U-manifolds

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We use non-perturbative U-duality symmetries of type II strings to construct new vacuum solutions. In some ways this generalizes the F-theory vacuum constructions. We find the possibilities of new vacuum constructions are very limited. Among them we construct new theories with $N = 2$ supersymmetry in 3-dimensions and $(1,1)$ supersymmetry in 2-dimensions.

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1. Introduction

It has now been appreciated that in many cases string theories enjoy more symmetries when quantum corrections are taken into account. A particular case of this is the enlargment of T-duality symmetries of type II strings to U-duality symmetries [1][2]. A special case of this involves type IIB in 10 dimensions which is conjectured to have $SL(2, \mathbf{Z})$ U-duality symmetry, or type IIA compactified on a circle which again is conjectured to have the $SL(2, \mathbf{Z})$ U-duality symmetry. Symmetries often have a geometric origin. It is natural to look for a way to 'geometrize' U-duality symmetries. For type IIA compactification to 9 dimensions this is done [3][4] by viewing the theory as coming from the 11 dimensional M-theory compactified on T^2 to 9 dimensions, where the $SL(2, \mathbf{Z})$ duality gets identified with the duality symmetries of the internal T^2. A similar motivation in geometrizing the U-duality of type IIB leads to the notion of a 12 dimensional F-theory [5] whose compactification on T^2 by definition gives rise to type IIB in 10 dimensions.

Quite aside from fundamental issues as to the physical meaning of such geometrizations, the power of geometrizing U-dualities lies in the fact that new vacuum solutions can be effectively constructed in this setup. If we allow the scalar fields (the 'U-fields') to vary over space and allow them to jump, consistent with U-dualities, this data will translate to a manifold whose base is the 'visible' space and the fiber being the geometrization of the U-duality. Questions of smoothness of compactifications are much easier to address in this geometric framework. In particular a criterion for having a 'good' vacuum is that the total space consisting of the base and the auxilliary space which is the fiber be smooth.

More can be said about usefulness of geometrization of U-dualities when the supercharges transform according to the spinor of both the uncompactified space as well as the internal fiber of the U-space. In particular the question of the number of supersymmetries one preserves gets mapped to the number of covariantly constant spinors on the *total space*; this is a subject which is of course well studied.

The vacuum constructions in F-theory can be viewed as utitilizing the $SL(2)$ symmetry of type IIB in 10 dimensions. Similarly, compactifications of M-theory on elliptic manifolds can be viewed as geometrizing the U-duality of type II strings (again $SL(2)$) in 9-dimensions. Given the power of M- and F-theory in constructing new vacua, it is natural to ask if one can use the U-dualities more generally to construct additional new vacua. The aim of this paper is to take a first step in this direction. Even though we will be able to construct some new vacua, in some sense the possibilities appear to be far more limited than in the case of M- or F-theory.

2. General setup

The moduli space of vacua for a string theory with enough supersymmetry and in particular for toroidal compactification of type II strings typically involves a coset space

$$\hat{G}/G(Z) \times H$$

where H is the maximal compact subgroup of $\hat{G}$, and $G(Z)$ is the U-duality group which one identifies as discrete gauge symmetry of the theory. The charges form representations of $\hat{G}$. This is also true for supercharges, but in that case by an appropriate redefinition of the fields we can take them to transform only under the compact subgroup $H \in \hat{G}$. In the following we will abbreviate $G(Z)$ by writing it simply as G. Once we fix a point p in the moduli space of the theory by setting the scalars to some expectation value, there will in general be some subgroup $G_p \subset G$ which preserves that point. We identify G_p as part of the symmetry of that particular background.

We can contemplate a number of ways to use this symmetry to construct new vacua:

1) Modding out by a subgroup $K \subset G_p$. This is the generalization of the orbifold idea and includes the standard orbifold compactifications as a special case. In particular in the case of compactifications of type II string on a d-dimensional torus, we have the perturbative symmetry $G_p \subset SO(d, d; Z)$. Constructions of orbifolds, symmetric as well as asymmetric ones, involve a choice of a subgroup of G_p. We can accompany the group action with a phase action depending on the charged states in the theory. In particular say the charges form a D-dimensional representation of $\hat{G}$. Let a be a vector in R^D. We can enlarge the group we mod out by including some action on the charged states according to

$$(\theta, a)|Q\rangle \rightarrow \exp(ia \cdot Q)|\theta Q\rangle$$

where θ denotes the action of the orbifold group element on the charge lattice. This generalization is also familiar from orbifold constructions where one introduces Wilson line.

2) We can further compactify on another space, say a d-dimensional torus and mod out by a symmetry which partly acts on the torus and partly on the internal U-duality symmetry H. This can of couse be reduced to the case 1) by considering the U-duality as to include that one obtains upon compactification on T^d, but will be useful to keep it as a separate case for reasons we shall explain later. Some examples of such orbifolds,

corresponding to the $SL(2,Z)$ S-duality symmetry of the type IIB theories, have been presented in [6].

3) We can consider the case where the fiber fields are varying smoothly over the space (base) except for some loci of singularities around which the fiber undergoes monodromies belonging to H. This case can be viewed as generalizing case 2) above.

The analogy with orbifold constructions is very helpful and points to some subtleties that have to be overcome. In discussing these theories we will use the terminology of orbifold constructions and in particular the notion of twisted sectors.

In cases of constructions 1) and 2) it is straightforward to deduce the massless modes which survive the projection in the untwisted sector. The difficulty lies in finding the spectrum and interactions involving the twisted states. In case 3) in general there is no division of states to twisted and untwisted sectors and we will have to find another way to find the low energy lagrangian.

The basic strategy in determining the spectrum we have is as follows: Consider further compactification on T^d and assume that for some d, upon conjugation by an element of the U-duality group, G can be viewed as a subgroup of the T-duality group in the compactified theory. In this way, assuming U-duality conjugation commutes with orbifolding, we can deduce the states of the twisted sectors using the standard orbifold techniques for cases 1) and 2) and using Kaluza-Klein compactification techniques to deduce the spectrum in case 3). Of course we may lose some information about the massless modes in the higher dimensional theory (such as chirality) with this procedure, but very often some knowledge of supersymmetry together with absence of anomalies can be used to reconstruct the massless modes of the original theory.

This trick of deducing massless modes upon dimensional reduction is not reliable for case 1) because in such cases there is evidence that U-duality and orbifolding does not commute; see for example [7]. However in cases 2) and 3) it appears to be reliable, as has been checked in many cases in [8]. There is a good reason why in cases 2) and 3) it should have worked: In such case the twisted sectors (or the loci where the fiber becomes singular) can be viewed as p-branes of the higher dimensional theory and in this sense the consistency of conjugating with U-duality amounts to checking the transformation property of the twisted sector p-branes under U-duality. This can thus be viewed as a check of the U-duality in the higher dimensional theory. Note that the fact that we cannot determine the spectrum in case 1) reliably does not imply that orbifolding does not make sense. It is simply a reflection of lack of a technique to deduce its properties. For this reason we will mainly concentrate on cases 2) and 3) in this paper.

In order to effectively use the geometrization of the U-duality symmetries we need the compact part of the U-duality be a subgroup of some holonomoy group and the fiber needed to construct the U-manifold is motivated by how the compact part of the U-duality group is to act on the fiber (recall that the supercharges transform according to a representation of H). For simplicity in this paper we will deal with the case where the compact part of the U-duality group H is $\otimes_i SO(n_i)$ for some n_i and that the supersymmetry charges transform in some spinor representation of $\otimes_i SO(n_i)$ and in particular we will restrict our attention in this paper to the U-dualities which arise upon toroidal compactification of type II strings to $d \geq 6$ spacetime dimensions. The basic idea of this paper clearly generalizes to the other cases as well. In fact an interesting case to consider may be the case with affine U-duality symmetry when we compactify down to $d = 2$ [9].

The auxilliary space we will use for the fiber of our construction will involve $\otimes_i T^{n_i}$. In this way when the base manifold uses some holonomies in the $\otimes SO(n_i)$ we can determine how much supersymmetry it preserves.

3. U-dualities

Let us start with U-dualities in higher dimensions and go down in dimension. We will limit ourselves in this paper to the U-dualities for type II compactifications to 6,7,8,9 and the uncompactified 10 dimensional case.

3.1. Dimension 10 and type IIB

In this case $H = SO(2)$. We can attach an auxilliary T^2. The supercharges transform in the spinor of $SO(2)$ of a given chirality:

$$Q = (\mathbf{S}_{10}^{+} \otimes \mathbf{1}_s^{+}) \oplus (\mathbf{S}_{10}^{+} \otimes \mathbf{1}_s^{-})$$

where $\mathbf{S}_{10}^{+}$ denotes the spinor of positive chirality of $SO(9,1)$ and $\mathbf{1}_s^{\pm}$ denote spinors of $SO(2)$ of $\pm$ chirality. If we go down on another T^2, this $SO(2)$ is contained, by U-duality conjugation, in the T-duality of string theory $SO(2,2)$. In particular it is conjugate to complex structure symmetry of type IIA on T^2. In fact, in this case we do not have to go all the way to string theory to find the structure of this theory: upon compactification on S^1, this $SO(2)$ can be identified with the symmetry of the M-theory compactification on

T^2, and so we get a connection with M-theory which at least for smooth compactifications of type 3) can be used to give the massless states. This chain of dualities implies that if we have a manifold K with T^2 fibers (with a section) we can construct new vacua. This was the original motivation for the introduction of F-theory [5]. Moreover the embedding of the U-dualities mentioned above implies that considering F-theory on $K \times S^1$ is the same as considering M-theory on K. Moreover considering F-theory on $K \times S^1 \times S^1$ is equivalent to type IIA on K.

Let us discuss for instance the compactification of F-theory on $K3$ [5] in the orbifold limits of $K3$ studied in [10][6], from the viewpoint of U-duality alone. We consider compactifying from 10 dimensions down to 8 on a T^2. Now the supercharges belong to

$$\mathbf{S}_8^{\pm} \otimes \mathbf{1}_s^{\pm} \otimes \mathbf{1}_s^{\pm}$$

where the first $\pm$ is correlated with the second one coming from the T^2 going down from 10 to 8. We will do a construction of type 2) in this setup. We mod out by a discrete rotation (ω, ω^{-1}) where ω is a rotation in the Kaluza-Klein $SO(2)$ and ω^{-1} is rotation in the U-group $SO(2)$ (as is well known the choices are very limited; ω^6=1). Then it is easy to see that there are two invariant 8-dimensional spinors $\mathbf{S}_8^+, \mathbf{S}_8^-$. By the chain of duality mentioned above upon compactification on a circle this becomes equivalent to M-theory on an orbifold $K3$ given by $T^2 \times T^2/(\omega, \omega^{-1})$ and upon compactification on an extra circle to type IIA on the same $K3$ orbifold.

3.2. d=9

In this case the U-duality group is still $SL(2)$ and this is geometrized in the context of M-theory as noted above.

3.3. d=8

In this case the compact part of U-duality group is $SO(3) \times SO(2)$. The $SO(2)$ part can be identified with the holonomy group of type IIB compactification on T^2, and the $SO(3)$ can be viewed as a holonomy group of compacitification of M-theory on T^3. The supercharges transform as

$$(\mathbf{S}_8^+ \otimes (2_s, 1_s^+)) + (\mathbf{S}_8^- \otimes (2_s, 1_s^-))$$

where 2_s denotes the spinor of $SO(3)$. Note that we can append a $T^2 \times T^3$ to 8 dimensions and think about spinors as being spinors on $R^8 \times T^3 \times T^2$, with the chirality of the fermions being correlated between the 8-dimensional Minkowski space and the internal T^2. This is also in line with thinking of the scalar moduli of the theory which are parameterized by

$$SL(2) \times SL(3)/SO(2) \times SO(3)$$

as corresponding to flat metrics on $T^2 \times T^3$, modulo an overall volume factor on each torus (i.e. the moduli of 'shapes' on each torus). We thus have a 13 dimensional theory whose compactifications involves manifolds of $T^2 \times T^3$ fibers. Let us call this S-theory (we do not know the connection of this theory to a 13-dimensional theory called S-theory in [11] but we will use the same notation–hopefully they are related!).

In order to talk about compactifications of this theory it is useful to connect it to F-theory, M-theory and type IIA compactifications in lower dimensions. To do this, note that if we go down one dimension the $SL(2) \times SL(3)$ is U-conjugate to the duality visible in F-theory, $SL(2)$ from the elliptic fiber and $SL(3)$ from compactification on an extra T^3. In other words we can identify the T^2 and T^3 of the F-theory with the $T^2 \times T^3$ we started with in 8 dimensions. Upon this reduction a 4-brane of S-theory wrapped around circle gets identified with the three brane of F-theory. Thus we learn that S-theory on $K \times S^1$ is equivalent to F-theory on K. The volume of the T^3 in F-theory is mapped to the inverse radius of the circle S^1. Of course we can continue the chain of dualities upon compactifications on another circle leading to M-theory on K where now the inverse radius of the extra circle is related to the volume of the T^2 fiber in M-theory. Continuing compactification on an extra circle will lead finally to type IIA on K.

The main difficulty with constructing interesting compactifications of S-theory is the fact that manifolds K which admit both a T^2 and T^3 fiber *in a non-trivial way* and which preserve some number of supersymmetries is *very limited*. In fact the only class, which is not already covered by other theories (in which only part of the $T^2 \times T^3$ structure is used) involves compactification on $K = CY_3 \times K3_e$ where CY_3 is a threefold Calabi-Yau which admits T^3 fibers, and $K3_e$ denotes a $K3$ which admits elliptic fibration.

First we ask if there are CY 3-folds admitting T^3 fibration. In fact a certain class of them was constructed in [12], where a concrete example of mirror symmetry was reduced to T-duality of the fiber T^3. The construction involves considering $T^3 \times T^3$ and modding out by an $SO(3)$ subgroup acting on each of the two T^3's. This will clearly give a Calabi-Yau, because $SO(3) \subset SU(3)$. In fact a much larger class of Calabi-Yau's apparently admit T^3 fibration and this has been conjectured to be the basis for mirror symmetry [13][14][15].

Upon compactification of S-theory on this 10 dimensional manifold, we are down to 3-dimensions. This theory has $N = 2$ supersymmetry in 3 dimensions (i.e. the same as reduction of $N = 1$ supersymmetry from 4 dimensions). Upon compactification on an extra circle this is dual to F-theory on $CY_3 \times K3_e$. Since F-theory on $K3_e$ is dual to heterotic strings on T^2, this means that in 2 dimensions we have a duality with heterotic strings on $CY_3 \times T^2$. We can thus push this up one step and obtain duality between heterotic strings on $CY_3 \times S^1$ and S-theory on $CY_3 \times K3_e$.

Note that for the heterotic compactifications on CY_3 we need to turn on 5-branes or put instantons. Similarly as was noted in [16] for F-theory compactification we put appropriate configuration of 3-branes for cancellation of anomalies. Similarly for S-theory we should put appropriate configuration of 4-branes.

3.4. d=7

In this case the U-duality group is $SL(5)$ and the spacetime supersymmetry charges belong to the spinor of $SO(5)$. We can append the 7-dimensional space with a 5-dimensional torus T^5, whose shape is free to change but its size is not dynamical, giving us the moduli space $SL(5)/SO(5)$ which is the moduli of scalars in 7-dimensions. Let us call this 12 dimensional thoery F'. If we only use compactifications which factors in the form of $T^2 \times T^3$ we will simply be getting the F-theory vacua. If we use a T^4 fibration of it, we will be getting M-theory vacua. New vacua will arise if we use the full T^5. If we compactify on an extra circle the $SO(5)$ we have is conjugate to the $SO(5)$ of M-theory compactification on T^5 where the volume of T^5 in M-theory is identified with the inverse radius of the circle we use for compactification. This implies that if we consider compactifications on F'-theory on K which is a manifold admitting T^5 fibration, upon further compactification on a circle we obtain M-theory on K.

Again, unfortunately as in the case of S-theory there are very limited possibilites of fully using the whole T^5 structure to get new vacua which preserve supersymmetry. In fact the only new class we are aware of involves compactifications on CY 5-folds which admit T^5 fibrations. These will give rise to $(1, 1)$ supersymmetries in $d = 2$. Upon compactification to $d = 1$ they are dual to M-theory on the same CY 5-fold. To compare, we note that the compactification of F-theory on the same CY 5-fold leads to a $(2, 0)$ supersymmetric theory in $d = 2$.

3.5. d≤6

As should be clear from the above discussion the possibilities of constructing new vacua using U-dualities are very limited. This will also be the case (and in fact more true) when we come down to lower dimensions.

For concreteness let us consider the case of $d = 6$. In this case the U-duality group is $SO(5,5)$ and the compact subgroup is $SO(5) \times SO(5)$. We can append a $T^5 \times T^5$ to the space; however now the supersymmetry charge is not a spinor of $SO(5) \times SO(5)$; rather it is a direct sum of spinors of each $SO(5)$:

$$[S_6 \otimes (4,1)] \oplus [S_6' \otimes (1,4)]$$

This implies that in studying the number of supersymmetries we have, we need to study each T^5 fiber separately. Note however that the moduli of scalars is not $SL(5) \times SL(5)/SO(5) \times SO(5)$, so the moduli of this theory is not the same as arbitrary metrics up to scale on $T^5 \times T^5$. It is more like the moduli of Narain compacitifations on T^5. In fact if we compactify further on a circle it is U-conjugate to the $SO(5,5)$ T-duality of type II on T^5. If we consider the diagonal $SO(5)$, i.e. if we consider the two T^5 fibers being the same it can be identified with the $SO(5)$ holonomy of M-theory on T^5. If we excite only the $SO(4,4)$ fields, then this can be identified with the T-duality in 6-dimensions and can be related to asymmetric orbifold constructions [17]. As an aside let us note an interesting asymmetric orbifold which acts on $T^4 \times T^4$: If we consider compactifications of type II on T^4, with Narain lattice corresponding to the SO(8) group (where left- and right-movers are $SO(8) \times SO(8)$ weight vectors with difference in root lattice), and mod out by an overall reflection on the left-movers and a translation by a fundamental weight $(1,0,0,0)$ on the right-movers, we obtain a theory in 6 dimension with $(4,2)$ supersymmetry (which in four dimensional terms has $N = 6$ supersymmetry). Note that by construction all the states in the twisted sector are massive. All the moduli consist of 4 scalars from R-R sector and 1 scalar, the dilaton, from the NS-NS sector. This theory has $SO(5,1)/SO(5)$ moduli space [9], and we conjecture that it also has an $SO(5,1;Z)$ U-duality symmetry which commutes with the element $g \in SO(5,5;\mathbf{Z})$ used in the orbifold construction.

Note that even though it is quite easy to construct this vacuum in string perturbation theory as an asymmetric orbifold it is not possible to give a geometric construction of it starting from either M-theory or F-theory. This example emphasizes how powerful string theory methods are, despite the appearance of more abstract theories such as M-theory or F-theory.

The connection of this U-duality group with the T-duality in one lower dimension is also a good guide for constructing vacua. In particular if we use the $SO(5,5)$ T-duality in construction of some vacuum for string theory then there is a strong coupling limit of it, where the theory grows one extra dimension and is equivalent to making use of this 6-dimensional U-duality group. Even so, the possibilities are very limited to get many new vacua which have no perturbative string equivalent, because in order to get something new we have to use the full $SO(5)$ holonomy, and this is not easy to do if one is also interested in preserving supersymmetries. The possibilities are thus rather limited.

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