{ce}$) will decrease at higher $m_i/m_e$, as indeed confirmed by the inset of Figure 5.4(a); second, lower peak anisotropies (and so, earlier onsets of efficient pitch angle-scattering) will be achieved at higher mass ratios, which explains the trend seen in Figure 5.4(b). In addition, since the energy of whistler waves ultimately comes from the free energy in electron anisotropy, higher mass ratios display weaker levels of whistler wave activity (panel (a)).

The electron entropy evolution in Figure 5.4(d) can be separated into two stages. In the first phase (which, for $m_i/m_e = 49$, occurs at $t \sim 0.45 \Omega_{ci}^{-1}$), the electron entropy grows
quickly. This stage corresponds to the late exponential phase of whistler wave growth (and so, we shall call it the “exponential phase”), when both the electron anisotropy (panel (b)) and the rate of breaking adiabatic invariance (panel (c)) — i.e., the two ingredients needed for efficient entropy production — are large. Since higher mass ratios reach lower levels of electron anisotropy, the entropy produced during this stage is a decreasing function of \( m_i/m_e \), as seen in Figure 5.4(d) (compare the purple line growth around \( t \sim 0.45 \Omega_{ci}^{-1} \) with the red line around \( t \sim 0.15 \Omega_{ci}^{-1} \)). After whistler waves have reached saturation, the electron entropy still increases, in a phase which we shall call “secular”. Here, the electron anisotropy stays close to the threshold of marginal stability (indicated in Figure 5.4(c) by the dotted lines, with the same color coding as the solid curves). Continuous pitch-angle scattering (and so, persistent violation of adiabatic invariance) is needed to oppose the steadily-driven compression and maintain the system close to marginal stability. It is then expected that entropy will continuously increase during the secular phase, albeit at a lower rate than in the exponential stage. For \( m_i/m_e \gtrsim 400 \), the electron anisotropy at late times is nearly insensitive to \( m_i/m_e \) (compare yellow, orange and red lines at \( \Omega_{ci} t \gtrsim 0.4 \) in panel (b)), which explains why the entropy growth in the secular phase is nearly the same for all \( m_i/m_e \gtrsim 400 \) (Figure 5.4(d)).

From Figure 5.4(d), we can infer how the entropy increase in the shock ramp should scale with mass ratio. Since the compression in the shock ramp lasts about one proton gyration time, we compare the entropy curves at \( \Omega_{ci} t \sim 1 \), as indicated by the vertical dotted black line in panel (d). When the mass ratio increases from \( m_i/m_e = 49 \) to \( m_i/m_e = 1600 \) (i.e., more than a factor of 32), the entropy produced until \( \Omega_{ci} t = 1 \) decreases from 0.065 to 0.048, only a \( \sim 30\% \) decrease. In fact, most of this 30% decrease is between \( m_i/m_e = 49 \) and 200. Beyond that, the entropy increase is virtually constant. The dependence on mass ratio
would be far more pronounced if we were only to consider the entropy produced during
the exponential phase. However, higher mass ratio runs have earlier onset times, as we
have explained above, so they spend more time (within the first $\Omega_{ci}^{-1}$) in the secular phase,
as compared to lower mass ratios. In summary, most of the entropy production at lower
mass ratios happens during the exponential phase, whereas at higher mass ratios the secular
phase lasts longer and thus compensates for the lower level of entropy generated during the
exponential stage. The net effect is that the entropy increase in our compressing box with
$m_i/m_e = 1600$ is only slightly smaller than for $m_i/m_e = 49$. The same conclusion should
hold also for our reference shock.

5.2 Electron Heating by Proton-Driven Waves

In the downstream region of our reference shock, at a distance of $\sim 2.5 r_{Li}$ from the shock
front, the electron entropy shows a second phase of rapid increase. Here, a large-scale
density compression co-exists with the growth of proton-driven waves, and both contribute
to magnetic field amplification and irreversible electron heating. The new concept here is
the effect of proton-driven waves, so we focus on that in this section. We demonstrate that
magnetic fluctuations induced by a proton temperature anisotropy can naturally lead to an
increase in electron entropy, even in the absence of a large-scale compression. We employ
periodic simulation domains with the standard form of the PIC equations (as opposed to
what we used in the previous section) and set up a population of anisotropic protons, with
a degree of anisotropy motivated by our reference shock run. We discuss the simulation
setup in Section 5.2.1, we discuss periodic box simulations applicable to our reference shock
run in Section 5.2.2, and we describe the dependence on mass ratio in Section 5.2.3.
5.2.1 Simulation setup

In order to study the role of anisotropy-driven proton modes in producing electron irreversible heating, we set up a periodic simulation box with anisotropic protons. The simulation is initialized to approximate the conditions right after the shock transition. The protons are initialized as a bi-Maxwellian distribution with $T_{i0,\perp}/T_{i0,\parallel} \sim 7$, as observed just behind the shock in Figure 4.1(g). The value of $T_{i0,\parallel}$ is the same as in the shock upstream (in fact, the parallel proton temperature is nearly uniform across the shock, see the orange line in Figure 4.1(f)). The electron temperature increases roughly by a factor of two across the shock (Figure 4.3(e)), so the electrons in the tests here are initialized with $T_{e0} \sim 2 \times 10^{-2}m_ec^2/k_B$ (i.e., two times the shock upstream electron temperature of $10^{-2}m_ec^2/k_B$). We take electrons to be isotropic, since the fast growth of whistler waves in the shock ramp ensures that the degree of downstream electron anisotropy is low (see the post-shock region in Figure 4.3(f)). We also take into account the fact that both density and magnetic field strength have increased by a factor of $\sim 2.5$ as compared to the shock upstream (Figure 4.1(a)).

We resolve the electron skin depth with 7 cells in order to (marginally) capture the electron Debye length. Since both the proton cyclotron instability, which dominates over the mirror mode in the downstream of our reference run, and the electron whistler instability have the fastest growing wavevector aligned with the background field, we employ 1D simulation domains with the box aligned with the $y$ direction of the background magnetic field. Thanks to the reduced dimensionality of our computational domain, we can employ a large number of particles per cell ($10^4$). Therefore, we have adequate statistics for the calculation of the electron specific entropy and we can properly control the effect of numerical heating.
In addition, in 1D simulations we can readily extend our results up to the realistic mass ratio. The length of the computational box is 1512 cells for \(m_i/m_e = 49\). Since the fastest growing mode of the proton cyclotron instability has a wavevector \(\sim \omega_{pi}/c\), we increase the number of cells in our computational domain proportional to \(\propto \sqrt{m_i/m_e}\), to include the same number of proton skin depths (and so, the same number of proton cyclotron wavelengths).\(^1\)

5.2.2 Application to the Reference Shock

In order to compare the results obtained from the periodic box simulations with the reference shock run, in Figure 5.5 and 5.6 we show the evolution of our periodic system for \(m_i/m_e = 49\). As a result of the initial proton temperature anisotropy, the proton cyclotron instability develops, generating exponentially-growing waves with \(\delta B_x\) and \(\delta B_z\) components (Figure 5.5(a)). As shown in Figure 5.6(b) and (c), the growing waves are dominated by modes with wavelength at the proton inertial scale (for \(m_i/m_e = 49\), the proton skin depth is \(c/\omega_{pi} = 7c/\omega_{pe}\)) and frequency comparable to the proton gyration frequency, as expected for the proton cyclotron instability.

At \(t \sim 4\Omega_{ci}^{-1}\), when the energy in proton cyclotron waves reaches a fraction \(\sim 10^{-1}\) of the background field energy, efficient pitch-angle scattering quickly reduces the proton temperature anisotropy (Figure 5.5(c)). During the isotropization process, the proton specific entropy increases (Figure 5.5(e) at \(t \sim 5\Omega_{ci}^{-1}\)).

The heating model described in Section 4.2 can be applied to protons, keeping in mind that in the current setup no perturbations in density or magnetic field exist on scales larger than the proton scales (so, no work is being done on the protons). It follows that the

\(^1\)Beyond \(m_i/m_e = 400\), we also increase the number of computational particles per cell proportional to \(\sqrt{m_i/m_e}\), in order to minimize numerical heating effects.
Figure 5.5: Time evolution of various space-averaged quantities in a 1D periodic box initialized with anisotropic protons, to mimic the shock conditions in the downstream. The background field is aligned with the box (in-plane configuration). We plot: (a) total energy in magnetic field fluctuations, normalized to the energy of the initial field; (b) energy in electron-scale fluctuations, extracted using the high-pass filter in frequency and wavenumber indicated by the red dashed lines in the power spectra of Figure 5.6(e) and (f); (c) proton and (d) electron temperature perpendicular (blue lines) and parallel (orange lines) to the background field, together with the mean temperature (green lines); (e) proton entropy change, measured from the proton distribution function (blue solid) or predicted from Equation (5.8) (orange dotted); (f) electron entropy change, measured from the electron distribution function (blue solid) or predicted from Equation (5.13) (orange dotted); (g) proton energy change in units of $k_B T_{i0}$, measured directly (green) or predicted using Equation (5.12) (red); (h) electron energy increase in units of $k_B T_{e0}$, measured directly (green) or predicted using Equation (5.14) (red). For other curves in panels (g) and (h), see the text (make sure all these extra curves are explained in the text – I didn’t catch them all).
Figure 5.6: Space-time diagrams and power spectra of a 1D periodic box initialized with anisotropic protons. The panels are the same as in Figure 4.5, with the only difference that the time unit here is $\Omega_{ci}^{-1}$ (and frequencies are normalized to $\Omega_{ci}$). As in Figure 4.5, since panels (a)-(c) are dominated by long-wavelength slowly-propagating proton modes, we isolate electron waves via a high-pass filter in the power spectra of panels (e) and (f), keeping only the high-$\omega$ high-$k_y$ region delimited by the red dashed lines. The resulting space-time wave pattern is shown in panels (h) and (i), whose insets clearly reveal the presence of electron whistler waves.
perpendicular and parallel energy per proton change as

\[
du_{i,\perp} = -dq_{i,\perp \rightarrow \parallel} - de_{w,i,\perp},
\]

(5.5)

\[
du_{i,\parallel} = dq_{i,\perp \rightarrow \parallel} - de_{w,i,\parallel},
\]

(5.6)

so that the total change in proton energy per particle is

\[
du_i = -de_{w,i,\perp} - de_{w,i,\parallel} \equiv -de_{w,i,\text{tot}},
\]

(5.7)

which simply states that the energy lost by protons is transferred to proton waves. Following Section 4.2, the change in specific proton entropy is

\[
\frac{ds_i}{T_i,\perp} = \left(\frac{1}{2} d\ln T_{i,\parallel}\right) \left(1 - \frac{T_{i,\parallel}}{T_{i,\perp}}\right) - \frac{de_{w,i,\text{tot}}}{T_{i,\parallel}},
\]

(5.8)

\[
\frac{ds_i}{T_i,\perp} = (-d\ln T_{i,\perp}) \left(\frac{T_{i,\perp}}{T_{i,\parallel}} - 1\right) - \frac{de_{w,i,\text{tot}}}{T_{i,\parallel}},
\]

(5.9)

where the two expressions are equivalent, as with Equation (4.13) and Equation (4.14). We now need to specify \(de_{w,i,\text{tot}}\), i.e., the energy per proton transferred to proton modes. As we anticipated in Section 4.2, this is not equal to the energy residing in proton waves, since some fraction of that is being used to perform work on the electrons. In the remainder of this section, we denote as \(n\) and \(B\) the density and magnetic field fluctuations induced by proton waves. Since the scale of the perturbations is much larger than the electron gyroradius, the fluctuations perform work on the electrons, so that Equation (4.4) for electrons becomes

\[
dw_e = T_{e,\perp} d\ln B + T_{e,\parallel} d\ln \left(\frac{n}{B}\right) \equiv dw_{e,\perp} + dw_{e,\parallel}.
\]

(5.10)
This energy increase in the electrons is at the expense of the energy in proton waves, so that the residual energy per particle residing in proton waves will be

\[ de_{w,i} = de_{w,i}^{\text{tot}} - dw_{e,\perp} - dw_{e,\parallel}, \]  

(5.11)

and the energy equation for protons reads

\[ du_i = -dw_{e,\perp} - dw_{e,\parallel} - de_{w,i}, \]  

(5.12)

where the three terms on the right hand side can be explicitly measured in our simulations.

Figure 5.5(e) and (g) demonstrate that our heating model works remarkably well for protons (later on, we will show that it also works for electrons). In Figure 5.5(e), the blue solid line is the proton entropy change measured directly from the simulation, using the distribution function as we did in Equation (4.27). It matches extremely well the prediction obtained by integrating the right hand side of the proton entropy equation, Equation (5.8) or Equation (5.9) (see the orange dotted line in Figure 5.5(e)). The agreement is also remarkable as regard to the proton energy equation, Equation (5.12). In Figure 5.5(g), the proton energy loss \(-\Delta u_i = -\int du_i\) is indicated as a green line. From Equation (5.12), this should be equal to \(\Delta e_{w,i} + w_\perp + w_\parallel\), where we have defined \(\Delta e_{w,i} = \int de_{w,i}\), \(w_\perp = \int dw_{e,\perp}\) and \(w_\parallel = \int dw_{e,\parallel}\). In fact, the green line nearly overlaps with the red curve.

The growth of proton cyclotron waves provides a source of field amplification and density perturbations that can perform work on the electrons. Indeed, Figure 5.5(h) shows that during the exponential phase of the proton cyclotron instability \((4 \lesssim \Omega_{ci} t \lesssim 7)\), the proton waves increase the electron perpendicular energy (i.e., \(dw_{e,\perp} > 0\), see the blue line in Figure 5.5(h)) and decrease the parallel component (i.e., \(dw_{e,\parallel} < 0\), see the orange line in Figure
5.5(h)). This leads to a temperature anisotropy $T_{e,\perp} > T_{e,\parallel}$ (compare the blue and orange lines in Figure 5.5(d) at $\Omega_{ci} t \sim 5$), which can be equivalently explained as a result of the conservation of the first and second adiabatic invariants in the growing fields of the proton cyclotron waves. The resulting electron anisotropy is sufficiently strong to trigger the growth of whistler waves.

While the presence of whistler waves is hard to identify by eye in the space-time diagrams of Figure 5.6(b) and (c), due to the dominance of proton cyclotron modes, we can extract their signature by applying a filter in frequency and wavenumber, as done in Section 4.4.2.1. Figure 5.6(e) and (f) show the power spectra of $\delta B_x$ and $\delta B_z$. Most of the power is concentrated near the origin at low frequencies and long wavelengths, associated with the proton cyclotron mode. However, we can still identify a significant peak around $\omega \simeq 13 \Omega_{ci} \simeq 0.3 \Omega_{ce}$ and $k_y \simeq 0.3 \omega_{pe}/c$. In analogy with the discussion in Section 4.4.2.1, we associate this peak with electron whistler waves. When applying a high-pass filter whose frequency and wavelength cuts are shown as red dashed lines in Figure 5.6(e) and (f), we recover in the space-time diagrams of Figure 5.6(h) and (i) the typical spatial and temporal patterns of electron whistler waves. As expected, most of the electron whistler activity takes place near the end of the exponential growth of proton waves, at $5 \lesssim \Omega_{ci} t \lesssim 7$ (see also the temporal evolution of the energy in whistler waves in Figure 5.5(b)). In this time interval, the electron anisotropy exceeds the threshold of whistler instability in the whole simulation domain (Figure 5.6(g)).

This period also corresponds to a rapid increase of the electron specific entropy, as measured directly from the electron distribution function (blue solid line in Figure 5.5(f)). This is expected, since whistler waves provide the pitch-angle scattering required to break adiabatic invariance, which (together with the sustained electron anisotropy, see Figure 5.5(d))
and Figure 5.6(d) at $5 \lesssim \Omega_{ci} t \lesssim 7$) drives efficient entropy generation. Based on our model in Section 4.2, the electron specific entropy should increase as

$$
\frac{d}{dt} s_e = \left[ \frac{1}{2} \frac{d}{dt} \ln \left( \frac{T_{e,||}}{(n/B)^2} \right) \right] \left( 1 - \frac{T_{e,||}}{T_{e,\perp}} \right) - \frac{d e_{w,e}}{T_{e,\perp}}, \tag{5.13}
$$

which follows from Equation (4.13) (an equivalent form can be obtained from Equation (4.14)). Here, we have used the condition $d e_{w,e,\text{tot}} = d e_{w,e}$ for electrons. The comparison of the measured entropy increase (blue solid line in Figure 5.5(f)) with the predicted entropy change (orange dotted line in Figure 5.5(f)) provides another validation of our heating model.

While most of the electron entropy production happens near the end of the exponential growth of proton waves, a moderate increase of the electron entropy also occurs during the secular stage (i.e., at $\Omega_{ci} t \gtrsim 10$). Here, the oscillating proton cyclotron fluctuations cause the electrons to slosh around and can occasionally excite local patches of electron anisotropy that exceed the whistler threshold (e.g., see Figure 5.6(g) at $\Omega_{ci} t \simeq 11$ and $y \simeq 175 c/\omega_{pe}$). In the same region, we can identify a short episode of electron whistler activity, particularly in $\delta B_z$ in Figure 5.6(i). These sporadic episodes of anisotropy-driven whistler waves further increase the electron entropy.

It is worth noticing, though, that at late times the box-averaged electron anisotropy switches sign, with $T_{e,||} \gtrsim T_{e,\perp}$ (in Figure 5.5(d), at $\Omega_{ci} t \gtrsim 9$), so the opportunities for whistler growth are fewer. This behavior is consistent with the conservation of the first and second adiabatic invariants in the decaying field of the proton cyclotron waves, leading to an increase in $T_{e,||}$ and a decrease in $T_{e,\perp}$ (as indeed seen in Figure 5.5(d) at late times). The same “inverted” anisotropy with $T_{e,||} \gtrsim T_{e,\perp}$ is seen in the far downstream of our reference
shock run (Figure 4.3(f)), accompanying the decay of the proton modes. We remark that electron entropy production is still possible when $T_{e,\parallel} \gtrsim T_{e,\perp}$, as long as the anisotropy is large enough to exceed the threshold of the firehose instability, which would then provide the mechanism for breaking adiabatic invariance. We have verified with expanding box simulations similar to the ones reported in Section 5.1 (not shown) that once the system exceeds the firehose threshold, the electron entropy rapidly increases.

Finally, by measuring directly the energy in whistler waves, we can also validate the electron energy equation

$$du_e = dw_{e,\perp} + dw_{e,\parallel} - dw_{w,e}.$$  \hspace{1cm} (5.14)

Once again, the time-integral of the left hand side matches very well the time-integral of the right hand side (compare green and red curves in Figure 5.5(h)), i.e., the change of electron internal energy can be accounted for by the total work done by the proton waves and the energy lost to generate electron whistler waves.

In summary, we have demonstrated that efficient electron entropy production can occur even in the absence of a large-scale compression. Magnetic and density fluctuations sourced by anisotropic protons drive electrons to become anisotropic, with $T_{e,\perp} > T_{e,\parallel}$. The electron anisotropy is relaxed by the growth of whistler waves, which break the electron adiabatic invariance and mediate the production of electron entropy. In the next subsection, we show that the resulting electron entropy increase is nearly independent of the proton-to-electron mass ratio.
Figure 5.7: Dependence on mass ratio (up to $m_i/m_e = 1600$) of various space-averaged quantities in a 1D periodic box with anisotropic protons (the legend is in panel (a)). The background field is aligned with the box (in-plane configuration). We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the initial field; (b) proton temperature anisotropy; (c) energy in electron-scale field fluctuations; (d) electron temperature anisotropy (solid lines) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (e) rate of violation of the electron adiabatic invariance $-d\ln(T_{e,\perp}/B)$; (f) electron entropy change, measured from the electron distribution function.
5.2.3 Dependence on the Mass Ratio

In this subsection, we explore with periodic boxes initialized with anisotropic protons how the development of the proton cyclotron instability and the subsequent electron irreversible heating depend on the mass ratio. We vary the mass ratio from 49 up to 1600, as indicated in the legend of Figure 5.7(a). Since the fastest growing mode of the proton cyclotron instability has wavevector $\sim \omega_{pi}/c$, we increase the number of cells in our computational domain as $\propto \sqrt{m_i/m_e}$, to include the same number of proton skin depths for all values of $m_i/m_e$ (and so, the same number of proton cyclotron wavelengths).

Figure 5.7 compares the runs. Panel (a) shows the time evolution of the wave magnetic energy, which is dominated by proton cyclotron modes. Panel (b) shows the evolution of the proton temperature anisotropy, which reduces strongly at $\Omega_{ci}t \gtrsim 4$ by pitch angle scattering off the strong cyclotron waves. Unsurprisingly, since these two quantities are related to protons, their evolution is almost identical for different mass ratios. As long as the mass ratio is sufficiently large to adequately separate electron and proton scales, the proton cyclotron instability — whose polarization is resonant with protons, but non-resonant with electrons — is not affected by electron physics.

As in the case $m_i/m_e = 49$ discussed above, the growth of the proton cyclotron instability induces an electron temperature anisotropy with $T_{e,\perp} > T_{e,\parallel}$ (Figure 5.7(d)) which excites electron whistler waves (Figure 5.7(c)), and facilitate electron entropy increase (Figure 5.7(f)) by violating the electron adiabatic invariance (Figure 5.7(e)). The dependence of the peak electron anisotropy on mass ratio for $m_i/m_e \lesssim 200$ can be understood from the same argument we have presented in Section 5.1: in units of the electron gyration period,

\footnote{We isolate the magnetic energy associated with whistler waves by applying a high-pass filter for frequency higher than $0.5\Omega_{ce}$ and wavelength shorter than $35c/\omega_{pe}$.}
the growth of proton waves (or the compressed field, for Section 5.1) is faster at lower $m_i/m_e$, which leads to an overshoot in electron anisotropy beyond the threshold of whistler marginal stability. The overshoot is more pronounced for lower $m_i/m_e$. Due to the higher electron anisotropy, more free energy is available for the growth of whistler waves at lower $m_i/m_e$ (see the trend from purple to green in Figure 5.7(c) at $\Omega_{ci}t \sim 6$). Because of the higher electron anisotropy and stronger whistler waves, the electron entropy increases slightly more at lower mass ratios (in particular, see the purple line in Figure 5.7(f) for $m_i/m_e = 49$).

On the other hand, the electron physics shows no appreciable dependence on mass ratio for $m_i/m_e \gtrsim 400$. The peak electron anisotropy at $5 \lesssim \Omega_{ci}t \sim 7$ saturates at the threshold of whistler marginal stability (indicated in Figure 5.7(c) by the dotted lines, with the same color coding as the solid lines). As a consequence, the peak strength of whistler waves is nearly independent of mass ratio (see Figure 5.7(b) in the same time interval), and the resulting entropy increase is the same for all mass ratios $m_i/m_e \gtrsim 400$ (Figure 5.7(f)). Even for $m_i/m_e = 49$, the electron entropy increase at the final time is only $\sim 30\%$ higher than for $m_i/m_e = 1600$.

### 5.3 Validation of the Electron Heating Physics in Shocks

We are now in a position to validate our heating model in full shock simulations. In Section 5.1 and Section 5.2, we have demonstrated that our heating model provides an excellent description of the change in electron energy and entropy for two physical scenarios: if electrons are subject to a large-scale compression, as in the shock ramp; and if electrons are driven to temperature anisotropy by the growth of proton-driven modes, as observed in the
Figure 5.8: Validation of the heating model in our reference shock simulation at $\Omega_{ci}t = 25.6$. In the top panel, we compare the $y$-averaged electron entropy change measured from the electron distribution function as in Equation (4.27) (blue solid line) with the predicted change based on Equation (4.14) (orange dashed line). The differential terms on the right hand side of Equation (4.14) are calculated by differencing neighboring cells along the $x$ direction. In the bottom panel, we compare the $y$-averaged electron energy change measured directly from our simulation (blue solid line) with the predicted increase based on Equation (4.7) (orange dashed line). For both entropy and internal energy, the agreement between the model and the simulation results is remarkably good.
far downstream. Since the two scenarios correspond to the two locations where the entropy profile in shocks shows the fastest increase, we expect that our model will properly capture the electron heating physics in the reference shock run described in Section 4.4.

In the top panel of Figure 5.8, we compare the electron entropy profile measured directly from the phase space distribution function as in Equation (4.27) (solid blue line), with the entropy change predicted by Equation (4.14) (dashed orange line). The differential terms on the right hand side of Equation (4.14) are calculated by taking differences of quantities in neighboring cells along the $x$ direction. The agreement between the measured entropy profile and the predicted one is remarkably good (with the exception of the far downstream region, where numerical heating of electrons might be responsible for the discrepancy, see Appendix 5.B). In particular, the theory correctly predicts the location and magnitude of the two sites of fastest entropy growth: in the shock ramp, where electron irreversible heating is induced by the shock-compression of density and magnetic field (in analogy to the scenario we have studied in Section 5.1); and at a distance of $\sim 2.5 \text{ } r_{\text{Li}}$ behind the shock, where a large-scale density and field compression co-exists with the growth of proton-driven waves, the latter contributing to further magnetic field amplification. The agreement of theory and measurement at this location is then a combined validation of the two scenarios described in Section 5.1 and Section 5.2, confirming that our model holds regardless of what drives the field amplification (and so, the resulting electron anisotropy).

In addition, in the bottom panel of Figure 5.8 we show that the change in electron energy per particle (blue solid line) is predicted extremely well by our heating model (orange dashed line, following Equation (4.7)).
Figure 5.9: Dependence on mass ratio (up to $m_i/m_e = 200$) of shock simulations at $t = 13.1 \Omega_p^{-1}$ (the legend is in panel (d)). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) excess of electron temperature beyond the adiabatic prediction for an isotropic gas; (h) change in electron entropy. The increase in electron entropy is nearly insensitive to the mass ratio.
5.3.1 Mass ratio dependence in the shock simulation

In the periodic box runs of Section 5.1 and Section 5.2, we have extended our study to realistic mass ratios, showing that the entropy increase at \( m_i/m_e = 1600 \) is only \( \sim 30\% \) smaller than for the choice \( m_i/m_e = 49 \) of our reference shock simulation. In Figure 5.9, we investigate the dependence of the electron physics in our full shock simulations on the mass ratio, from \( m_i/m_e = 25 \) up to \( m_i/m_e = 200 \) (as indicated by the legend in panel (d)). We typically employ 32 computational particles per cell, with the exception of \( m_i/m_e = 200 \), where we use 64 particles per cell to keep numerical heating under control. We keep the upstream electron temperature fixed at \( k_B T_{e0} = 10^{-2} m_e c^2 \), so that electrons stay safely non-relativistic. This implies that the plasma inflow velocity is slower with increasing mass ratio, as \( \propto \sqrt{m_e/m_i} \).

The proton physics is expected to be the same regardless of mass ratio, and in fact the profiles of density (panel (a)), proton temperature (panel (c)) and proton anisotropy (panel (d)) are nearly the same for all mass ratios. The same holds for the wave magnetic energy at \( (x - x_{sh})/r_{Li} \lesssim -0.5 \) (panel (b)), where proton-driven modes dominate.

On the other hand, the peak electron anisotropy at the shock (panel (f)) is systematically lower for higher mass ratios, in perfect agreement with the trend observed in the periodic box experiments of Figure 5.4. Despite the pronounced difference in peak anisotropy, Figure 5.4(d) showed that the entropy increase until \( t \sim \Omega_{ci}^{-1} \) was only marginally lower at higher \( m_i/m_e \). This trend (and the weak mass ratio dependence) is confirmed by the profiles of electron entropy in the shock ramp shown in Figure 5.9(h). Overall, Figure 5.9(h) confirms the results of our periodic box experiments, namely, the electron entropy increase is nearly independent of mass ratio (with the exception of the lowest mass ratio \( m_i/m_e = 25 \)).
presented in Figure 5.9). Even though these full shock simulations only extend up to $m_i/m_e = 200$, the results of our periodic runs in Section 5.1 and Section 5.2 suggest that the same conclusions should hold up to the realistic mass ratio.

5.4 Summary and Discussion

In this and previous chapter, we have investigated by means of analytical theory and 2D PIC simulations the electron heating physics in low Mach number perpendicular shocks, in application to merger shocks in galaxy clusters. While most of the electron heating is adiabatic — induced by shock-compression of the upstream magnetic field — we direct our attention to the electron entropy increase, i.e., to the production of irreversible electron heating.

We find that, in analogy to the so-called “magnetic pumping” mechanism, two basic ingredients are needed for electron irreversible heating: (i) the presence of a temperature anisotropy, induced by field amplification coupled to adiabatic invariance; and (ii) a mechanism to break the adiabatic invariance itself.

We have demonstrated that, in our reference shock with sonic Mach number $M_s = 3$ and plasma beta $\beta_{p0} = 16$, efficient electron entropy production occurs at two major sites: at the shock ramp, where density compression coupled to flux freezing leads to field amplification and a high degree of electron anisotropy; and farther downstream, where density compression and long-wavelength magnetic waves induced by the proton temperature anisotropy are both contributing to magnetic field growth. Regardless of the origin of field amplification, electrons are driven to a large degree of temperature anisotropy, exceeding the threshold of the electron whistler instability. The resulting growth of electron whistler waves — whose
presence is one of the common denominators of the two sites mentioned above — causes the violation of the electron adiabatic invariance, and allows for efficient entropy production.

Our model is in excellent agreement with the measured electron entropy increase, which can be quantified directly from the electron distribution function in our simulations. The agreement holds for our reference shock simulation, as well as for controlled periodic box experiments meant to reproduce the shock conditions at the two major sites of entropy production. In particular, the shock physics in the ramp can be replicated in a periodic box where the PIC equations are modified to allow for a continuous large-scale compression, as in Sironi & Narayan (2015); Sironi (2015). Also, the physics of anisotropy-driven proton waves, and the resulting electron irreversible heating, can be conveniently studied in a periodic box initialized with anisotropic protons, with a degree of anisotropy inspired by the shock simulation. The advantage of the periodic domains is twofold: (i) they allow for a more direct control of the relevant physics; (i) and, due to less demanding computational requirements, they permit to extend our investigation up to the realistic mass ratio. We have then be able to ascertain that the entropy increase has only a weak dependence on mass ratio (less than $\sim 30\%$ decrease, as we increase the mass ratio from $m_i/m_e = 49$ up to $m_i/m_e = 1600$).

Finally, we remind the reader that in this and previous chapter, we have only focused on one representative set of shock parameters, fixing the Mach number $M_s = 3$ and the plasma beta $\beta_{p0} = 16$. For this case, the post-shock electron temperature $T_e$ in the far downstream (where the proton anisotropy has settled to the marginal stability threshold for mirror and proton cyclotron modes, with $T_{i,\perp}/T_{i,\parallel} \sim 0.4$) exceeds the adiabatic expectation $T_{e,\text{ad}} \simeq 2T_0$ by $\approx 15\%$, i.e., $T_e \approx 2.3T_0$, as a result of entropy production at the shock (here, $T_0$ is the pre-shock temperature). The downstream proton temperature $T_i \approx 5T_0$ is much
larger than the adiabatic expectation $T_{i,\text{ad}} \simeq 2T_0$, so most of the entropy produced by the shock goes to the protons (a factor of $\sim 10$ more than to electrons). The resulting post-shock temperature ratio for our reference case is $T_e/T_i \simeq 0.45$. In the forthcoming chapters, we will explore the dependence of our conclusions (and in particular, the efficiency of electron heating and the resulting downstream electron-to-proton temperature ratio) on sonic Mach number and plasma beta.

5.A Comparison between in-plane and out-of-plane magnetic field geometries

In the 2D shock simulations presented in the main body of the paper, we have initialized the upstream field in the $xy$ plane of the simulation (“in-plane” geometry). As we have discussed, this is instrumental in capturing the dominant wavevector of both proton and electron waves: the fastest growing mode of the proton cyclotron instability is aligned with the background field, and mirror modes are also naturally resolved if the magnetic field lies in the simulation plane; similarly, the dominant mode of the electron whistler instability is nearly parallel to the background field.

Given that the heating mechanism that we propose relies on such waves for breaking the electron adiabatic invariance (in the case of whistler waves) or for amplifying the magnetic field, thus leading to irreversible electron heating (in the case of proton modes), we expect that the alternative “out-of-plane” geometry, in which the field is initialized along the $z$ direction, will lead to weaker electron heating. This is confirmed by Figure 5.10: there, orange lines refer to our reference 2D simulation with in-plane fields, blue lines to a 2D simulation with out-of-plane fields, and green lines to a 1D simulation. The physical and
Figure 5.10: Comparison at $\Omega_c t = 23.1$ between two 2D simulations with in-plane (orange) or out-of-plane (blue) fields and a 1D simulation (green), as indicated in the legend of panel (c). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature perpendicular (solid) and parallel (dashed) to the background field; (g) change in electron entropy.
numerical parameters of the two 2D runs are the same as in our reference run (of course, apart from the field orientation). The 1D simulation has the same physical parameters, but a higher number of particles per cell (5000 per species).

As expected, the 2D out-of-plane case is remarkably similar to 1D results (compare blue and green lines). In both cases, both protons and electrons stay highly anisotropic (panels (d) and (f)), due to the lack of anisotropy-driven waves (and in fact, the wave energy in panel (b) does not appreciably exceed noise levels). This should be contrasted with the in-plane case (orange lines), where both electron and proton anisotropies get reduced by the effect of strong self-generated waves. As a consequence, the entropy increase in the in-plane case (orange line in panel (g)) is much more pronounced than in the out-of-plane run (blue), which in turn is fairly similar to the 1D result (green).  

We remind the readers that while the test here shows 2D simulations are essential to capture the heating physics in shocks, 1D simulations suffice for our controlled experiments in Sections 5.1 and 5.2. This is because for the controlled experiments, we only need to resolve the direction parallel to the ambient magnetic field to allow the growth of the relevant plasma instabilities, while for shock simulations, we need to resolve the additional direction parallel to the shock propagation, which is orthogonal to the ambient magnetic field.

3The deviation of the blue and green lines in the far downstream region of panel (g) is likely to come from numerical noise in the 2D out-of-plane simulation.
Figure 5.11: Comparison at $\Omega_{ci}t = 15.8$ of three runs with the same physical parameters as in our reference shock run, but with different numbers of particles per cell, as indicated in the legend of panel (d). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) excess of electron temperature beyond the adiabatic prediction for an isotropic gas; (h) change in electron entropy.
5.B Dependence on the number of computational particles per cell

In a two-temperature plasma, with protons hotter than electrons, numerical noise will tend to heat the electrons, even in the absence of any physical effect. It is therefore important to check that our results are converged with respect to the number of computational particles per cell, whose value controls the noise level of PIC simulations, and so the rate of numerical electron heating. In Figure 5.11, we compare our results for three choices of the number of particles per cell (including both species), from 8 (light blue) to 128 (dark blue), as shown in the legend of panel (d). We see that the proton physics is largely independent of the number of particles per cell (panels (a), (c) and (d)). On the other hand, panel (b) shows that for 8 particles per cell the noise level of field fluctuations is not negligible, as compared to the physical fields (see the light blue line in panel (b) ahead of the shock). As a result, electrons are heated due to numerical artifacts (light blue line in panels (g) and (h)), to a temperature much larger than in runs with a higher number of particles per cell. The comparison of the runs with 32 and 128 particles per cell shows solid evidence of convergence, even in the profiles of irreversible electron heating (panels (g) and (h)), which are most sensitive to numerical noise. Still, a small difference persists between the entropy profiles obtained with 32 and 128 particles per cell (compare the medium-blue with the dark-blue line in panel (h)). We argue that the small deviation of our model from the measured entropy profile far downstream in the top panel of Figure 5.8 might be largely explained by numerical effects acting far behind the shock.
Chapter 6

Electron heating in low Mach number perpendicular collisionless shocks. III. Parameter dependence of the heating mechanism in idealized conditions

The following two thesis chapters are combined and submitted for publication as


In the previous two chapters, we used one reference shock simulation \((M_s = 3, \beta_p = 16)\) to illustrate that irreversible electron heating in the shock downstream is accompanied by the occurrence of electron (whistler) instability. We developed a theory for heating of anisotropic plasma. It relies on the presence of two basic ingredients: \((i)\) temperature anisotropy; and \((ii)\) a mechanism to break the adiabatic invariance. The temperature anisotropy is usually induced by external field amplification, which affects temperature of
the plasma parallel and perpendicular to the magnetic field in different manners. The
temperature anisotropy then leads to plasma instability which provides means of pitch-
angle scattering that isotropize the plasma distribution. During the isotropization process,
the plasma no longer behaves adiabatically. The adiabatic breaking leads to irreversible
heating of the plasma. Specifically, the entropy increase of the plasma can be described by
two equivalent equations

\[
\begin{align*}
ds &= \left[ \frac{1}{2} \ln \left( \frac{T_\parallel (n/B)^2}{} \right) \right] \cdot \left[ 1 - \frac{T_\parallel}{T_\perp} \right] - \frac{d_{\text{w tot}}}{T_\perp}, \\
ds &= - \left[ \ln \left( \frac{T_\perp}{B} \right) \right] \cdot \left[ \frac{T_\perp}{T_\parallel} - 1 \right] - \frac{d_{\text{w tot}}}{T_\parallel}.
\end{align*}
\]

We have demonstrated the principle of the theory using two kinds of idealized experiments,
the first where the field amplification is sourced by continuous uniform density compression,
the second where the field amplification is source by the proton temperature anisotropy
instability. We will refer to the first kind experiments as compressing box experiments
and the second kind as undriven box experiments. Our electron heating model applies in
both idealized conditions. When both continuous compression and proton instabilities are
present in the complicated shock downstream, our theory still holds.

Our next goal is to understand the dependence of the electron heating mechanism on
parameters of (perpendicular) collisionless shocks, specifically, on \( M_s \) and \( \beta_{p0} \). Before pre-
senting results from full shock simulations covering a wide range of parameters, in this
chapter, we utilize again periodic box simulations to understand in detail the effect of \( M_s \)
and \( \beta_{p0} \) under idealized conditions. In the next chapter, we will use the lessons learned
from this chapter to interpret the results from full shock simulations.

The structure of this chapter is the following. In Section 6.1, we discuss a few key
properties of the electron whistler instability from linear theory. Since the electron whistler
instability plays a key role in facilitating electron entropy production, we will refer to
these properties frequently when we discuss the results from numerical experiments in the
following sections. We then discuss the parameter dependence of the heating mechanism
in compressing box experiments in Section 6.2. In Section 6.3, we explore the parameter
dependence of the heating mechanism in undriven box experiments.

6.1 Linear Properties of the Electron Whistler Instability

According to our electron heating model, the presence of a mechanism to break the electron
adiabatic invariance is essential for generating electron entropy. As we have seen from
previous chapters, the electron whistler instability usually serves as the agent that breaks
the electron adiabatic invariance. Thus understanding the linear properties of the instability
would be helpful for interpreting the results described in upcoming sections.

Following Gary & Madland (1985), we solve the following dispersion relation for the
electromagnetic mode propagating parallel to the background magnetic field induced by
electron temperature anisotropy,

\[
0 = D^\pm(k_y, \Omega) \\
= \Omega^2 - c_e^2 k_y^2 + \omega_{pi}^2 \frac{\Omega}{k_y v_i} Z(\zeta_i^\pm) + \omega_{pe}^2 \frac{\Omega}{k_y v_{e,\|}^2} Z(\zeta_e^\pm) \\
+ \omega_{pe}^2 \left( \frac{T_{e,\perp}}{T_{e,\|}} - 1 \right) \left[ 1 + \zeta_e^\pm Z(\zeta_e^\pm) \right],
\]

where \( \zeta_e^\pm = (\Omega \pm \omega_{ce})/k_y v_{e,\|} \), \( v_{e,\|} = (2k_B T_{e,\|}/m_e)^{1/2} \), \( \zeta_i^\pm = (\Omega \pm \omega_{ci})/k_y v_i \),

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\( v_i = (2k_B T_i/m_i)^{1/2} \), and \( Z(\zeta) \) is the plasma dispersion function

\[
Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{\exp(-x^2)}{x - \zeta}.
\]  

(6.4)

In particular, we explore how the dispersion relation, i.e. the growth rate \( \gamma \), which is the imaginary part of the wave frequency \( \Omega \), varies as a function of the wavevector \( k_y \), under different levels of electron temperature anisotropy, \( T_{e,\perp}/T_{e,\parallel} \), and initial electron plasma beta parallel to the magnetic field \( \beta_{e,\parallel} \). For all calculations, we fix \( m_i/m_e = 49 \), \( v_i = 0.02 \).

Figure 6.1(a) compares the dispersion relation at fixed electron temperature anisotropy \( T_{e,\perp}/T_{e,\parallel} = 2 \) but different \( \beta_{e,\parallel} \), ranging from 2 to 32. For a given temperature anisotropy, the growth rate of the electron whistler instability increases monotonically with \( \beta_{e,\parallel} \), and the wavelength of the maximally growing mode increases, or equivalently the wave vector of the maximally growing mode decreases. Figure 6.1(b) depicts the trend of the dispersion relation at fixed \( \beta_{e,\parallel} = 8 \) but different levels of electron temperature anisotropy. We see that the growth rate of the electron whistler instability increases monotonically with increasing temperature anisotropy \( T_{e,\perp}/T_{e,\parallel} \), and the wavelength of the maximally growing mode decreases. It follows that for reaching a given growth rate of the instability, the temperature anisotropy needed is less for higher \( \beta_{e,\parallel} \) and the wavelength of the maximally growing mode is also longer. Indeed, Figure 6.1(c) shows that in order to reach a maximum growth rate of \( \sim 0.28\Omega_{ce} \), the necessary temperature anisotropy decreases from 2.5 for \( \beta_{e,\parallel} = 2 \) to 1.5 for \( \beta_{e,\parallel} = 32 \). The maximally growing wave vector decreases from \( 0.7\omega_{pe}/c \) to \( 0.35\omega_{pe}/c \).
Figure 6.1: Dispersion relation of the electron whistler instability, i.e. solution to Equation (6.3). Panel (a) shows dependence on $\beta_e, ||$ at fixed electron temperature anisotropy $T_{e, \perp}/T_{e, ||} = 2$; Panel (b) explores the dependence on temperature anisotropy at fixed $\beta_e, || = 8$; Panel (c) shows the dispersion relation of different combinations of $\beta_e, ||$ and $T_{e, \perp}/T_{e, ||}$ for a fixed maximum growth rate of $\gamma_{\text{max}} = 0.28\Omega_{ce}$. 
6.2 Parameter Dependence of the Heating Mechanism in Compressing Box Experiments

6.2.1 Dependence on $M_{s,sh}$

For a shock, the Mach number $M_s$ determines the density compression from upstream to downstream (in the high $\beta_p$ regime that we are focusing on, $\beta_p$ has very little additional effect). In the hydrodynamic limit ($\beta_p \to \infty$), the density jump across the shock is simply

\[
\frac{n_2}{n_1} = \frac{(\Gamma + 1)M_s^2}{(\Gamma - 1)M_s^2 + 2},
\]

which is a monotonically increasing function of $M_s$. For an isotropic gas with three degrees of freedom ($\Gamma = 5/3$), the density compression saturates at 4 as $M_s \to \infty$. In addition to the Rankine-Hugoniot relation that determines the density in the far downstream, the density overshoot at the shock front also increases with $M_s$ (Leroy, 1983). When translating to compressing box experiment setups, the effect of $M_s$ comes through the compression rate, $q \equiv \partial(n/n_0)/\partial t$. Intuitively, since the shock width is always on the order of $\sim 1r_{Li}$, the more the density jumps across the fixed shock width, the higher the compression rate. Quantitatively, we show in Figure 6.2 the density profiles as a function of electron comoving time, taken from shock simulations with $M_s$ ranging from 2 to 5 at fixed $\beta_{p0} = 16$. Indeed, the compression rate increases monotonically with $M_s$. For setting up the compressing box, as in the previous chapter, we adopt again a linear approximation of the compression profile and overplot the linear model in dashed lines for comparison (Figure 6.2).

In this subsection we investigate the role of the shock Mach number $M_{s,sh}$, which determines the compression rate $q$, in compressing box experiments. (We add the subscript
sh to distinguish that there is no shock in the periodic box experiments and $M_{s,sh}$ is just entering through the setup of initial conditions.) All boxes have fixed $\beta_{e0} = \beta_{i0} = 8$, and thus $\beta_{p0} = 16$, $kT_{e0} = 10^{-2}m_ec^2$, $m_i/m_e = 49$. As in the previous chapter, the motion of protons is frozen. We resolve the plasma skin depth with 10 cells. The simulation box is one-dimensional and has 864 cells along the $y$-direction. The background magnetic field is also aligned with $y$ to allow the growth of the electron whistler instability. We employ 1600 particles per cell per species. The numerical parameters are summarized in Table 6.1.

Figure 6.3 compares the results of compressing box simulations initialized with different $q$ at the same $\beta_{p0}$ (i.e. runs $Ms2c$, $Ms3c$, $Ms4c$, $Ms5c$ in Table 6.1).

Initially, the electron temperature anisotropy (Figure 6.3(b)) grows linearly with time due to the continuous density compression, i.e. $T_{e,\perp}/T_{e,\parallel} - 1 = qt$. Thus the electrons are driven to anisotropy faster with higher $q$, or equivalently higher $M_{s,sh}$. The electron temperature anisotropy excites the electron whistler waves. After the wave energy (Figure 6.3(a)) reaches $\sim 1\%$ of the background magnetic field energy, wave-particle scattering becomes efficient in isotropizing the electrons and breaking their adiabatic invariance. As a result, the electron temperature anisotropy reduces quickly to the marginal stability threshold of the electron whistler instability (Gary, 2005),

$$T_{e,\perp}/T_{e,\parallel} - 1 = \frac{0.21}{\beta_{e,\parallel}^{0.6}},$$

(dotted lines in Figure 6.3(b)), which is similar across all runs, since they have comparable $\beta_{e,\parallel}$. The time when the electron adiabatic invariance is most violently broken (Figure 6.3(c)), always corresponds to the time when the electron anisotropy shows the sharpest decrease (Figure 6.3(b)). Consistent with our electron heating model (Equation (6.2)), a steep
Table 6.1: Parameters for compressing box experiments discussed in Section 6.2.

<table>
<thead>
<tr>
<th>run name</th>
<th>$M_{s,sh}$</th>
<th>$q$</th>
<th>$\beta_{p0}$</th>
<th>$k_B T_{e0} / m_e c^2$</th>
<th>$N_{ppc}$</th>
<th>$L$</th>
<th>$c/\omega_{pe}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.5</td>
<td>16</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Ms3c/beta16c</td>
<td>3</td>
<td>2.5</td>
<td>16</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Ms4c</td>
<td>4</td>
<td>3.5</td>
<td>16</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Ms5c</td>
<td>5</td>
<td>4</td>
<td>16</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>beta4c</td>
<td>3</td>
<td>2.5</td>
<td>4</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>beta8c</td>
<td>3</td>
<td>2.5</td>
<td>8</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>beta32c</td>
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<td>2.5</td>
<td>32</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>130</td>
<td></td>
</tr>
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<td>beta64c</td>
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<td>2.5</td>
<td>64</td>
<td>$10^{-2}$</td>
<td>3200</td>
<td>173</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.2: The density profiles experienced by electrons as a function of the comoving time of the electron fluid, during propagation from upstream to downstream, for shock simulations with $M_s = 2, 3, 4, 5$. The dotted lines show our linear approximations to the shock compression profiles, $n_e/n_{e0} = (1 + qt)$ with $q = 1.5, 2.5, 3.5, 4$, used in the compression box experiments.
increase in entropy (Figure 6.3(d)) accompanies the rapid breaking of the electron adiabatic invariance and decrease of electron temperature anisotropy. Quantitatively, the adiabatic breaking rate is higher for higher $q$, as the electron temperature anisotropy reaches higher peak values in those runs before being reduced to the marginal stability level. This leads to a monotonically increasing entropy production during the exponential stage of the instability with increasing $q$. We can understand the trend of peak electron anisotropy, as before, from the competition between the compression rate (which drives the electron anisotropic) and the growth rate of the electron whistler instability (which drives the anisotropic electrons back to isotropy). Indeed, the growth rate of the instability (Figure 6.3(a)) increases with increasing $q$ (Figure 6.3(b)). In order to reach a faster growth rate of the electron whistler instability at comparable $\beta_{e,\parallel}$, the electron needs to accumulate a higher level of temperature anisotropy (recall Figure 6.1(b)). Thus the runs with higher $q$ reach higher peak electron anisotropy (Figure 6.3(c)). Incidentally, since the energy of the whistler waves is sourced by the free energy of the anisotropic electrons, the wave energy also increases with increasing $q$ (Figure 6.3(a)).

In the secular phase, all runs reach a similar level of temperature anisotropy ($T_{e,\perp}/T_{e,\parallel} \sim 1.2$). The adiabatic breaking rate also tends to a similar value (Figure 6.3(c)). According to Equation (6.2), both factors lead to a similar growth rate of electron entropy in the secular phase (Figure 6.3(d)). We emphasize that the evolution of electron entropy is again captured excellently by our electron heating model (see the thin dashed lines which show the integral version of the right hand side of Equation (6.1) and its agreement with the direct measurement of the integral version of the left hand side of the equation in Figure 6.3(d)).

In summary, at fixed $\beta_{p0}$, the effect of an increasing $M_{s,sh}$ is a faster compression rate,
The faster compression rate leads to a higher peak anisotropy and a higher adiabatic breaking rate of the electrons. As a result, the electron entropy increases monotonically with $q$. In shock simulations, we thus expect that, in the first ramp where the field amplification is provided by compression alone, the electron entropy will increase monotonically with $M_s$. Incidentally, in full shock simulations, at higher $M_s$, proton instabilities are present even at the first ramp, providing additional field amplification, which leads to even more efficient electron entropy production.

### 6.2.2 Dependence on $\beta_{p0}$

In this subsection, we study the dependence of electron heating in compressing box experiments on $\beta_{p0}$. In order to do so, we fix the compression rate at $q = 2.5\Omega_{ci}$ and vary the initial plasma beta $\beta_{p0}$ from 4 to 64. In anticipation of a longer wavelength of the relevant electron whistler mode, as suggested by the study on linear properties of the electron whistler instability in Section 6.1, we increase the box size proportional to $\sqrt{\beta_{p0}}$ for $\beta_{p0} = 32, 64$, as summarized in Table 6.1.

Figure 6.4 compares the results of the compressing box simulations beta4c, beta8c, beta16c, beta32c, beta64c. Since all the simulations use the same compression rate $q = 2.5$, the electron temperature anisotropies evolve initially at the same rate (Figure 6.4(b)) given by $T_{e,\perp}/T_{\parallel} - 1 = qt$. Under the same compression rate, the growth rate of the electron whistler instability needed to compete with the compression is also similar across different $\beta_{p0}$. Indeed, we observe similar growth rates of the electron whistler instability during the exponential phase (Figure 6.4(a)). As we have learned in Section 6.1, for a given growth rate of the instability, the amount of temperature anisotropy ($T_{e,\perp}/T_{\parallel}$) required to excite it decreases with increasing plasma beta. As a result, we find that the peak temperature
Figure 6.3: Dependence on $M_{s,ni}$, or equivalently the compression rate $q$, of various quantities in compressing box experiments $Ms_{2c}$, $Ms_{3c}$, $Ms_{4c}$, $Ms_{5c}$ (Table 6.1). We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the compressed field; (b) electron temperature anisotropy (solid lines) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (c) rate of violation of adiabatic invariance $-d\ln(T_{e,\perp}/B)$; (d) electron entropy change, measured from the electron distribution function as in Equation (4.27) by solid lines, and predicted from our heating model by thin dashed lines.
anisotropy during the exponential phase of the instability decreases monotonically with increasing $\beta_{p0}$ (Figure 6.4(b)). After the whistler instability grows strong enough to provide efficient pitch-angle scattering and break the electron adiabatic invariance, the electron anisotropy quickly reduces to the marginally stable threshold level (dotted lines in Figure 6.4(b)). During the same time, the adiabatic breaking rate reaches its maximum and electron entropy production is most efficient. Quantitatively, since the difference between the peak anisotropy and the marginal instability level decreases with $\beta_{p0}$, the electron entropy growth decreases monotonically with increasing $\beta_{p0}$ during the exponential phase.

During the secular phase, the electron temperature anisotropy saturates at the marginally unstable level, which decreases monotonically with increasing $\beta_{p0}$, as indicated by the linear theory of the instability. As a result, during the secular phase, both the temperature anisotropy and the adiabatic breaking rate are kept at a lower level with increasing $\beta_{p0}$, which further leads to less efficient electron entropy production.

We remark that the wave energy, when normalized to electron thermal energy (in the parallel direction), decreases with $\beta_{p0}$ (shown in the inset of panel (a)). Intuitively, since there is less energy in the wave to serve as the agent to break electron adiabatic invariance, the adiabatic breaking rate is smaller for higher $\beta_{p0}$.

In summary, when the field amplification is due to compression alone, the electron entropy production efficiency decreases monotonically with increasing plasma beta. However, as we shall see in the next section, the trend with $\beta_{p0}$ is opposite when the field amplification is provided by proton instability alone. As a result, in full shock simulation, where both compression and proton instability are sources of field amplification, there can be a weaker overall dependence on $\beta_{p0}$. 

\[ \text{206} \]
Figure 6.4: Dependence on $\beta_{p0}$ of various quantities in compressing box experiments beta4c, beta8c, beta16c, beta32c and beta64c. We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the compressed field; in the inset we also plot the energy in magnetic field fluctuations normalized by electron thermal energy in the parallel direction; (b) electron temperature anisotropy (solid lines) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (c) rate of violation of adiabatic invariance $-d \ln (T_{e, \perp}/B)$; (d) electron entropy change, measured from the electron distribution function as in Equation (4.27) in solid lines and predicted from our heating model in thin dashed lines.
6.3 Parameter Dependence of Heating in Undriven Box Experiments with Anisotropic Protons

In this section, we explore the parameter dependence of the heating mechanism when the source of field amplification is proton anisotropy alone. For this we use undriven box experiments with anisotropic protons.

6.3.1 Model of the Undriven Box Setup

We first describe how we set up the initial condition of the undriven box given the shock parameters $M_{s,sh}$ and $\beta_{p0,sh}$.

Our goal is to mimic the immediate shock downstream where proton anisotropy can drive instabilities that lead to field amplification on scales larger than that of the electrons. Since the electrons isotropize on a much more rapid time scale than the protons, we assume that the electrons are initially isotropic for simplicity.

We need to set the following parameters $T_{i0,\perp}, T_{i0,\parallel}, T_{e0}, \beta_{e0}$. Without resorting to measuring these values directly from full shock simulation, we can already get a good estimate from our analytic shock model.

When there are no plasma instabilities to isotropize the electrons and protons, the behavior of the plasma is decoupled in the directions perpendicular and parallel to the background magnetic field. The particles in the plasma essentially have only two degrees of freedom (only perpendicular to the magnetic field), corresponding to an adiabatic index of
2. This is demonstrated from the 2D out-of-plane and 1D shock simulations we presented in the Appendix of chapter 5. In our analytic model for initial conditions, we treat the motion perpendicular to the magnetic field as a gas with $\Gamma = 2$, whose density and mean temperature jump obey the Rankine-Hugoniot condition (see 7.A for a derivation of the Rankine-Hugoniot relation for magnetized shocks with arbitrary adiabatic index and shock obliquity angle)

$$
\frac{n_2}{n_1} = \frac{3\beta_p M_s^2}{2 + \beta_p (2 + M_s^2)},
$$

(6.7)

$$
t_{2RH, \Gamma=2} \equiv \frac{T_{2e, \perp} + T_{2i, \perp}}{T_{1e} + T_{1i}} = M_s^2 \left( 2 - \frac{2}{r_{2RH}} \right) + \frac{4 - 4r_{RH}}{\beta_p} + 4.
$$

(6.8)

We then assume that the electrons are first heated adiabatically in the perpendicular direction and that the plasma obeys the flux freezing condition, so that $T_{e, \perp} = r_{RH, \Gamma=2} T_{1,sh}$, where $T_{1,sh}$ is the plasma temperature in the upstream of the corresponding shock simulation. By requiring that the jump of the mean temperature obeys the Rankine-Hugoniot relation, we find that the initial proton perpendicular temperature in the undriven box must be

$$
T_{i0, \perp} = T_{1,sh} (2t_{2RH, \Gamma=2} - r_{RH, \Gamma=2}).
$$

(6.9)

The parallel direction of the proton and electron temperatures are assumed not to have changed across the shock. Thus we set

$$
T_{i0, \parallel} = T_{1,sh}.
$$

(6.10)

We then assume that the electrons isotropize rapidly via their own (whistler) instability without exchanging energy with the protons, so that for the purpose of the undriven box
initial condition, we obtain

\[ T_{e0} = T_{e0,\perp} = T_{e0,\parallel} = \frac{2r_{RH,\Gamma=2} + 1}{3} T_{1,sh}. \] (6.11)

The value of \( \beta_{e0} \) can then be set by assuming flux freezing and setting the density compression to be \( r_{RH,\Gamma=2} \), i.e.

\[ \beta_{e0} = \frac{2r_{RH,\Gamma=2} + \frac{1}{2} \beta_{p0,sh}}{3r_{RH,\Gamma=2}}. \] (6.12)

The parameters used for the undriven box experiments in this section are summarized in Table 6.2. We can see that the main effect of \( M_{s,sh} \) is on the proton initial anisotropy. According to Equation (6.9), because \( t_{2RH} \sim O(M_{s}^2) \), while \( r_{RH} \sim O(1) \), we have \( T_{i0,\perp}/T_{i0,\parallel} \sim O(M_{s,sh}^2). \)

### 6.3.2 Linear Properties of the Proton Cyclotron Instability

Since the source of field amplification in the undriven box simulations is provided by the proton cyclotron instability, it helps to understand the properties of this instability and its dependence on plasma beta and proton temperature anisotropy.

Following Davidson & Ogden (1975), we solve the dispersion relation for the electromagnetic mode propagating parallel to the background magnetic field (Equation (3) of Davidson & Ogden 1975)

\[
0 = D^\pm(k_y, \Omega) = \Omega^2 - c^2k_y^2 + \omega_{pe}^2 \frac{\Omega}{k_y v_e} Z(\zeta_e^+) + \omega_{pi}^2 \frac{\Omega}{k_y v_{i,\parallel}^+} Z(\zeta_i^+) \\
- \omega_{pi}^2 \left( 1 - \frac{T_{i,\perp}}{T_{i,\parallel}} \right) \left[ 1 + \zeta_i^+ Z(\zeta_i^+) \right],
\] (6.13)
Table 6.2: Parameters for undriven box experiments discussed in Section 6.3. All runs are initialized with $kT_{i,\parallel} = 2 \times 10^{-4} m_i c^2$.

<table>
<thead>
<tr>
<th>run name</th>
<th>$M_{s,sh}$</th>
<th>$\beta_{p1,sh}$</th>
<th>$T_{i0,\perp}/T_{i0,\parallel}$</th>
<th>$T_{e0}/T_{i0,\parallel}$</th>
<th>$\beta_{e0}$</th>
<th>$N_{ppc}$</th>
<th>$L , [c/\omega_{pe}]$</th>
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<td>$10^4$</td>
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<td>25.70</td>
<td>$2 \cdot 10^4$</td>
<td>46.3</td>
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</table>
where \( \zeta_{\pm} = (\Omega \pm \omega_{ce})/k_y v_e \), \( v_e = (2k_B T_e/m_e)^{1/2} \), \( \zeta_{\pm} = (\Omega \pm \omega_{ci})/k_y v_{i,\parallel} \), \( v_{i,\parallel} = (2T_{i,\parallel}/m_i)^{1/2} \), and \( Z(\zeta) \) is the plasma dispersion function

\[
Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \frac{\exp(-x^2)}{x - \zeta}.
\]

To set up a dispersion relation calculation of Equation 6.13, one choice of independent set of parameters is

\[
T_{i,\perp}/T_{i,\parallel}, T_{i,\parallel}, T_e, \beta_{e,0}, m_i/m_e.
\]

We employ the analytic model to compute quantities in the immediate shock downstream, as described in the previous subsection. This gives the values of \( T_{i,\perp}/T_{i,\parallel}, T_{i,0}, T_e, \beta_{e,0} \). for a a variety of \( M_{s,sh} \) and \( \beta_{p0,sh} \). We use both \( m_i/m_e = 49 \) and realistic \( m_i/m_e = 1836 \) to show that the linear property of the proton cyclotron instability depends very weakly on mass ratio.

Figure 6.5 shows the results. We see that with increasing \( M_{s,sh} \), which implies increasing \( T_{i,\perp}/T_{i,\parallel} \), the growth rate (\( \gamma \)) and the wavevector (\( k \)) of the fastest growing mode increases monotonically at fixed \( \beta_{p0} \). With fixed \( T_{i,\perp}/T_{i,\parallel} \) (i.e. fixed \( M_{s,sh} \)), the growth rate (\( \gamma \)) and the wavevector (\( k \)) of the fastest growing mode increases moderately with \( \beta_{p0} \). Comparing the dispersion relations corresponding to the two mass ratios, we see that the linear proton cyclotron mode is insensitive to the mass ratio, as long as there is a reasonable separation between the proton and electron scales. This again justifies our use of a reduced mass ratio \( m_i/m_e = 49 \) in various parts of this thesis. We remark however that, although the dispersion relations plotted in proton frequency (\( \Omega_{ci} \)) and length units (\( c/\omega_{pi} \)) is insensitive to mass ratio, the ratio of \( \Omega_{ce}/\Omega_{ci} \) and \( \omega_{pe}/\omega_{pi} \), scales with mass ratio as \( \propto m_i/m_e \) and \( \propto \sqrt{m_i/m_e} \). Thus the separation of space and time scale between proton instability and electron instability will be less clear when we use an artificially reduced mass ratio.
Figure 6.5: Dependence of the dispersion relation (growth rate $\gamma$ as a function of wavevector $k$) of the proton cyclotron instability on initial proton temperature anisotropy and plasma beta. The initial conditions for computing the various dispersion relations are generated by the method described in Section 6.3.1. The first row is computed using $m_i/m_e = 49$. The second row is computed using a realistic mass ratio $m_i/m_e = 1836$, while keeping the electron temperature fixed. The dispersion relation has very weak dependence on mass ratio due to the fact that the proton cyclotron instability is not resonant with the electrons.
6.3.3 Dependence on $M_{s,sh}$

In this subsection, we compare undriven box simulations $Ms2ud$, $Ms3ud$, $Ms4ud$, $Ms5ud$, which are initialized with fixed $\beta_{p0,sh} = 16$, but varying $M_{s,sh}$ from 2 to 5. The numerical parameters are summarized in Table 6.2. Figure 6.6 shows the results.

As expected from the scheme we use to set up the initial conditions, increasing $M_{s,sh}$ leads to increasing initial proton temperature anisotropy (Figure 6.6(b)). With the increasing availability of free energy from the proton temperature anisotropy, the proton wave energy increases monotonically as a result (Figure 6.6(a)). The large temperature anisotropy at similar $\beta_{0.\parallel}$ also leads to faster growth rate of the proton instability, as expected from the linear theory discussed in Section 6.3.2. As a result, the protons isotropize faster with increasing $M_{s,sh}$, despite the act that the higher $M_{s,sh}$ runs start with higher proton temperature anisotropy (Figure 6.6(b)).

In all cases except $M_{s,sh} = 2$, the proton waves are strong enough to drive electrons beyond the electron whistler threshold (dotted lines in Figure 6.6(d)). The electron waves are excited earlier with increasing $M_{s,sh}$ (Figure 6.6(c)), due to faster and stronger driving by proton waves.

The electron temperature anisotropy evolution is more complicated in the undriven box simulations than in the compressing box simulations we considered earlier. There, the field amplification was spatially uniform, but now, because of the spatially non-uniform nature of the proton cyclotron instability, the situation is a little more complicated. In Figure 6.6(d), we use solid linea to plot the time profilea of rhw spatial average of the electron temperature anisotropy. We also add a shaded region, to represent the 50%–90% percentile of the electron temperature anisotropy in different computational cells at a given time.
Figure 6.6: Dependence on $M_{s,\text{sh}}$ of various quantities in the undriven box experiments $M_{s2,\text{ud}}$, $M_{s3,\text{ud}}$, $M_{s4,\text{ud}}$, $M_{s5,\text{ud}}$. We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the initial field; (b) proton temperature anisotropy; (c) energy in electron-scale field fluctuations normalized to the electron parallel thermal energy; (d) electron temperature anisotropy (solid lines, shaded region represents 50% – 90% percentile of the value of electron temperature anisotropy in computational cells) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (e) rate of violation of the electron adiabatic invariance $-d\ln(T_{e,\parallel}/B)$; (f) electron entropy change measured from the electron distribution function, shown by thick solid lines, and predicted by the integral version of the right hand side of Equation (6.1), shown by thin dashed lines. The time axis of the $M_{s,\text{sh}} = 2$ run is squeezed by a factor of 2 for better comparison. For the electron wave power plot in panel (c), we apply high-pass filters for frequencies higher than $0.064\omega_{ce}$ and wavelengths shorter than $25c/\omega_{pe}$ for all runs.
With stronger proton waves (in higher $M_{s,sh}$ runs), the variations in electron temperature anisotropy from cell to cell is larger. Overall, faster and stronger proton waves induce more electron anisotropy, which leads to stronger electron waves (Figure 6.6(c)), which in turn helps break electron adiabaticity (Figure 6.6(e)). Combining the above mentioned effects, we see that electron entropy generation becomes more efficient with $M_{s,sh}$ (Figure 6.6(f)). The evolution of the entropy also agrees well with our quantitative prediction (integral version Equation (6.1)), shown as thin dashed lines in panel (f).

### 6.3.4 Dependence on $\beta_{p0,sh}$

In this subsection, we compare undriven box simulations beta8ud, beta16ud, beta32ud, beta64ud, which are initialized with fixed $M_{s,sh} = 3$, but varying $\beta_{p0,sh}$ from 8 to 64. Figure 6.7 shows the results.

At similar initial proton temperature anisotropy prescribed by our analytic setup model (Figure 6.7(b)), the plasma beta now plays the major role in determining the growth rate and strength of the proton instability. Naturally, with higher thermal content (higher $\beta_{p0,sh}$) and similar proton temperature anisotropy to supply the free energy of the proton waves, the proton wave energy with respect to the background magnetic energy (Figure 6.7(a)) increases monotonically with $\beta_{p0,sh}$. In accord with linear theory, the growth rate of the proton cyclotron instability also increases monotonically with $\beta_{p0,sh}$, leading to faster isotropization of the protons.

In all cases, the growth of proton waves drives electrons anisotropic above the whistler threshold (Figure 6.7(d)). With higher $\beta_{p0,sh}$, both the proton waves grow faster and the electron whistler threshold is lower, thus electrons become unstable earlier (Figure 6.7(c)). The strong driving from the proton waves, combined with the ease of exceeding the whistler
Figure 6.7: Dependence on $\beta_{p0,sh}$ of various quantities in the undriven box experiments beta8ud, beta16ud, beta32ud, beta64ud. We plot: (a) energy in magnetic field fluctuations, normalized to the energy of the initial field; (b) proton temperature anisotropy; (c) energy in electron-scale field fluctuations normalized to the electron parallel thermal energy; (d) electron temperature anisotropy (solid lines, shaded region represents 50% – 90% percentile of the value of electron temperature anisotropy in computational cells) and threshold condition for the electron whistler instability (dotted lines with the same color coding as the solid lines); (e) rate of violation of the electron adiabatic invariance $-d\ln(T_{e,\parallel}/B)$; (f) electron entropy change measured from the electron distribution function, shown by thick solid lines, and predicted by the integral version of the right hand side of Equation (6.1), shown by thin dashed lines. For the electron wave power plot in panel (c), we apply high-pass filters for frequencies higher than 3.1, 3.1, 1.9, 1.95$c_{ei}$ and wavelengths shorter than 20, 25, 35, 35$c_{ei}$/$\omega_{pe}$ for runs beta8ud, beta16ud, beta32ud, beta64ud, respectively. The decreasing cut in frequency and increasing cut in wavelength is because, with higher beta, the electron whistler instability is excited at weaker anisotropy and leads to lower frequency and longer wavelength of the maximally growing mode.
threshold, lead to higher adiabatic breaking rate of the electrons during the growth of the proton waves (Figure 6.7(e)). Overall, the electron entropy generation becomes more efficient with increasing $\beta_{p0,sh}$, when the field amplification is provided solely by proton instability.

### 6.4 Summary

In this chapter, we studied the parameter dependence of the electron heating mechanism under idealized conditions using compressing box experiments and undriven box experiments. These experiments are designed to mimic, respectively, conditions at the shock ramp, where the field amplification is mainly provided by density compression of the shock, and conditions in the shock downstream, where ion temperature anisotropy driven instability is an additional source of field amplification. We began in Section 6.1 by first reviewing the linear properties of the electron whistler instability, the key agent for irreversible heating of electrons. We find that for a fixed electron temperature anisotropy, the growth rate of the instability is larger for higher electron beta. At fixed electron beta, the growth rate is higher for higher temperature anisotropy. Thus for reaching a given growth rate, the electron temperature needed is less for higher electron beta. We then studied the dependence on $M_{s,sh}$ and $\beta_{p0}$ in compressing box experiments in Section 6.2. We find that the electron entropy production is more efficient with increasing $M_{s,sh}$ and decreasing $\beta_{p0}$ when the field amplification is provided by density compression alone. In Section 6.3, we find that the electron entropy production is also more efficient with increasing $M_{s,sh}$ but less efficient with decreasing $\beta_{p0}$. In all cases, our heating model (Section 4.2) continues to apply remarkably well. Since both density compression and proton driven waves are present in the shock
downstream, we expect that in full shock simulations, the electron entropy production will be increasingly efficient with $M_{s,\text{sh}}$ and not very sensitive to $\beta_{p0}$ due to the two opposing trends we find here. We will verify this in the next chapter.
Chapter 7

Electron heating in low Mach number perpendicular collisionless shocks. IV. Parameter dependence of electron temperature in shock simulations

In the previous three chapters, we developed an electron heating model that applies well to the reference shock simulation with $M_s = 3, \beta_{p0} = 16$. We showed that the key ingredients for irreversible heating of electrons are i) the existence of electron temperature anisotropy, and ii) a mechanism that provides a means to isotropize the electron distribution and thus break the adiabatic invariant. Magnetic field amplification during the sudden compression at the shock front leads to electron temperature anisotropy with $T_{e,\perp} > T_{e,\parallel}$. This excited the electron whistler instability, which provides the mechanism to break the electron adiabatic invariance. In the shock downstream, two different channels provide
magnetic field amplification. One is the density compression that is a natural consequence of oscillations in the post-shock fluid. The other is the proton temperature anisotropy-driven instabilities. We have utilized periodic box experiments that mimic the features of both shock compression and proton temperature anisotropy instability to study the parameter dependence of the electron heating mechanism. We learned that with increasing $M_s$, the compression is faster and the proton waves are stronger, thus we expect electron heating to be more efficient with increasing $M_s$. With increasing $\beta_p \rho_0$, electron heating is less efficient when the field amplification is due to compression alone at a fixed rate. However, the proton instability becomes stronger and grows faster with increasing $\beta_p \rho_0$, compensating for the loss of efficiency from the compression channel.

In this chapter, we apply the lessons learned from the above controlled experiments to understand the parameter dependence of electron heating in full shock simulations. We then summarize the results from our parameter scan and provide an empirical fitting function for future comparison with observations.

### 7.1 Simulation Setup

The set up of the shock simulations presented in this chapter largely follows the setup of the reference run described in Chapter 4. The shock moves in the $+\vec{x}$ direction and the background magnetic field is initialized perpendicular to it in the $+\vec{y}$ direction. The simulation plane lies in the $x - y$ plane, allowing the growth of both proton and electron temperature anisotropy instabilities that are crucial for electron heating. We vary $M_s$ and $\beta_p \rho_0$ as listed in Table 7.1. We point out that we only have control of the upstream velocity in the simulation frame (downstream rest frame), $V_0$, while the resulting shock velocity
is self-consistently determined by the shock property, most importantly by the effective
adiabatic index which is closely related to the asymptotic temperature anisotropy. Because
of this, the actual Mach number of the simulation may deviate slightly from our targeted
Mach number. We thus also list in Table 7.1 the measured Mach number,

\[ M_{\text{measured}} = \frac{V_{1,\text{measured}}}{c_s} = \frac{V_0 + V_{\text{sh,measured}}}{c_s (1 + V_0 \cdot V_{\text{sh,measured}})} . \] (7.1)

Note that, in most cases, the measured Mach number is within 2% of the targeted Mach
number. All simulations are initialized with \( T_{i0} = T_{e0} = T_0 = 10^{-2}m_ec^2/k_B \). The Debye
length of all the simulations is the same \( = 0.1c/\omega_{pe} \), and we use 10 computational cells
to resolve the electron skin depth. The transverse box size \( L_y \) spans \( \sim 21c/\omega_{pi} \) to allow
the full growth of proton instabilities in the downstream. For the purpose of running
long simulations to scan the parameter space, we use the reduced mass ratio \( m_i/m_e = 49 \), which we have shown in Chapter 5 agrees well with results obtained with a higher
mass ratio \( m_i/m_e = 200 \). We also run a few additional \( m_i/m_e = 200 \) simulations here to
confirm that the convergence applies for other Mach number as well. In addition, the better
scale separation in those runs help us visually separate the coexisting proton and electron
instabilities in the higher Mach number runs, as we shall see soon.

### 7.2 Dependence on \( M_s \)

In this subsection, we compare results from shock simulations with fixed \( \beta_{p0} = 16 \) but
varying \( M_s = 2 \) to 5, namely runs Ms2beta16, Ms3beta16, Ms4beta16 and Ms5beta16.

In Figure 7.1(a) we show the compression profile of plasma density (thick solid lines) and
magnetic field strength (thin solid lines). As expected from the Rankine-Hugoniot relation,
the compression across the shock increases with $M_s$ in the low Mach number regime we are interested in. The deviation between magnetic field compression (thin lines in Figure 7.1(a)) and density compression (thick lines in panel (a)) is more severe with increasing $M_s$, due to the increasing contribution to the magnetic field energy from plasma instabilities (Figure 7.1(b)). In Figure 7.2 we show the 2D pattern of the fluctuating component of $B_x$.

For the lower Mach number runs, $Ms2\beta16$ and $Ms3\beta16$, the first ramp is dominated by short wavelength electron whistler instability as shown in Chapter 4, while the proton waves slowly develop in the downstream. For the higher Mach number runs, $Ms4\beta16$ and $Ms5\beta16$, the proton instabilities already start growing at the shock foot, leading to a stronger wave energy that is comparable to the background magnetic field energy (red and orange lines in Figure 7.1(b)). We also notice that the wavelength of the proton instability keeps decreasing as $M_s$ increases. The trend of faster growth rate and decreasing wavelength at larger $M_s$ is a natural consequence of linear theory, since the proton temperature anisotropy increases with increasing $M_s$ (Figure 7.1(d)). The increase of initial proton temperature anisotropy was predicted by our analytic model for the immediate downstream environment (Section 6.3.1). Namely, immediately behind the shock, before pitch-angle scattering becomes efficient, only the perpendicular temperature of the plasma increases. The increase of the mean perpendicular temperature has to obey the Rankine-Hugoniot relation for a gas with $\Gamma = 2$. Since protons are much more massive than electrons, they gain most of the initial perpendicular temperature jump, which roughly scales as $\propto M_s^2$.

In addition, in the far downstream, where the protons have been isotropized by their own instability, thus having an adiabatic index close to $5/3$, the total proton temperature $T_i$ still shows a quadratic trend with respect to $M_s$ (Figure 7.1(c)). This is because the energy given up by protons to the waves is insignificant compared to their thermal energy and
Table 7.1: Parameters for shock simulations presented in this chapter. All runs are initialized with $T_{i0} = T_{e0} = T_0 = 10^{-2}m_ec^2/k_B$.

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thus we can use the argument of protons contributing to most of the Rankine-Hugoniot temperature jump for a gas of $\Gamma = 5/3$, which also scales like $M_s^2$. Incidentally, the peak proton anisotropy in run $\text{Ms5beta16}$ is roughly the same as run $\text{Ms4beta16}$. This is because the proton instability in run $\text{Ms5beta16}$ grows so rapidly that it quickly reduces the proton temperature anisotropy. In all cases except $\text{Ms2beta16}$, the proton cyclotron instability reduces the proton temperature anisotropy below the threshold far downstream of the shock (dotted lines in Figure 7.1(d)). In run $\text{Ms2beta16}$, the proton instability only starts to grow at a distance $\sim 10r_{Li}$ in the shock downstream (Figure 7.2(a)) and it grows to only a small amplitude. Correspondingly, the isotropization proceeds at a very slow pace, and the anisotropy has not yet decreased below the instability threshold at the end of our simulation.

As for the protons, the electron temperature also increases monotonically with $M_s$ (Figure 7.1(e)). This is to be expected just from adiabatic heating from compression alone, as the density compression increases with $M_s$ (Figure 7.1(a)). In addition, the irreversible heating of the electrons, quantified with super-adiabatic heating (Figure 7.1(g)) and electron entropy increase (Figure 7.1(h)), also become more efficient with increasing $M_s$. The reasons are two-fold. First, the compression rate is faster with increasing $M_s$. As we learned from Section 6.2.1, this drives the electrons to higher anisotropy and higher adiabatic breaking rate, and thus more efficient entropy production. Secondly, with increasing $M_s$, the proton waves also become stronger in the downstream, providing additional field amplification that drives electron anisotropic and thus induces adiabatic breaking behavior. In the case of $\text{Ms2beta16}$, the proton waves are very weak in the downstream due the low proton temperature anisotropy. From lessons learned from the undriven box experiments, the weak proton waves are barely capable of driving electrons above threshold. Indeed, we do
Figure 7.1: Dependence of shock properties as a function of $M_s$ at fixed $\beta_{p0} = 16$, as seen in shock simulations Ms2beta16, Ms3beta16, Ms4beta16 and Ms5beta16, at $t = 22 \Omega_{ci}^{-1}$ (the legend is in panel (d)). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density in thick lines and magnetic field strength in thin lines; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) excess of electron temperature beyond the adiabatic prediction for an isotropic gas; (h) change in electron entropy. The net increase in electron entropy increases monotonically with $M_s$. 
Figure 7.2: Dependence on $M_s$ of 2D structure of magnetic field fluctuation $\delta B_x$ in shock simulations Ms2beta16, Ms3beta16, Ms4beta16, Ms5beta16 at $t = 22 \Omega_{ci}^{-1}$. The $x$ coordinate is measured relative to the shock location $x_{sh}$, and both $x$ and $y$ are normalized to the proton Larmor radius $r_{Li}$. 
not notice electron entropy production beyond the first ramp, where the field amplification is provided by compression alone. In the case of \texttt{Ms4beta16} and \texttt{Ms5beta16}, the proton waves already grow to their peak strength in the first ramp, and the field amplification is further enhanced as compared to \texttt{Ms2beta16} and \texttt{Ms3beta16}, where density compression provides most of the field amplification in the first ramp. As a result, we see significantly more electron entropy production at the first ramp in \texttt{Ms4beta16} and \texttt{Ms5beta16}. In addition, in the high Mach number runs \texttt{Ms4beta16}, \texttt{Ms5beta16}, the proton waves sustain their strength for a few proton Larmor radii immediately behind the shock. In the same region, the protons waves further drive electrons anisotropic locally and cause continuous electron entropy production behind the first ramp. This is again in contrast with the lower Mach number runs. In \texttt{Ms3beta16}, there is a plateau of electron entropy production after the first ramp due to the delayed growth of proton waves while the plasma is undergoing expansion. In \texttt{Ms2beta16}, as mentioned before, electrons barely experience any additional non-adiabatic heating beyond the first density ramp due to the weakness of proton waves.

Though the existence of electron whistler instability may be hard to discern by eye in \texttt{Ms4beta16} and \texttt{Ms5beta16} from Figure 7.2(c)-(d), they definitely exist and facilitate entropy generation in electrons. The difficulty of discerning electron whistler modes by eye in the first shock ramp is mainly due to the reduced mass ratio. As we have pointed out in Section 6.3.2, with increasing $M_s$, the protons become more anisotropic and the wavelength of their instability decreases accordingly, approaching the electron length scale. In Figure 7.4 we show the same quantities as in Figure 7.2, but for the higher mass ratio runs \texttt{mi200Ms2}, \texttt{mi200Ms3}, \texttt{mi200Ms4}, \texttt{mi200Ms5}, at an earlier time $t\Omega_{ci} = 12.9$. Comparing Figures 7.2 and 7.4, we see that the wavelength of the proton modes in units of proton Larmor radius is unchanged from low to high mass ratio, as it should. But thanks to the better scale
separation with increasing mass ratio, we can clearly identify by eye the coexistence of proton modes and shorter wavelength electron modes at the shock front in the higher Mach number runs mi200Ms4 and mi200Ms5.

For completeness, we also show the 1D averaged shock profiles of mi200Ms2, mi200Ms3, mi200Ms4, mi200Ms5 at $t = 12.9\Omega_{ci}^{-1}$ in Figure 7.3. Comparing with Figure 7.1, we see that the profiles are almost identical, reassuring us again that the heating physics is insensitive to mass ratio, once there is reasonable scale separation, as we have already shown in Chapter 5.

### 7.3 Dependence on $\beta_{p0}$

In this subsection we investigate the dependence of electron heating on plasma beta at fixed Mach number. We focus our discussion on comparisons at fixed $M_s = 3$ in Section 7.3.1. We briefly discuss the results at fixed $M_s = 5$, which lead to similar conclusions, in Section 7.3.2.

#### 7.3.1 $M_s = 3$ runs

Figure 7.5 compares the results from runs Ms3beta4, Ms3beta8, Ms3beta16 and Ms3beta32. The density compression across the shock shows a very weak increasing trend with plasma beta (Figure 7.5(a)). There are two reasons contributing to this. First, at fixed $M_s$, the Rankine-Hugoniot relation predicts a weak dependence of the compression ratio on $\beta_{p0}$ in the high beta ($\beta_{p0} \gg 1$) regime. In addition, as $\beta_{p0}$ increases, the marginal proton temperature anisotropy in the downstream decreases (Figure 7.6(d)). This implies that the total electron-proton plasma is approaching having full 3 degrees of freedom (corresponding
Figure 7.3: Dependence on $M_s$ of shock simulations with $m_i/m_e = 200$, i.e. $m_{i200ms2}$, $m_{i200ms3}$, $m_{i200ms4}$ and $m_{i200ms5}$, at $t = 12.9 \Omega_{ci}^{-1}$ (the legend is in panel (d)). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density in thick lines and magnetic field strength in thin lines; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) electron temperature excess above the adiabatic prediction for an isotropic gas; (h) change in electron entropy. Comparing with Figure 7.1, which shows the same quantities from runs with $m_i/m_e = 49$, we confirm that electron heating depends only weakly on the mass ratio.
Figure 7.4: Dependence on $M_s$ of 2D structure of magnetic field fluctuation $\delta B_x$ in shock simulations $\text{mi200Ms2}, \text{mi200Ms3}, \text{mi200Ms4}, \text{mi200Ms5}$ at $t = 12.9 \Omega^{-1}_p$. The $x$ coordinate is measured relative to the shock location $x_{sh}$, and both $x$ and $y$ are normalized to the proton Larmor radius $r_{Li}$. Compared to Figure 7.2, which shows equivalent results for lower mass ratio runs, here we can identify the electron modes more easily thanks to the better separation of scales with increasing mass ratio.
to $\Gamma = 5/3$), as compared to only 2 degrees of freedom (corresponding to $\Gamma = 2$) if the parallel temperature does not change at all across the shock. According to the Rankine-Hugoniot relation, the compression ratio also increases from $\Gamma = 2$ to $\Gamma = 5/3$.

Figure 7.5(b) shows the magnetic wave energy of the runs under discussion. At relatively low proton temperature anisotropy ($T_{i,\perp}/T_{i,\parallel} \sim 7$), proton waves grow rather slowly in the downstream and the first ramp is still dominated by small amplitude ($\delta B^2/B_{\parallel}^2 < 10^{-2}$) electron whistler waves, whose 2D pattern in $\delta B_x$ can be seen in Figure 7.6. After the first ramp, proton waves grow sooner and reach higher peak amplitude with increasing $\beta_{p0}$. This is consistent with our expectation from undriven box experiments (Section 6.3) and linear theory (Section 6.3.2): with increasing $\beta_{p0}$, the proton instability threshold is lower and is crossed earlier in time. In addition, for similar proton temperature anisotropy, the growth rate of the proton stability is larger at higher $\beta_{p0}$. Also, with increasing beta, the free energy from proton temperature anisotropy becomes more significant compared to the background magnetic energy, thus leading to higher wave energy normalized by background magnetic energy. The stronger proton wave energy with respect to increasing $\beta_{p0}$ is also manifested in the additional magnetic field amplification (thin solid lines in Figure 7.5(a)) on top of the density compression (thick solid lines in Figure 7.5(a)). The proton waves quickly reduce the temperature anisotropy of protons below threshold values (dotted lines in Figure 7.5(d)).

The total proton temperature jump displays a very weak increasing trend with increasing $\beta_{p0}$ (Figure 7.5 (c)). This can be understood in a similar manner as the trend in density compression. The total temperature jump of the Rankine-Hugoniot relation at fixed adiabatic index $\Gamma$ increases weakly with $\beta_{p0}$ in the high beta regime we are exploring.

From the weak dependence of density compression on $\beta_{p0}$, we expect also weak depen-
dence of adiabatic heating of the electrons. In addition, our simulations show that the total heating of electrons depends very weakly on $\beta_{p0}$ (Figure 7.5(e)). This is because the non-adiabatic heating of electrons also depends very weakly on $\beta_{p0}$, as shown in Figure 7.5(g)-(h). We can understand this from the lessons we learned from controlled experiments in the previous chapter. When the field amplification is from uniform density compression alone, at fixed compression rate and increasing $\beta_{p0}$, we expect that the electron whistler instability can be triggered at lower electron temperature anisotropy and produce lower entropy. This is indeed happening in the first ramp of the shock simulations presented here, where we see that the proton waves have not yet grown. There, the electron temperature anisotropy is weaker with increasing $\beta_{p0}$ and the entropy production is decreasing as well, consistent with our conclusions from compressing box experiments. However, when proton waves also participate in field amplification, higher $\beta_{p0}$ actually helps the proton to grow stronger waves, as we see in Figure 7.5(b). The additional driving from the proton waves is stronger with increasing $\beta_{p0}$, which pushes electrons above whistler threshold and break adiabatic behavior more frequently, leading to more efficient entropy production. Indeed, in the second ramp, where proton waves have grown to their peak, runs with higher $\beta_{p0}$, i.e. runs with stronger proton waves, are more efficient in electron entropy production. When the two opposing trends are combined, the total non-adiabatic heating of electrons depends very weakly on $\beta_{p0}$. The comparison between Ms3beta16 (orange) and Ms3beta4 (blue) illustrates this point quite well. At the first ramp, where the field amplification is provided by compression only, the entropy production in Ms3beta16 is less efficient than Ms3beta4. However, as the proton waves in Ms3beta16 grow stronger than Ms3beta4 at $\sim 1.5$ to $3r_{Li}$ in the downstream, the entropy production in Ms3beta16 catches up. Finally, when combined with the fact that adiabatic heating of electrons also depends weakly on $\beta_{p0}$, we conclude
Figure 7.5: Dependence on $\beta_{p0}$ of shock simulations Ms3beta4, Ms3beta8, Ms3beta16 and Ms3beta32, at $t = 20.79_{\Omega_{ci}}^{-1}$ (the legend is in panel (d)). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density in thick lines and magnetic field strength in thin lines; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) electron temperature excess above the adiabatic prediction for an isotropic gas; (h) change in electron entropy. The increase in electron entropy is insensitive to $\beta_{p0}$. 

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Figure 7.6: Dependence on $\beta_{p0}$ of 2D structure of magnetic field fluctuation $\delta B_x$ in shock simulations Ms3beta4, Ms3beta8, Ms3beta16 and Ms3beta32 at $t = 2.7\Omega_{ci}^{-1}$. The $x$ coordinate is measured relative to the shock location $x_{sh}$, and both $x$ and $y$ are normalized to the proton Larmor radius $r_{Li}$. 
that the electron heating depends very weakly on $\beta_{p0}$ when $M_s$ is fixed.

### 7.3.2 $M_s = 5$ runs

Here, we show, supplementary to Section 7.3.1, the weak dependence of electron heating on $\beta_{p0}$ when the sonic Mach number is fixed at $M_s = 5$.

Figure 7.7 shows the same quantities as in Figure 7.5 but for simulations $\text{Ms5beta4}, \text{Ms5beta8}, \text{Ms5beta16}, \text{Ms5beta32}$. Our conclusions from Section 7.3 hold in general. Here we point out the main difference between $M_s = 3$ and $M_s = 5$ runs. At $M_s = 5$, the initial proton anisotropy (Figure 7.7(d)) is much stronger than at $M_s = 3$. This leads to stronger and faster growing proton waves (Figure 7.7(b)). In all cases, the proton waves already grow to the peak at the first density ramp. As a result, we no longer see the decreasing trend with $\beta_{p0}$ of electron non-adiabatic heating in the first ramp as in the $M_s = 3$ runs (Figure 7.5(h)). Instead, the compensating effect of stronger proton wave driving with higher $\beta_{p0}$ is already taking place in the first ramp. Consequently, the electron entropy production in $M_s = 5$ runs also show very weak $\beta_{p0}$ dependence.

### 7.4 Summary and Discussion

Finally, we summarize the results of the many simulations through which we explored the dependence of electron heating on $M_s$ and $\beta_{p0}$. For all measurements shown in this section, we first take the spatial average of the quantity in the far downstream, which is defined as where proton anisotropy has fallen below the instability threshold value. We then take time average of the spatial averages. We note that for runs with $M_s = 2$, the proton anisotropy has not decreased below the threshold even after $40\Omega_{ci}^{-1}$, due to the very slow growth of
Figure 7.7: Dependence on $\beta_{p0}$ of shock simulations Ms5beta4, Ms5beta8, Ms5beta16 and Ms5beta32, at $t = 22\Omega_{ci}^{-1}$ (the legend is in panel (d)). Along the shock direction of propagation, we plot the $y$-averaged profiles of: (a) number density in thick lines and magnetic field strength in thin lines; (b) energy in magnetic fluctuations, normalized to the energy of the frozen-in field; (c) mean proton temperature; (d) proton temperature anisotropy; (e) mean electron temperature; (f) electron temperature anisotropy; (g) excess of electron temperature beyond the adiabatic prediction for an isotropic gas; (h) change in electron entropy. The increase in electron entropy is insensitive to $\beta_{p0}$. 

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proton waves. We thus report the spatial averages starting from \( \sim 20r_{Li} \) downstream of the shock. However, we point out that this does not affect our estimate of the electron heating much because the electron heating in \( M_s = 2 \) runs are largely adiabatic anyway as the proton waves are too weak to further drive electron anisotropy and induce entropy production downstream.

Figure 7.8(a) summarizes the marginal proton temperature anisotropy in the far shock downstream. As mentioned above, since the \( M_s = 2 \) runs has not yet reached its asymptotic state in terms of proton anisotropy, our measurements are merely upper bounds (indicated by the downwards arrows). The residual proton anisotropy decreases with increasing \( \beta_{p0} \) and increasing \( M_s \). We can understand this trend by invoking our analytic model for the immediate shock downstream (Section 6.3.1) and assuming that the energy of the proton waves is insignificant compared to the proton thermal energy. We assume that, immediately after the shock,

\[
T_{i0,\perp} = T_0(2t_{2RH,\Gamma=2} - r_{RH,\Gamma=2}) ,
\]

and

\[
T_{i0,\parallel} = T_0 ,
\]

where

\[
r_{RH,\Gamma=2} \equiv \frac{n_2}{n_1} = \frac{3\beta_{p0}M_s^2}{2 + \beta_{p0}(2 + M_s^2)} ,
\]

\[
t_{2RH,\Gamma=2} \equiv \frac{T_{2e,\perp} + T_{2i,\perp}}{T_{1e} + T_{1i}} = M_s^2 \left( 2 - \frac{2}{r_{RH}^2} \right) + \frac{4 - 4r_{RH}}{\beta_{p0}} + 4 .
\]
We then assume that the total proton temperature,

\[ T_i \equiv \frac{2T_{i,\perp} + T_{i,\parallel}}{3}, \quad (7.6) \]

equals

\[ T_i = \frac{4t_{2RH,\Gamma=2} - 2t_{RH,\Gamma=2} + 1}{3} T_0, \quad (7.7) \]

and is conserved. Though this is not strictly speaking true due to the loss of energy of protons to the proton waves, in the high beta regime we are exploring, this is not a bad assumption. Correspondingly, the total proton plasma beta

\[ \beta_i \equiv \frac{8\pi nT_i}{B^2} = \beta_{p0} \frac{4t_{2RH,\Gamma=2} - 2t_{RH,\Gamma=2} + 1}{6} \frac{1}{\rho_{RH,\Gamma=5/3}}, \quad (7.8) \]

where we assume that the jump of density and magnetic field strength is the same by flux freezing, and is equal to \( \rho_{RH,\Gamma=5/3} \), which is the density jump of a shocked gas with \( \Gamma = 5/3 \). Since the upper bound of proton anisotropy regulated by proton cyclotron and mirror instabilities is given by

\[ \frac{T_{i,\perp}}{T_{i,\parallel}} - 1 \lesssim \frac{1.1}{\beta_{i,\parallel}^{0.55}}, \quad (7.9) \]

we solve for \( T_{i,\perp}/T_{i,\parallel} \) at the equality of Equation (7.9),

\[ \frac{T_{i,\perp}}{T_{i,\parallel}} - 1 - \frac{1.1}{\beta_{i,\parallel}^{0.55}} = 0, \quad (7.10) \]

by using Equations (7.8),(7.4), (7.5).

The solutions to Equation (7.10) at different \( \beta_{p0} \) as a function of \( M_s \) are plotted as dash dotted lines in Figure 7.8(a). They capture the trend of decreasing asymptotic proton
temperature anisotropy with increasing $\beta_{p0}$ and $M_s$. Essentially, this is because $\beta_i$ in the downstream increases with $\beta_{p0}$ and $M_s$ as a result of the Rankine-Hugoniot relations. In addition, all the simulations that have reached the asymptotic state in terms of proton anisotropy (i.e. runs with $M_s > 2$), do indeed stay below our prediction of the upper bound of the proton anisotropy.

Figure 7.8(b) shows the density compression across the shock. For comparison, we over-plot the Rankine-Hugoniot density jump for a gas with $\Gamma = 2$ (or 2 degrees of freedom, dash-dotted lines) and $\Gamma = 5/3$ (or 3 degrees of freedom, dashed lines) for different $\beta_{p0}$ (differentiated by colors). Since, in all of our simulations, both protons and electrons isotropize to a large extent (i.e. they end up with more than 2 degrees of freedom), the measured shock density jump is always larger than the prediction for a gas with $\Gamma = 2$. At fixed $M_s$, the density jump of the shock approaches the Rankine-Hugoniot prediction for $\Gamma = 5/3$ as $\beta_{p0}$ increases. This is because the residual proton anisotropy diminishes with increasing $\beta_{p0}$ (Figure 7.8(a)) and the gas comes closer to attaining the full 3 degrees of freedom. The increase with respect to $M_s$ is simple to understand from the Rankine-Hugoniot relation alone.

Figure 7.9(a) shows the total electron temperature jump across the shock and Figure 7.9(b) shows the non-adiabatic heating of electrons quantified by $(T_{e,2} - T_{e,ad}) / T_{e0}$. The trend of non-adiabatic electron heating in panel (b) reflects our analysis in Section 7.2 and Section 7.3: it increases monotonically with increasing $M_s$ and is nearly independent of $\beta_{p0}$. In fact, the data can be well fit by a quadratic model that depends only on $M_s$

$$\frac{T_{e,2} - T_{e,ad}}{T_{e0}}(M_s) = 0.044 \cdot M_s \cdot (M_s - 1),$$

(7.11)
Figure 7.8: Summary of time- and space-averaged quantities far downstream of the shock in simulations with $m_i/m_e = 49$ listed in Table 7.1. The symbols in Panel (a) show the measured asymptotic proton temperature anisotropy. In dashed dotted lines we plot the predicted upper bound on the proton anisotropy from Equation (7.10). The symbols in Panel (b) show the density compression. For comparison, we show the Rankine-Hugoniot density jump predictions for a gas with 3 degrees of freedom ($\Gamma = 5/3$) in dashed lines and with 2 degrees of freedom ($\Gamma = 2$) in dash-dotted lines.
which is shown as the black dashed line in Figure 7.9(b). We note that this fitting formula exhibits the correct boundary behavior, viz., when \( M_s = 1 \) (no shock), there is no non-adiabatic heating.

The total electron temperature (Figure 7.9(a)) increases very weakly with increasing \( \beta_{p0} \). This trend is mostly due to the adiabatic heating

\[
\frac{T_{e,\text{ad}}}{T_{e0}} = (n_2/n_1)^{2/3},
\]

which depends on the density jump \( n_2/n_1 \) and therefore increases with \( \beta_{p0} \) (see Figure 7.8(b), which shows that the density jump increases with increasing \( \beta_{p0} \), because of increasing isotropization). However, we expect the trend to saturate at yet larger \( \beta_{p0} \) as the protons are nearly isotropic at \( \beta_{p0} = 32 \) (they have their full 3 degrees of freedom) for all runs with \( M_s > 2 \).

Finally, we can combine the empirical fit for the non-adiabatic electron heating given in Equation 7.11 with the adiabatic heating as predicted by the Rankine-Hugoniot density jump to predict the total electron temperature jump across the shock:

\[
\frac{T_{e2}}{T_{e0}} = \frac{r^{2/3}_{\text{RH,G=5/3}} + 0.044 \cdot M_s \cdot (M_s - 1)}{2t_{2,\text{RH,G=5/3}} - r^{2/3}_{\text{RH,G=5/3}} - 0.044 \cdot M_s \cdot (M_s - 1)}. \tag{7.12}
\]

Then, using the mean temperature jump from the Rankine-Hugoniot relation and Equation (7.12), we can deduce the proton temperature jump, or equivalently the electron-to-ion temperature ratio in the shock downstream:

\[
\frac{T_{e2}}{T_{i2}} = \frac{r^{2/3}_{\text{RH,G=5/3}} + 0.044 \cdot M_s \cdot (M_s - 1)}{2t_{2,\text{RH,G=5/3}} - r^{2/3}_{\text{RH,G=5/3}} - 0.044 \cdot M_s \cdot (M_s - 1)}. \tag{7.13}
\]
Figure 7.9: Summary of time- and space-averaged shock downstream quantities from simulations with $m_i/m_e = 49$. Panel (a) shows the total electron temperature jump across the shock. Panel (b) shows the non-adiabatic heating of the electrons. For comparison, we show the quadratic best-fit model (Equation (7.11)) as the dashed line. Panel (c) shows the post-shock electron-to-proton temperature ratio. For comparison, the predicted ratio (Equation (7.13)) is shown by the dashed line.
In Figure 7.9(c), we compare the prediction from Equation (7.13) (black dashed line, calculated using $\beta_{p0} = 32$ for the Rankine-Hugoniot related quantities) with data from our simulations. We see an excellent agreement.

### 7.4.1 Comparison to Observations

Currently, due to the lack of diagnostics of proton temperature from cluster merger shocks, we cannot directly compare our simulation results of post-shock electron-to-ion temperature ratio to corresponding observations in high beta shocks. However, the geophysics community has direct measurements of the particle distributions of both electrons and protons downstream of planetary shocks using satellite data. The main caveat is that planetary shocks usually correspond to low plasma beta $\beta_{p0} \lesssim 1$. Thus the most appropriate simulations in our work for comparison are the runs Ms2beta4, Ms3beta4, Ms3beta4, Ms5beta4, which we show as blue stars in Figure 7.10. The satellite data are taken from the compilation in Ghavamian et al. (2013), with original data from Schwartz et al. (1988) and Masters et al. (2011), which measured the electron-to-ion temperature ratio in Earth’s bow shock and Saturn’s bow shock, respectively. We note that the data are usually presented in terms of the magneto-sonic Mach number,

$$M_{ms} \equiv \frac{V_1}{\sqrt{v_A^2 + c_s^2}} \quad (7.14)$$

where $v_A$ is the Alfvenic velocity. This is related to the sonic Mach number and plasma beta by

$$M_{ms} = M_s \frac{1}{\sqrt{1 + 2/(\Gamma \beta_{p0})}} \quad (7.15)$$

Figure 7.10 shows that our simulation results (blue stars) are consistent with the obser-
Figure 7.10: Comparison of the prediction of the shock heating model with post-shock electron-to-proton temperature ratios observed in planetary shocks. The green open circles are measurements made by Schwartz et al. (1988) in Earth’s bow shock, and black open circles are measurements made by Masters et al. (2011) in Saturn’s bow shock. The large filled blue stars show the shock downstream $T_e/T_i$ measured in our simulations with $\beta_p = 4$: Ms2beta4, Ms3beta4, Ms4beta4, Ms5beta4. The temperature ratio predicted by Equation (7.13) is shown by the blue solid line, which is consistent with the observational data. For contrast, we also show by the blue dotted line the prediction if electrons do not experience any irreversible heating.
vations (open circles). Furthermore, the predicted post-shock electron-to-ion temperature ratio from our Equation (7.13) (blue solid line in Figure 7.10) correctly captures the trend of the data in the higher $M_{\text{ms}}$ regime. Specifically, both the data and our prediction suggest that $T_{e2}/T_{i2}$ levels off at a constant value in the high Mach number regime, because both $T_{e2}$ and $T_{i2}$ scale like $M_{\text{ms}}^2$. For contrast, we also show the predicted electron-to-ion temperature ratio if we assume that electrons are only heated adiabatically (blue dotted line). This is clearly not consistent with the data.

We point out that the large spread in the data is likely due to the dependence on shock obliquity angle, which we have not explored in this work (we only considered perpendicular shocks).

In conclusion, in this chapter, we built on the lessons learned from controlled experiments in previous chapters, and studied the dependence of electron heating on Mach number $M_{\text{s}}$ and plasma beta $\beta_{p0}$ from full 2D shock simulations. The results are summarized in Figure 7.9. We find that the non-adiabatic electron heating increases monotonically with $M_{\text{s}}$ and is fairly insensitive to $\beta_{p0}$. We provide a quadratic fitting formula for the non-adiabatic electron heating (Equation (7.11)). When combined with the Rankine-Hugoniot shock jump conditions, this allows us to predict the post-shock electron-to-ion temperature ratio by Equation (7.13). We compare our simulation results and the prediction from the fitting function to observational data on planetary shocks and find good agreement.
7.A Shock jump condition for oblique magnetized shocks

Following similar normalization conventions as in Tidman & Krall (1971)\(^1\), we derive the jump conditions for non-relativistic magnetic shocks with arbitrary adiabatic index $\Gamma$ and shock obliquity angle.

We set the shock to be moving along $x$-direction and the background magnetic field lying along $z$-direction. We make the reasonable assumptions that far away from the shock transition region, there is no current and net charge, so the density and velocity of two species are the same, i.e. $N_{e1,2} = N_{i1,2}$, $\vec{v}_{e1,2} = \vec{v}_{i1,2}$ where the subscript 1, 2 denotes far upstream and far downstream respectively. By conservation of mass, momentum and energy of the fluid consisting of ion and electron, we have

$$N_1 V_1 = N_2 V_{2x} \quad (7.16)$$

$$(m_i + m_e) N_1 V_1^2 + N_1 k(T_{1i} + T_{1e}) + \frac{B_{1z}^2}{8\pi} = (m_i + m_e) N_2 V_{2x}^2 + N_2 k(T_{2i} + T_{2e}) + \frac{B_{2z}^2}{8\pi} \quad (7.17)$$

$$N_1 V_1 \left( (m_i + m_e) \frac{V_1^2}{2} + \frac{\Gamma}{\Gamma - 1} k(T_{1i} + T_{1e}) \right) + \frac{V_1 B_{1z}^2}{4\pi} = N_2 V_{2x} \left( (m_i + m_e) \frac{V_{2x}^2}{2} + \frac{\Gamma}{\Gamma - 1} k(T_{2i} + T_{2e}) \right) + \frac{V_{2x} B_{2z}^2}{4\pi} \quad \frac{4\pi}{4\pi} - \frac{V_{2x} B_{2x} B_{2z}}{4\pi} \quad (7.18)$$

\(^1\)We also do not assume $m_i \gg m_e$ and keep both $m_i$ and $m_e$ in the expressions.
In addition, we also assume that far away from the shock transition region, there is neither electric field nor any turbulent field and the fluid is not experiencing any net force so that

\[ V_1 B_{1z} = V_{2x} B_{2z} - V_{2z} B_x. \] (7.19)

\[ \frac{B_{1z} B_x}{4\pi (m_i + m_e)} = \frac{B_{2z} B_x}{4\pi (m_i + m_e)} - N_2 V_{2x} V_{2z} \] (7.20)

Defining dimensionless variables

\[ n_2 = \frac{N_2}{N_1}, \quad u_{2x} = \frac{V_{2x}}{V_1}, \quad v_{1,2} = \sqrt{\frac{k(T_{1,2z} + T_{1,2e})}{(m_i + m_e) V_1^2}}, \quad b_{1,2z} = \sqrt{\frac{B_{1,2z}^2}{4\pi (m_i + m_e) N_1 V_1^2}}. \] (7.21)

we obtain

\[ 1 = n_2 u_{2x}, \] (7.22)

\[ 2 \left( v_1^2 + 1 \right) + b_{1z}^2 = 2 n_2 \left( v_2^2 + u_{2z}^2 \right) + b_{2z}^2, \] (7.23)

\[ 1 + \frac{2\Gamma}{\Gamma - 1} v_1^2 + 2 b_{1z}^2 = \frac{2\Gamma}{\Gamma - 1} v_2^2 + u_{2x}^2 + u_{2z}^2 + 2 b_{2x} b_{1z}, \] (7.24)

\[ b_{1z} = u_{2x} b_{2z} - u_{2z} b_x. \] (7.25)

\[ b_{1z} b_x = b_{2z} b_x - u_{2z}. \] (7.26)
Using Equation (7.25) and (7.26), we can solve for $b_{2z}$ and $u_{2z}$, obtaining

$$b_{2z} = \frac{b_{1z} \left(1 - b_{2}^2\right)}{(u_{2x} - b_{2}^2)}.$$  \hspace{1cm} (7.27)

$$u_{2z} = \frac{b_{1z} b_{2} (1 - u_{2x})}{(u_{2x} - b_{2}^2)}.$$  \hspace{1cm} (7.28)

One can then use Equation (7.22) to eliminate $n_{2}$ in Equation (7.23), and use Equation (7.24) to eliminate $v_{2}$ in Equation (7.23), and use Equation (7.27), (7.28) to eliminate $b_{2z}$ and $u_{2z}$ in Equation (7.23). This results in a forth order equation for $u_{2x}$, which has a trivial root at $u_{2x} = 1$, corresponding to no shock at all. Factoring out the trivial solution and a few multiplicative constants, we obtain

$$\frac{b_{1z}^2 (u_{2x} (\Gamma(u_{2x} - 1) + 2) - b_{2}^2 (\Gamma(u_{2x} - 1) + u_{2x} + 1))}{(b_{2}^2 - u_{2x})^2} + \Gamma (-u_{2x} + 2v_{1}^2 + 1) - u_{2x} - 1 = 0.$$  \hspace{1cm} (7.29)

The explicit form of the cubic equation for $\Gamma = 5/3$ has been given by Tidman & Krall (1971), which we reproduce here

$$4u_{2x}^3 - \frac{1}{2}u_{2x}^2 (5b_{1z}^2 + 16b_{x}^2 + 10v_{1}^2 + 2)$$
$$- \frac{1}{2}u_{2x} (-8b_{1z}^2 b_{x}^2 + b_{1z}^2 - 8b_{x}^4 - 20b_{x}^2 v_{1}^2 - 4b_{x}^2) - b_{x}^2 (5b_{1z}^2 + 5b_{x}^2 v_{1}^2 + b_{x}^2) = 0.$$  \hspace{1cm} (7.30)

In addition, we supply the explicit form of the cubic equation when $\Gamma = 2$

$$3u_{2x}^3 - u_{2x}^2 (2b_{1z}^2 + 6b_{x}^2 + 4v_{1}^2 + 1)$$
$$+ u_{2x} (3b_{1z}^2 b_{x}^2 + 3b_{x}^4 + 8b_{x}^2 v_{1}^2 + 2b_{x}^2) - b_{x}^2 (4b_{x}^2 v_{1}^2 + b_{x}^2 + b_{1z}^2) = 0.$$  \hspace{1cm} (7.31)
### 7.A.1 The special case of perpendicular shocks

In the special case of perpendicular shocks, i.e. \( b_x = 0 \), Equation (7.29) reduces to a simple quadratic equation

\[
\begin{align*}
[\Gamma + 1] u_{2x}^2 - u_{2x} [\Gamma b_{1z}^2 + 2 \Gamma v_1^2 + \Gamma - 1] + b_{1z}^2 [\Gamma - 2] &= 0. 
\end{align*}
\] (7.32)

For gas with 2 degrees of freedom, i.e. \( \Gamma = 2 \), we only have one simple non-trivial solution

\[
u_{2x} = \frac{1 + 2 b_{1z}^2 + 4 v_1^2}{3},
\] (7.33)

or

\[
r_{RH} \equiv u_{2x}^{-1} = \frac{3}{1 + 2 b_{1z}^2 + 4 v_1^2}.
\] (7.34)

The total temperature jump is

\[
t_{2RH} = \frac{T_{2e} + T_{2i}}{T_{1e} + T_{1i}} = \frac{v_{2}^2}{v_{1}^2} = \frac{1 + 4 v_1^2 + 2(1 - r_{RH}) b_{1z}^2 - r_{RH}^{-2}}{4 v_1^2}.
\] (7.35)

To make it easier to use the formula, we provide conversions of the dimensionless parameters from the usual shock parameters, the sonic Mach number

\[
M_s \equiv \frac{V_1}{C_s} = \sqrt{\frac{V_1^2 (m_i + m_e)}{\Gamma k (T_{1i} + T_{1e})}},
\] (7.36)

and plasma beta

\[
\beta_p \equiv \frac{8 \pi N_1 k (T_{1i} + T_{1e})}{B_1^2},
\] (7.37)
which follows

\[ v_1^2 = \frac{1}{\Gamma M_s^2}, \]  

(7.38)

\[ b_{1z}^2 = \frac{2}{\Gamma M_s^2 \beta_p}. \]  

(7.39)

In summary, for given \( M_s \) and \( \beta_p \), one can utilize Eqs. (7.38) and (7.39) to plug in to Eq. (7.34) and then Eq. (7.35) to determine the density and total temperature jump, which we write out explicitly for \( \Gamma = 2 \)

\[ r_{RH, \Gamma=2} = \frac{3 \beta_p M_s^2}{2 + \beta_p (2 + M_s^2)}, \]  

(7.40)

\[ t_{2RH, \Gamma=2} = M_s^2 \left( 2 - \frac{2}{r_{2RH}^2} \right) + \frac{4 - 4r_{RH}}{\beta_p} + 4. \]  

(7.41)

We notice that as \( M_s \rightarrow \infty, r_{RH} \rightarrow 3, t_{2RH} \propto M_s^2 \gg r_{RH}. \)

When \( \Gamma = 5/3, u_{2x} \) is the positive root of the quadratic equation

\[ 8u_{2x}^2 - u_{2x} \left( 5b_{1z}^2 + 10v_1^2 - 2 \right) + b_{1z}^2 = 0. \]  

(7.42)

The density jump is the inverse of \( u_{2x} \) and the temperature jump can be obtained from substitution to Equation (7.23).
Chapter 8

Conclusions and Future Directions

In this thesis, I studied particle acceleration and heating in low Mach number collisionless shocks by means of \textit{ab initio} particle-in-cell (PIC) simulations. The primary astrophysical application of my results is to merger shocks in galaxy clusters. Thermal and non-thermal radiation from these shocks has been observed extensively in X-ray and radio bands. However, an understanding of the microphysics that governs particle heating and acceleration in these low Mach number shocks has been lacking.

In the first part of this thesis, Chapters 2 and 3, I focused on addressing the problem of particle acceleration. In Chapter 2, I showed that electrons are efficiently accelerated in low Mach number quasi-perpendicular shocks. I identified a Fermi-like electron acceleration mechanism in which particle injection is governed by the shock drift acceleration (SDA) mechanism. The non-thermal electrons bounce between the upstream region and the shock front, with each reflection at the shock resulting in energy gain via SDA. The upstream scattering is provided by magnetic waves associated with the oblique electron firehose instability. These waves are self-generated by the electrons escaping ahead of the shock.

In Chapter 3, I explored how the efficiency of electron acceleration depends on pre-shock conditions of the plasma. I found that the mechanism I identified works for shocks with
high plasma beta at nearly all magnetic field obliquities, and for electron temperatures in the range relevant for galaxy clusters. My findings offer a natural solution to the conflict between the bright radio synchrotron emission observed from the outskirts of galaxy clusters and the low electron acceleration efficiency previously assumed for low Mach number shocks.

The above studies leave several questions open, suggesting directions for future research. A direct extension would be to follow the simulations to even longer times in order to address the following questions.

First, in the Fermi-type acceleration mechanism I identified, the electrons bounce between the upstream and the shock front. The upstream scattering is provided by electron firehose waves and the bounce at the shock is by SDA. This bypasses the problem that electrons need to have Larmor radii comparable to that of thermal ions in order to participate in Fermi-type acceleration, and thus solves the electron injection problem. However, as the non-thermal electrons continue to reach higher energies through the Fermi-type acceleration identified in this thesis, their Larmor radii will eventually grow larger than the shock thickness. At that point, they will penetrate to the downstream during their gyrations. It remains an open question what kind of plasma instability in the downstream might scatter these non-thermal electrons and whether they will continue to be accelerated to yet higher energies.

Secondly, one key assumption for deriving the universal power-law particle momentum distribution from DSA theory (Equation (1.6)) is that the particle distribution is isotropic. The Fermi-type acceleration mechanism identified in this thesis does not guarantee isotropy, since SDA always accelerates the parallel momenta of particles more than their perpendicular momenta. It remains an open question whether, once the non-thermal electrons start to diffuse between both sides of the shock, the slope of their momentum distribution would actually match the predictions from DSA theory (i.e. Equation (1.6)).
In the second part of the thesis, Chapters 4 - 7, I addressed the problem of particle heating in low Mach number perpendicular shocks. In Chapter 4 and 5, I focused on a representative shock with $M_s = 3$ and $\beta_p = 16$. In this reference simulation, the post-shock electron temperature exceeds the prediction from pure adiabatic compression by $\simeq 15\%$. This means that the electrons gain a modest amount of entropy as they cross the shock. The resulting downstream electron-to-proton temperature ratio is $\simeq 0.45$. I found that two basic ingredients are needed for electron entropy production: (i) an electron temperature anisotropy, induced by field amplification coupled to adiabatic invariance; and (ii) a mechanism to break the electron adiabatic invariance itself. In shocks, I found that field amplification happens through two channels: first, the density compression of the shock leads to an increase in the frozen-in field; second, proton temperature anisotropy in the shock downstream generates strong proton cyclotron and mirror instabilities. Furthermore, I showed that the electron temperature anisotropy induced by field amplification exceeds the threshold of the electron whistler instability. The resulting growth of whistler waves breaks the electron adiabatic invariance, and allows for efficient entropy production. I developed an analytical model for electron irreversible heating and showed that it is in excellent agreement with the simulation results.

In Chapter 6 and 7, I explored the dependence of the electron heating mechanism on the two principal shock parameters, the sonic Mach number $M_s$ and the upstream plasma beta $\beta_p$. I found that electron entropy production becomes more and more efficient with increasing Mach number, but is insensitive to plasma beta in the high beta regime I focused on. I obtained a fitting formula for irreversible heating in electrons and used it to predict the electron-to-proton temperature ratio in the shock downstream. I showed that the predictions from this model are in good agreement with observational data.
One direct extension of the work reported in this part of the thesis would be to explore the
dependence of the electron heating mechanism I identified on the magnetic obliquity angle,
which will almost certainly introduce some changes in the downstream electron temperature
at the same Mach number and plasma beta. As a starting point, in this thesis, I restricted
my attention to perpendicular shocks, so I that I could circumvent the complications from
the non-thermal particles that naturally arise in oblique shocks (shown in the first part of
the thesis). The heating model I developed in Chapter 4.2 assumes that the electrons
consist of a single thermal population with a bi-Maxwellian distribution. Appropriate mod-
ifications will need to be incorporated in the heating model to account for the existence of
an additional non-thermal population. As we have shown in the first part of the thesis, the
non-thermal electrons tend to have $T_{e,\parallel} > T_{e,\perp}$ under the action of SDA, and they trigger
the electron firehose instability in the upstream. On the other hand, the thermal electrons
usually have the opposite temperature anisotropy $T_{e,\parallel} < T_{e,\perp}$ as a result of field amplifica-
tion and they generate the electron whistler instability near the shock transition region. It
would be interesting to see the interplay between these opposing temperature anisotropies
and their resulting instabilities. I am fairly confident that the electron heating mechanism
I identified for strictly perpendicular shocks will carry over to quasi-perpendicular shocks.
Even the fitting function I obtained for the downstream temperature ratio will probably
survive with minor quantitative changes. However, it is likely that the story will be very
different for quasi-parallel shocks where the field obliquity is large.

In the study of particle acceleration and heating in collisionless shocks, there is still the
unexplored frontier of high Mach number high plasma beta shocks. Such shocks are too
expensive to simulate at the present time. This is because the ratio between the proton
Larmor radius, which governs the shock dynamics, and the electron skin depth, which
needs to be resolved in order to capture the electron kinematics, is proportional to $M_s \sqrt{\beta_{\rho_0}}$. For large values of $M_s$, the range of scales that will need to be covered is prohibitively expensive. On the other hand, understanding the microphysics of these shocks can have important astrophysical applications. In particular, accretion shocks that form during the formation of galaxy clusters fall into this regime. These accretion shocks are abundant in cosmological simulations (e.g. Miniati et al., 2000; Ryu et al., 2003; Pfrommer et al., 2006; Skillman et al., 2008), and have Mach numbers ranging from 10 to a few $10^3$. The shocks are located beyond the virial radius of galaxy clusters. Due to the low plasma density at these locations, current observational facilities have not yet been able to detect these shocks, but there is an active ongoing hunt (see recent progress in Akamatsu et al., 2017a). Theoretical understanding of these shocks could guide future observations.
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