Holography Beyond AdS/CFT: Explorations in Kerr/CFT and Higher Spin DS/CFT

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Abstract

In this thesis we explore holography beyond the much studied case of AdS/CFT by considering two less well understood dualities: the Kerr/CFT and dS/CFT conjectures. Conformal symmetry continues to appear as a central organizing principle in both cases. In the first part of the thesis we use Kerr/CFT to shed light on the microscopic origin of black hole entropy. Specifically we consider corrections, logarithmic in the horizon area, to the microcanonical entropy of spinning black holes in dimensions four and greater. The logarithmic corrections to the black hole area/entropy law are entirely determined macroscopically by the massless particle spectrum, and therefore serve as powerful consistency checks on any proposed enumeration of black hole microstates. We compute these corrections microscopically using the Kerr/CFT correspondence and provide support for the conjecture by successfully matching the macroscopic computation in the simpler case of odd dimensions. The match depends sensitively on the values of the CFT central charge and the levels of the Kac Moody current algebras corresponding to spacetime and gauge isometries and sheds light on the statistical ensembles occurring in Kerr/CFT. We also take the extremal limit of the non-extremal microscopic computation and reproduce the macroscopic logarithmic corrections to the extremal black hole.

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In the second part of the thesis we discuss the simplest known explicit examples of $\mathrm{dS}_4/\mathrm{CFT}_3$: the duality between Vasiliev's higher spin gravity in dS_4 and three dimensional Euclidean vector model CFT's. In particular we conjecture that the level k $U(N)$ Chern-Simons theory coupled to free anticommuting scalar matter in the fundamental is dual to non-minimal higher-spin Vasiliev gravity in dS_4 with parity-violating phase $\theta_0 = \frac{\pi N}{2k}$ and Neumann boundary conditions for the scalar. Related conjectures are made for fundamental commuting spinor matter and critical theories. This generalizes a recent conjecture relating the minimal Type A Vasiliev theory in dS_4 to the $Sp(N)$ model with fundamental real anti-commuting scalars.

Contents

To My Parents.

Citations to Previously Published Work

The results of Chapter 1 were partly obtained in collaboration with Achilleas Porfyriadis, Andrew Strominger and Oscar Varela. The text has partly appeared previously in

> A. Porfyriadis, A. Pathak, A. Strominger and O. Varela.
> "Logarithmic corrections to black hole entropy from Kerr/CFT"
> JHEP **1704**, 090 (2017) [arXiv:1612.04833 [hep-th]].

The results of Chapter 3 were obtained in collaboration with Chi Ming Chang and Andrew Strominger. The text has partly appeared previously in

> C-M. Chang, A. Pathak and A. Strominger. "Non-Minimal Higher-Spin DS_4/CFT_3" [arXiv:1309.7413 [hep-th]].

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0 Introduction

A holographic duality is an equivalence between a theory of quantum gravity and a lower dimensional quantum mechanical theory without gravity. The study of holographic dualities was revolutionized about twenty years ago with the conjecture of the AdS/CFT correspondence relating quantum gravity in Anti de Sitter spacetimes with a conformal field theory living on the boundary of AdS. Explorations of AdS/CFT have inspired attempts to formulate holographic dualities in other settings such as the dS/CFT correspondence for cosmologically relevant expanding de Sitter universes [36] and the Kerr/CFT correspondence in the context of astrophysically relevant Kerr black holes [21]. This thesis works towards flushing out both the dS/CFT and Kerr/CFT dualities albeit by taking a different tack in each case. Understanding of AdS/CFT was greatly aided by the study of explicit examples in string theory, a microscopic theory of gravity, but dS/CFT has suffered for a lack of explicit microscopically complete examples. An important piece of progress in this direction was the conjecture of a duality between a particular Vasiliev higher spin gravity in dS_4 and the three dimensional Sp(N) CFT [6]. In this thesis we generalize this conjecture and propose explicit duals to the entire family of Vasiliev higher spin gravities in dS_4 - they are Chern-Simons vector models with "wrong"-statistics scalars or spinors. On the other hand with Kerr/CFT we take a bottom up, effective theory approach and explore universal constraints on the microscopic theory. The constraints arise from demanding a match of certain universal corrections to black hole entropy that can be reliably computed on the gravity side using low energy effective theory.

In the remainder of this chapter we provide separate detailed introductions to the two parts of this thesis. In the first part we discuss logarithmic corrections to black hole entropy and in the second non-minimal higher spin dS/CFT dualities.

0.1 Part 1: Understanding logarithmic corrections to black hole entropy using Kerr/CFT

In classical General Relativity, a black hole is a solution to Einstein's equations with an event horizon. No matter can escape from inside the event horizon and so a black hole is a perfect black body at zero temperature in classical physics. However taking into account quantum mechanical effects, black holes behave like thermodynamic systems with a finite thermodynamic entropy and temperature

$$S_{BH} = \frac{A}{4L_p^2} = \frac{A}{4G_N}, \quad T = \frac{\kappa}{2\pi}, \tag{1}$$

which obey the first law,

$$dM = TdS_{BH} + \cdots \tag{2}$$

Here A is the area of the event horizon and κ is the surface gravity. This raises the question of whether this thermodynamic entropy has a statistical interpretation.

In other words can we associate a set of quantum states with the black hole whose degeneracy (d_m) for fixed black hole charges (mass, angular momentum, etc.) yields,

$$S_{BH} \stackrel{?}{=} \ln(d_m) \tag{3}$$

Answering this question requires knowledge of the quantum theory of gravity. Turning the logic around, any self consistent theory of quantum gravity must reproduce S_{BH}.

Now if the microscopic theory of gravity is known the quantum black hole states can in principle be explicitly identified and counted. This has been done for a variety of supersymmetric black holes in string theory starting with the work of Strominger and Vafa [38]. However the universal Bekenstein-Hawking entropy follows from the low energy behavior of gravity and is insensitive to the specific UV completion of Einstein's theory. This suggests that all the nitty-gritty details of the microscopic theory governing quantum microstates are not needed to reproduce S_{BH}. Rather any putative microscopic theory should possess universal properties which are responsible for reproducing the universal entropy area relation.

For rotating black holes the Kerr/CFT conjecture proposes an answer to this question [7, 14, 18, 21, 35, 37]. There are different formulations of the conjecture but it generally states that the theory governing the quantum microstates of a rotating black hole is a two dimensional quantum field theory with Virasoro and Kac Moody symmetries in the IR limit. These infinite dimensional symetries constrain the microscopic degeneracy and are responsible for reproducing the Bekenstein-Hawking entropy. The Kerr/CFT conjecture, as well as the origin of S_{BH} are better understood for near extremal black holes which is the case we consider in this thesis.

The challenge of reproducing the Bekenstein Hawking formula microscopically has spurred many developments in quantum gravity including the Kerr/CFT correspondence. However this formula is approximate since it ignores higher derivative corrections to Einstein's equations as well as quantum loop effects due to fields fluctuating about the black hole. S_{BH} is the leading contribution when the area of the black hole is large. It receives subleading corrections suppressed by inverse powers of A from higher derivative terms as well as subleading contributions proportional to Log(A). Such logarithmic corrections arise from one loop contributions to the black hole partition function whose Legendre transform in turn yields the statistical entropy. Crucially they only arise from loops of massless fields and from integrating over loop momenta much smaller than the Planck scale. They can therefore be evaluated using only low energy data- the spectrum of massless fields and their coupling to the black hole background. Matching logarithmic corrections therefore imposes a consistency check on any proposed enumeration of black hole microstates that goes beyond the match of the Bekenstein-Hawking entropy. Writing the corrected entropy as

$$S = S_{BH} + n\, LogA + \cdots\ , \tag{4}$$

our goal is to reproduce the number n from the microscopic theory of black hole states. We will do so for a general rotating black hole in D dimensions using the framework of the Kerr/CFT correspondence.

Before exploring the Kerr/CFT dictionary further let us spell out more details regarding the origin of the logarithmic correction. For a non-extremal, rotating, charged black hole in D dimensions, the logarithmic correction to its microcanonical entropy reads,

$$S_{mc}\left(M,\vec{J},\vec{Q}\right) = S_{BH}\left(M,\vec{J},\vec{Q}\right) + \log a \left(C_{local} - \frac{D-4}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V \right) \tag{5}$$

where a is the black hole radius with $A \sim a^{D-2}$[33]. The black hole is labelled by specifying $N_C = [(D-1)/2]$ angular momenta, (N_C is the number of Cartan generators of the spatial rotation group) and by n_V U(1) gauge charges. The term

C_{local} arises form the contribution of the determinants of kinetic operators of massless fields to the Euclidean (thermal) partition function and is proportional to the trace anomaly of the massless fields. In particular C_{local} vanishes in odd spacetime dimensions. The remaining contributions to arise from path integrating over the zero modes of the kinetic operator and from the Laplace transform relating the microcanonical entropy to the partition function. For extremal black holes the formula (1.46) is slightly modified to,

$$S_{mc}\left(\vec{J},\vec{Q}\right) = S_{BH}\left(\vec{J},\vec{Q}\right) + \log a\left(C_{local} - 3\frac{D-2}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V\right) \tag{6}$$

The second term multiplying *log a* has changed from $-\frac{D-4}{2}$ to $-3\frac{D-2}{2}$. This is not a contradiction because the precise definition of S_{mc} differs in the non-extremal and extremal cases as observed in [33]. In the non-extremal case $S_{mc}(M,J,Q)$ is a density in the sense that $e^{S_{mc}(M,J,Q)}\Delta M$ equals the total number of states in the mass interval ranging from M to $M+\Delta M$. However in the extremal case $S_{mc}(J,Q)$ is a degeneracy, not a density, because the extremal black hole is labelled by quantized (discrete) charges. Therefore it makes sense to count the total number of states associated with a fixed set of charges. On the other hand the mass M is not quantized in the low energy theory and assumes continuous values, thereby forcing us to consider a density in the non-extremal case. Nevertheless it is possible to deduce the extremal formula (1.46) from a limit of the non-extremal formula: the argument is presented in chapter 3.

We now survey the Kerr/CFT dictionary which prescribes features of the microscopic theory governing black hole states that we will need to deduce the entropy microscopically. It states that the "CFT" is a two dimensional quantum field theory living on the space spanned by the coordinates t and φ_i. Here t is near horizon time (for extremal and near extremal black holes it is the AdS_2 time) while φ_i is one of the azimuthal angles in the geometry along which the black hole has angular momentum J_i. We henceforth restrict ourselves to extremal and near extremal black holes whose near horizon geometries have a $SL(2,R)\times U(1)$

isometry subgroup arising from the isometries of the near horizon AdS_2 and the rotation isometry along the CFT direction φ_i. Kerr/CFT proposes different infinite dimensional enhancements of these isometries. Explicit examples from string theory [39] suggest that the most plausible enhancement is

$$SL(2,R)_R \times U(1)_L \rightarrow Vir_R \times \widehat{U(1)}_L \,, \tag{7}$$

where the R index refers to the AdS_2 time (t) direction while the L index refers to the angluar direction (φ_i). Both the Virasoso and Kac-Moody extended symmetries appearing on the righthand side are rightmoving. Another proposed enhancement is [11],

$$SL(2,R)_R \times U(1)_L \rightarrow Vir_R \times Vir_L \;, \tag{8}$$

In this proposal Vir_R is rightmoving while Vir_L is left moving. We show in chapter 2 that both extensions yield the same logarithmic corrections. Since the second extension is more familiar - its symmetries are the same as those of a CFT_2 - we preview it here while leaving the details of the other extension to chapter 2.

The Kerr/CFT dictionary for the enhancement (1.32) states that the central charges of the Virasoro algebra scale as,

$$c_L = c_R \equiv c \sim A \sim a^{D-2} \tag{9}$$

Additional spatial rotation isometries lead to corresponding Kac-Moody algebras in the CFT with levels (k_J) that scale as,

$$k_J \sim c \sim A \sim a^{D-2} \tag{10}$$

Finally the n_V $U(1)$ gauge fields correspond to n_V additional $U(1)$ Kac-Moody current algebras in the microscopic theory with levels (k_Q),

$$k_Q \sim a^{D-4} \tag{11}$$

Note that all these central charges and levels are microscopic quantities defined in the theory governing black hole states. They are not detected directly by probe fields in the macroscopic black hole spacetime. With the central charges and levels in hand we can give a discussion of the microscopic computation of black hole entropy.

In the microscopic theory the entropy is extracted from the CFT partition function

$$Z(\tau, \bar{\tau}) = \mathrm{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0}\,, \tag{12}$$

which is approximated using its modular transformation property,

$$Z(\tau, \bar{\tau}, \vec{\mu}) = e^{-\frac{2\pi i \mu^2}{\tau}} Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}, \frac{\vec{\mu}}{\tau}\right)\,. \tag{13}$$

The μ_i are chemical potentials for the Kac-Moody charges. The density of states is in turn related to the approximate partition function as,

$$\rho(E_L, E_R, \vec{p}) \simeq \int d\tau d\bar{\tau}\, d^n\mu\, e^{2\pi i\left(-\frac{\mu^2}{\tau} - \frac{E_L^v}{\tau} + \frac{E_R^v}{\bar{\tau}} - E_L\tau + E_R\bar{\tau} - \mu_i p^i\right)}\,, \tag{14}$$

as elaborated in chapter 2. The integral can be performed using the saddle point approximation. The saddle point result yields the Bekenstein-Hawking entropy upon applying the Kerr/CFT dictionary. Gaussian fluctuations about the saddle yield logarithmic corrections. Using the scalings for the various quantities prescribed bby the Kerr/CFT dictionary, we recover the logarithmic correction (1.46). The reason we get the non-extremal result is because (1.56) is a density.

The rest of the thesis is organized as follows. In Chapter 2, the logarithmic correction to the density (1.46) is obtained microscopically. Chapter 3 considers extremal black holes and deduces the formula (6) for the degeneracy.

0.2 Part 2: Explicit examples of non-minimal higher spin dS/CFT

Astronomical hints of a positive cosmological constant suggest our universe may asymptote to de Sitter space in the future [8, 9]. The dS/CFT conjecture was formulated precisely for such asymptotically de Sitter spacetimes. It posits that quantum gravity in a de Sitter spacetime (dS) is holographically dual to a conformal field theory (CFT) living on the spacelike boundary of dS at future infinity [22, 36, 44]. However basic conceptual issues in the duality, for example the microscopic interpretation of the entropy associated with the de Sitter cosmological horizon, still remain mysterious. Exploring concrete examples of the dS/CFT conjecture may help facilitate our understanding of such issues. A few years ago an explicit example was proposed which involved one of Vasiliev's higher spin gravity theories in dS_4: Vasiliev type A theory in dS_4 is conjecturally dual to the $Sp(N)$ CFT_3 with anti-commuting scalars [6]. This duality was deduced by analytically continuing a previously established AdS/CFT duality between Vasiliev type A theory in AdS_4 and the $O(N)$ CFT_3. The

Vasiliev actually constructed families of classical higher-spin gravity theories in dS_4 labelled by a parity-violating phase θ_0 and, at the quantum level, a loop counting parameter g_{dS}^2. Indeed in AdS/CFT there is a whole family of dualities with the microscopic theories being U(N) Chern-Simons theories coupled to free or critical bosons or fermions in the fundamental representation of U(N)[13, 20, 25, 30]. In this thesis we construct the analogous family of dualities in dS_4 again utilizing the tool of analytic continuation from AdS_4 to dS_4. The dualities are displayed in the table below. The existence of such dualities was anticipated in [5, 28].[1]

Let us discuss the general structure of our dS/CFT claim. We will conjecture a minimal form of the correspondence which starts with correlation functions of fields in the gravitational theory. Correlation functions of operators (O_φ) in the dual CFT are obtained simply by pushing points in bulk correlation functions of

[1] The complexification of k considered in [5] is not realized in our construction.

Table 0.2.1: **CONJECTURED dS_4/CFT_3 DUALITIES**

VASILIEV dS_4 THEORY	**BOUNDARY CFT_3**	**SPECTRUM**
Non-minimal; $\theta_0 = \frac{\pi}{2}\lambda$ Neumann scalar	$U(N)_k$ Chern-Simons; free anticommuting scalar	2*All integer spins
Non-minimal; $\theta_0 = \frac{\pi}{2}(1-\lambda)$ Dirichlet scalar	$U(N)_k$ Chern-Simons; free commuting spinor	2*All integer spins
Non-Minimal; $\theta_0 = 0$ Neumann scalar	$U(N)_\infty$ Chern-Simons free anticommuting scalar	2*All integer spins
Non-minimal; $\theta_0 = \frac{\pi}{2}$ Dirichlet scalar	$U(N)_\infty$ Chern-Simons; free commuting spinor	2*All integer spins
Minimal; $\theta_0 = \frac{\pi}{2}\lambda$ Neumann scalar	$Sp(N)_k$ Chern-Simons; free anticommuting scalar	2*Even spins
Minimal; $\theta_0 = \frac{\pi}{2}(1-\lambda)$ Dirichlet scalar	$Sp(N)_k$ Chern-Simons; free commuting spinor	2*Even spins
Minimal $\theta_0 = 0$ Neumann scalar	$Sp(N)_\infty$ Chern-Simons; free anticommuting scalar	2*Even spins
Minimal; $\theta_0 = \frac{\pi}{2}$ Dirichlet scalar	$Sp(N)_\infty$ Chern-Simons; free commuting spinor	2*Even spins

$g_{dS}^2 = \frac{1}{N}$; $\lambda = \frac{N}{k}$; Dirichlet (Neuman) bulk scalars have $\Delta = 2$ ($\Delta = 1$).
Free (critical) anticommuting scalars are dual to critical (free) commuting spinors.

the corresponding fields ("φ) to the future boundary and stripping of appropriate powers of the dS time coordinate.

$$\langle \varphi(x_1) \cdots \varphi(x_n) \rangle_{\mathrm{dS}} \leftrightarrow \langle O_\varphi(x_1) \cdots O_\varphi(x_n) \rangle_{\partial \mathrm{dS}} \tag{15}$$

We do not discuss the interesting formulation of the correspondence involving the equality of the de Sitter wavefunction and the CFT partition function.

The spectrum of fields in a general bosonic (non-minimally truncated) Vasiliev higher spin gravity consists of massless symmetric tensor fields of all integer spins starting with spin 0. These fields are in one to one correspondence with higher spin symmetry currents in the dual vector model CFT's. In AdS_4, the boundary three point functions of the higher spin fields were explicitly computed for several of Vasiliev's higher spin models and shown to equal the correponding CFT three point corelators in [13, 25, 30] thus providing evidence for the duality. We will demonstrate agreement of the corresponding three point functions in dS_4 using analytic continuation.

It is a special feature of Vasiliev theory that the dS and AdS theories are related by a simple analytic continuation which involves taking the cosmological constant to minus itself,

$$\Lambda \to -\Lambda, G_N \ \to \ \text{fixed} \tag{16}$$

The duality set up in [13, 30] identifies,

$$N \ \sim \ \frac{1}{\Lambda G_N} \tag{17}$$

and thus this analytic continuation corresponds to $N \to -N$ in the dual theories. We will argue in chapter 3 that taking $N \to -N$ is equivalent to reversing the statistics of the matter fields coupled to the Chern-Simons gauge fields.

The analytic continuation from AdS to dS in Vasiliev theory stands in stark contrast with familiar examples of AdS/CFT in string theory such as the duality between $N = 4$ super Yang-Mills and string theory of $AdS_5 \times S_5$. There taking $\Lambda \to -\Lambda$ leads to severe problems such as imaginary flux (N) in the bulk as well as unstable bulk fields with wrong sign kinetic terms.

The outline of part 2 is as follows. In Chapter 3 Section 2, we review the Chern-Simons theories coupled to bosonic scalar or fermionic spinor matter in the fundamental of $U(N)$. We also discuss the statistics-reversed versions of these theories, namely Chern-Simons coupled to fundamental anticommuting scalar or commuting spinor matter, as well as Wick rotation from Minkowski to Euclidean space. In Section 3, we review the parity-violating Vasiliev theories in the AdS_4 and dS_4 vacua. In section 4, we present an analytic continuation that relates them. In particular, we show how the n-point correlation functions in AdS_4 and in dS_4 are related by this analytic continuation. In section 5, higher-spin bulk duals are conjectured for the various wrong-statistics Chern-Simons-matter theories. Formulae are given relating the bulk coupling constants and boundary conditions to the boundary level, gauge group and interactions. Spinor conventions are in the appendix.

1

Microscopic computation of logarithmic corrections to the density of states

Our goal in this chapter is to use the Kerr/CFT correspondence to microscopically reproduce macroscopically computed logarithmic corrections to the entropy of near a near extremal, charged, rotating black hole in D spacetime dimensions. We start by considering elements of Kerr/CFT for the specific example of the five dimensional Kerr/Newman black hole and follow this up with a discussion of Kerr/CFT for general D dimensional black holes. After this we review the macroscopic computation of the logarithmic corrections (1.46) and finally end with a microscopic, Kerr/CFT, derivation of the same.

Let us start by considering aspects of Kerr/CFT for a charged and rotating black hole solution of five-dimensional Einstein gravity minimally coupled to a gauge field. The dynamics of the latter is specified by the

Yang-Mills-Chern-Simons Lagrangian, so that the complete action is,[1]

$$S_5 = \frac{1}{4\pi^2} \int d^5x \left(\sqrt{-g} \left(R - \frac{3}{4} F^2 \right) + \frac{1}{4} \varepsilon^{abcde} A_a F_{bc} F_{de} \right) . \qquad (1.1)$$

Specifically, we are interested in the following Kerr-Newman black hole solution to (1.1) considered in [14],

$$\begin{aligned} ds_5^2 = &-\frac{(a^2+\hat{r}^2)(a^2+\hat{r}^2-M_0)}{\Sigma^2} d\hat{t}^2 + \Sigma \left(\frac{\hat{r}^2 d\hat{r}^2}{f^2 - M_0 \hat{r}^2} + \frac{d\theta^2}{4} \right) - \frac{M_0 F}{\Sigma^2} \left(d\hat{\psi} + \cos\theta \, d\hat{\varphi} \right) d\hat{t} \\ &+ \frac{\Sigma}{4} \left(d\hat{\psi}^2 + d\hat{\varphi}^2 + 2\cos\theta \, d\hat{\psi} \, d\hat{\varphi} \right) + \frac{a^2 M_0 B}{4\Sigma^2} \left(d\hat{\psi} + \cos\theta \, d\hat{\varphi} \right)^2 , \end{aligned} \qquad (1.2)$$

$$A = \frac{M_0 \sinh 2\delta}{2\Sigma} \left(d\hat{t} - a e^{\delta} \left(d\hat{\psi} + \cos\theta \, d\hat{\varphi} \right) \right) , \qquad (1.3)$$

where we have defined the quantities

$$\begin{aligned} B = a^2 + \hat{r}^2 - 2M_0 s^3 c^3 - M_0 s^4 (2s^2+3) , \qquad & F = a(\hat{r}^2 + a^2)(c^3 + s^3) - a M_0 s^3 , \\ \Sigma = \hat{r}^2 + a^2 + M_0 s^2 , \qquad & f = \hat{r}^2 + a^2 , \end{aligned} \qquad (1.4)$$

and $s \equiv \sinh\delta$, $c \equiv \cosh\delta$. The geometry depends on three independent parameters (a, M_0, δ) and the physical quantities of the black hole, *i.e.*, its mass, angular momentum and electric charge, are given in terms of those parameters by

$$M = \frac{3M_0}{2} \cosh 2\delta , \qquad J_L = a M_0 \left(c^3 + s^3 \right) , \qquad Q = M_0 sc . \qquad (1.5)$$

In five dimensions, it is possible to have a second angular momentum, J_R, but we set $J_R = 0$. Note that the $SU(2)_L$ angle is identified $\hat{\psi} \sim \hat{\psi} + 4\pi$.

[1] This coincides with the bosonic sector of minimal supergravity in five dimensions.

This black hole displays inner and outer horizons located at

$$r_\pm^2 = \tfrac{1}{2}(M_0 - 2a^2) \pm \tfrac{1}{2}\sqrt{M_0(M_0 - 4a^2)} \ . \tag{1.6}$$

At the (outer) horizon, the angular velocities are

$$\Omega_L \equiv \Omega_{\hat\psi} = \frac{4a}{M_0}\,\frac{1}{(c^3 - s^3) + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} \ , \qquad \Omega_R \equiv \Omega_{\hat\varphi} = 0 \ , \tag{1.7}$$

and the electric potential is

$$\Phi = \frac{c^2 s - s^2 c + (c^2 s + s^2 c)\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} \ . \tag{1.8}$$

Finally, the Hawking temperature is given by

$$T_H = \frac{1}{\pi\sqrt{M_0}}\,\frac{\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} \ , \tag{1.9}$$

and the Bekenstein-Hawking entropy is

$$S_{BH} = \pi\sqrt{2}\,M_0\sqrt{(c^6 + s^6)M_0 - 2(c^3 + s^3)^2 a^2 + (c^4 + c^2 s^2 + s^4)\sqrt{M_0(M_0 - 4a^2)}} \ . \tag{1.10}$$

The black hole approaches extremality in the limit $M_0 \to 4a^2$. In this limit, the two horizons (1.6) coalesce at $r_+ = a$ and the Hawking temperature (1.9) vanishes. The charges (1.5) become

$$M_{\text{ext}} = 6a^2 \cosh 2\delta \ , \qquad J_{L\,\text{ext}} = 4a^3\,(c^3 + s^3) \ , \qquad Q_{\text{ext}} = 4a^2 sc \ , \tag{1.11}$$

and the angular velocity (1.7) and electric potential (1.8) become

$$\Omega_L = \frac{1}{a(c^3 - s^3)} \ , \qquad \Phi_{\text{ext}} = \frac{c^2 s - s^2 c}{c^3 - s^3} \ . \tag{1.12}$$

At extremality, the Bekenstein-Hawking entropy (1.10) reduces to

$$S_{BH} = 8\pi a^3(c^3 - s^3) \ . \tag{1.13}$$

In this paper we are interested in the near-extreme case so we introduce a small parameter $\hat{\kappa}$ that measures the deviation from extremality and write $M_0 = 4a^2 + a^2\hat{\kappa}^2$. Substituting this into (1.10) and keeping terms up to linear order in $\hat{\kappa}$, the near extremal entropy is

$$\begin{aligned} S_{BH\,\text{near ext}} &= 8\pi a^3(c^3 - s^3) + 4\pi a^3(c^3 + s^3)\,\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^2) \\ &= \frac{\pi^2}{3}\,(6J_L)\left(\frac{1}{\pi}\frac{c^3 - s^3}{c^3 + s^3} + \frac{\hat{\kappa}}{2\pi}\right) + \mathcal{O}(\hat{\kappa}^2) \ . \end{aligned} \tag{1.14}$$

1.0.1 Near horizon, near extremal limit

Consider the coordinate transformation

$$t = \tfrac{1}{2}\,\varepsilon\,\Omega_L\,\hat{t} \ , \qquad r = \frac{\hat{r}^2 - r_+^2}{\varepsilon\, r_+^2} \ , \qquad \psi = \hat{\psi} - \Omega_L\,\hat{t} \ , \qquad \varphi = \hat{\varphi} \ . \tag{1.15}$$

Here, r_+ is the location of the outer horizon given in (1.6) and Ω_L is the extremal angular velocity (1.12). Making this coordinate transformation in the five-dimensional geometry (1.2), (1.3), with M_0 fixed to its extremal value, $M_0 = 4a^2$, and letting $\varepsilon \to 0$, one obtains the extremal near horizon geometry given in [14].

Here, we are interested in reaching the near horizon geometry of the black hole close, but not exactly at, extremality. This is the analog of the so-called near-NHEK limit for 4D Kerr considered in [18]. In order to do this, we still make the coordinate transformation (1.15), but now parametrize deviations from extremality with a parameter κ defined by

$$M_0 = 4a^2 + a^2\varepsilon^2\kappa^2 \ . \tag{1.16}$$

Then the metric (1.2) gives rise to

$$ds_5^2 = \frac{M}{12}\left[-r(r+2\kappa)dt^2 + \frac{dr^2}{r(r+2\kappa)} + d\theta^2 + \sin^2\theta d\varphi^2 + \frac{27 J_L^2}{M_3\left(\pi T_L(d\psi + \cos\theta\, d\varphi) + (r+\kappa)dt\right)^2}\right] \tag{1.17}$$

in the $\varepsilon \to 0$ limit. Here, we have defined

$$T_L \equiv \frac{1}{\pi}\frac{c^3 - s^3}{c^3 + s^3} \ . \tag{1.18}$$

This notation will be clarified in the next subsection. The location of the horizon in (1.17) is at $r = 0$ and the associated surface gravity is κ. We denote the corresponding Hawking temperature by

$$T_R \equiv \frac{\kappa}{2\pi} \ . \tag{1.19}$$

When we identify κ with the parameter $\hat{\kappa}$ introduced in (1.14), the metric (1.17) corresponds to the near horizon geometry of the black hole (1.2) close to extremality in the following complementary sense as well. Making the coordinate transformation (1.15) with $\varepsilon = 1$ and expanding the metric components in (1.2) to leading order in $r \sim \hat{\kappa} \ll 1$ we obtain (1.17) with $\kappa = \hat{\kappa}$. In the rest of the paper we make this identification throughout.

The gauge field corresponding to the near horizon, near extremal geometry is obtained by accompanying the coordinate transformation (1.15) with the gauge transformation

$$A \to A - d\Lambda \ , \qquad \text{with} \quad \Lambda \equiv \Phi \hat{t}. \tag{1.20}$$

Then the gauge field (1.20) becomes

$$A = -ae^{\delta}\tanh 2\delta \left(d\psi + \cos\theta d\varphi + e^{-2\delta}(r+\kappa)dt\right) \tag{1.21}$$

in the $\varepsilon \to 0$ limit.

1.0.2 Frolov-Thorne temperatures

We now move on to compute the Frolov-Thorne temperatures corresponding to the near-extremal Kerr-Newman black hole, by adapting the strategy of [18] to our present context. Consider a scalar field

$$\phi = e^{-i\omega\hat{t}+im\hat{\psi}}\,\hat{R}(\hat{r})\,S(\theta)\,T(\hat{\varphi}) \tag{1.22}$$

on the the black hole background (1.2), with charge q under the gauge field (1.3). Zooming into the near horizon region requires performing the coordinate transformation (1.15) combined with the gauge transformation (1.20). The charged scalar (1.22) thus becomes

$$\phi = e^{iq\Lambda}e^{-in_R t+in_L\psi}\,R(r)\,S(\theta)\,T(\varphi) \tag{1.23}$$

with

$$\Lambda = \frac{2\Phi}{\varepsilon\,\Omega_L}\,t\,, \quad m = n_L\,, \quad \omega = \frac{1}{2}\varepsilon\,\Omega_L\left(n_R + \frac{2}{\varepsilon}\,n_L - \frac{2q\Phi}{\varepsilon\,\Omega_L}\right)\,. \tag{1.24}$$

Now, the scalar field is in a mixed quantum state whose density matrix has eigenvalues given by the Boltzmann factor $e^{-\frac{1}{T_H}(\omega - m\Omega_L + q\Phi)}$, where T_H is the Hawking temperature (1.9). Identifying

$$e^{-\frac{1}{T_H}(\omega - m\Omega_L + q\Phi)} = e^{-\frac{2n_L}{T_L}-\frac{n_R}{T_R}-\frac{q}{T_Q}} \tag{1.25}$$

and using (1.24) we find the following Frolov-Thorne temperatures:

$$T_R = \frac{2}{\varepsilon\,\Omega_{L\mathrm{ext}}} T_H = \frac{2a}{\varepsilon\pi\sqrt{M_0}} \frac{(c^3 - s^3)\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} \,, \tag{1.26}$$

$$T_L = -\frac{2}{\Omega_L - \Omega_{L\mathrm{ext}}} T_H = \frac{2a}{\pi\sqrt{M_0}} \frac{(c^3 - s^3)}{c^3 + s^3 + (c^3 - s^3)\sqrt{1 - 4a^2/M_0}} \,, \tag{1.27}$$

$$T_Q = \frac{1}{\Phi - \Phi_{\mathrm{ext}}} T_H = \frac{1}{2\pi\sqrt{M_0}} \frac{c^3 - s^3}{s^2 c^2} \,. \tag{1.28}$$

Near extremality, M_0 is given by (1.16) and (1.26)–(1.28) become, in the $\varepsilon \to 0$ limit,

$$T_R = \frac{\kappa}{2\pi} \,, \qquad T_L = \frac{1}{\pi} \frac{c^3 - s^3}{c^3 + s^3} \,, \qquad T_Q = \frac{1}{4\pi a} \frac{c^3 - s^3}{s^2 c^2} \,. \tag{1.29}$$

Recall that both T_R and T_L have already appeared in our discussion: the former as the Hawking temperature (1.19) of the near-horizon, near-extremal metric (1.17) and the latter as a parameter, (1.18), in that metric. The present analysis elucidates the names given previously to those quantities.

At this stage instead of continuing to restrict to $D = 5$, we lay out the Kerr/CFT dictionary for a general rotating charged black hole in D dimensions. The $D = 5$ case is a subcase of the results presented below.

1.1 The Kerr/CFT dictionary in general dimensions ($D \geq 4$)

Consider a general black hole in D spacetime dimensions. This solution is labelled by $N_C = \left[\frac{D-1}{2}\right]$ angular momenta ($\vec{J} = \{J_i\}$) and n_V $U(1)$ gauge charges ($\vec{Q}$). Here N_C is the number of Cartan generators of the spatial rotation group and n_V is the number of gauge fields in the theory. If the black hole is extremal then these are all the labels while non extremal black holes are additionally labelled by their mass M. Note that for a given black hole solution some of the N_C

angular momenta and some of the n_V $U(1)$ gauge charges may be zero.

Suppose we scale the horizon radius of the black hole by a so that the horizon area scales as,

$$A \sim a^{D-2} \tag{1.30}$$

Then the non zero angular momenta $\vec{J}$, charges $\vec{Q}$ and mass M scale as [33],

$$\vec{J} \sim a^{D-2}, \;\; \vec{Q} \sim a^{D-3} \tag{1.31}$$

We now turn to the Kerr/CFT dictionary for such general black holes. We will only consider extremal and near extremal black holes since the dictionary is better understood in these cases. For a D dimensional Kerr-Newman black hole, the dual "CFT" lives on the two dimensional space spanned by the coordinates t and φ_i. Here t is the near horizon time - in the case of extremal and near-extremal black holes it is the AdS_2 time - while φ_i is one of the azimuthal anlges in the geometry along which the black hole has a non-zero angular momentum J_i. Indeed a black hole with multiple angular momenta may be holographically described using one of multiple CFT's corresponding to the multiple anlges φ_i. It has been checked in a wide variety of examples that at least for the leading entropy, the precise angle φ_i chosen for the CFT to live on does not matter- each CFT yields the same answer for the entropy [3].

The near horizon geometry of a general D dimensional near extremal black hole has a $SL(2,R)_R \times U(1)_L$ isometry subgroup coming from the isometries of the near horizon AdS_2 submanifold and the unbroken $U(1)$ rotation isometry corresponding to shifts in the angle φ_i on which the CFT lives respectively [16]. Various infinite-dimensional enhancements of this global isometry, involving different boundary conditions, have been extensively considered in the literature, and may be relevant in different circumstances or for different computations. See [11] for a recent discussion. We consider two of them which turn out to both give the same log corrections.[2]

[2]Had they been different, the matching of logarithmic corrections would have singled one out.

1.1.1 $Vir_R \times Vir_L$

In this subsection we consider a microscopic theory in which the global symmetries are enhanced as

$$SL(2,R)_R \times U(1)_L \to Vir_R \times Vir_L \ , \tag{1.32}$$

where Vir_L and Vir_R are left and right moving Virasoro algebras with generators L_n and $\bar{L}_n$ respectively. L_0 generates ψ rotations and $\bar{L}_0$ generates AdS_2 time translations. Let $c_L = c_R = c$ denote the equal central charges of Vir_R and Vir_L. Using the results of [11],[4], we have,

$$c_L = c_R = c \sim A \sim a^{D-2} \tag{1.33}$$

The fact that the central charges are equal follows from the asymptotic symmetry analysis in [11]. The Frolov Thorne temperatures are,

$$T_L, T_R \sim a^0 \tag{1.34}$$

Furthermore suppose that a subset, $\vec{J}'$ of the N_C angular momenta, $\vec{J}$ are zero while the remainder are non-zero. Then the near horizon geometry has an additional isometry subgroup $U(1)^{[J-J']} \times SO(D-1-[J'])$. Here $[J]$ is defined to be the dimension of the vector $\vec{J}$ and similarly for $[J-J']$. In Kerr/CFT we expect this to enhance to the corresponding Kac-Moody current algebra [23]

$$U(1)^{[J-J']} \times SO(D-1-[J']) \to \widehat{U(1)}^{[J-J']} \times \widehat{SO(D-1-[J'])} \tag{1.35}$$

Furthemore the result of [23] in $D = 4, 5$ for the levels of these current algebras straightforwardly generalizes to,

$$k_{U(1)} \sim k_{SO} \sim c \sim A \sim a^{D-2} \tag{1.36}$$

Finally the n_V $U(1)_Q$ gauge fields in the gravity theory correspond to n_V additional $U(1)$ Kac Moody current algebras in the microscopic theory. In Appendix 1.5 the $U(1)_Q$ level has been computed for $D = 5$ dimensions. While we do not explicitly do so, we expect the analysis of Appendix 1.5 to generallize straightforwardly to general D and yield,

$$k_Q \sim a^{D-4} \tag{1.37}$$

1.2 Logarithmic correction to entropy

In this section we review the computation of the logarithmic corrections to the microcanonical entropy of a non-extremal, rotating charged black hole in general spacetime dimension (D) done by Sen in [33]. His results apply to the near extremal black holes considered in this chapter, which have a small but non zero Hawking temperature.

Consider an asymptotically flat black hole solution of Einstein's theory minimally coupled to a set of massless fields including scalars, vectors, Dirac fermions and Rarita-Schwinger fields in D dimensions with canonical kinetic terms for all of these fields. We will use the Euclidean path integral approach to compute the quantum corrected black hole entropy. The Euclidean partition function is macroscopically defined as,

$$Z(\beta,\ \vec{\omega},\ \vec{\mu}) \ = \ \int D\Upsilon\, e^{-S_E(\Upsilon)} \tag{1.38}$$

where Υ includes all fields in the macroscopic theory and $S_E(\Upsilon)$ is the Euclidean action. β is the period of the Euclidean time coordinate while μ^i/β and ω^j/β denote the asymptotic values of the time component of the corresponding gauge field and the $t - \varphi^j$ component of the metric respectively. The Euclidean path integral over macroscopic fields (1.38) is interpreted microscopically as a statistical partition function involving a trace over black hole microstates,

$$Z(\beta,\ \vec{\omega},\ \vec{\mu}) = Tr\left(e^{-\beta E-\vec{\omega}.\vec{J}-\vec{\mu}.\vec{Q}}\right) \tag{1.39}$$

When considering extremal black holes in the next chapter we will relate the microscopic statistical partition function to a Euclidean path integral over the near horizon geometry of the black hole. However for the non extremal black holes in this chapter the path integral is over the entire asymptotically flat region outside the horizon. The statistical interpretation of the partition function (1.39) means that the density of microscopic states can be extracted from Z by performing a Laplace transform.

$$\rho(M,\ \vec{J},\ \vec{Q}) = \int\ d\beta\ d\omega\ d\mu\ e^{\,\mathrm{Log}\ Z(\beta,\vec{\omega},\vec{\mu})+\beta M+\vec{\omega}.\vec{J}+\vec{\mu}.\vec{Q}} \tag{1.40}$$

The Euclidean partition function (1.38) can be evaluated using the saddle point approximation which in our case consists of evaluating the action on the black hole solution. However the Euclidean black hole saddle point contribution to (1.38) corresponds to a black hole in thermal equilibrium with a thermal gas of particles. Because we are interested in the entropy of the black hole we should remove the contribution to log(Z) from the thermal gas. Since the thermal gas contribution diverges with volume we first put the system in a box of size L. The thermal gas contribution can now be removed by considering another black hole solution of size a_0 instead of a, putting it in a box of size $L_0 = (a_0/a)L$ and subtracting Log Z_0 from Log Z, where Z_0 is the partition function of the new black hole. This works because the the leading contribution to log(Z) from the thermal gas takes the form $L^{D-1} f(\beta, \vec{\omega}, \vec{\mu})$. By dimensional analysis upon scaling the size a of a black hole by a_0/a, β and μ also scale as a_0/a (ω does not scale) and the function f scales as $(a_0/a)^{1-D}$. Therefore the new black hole of size a_0, in a box of size $(a_0/a)L$, will have the same leading thermal gas contribution as the original black hole and it will cancel in the difference Log Z − Log Z_0. Henceforth we will drop the explicit Log Z_0 everywhere for brevity of notation.

Moving on, since we are in the thermodynamic limit we will use the saddle point approximation when extracting the entropy from the partition function using the above laplace integral: the saddle point values of β, $\vec{\omega}$ and $\vec{\mu}$ extremize the exponent in (1.40). Additionally the gaussian integral over the potentials about their saddle point values gives a logarithmic contribution to the entropy. The remaining logarithmic corrections to the entropy originate from multiplicative corrections to the Euclidean partition function itself from the quadratic (one-loop) fluctuations of quantum fields about the black hole solution. The one loop path integral about the black hole background can be divided into an integral over zero modes (Υ_z) and non-zero modes (Υ_{nz}) of the fields. These modes are defined by,

$$\Box \Upsilon_z = 0, \;\; \Box \Upsilon_{nz} = \lambda \Upsilon_{nz} \tag{1.41}$$

Here $\Box$ denotes the kinetic operators of the fields Υ and λ denotes the non-zero eigenvalues of $\Box$. The multiplicative one-loop contribution to the path integral

$$Z \approx Z_{\text{saddle}} \, Z_{1-\text{loop}}, \tag{1.42}$$

decomposes as,

$$Z_{1-\text{loop}} = (det \, \Box)^{-\frac{1}{2}} \, Z_z \tag{1.43}$$

where $det \, \Box$ is the determinant of $\Box$ over the non zero mode subspace,

$$det \, \Box = \Pi_{\lambda_n \neq 0} \, \lambda_n, \tag{1.44}$$

while Z_z is the remaining path integral over the zero modes,

$$Z_z = \int D\Upsilon_z \tag{1.45}$$

Evaluating $Z_{1-\text{loop}}$ about the general black hole background and plugging into

(1.40) yields the following logarithmic correction to the statistical entropy [33],

$$S_{mc}\left(M,\vec{J},\vec{Q}\right) = S_{BH}\left(M,\vec{J},\vec{Q}\right) + \log a\left(C_{local} - \frac{D-4}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V\right) \tag{1.46}$$

where $N_C = \left[\frac{D-1}{2}\right]$ is the number of Cartan generators of the spatial rotation group and n_V is the number of vector fields in the theory.

In the above formula C_{local} arises from the path integral over the non-zero modes and vanishes in odd dimensions. We will not consider it further in this thesis. The remaining logarithmic correction comes from gaussian integrals over β, $\vec{\omega}$ and $\vec{\mu}$ about their saddle point values in (1.40) and from the path integral over translational zero modes of the black hole. These zero modes correspond to the (asymptotic) translational symmetries broken by the black hole. Actually only the zero modes corresponding to translations which commute with the rotation isometry of the black hole contribute to the Euclidean path integral. This is because all the other translational zero modes as well as all rotational zero modes do not have the required periodicity in Euclidean time as carefully explained by Sen in [33]. In the next chapter we will carefully describe how the integration over a zero mode corrects the Euclidean partition function. For now we proceed towards our main goal of using Kerr/CFT to reproduce the result (1.46) microscopically.

One loop exactness

Before proceeding let us briefly discuss, for completeness, the robustness of the logarithmic correction to the entropy, specifically its one-loop exactness by reviewing the discussion in [33] . Consider a vacuum diagram of massless fields with l loops in a low energy theory with a two derivative lagrangian. The mass dimension of this graph is $(D-2)l+2$. A contribution to the lagrangian desity (L) must have dimension D, so the remaining dimension is made up by the appropriate power of the Planck length (the fundamental scale in the theory), namely l_p^{D-2}. Log Z involves an additional integral $\int dx^D \sqrt{g}$ which gives a factor of a^D. Putting the various factors together, the l loop contribution takes the form,

$$log\, Z \supset l_p^{(D-2)(l-1)} \int^{\Lambda} d^D k\, k^{2-2l} F(ka). \tag{1.47}$$

Λ is a momentum cutoff. The function $F(ka)$ accounts for the fact that the propagators deviate from their flat space expression in the black hole back ground. It has the property that

$$F(ka)|_{k\to\infty} \to 1 \tag{1.48}$$

since space looks flat at short distances. Scaling out the factors of a we can write,

$$Log\, Z \supset l_p^{(D-2)(l-1)}\, a^{-(D-2)(l-1)} \int^{a\Lambda} d^D k\, k^{2-2l} F(k) \tag{1.49}$$

Assuming all the loop momenta are of the same order, expand the integrand in powers of k^{-1} for large k where the function F asymptotes to 1. The only $Log(a\Lambda)$ term from the momentum integral can come from the $k^{-(D-2)l-2}$ term in the expansion of the integrand. But this term will be multiplied by an additional factor of $a^{-(D-2)(l-1)}$ unless $l = 1$. Finally if some of the loop momenta are small (soft), the effect of the hard part may be thought of as renormalizing the propagators and vertices of the soft part. Assuming this renormalization does not change the low energy effective action, for instance massless particles remain massless, we can integrate out the hard part and run the same argument as before for the soft part. For example this assumption holds for the graviton which remains massless in the face of quantum corrections. Finally, higher derivative corrections generated by integrating out the hard part will be suppressed by powers of l_p/a and will not contribute to the pure logarithmic term.

1.3 Microscopic computation

We now change gears and compute the logarithmic correction to the entropy of the microscopic theory dual to the charged rotating black holes. In [14] the five dimensional Kerr/Newman example was embedded into string theory and the microscopic dual thereby shown to be the infrared fixed point of a 1+1 field theory living on the brane intersection. This fixed point is a possibly non-local deformation of an ordinary 1+1 conformal field theory which preserves at least one infinite-dimensional conformal symmetry. While the string theoretic construction implies the existence of the fixed point theory, it exhibits a new kind of 1+1 dimensional critical behavior and is only partially understood.

Since we have already computed the scaling of the microscopic central charges and levels with the horizon area of the black hole in hand we can proceed with the microscopic computation of black hole entropy.

We start by putting the CFT on a circle along $\psi - t$ and consider the ensemble

$$Z(\tau, \bar{\tau}) = \mathrm{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0} \,. \tag{1.50}$$

We assume that

$$4\pi\tau = \beta_L - \beta_R + i(\beta_L + \beta_R) \tag{1.51}$$

and $4\pi\bar{\tau} = \beta_L - \beta_R - i(\beta_L + \beta_R)$. Standard modular invariance of this partition function is $Z(\tau, \bar{\tau}) = Z(-1/\tau, -1/\bar{\tau})$. The microscopic dual to the Kerr-Newman black hole we are considering in this paper has additional global symmetries, corresponding to rotation isometries and $U(1)$ gauge symmetries. Turning on the associated chemical potentials, the partition function becomes

$$Z(\tau, \bar{\tau}, \vec{\mu}) = \mathrm{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0 + 2\pi i \mu_i P^i} \tag{1.52}$$

and it obeys the modular transformation rule

$$Z(\tau, \bar{\tau}, \vec{\mu}) = e^{-\frac{2\pi i \mu^2}{\tau}} Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}, \frac{\vec{\mu}}{\tau}\right) \,. \tag{1.53}$$

Here μ_i are left chemical potentials associated with the left moving conserved charges P^i and $\mu^2 \equiv \mu_i \mu_j k^{ij}$ with k^{ij} the matrix of Kac-Moody levels of the left moving currents. i, j run from 1 to n. This partition function is related to the density of states, ρ, at high temperatures by

$$Z(\tau, \bar{\tau}, \vec{\mu}) = \int dE_L \, dE_R \, d^n p \, \rho(E_L, E_R, \vec{p}) \, e^{2\pi i \tau E_L - 2\pi i \bar{\tau} E_R + 2\pi i \mu_i p^i} \,, \tag{1.54}$$

where E_L, E_R, p^i are the eigenvalues of $L_0, \bar{L}_0, P^i$ respectively. For small τ, (1.53) implies that

$$Z(\tau, \bar{\tau}, \vec{\mu}) \approx e^{-\frac{2\pi i \mu^2}{\tau}} e^{-\frac{2\pi i E_L^v}{\tau} + \frac{2\pi i E_R^v}{\bar{\tau}} + \frac{2\pi i \mu_i p_v^i}{\tau}} \,. \tag{1.55}$$

Then, inverting (1.54), we obtain the following expression for the density of states:

$$\rho(E_L, E_R, \vec{p}) \simeq \int d\tau d\bar{\tau} \, d^n \mu \, e^{2\pi i \left(-\frac{\mu^2}{\tau} - \frac{E_L^v}{\tau} + \frac{E_R^v}{\bar{\tau}} - E_L \tau + E_R \bar{\tau} - \mu_i p^i \right)} \,, \tag{1.56}$$

where we have assumed that the vacuum is electrically neutral, $p_v^i = 0$. This integral may be evaluated by saddle point methods. The integrand reaches an extremum at

$$\tau_0 = \sqrt{\frac{4E_L^v}{4E_L - \mathcal{P}^2}} \,, \qquad \bar{\tau}_0 = -\sqrt{\frac{E_R^v}{E_R}} \,, \qquad \mu_{0i} = -k_{ij} p^j \sqrt{\frac{E_L^v}{4E_L - \mathcal{P}^2}} \,, \tag{1.57}$$

where the matrix k_{ij} is the inverse of k^{ij} and $\mathcal{P}^2 \equiv p^i p^j k_{ij}$. The leading contribution to the entropy is obtained by evaluating (1.56) at the saddle (1.57). This gives

$$S = \log \rho_0 = 2\pi \sqrt{-E_L^v \left(4E_L - \mathcal{P}^2\right)} + 2\pi \sqrt{-E_R^v \left(4E_R\right)} \,. \tag{1.58}$$

Putting

$$E_L^v = E_R^v = -\frac{c}{24} \,, \quad E_L - \frac{\mathcal{P}^2}{4} = \frac{\pi^2}{6} c \, T_L^2 \,, \quad E_R = \frac{\pi^2}{6} c \, T_R^2 \,, \tag{1.59}$$

we have

$$S = \frac{\pi^2}{3} c\, T_L + \frac{\pi^2}{3} c\, T_R\,. \tag{1.60}$$

The analysis of [11, 14] yields $c = 6J_{L\,\mathrm{ext}}$ and using the values for T_L, T_R obtained in (1.29), we see that (1.60) matches the near-extremal Bekenstein-Hawking entropy (1.14) to linear order in κ. This extends the match of [14] from the extremal to the near-extremal regime.

The logarithmic correction ΔS to the leading entropy (1.58) is generated by Gaussian fluctuations of the density of states (1.56) about the saddle (1.57):

$$\Delta S = -\frac{1}{2} \log \frac{\det \mathcal{A}}{(2\pi)^{n+2}}\,, \tag{1.61}$$

where $\mathcal{A}$ is the determinant of the matrix of second derivatives of the exponent in the integrand of (1.56) with respect to τ, μ_i, $\bar{\tau}$. We find

$$\det \mathcal{A} = \frac{(2\pi)^{n+2}}{16}\, \left(-E_L^v\right)^{-\frac{n+1}{2}} \left(4E_L - \mathcal{P}^2\right)^{\frac{n+3}{2}} \left(-E_R^v\right)^{-\frac{1}{2}} \left(4E_R\right)^{\frac{3}{2}} \det k^{ij}\,. \tag{1.62}$$

We now fix $n = N_C + n_V$ for the Cartan currents of the left moving $U(1)^{[J-J']} \times SO(D-1-[J'])$ current algebra corresponding to $SU(2)$ rotations and those corresponding to the n_V U(1) gauge fields. $p^2 \propto Q$. The scalings of all central charges and levels are given in equations (1.33), (1.36) and (1.37). Taking into account (1.11), we thus have the following scalings,

$$E_L^v\,, E_R^v\,, E_L - \mathcal{P}^2/4\,, E_R \sim a^{D-2}\,, \quad k_Q \sim a^{D-4}\,, \quad k_J \sim a^{D-2}\,. \tag{1.63}$$

Bringing (1.63) to (1.61, 1.62), we obtain

$$\Delta S = \left(-\frac{D-2}{2} - \frac{D-2}{2} N_C - \frac{D-4}{2} n_V\right) \log a\,. \tag{1.64}$$

which almost matches (1.46) with $C_{local} = 0$. We explain the resolution of this descrepancy shortly in subsection 1.4.

$Vir_R \times \widehat{U(1)}_L$

In this subsection we consider a warped CFT, in which the global symmetries are enhanced as

$$SL(2,R)_R \times U(1)_L \to Vir_R \times \widehat{U(1)}_L . \tag{1.65}$$

Here $\widehat{U(1)}_L$ is a right moving Kac-Moody algebra whose zero mode $\tilde{R}_0$ generates the left sector $U(1)_L$ rotational isometry and Vir_R is a right moving Virasoro algebra whose zero mode $\tilde{L}_0$ generates time translations in AdS$_2$.

Let c be the central charge of Vir_R and k_R be the level of the $U(1)$ Kac Moody $\widehat{U(1)}_L$. The analysis of [11] yields,

$$c \sim k_R \sim A \sim a^{D-2} \tag{1.66}$$

The levels of additional current algebras arising from the spatial rotation group and from any $U(1)$ gauge fields scale in the same way as (1.36), (1.37). Once again with these results in hand we can proceed to the microscopic computation of the entropy.

The symmetry algebra of our warped CFT is

$$\begin{aligned}
\left[\tilde{L}_m, \tilde{L}_n\right] &= (m-n)\tilde{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n} , \\
\left[\tilde{R}_m, \tilde{R}_n\right] &= \frac{k_R}{2} m\delta_{m+n} , \quad \left[\tilde{L}_m, \tilde{R}_n\right] = -n\tilde{R}_{m+n} ,
\end{aligned}$$

where $\tilde{L}_m$ and $\tilde{R}_m$ are the Virasoro and Kac-Moody generators respectively. Putting the theory on a circle along ψ, the partition function at inverse temperature β and angular potential θ is given by $Z(\beta,\theta) = \mathrm{Tr}\, e^{-\beta\tilde{R}_0 + i\theta\tilde{L}_0}$. On the other hand, in [29] it was shown that by redefining the charges as

$$L_n = \tilde{L}_n - \frac{2}{k_R}\tilde{R}_0\tilde{R}_n + \frac{1}{k_R}\tilde{R}_0^2\delta_n , \quad R_n = \frac{2}{k_R}\tilde{R}_0\tilde{R}_n - \frac{1}{k_R}\tilde{R}_0^2\delta_n , \tag{1.67}$$

and putting the theory on the same circle but in the different ensemble[3]

$$Z(\tau, \bar{\tau}) = \mathrm{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} R_0}\,, \tag{1.68}$$

the partition function obeys the usual CFT modular invariance:

$$Z(\tau, \bar{\tau}) = Z(-1/\tau, -1/\bar{\tau})\,. \tag{1.69}$$

Assuming

$$4\pi\tau = \beta_L - \beta_R + i(\beta_L + \beta_R) \tag{1.70}$$

and $4\pi\bar{\tau} = \beta_L - \beta_R - i(\beta_L + \beta_R)$ we may then proceed as in the previous section replacing $\bar{L}_0$ with R_0 everywhere starting from equation (1.52) onwards [4]. We thus arrive at the same results for the leading entropy and its logarithmic correction.

1.4 Match of the macroscopic and microscopic computations

We have already exhibited the match, in the near-extremal regime, of the bulk and microscopic results for the leading term of the entropy the $D = 5$ Kerr-Newman black hole under consideration: the Cardy formula (1.60) reproduces the near-extremal Bekenstein-Hawking entropy (1.14). This match can be straightfowardly generalized to higher dimensions.

We will now show that the logarithmic corrections in general D agree. In order to furnish a sensible comparison, one must ensure that both results are given in the same ensemble. This is not the case for the macroscopic, (1.46), and microscopic, (1.64), results given above. The former assumes the entropy to be a function of the energy $Q[\partial_{\hat{t}}]$ conjugate to the asymptotic time which features in

[3] A change of ensemble may result in different logarithmic corrections to the entropy. However, as explained in Appendix 1.6, the change of ensemble corresponding to the charge redefinitions (1.67) here does not imply any change in the logarithmic correction to the entropy.

[4] The derivation of the leading entropy with the current enhancement is subtle and to the authors knowledge has not appeared previously in the literature. However it has appeared in the unpublished works [17, 27].

the full black hole solution (1.2), while the latter is instead a function of the energy $Q[\partial_t]$ conjugate to the near horizon time which appears in (1.17). The transformation between the macroscopic and microscopic density of states requires a Jacobian factor (Appendix 1.6),

$$\rho_{\text{bulk}} = \frac{\delta Q[\partial_t]}{\delta Q[\partial_{\hat{t}}]}\,\rho\ . \tag{1.71}$$

Now, from the change of coordinates (1.15) and the expression for the extremal angular velocity in (1.12), we see that this Jacobian scales like

$$\frac{\delta Q[\partial_t]}{\delta Q[\partial_{\hat{t}}]} \sim a\ . \tag{1.72}$$

Thus

$$\Delta S_{\text{bulk}} = \Delta S + \log a\ , \tag{1.73}$$

which indeed is satisfied by (1.64) and (1.46).

1.5 Appendix: Computation of k_Q

In this appendix we compute the level k_Q of the $U(1)$ Kac-Moody algebra associated with the gauge field A_μ. We do not perform a full asymptotic symmetry group analysis here. We expect that with appropriate boundary conditions on the gauge field this Kac-Moody is consistent with the rest of the asymptotic symmetries used in Section 3. Here we are particularly interested in deriving the scaling of the level k_Q with a.

Thus we assume the $U(1)$ current algebra is generated by

$$\Lambda_\eta = \eta(\tilde{y})\ , \tag{1.74}$$

where $\tilde{y} = \pi T_L\, \psi$. In modes, the generators

$$p_n = -2\pi T_L\, e^{-in\tilde{y}/(2\pi T_L)}\ , \tag{1.75}$$

satisfy the algebra

$$[p_m, p_n] = 0\,. \tag{1.76}$$

Using the formulas in [10], one can compute the central extension in the corresponding Dirac bracket algebra. We find:

$$\left\{Q_{p_m}, Q_{p_n}\right\} \quad = \quad -im\, 24\pi^2 T_L^2 a e^\delta \tanh 2\delta\, \delta_{m+n}\,. \tag{1.77}$$

The central extension comes entirely from the Chern-Simons term in the action (1.1). Passing to the commutators $\{\,,\} \to -i[\,,]$ we obtain the current algebra,

$$[P_m, P_n] = \frac{k_Q}{2} m\delta_{m+n}\,, \tag{1.78}$$

with level given by

$$k_Q = 12\,(2\pi T_L)^2\, ae^\delta \tanh 2\delta\,. \tag{1.79}$$

1.6 Appendix: Change of ensemble

Under a charge redefinition, $\vec{q} = \vec{q}\,(\vec{q}\,')$, the density of states, $\rho(\vec{q})$, transforms with the appropriate Jacobian factor as

$$\rho'(\vec{q}\,') = \frac{\partial(q_1, q_2, \ldots)}{\partial(q_1', q_2', \ldots)}\rho(\vec{q})\,. \tag{1.80}$$

The leading piece of the entropy $S = \log \rho$ typically scales like a^{D-2} for large $q \sim a$ and is therefore independent of the change of ensemble. However, the logarithmic correction, which scales like $\log a$, often picks up contributions from the Jacobian factor above. We have seen this explicitly in section 1.4 where the Jacobian (1.72) scales with a.

Another instance of a change of ensemble was mentioned in relation to the charge redefinitions in (1.67). In this case the Jacobian is

$$\frac{\partial(L_0, R_0)}{\partial(\tilde{L}_0, \tilde{R}_0)} = 2\frac{\tilde{R}_0}{k_R} = 2\sqrt{\frac{R_0}{k_R}}\,. \tag{1.81}$$

However, $k_R \propto c \sim a^3$ [11] and $R_0 \sim a^3$ so in this instance the Jacobian does not scale with a and therefore the logarithmic correction to the entropy is left intact by this particular change of ensemble.

2

Microscopic computation of logarithmic corrections to the extremal degeneracy

In this chapter we consider logarithmic corrections to the entropy of exactly extremal black holes. We will use the quantum entropy function formalism which equates the microcanonical microstate degeneracy with a macroscopic Euclidean path integral over the near horizon region of the black hole. This is unlike our macroscopic computations in the previous chapter where the Euclidean path integral yielded the statistical partition function, not the degeneracy. The main non-trivial result of this chapter is that the multiplicative correction to this macroscopic Euclidean path integral from the integral over zero modes matches the correction on the microscopic side coming from the laplace transform of the microscopic canonical partition function required to extract the microcanonical degeneracy.

We start this chapter with a brief review of the computation of the degeneracy within the quantum entropy formalism. We follow this up by taking a subtle extremal limit of the macroscopic computation of the previous section and reproduce the logarithmic corrections coming from macroscopic zero modes.

2.1 Computation of the extremal degeneracy using the quantum entropy function formalism

In this section we review the argument supporting the equality between the extremal degeneracy and a Euclidean AdS_2 partition function. As discussed in the introduction, the near horizon region of extremal black holes contains an AdS_2 factor plus additional compact directions. Dimensionally reducing, we get two dimensional scalars, vectors and two tensors propagaing on AdS_2. Additionally there may be fermions but we restrict ourselves to bosonic fields in this chapter. The emergent AdS_2 factor plays a central role in the entropy function formalism for extremal black holes. We will work in Euclidean signature where the AdS_2 space has the topology of a disc with a metric,

$$ds^2 = \frac{l^2}{(1-\rho^2)^2}(d\rho^2 + \rho^2 d\tau^2) \tag{2.1}$$

We will impose an infrared cutoff in the AdS_2 space so that the coordinates have the ranges,

$$0 \leq \rho < 1 - \varepsilon < 1, \quad \tau \sim \tau + 2\pi \tag{2.2}$$

Next consider the following partition function on this regulated space,

$$Z_{\mathrm{AdS}_2} = \left\langle e^{\left(-iQ_j \int_{\rho=1-\varepsilon} d\tau A^j_\tau\right)} \right\rangle \tag{2.3}$$

The expectation values imply a path integral over all macroscopic fields propagating on AdS_2 weighted by the exponential of the Euclidean action. The set $\{Q_j\}$ includes all the charges labelling the black hole, namely its N_C Cartan angular momenta and n_V electric charges. From the AdS_2 point of view these charges equal the fluxes of the corresponding AdS_2 electric fields A^j.

In order to define the Euclidean path integral (2.3) we have to specify boundary conditions on the AdS_2 gauge fields A^j. We will work in Fefferman-Graham gauge with $A_\rho = 0$. Maxwell's equations in two dimensions imply that time independent solutions take the asymptotic form,

$$A_\tau = A^{(0)} + \frac{A^{(1)}}{\varepsilon} \tag{2.4}$$

The coefficient of the non-normalizable mode $A^{(1)}$ equals the electric charge. Thus if we fix the non-normalizable coefficient $A^{(1)}$ while carrying out the path integral the resulting Euclidean partition function is labelled by the black hole charges $\{Q_j\}$.

$$Z_{\mathrm{AdS}_2} = Z_{\mathrm{AdS}_2}(\{Q_j\}) \tag{2.5}$$

Note the contrast with higher dimensional (≥ 4) AdS spaces where the constant mode of the gauge field, equal to the chemical potential, is dominant near the boundary and held fixed while the mode equalling the electric charge is sub-dominant and integrated over. Thus in that case the corresponding Euclidean path integral computes a partition function labelled by chemical potentials conjugate to the charges. Finally, the extra term $e^{\left(-iQ_j \int_{\rho=1-\varepsilon} d\tau A^j_\tau\right)}$ in (2.3) serves to make the variational principle well defined when the divergent mode $A^{(1)}$ is held fixed.

So far we have not argued why the Euclidean partition function $Z_{\mathrm{AdS}_2}(Q_j)$ equals the degeneracy of extremal black hole microstates,

$$Z_{\mathrm{AdS}_2}(\{Q_j\}) \stackrel{?}{=} d_{\mathrm{ext}}(\{Q_j\}) = e^{S_{\mathrm{ext}}(\{Q_j\})} \tag{2.6}$$

To do so we will use the AdS_2/CFT_1 conjecture which equates the Euclidean AdS_2 partition function to a thermal partition function in the dual quantum mechanics. First as discussed in [19], the theory dual to the black hole has a gap in its spectrum separating the ground states from the first excited (non-extremal) states. Since the extremal black hole has zero temperature it is captured by the infrared limit of the dual quantum mechanics - due to the gap in the spectrum only the ground states survive the zero temperature limit. Hence, the thermal partition function of the quantum mechanics becomes,

$$Z_{\rm QM} \;=\; Tr(e^{-\beta H}) \;\to\; d(0)\, e^{-\beta E_0} \tag{2.7}$$

Here H is the Hamiltonian of the quantum mechanics and $d(0)$ is the degeneracy of ground states. E_0 is the ground state energy which can be arbitrarily shifted, while β is the inverse temperature.

Using the $\mathrm{AdS}_2/\mathrm{CFT}_1$ conjecture we equate the AdS_2 partition function Z_{AdS_2} with the limit of the partition function (2.7). This implies that the finite parth of the Euclidean AdS_2 partition function Z_{AdS_2} equals the degeneracy of extremal states $d(0)$.

$$Z_{\mathrm{AdS}_2} = e^{\mathrm{C\,L}}\, d(0), \;\; \mathrm{L} \to \infty, \tag{2.8}$$

where L is length of the AdS_2 time boundary and C is an arbitrary, counterterm dependent, constant.

Finally note that all quantum mechanical states involved in the thermal trace possess the same charges $\{Q_j\}$. The reason is that we have fixed the non-normalizable mode of the gauge field $A^{(1)}$ on the gravity side and thus restricted to a fixed charge sector in the microscopic theory. Thus the degeneracy of ground states $d(0)$ is the degeneracy in a fixed charge sector where all states have the same charges $\{Q_j\}$. In other words, because of the boundary conditions on the AdS_2 path integral, which hold fixed the asymptotic AdS_2 electric fluxes, Z_{AdS_2} equals the degeneracy in the microcanonical ensemble in which all the charges are specified. Thus unlike in higher dimensions we have an Euclidean partition function which equals the degeneracy rather than the microscopic canonical partition function.

$$d(0) = d(\{Q_j\}) = e^{S_{\rm ext}(\{Q_j\}} \tag{2.9}$$

2.2 Logarithmic corrections to the extremal entropy

In this section we discuss logarithmic corrections to the extremal entropy focusing on those arising from zero modes on the macroscopic side. We start by discussing the macroscopic computation in the first subsection and follow with the microscopic results in the second subsection.

2.2.1 Macroscopic computation of logarithmic corrections

In the previous section we argued that the microcanonical extremal degeneracy $d(\{Q_j\})$ equals the Euclidean partition function Z_{AdS_2}. Hence we can now compute the logarithmic correction to the entropy by computing the one-loop correction to the partition function about in the near horizon black hole background. The one-loop computation is the same as that outlined in section 1.2 in chapter 1. Since in this thesis we are interested in connecting the corrections coming from zero modes to those arising from a change of ensemble in the microscopic theory, we will now elaborate on the macroscopic zero mode path integral following [32].

As discussed in 1.2 we are interested in determining the a ($A_{\text{hor}} \sim a^{D-2}$) dependence of,

$$Z_z \;=\; \int D\Psi_z = \int \Pi\, Da_n, \tag{2.10}$$

where a general zero mode is decomposed as,

$$\Psi_z = \sum_n a_n \Psi_z^n \tag{2.11}$$

We will dimensionally reduce the near horizon geometry to two dimensions and consider zero modes of fields propagating on AdS_2. The zero modes Ψ_z are associated with asymptotic symmetries, large gauge transformations which act non trivially near the boundary. For example the two dimensional metric zero modes correspond to reparameterizations of the AdS_2 time τ. In this case, the general metric zero mode is,

$$h^z_{\mu\nu} = D_{(\mu}\varepsilon_{\nu)} \tag{2.12}$$

where ε_ν corresponds to the reparameterization,

$$\tau \to \tau + \varepsilon_\tau(\tau) \tag{2.13}$$

and is expanded as,

$$\varepsilon_\tau(\tau) = \sum_n b_n\, e^{in\tau} \tag{2.14}$$

Our strategy for evaluating (2.10) will be to perform a change of variables from the coefficients a_n of the zero modes to the parameters labelling the asymptotic symmetry group b_n. The ranges of the asymptotic symmetry group parameters are independent of the black hole scale a. Suppose the Jacobian for the change of variables from the ith field species ($i = m$ for the metric, $i = v$ for a vector field, etc.) to the corresponding asymptotic symmetry group parameters gives a factor of a^{β_i} for each zero mode. For reasons that will become clear momentarily the index i refers to a field in the entire $\mathrm{AdS}_2 \times \mathrm{M}$ near horizon geometry and not AdS_2 fields. We have,

$$\Pi\, Da^i_n = a^{\beta_i\, N^i_{zm}}\, \Pi\, Db^i_n \tag{2.15}$$

where N^i_{zm} is the number of zero modes of the ith field species. Hence,

$$D\Psi_z \;=\; a^{\sum_i \beta_i\, N^i_{zm}}\, \Pi\, Db^i_n \tag{2.16}$$

The entire a dependence is now isolated in the prefactor on the right hand side of (2.16). Our remaining task is to determine the numbers N^i_{zm} and β_i.

Determination of β_i

For concreteness we first perform the computation of β_i for a $\mathrm{U}(1)$ vector field A_μ ($i = v$) propagating on the near horizon geometry $\mathrm{AdS}_2 \times M$. The path integral measure for A_μ is normalized as,

$$\int DA_\mu \, e^{[-\int d^D x \sqrt{g}\, g^{\mu\nu} A_\mu A_\nu]} = 1 \tag{2.17}$$

The integral in the exponent is over $AdS_2 \times M$. Noting that

$$g_{\mu\nu} = a^2 \, g^{\circ}_{\mu\nu}, \tag{2.18}$$

where $g^{\circ}_{\mu\nu}$ is independent of a, we have,

$$\int DA_\mu \, e^{[-a^{D-2} \int d^D x \sqrt{g^{\circ}}\, g^{\circ\,\mu\nu} A_\mu A_\nu]} = 1. \tag{2.19}$$

This requires,

$$DA_\mu = \Pi_{(\mu\, x)} \, d(a^{D-2} A_\mu(x)) \tag{2.20}$$

We noted earlier that the vector field zero modes are associated with large gauge transformations under which A_μ changes as

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \alpha(x). \tag{2.21}$$

The function $\alpha(x)$ parameterizes the asymptotic symmetry group and its integration range is independent of a. Thus upon changing integration variables from the $A_\mu(x)$ zero modes to the large gauge parameters $\alpha(x)$ we pick up a factor of $a^{(D-2)/2}$ for each independent zero mode A^n_μ or equivalently each independent gauge transformation α^n, that is,

$$\beta_v = \frac{D-2}{2} \tag{2.22}$$

Performing the analogous analysis for metric fluctuations $h_{\mu\nu}$ we find,

$$\beta_m = \frac{D}{2} \tag{2.23}$$

Though we will not need it in this thesis the analogous analysis for gravitino zero modes yields,

$$\beta_f = D - 1 \tag{2.24}$$

Determination of N^i_{zm}

In order to compute the number of zero modes for each field species it is convenient to define the following covariant Kernel,

$$K^i(x) \;=\; \sum_n G^i_{pq} \Psi_n^{(i,p)}(x)\, \Psi_n^{(i,q)}(x). \tag{2.25}$$

Here G^i_{pq} is a non-degenerate metric on the space of the ith field species. For example for the vector field A_μ we have,

$$G^{v}_{pq} \;=\; g_{pq} \tag{2.26}$$

The index n runs over all zero modes of the ith field species. The zero modes $\Psi_n^{(i,p)}(x)$ are normalized such that,

$$\int d^D x\, \sqrt{g}\, G^i_{pq}\, \Psi_n^{(i,p)}(x)\, \Psi_{n'}^{(i,q)}(x) \;=\; \delta_{nn'} \tag{2.27}$$

Because AdS_2 is a homogeneous space the kernel $K^i(x)$ does not depend on AdS_2 coordinates. Denoting the coordinates of the internal space M by y we have,

$$N^i_{zm} \;=\; \int dx\, \sqrt{g}\, K^i(x) \;=\; 2\pi \left(\frac{1}{\epsilon} - 1\right) \int d^{D-2}y\, \sqrt{g_y}\, K^i(y) \tag{2.28}$$

The infinite pre-factor of $2\pi\left(\frac{1}{\epsilon} - 1\right)$ arises from the AdS_2 volume integral which we could pull out since $K^i(x)$ depends only on the internal coordinates. The problem has now been reduced to computing $\int d^{D-2}y\, \sqrt{g_y}\, K^i(y)$. This is most conveniently done by first dimensionally reducing each field species to AdS_2. As an example consider the metric. Upon dimensional reduction over M we get scalars, vectors and a two tensor on AdS_2. The number of metric zero modes on $AdS_2 \times M$ equals the total number of zero modes from the AdS_2 fields obtained after dimensionally reducing the metric fluctuations on M. Thus N^i_{zm} can be written as,

$$N^i_{zm} \;=\; \sum l\, N^{(i,l)}_{zm} \sum l \int_{\mathrm{AdS}_2} dx^2\, \sqrt{g}\, K^{(i,l)}_{\mathrm{AdS}_2}(x), \tag{2.29}$$

where $K^{(i,l)}_{\mathrm{AdS}_2}(x)$ is the Kernel for the the lth AdS$_2$ field coming from the reduction of the the ith field on AdS$_2$ $\times$ M. The computation of $N^{(i,l)}$ for $l = v,\ m$ i.e. for AdS$_2$ vectors and two tensors has been done in [31, 32] where it is also shown that AdS$_2$ scalar fields have no zero modes. The results are,

$$N^{(i,v)} \;=\; \frac{1}{\varepsilon} - 1,\;\; N^{(i,m)} \;=\; \frac{3}{\varepsilon} - 3 \tag{2.30}$$

Because AdS$_2$ is a non-compact space with infinite volume the number of zero modes is infinite which is reflected in the volume divergence in (2.30). However writing the a dependence of the zero mode contribution Z_z to Z_{AdS_2} as,

$$Z_z \;\sim\; a^{\sum_i \beta_i\, N^i_{zm}} = e^{\log a\left(\sum_i \beta_i\, N^i_{zm}\right)}, \tag{2.31}$$

we see that any volume divergences in $N^{(i,v)}$ and hence N^i_{zm} can be absorbed into into the arbitrary ground state energy E_0.

Putting together the above results for a general extremal black hole labelled by N_C Cartan angular momenta in a theory with n_V D dimensional gauge fields we get the following finite a dependent contribution to Log Z_z and hence also to the microcanonical degeneracy $d(\{Q_j\}$,

$$\mathrm{Log}\, Z_z \;\sim\; \log a\left(-3\frac{D-2}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V\right) \tag{2.32}$$

2.2.2 Microscopic computation of logarithmic corrections

In this section we discuss the microscopic computation of the logarithmic corrections to the extremal entropy (2.32). Much of the computation can be directly imported from the microcopic calculation in section 1.3 of the previous chapter. First, consider the correction coming from a change of ensemble in the microscopic theory with respect to the chemical potentials corresponding to the N_C Cartan generators of the spatial rotation group as well as those corresponding to the n_V D dimensional U(1) vector fields. According to (1.62) they give a contribution,

$$\Delta S_{ext}\left(\vec{J},\vec{Q}\right) \;\supset\; \log a\left(-\frac{D-2}{2}N_C-\frac{D-4}{2}n_V\right) \tag{2.33}$$

Thus we have shown that the logarithmic contribution to $S_{ext}\left(\{Q_j\}\right)=\mathrm{Log}\, d(\{Q_j\})$ from the n_V U(1) vector zero modes as well as that from the metric zero modes associated with the rotating $t-\varphi^i$ components of $g_{\mu\nu}$ (i.e. those which upon dimensional reduction become AdS_2 gauge fields) is manifested on the microscopic side as the correction coming from the laplace transform with respect to the corresponding chemical potentials involved in going from the canonical to the microcanonical ensemble.

It remains only to reproduce the $-3\frac{D-2}{2}$ contribution in (2.32) which comes from the integral over AdS_2 symmetric two-tensor (metric) zero modes. To do so note that after performing the laplace transfom with respect to T_R in (1.56) we get the density of states $\rho(E_R,\{Q_j\})$. In order to compute the extremal degeneracy we would like to set $E_R=0$. However this is too quick because $\rho(E_R,\{Q_j\})$ is a density in the sense that $\rho(E_R,\{Q_j\})\Delta E_R$ equals the total number of states in the interval ranging from E_R to $E_R+\Delta E_R$. However in the extremal case $d(0,\vec{J},\vec{Q})=e^{S(J,Q)_{\rm ext}}$ is a degeneracy, not a density, because the extremal black hole is labelled by charges which are quantized (discrete). In other words we are counting the total number of extremal states associated with a fixed set of charges. We now explain how to go from the microscopic density computed in 1.3 to the degeneracy in the limit $E_R\to 0$.

Let the number of quantum microstates in the interval from E_R to $E_R+\Delta E_R$ be Δn. Then,

$$\Delta n=\rho(E_R,\{Q_j\})\,\Delta M \tag{2.34}$$

Since we are interested in the extremal limit, the temperature of non-extremal excitations, T_R, is infinitesimal and we work at leading order in T_R. As discussed in chapter 2, T_R does not scale with a. To first order in T_R the number of near-extremal states in the range dE_R is,

$$dn = e^{S_{mc}(J,Q)_{\text{ext}}} \times \left(\frac{\partial S}{\partial T_R} T_R \right) dE_R \tag{2.35}$$

From equation (1.60) we see that,

$$\frac{\partial S}{\partial T_R} \sim c \sim a^{D-2} \tag{2.36}$$

Hence in addition to the extremal degeneracy $e^{S(J,Q)_{\text{ext}}}$, (2.34) also includes a number of near-extremal (excited) states that scales with c at first order in T_R. Since we are working in a large c expansion it does not make sense to discard this $O(c)$ number of states in taking the extremal limit $T_R = 0$. In order to obtain the extremal degeneracy we must first change variables and express the density in terms of a charge X and take the extremal limit such that there are an $O(1)$ number of excited states about the extremal point $X = 0$ for $DeltaX \sim O(1)$.The result will equal the extremal degeneracy upto irrelevant numerical factors independent of a. Defining,

$$E_R \equiv \frac{X}{c} \tag{2.37}$$

$$\rho(E_R) = e^{S(E_R,\{Q_j\})} \text{ and } \tilde{\rho}(X) = e^{\tilde{S}_{mc}(X,\{Q_j\})} \tag{2.38}$$

Since ρ is a density (not a scalar degeneracy) it transforms as,

$$\tilde{\rho}(X) = \frac{\partial E_R}{\partial X} \rho(E_R) = \frac{\rho(E_R)}{c}. \tag{2.39}$$

We identify the extremal degeneracy as a limit of the density as

$$e^{S(\vec{J},\vec{Q})_{\text{ext}}} = \tilde{\rho}(X)|_{X\to 0} = \frac{\rho(E_R)}{c}|_{E_R\to 0}, \tag{2.40}$$

Adding this extra logarithmic correction to the correction coming from the change of ensemble with respect to T_R in (1.62) we reproduce the remaining macroscopic correction of $-3\frac{D-2}{2}$ coming from the integral over AdS_2 metric zero modes.

Let us elaborate a bit more on the above reasoning. As discussed in [19] the spectrum of black hole microstates has a gap of order $1/c$ in the energy above extremality (E_R) at $E_R = 0$. Furthermore there are an $O(1)$ number of states at the edge of this gap with energies of order $1/c$. Since our results for the entropy are organized as a large c expansion and the gap closes as $c \to \infty$, we expect the statistical fluctuations of the $O(1)$ number of states present at the gap to be relevant when extracting the extremal density $\rho(0, \{Q\})$ from the canonical partition function. Since we are interested in counting only extremal states (in a c expansion) we do not expect statistical fluctuations involving a large -i.e. scaling with c - number of excited states to affect $\rho(0, \{Q\})$. It is to systematically retain only the $O(1)$ number of states with $E_R \sim 1/c$ at the gap that we changed variables from E_R to X. The gap in the X spectrum at $X = 0$ is $O(1)$ and there are are an $O(1)$ number of excited states for any $O(1)$ value of X. Thus in computing the density $\tilde{\rho}(X = 0, \{Q\})$ via a transform from the canonical ensemble only statistical fluctuations of the $O(1)$ number of states at the gap are relevant.

3

A family of dS/CFT dualities

In this chapter we analytically continue the conjectured dualities between Chern-Simons theories coupled to vector matter and Vasiliev higher spin gravities in AdS_4 to deduce the analogous dualities in dS_4. We start by reviewing the relevant $U(N)$ Chern-Simons theories coupled to fundamental matter as well as their statistics reversed versions in section 2. In Section 3, we review the parity-violating Vasiliev theories in the AdS_4 and dS_4 vacua. In section 4, we present an analytic continuation that relates them. In particular, we show how the n-point correlation functions in AdS_4 and in dS_4 are related by this analytic continuation. In section 5, higher-spin bulk duals are conjectured for the various wrong-statistics Chern-Simons-matter theories. Formulae are given relating the bulk coupling constants and boundary conditions to the boundary level, gauge group and interactions. Spinor conventions are in the appendix.

3.1 $U(N)$ Chern-Simons theories

In this section, we briefly review the Chern-Simons scalar and Chern-Simons fermion theories in three dimensions, which are conjectured to be dual to the parity-violating Vasiliev theories in AdS_4 [25, 30]. We also discuss the statistics reversed versions of these theories which are conjectured, in Section 4, to be dual to the parity-violating Vasiliev theories in dS_4.

3.1.1 Spinors

The action for a complex anticommuting fermion ψ in the fundamental representation of $U(N)$ coupled to a gauge field A_i with a level k Chern-Simons interaction in three Lorentzian dimensions is

$$S = \frac{k}{4\pi}\int tr\left(AdA + \frac{2}{3}A^3\right) + \int d^3x\,\bar{\psi}\gamma^i D_i\psi. \tag{3.1}$$

According to [30], the spectrum of primary operators in this theory consists of a spin-s single trace operator $J^{(s)}$ for each $s > 0$, which take the schematic form

$$J^{(s)}_{i_1\cdots i_s} = i^s\bar{\psi}\gamma_{i_1}D_{i_2}\cdots D_{i_s}\psi + \cdots\,. \tag{3.2}$$

$J^{(s)}_{i_1\cdots i_s}$ are almost conserved, in the sense that the violation of current conservation is suppressed by a power of $1/N$, and the conformal dimensions of these operators are given by the unitarity bound up to $1/N$ corrections, i.e. $\Delta = s + 1 + \mathcal{O}(1/N)$. In additional to the spin-s primary operators, there is a spin zero primary operator:

$$J^{(0)} = \bar{\psi}\psi, \tag{3.3}$$

with conformal dimension $\Delta = 2 + O(1/N)$. All other primaries are products of these "single-trace" operators. This theory is conjectured by [30] to be dual to a Vasiliev theory with parity-violating phase θ_0.[1] The higher-spin currents $J^{(s)}$ are

[1] We review parity-violating Vasiliev theories in section 3.

dual to the higher-spin gauge fields in the bulk, and the spin zero operator $J^{(0)}$ is dual to a bulk scalar with $\Delta = 2$ (Dirichlet) boundary condition. The 't Hooft coupling

$$\lambda = \frac{N}{k} \tag{3.4}$$

is mapped by the duality to the parity-violating phase θ_0 by

$$\theta_0 = \frac{\pi}{2}(1 - \lambda). \tag{3.5}$$

The planar three-point functions for the spin-0, 1 currents have been computed in [15], which exactly match with the tree level correlation functions in the bulk Vasiliev theory. In general, the N dependence of a Feynman diagram is given by N^{2-2g-h}, where h is the number of fermion loops, and g is the genus of the diagram. More explicitly, the n-point function takes the form as $N^{2-2g-h} f_{g,h}(\lambda, x_k^i)$, where x_k^i for $k = 1, \cdots, n$ are the positions of the n points.

Now consider the same theory, but with opposite statistics for the spinors. The action for the theory is

$$S = \frac{k}{4\pi} \int tr\left(AdA + \frac{2}{3}A^3\right) + \int d^3x\, \bar{\xi}\gamma^i D_i \xi, \tag{3.6}$$

where ξ is a *commuting* Dirac spinor in the fundamental representation of $U(N)$. In the 't Hooft large-N limit, the spectrum of single-trace primary operators contains the spin-s operators $J^{(s)}$ for each $s \geq 0$. These take the schematic form:

$$J^{(s)}_{i_1 \cdots i_s} = i^s \bar{\xi}\gamma_{i_1} D_{i_2} \cdots D_{i_s} \xi + \cdots, \text{ for } s > 0, \text{ and } J^{(0)} = \bar{\xi}\xi. \tag{3.7}$$

By the same argument as in [30], these spin-s operators are almost conserved and have an anomalous dimension of order $1/N$. The correlation functions of these operators can be computed by the exact same diagrams as in the theory (3.1) with the anticommuting spinors. The only change is that there is an extra minus sign associated with every independent matter loop by Bose statistics. As a result, the correlation functions with h matter loops at genus g take the form

$(-1)^h N^{2-2g-h} f_{g,h}(\lambda, x_k^i)$, where $f_{g,h}(\lambda, x_k^i)$ is the same function as in the theory with anticommuting spinors. So to obtain the current correlation functions in the reversed statistics theory, we simply have to flip the sign of N, while keeping λ fixed.

3.1.2 Scalars

The Lorentzian action for a three-dimensional complex scalar φ in the fundamental representation of $U(N)$ coupled to a gauge field A_i with a Chern-Simons interaction at level k is

$$S = \frac{k}{4\pi} \int tr \left(AdA + \frac{2}{3} A^3 \right) + \int d^3x \left(|D_i \varphi|^2 + \frac{\lambda_6}{3! N^2} (\varphi^\dagger \varphi)^3 \right), \qquad (3.8)$$

where $D_i = \partial_i + A_i$, and $k \in \mathbb{Z}$. We are interested in the 't Hooft large-N limit, keeping $\lambda = \frac{N}{k}$ and λ_6 fixed. According to [25], conformality constrains the parameter λ_6 to be a function of λ. The spectrum of operators in the theory includes a single primary operator for each integer spin $s \geq 0$. Each $J^{(s)}$ can be written as a symmetric, traceless tensor that is schematically given by

$$J^{(s)}_{i_1 \cdots i_s} = i^s \varphi^\dagger D_{i_1} \cdots D_{i_s} \varphi + \cdots . \qquad (3.9)$$

As in the fermion case, the $J^{(s)}_{i_1 \cdots i_s}$ are almost conserved currents. This theory is conjectured [25] to be dual to a parity-violating Vasiliev theory with $\Delta = 1$ (Neumann) boundary condition for the bulk scalar, and the higher-spin operators $J^{(s)}$ are dual to the higher-spin gauge fields in the bulk. The planar three-point functions for spin-0, 1, 2 currents have been computed in [26], which exactly match with the tree level correlation functions in the bulk Vasiliev theory with $\theta_0 = \frac{\pi}{2}\lambda$. As in the previous subsection, we want to have a formula for the N dependence of general Feynman diagrams. For this purpose, it is convenient to introduce the auxiliary fields D and σ. The equivalent action with the auxiliary fields is given by

$$S = \frac{k}{4\pi}\int tr\left(AdA + \frac{2}{3}A^3\right) + \int d^3x\left(|D_i\varphi|^2 + \varphi^\dagger(\sigma^2 - D)\varphi + \sqrt{\frac{3!N^2}{\lambda_6}}\mathrm{tr}(D\sigma)\right). \tag{3.10}$$

In this form it is evident that the N dependence of the Feynman diagrams with h matter loops at genus g is given by N^{2-2g-h}.

Now consider the same theory, but with opposite statistics for the scalar field. The action for the theory is

$$S = \frac{k}{4\pi}\int tr\left(AdA + \frac{2}{3}A^3\right) + \int d^3x\left(|D_i\chi|^2 + \frac{\lambda_6}{3!N^2}(\chi^\dagger\chi)^3\right), \tag{3.11}$$

where χ is an anticommuting scalar in the fundamental representation of $U(N)$. In the 't Hooft large-N limit, the spectrum of single-trace primary operators contains the spin-s operators $J^{(s)}$ for each $s \geq 0$. These take the schematic form:

$$J^{(s)}_{i_1\cdots i_s} = i^s\chi^\dagger D_{i_1}\cdots D_{i_s}\chi + \cdots . \tag{3.12}$$

By the same argument as in [25], these spin-s operators are almost conserved and have an anomalous dimension of order $1/N$. The correlation functions of these operators can be computed by the exact same diagrams as in the theory with the commuting scalar. The only change is that there is an extra minus sign associated with every matter loop by Fermi statistics. The net effect is to flip the sign of N while keeping λ fixed.

3.1.3 Wick rotation

The future boundary of dS_4 has Euclidean signature, so we are interested in Euclidean CFT_3s. Let us consider the analytic continuation of the statistics reversed $U(N)$ Chern-Simons spinor and Chern-Simons scalar theories from Lorentzian signature to Euclidean signature. This can easily be done by an analytic continuation of the coordinate of the function $f_{g,h}(\lambda, x^i_a)$ from x^0_a to $-ix^3_a$.

More explicitly, the analytic continuation of the higher-spin currents are

$$J^{(s),E}_{i_1\cdots i_s} = i^n J^{(s),L}_{i_1\cdots i_s}\Big|_{x^0\to -ix^3}, \tag{3.13}$$

where the superscripts L, E distinguish the operators in Lorentzian or Euclidean signature, and n is the number of the indices of $J^{(s),L}_{i_1\cdots i_s}$ that are 0. It is convenient to define generating functions

$$\begin{aligned} J^{(s)}_L(x|y) &= \sum_{i_1,\cdots,i_s} J^{(s),L}_{i_1\cdots i_s}(y\sigma^{i_1}_L y)\cdots(y\sigma^{i_s}_L y), \\ J^{(s)}_E(x|y) &= \sum_{i_1,\cdots,i_s} J^{(s),E}_{i_1\cdots i_s}(y\sigma^{i_1}_E y)\cdots(y\sigma^{i_s}_E y), \end{aligned} \tag{3.14}$$

where y^α is an auxiliary bosonic spinor variable,

$$y\sigma^i_L y = y_\alpha(\sigma^i_L)^\alpha{}_\beta y^\beta, \;\; y\sigma^i_E y = y_\alpha(\sigma^i_L)^\alpha{}_\beta y^\beta, \tag{3.15}$$

$(\sigma^i_E)^\alpha{}_\beta = (\sigma^1, \sigma^3, \sigma^2)$ are Pauli matrices, and $(\sigma^i_L)^\alpha{}_\beta = (\sigma^1, \sigma^3, i\sigma^2)$. In terms of the generating functions, the analytic continuation of the higher-spin currents can be simply stated as

$$J^{(s)}_E(x|y) = J^{(s)}_L(x|y)\Big|_{x^0\to -ix^3}, \tag{3.16}$$

which accounts for the i^n in (3.13). The analytic continuation of the correlators is simply

$$J^{(s_1)}_E(x_1|y)\cdots J^{(s_n)}_E(x_n|y) = J^{(s_1)}_L(x_1|y)\cdots J^{(s_n)}_L(x_n|y)\Big|_{x^0_k\to -ix^3_k}. \tag{3.17}$$

3.2 Vasiliev theories in AdS$_4$ and dS$_4$

In this section, we review the Vasiliev theory in AdS$_4$ and dS$_4$ backgrounds [1, 40–42]. We will start with a background independent formalism, and then specify the vacuum solutions and reality conditions. The fields in the Vasiliev theory are functions of bosonic variables $(x, Y, Z) = (x^\mu, y^a, z^a, \bar{y}^{\dot{a}}, \bar{z}^{\dot{a}})$. Here x^μ are an arbitrary set of coordinates on the four dimensional spacetime manifold with signature $(-,+,+,+)$. $(y^a, z^a, \bar{y}^{\dot{a}}, \bar{z}^{\dot{a}})$ are commuting $SO(1,3)$ spinors. Our spinor conventions for AdS_4 and dS_4 are given in the Appendix A. The Vasiliev master fields consist of an x-space 1-form

$$W = W_\mu dx^\mu, \tag{3.18}$$

a Z-space 1-form

$$S = S_a dz^a + S_{\dot{a}} d\bar{z}^{\dot{a}}, \tag{3.19}$$

and a scalar B, all of which depend on all the bosonic variables introduced above. The master fields are truncated by the condition

$$\pi\bar{\pi}(W) = W, \quad \pi\bar{\pi}(S) = S, \quad \pi\bar{\pi}(B) = B, \tag{3.20}$$

where the π-action is defined as

$$\pi : (y, z, dz, \bar{y}, \bar{z}, d\bar{z}) \mapsto (-y, -z, -dz, \bar{y}, \bar{z}, d\bar{z}), \tag{3.21}$$

and the $\bar{\pi}$-action is given by the π-action with exchanging the barred and unbarred variables. It is easy to check that the equation of motion is consistent with the truncation (3.20).

The master field equation of motion is [1, 2]:

$$d_x\hat{A} + \hat{A} * \hat{A} = \left(\frac{1}{4} + B * K\, e^{i\theta_0}\right) dz^2 + \left(\frac{1}{4} + B * \overline{K}\, e^{-i\theta_0}\right) d\bar{z}^2, \tag{3.22}$$

where

$$\hat{A} = W + S - \frac{1}{2} z dz \tag{3.23}$$

and $K = e^{zy}, \overline{K} = e^{\bar{z}\bar{y}}$ and θ_0 is a coupling constant and d_x is the exterior derivative with respect to spacetime coordinate x^μ. Here the Vasiliev's $*$-product is defined by

$$f*g = f(Y,Z) exp\left[\epsilon^{\alpha\beta}\left(\overleftarrow{\partial}_{y^\alpha}+\overleftarrow{\partial}_{z^\alpha}\right)\left(\overrightarrow{\partial}_{y^\beta}-\overrightarrow{\partial}_{z^\beta}\right)+\epsilon^{\dot\alpha\dot\beta}\left(\overleftarrow{\partial}_{y^{\dot\alpha}}+\overleftarrow{\partial}_{z^{\dot\alpha}}\right)\left(\overrightarrow{\partial}_{y^{\dot\beta}}-\overrightarrow{\partial}_{z^{\dot\beta}}\right)\right]g(Y,Z). \tag{3.24}$$

In the parity invariant A-type and B-type theories, θ_0 takes the values $\theta_0 = 0$ and $\theta_0 = \frac{\pi}{2}$, respectively. Parity is not conserved for generic θ_0. In addition to θ_0, the quantum Vasiliev theory has an additional coupling constant g which measures the strength of quantum corrections. For the Vasiliev theory on AdS$_4$ and dS$_4$ background, we will denote this coupling as g_{AdS} or g_{dS}, respectively.

The Vasiliev master fields are, a priori, complex-valued fields. There are several different consistent reality conditions that can be imposed on the master fields. Different reality conditions preserve different vacuum solutions. In the following two subsections, we review the Vasiliev theory on AdS$_4$ and dS$_4$ backgrounds, and specify the reality conditions that preserve these two backgrounds.

3.2.1 AdS$_4$

Let us consider the Vasiliev theory with the spacetime signature $(+,+,+,-)$, with coordinates denoted $x^\mu = (z, x^1, x^2, x^0)$. The AdS$_4$ vacuum solution is

$$W = W_0 = \omega_0(x|Y) + e_0(x|Y), \quad B = 0, \quad S = 0, \tag{3.25}$$

where

$$\begin{aligned} \omega_0(x|Y) &= -\frac{1}{8}\frac{dx_i}{z}\left(y\sigma^{iz}_{AdS}y + \bar{y}\sigma^{iz}_{AdS}\bar{y}\right), \\ e_0(x|Y) &= -\frac{1}{4}\frac{dx_\mu}{z}y\sigma^\mu_{AdS}\bar{y}, \end{aligned} \tag{3.26}$$

and the σ-matrices are defined in the (3.59). The metric or vielbein are not fundamental quantities in Vasiliev theory. They can be extracted from the vacuum solution W_0. The vielbein e^a_{AdS} can be extracted from $e_0(x|Y)$ by

$$e_0(x|Y) = -\frac{1}{4}\eta_{ab}e^a_{AdS}(y\sigma^b_{AdS}\bar{y}), \tag{3.27}$$

and the AdS$_4$ metric is then given by

$$g^{AdS}_{\mu\nu}dx^\mu dx^\nu = \eta_{ab}e^a_{AdS}e^b_{AdS} = (\eta_{\mu\nu}/z^2)dx^\mu dx^\nu \tag{3.28}$$

The reality condition on the Vasiliev's master fields that preserves the AdS$_4$ vacuum solution is[2]

$$-\iota(\hat{A})^* = \hat{A}, \quad \iota\pi(B)^* = B, \tag{3.29}$$

where the reality condition on the auxiliary variables (Y, Z) are defined in appendix A, and the ι-action is defined as

$$\iota : (y, \bar{y}, z, \bar{z}, dz, d\bar{z}) \mapsto (iy, i\bar{y}, -iz, -i\bar{z}, -idz, -id\bar{z}) \tag{3.30}$$

It follows that the ι-action would reverse the $*$-product, i.e.

$$\iota(f(Y,Z) * g(Y,Z)) = \iota(g(Y,Z)) * \iota(f(Y,Z)). \tag{3.31}$$

At the linear level, after an appropriate gauge fixing and eliminations of the auxiliary fields, the Vasiliev's equation of motion on the background (3.26) reduces to the Fronsdal's equation of motion [12, 13, 43, 45]. The Fronsdal equation in the traceless gauge is,

$$-(\Box - m^2)\phi^{AdS}_{\mu_1\cdots\mu_s} + s\nabla_{(\mu_1}\nabla^\nu\phi^{AdS}_{\mu_2\cdots\mu_s)\nu} - \frac{s(s-1)}{2(d+2s-3)}g^{AdS}_{(\mu_1\mu_2}\nabla^{\nu_1}\nabla^{\nu_2}\phi^{AdS}_{\mu_3\cdots\mu_s)\nu_1\nu_2} = 0, \tag{3.32}$$

where $m^2 = s(s-2) - 2$, and $\phi^{AdS}_{\mu_1\cdots\mu_s}$ is traceless symmetric spin-s gauge field. It appears in the components of the Vasiliev master fields W, B. More explicitly, the spin-s higher-spin gauge field $\phi^{AdS}_{\mu_1\mu_2\cdots\mu_s}$ is an expansion coefficient of the master field W (equation (3.59) in [12])

$$W(x, Y, Z = 0)\big|_{y^{s-1},\bar{y}^{s-1}} \propto (iz)^{s-1}\phi^{AdS}_{\mu_1\mu_2\cdots\mu_s}(y\sigma^{\mu_2}_{AdS}\bar{y})\cdots(y\sigma^{\mu_s}_{AdS}\bar{y}). \tag{3.33}$$

[2] The $\iota(W)^*$ is to be understood as first acting the ι on W then taking the complex conjugate.

The spin zero field ϕ_{AdS} sits in the (Y, Z) independent part of master field B as

$$B(x|Y,Z)\Big|_{Y=Z=0} = \phi_{AdS}. \tag{3.34}$$

At the nonlinear level, one can, in principle, extract the corrections to the right hand side of the linear equation (3.32) from the Vasiliev equation order by order in the number of higher spin gauge fields. A systematic procedure for this was discussed in [12, 34].

The reality condition (3.29) for the master fields, hence, gives the reality condition on the physical higher-spin gauge fields. More explicitly, by imposing the reality condition on equation (3.33), we find

$$(\phi^{AdS}_{\mu_1\cdots\mu_s})^* = \phi^{AdS}_{\mu_1\cdots\mu_s}. \tag{3.35}$$

The scalar field, on the other hand, is the bottom component of B according to (3.34). Imposing the reality condition on (3.34) gives $\phi^*_{AdS} = \phi_{AdS}$ for the spin-0 field.

The scalar has mass square $m^2 = -2$. Depending on the boundary condition for this scalar, its dual operator has either dimension $\Delta = 1$ or $\Delta = 2$, classically. We will refer to the two different boundary conditions as $\Delta = 1$ (Neuman) and $\Delta = 2$ (Dirichlet) boundary conditions, respectively.

3.2.2 dS_4

In dS_4, we label the coordinates by (η, x^1, x^2, x^3) with the signature $(-,+,+,+)$. The dS_4 vacuum solution to Vasiliev's equation of motion is given by

$$W = W_0 = \omega_0(x|Y) + e_0(x|Y), \quad B = 0, \quad S = 0, \tag{3.36}$$

and

$$\begin{aligned}\omega_0(x|Y) &= -\frac{1}{8}\frac{dx_i}{\eta}\left(y\sigma^{i\eta}_{dS}y + \bar{y}\sigma^{i\eta}_{dS}\bar{y}\right),\\ e_0(x|Y) &= -\frac{i}{4}\frac{dx_\mu}{\eta}y\sigma^{\mu}_{dS}\bar{y},\end{aligned} \tag{3.37}$$

where the σ^{μ}_{dS}, $\sigma^{i\eta}_{dS}$ are the σ-matrices defined in (3.64). The vielbein e^a_{dS} is extracted from $e_0(x|Y)$ according to

$$e_0(x|Y) = -\frac{1}{4}\eta^{dS}_{ab}e^a_{dS}(y\sigma^b_{dS}\bar{y}). \tag{3.38}$$

The metric is then

$$g^{dS}_{\mu\nu}dx^\mu dx^\nu \;=\; \eta_{ab}e^a_{dS}e^b_{dS} \;=\; -\,(\eta_{\mu\nu}/\eta^2)dx^\mu dx^\nu \tag{3.39}$$

A reality condition on the Vasiliev's master fields that preserves the dS_4 vacuum solution (3.37) is

$$\pi(\hat{A})^* = \hat{A}, \quad \pi(B)^* = B, \tag{3.40}$$

which is also compatible with the equation of motion (3.2.1) and truncation (3.20).[3]

As in the AdS_4 case, the linearized Vasiliev equation of motion on the dS_4 background (3.37) is reduced to the Fronsdal equation (3.32) with all the subscripts and superscripts AdS replaced by dS. The spin-s higher-spin gauge field $\phi^{dS}_{\mu_1\cdots\mu_s}$ is the expansion coefficient of the master fields W and B:

$$\begin{aligned}W(x,Y,Z=0)\Big|_{y^{s-1},\bar{y}^{s-1}} &\propto \eta^{s-1}\phi^{dS}_{\mu_1\mu_2\cdots\mu_s}(y\sigma^{\mu_2}_{dS}\bar{y})\cdots(y\sigma^{\mu_s}_{dS}\bar{y}),\\ B(x|Y,Z)\Big|_{Y=Z=0} &= \phi_{dS}.\end{aligned} \tag{3.41}$$

The reality condition (3.40) implies

$$(\phi^{dS}_{\mu_1\cdots\mu_s})^* = (-1)^s\phi^{dS}_{\mu_1\cdots\mu_s}. \tag{3.42}$$

[3] This reality condition agrees with [1] when reduced to the minimal theory.

Note that, for the odd spin gauge fields, this reality condition differs from the reality condition (3.35) in AdS_4. However we will find below that they are mapped into one another by our analytic continuation procedure.

3.3 Analytic continuation from AdS_4 to dS_4

In this section, we describe the analytic continuation of Vasiliev theory from AdS_4 to dS_4.[4] Let us start with the Vasiliev equation - with any value of θ_0 - for the master fields expanded about the AdS_4 background. Before imposing reality conditions on either the auxilliary spinor variables or the master fields, the analytic continuation of the coordinates:

$$(z, x^1, x^2, x^0)_{AdS} = (-i\eta, x^1, x^2, -ix^3)_{dS}, \tag{3.43}$$

maps the AdS_4 background solution (3.26) to the dS_4 background solution (3.37). This gives the first-order Vasiliev master field equations expanded about the dS_4 background. It follows that the second-order equation of motion for the physical higher-spin component fields in dS_4 obtained by continuing the second-order Vasiliev equation in AdS_4 will match the second-order equation obtained directly from the Vasiliev equation expanded about the dS_4 background. Hence the AdS_4 and dS_4 theories are simply related by analytic continuation. We will describe our prescription for the analytic continuation of the higher-spin gauge fields in subsection 3.3.1, and of the correlation functions in subsection 3.3.2.

[4] We could have obtained the dualities for the parity invariant Type A and B models by analytically continuing the corresponding results from $EAdS_4$ instead of AdS_4. This was done for the minimal Type A model in [6]. But because of the (4, 0) signature of EAdS_4 we cannot impose reality conditions on the auxiliary Weyl spinors $(y, \bar{y}, z, \bar{z})$. As noted in [1] this means that the reality conditions on the master fields in $EAdS_4$ are not compatible with Vasiliev's equation for the parity-violating models. Here we have circumvented this Euclidean problem by directly continuing from Lorentzian AdS_4 to Lorentzian dS_4.

3.3.1 Fields

In this subsection, we give the analytic continuation of higher-spin gauge fields. First of all, applying the analytic continuation (3.43) to the background solutions (3.26) and (3.37), we find that the AdS$_4$ metric $g^{AdS}_{\mu\nu}$ is indeed related to the dS$_4$ metric $g^{dS}_{\mu\nu}$ by

$$g^{dS}_{\mu\nu}dx^\mu dx^\nu = g^{AdS}_{\mu\nu}dx^\mu dx^\nu\Big|^{x^0\to -ix^3}_{z\to -i\eta}. \tag{3.44}$$

The prescription for continuing the higher-spin fields is

$$\phi^{dS}_{\mu_1\cdots\mu_s}(\eta, x^1, x^2, x^3) = i^n\phi^{AdS}_{\mu_1\cdots\mu_s}(-i\eta, x^1, x^2, -ix^3), \tag{3.45}$$

where n is the total number of 0 and z indices. By the reality conditions (3.35) and (3.42), the odd spin fields are pure imaginary on the left hand side, while the odd spin fields are real on the right hand side.

At first sight, this analytic continuation might seem to lead to bunch of unwanted i's in the Vasiliev equation of motion, but this is actually not the case. Note that there are no explicit indices in Vasiliev's equation of motion, that is, every free index on a higher-spin gauge field must be contracted with $y\sigma^\mu_{AdS}\bar{y}$, $y\sigma^{\mu\nu}_{AdS}y$, $\bar{y}\sigma^{\mu\nu}_{AdS}\bar{y}$ or similar terms with $y, \bar{y}$ replaced by $z, \bar{z}$. When we perform the analytic continuation from AdS$_4$ to dS$_4$, the σ-matrices for AdS$_4$ absorb the i's and turn into the σ-matrices for dS$_4$. More explicitly, the first and last components of σ^μ_{AdS} in

$$\sigma^\mu_{AdS,\alpha\dot{\beta}} = (i1, \boldsymbol{\sigma}^1, \boldsymbol{\sigma}^3, \mathbf{i}\boldsymbol{\sigma}^2) \tag{3.46}$$

absorb an $-i$ and turn into the first and last components of σ^μ_{dS} in

$$\sigma^\mu_{dS,\alpha\dot{\beta}} = (1, \sigma^1, \sigma^3, \sigma^2). \tag{3.47}$$

This suggests that we focus on the generating functions:

$$\Phi^s_{AdS}(x|y,\bar{y}) = \phi^{AdS}_{\mu_1\cdots\mu_s}(y\sigma^{\mu_1}_{AdS}\bar{y})\cdots(y\sigma^{\mu_s}_{AdS}\bar{y}), \tag{3.48}$$

and

$$\Phi^s_{dS}(x|y,\bar{y}) = \phi^{dS}_{\mu_1\cdots\mu_s}(y\sigma^{\mu_1}_{dS}\bar{y})\cdots(y\sigma^{\mu_s}_{dS}\bar{y}). \tag{3.49}$$

For these the analytic continuation procedure takes the very simple form

$$\Phi^s_{dS}(\eta, x^1, x^2, x^3|Y) = \Phi^s_{AdS}(-i\eta, x^1, x^2, -ix^3|Y). \tag{3.50}$$

3.3.2 Correlators

Now we give the prescription for analytically continuing the AdS_4 correlators to the dS_4 ones. In order to go from the classical equations of motion to the quantum ones one must specify an additional coupling (essentially $\hbar$), which we have denoted g_{AdS} (g_{dS}) for AdS_4 (dS_4). These couplings may be defined as the coefficient of the singularity in the scalar two point function: More explicitly, one needs to associate a factor g^{-2}_{AdS} with each internal or external line, and a factor g^2_{AdS} with each (cubic) vertex. This gives a factor $g^{2n+2\ell-2}_{AdS}$ for the ℓ-loop, n-point function. For example, the bulk scalar two point function takes the form

$$\phi^0_{AdS}(x^\mu_1)\phi^0_{AdS}(x^\mu_2)_{AdS} \approx g^2_{AdS}\frac{z_1z_2}{-(x^0_1-x^0_2)^2+(x^1_1-x^1_2)^2+(x^2_1-x^2_2)^2+(z_1-z_2)^2}, \tag{3.51}$$

$$\phi^0_{dS}(x^\mu_1)\phi^0_{dS}(x^\mu_2)_{dS} \approx g^2_{dS}\frac{\eta_1\eta_2}{-(\eta_1-\eta_2)^2+(x^1_1-x^1_2)^2+(x^2_1-x^2_2)^2+(x^3_1-x^3_2)^2}, \tag{3.52}$$

in the limit when the two points are very close to each other, i.e. $(z_1, \vec{x}_1) \to (z_2, \vec{x}_2)$. Once this normalization is specified, the dependence of higher point correlators on the coupling is determined by unitarity. By our analytic continuation procedure, the z_1z_2 in the numerator of (3.53) becomes $-\eta_1\eta_2$ in the numerator of (3.52). Hence, as in [6], the bulk coupling constant must continue as $g^2_{AdS} \to -g^2_{dS}$ at the same time to maintain the positivity of the kinetic term.

We now examine the short distance singularity in the two point function for fields of higher spin gauge fields. For $s > 0$, the two point functions for the physical transverse components of two higher spin gauge fields have the singularity

$$
\begin{aligned}
\phi^{AdS}_{i_1\cdots i_s}(x^\mu_1)\phi^{AdS}_{i_1\cdots i_s}(x^\mu_2)_{AdS} &\approx g^2_{AdS}\frac{(z_1 z_2)^{-s+1}}{-(x^0_1-x^0_2)^2+(x^1_1-x^1_2)^2+(x^2_1-x^2_2)^2+(z_1-z_2)^2},\\
\phi^{dS}_{i_1\cdots i_s}(x^\mu_1)\phi^{dS}_{i_1\cdots i_s}(x^\mu_2)_{dS} &\approx g^2_{dS}\frac{(-1)^s(\eta_1\eta_2)^{-s+1}}{-(\eta_1-\eta_2)^2+(x^1_1-x^1_2)^2+(x^2_1-x^2_2)^2+(x^3_1-x^3_2)^2},
\end{aligned}
\tag{3.53}
$$

in the limit when the two points are very close to each other, where $i_1, i_2, \cdots, i_s = 1, 2$. They are related by our analytic continuation procedure and the analytic continuation of coupling constant: $g^2_{AdS} \to -g^2_{dS}$. Recalling that the reality condition implies that the odd spin component fields are purely imaginary in dS_4, the important factor of $(-)^s$ in the second line of (3.53) implies positivity of the kinetic term (in terms of real fields) is maintained by the analytic continuation.

The rule for the analytic continuation of the bulk correlation function is

$$
\Phi^{s_1}_{dS}(x^\mu_1|Y)\cdots\Phi^{s_n}_{dS}(x^\mu_n|Y)_{dS} = \Phi^{s_1}_{AdS}(x^\mu_1|Y)\cdots\Phi^{s_n}_{AdS}(x^\mu_n|Y)_{AdS}\Big|^{g^2_{AdS}\to -g^2_{dS}}_{x^0\to -ix^3,\, z\to -i\eta}.
\tag{3.54}
$$

The boundary correlation functions can be extracted from the bulk correlation functions by taking the scaled boundary limit [12, 13][5]:

$$
J^{(s)}_{AdS}(\vec{x}|y) = \lim_{z\to 0}\frac{1}{g^2_{AdS}z}\Phi^s_{AdS}(x^\mu|y, \bar{y} = -i\sigma^z_{AdS}y),
\tag{3.55}
$$

and similarly

$$
J^{(s)}_{dS}(\vec{x}|y) = \lim_{\eta\to 0}\frac{1}{ig^2_{dS}\eta}\Phi^s_{dS}(x^\mu|y, \bar{y} = \sigma^\eta_{dS}y).
\tag{3.56}
$$

Therefore, we have

$$
J^{(s_1)}_{dS}(\vec{x}_1|y)\cdots J^{(s_n)}_{dS}(\vec{x}_n|y)_{dS} = J^{(s_1)}_{AdS}(\vec{x}_1|y)\cdots J^{(s_n)}_{AdS}(\vec{x}_n|y)_{AdS}\Big|^{g^2_{AdS}\to -g^2_{dS}}_{x^0\to -ix^3}.
\tag{3.57}
$$

[5] Notice that $-iy\sigma^\mu_{AdS}\sigma^z_{AdS}y = \delta_{\mu,i}y\sigma^i_L y$, and $y\sigma^\mu_{dS}\sigma^\eta_{dS}y = \delta_{\mu,i}y\sigma^i_E y$.

3.4 dS$_4$/CFT$_3$

In section 2, we showed that for both the Chern-Simons scalar and Chern-Simons fermion theories, the net effect of reversing the statistics of the matter fields is flipping the sign of N while keeping λ fixed. Correlators of the statistics reversed theories can be further transformed to the corresponding correlators in Euclidean signature by analytic continuation $x^0 \to -ix^3$. In section 3, we showed that the correlators in the Vasiliev theory in AdS$_4$ and dS$_4$ are related by the analytic continuation (3.57). In particular, the correlators in dS$_4$ are given by the correlators in AdS$_4$ by flipping the sign of the squared coupling, i.e. $g^2_{AdS} \to -g^2_{dS}$ together with the analytic continuation on the coordinates. Using the conjectures in [25, 30], the parity-violating Vasiliev theory in AdS$_4$, with the $\Delta = 1$ or $\Delta = 2$ boundary condition for the scalar, is dual to the Chern-Simons scalar or Chern-Simons fermion theory, respectively, with $N = g^{-2}_{AdS}$. For the case $\Delta = 1$ ($\Delta = 2$), the bulk parity-violating phase θ_0 and boundary 'tHooft coupling $\lambda = \frac{N}{k}$ are related by $\theta_0 = \frac{\pi}{2}\lambda$ ($\theta_0 = \frac{\pi}{2}(1-\lambda)$). Hence, if the conjectures in [25, 30] are correct, the parity-violating Vasiliev theory in dS$_4$, with either boundary condition, is dual to the statistics reversed Chern-Simons scalar or Chern-Simons spinor theories, respectively, with $N = g^{-2}_{dS}$, with θ_0 and λ obeying the same boundary-condition-dependent relation as in the AdS$_4$ theory.

In the special case $k \to \infty$ of our conjecture, we obtain that the Type A theory in dS$_4$ with $\Delta = 1$ boundary condition is dual to the free $U(N)$ anticommuting scalar theory, and the Type B theory in dS$_4$ with $\Delta = 2$ boundary condition is dual to the free $U(N)$ commuting spinor theory. Our conjecture can also be generalized to the $Sp(N)$ Chern-Simons anticommuting scalar or commuting spinor theories.[6] The bulk dual of these theories is the Vasiliev theory in dS$_4$ background with minimal truncation: $-\iota(\hat{A}) = \hat{A}$, $\iota\pi(B) = B$. The Chern-Simons critical scalar and Chern-Simons critical spinor theories are also

[6]The correlators in the $Sp(2N)$ Chern-Simons theory with wrong-statistics matter are equal to the correlators in the $SO(2N)$ Chern-Simons matter theory with N replaced by $-N$. [24]

dual to the parity-violating Vasiliev theory with the $\Delta = 2$ or $\Delta = 1$ boundary conditions, respectively. On the CFT side, by the bosonization duality [26], the Chern-Simons critical scalar theory is dual to the Chern-Simons non-critical spinor theory, and the Chern-Simons critical spinor theory is dual to the Chern-Simons non-critical scalar theory. We expect the bosonization duality still holds after reversing the statistics of the matter fields.

3.5 Appendix

3.5.1 Conventions

In this appendix, we give our conventions for the σ-matrices and the auxilliary spinor variables $(y, \bar{y}, z, \bar{z})$ for the theories in dS_4 and AdS_4.

3.5.2 AdS_4

AdS_4 has signature $(+,+,+,-)$. It is parametrized by the coordinate (z, x^1, x^2, x^0) in Poincare patch. The σ-matrices in AdS_4 are defined by

$$(\sigma^{\mu}_{AdS})_{\alpha}{}^{\dot{\beta}}(\sigma^{\nu}_{AdS})^{\gamma}{}_{\dot{\beta}} + (\sigma^{\nu}_{AdS})_{\alpha}{}^{\dot{\beta}}(\sigma^{\mu}_{AdS})^{\gamma}{}_{\dot{\beta}} = 2\delta^{\gamma}_{\alpha}\eta^{\mu\nu}_{AdS}, \tag{3.58}$$

where $\eta^{\mu\nu}_{AdS} = \mathrm{diag}(1,1,1,-1)$. An explicit representation[7] of the σ-matrices is

$$(\sigma^{\mu}_{AdS})_{\alpha\dot{\beta}} = (i1, \sigma^1, \sigma^3, i\sigma^2), \tag{3.59}$$

where the $\sigma^1, \sigma^2, \sigma^3$ are Pauli matrices. In this representation the complex conjugate of the σ-matrices are given by

$$(\sigma^{\mu}_{AdS,\alpha\dot{\beta}})^* = -(\sigma^{\mu}_{AdS})^{\beta\dot{\alpha}}. \tag{3.60}$$

[7] This is not the conventional representation for the σ-matrices, but it is related by analytic continuation to the conventional representation for the σ-matrices in dS_4.

The reality condition for the bosonic spinor variables (Y, Z) is defined as

$$(y^{\alpha})^* = \bar{y}_{\dot{\alpha}}, \quad (\bar{y}_{\dot{\alpha}})^* = y^{\alpha}, \quad (z^{\alpha})^* = \bar{z}_{\dot{\alpha}}, \quad (\bar{z}_{\dot{\alpha}})^* = z^{\alpha}, \tag{3.61}$$

such that $y\sigma^{\mu}_{AdS}\bar{y}$ is real and

$$(y\sigma^{\mu}_{AdS}\sigma^{\nu}_{AdS}y)^* = \bar{y}\sigma^{\mu}_{AdS}\sigma^{\nu}_{AdS}\bar{y} \tag{3.62}$$

We also define

$$(\sigma^{\mu\nu}_{AdS})_{\alpha\beta} = (\sigma^{[\mu}_{AdS})_{\alpha}{}^{\dot{\gamma}}(\sigma^{\nu]}_{AdS})_{\beta\dot{\gamma}} \tag{3.63}$$

3.5.3 dS_4

dS_4 has signature $(-,+,+,+)$. It is parametrized by the coordinate (η, x^1, x^2, x^3) in Poincare patch. Our definition of the σ-matrices in dS_4 is different from the σ-matrices in AdS_4. Hence, we denote the σ-matrices in dS_4 by σ_{dS} to avoid confusion. The σ-matrices in dS_4 are defined by the same algebra as in AdS_4:

$$(\sigma^{\mu}_{dS})_{\alpha}{}^{\dot{\beta}}(\sigma^{\nu}_{dS})^{\gamma}{}_{\dot{\beta}} + (\sigma^{\nu}_{dS})_{\alpha}{}^{\dot{\beta}}(\sigma^{\mu}_{dS})^{\gamma}{}_{\dot{\beta}} = 2\delta^{\gamma}_{\alpha}\eta^{\mu\nu}_{dS}, \tag{3.64}$$

however, with a different representation:

$$(\sigma^{\mu}_{dS})_{\alpha\dot{\beta}} = (1, \sigma^1, \sigma^3, \sigma^2), \tag{3.65}$$

and $\eta^{\mu\nu}_{dS} = \mathrm{diag}(-1, 1, 1, 1)$. In this representation, the complex conjugate of the σ-matrices is given by

$$(\sigma^{\mu}_{dS,\alpha\dot{\beta}})^* = (\sigma^{\mu}_{dS})_{\beta\dot{\alpha}}. \tag{3.66}$$

The reality condition for the auxiliary variables is defined as

$$(y_{\alpha})^* = \bar{y}_{\dot{\alpha}}, \quad (\bar{y}_{\dot{\alpha}})^* = y_{\alpha}, \quad (z_{\alpha})^* = \bar{z}_{\dot{\alpha}}, \quad (\bar{z}_{\dot{\alpha}})^* = z_{\alpha}, \tag{3.67}$$

such that $y\sigma^{\mu}_{dS}\bar{y}$ is real and

$$(y\sigma^{\mu}_{dS}\sigma^{\nu}_{dS}y)^* = \bar{y}\sigma^{\mu}_{dS}\sigma^{\nu}_{dS}\bar{y} \tag{3.68}$$

We also define

$$(\sigma^{\mu\nu}_{dS})_{\alpha\beta} = (\sigma^{[\mu}_{dS})_{\alpha}{}^{\dot{\gamma}}(\sigma^{\nu]}_{dS})_{\beta\dot{\gamma}} \tag{3.69}$$

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