N = 1 dualities of SO and USp gauge theories and T-duality of string theory

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N=1 Dualities of SO and USp Gauge Theories and T-Duality of String Theory

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ABSTRACT

Extending recent work on SU gauge theory, we engineer local string models for $N = 1$ four-dimensional SO and USp gauge theories coupled to matter in the fundamental. The local models are type IIB orientifolds with D7 branes on a curved orientifold 7-plane, and matter realized by adding D3 branes on the orientifold plane. The Higgs branches of the SO and USp theories can be matched with the moduli spaces of SO and USp instantons on the compact four-dimensional part of the D7 branes worldvolume. The R-charge of the gauge theories is identified with a $U(1)$ symmetry on the worldvolume of an Euclidean D3 brane instanton. We argue that the quantum field theory dualities of these gauge theories arise from T-dualities of type IIB strings exchanging D7 and D3 charges. A crucial role is played by the induced D3 charge of D7 branes and an orientifold 7-plane, both partially compactified on a Z_2 orbifold of $K3$.

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1. Introduction

It has become increasingly clear over the last year that many results in supersymmetric gauge field theories can be derived rather efficiently using string theory. In particular T-duality symmetries of perturbative strings has emerged as an extremely powerful tool in this connection. For the case of $N = 4$ supersymmetric field theories strong-weak coupling duality is argued to arise from T-duality[1][2][3]. The non-perturbative physics of $N = 2$ supersymmetric field theories can be derived from mirror symmetry [3], a symmetry that amounts to T-duality of type II strings [4][5][6]. In many of these works the basic idea is building a local string model for the gauge field theory. This is done by isolating the part of the compactification data which is relevant for field theory questions and taking the limit where gravity is turned off. In the resulting local model the compactification data is replaced by the relevant non-compact piece of the internal space.

$N = 1$ supersymmetric theories and their dualities, have also begun to be understood in the context of string theories. In particular, $N = 1$ pure Yang-Mills was engineered in [7] in the context of F-theory compactification on elliptically fibered Calabi-Yau fourfolds. This amounts to considering a space S of complex dimension two (a codimension one subspace of the base), over which the elliptic fibration acquires an ADE singularity. Moreover it was shown that (to avoid adjoint matter) the space S must satisfy $h^{2,0}(S) = h^{1,0}(S) = 0$. More recently, and for the case of $SU(N_c)$, this was extended to include N_f fundamentals and anti-fundamentals [8], by adding N_f D3 branes filling four-dimensional spacetime and bringing them close to S. It was checked that this local string model does indeed reproduce some of the well known results concerning these gauge theories. Higgs branches of these gauge theories were correctly identified with the moduli spaces of $SU(N_c)$ instantons on S. Nonperturbative generation of superpotentials, and quantum corrections to moduli spaces of the field theories were seen to arise when expected through the effects of euclidean three branes in the local models. Moreover, it was argued that both the quantum field theory dualities between $N = 2$ supersymmetric $SU(N_c)$ gauge theory with N_f flavors and $SU(N_f - N_c)$ gauge theory with N_f flavors [9], and the corresponding $N = 1$ dualities, that hold upon addition of some neutral matter [10], arise in the string models as T-duality transformations of type IIB. This makes sense even though one is discussing F-theory vacua which generically have no T-duality symmetries. This is because in the context of SU gauge groups one can realize the F-theory local model by D7 branes of a fixed perturbative type IIB string [11] for which the T-duality symmetry applies. The

[3] The original discussions were in the context of string-string dualities, but such non-trivial string dualities are not strictly necessary for this connection. A local model can be constructed as type IIA strings in the background of ALE spaces times a two-torus. A T-duality transformation of the two-torus maps to an S-duality of the $N = 4$ Yang-Mills theory. The only non-trivial ingredient in this argument, is the fact that ALE spaces give rise to non-perturbative enhancement of gauge symmetry through wrapped D2 branes.

T-duality transformation roughly speaking inverts the volume of S. This, in particular, exchanges D3 and D7 brane charges. Due to an induced D3 brane charge on the curved D7 worldvolume, this symmetry ends up exchanging $N_c \leftrightarrow N_f - N_c$ instead of $N_c \leftrightarrow N_f$ [8].

It is natural to ask if the results of [8] can be extended to other groups and in particular to SO and USp groups. In fact a similar story should repeat at least for SO, because F-theory backgrounds with SO gauge symmetry can be viewed as type IIB orientifolds [12] for which T-dualities apply and can shed light on $N = 1$ dualities. Similarly, since the difference between the SO and USp theories at the string perturbative level is the choice of the sign for the diagrams with odd number of crosscaps (as we will review in section 2) the arguments work with equal ease for the USp theories. In the present paper we will extend the results of [8] to the case of $N = 1$, four dimensional $SO(N_c)$ gauge field theories with $2N_f$ flavors of matter in the fundamental, and to the case of $USp(N_c)$ gauge theory with $2N_f$ flavors in the fundamental.[4] In particular we show that the Higgs moduli is mapped to the instanton moduli space on S. Moreover we argue that the gauge symmetry duality is mapped to T-duality which exchanges $SO(N_c)$ with $SO(2N_f - N_c + 4)$. The dual group differs from the naive expectation $SO(2N_f)$, due to the contribution of induced D charges by D7 branes and the orientifold. As in the SU case the curved worldvolume of the D7 brane induces $-N_c/2$ units of D3 charge (accounting for the $-N_c$ shift). The orientifold 7-plane carries D7 charge, as is familiar, and due to its curvature induces D3 charge (giving rise to a net $+4$ shift), as will be discussed at length below. Similar statements apply for $USp(N_c)$ for which the dual group is $USp(2N_f - N_c - 4)$. These results are in accord with field theory dualities for $SO(N_c)$ discussed in Refs.[10][13], and field theory dualities for $USp(N_c)$ discussed in Ref.[14]. These field theory dualities can be 'derived' by flowing from the $N = 2$ versions of the above gauge theories [15][16]. In fact the stringy realization of these $N = 1$, SO and USp dualities also starts from an $N = 2$ situation, as was the case for SU [8].

Even though the action of T-duality on D-brane charges is relatively simple, the action of T-duality on curved spaces with wrapped D-branes has not been investigated before. This is an important subject to study further, perhaps using techniques familiar from mirror symmetry, in order to shed further light on field theory dualities. Nevertheless, the fact that $N = 1$ dualities for SU, SO, and USp gauge theories with fundamental matter

[4] In our conventions $USp(N)$, with N necessarily even, is the compact gauge group defined by the set of $N \times N$ complex matrices that are both unitary (belong to $U(N, \mathbf{C})$) and symplectic (belong to $Sp(N, \mathbf{C})$). The corresponding Lie algebra is the real Lie algebra $usp(N)$ defined by the set of $N \times N$ antihermitian matrices A that in addition satisfy the symplectic condition $JA = -A^t J$ with $J = \begin{pmatrix} 0 & I_{N/2} \\ -I_{N/2} & 0 \end{pmatrix}$. The $usp(N)$ algebra is of rank $N/2$, and real dimension $N(N+1)/2$. The fundamental representation of this algebra is of dimension N and it is pseudoreal, and the adjoint representation appears in the symmetric part of the product of two fundamentals.

can all be *derived* in a simple and unified way suggests strongly that our assumptions about T-dualities are valid.

2. The local model for SO and USp groups

As mentioned in the previous section, local models of F-theory compactified on elliptic CY_4's which admit a perturbative string description are a natural setup to gain insight into $N = 1$ dualities. In particular for the SO models we can replace the F-theory description with a type IIB orientifold. The type IIB orientifold description also allows us to easily construct the USp series. In this section we discuss the local models we need in detail.

We start by constructing a local model for type IIB string on a Calabi-Yau threefold. The local model for the Calabi-Yau threefold is the total space of a complex 2-surface K together with its canonical line bundle. This gives a non-compact threefold with $c_1 = 0$. We then consider an orientifold of the above model by letting the space reflection part act trivially on K but as an inversion on the line bundle over K. The zero section of the line bundle, which can be identified with K, times the Minkowski space M^4 can be viewed as an orientifold 7-plane, or more briefly, an O7 plane.

We now put $N_c/2$ *physical* D7 branes on the O7 plane.[5] Depending on the choice of weight factor for string diagrams involving crosscaps the branes give rise to $SO(N_c)$ or $USp(N_c)$ gauge theory. To see this recall that in open strings we start with N_c Chan-Paton factors. In oriented strings this gives in the open string sector a $U(N_c)$ gauge symmetry. For non-orientable strings, however, we have to take into account the action of orientifolding on the open string sector. This amounts to symmetrization or anti-symmetrization of the Chan-Paton indices, leading to $USp(N_c)$ or $SO(N_c)$ respectively. The difference between these two is the choice of the sign for the action of the orientifold (twist) operator Ω on the open string sector. The two possible choices of sign correspond, for the case of the open string one loop vacuum graphs, to either adding or subtracting the diagrams for the annulus and the Moebius strip. Since the Moebius strip is a disk with a crosscap, the net effect is to weight string diagrams having no external open strings with a weight factor $(-1)^c$ where c denotes the number of crosscaps [17][6]. We learn that to exchange SO and USp all that needs to be done is to change the sign of the contribution of the crosscap. In particular a diagram with a single crosscap comes with opposite signs in $SO(N)$ and $USp(N)$ theories. This fact will be used below.

[5] If the branes are moved away from the orientifold plane, branes will have images under the space reflection. The total number of branes would be N_c, out of which $N_c/2$ branes are called physical because their positions can be adjusted at will. While the total number of Chan-Paton indices is N_c, the number of physical branes coincides with the rank of the group.

[6] This is a well-defined weight factor since the number of crosscaps is well defined except for the fact that one can trade three crosscaps for a single crosscap plus a handle. Thus any surface can be said to have either two, one or no crosscaps. Only surfaces without crosscaps are orientable.

If $K = T^4$ one finds $N = 4$ gauge theory in four dimensions. If $K = K3$ one finds $N = 2$, and if K is generic, one finds $N = 1$ gauge theory in four dimensions, with $h^{2,0}(K) + h^{1,0}(K)$ matter multiplets in the adjoint of the gauge group [7]. We will mainly consider in the latter case the situation with $h^{2,0} = h^{1,0} = 0$. The generalization when they are non-vanishing is exactly as in [8] and will not be repeated here.

If we wish to obtain matter in the fundamental representation of the gauge group of the seven branes we have to bring in some D3 branes filling the spacetime, and put them on K, where they appear as points. This case was studied in the context of D5 branes and D9 branes in [18][19] a case equivalent to ours by T-duality. In the case of $SO(N_c)$ if we bring in N_f D3 branes and put them all at the same point on K we obtain an $N = 2$, $USp(2N_f)$ symmetry in four dimensions, with extra matter in the *antisymmetric* representation $N_f(2N_f - 1)$, as well as half-hypermultiplets in the mixed representation $(N_c, 2N_f)$. If we choose the other sign factor for diagrams with odd number of crosscaps, the roles of SO and USp are exchanged. In particular, we get $USp(N_c)$ from D7 branes and the extra sector we obtain from D3 branes is an $N = 2$ system with $SO(2N_f)$ gauge symmetry. Moreover, in addition to the mixed matter half-hypermultiplets in $(N_c, 2N_f)$, we get matter in the *symmetric* representation of $SO(2N_f)$, i.e., in the $N_f(2N_f + 1)$ dimensional representation.

Summarizing, the two gauge groups we have engineered are

$$\begin{aligned} &SO(N_c) \times USp(2N_f)\,, \\ &USp(N_c) \times SO(2N_f)\,, \end{aligned} \tag{2.1}$$

and the matter content is given as follows. For $K = T^4$, in $N = 2$ hypermultiplets

$$\tfrac{1}{2} \cdot [N_c\,,\, 2N_f] \;+\; [1\,,\, N_f(2N_f \mp 1)] \;+\; [N_c(N_c \mp 1)/2\,,\, 1]\,, \tag{2.2}$$

where the top sign refers to $SO(N_c) \times USp(2N_f)$ and the lower sign refers to $USp(N_c) \times SO(2N_f)$. The last $N = 2$ hypermultiplet arises by noting that the $N = 4$ theory has an extra adjoint hypermultiplet in the $N = 2$ terminology. For $K = K3$, we find in $N = 2$ hypermultiplets

$$\tfrac{1}{2} \cdot [N_c\,,\, 2N_f] \;+\; [1\,,\, N_f(2N_f \mp 1)]\,, \tag{2.3}$$

i.e. the adjoint is lost due to having less supersymmetry preserved on the D7 worldvolume. For $K = S$, in terms of four-dimensional $N = 1$ chiral multiplets we get

$$2 \cdot \tfrac{1}{2} \cdot [N_c\,,\, 2N_f] \;+\; 2 \cdot\, [1\,,\, N_f(2N_f \mp 1)] \,+\, [1\,,\, N_f(2N_f \pm 1)]\,, \tag{2.4}$$

where the factors of 2 arise from writing an $N = 2$ hypermultiplet as two $N = 1$ chiral multiplets, and the last representation arises from writing the $N = 2$ vector multiplet on D3 branes in terms of an adjoint $N = 1$ chiral multiplet.

3. Matching of Higgs branches to Instanton moduli spaces

For concreteness let us first consider the case with $SO(N_c)$ gauge group for the D7 branes. The Higgs phase of the gauge theory models arise in the local string models as follows. The N_f D3 branes, living inside the D7 branes, fill four dimensional Minkowski space and appear as N_f points in K. When these N_f points coincide the $USp(2N_f)$ symmetry is unbroken and the D3 branes can be viewed as N_f coinciding zero size instantons of $SO(N_c)$. Each three brane contributes an instanton number of $+1$, giving a total instanton number $k = N_f$. In this section we test our construction of local models by checking that the dimensionality of the Higgs branches of the field theories match the dimensionality of the corresponding moduli space of instantons on K in the local string model. This can be viewed as an extension of Ref.[18]. The fact that not only the dimension but also the local structure of the instanton moduli space matches the Higgs description follows from [18] for the $N = 2$ case. For the $N = 1$ case presumably an extension of the $N = 2$ result works; such extension was established for $N = 1$ supersymmetric $SU(N)$ gauge theories in [8].

We begin by recalling the dimensionalities of moduli spaces of instantons. Consider a gauge theory based on the gauge group G and living on a four dimensional manifold K. The moduli space $\mathcal{M}_k(G, K)$ of instantons with total instanton number k has complex dimension

$$\tfrac{1}{2}\dim_{\mathbf{C}}\mathcal{M}_k(G,K) = kc_2(G) \, - \tfrac{1}{8}\left[\chi(K) + \sigma(K)\right] \dim(G)\,, \tag{3.1}$$

where $c_2(G)$ is the dual Coxeter number of the group G and $\chi(K)$ and $\sigma(K)$ denote the Euler characteristic and the signature of K respectively. As mentioned earlier, we have three cases of interest: $K = T^4, K3$ and S. For T^4 we have $\chi + \sigma = 0$, for $K3$ we have $\chi + \sigma = 8$, and for S we have $\chi + \sigma = 4$. We therefore write

$$\begin{aligned} \tfrac{1}{2}\dim_{\mathbf{C}}\mathcal{M}_k(G,T^4) &= kc_2(G)\,, \\ \tfrac{1}{2}\dim_{\mathbf{C}}\mathcal{M}_k(G,K3) &= kc_2(G) \, - \, \dim(G)\,, \\ \tfrac{1}{2}\dim_{\mathbf{C}}\mathcal{M}_k(G,S) &= kc_2(G) \, - \tfrac{1}{2}\dim(G)\,. \end{aligned} \tag{3.2}$$

We also list, for easy reference, the dual Coxeter numbers of the groups we will be dealing with

$$c_2(SU(N)) = N\,, \quad c_2(SO(N)) = N-2\,, \quad c_2(USp(N)) = \tfrac{1}{2}N + 1\,. \tag{3.3}$$

We can now consider the Higgs branches of the supersymmetric field theories. In $N = 2$ four dimensional gauge theory we count the (real) dimensionality of the Higgs branch $\mathcal{M}_H$ using the equation

$$\tfrac{1}{4}\dim_{\mathbf{R}}\mathcal{M}_H = \#\text{hyp.} - \dim(G)\,. \tag{3.4}$$

Consider first the case of T^4, which gives rise to the gauge group $SO(N_c) \times USp(2N_f)$, with the matter representations listed in (2.2). We find

$$\begin{aligned} \tfrac{1}{4}\mathrm{dim}_{\mathbf{R}}\mathcal{M}_H &= N_c N_f + N_f(2N_f - 1) + \tfrac{1}{2}N_c(N_c - 1) \\ &\quad - \tfrac{1}{2}N_c(N_c - 1) - N_f(2N_f + 1)\,, \\ &= (N_c - 2)N_f\,, \end{aligned} \tag{3.5}$$

in agreement with the expectation from the first equation in (3.2), for the case of $G = SO(N_c)$. Had the $USp(2N_f)$ part of the gauge symmetry been completely broken down to $[USp(2)]^{N_f}$ by pulling the D3 branes apart on K, the dimensionality of the Higgs branch in the resulting theory would have still been the same.

For the case when $K = K3$ we still have an $N = 2$ gauge field theory. We must now use the matter representations listed in (2.3) which now give

$$\begin{aligned} \tfrac{1}{4}\mathrm{dim}_{\mathbf{R}}\mathcal{M}_H &= N_c N_f + N_f(2N_f - 1) - \mathrm{dim}SO(N_c) - N_f(2N_f + 1)\,, \\ &= (N_c - 2)N_f - \mathrm{dim}SO(N_c)\,, \end{aligned} \tag{3.6}$$

in agreement with the expectation from the second equation in (3.2), for the case of $G = SO(N_c)$.

Finally, we consider the case when $K = S$. Let us first note that the last representation listed in (2.4) which arises from an $N = 2$ vector multiplet comes with a superpotential term W induced from $N = 2$ supersymmetry. Solving the $dW = 0$ constraint implies that this field appears as a constraint on the Higgs branch and thus acts to decrease the dimension of Higgs branch, rather than add to it. We thus find (counting in chiral multiplets which correspond to computing $\frac{1}{2}\mathrm{dim}_{\mathbf{R}}\mathcal{M}_H$)

$$\begin{aligned} \tfrac{1}{2}\mathrm{dim}_{\mathbf{R}}\mathcal{M}_H &= 2N_f N_c + 2N_f(2N_f - 1) - N_f(2N_f + 1) \\ &\quad - \mathrm{dim}(USp(2N_f)) - \mathrm{dim}(SO(N_c))\,, \\ &= 2N_f(N_c - 2) - \mathrm{dim}(SO(N_c))\,, \end{aligned} \tag{3.7}$$

in agreement with the third equation in (3.2), for the case of $G = SO(N_c)$ (note that one equation refers to complex dimension while the other refers to real dimension).

For the D7 branes giving USp the story is similar. The main difference is that now the instanton number for USp is related to N_f by $k = 2N_f$. This is in agreement with [20] where it was explained that the k-instanton moduli space of USp theories is governed by an $SO(k)$ gauge theory. Indeed, in our case the SO group induced by the three branes is $SO(2N_f)$. To understand this counting explicitly note that we can have integral or half-integral D-brane charges for the SO group. Only pairs of half-integral D-branes can be moved off the orientifold plane. In the case we have been considering we have brought D3 branes onto the orientifold plane O7, and thus each one will count as two instantons

of zero size on top of each other. Once they are on O7 they can split into two half-integral D3 charges, each of which would correspond to one zero-size instanton of $USp(N_c)$. Note in particular that for one instanton of zero size in $USp(N_c)$ we would get an $SO(1)$ group corresponding to a half D3 brane stuck on the orientifold plane. The absence of gauge group in this case implies the absence of a scalar degree of freedom which can move the D3 brane off the orientifold plane. We thus conclude that for N_f D3 branes the instanton number for $USp(N_c)$ is $k = 2N_f$. It is now straightforward to repeat the above analysis for the USp D7 branes and show that the dimensions for the instanton moduli spaces match the dimensions of the Higgs branches.

4. R-charge and Instanton Corrections

It was shown in [8] that point instantons of $SU(N_c)$ supersymmetric QCD can be identified with Euclidean D3 branes wrapped around S. Moreover, following [21] an R-charge Q was defined so that instantons contribute to the superpotential if there is a charge violation $\Delta Q = 1$. In this section we will extend these considerations to the local models for SO and USp gauge theories. With this end in mind, let us first review the calculation of charge violation in the $SU(N_c)$ local model of [8].

We have an Euclidean D3 brane wrapping S and we assume we are in the Higgs branch, namely, there are $SU(N_c)$ background gauge fields on S of instanton number N_f. The total charge violation arises from additive contributions from two sectors. The first sector, arising from open strings stretched from the D7 branes to the Euclidean D3 brane, represents matter in the fundamental of $SU(N_c)$, living on S and interacting with the background $SU(N_c)$ gauge fields. The charge violation in this sector is given by the index, in the fundamental representation, of the twisted Dirac operator $\overline{\partial}_A$, with A denoting the $SU(N_c)$ background gauge fields. This index is given by

$$\Delta Q|_{D3-D7} = \text{index } \overline{\partial}_A = N_c - N_f. \tag{4.1}$$

The second sector arises from open strings stretched from the Euclidean D3 brane onto itself, and corresponds to a twisted $N = 4$ supersymmetric $U(1)$ theory on the Euclidean D3 brane. Certain aspects of the field theory on this Euclidean instanton have been studied recently in [22]. The contribution to ΔQ from this sector is zero, because the fermions in the hypermultiplet and vector multiplet carry opposite Q charge[7] and they have equal number of zero modes, i.e. from this sector we have

$$\Delta Q|_{D3-D3} = \#\text{hyp.} - \#\text{vect.} = 0\,. \tag{4.2}$$

[7] This follows because the twisting of $N = 4$ is related to the Q charge. In particular four scalars in the hypermultiplet as well as the gauge field of the $N = 4$ are neutral under Q which forces the fermions in the vector and hypermultiplet to carry opposite Q charge.

The total charge violation is thus given by $\Delta Q = N_c - N_f$. One expects superpotential generation for $\Delta Q = 1$, or equivalently, for $N_f = N_c - 1$. This expectation is confirmed by field theory analysis.

Now we come to the case of SO and USp gauge theories. Let us consider the SO case first. The $SO(N_c)$ background gauge fields on S define a background with instanton number N_f, and we have an Euclidean D3 brane wrapped around S. Once again, we have two sectors contributing to charge violation. From D3−D3 open strings, the Euclidean D3 brane will carry an $N = 2$ supersymmetric $USp(2)$ gauge theory with an additional singlet hypermultiplet. This sector gives a contribution

$$\Delta Q|_{D3-D3} = \#\text{hyp.} - \#\text{vect.} = 1 - 3 = -2\,. \tag{4.3}$$

From D3−D7 open strings, we have a half-hypermultiplet in the $[2, N_c]$ representation of $USp(2) \times SO(N_c)$. As before, the violation of Q charge in this sector is given by the index, in the $[2, N_c]$ representation, of $\overline{\partial}_A$, where A is the nontrivial $SO(N_c)$ background. We find

$$\Delta Q|_{D3-D7} = N_c - 2N_f\,. \tag{4.4}$$

We thus find that the total violation is given by

$$\Delta Q = N_c - 2N_f - 2\,. \tag{4.5}$$

We thus expect that for $\Delta Q = 1$, i.e. when the number of flavors $2N_f$ is given by $2N_f = N_c - 3$ we have a superpotential generated. Moreover, as in [8], one can see that the superpotential must have a first order pole. This is in accord with field theory expectations [13]. The situation, however, is more complicated than in the SU case, because for $\Delta Q = 1$ we now have an unbroken $SO(3)$ gauge theory with non-trivial infrared dynamics and gaugino condensates will play a role in the creation of the superpotential. Similarly, for $\Delta Q = 0$, corresponding to $2N_f = N_c - 2$, there could be a quantum correction to the moduli space. As it turns out, in this case there are instanton corrections to the coupling of the unbroken $U(1)$ on the moduli space. These have been computed exactly in [13] and it would be interesting to see how the above Euclidean D3 instanton (and multi-instantons) reproduce those results.[8]

The situation for the $USp(N_c)$ gauge group is similar, modulo a small twist, as in the previous section. If we consider one Euclidean D3 brane wrapped around S, as noted in the previous section, it corresponds to *two* zero-size instantons on top of each other. Since this is most similar to the case considered above let us first compute the Q charge violation for two instantons and at the end divide the result by two to obtain the charge violation for *one* Euclidean D3 instanton. For two instantons we have an $SO(2)$ gauge group living

[8] It would be interesting to see what happens if we turn on $USp(2)$ instantons on the Euclidean D3 brane and sum over all possible values of instanton number.

on the Euclidean D3 brane. The R-charge violation from the mixed sector is the same as before. But the violation from the D3-D3 sector is different, because now we have three hypermultiplets (symmetric representation of $SO(2)$) and one vector multiplet (adjoint of $SO(2)$) and we find

$$2\Delta Q|_{D3-D3} = 3 - 1 = 2\,. \tag{4.6}$$

Thus the net violation of the Q charge is

$$2\Delta Q = N_c - 2N_f + 2 \quad \rightarrow \quad \Delta Q = \tfrac{1}{2}N_c - N_f + 1\,. \tag{4.7}$$

Note that we could have also done the computation directly in terms of a *single* instanton corresponding to an Euclidean half D3 brane. This leads to an $SO(1)$ theory on the Euclidean brane with one hypermultiplet in the D3−D3 sector, and no vector multiplet. This gives the contribution $1 - 0 = 1$ to the charge violation. The mixed D3−D7 sector will give a single half-hypermultiplet contributing $\frac{1}{2}N_c - N_f$ charge violation. These two contributions add up to the result in (4.7).

For $\Delta Q = 1$ we thus expect, as before, a superpotential with a first order pole be generated by point-like instantons. This corresponds to the case $\frac{1}{2}N_c = N_f$, where indeed there is a superpotential generated with a first order pole [14].[9] We would also expect that for $\Delta Q = 0$ there could be an instanton correction to the moduli space. This is indeed the case [14].

5. $N = 1$ duality as T-duality

In this section, as a useful preliminary to the analysis of $N = 1$ supersymmetric SO and USp gauge theories we first review the results of [8] dealing with SU gauge theories. We then turn to the computation of induced charges by orientifold planes and D-branes, and finally derive the dualities of $N = 1$ supersymmetric SO and USp theories from T-dualities of the local models.

5.1. Review of the SU case

In this case one starts with $K = K3$ with N_c D7 branes wrapped on it and N_f D3 branes appearing as points on it. The N_c D7 branes wrapped on $K3$ induce $-N_c$ units of D3 brane charge [23] [24]. For later application let us note that, by T-duality, the above statement implies that a D-brane wrapped on $K3$ gives rise to a codimension-four D-brane

[9] In comparing with [14] note that the authors work with $SP(N_c)$, a group that in our notation is $USp(2N_c)$. The number of flavors, $2N_f$, is the same as in our case. To compare with our results simply let $N_c \rightarrow N_c/2$ in the results of [14].

charge of (-1). Thus in this configuration the total seven brane charge Q_7 and the total three-brane charge Q_3 are given by

$$\begin{aligned} Q_7 &= N_c\,, \\ Q_3 &= N_f - N_c\,. \end{aligned} \tag{5.1}$$

If the volume of $K3$ is small we are in a regime which by T-duality is equivalent to a large volume $K3$. Note that perturbatively the volume of $K3$ is related to the bare coupling of the D7 gauge group $SU(N_c)$ by $V(K3) = 1/g^2$. Thus, when we enter the small volume regime we are entering a large coupling region of the original theory. Once we dualize we are back to a weak coupling description. This, however, exchanges the D3 and D7 charges. In particular now we have

$$\begin{aligned} Q'_7 &= Q_3 = N'_c\,, \\ Q'_3 &= Q_7 = N'_f - N'_c\,. \end{aligned} \tag{5.2}$$

Solving for N'_c and N'_f we find

$$\begin{aligned} N'_c &= N_f - N_c\,, \\ N'_f &= N_f\,. \end{aligned} \tag{5.3}$$

This means we now have a weak coupling description of $SU(N_f - N_c)$ with N_f flavors in the fundamental of this group. This is still an $N = 2$ theory. To obtain an $N = 1$ theory we assume $K3$ has an extra Z_2 symmetry which inverts the sign of the holomorphic 2-form. We can now mod out the $K3$ by this Z_2 symmetry to obtain a space $S = K3/Z_2$ which has $h^{2,0} = 0$. Note that this Z_2 acts only on the middle cohomologies of $K3$. This does not interfere with the D7 and D3 brane charges as they come from zero and four dimensional cycles of $K3$. So we expect the same duality to continue to hold by the time we get to $N = 1$. This is very much in the same spirit as the flow from $N = 2$ microscopic/macroscopic theories to the dual pairs of $N = 1$ theories [15]. Moreover, the meson field of the dual magnetic theory can naturally arise if the N_f D3 branes in the dual theory are forced to be on top of each other.

5.2. Computation of induced charges for orientifold planes and D-branes

We now wish to repeat the same count of D-brane charge for the SO and USp theories, and see if they are in accord with the expectations. Again we start with $K = K3$ and assume that modding $K3$ by the extra Z_2 will not modify the D-brane charge count. This would then be in the same spirit as the flow from the $N = 2$ microscopic/macroscopic theories to the dual $N = 1$ theories [16]. We will assume that the D3 branes are separate. In the case of SO D7 branes this implies that we have $USp(2)^{N_f}$. But the dynamics of these gauge factors will be infrared trivial if $N_c > 4$, which we will be assuming. Therefore, these factors will not affect the field theory dualities. A similar comment applies to the case of USp D7 branes.

The main complication for the SO and USp cases compared to the SU case is that, in addition to the D7 branes, the orientifold planes can also induce D-brane charges. The D7 and O7 induce 3-brane charges through an interaction of the form $\int B^4 \wedge tr(R \wedge R)$ where B^4 is the gauge field coupling to D3 charges. This arises for the D7 case from a disk diagram with the boundary on the D7 branes. For the O7 this term arises from the sphere with a single crosscap (RP^2). That these are generated follows from the anomaly cancellation considerations for type I strings (which by T-duality is related to the above interactions). In addition, there is the familiar D7 charge induced by the O7. Let us study these contributions in detail.

<u>D3-brane charge induced by curved D7-branes</u> Each physical D7 brane will contribute some Q_3 charge. This is because the worldvolume of the seven brane is curved and the curvature of $K3$ is responsible for generating an effective D3 brane charge, as in the SU case. As mentioned above the string diagram computation involves a disk worldsheet. Being an orientable diagram it is also present with exactly the same value in the non-orientable case we are now dealing with. Thus, as before, each D7 brane will contribute a Q_3 charge

$$Q_3(\mathrm{D7}) = -1\,, \tag{5.4}$$

irrespective of whether we are dealing with SO or USp seven branes. Note that for $N_c/2$ D7 branes we will end up getting $-N_c/2$ units of D3 charge.

<u>D7-brane charge of O7 planes</u> The contribution of the O7 plane to the seven brane charge Q_7 is easily determined once we note that O9 plane carries (-16) units of D9 charge (the explanation of $SO(32)$ gauge symmetry as orientifold of type IIB in 10 dimensions [25]). Once we compactify on a two-torus T^2 and do T-duality we obtain four O7 planes with the same total charge as before, namely (-16) units of D7 charge. Thus each O7 plane carries (-4) units of seven brane charge Q_7. Note that since the contribution of the O7 plane to D7 charge comes from a diagram with a single crosscap (RP^2) and going from SO to USp involves the change in sign for diagrams with odd number of crosscaps, the O7 plane will carry $+4$ units of seven brane charge Q_7 for the USp case. Thus we write

$$Q_7(\mathrm{O7}) = \mp 4\,, \tag{5.5}$$

where the top sign refers to SO D7 branes and the lower sign refers to USp D7 branes.

<u>D3-brane charge of curved O7 planes</u> We now wish to compute the D3 charge induced by an O7 wrapped around $K3$. As noted above, this comes from an RP^2 diagram producing an interaction of the form $\int B^4 \wedge tr(R \wedge R)$. To get the normalization, it is convenient to proceed as follows. Let us first compute the D5 charge of an O9 wrapped around $K3$. In the ten dimensional type I theory compactified on $K3$ the 16 D9 branes and the single O9 plane both contribute to the five brane charge Q_5. As is familiar [18], upon $K3$ compactification in order to cancel the D5 charge induced by O9 and D9 we require the addition of 24 five-branes. This means that the induced charges satisfy

$$16 \cdot Q_5(\mathrm{D9}) + Q_5(\mathrm{O9}) = -24\,. \tag{5.6}$$

It follows from our earlier comments that $Q_5(\mathrm{D9}) = -1$, and therefore from (5.6) we find that $Q_5(\mathrm{O9}) = -8$.

To find the D3 brane charge of O7 planes wrapped on $K3$ we use the same trick as before, namely compactify on T^2 and dualize. In this process we obtain four O7 planes wrapped on $K3$, and therefore each O7 plane wrapped on $K3$ will induce $-8/4 = -2$ units of D3 brane charge. Note that this is for the case of SO gauge groups. When we deal with the USp groups, as noted earlier, the contribution of the diagrams with a single crosscap change. This implies that for the USp case the induced D3 brane charge is $+2$ units. All in all we find

$$Q_3(\mathrm{O7}) = \mp 2\,, \tag{5.7}$$

where the top sign refers to SO and the lower sign refers to USp.

<u>Total three brane and seven brane charges</u> In the local models we have been considering we have focused on a single orientifold O7 plane and have placed a total of $N_c/2$ physical D7 branes on top of it giving rise to the gauge group $SO(N_c)$ or $USp(N_c)$. It follows that the total amount Q_7 of D7 brane charge of this local configuration is given by the number of branes plus the orientifold contribution (5.5)

$$Q_7 = \tfrac{1}{2}N_c \mp 4, \tag{5.8}$$

where the top and lower signs in the above refer to $SO(N_c)$ and $USp(N_c)$ respectively.

The total three brane charge Q_3 arises from the N_f physical $D3$ branes, the D7 brane contribution indicated in (5.4) multiplied by the number of seven branes $\frac{1}{2}N_c$, and the O7-plane contribution indicated in (5.7)

$$Q_3 = N_f - \tfrac{1}{2}N_c \mp 2\,. \tag{5.9}$$

5.3. T-Duality transformations for the SO and USp local models

We are now ready to see the effect of T-duality on the above charge assignments. First of all we need to argue that T-duality brings the $K3$ orientifold back to itself. This would *not* be the case if we were dealing with T^4 where T-duality would have turned the O7 plane into O3 planes. To see that it is reasonable for $K3$ note that, if we realize $K3$ as T^4/Z_2 the orbifold group is generated by the elements $\{1, \Omega, I, I\Omega\}$ where Ω refers to the orientifold action and I refers to the Z_2 inversion of T^4. Under T-duality, I goes to itself whereas Ω and $I\Omega$ get interchanged, so we end up again with a $K3$ orientifold. This argument leads us to believe that T-duality will bring the $K3$ orientifold back to itself even away from the orbifold limit of $K3$ (which we need to assume in order to avoid the zero size instantons concentrated at the fixed points of the orbifold, as discussed in [26]). At any rate we shall assume that for a smooth $K3$ orientifold the T-dual is still a $K3$ orientifold.

Now let us concentrate on the total D-brane charges in our local models. These were given in eqns.(5.8) and (5.9), which we reproduce for convenience here

$$\begin{aligned} Q_7 &= \tfrac{1}{2}N_c \mp 4\,, \\ Q_3 &= N_f - \tfrac{1}{2}N_c \mp 2\,. \end{aligned} \tag{5.10}$$

Since T-duality exchanges the D3 and D7 charges, in the dual model, the seven brane charge Q'_7 and the three brane charge Q'_3 are given by

$$\begin{aligned} Q'_7 &= Q_3 = \tfrac{1}{2}N'_c \mp 4\,, \\ Q'_3 &= Q_7 = N'_f - \tfrac{1}{2}N'_c \mp 2\,, \end{aligned} \tag{5.11}$$

where N'_c and N'_f denote respectively the number of D7 branes and D3 branes in the dual model. Solving for N'_c and N'_f we find

$$\begin{aligned} N'_c &= 2N_f - N_c \pm 4\,, \\ N'_f &= N_f\,. \end{aligned} \tag{5.12}$$

This implies that under a T-duality transformation the gauge groups in the local models get exchanged as

$$\begin{aligned} SO(N_c) \;&\leftrightarrow\; SO(2N_f - N_c + 4)\,, \\ USp(N_c) &\leftrightarrow USp(2N_f - N_c - 4)\,, \end{aligned} \tag{5.13}$$

each theory with $2N_f$ flavors in the fundamental representation of the corresponding group. These are the familiar $N = 1$ dualities for the SO and USp groups with matter in the fundamental [13][14]. The additional fundamental meson fields appearing in the magnetic side may arise in the local models (as in [8]) if in the T-dual theory the N_f D3 branes are forced to be near each other. In this case, the superpotential term involving the fundamental meson field on the magnetic side will also arise as expected.

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