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MACROSCOPIC AND MICROSCOPIC ENTROPY OF NEAR-EXTREMAL SPINNING BLACK HOLES

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Abstract

A seven parameter family of five-dimensional black hole solutions depending on mass, two angular momenta, three charges and the asymptotic value of a scalar field is constructed. The entropy is computed as a function of these parameters both from the Bekenstein-Hawking formula and from the degeneracies of the corresponding D-brane states in string theory. The expressions agree at and to leading order away from extremality.

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1. Introduction

In recent months string theory has demonstrated a remarkable and detailed knowledge of black hole thermodynamics. In [1] it was found that the newly-understood rules [2–8] for counting degeneracies of BPS-saturated, D-brane soliton bound states precisely reproduces the Bekenstein-Hawking entropy for a certain five-dimensional extremal Reissner-Nordstrom black hole. These results were extended to leading order above extremality in [9,10], solidifying the identification of the microscopic states responsible for the entropy. In [11] it was shown that the stringy degeneracies continue to match the extremal Bekenstein-Hawking entropy when rotation is added. Ordinarily the addition of rotation (without energy) to an extremal Reissner-Nordstrom black hole destabilizes the horizon and yields a naked singularity. However string theory, in order to avoid a conflict with the microscopic counting, cleverly stabilizes the horizon with the help of a Chern-Simon coupling in the low-energy field theory. In the process a qualitatively new class of supersymmetric spinning black hole solutions was found[11].

In this paper we will combine the analyses of [10,11] and consider these new spinning solutions just above extremality. Again we will find perfect agreement - a seven parameter fit - between the detailed thermodynamic behavior predicted by the Bekenstein-Hawking entropy and by the microscopic state counting.

2. The Rotating Nonextremal Black Hole

The low-energy action for six-dimensional type IIB string theory contains the terms

$$\frac{1}{16\pi}\int d^6x\sqrt{-g}(R-(\nabla\phi)^2-\frac{1}{12}e^{2\phi}H^2) \tag{2.1}$$

in the six-dimensional Einstein frame. H denotes the RR three form field strength. We adopt conventions in which $G_N = 1$. The scalar ϕ here is the logarithm of the volume of the internal four-manifold in the string frame. The ten-dimensional string dilaton is an arbitrary constant for our solutions and is suppressed. We will further compactify to five dimensions by periodically identifying $y \sim y + L$, where y denotes the fifth spatial coordinate. We take the asymptotic length L of the compact dimension to be very large.

The solutions of interest to us are most simply represented as six-dimensional black string solutions to (2.1), which wind around the y direction and hence are black holes in five dimensions. The six-dimensional black string can can carry both electric and magnetic charge with respect to H :

$$\begin{aligned} Q_+ &\equiv \frac{1}{8}\int_{S^3} e^{2\phi}\ \ *H, \\ Q_- &\equiv \frac{1}{4\pi^2}\int_{S^3} H\ . \end{aligned} \tag{2.2}$$

It may also carry total ADM momentum P along the y direction which appears in five dimensions as an electric charge [10]:

$$P \equiv \frac{2\pi n}{L}\ . \tag{2.3}$$

We have chosen our conventions so that n and $Q_-Q_+ \equiv \frac{1}{2}Q^2$ are integers. In five spacetime dimensions the spatial rotation group is $SU(2) \times SU(2)$. Hence solutions are in addition labeled by two angular momenta.

Black string solutions are also characterized by the asymptotic value of ϕ. We are primarily interested in the entropy which cannot depend on the asymptotic value of ϕ [12–15]. For a special asymptotic value ϕ_h, the sources for ϕ cancel exactly and the equations of motion imply ϕ is constant everywhere. This special value is

$$e^{2\phi_h} = \frac{2Q_+}{\pi^2 Q_-} \ . \tag{2.4}$$

In order to compute the entropy it is sufficient to consider the solutions with $\phi = \phi_h$.

Reduction from six to five dimensions yields a second five dimensional scalar field whose asymptotic value is L, the size of the S^1 parameterized by y. This scalar could also be frozen to a value which would be proportional to n/Q. However it is important not to freeze this field because we will need to compute how the entropy varies as a function of both the energy and n with all other quantities - in particular the asymptotic values of the fields - held fixed. This is impossible to do if the value of the scalar field is tied to n/Q. This problem does not arise for the scalar ϕ because, once the behavior of the entropy is known for any value of the ratio Q_+/Q_-, it is determined for any other value by duality which implies that it can depend only on the product $Q^2/2$.

The solutions of interest can be generated by beginning with the five dimensional Kerr solution which spins in two independent planes. This may be lifted to a black string solution of heterotic string theory in six dimensions by adding a trivial flat direction y. A boost is performed mixing the time direction t with an internal direction to yield a nontrivial right-handed gauge field. Next, by applying string-string duality followed by a T-duality transformation, one obtains a black string solution of Type IIB string theory in six dimensions. Lastly, a boost is performed along the string yielding the following solution

$$
\begin{aligned}
ds_6^2 = &-\left[1-\frac{(r_+^2\cosh^2\alpha - r_-^2\sinh^2\alpha)}{\rho^2}\right]dt^2 + \left[1-\frac{(r_-^2\cosh^2\alpha - r_+^2\sinh^2\alpha)}{\rho^2}\right]dy^2 \\
&+\sin^2\theta\left[r^2+a^2+\frac{(a^2r_+^2-b^2r_-^2)\sin^2\theta}{\rho^2}\right]d\varphi^2 \\
&+\cos^2\theta\left[r^2+b^2+\frac{(b^2r_+^2-a^2r_-^2)\cos^2\theta}{\rho^2}\right]d\psi^2 \\
&+\rho^2 d\theta^2 + \frac{\rho^2}{r^2}\left[\left(1-\frac{r_-^2}{r^2}\right)\left(1-\frac{r_+^2-a^2-b^2}{r^2}\right)+\frac{a^2b^2}{r^4}\right]^{-1}dr^2 \\
&+2\sin^2\theta\frac{(ar_+^2\cosh\alpha - br_-^2\sinh\alpha)}{\rho^2}dtd\varphi \\
&+2\sin^2\theta\frac{(ar_+^2\sinh\alpha - br_-^2\cosh\alpha)}{\rho^2}dyd\varphi \\
&+2\cos^2\theta\frac{(br_+^2\cosh\alpha - ar_-^2\sinh\alpha)}{\rho^2}dtd\psi \\
&+2\cos^2\theta\frac{(br_+^2\sinh\alpha - ar_-^2\cosh\alpha)}{\rho^2}dyd\psi \\
&+2\cosh\alpha\sinh\alpha\frac{(r_+^2-r_-^2)}{\rho^2}dtdy + 2\cos^2\theta\sin^2\theta\frac{ab(r_+^2-r_-^2)}{\rho^2}d\varphi d\psi\ , \\
\phi = &\ \phi_h\ ,
\end{aligned}
\tag{2.5}
$$

where $\rho^2 \equiv r^2 + a^2\cos^2\theta + b^2\sin^2\theta$, α is a boost parameter, and a,b are components of the angular momentum per unit mass of the original Kerr solution. A nontrivial RR three-form field strength is present, but its precise form will not be needed in the following. The parameters $r_\pm$ are related to the charge by $Q^2 \equiv 2Q_+Q_- = (\pi r_+ r_-)^2$. The outer and inner event horizons are located at

$$
r^2 = \frac{1}{2}\left(r_+^2 + r_-^2 - a^2 - b^2 \pm \sqrt{(r_+^2 - r_-^2 - a^2 - b^2)^2 - 4a^2b^2}\right)\ . \tag{2.6}
$$

The six-dimensional ADM energy of this solution is

$$
E = \frac{L\pi}{8}(2(r_+^2 + r_-^2) + (r_+^2 - r_-^2)\cosh 2\alpha)\ . \tag{2.7}
$$

The ADM momentum along the string is given by

$$
P = \frac{\pi L}{8}\sinh 2\alpha (r_+^2 - r_-^2)\ . \tag{2.8}
$$

The angular momenta in the independent planes defined by φ and ψ are

$$
\begin{aligned}
J_1 \equiv J_\varphi &= \frac{\pi L}{4}(ar_+^2\cosh\alpha - br_-^2\sinh\alpha)\ , \\
J_2 \equiv J_\psi &= \frac{\pi L}{4}(br_+^2\cosh\alpha - ar_-^2\sinh\alpha)\ .
\end{aligned}
\tag{2.9}
$$

Following [10], we expect the Bekenstein-Hawking entropy to agree with the D-brane counting away from the supersymmetric extremal limit [11] provided the momentum density P/L and the excitation energy density $\delta E/L$ are small. To study this limit we expand

$$r_\pm = r_0 \pm \epsilon \ , \tag{2.10}$$

with $\epsilon \ll 1$, and α finite. Note we need to take the limit in such a way that $r_+^2 - (|a|+|b|)^2 > r_-^2$ in order to avoid naked singularities. This implies that a^2 and b^2 are of order ϵ. The longitudinal size of the string near the horizon is finite in this limit. To first order in ϵ, the excitation energy is

$$\delta E = \frac{L\pi r_0 \epsilon}{2} \cosh 2\alpha \ , \tag{2.11}$$

and the entropy is

$$\begin{aligned} S = \frac{1}{2} L\pi^2 r_0^2 \Big(&(4r_0\epsilon - a^2 - b^2)\cosh^2\alpha + 2ab\cosh\alpha\sinh\alpha - \frac{1}{2}(4r_0\epsilon - a^2 - b^2) \\ &+ \frac{1}{2}\sqrt{(4r_0\epsilon - a^2 - b^2 - 2ab)(4r_0\epsilon - a^2 - b^2 + 2ab)}\Big)^{1/2} \ . \end{aligned} \tag{2.12}$$

Now define the following quantities

$$\begin{aligned} \tilde{n}_R &= \frac{L}{4\pi}(\delta E + P) - \frac{(J_1 - J_2)^2}{2Q^2} \ , \\ \tilde{n}_L &= \frac{L}{4\pi}(\delta E - P) - \frac{(J_1 + J_2)^2}{2Q^2} \ . \end{aligned} \tag{2.13}$$

The entropy (2.12) may then be expressed as

$$S = \pi Q(\sqrt{2\tilde{n}_L} + \sqrt{2\tilde{n}_R}) \ . \tag{2.14}$$

3. Matching to D-brane Degeneracies

As discussed in [1] the $P = 0$ black hole ground state is a bound state of Q_+ RR onebranes wound around the S^1 in the y direction with Q_- RR fivebranes wound around both the S^1 and the internal four-manifold. In the limit of large radius L for S^1, the excitations of this system are described by a supersymmetric sigma model on a manifold of real dimension $2Q^2$ [6–8] . In the regime of charges we are interested in, to leading order the degeneracy comes from string modes with short wavelengths and hence the curvature of the manifold is irrelevant. Thus we have the same leading degeneracy as the excitations of $2Q^2$ species of massless bosons and $2Q^2$ species of massless fermions which move around the S^1. Ignoring angular momentum, the entropy of N_B (N_F) species of right-moving bosons (fermions) with total energy E_R in a box of length L is given by the standard thermodynamic formula

$$S = \sqrt{\frac{\pi(2N_B + N_F)E_R L}{6}} \ . \tag{3.1}$$

At low energies and large L the system is dilute, interactions can be ignored, and the entropy is additive. Hence, using $N_F = N_B = 2Q^2$ and $E_{R,L} = 2\pi n_{R,L}/L$, (3.1) becomes [1,10]

$$S = \pi Q(\sqrt{2n_L} + \sqrt{2n_R}) \ , \tag{3.2}$$

where $n_{L,R}$ are given by (2.13) with $J_1 = J_2 = 0$.

Now let us correct for the angular momentum. As argued in [11] $J_1 + J_2$ is carried by left-movers, while $J_1 - J_2$ is carried by right movers. Fixing the total angular momentum carried by the right movers decreases the number of states available for fixed energy. As shown in [11] the effect of this on the entropy for left-movers only is to replace n_L with $\tilde{n}_L$. However since the entropy of left and right movers is additive we have simply

$$S = \pi Q(\sqrt{2\tilde{n}_L} + \sqrt{2\tilde{n}_R}) \ , \tag{3.3}$$

in agreement with the black hole calculation (2.14).

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