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Enhancing Acceleration Radiation from Ground-State Atoms via Cavity Quantum Electrodynamics

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When ground state atoms are accelerated through a high Q microwave cavity, radiation is produced with an intensity which can exceed the intensity of Unruh acceleration radiation in free space by many orders of magnitude. The cavity field at steady state is described by a thermal density matrix under most conditions. However, under some conditions gain is possible, and when the atoms are injected in a regular fashion, the radiation can be produced in a squeezed state.

One of the most intriguing results of modern quantum field theory is the proof by Davies, Fulling, Unruh and DeWitt 1, and others 2 that ground state atoms, accelerated through vacuum, are promoted to an excited state just as if they were in contact with a blackbody thermal field. These studies [1, 2] predict that a (two-level) ground state atom, having transition frequency \( \omega \), and experiencing a constant acceleration \( a \), will be excited to its upper level with a probability governed by the Boltzmann factor \( \exp(-2\pi\omega/\alpha) \), where \( \alpha = a/c \), \( c \) is the speed of light in vacuum. Unfortunately, even for very large acceleration “frequency” \( \alpha \approx 10^8 \) Hz, and microwave frequency \( \omega \approx 10^{10} \) Hz 4, this factor is exponentially small, \( \sim 10^{-200} \); and is not of experimental interest.

Thus we were motivated to study a simple gedanken experiment based on a model consisting of a high Q “single mode” cavity through which we pass accelerated two-level atoms as in Fig. 1a. We find that the radiation is thermal (in the typical case) and the effective “Boltzmann factor” is now given by \( \alpha/2\pi\omega \). For the above example, \( \alpha/2\pi\omega \sim 10^{-3} \), hence, it is many orders of magnitude larger than that for the usual Unruh effect and is potentially observable.

The envisioned experiment can be described as a kind of “acceleration radiation” mazer 3, 6. In the ordinary mazer, stimulated emission is the mechanism for the production of radiation. In the present case, the physics of the emission process is intimately association with the center-of-mass motion (taken in the z direction).

One scheme for accelerating 6 the atoms uses a particle accelerator with, e.g., hydrogen like ions. In such a case, ordinary (i.e. not Unruh) radiation emitted by accelerated charged particles must be taken into account. Alternatively, we could envision atoms accelerated in a strong gravitational field through a cavity. Other means of operation via periodically driven atoms are also possible as in Figs. 1(b,c) and are discussed later. For the moment, we simply assume the trajectories given by, e.g., Eq. (2) and neglect the quantization of translational motion and recoil effect.

Our main results are contained in Eqs. (4)-(9). We find that the acceleration radiation is generated by a kind of parametric process 3 in which both the atomic polarization (the idler) and the radiation (the signal) are excited by extracting energy from the atomic center-of-mass motion (the pump). Such processes are intimately related to the so-called counter-rotating terms in the atom-field interaction Hamiltonian and are discarded in the rotating wave approximation (RWA).

This provides a simple picture for the generation of acceleration radiation. The photons emitted are real. The generation of radiation by the counter-rotating terms is interesting; but, perhaps, no more bizarre than the ear-
lier demonstration of mazer emission due to scattering of atoms off the cavity interface. Furthermore, we find that the radiation may even be squeezed when $S_{1,2}$ in Eq. (8) are nonvanishing. Calculation details will be given elsewhere as will experimental implications.

As in the quantum theory of the laser, the (microscopic) change in the density matrix of a cavity mode due to any one atom, $δρ$, is small. The (macroscopic) change due to $ΔN$ atoms is then $Δρ = ∑_i δρ_i = ΔN δρ_i$. Writing $ΔN = r Δt$, where $r$ is the atomic injection rate, we have a coarse grained equation of motion: $Δρ/Δt = r δρ$. The change $δρ_i$ due to an atom injected at time $τ_i$ in the atomic rest frame is

$$δρ_i = -\frac{1}{ℏ^2} \int_{τ_i}^{τ_i+T} \int_{τ_i}^{τ_i+τ′} tr_{atom} \times \left[ \hat{V}(τ′), \left[ \hat{V}(τ″), ρ_{atom}(τ_i) ⊗ ρ(t(τ_i)) \right] \right] dτ′ dτ″,$$

where $T$ is the proper time of flight through the cavity and $tr_{atom}$ denotes the trace over atom states. The time $τ$ is the atomic proper time, i.e., the time measured by an observer riding along with the atom. The cavity proper time $t(τ)$ and the atomic trajectory of the atom as it passes through the cavity, $z(τ)$, are given by

$$t(τ) = t_0 + \frac{1}{c} \sinh(ατ), \quad z(τ) = \frac{c}{α} \left[ \cosh(ατ) - 1 \right], \quad (2)$$

where $t_0 = t(τ = 0)$ is the moment of time in the laboratory (cavity) frame when the atom starts its acceleration. The distinction between atomic and cavity field proper times is important. It is most convenient to calculate $δρ_i$ in the atomic frame. In the case of a running wave with a wave vector $k$, $k_z = k \cdot V/ν$, the atom-field interaction Hamiltonian in the atomic frame is given by

$$\hat{V}(τ) = h g(τ) \left( \hat{a}_k e^{-iωτ + i k_z z(τ)} + h.c. \right) \left[ \hat{σ} e^{-iωτ} + h.c. \right]. \quad (3)$$

Here $g(τ) = μE′/ℏ$ is the atom-field coupling frequency which depends on the atomic dipole moment $μ$ and the electrical field $E′$ in the frame of the atom. For simplicity, consider the case of the co-propagating atom and field, $k_z = |k| = ν/c$, so that $E′ = √{(c−ν)/(c+ν)} E$. Since $ν = c \tanh(ατ)$ for a uniformly accelerated particle, we have $E′ = e^{-ατ} E$ and $g(τ) = ge^{-ατ}$. The operator $\hat{a}_k$ is the annihilation operator for the running wave, while $\hat{σ}$ is the atomic lowering operator. Inserting Eq. (3) into Eq. (1) and using Eq. (2), we obtain the results given in Eqs. (4)-(8) below.

In the case of random injection times, the equation of motion for the density matrix of the field is

$$dρ_{nn}/dt = \frac{i ν}{ν} \left( \frac{α}{ω} \right)^{-iω} e^{iωτ} \hat{σ} \hat{σ} \left[ Γ(z, we^{-αT}) - Γ(z, u) \right], \quad (7b)$$

where $z = 1 \mp i\frac{α}{ω}$, $u = -i\frac{α}{ω} e^{-ατ}$, and $Γ(z, u) = \int_{u}^{∞} e^{-z x^2 + 1} dx$ is the incomplete gamma function.

$$\hat{V}(τ) = h g(τ) \left( \hat{a}_k e^{-iωτ + i k_z z(τ)} + h.c. \right) \left[ \hat{σ} e^{-iωτ} + h.c. \right]. \quad (3)$$

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The above analysis clearly shows that the mechanism of the fields and atom excitation in cavity quantum elec-

drodynamics is the same as for the Unruh effect in free space and is nothing but a nonadiabatic transition due to 

the counter-rotating term $\hat{a}_1^\dagger \hat{a}_2^\dagger$ in the interaction Hamiltonian (3), i.e. $V_2$. The reason for an enhanced excita-

tion in the cavity is the relatively large amplitude for a quantum transition $|b,0\rangle \rightarrow |a,1\rangle$ due to the sudden 

nonadiabatic switching on of the interaction. As a result of this rapid turn on, the initial state $|b,0\rangle$ is no longer 

an eigenstate of the Hamiltonian. Now, a linear super-

position of the excited states of atom and field makes up 

the dressed [14] ground state of the interacting system 

$\psi_0 = |b,0\rangle - \frac{\phi(\tau)}{\phi(\tau)}, |a,1\rangle$ as well as the dressed excited 

state $\psi_1 = |a,1\rangle + \frac{\phi(\tau)}{\phi(\tau)} |b,0\rangle$.

In particular, the amplitude of the bare excited state 

$|a,1\rangle$ in $\psi_0$ is of the order of $C \sim \mu E'/\hbar(\omega + \nu')$. 

The latter corresponds to the atomic excitation probability 

$\rho_{\text{atom}} = |C|^2 \sim |\mu E'/\hbar(\omega + \nu')|^2 \sim |gI_2|^2$. This can be 

also found directly from the density matrix equation for 

the atom, via the atomic counterpart to Eq. (1) with a 

trace over the photon states instead of the $\text{tr}_{\text{atom}}$. This probability has the same origin and value as the well-

known Bloch-Siegert shift of a two-level atomic transition 

[14], $\Delta \omega = (\mu E'/\hbar(\omega + \nu'))^2$, due to counter-rotating 

terms in the interaction Hamiltonian.

Clearly, the second term in Eqs. (6b) and (7a) rep-

resents the contributions from boundaries to the non-

adiabatic transition amplitudes. In the absence of the 

boundary contributions, the emission integral $I_2(\omega) = 

I_1(-\omega)$ in Eqs. (6) and (7) becomes exponentially small 

$\sim \exp(-\pi \omega/\alpha)$ for the small parameter $\alpha/2\pi \omega \ll 1$ 

since there are no stationary phase points in the inte-

gration interval. The absorption integral $I_1$ does have 

a point of stationary phase when the atomic frequency 

$\omega$ is brought into resonance with the field due to the 

time-dependent Doppler shift of the mode frequency 

$\nu(\tau) = \nu \exp(-\alpha \tau)$. This fact explains why the related 

exponential factor effectively disappears from the absorp-

tion integral (7a), $|\frac{\phi(\tau)}{\phi(\tau)}(1 - i\omega/\alpha)| \approx (2\pi \omega/\alpha)^1/2$, when 

$\alpha \ll 2\pi \omega$. As a result, if there are no edge effects, we ob-

tain the same excitation factor $R_2/R_1 = \exp(-2\pi \omega/\alpha)$ 

as in the Unruh effect (in free space). This means that in 

order to observe the standard Unruh result one has to ex-

tend the mode profile $g(z)$ near the boundaries, i.e., elimi-

nate nonadiabatic boundary contributions corresponding 

to the second term in Eq. (6b).

The nonadiabatic nature of the Unruh effect can be 

demonstrated most clearly by following explicit deriv-

ation of the Unruh factor as a probability of the non-

adiabatic transition $|0\rangle \rightarrow |a\rangle$ from the dressed ground 

state. Indeed, the Schrödinger equation $i\hbar \psi/\psi/\tau = 

H\psi$ in the two-level case $\psi = c_0|\psi_0\rangle + c_1|\psi_1\rangle$ yields 

d$\psi_0/\psi_1/\tau + i(E_1/H + |\psi_0/\psi_1\rangle)c_1 = -c_0/|\psi_0/\psi_1\rangle$. The 

difference between the eigenenergies is, to the first order, 

$E_1 - E_0 = \hbar(\omega + \nu')$. For small nonadiabatic coupling 

$-\langle\psi_0/\psi_1\rangle = \frac{d}{\tau} \frac{\phi(\tau)}{\phi(\tau)} \ll \omega + \nu'$, the perturbation solu-

tion is $|c_1|^2 = \int_\tau^{\tau'} \exp[i \int_{\tau}^{\tau'} (\nu' + \omega)\tau' \frac{d}{\tau} \left( \frac{\phi(\tau)}{\phi(\tau)} \right) d\tau']^2$.

If we now make the assumption of an adiabatic switching 

(on and off) of the interaction $g(\tau)$ as in standard Unruh 

effect treatments, then after integration by parts the lat-

ter integral is reduced to the integral $I_2(\omega) = I_1(-\omega)$ 

in Eqs. (6) but in the infinite limits, i.e. without 

edge effects. This yields the standard Unruh factor 

$|c_1|^2 \propto \exp(-2\pi \omega/\alpha)$. This derivation clearly shows the 

dramatic effect of boundary contributions leading to a 

large amplitude $g(\tau)(\omega + \nu')$ of the atomic excited state $|a\rangle$. Only if we eliminate the edge effects by adi-

abatic switching of the interaction, do we retrieve the 

exponentially small excitation factor.

The surprising result is that in the cavity, the excitation 

factor $\exp(-\hbar \nu/k_B T) \equiv R_2/R_1 = \alpha/2\pi \omega$ is deter-

mined by the first power of the same nonadiabaticity 

parameter $\alpha/2\pi \omega$. The reason for this effect is the existence of a true resonance, i.e., a stationary-phase point, in the 

absorption coefficient (the first term in the integral $I_1$ 

in Eqs. (6b) and (7a)). As mentioned earlier, this yields a 

resonance between the atomic transition frequency and 

the Doppler-shifted frequency of the field seen by the 

atom, $\omega + \frac{\phi(\tau)}{\phi(\tau)}(\nu - \gamma) \approx 0$, and is responsible for the 

aforementioned effect.

Another surprise of the cavity acceleration radiation is 

squeezing. If the atoms are injected at regular intervals of 

times, $t_i = \pi m_i/\nu + t_0$, where $m_i$ is an integer, all atoms 

have the same phase with respect to the cavity mode, 

$\Phi = \sum_{n=1}^{\Delta \mathcal{N}} e^{-2\pi m_1/2\nu t_0}/\mathcal{N} = e^{-2\pi t_0/\alpha}$, and instead of 

Eq. (4) we find

\begin{align}
\dot{\rho}_{n,n} &= -R_1 [n\rho_{nn} - (n + 1)\rho_{n+1,n+1}] \\
&\quad -R_2 [(n + 1)\rho_{nn} - n\rho_{n-1,n-1}] \\
&\quad +[-S_1 \sqrt{(n + 1)(n + 2)}\rho_{n+2,n} - S_2 \sqrt{(n - 1)n}\rho_{n,n-2} \\
&\quad +\{S_1 + \sqrt{(n + 1)n}\rho_{n+1,n+1} + n \text{ h.c.}\}].
\end{align}

In this case the analysis is similar to the analysis of a polarization injected laser [12], and the radiation density 

matrix is far from being thermal due to squeezing factors

\begin{align}
S_{1,2} &= r g^2 \Phi e^{-2\pi t_0/\alpha} \int_{t_1}^{t+n+1} d\tau' \int_{t_1}^{t+n+1} d\tau'' \times \\
&\quad e^{i\frac{\pi}{2} e^{-\alpha \tau'} \pm i\omega \tau' - \alpha \tau''} e^{i\frac{\pi}{2} e^{-\alpha \tau''} \pm i\omega \tau'' - \alpha \tau''}.
\end{align}

It is also possible to implement a more powerful res-

onant emission by ground state atoms in a cavity, e.g. 

when the center of mass of the atom is oscillating as 

$z(\tau) = z_0 \cos(\omega_0 \tau)$, Fig. 1(b). This can be viewed as 

another example of mazer action. In such a case, the 

density matrix of a cavity mode is again found to obey 

Eq. (8) but now

\begin{align}
R_{1,2} &\cong \frac{r g^2}{(\gamma + \alpha)^2 J_p(k z_0)} J_p(k z_0),
\end{align}

where $p$ is an integer, $J_p(x)$ the Bessel function, $\gamma$ the 

effective atomic decay rate; and the squeezing terms
$S_{1,2}$ are governed by cross terms which go as $(rg^2/\alpha)J_pJ_0$. Since this is a resonant parametric process, the absorption ($p = 0$, $\omega = \nu$) and emission ($p \neq 0$, $\omega + p = \nu\omega_0$) coefficients (9) are larger than for counter-rotating interactions, Eqs. (6) and (7a), by a resonant factor $|\nu/\alpha|^2$. In this case, parametric generation is possible [18] (see [11] for details).

Concluding remarks. Our simple model clearly demonstrates that the ground state atoms accelerated through a vacuum-state cavity radiate real photons. For relatively small acceleration $\alpha < 2\pi\omega_c$, the excitation Boltzmann factor $\exp(-\hbar\nu/kBT_c) \sim \alpha/2\pi\omega$ is much larger than the standard Unruh factor $\exp(-2\pi\omega/\alpha)$. The physical origin of the field energy in the cavity and of the real internal energy in the atom is, of course, the work done by an external force driving the center-of-mass motion of the atom against the radiation reaction force. Both the present effect (in a cavity) and standard Unruh effect (in free space) originate from the transition of the ground state atom to the excited state with simultaneous emission of photon due to the counter-rotating term $\hat{a}_+^\dagger\hat{\sigma}^+$ in the time-dependent Hamiltonian (3). The enhanced rate of emission into the cavity mode comes from the second term in Eqs. (6b) and (7a)–the nonadiabatic transition at the boundaries of the cavity; the standard Unruh excitation comes from the first term in Eqs. (6b) and (7a)– the nonadiabatic transition in free space due to the time dependence of the Doppler-shifted field frequency $\nu' = \nu e^{-\alpha\tau}$, as seen by the atom in the course of acceleration.

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[3] For example, the acceleration experienced by He$^+$ in a particle acellerator yielding a field $\sim 10^9$ V/m.

[4] The frequency $\omega$ is lower bounded (for a possible experiment) by cryogenic technology and the requirement that the effect should not be obscured by “hot” walls.


[10] M. Scully et al., to be published.

[11] V. Kocharovsky et al., to be published.


[15] For brevity, we keep only the main terms in the expressions for the eigenstates. For details, see, e.g., S. Swain, J. Phys. A6, 1919 (1973).

[16] The Doppler-shifted frequency of the field, as it is seen by the atom is $\nu' = d(\nu(t(\tau) - z(\tau)))/d\tau = \nu e^{-\alpha\tau}$. For the case of a counter-propagating wave one has to change the sign of $\alpha$ in $e^{-\alpha\tau}$ and related Eq. (6).

[17] For the review on nonadiabatic transitions see, e.g., V.V. Zheleznyakov, V.V. Kocharovsky, and V.I. Kocharovsky, Sov. Phys. Usp. 26, 877 (1983).

[18] Remarkably, as the analysis of the result (6) shows, parametric gain is possible in cavity QED via counter-resonant emission by ground state atoms with random injection times. For such gain to occur, the time of flight $T$ should be tuned to a set of specific values to ensure that the atom emits into the cavity mode more energy than it takes away, $R_2 > R_1$. The counter-propagating mode is more favorable for gain since the absorption can then be anomalously small while the gain remains as large as for the co-propagating mode. The gain can be easily seen in the case of a constant velocity:

$$\frac{R_2}{R_1} = \left| \frac{\nu' - \omega}{\nu' + \omega} \right|^2 \left| \frac{1 - e^{-i(\nu' + \omega)T}}{1 - e^{-i(\nu' - \omega)T}} \right|^2, \quad \nu' = \nu - \frac{k \cdot v}{\nu + k \cdot v} \right|^{1/2}.$$
$|e^{-i(\nu' + \omega)T} - 1| \sim 1$. A similar time of flight tuning is used in some electronic devices, e.g., klystrons.