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Estimating the Spin of Stellar-Mass Black Holes via Spectral Fitting of the X-ray Continuum

Rebecca Shafee¹, Jeffrey E. McClintock², Ramesh Narayan², Shane W. Davis³, Li-Xin Li⁴,
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ABSTRACT

We fit X-ray spectral data in the thermal dominant or high soft state of two dynamically confirmed black holes, GRO J1655-40 and 4U 1543-47, and estimate the dimensionless spin parameters $a_* \equiv a/M$ of the two holes. For GRO J1655-40, using a spectral hardening factor computed for a non-LTE relativistic accretion disk, we estimate $a_* \sim 0.75$ and $\sim 0.65 - 0.75$, respectively, from *ASCA* and *RXTE* data. For 4U 1543-47, we estimate $a_* \sim 0.75 - 0.85$ from *RXTE* data. Thus, neither black hole has a spin approaching the theoretical maximum $a_* = 1$.

Subject headings: X-ray: stars — binaries: close — accretion, accretion disks — black hole physics — stars: individual (4U 1543-47, GRO J1655-40)

1. INTRODUCTION

An astrophysical black hole is completely defined by two numbers that specify its mass and spin. The masses of 20 accreting black holes located in X-ray binary systems have been determined or constrained by dynamical optical studies (McClintock & Remillard 2005, hereafter MR05; Casares et al. 2004; Orosz et al. 2004). However, no reliable measurements have been reported so far of the dimensionless black hole spin parameter $a_* \equiv a/M$, where $a = J/cR_g$, J is the angular momentum of the black hole, M is its mass, and $R_g = GM/c^2$.

In this work, we estimate a_* of two black hole binaries by fitting their X-ray thermal continuum spectra using a fully relativistic model of a thin accretion disk around a Kerr

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black hole (Li et al. 2005). The model includes all relativistic effects such as frame-dragging, Doppler boosting, gravitational red shift, and bending of light by the gravity of the black hole. It also includes self-irradiation of the disk, the effects of limb darkening, and a spectral hardening factor f_{col} to relate the color temperature T_{col} and the effective temperature T_{eff} of the disk emission: $f_{\text{col}} = T_{\text{col}}/T_{\text{eff}}$ (Shimura & Takahara 1995, hereafter ST95; Merloni, Fabian & Ross 2000; Davis et al. 2005, hereafter D05).

In order to estimate the black hole spin by fitting the broadband X-ray spectrum, one must know the mass M of the black hole, the inclination i of the accretion disk (which we assume is the same as the inclination of the binary system, but see Maccarone 2002), and the distance D to the binary (Zhang, Cui & Chen 1997). Accordingly, we have selected two black hole binaries, GRO J1655-40 (hereafter J1655) and 4U 1543-47 (hereafter U1543), for which all three of these quantities have been well determined from optical observations: for J1655, $M = 6.30 \pm 0.27 M_{\odot}$, $i = 70^{\circ}2 \pm 1^{\circ}2$, $D = 3.2 \pm 0.2$ kpc, and for U1543, $M = 9.4 \pm 1.0 M_{\odot}$, $i = 20^{\circ}7 \pm 1^{\circ}5$, $D = 7.5 \pm 1.0$ kpc (Orosz et al. 2003, and private communication; Hjellming & Rupen 1995).

2. OBSERVATIONS AND DATA REDUCTION

We consider only those X-ray data that were obtained in the thermal dominant state, or TD state (formerly high soft state), for which $> 75\%$ of the 2–20 keV flux is supplied by the accretion disk (MR05). This state is consistent with a simple multicolor blackbody model (Gierlinski & Done 2004), making it amenable to theoretical modeling. We carried out all data analysis and model fits using XSPEC *version 12.2.0* and HEASOFT *version 5.2*.

The continuum X-ray spectrum of J1655 was observed by the *Advanced Satellite for Cosmology and Astrophysics (ASCA)* on 1995 August 15 (Ueda et al. 1998; hereafter U98) and on 1997 between February 25 and 28 (Yamaoka et al. 2001; hereafter Y01), with total exposure times of ≈ 19 ks and ≈ 100 ks, respectively. The source was very bright during both observations (2.3 Crab in 1995 and 1.2 Crab in 1997), and we therefore analyzed only the data from the GIS2 and GIS3 detectors. Starting with the unscreened *ASCA* data files obtained from the HEASARC, we followed as closely as possible the data reduction procedures and criteria mentioned in U98 and Y01. As in Y01, we added a systematic error of 2% to the GIS spectra to account for calibration uncertainties. As a check on our reduction procedures, we fitted both the 1995 and 1997 spectra using the disk blackbody models employed by U98 and Y01, and we succeeded in closely reproducing all of their published results.

J1655 was also observed with the *Rossi X-ray Timing Explorer (RXTE)* in 1997 (Sobczak

et al. 1999), including an observation on February 26 which was performed simultaneously with ASCA. Our second source, U1543, was observed with *RXTE* during its 2002 outburst (Park et al. 2004; hereafter P04). For both *RXTE* data sets, the data reduction procedures are identical to those described by P04. In brief, we used the “Standard 2 mode” data from PCU-2 only. The event files and spectra were screened and the background spectra and a response files created. Systematic errors of 1% were added to all the PCU-2 energy channels. Referring to Figure 2 in Remillard (2005), we selected the contiguous group of 31 observations extending from MJD 50453.6 to MJD 50663.7 for which the source J1655 was in the TD state. Because the data span 210 days, we created and used several different response files. For U1543, from among the 49 observations considered by P04 (see their Table 1), we selected the 34 observations (Obs. nos. 1-3, 5-19, 27-42) for which the source was in the TD state (MR05).

3. DATA ANALYSIS

RXTE long fits: First we fitted the *RXTE* pulse-height spectra of J1655 and U1543 in the 2.8–25.0 keV range. These fits, which we refer to as the “long fits,” were made using a spectral model comprised of three principal components: *kerrbb*, which models a relativistic accretion disk (Li et al. 2005), a standard low-energy absorption component (*phabs*), and a simple power-law component (*power*). In addition, following the work of Y01, we found it necessary to add three edge/line features in order to obtain acceptable fits: (1) a smeared Fe edge (*smedge*) with edge energy restricted to the interval 6.8–9.0 keV and width fixed at 7 keV; (2) a sharp absorption edge with edge energy restricted to the interval 9–11 keV; and (3) a Gaussian absorption line that was used solely for J1655. The central energy of this line was restricted to the range 6.4–7.0 keV, and its width was fixed at 0.5 keV. (The width was determined by our analysis of the *ASCA* data, see below). We fixed the equivalent neutral hydrogen column density N_H at $0.7 \times 10^{22} \text{cm}^{-2}$ for J1655 (Y01) and $0.4 \times 10^{22} \text{cm}^{-2}$ for U1543 (P04).

RXTE short fits: Since we are interested in the disk component of the spectrum, we analyzed the same data over restricted energy ranges that are dominated by the thermal component, thereby generating “short fits.” For all these short fits, we used the three principal componenets used in the long fits. For the J1655 data we also used the gaussian line feature mentioned above. We explored several different upper energy limits for the *RXTE* spectra, finally choosing 2.8–7.5 keV for J1655 and 2.8–7.0 keV for U1543. We determined the upper limit by the highest energy that allowed the exclusion of the *smedge* component required by the long fits (for which this feature had a mean edge energy of 7.7 keV for

J1655 and 7.0 keV for U1543). The 7.5 keV limit for J1655 (vs. 7.0 keV) was required to ensure that the parameters of the Fe absorption line feature present between 6.4 and 7.0 keV were well-determined. Twenty-six out of 31 of the J1655 short fits (including the 1997 Feb 26 spectrum featured in Fig. 1) succeeded without a power-law component (i.e., $\chi^2_\nu < 1$); furthermore, its inclusion did not improve these fits significantly. However, in the case of U1543 (which on average had a power-law to total flux ratio of 0.15, compared to 0.06 for J1655), the power-law component was essential in fitting all 34 of the spectra. In order to be consistent, the power-law component was included in all short fits of both sources with parameters fixed to the values determined from the long fits.

ASCA short fits: The *ASCA* data for J1655 were analyzed only over a short energy range, which was chosen to be 1.2–7.5 keV, because the bandwidth of the GIS detectors is too limited to constrain the power-law component. For 1997 *ASCA* we modelled the spectra using *kerrbb*, *phabs* and the gaussian absorption line mentioned earlier. For the 1995 data we used only *kerrbb* and *phabs*.

In the analysis of the *RXTE* data, we found it necessary to correct the fluxes downward as follows. For U1543, the Crab flux calculated using the P04 response file exceeded the flux predicted by the standard Crab spectrum of Koyama et al. (1984) by a factor of 1.174. For J1655, we used a current response matrix and found a much smaller correction factor of 1.034.

In fitting the data using *kerrbb*, we fixed M , i and D to their mean observed values (§1). Also, we switched on limb-darkening (lflag=1) and returning radiation effects (rflag=1). We set the torque at the inner boundary of the accretion disk to zero, fixed the normalization to 1 (as appropriate when M , i and D are held fixed), and allowed the mass accretion rate \dot{M} to vary freely. Of the two remaining parameters, viz., the spectral hardening factor f_{col} and the black hole spin a_* , we held one or the other fixed and fitted the other.

4. RESULTS

Figure 1 shows the results of analyzing the 1995 and 1997 *ASCA* data on J1655 with *kerrbb*. In this analysis, we kept f_{col} fixed at selected values and fitted a_* and \dot{M} . We see that, for a given value of f_{col} , the data are able to determine a_* accurately. The χ^2_ν values are acceptable, and there is reasonable agreement between the GIS2 and GIS3 results and between the 1995 and 1997 data. There is agreement also with the simultaneous *RXTE* observation done on 1997 Feb 26. However, since the χ^2_ν is acceptable for all the values of f_{col} , it is clear that the data by themselves cannot constrain f_{col} . We thus need an independent

theoretical determination of f_{col} if we wish to estimate a_* .

ST95 were among the first to calculate model disk atmospheres for black hole binaries, including full radiative transfer and Comptonization. From these models they estimated f_{col} as a function of the bolometric disk luminosity L_{bol} . Roughly, they found $f_{\text{col}} \approx 1.7 + 0.2(\log \ell + 1.25)$, where $\ell \equiv L_{\text{bol}}/L_{\text{Edd}}$ is the Eddington-scaled disk luminosity and $L_{\text{Edd}} = 1.5 \times 10^{38} (M/M_{\odot})$. Using the observed luminosities of J1655 during the *ASCA* 1995 and 1997 observations, viz., $\log \ell \sim -0.85, -1.02$, we have estimated the respective values of f_{col} according to the ST95 model. These are shown by the dotted lines marked ST95 in Figure 1. The last column of Table 1 shows the corresponding estimates of a_* .

Recently, D05 computed more detailed disk atmosphere models which improve upon the pure hydrogen atmospheres of ST95 by including metal opacities. Metals tend to reduce the spectral hardening. Therefore, the D05 values of f_{col} are generally smaller than those of ST95. For the analysis presented in this paper, we used D05’s code to calculate a grid of values of f_{col} as a function of ℓ and a_* . The calculations were done for the specific inclinations of J1655 and U1543, and for two values of the disk viscosity parameter, $\alpha = 0.01, 0.1$. Figure 1 shows the range of f_{col} values predicted by the D05 model for the 1995 and 1997 *ASCA* observations, and the second to last column of Table 1 gives the corresponding estimates of a_* . While we present in this paper results for both the ST95 and D05 models of spectral hardening, we view the latter as more reliable.

Figure 2 shows the short-fit results (§3) of all the 31 *RXTE* observations of J1655 in the TD state. Here we have assumed various values of a_* , and computed for each observation the best-fit values of f_{col} and ℓ (the latter is obtained from \dot{M} and a_*). We then compare the data-fitted values of f_{col} with the model predictions of D05 and ST95. The comparison with the D05 model indicates that the spin of J1655 is likely to lie in the range $a_* \sim 0.65 - 0.75$; this result is entered in Table 1. It is interesting to note that the D05 model gives nearly identical results for $\alpha = 0.01$ and 0.1 so long as $\log \ell \lesssim -1$, but shows noticeable variations with α when the disk is more luminous. Since the true α of the disk is not known (it is likely to be in the range 0.01 to 0.2), and moreover since disks tend to be radiation pressure dominated at higher luminosities thereby introducing additional uncertainties, we give greater weight to the observations for which $\log \ell < -1$. Thus, we ignore the 1995 *ASCA* data ($\log \ell \sim -0.85$), and give more weight to the 1997 data ($\log \ell \sim -1.02$). We also favor the GIS2 results because the GIS3 spectra have a prominent feature between 1 and 2 keV that cannot be eliminated either by using the XSPEC “gain” command and/or by adding an ad hoc absorption edge to the fit.

Figure 3 shows a similar analysis for U1543. Here we found that 7 of the 34 observations in the TD state (MJD 52444.515-52449.11) gave poor fits for the short range due to an edge-

like feature between 4 and 5 keV which is not present in other observations. The long fits for these data still gave reasonable fits with $\chi^2_\nu < 2$. Comparing the fitted values of f_{col} with the D05 model, we estimate $a_* \sim 0.75 - 0.85$. Once again we focus on the lower-luminosity data with $\log \ell < -1$.

The disk parameters obtained from the short and long fits agree very well in all cases, with typical differences in $\log \ell$ and f_{col} of order 0.003 and 0.005 respectively; the uncertainties in the fitted values are considerably larger (~ 0.01 and $\sim .02$). Also to check whether the different energy ranges of the *ASCA* and *RXTE* data is an issue, we reanalyzed the *ASCA* data using only the restricted energy range 2.8–7.5 keV and found the results agree. We also explored the effect of varying the values of M , D and i over one standard deviation in either direction (§1). We find that the derived a_* values lie within the ranges given in Table 1. We note that system parameters need to be measured with high accuracy for this method to succeed. For example, we attempted to analyze the BH candidate XTE J1550-564 using the methods described here, but since the distance to this source is quite uncertain, $D = 5.9^{+1.7}_{-3.1}$ (Orosz et al. 2002), we were unable to obtain any useful constraint on a_* .

5. SUMMARY AND CONCLUSIONS

The method of estimating spin that we have employed was pioneered by Zhang et al. (1997; see also Gierlinski et al. 2001). However, only recently have the necessary data analysis tools (*kerrbb*, Li et al. 2005) and disk atmosphere models (D05) been developed to the point where the method may be applied with some confidence.

Effectively, in this technique, one determines the radius R_{in} of the inner edge of the accretion disk and assumes that this radius corresponds to the innermost stable circular orbit (R_{ISCO}). Since R_{ISCO}/R_g is a monotonic function of a_* , a measurement of R_{in} and M directly gives a_* . Provided that (i) i and D are known to sufficient accuracy, (ii) the X-ray flux and spectral temperature are measured from well calibrated X-ray data in the TD state, and (iii) the disk radiates as a blackbody, it is clear that R_{in} can be estimated. However, the disk emission is not a true blackbody but a modified blackbody with a spectral hardening factor f_{col} . Therefore, the observations only give the quantity $R_{\text{in}}/f_{\text{col}}^2$, and we need an independent estimate of f_{col} in order to estimate a_* . We have tailored the state-of-the-art disk atmosphere model of D05 to obtain estimates of f_{col} for this work.

Our results in brief are as follows. By fitting *ASCA* and *RXTE* spectral data on the black hole X-ray binary J1655, we estimate the dimensionless spin parameter of the black hole to be $a_* \sim 0.65 - 0.75$ (Table 1). In the case of U1543 we estimate $a_* \sim 0.75 - 0.85$,

though this is based on only *RXTE* data. To obtain these estimates, we have focused on observations for which the disk luminosity was relatively low ($\log \ell < -1$), and we have used the D05 model for f_{col} (though for completeness we give results also for the ST95 model in Table 1 and in the Figures).

Based on these results, we consider it unlikely that either J1655 or U1543 has a spin close to the theoretical maximum for a rotating black hole, $a_* = 1$. Even $a_* = 0.85$, which is the largest value we find, corresponds to a quite moderate spin, as one can see by considering the binding energy per unit mass of a particle in the last stable circular orbit: For a_* in the range 0 to 1, this quantity varies from 5.7% to 42.3%, whereas it is only 13.6% for $a_* = 0.85$. Moreover, most systematic effects that one might consider only push our estimates of a_* down. If α is larger than 0.1, as suggested by some studies of white dwarf disks, it would cause our estimates of a_* to decrease (see Figs. 2, 3). Similarly, if we allow a non-zero torque at the inner edge of the disk and/or allow the disk to radiate inside R_{ISCO} (Krolik 1999), a_* would decrease still further.

What spin might we expect a black hole to accrue due to disk accretion alone? If the hole accretes long enough to achieve spin equilibrium, then the limiting a_* is likely to be in the range $\sim 0.9 - 0.998$ (Gammie, Shapiro & McKinney 2004, and references therein). However, X-ray binaries rarely live long enough for such equilibrium to be established. U1543, for instance, contains a $\approx 2.5M_{\odot}$ main-sequence secondary (Orosz et al. 1998), and thus the age of the system is $\lesssim 1$ Gyr. Based on the X-ray fluences from the 1971, 1983, 1992 and 2002 outbursts, the average mass accretion rate is $\sim 1 \times 10^{-9}M_{\odot} \text{ yr}^{-1}$ for $D = 7.5$ kpc (Chen, Shrader, & Livio 1997; P04). At this rate, the black hole will accrete at most $1 M_{\odot}$ during the lifetime of the system and thus, assuming that its natal spin is zero, the spin today should be $a_* \lesssim 0.35$, considerably less than our estimate of $a_* \sim 0.75 - 0.85$. This suggests that our measurements are sensitive to the natal spins of these black holes. It is then interesting that neither of the two holes has a spin close to either 0 or 1.

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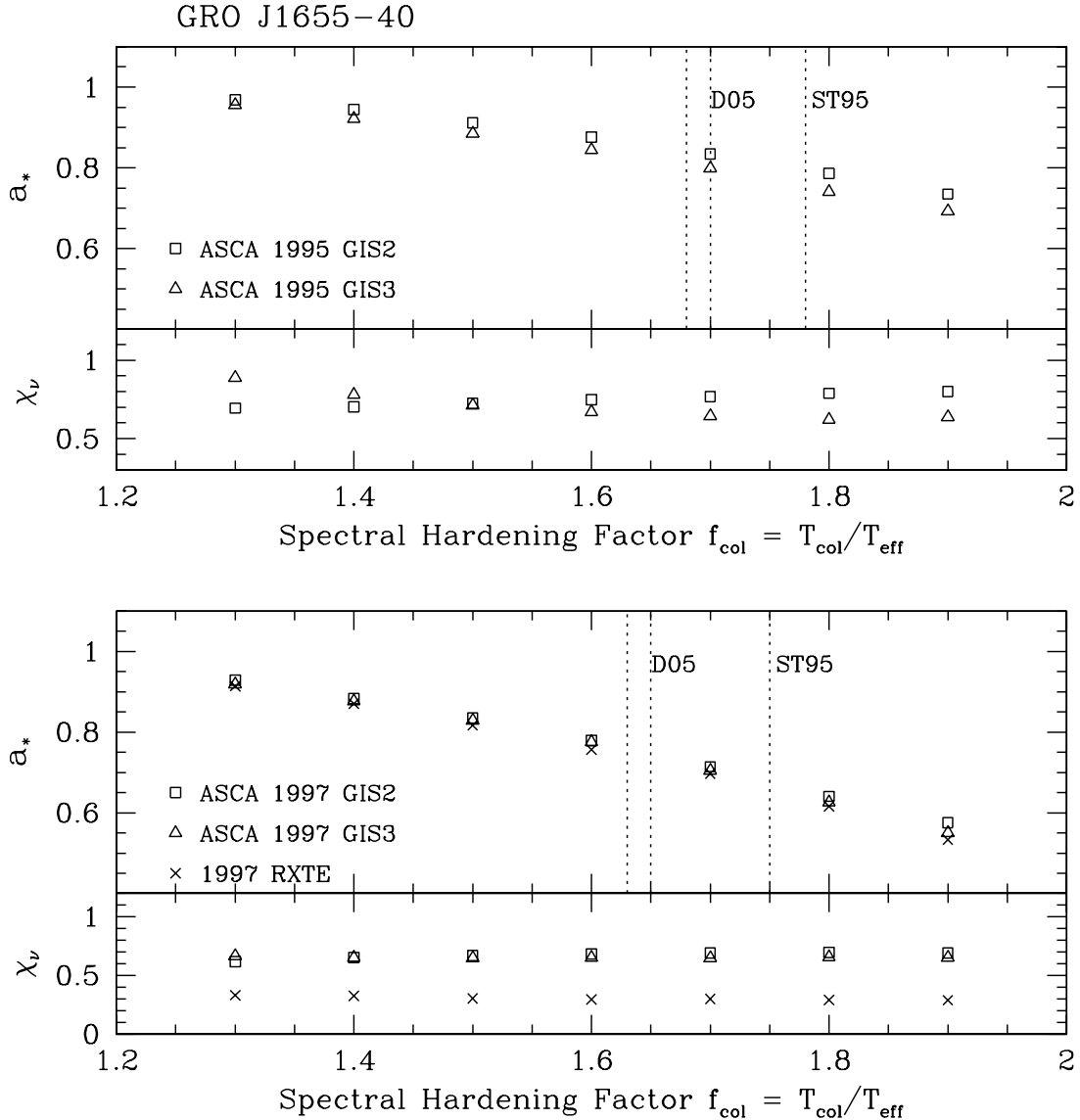


Fig. 1.— [Top] GRO J1655-40 *ASCA* 1995 spectra fitted with different fixed values of the spectral hardening factor f_{col} . The upper panel shows the values of a_* obtained for each value of f_{col} , and the lower panel shows the corresponding reduced χ_r^2 (84 dof). The vertical dotted lines indicate the likely values of f_{col} according to the D05 and ST95 models. [Bottom] Corresponding results for *ASCA* 1997 and *RXTE* 1997 Feb 26 spectra. Errorbars in a_* are very small on average ($\sim .005$) and are not shown for clarity. The *ASCA* fits are for 81 dof and the *RXTE* fits are for 8 dof.

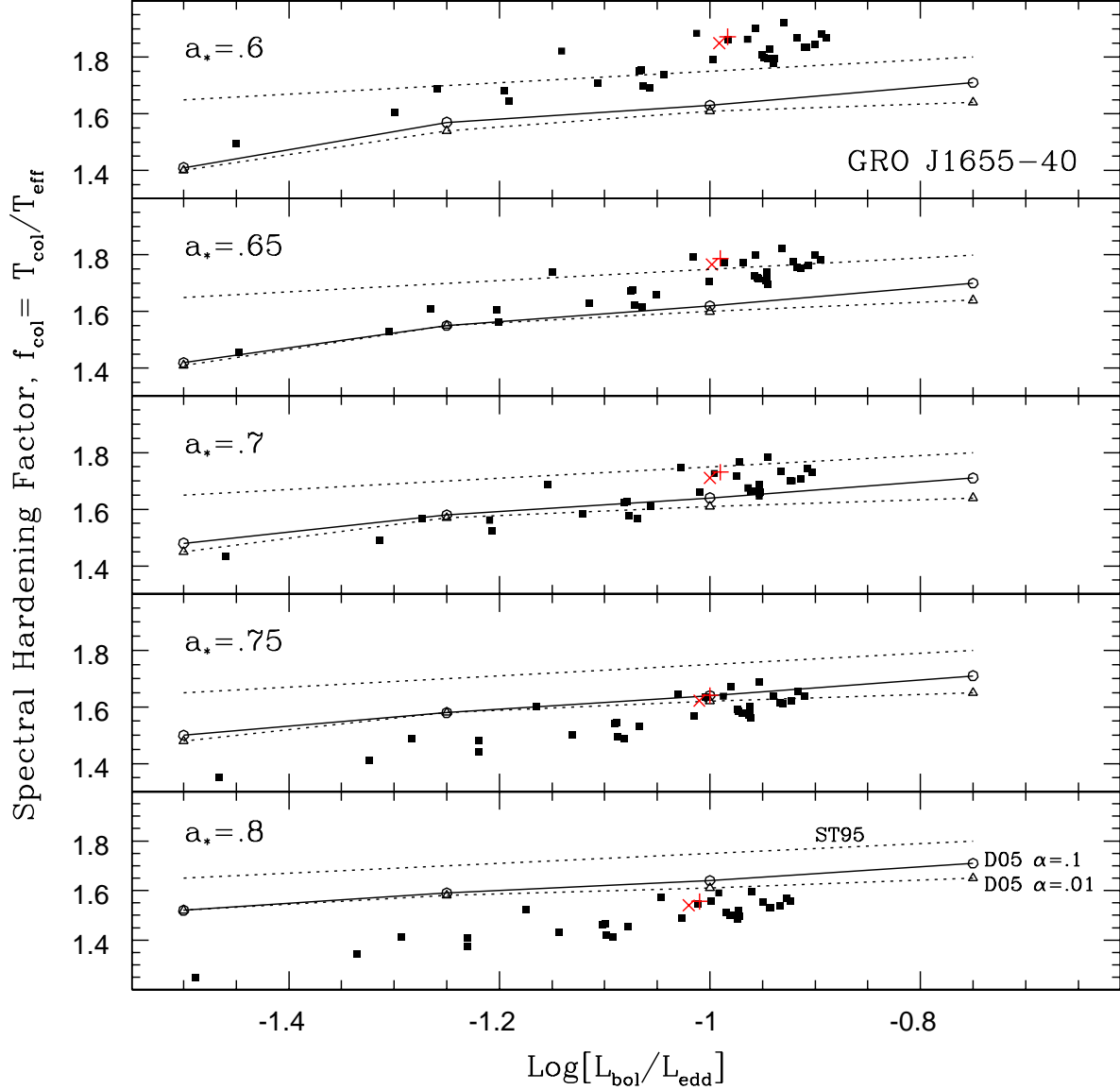


Fig. 2.— J1655 *RXTE* spectra fitted for five different fixed values of a_* . The solid squares show the fitted values of the spectral hardening factor f_{col} and the dimensionless luminosity ℓ obtained with short fits. The *ASCA* 1997 GIS2 point is shown as a red plus and the GIS3 point as a red cross. The lines show the calculated values of f_{col} from the D05 model for $\alpha = 0.01$ and 0.1 and from the ST95 model. The solid line is for $\alpha = 0.1$, the value we emphasize. The D05 model constrains a_* of J1655 to lie in the range $0.65 - 0.75$. Errorbars in f_{col} (typically ~ 0.02) are not shown. The fits are for 8 dof with χ^2_ν typically ~ 0.3 .

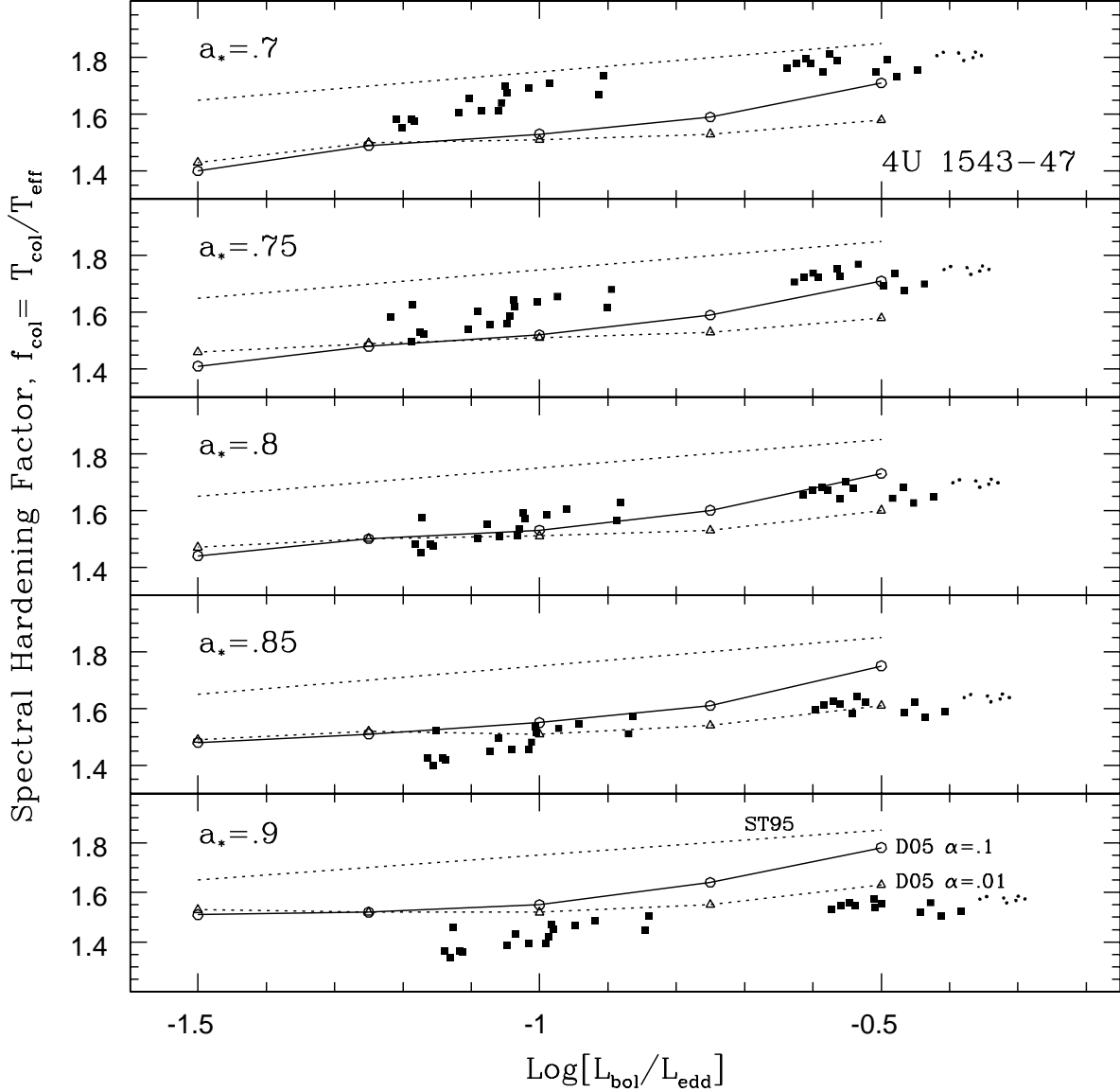


Fig. 3.— U1543 *RXTE* spectra fitted for five different fixed values of a_* . The data points and lines have the same meanings as in Fig. 2. The 7 dots on the extreme right of each panel correspond to spectra that contained an absorption edge like structure between 4 and 5 keV and gave fits with $\chi^2_\nu > 2$. The D05 model constrains a_* of U1543 to lie in the range 0.75 – 0.85. The fits are for 7 dof with χ^2_ν typically ~ 0.3 .

TABLE 1

Black Hole Spin Estimates Using The Mean Observed Values of M , D , and i

Candidate	Observation Date	Satellite	Detector	a_* (D05)	a_* (ST95)
GRO J1655-40	1995 August 15	ASCA	GIS2	~ 0.85	~ 0.8
GRO J1655-40	1995 August 15	ASCA	GIS3	~ 0.80	~ 0.75
GRO J1655-40	1997 February 25-28	ASCA	GIS2	$\sim \mathbf{0.75}$	~ 0.70
GRO J1655-40	1997 February 25-28	ASCA	GIS3	$\sim \mathbf{0.75}$	~ 0.7
GRO J1655-40	1997 February 26	RXTE	PCA	$\sim \mathbf{0.75}$	~ 0.65
GRO J1655-40	1997(several)	RXTE	PCA	$\mathbf{0.65-0.75}$	0.55–0.65
4U 1543-47	2002(several)	RXTE	PCA	$\mathbf{0.75-0.85}$	0.55–0.65