Essays in Macro-Finance

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Essays in Macro-Finance

A dissertation presented
by

Sarita Bunsupha

to

The Department of Economics

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Economics

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Abstract

This dissertation focuses on three different topics in the intersection between financial economics and macroeconomics. In the first chapter, I embed the standard regime-change games prevalent in macroeconomics and financial economics with strategic substitutability. I extend existing regime-change games by allowing payoffs to vary monotonically with the size of attacking coalition. I characterize equilibrium outcomes under different information structures, thereby extending global games results outside the super-modular setting. In the second chapter, I revisit the relationship between interest rate differentials and differential returns on domestic and foreign bonds over time horizon, using board data samples. I find that countries with higher contemporaneous interest rates earn excess positive bond returns in the short run but reverse to earn excess negative return in the medium run. In the long run, interest differentials have no excess return predictability. I then propose a behavioral model, in which investors rely not only on fundamentals (interest differentials) but also extrapolate past exchange rates when forming future exchange forecasts. Using survey data, I show that the proposed extrapolative model is consistent with both excess return patterns and survey evidence. In the last chapter, I document the price and return term structures of equity using different sources of dividend data on various equity indices around the world. I propose the supply-based asset pricing model and argue that implied index dividends are sensitive to structural flows from equity structured products. Dividend risks, born by intermediaries from the issuance of exotic products, result in the variation of the dividend yield term structure both in the time-series and in the cross-sectional data.
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To my parents, Saengarun and Pimrampai Bunsupha, without whom none of my success would be possible.
Introduction

The first chapter embeds existing regime-change games with a realistic substitutability feature by allowing payoffs to vary monotonically with the size of attacking coalition. I characterize equilibrium outcomes under different information structures, thereby extending global games results outside the super-modular setting. I prove that, in the private signal case, the equilibrium is unique under a class of strategies with monotone aggregate action. As noise vanishes, this equilibrium converges to a monotone equilibrium. The limiting monotone equilibrium strategy is a threshold strategy with the cutoff that is governed by the substitutability feature. The paper highlights the role of liquidity in coordination games. Regarding speculative currency attacks, the model highlights the trade-off between rewards and liquidity and sheds light upon differential trading patterns of developed versus emerging currencies.

Following Engel (2016) and Valchev (2015), the second chapter documents the relationship between interest rate differentials and differential returns on domestic and foreign bonds over time horizon. Using more comprehensive data, I confirm that countries with higher contemporaneous interest rates fail to depreciate against lower interest currencies initially, causing excess positive returns in holding higher interest bonds. Nevertheless, in the medium run, higher contemporaneous interest currencies over-depreciate, resulting in excess negative returns. In the long run, interest differentials have no excess return predictability. The pattern of under- and over-depreciation in exchange rates naturally points to the bubble phenomenon. In particular, I propose the extrapolative model, in which investors rely not only on current and future interest differentials but also extrapolate
past exchange rates to form exchange forecasts. The extrapolative force interacts with the dynamics of interest differentials, resulting in the observed excess return patterns. Using survey data, I confirm that investors indeed extrapolate past depreciation, when predicting future exchange rates.

The third chapter studies the equity term structure. Recent empirical evidence of a downward-sloping term structure of equity risk premia in Van Binsbergen et al. (2012) and Van Binsbergen et al. (2013) challenges many leading asset pricing models. This chapter discusses possible methods to measure $T$-maturity asset prices and reassesses empirical facts using different sources of implied dividend data. It is well known that dividend markets are initiated by intermediaries’ needs to offload their dividend risks. I investigate where such dividend risks come from. After issuing long-dated structured retail products, exotic trading desks are left with long dividend exposure. As these traders cannot warehouse all the residual risks, they need to offload some of these risks in the market. I apply the demand-based option pricing model of Garleanu et al. (2009) to dividend markets and argue that exogenous dividend supply from the structured product issuance impacts dividend pricing. Empirically, I use the issuance data of structured trades to show that the implied index dividend term structure is somewhat sensitive to structural flows from equity structured product issuances. When there is a lot of structured product issuance, the dividend yield term structure is likely to be inverted. I conclude that exotic derivative products are sources of dividend supply shocks, resulting in the variation in implied dividends both in the time-series and in the cross-sectional data.
Chapter 1

Competing to Coordinate

1.1 Introduction


While these stylized models generate meaningful economic implications, they miss one key realistic feature. They feature constant individual payoffs regardless of the size of attacking coalition. In real life applications, the baseline substitutability force always exists. Larger aggregate attacking size should lead to lower individual returns. For instance, investors attack a pegged currency only when they anticipate enough selling pressure that can exhaust the country’s reserve. Once the regime changes to floating, the investors who launched an attack need to buy back the sold currency to capture their profits. A higher aggregate attacking force translate to a greater buying pressure resulting in higher buying prices and lower returns. In this example, coordination is required to complete the mission. However, given the success, standard crowding-out effects apply.

\footnote{Co-authored with Saran Ahuja}
The coexistence of strategic complementarity and strategic substitutability prompts two natural questions: how do these two forces interact? What are equilibria outcomes with the presence of substitutability? In the currency attack setting, one may have a choice of attacking two currencies. The first currency has high potential rewards but is illiquid. The second one has lower total rewards but is more tradable. Suppose these two currencies have the same level of fundamentals, which of the two should traders attack? Which currency will experience regime shifts with higher probability? High rewards aid coordinated attacks, but illiquidity implies that trading sizes have big price impacts. It is unclear ex-ante whether a coordinating force or a competing force is dominant.

To analyze such problems, this paper extends standard regime-change games to allow for variable payoffs. In particular, our setup assumes that individual payoffs decrease monotonically with the aggregate attacking size. This simple modification not only makes existing regime-change games more realistic but also has interesting implications.

We study the modified regime-change games under different information structures. Results are as follows. Under complete information, equilibria have similar characteristics to standard regime-change cases. No one attacks when the fundamental is sufficiently strong. When the fundamental is weak, everyone attacks with the highest positive probability that yields non-negative individual payoffs. In this crisis region, agents either do not attack or attack with the aforementioned positive probability.

Under public signal, at least one additional mixed-strategy equilibrium appears in the crisis region. Agents face not only strategic uncertainty from guessing other people’s actions but also fundamental uncertainty. Any mixed strategy resulting in the expected utility of zero can be sustained as an equilibrium strategy.

In both the complete information and the public signal cases, common knowledge aids coordination among agents, resulting in multiple equilibria.

The most interesting and most realistic case is the private information case. Global games perturbation outside super-modular payoffs remains an open question. This paper offers a leading example of how to solve such a problem. We characterize an equilibrium
strategy in the setting with uniform private noises. The proposed equilibrium strategy may appear to behave differently depending on the degree of substitutability, but the corresponding convergent equilibrium shares a common monotone feature and has an interesting monotone continuity, as discussed in the paper.

When the substitutability is weak, both the right and left dominance regions remain. There exists a monotone pure-strategy Nash equilibrium. Once the substitutability force grows stronger, any finite-switching pure strategy (including monotone strategy) can never be sustained. In particular, the left dominance region vanishes. Starting from the left, any intervals mapping to a constant action have an upper bound on their lengths. Intuitively, each agent can infer a posterior of fundamentals and can subsequently deduce the posterior of possible observed signals. When receiving extremely weak signals, he is aware that any attack will be successful. If no one attacks, he has an incentive to attack. On the other hand, if all agents attack, payoffs from attacking are negative. Agents thus have an incentive to deviate if the same action is taken over a wide range of possible observed signals.

The above thought exercise alludes to the confounding effect added by substitutability. Unlike games with only strategic complementarity, agents’ strategies no longer admit natural orderings. By giving a full characterization of a pure-strategy equilibrium, this paper proves a non-trivial existence result. The single crossing property as in Milgrom and Shannon (1994) does not apply in our setting. Therefore, we cannot use sufficient conditions from Athey (2001) to guarantee the existence of pure-strategy Nash equilibria.

The constructed equilibrium strategy can be described as follows. No attack is dominant when seeing a private signal above a specified cutoff. In this region, the probability of successful attack is too low to cover the attacking cost. As signals weaken, a high potential reward due to a small attacking force will eventually compensate a probability of failed attack. Agents will have strict incentives to attack, as signals move to the left as long as the attacking size has not become big enough for the crowding-out effect to be prevalent. Eventually, as more agents attack, an individual payoff is driven lower and can just balance out the attacking cost. To the right of this signal, agents start to switch between no attack
and attack with a specific ratio leaving them indifferent over the whole region. Although an individual equilibrium strategy seems erratic, the resulting aggregate attacking size is monotone in fundamentals.

Furthermore, we prove that the constructed infinitely-switching pure strategy is a unique equilibrium strategy under a class of strategies with monotone aggregate actions. The uniqueness result and the complete characterization of an equilibrium strategy enable us to study equilibrium properties in details. As the noise of private signal shrinks, the constructed equilibrium strategy becomes more chaotic. The optimal strategy switches more frequently, as noises become more precise. In the limit of revealing private signals, the equilibrium strategy converges in distribution to a monotone mixed strategy.

Games with almost precise signals are perturbations of complete information games. This perturbation uniquely selects an equilibrium strategy in the crisis region. There exists a threshold below which a mixed strategy is played. That is, a mixed strategy is selected over no attack over some range in the crisis region. This may initially appear counterintuitive. It is commonly known that pure strategies are strict under best-response correspondences. We can rationalize our results by the conventional wisdom from Weinstein and Yildiz (2007). They illustrate that multiplicity is not a robust outcome. With rich enough structures, type beliefs can be perturbed so that a chosen rationalizable action is uniquely rationalizable under new types. This unique outcome is subsequently robust to further small perturbations. In our setting, global games perturbation uniquely rationalizes the mixed strategy for some range in the crisis region.

Moreover, our analysis reveals complete determinants of the right-dominant cutoff. For a given reward function and cost, this threshold can be precisely calculated for any level of noises. We are now ready to answer all kinds of questions. In the currency attack example, we can examine how two trade-offs, the potential reward and the liquidity, intertwine.

Naturally, both the level and the rate of substitutability play essential roles in pinning down the equilibrium. Endowing reward functions with supremum norms, the cutoff is continuous in reward functions for any level of noises. Furthermore, the cutoff is
monotonically increasing in reward functions under the point-wise dominance order.

The paper makes theoretical contributions to existing global games literature. While global games emerge as an equilibrium selector for games with super-modular payoffs, the uniqueness of more generalized games cannot be easily established. Morris and Shin (2009) demonstrate, using a Cobweb model, that uniqueness fails commonly in games with strategic substitutability.

Even though our paper is not the earliest work featuring both strategic complementarity and strategic substitutability, it is the first to successfully categorize equilibrium outcomes. Karp et al. (2007), henceforth KLM, study equilibrium properties of incomplete information games where players’ action can be both complements and substitutes. In their setup, an aggregate attacking size and a strength of fundamentals enter payoff functions separately and additively. They use Glickberg’s fixed point theorem to show that a distributional-strategy equilibrium exists. Hoffmann and Sabarwal (2015) later discover the gap in the proof and argues that KLM’s arguments apply only in the finite-player setting.

In addition to their existence claim, KLM documents that, in the class of symmetric pure strategies, monotone switching strategy equilibria exist only when congestion is sufficiently small. When the substitutability force is adequately strong, monotone equilibria vanish. KLM cannot prove the existence of non-monotone pure-strategy equilibria but can deduce that, if such equilibria exist, signal spaces over which one action is optimal must alternate frequently with intervals in which the other action is optimal. This result lays the foundation for our proposition that the length of intervals in which agents take the same action cannot be too large.

This paper builds a tractable model for games with both strategic complementarity and strategic substitutability. Our proposed setup nests standard regime-change games commonly used in economic applications. We analyze equilibrium outcomes under different information structures. Compared to KLM, we show the existence result by giving the full characterization of an equilibrium strategy. We prove that such a strategy is unique under the class of strategies with monotone aggregate attacking size. KLM cannot show such a
strong uniqueness result.

Embedding substitutability into regime-change games make economic models more realistic. Goldstein and Pauzner (2005) note that an agent’s incentive to withdraw early is the highest when the number of agents demanding withdrawal reaches the level at which the bank just goes bankrupt. Recent applications of games with similar structure include He et al. (2015). Investing in a sovereign debt is both strategic complementary and strategic substitute. Sovereign bonds need enough buyers to eliminate the rollover risk. On the other hand, higher investment demands drive up bond prices and lower bond yields.

This paper works out the problem of how the two counteracting forces interact. Adding substitutability in coordination games allows us to fit boarder empirical patterns. For the case of currency attacks, more extreme news is required to trigger trades in less liquid currencies. Intuitively, investors are aware of price impacts when trading. It is harder to justify investors’ trading costs and slippages in selling lower liquid currencies. These trading patterns have implications on price volatilities.

Another direct implication of this model is erratic individual trading strategies among illiquid currencies. Investors often have conforming strategies when trading liquid assets. For illiquid assets, an investor with bad news may not trade because he is rightly afraid that big sell orders will crowd out his returns. Thus, investors with worse news might avoid trading while those with better news dump assets. These are some of implications from the paper. Our setup can be applied to other applications in various branches of economics.

The paper proceeds as follows. Section 1.2 describes the model setup. Section 1.3 discusses equilibria under common knowledge. Section 1.4 analyzes equilibrium outcomes under private signals first for the high substitutability case and then for the low substitutability case for completeness. Section 1.5 and Section 1.6 discuss model implications and relevant applications to economics, respectively. We conclude our findings and contributions in Section 1.7.
Successful \((A \geq \theta)\)  |  Unsuccessful \((A < \theta)\) \\
--- | --- | --- \\
Attack \((a_i = 1)\) | \(b(A) - c\) | \(-c\) \\
Not attack \((a_i = 0)\) | 0 | 0 \\

Table 1.1: *Individual Payoffs*

### 1.2 Model

We consider the following binary-action game of regime changes. There is a continuum of players indexed by \(i \in [0, 1]\). Each player chooses whether to attack \((a_i = 1)\) or not attack \((a_i = 0)\). We denote by \(A = \int_0^1 a_i \, di\) a fraction of players choosing to attack. *Fundamental* parametrized by \(\theta\) governs the regime’s strength. The regime changes if and only if the aggregate attacking size is greater than or equal to the regime’s fundamental, i.e. \(A \geq \theta\). No attack always results in a payoff of zero. If a player attacks, he pays a fixed cost \(c > 0\) and receives a benefit of \(b(A)\) if the regime changes and 0 otherwise. We call \(b : [0,1] \to \mathbb{R}_+\) the *reward function*.

The payoff of a player taking action \(a_i\) when the aggregate attacking size is \(A\) and the fundamental’s strength is \(\theta\), denoted by \(u(a_i, A, \theta)\), can be written as:

\[
u(a_i, A, \theta) = a_i \, b(A) \mathbf{1}_{A \geq \theta} - c,\]

where \(b(\cdot), c, \text{ and } \theta\) are model parameters. Table 1.1 summarizes an individual payoff when he takes action according to the row.

Existing literature on regime-change models often assumes constant individual rewards irrespective of the size of aggregate attack, i.e. \(b(A) \equiv B\), for some constant \(B \in \mathbb{R}_+\). However, in many practical applications, individual rewards are not constant but depend on the size of attacking coalition. When the regime switches, a fixed total amount of rewards are shared among attacking participants. This dilution effect adds the strategic substitutability feature into players’ actions. This paper makes the following assumption on the reward function \(b(\cdot)\).

**Assumption 1** \(b(\cdot)\) is continuous and monotonically decreasing with \(b(0) > c\).
Assumption 1 adopts the continuity in the reward function for the simplicity of the proof. The monotone decreasing property of the reward function depicts the dilution effect. Bigger attacking coalition results in lower individual rewards from successful attacks. $b(0) > c$ ensures that the total reward is strictly higher than the attacking cost, i.e. the problem is not innocuous.

We now define $\rho \in (0, 1]$ as the total attacking size resulting in individual rewards that equal to the attacking cost, namely

$$\rho = \{ A \in [0, 1] ; b(A) = c \text{ if } b(1) \leq c \text{ and } 1 \text{ if } b(1) > c \}.$$  \hspace{1cm} (1.2)

If $b(1) > c$, each individual player finds it profitable to attack even when everyone else attacks. The substitution effect is weak. In this case, $\rho$ is unique and is equal to 1. If $b(1) \leq c$, individual rewards are no greater than the attacking cost when everyone else attacks. The monotonicity and continuity of $b(\cdot)$ imply that $\rho$ will be bigger than 0. There can be multiple $\rho$ satisfying equation (1.2), however strictly decreasing $b(\cdot)$ will feature a unique $\rho$.

In a way, the set of $\rho$ tells us some information about the degree of substitutability. Lower $\rho$ implies higher degree of substitution. As discussed in Section 1.5, the shape of $b(\cdot)$ governs another dimension of substitutability.

Our slight modification in the reward function $b(\cdot)$ extends the standard model in a natural way. Concretely, consider an example of reward function below:

**Example 2**

$$b(A) = \frac{d}{(1 + A)^a}$$

where $d > 0$ represents a total reward, and $a \in [0, \infty)$ governs a degree of substitutability.

In this case, we have

$$\rho = \min \left( \left( \frac{d}{c} \right)^{1/a} - 1, 1 \right).$$

That is, higher $a$ leads to lower $\rho$, i.e. higher degree of substitutability.

Figure 1.1 illustrates an individual attacking payoff as a function of aggregate attacking size $A$ for a given fundamental $\theta$. This reward function in Example 2 can be applied to many
settings. In the case of social media platforms, given I am connected to all my network, an extra online user barely reduces my benefits. The crowding-out effect is negligible, and $\alpha$ in the reward function is close to 0. On the contrary, higher $\alpha$ is more fitting to applications in political economy. A political party needs a certain number of votes to win the campaign. However, a negotiation among a larger group of people hinders agreements. Benefits for each advocate in the winning party decline with the number of people in the winning coalition.

1.3 Equilibrium under Common Knowledge

We first analyze outcomes under common knowledge, as the limiting private information case will pick a unique strategy out of multiple equilibrium strategies presented in the common knowledge case.

1.3.1 Complete Information

When each player observes the underlying fundamental $\theta$, the equilibrium set can be explicitly solved and stated. For notational conveniences, a player adopts a strategy $p \in [0,1]$ whenever his equilibrium strategy is attacking with probability $p$. Figure 1.2 summarizes equilibrium strategies over the range of possible fundamental levels. A formal characterization of the equilibrium set is stated in Appendix A.1.
For sufficiently strong fundamentals ($\theta$ is no less than $\rho$), no attack ($a_i = 0$) is a dominant strategy. At least $\rho$ fraction of the people needs to coordinate their attack to overthrow the regime, resulting in non-positive payoffs. We call this region with a high level of fundamentals "the right dominance region".

Standard regime-change models feature another dominant region when $\theta$ is sufficiently low, i.e. $\theta \leq 0$. The fundamental is so weak that any size of aggregate action will lead to a successful attack. In that model, attacking ($a_i = 1$) is strictly dominant. In our setting, attacking remains a dominant strategy whenever the degree of substitutability remains low. In particular, attacking remains a dominant strategy if and only if $\rho = 1$. A high degree of substitutability ($\rho < 1$) lowers individual benefits making "all attack" unprofitable. The only equilibrium sustainable in this low-fundamental region is attacking with probability $\rho$, which yields a payoff of zero.

For a fundamental that is neither too strong nor too weak, i.e. $\theta \in (0, \rho]$, two equilibria are sustainable. Players coordinate either to take no action or to attack with probability $\rho$. Individual payoffs from both strategies are equal to 0, while resulting regimes are different. The regime persists in the no-attack equilibrium, while the regime switches in the $\rho$-attacking equilibrium. This intermediate region is called "the crisis region". Multiple equilibria in this region with two distinct resulting regimes capture the idea of self-fulfilling attacking, which explains many phenomena in macroeconomics.

1.3.2 Public Signal

Information frictions are prevalent in the real world. In each economic setting, agents rarely observe the exact level of fundamentals. Instead, they observe news that is related in a certain way to true fundamentals. In this section, we analyze equilibrium outcomes when agents in the economy cannot observe fundamentals directly.

Formally, this section considers the following incomplete information setting. Each player observes a public signal $y$, which is a noisy signal of the fundamental:
where $e$ is a uniformly distributed noise. Each player is equipped with (improper) uniform prior on the real line so that the posterior distribution of $\theta$ is uniformly distributed with mean $y$ and the same variance as that of the noise.

**Remark 3** Assumptions of a uniform prior and a uniform posterior enable us to obtain explicit solutions for equilibrium outcomes. As we shall see, results in this section extend to more general noise structures without loss of generality.

Without loss of generality, we restrict our attention to symmetric Nash equilibria, where each player takes the same action.\(^2\) A symmetric strategy with each player attacking with probability $p \in [0, 1]$ leads to an aggregate attacking size $p$. Given that such strategy is played in an equilibrium, the expected payoff from attacking conditional on observing public signal $y$ is given by

\[^2\text{Suppose equilibrium strategies are asymmetric. All players receive same expected payoffs, so they must be indifferent among asymmetric strategies. We can always replace asymmetric strategies with an equivalent symmetric mixed strategy.}\]
\[ \mathbb{E}_y[u(1, p, \theta) \mid y] = b(p)\mathbb{P}(\theta \leq p \mid y) - c, \]  
(1.3)

where \( \mathbb{P}(\cdot \mid y) \) denotes a conditional probability given a public signal \( y \).

No attacking is an equilibrium strategy if and only if the expected payoff from attacking when the aggregate attacking size is zero is no greater than the expected payoff from no attack. Equivalently, \( \mathbb{E}[u(1, 0, \theta) \mid y] \leq 0 \). Similarly, attacking is an equilibrium strategy if and only if the expected payoff from attacking when an aggregate attacking size is one is no less than the expected payoff from no attack. That is, \( \mathbb{E}[u(1, 1, \theta) \mid y] \geq 0 \). A player will play strictly mixed strategy \( p \in (0, 1) \) if and only if he is indifferent between attacking and no attack, i.e. when the following indifference condition holds

\[ \mathbb{E}u(1, p, \theta) \mid y] = 0. \]  
(1.4)

We can fully characterize equilibrium outcomes under the public signal case. For arbitrary payoff functions \( b(\cdot), c \), the public signal \( y \), and the precision of the public signal’s noise \( e \), there exists \( p \) such that the indifference condition (1.4) holds. That is,

\[ F_\theta(p \mid y) = \mathbb{P}(\theta \leq p \mid y) = \frac{c}{b(p)}. \]  
(1.5)

The solution \( p \) to equation (1.5) yields a mixed-strategy equilibrium strategy. When the noise \( e \) is large, there can be an arbitrary number of solutions \( p \in (0, 1) \). However, when the noise goes to zero, most of these equilibria vanish. Figure 1.3 provides an example of plots of \( f(p) = \mathbb{P}(\theta \leq p \mid y) \) and \( g(p) = \frac{c}{b(p)} \) for a given \( y \). Intersections of \( f \) and \( g \) yield mixed-strategy equilibrium strategies.

The right-hand side of Figure 1.3 illustrates that there is at most one intersection between \( f \) and \( g \) for \( p \) in \((y - e, y + e)\) when the noise is sufficiently small. We denote such point by \( \delta(y) \).

Figure 1.4 illustrates an equilibrium correspondence when the noise is sufficiently small. A formal proposition characterizing the set of equilibrium strategies for such case can be found in Appendix A.1.
Figure 1.3: Plot of $f(p) = \Pr(p \geq \theta \mid y)$ (dash) and $g(p) = \frac{c}{E(p)}$ (solid) for $p$ in $(0, 1)$ when $y = 0.5$

Figure 1.4: Equilibria for the Public Signal Case when Noise is Sufficiently Small

Compared to the complete information case, we identify an extra equilibrium strategy $\delta(y)$. This equilibrium strategy arises from the common uncertainty on the strength of fundamentals. There exists another mixed strategy, in which the expected gain from a successful attack is equal to the expected loss due to failed attempts. The emergence of
extra equilibria is due to a structure of regime-change models and is not a result of the added substitutability feature. The strategic substitute only affects an exact level of \( \delta(y) \). The higher the level of substitutability, the lower the probability of attack \( \delta(y) \) so that the expected reward conditional on successful attacks is higher.

### 1.4 Equilibrium with Private Signals

Common knowledge is a restrictive assumption with strong implications. This assumption also aids the equilibrium analysis. In real life applications, agents in the economy are exposed to different sets of information and/or use distinctive technologies in interpreting data. In this section, each player sees neither the fundamentals nor a public signal but observes his own private signal.

Formally, \( \theta \) is unobservable. Each player receives a private signal \( x_i \):

\[
x_i = \theta + \epsilon_i, \quad i \in [0, 1],
\]

where \( (\epsilon_i)_{i \in [0, 1]} \) is independently uniformly distributed over \([-e, e]\). The posterior distribution of \( \theta \) given a signal \( x_i \) is assumed to be uniform over \([x_i - e, x_i + e]\).

Observing individual-specific signals, agents form beliefs not only on fundamentals but also on the distribution of signals observed by the rest of players. The posterior distribution of other players’ signals conditional on observing \( x_i \) is as follows:

\[
x_j = \theta + \epsilon_j = x_i - \epsilon_i + \epsilon_j.
\]

In the uniform noise case, the posterior distribution of other players’ signals spans over an interval \([x_i - 2e, x_i + 2e]\) as displayed in Figure 1.5.

We focus on symmetric-strategy equilibria in the common knowledge case because everyone sees the same signal (either the fundamental or the common public signal). The strategy profile is conditional on the one observed signal. Under private signals, payoffs depend on both fundamentals and actions of others. As each agent observes different signals, the strategy profile depends on the whole posterior distribution of signals observed.
by all players. This paper focuses on distributional-strategy equilibria, i.e. equilibria in which players with the same private signal take the same action.

A measurable function $s : \mathbb{R} \to [0, 1]$ denotes a strategy profile of a player. $s(x_i)$ indicates a probability of attacking conditional on observing a private signal $x_i$. We define the corresponding aggregate action $A^s : \mathbb{R} \to [0, 1]$ by

$$A^s(\theta) = \frac{1}{2e} \int_{\theta-e}^{\theta+e} s(u)du. \quad (1.6)$$

Assuming players use a strategy profile $s$, $A^s(\theta)$ reveals a fraction of players attacking when the actual realized fundamental is $\theta$. Note that $A^s(\theta)$ is unobservable since $\theta$ is unobservable.

Given that other players employ $s$, an action of an individual player $i$ does not affect $A^s$. The attacking payoff of player $i$ with a private signal $x_i$ can be written as:
\[ F^s(x_i) = \mathbb{E}[u(1, A^s(\theta), \theta)|x_i] \]
\[ = \int_{\mathbb{R}} \left( b(A^s(\theta))1_{A^s(\theta) \geq \theta} - c \right) f(\theta|x_i) d\theta \]
\[ = \frac{1}{2\epsilon} \int_{x_i-\epsilon}^{x_i+\epsilon} \left( b(A^s(\theta))1_{A^s(\theta) \geq \theta} - c \right) d\theta \]
\[ = \frac{1}{2\epsilon} \int_{x_i-\epsilon}^{x_i+\epsilon} \left( b(A^s(\theta))1_{A^s(\theta) \geq \theta} \right) d\theta - c, \quad (1.7) \]

when \( f(\theta|x_i) \) denotes the posterior density of \( \theta \) conditional on observing \( x_i \). We will omit the superscript \( s \) in \( A^s \) when it is clear in the contexts.

1.4.1 Strong Substitutability: \( \rho < 1 \)

This section proves by construction the existence of a pure-strategy equilibrium and shows that the established equilibrium is unique under a class of equilibria with monotone aggregate attacking size. We then discuss properties of this equilibrium. In particular, this equilibrium converges to a monotone mixed-strategy as noise vanishes. We conclude this section by commenting on results and comparing them to existing global games literature.

Existence

We first restrict our search to pure-strategy equilibria. Sufficiently strong private signals make no attack dominant. The right dominance region persists. It is however suboptimal to not attack when observing low private signals. When the fundamental is too low, any single player is capable of overthrowing the regime and reaps high benefits. On the other hand, attacking is also unsustainable. A sizable aggregate attacking size leads to inadequate individual rewards. The following lemma formalizes that the length of signal intervals with constant action cannot be too large compared to the signal noise.

Lemma 4 (Infinite-Switching Pure-Strategy) For \( \rho < 1 \), consider any pure-strategy Nash equilibrium profile \( s: \mathbb{R} \rightarrow \{0,1\} \). If \( s \) is constant over any interval \( I = (\bar{x}, \bar{x}) \) with \( \bar{x} \leq 0 \), then \( |I| = \bar{x} - \bar{x} < 4\epsilon \).
Proof. We prove by contradiction. Suppose there exists a pure-strategy Nash equilibrium profile \( s \) and an interval \( I = (x, \bar{x}) \) with \( \bar{x} \leq 0 \) and \( |I| \geq 4\epsilon \). Let \( x_m = \frac{x + \bar{x}}{2} \). If \( s(x) \equiv 0 \) on \( I \), then \( A(\theta) = 0 \geq \theta \) for all \( \theta \in [x_m - e, x_m + e] \) and \( F^s(x_m) = b(0) - c > 0 \). Attacking strictly dominates not attacking, contradicting the fact that \( s \) is an equilibrium strategy. Similarly, if \( s(x) \equiv 1 \) on \( I \), then \( A(\theta) = 1 \geq \theta \) for all \( \theta \in [x_m - e, x_m + e] \). Therefore, \( F^s(x_m) = b(1) - c < 0 \). Attacking is strictly dominated by no attack, again contradicting the fact that \( s \) is an equilibrium strategy.

Lemma 4 is similar to Proposition 3 in Karp et al. (2007). It states that for any pure-strategy Nash equilibrium, the length of signal intervals over which the strategy is constant must be small compared to the noise’s variance. This restricts behaviors of pure-strategy equilibria. Since a signal space spans the real line, this lemma implies that any pure-strategy equilibrium profile must switch between attacking and no attack infinitely many times. That is, all pure-strategy equilibria must be non-monotone.

We next state an existence result by characterizing an infinite-switching pure strategy that can sustain a Nash equilibrium.

**Theorem 5 (Existence of Pure-Strategy Equilibrium)** The following pure-strategy profile describes a Nash equilibrium of regime-change games with \( \rho < 1 \).

\[
\begin{align*}
    s^e(x_i) = \begin{cases} 
        0 & ; x_i \geq \bar{x}^e \\
        1 & ; \bar{x}^e - 2ek - 2e\rho \leq x_i < \bar{x}^e - 2ek, \quad k = 0, 1, 2, \ldots \\
        0 & ; \bar{x}^e - 2e(k+1) \leq x_i < \bar{x}^e - 2ek - 2e\rho, \quad k = 0, 1, 2, \ldots 
    \end{cases}
\end{align*}
\]

(1.8)

when \( \bar{x}^e \) is given by

\[
\bar{x}^e = -e + (1 + 2e)(\rho - \bar{u}),
\]

(1.9)

and \( \bar{u} \) is the unique value \( u \in (0, \rho) \) satisfying

\[
\frac{1}{\rho} \int_0^u b(\rho - y)dy = c.
\]

Proof. See Appendix A.2
Figure 1.6 illustrates the strategy profile from Theorem 5. Actions are clearly non-monotonic in private signals. For all signals greater than the threshold $\bar{x}^e$, players never attack. To the left of this threshold, attacking intervals alternate with no-attack intervals with lengths of $2e\rho$ and of $2e(1 - \rho)$, respectively. Notice that both sets of intervals have lengths less than $4e$, in compliance with Lemma 4.

![Figure 1.6: Strategy Profile $s(x_i)$ as a Function of Private Signal $x_i$ (The length of each interval is shown on top)](image)

The constructed strategy profile results in an aggregate attacking size as shown in Figure 1.7.

Any player can deduce that a regime will be overthrown if and only if $A(\theta) \geq \theta$. Equipped with this information, a player can evaluate when the expected reward from attacking is worth the cost. Players with a private signal $x_i$ calculate a payoff from attacking using equation (1.7). If this payoff is positive (negative), attack (no attack) is dominant. Players will be indifferent if the payoff is zero. Figure 1.8 illustrates the expected payoff from attacking given that all other players follow strategy $s^e$.

Attack is never successful whenever $x_i \geq \hat{\theta} + e$. As a player sees a slightly lower private signal, a probability of successful attack becomes non-zero. When $x_i = x^e \in [\hat{\theta} - e, \hat{\theta} + e]$, a probability of regime changes is high enough to compensate for the attacking cost. We can therefore conclude that a probability of successful attack is too low for $x_i \in (x^e, \hat{\theta} + e)$.
making no attack dominant.

For private signals slightly lower than $\bar{x}^e$, a probability of regime changes increases without having to compensate for negative payoffs from crowding out. The expected payoff from attacking is positive for all $x_i \in (\hat{\theta} - e, \bar{x}^e)$. Attacking strictly dominates in this region.

For any private signal $x_i < \hat{\theta} - e$, attack succeeds with probability 1. Attacking players with $x_i \in (\bar{x}^e - 2e\rho, \hat{\theta} - e]$ know with certainty that an attack will be successful while enjoying higher rewards since less people launches an attack. Attacking remains dominant for this region. Observing a private signal no greater than $\bar{x}^e - 2e\rho + e \leq \bar{x}^e - 2e(\rho - \bar{u}) + e = \hat{\theta}$, a player knows that exactly $\rho$ fraction of players launches an attack and is indifferent between attacking and no attack.

An existence of a pure-strategy Nash equilibrium for the case of strong substitututability is non-trivial. While the right dominance region pertains, the left dominance region vanishes. Due to the severe crowding out, neither all attack nor no attack is optimal when receiving exceptionally low private signals. The iterative deletion of strictly dominated strategy used in standard global games is not of much help here.
Uniqueness

Proposition 4 rules out all monotone pure-strategy equilibria. However, we have yet to explore an entire universe of non-monotone pure-strategy equilibria as well as mixed-strategy equilibria. As we prove by construction an existence result, readers may wonder whether our setup has spurious equilibria. The following theorem demonstrates that the constructed equilibrium is indeed unique under a large class of strategy profiles resulting in monotone aggregate action.

First, we define

$$\Gamma = \{ s : \mathbb{R} \rightarrow [0, 1]; A^s(\theta) \text{ is monotonic in } \theta \}.$$ 

$\Gamma$ consists of all strategy profiles with aggregate attacking sizes that are monotone in the fundamental $\theta$. Strategy profiles in $\Gamma$ include not only all profiles with monotone strategies (both pure and mixed) but also a substantial number of non-monotone strategies. The constructed equilibrium profile $s^e$ from equation (1.8) is obviously non-monotone but is in a class of $\Gamma$, as illustrated by Figure 1.7.

Limiting an equilibrium search to $\Gamma$ is not restrictive. In addition, it is natural to expect an aggregate attacking size to be non-increasing in fundamentals. Players know that stronger
fundamentals will result in higher chances of failed attempts and stay clear from attacking.

**Theorem 6 (Uniqueness under $\Gamma$)** For $e > 0$, the strategy profile $s^e$ described in equation (1.8) is a unique equilibrium in $\Gamma$.

**Proof.** See Appendix A.3

Theorem 6 stresses that there is exactly one equilibrium with monotone aggregate attacking size. Any other equilibria, if exists, must have an erratic relationship between an aggregate attacking size and the fundamental.

This (local) uniqueness hints that private signals still work as an equilibrium selector. Common knowledge results in multiple equilibria, all of which has monotone aggregate attacking size. Taking away common signals deprives players of their coordinating device and eliminates multiple equilibria.

**Convergence**

The constructed equilibrium strategy looks somewhat distinct from equilibrium strategies under the common knowledge assumption. This subsection will show that as the variance of noise goes to zero, the equilibrium strategy converges to a monotone equilibrium strategy and uniquely picks from equilibrium strategies from common information case.

Figure 1.9 shows that the switching between attack intervals and no-attack intervals becomes more rapid with more precise signals.

![Figure 1.9: Pure-Strategy Equilibria as Characterized by Equation (1.8) for Different Levels of Noise](image-url)
In the limit of $e$ going to 0, a threshold $\bar{x}^e$ converges to $\bar{x}^0 = \rho - \bar{u}$. For all sufficiently small $e$, a fraction of attacking intervals in any small neighborhood around $x_i$ has a measure of $\rho$ for $x_i < \bar{x}^e$ and 0 for $x_i > \bar{x}^e$. Define $s^0(x_i)$ as followed:

$$
\begin{align*}
  s^0(x_i) &= \begin{cases} 
    \rho, & \text{if } x_i \leq \bar{x}^0 \\
    0, & \text{if } x_i > \bar{x}^0 
  \end{cases} 
\end{align*}
$$

(1.10)

Figure 1.10 overlays equilibrium strategies with a monotone mixed-strategy $s^0(x_i)$ and convincingly conveys that $s^e$ converges to $s^0$ in some sense as $e \to 0$. Proposition 7 states a formal convergence proposition.

**Figure 1.10: Pure-Strategy Equilibrium Profiles for Different Levels of Noise and The Limiting Monotone Mixed-Strategy Profile**

**Proposition 7 (Convergence to Monotone Equilibrium)** The strategy $s^e$ defined by equation (1.8) converges in distribution to $s^0$ as $e \to 0$. That is, for all $e > 0$, there exists $\delta > 0$ such that $e \in [0, \delta) \Rightarrow \int_{\theta \in \mathbb{R}} |A^e(\theta) - A^0(\theta)| \, d\theta < e$.

**Proof.** It is straightforward to check that $A^e(\theta) = A^0(\theta) = \rho$ for $\theta < \min(\bar{x}^e + e - 2\epsilon, \bar{x}^0)$, $A^e(\theta) = A^0(\theta) = 0$ for $\theta > \max(\bar{x}^e + e, \bar{x}^0)$, and $0 \leq A^e(\theta), A^0(\theta) \leq \rho$ otherwise. Thus,

$$
\int_{\theta \in \mathbb{R}} |A^e(\theta) - A^0(\theta)| \leq (2\epsilon)\rho = 2\epsilon \rho^2
$$

which converges to 0 as $e \to 0$ as desired. □

Proposition 7 states that in the limit of a precise private signal, i.e. the variance of noise is zero, a unique equilibrium under $\Gamma$ converges to a monotone mixed-strategy equilibrium $s^0$ defined in equation (1.10).
The private signal case is thought of as a perturbation of common knowledge cases. In this sense, such perturbation uniquely selects an optimal strategy profile out of multiple equilibrium strategies. Precisely, there is a threshold below (above) which a mixed strategy (no attack) is selected. Figure 1.11 displays an equilibrium selection by private signals.

![Equilibrium Selection by Private Signals](image)

**Figure 1.11: Equilibrium Selection by Private Signals**

### 1.4.2 Weak substitutability: $\rho = 1$

This section discusses equilibrium outcomes for the case $\rho = 1$ for completeness. With a low degree of substitutability, rewards from attacking remain high enough to cover an attacking cost even when everyone else also attacks. We regain the left dominance region, where attacking is dominant when observed private signals are extremely low. The theorem below is parallel to Theorem 5 from the strong substitutability case. In this case, there exists a monotone pure-strategy equilibrium.

**Theorem 8 (Existence of Monotone Pure-Strategy Equilibrium)** When $\rho = 1$, the following
single switching strategy can be sustained in a Nash equilibrium:

\[
 s(x_i) = \begin{cases} 
 0, & \text{if } x_i > \bar{x}^e \\
 1, & \text{if } x_i \leq \bar{x}^e 
\end{cases}, \tag{1.11}
\]

where the threshold \( \bar{x}^e \) is given by

\[
\bar{x}^e = -e + (1 + 2e)(1 - \bar{u}), \tag{1.12}
\]

and \( \bar{u} \) is a unique value \( u \in (0, 1) \) satisfying

\[
\int_0^{\bar{u}} b(1-y)dy = c.
\]

**Proof.** See Appendix A.4  ■

A strategy profile in Theorem 8 in exactly the same as a strategy profile in Theorem 5 when setting \( \rho = 1 \). In this case, all no-attack intervals to the left of \( \bar{x}^e \) have a measure of zero. The infinite switching disappears. Instead, we have a monotone pure-strategy equilibrium. Figure 1.12 displays an equilibrium strategy profile for the case of weak substitutability.

The constructed monotone pure-strategy equilibrium is similar to a unique monotone pure-strategy equilibrium from classical regime-change games with constant rewards (\( b(A) \equiv B \)). The substitutability affects only a threshold \( \bar{x}^e \) of equilibrium outcomes. In
particular, the higher degree of substitution results in the lower threshold $\bar{x}^e$. We formally
discuss this in Section 1.5.

One may again wonder whether the constructed monotone-pure strategy equilibrium
in Theorem 8 is unique. In classical regime-change games, an iterate deletion of strictly
dominated strategies pins down a unique equilibrium strategy. As both the left and right
dominance regions persist in the weak substitutability case, we conjecture that the same
iterated deletion method should apply and result in a similar uniqueness outcome.

Regardless, Theorem 6 still applies. That is, the constructed monotone pure-strategy
stated by equation (1.11) is unique under a class of strategies with monotone aggregate
action.

The level of noise affects the switching frequency in the case of strong substitutability
($\rho < 1$). When the substitutability is weak, the shape of equilibrium strategy is independent
of noises. An equilibrium strategy always has a single switching regardless of the size of
the noise variance. The noise precision affects only the switching threshold. Again, taking
the limit of precise private signals uniquely picks an equilibrium strategy from the common
knowledge case. The resulting equilibrium outcome retains a monotone switching shape,
and players attack below a certain cutoff and do not attack otherwise.

1.5 Model Implications

This paper extends standard regime-change games in a natural way. Inspired by the demand-
driven price determination, we allow individual payoffs to vary with aggregate attacking
size. We analyze equilibrium outcomes under different information structures. Our biggest
contribution is an analysis under private signals.

We show that any pure-strategy Nash equilibrium must be infinitely switching in private
signals whenever the substitutability force is strong. We construct a pure-strategy Nash
equilibrium and prove that it is unique under a class of equilibrium strategies with monotone
aggregate action.

The constructed equilibrium strategy looks erratic. The infinitely switching implies that,
when the crowding out is strong, better signals can induce attack while worse signals may not. Individual strategies become more erratic as noise gets smaller. Despite its unstable behaviors, the constructed equilibrium strategy has nice aggregate properties. A resulting aggregate attacking size is monotone in realized but unobservable fundamentals.

Furthermore, this equilibrium strategy converges to a monotone mixed-strategy equilibrium in the limit of precise private signals. That is, a mixed-strategy is selected over a pure strategy over a range in the crisis region. This result may sound surprising because mixed-strategies are often thought to be non-robust under best-respond correspondences. However, we can reconcile this result using wisdoms from Weinstein and Yildiz (2007). A private signal perturbation is a perturbation that can rationalize a mixed strategy.

Another nice property of the current setup is that it nests standard regime-change games. As mentioned in Section 1.4, the substitutability force affects the threshold above which no attack is dominant. In particular, lower substitutability induces attacking and results in higher threshold as well as higher probability of regime changes. We formalize the monotonicity property in the proposition below.

**Proposition 9 (Monotonicity and Continuity)** For a given \( e \geq 0 \) and \( b(\cdot) \) with a unique \( \rho \) such that \( b(\rho) = c \), \( \bar{x}^e \) is continuous in \( b(\cdot) \) endowed with the supremum norm and monotonically increasing in \( b(\cdot) \) under the pointwise dominance order.

**Proof.** See Appendix A.5.

**Corollary 10** The cutoff threshold \( \bar{x}^0 \) from a monotone equilibrium strategy \( s^0 \) is monotonically increasing in \( b(\cdot) \) under the pointwise dominance order.

Figure 1.13 displays the continuity and the monotonicity of \( \bar{x}^0 \) from an illustrative case: \( b(A) = \frac{d}{(1+A)^\alpha} \) for all \( \alpha \in [0, \infty) \). The lower \( \alpha \) means lower substitutability and lower \( \rho \). The resulting \( \bar{x}^0 \) is higher.

In fact, with a complete characterization of an equilibrium strategy, our paper enables readers to calculate the cutoff threshold \( \bar{x}^e \) for any given level of noise, reward function \( b(\cdot) \), and cost \( c \).
Moving away from the pointwise dominance ordering, Figure 1.14 illustrates that both the level and the rate of substitutability matter. The first and second reward functions feature weak substitutability. However, both have the switching thresholds $x^0$ that are lower than that of the third reward function. Even though the third reward function features a strong substitutability, the substitutability force kicks in very late over a small range of an aggregate attacking size.
1.6 Model Applications

Our setup applies to various branches of economics. In macroeconomics and financial economics, it explains investor trading patterns. If we define illiquid stocks as those with high price impacts from trading sizes, we can explain why erratic trading patterns are prevalent among illiquid stocks. For liquid assets, investors with worse news sell whenever those with better news do. However, with less liquid security, investors with worse news may decide to hold on to illiquid stocks even when those with better news offload them. Intuitively, traders are aware that an execution of illiquid stocks has high slippage costs. An executed selling price will be so low that positive returns from selling are no longer warranted. Additionally, the monotonicity from Proposition 9 states that extreme news is needed to trigger trades of illiquid stocks compared to liquid stocks.

Other macro-finance implications includes an explanation of why bigger debt pools might be more attractive than smaller ones once controlled for fundamentals. In the context of corporate bankruptcy, it explains why companies with more assets may face a higher risk of default.

In the political economy setting, the model rationalizes the concept of minimum winning coalition and explains formation patterns of treaty organizations or unions around the world. In this case, players in our setting are electorates voting for a party or countries debating whether to join a union.

Related to urban economics, this model extends the "big push” model and ties a city size to a probability of industrialization. The movement of people and firms across borders affects the distribution of country-specific wealth. One may embed this model to enrich existing literature in international trade.

1.7 Conclusion

This paper studies regime-change games with payoffs that incorporate the strategic substitutability force.
Under common knowledge, there are multiple equilibria in the crisis region, where the fundamental is neither high nor low. Pure strategies are played when the degree of substitutability is weak, while mixed strategies pertain when the substitutability becomes strong.

Under private signals, each player needs to form beliefs not only about fundamentals but also about what other players believe. Higher-order beliefs complicate the equilibrium analysis. Standard regime-change games manage to pin down equilibrium outcomes by the iterated deletion of dominated strategies.

Once the substitutability is introduced, a player’s action no longer admits a natural ordering. While the iterated deletion of dominated strategies restricts possible equilibrium outcomes, it fails to characterize them precisely. The paper proves the existence of equilibrium by carefully constructing a strategy profile sustainable in the Nash equilibrium. The constructed strategy profile is shown to be unique under a class of strategy profiles with monotone aggregate attacking size. Taking away an assumption of common belief foundation helps eliminating the indeterminacy of equilibrium outcomes as per an insight from Morris and Shin (2000).

While we are unable to show global uniqueness, we argue that searching over a class of strategies with monotone aggregate attacking size is not at all restrictive. Such class includes both pure and mixed strategies that can be either monotonic or not. Even though individual strategies appear erratic, they aggregate up nicely. Our uniqueness result states that either players follow the constructed strategies or they follow some equilibrium strategies that will result in an aggregate attacking size that is non-monotone with respect to fundamentals.

Contrasting to previous literature on the intersection of the strategic complementarity and the strategic substitutability, this paper is the first to successfully characterize an equilibrium set. Our setup is also appealing in a sense that it is a generalization of standard regime-change games.

From our analysis, strong substitutability invalidates all monotone strategy profiles. The only sustainable equilibrium is an infinitely-switching pure-strategy one. This strategy
profile behaves more erratically, as noises get smaller. As noises vanish, such strategy profile converges to a monotone mixed-strategy equilibrium. That is, in some range of the crisis region, a mixed strategy is uniquely picked against no attack. This might seem like a challenge to the conventional wisdom that mixed strategies are not robust under best-response correspondences. We reconcile this finding by using Weinstein and Yildiz (2007) and argue that, in fact, this mixed-strategy equilibrium is a natural outcome from extending standard regime-change games.

A study of the setup in this paper is essential because, in real life applications, players’ actions have both the strategic complement and the strategic substitute components. The model implies economic intuitions and applications that the standard model falls short to talk about. For example, the paper sheds light on trade-offs between high total rewards and severe crowding out.

This paper focuses on theoretical contributions, but we acknowledge various potential promising applications to economics. As a matter of fact, He et al. (2015), henceforth HKM, use a variation of our more generalized reward functions to explain determinants of reserve assets. The main differences between HKM and our paper are as follows. First, HKM study the competition between two countries. Each investor either invests in one country’s debt or the other. The analysis performed in this paper is more suitable to the setup when players either invest or do not invest. Second, HKM focus mainly on applications and makes certain simplifying assumptions on information structures. HKM assume that each player believes his private signal fully reveals the fundamental but is uncertain about what other players observe. This setting features only strategic uncertainty with no fundamental uncertainty. Our paper considers standard private information structures with both fundamental and strategic uncertainty.

Beyond sovereign debts, the model proposed in this paper is well suited to explain any interaction in group settings. Such settings naturally speak to the field of political economy. Reward functions with both strategic complementarity and strategic substitutability represent individual payoffs from joining a group.
While the paper focuses on the classical regime-change setting in which players’ action is binary, an extension to multiple regimes is interesting. Without substitutability, a higher number of regimes intuitively hinders coordination. With substitutability, more regimes potentially alleviate the crowding out. Players may receive higher payoffs if they can coordinate and distribute their forces across different regimes. We leave a proper analysis of multiple regimes for further research.

Other extensions include (1) a robustness check of results with respect to different noise structures (normally distributed noise instead of uniform noise). We conjecture that the existence, the uniqueness, and limiting properties stated in this paper should hold for more general distributions; (2) a generalization of payoff functions, for example, reward functions may depend not only on aggregate attacking size but also a level of fundamentals; (3) a micro-foundation of applications.
Chapter 2

Extrapolative Beliefs and Exchange Rate Markets

2.1 Introduction

There are two major approaches in valuing currencies: the demand-based approach and the fundamental approach. Standard finance theories follow the second approach and believe that the correct valuation of any asset is its fundamental value.

The fundamental of assets can be decomposed into two components: the flow utility and the future valuation (the expectation of future derived utilities). For stocks, prices fundamentally relate to current dividends and future expected dividend streams. Bond prices depend on expected interest accrued. Analogously, exchange rates are fundamentally pinned down by differences between interests on the long and short legs of a currency pair.

The relationship between interest rate differentials and bilateral exchange rates allows economists to model behaviors of exchange rates. Formally, let $s_{fh,t}$ be the log of the exchange rate at time $t$ in terms of home ($h$) currency per foreign ($f$) currency, $i_t$ and $i_t^*$ be respective 1-period home and foreign nominal interest rates of default-free bonds at time $t$, $x_{fh,t} = i_t - i_t^*$ be time-$t$ interest rate differential, $E_t(s_{fh,t+k})$ be the log of time-$t$ expectation of $k$-period-ahead spot rate, and $t$ be time with the unit equals to 1 period.
Home bonds yield an interest of $i_t$, while foreign bonds expose investors to additional exchange depreciation risk. In particular, 1-period return of holding foreign bonds is equal to $i_t^* + \mathbb{E}_t(s_{fh,t+1}) - s_{fh,t}$. Returns earned from home bonds should be equalized to returns from foreign bonds. That is, the following hold:

$$s_{fh,t} = \mathbb{E}_t(s_{fh,t+1}) - (i_t - i_t^*)$$

$$= \mathbb{E}_t(s_{fh,t+1}) - x_{fh,t}$$

$$= \mathbb{E}_t(s_{fh,t+T}) - \sum_{j=0}^{T-1} \mathbb{E}_t(x_{fh,t+j}). \quad (2.1)$$

The above relationship implies that interest rate differentials are fundamentals pinning down exchange levels. Yet, Meese and Rogoff (1983) and previous literature find that a random walk predicts exchange rates better than macroeconomic models (including an interest rate path, an inflation path, etc.) in the short run. Lyons et al. (2001) call this weak explanatory power of macroeconomic fundamentals "the exchange rate determination puzzle".

Equation (2.1) also links the volatility of exchange rates with the volatility of interest rate differentials. However, Backus et al. (1993), Bekaert (1996), and Moore and Roche (2002) document an excess exchange volatility beyond a movement in interest rate differentials. This stylized fact registers yet another puzzle called "the excess volatility puzzle".

Let $\rho_{fh,t+1} = s_{fh,t+1} - s_{fh,t} - x_{fh,t}$ be the realized 1-period excess return on holding foreign over home bonds. The second equality from equation (2.1) implies that this expected excess return should be zero. High interest currencies should depreciate against low interest ones to equalize bond returns. This is called the uncovered interest parity (UIP).

Empirical studies unanimously find that the UIP does not hold in the data. Bilson (1981) and Fama (1984) run the following regression:

$$s_{fh,t+1} - s_{fh,t} = a + b(i_t - i_t^*) + u_{t+1}. \quad (2.2)$$

Under the null hypothesis that the UIP holds, regression coefficients from equation (2.2) should have $a = 0$ and $b = 1$. Empirically, $b$ is estimated to be consistently less than 1.
and usually even lower than 0. This poses the UIP puzzle. Froot and Thaler (1990) and Engel (1996) are examples of older empirical surveys. Such patterns are robust even in contemporary studies.

Recently, Engel (2016) and Valchev (2015) extend to look at the relationship of interest differentials and exchange rates over time horizon. Both studies document that patterns of the UIP deviation are a function of time horizon. Higher interest rates predict positive excess returns of holding higher interest bonds initially. Such patterns reverse in the medium run when higher interest rates predict negative excess returns. In the long run, there is no predictable excess return from interest rate differentials.

Formally, following Valchev (2015), I define $r_{fh,t+k} = r_{fh,t} - r_{fh,t+k}$ as the $k$-period-ahead realized excess return of holding foreign over home bonds. I consider the following regression when the period is set to monthly for $k = 1, 2, ..., 180$.

$$r_{fh,t+k} = a_k + b_k x_{fh,t} + e_{t+k}$$

(2.3)

I confirm patterns found in Engel (2016) and Valchev (2015) and note that (1) $b_1 > 0$, i.e. higher interest currencies do not depreciate as much as predicted by forward premiums over the next period, (2) there exists $h \geq 2$ such that $b_h > 0$. That is, higher interest currencies eventually earn negative excess returns with respect to the UIP benchmark at some point in the future period, (3) $\lim_{k \to \infty} b_k = 0$, which implies that there is no excess return in the long run, and (4) $\sum_{k=1}^{\infty} b_k = 0$. Higher interest currencies have levels as strong as implied by interest differentials.

International economists attempt to rationalize the UIP deviation as well as other exchange rate puzzles using two main methods. The first approach is the risk-based explanation with the key underlying idea that currencies with higher interests are riskier and require higher returns to compensate for such risk. Verdelhan (2010) uses the external habit model as in Campbell and Cochrane (1999) to argue that investing in foreign currency is riskier in bad times precisely when the foreign interest rates are low relative to those of domestic. Colacito and Croce (2011), Bansal and Shaliastovich (2012), and Colacito and
Croce (2013) resort to long-run risk models, while Farhi and Gabaix (2015) focus on the rare disaster risk.

The second approach ignores higher-order cumulants resulting in risk and departs instead from the rational expectation assumption. For example, Gourinchas and Tornell (2004) assume investors confuse trend changes in interest rate differentials for level changes. Investors then learn and slowly update their beliefs about interest rates resulting in some predictability in excess returns.

Notably, most of current risk-based and deviation-from-rationality explanations fail to reconcile new empirical patterns. They are unable to explain the reversal in the sign of excess returns. Some exceptions include Engel (2016) and Itskhoki and Mukhin (2017), where an extra exogenous liquidity shock is introduced into the system. Valchev (2015) endogenizes this added shock by introducing an interaction between monetary and fiscal policies to create an endogenous convenience yield.

Previous literature decomposing the forward discount bias into the risk premium and the expectational error components finds that risk alone cannot fully capture deviations from the UIP. Prominently, Froot and Frankel (1989) use survey data to decompose the bias and reject that all bias is due to the risk premium. They cannot reject that all bias is attribute to expectational errors. Bacchetta et al. (2009) argue that the excess return predictability in foreign exchanges (and in other financial markets) is related to the predictability of expectational errors.

Additionally, there is evidence that human expectations does not follow rational expectations. A controlled experiment in Hommes et al. (2008) show that expectations of risky assets deviate from rational expectation and seem to be driven by trend-chasing behaviors. Greenwood and Shleifer (2014) document discrepancies between expected returns and return expectations and uses mutual fund flows to suggest that investors act according to their expectations. Barberis et al. (2015) use survey data from Greenwood and Shleifer (2014) to parametrize the functional form of extrapolation that can fit stock market returns.

Focusing on exchange rate expectations, Frankel and Froot (1987) use different surveys
of the yen/dollar exchange and conclude that expectations exhibit bandwagon effects in the short horizon. Ito (1990) also finds that investors’ expectations of the yen/dollar rate violate the rational expectation hypothesis. Expanding the scope of exchange rate pairs, Chinn and Frankel (1994) find that forecasts of minor currencies exhibit smaller biases than those of major currencies. Chinn and Frankel (2002) widen the scope of survey data sources and find that forecasts are biased, and the risk premium is less variable than expected depreciations.

The above evidence suggests that deviations from rational expectations deserve more attention. To the best of my knowledge, this paper is the first attempt of using a behavioral-based model in fitting newly observed UIP patterns over time.

The baseline model features investors with extrapolative beliefs. These investors are aware of fundamentally-implied exchange levels but still incorporate past exchange depreciations when forming their expectations.

Higher interest currencies have stronger-than-average exchange rates. Such elevated levels lead investors to form overly optimistic beliefs of such currencies in the next period. This extrapolation leads higher interest currencies to not depreciate as much as implied by interest differentials initially. The magnitude of this extrapolative force diminishes over time. In the medium run, mean-reverting interest differentials dominate and drive exchange levels lower. Investors then extrapolate the depreciation causing higher interest currencies to over-depreciate during some periods in the future. Eventually, interest differentials along with the extrapolative force vanish. The UIP holds in the long run.

The paper is structured as follows. Section 2.2 documents empirical patterns of deviations from the UIP over time. Both Engel (2016) and Valchev (2015) use the US Dollar as a home currency and focus on only most developed currency pairs. I expand the scope of the test by including developed and developing currencies from different regions around the world. I then test whether the choice of base currencies matters. Section 2.3 discusses what observed patterns say about the relationship between interest rates and exchange rates over time. Section 2.4 presents an extrapolative model with predictions in accordance to the observed foreign exchange dynamics. Section 2.5 tests some of the model assumptions and
implications. In particular, survey data is used to check whether investors indeed have extrapolative beliefs. I summarize and reiterate my findings in Section 2.6.

2.2 UIP Over Time Horizon

This section revisits stylized facts on exchange rates. The uncovered interest parity (UIP) states that high interest currencies should depreciate against low interest ones in order to equalize bond returns. Since expectations of spot rates are not tradable, the UIP needs not always hold.

Many trading strategies let investors bet on return differentials between home and foreign bonds. Investors can long/short country-specific bond series leaving themselves exposed to interest differentials between two currencies in a particular currency pair. Alternatively, traders can trade spot rates against forward rates to expose themselves to interest differentials. Formally, let $F_{fh,t+1}$ be a forward rate that investors agree at time $t$ to exchange currencies at time $t+1$ in term of home per foreign currency. No arbitrage implies

$$F_{fh,t+1} = S_{fh,t} \cdot e^{i_t} = S_{fh,t} \cdot e^{x_{fh,t}}. \tag{2.4}$$

Equation (2.4) conveys that spot and forward rates can convey interest differentials. This method of constructing interest differentials has advantages over subtracting two interest rate series since it abstracts away from choosing interest rate series. Since money markets are structured differently in different countries, conventional benchmarks for each country vary. Previous studies of empirical UIP patterns use eurocurrency rates, which are interests on bonds deposited in banks outside the home market, as benchmark rates. However, eurocurrency data is limited, especially in emerging markets.

This paper uses two main datasources for exchange rates: Bloomberg and Datastream. Spot and forward rates in Bloomberg are 5pm New York close (21:00 GMT) levels. Datastream contains two series: the World Markets PLC/Reuters (WM/R) series and the Thom-
son/Reuters (T/R) series. WM/R provides 4pm London fixing (15:00 GMT) rates and has more comprehensive currency coverages. T/R is more limited in terms of currency coverages but has time series that go back further in the past history.

The paper pulls interest rate data solely from Datastream. Eurocurrency rates are used when available. Alternative rates such as deposit rates are used as supplements whenever eurocurrencies are unavailable.¹

The paper constructs five sets of time-series data for exchange rates and interest rate differentials from the earliest available to 7 June 2017.

**Table 2.1: Sources of Exchange Rates Data and Interest Rate Differentials Data for Each Dataset**

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Exchange Rates</th>
<th>Interest Rate Differentials</th>
</tr>
</thead>
<tbody>
<tr>
<td>WM</td>
<td>Spot rates from WM/R</td>
<td>Constructed using spot and forward rates from WM/R</td>
</tr>
<tr>
<td>BBG</td>
<td>Spot rates from Bloomberg</td>
<td>Constructed using spot and forward rates from Bloomberg</td>
</tr>
<tr>
<td>TR</td>
<td>Spot rates from T/R</td>
<td>Constructed using spot and forward rates from T/R</td>
</tr>
<tr>
<td>i</td>
<td>Combine spot series from WM/R, Bloomberg, and T/R</td>
<td>Interest rate data from Datastream</td>
</tr>
<tr>
<td>Combine</td>
<td>Spot rates from WM/R, BBG, T/R, and i</td>
<td>Corresponding interest differentials from respective datasets</td>
</tr>
</tbody>
</table>

The remainder of this section focuses on the last dataset, i.e. the "Combine" method. This dataset contains the most comprehensive cross sectional sample and the longest time-series. Detailed construction can be found in Appendix B.1.

I consider the following set of regressions.

\[
s_{fh,t+k} - s_{fh,t+k-1} = a_k^1 + \gamma_k x_{fh,t} + \epsilon_{t+k}^1 \quad (2.5)
\]

\[
x_{fh,t+k-1} = a_k^2 + \lambda_k x_{fh,t} + \epsilon_{t+k-1}^2 \quad (2.6)
\]

\[
s_{fh,t+k} - s_{fh,t+k-1} - x_{fh,t+k-1} = a_k^3 + \beta_k x_{fh,t} + \epsilon_{t+k}^3 \quad (2.7)
\]

By construction, \( \gamma_k - \lambda_k = \beta_k \). This section discusses regression results when \( t \) is monthly and \( k = 1, 2, 3, ..., 180 \) in the unit of month.

The analysis includes 52 currencies: Argentine Peso, Australian Dollar, Austrian Schilling,

¹Appendix B.1 provides comprehensive discussion of interest rate series from each country.

For countries that have since joined the European currency union, fixed conversion factors against the Euro are used to construct hypothetical levels. These factors were set when the respective European legacy currency was fixed to the Euro.

I run the set of regressions (2.5), (2.6), and (2.7) on both country-specific time series and pooled panels. Standard errors are adjusted for heteroskedasticity, serial correlation, and cross-country correlation using Newey and West (1987) for time-series regressions and Driscoll and Kraay (1998) for panel regressions.

I assume and later verify that interest rate differentials follow an autoregressive process of order 1 with an autocorrelation of \( \lambda \in [0, 1] \) with independent and identically distributed innovations \( \epsilon_t \) that is normally distributed with mean 0 and variance \( \sigma^2 \). That is,

\[
x_{fh,t} = \lambda x_{fh,t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2) \quad \text{and} \quad \text{cov}(\epsilon_t, \epsilon_{t-1}) = 0
\]  

(2.8)

Figure 1 plots coefficients \( \gamma_k \), \( \lambda_k \), and \( \beta_k \) for different \( k \) if the UIP were to hold.

An assumption of AR(1) interest differentials means that the coefficient \( \lambda_k \) from the regression equation (2.6) is equal to \( \lambda^k \). The middle plot of Figure 2.1 illustrates the evolution of \( \lambda_k \) over time.

In the world where the UIP holds, exchange rates should move to offset differentials in interest rates and nullify excess returns in holding foreign versus home bonds. That is, \( \gamma_k \)
must equal to \(-\lambda_k\) so that \(\beta_k = 0\).

Moving away from the hypothetical world, I next display patterns observed in the data. I analyze 4 different ways of pooling the data.

1. Pooled panel: includes all data from 52 countries.

2. Rich panel: includes data from countries whose gross domestic product (GDP) based on the purchasing power parity (PPP) per capita is not less than the median in each respective fiscal year.\(^2\)

3. Poor panel: includes data from countries whose GDP per capita is below the median in each respective fiscal year.

4. G7: includes data from the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States).

\(^2\)GDP based on PPP per capita is in the unit of current international dollars. Details on the GDP-based categorization are in Appendix B.1.
2.2.1 Bilateral Exchange Rates

US Dollar as a Base Currency

Following existing literature, the paper first looks at estimated regression coefficients when the United States is a home country. Figure 2.2 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (2.5), (2.6), and (2.7) using the "Combine" data for the G7 countries.

I make the following observations from Figure 2.2.

1. $\lambda_k$ decays smoothly implying that interest rate differentials roughly follow an AR(1) process.

2. Exchange rates are mostly unpredictable. $\gamma_k$ is almost always indistinguishable from 0.
except for medium $k$. $\gamma_k$ is significantly positive, implying that higher contemporaneous interest rates predict exchange depreciations some time in the future.

3. $\beta_1$ is negative, reiterating the standard UIP puzzle. Higher interest currencies fail to depreciate as much as implied by forward premiums resulting in positive excess returns in holding higher interest bonds.

4. $\beta_k$ stays positive initially. There are positive excess returns in holding bonds of higher interest currencies initially.

5. $\beta_k$ turns negative for some $k$ around 70 - 90. Higher contemporaneous interest rates predict negative excess returns of holding such bonds around 3 months after.

6. $\beta_k$ reverts back to 0 and stays indistinguishable from 0 for all $k \geq 100$. There is no predictable excess returns from interest differentials eventually.

7. The sign of $\sum_{k=1}^{\infty} \beta_k$ appears indistinguishable from 0.

Such patterns are robust across different samples. Figure 2.3 plots point estimates from the same regression equations for four different ways of pooling the data. An additional robustness check can be found in Appendix B.2.

Figure 2.3 confirms that interest rate differentials seem to follow an AR(1) process and reiterates the cycle of $\beta$ (starting with negative beta, turning positive in the medium run, and converged to 0 eventually).

**Alternative Home Countries**

This section explores the robustness of the above empirical patterns by looking at alternative home countries. Figure 2.4 plots estimated coefficients along with 95% confidence bands when Euro is used as the base currency.

Similar patterns unfold when EUR instead of USD is used as home currency albeit the statistical significance is compromised for the standard UIP puzzle.
2.2.2 Absolute Exchange Rates

This section attempts to control for the base currency effect. Let $S_{j,t}$ be the country-$j$ absolute exchange rate at time $t$. I define the absolute exchange rate for country $j$ as the relative price of non-tradable to tradable goods for country $j$, i.e.

$$S_{j,t} = \frac{U_{c,NT}^{j,t}}{U_{c,T}^{j,t}},$$  \hspace{1cm} (2.9)$$

where $U_{c,NT}^{j,t}$ is the marginal consumption utility of nontradable goods, and $U_{c,T}^{j,t}$ is the marginal consumption utility of tradable goods respectively.

I do not observe the marginal consumption utility directly, but there exists a relationship between absolute and bilateral exchange rates. I can therefore construct a proxy for the
absolute exchange rates using the bilateral rates data. In particular, the bilateral exchange rate $S_{fh,t}$ is defined as:

$$S_{fh,t} = \frac{S_{f,t}}{S_{h,t}}. \quad (2.10)$$

I proxy for the absolute exchange rate using $\tilde{s}_{j,t}$, where $\tilde{s}_{j,t} = s_{jh,t} - \frac{1}{n} \sum_{k=1}^{n} s_{kh,t}$, when $n$ is a total number of currency pairs, and the smaller cases represent the logarithm of the upper-case variables. Combining equations (2.9) and (2.10) yields

$$\tilde{s}_{j,t} = s_{j,t} - \frac{1}{n} \sum_{j=1}^{n} \ln \left( \frac{U_{cjt}^{NT}}{U_{cjt}^{T}} \right). \quad (2.11)$$
If \( \frac{1}{n} \sum_{j=1}^{n} \ln \left( \frac{U_{NT}}{U_{c_jT}} \right) \) from equation (2.11) is close to 0, \( \tilde{s}_{j,t} \) will be a good proxy of \( s_{j,t} \). This happens when the country-average log marginal consumption utility from nontradable good is roughly the same as the country-average log marginal consumption utility from each respective country tradable good.

Let \( \tilde{x}_{j,t} \) be a proxy for the country-\( j \) absolute interest differentials defined analogously by

\[
\tilde{x}_{j,t} = x_{jh,t} - \frac{1}{n} \sum_{j=1}^{n} x_{kh,t}.
\] (2.12)

When using the US Dollar as a base currency, proxies for absolute exchange rates and absolute interest differentials tell us where country \( j \)'s currency and interest rate stand relative to the equally-weight basket of currencies and interest rates respectively.

This section pools all 51 bilateral exchange rates against the US Dollar and defines absolute exchange rates to be deviations from the mean. That is, \( n = 51 \) in my sample. Figure 2.5 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (2.5), (2.6), and (2.7) using the "Combine" data for the G7 countries.

Compared to patterns from bilateral exchange rates, absolute exchange rates lose the statistical significance of the reversal in the sign of excess returns.
2.3 Excess Return Predictability and Exchange Rate Dynamics

Results from Section 2.2 confirm recent empirical findings in Engel (2016) and Valchev (2015). The patterns are robust and are only partially affected by the choice of a base currency. From here onwards, the paper focuses on the US Dollar as a base currency case and drops a $f$ subscript on $S, s, F, f$, and $x$ whenever it generates no possible confusion.

This section discusses the implications of excess return predictability observed in the data. The significantly negative $\hat{\beta}_1$ implies that higher interest currencies do not depreciate as much as implied by forward premiums, creating positive excess returns in holding higher (versus lower) interest bonds initially. This reiterates the classical UIP puzzle.

The coefficient $\hat{\beta}_k$ stays negative for a while before turning positive for medium $k$ around
This implies that higher interest currencies over-depreciate roughly 3 months later. In other words, higher contemporaneous interest rates forecast significantly negative returns in holding higher interest bonds in the medium run.

Eventually, \( \lim_{k \to \infty} \hat{\beta}_k = 0 \). There is no predictable excess return from interest differentials in the long run.

The documented empirical patterns imply that with respect to the UIP benchmark, higher interest currencies under-depreciate, then over-depreciate before reverting back to the implied movement pattern. Such dynamics reiterate previously known puzzles in exchange-rate economics.

The initial depreciation underpins the UIP puzzle. On the other hand, the close-to-zero estimated \( \gamma_1 \) emphasizes the exchange rate disconnect puzzle. Interest differentials indeed fail to predict any movement on exchange rates. The cycle of under- and over-depreciation of exchange rates with respect to fundamentals highlights the excess volatility puzzle.

Most importantly, the reversal in the sign of excess returns begs a quest for new models. Existing theoretical UIP literature lacks forces that drive the change in the sign of excess returns.

Risk-based models rely on the argument that higher interest currencies are riskier and thus demand higher returns to compensate for the risk. Most of this class of model contain only one risk and can only explain why \( \hat{\beta}_k \) is negative.

Models with deviations from rational expectations rely on diverse explanations. Most papers feature frictions that result in the sluggishness in an exchange rate adjustment. The slow adjustment, however, fails to explain the reversal in signs.

Unlike Engel (2016), this paper does not find strong evidences of \( \sum_{k=1}^{\infty} \hat{\beta}_k \geq 0 \). Our samples indicate that the sum seems to be indistinguishable from zero. The discussion below illustrates how the sign of the sum has an implication on the level of exchange rates.
Taking equation (2.3) as given and summing across \( k \), I have

\[
\sum_{k=1}^{\infty} \alpha_k + \sum_{k=1}^{\infty} \beta_k x_t + \sum_{k=1}^{\infty} \epsilon_{t+k} = \sum_{k=1}^{\infty} \rho_k \\
= \sum_{k=1}^{\infty} (s_{t+k} - s_{t+k-1} - x_{t+k-1}) \\
= \lim_{h \to \infty} s_{t+h} - s_t - \sum_{k=1}^{\infty} x_{t+k-1}
\]

Assuming \( \sum_{k=1}^{\infty} \alpha_k = 0, \sum_{k=1}^{\infty} \beta_k \geq 0, \) and \( \sum_{k=1}^{\infty} \epsilon_{t+k} = 0 \) implies that, for \( x_t \geq 0 \),

\[
\sum_{k=1}^{\infty} (s_{t+k} - s_{t+k-1} - x_{t+k-1}) = \sum_{k=1}^{\infty} \beta_k x_t \\
\geq 0 \\
= \sum_{k=1}^{\infty} (s_{t+k}^{\text{UFP}} - s_{t+k-1}^{\text{UFP}} - x_{t+k-1}) \\
= \lim_{h \to \infty} s_{t+h}^{\text{UFP}} - s_t^{\text{UFP}} - \sum_{k=1}^{\infty} x_{t+k-1} \\
\Leftrightarrow s_t^{\text{UFP}} \geq s_t.
\]

The last condition follows from the assumption that there is no confusion on long-run exchange rates, i.e. \( \lim_{h \to \infty} s_{t+h} = \lim_{h \to \infty} s_{t+h}^{\text{UFP}} \).

The above discussion illustrates that the sign of \( \sum_{k=1}^{\infty} \beta_k \) indicates the strength of \( s_t \) compared to the level implied by the UIP \( s_t^{\text{UFP}} \). Higher contemporaneous interest currencies are at least as strong (weak) as levels implied by interest differentials if the sum of excess return regression coefficients is non-negative (non-positive).

Evidence from Engel (2016) conveys that the sum is positive, indicating that there is a level puzzle, i.e. higher interest currencies are too strong.

As with a reversal in the sign of excess returns, existing strands of theoretical UIP literature cannot explain this level puzzle. If higher interest currencies are riskier, their currencies should be weaker than implied by forward premiums. On the other hand, slow adjustments mean that higher interest currencies do not appreciate enough initially.

This section has argued that newly documented patterns invalidate most of existing
theoretical UIP models and thus warrant a search for new models.

2.4 Extrapolative Model

2.4.1 Bubbles and Exchange Rates

Embedding the bubble framework in exchange rates can reconcile most puzzles in exchange-rate economics. Here, I refer to bubbles as price deviations from underlying asset’s intrinsic values.

Viewing exchange rates as an asset class, the exchange rate disconnect puzzle is just a bubble phenomenon in exchange markets. Traditional bubble episodes are often accompanied by excess price and return volatilities. The over- and under-valuation of exchange rates with respect to forward premiums draw close parallels to patterns of a bubble’s boom and bust.

The evolution of excess returns resembles the typical bubble episode. Initial positive excess returns represent an emerging phase of the bubble. These positive excess returns last for a while. At a certain point, the bubble bursts. Excess returns turn negative before adjusting slowly toward fundamentals.

Exchange rate dynamics evidently point to the existence of bubbles in exchange rate markets. There are two main types of bubbles in the finance literature: rational bubbles and behavioral bubbles. While models of rational bubbles can potentially explain the life cycle patterns of exchange rates, I focus mainly on behavioral bubbles.

Under rational bubble regimes, little is known about what governs the evolution of price movements and which factors contribute toward extra volatility components. In contrast, behavioral bubbles offer more structures, often specifying the origin of the bubble development.
2.4.2 Extrapolative Beliefs

This paper acknowledges many sources of biases in beliefs but will focus on extrapolative beliefs.


In exchange rate economics, investors can extrapolate two main objects: interest rates and exchange rates. Investors learn information regarding short- and medium-term interest differentials from forward rates. As information on interest rates is readily available, I assume that investors have rational expectations about interest differentials but are subjected to behavioral biases when forming their expectations on exchange rates.

This section presents the baseline model with a large home country and an infinitesimally small foreign country. Bond market equilibrium is therefore entirely determined by investors in the large home country.\(^3\) I index the continuum of home investors by \(j \in [0, 1]\). Each investor has a wealth of \(W^j_t\) denominated in the home currency at time \(t\) and makes an investment decision of whether to invest in home or foreign bonds. I normalize the unit of bonds in both countries such that their prices in the home currency are equal to 1.

Assume that the return on home (foreign) bonds are exogenously given by \(i_i(i^*_i)\). Each investor holds \(B^{h,i}_t(B^{f,i}_t)\) units of home (foreign) bonds respectively to maximize the next-

---

\(^3\)This assumption allows us to work around the famous Siegel’s paradox from Siegel (1972).
period consumption \( C_{t+1}^j \). The optimization problem of each home investor \( j \) is as follows:

\[
\text{max } B_{H,t}^j, B_{F,t}^j \quad \text{subject to } \quad W_t^j = B_{H,t}^j + B_{F,t}^j \quad \text{and } \quad C_{t+1}^j = B_{H,t}^j \exp(i_t) + B_{F,t}^j [\exp(i_t^\ast) \frac{S_{t+1}}{S_t} - \exp(i_t)].
\]

The solution to the above optimization problem is

\[
B_{F,t}^j = \begin{cases} 
\infty, & \text{for } s_t < E_t^{\ast}\{(s_t+1) - x_t 
\end{cases} \]

Expectations of exchange depreciations affect individual holding of foreign bonds.

Let \( E_t^{\ast}(s_t+k) \) be the time-\( t \) rationally-expected \( k \)-horizon-ahead exchange rate. Rationally-expected exchange rates are pinned down by interest rate differentials as per below:

\[
E_t^{\ast}(s_t+k) = \lim_{T \to \infty} E_t(s_t+T) - \sum_{h=0}^{\infty} E_t(x_{t+k+h}). \quad (2.13)
\]

Next, denote the time-\( t \) extrapolative \( k \)-horizon-ahead exchange rate by \( E_t^{X}(s_t+k) \). I define,

\[
E_t^{X}(s_t+k+1) = E_t^{\ast}(s_t+k+1) + \gamma(E_t^{X}(s_t+k-1) - E_t^{X}(s_t+k-2)), \quad (2.14)
\]

where \( \gamma \in [0, \infty) \) governs the degree of behavioral bias.

Equation (2.14) implies that extrapolative investors are aware of fundamentals affecting exchange rates but at the same time are subjected to some degree of behavioral bias. This bias induces investors to incorporate past depreciations when forming exchange expectations.

The gap between extrapolative expectations and fundamental levels is a function of expected recent depreciations. Positive \( \gamma \) means that past depreciations result in weaker expectations. \( \gamma = 0 \) reflects the complete rational case.

I assume that all investors have homogenous extrapolative beliefs regarding next-period
exchange rates, i.e.

\[ E_t^j(s_{t+1}) = E_t^X(s_{t+1}) \quad \forall j \in [0,1]. \tag{2.15} \]

Market clearing conditions for non-zero fixed-supply home and foreign bonds require

\[ s_t = E_t^X(s_{t+1}) - x_t. \tag{2.16} \]

For simplicity, assume that interest differentials follow a stationary autoregressive process of order 1 with an autocorrelation coefficient of \( \lambda \in [0,1] \) and an independently identically distributed innovation \( \epsilon_t \) normally distributed with mean zero and variance \( \sigma^2 \), i.e.

\[ x_t = \lambda x_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2). \tag{2.17} \]

Investors have rational beliefs on interest differential process \( x_t \).

### 2.4.3 Equilibrium Exchange Rate

**Proposition 11 (Equilibrium Exchange Rate)** *The equilibrium exchange rate \( s_t \) satisfies*

\[ s_t = \lim_{T \to \infty} E_t(s_{t+T}) - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2}). \tag{2.18} \]

Proposition 11 states that the equilibrium exchange rate is pinned down by the current interest differential as well as the 1-period lagged depreciation. \( \gamma = 0 \) recovers the fundamental exchange level. When \( \gamma > 0 \), the recent past exchange change affects the equilibrium exchange level. If the foreign currency recently depreciates against the home (\( s_{t-1} > s_{t-2} \)), foreign currency will be weaker than the fundamentally implied level in equilibrium.

I make another simplifying assumption. Following Gourinchas and Tornell (2004), I assume that the nominal exchange rate is conditionally stationary, i.e. \( \lim_{T \to \infty} E_t(s_{t+T}) \) is well-defined and denoted by \( \bar{s}_t \). Investors have the correct belief regarding this long-run level. The stationary assumption is made to aid the mathematical analysis.

Under the stationary assumption, the equilibrium exchange rate \( s_t \) is defined as below:

\[ s_t = \bar{s}_t - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2}). \tag{2.19} \]
For the remainder of the paper, I follow the unconditional stationary assumption, i.e. 
$s_t = s$, unless otherwise specified. This assumption implies that $\text{cov}(s_t, x_t) = 0$, simplifying 
the computation. Appendix B.5 discusses the case in which the covariance between the 
contemporaneous long-run exchange rate expectation and the contemporaneous interest 
rate differentials is non-zero. I show that as long as some sufficient conditions are met, the 
baseline results will remain.

2.4.4 Model Implications

This section illustrates how exchange rate dynamics evolve under extrapolative beliefs. 
In particular, I explore what the above model has to say about foreign exchange market 
anomalies.

**Proposition 12 (The Level Puzzle)** For $g < \frac{1}{\lambda(1-\lambda)}$, $\text{cov}(s_t, x_t) \leq \text{cov}(s_t^{\text{UIP}}, x_t)$. This implies 
$\sum_{k=1}^{\infty} \beta_k \geq 0$.

Currencies with higher contemporaneous interest rates are at least as strong as implied 
by interest differentials (under the UIP). The equality holds when $g = 0$. That is, equilibrium 
exchange rates are completely pinned down by the interest differential path.

Proposition 12 can reconcile the finding in Engel (2016) that currencies with higher 
contemporaneous interest rates are at least as strong as implied by the UIP.

**Proposition 13 (The UIP Puzzle)** The regression coefficient $\beta_1$ from the regression equation (2.7) 
is as follows:

$$
\beta_1 = \begin{cases} 
0 & \text{for } g = 0 \\
< 0 & \text{for } g \in (0, \frac{1}{\lambda(1-\lambda)})
\end{cases}
$$

Proposition 13 states that as long as investors do not extrapolate excessively, extrapolation 
leads exchange rates to deviate from the UIP. In particular, higher interest currencies do not 
depreciate enough over the following period to nullify excess returns. The UIP is recovered 
whenever investors are rational and do not extrapolate, i.e. when $g = 0$. 

55
Proposition 14 (Reversion in Excess Returns) For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, there exists $h \geq 2$ such that $\beta_h > 0$. For $\gamma = 0$, $\beta_k \equiv 0$ for all $k$.

Proposition 14 follows directly from combing Proposition 12 and Proposition 13. When investors do not extrapolate, i.e. $\gamma = 0$, there is no predictable excess return from interest differentials at any horizon. Investors with non-explosive extrapolative beliefs, on the other hand, experience the reversal in the sign of excess returns. Higher contemporaneous interest rates under-depreciate initially but will over-depreciate at some later period.

Proposition 15 (Long-Run Reversion to the UIP) For $\gamma \in [0, 1)$. $\lim_{k \to \infty} \beta_k = 0$.

In any case, interest rate differentials have no predictive power of excess returns in the long run.

Proposition 16 (Excess Volatility Puzzle) For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $\text{var}(s_t) \geq \text{var}(s_t\text{UIP})$.

Extrapolative beliefs potentially contribute to higher volatility of exchange rates (in excess of variations in interest differentials).

I now discuss key mechanisms driving the results. Investor beliefs affect their trading behaviors, which in turn pin down equilibrium exchange rates. When home interest rates are higher than average, home currencies are unusually strong. With extrapolative beliefs, investors form even more optimistic forecasts of next-period home levels resulting in even stronger equilibrium home currencies in the current period. Higher contemporaneous home levels increase extrapolators’ expectations even more. This chain reaction results in initial positive excess returns in holding higher interest currencies.

It is not surprising that a sufficiently high extrapolative coefficient may result in an explosive path of exchange rates. An initial appreciation may lead investors that extrapolate excessively to form extremely optimistic forecasts. As the recent appreciation feeds into the belief formation process, this initial appreciation may lead to everlasting appreciations.

\textsuperscript{4}All proofs are in Appendix B.4.
Readers may wonder what excessive extrapolations entail. Counteracting extrapolative beliefs in the above model is the depreciating force from the stationary AR(1) assumption of interest differentials. In an absence of extrapolation, there is a natural force pulling high interest currencies back to their long-run levels. Extrapolative behaviors add another force governing exchange rate changes.

The interaction between the extrapolative force and the interest differential force is as follows. Initially, the extrapolative force counteracts the interest differential force. Investors extrapolate the recent appreciation of high interest currencies. Such action dampens the supposed depreciation and results in initial positive excess returns.

Non-explosive extrapolation guarantees the existence of equilibrium exchange rates as well as the eventual reversal in the sign of excess returns. As time passes, the extrapolative force will get weaker in magnitude and becomes dominated by the interest differential force. Immediately after that point in time, the extrapolative force reinforces the interest differential force, leading to over-depreciation of high contemporaneous interest currencies. Negative excess returns are registered, as observed in the empirical data.

Eventually, both the interest differential force and the extrapolative force die off. Minimal extrapolation means the eventual reversion to the UIP.

As the extrapolative force makes exchange levels more dispersed, it naturally results in excess volatilities. In addition to the interest differential variation, there are two added components of the exchange rate variation. The first component is the exchange depreciation entering the price volatility with a magnifying factor that is equal to the square of the extrapolative coefficient ($\gamma^2$). This term always contributes to higher resulting volatilities. The second component is the interaction between the interest differential force and the extrapolative force mentioned earlier. As discussed, these two forces sometimes cancel each other and at the other time reinforce each other. As shown in Appendix B.4, the interaction also contributes to higher volatilities.

I illustrate the simulated exchange rate path in Figure 2.6. Without extrapolation, exchange rates will mirror the path of interest differentials. With extrapolative investors,
exchange rates become more volatile. Momentum in investor expectations causes exchange rates to fluctuate around their fundamental levels.

![Figure 2.6: Upper: Interest Differential Path under an AR(1) Assumption with $\lambda = 0.95$ (upper). Lower: Exchange Rates Path under the UIP (dashed) and under Extrapolative Beliefs with $\gamma = 0.5$ (solid)](image)

2.4.5 Model Discussion

The proposed model is fairly tractable with the expectation formation process that nests the complete fundamental case. The baseline model can generate patterns in excess returns as observed in the data. Section 2.5 provides empirical evidence to support some of the model assumptions. In particular, survey data is used to test whether investors indeed extrapolate.
2.5 Testing Model Assumptions and Implications

This section attempts to support some key assumptions made in the baseline model in Section 2.4. I begin by examining the AR(1) assumption of interest rate differentials and then focus on the essential question of whether investors indeed extrapolate. I conclude this section by comparing my findings to existing extrapolation literature.

2.5.1 The AR(1) Assumption of Interest Rate Differentials

This section checks the validity of the AR(1) assumption of the 1-month interest rate differentials. Each country’s daily time series data on the 1-month interest differentials against the United States is tested for whether it follows an autoregressive process of order 1. I proceed by first testing for the stationary of the process using the Dickey-Fuller test and then use the Akaike Information Criterion (AIC) to choose the order of the autoregressive model.

Table B.1 in Appendix B.3 indicates that a majority of countries have an augmented Dickey-Fuller p-value that is less than 0.05. I reject the null hypothesis of a unit root with 95% confidence level for these countries. The null of a unit root can only be rejected with 90% confidence level for Argentina and Turkey. The high p-value for Austria, the EU, and Colombia makes it impossible to reject the null of a unit root in those countries.

The last column of Table B.1 in Appendix B.3 shows that the AIC criterion picks the lag order of 1 for all countries.

Combining the p-value with the optimal order overwhelmingly points to an evidence of a stationary AR(1) structure of interest differentials. I also plot the autocorrelation function (ACF) and the partial autocorrelation function (PACF) to confirm the AR/MA structure of interest differentials.

Figure 2.7 illustrates the ACF and PACF plots using the Switzerland data. The ACF plot slowly decays over time ruling out the pure MA structure as well as suggesting a relatively high autoregressive coefficient. The PACF plot spikes at 1 and cuts off completely thereafter, strongly supporting the AR(1) structure.
Figure 2.7: ACF and PACF of US-Switzerland Interest Rate Differentials. Monthly Data. “i” Method

Results from other countries have exactly identical patterns (decaying ACF and cut-off-after-1 PACF). Such prominent features serve as clear evidence for the AR(1) structure of interest rate differentials.

I perform the same analysis for the 1-, 3-, 6-, 12-, and 24-month interest rate differentials using both daily and monthly data. The results from the ADF test, the AR fitting, the ACF plot, and the PACF plot retain the same patterns in all these different samples.

I conclude that the assumption of an autoregressive process of order 1 for the interest differentials is valid.

2.5.2 Evidence from Survey Data

This section explores evidence of irrationality in exchange rate markets. The study of investor beliefs requires data on expectations since individual beliefs are rarely elicited. I obtain consensus forecasts from the Forecasts Unlimited Inc. (FX4casts.com). Appendix B.1 describes this dataset in more detail. In short, FX4casts.com gathers survey consensus from large financial institutions. The data contains monthly historical spots as well as 1-,
3-, 6-, 12-, and 24-month-ahead spot forecasts of 32 currencies along with their confidence intervals.

Patterns of excess returns displayed in Section 2.2 are robust to the choice of the period step as shown in Appendix B.3. This section provides empirical evidence from 3-month forecasts instead of 1-month forecasts, as the 3-month data starts in August 1986 while the 1-month data only starts in July 2008. I complement spots and forecasts with interest rate differentials data from the "i" method.

**Survey-Expected Excess Returns**

Analogous to the analysis performed in section 2.2, this subsection examines deviations from the UIP when expected depreciations are used instead of realized depreciations. In particular, I analyze the following regressions.

\[
\rho_{t+h} = s_{t+h} - s_t - x_t = \kappa_1 + \eta_1 x_t + \tilde{\xi}_{1,t+h}
\]

\[
E_t(\rho_{t+h}) = E_t(s_{t+h}) - s_t - x_t = \kappa_2 + \eta_2 x_t + \tilde{\xi}_{2,t}
\]

(2.20) (2.21)

The realized excess return from holding foreign bonds from time \( t \) for \( h \) periods is denoted by \( \rho_{t+h} = s_{t+h} - s_t - x_t \). Investor’s expected excess return of holding foreign instead of home bonds is denoted by \( E_t(\rho_{t+h}) = E_t(s_{t+h}) - s_t - x_t \).

Regression equation (2.20) is the standard UIP regression, while regression equation (2.21) tests whether the UIP holds when investor forecasts are used instead of realized rates.

Estimated \( \hat{\eta}_2 \) is less negative than estimated \( \hat{\eta}_1 \) in Table 2.2. This implies that deviations from the UIP are less severe in the survey data. Investors are aware that higher interest currencies should depreciate over the next period and form their forecasts to reflect weaker exchange levels than next-period realized rates. Significantly positive \( \hat{\eta}_2 \) indicates that there are still positive excess returns in holding higher-interest currencies in their expectations.

**Survey Exchange Rates**

The proposed model argues that investors extrapolate by incorporating past deprecia-
Table 2.2: Excess Returns when the Period Step is 3 Months ($h = 3$)

<table>
<thead>
<tr>
<th></th>
<th>All $\rho_{t+h}$</th>
<th>$E_t'(\rho_{t+h})$</th>
<th>G7 $\rho_{t+h}$</th>
<th>$E_t'(\rho_{t+h})$</th>
<th>Rich $\rho_{t+h}$</th>
<th>$E_t'(\rho_{t+h})$</th>
<th>Poor $\rho_{t+h}$</th>
<th>$E_t'(\rho_{t+h})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>-1.070***</td>
<td>-0.452*</td>
<td>-1.392</td>
<td>-0.963***</td>
<td>-1.299</td>
<td>-0.579*</td>
<td>-1.010**</td>
<td>-0.419*</td>
</tr>
<tr>
<td></td>
<td>(-3.65)</td>
<td>(-2.55)</td>
<td>(-1.88)</td>
<td>(-3.45)</td>
<td>(-1.83)</td>
<td>(-2.23)</td>
<td>(-3.18)</td>
<td>(-2.04)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.0004</td>
<td>-0.0012</td>
<td>0.0015</td>
<td>-0.0024</td>
<td>0.0028</td>
<td>-0.0001</td>
<td>-0.0056</td>
<td>-0.0028</td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(-0.61)</td>
<td>(0.49)</td>
<td>(-1.87)</td>
<td>(0.78)</td>
<td>(-0.11)</td>
<td>(-0.81)</td>
<td>(-0.65)</td>
</tr>
<tr>
<td>$N$</td>
<td>4311</td>
<td>4311</td>
<td>927</td>
<td>927</td>
<td>2784</td>
<td>2784</td>
<td>1527</td>
<td>1527</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0173</td>
<td>0.0057</td>
<td>0.0139</td>
<td>0.0530</td>
<td>0.0112</td>
<td>0.0195</td>
<td>0.0231</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Equations when forming forecasts of future exchange rates. Consider the following regressions:

$$E_t'(s_{t+h}) = \kappa_3 + \eta_3 x_t + \zeta_3,t$$  \hspace{1cm} (2.22)

$$E_t'(s_{t+h}) = \kappa_4 + \eta_4 x_t + \gamma_4 (s_{t-h} - s_{t-2h}) + \zeta_{4,t}.$$ \hspace{1cm} (2.23)

Equation (2.22) regresses exchange forecasts on interest rate differentials, while equation (2.23) tests whether past depreciations have any additional predictive power in addition to interest differentials.\(^5\)

Coefficients $\eta_3$ and $\eta_4$ from regression equations (2.22) and (2.23) are significantly negative in both the G7 and the rich-country samples. Currencies of these countries are generally stronger when their interest rates are higher. The poor-country sample, on the other hand, has insignificantly positive estimates of $\eta_3$ and $\eta_4$. In the lower-than-median GDP per capita countries, higher interest rates may correlate with other characteristics associated with weaker currencies. For example, higher interest rates among poor countries may signal the inflation problem, leading investors to form less optimistic forecasts of such

\(^5\)The Frisch-Waugh-Lovell theorem states that the estimated coefficient $\gamma_4$ from the regression equation (2.23) is the same as the estimate from the following regression:

$$E_t'(s_{t+h}) - E_t'(\tilde{s}_{t+h}) = \kappa_4 + \gamma_4 (s_{t-h} - s_{t-2h}) + \zeta_{4,t},$$

where $E_t'(s_{t+h})$ is the predicted forecast levels from interest differentials. That is, $E_t'(\tilde{s}_{t+h}) = \kappa_3 + \theta_3 x_t$, where $\kappa_3$ and $\theta_3$ are regression coefficients from equation (2.22).
Table 2.3: Survey Exchange Rates when the Period Step is 3 Months ($h = 3$)

<table>
<thead>
<tr>
<th></th>
<th>All $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>G7 $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>Rich $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>Poor $\mathbb{E}<em>t(s</em>{t+h})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>-0.208 (-0.13)</td>
<td>-7.558** (-3.14)</td>
<td>-6.595* (-2.18)</td>
<td>1.852 (0.78)</td>
</tr>
<tr>
<td></td>
<td>-0.251 (-0.16)</td>
<td>-6.793** (-2.86)</td>
<td>-5.813* (-1.99)</td>
<td>1.482 (0.66)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.543*** (3.40)</td>
<td>0.443*** (3.98)</td>
<td>0.450*** (3.35)</td>
<td>0.554** (2.77)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.365*** (-103.66)</td>
<td>-1.237*** (-85.03)</td>
<td>-1.639*** (-103.35)</td>
<td>-3.670*** (-72.11)</td>
</tr>
<tr>
<td></td>
<td>-2.362*** (-106.37)</td>
<td>-1.236*** (-86.29)</td>
<td>-1.638*** (-104.45)</td>
<td>-3.668*** (-73.91)</td>
</tr>
<tr>
<td>$N$</td>
<td>4272</td>
<td>4272</td>
<td>920</td>
<td>920</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0000</td>
<td>0.0251</td>
<td>0.0544</td>
<td>0.0850</td>
</tr>
<tr>
<td></td>
<td>0.0348</td>
<td>0.0613</td>
<td>0.0337</td>
<td>0.0233</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Currencies. This signaling channel is absent or less prominent in more developed countries with better reputations on the inflation management.

Notably, Table 2.3 displays significantly positive $\hat{\gamma}_4$ in all samples. Controlling for interest rate differentials, a 1% past depreciation leads to between 0.44% to 0.55% decrease in forecast levels. I view this as evidence for extrapolative beliefs among investors.

Equilibrium Exchange Rates

As investor beliefs affect their trading behaviors, exchange forecasts should have an impact on contemporaneous equilibrium exchange rates. This section explores whether extrapolative beliefs leave some traces on realized exchange rates.

Replacing survey exchange rates with equilibrium exchange rates yields analogs of regression equations (2.22) and (2.23) as below:

$$s_t = \kappa_5 + \eta_5 x_t + \xi_{5,t}$$

(2.24)

$$s_t = \kappa_6 + \eta_6 x_t + \gamma_6 (s_{t-h} - s_{t-2h}) + \xi_{6,t}$$

(2.25)

Table 2.4 shows that estimated $\hat{\eta}_5$ and $\hat{\eta}_6$ from regression equations (2.24) and (2.25) have similar patterns when realized exchange rates are used instead of survey levels as a
Table 2.4: Equilibrium Exchange Rates when the Period Step is 3 months ($h = 3$)

<table>
<thead>
<tr>
<th></th>
<th>All $s_t$</th>
<th>G7 $s_t$</th>
<th>Rich $s_t$</th>
<th>Poor $s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>-0.643</td>
<td>-0.686</td>
<td>-7.534**</td>
<td>-6.823**</td>
</tr>
<tr>
<td></td>
<td>(-0.44)</td>
<td>(-0.49)</td>
<td>(-3.18)</td>
<td>(-2.91)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.531***</td>
<td>0.412***</td>
<td>0.418**</td>
<td>0.560**</td>
</tr>
<tr>
<td></td>
<td>(3.40)</td>
<td>(3.60)</td>
<td>(3.04)</td>
<td>(2.88)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.363***</td>
<td>-2.360***</td>
<td>-1.234***</td>
<td>-1.234***</td>
</tr>
<tr>
<td></td>
<td>(-110.04)</td>
<td>(-113.11)</td>
<td>(-85.08)</td>
<td>(-86.06)</td>
</tr>
<tr>
<td>$N$</td>
<td>4272</td>
<td>4272</td>
<td>920</td>
<td>920</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0005</td>
<td>0.0288</td>
<td>0.0531</td>
<td>0.0790</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The above evidence supports the model assumption of extrapolative beliefs among investors. Among the G7 countries and the rich countries, a 1% increase in the foreign against home interest rates leads to around 7% - 7.5% stronger foreign currencies. Again, the estimates of $\eta_5$ and $\eta_6$ have opposite signs (positive instead of negative) and are no longer significant in the poor-country sample.

The significantly positive estimates of $\gamma_6$ from Table 2.4 indicate that the 1-period lag depreciation has an additional predictive power beyond interest differentials on equilibrium exchange rates. Specifically, a 1% recent depreciation leads to around 0.41% - 0.55% weaker exchange levels. Past depreciations have roughly the same effect on equilibrium exchange rates as on survey expected levels.

**Discussion**

The above evidence supports the model assumption of extrapolative beliefs among investors. Investors appear to take into account not only fundamentals (interest rate differentials) but also past exchange changes when forming forecasts. In particular, investors extrapolate the 1-period lagged depreciation. As investor beliefs affect trading behaviors, past depreciations lead to lower equilibrium exchange rates.

The results of extrapolations are robust to different specifications, as discussed in Appendix B.3.
2.5.3 Relation to Existing Extrapolation Literature

Using survey data to fit regression equations (2.23) and (2.25) derive an extrapolative coefficient $\gamma$ around 0.41% - 0.55% (when the period step is 3 months). This subsection compares the proposed model with previous extrapolative literature.

Greenwood and Shleifer (2014) document discrepancies between expected returns and return expectations and use mutual fund flows to conclude that investors act according to their expectations. Empirical tests in this paper point to the same conclusion that investor expectations affect equilibrium exchange rates. Extrapolative coefficients $\gamma$ are roughly the same in both the expectation formation regression (2.23) and the equilibrium exchange regression (2.25).

Barberis et al. (2015) use survey data from Greenwood and Shleifer (2014) to parametrize the functional form of extrapolation to fit stock market returns. Their underlying mechanism is similar to the proposed model. When past price changes are positive, extrapolators expect stock markets to perform well in the future, pushing the current price even higher. Their model features heterogenous agents with rational investors trading with extrapolators. Their parametrization results in 50% of each group. The proposed baseline model needs only one type of investors since the model has a built-in depreciating force from the an AR(1) assumption of interest differentials. Future research may extend the baseline model to include heterogenous agents, but the key underlying idea will apply.

Jin and Sui (2017) model different functional forms of extrapolation and use survey expectations from Greenwood and Shleifer (2014) to get the parametrized weight between fundamental and behavioral beliefs around 0.5 (The weight of 0 indicates complete rationality, while the weight of 1 indicates pure extrapolation).

As there are different ways of modeling extrapolation, it is difficult to reconcile extrapolative coefficients across different models. Previous discussion centers around the sign and not the magnitude of extrapolation. Therefore, there exists no consensus extrapolative coefficient readily available.

As pointed out by previous studies, equilibrium exists only when the extrapolative
component is not too high. Otherwise, optimistic future prices will push the current price higher and so on. The infinite feedback loop makes equilibrium vanish.

The baseline model includes only the 1-period lagged depreciation. In this sense, extrapolators quickly forget all but the most recent changes. There are ongoing debates on whether this is realistic. Greenwood and Shleifer (2014) argue that investor expectations depend mostly on recent returns, while Malmendier and Nagel (2011) and Malmendier and Nagel (2015) suggest that distant past events might also play a role.

The framing of survey questions as well as the forecast horizon seem to affect how far back investors look into the past. Investors look back only for a couple months when forming short-term forecasts but incorporate almost their entire experiences when forming long-term forecasts. It is possible to extend my current model to include more lags at the cost of computational complexity.

2.6 Conclusion

The paper revisits the relationship between interest rates and exchange rates. As documented in Engel (2016) and Valchev (2015), deviations from the UIP vary with time horizon. The paper decomposes excess returns in holding higher versus lower interest bonds into two main components: exchange depreciations and interest rate differentials. Using a large scope of currencies, the paper finds that exchange rate changes are mostly unpredictable by interest rate differentials. While the interest rate differentials appear to follow an autoregressive process of order 1, exchange rates behave much more like a random walk. The failure of exchange depreciation to offset interest differentials results in excess return predictabilities.

The paper confirms recent findings that there are positive excess turns in holding higher interest bonds initially. Such excess returns reverse to negative at some period in the future. In the long run, the UIP appears to hold. Such patterns are robust when expanding the currency scope to cover both developed and developing currencies across different continents. The patterns persist regardless of whether the US Dollar is used as the base currency.
Observed empirical patterns, especially the reversal in the sign of excess returns, invalidate many of existing theoretical UIP models. The paper proposes a simple behavioral model based on extrapolation that is consistent with observed patterns of excess returns.

Higher interest rates are associated with stronger-than-average currencies. Extrapolative investors then form optimistic views of next-period levels, resulting in even stronger contemporaneous exchange rates. Momentum in investor beliefs leads to initial persistent positive excess returns in holding higher interest bonds.

As interest differentials follow a stationary autoregressive process of order 1, there exists a built-in depreciating force that pulls exchange rates back to their long-run levels. The interaction between the depreciating force and the extrapolative force results in the eventual reversal in the sign of excess returns. Both forces lose magnitude with time, leading exchange rates to revert back to the UIP level in the long run.

I use survey data to show that investors indeed extrapolate exchange rates. The proposed extrapolative model is consistent with patterns of excess returns and evidence from survey data.
3.1 Introduction

Bonds with different maturities have varying yields. Similarly, risky assets do not have flat term structure but also have returns varying with time horizon. Unlike its riskless counterpart, the study of differential returns of risky assets with varying maturities is much less established.

Using dividend strips, Van Binsbergen et al. (2012) find that short-term dividends have higher risk premia than long-term dividends. Van Binsbergen et al. (2013) use a new dataset of dividend futures to construct equity yields and finds that the slope of the term structure of risk premia is procyclical. Giglio et al. (2014) exploit a feature of leaseholds versus freeholds in housing markets in England and Singapore and find low long-run risk premia.

Downward-sloping risky term structure is inconsistent with most macro-finance models. Stocks are intertemporally risky in Bansal and Yaron (2004)’s long-run risk model, resulting in upward-sloping equity term structure. External habit formation, as in Campbell and Cochrane (1999), features the persistent increase in the price of consumption risk. Long-
maturity assets are thus riskier and demand higher risk premia. Enhancing the rare disaster model with a time-varying disaster probability and a time-varying disaster magnitude, as in Gabaix (2012), can at most generate flat term structure of risk premia.

This paper starts by reassessing empirical facts about equity term structure. We use finite-horizon equity-linked instruments to construct equity price, yield, and return term structures. We find that there is heterogeneity in the shape and slope of risk term structure across indices and across time.

We propose the market specialization story that might explain variation in prices and returns of assets with different maturities. In our setup, there are two main types of dividend investors: end-users and intermediaries. End-users demand an exogenous amount of structured retail products. The structured retail product issuance generates dividend supply shocks. Intermediaries must bear risk associated with such supply shocks. Time-varying dividend supply coupled with the limited risk capacity of intermediaries results in the price and return variation of dividend claims. During the period in which there are a lot of structured retail product issuances, intermediaries must absorb a high volume of dividends. As intermediaries have certain risk limits, high dividend supplies mean low dividend prices. Dividend returns must also be high to compensate intermediaries for absorbing market demands.

The proposed market specialization story is closely related to existing literature on demand-based asset pricing. Garleanu et al. (2009) propose the demand-based option pricing model and show that demand pressures from the put-call imbalance indeed explain cross-sectional variation in volatility skewness across U.S. equity options. Vayanos and Vila (2009) and Greenwood and Vayanos (2014) use the preferred-habitat model to explain the term structure of riskless returns. Risk-averse intermediaries trade with end clients with strong preferences for specific-maturity bonds driving the price and return variation across different maturity.

In order to evaluate our market segmentation story, we obtain structured product issuance data and use it as a proxy of dividend supply. We then test whether dividend
supply risk can explain some variation in equity term structure.

The paper is structured as follows. Section 3.2 discusses different ways of constructing equity term structures and provides empirical patterns of equity term structure across different indices. Then, we move on to provide an overview of equity derivative markets and discuss their relation to dividend markets in Section 3.3. Section 3.4 proposes the supply-based dividend pricing model along with its implications. We discuss empirical strategies used in verifying our proposed mechanism along with results from structured product issuance data in Section 3.5. We conclude our findings in Section 3.6.

3.2 The Equity Term Structure

The equity return term structure is the relationship between returns from holding $T$-maturity assets and time to maturity $T$. Stocks and indices are infinitely-lived assets. Holders of such assets are entitled to any dividends paid in the future. One way to construct $T$-maturity assets is to look at assets that paid dividends only up to a certain period in the future. That is, dividends are one of the most straightforward instruments that can be used to calculate equity term structures.

3.2.1 The Pricing of $T$-Maturity Assets

No arbitrage assumption implies that the price of the $T$-maturity assets linked to a specific underlying must be equal to the price of dividends paid up to $T$ periods from now. We discuss different ways used in pricing equity dividends as follows.

Spots versus Forwards or Futures

In order to be entitled to dividends paid, investors must pay spot prices to own shares of related underlyings. Forwards and futures are contracts that bind the buyers to buy a certain quantity of a security at a specified price at a specified date in the future. Holding a long position on forwards or futures gives no exposure to dividends paid in between. On
the other hand, buyers of forwards or futures contract do not have to pay the full price of the security in advance. We can deduce dividend exposures from trading spots versus forwards or futures by hedging out interest rates (and any associated repurchase costs).

Formally, let $P_{t,T}$ be the price of $T$-maturity asset at time $t$; $S_t$ be the stock price at time $t$; $F_{t,T}$ be the forward price at time $t$ for an exchange at time $t + T$; $r_{t,T}$ be the per-period interest rate from time $t$ to $t + T$; and $\delta_{t,T}$ be the dividend yield of dividends paid between time $t$ and $t + T$. We have the following equations:

\begin{align*}
F_{t,T} &= S_t \cdot e^{(r_{t,T} - \delta_{t,T})T} \quad (3.1) \\
\delta_{t,T} &= r_{t,T} - \frac{\log\left( \frac{F_{t,T}}{S_t} \right)}{T} \quad (3.2) \\
P_{t,T} &= \delta_{t,T} \cdot S \cdot T. \quad (3.3)
\end{align*}

Equation (3.1) comes directly from the no-arbitrage condition. It should cost investors exactly the same whether they buy securities now or enter in the forward agreement to buy in the future. As long as the forward is priced correctly, equation (3.1) should hold.

Rearranging equation (3.1) yields equation (3.2). That is, dividend yields can be implied from spots versus forwards after filtering out the interest rate component.

We can then construct the price of $T$-maturity asset from dividend yields using equation (3.3).

**Synthetic Forwards**

Most equity markets only quote and trade near-dated futures with mostly quarterly expiries. Constructing synthetic forwards from options allows for greater coverages with a bigger range of maturities. Specifically, investors can create synthetic long forwards by buying European calls and selling European puts. Formally, let $c_{t,T}$ be the price of the at-the-money call expiring at time $t + T$; $p_t$ be the price of the at-the-money put expiring at time $t + T$; $D_i$ be the dividend payable at time $t + i$; and $N$ be the biggest number such that $t_N \leq T$ when
$t_i$ is weakly increasing in $i$.\(^2\) The put-call parity yields

\[
(c_{t,T} - p_{t,T}) \cdot e^{r_{t,T} \cdot T} = F_{t,T} - S_t.
\]

(3.4)

Figure 3.1: Put-Call Parity

Figure 3.1 illustrates how a combination of long calls and short puts is equivalent to being long forwards.

The relationship in equation (3.4) allows us to derive the $T$-maturity dividend yields and $T$-maturity asset price accordingly.

Dividend Swaps

Dividend swaps are trading instruments that give investors pure dividend risk. Such instruments were created back in the late 1990s first with index dividend swaps. Dividend swaps on single stocks emerged around the year 2000.

The buyer of a dividend swap agrees to pay a fixed amount (called the fixed leg) in exchange for the sum of all qualifying dividends paid during the life of the swap. Such sum

\(^{2}\)At-the-money options have the strike price that is equal to the current spot price $S_t$. 
is called the floating leg.\footnote{As dividends are summed, the exact ex-date within the period become irrelevant.}

Let $SW_{t,i}$ be the price of the dividend swap (i.e. the fixed leg) at the time $t$ of cumulative dividend points payable between time $t + i - 1$ and time $t + i$. That is, buying such dividend swap yields the buyer a net profit of whatever dividends accrued from time $t + i - 1$ to time $t + i$ less the cost that is equal to the fixed leg. We can then calculate the price of $T$-maturity asset by summing over the price of dividends paid between time $t$ to time $t + T$, i.e.

$$P_{t,T} = \sum_{i=1}^{T} SW_{t,i}.$$  

**Dividend Futures**

Dividend futures, like dividend swaps, expose investors only to dividend risk. Dividend futures were created in 2008 as listed alternatives to dividend swaps (which are traded over the counter). The first dividend futures were linked to the SX5E index. European dividend futures entered the markets in 2009. SX5E single stocks and Japanese dividend futures followed in 2010.

Again, let $F^d_{t,i}$ be the price (the fixed leg) at time $t$ of cumulative dividend points payable between time $t + i - 1$ and $t + i$. It follows that

$$P_{t,T} = \sum_{i=1}^{T} F^d_{t,i}.$$  

Four different ways of pricing $T$-maturity assets have their own pros and cons. Forwards and futures have the most comprehensive data (as both are traded heavily in the markets). However, they have other confounding risks (such as interest rates and repo risk) in addition to dividend exposures. Options have the same advantage and drawback as forwards and futures.

Dividend swaps and dividend futures are ideal instruments to price $T$-maturity assets since both only have direct dividend exposures. However, data on such instruments is less abundant. Dividend swaps are traded over-the-counter with no systematic record keeping.
Each trading entity maintains its own records of dividend swap levels. On the other hand, dividend futures are listed in the market and thus are more transparent. However, dividend futures started trading much later. Low liquidity and sparse trades among dividend futures also limit the data availability.

3.2.2 Data and Methods

Since different ways of pricing T-maturity assets have their own pros and cons, this section discusses how the paper conducts an analysis of equity term structure.

Data

We focus on the following equity indices: the SX5E Index, the UKX Index, the SPX Index, the NKY Index, the KOSPI2 Index, the HSI Index, and the HSCEI Index.

Bloomberg

We obtain spot prices, index futures, total return index, dividend points, and option data associated with the above indices from Bloomberg. We construct the zero rate curves associated with each index by bootstrapping the yield curve and the interest rate swap curve. We obtain dividend future levels for above indices (except KOSPI2) from the earliest available to January 2017.

Proprietary Option Prices

We obtain the proprietary data from one of the investment banks containing call and put premiums for 3-, 6-, 9-, and 12-month and 2-, 3-, 4-, 5-, 7-year at-the-money options for each index in the list.

Proprietary Dividend Swaps

BNP Paribas kindly provides fixed-maturity dividend swap levels (2005 - 2018 dividend swaps) for SX5E, UKX, SPX, NKY, and HSCEI.

Van Binsbergen et al. (2012)

We will compare the construction from 3 different methods (spot versus forwards and futures, synthetic forwards, and dividend swaps and dividend futures) to results from
Methods

Recall that $P_{t,T}$ is the price at time $t$ of dividends paid between time $t$ and $t + T$. Similarly, we denote $P_{t,j,T}$ to be the price at time $t$ of dividends paid between time $j$ and $t + T$. At a given time $t$, the dividend contract with price $t, j, T$ is called a front-month contract if $j < t$. If $j \geq t$, such contract is not front-month, and the realization of dividends has not yet occurred. Let $R_{t+1,j,T}$ be the simple 1-period return of dividends paid between time $j$ and $t + T$. We then have that

$$R_{t+1,j,T} = \frac{P_{t+1,j, T-1} + D_{t,t+1}}{P_{t,j,T}} - 1,$$

where $D_{t,t+1}$ is the dividends paid between time $t$ and $t + 1$.

The majority of our data has a fixed maturity, i.e. the maturity is December 2018 instead of 6 months from the inception. Let $c$ be the constant maturity of our interest. Suppose that $t + c$ falls in between two dividend contracts with consecutive maturities $t + T_1$ and $t + T_2$ such that $t + T_1 \leq t + c \leq t + T_2$. We calculate the constant maturity $c$ dividend price using the linear interpolation as below:

$$P_{t,c} = F_{t,T_1-1,T_1} + \frac{F_{t,T_2-1,T_2} - F_{t,T_1-1,T_1}}{T_2 - T_1} \cdot (c - T_1).$$

Similarly, we can linearly interpolate and construct the constant maturity dividend return by again looking at the two contracts with consecutive maturities $t + T_1$ and $t + T_2$ such that $t + T_1 \leq t + c \leq t + T_2$. At each valuation date $t$, the constant maturity $c$ dividend return is equal to:

$$R_{t+1,t,c} = w_1 R_{t+1,T_1-1,T_1} + (1 - w_1) R_{t+1,T_2-1,T_2},$$

where $w_1 = \frac{T_2-c}{T_2-T_1}$. 

Van Binsbergen et al. (2012).
3.2.3 Comparison across Different Methods of the Pricing of T-Maturity Assets

We use the aforementioned data to construct and compare prices of T-maturity assets from different methods: bbk, divfut, equityfut, and opt represent data from Van Binsbergen et al. (2012), the merge data of dividend swaps and dividend futures, spot versus forwards and futures, and synthetic forwards, respectively.

Figure 3.2: 1-Year Dividend Yield of SPX Index over Time

Figure 3.2 shows that 1-year dividend prices of S&P 500 Index from different constructing methods are roughly aligned. Appendix C.1 shows the same comparison for different maturities and different indices.

Table 3.1: Summary Statistics of 1-Year S&P Dividend Yields

<table>
<thead>
<tr>
<th></th>
<th>eqfut</th>
<th>opt</th>
<th>divfut</th>
<th>bbk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0212</td>
<td>0.0209</td>
<td>0.0208</td>
<td>0.0210</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.0028</td>
<td>0.0028</td>
<td>0.0017</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

Notes: Monthly data from Jan 2005 to Oct 2009 (Maximum period coverage in which all four datasets commonly exist). The total number of observation is 58.
Table 3.1 displays the mean and the standard deviation of 1-year S&P 500 dividend yields from different methods during the same period of time. It shows that, during overlapping periods, different constructing methods have in-line dividend prices. That is, different ways of pricing $T$-maturity assets work equally well, and we may focus on the most comprehensive source.

3.2.4 $T$-Maturity Dividend Yields over Time

This section displays $T$-maturity dividend yields for different equity indices over time when $T$ is equal to 1, 2, 3, and 5 years along with the underlying index level. Dividend prices in this section are constructed from synthetic forwards unless otherwise stated.

Figure 3.3: T-Maturity Dividend Yield of S&P 500

Figure 3.3 displays the evolution of 1-, 2-, 3-, and 5-year dividend yields of S&P 500 along with the SPX index level. We observe substantial variation of the dividend yield structure across time. Similar to Van Binsbergen et al. (2013), S&P 500 dividend yield term structure seems to be somewhat pro-cyclical. In this sense, dividend yields in the US markets may partially reflect the expectation of economic growth.
Conversely, Figure 3.4 conveys that the dividend yield term structure stays inverted throughout time for Euro Stoxx 50. 5-year dividend yields are always compressed relative to the front-year yields. Coincidentally, European markets are those with the highest concentration of structured product issuance.

For Japan, Figure 3.5 depicts the variation of the dividend yield term structure across time. Similar to S&P 500, the yield term structure seems to be somewhat pro-cyclical.

Appendix C.2 shows the evolution of the dividend yield term structure over time for other equity indices.

### 3.2.5 Cumulative Returns of Assets with Different Maturities

After looking at the price/yield patterns in the previous section, we now turn to an analysis of the dynamics of $T$-maturity cumulative returns over time. Again, we use synthetic forwards to construct dividend return data unless otherwise specified.

Figure 3.6 illustrates the S&P 500 cumulative return over time. After the 2008 global financial crisis, longer-maturity assets have outperformed shorter-maturity ones.
Some financial analysts argued that one of the contributors to the Great Recession was the increasing complexity of traded financial instruments. In the aftermath of the Great
Recession, policy makers required intermediaries to follow stricter guidelines (more trade disclosures and more comprehensive term sheets, etc). Exotic products that were booming suddenly fell out of favor with investors.

Shortly after the crisis, investors’ scars were fresh. They shied away from structured product trading. Banks themselves found it more costly to offer customized solutions because of the associated legal risks. All in all, there were not much new trades. The volume of exotic flows stayed low, and most banks merged their exotic trading teams with their flow trading teams as a result.

This observation ties well with our proposed story. During the short period after the financial crisis, the role played by structured retail products was minimal, and the equity return term structure retained its normal upward sloping shape during that time.

![Cumulative Returns of Euro Stoxx 50 T-Maturity Asset](image)

**Figure 3.7: Cumulative Returns of Euro Stoxx 50 T-Maturity Asset**

However, Figure 3.7 shows that the Euro Stoxx 50 risky term structure remains inverted most of the time. It is well known by intermediaries that European clients are much more familiar with complex structures than their American counterpart. In fact, European investors often have preferences for structured products as they search for yield.
Similar to the SX5E Index, the Nikkei 225 risky term structure is also almost always downward sloping from Figure 3.8.

Outside of European markets, the Japanese market is also well known for the trading of structured products. Similar to European investors, Japanese traders also have high appetites for yield enhancers. Intermediaries respond by offering structured notes called "Uridashi" to the markets.

Appendix C.3 shows the same comparison of cumulative returns of $T$-maturity assets associated with other equity indices.

Cumulative returns of different equity indices reflect that the risky return term structure varies not only with time but also with underlying equities. While cumulative returns of S&P 500-linked assets increase with maturity, the same does not hold for other European and Asian indices.

The patterns of equity return term structure are somewhat related to the structured product issuance. During the time when demands for exotic structures remain muted, longer-maturity assets earn higher returns. The risky return term structure tends to be
more inverted in the markets where structured products are popular (such as European and Japanese markets). We will establish the relationship between structured product issuance and the risky term structure formally in Section 3.5.

3.2.6 Properties of $T$-Maturity Asset Returns

Summary Statistics of Returns

Table 3.2: *Summary Statistics of $T$-Maturity Asset Returns*

<table>
<thead>
<tr>
<th>Maturity in Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SPX</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.5648</td>
<td>0.6848</td>
<td>0.6606</td>
<td>0.7041</td>
<td>0.7286</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.1765</td>
<td>0.1875</td>
<td>0.1749</td>
<td>0.1743</td>
<td>0.1817</td>
</tr>
<tr>
<td><strong>SX5E</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.5042</td>
<td>0.8279</td>
<td>0.8432</td>
<td>1.0706</td>
<td>0.5421</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>8.7567</td>
<td>4.5229</td>
<td>4.9631</td>
<td>6.0378</td>
<td>4.8796</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.0576</td>
<td>0.1830</td>
<td>0.1699</td>
<td>0.1773</td>
<td>0.1111</td>
</tr>
<tr>
<td><strong>NKY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>1.389</td>
<td>1.8059</td>
<td>2.7967</td>
<td>5.27</td>
<td>0.6128</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>9.6769</td>
<td>9.9611</td>
<td>19.8897</td>
<td>41.2723</td>
<td>5.6543</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.1436</td>
<td>0.1813</td>
<td>0.1406</td>
<td>0.1279</td>
<td>0.1083</td>
</tr>
</tbody>
</table>

*Notes:* Monthly data from Apr 2004 to Jan 2017 using the synthetic forward method. The total number of observation per index is 154.

Table 3.2 summarizes the mean, the standard deviation, and the sharpe ratio of the 1-, 2-, 3-, and 5-year asset returns along with the associated equity returns. The mean return and the sharpe ratio rise almost monotonically in maturity for S&P 500. On the other hand, Euro Stoxx 50 and Nikkei 225 asset returns are higher with higher sharpe ratio for shorter-maturity assets. Table C.1 in Appendix C.4 exhibits properties of $T$-maturity asset returns for other indices including UKX, KOSPI2, HSI, and HSCEI.
Alpha and Beta of Excess Returns

This section looks at the correlation of \( T \)-maturity excess returns with market excess returns for each underlying index.

### Table 3.3: Regression of Excess Returns on Market Excess Returns

<table>
<thead>
<tr>
<th>Maturity in Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SPX</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>0.0031</td>
<td>0.0031</td>
<td>0.0025</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>(1.3879)</td>
<td>(1.4154)</td>
<td>(1.1026)</td>
<td>(0.9988)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.1764</td>
<td>0.3711**</td>
<td>0.4074**</td>
<td>0.4765***</td>
</tr>
<tr>
<td></td>
<td>(1.9426)</td>
<td>(2.8241)</td>
<td>(2.9379)</td>
<td>(3.7704)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0491</td>
<td>0.1660</td>
<td>0.1873</td>
<td>0.2240</td>
</tr>
<tr>
<td><strong>SX5E</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>0.0028</td>
<td>0.0051</td>
<td>0.0048</td>
<td>0.0062</td>
</tr>
<tr>
<td></td>
<td>(0.4095)</td>
<td>(1.9348)</td>
<td>(1.5343)</td>
<td>(1.7720)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.2473*</td>
<td>0.5057***</td>
<td>0.6372***</td>
<td>0.7873***</td>
</tr>
<tr>
<td></td>
<td>(2.1797)</td>
<td>(7.3649)</td>
<td>(7.6661)</td>
<td>(8.4725)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0190</td>
<td>0.2991</td>
<td>0.3913</td>
<td>0.4034</td>
</tr>
<tr>
<td><strong>NKY</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>0.0135*</td>
<td>0.0138</td>
<td>0.0217</td>
<td>0.0437</td>
</tr>
<tr>
<td></td>
<td>(2.1717)</td>
<td>(1.6770)</td>
<td>(1.5013)</td>
<td>(1.4790)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.1545</td>
<td>0.6351**</td>
<td>0.9800**</td>
<td>1.5190*</td>
</tr>
<tr>
<td></td>
<td>(0.8371)</td>
<td>(3.2085)</td>
<td>(2.8522)</td>
<td>(2.1257)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.0082</td>
<td>0.1294</td>
<td>0.0773</td>
<td>0.0431</td>
</tr>
</tbody>
</table>

**Notes:** Monthly data during Apr 2004 to Jan 2017 from the synthetic forward method. The total number of observation per index is 154. \( t \) statistics using Newey-West standard error in parentheses. * \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)

From Table 3.3, short-dated assets have low beta. Near-end dividend yields are partially determined by announced dividends along with analyst forecasts, and their returns do not vary much with equity returns. Assets with longer maturities rely more on the long-term expectations. For S&P 500, Euro Stoxx 50, and Nikkei 225, the beta of assets increase with
their maturities. There is no prominent excess alpha except for 1-year NKY.\textsuperscript{4}

3.3 Equity Derivative Markets

Section 3.2 discussed empirical facts about equity term structure. As briefly mentioned, equity return term structure appears to be more inverted in markets with more structured product issuances or during the time when the total issuance is higher.

We discuss equity derivative markets and their potential relationship to the equity term structure in this section.

3.3.1 Three Main Types of Instruments

There are three main products tradable in the equity derivative markets: forwards, vanilla options, and exotic options. Trading desks in typical investment banks are divided according to who is responsible for trading which products among the three.

The delta\textsubscript{1} trading desk mainly covers any instrument with delta of 1 (Delta is the sensitivity of an instrument’s price to the spot price of underlying assets). Products going to the trading book of this group include forwards, futures, baskets of stocks, equity swaps, and dividend swaps and futures.

The flow trading desk has traders who are responsible for option market-making, the pricing of vanilla options, and warrant market-making. Instruments going to the trading book of this group involve mainly vanilla calls and puts.

The exotic trading desk covers everything else. Traders in this group focus on the pricing of exotic options ranging from light exotic to heavy exotic. Instruments going to the risk book of this group include structured retail products and more customized solution-based structures.

Typically, these three trading desks in the equity derivative trading group maintain separate trading books and have divided risk management systems. With the decline in the

\textsuperscript{4}Table C.2 in Appendix C.4 shows the regression results for UKX, KOSPI2, HSI, and HSCEI.
volume of non-vanilla products, there has been an increasing trend in combining the flow and exotic trading desks.

For the purpose of this paper, we divide the players in equity derivative markets into two types: intermediaries (investment banks, market makers, etc) and end-users (hedge funds, retailers, etc). Typical instruments traded by institutional clients (hedge funds) are over-the-counter (OTC) and listed options, while those traded by retail clients include structured products and warrants. We do not model end-user investment decisions and take their demands as being exogenous.

Intermediaries in our paper have limited risk-bearing capacities. Their main function is to price all tradable instruments.

### 3.3.2 Dividends and Equity Derivatives

There are many factors affecting the price of equity derivatives such as volatility curve, dividend curve, interest rate curve, etc.

This paper focuses on the structured retail products (SRP) because vanilla OTC or listed options are usually of shorter maturities with lower dividend risk. On the other hand, structured retail products normally have long maturities, and their pricings are decently sensitive to dividend exposure.

End users often long calls to get asymmetric upside exposure or short puts from buying reverse convertibles (autocallables). Performance of instruments is almost always given on a price return index and a total return index. As intermediaries short forward exposures from being long calls and short puts, they hedge by buying equities and become net long dividends.

Autocallables (reverse convertible securities) first gained popularity in Japan around 1990s, as a way to enhance yields. Investors use structured notes to sell insurance (by selling puts) to the markets. Such notes are designed to provide investors with higher yields under a mechanism that is similar to that of bonds. As the price of underlying securities falls, the

---

5 A sample term sheet is in Appendix C.5.
yield on the product rises. However, if the market falls too much, the product converts from a fixed income to an equity.

There are also other types of structured products tradable in the markets. The vast majority of structured products are saving products and consequently involve investors wanting to long exposure to equity markets at some future date. These investors’ long forward exposure leaves banks with the short forward exposure.

To hedge the short forward position, banks would long cash equity and subsequently become long dividends. Banks can offload their long dividend exposure by selling dividend swaps to institutional investors. That is, the demand of structured retail products leaves some structural imbalances in dividend markets. In particular, higher structured product demands lead to a bigger supply of dividends, suppressing dividend strips’ prices and generating excess dividend returns.

3.3.3 Dividend Markets

Trading spot versus forwards/futures and trading options are two indirect ways of getting the dividend exposure. The creation of pure dividend instruments emerged in the late 1990s. Dividend swaps were originally created by JP Morgan as a way to offload dividend risks that dealers hold from selling equity-linked structured notes.

Dividend futures were later created as listed alternatives to OTC dividend swaps. The first dividend futures started trading in 2008 with Euro Stoxx 50 market, which is the world’s biggest structured product markets. Dividend futures linked to the Japanese index and the US index were subsequently created in 2010 and 2015, respectively.

Primary users of listed dividends or any dividend derivatives are exotic trading desks, as they issue a vast majority of autocallable products. In fact, the liquidity of index dividend swaps is primarily driven by the presence of structured products linked with such an index. As structured products are more common on non-US indices, the implied dividend market is significantly less liquid in the US (especially when compared to the size of the equity market).
Beside the structure retail product, there are other possible factors affecting the structural demand and supply of dividends. During certain periods (SX5E in 2012), dealers hold excess inventory of puts, leaving them long dividend exposure. There was also an increasing demand for upside exposure in Europe and Japan via long-dated calls in 2013 fueled partially by low implied option volatilities. Dealers selling calls and shorting forward depressed dividend prices in such markets during that period.

Intermediaries’ needs in hedging their dividend risk are due to their limited risk-bearing capacities. Trading desks in investment banks are not only concerned about daily profits and losses along with their fluctuations, but also subjected to various risk evaluation from their internal risk departments. On top of that, banks as a whole are also subjected to external pressures, such as balance sheet constraints from Basel III.

3.4 Model and Model Implications

Actual dividend markets have two active participants: banks and institutional investors. Fund and proprietary trading desks still account for roughly 80% of the dividend markets.

This paper models two types of players in the dividend markets: end-users and dealers. End-users include retailers demanding structured products. They act as an exogenous supplier of dividends. Dealers include banks and institutional investors whose risk capacities will determine dividend prices.

3.4.1 Model Setup

We will propose a supply-based dividend pricing model that closely resembles the demand-based option pricing model of Garleanu et al. (2009).

Consider an infinite-horizon discrete-time economy with two types of players: end-user and dealers. There are two types of assets: riskfree and risky assets. Riskfree assets have constant returns of \( R_f \), while risky assets are equities with exogenous strictly positive price \( P^e_t \), dividend \( D_t \), and excess returns of \( R^e_t = \frac{P^e_t + D_t}{P^e_{t-1}} - R_f \). The distribution of future prices and returns are governed by Markov state variables \( X_t \) such that \( X^e_t = P^e_t \). Assume that
\((R_t', X)\) satisfies a Feller-type condition as discussed in Appendix C.6 and \(X_t\) is bounded for all \(t\).

This economy has a number of dividend securities indexed by \(n \in N\), where \(n\) contains information such as maturity. The prices of these securities are denoted by \(p^n\) and will be determined endogenously. The set of derivatives tradable at time \(t\) is indexed by \(N_t\).

End-users will exogenously supply dividends of \(q_t = (q^n_t)_{n \in N_t}\) at time \(t\). The distribution of future supplies is assumed to be characterized by \(X_t\).

Dealers are competitive with a representative deal with discount factor \(\rho\) and constant absolute risk aversion \(\gamma\). This representative dealer faces the following utility maximizing problem:

\[
\max_{C_t, \theta_t, d_t = (d^n_t)_{n \in N_t}} u(C_t, C_{t+1}, ...) = \mathbb{E}_t \left[ \sum_{v=t}^{\infty} \rho^{v-t} u(c_v) \right], \quad (3.8)
\]

where \(d_t\) is the number of dividends held, \(\theta_t\) is the dollar investment in the underlying equity, \(u(C) = -\frac{e^{-\gamma C}}{\gamma}\), and such that the following hold:

\[
W_{t+1} = (W_t - C_t) R_f + d_t (p_{t+1} - R_f R_t) + \theta_t R_{f+1}^t, \quad (3.9)
\]

\[
\lim_{T \to \infty} \mathbb{E}_t [\rho^{-T} e^{-kW_t}] = 0. \quad (3.10)
\]

Dividend prices will be determined in a competitive equilibrium.

**Definition 17** \(p_t = p_t(q_t, X_t)\) is a (competitive Markov) equilibrium if, given \(p\), dealer’s optimal holding of dividends clears the market, i.e. \(d = q\).

### 3.4.2 Solving Dealers’ Problem

Let \(J(W_t; t, x)\) be the value function at time \(t\) of the dealer with wealth \(W_t\) and facing the state of nature \(X_t\). We can rewrite the dealer’s optimization problem in (3.8) to

\[
\max_{C_t, \theta_t, d_t} -\frac{1}{\gamma} e^{\gamma C} + \rho \mathbb{E}_t [J(W_{t+1}; t+1, X_{t+1})], \quad (3.11)
\]

such that the resource constraint (3.9) holds.
Lemma 18 If \( p_t = p_t(q_t, X_t) \) is the equilibrium price process and \( k = \frac{\gamma(R_f - 1)}{R_f} \). Then, dealer's value function and optimal consumption are given by

\[
J(W_t; t, X_t) = -\frac{1}{k}e^{-k(W_t + G_t(q_t, X_t))}
\]

\[
C_t = \frac{R_f - 1}{R_f}(W_t + G_t(q_t, X_t)),
\]

and stock and dividend holdings are characterized by the respective first-order conditions:

\[
0 = \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + d_t(p_{t+1} - R_f p_t) + G_{t+1}(q_{t+1}, X_{t+1}))} R_{t+1}^e] \tag{3.14}
\]

\[
0 = \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + d_t(p_{t+1} - R_f p_t) + G_{t+1}(q_{t+1}, X_{t+1}))} (p_{t+1} - R_f p_t)], \tag{3.15}
\]

where, for \( t \leq T \), \( G_t(q_t, X_t) \) is derived recursively using equations (3.14) and (3.15) and

\[
e^{-kR_fG_t(q_t, X_t)} = R_f \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + d_t(p_{t+1} - R_f p_t) + G_{t+1}(q_{t+1}, X_{t+1}))}]. \tag{3.16}
\]

For \( t > T \), the function \( G_t(q_t, X_t) = \tilde{G}(x_t) \), where \( (\tilde{G}(x_t), \tilde{\theta}(x_t)) \) solves

\[
e^{-kR_fG(x)} = R_f \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + \tilde{G}(x_{t+1}))}] \tag{3.17}
\]

\[
0 = \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + \tilde{G}(x_{t+1}))} R_{t+1}^e]. \tag{3.18}
\]

The optimal consumption is unique. The optimal dividend security holdings are unique provided that their payoffs are linearly independent.

The proof of Lemma 18 is in Appendix C.6.

3.4.3 Price Effects of Supply Pressure

At maturity \( T \), dividends have a known price of \( p_T \). At any prior date \( t \), the price \( p_t \) can be found recursively by inverting the equation (3.15) to get

\[
p_t = \frac{\mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + q_{t+1} + G_{t+1})} p_{t+1}]}{R_f \mathbb{E}_t[e^{-k(\bar{R}_{t+1}^e + q_{t+1} + G_{t+1})}]} \tag{3.19}
\]
where the hedge position in the underlying $\theta_t$ solves

$$0 = \mathbb{E}_t[e^{-k(\theta_t^c R_{t+1}^c + q_t p_{t+1}^n + G_{t+1}^c)} R_{t+1}^c],$$

(3.20)

and $G$ is computed recursively as in Lemma 18.

Equations (3.19) and (3.20) can be written in terms of supply-based pricing kernels.

**Theorem 19** Price $p$ and the hedge position $q$ satisfy

$$p_t = \mathbb{E}_t(m_{t+1}^q p_{t+1}) = \frac{1}{R_f} \mathbb{E}_t^q(p_{t+1})$$

(3.21)

$$0 = \mathbb{E}_t(m_{t+1}^q R_{t+1}^c) = \frac{1}{R_f} \mathbb{E}_t^q(R_{t+1}^c),$$

(3.22)

where the pricing kernel $m^q$ is a function of supply pressure $q$:

$$m_{t+1}^q = \frac{e^{-k(\theta_t^c R_{t+1}^c + q_t p_{t+1}^n + G_{t+1}^c)}}{R_f \mathbb{E}_t[e^{-k(\theta_t^c R_{t+1}^c + q_t p_{t+1}^n + G_{t+1}^c)}]},$$

(3.23)

and $\mathbb{E}_t^q$ is the expected value with respect to the corresponding risk-neutral measure, i.e. the measure with a Radon-Nikodym derivative with respect to the objective measure of $R_f m_{t+1}^q$.

Note that the pricing kernel is small whenever the unhedgeable part $d_t p_{t+1} + \theta_t R_{t+1}^c$ is large.

**Definition 20** The unhedgable price change $\bar{p}_{t+1}^n$ of any dividend strips $n$ is defined as its excess return $p_{t+1}^n - R_f p_{t}^n$ optimally hedged with underlying equity position $\frac{\text{cov}_t^q(p_{t+1}^n, R_{t+1}^c)}{\text{var}_t^q(R_{t+1}^c)}$:

$$\bar{p}_{t+1}^n = R_f^{-1}(p_{t+1}^n - R_f p_{t}^n - \frac{\text{cov}_t^q(p_{t+1}^n, R_{t+1}^c)}{\text{var}_t^q(R_{t+1}^c)} R_{t+1}^c).$$

(3.24)

**Theorem 21** The sensitivity of the price of dividend security $n$ to supply pressure in dividend security $o$ is proportional to the covariance of their unhedgeable risks:

$$\frac{\partial p_{t+1}^n}{\partial q_{t+1}^o} = -\gamma (R_f - 1) \mathbb{E}_t^q(\bar{p}_{t+1}^n \bar{p}_{t+1}^o) = -\gamma (R_f - 1) \text{cov}_t^q(\bar{p}_{t+1}^n, \bar{p}_{t+1}^o).$$

(3.25)

Theorem 21 says that the supply of dividend security $o$ has an effect on the price of dividend security $n$ as long as the unhedgeable risks of dividend securities $o$ and $n$ correlate with each other.
If we set $o = n$ in Theorem 21, we have that higher exogenous supply of dividend security $n$ will lead to lower dividend prices.

3.5 The Effect of Structured Products on $T$-Maturity Asset Prices, Yields, and Yield Term Structure

According the the supply-based asset pricing model, exogenous dividend supply of certain maturities affects asset prices and returns. In our context, structured retail products will affect the equity term structure if the following key identifying assumption is true: exotic products are a proxy for exogenous dividend supply.

A priori, dividend yields may affect the demand of structured product issuance. The risk-reward profile of structured products depends on many factors, one of which is the dividend curve. That is, dividend levels can affect the amount of exotic trading. This potential simultaneity issue may confide our empirical results.

This section aims to evaluate our proposed mechanism by testing the effect of structured product issuance on asset prices, returns, and price term structure.

3.5.1 Data

We obtain structured product issuance data from mtn-i.com. This database contains precise and consistent data on underlyings for the US SEC registered market from 2014 onward and for the Canadian and Japanese domestic retail markets from 2015 onward. Data for other markets or further back in time is patchier.

Data is presented trade by trade with ISIN (the International Securities Identification Number) along with dealer name, issuer name, trade size, trade coupon, asset class, product type, settlement date, and maturity date.

The raw data contains all trades with SX5E, UKX, SPX, KOSPI2, NKY, HSI, and HSCEI as underlyings. I restrict the sample to only equity-linked trades. For trades with multiple underlyings (basket or worst-of structures), the total notional is divided equally across each
associated underlying index.

### 3.5.2 Empirical Strategy

Since structured products are complicated instruments with complex risk, we make certain simplifying assumptions to connect the issuance data to the amount of dividend supplied. Ideally, we need not just the notional of each trade but also dividend exposure associated with each product. Each instrument has different dividend risk. Dissecting dividend risk for each trade is possible if we have pricing models similar to those in investment banks. Since there is no easy way to extract actual dividend exposure, we assume that dividend risk is just proportional to the issuance amount.

In addition, without investment bank risk management tools, it becomes impossible to categorize dividends for each maturity bucket. 5-year structured products may have higher sensitivity to 3-year dividends than to 5-year dividends. This paper will not attempt to bucket dividend risk. We leave refinement of this process to future research.

In addition, the dividend exposure for each trade is spot- and time-dependent. Dividend risk for each instrument depends on the performance of underlying equities. This paper assumes constant dividend exposure throughout the life of each trade.

Relatedly, some products might actually knock out early or get unwound and cease to exist. Since we only have data on settlement date and maturity date, we cannot keep track of which instrument has knocked out (Our data does not contain specific terms such as knocked out level and strike level, etc.).

Given the above assumptions, I aggregate the total issuance for each respective date and underlying index by adding up the notional of all trades that have already settled but have not yet matured.

Formally, let $q_i^t$ be the amount of dividends related to index $i$ at time $t$. According to our model, the part driving dividend prices is the unhedgable risk part. As long as the market is extremely liquid, a high volume of dividend supply can be absorbed. We therefore construct the normalized dividend amount $nq_i^t$ to be the total dividend divided by the
total volume of index traded in the market. Here, we use the volume of index traded as
a proxy for the depth of dividend markets. Better normalization can use the total traded
volume of dividend swaps and dividend futures. However, dividend swaps are traded
over-the-counter, and we currently do not have access to this data.

We will assume that the aggregate structured product issuance is a proxy of the dividend
supply. From the data, both the total outstanding issuance and the normalized total
outstanding issuance are nonstationary. To get around such issues, we group the data by
quarter and stationalize each series by taking first difference or log difference depending
on whether the series has a linear or exponential trend. We then run the analysis on
these differenced series. Appendix C.7 displays the constructed total outstanding issuance
notional related to each index along with the quarter-by-quarter percentage change in
notional outstanding.\footnote{There appears to be a structural break in the notional issuance around 2014, which may be due to the fact that data is less comprehensive prior to 2014. We check the robustness of the results in this section by dividing the data into the pre- and post-2014 samples. Results appear to be robust as shown in Appendix C.8.}

### 3.5.3 Price & Yield Regression

Let \( P_{i,t} \) be the price of \( T \)-maturity asset related to index \( i \) at time \( t \), \( \delta_{i,T} \) be the index \( i \) \( T \)-maturity dividend yield at time \( t \), \( S_i^t \) be the index \( i \) level at time \( t \), and \( R_i^t \) be the monthly
return of index \( i \) at time \( t \).

Denote the quarterly-average price, yield, index level, and index return by \( P_{i,qtr}^t \), \( \delta_{i,qtr}^t \),
\( S_i^{qtr} \), \( R_i^{qtr} \) respectively. Let \( nq_{i,qtr}^t \) and \( niss_{i,qtr}^t \) be the quarterly sum of the normalized
dividend amount and the quarterly sum of the normalized structured product issuance,
where \( t \) is now in the unit of quarterly.

We assume that \( nq_{i,qtr}^t \propto niss_{i,qtr}^{t-1} \), i.e. the sum of the normalized dividend amount in the
current quarter is proportional to the sum of normalized structured product issuance from
the previous quarter. We make such an assumption for the following reasons. First, we try
to work around the simultaneity issue. Structured product issuance is likely higher when
dividend yields are higher or dividend term structure is more inverted, as investor payoffs
will become more attractive. We believe that the price effect of supply persists for some quarters in the future. Therefore, using the lagged percentage change can provide a lower bound on the exact magnitude of the impact. Second, intermediaries usually take some time in recycling their risk from exotic trades.

We consider the following time series regressions for $T = 1, 2, 3,$ and 5 years and $i$ represents SPX, SX5E, NKY, UKX, NKY, KOSPI2, HSI, and HSCEI as well as their panel regression analogs.

\[
\Delta P_{i,T}^{Qtr} = \kappa_i^{i,p} + \eta_i^{i,p} \Delta \log n_{q^{i,qt}}^i + \epsilon_T^p 
\]

\[
\Delta P_{i,T}^{Qtr} = \kappa_i^{i,p} + \eta_i^{i,p} \Delta \log n_{q^{i,qt}}^i + \lambda_i^S \Delta \log S_{q^{i,qt}}^i + \epsilon_T^p 
\]

(3.26) (3.27)

Regression equations (3.27) and (3.26) test for the effect of the percentage change of 1-quarter-lag normalized structured product issuance on the change of dividend yields with and without controlling for the percentage change in spot level respectively.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\Delta \log n_{q^{i,qt}}^i$</th>
<th>$\Delta \log S_{q^{i,qt}}^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12.32 (-1.07)</td>
<td>105.8* (2.66)</td>
</tr>
<tr>
<td>2</td>
<td>-32.09 (-1.26)</td>
<td>297.8*** (3.88)</td>
</tr>
<tr>
<td>3</td>
<td>-53.18 (-1.31)</td>
<td>533.4*** (4.76)</td>
</tr>
<tr>
<td>5</td>
<td>-98.25 (-1.40)</td>
<td>1035.6*** (5.78)</td>
</tr>
</tbody>
</table>

Notes: Data from Q4 2004 - Q4 2016. Panel Regression with Fixed Effect. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 3.4 shows the panel regression results controlling for both country-specific and time-specific effects. Standard errors are robust and clustered to address potential heteroskedasticity and autocorrelation issues. Even though the coefficient in front of the log difference of dividend amount is negative, it is insignificantly so. 1-quarter lagged issuance does not seem to have a noticeable effect on dividend prices.

Regression equation (3.27) controls for the level effect (dividend prices might track the spot level). The coefficient in front of the log difference of spot level is indeed significantly
positive. Whenever equity index goes higher, finite-maturity dividend prices also go up.

As prices of longer-maturity assets should always be higher than those of assets that cover less dividend payments, it is hard to do a comparison across maturity using the price data. To enable the comparison across maturity, we turn to the yield regression. Dividend yields are dividend prices normalized by the spot level as well as the maturity.

In particular, we consider the following regression equations:

\[
\Delta \delta_{i,T}^{i,qtr} = \kappa_i \delta + \eta_i \delta \Delta \log nq_{i,T}^{i,qtr} + \epsilon_T
\]

(3.28)

\[
\Delta \delta_{i,T}^{i,qtr} = \kappa_i \delta + \eta_i \delta \Delta \log nq_{i,T}^{i,qtr} + \lambda_i \Delta R_{i,T}^{i,qtr} + \epsilon_T.
\]

(3.29)

Regression equation (3.28) looks at the impact of lagged structured product issuance on dividend yields. Equation (3.29) tests for the effect of a percentage increase in the dividend supply on the percentage change in dividend yields controlling for the percentage change in index return.

<table>
<thead>
<tr>
<th></th>
<th>(T = 1)</th>
<th>(T = 2)</th>
<th>(T = 3)</th>
<th>(T = 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \log nq_{i,T}^{i,qtr})</td>
<td>(0.0002)</td>
<td>(-0.0002)</td>
<td>(-0.0002)</td>
<td>(-0.0004)</td>
</tr>
<tr>
<td></td>
<td>((0.66))</td>
<td>((-0.46))</td>
<td>((-0.33))</td>
<td>((-1.03))</td>
</tr>
<tr>
<td>(\Delta R_{i,T}^{i,qtr})</td>
<td>(-0.0010)</td>
<td>(-0.0025)</td>
<td>(-0.0030)</td>
<td>(-0.0037)</td>
</tr>
<tr>
<td></td>
<td>((-0.43))</td>
<td>((-1.07))</td>
<td>((-1.22))</td>
<td>((-1.79))</td>
</tr>
<tr>
<td>(N)</td>
<td>319</td>
<td>319</td>
<td>319</td>
<td>319</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.0010</td>
<td>0.0023</td>
<td>0.0009</td>
<td>0.0054</td>
</tr>
<tr>
<td></td>
<td>0.0135</td>
<td>0.0254</td>
<td>0.0085</td>
<td>0.0296</td>
</tr>
</tbody>
</table>

Notes: Data from Q4 2004. Panel Regression with Fixed Effect. \(t\) statistics using Driscoll-Kraay standard error in parentheses. * \(p < 0.05\), ** \(p < 0.01\), *** \(p < 0.001\).

Results from the panel regression controlling for both index-specific and time-specific effects is shown in Table 3.5. The coefficient in front of the log difference of dividend amount is insignificantly negative. The 1-quarter lagged issuance does not seem to affect contemporaneous dividend yields.

Controlling for the change in index return does not change the effect of the percentage increase in dividend supply on changes in dividend yields. The coefficient in front of the change in index return is negative but insignificantly so.
So far, we have shown that structured product issuance seems to have no effect on dividend prices and dividend yields. The flow of structured product issuance does not predict dividend prices and dividend yields. Next, we investigate whether the percentage increase in the notional outstanding of structured products affects any changes in the yield term structure. We consider the following regressions:

\[
\Delta (d_{i,T_2}^{\text{ltr}} - d_{i,T_1}^{\text{ltr}}) = \kappa_{T_2,T_1}^{i,d} + \eta_{T_2,T_1}^{i,d} \Delta \log n_{i}^{\text{ltr}} + \epsilon_{T_2,T_1}^{i,d} 
\]  (3.30)

\[
\Delta (d_{i,T_2}^{\text{ltr}} - d_{i,T_1}^{\text{ltr}}) = \kappa_{T_2,T_1}^{i,d} + \eta_{T_2,T_1}^{i,d} \Delta \log n_{i}^{\text{ltr}} + \lambda_i \Delta R_{T_2,T_1}^{R} + \epsilon_{T_2,T_1}^{i,d}. 
\]  (3.31)

Regression equation (3.30) tests whether the percentage change in the total structured product outstanding affects the change in the yield term structure. Higher structured product issuance should depress dividend prices. Since the 1-year dividend is either announced or well-forecasted, the major dividend risk from structured products is around 2, 3, or 5 years depending on the maturity of the trade. We expect that when \(T_2 = 2, 3\) or \(5\), the coefficient \(\eta_{T_2,T_1}^{i,d}\) should be negative (the yield term structure should be inverted).

\[\text{Table 3.6: Yield Term Structure Regression}\]

<table>
<thead>
<tr>
<th>(T_1, T_2 = 1, 2) (3.30)</th>
<th>(T_1, T_2 = 1, 3) (3.31)</th>
<th>(T_1, T_2 = 2, 3) (3.30)</th>
<th>(T_1, T_2 = 1, 5) (3.30)</th>
<th>(T_1, T_2 = 2, 5) (3.30)</th>
<th>(T_1, T_2 = 3, 5) (3.30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \log n_{i}^{\text{ltr}})</td>
<td>-0.0004 (1)</td>
<td>-0.0006* (2)</td>
<td>-0.0003* (3)</td>
<td>-0.0008* (4)</td>
<td>-0.0004 (5)</td>
</tr>
<tr>
<td>(\Delta R_{i}^{\text{ltr}})</td>
<td>-0.0015* (6)</td>
<td>-0.0020* (7)</td>
<td>-0.0004 (8)</td>
<td>-0.0005 (9)</td>
<td>-0.0001 (10)</td>
</tr>
<tr>
<td>(N)</td>
<td>319</td>
<td>319</td>
<td>319</td>
<td>319</td>
<td>319</td>
</tr>
<tr>
<td>\text{Adj. } R^2</td>
<td>0.0136 0.0277</td>
<td>0.0188 0.0315</td>
<td>0.0149 0.0186</td>
<td>0.0183 0.0264</td>
<td>0.0126 0.0140</td>
</tr>
</tbody>
</table>

* \(p < 0.05\), ** \(p < 0.01\), *** \(p < 0.001\).

Notes: Data from Q4 2004. Panel Regression with Fixed Effect. \(t\) statistics using Driscoll-Kraay standard error in parentheses.

From Table 3.6, the coefficient in front of \(\Delta \log n_{i}^{\text{ltr}}\) is significantly negative whenever \((T_2, T_1) = (3, 1), (3, 2),\) or \((5, 1)\) whether we control for the change in index return or not. When there is an increase in the structured product issuance, the 3y1y, 3y2y, and 5y1y term structure becomes more inverted. The 3-year and 5-year dividend yields become more compressed with respect to front-year dividend yields. We view this as the evidence supporting the supply-based asset pricing model. Specific equity index time-series regression results

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can be found in Appendix C.9.

3.6 Conclusion

This paper have discussed different ways of pricing T-maturity assets. Using comprehensive data for various equity indices around the world, we documented the patterns of T-maturity asset prices and returns. We find that the price and return term structure vary across indices and with time.

The feature of downward-sloping return term structure is discordant with standard macro-finance theories. This paper argues that the key determinant of dividend prices is dividend supply. Looking into the origin of dividend markets, we find that such markets were created due to the need of intermediaries to offload their dividend risks. Banks accumulate dividends from structured product issuance. Inspired by this fact, we propose the market specialization story to explain the variation in T-maturity asset prices.

We argue that structured products present a significant dividend risk to exotic trading desks. Exotic traders have limited risk-bearing capacity and will offload some of the dividend risk to the markets. The dividend supply from exotic traders hence suppresses dividend prices and dividend yields, especially around those with 2- to 5-year maturity. That is, the term structure of implied dividends is more inverted when the volume of structured product issuance is higher.

The proposed supply-based asset pricing model follows closely the demand-based option pricing model of Garleanu et al. (2009). Based on this theory, exogenous dividend supply from structured retail products will impact risky asset prices, returns, and term structure.

Using issuance data from a private vendor, we find that the 3y1y, 3y2y, 5y1y dividend yield term structure is indeed more inverted when the total outstanding issuance of structured products is higher. This provides an empirical basis to our story that structured products indeed impact equity term structure.

Based on our story, investors may take advantage of long dividend (and short equity) trading strategies due to the abnormality of the equity return term structure caused by the
structural imbalance from exotic products.

This paper also speaks to policy makers. Exogenous dividend supply has a big impact on dividend prices whenever intermediaries have trouble recycling their risks. In order to eliminate this market anomaly, policy makers should promote greater liquidity in dividend markets. Another debatable approach to eliminate market inefficiency is to allow for greater risk-taking limits in intermediaries.
References


Appendix A

Appendix to Chapter 1

A.1 Formal Equilibrium Characterization under Common Knowledge

A.1.1 Complete Information

For a given payoff function governed by \( b(\cdot) \) and \( c \), the set of equilibrium strategies \( \Gamma(\theta) \) is governed by

\[
\Gamma(\theta) = \begin{cases} 
\{\rho\} & ; \theta < 0 \\
\{0, \rho\} & ; 0 \leq \theta \leq \rho \\
\{0\} & ; \rho < \theta
\end{cases}
\] (A.1)

for each \( \rho \) that is a solution to \( b(\rho) = c \).

A.1.2 Public Signal

As mentioned in the main paper, a proposition below characterizes the set of equilibrium strategies when the noise is sufficiently small.

Proposition 22 Consider a given payoff function governed by strictly decreasing \( b(\cdot) \) and \( c \). Assume the noise of public signal is sufficiently small, i.e. \( e < \frac{1}{2M} \), when \( M = \sup_{p \in [0,1]} \frac{|b'(p)|}{b(p)^2} \), the set of
equilibrium strategy $\Gamma(y)$ is then given by

$$
\Gamma(y) = \begin{cases} 
\{\rho\} & ; y < y_L \\
\{0, \delta(y), \rho\} & ; y_L \leq y \leq y_H(\rho) , \text{ when } \rho = 1 \\
\{0\} & ; y_H(\rho) < y
\end{cases}
$$

(A.2)

\[ y_L = 2e \left( \frac{1}{2} - \frac{c}{b(0)} \right) , \quad y_H(\rho) = \begin{cases} 
1 + 2e \left( \frac{1}{2} - \frac{c}{b(0)} \right) & ; \rho = 1 \\
\rho - e & ; \rho < 1
\end{cases} \]

Remark 23 $y_H(\rho)$ is continuous in $\rho$.

Proof. case I $\rho = 1$:

No attack is an equilibrium if and only if $E_y[u(1,0,\theta) \mid y] = b(0)P(\theta \leq 0 \mid y) - c \leq 0$, which holds whenever $y \geq 2e \left( \frac{1}{2} - \frac{c}{b(0)} \right) = y_L$. On the other hand, attack is an equilibrium if and only if $E_y[u(1,1,\theta) \mid y] = b(1)P(\theta \leq 1 \mid y) - c \geq 0$, which holds whenever $y \leq 1 + 2e \left( \frac{1}{2} - \frac{c}{b(0)} \right)$.

A strictly mixed strategy $p \in (0,1)$ is sustainable in equilibrium if and only if $E_y[u(1,p,\theta) \mid y] = b(p)P(\theta \leq p \mid y) - c = 0$.

Let $f(p) = P(\theta \leq y), g(p) = \frac{c}{b(p)},$ and $h(p) = f(p) - g(p)$. Note that

$$
h(p) = \begin{cases} 
1 - \frac{c}{b(p)} & ; y + e < p \\
p - \frac{(y - e)}{2e} - \frac{c}{b(p)} & ; y - e \leq p \leq y + e \\
- \frac{c}{b(p)} & ; p < y - e
\end{cases}
$$

For $p < y - e, h(p) < 0$. For $p > y + e, h(p) = 1 - \frac{c}{b(p)} > 1 - \frac{c}{b(1)} \geq 0$. The inequality follows from the strict monotonicity of $b(\cdot)$. Therefore, the solution to $h(p) = 0$ must be $p \in [y - e, y + e]$.

For $p \in [y - e, y + e]$, $\frac{dh(p)}{dp} = \frac{1}{2e} + \frac{c}{b(p)P(\theta \leq p \mid y)} \geq \frac{1}{2e} - cM > 0$. That is, $h(p)$ is strictly increasing in $p$. $h(0) \leq 0$ if and only if $y \geq 2e \left( \frac{1}{2} - \frac{c}{b(0)} \right) = y_L$, while $h(1) \geq 0$ if and only if $y \leq 1 + 2e \left( \frac{1}{2} - \frac{c}{b(1)} \right) = y_H$. There is no intersection for $y \notin (y_L, y_H)$ and is exactly one intersection, namely $\delta(y)$, for $y \in (y_L, y_H)$ by the intermediate value theorem.
case II $\rho < 1$:

Recall that $\rho \in (0,1)$ is such that $b(\rho) = c$. Similar to the previous case, no attack is an equilibrium if and only if $y \geq y_L$. Attack can never be sustained as an equilibrium as $b(1)P(\theta \leq p \mid y) - c \leq b(1) - c < 0$ for all $y \in \mathbb{R}$.

Any strictly-mixed strategy $p \in (0,1)$ is played in an equilibrium if and only if $y \in y_L$. Attack can never be sustained as an equilibrium as $b(1)P(\theta \leq p \mid y) - c \leq b(1) - c < 0$ for all $y \in \mathbb{R}$.

Any strictly-mixed strategy $p \in (0,1)$ is played in an equilibrium if and only if it is a solution to $h(p) = 0$. As before, we consider three different ranges of $p$ with respect to $y$.

For $p < y - e$, $h(p) < 0$. For $p > y + e$, $h(p) = 1 - \frac{c}{b(p)} = 0$ if and only if $p = \rho$. That is, attacking with probability $\rho$ is an equilibrium strategy whenever $y < \rho - e$.

For $p \in [y - e, y + e]$, $h(p) = \frac{p - (y - e)}{2e} - \frac{c}{b(p)}$, and thus, $\frac{dh(p)}{dp} > 0$. Note that for $p > \rho$, $h(p) = \frac{p - (y - e)}{2e} - \frac{c}{b(p)} \leq 1 - \frac{c}{b(p)} < 1 - \frac{c}{b(\rho)} < 0$. For $p = \rho$, $h(p) = 0$ if and only if $y = \rho - e$. Therefore, an extra mixed strategy $\delta(y)$ must be in $(0, \rho)$. Because $h(0) \leq 0$ if and only if $y \geq 2e\left(\frac{1}{2} - \frac{c}{b(0)}\right) = y_L$ and $h(\rho) \geq 0$ if and only if $y \leq \rho - e$, there is an extra intersection in addition to $\rho$, denoted by $\delta(y)$, if and only if $y \in (y_L, \rho - e]$.

\[\blacksquare\]

### A.2 Proof of Theorem 5

**Proof.** Recall from (1.2) that $\rho$ satisfies $b(\rho) = c$. Let $\bar{u} \in (0, \rho)$ denotes a unique solution to

$$f(u) := \frac{1}{\rho} \int_0^u b(\rho - y)dy = c. \quad (A.3)$$

Note that such $\bar{u}$ exists and is unique since $f$ is strictly increasing with $f(0) = 0$, and

$$f(\rho) = \frac{1}{\rho} \int_0^\rho b(\rho - y)dy > \frac{1}{\rho} \int_0^\rho b(\rho)dy = c.$$

Next, we let

$$x^e = -e + (1 + 2e)(\rho - \bar{u}). \quad (A.4)$$
Consider the following strategy

\[
\begin{align*}
  s(x_i) = \begin{cases} 
    0 & ; x_i \geq \bar{x}^e \\
    1 & ; \bar{x}^e - 2ek - 2ep \leq x_i < \bar{x}^e - 2ek, \quad k = 0, 1, 2, \ldots \\
    0 & ; \bar{x}^e - 2e(k+1) \leq x_i < \bar{x}^e - 2ek - 2ep, \quad k = 0, 1, 2, \ldots 
  \end{cases} 
\end{align*}
\]  

(A.5)

We now verify that \((s(x))_x \in \mathbb{R}\) is an equilibrium strategy profile. Assume that other players adopt the strategy \((s(x))_x \in \mathbb{R}\). Under a continuum of agents assumption, \(A^x(\theta)\) is independent of a chosen action of player \(i\) and is given deterministically by

\[
A(\theta) = \int_{\theta-e}^{\theta+e} 1_{\{s(u)=1\}} du \biggl/ 2e = \begin{cases} 
  \rho & ; \theta < \bar{x}^e + e - 2ep \\
  \frac{1}{2e} (\bar{x}^e + e - \theta) & ; \bar{x}^e + e - 2ep \leq \theta \leq \bar{x}^e + e \\
  0 & ; \theta > \bar{x}^e + e
\end{cases} \] (A.6)

We observe that \(A(\theta) - \theta\) is strictly decreasing over \(\mathbb{R}\). Thus, there is a unique \(\hat{\theta} \in \mathbb{R}\) such that \(A(\hat{\theta}) = \hat{\theta}\). By (A.4) and (A.6), we have

\[
A(\bar{x}^e + e - 2ep + 2e\bar{u}) - \bar{x}^e + e - 2ep + 2e\bar{u} = (\rho - \bar{u}) - (\rho - \bar{u}) = 0 \] (A.7)

Therefore, a unique solution to \(A(\theta) = \theta\) is

\[
\hat{\theta} = \bar{x}^e + e - 2ep + 2e\bar{u}. \] (A.8)

Note also that \(\hat{\theta} \in (\bar{x}^e + e - 2ep, \bar{x}^e + e)\).

Next, we verify that \(s\) is indeed a best response correspondence. From equations (A.3), (A.4), and (A.8), it follows that

\[
F^A(\bar{x}^e) = \frac{1}{2e} \int_{\bar{x}^e-e}^{\bar{x}^e+e} b(A(\theta)) \mathbf{1}_{A(\theta) \geq \rho} d\theta - c = \frac{1}{2e} \int_{\bar{x}^e-e}^{\hat{\theta}} b(A(\theta)) d\theta - c
\]

\[
= \frac{1}{2e} \int_{\bar{x}^e-e}^{\bar{x}^e+e} b(A(\theta)) d\theta + \frac{1}{2e} \int_{\bar{x}^e+e-2ep+2e\bar{u}}^{\bar{x}^e+e-2ep} b(A(\theta)) d\theta - c
\] (A.9)

\[
= (1-\rho)c + \int_{0}^{\bar{u}} b(\rho - y) dy - c = (1-\rho)c + \rho c - c = 0.
\]

That is, a player observing signal \(\bar{x}^e\) is indifferent between attacking and no attack. Since
there is no reward beyond \( \hat{\theta} \), a direct comparison yields \( F(x_i) < F(\bar{x}) = 0 \) for all \( x_i > \bar{x} \). That is, no attack is dominant in such region, and \( s(x_i) = 0 \) for all \( x_i > \bar{x} \).

For \( x_i \in [\bar{x} - 2\epsilon, \bar{x}] \), equation (A.8) implies that \( x_i + \epsilon \geq \hat{\theta} \) and that
\[
F(x_i) = \frac{1}{2\epsilon} \int_{x_i-e}^{x_i+e} b(A(\theta)) \chi_{A(\theta) \geq \hat{\theta}} d\theta - c = \frac{1}{2\epsilon} \int_{x_i-e}^{\hat{\theta}} b(A(\theta)) d\theta - c \geq \frac{1}{2\epsilon} \int_{x_i-e}^{x_i+e} b(\hat{\theta}) d\theta - c = 0. \tag{A.10}
\]

Attacking is dominant, and \( s(x_i) = 1 \) for \( x_i \in [\bar{x} - 2\epsilon, \bar{x} - 2\epsilon \bar{u}, \bar{x}] \).

Next, for \( x_i \in [\bar{x} - 2\epsilon, \bar{x} - 2\epsilon \bar{u}] \), we have \( x_i + \epsilon < \hat{\theta} \), so \( A(\theta) > \theta \) for all \( \theta \in [x_i - \epsilon, x_i + \epsilon] \). From equation (A.6), we also have that \( A(\theta) \leq \rho \) for all \( \theta \in [x_i - \epsilon, x_i + \epsilon] \).

It follows that
\[
F(x_i) = \frac{1}{2\epsilon} \int_{x_i-e}^{x_i+e} b(A(\theta)) \chi_{A(\theta) \geq \hat{\theta}} d\theta - c \geq \frac{1}{2\epsilon} \int_{x_i-e}^{x_i+e} b(\theta) d\theta - c = 0. \tag{A.11}
\]

Attack is again dominant and \( s(x_i) = 1 \) for \( x_i \in [\bar{x} - 2\epsilon, \bar{x} - 2\epsilon \bar{u}, \bar{x}] \).

Lastly, for \( x_i \in (-\infty, \bar{x} - 2\epsilon) \), \( A(\theta) = \rho \) and \( A(\theta) > \theta \) for all \( \theta \in [x_i - \epsilon, x_i + \epsilon] \).

Therefore,
\[
F(x_i) = \frac{1}{2\epsilon} \int_{x_i-e}^{x_i+e} b(\rho) d\theta - c = c - c = 0. \tag{A.12}
\]

That is, a player is indifferent in this signal interval.

From equations (A.9)-(A.12), we see that \( (s(x))_{x \in \mathbb{R}} \) given by (A.5) is indeed a best response. ■

### A.3 Proof of Theorem 6

**Proof.** First, note that there exists a unique \( \hat{\theta} \) such that \( A^c(\hat{\theta}) = \hat{\theta} \) due to the monotone assumption on \( A^c \) and \( A^c(\theta) - \theta < 0 > 0 \) for sufficiently large (small) \( \theta \).

Second, we show that \( A^c(\theta) \leq \rho \) for all \( \theta \). Suppose there exists \( \theta \) with \( A^c(\theta) > \rho \), then, \( \forall x_i \leq \theta_i - e \),
\[
F(x_i) = \frac{1}{2\epsilon} \left( \int_{x_i-e}^{x_i+e} b(A^c(\theta)) \chi_{A^c(\theta) \geq \hat{\theta}} d\theta \right) - c < \frac{1}{2\epsilon} \left( \int_{x_i-e}^{x_i+e} b(\rho) \right) - c = 0.
\]
That is, $F(x_i) < 0$, and $s(x_i) = 0 \forall x_i < \tilde{\theta} - e$. This conflicts with Lemma 4.

Third, we show that there exists $\theta_0$ such that $A^s(\theta_0) = \rho$. Suppose $A^s(\theta) < \rho \forall \theta$, then, $\forall x_i \leq \hat{\theta} - e$,

$$F(x_i) = \frac{1}{2e} \left( \int_{x_i-e}^{x_i+e} b(A^s(\theta)) \mathbf{1}_{A(\theta) \geq \theta} d\theta \right) - c > \frac{1}{2e} \left( \int_{x_i-e}^{x_i+e} b(\rho) d\theta \right) - c = 0.$$ 

That is, $F(x_i) > 0 \forall x_i \leq \hat{\theta} - e$. In particular, $A^s(\hat{\theta} - 2e) = 1 > \rho$, contradicting with $A^s(\theta) \leq \rho \forall \theta$.

Now, we let

$$\hat{x}^e = \sup \{ x \in [\hat{\theta} - e, \hat{\theta} + e] ; \frac{\int_{\hat{\theta} - 2e}^{\hat{\theta}} b(A(\theta)) d\theta}{2e} = c \} , \quad \theta_\rho = \sup \{ \theta \in \mathbb{R} ; A^s(\theta) = \rho \}.$$ 

This $\hat{x}^e$ is well-defined since $\frac{\int_{\hat{\theta} - 2e}^{\hat{\theta}} b(A(\theta)) d\theta}{2e} \geq c$ and $\hat{\theta} - 2e < x - e$. That is, $\hat{x}^e$ is the rightmost indifference point after which attacking becomes suboptimal. We show that $\theta_\rho < \hat{\theta}$. Suppose that $\hat{\theta} \leq \theta_\rho$, then, $\forall x_i > \hat{\theta} - e$,

$$F(x_i) = \frac{1}{2e} \left( \int_{x_i-e}^{x_i+e} b(A^s(\theta)) \mathbf{1}_{A^s(\theta) \geq \theta} d\theta \right) - c = \frac{1}{2e} \left( \int_{x_i-e}^{\hat{\theta}} b(A^s(\theta)) d\theta \right) - c \leq \frac{1}{2e} (\hat{\theta} - x_i + e) c - c < 0.$$ 

That is, $F(x_i) < 0 \forall x_i > \hat{\theta} - e$ contradicting with $A^s(\hat{\theta}) \geq \rho$. It is then straightforward to check the following properties:

1. $F(x_i) < 0$ for all $x_i > \hat{x}^e$

2. $F(\hat{x}^e) = 0$

3. $F(x_i) > 0$ for all $x_i \in (\theta_\rho - e, \hat{x}^e)$

4. $F(x_i) = 0$ for all $x_i \leq \theta_\rho - e$

5. $A(\theta) = \rho$ for all $\theta \leq \theta_\rho$

For a fixed $\hat{\theta}$, the threshold $\hat{x}^e$ and consequently $(A^s(\theta))_{\theta \in \mathbb{R}}$ and $(s(x_i))_{x_i \in \mathbb{R}}$ are uniquely determined. We will now show that for a given $b(\cdot)$ and $c$, $\hat{\theta}$ is uniquely determined.
The characterization of $\hat{\theta}$: Note that $A^s$ is given by

$$A^s(\theta) = \begin{cases} 
\rho & \theta \leq \theta_\rho \\
\frac{1}{2e}(\bar{x}^e + e - \theta) & \theta_\rho \leq \theta \leq \bar{x}^e + e \\
0 & \theta \geq \bar{x}^e + e
\end{cases}.$$  \hfill (A.13)

Therefore, we have $\theta_\rho = \bar{x}^e + e - 2ep$. From $F(\bar{x}^e) = 0$, we have

$$c = \frac{1}{2e} \int_{\bar{x}^e - e}^{\bar{x}^e + e} b(A^s(\theta)) \mathbf{1}_{A^s(\theta) \geq \theta} d\theta$$

$$= \frac{1}{2e} \int_{\bar{x}^e - e}^{\theta} b(A^s(\theta)) d\theta$$

$$= \frac{1}{2e} \left( \int_{\bar{x}^e - e}^{\bar{x}^e + e - 2ep} b(A(\theta)) d\theta + \int_{\bar{x}^e + e - 2ep}^{\theta} b(A(\theta)) d\theta \right)$$

$$= \frac{1}{2e} \left( 2e(1 - \rho)b(\rho) + 2e \int_{\frac{\theta - (\bar{x}^e + e - 2ep)}{2e}}^{\frac{\theta - (\bar{x}^e + e - 2ep)}{2e}} b(\rho - y) dy \right).$$

We use the change of variable $y = \frac{\theta - (\bar{x}^e + e - 2ep)}{2e}$. It follows that

$$\rho c = \int_{0}^{\frac{\theta - (\bar{x}^e + e - 2ep)}{2e}} b(\rho - y) dy. \hfill (A.14)$$

Let $\bar{u}$ be the unique value in $(0, \rho)$ such that $\frac{\int_{0}^{\bar{u}} b(\rho - y)dy}{\rho} = c$. We have $\hat{\theta} = \bar{x}^e + e - 2e(\rho - \bar{u})$.

From equation (A.13),

$$\hat{\theta} = A(\hat{\theta}) = A(\bar{x}^e + e - 2e(\rho - \bar{u})) = \rho - \bar{u}.$$  

That is, $\hat{\theta}$ is uniquely determined. \hfill $\blacksquare$

A.4 Proof of Theorem 8

Proof. Denote $\bar{u} \in (0, 1)$ a unique solution to

$$f(u) := \int_{0}^{u} b(1 - y) dy = c.$$  

Such $\bar{u}$ exists and is unique because
1. $f(u)$ is strictly increasing $\forall u \in [0, 1]$.

2. $f(0) = 0$.

3. $f(1) = \int_0^1 b(1-y)dy > \int_0^1 b(1)dy \geq c$.

Consider the switching strategy

$$s(x_i) = \begin{cases} 
1, & \text{if } x_i \leq \bar{x}^e \\
0, & \text{if } x_i > \bar{x}^e 
\end{cases}$$

when $\bar{x}^e = -e + (1 + 2e)(1 - \bar{u})$.

Assuming all other players use the described strategy $s(x_i)$, we have

$$A(\theta) = \int_{\theta-e}^{\theta+e} s(x_i)dx_i = \begin{cases} 
1, & \text{if } \theta \leq \bar{x}^e - e \\
\frac{(\bar{x}^e + \theta - \theta)}{2e}, & \text{if } \theta \in [\bar{x}^e - e, \bar{x}^e + e] \\
0, & \text{if } \theta \geq \bar{x}^e + e 
\end{cases}$$

The expected payoff from attacking conditional on a private signal $\bar{x}^e$ is

$$F^A(\bar{x}^e) = \frac{1}{2e} \left( \int_{\bar{x}^e - e}^{\bar{x}^e + e} b(A^s(\theta))1_{A^s(\theta) \geq \theta}d\theta \right) - c$$

$$= \frac{1}{2e} \left( \int_{\bar{x}^e - e}^{\bar{x}^e + e} b(A^s(\theta))d\theta \right) - c = \int_0^\theta b(1-y)dy - c = 0.$$

The second equality follows from $A^s(\theta) \geq \theta \leftrightarrow \frac{(\bar{x}^e + \theta - \theta)}{2e} \geq \theta \leftrightarrow \frac{\bar{x}^e + e}{1 + 2e} \geq \theta$. The third equality is a result of a change of variable with $y = 1 - A^s(\theta) = 1 - \left( \frac{(\bar{x}^e + \theta)}{2e} \right)$.

Since $A^s(\theta) - \theta$ is strictly decreasing, and $A^s(\theta) - \theta < (>) 0$ for sufficiently large (small) $\theta$, there exists a unique $\theta$ satisfying $A^s(\theta) - \theta = 0$, denoted by $\hat{\theta}$. We then note that this $\hat{\theta} \in (\bar{x}^e - e, \bar{x}^e + e)$. Otherwise if $\hat{\theta} \leq \bar{x}^e - e$, $F^A(\bar{x}^e) = -c \neq 0$. Else if $\hat{\theta} \geq \bar{x}^e + e$, then $F^A(\bar{x}^e) = 0 \Rightarrow c = \int_0^1 b(1-y)dy < c$. Both cases result in a contradiction.

Now consider $x_i < \bar{x}^e$. First, if $x_i \leq \hat{\theta} - e < \bar{x}^e$, then $F^A(x_i) \geq b(1) - c \geq 0$. If
\(\hat{\theta} - e < x_i \leq \hat{x}^e\), then

\[
F^A(x_i) = \frac{1}{2e} \left( \int_{x_i - e}^{x_i + e} b(A^s(\theta)) \mathbf{1}_{A^s(\theta) \geq 0} d\theta \right) - c = \frac{1}{2e} \left( \int_{x_i - e}^{\hat{\theta}} b(A^s(\theta)) d\theta \right) - c
\]

\[
\geq \frac{1}{2e} \left( \int_{\hat{x}^e - e}^{\hat{x}^e + e} b(A^s(\theta)) d\theta \right) - c = \frac{1}{2e} \left( \int_{\hat{x}^e - e}^{x_i} b(A^s(\theta)) \mathbf{1}_{A^s(\theta) \geq 0} d\theta \right) - c
\]

\[
= F^A(\hat{x}^e) = 0.
\]

That is, \(F^A(x_i) \geq 0\), and hence \(s(x_i) = 1\) is sustainable \(\forall x_i \leq \hat{x}^e\). Similarly, we can prove that \(F^A(x_i) < F^A(\hat{x}^e) = 0\) when \(x_i > \hat{x}^e\), and hence \(s(x_i) = 0\) is rationalizable \(\forall x_i > \hat{x}^e\).

\[\blacksquare\]

A.5 Proof of Proposition 9

Proof. Consider \(e > 0\) and a reward function \(b_1\). We would like to show

1) Continuity: We want to show that for any \(e > 0\), there exist \(\delta > 0\) such that for any reward function \(b_2\) satisfying \(\sup_{A \in [0,1]} \| b_1(A) - b_2(A) \| < \delta\), we have \(|\hat{x}^e - \hat{x}_2^e| < e\). Here, \(\hat{x}_1^e\) and \(\hat{x}_2^e\) are as defined in (1.9) corresponding to \(b_1\) and \(b_2\) respectively. Fix \(e > 0\), let \(\rho_1\) and \(\rho_2\) be the unique values such that \(b_1(\rho_1) = b_2(\rho_2) = c\), it follows that

\[
\sup_{A \in [0,1]} \| b_1(A) - b_2(A) \| < \delta \Rightarrow |b_1(\rho_1) - b_2(\rho_1)|, |b_1(\rho_2) - b_2(\rho_2)| < \delta
\]

\[
\Rightarrow c - \delta < b_2(\rho_1), b_1(\rho_2) < c + \delta
\]

Let \(m = \min_{y \in [0,1]} |b'_1(y)| > 0\), then by the Mean Value Theorem,

\[
b_1(\rho_1) - b_1(\rho_2) = b'_1(\rho^*)(\rho_1 - \rho_2).
\]

Thus,

\[
|\rho_1 - \rho_2| \leq \frac{\delta}{m}.
\]

Therefore, for any \(e' > 0\), we can select \(\delta\) small enough that \(|\rho_1 - \rho_2| < e'\). Next, from the
definition of $\bar{u}_1$ and $\bar{u}_2$, we have that
\[ \int_{\rho_1 - \bar{u}_1}^{\rho_1} b_1(A)dA = \rho_1 c, \quad (A.15) \]
and
\[ \int_{\rho_2 - \bar{u}_2}^{\rho_2} b_2(A)dA = \rho_2 c. \quad (A.16) \]
Without loss of generality, assume that $\rho_1 > \rho_2$. We can ensure that the selected $\delta$ is sufficiently small that $\rho_2 > \rho_1 - \bar{u}_1$. Subtracting equation (A.16) from equation (A.15), we have
\[ (-1)^{i+1} \int_{\min(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)}^{\max(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)} b_i(A)dA = - \int_{\max(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)}^{\rho_2} (b_1(A) - b_2(A))dA \]
\[ = - \int_{\rho_2}^{\rho_1} b_1(A)dA + (\rho_1 - \rho_2)c, \]
where $i$ is such that $\min(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2) = \rho_i - \bar{u}_i$. Let $M = b_i(\max(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2))$, we have, by the triangle’s inequality, the monotonicity of reward functions, and the definition of supremum norm, that
\[ | (\rho_1 - \bar{u}_1) - (\rho_2 - \bar{u}_2) | M \leq | (-1)^{i+1} \int_{\min(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)}^{\max(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)} b_i(A)dA | \]
\[ \leq | \int_{\max(\rho_1 - \bar{u}_1, \rho_2 - \bar{u}_2)}^{\rho_2} (b_1(A) - b_2(A))dA | \]
\[ + | \int_{\rho_2}^{\rho_1} b_1(A)dA | + | (\rho_1 - \rho_2)c | \]
\[ \leq \rho_2 \delta + (\rho_1 - \rho_2)b_1(\rho_2) + (\rho_1 - \rho_2)c. \]
That is,
\[ | \bar{x}_1^e - \bar{x}_2^e | = (2\varepsilon + 1) | (\rho_1 - \bar{u}_1) - (\rho_2 - \bar{u}_2) | \]
\[ \leq (2\varepsilon + 1) \frac{\rho_2 \delta + (\rho_1 - \rho_2)b_1(\rho_2) + (\rho_1 - \rho_2)c}{M} \]
\[ < \varepsilon \]
for sufficiently small $\delta$.

2) Monotonicity: We want to show that $b_1 \geq b_2 \Rightarrow \bar{x}_1^e \geq \bar{x}_2^e$. 

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Assume $b_1 \geq b_2$, we have that $\rho_1 \geq \rho_2$. We need to show that $\rho_1 - \bar{a}_1 \geq \rho_2 - \bar{a}_2$. Suppose otherwise that $\rho_1 - \bar{a}_1 < \rho_2 - \bar{a}_2$. By definition of $\bar{a}_1$ and $\bar{a}_2$ (see equations (A.15) and (A.16)), it follows that

$$\rho_1 c = \int_{\rho_1 - \bar{a}_1}^{\rho_1} b_1(A) dA$$

$$= \int_{\rho_2}^{\rho_1} b_1(A) dA + \int_{\rho_2 - \bar{a}_2}^{\rho_2} b_1(A) dA + \int_{\rho_1 - \bar{a}_1}^{\rho_2 - \bar{a}_2} b_1(A) dA$$

$$\geq (\rho_1 - \rho_2) c + \int_{\rho_2 - \bar{a}_2}^{\rho_2} b_2(A) dA + \int_{\rho_1 - \bar{a}_1}^{\rho_2 - \bar{a}_2} b_1(A) dA$$

$$\geq (\rho_1 - \rho_2) c + \rho_2 c + \int_{\rho_1 - \bar{a}_1}^{\rho_2 - \bar{a}_2} b_1(A) dA.$$

That is,

$$\int_{\rho_1 - \bar{a}_1}^{\rho_2 - \bar{a}_2} b_1(A) dA \leq 0,$$

which is a contradiction. Therefore, $\rho_1 - \bar{a}_1 \geq \rho_2 - \bar{a}_2$ which implies $\bar{x}_1^c \geq \bar{x}_2^c$ as desired.

\[ \blacksquare \]
Appendix B

Appendix to Chapter 2

B.1 Data Appendix

B.1.1 Daily versus Monthly Data

"Daily" data pulls information from every trading day, while "Monthly" data picks only the last trading day of each month to construct the month-end data.

B.1.2 Exchange Rates Data and Interest Rates Data

WM

World Markets PLC/Reuters (WM/R) provides daily 4pm London fixing (15:00 GMT) spot and forward rates. I combine bilateral exchange rates with US Dollar (USD) as a base currency with those with British Pound (GBP) as a base currency. Most GBP series are longer except for the Euro.

Bloomberg

Bloomberg provides daily 5pm New York Close (21:00 GMT) spot and forward rates for a majority of currencies in the study. The data ranges from 1 December 1983 to 7 June 2017. The FXTF function on Bloomberg terminal reveals a list of currencies (AUD, EUR, IEP, NZD, and GBP) with special forward-points convention. Pakistani Rupee only has data of onshore forward points.
TR

Thomson Reuters (T/R) provides daily 5pm New York Close (21:00 GMT) spot and forward rates. Again, I complement the USD series with the GBP ones.

For above datasets, I calculate implied interest rate differentials $x_t$ using the following formula:

$$x_t = \log \left( \frac{S_t}{F_t} \right).$$

This method pulls daily data of annual Eurocurrency rates provided by Intercapital from Datastream. The data covers the period from 2 Jan 1970 to 7 June 2017. The mnemonics for the Eurocurrency rates are ECxxxyy, where xxx is the country code and yy represents the horizon (for example, 1M for 1 month). As the eurocurrency rates are often missing or incomplete for non-OECD countries, the paper uses the following alternatives in the empirical studies.

1-month VIBOR and Real 1-month implied rates are used for Austria and Brazil respectively, as these rates are roughly in line with forward-implied rates. The paper uses TR Chinese Yuan 1-month deposit for China, as the TR deposit rate is quite compatible with the discontinued Eurocurrency rates. Finland Euro-Markka 1-month ICAP/TR rate is used for Finland. For Greece, I combine the ECGRD1M with earlier observations from the Greek deposit rate. From the year 2002 onwards, interest rates for Greece follow the common Euro 1-month rate. The TR deposit rate is combined with earlier observations from The Taiwan deposit rate for Taiwan. For Thailand, I complement the ECTHB1M with later observations from the TR deposit rate.

All interest rates are annually adjusted and are in percentage. The paper calculates interest rate differentials using the following formula:

$$x_t = i_t - i_t^* = (1 + \frac{i^\text{raw}_t}{100})^{\frac{1}{12}} - (1 + \frac{i^*\text{raw}_t}{100})^{\frac{1}{12}}.$$
Combine

The paper ranks the data quality in the following order from the most reliable to the least reliable: WM, BBG, TR, and i. WM is ranked first because it appears to be the most accurate and the most recent. BBG used by a majority of active currency investors is augmented to the WM series whenever the WM data is missing. TR with more sparse data is then used. I rank the spot and forward pairs above the "i" method as both come from the same source. The "i" method combines the spot series from the previous three methods and calculates interest rate differentials from interest rate series from Datastream. I note that there are slight discrepancies of spot rates and interest differentials among each dataset due to different recording times. These differences appear to be minimal.

Even though WM is expansive in term of the currency coverage, its forward data only starts in the early 90s. Bloomberg data is as extensive as WM with an addition of Uruguay forward data. Data from TR is sparse in term of coverage but goes back earlier in time. The interest differentials from the "i" method cover all countries of interests and run the furthest back.

B.1.3 GDP-based Categorization

The paper uses the time series of the GDP per capita, current prices (purchasing power parity in the unit of international dollars per capita) provided by the International Monetary Fund (http://www.imf.org/external/datamapper/PPPPC@WEO/ARG/AUS/AUT/BEL/BRA/CAN/CHL/CHN/CZE/DNK/EGY/EU).

This data has an annual frequency dating back to 1980. I use the 1980 level to proxy for levels prior to the year 1980. There is no available data for the Euro area, so the paper uses the "whole European union" series as a proxy.

Another popular measure of categorization is MSCI market classification of countries into developed and emerging countries. The paper does not explore this method.
B.1.4 Survey Data from the Forecasts Unlimited Inc.

Background of the Forecasts Unlimited Inc. (FX4casts.com)

The Currency Forecasters’ Digest was started in August 1984. It was sold to the Financial Times in September 1994 and was renamed as the Financial Times Currency Forecaster. The company was repurchased and renamed Biz4casts.com in January 1999. It has been known as FX4casts.com since December 2002. Throughout the change in the company’s ownership, the production staff remained the same, with the addition of Marsha Kameron in January 1988. This ensures the consistency of the data collected.

Contributors of Consensus Forecasts


The list has changed over the 30-year period to reflect mergers among banks and financial institutions but always contains major intermediaries in the exchange rate markets.

Data

Data contains monthly spot rates and consensus 3-, 6-, and 12-month forecasts for 32 currencies. The series start on August 1986 for 10 currencies: British Pound, Danish Krone, Euro (with Deutsche Mark prior to January 1999), Norwegian Krone, Swedish Krona, Swiss Franc, Australian Dollar, New Zealand Dollar, Japanese Yen, and Canadian Dollar. The series start on October 2001 for the remaining 22 countries: Czech Koruna, Hungarian Forint, Polish Zloty, Russian Rouble, Turkish Lira, Chinese Renminbi, Hong Kong Dollar,
Indian Rupee, Indonesian Rupiah, New Zealand Dollar, Philippine Peso, Singapore Dollar, South Korean Won, Taiwan Dollar, Thai Baht, Argentine Peso, Brazilian Real, Chilean Peso, Colombian Peso, Mexican Peso, Venezuelan Bolivar, and South African Rand.

The 95% confidence intervals for all 32 currencies are available starting October 2001. The 1- and 24-month forecasts data for all 32 currencies became available starting July 2008. The data only contains consensus forecasts for the Euro with no data for each individual European currency.

I note that this dataset from FX4casts.com has been used previously in academic research. Bacchetta et al. (2009) and Gourinchas and Tornell (2004) are examples of previous articles using the consensus forecasts from this source.

### B.2 Empirical Patterns Appendix

The main paper focuses on results for the G7 sample using monthly data from the "Combine" method when the period step is 1 month. This section provides some robustness checks by looking at different ways of pooling the data.

#### B.2.1 Different Samples

Figure B.1, Figure B.2, Figure and B.3 plot estimated coefficients along with 95% confidence bands from panel regressions of equations (2.5), (2.6), and (2.7) using the "Combine" data for the higher-than-median GDP per capita sample, the lower-than-median GDP per capita sample, and the entire sample, respectively.

Key patterns hold across different samples.

#### B.2.2 Data Frequency

Figure B.4 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (2.5), (2.6), and (2.7) using the "Combine" data for the G7 sample when USD is a base currency.
Instead of month-end data, daily data is now used. For a comparison, the numbers of observations per country are 11,049 and 508 for the daily and the monthly data, respectively.

Empirical patterns observed in the paper are robust when daily data is used instead of the month-end data.

**B.2.3 Alternative Datasets**

The main paper displays results from the "Combine" method. This section illustrates empirical patterns from alternative datasets.

**WM**

Figure B.5, B.6 B.7, and B.8 plot estimated coefficients along with 95% confidence bands from panel regressions of equations (2.5), (2.6), and (2.7) for the G7 countries using the
This section checks the robustness of empirical patterns emphasized in the main paper by looking at alternative datasets for exchange rates and interest rate differentials. Point estimates from alternative datasets show the same patterns. The statistical significance is lost in all but the "i" method. This is potentially due to the shorter interest differentials samples.

### B.2.4 Varying Period Step

The main paper displays empirical patterns when the period step is fixed at 1 month. This section checks whether such patterns are robust when the period step is different. In particular, Figure B.9, Figure B.10, and Figure B.11 plot estimated coefficients along with
Figure B.3: \( k = 1 \) Panel Coefficients for All 51 Currencies Using Monthly Data and “Combine” Method

95% confidence bands from panel regression of equations (2.5), (2.6), and (2.7) for the G7 countries when the period step is 3, 6, and 12 months, respectively.

Patterns of excess returns are robust to varying period steps. As expected, plots become smoother as the period step gets longer. The reversal in the sign of \( \beta \) remains but loses some statistical significance when \( k = 6 \) and 12.
Figure B.4: $k = 1$ Panel Coefficients for G7 Using Daily Data and "Combine" Method
Figure B.5: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "WM" Method
Figure B.6: \( k = 1 \) Panel Coefficients for G7 Using Monthly Data and "BBG" Method
Figure B.7: $k = 1$ Panel Coefficients for G7 Using Monthly Data and “TR” Method
Figure B.8: $k = 1$ Panel Coefficients for G7 Using Monthly Data and “i” Method
Figure B.9: $k = 3$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method
Figure B.10: $k = 6$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method
Figure B.11: $k = 12$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method
B.3 Empirical Evidence Appendix

B.3.1 The AR(1) Assumption of Interest Rate Differentials

Table B.1: The Unit Root Test and Optimal Order of the Autoregressive Model for Each Country’s 1-month Interest Rate against the US Rate

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<th>Country</th>
<th>P-Value</th>
<th>Order</th>
<th>Country Code</th>
<th>Country</th>
<th>P-Value</th>
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</table>

The "i" method provides no available interest rate data for 4 countries: Egypt, Ireland, Mexico, and Uruguay. Most of the remaining countries have the Augmented Dickey-Fuller test’s p-value that is less than 0.05.
B.3.2 Implied Interest Rate Differentials

Section 2.5 in the paper displays the results using the 3-month interest rate differentials data from the "i" method. This section replicates the analysis using the constructed interest rate differentials data from the "Combine" method.

Survey-Expected Excess Returns

Table B.2 replicates Table 2.2 in the main paper.

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</thead>
<tbody>
<tr>
<td>$\rho_{t+h}$</td>
<td>$E_t^r(\rho_{t+h})$</td>
<td>$E_t^r(\rho_{t+h})$</td>
<td>$E_t^r(\rho_{t+h})$</td>
<td>$E_t^r(\rho_{t+h})$</td>
</tr>
<tr>
<td>$x_t$</td>
<td>-0.573***</td>
<td>-1.832***</td>
<td>-1.197*</td>
<td>-0.555***</td>
</tr>
<tr>
<td></td>
<td>(-6.59)</td>
<td>(-3.73)</td>
<td>(-2.36)</td>
<td>(-6.59)</td>
</tr>
<tr>
<td>constant</td>
<td>0.0022</td>
<td>0.0012</td>
<td>0.0020</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.44)</td>
<td>(0.58)</td>
<td>(0.31)</td>
</tr>
<tr>
<td>$N$</td>
<td>5075</td>
<td>989</td>
<td>3120</td>
<td>1955</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0490</td>
<td>0.0273</td>
<td>0.0136</td>
<td>0.0853</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Survey-expected excess returns point to less severe UIP deviations in the G7 and the rich-country samples but more severe deviations in the poor-country sample.

Survey Exchange Rates and Equilibrium Exchange Rates

The analysis in this section is crucial in establishing extrapolative beliefs among investors. Controlling for fundamentals, I test whether past depreciations have any additional effect on survey forecasts and equilibrium exchange rates.

Table B.3 and Table B.4 replicate Table 2.3 and Table 2.4, respectively.

Using the interest rate data from the "Combine" method yields similar results as in the main paper. Investors expect currencies to generally be stronger when their interest rates are higher except in the poor-country sample when the signaling channel confounds the results.

Coefficients in front of past depreciations are significantly positive. Past depreciations
Table B.3: Survey Exchange Rate when the Period Step is 3 Months ($h = 3$)

<table>
<thead>
<tr>
<th></th>
<th>All $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>G7 $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>Rich $\mathbb{E}<em>t(s</em>{t+h})$</th>
<th>Poor $\mathbb{E}<em>t(s</em>{t+h})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>0.626 (0.92) 0.333 (0.45)</td>
<td>-8.998*** (-5.30) -8.425*** (-5.16)</td>
<td>-3.441* (-2.02) -3.066 (-1.92)</td>
<td>0.948 (1.20) 0.613 (0.73)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.580*** (3.54)</td>
<td>0.484*** (4.34)</td>
<td>0.547*** (4.07)</td>
<td>0.564* (2.39)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.343*** (-110.33)</td>
<td>-1.100*** (-83.94)</td>
<td>-1.680*** (-110.50)</td>
<td>-3.412*** (-88.82)</td>
</tr>
<tr>
<td></td>
<td>0.431 0.145 (0.65) (0.20)</td>
<td>-8.665*** (-5.23) -8.135*** (-5.10)</td>
<td>-3.806* (-2.21) -3.453* (-2.12)</td>
<td>0.764 (0.98) 0.429 (0.51)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.565*** (3.52)</td>
<td>0.446*** (3.92)</td>
<td>0.514*** (3.78)</td>
<td>0.564* (2.48)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.341*** (-116.37)</td>
<td>-1.099*** (-83.76)</td>
<td>-1.680*** (-108.17)</td>
<td>-3.405*** (-95.96)</td>
</tr>
<tr>
<td>$s_{t}$</td>
<td>0.0020 0.0292</td>
<td>0.0821 0.112</td>
<td>0.0144 0.0544</td>
<td>0.0060 0.0248</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0020 0.0292</td>
<td>0.0821 0.112</td>
<td>0.0144 0.0544</td>
<td>0.0060 0.0248</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table B.4: Equilibrium Exchange Rate when the Period Step is 3 Months ($h = 3$)

<table>
<thead>
<tr>
<th></th>
<th>All $s_t$</th>
<th>G7 $s_t$</th>
<th>Rich $s_t$</th>
<th>Poor $s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>0.431 0.145 (0.65) (0.20)</td>
<td>-8.665*** (-5.23) -8.135*** (-5.10)</td>
<td>-3.806* (-2.21) -3.453* (-2.12)</td>
<td>0.764 (0.98) 0.429 (0.51)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.565*** (3.52)</td>
<td>0.446*** (3.92)</td>
<td>0.514*** (3.78)</td>
<td>0.564* (2.48)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.341*** (-116.37)</td>
<td>-1.099*** (-83.76)</td>
<td>-1.680*** (-108.17)</td>
<td>-3.405*** (-95.96)</td>
</tr>
<tr>
<td>$s_{t}$</td>
<td>0.0011 0.0308</td>
<td>0.0771 0.103</td>
<td>0.0173 0.0517</td>
<td>0.0048 0.0282</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0011 0.0308</td>
<td>0.0771 0.103</td>
<td>0.0173 0.0517</td>
<td>0.0048 0.0282</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

lead investors to have more pessimistic forecasts with estimated effects in the same ballpark as in the main paper. That is, controlling for interest rate differentials, a 1% past depreciation leads to between 0.48% to 0.58% lower level forecasts.

Effects of interest rate differentials and past depreciations on equilibrium exchange rates are similar to those on survey rates. Estimated coefficients from Table B.3 and Table B.4
have similar magnitudes. Controlling for interest rate differentials, a 1% past depreciation leads to between 0.45% to 0.57% lower current-period equilibrium exchange rates.

The above analysis confirms that the results in Section 2.5 are robust to different sources of interest rate data. Using implied interest differentials supports that currency investors hold extrapolative beliefs.

B.3.3 Varying Period Step

This section explores whether survey data yields consistent evidence across different period steps.

Section 2.2 illustrates results when the period step is set to 1 month, while Section 2.5 switches to use the period step of 3 months due to the limited data availability for the 1-month forecasts.¹

Excess Returns

Table B.5, Table B.6, and Table B.7 compare realized excess returns with survey-expected excess returns when the time steps are 1, 6, and 12 months, respectively.

**Table B.5**: Excess Returns when the Period Step is 1 Month (h = 1)

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>G7</th>
<th>Rich</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ρᵣₜ₊ₜ</td>
<td>Eᵣₜ₊ₜ(ρᵣₜ₊ₜ)</td>
<td>ρᵣₜ₊ₜ</td>
<td>Eᵣₜ₊ₜ(ρᵣₜ₊ₜ)</td>
</tr>
<tr>
<td>x₁</td>
<td>-0.0965</td>
<td>-0.974***</td>
<td>12.63</td>
<td>-3.508**</td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(-3.88)</td>
<td>(2.01)</td>
<td>(-3.02)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.0021</td>
<td>-0.0035**</td>
<td>-0.0019</td>
<td>-0.0019**</td>
</tr>
<tr>
<td></td>
<td>(-0.71)</td>
<td>(-3.29)</td>
<td>(-0.64)</td>
<td>(-2.94)</td>
</tr>
<tr>
<td>N</td>
<td>2141</td>
<td>2141</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0000</td>
<td>0.0089</td>
<td>0.0356</td>
<td>0.0226</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: July 2008 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * p < 0.05, ** p < 0.01, *** p < 0.001.

¹FX4casts.com starts collecting the 1-month forecasts only in July 2008, roughly 22 years after the earliest observations for the 3-, 6-, and 12-month forecasts.
<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>G7</th>
<th>Rich</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{t+h}$</td>
<td>$E_t^r(\rho_{t+h})$</td>
<td>$\rho_{t+h}$</td>
<td>$E_t^r(\rho_{t+h})$</td>
</tr>
<tr>
<td>$x_t$</td>
<td>-1.330***</td>
<td>-0.482***</td>
<td>-1.458*</td>
<td>-0.356</td>
</tr>
<tr>
<td></td>
<td>(-5.64)</td>
<td>(-3.61)</td>
<td>(-2.27)</td>
<td>(-1.53)</td>
</tr>
<tr>
<td>constant</td>
<td>-0.0032</td>
<td>0.0036</td>
<td>0.0027</td>
<td>-0.0051*</td>
</tr>
<tr>
<td></td>
<td>(-0.43)</td>
<td>(1.42)</td>
<td>(0.48)</td>
<td>(-2.13)</td>
</tr>
<tr>
<td>$N$</td>
<td>4080</td>
<td>4080</td>
<td>916</td>
<td>916</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0585</td>
<td>0.0627</td>
<td>0.0289</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The predictability of excess returns in both the 6- and 12-month samples shares the same patterns as in the 3-month sample. The standard UIP puzzle is recovered with more comprehensive data. There is no survey-expected excess returns in the G7 and the rich-country samples. In the poor-country samples, investors expect positive excess returns in holding higher interest currencies. The magnitude of excess returns is lower in the survey.

coefficients using realized excess returns. That is, the standard UIP puzzle is absent. Survey-expected returns, on the other hand, suggest that holding higher interest bonds yields significantly positive excess returns over holding lower interest bonds. Coefficients in front of interest differentials are significantly negative in all samples.
than in the realized data. Investors are aware that high interest currencies should depreciate over the next period, and deviations from the UIP are mitigated in the survey data.

**Survey Exchange Rates**

Again, this section attempts to understand how investors form forecasts. I test whether investors incorporate past depreciations into their expectations.

Table B.8, Table B.9, and Table B.10 examine potential auxiliary effects of past depreciations on expected levels when the period steps are 1, 6, and 12 months respectively.

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>G7</th>
<th>Rich</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathbb{E}<em>t^c(s</em>{t+h})$</td>
<td>$\mathbb{E}<em>t^c(s</em>{t+h})$</td>
<td>$\mathbb{E}<em>t^c(s</em>{t+h})$</td>
<td>$\mathbb{E}<em>t^c(s</em>{t+h})$</td>
</tr>
<tr>
<td>$x_t$</td>
<td>35.08*** (5.55)</td>
<td>-133.2*** (-4.08)</td>
<td>-134.1*** (-4.20)</td>
<td>-82.72*** (-4.48)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>34.47*** (5.41)</td>
<td>-134.06*** (-4.20)</td>
<td>-82.06*** (-4.48)</td>
<td>45.88*** (6.38)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>45.12*** (6.19)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.489* (2.09)</td>
<td>0.509* (2.54)</td>
<td>0.311 (1.86)</td>
<td>0.487 (1.53)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.527*** (-95.20)</td>
<td>-1.019*** (-67.28)</td>
<td>-1.769*** (-122.89)</td>
<td>-3.372*** (-96.66)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0813 0.0888 0.319 0.337 0.269 0.278 0.148 0.153</td>
<td>0.148 0.153</td>
<td>0.148 0.153</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Investors incorporate fundamentals into the exchange calculation in the expected way for the G7 and the rich-country samples. The estimated coefficients are most negative in the 1-month samples and increase monotonically to close to zero as the period step lengths.

The reduction in absolute magnitudes with the length of period step arises naturally. An AR(1) structure implies that the 3-month autoregressive coefficient is roughly the 1-month coefficient to the power of 3. For stationary processes, the 1-month coefficient is less than 1. Any positive integer power of a number less than 1 is declining with the size of the power.

Coefficients in front of interest differentials have the opposite sign in the poor-country sample. They are all positive, but only that of the 1-month is significantly so. This suggests
Table B.9: **Survey Exchange Rates when the Period Step is 6 Months** (h = 6)

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>G7</th>
<th>Rich</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
</tr>
<tr>
<td>$x_t$</td>
<td>-0.217 (-0.35)</td>
<td>-3.290** (-2.61)</td>
<td>-3.254* (-2.17)</td>
<td>0.811 (1.24)</td>
</tr>
<tr>
<td></td>
<td>-0.0724 (-0.12)</td>
<td>-2.822* (-2.30)</td>
<td>-2.613 (-1.81)</td>
<td>0.712 (1.19)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.504** (2.76)</td>
<td>0.354** (3.27)</td>
<td>0.393** (2.87)</td>
<td>0.558 (1.97)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.376*** (-125.86)</td>
<td>-1.229*** (-84.21)</td>
<td>-1.504*** (-101.29)</td>
<td>-3.926*** (-108.38)</td>
</tr>
<tr>
<td></td>
<td>0.0962 (0.23)</td>
<td>-1.207 (-1.89)</td>
<td>-1.019 (-1.73)</td>
<td>-1.216 (-1.49)</td>
</tr>
<tr>
<td></td>
<td>0.197 (0.57)</td>
<td>-1.019 (-1.73)</td>
<td>-0.959 (-1.49)</td>
<td>0.805* (2.17)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.584*** (4.37)</td>
<td>0.416*** (6.09)</td>
<td>0.371*** (3.68)</td>
<td>0.800*** (3.80)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.314*** (-125.54)</td>
<td>-1.223*** (-82.15)</td>
<td>-1.454*** (-104.54)</td>
<td>-3.901*** (-111.37)</td>
</tr>
<tr>
<td>$N$</td>
<td>4053</td>
<td>3941</td>
<td>911</td>
<td>1367</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0003</td>
<td>0.0426</td>
<td>0.0359</td>
<td>0.0146</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * p < 0.05, ** p < 0.01, *** p < 0.001.

Table B.10: **Survey Exchange Rates when the Period Step is 12 Months** (h = 12)

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>G7</th>
<th>Rich</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
<td>$E_t^f(s_{t+h})$</td>
</tr>
<tr>
<td>$x_t$</td>
<td>0.0962 (0.23)</td>
<td>-1.207 (-1.89)</td>
<td>-1.216 (-1.79)</td>
<td>0.902 (1.85)</td>
</tr>
<tr>
<td></td>
<td>0.197 (0.57)</td>
<td>-1.019 (-1.73)</td>
<td>-0.959 (-1.49)</td>
<td>0.805* (2.17)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.584*** (4.37)</td>
<td>0.416*** (6.09)</td>
<td>0.371*** (3.68)</td>
<td>0.800*** (3.80)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.314*** (-125.54)</td>
<td>-1.223*** (-82.15)</td>
<td>-1.454*** (-104.54)</td>
<td>-3.901*** (-111.37)</td>
</tr>
<tr>
<td>$N$</td>
<td>3941</td>
<td>3941</td>
<td>887</td>
<td>1367</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0002</td>
<td>0.0243</td>
<td>0.0225</td>
<td>0.0146</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * p < 0.05, ** p < 0.01, *** p < 0.001.

that only shorter rates are used as a signal for the inflation management problem.

Significantly positive estimated coefficients in front of past depreciations suggest that investors indeed extrapolate.

**Equilibrium Exchange Rates**

Table B.11, Table B.12, and Table B.13 display the predictive power of interest differentials
and past depreciations on realized exchange rates when the period steps are 1, 6, and 12 months, accordingly.

Table B.11: Equilibrium Exchange Rates when the Period Step is 1 Month ($h = 1$)

<table>
<thead>
<tr>
<th></th>
<th>All $s_t$</th>
<th>G7 $s_t$</th>
<th>Rich $s_t$</th>
<th>Poor $s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>12.98*</td>
<td>12.91*</td>
<td>-21.43**</td>
<td>-20.91**</td>
</tr>
<tr>
<td></td>
<td>(2.00)</td>
<td>(2.02)</td>
<td>(-3.05)</td>
<td>(-2.99)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.627**</td>
<td>0.550**</td>
<td>0.468*</td>
<td>0.638*</td>
</tr>
<tr>
<td></td>
<td>(3.20)</td>
<td>(3.22)</td>
<td>(2.51)</td>
<td>(2.40)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.279***</td>
<td>-2.279***</td>
<td>-1.235***</td>
<td>-1.235***</td>
</tr>
<tr>
<td></td>
<td>(-105.54)</td>
<td>(-107.12)</td>
<td>(-85.54)</td>
<td>(-85.92)</td>
</tr>
<tr>
<td>$N$</td>
<td>4467</td>
<td>4467</td>
<td>928</td>
<td>928</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0227</td>
<td>0.0322</td>
<td>0.0473</td>
<td>0.0588</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: July 2008 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table B.12: Equilibrium Exchange Rates when the Period Step is 6 Months ($h = 6$)

<table>
<thead>
<tr>
<th></th>
<th>All $s_t$</th>
<th>G7 $s_t$</th>
<th>Rich $s_t$</th>
<th>Poor $s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>-0.570</td>
<td>-0.435</td>
<td>-3.858**</td>
<td>-3.403**</td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td>(-0.76)</td>
<td>(-3.23)</td>
<td>(-2.92)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.470**</td>
<td>0.344**</td>
<td>0.354*</td>
<td>0.519</td>
</tr>
<tr>
<td></td>
<td>(2.69)</td>
<td>(3.23)</td>
<td>(2.49)</td>
<td>(1.95)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.377***</td>
<td>-2.375***</td>
<td>-1.223***</td>
<td>-1.224***</td>
</tr>
<tr>
<td></td>
<td>(-130.52)</td>
<td>(-136.11)</td>
<td>(-83.79)</td>
<td>(-85.63)</td>
</tr>
<tr>
<td>$N$</td>
<td>4053</td>
<td>4053</td>
<td>911</td>
<td>911</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0018</td>
<td>0.0558</td>
<td>0.0561</td>
<td>0.0934</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Results on equilibrium exchange rates share almost exactly same patterns as results on survey forecasts. The estimated coefficients in front of interest differentials are significantly negative in all the G7 and the rich-country samples. The estimates' magnitude declines
Table B.13: Equilibrium Exchange Rates when the Period Step is 12 Months ($h = 12$)

<table>
<thead>
<tr>
<th></th>
<th>All $s_t$</th>
<th>G7 $s_t$</th>
<th>Rich $s_t$</th>
<th>Poor $s_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>-0.401</td>
<td>-0.303</td>
<td>-2.006***</td>
<td>-1.797**</td>
</tr>
<tr>
<td></td>
<td>(-1.02)</td>
<td>(-0.90)</td>
<td>(-3.36)</td>
<td>(-3.31)</td>
</tr>
<tr>
<td>$s_{t-h} - s_{t-2h}$</td>
<td>0.572***</td>
<td>0.462***</td>
<td>0.395***</td>
<td>0.739***</td>
</tr>
<tr>
<td></td>
<td>(4.62)</td>
<td>(6.38)</td>
<td>(3.53)</td>
<td>(3.77)</td>
</tr>
<tr>
<td>constant</td>
<td>-2.320***</td>
<td>-2.316***</td>
<td>-1.214***</td>
<td>-1.213***</td>
</tr>
<tr>
<td></td>
<td>(-132.25)</td>
<td>(-161.74)</td>
<td>(-80.52)</td>
<td>(-87.17)</td>
</tr>
<tr>
<td></td>
<td>-1.457***</td>
<td>-1.458***</td>
<td>(-95.83)</td>
<td>(-100.52)</td>
</tr>
<tr>
<td></td>
<td>(-121.55)</td>
<td>(-167.73)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>3941</td>
<td>3941</td>
<td>887</td>
<td>887</td>
</tr>
<tr>
<td>Adj. R-Square</td>
<td>0.0026</td>
<td>0.158</td>
<td>0.0611</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0597</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0098</td>
<td>0.248</td>
</tr>
</tbody>
</table>

Notes: Monthly Data: August 1986 - June 2017. "i" Method. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

with the length of the period step. As before, the estimated coefficients in front of interest differentials are positive in all the poor-country samples. These results point to the perceived inflation risk among developing countries.

Across all samples, past depreciations affect equilibrium exchange rates even after controlling for interest differentials. Investors extrapolate in a way that past depreciations weaken realized exchange rates.

B.4 Mathematical Proofs

B.4.1 Proof of Proposition 11

Proof.
∀k ≥ 0,

\[ s_t = \mathbb{E}_t^X (s_{t+1}) - x_t \]
\[ = \mathbb{E}_t^F (s_{t+1}) + \gamma (s_{t-1} - s_{t-2}) - x_t \]
\[ = \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \sum_{h=0}^{\infty} \mathbb{E}_t (x_{t+1+h}) + \gamma (s_{t-1} - s_{t-2}) - x_t \]
\[ = \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \frac{x_t}{1 - \lambda} + \gamma (s_{t-1} - s_{t-2}) \]

The first and second equalities follow from the market clearing condition (2.16) and the relationship between extrapolative and fundamental beliefs (2.14) respectively. The third equality is a direct result of the definition of fundamental exchange rates from the equation (2.13). The last equality uses the AR(1) assumption of interest differentials.

**B.4.2 Proof of Proposition 12**

**Proof.** Using the time-series lag operator $L$ to rewrite the equilibrium exchange rate equation (2.18) yields the first equality as per below.

\[ s_t = \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \frac{x_t}{1 - \lambda} + \gamma (Ls_t - L^2 s_t) \]
\[ (1 - \gamma (L - L^2)) s_t = \bar{s}_t - \frac{x_t}{1 - \lambda} \]
\[ s_t = \frac{1}{1 - \gamma (L - L^2)} \left( \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \frac{x_t}{1 - \lambda} \right) \]
\[ = (1 + \gamma (L - L^2) + \gamma^2 (L - L^2)^2 + \ldots) \left( \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \frac{x_t}{1 - \lambda} \right) \]
\[ \text{cov}(s_t, x_t) = \text{cov}((1 + \gamma (L - L^2) + \gamma^2 (L - L^2)^2 + \ldots) \left( \lim_{T \to \infty} \mathbb{E}_t (s_{t+T}) - \frac{x_t}{1 - \lambda} \right), x_t) \]

The stationary assumption of exchange rates implies that $\text{cov}(\lim_{T \to \infty} \mathbb{E}_t (s_{t+T}), x_t) = 0$
 resulting in:

\[
\text{cov}(s_t, x_t) = \text{cov}\left((1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...)(-\frac{x_t}{1 - \lambda}), x_t\right)
\]

\[
= -\text{cov}(x_t + \gamma(x_{t-1} - x_{t-2}) + \gamma^2((x_{t-2} - x_{t-3}) - (x_{t-3} - x_{t-4})) + ..., x_t)
\]

\[
= -\frac{1 + \gamma \lambda (1 - \lambda) + \gamma^2 \lambda^2(1 - \lambda)^2 + ...}{1 - \lambda} \text{var}(x_t)
\]

\[
= -\frac{1}{1 - \lambda} \cdot \frac{1}{1 - \gamma \lambda (1 - \lambda)} \text{var}(x_t)
\]

\[
\leq -\frac{1}{1 - \lambda} \text{var}(x_t)
\]

\[
= \text{cov}(\frac{-x_t}{1 - \lambda}, x_t)
\]

\[
= \text{cov}(\lim_{T \to \infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1 - \lambda}, x_t)
\]

\[
= \text{cov}(s_t^{\text{UIP}}, x_t).
\]

The above infinite geometric series has a finite sum only if \(\gamma \lambda (1 - \lambda) < 1\). The inequality holds with equality whenever \(\gamma = 0\), i.e. when investors have no behavioral bias.

Below shows the relationship between the covariance inequality and the exchange level compared to the UIP-implied level.

\[
\text{cov}(s_t, x_t) \leq \text{cov}(s_t^{\text{UIP}}, x_t)
\]

\[
\text{cov}(\mathbb{E}_t(s_{t+T}) - s_t - \sum_{k=1}^{\infty}(x_{t+k-1}, x_t) \geq \text{cov}(\mathbb{E}_t(s_{t+T}) - s_t^{\text{UIP}} - \sum_{k=1}^{\infty}(x_{t+k-1}, x_t)
\]

\[
\frac{\text{cov}(\sum_{k=1}^{\infty} \mathbb{E}_t(s_{t+k} - s_{t+k-1} - x_{t+k-1}), x_t)}{\text{var}(x_t)} \geq \frac{\text{cov}(\sum_{k=1}^{\infty} \mathbb{E}_t(s_{t+k}^{\text{UIP}} - s_{t+k-1}^{\text{UIP}} - x_{t+k-1}), x_t)}{\text{var}(x_t)}
\]

\[
\sum_{k=1}^{\infty} \frac{\text{cov}(\mathbb{E}_t(\rho_k), x_t)}{\text{var}(x_t)} \geq 0
\]

\[
\sum_{k=1}^{\infty} \beta_k \geq 0
\]

The covariance inequality indicates the sign of the sum \(\sum_{k=1}^{\infty} \beta_k\), which in turn relates
the equilibrium exchange rate ($s_t$) with the level implied by the UIP ($s_t^{UIP}$).

\[ \]

**B.4.3 Proof of Proposition 13**

**Proof.**

From Proposition 12, \( \frac{\text{cov}(s_t, x_t)}{\text{var}(x_t)} = \frac{1}{1-\lambda} \cdot \frac{1}{1-\gamma\lambda(1-\lambda)} \). Similarly, calculate \( \frac{\text{cov}(\mathbb{E}_t(s_{t+1}), x_t)}{\text{var}(x_t)} \).

\[ s_{t+1} = \mathbb{E}_{t+1}^X(s_{t+1}) \]
\[ = \mathbb{E}_{t+1}^F(s_{t+1}) + \gamma(s_t - s_{t-1}) \]
\[ = s_{t+1} - \frac{x_{t+1}}{1-\lambda} + \gamma(s_t - s_{t-1}) \]

\[ (1 - \gamma(L - L^2))s_{t+1} = s_{t+1} - \frac{x_{t+1}}{1-\lambda} \]

\[ s_{t+1} = \frac{1}{1-\gamma(L-L^2)}(s_{t+1} - \frac{x_{t+1}}{1-\lambda}) \]

\[ = (1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + \ldots)(s_{t+1} - \frac{x_{t+1}}{1-\lambda}) \]

\[ \text{cov}(\mathbb{E}_t(s_{t+1}), x_t) = - \frac{\text{cov}(\mathbb{E}_t[(1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + \ldots)x_{t+1}], x_t)}{1-\lambda} \]

\[ = - \frac{\text{cov}(\mathbb{E}_t[x_{t+1} + \gamma(x_t - x_{t-1}) + \gamma^2((x_{t-1} - x_{t-2}) - (x_{t-2} - x_{t-3})) + \ldots], x_t)}{1-\lambda} \]

\[ = \left[ -\lambda + \gamma(1-\lambda) + \gamma^2\lambda(1-\lambda)^2 + \ldots \right] \frac{1}{1-\lambda} \text{var}(x_t) \]

\[ \frac{\text{cov}(\mathbb{E}_t(s_{t+1}), x_t)}{\text{var}(x_t)} = - \frac{1}{1-\lambda} \left( \lambda + \frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)} \right) \]

\[ \frac{\text{cov}(\mathbb{E}_t(s_{t+1}) - s_t - x_t, x_t)}{\text{var}(x_t)} = - \frac{1}{1-\lambda} \left( \lambda + \frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)} \right) + \frac{1}{1-\lambda} \cdot \frac{1}{1-\gamma\lambda(1-\lambda)} - 1 \]

\[ \frac{\text{cov}(\mathbb{E}_t(x_1), x_t)}{\text{var}(x_t)} = - \frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)} \]

When \( 0 < \gamma < \frac{1}{\lambda(1-\lambda)} \), \( \beta_1 = \frac{\text{cov}(\mathbb{E}_t(x_1), x_t)}{\text{var}(x_t)} < 0 \). If \( \gamma = 0 \), \( \beta_1 = 0 \), and UIP holds.

\[ \]

**B.4.4 Proof of Proposition 14**

**Proof.**
For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $\sum_{k=1}^{\infty} \beta_k \geq 0$ and $\beta_1 < 0$, therefore there exists $h \geq 2$ such that $\beta_h > 0$.

For $\gamma = 0$, $s_t = s_t - \frac{s_t}{1-\lambda}$, therefore for all $k \geq 1$

$$
\beta_k = \frac{\text{cov}(\mathbb{E}_t(s_{t+k} - s_{t+k-1} - x_{t+k-1}), x_t)}{\text{var}(x_t)}
= \frac{\text{cov}(s_{t+k} - s_{t+k-1} + \frac{\lambda^{t+k-1}x_t - \lambda^{t+k}x_t}{1-\lambda} - \lambda^{t+k-1}x_t, x_t)}{\text{var}(x_t)}
= 0
$$

B.4.5 Proof of Proposition 15

Proof.

I first calculate $\frac{\text{cov}(s_{t-1}, x_t)}{\text{var}(x_t)}$.

$$
\frac{\text{cov}(s_{t-1}, x_t)}{\text{var}(x_t)} = \frac{\text{cov}((1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...)(\bar{s}_{t-1} - \frac{x_t}{1-\lambda}), x_t)}{\text{var}(x_t)}
= -\frac{\lambda + \lambda \gamma \lambda (1-\lambda) + \lambda \gamma^2 (\lambda(1-\lambda))^2 + ...}{1-\lambda}
= -\frac{\lambda}{(1-\lambda)(1-\gamma \lambda (1-\lambda))}
$$
Define $A_k = \frac{\text{cov}(E_t(s_{t+k} - s_{t+k-1}), x_t)}{\text{var}(x_t)}$ and $B_k = \frac{\text{cov}(E_t(s_{t+k} - s_{t+k-1} - x_{t+k-1}, x_t))}{\text{var}(x_t)}$.

$A_0 = \frac{\text{cov}(s_t - s_{t-1}, x_t)}{\text{var}(x_t)}$

$= \frac{\text{cov}(s_t, x_t) - \text{cov}(s_{t-1}, x_t)}{\text{var}(x_t)}$

$= -\frac{1}{\text{var}(x_t)}$

$= 1 - \gamma \lambda (1 - \lambda)$

$B_0 = A_0 - \frac{\text{cov}(x_{t-1}, x_t)}{\text{var}(x_t)}$

$= A_0 - \lambda$

$= -1 - \lambda + \gamma \lambda^2 - \gamma \lambda^3$

$= \frac{1 - \gamma \lambda (1 - \lambda)}{\text{var}(x_t)}$

$A_1 = \frac{\text{cov}(E_t(s_{t+1}) - s_t, x_t)}{\text{var}(x_t)}$

$= \frac{\text{cov}(E_t(s_{t+1}) - s_t, x_t) - \text{cov}(s_t, x_t)}{\text{var}(x_t)}$

$= \frac{1 - \gamma + \gamma \lambda^2}{1 - \gamma \lambda (1 - \lambda)}$

$B_1 = A_1 - \frac{\text{cov}(x_t, x_t)}{\text{var}(x_t)}$

$= A_1 - 1$

$= \frac{-\gamma (1 - \lambda)}{1 - \gamma \lambda (1 - \lambda)}$
For \( k \geq 2 \),

\[
\begin{align*}
    s_{t+k} &= s_{t+k} - \frac{x_{t+k}}{1-\lambda} + \gamma(s_{t+k-1} - s_{t+k-2}) \\
    s_{t+k-1} &= s_{t+k-1} - \frac{x_{t+k-1}}{1-\lambda} + \gamma(s_{t+k-2} - s_{t+k-3}) \\
    \mathbb{E}_t(s_{t+k} - s_{t+k-1}) &= s_{t+k} - s_{t+k-1} + \frac{\mathbb{E}_t(x_{t+k-1} - x_{t+k})}{1-\lambda} \\
    &\quad+ \gamma[(\mathbb{E}_t(s_{t+k-1} - s_{t+k-2})) - (\mathbb{E}_t(s_{t+k-2} - s_{t+k-3}))] \\
    &= s_{t+k} - s_{t+k-1} + \frac{\lambda^{-1}x_t - \lambda^2x_t}{1-\lambda} + \gamma[\mathbb{E}_t(s_{t+k-1} - s_{t+k-2}) - \mathbb{E}_t(s_{t+k-2} - s_{t+k-3})] \\

\end{align*}
\]

\[
\text{cov}(\mathbb{E}_t(s_{t+k} - s_{t+k-1}), x_t) = \lambda^{k-1}\text{var}(x_t) + \gamma(\text{cov}(\mathbb{E}_t(s_{t+k-1} - s_{t+k-2}), x_t) - \text{cov}(\mathbb{E}_t(s_{t+k-2} - s_{t+k-3}), x_t))
\]

\[
\begin{align*}
    A_k &= \lambda^{k-1} + \gamma(A_{k-1} - A_{k-2}) \\
    B_k &= A_k - \frac{\text{cov}(\mathbb{E}_t(x_{t+k-1}), x_t)}{\text{var}(x_t)} \\
    &= A_k - \lambda^{k-1} \\
    &= \gamma(A_{k-1} - A_{k-2})
\end{align*}
\]

**Lemma 24** \( \gamma \in [0, 1) \) is a sufficient condition for \( \lim_{k \to \infty} A_k = 0 \)

**Proof** From the recurrence relation \( A_k = \gamma(A_{k-1} - A_{k-2}) + \lambda^{k-1} \), I solve for the close-form solution of \( A_k \) using characteristic polynomials.

\[
\begin{align*}
    A_k &= \gamma(A_{k-1} - A_{k-2}) + \lambda^{k-1} \quad (\text{B.1}) \\
    A_{k+1} &= \gamma(A_k - A_{k-1}) + \lambda^k \quad (\text{B.2})
\end{align*}
\]

\[ (\text{B.2}) - \lambda (\text{B.1}) : A_{k+1} - \lambda A_k = \gamma(A_k - A_{k-1}) - \gamma \lambda (A_{k-1} - A_{k-2}) \]

\[
\begin{align*}
    x^3 - \lambda x^2 - \gamma x^2 + \gamma x + \gamma \lambda x - \gamma \lambda &= 0 \\
    (x - \lambda)(x^2 - \gamma x + \gamma) &= 0 \\
    x &= \lambda, \frac{\gamma \pm \sqrt{\gamma^2 - 4\gamma}}{2}
\end{align*}
\]

Write \( A_k \) in term of 3 roots with \( x_1 = \lambda, x_2 = \frac{\gamma + \sqrt{\gamma^2 - 4\gamma}}{2} \), and \( x_3 = \frac{\gamma - \sqrt{\gamma^2 - 4\gamma}}{2} \), i.e. \( A_k = ax_1^k + bx_2^k + cx_3^k \) for some constants \( a, b \) and \( c \).
It is sufficient to show that $\gamma \in [0, 1)$ implies $\|x_i\| < 1$ for all $i = 1, 2, \text{and} 3$ because $\lim_{k \to \infty} A_k = a \lim_{k \to \infty} x_1^k + b \lim_{k \to \infty} x_2^k + c \lim_{k \to \infty} x_3^k = 0$. Recall $\lambda \in [0, 1)$, so $\|x_1\| < 1$.

For $\gamma \in [0, 1)$, $\gamma^2 - 4\gamma \leq 0$. Therefore, $\sqrt{\gamma^2 - 4\gamma} = \sqrt{4\gamma - \gamma^2i}$.

$$\frac{\gamma + \sqrt{\gamma^2 - 4\gamma}}{2} = \frac{\gamma + \sqrt{4\gamma - \gamma^2i}}{2}$$

$$\frac{\gamma - \sqrt{\gamma^2 - 4\gamma}}{2} = \frac{\gamma - \sqrt{4\gamma - \gamma^2i}}{2}$$

Therefore $\|\frac{\gamma + \sqrt{\gamma^2 - 4\gamma}}{2}\| = \|\frac{\gamma - \sqrt{\gamma^2 - 4\gamma}}{2}\| = \sqrt{\frac{\gamma^2 + \gamma(4-\gamma)}{4}} = \sqrt{\frac{4\gamma}{4}} < 1$. That is, $\gamma \in [0, 1)$ is a sufficient condition for $\lim_{k \to \infty} A_k = 0$.

So, for $\gamma \in [0, 1)$, $\lim_{k \to \infty} \beta_k = \lim_{k \to \infty} B_k = \lim_{k \to \infty} (A_k - \lambda^{k-1}) = \lim_{k \to \infty} A_k = 0$.

\[\framebox{\textbf{B.4.6} Proof of Proposition 16} \]

\textbf{Proof.} First, recall from proposition 15 that $\frac{\text{cov}(s_{i+1}, x_i)}{\text{var}(x_i)} = -\frac{\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}$. Similarly, we can calculate $\frac{\text{cov}(s_{i-2}, x_i)}{\text{var}(x_i)}$.

$$\frac{\text{cov}(s_{i-2}, x_i)}{\text{var}(x_i)} = \frac{\text{cov}((1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...)(\xi_{i-2} - \frac{x_i + \gamma}{1-\lambda}), x_i)}{\text{var}(x_i)}$$

$$= -\frac{\lambda^2 + \lambda^2\gamma\lambda(1-\lambda) + \lambda^2\gamma^2\lambda^2(1-\lambda)^2 + ...}{1-\lambda}$$

$$= -\frac{\lambda^2}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}$$

Now, we have

$$\text{var}(s_i) = \text{var}(\xi_i - \frac{x_i}{1-\lambda} + \gamma(s_{i-1} - s_{i-2}))$$

$$= \frac{\text{var}(x_i)}{(1-\lambda)^2} + \gamma^2\text{var}(s_{i-1} - s_{i-2}) - \frac{\gamma}{1-\lambda}\text{cov}(s_{i-1} - s_{i-2}, x_i)$$

$$= \frac{\text{var}(x_i)}{(1-\lambda)^2} + \gamma^2\text{var}(s_{i-1} - s_{i-2}) - \frac{\gamma}{1-\lambda}[-\frac{\lambda}{(1-\lambda)(1-\lambda^2)} + \frac{\lambda^2}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}]\text{var}(x_i)$$

$$= \frac{\text{var}(x_i)}{(1-\lambda)^2} + \gamma^2\text{var}(s_{i-1} - s_{i-2}) + \frac{\gamma\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}\text{var}(x_i).$$

Whenever $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $\text{var}(s_i) > \frac{\text{var}(x_i)}{1-\lambda^2} = \text{var}(s_i^{\text{upid}})$. 

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B.5 The Stationary Assumption of Long-Run Nominal Exchange Rates

The baseline model displays results when nominal exchange rates are assumed to be stationary. This section shows that such assumption may be relaxed. In fact, it is clear from Appendix ?? that all proofs follow as long as the equilibrium exchange rate $s_t$ can be written as:

$$s_t = \lim_{T \to \infty} E_t(s_{t+T}) - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2})$$

$$= q_t - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2}),$$  \hspace{1cm} (B.3)

where $q_t$ is any process such that $\text{cov}(q_t, x_t) = 0$.

With the stationary assumption, $q_t$ is obviously uncorrelated with the contemporaneous exchange rate.

$\text{cov}(q_t, x_t) = 0$ is a sufficient, not a necessary, condition. In fact, the condition can be relaxed even further. All of our results follow as long as $\text{cov}(\lim_{T \to \infty} E_t(s_{t+T}), x_t) = \text{cov}(q_t, x_t)$ is less than $\frac{1}{1-\lambda}$. 

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Appendix C

Appendix to Chapter 3

C.1 Comparison across Different Methods of the Pricing of $T$-Maturity Assets

C.1.1 Different Constant Maturities of S&P 500 Index Dividend Yield over Time

This paper has shown that different methods yield roughly in-line 1-year S&P 500-related prices. This section focuses on S&P 500 dividend yields with 2-, 3-, and 5-year maturity.

According to Figure C.1, Figure C.2, and Figure C.3, different data constructing methods for 2-, 3-, and 5-year S&P 500 dividend points align roughly with one another. Data from Van Binsbergen et al. (2012) does not cover dividends longer than 2 years, and data from equity futures does not cover those longer than 3 years.

C.1.2 2-Year Dividend Yield for Different Equity Indices over Time

This section now explores beyond S&P 500 dividend yields and tests whether different pricing methods result in comparable 2-year asset prices for Euro Stoxx 50, Financial Time Stock Exchange 100, Nikkei 225, KOSPI 200, Hang Seng Index, and Hang Seng China Enterprises Index.

Moving beyond S&P 500, the most comprehensive construction in our paper is the use
of synthetic forwards. Dividend futures and swaps cover most of the indices except KOSPI 200.
Figure C.3: 5-Year Dividend Yield of SPX Index over Time

Figure C.4: 2-Year Dividend Yield of SX5E Index over Time

Figure C.4, Figure C.5, Figure C.6, Figure C.7, Figure C.8, and Figure C.9 show that, whenever data is available, different methods of construction lead to similar $T$-maturity
Figure C.5: 2-Year Dividend Yield of UKX Index over Time

Figure C.6: 2-Year Dividend Yield of NKY Index over Time

asset prices.
**Figure C.7:** 2-Year Dividend Yield of KOSPI2 Index over Time

**Figure C.8:** 2-Year Dividend Yield of HSI Index over Time
Figure C.9: 2-Year Dividend Yield of HSCEI Index over Time
C.2 *T*-Maturity Dividend Yields over Time for Different Equity Indices

![Graph](image)

**Figure C.10: T-Maturity Dividend Yield of FTSE 100**

Similar to SX5E index, the FTSE 100 dividend yield term structure stays inverted most of the time and is not prominently pro-cyclical.

Figure C.11 conveys that KOSPI 200 dividend yield term structure is inverted most of the time like those of European indices. There was a significant drop in dividend yields around 2012 - 2013, which most likely reflects the change in the dividend payout policy among Korean stocks.

From Figure C.12, the dividend yield term structure of Hang Seng Index used to be inverted up until around 2008 - 2009, after which the HSI yield term structure has the upward sloping shape throughout.

Figure C.13 reflects the dividend yield term structure of HSCEI index over time. HSCEI dividend prices were constructed from dividend futures/dividend swaps instead of synthetic forwards due to data limitability. We have that the HSCEI dividend yield term
structure is usually upward sloping with some periods of the backwardation.

In conclusion, the dividend yield term structure is dynamic. It varies with time and
Figure C.13: T-Maturity Dividend Yield of Hang Seng China Enterprises. Dividend Prices Constructed from Dividend Futures/Dividend Swaps

across different equity indices.
C.3 Cumulative Returns of $T$-Maturity Assets associated with Different Equity Indices

Similar to SX5E Index, the FTSE 100 term structure stays inverted throughout the period of our study as depicted in Figure C.14. Coincidentally, UKX Index is also a popular underlying for exotic structures.

Figure C.15 conveys that the KOSPI 200 risky term structure varies with time. There was also a significant drop in the finite-maturity asset prices during 2012 - 2013 due to a change in the dividend payout policy of Korean stocks.

Figure C.16 and Figure C.17 show that HSI-linked and HSCEI-linked assets have return patterns that vary over time.

Overall, cumulative returns across different equity indices over time confirm that the risky term structure is not static. Such term structure is sometimes upward sloping, while it is downward sloping in other periods.
Figure C.15: Cumulative Returns of KOSPI 200 T-Maturity Asset

Figure C.16: Cumulative Returns of Hang Seng T-Maturity Asset
Figure C.17: Cumulative Returns of Hang Seng China Enterprises T-Maturity Asset. Dividend Prices Constructed from Dividend Futures/Dividend Swaps
Table C.1: Summary Statistics of T-Maturity Asset Returns

<table>
<thead>
<tr>
<th>Maturity in Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>UKX (Apr 2004 - Jan 2017, NObs = 154)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.9068</td>
<td>1.2111</td>
<td>1.2112</td>
<td>0.8039</td>
<td>0.3705</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>3.8493</td>
<td>4.3114</td>
<td>5.1841</td>
<td>5.2137</td>
<td>3.7047</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.2356</td>
<td>0.2809</td>
<td>0.2337</td>
<td>0.1542</td>
<td>0.0990</td>
</tr>
<tr>
<td>KOSPI2 (Apr 2004 - Jan 2017, NObs = 154)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>1.0253</td>
<td>0.9974</td>
<td>1.0749</td>
<td>1.4144</td>
<td>0.7730</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>20.1911</td>
<td>13.8770</td>
<td>12.5844</td>
<td>14.0654</td>
<td>4.8757</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.0508</td>
<td>0.0788</td>
<td>0.0854</td>
<td>0.1006</td>
<td>0.1585</td>
</tr>
<tr>
<td>HSI (Nov 2004 - Jan 2017, NObs = 147)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.6719</td>
<td>0.3532</td>
<td>0.1982</td>
<td>0.3018</td>
<td>0.8219</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>8.7169</td>
<td>6.9001</td>
<td>6.9189</td>
<td>6.9367</td>
<td>6.1261</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.0771</td>
<td>0.0512</td>
<td>0.0287</td>
<td>0.0435</td>
<td>0.1342</td>
</tr>
<tr>
<td>HSCEI (Jan 2005 - Nov 2013, NObs = 107)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>1.2964</td>
<td>1.2203</td>
<td>1.2856</td>
<td>1.4242</td>
<td>1.2507</td>
</tr>
<tr>
<td>Stdev (%)</td>
<td>5.6395</td>
<td>5.5930</td>
<td>5.9390</td>
<td>6.6023</td>
<td>9.1358</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.2299</td>
<td>0.2182</td>
<td>0.2165</td>
<td>0.2157</td>
<td>0.1380</td>
</tr>
</tbody>
</table>

Notes: Monthly data from the synthetic forward method (except for HSCEI, which uses the dividend future and swap method). NObs represents the total number of observation.

From Table C.1, UKX assets with shorter maturity have higher mean returns with comparable standard deviations. This results in the downward-sloping Sharpe ratio. This fits well with the observation that FTSE 100 is one of the popular underlyings for structured product issuance.

KOSPI 200 shorter-maturity assets have high returns but are also highly volatile. The
resulting sharpe ratio is monotonically increasing with maturity. Hang Seng mean returns are monotonically increasing with maturity with comparable standard deviations. Therefore, the resulting sharpe ratio also increases with maturity. For Hang Send China Enterprises, returns are roughly similar for all maturities with lower standard deviations for shorter maturities. The sharpe ratio associated with HSCEI therefore has an inverted term structure.

Table C.2 is the analog of Table 3.3 in the main paper.

From Table C.2, beta is increasing in maturity for all UKX, KOSPI, HSI, and HSCEI Index. There is no excess alpha except for UKX, where alpha for 1-, 2-, 3-, and 5-year assets are significantly positive.
C.5 Sample Term Sheet for Autocallables
Worst-of European Barrier Quanto Autocallable Notes linked to a Basket of Equity Indices and ETFs

CAPITAL AT RISK

NO PUBLIC OFFERS PERMITTED. THE DOCUMENTATION FOR THE ISSUANCE OF THESE SECURITIES HAS NOT BEEN REVIEWED OR APPROVED BY ANY REGULATORY AUTHORITY, AND SUCH SECURITIES MAY NOT BE OFFERED, SOLD, OR RE-SOLD TO THE PUBLIC IN THE EUROPEAN ECONOMIC AREA ("EEA") OR ANY OTHER REGION. ANY OFFER, SALE OR RE-SALE OF THESE SECURITIES IN THE EEA OR ANY OTHER REGION MAY ONLY BE MADE PURSUANT TO AN EXEMPTION FROM THE REQUIREMENT TO PUBLISH A PROSPECTUS UNDER THE PROSPECTUS DIRECTIVE AND IN COMPLIANCE WITH ALL OTHER RELEVANT LAWS AND REGULATIONS.

FURTHER, THESE SECURITIES HAVE NOT BEEN AND WILL NOT BE REGISTERED UNDER THE U.S. SECURITIES ACT OF 1933, AS AMENDED, AND MAY NOT BE OFFERED, SOLD, RE-SOLD OR DELIVERED WITHIN THE UNITED STATES OR TO, OR FOR, THE BENEFIT OF, UNITED STATES PERSONS. THIS TERM SHEET MAY NOT BE DISTRIBUTED IN THE UNITED STATES.

SUMMARY TERMS

This Term Sheet is a non-binding summary of the economic terms and does not purport to be exhaustive. The binding terms and conditions will be set out in the pricing supplement which amends and supplements the terms and conditions in the offering circular. Investors must read all of these documents and copies are available from the Issuer and the issue and paying agent.

• The Risk Factors set out in the Offering Circular and this Term Sheet highlight some, but not all, of the risks of investing in this investment product.
• The Issuer makes no representations as to the suitability of this investment product for any particular investor nor as to the future performance of this investment product.
• Prior to making any investment decision, investors should satisfy themselves that they fully understand the risks relating to this investment product and seek professional advice as they deem necessary.

SUMMARY DESCRIPTION

The product is issued as Notes in USD and aims to pay conditional coupons on a periodic basis for the life of the Securities. Whether or not the coupons are payable will be determined based on the performance of each Basket Constituent, as described below. The Securities have an early redemption feature whereby, depending on the performance of each Basket Constituent which is evaluated on a periodic basis, the Securities may redeem early and Securityholders will receive 100% of the Calculation Amount in such circumstance.

If the Securities have not redeemed early, the amount payable at maturity for each Note (the "Redemption Amount") will be determined by reference to the price of the Worst Performing Basket Constituent on the Final Valuation Date. Therefore, the Redemption Amount will be either a cash amount equal to 100% of the Calculation Amount or a cash amount determined by reference to the performance of the Worst Performing Basket Constituent, as described below.

PRODUCT DETAILS

Issuer: Barclays Bank PLC ("Barclays")
Type of Security: Note
Issue Currency: United States Dollar ("USD")
Aggregate Nominal Amount: USD 2,000,000
Specified Denomination: USD 1,000
Minimum Tradable Amount: USD 1,000 (and USD 1,000 thereafter)

During the life of the Securities, there may be no sales or partial redemptions of Securities in amounts less than the Minimum Tradable Amount.

Calculation Amount per Security: USD 1,000
Issue Price: 100.00% of par

The Issue Price relates to the Securities the Issuer sells initially on the Trade Date. The Issuer may decide, after the Trade Date, to issue additional Securities that will become immediately fungible, when issued, with the Securities described in this Term Sheet. Any such securities may be sold at varying prices to be determined at the time of each sale, which may be at market prices prevailing, at prices related to such prevailing prices or at negotiated prices.

Trade Date: 12 August 2015
Issue Date: 19 August 2015
Redemption Date: 19 February 2019

Reference Assets: A basket comprised of 1 Equity Index and 1 ETF, each of which is set out in the Appendix (each, a "Basket Constituent" and together, the "Basket of Equities"). Any Basket Constituent stated as being an "Index" represents a notional investment in such index with a notional investment size of 1 Reference Asset Currency per index point. Any Basket Constituent stated as being an "ETF" is an Exchange Traded Fund, which is a "Share" for the purposes of this Security.

Settlement Method: Cash
Settlement Currency: USD

<table>
<thead>
<tr>
<th>INTEREST</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest (coupon(s))</td>
<td>Provided that a Specified Early Redemption Event has not occurred prior to the relevant Interest Valuation Date, as determined by the Determination Agent, in respect of the relevant Interest Payment Date:</td>
</tr>
<tr>
<td>(i) If the Valuation Price of each Basket Constituent on the relevant Interest Valuation Date is at or above its Interest Barrier:</td>
<td></td>
</tr>
<tr>
<td>2.00% x Calculation Amount; or</td>
<td></td>
</tr>
<tr>
<td>(ii) Otherwise, zero.</td>
<td></td>
</tr>
<tr>
<td>Where:</td>
<td></td>
</tr>
<tr>
<td>&quot;Interest Barrier&quot; means, in respect of a Basket Constituent, 70% of the Initial Price of that Basket Constituent, as specified in the Appendix.</td>
<td></td>
</tr>
<tr>
<td>&quot;Initial Price&quot; means, in respect of a Basket Constituent, the price of that Basket Constituent at the Valuation Time on the Initial Valuation Date as specified in the Appendix.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interest Valuation Dates</th>
<th>Each date set out in the table below in the column entitled “Interest Valuation Dates”.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Valuation Date(s)</td>
<td>Interest Payment Date(s)</td>
</tr>
<tr>
<td>12 November 2015</td>
<td>19 November 2015</td>
</tr>
<tr>
<td>12 February 2016</td>
<td>22 February 2016</td>
</tr>
<tr>
<td>12 May 2016</td>
<td>19 May 2016</td>
</tr>
<tr>
<td>12 August 2016</td>
<td>19 August 2016</td>
</tr>
<tr>
<td>14 November 2016</td>
<td>21 November 2016</td>
</tr>
<tr>
<td>13 February 2017</td>
<td>21 February 2017</td>
</tr>
<tr>
<td>12 May 2017</td>
<td>19 May 2017</td>
</tr>
<tr>
<td>14 August 2017</td>
<td>21 August 2017</td>
</tr>
<tr>
<td>13 November 2017</td>
<td>20 November 2017</td>
</tr>
<tr>
<td>12 February 2018</td>
<td>20 February 2018</td>
</tr>
<tr>
<td>14 May 2018</td>
<td>21 May 2018</td>
</tr>
<tr>
<td>13 August 2018</td>
<td>20 August 2018</td>
</tr>
<tr>
<td>12 November 2018</td>
<td>19 November 2018</td>
</tr>
<tr>
<td>11 February 2019</td>
<td>19 February 2019</td>
</tr>
</tbody>
</table>

| Interest Payment Dates | Each date set out in the table above in the column entitled “Interest Payment Dates”. |

<table>
<thead>
<tr>
<th>REDEMPTION</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Cash Settlement Amount</td>
<td>Provided that no event that may lead to the early redemption or termination of the Securities has occurred prior to the Redemption Date as determined by the Determination Agent, on the Redemption Date, each Security will be redeemed by the Issuer at a cash amount determined by the Determination Agent in accordance with the following:</td>
</tr>
<tr>
<td>(a) If, in respect of the Worst Performing Basket Constituent, the Valuation Price on the Final Valuation Date is at or above the relevant Knock-in Barrier Price, a cash amount equal to the Calculation Amount; or</td>
<td></td>
</tr>
</tbody>
</table>

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(b) If, in respect of the Worst Performing Basket Constituent, the Valuation Price on the Final Valuation Date is below the relevant Knock-In Barrier Price, a cash amount equal to the Calculation Amount multiplied by the Valuation Price of the Worst Performing Basket Constituent on the Final Valuation Date and divided by the Strike Price of the Worst Performing Basket Constituent.

Where:

"Knock-in Barrier Price" means, in respect of a Basket Constituent, 70% of the Initial Price of that Basket Constituent, as specified in the Appendix.

"Strike Price" means, in respect of a Basket Constituent, 100.00% of the Initial Price of that Basket Constituent as specified in the Appendix.

"Initial Price" means, in respect of a Basket Constituent, the price of that Basket Constituent at the Valuation Time on the Initial Valuation Date as specified in the Appendix.

"Initial Valuation Date" means 12 August 2015.

"Valuation Price" means, in respect of a Valuation Date and any relevant Scheduled Trading Day, the price of the Basket Constituent at the Valuation Time on such day, as determined by the Determination Agent.

"Final Valuation Date" means 11 February 2019.

"Valuation Time" means in respect of each Basket Constituent which is an Index the time at which the official level of the Index is calculated and published by the Index Sponsor; for all other Basket Constituents the time at which the official closing price of the Basket Constituent is published by the relevant Exchange.

"Worst Performing Basket Constituent" means the Basket Constituent with the lowest performance calculated as follows:

\[
\frac{V_{\text{Final}}(i)}{V_{\text{Initial}}(i)}
\]

Where:

\(V_{\text{Final}}(i)\) is the Valuation Price of Basket Constituent on the Final Valuation Date.

\(V_{\text{Initial}}(i)\) is the Initial Price of Basket Constituent.

Provided that where more than one Basket Constituent has the same lowest performance, the Determination Agent shall in its sole discretion select which of the Basket Constituents with the same lowest performance shall be the Worst Performing Basket Constituent.

**EARLY REDEMPTION FOLLOWING A SPECIFIED EARLY REDEMPTION EVENT**

<table>
<thead>
<tr>
<th>Specified Early Redemption Event</th>
<th>Applicable, and Automatic Early Redemption Applicable.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the Valuation Price of each Basket Constituent on any Autocall Valuation Date is at or above its respective Autocall Barrier, the Issuer shall notify the Securityholder upon the occurrence of such event and shall redeem all of the Securities (in whole only) early at the Specified Early Cash Settlement Amount on the Specified Early Cash Redemption Date.</td>
<td></td>
</tr>
<tr>
<td>Where:</td>
<td></td>
</tr>
<tr>
<td>&quot;Autocall Barrier&quot; means, in respect of a Basket Constituent, 100.00% of the Initial Price of that Basket Constituent, as specified in the Appendix.</td>
<td></td>
</tr>
<tr>
<td>&quot;Initial Price&quot; means, in respect of a Basket Constituent, the price of that Basket Constituent at the Valuation Time on the Initial Valuation Date as specified in the Appendix.</td>
<td></td>
</tr>
<tr>
<td>&quot;Autocall Valuation Date&quot; means each date set out in the table below in the column entitled &quot;Autocall Valuation Dates&quot;.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autocall Valuation Date(s)</th>
<th>Specified Early Cash Redemption Date(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 November 2015</td>
<td>19 November 2015</td>
</tr>
<tr>
<td>12 February 2016</td>
<td>22 February 2016</td>
</tr>
<tr>
<td>12 May 2016</td>
<td>19 May 2016</td>
</tr>
<tr>
<td>12 August 2016</td>
<td>19 August 2016</td>
</tr>
<tr>
<td>14 November 2016</td>
<td>21 November 2016</td>
</tr>
<tr>
<td>13 February 2017</td>
<td>21 February 2017</td>
</tr>
<tr>
<td>12 May 2017</td>
<td>19 May 2017</td>
</tr>
<tr>
<td>14 August 2017</td>
<td>21 August 2017</td>
</tr>
</tbody>
</table>
Specified Early Cash Settlement Amount

In respect of each Security, the Calculation Amount.

Specified Early Cash Redemption Date

Each date set out in the table above in the column entitled “Specified Early Cash Redemption Dates”.

Valuation Date

The Initial Valuation Date, Final Valuation Date, each Interest Valuation Date and each Autocall Valuation Date.

---

### ADDITIONAL DISRUPTION EVENT AND ADJUSTMENT OR EARLY REDEMPTION

**Additional Disruption Event**

The Issuer may either (i) require the Determination Agent to make an adjustment to the terms of the Securities or (ii) on giving not less than 10 Business Days notice to the Securityholders, redeem all of the Securities early at the Early Cash Settlement Amount on the Early Cash Redemption Date if any of the following events occur:

- Change in Law, Currency Disruption Event, Issuer Tax Event, Extraordinary Market Disruption, Hedging Disruption
- Insolvency Filing, Merger Event, Nationalisation, Insolvency, Delisting, Tender Offer, Fund Disruption Event

**Other Additional Disruption Event(s) in respect of Share Linked Securities**

Insolvency Filing, Merger Event, Nationalisation, Insolvency, Delisting, Tender Offer, Fund Disruption Event

**Other Additional Disruption Event(s) in respect of Index Linked Securities**

Index Adjustment Event - provided that an Index Adjustment Event shall only constitute an Additional Disruption Event if the Determination Agent determines that it can no longer continue to calculate such Index.

**Delay or Postponement of Payments and Settlement**

If the determination of a price or level used to calculate any amount payable or deliverable on any payment or settlement date is delayed or postponed pursuant to the terms and conditions of the Securities, payment or settlement will occur on the later of either (i) the scheduled payment or settlement date or (ii) the second Business Day following the date on which such price or level is determined.

No additional amounts shall be payable or deliverable by the Issuer because of such postponement.

**Substitution of Shares**

Substitution of Shares – ETF underlying is applicable. If any Share is affected by a Fund Disruption Event, Merger Event, Tender Offer, Nationalisation, Insolvency Filing, Insolvency or Delisting, or if the Share is cancelled or there is an announcement for it to be cancelled then, in addition to the Issuer’s right to adjust or redeem the Securities, the Issuer or the Determination Agent has the discretion to substitute such Shares with shares, units or other interests of an exchange-traded fund or other financial security, index or instrument (each a "Replacement Security") that the Determination Agent determines is comparable to the discontinued Share (or discontinued Replacement Security). Upon substitution of a Replacement Security, the Determination Agent may adjust any variable in the terms of the Securities (including, without limitation, any variable relating to the price of the shares, units or other interests in the Share, the number of such shares, units or other interests outstanding, created or redeemed or any dividend or other distribution made in respect of such shares, units or other interests), as, in the good faith judgment of the Determination Agent, may be and for such time as may be necessary to render the Replacement Security comparable to the shares or other interests of the discontinued Share (or discontinued Replacement Security). The Determination Agent shall notify the Securityholders as soon as practicable after the selection of the Replacement Security.

**Adjustments and Early Redemption**

**Successor Index Sponsor and Successor Index:** In respect of an Equity Index, in the event that the Index Sponsor ceases to calculate and announce the Index but the Index is calculated and announced by a successor index sponsor or the Index is replaced by a successor index which is the same as, or substantially similar to the Index (as determined by the Determination Agent), the level of the Index will be determined with reference to the calculations of the successor index sponsor or the level of that successor index.

**Index Adjustment Events:** In respect of an Equity Index, if there occurs an Index Modification, Index Cancellation or Index Disruption (each an "Index Adjustment Event"), the Determination Agent may (i) calculate the level of the Index using the formula for and method of calculating the Index last in effect prior to the Index Adjustment Event, or (ii) if the Determination Agent determines that it can no longer continue to calculate the level of the Index, deem such Index Adjustment Event to constitute an Additional Disruption Event and the Issuer may either (x) require the Determination Agent to make an adjustment to the terms of the Securities, or (y) redeem all of the Securities at the Early Cash Settlement Amount on the Early Cash Redemption Date.

**Potential Adjustment Event:** In respect of Shares, if (i) there occurs a subdivision, consolidation or reclassification of the Share, or (ii) a distribution, dividend, extraordinary dividend, repurchase of the Shares or similar corporate action is declared by the Share Company (each, a "Potential Adjustment Event"), in any case that the Determination Agent determines has a diluting or concentrative effect on the theoretical value of the Share, (x) the Determination Agent may make an adjustment to the Share, any amounts payable under the Securities and/or any of the other terms of the Securities, taking into account any costs incurred by or on behalf of the Issuer as a result of such Potential Adjustment Event, as determined in good faith by the Determination Agent, or (y) the Issuer may deliver to the Securityholder one or more additional Securities and/or pay to the Securityholder a cash amount, which aggregate value shall be equal to the value of the concentrative effect of such Potential Adjustment Event on the theoretical value of the relevant Shares.
Early Cash Settlement Amount

An amount per Calculation Amount in the Settlement Currency determined as the pro rata proportion of the market value of the Securities following the event triggering the early redemption or cancellation (including the value of accrued interest (if applicable)). Such amount shall be determined as soon as reasonably practicable following the event giving rise to the early redemption or cancellation of the Securities by reference to such factors as the Determination Agent considers to be appropriate including, without limitation:

(a) market prices or values for the reference asset(s) and other relevant economic variables (such as interest rates and, if applicable, exchange rates) at the relevant time;

(b) the remaining term of the Securities had they remained outstanding to scheduled maturity or expiry and/or any scheduled early redemption or exercise date;

(c) the value at the relevant time of any minimum redemption or cancellation amount which would have been payable had the Securities remained outstanding to scheduled maturity or expiry and/or any scheduled early redemption or exercise date;

(d) internal pricing models; and

(e) prices at which other market participants might bid for securities similar to the Securities,

provided that the Determination Agent may adjust such amount to take into account deductions for any costs, charges, fees, accruals, losses, withholdings and expenses, which are or will be incurred by the Issuer or its Affiliates in connection with the unwinding of any Hedge Positions and/or related funding arrangements, when determining such market value.

“Affiliate” means, in relation to any entity (the “First Entity”), any entity controlled, directly or indirectly, by the First Entity, any entity that controls, directly or indirectly, the First Entity or any entity, directly or indirectly, under common control with the First Entity. For these purposes, “control” means ownership of a majority of the voting power of an entity.

“Hedge Positions” means any purchase, sale, entry into or maintenance of one or more (a) positions or contracts in securities, options, futures, derivatives or foreign exchange, (b) stock loan transactions or (c) other instruments or arrangements (however described) by the Issuer or any of its Affiliates in order to hedge individually, or on a portfolio basis, the Issuer’s obligations in respect of the Securities.

Early Cash Redemption Date

In respect of an early redemption following an Additional Disruption Event, the 10th Business Day after the giving of the redemption notice by or on behalf of the Issuer or the Determination Agent to the Securityholders.

OTHER TERMS

Disruption

In respect of Shares in a Basket, in the event that any Valuation Date is a Disrupted Day (as described in the Offering Circular) in relation to each Share affected by the occurrence of a Disrupted Day (each an “Affected Share”), the relevant valuation will be postponed for up to eight Scheduled Trading Days. After this time, (1) the eighth Scheduled Trading Day shall be deemed to be the Valuation Date; and (2) the Determination Agent will make the relevant determination by estimating the price of the Affected Share that would have prevailed on such eighth Scheduled Trading Day. In respect of each Share not affected by the occurrence of a Disrupted Day, the Valuation Date shall be the Scheduled Valuation Date.

In respect of an Equity Index in a Basket, in the event that any Valuation Date is a Disrupted Day (as described in the Offering Circular), in relation to each Index affected by the occurrence of a Disrupted Day (each an “Affected Index”), the relevant valuation in respect of such Affected Indices will be postponed for up to eight Scheduled Trading Days. After this time, (1) the eighth Scheduled Trading Day shall be deemed to be the Valuation Date; and (2) the Determination Agent shall determine the level of the Affected Indices using the level of the index set out in the applicable Pricing Supplement, or in the event that no index level is provided in the Pricing Supplement, by using the index level on the eighth Scheduled Trading Day determined in accordance with the formula and method of calculating that index in effect immediately prior to the occurrence of first Disrupted Day using the Exchange traded or quoted price on the eighth Scheduled Trading Day of each component of that Index. In respect of each Index that is not affected by the occurrence of a Disrupted Date, the Valuation Date shall be the Scheduled Valuation Date.

Unlawfulness and impracticability

If the Issuer determines that the performance of any of its obligations under the Securities has become, or there is a substantial likelihood that it will become, unlawful or impracticable, in whole or in part, as a result of (i) any change in financial, political or economic conditions or currency exchange rates, or (ii) compliance in good faith by the Issuer or any relevant subsidiaries or Affiliates with any applicable present or future law, rule, regulation, judgement, order or directive of any governmental, administrative or judicial authority or power or in interpretation thereof, the Issuer may, as its option, redeem or cancel the Securities by giving notice to Securityholders.

If the Issuer elects to redeem or cancel the Securities, then each Security shall become due and payable at its Early Cash Settlement Amount.

Notices

The Issuer or Determination Agent shall give notice to the Securityholders of any adjustment or redemption as soon as practicable following the occurrence of the event triggering such adjustment or redemption. Failure by the Issuer or Determination Agent to publish or give notice shall not affect the validity or effectiveness of any such adjustment or redemption.

GENERAL INFORMATION

Programme

Barclays Bank PLC Global Structured Securities Programme

Offering Circular

Offering Circular dated 24 June 2015 pursuant to the Programme. The Offering Circular is available at:
REGULATORY REVIEW AND IMPORTANT INFORMATION FOR PROSPECTIVE INVESTORS:

THE OFFERING CIRCULAR HAS NOT BEEN SUBMITTED TO, REVIEWED BY OR APPROVED BY THE UNITED KINGDOM FINANCIAL CONDUCT AUTHORITY IN ITS CAPACITY AS COMPETENT AUTHORITY UNDER THE FINANCIAL SERVICES AND MARKETS ACT 2000 (THE “FSMA”) OR ANY OTHER REGULATORY AUTHORITY IN ITS CAPACITY AS COMPETENT AUTHORITY IN THE EU OR THE LONDON STOCK EXCHANGE PLC OR ANY OTHER STOCK EXCHANGE WHICH CONSTITUTES A REGULATED MARKET FOR THE PURPOSES OF DIRECTIVE 2004/39/EC (THE “MARKETS IN FINANCIAL INSTRUMENTS DIRECTIVE”).

THIS MEANS THAT THE OFFERING CIRCULAR DOES NOT COM普RISE (I) A BASE PROSPECTUS FOR THE PURPOSES OF ARTICLE 5.4 OF DIRECTIVE 2003/71/EC (AND AMENDMENTS THERETO) (THE “PROSPECTUS DIRECTIVE”) OR ANY UK OR OTHER IMPLEMENTING LEGISLATION RELATED TO THE PROSPECTUS DIRECTIVE, OR (II) LISTING PARTICULARS FOR THE PURPOSES OF SECTION 79 OF THE FSMA OR ANY OTHER RULES OR REGULATIONS RELATED TO A LISTING ON ANY REGULATED MARKET OF ANY STOCK EXCHANGE.

<table>
<thead>
<tr>
<th>Relevant Annex</th>
<th>Equity Linked Annex</th>
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<tr>
<td>Issuer Rating (Long Term)</td>
<td>As of the date of this Term Sheet, A2/A-/A (Moody's/S&amp;P/Fitch)</td>
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<td>Status</td>
<td>Unsecured and Unsubordinated</td>
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<td>Form</td>
<td>Global Bearer Securities: Permanent Global Security</td>
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<td>Manager</td>
<td>Barclays Bank PLC</td>
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<tr>
<td>Issue and Paying Agent</td>
<td>The Bank of New York Mellon</td>
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<tr>
<td>Business Days</td>
<td>With regard to payments: London, New York City and a Clearing System Business Day.</td>
</tr>
<tr>
<td>Business Day Convention</td>
<td>With regard to all payment dates in this Term Sheet, unless otherwise specified: Modified Following</td>
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<td>Listing and Admission to Trading</td>
<td>None</td>
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<td>Relevant Clearing Systems</td>
<td>Euroclear</td>
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<td>Clearstream</td>
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<td>Governing Law</td>
<td>English Law</td>
</tr>
<tr>
<td>Jurisdiction</td>
<td>Courts of England</td>
</tr>
<tr>
<td>Documentation</td>
<td>The full terms and conditions of the Securities (including Terms used but not defined in this Term Sheet) will be set out in the Offering Circular as supplemented and amended by the Pricing Supplement.</td>
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SELLING RESTRICTIONS AND TAX

Selling Restrictions  Investors are bound by all applicable laws and regulations of the relevant jurisdiction(s) in which the Securities are to be offered, sold and distributed, including the selling restrictions set out in this document and the Offering Circular. Investors in this Product should seek specific advice before on-selling this Product.

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Third Party Fees  The Issue Price for any Securities (whether issued on or after the Issue Date) includes a commission element shared with a third party, which will not exceed 1.63% of the Calculation Amount per Security. Further details of the commission element are available upon request.

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CAPITAL AT RISK

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CAPITAL AT RISK ON EARLY REDEMPTION

THE PRODUCT MAY BE REDEEMED BEFORE ITS SCHEDULED MATURITY DATE. IF THE PRODUCT IS REDEEMED EARLY, INVESTORS MAY RECEIVE BACK LESS THAN THEIR ORIGINAL INVESTMENT IN THE PRODUCT, OR EVEN ZERO. The amount payable to an investor on an early redemption may factor in Barclays' costs of terminating hedging and funding arrangements associated with the Product.

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AN INVESTOR MAY NOT BE ABLE TO FIND A BUYER FOR THE PRODUCT SHOULD THE INVESTOR WISH TO SELL THE PRODUCT. If a buyer can be found, the price offered by that buyer may be lower than the price that an investor paid for the Product or the amount an investor would otherwise receive at the maturity of the Product.

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THE ISSUER MAY ISSUE MORE SECURITIES THAN THOSE WHICH ARE TO BE INITIALLY SUBSCRIBED OR PURCHASED BY INVESTORS. The Issuer (or the Issuer's affiliates) may hold such Securities for the purpose of meeting any future investor interest or to satisfy market making requirements. Prospective investors in the Securities should not regard the size issue of any Series as indicative of the depth or liquidity of the market for such Series or of the demand for such Series.

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AN INVESTMENT IN THE PRODUCT IS NOT THE SAME AS AN INVESTMENT IN THE UNDERLYING ASSETS REFERENCED BY THE PRODUCT. An investor in the Product has no ownership of, or rights to, the underlying assets referenced by the Product. The market value of the Product may not reflect movements in the price of such underlying assets. Payments made under the Product may differ from payments made under the underlying assets.

ADJUSTMENTS

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SMALL HOLDINGS

SMALL HOLDINGS MAY NOT BE TRANSFERABLE. Where the Product terms specify a minimum tradable amount, investors will not be able to sell the Product unless they hold at least such minimum tradable amount.

INTEREST RATE RISK

INVESTORS IN THE PRODUCT WILL BE EXPOSED TO INTEREST RATE RISK. Changes in interest rates will affect the performance and value of the Product. Interest rates may change suddenly and unpredictably.

PAYMENTS

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INTERACTION RISK

THIS PRODUCT COMBINES DIFFERENT FINANCIAL COMPONENTS AND EXPOSURES WHICH MAY INTERACT UNPREDICTABLY AND COULD AFFECT THE PERFORMANCE OF THE PRODUCT.

PERFORMANCE OF SHARE INDICES

THE PERFORMANCE OF SHARES IN AN INDEX IS UNPREDICTABLE. It depends on financial, political, economic and other events as well as the share issuers' earnings, market position, risk situation, shareholder structure and distribution policy.

INDEX RETURN

AN INDEX RETURN MAY BE LOWER THAN THE ACTUAL RETURN ON THE COMPONENTS COMPRISING SUCH INDEX. Indices may deduct fees, costs and commissions. An investment in an index may be taxed differently to a direct investment in the components of the index.

ADJUSTMENTS, SUSPENSION AND TERMINATION OF AN INDEX

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INDEX SUBSTITUTION

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TRACKING RISK

There may be a difference between the performance of the underlying ETF and the performance of the asset pool or index that the ETF is designed to track as a result of, for example, failure of the tracking strategy, currency differences, fees and expenses.

DERIVATIVE RISK

The ETF may invest in financial derivative instruments which expose the ETF and an investor to the credit, liquidity and concentration risks of the counterparties to such financial derivative instruments.

PERFORMANCE OF AN ETF

THE PERFORMANCE OF SHARES IN AN ETF IS UNPREDICTABLE. It depends on financial, political, economic and other events as well as the ETF's earnings, market position, risk situation, shareholder structure and distribution policy.

ETF ISSUER ACTION

THE ETF ISSUER IS NOT INVOLVED IN THE PRODUCT. The ETF Issuer may take actions that adversely affect the value and performance of the Product.
YOU MAY NOT RECEIVE ANY INTEREST PAYMENTS. Barclays will not necessarily make interest payments under the terms of the Product. If the Valuation Price of at least one of the Basket Constituents on the relevant Interest Valuation Date is less than the Interest Barrier, Barclays will not make the interest payment applicable to such Interest Valuation Date. If the Valuation Price of at least one of the Basket Constituents is less than the Interest Barrier on each of the Interest Valuation Dates, Barclays will not make any interest payments during the term of the Product, and you will not receive a positive return on your initial investment.

There is no guarantee that you would be able to reinvest the proceeds from an investment in the Product in a comparable investment with a similar level of risk in the event the Securities are called prior to the Redemption Date.

The return potential of the Securities is limited to the interest payments based on the pre-specified interest rate, regardless of the appreciation of the Basket Constituents. In addition, the total return on the Securities will vary based on the number of Interest Valuation Dates on which the Valuation Price of each Basket Constituent has equaled or exceeded its respective Interest Barrier price prior to the Redemption Date or a Specified Early Redemption Event. Further, if the Securities are called due to a Specified Early Redemption Event, you will not receive any interest payments in respect of any Interest Valuation Dates after the applicable Specified Early Redemption Date. Because the Securities could be called as early as the first Autocall Valuation Date, the total return on the Securities could be minimal. If the Valuation Price of the Worst Performing Basket Constituent on the Final Valuation Date is below its Knock-in Barrier Price and no event that may lead to the early redemption or termination of the Securities has occurred prior to the Redemption Date as determined by the Determination Agent, the Securities will be fully exposed to the decline in the price of the Worst Performing Basket Constituent and a Securityholder will lose some or all of their principal investment in the Securities.

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APPENDIX

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<tr>
<th></th>
<th>Basket Constituent</th>
<th>Type</th>
<th>Bloomberg Code (for identification purposes only)</th>
<th>ISIN/Index Sponsor</th>
<th>Exchange</th>
<th>Related Exchange</th>
<th>Reference Asset Currency</th>
<th>Initial Price</th>
<th>Strike Price (100.00% of Initial Price displayed to 4 d.p.)</th>
<th>Interest Barrier (70.00% of Initial Price displayed to 4 d.p.)</th>
<th>Autocall Barrier (100.00% of Initial Price displayed to 4 d.p.)</th>
<th>Knock-in Barrier Price (70.00% of Initial Price displayed to 4 d.p.)</th>
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<td>2</td>
<td>Euro Stoxx 50® Index</td>
<td>Index</td>
<td>SXSE</td>
<td>Stoxx Ltd.</td>
<td>Multi-exchange</td>
<td>All Exchanges</td>
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<td>3,484.4100</td>
<td>2,439.0870</td>
<td>3,484.4100</td>
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'Multi-exchange' means, in respect of each component security of the Index (each, a 'Component Security'), the stock exchange on which such Component Security is principally traded, as determined by the Determination Agent.
### Table C.2: Regression of Excess Returns on Market Excess Returns

<table>
<thead>
<tr>
<th>Maturity in Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
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<tr>
<td><strong>UKX (Apr 2004 - Jan 2017, NObs = 154)</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Alpha</td>
<td>0.0069***</td>
<td>0.0094***</td>
<td>0.0090***</td>
<td>0.0046**</td>
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<tr>
<td></td>
<td>(3.1887)</td>
<td>(3.2251)</td>
<td>(6.9208)</td>
<td>(3.0348)</td>
</tr>
<tr>
<td>Beta</td>
<td>0.2835***</td>
<td>0.4810***</td>
<td>0.6319***</td>
<td>0.7854***</td>
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<tr>
<td></td>
<td>(5.1297)</td>
<td>(6.2627)</td>
<td>(8.1120)</td>
<td>(9.6202)</td>
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<tr>
<td>$R^2$</td>
<td>0.0775</td>
<td>0.1740</td>
<td>0.2076</td>
<td>0.3172</td>
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<tr>
<td><strong>KOSPI2 (Apr 2004 - Jan 2017, NObs = 154)</strong></td>
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<tr>
<td>Alpha</td>
<td>0.0054</td>
<td>0.0049</td>
<td>0.0046</td>
<td>0.0085</td>
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<tr>
<td></td>
<td>(0.3268)</td>
<td>(0.4454)</td>
<td>(0.4065)</td>
<td>(0.6551)</td>
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<tr>
<td>Beta</td>
<td>0.3935</td>
<td>0.4614**</td>
<td>0.6838***</td>
<td>0.5812***</td>
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<td>(1.4947)</td>
<td>(2.3153)</td>
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<td>$R^2$</td>
<td>0.0090</td>
<td>0.0261</td>
<td>0.0697</td>
<td>0.0403</td>
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<td><strong>HSI (Nov 2004 - Jan 2017, NObs = 147)</strong></td>
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<tr>
<td>Alpha</td>
<td>0.0037</td>
<td>-0.0008</td>
<td>-0.0031</td>
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<td></td>
<td>(0.4660)</td>
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<td>Beta</td>
<td>0.2226</td>
<td>0.3721**</td>
<td>0.4683***</td>
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<td>(1.5021)</td>
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<td>(3.3205)</td>
<td>(4.0029)</td>
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<td>$R^2$</td>
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<td>0.1091</td>
<td>0.1719</td>
<td>0.2673</td>
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<tr>
<td><strong>HSCEI (Jan 2005 - Nov 2013, NObs = 107)</strong></td>
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<tr>
<td>Alpha</td>
<td>0.0092</td>
<td>0.0088</td>
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<td></td>
<td>(1.7232)</td>
<td>(1.4465)</td>
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<tr>
<td>Beta</td>
<td>0.1443</td>
<td>0.1736**</td>
<td>0.2253***</td>
<td>0.3492***</td>
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<td></td>
<td>(1.7415)</td>
<td>(2.6450)</td>
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<tr>
<td>$R^2$</td>
<td>0.0552</td>
<td>0.0801</td>
<td>0.1208</td>
<td>0.2348</td>
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*Notes*: Monthly data from the synthetic forward method (except for HSCEI, which uses the dividend future and swap method). NObs represents the total number of observation. $t$ statistics using Newey-West standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

### C.6 Mathematical Proof

#### C.6.1 Proof of Lemma 18

Assume the following (Feller-like) conditions on $(R_t^c, X)$:
• $q, X$ have compact supports.

• For any continuous function $f$, $\mathbb{E}[f(R_{t+1}^e, X_{t+1}) \mid X_t = x]$ is continuous in $x$. This is a weak requirement on the dependence of $(R_t^e, X_t)$ in $X_t$.

• $R_{t+1}^e$ is bounded so that all expectations are well-defined.

Consider the proposed-form of the value function $J(W_t; t, X_t) = \frac{1}{k} e^{-k(W_t + G_t(q_t, X_t))}$.

Recall that

$$\max_{C_t, d_t, \theta_t} \frac{1}{\gamma} e^{\gamma C_t} + \rho \mathbb{E}_t[J(W_{t+1}; t + 1, X_{t+1})]. \quad (C.1)$$

Taking the first-order condition with respect to $C_t$ yields:

$$0 = e^{-\gamma C_t} + kR_f \rho \mathbb{E}_t[J(W_{t+1}; t + 1, X_{t+1})] \quad (C.2)$$

$$= e^{-\gamma G_t} + kR_f [J(W_t; t, x_t) + \frac{1}{\gamma} e^{-\gamma C_t}] \quad (C.3)$$

$$e^{-\gamma C_t} = e^{-k(W_t + G_t(q_t, X_t))}. \quad (C.4)$$

That is, we have equation (3.13). Taking the first-order conditions with respect to $\theta_t$ and $d_t$ will yield equations (3.14) and (3.15), respectively.

For the derivation of $G$, first let $G(t + 1, \cdot)$ be given. $\theta_t$ and $d_t$ are given as unique solutions to equations (3.14) and (3.15).

### C.6.2 Proof of Theorem 21

Differentiating the pricing kernel yields:

$$\frac{\partial m^q_{t+1}}{\partial d_t} = km^q_{t+1} (p^o_{t+1} - R_f p^o_t + \frac{\partial \theta_t}{\partial d_t} R^e_{t+1}), \quad (C.5)$$

where we use the facts that $\frac{\partial G_{t+1}(q_{t+1}, X_{t+1})}{\partial d_t} = 0$ and $\frac{\partial p^o_{t+1}}{\partial d_t} = 0$. Differentiating (3.22) results in

$$0 = \mathbb{E}_t (m^q_{t+1} (p^o_{t+1} - R_f p^o_t - \frac{\partial \theta_t}{\partial d_t} R^e_{t+1}) R^e_{t+1}), \quad (C.6)$$
implying that the marginal hedge position is

$$\frac{\partial \theta_t}{\partial q^o_{t}} = - \frac{\partial \theta_t}{\partial d^o_{t}} = - \frac{\mathbb{E}_t(m^q_{t+1}(p^o_{t+1} - R_f p^o_t) R^e_{t+1})}{\mathbb{E}_t(m^q_{t+1}(R^e_{t+1})^2)}$$

$$= - \frac{\text{cov}^q_t(p^o_{t+1}, R^e_{t+1})}{\text{var}^q_t(R^e_{t+1})}. \quad (C.7)$$

The price sensitivity comes from differentiating (3.21) as follows:

$$\frac{\partial p^n_t}{\partial q^o_{t}} = -k \mathbb{E}_t[m^q_{t+1}(p^o_{t+1} - R_f p^o_t + \frac{\partial \theta_t}{\partial q^o_{t}} R^e_{t+1}) p^n_{t+1}]$$

$$= - k \frac{\mathbb{E}_t[(p^o_{t+1} - R_f p^o_t) - \frac{\text{cov}^q_t(p^o_{t+1}, R^e_{t+1})}{\text{var}^q_t(R^e_{t+1})} R^e_{t+1}) p^n_{t+1}]}{R_f}$$

$$= - \gamma(R_f - 1) \mathbb{E}_t[p^o_{t+1} p^n_{t+1}]$$

$$= - \gamma(R_f - 1) \text{cov}^q_t(\bar{p}^o_{t+1}, \bar{p}^n_{t+1}). \quad (C.8)$$

C.7 Issuance Data
C.8 Robustness Check on the Effect of Structured Products on $T$-Maturity Asset Prices

The issuance data in Figure C.18 suggests the potential structural change in structured product issuance around 2014. This may be due to the fact that some of the trades before 2014 are not reported. Our data provider, mtn-i.com, confirms that pre-2014 is less comprehensive.

This section checks the robustness of our empirical results in Section 3.5 by dividing the sample into 2 subsamples: pre-2014 and post 2014.

Table C.3, Table C.4, and Table C.5 are analogs of Table 3.4, Table 3.5, and Table 3.6 in the main paper, respectively. The notional issuance has no significant effect on dividend yields and dividend prices. Nevertheless, the issuance affects the dividend yield term structure. The higher dividend supply leads to more inverted 2y1y, 3y1y, 3y2y, 5y1y dividend yield term structure in the pre-2014 sample and more inverted 3y2y and 5y2y dividend yield term structure in the post-2014 sample.
### Table C.3: Pre- and Post-2014 Price Regression

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<tr>
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<th>$T = 1$ (3.26)</th>
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<th>$T = 5$ (3.26)</th>
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<td><strong>Pre-2014</strong></td>
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<td></td>
</tr>
<tr>
<td>$\Delta \log n_{it}^{qtr}$</td>
<td>-13.68</td>
<td>-7.053</td>
<td>-35.64</td>
<td>-18.29</td>
</tr>
<tr>
<td></td>
<td>(-1.05)</td>
<td>(-0.85)</td>
<td>(-1.24)</td>
<td>(-1.13)</td>
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<tr>
<td>$\Delta \log S_{it}^{qtr}$</td>
<td>121.5**</td>
<td>318.1***</td>
<td>555.4***</td>
<td>1050.8***</td>
</tr>
<tr>
<td></td>
<td>(3.29)</td>
<td>(4.31)</td>
<td>(5.03)</td>
<td>(5.70)</td>
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<td>247</td>
<td>247</td>
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<tr>
<td>Adj. $R^2$</td>
<td>0.0132</td>
<td>0.109</td>
<td>0.0211</td>
<td>0.175</td>
</tr>
<tr>
<td><strong>Post-2014</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \log n_{it}^{qtr}$</td>
<td>-5.776</td>
<td>-8.552</td>
<td>-13.66</td>
<td>-11.60</td>
</tr>
<tr>
<td></td>
<td>(-0.67)</td>
<td>(-0.89)</td>
<td>(-0.63)</td>
<td>(-0.48)</td>
</tr>
<tr>
<td>$\Delta \log S_{it}^{qtr}$</td>
<td>-84.88</td>
<td>63.11</td>
<td>286.7</td>
<td>855.7</td>
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<td></td>
<td>(-1.53)</td>
<td>(0.58)</td>
<td>(1.38)</td>
<td>(2.05)</td>
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<td>$N$</td>
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<tr>
<td>Adj. $R^2$</td>
<td>0.0051</td>
<td>0.0547</td>
<td>0.0067</td>
<td>0.0132</td>
</tr>
</tbody>
</table>

Notes: Data from Q4 2004 - Q4 2016. Panel Regression with Fixed Effect. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

### Table C.4: Pre- and Post-2014 Yield Regression

<table>
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<th>$T = 1$ (3.28)</th>
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<th>$T = 3$ (3.29)</th>
<th>$T = 5$ (3.29)</th>
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<tr>
<td><strong>Pre-2014</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \log n_{it}^{qtr}$</td>
<td>0.0002</td>
<td>0.0004</td>
<td>-0.0003</td>
<td>-0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(1.24)</td>
<td>(-0.69)</td>
<td>(-0.21)</td>
</tr>
<tr>
<td>$\Delta R_i^{qtr}$</td>
<td>-0.0052</td>
<td>-0.0053*</td>
<td>-0.0044*</td>
<td>-0.0030</td>
</tr>
<tr>
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<td>(-2.19)</td>
<td>(-1.60)</td>
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<td>247</td>
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<td>Adj. $R^2$</td>
<td>0.0009</td>
<td>0.0408</td>
<td>0.0025</td>
<td>0.0639</td>
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<tr>
<td><strong>Post-2014</strong></td>
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<td></td>
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<tr>
<td>$\Delta \log n_{it}^{qtr}$</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>(-0.13)</td>
<td>(-0.15)</td>
<td>(0.80)</td>
<td>(0.80)</td>
</tr>
<tr>
<td>$\Delta R_i^{qtr}$</td>
<td>0.0023</td>
<td>0.0019</td>
<td>0.0012</td>
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<tr>
<td></td>
<td>(0.99)</td>
<td>(0.91)</td>
<td>(0.67)</td>
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<td>$N$</td>
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<tr>
<td>Adj. $R^2$</td>
<td>0.0002</td>
<td>0.0140</td>
<td>0.0074</td>
<td>0.0207</td>
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Notes: Data from Q4 2004. Panel Regression with Fixed Effect. $t$ statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 

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Table C.5: Pre- and Post-2014 Yield Term Structure Regression

<table>
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<tr>
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<th>( T_1, T_2 = 1, 2 )</th>
<th>( T_1, T_2 = 1, 3 )</th>
<th>( T_1, T_2 = 2, 3 )</th>
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<th>( T_1, T_2 = 3, 5 )</th>
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<td><strong>Pre-2014</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \log nq_{it}^{ERP} )</td>
<td>-0.0005* (-2.58)</td>
<td>-0.0005* (-2.32)</td>
<td>-0.0007* (-2.55)</td>
<td>-0.0008* (-2.41)</td>
<td>-0.0002 (-2.06)</td>
<td>-0.0003* (-2.24)</td>
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<td>-0.0009* (-2.27)</td>
<td>-0.0010* (-2.12)</td>
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<td>-0.0004 (-1.68)</td>
<td>-0.0004 (-1.71)</td>
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<td></td>
<td>-0.0002 (-1.28)</td>
<td>-0.0002 (-1.24)</td>
</tr>
<tr>
<td>( \Delta R^e_{it} )</td>
<td>-0.0001 (-0.09)</td>
<td>0.0008 (0.62)</td>
<td>0.0009 (1.37)</td>
<td>0.0013 (0.68)</td>
<td>0.0013 (1.18)</td>
<td>0.0005 (0.66)</td>
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</tr>
<tr>
<td><strong>Post-2014</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \log nq_{it}^{ERP} )</td>
<td>0.0004 (1.63)</td>
<td>0.0002 (0.79)</td>
<td>0.0002 (-3.43)</td>
<td>-0.0002** (-3.58)</td>
<td>-0.0002** (0.09)</td>
<td>-0.0004* (-2.82)</td>
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<td></td>
<td>-0.0004** (-3.31)</td>
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<td></td>
<td>-0.0002 (-1.74)</td>
</tr>
<tr>
<td>( \Delta R^e_{it} )</td>
<td>-0.0004 (-0.74)</td>
<td>-0.0011 (-1.32)</td>
<td>-0.0007 (-1.66)</td>
<td>-0.0028 (-2.23)</td>
<td>-0.0024* (-2.88)</td>
<td>-0.0017* (-3.08)</td>
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<td><strong>Notes</strong></td>
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Notes: Data from Q4 2004. Panel Regression with Fixed Effect. t statistics using Driscoll-Kraay standard error in parentheses. * \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \).
C.9 Time-Series Regression Results of the Effect of Structured Products on $T$-Maturity Asset Prices

This section shows the individual time series regression for each equity index. Table C.6 is an analog of Table 3.4 in the main paper.

Table C.6 shows time series regression results for each underlying index. Similar to the pooled panel, the coefficient in front of the log difference of notional issuance is mostly insignificantly negative. The exceptions include NKY and UKX.

For NKY, a percentage increase in the normalized notional issuance prominently decreases the price of 5-year dividends. The statistical significance is lost once we control for the percentage change in spot levels. This implies that the notional issuance is negatively correlated with spot levels. There are many possible explanations. Investors might have less appetite for structured products when the index level is high. Index volatility may be low when spots are high, making structured products less attractive, etc.

A percentage increase in the normalized notional issuance decreases the price of 2-, 3-, and 5-year UKX dividends. After controlling for the change in spots, the impact remains significant only for 5-year prices.

Next, we explore the effect of structured product issuance on dividend yields within each equity index. Table C.7 displays the individual time-series results from regression equations (3.28) and (3.29).

From Table C.7, the impact of structured products on dividend yields are muted. A percentage increase in the notional issue only significantly decreases 5-year UKX yields.

We fail to detect the impact of structured product issuance on dividend yields. We then explore whether the structured product issuance affects the slope of dividend yield term structure.

Looking at time series regression for each individual equity index, we lose some predictive power, as evident in Table C.8. The coefficient in front of the log difference in dividend supply is mostly negative but insignificantly so. The loss in statistical significance may be
due to small sample size for each individual underlying.

A percentage increase in the structured product issuance makes the 2y1y and 3y1y UKX dividend yield term structure and the 2y1y NKY dividend term structure more inverted.
Table C.6: Individual Equity Index Time-Series Price Regression

<table>
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<tr>
<th></th>
<th>$T = 1$</th>
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<th>$T = 3$</th>
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<tr>
<td></td>
<td>$(3.26)$</td>
<td>$(3.27)$</td>
<td>$(3.26)$</td>
<td>$(3.27)$</td>
</tr>
<tr>
<td><strong>SPX (Q4 2004 - Q4 2016)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-0.0545 (0.23)</td>
<td>0.158 (0.49)</td>
<td>-0.747 (1.12)</td>
<td>-0.102 (0.17)</td>
</tr>
<tr>
<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>7.411* (2.15)</td>
<td>22.52*** (5.42)</td>
<td>37.73*** (7.96)</td>
<td>71.80*** (12.39)</td>
</tr>
<tr>
<td>$N$</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td><strong>SX5E (Q4 2004 - Q4 2016)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>0.0821 (0.04)</td>
<td>1.076 (0.78)</td>
<td>-3.603 (0.57)</td>
<td>-0.0555 (0.02)</td>
</tr>
<tr>
<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>34.29** (2.84)</td>
<td>122.3*** (5.41)</td>
<td>218.0*** (6.17)</td>
<td>411.3*** (8.07)</td>
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<tr>
<td>$N$</td>
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<td>49</td>
<td>49</td>
</tr>
<tr>
<td><strong>NKY (Q4 2004 - Q4 2016)</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-1.729 (0.34)</td>
<td>0.647 (0.13)</td>
<td>-13.45 (0.24)</td>
<td>-3.518 (0.36)</td>
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<tr>
<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>36.27 (1.05)</td>
<td>151.6* (2.22)</td>
<td>303.0** (3.09)</td>
<td>611.5*** (4.21)</td>
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<td><strong>UKX (Q2 2006 - Q4 2016)</strong></td>
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<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-9.997 (1.36)</td>
<td>-4.289 (0.56)</td>
<td>-33.82* (2.18)</td>
<td>-15.54 (0.91)</td>
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<tr>
<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>59.20*** (4.64)</td>
<td>189.6** (3.40)</td>
<td>382.4*** (5.28)</td>
<td>724.0*** (24.49)</td>
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<td>$N$</td>
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<td><strong>KOSPI2 (Q4 2004 - Q4 2016)</strong></td>
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<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-0.426 (-1.62)</td>
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<td>-0.173 (-0.55)</td>
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<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>4.929** (3.48)</td>
<td>9.607*** (4.23)</td>
<td>14.45*** (4.48)</td>
<td>19.10*** (3.57)</td>
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<td><strong>HSI (Q3 2005 - Q4 2016)</strong></td>
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<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-82.87 (-1.13)</td>
<td>-51.83 (-1.17)</td>
<td>-187.8 (-1.22)</td>
<td>-106.4 (-1.35)</td>
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<tr>
<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>444.0* (2.31)</td>
<td>1164.1*** (3.68)</td>
<td>2041.1*** (4.89)</td>
<td>3954.1*** (6.23)</td>
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<td><strong>HSCEI (Q3 2005 - Q4 2013)</strong></td>
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<td>$\Delta \log n_{i,t}^{q_{t}}$</td>
<td>-0.256 (-0.05)</td>
<td>1.870 (0.68)</td>
<td>-11.53 (-0.80)</td>
<td>-4.92 (-0.60)</td>
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<td>$\Delta \log S_{i,t}^{e_{t}}$</td>
<td>39.89* (2.35)</td>
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<td>227.7** (2.84)</td>
<td>450.4** (3.63)</td>
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Notes: $t$ statistics using Newey-West standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 

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### Table C.7: Individual Equity Index Time-Series Yield Regression

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<th>( T = 4 )</th>
<th>( T = 5 )</th>
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<td><strong>SPX (Q4 2004 - Q4 2016)</strong></td>
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<td></td>
</tr>
<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>0.0006 (0.99)</td>
<td>0.0006 (1.09)</td>
<td>0.0020 (0.51)</td>
<td>0.0002 (0.51)</td>
<td>0.0001 (0.28)</td>
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<tr>
<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>-0.0057 (-1.88)</td>
<td>-0.0023 (-1.44)</td>
<td>-0.0009 (-0.76)</td>
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<td>0.0001 (0.09)</td>
</tr>
<tr>
<td>( N )</td>
<td>49 49 49 49 49</td>
<td>49 49 49 49 49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SX5E (Q4 2004 - Q4 2016)</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>0.0011 (0.86)</td>
<td>0.0011 (0.78)</td>
<td>0.0001 (0.21)</td>
<td>0.0002 (0.25)</td>
<td>-0.0002 (-0.38)</td>
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<tr>
<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>0.0056 (1.11)</td>
<td>-0.0016 (-0.50)</td>
<td>-0.0046 (-1.52)</td>
<td></td>
<td>-0.0072* (-2.37)</td>
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<tr>
<td>( N )</td>
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<td>49 49 49 49 49</td>
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<tr>
<td><strong>NKY (Q4 2004 - Q4 2016)</strong></td>
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<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>0.0004 (0.59)</td>
<td>0.0006 (0.72)</td>
<td>0.0000 (0.00)</td>
<td>0.0002 (0.27)</td>
<td>-0.0003 (-0.44)</td>
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<tr>
<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>-0.0031 (-1.39)</td>
<td>-0.0043 (-1.32)</td>
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<td>49 49 49 49 49</td>
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<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>0.0008 (0.57)</td>
<td>0.0007 (0.57)</td>
<td>-0.0006 (-0.42)</td>
<td>-0.0006 (-0.44)</td>
<td>-0.0013 (-1.56)</td>
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<tr>
<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>0.0094* (2.12)</td>
<td>0.0049 (1.31)</td>
<td>-0.0011 (-0.51)</td>
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<td>-0.0029 (-1.00)</td>
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<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>-0.0008 (-0.86)</td>
<td>-0.0005 (-0.55)</td>
<td>-0.0005 (-0.73)</td>
<td>-0.0003 (-0.35)</td>
<td>-0.0013 (-1.08)</td>
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<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>-0.0082 (-1.48)</td>
<td>-0.0095 (-1.74)</td>
<td>-0.0071 (-1.41)</td>
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<td>-0.0041 (-1.03)</td>
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<td><strong>HSI (Q3 2005 - Q4 2016)</strong></td>
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<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
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<td>-0.0006 (-0.58)</td>
<td>-0.0009 (-0.88)</td>
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<td>( \Delta R_{it}^{\text{eqtr}} )</td>
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<td>0.0003 (0.18)</td>
<td>0.0006 (0.47)</td>
<td>0.0009 (0.70)</td>
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<td><strong>HSCEI (Q3 2005 - Q4 2013)</strong></td>
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<tr>
<td>( \Delta \log q_{it}^{\text{eqtr}} )</td>
<td>0.0009 (1.01)</td>
<td>0.0009 (0.92)</td>
<td>0.0004 (0.47)</td>
<td>0.0004 (0.46)</td>
<td>0.0002 (0.20)</td>
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<tr>
<td>( \Delta R_{it}^{\text{eqtr}} )</td>
<td>0.0013 (0.37)</td>
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<td>-0.0022 (-0.53)</td>
<td>-0.0058 (-1.49)</td>
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</table>

Notes: \( t \) statistics using Newey-West standard error in parentheses. * \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \).
### Table C.8: Individual Equity Index Time-Series Yield Term Structure Regression

<table>
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<th>$T_1, T_2 = 1, 2$ (3.30)</th>
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<td>$\Delta \log nq_i^{\text{eff}}$</td>
<td>-0.0004 (-1.42)</td>
<td>-0.0004 (-1.69)</td>
<td>-0.0005 (-1.55)</td>
<td>-0.0001 (-1.10)</td>
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<td>-0.0005 (-1.49)</td>
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<tr>
<td>$\Delta R_i^{\text{eff}}$</td>
<td>0.0034 (1.85)</td>
<td>0.0048 (1.76)</td>
<td>0.0014 (1.51)</td>
<td>0.0058 (1.66)</td>
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<td>0.0011 (1.20)</td>
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<td>$\Delta \log nq_i^{\text{eff}}$</td>
<td>-0.0010 (-1.12)</td>
<td>-0.0009 (-1.06)</td>
<td>-0.0013 (-1.11)</td>
<td>-0.0003 (-0.86)</td>
<td>-0.0017 (-1.12)</td>
<td>-0.0015 (-1.03)</td>
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<td>$\Delta R_i^{\text{eff}}$</td>
<td>-0.0073* (-2.27)</td>
<td>-0.0102* (-2.39)</td>
<td>-0.0030* (-2.37)</td>
<td>-0.0129* (-2.32)</td>
<td>-0.0056* (-2.22)</td>
<td>-0.0026 (-1.69)</td>
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<td>$\Delta \log nq_i^{\text{eff}}$</td>
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<td>-0.0004 (-1.76)</td>
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<td>-0.0002 (-1.54)</td>
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<td>-0.0009 (-1.86)</td>
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<td>$\Delta R_i^{\text{eff}}$</td>
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<td>-0.0023 (-0.79)</td>
<td>-0.0011 (-1.09)</td>
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<td>-0.0027 (-1.50)</td>
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<tr>
<td>$\Delta \log nq_i^{\text{eff}}$</td>
<td>-0.0014** (-2.71)</td>
<td>-0.0014** (-2.97)</td>
<td>-0.0021* (-2.43)</td>
<td>-0.0020** (-2.77)</td>
<td>-0.0007 (-1.48)</td>
<td>-0.0007 (-1.34)</td>
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<td>$\Delta R_i^{\text{eff}}$</td>
<td>-0.046 (-1.42)</td>
<td>-0.0105* (-2.16)</td>
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<tr>
<td>$\Delta \log nq_i^{\text{eff}}$</td>
<td>0.0003 (0.52)</td>
<td>0.0003 (0.56)</td>
<td>0.0000 (0.06)</td>
<td>-0.0003 (-1.43)</td>
<td>-0.0003 (-1.56)</td>
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<td>$\Delta R_i^{\text{eff}}$</td>
<td>-0.0013 (-0.82)</td>
<td>0.0011 (0.53)</td>
<td>0.0024 (1.75)</td>
<td>0.0041 (1.27)</td>
<td>0.0054* (2.08)</td>
<td>0.0030 (1.65)</td>
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</tr>
<tr>
<td>$\Delta \log nq_i^{\text{eff}}$</td>
<td>-0.0003 (-1.73)</td>
<td>-0.0003 (-1.79)</td>
<td>-0.0003 (-1.21)</td>
<td>-0.0000 (-0.10)</td>
<td>-0.0004 (-0.94)</td>
<td>-0.0004 (-0.27)</td>
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<td>$\Delta R_i^{\text{eff}}$</td>
<td>0.0006 (0.87)</td>
<td>0.0010 (0.95)</td>
<td>0.0004 (0.95)</td>
<td>0.0013 (0.83)</td>
<td>0.0007 (0.68)</td>
<td>0.0003 (0.46)</td>
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<tr>
<td>$N$</td>
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<td>-0.0005* (-2.04)</td>
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<td>-0.0002 (-0.88)</td>
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<tr>
<td>$\Delta R_i^{\text{eff}}$</td>
<td>-0.0022** (-2.79)</td>
<td>-0.0035** (-3.54)</td>
<td>-0.0013*** (-4.92)</td>
<td>-0.0041*** (-4.39)</td>
<td>-0.0019** (-2.55)</td>
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**Notes:** t statistics using Newey-West standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. 

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