



# Theoretical Modeling of ISO Results on Planetary Nebula NGC 7027

The Harvard community has made this  
article openly available. [Please share](#) how  
this access benefits you. Your story matters

Citation	Yan, M., S. R. Federman, A. Dalgarno, and J. E. Bjorkman. 1999. "Theoretical Modeling of ISO Results on Planetary Nebula NGC 7027." <i>The Astrophysical Journal</i> 515 (2): 640–48. <a href="https://doi.org/10.1086/307047">https://doi.org/10.1086/307047</a> .
Citable link	<a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:41397397">http://nrs.harvard.edu/urn-3:HUL.InstRepos:41397397</a>
Terms of Use	This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA">http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA</a>

## THEORETICAL MODELING OF *ISO* RESULTS ON PLANETARY NEBULA NGC 7027

M. YAN<sup>1</sup> AND S. R. FEDERMAN<sup>2</sup>

Department of Physics and Astronomy, University of Toledo, Toledo, OH 43606

A. DALGARNO<sup>3</sup>

Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138

AND

J. E. BJORKMAN<sup>4</sup>

Department of Physics and Astronomy, University of Toledo, Toledo, OH 43606

Received 1998 June 24; accepted 1998 December 1

### ABSTRACT

We present a thermal and chemical model of the neutral envelope of planetary nebula NGC 7027. In our model, the neutral envelope is composed of a thin dense shell of constant density and an outer stellar wind region with the usual inverse-square law density profile. The thermal and chemical structure is calculated with the assumption that the incident radiation field on the inner surface equals  $0.5 \times 10^5$  times Draine's fit to the average interstellar far-ultraviolet field. The rate coefficient for  $H_2$  formation on grains is assumed to be  $\frac{1}{5}$  the usual value to take into account the lower dust-gas mass ratio in the neutral envelope of NGC 7027. The calculated temperature in the dense shell decreases from 3000 to under 200 K. Once the temperature drops to 200 K, we assume that it remains at 200 K until the outer edge of the dense shell is reached, so that the observed intensities of CO  $J = 16-15$ ,  $15-14$ , and  $14-13$  lines can be reproduced. The 200 K temperature can be interpreted as the average temperature of the shocked gas just behind the forward shock front in the framework of the interacting stellar wind theory. We calculate the intensities of the molecular far-infrared rotational lines by using a revised version of the escape probability formalism. The theoretical intensities for rotational lines of CO (from  $J = 29-28$  to  $J = 14-13$ ),  $CH^+$ , OH, and CH are shown to be in good agreement with *ISO* observations. The  $H_2$  rovibrational line intensities are also calculated and are in agreement with available observations.

*Subject headings:* infrared: ISM: lines and bands — ISM: molecules — molecular processes — planetary nebulae: individual (NGC 7027)

### 1. INTRODUCTION

Planetary nebula NGC 7027 has been studied extensively at optical, infrared, submillimeter, millimeter, and radio wavelengths (Middlemass 1990; Hoare, Roche, & Clegg 1992; Graham et al. 1993; Volk & Kwok 1997). Its very bright and rich optical spectrum originates from the ionized nebula and is well characterized by the semiempirical photoionization model of Middlemass (1990). The ionized nebula is surrounded by an expanding molecular envelope, which was first discovered by Mufson, Lyon, & Marioni (1975) through detection of CO rotational lines in the millimeter waveband. Photometric studies reveal that NGC 7027 has a large midinfrared excess and a large submillimeter excess that can be attributed to hot and cold grains, respectively (Hoare et al. 1992). The launch of the *Infrared Space Observatory* (*ISO*) made it possible to do far-infrared spectroscopic studies of planetary nebulae. Extensive observations on NGC 7027 with *ISO* showed that NGC 7027 is also very rich in far-infrared emission lines. Liu et al. (1996, 1997) and Cernicharo et al. (1997) reported detections of the pure rotational lines of CO from  $J = 14-13$  up to  $J = 29-28$ , three of CH, five  $CH^+$  lines, and five for OH, and they report the possible detection of one  $H_2O$  line. Although the observational characteristics of the ionized region of NGC 7027 are explained by the photoionization model in the framework of the interacting stellar

wind theory (Kwok, Purton, & FitzGerald 1978; Kwok 1994), those of the molecular envelope are not. Recently, Volk & Kwok (1997) presented a model of the neutral envelope of NGC 7027 based on *ISO* results of CO  $J = 14-13$ ,  $15-14$ ,  $16-15$ , and ortho- $H_2O$   $179.5 \mu m$  lines, as well as previous radio and infrared observations. Although their model can account for *ISO* results available to them, it fails to explain the vast amount of other molecular lines now detected by *ISO*. In this paper, we present a thermal-chemical model that is consistent with currently available *ISO* results on NGC 7027 as well as previous results in other wave bands—in particular, the observations for a number of prominent  $H_2$  infrared lines (Treffer et al. 1976; Beckwith et al. 1980; Graham et al. 1993). In addition, we make predictions for the intensities of other  $H_2$  infrared emission lines that may be detectable.

### 2. THE THERMAL AND CHEMICAL STRUCTURE OF THE NEUTRAL ENVELOPE

Molecular rovibrational lines and atomic fine structure lines are good diagnostics of dense warm neutral clouds (Tielens & Hollenbach 1985; Hollenbach & McKee 1989). For NGC 7027,  $H_2$  rovibrational lines were detected by Treffer et al. (1976) and Beckwith et al. (1980). Subsequently O I  $63 \mu m$  and C II  $157 \mu m$  lines (Ellis & Werner 1984) were also seen. The  $H_2$   $1-0$  S(1) line emission was mapped by Gatley, Depoy, & Fowler (1988) and Graham et al. (1993). Although originally thought to be due to shock excitation (Beckwith et al. 1980), these infrared lines can be interpreted as originating from a dense photodissociation region (Burton, Hollenbach, & Tielens 1990; Graham et al. 1993;

<sup>1</sup> Email: myan@astro1.panet.utoledo.edu.

<sup>2</sup> Email: sfederm@uoft02.utoledo.edu.

<sup>3</sup> Email: adalgarno@cfa.harvard.edu.

<sup>4</sup> Email: jon@physics.utoledo.edu.

Liu et al. 1996). Since the neutral envelope surrounds the ionized nebula, it is subjected to the far-ultraviolet photons from the nebula. It is also influenced by a forward shock in the framework of the largely successful interacting stellar wind theory (Kwok 1994; Volk & Kwok 1997). Thus we construct a model for the neutral envelope in which both photodissociation and a shock play important roles.

The shape of the neutral envelope of NGC 7027 on the sky is largely rectangular (Graham et al. 1993). To simplify our treatment, we use the spherical approximation; the geometry of our model is the same as that of Volk & Kwok (1997). In our model NGC 7027 has a three-component structure: a central ionized region; a thin, neutral, high-density shell of constant density; and a stellar wind region with the usual inverse-square density profile. We assume that the radiation field incident upon the neutral envelope is represented by a  $\chi = 0.5 \times 10^5$  Draine field (Draine 1978; Burton et al. 1990; Liu et al. 1996). The density and thickness of the high-density shell are taken to be  $n_{\text{H}} = 2 \times 10^6 \text{ cm}^{-3}$  and  $3 \times 10^{15} \text{ cm}$ , respectively. The density at the inner edge of the stellar wind region is taken to be  $10^5 \text{ cm}^{-3}$ . According to the interacting stellar wind theory, both the high-density shell and the outer stellar wind belong to the same asymptotic giant branch wind (Kwok 1994; Volk & Kwok 1997). Thus the elemental abundances for the dense shell should be the same as those of the stellar wind region, and they are taken to be the standard interstellar medium (ISM) values (Tielens & Hollenbach 1985), since the carbon abundance in the stellar wind region may be much smaller than that of the ionized region (Middlemass 1990) and close to the standard ISM value (Jaminet et al. 1991). The adopted parameters are similar to those of Burton et al. (1990), Liu et al. (1996), and Volk & Kwok (1997).

We adopted the chemical reaction network and rate coefficients from the compilation of Millar, Farquhar, & Willacy (1997) except the photoprocesses, which were taken from the compilation of Sternberg & Dalgarno (1995). We included H, He, C, N, O, Si, S, Cl, and Fe in our model. Except for  $\text{H}_3\text{O}^+$  and  $\text{CH}_3^+$ , only three-constituent and smaller species in the Millar et al. (1997) compilation were involved. In all, 1389 reactions and 134 species are considered. We used the  $\text{H}_2$  self-shielding formula of Federman, Glassgold, & Kwan (1979). The CO self-shielding and mutual-shielding formula was taken from van Dishoeck & Black (1988); for the high-density shell, the shielding effect was reduced by a factor of about 2.3 (Federman & Yan 1998), since the CO excitation temperature there is much higher than the 10 K adopted by van Dishoeck & Black (1988). Comprehensive studies of CO self-shielding and mutual-shielding in warm, dense clouds are needed for a more detailed study of NGC 7027. They may also be needed for studies of other warm, dense, shocked regions and photodissociation regions. We computed the depth-dependent fractional abundances of chemical species by solving equilibrium chemical equations of the form

$$\begin{aligned} & \sum_{jl} n_{\text{H}} k_{ijl}(T) x_j x_l + \sum_j (\Gamma_{ij} + \zeta_{ij}) x_j \\ & = x_i \left\{ \sum_{jl} n_{\text{H}} k_{jil} x_l + \sum_j [\Gamma_{ji}(z) + \zeta_{ji}] \right\}, \end{aligned} \quad (1)$$

where  $x_i = n_i/n_{\text{H}}$  is the abundance of atom or molecule  $i$ ,  $k_{ijl}(T)$  are the (temperature-dependent) rate coefficients

( $\text{cm}^3 \text{ s}^{-1}$ ) for chemical reactions between species  $j$  and  $l$  that lead to the production of  $i$ , and  $\Gamma_{ij}(z)$  and  $\zeta_{ij}$  are the photon and cosmic-ray destruction rates ( $\text{s}^{-1}$ ) of species  $j$  with products  $i$ .

Since rovibrationally excited  $\text{H}_2$  plays an important role in the thermal and chemical processes (Tielens & Hollenbach 1985; Sternberg & Dalgarno 1989), we solved the fractional abundance of each  $\text{H}_2$  rovibrational level through statistical equilibrium equations as follows:

$$\begin{aligned} & x_{vj} \left\{ \sum_{v'j'} [A(vj \rightarrow v'j') + \sum_i n_i Q_i(T; vj \rightarrow v'j')] \right. \\ & \quad \left. + \sum_{v^*j^*} P(vj \rightarrow v^*j^*) + \zeta + \sum_i k_{ivj}(T) n_i \right\} \\ & = \sum_{v'j'} x_{v'j'} \left[ A(v'j' \rightarrow vj) + \sum_i n_i Q_i(T; v'j' \rightarrow vj) \right. \\ & \quad \left. + \sum_{v^*j^*} P(v'j' \rightarrow v^*j^*) b(v^*j^* \rightarrow vj) \right] \\ & \quad + R\beta(vj) n_{\text{H}} x_1 + \sum_{il} k_{ilvj}(T) n_{\text{H}} x_i x_l. \end{aligned} \quad (2)$$

In these equations  $A(vj \rightarrow v'j')$  are quadrupole radiative transition probabilities from levels  $vj$  to levels  $v'j'$ , and  $P(vj \rightarrow v^*j^*)$  are the UV excitation rates out of level  $vj$  of the ground electronic state to level  $v^*j^*$  of an excited electronic state. The parameters  $b(v^*j^* \rightarrow vj)$  are the branching ratios of the transitions between levels  $v^*j^*$  and  $vj$  of the excited and ground electronic states. We followed the approach of Sternberg & Dalgarno (1989) in calculating  $P(vj \rightarrow v^*j^*)$  and  $b(v^*j^* \rightarrow vj)$ ;  $\zeta$  is the cosmic-ray destruction rate. The  $Q_i$  are the temperature-dependent rate coefficients for collisions with particles of density  $n_i$  that induce transitions from  $vj$  to  $v'j'$ . We use values of  $Q_i$  for collisions with H,  $\text{H}_2$ , He, and electrons from Sternberg & Dalgarno (1989) and Tine et al. (1997). The value  $R$  is the rate coefficient for  $\text{H}_2$  formation on grains; we adopt a value of  $6 \times 10^{-18} \text{ cm}^3 \text{ s}^{-1}$ , since the dust-gas mass ratio in the neutral envelope of NGC 7027 is about 20% the standard value for the ISM (Hoare et al. 1992). The value  $x_1$  is the atomic hydrogen fractional abundance, and  $\beta(vj)$  are the fractions of molecules populating levels  $vj$  in the process of  $\text{H}_2$  formation on grains. We adopt the formula for  $\beta(vj)$  given by Sternberg & Dalgarno (1989). The values  $k_{ivj}(T)$  are the rate coefficients for chemical reactions between species  $i$  and  $\text{H}_2$  in level  $vj$  that lead to the destruction of  $\text{H}_2$ ;  $k_{ilvj}(T)$  are the rate coefficients for chemical reactions between species  $i$  and  $l$  that lead to the formation of  $\text{H}_2$  in level  $vj$ . We followed the approach of Sternberg & Dalgarno (1995) when calculating  $k_{ivj}(T)$  and  $k_{ilvj}(T)$ , except that we replaced the  $\text{H}_2$  vibrational levels they used by  $\text{H}_2$  rovibrational levels.

The relative roles of different chemical processes depend on the gas temperature. The depth-dependent gas temperature was calculated under the assumption of thermal equilibrium:

$$G(T) - L(T) = 0, \quad (3)$$

where  $G(T)$  and  $L(T)$  are the total heating and cooling rates. The major thermal processes considered by us are grain photoelectric heating, heating from collisional quenching of  $\text{H}_2^*$ , gas-grain collisional heating (or cooling), cooling through atomic and ionic metastable and fine-structure emission, and molecular rovibrational emission. Our

heating and cooling rate formulas are taken from Sternberg & Dalgarno (1995) and Yan (1996). We adopted a turbulence velocity of  $2 \text{ km s}^{-1}$  (Jamiet et al. 1991) for the line width in calculating the cooling rates.

Gas-grain collisions are an important cooling channel for dense, warm gases (Hollenbach & McKee 1989). The dust temperature is needed in calculating the rate of this cooling process. It was calculated from the thermal balance between dust absorption and dust emission. We used a formula given in Sternberg & Dalgarno (1989) to calculate the dust temperature except that we assume a minimum dust temperature of 20 K to account for the dust absorption of infrared emission (Tielens & Hollenbach 1985).

We assumed the minimum temperature in the high-density shell to be 200 K, which occurs at the outer boundary. The temperature was needed for fitting of the observed CO  $J = 14-13$ ,  $15-14$ , and  $16-15$  lines and can be interpreted as due to the heating by the forward shock, which is produced by the two interacting stellar winds (Kwok 1994).

The thermal and chemical structure of the neutral envelope was solved by integrating

$$\frac{dN_i(z)}{dz} = n_H x_i(z) \quad (4)$$

through a Bulirsch-Stoer adaptive step size integrator (Press et al. 1992), where  $x_i(z)$  and  $N_i(z)$  are the fractional abundance and column density for species  $i$ . The  $x_i(z)$  was

derived by solving equations (1), (2), and (3) together. Numerically, this was done through the following procedure. First, we solved equations (1) and (2) for fractional abundances using the temperature from the previous step (or an assumed temperature for the first step). Second, we solved equation (3) for the temperature using the derived fractional abundances. We then iterated the two processes until the temperature and fractional abundances converge.

The major results of our calculation are illustrated in Figures 1–3. We note that the photodissociation and photoionization of atomic and molecular species by the diffuse interstellar UV radiation field (from outside the system) are not considered in the model. The  $\text{H}_2$  photodissociation rate  $\xi$ , which is derived by summing up the products of the UV pumping rates (Sternberg & Dalgarno 1989) and the corresponding dissociation fractions (Stephens & Dalgarno 1972) and the kinetic temperature  $T$ , are presented as functions of the radial distance in Figure 1. Within the high-density shell,  $\xi$  decreases monotonically from about  $10^{-6} \text{ s}^{-1}$  at the inner surface to about  $10^{-15} \text{ s}^{-1}$  at the outer edge, while  $T$  also decreases from about 3000 to 200 K. In the stellar wind region,  $\xi$  continues to decrease and becomes far less than the cosmic-ray ionization rate at a radial distance of  $4 \times 10^{17} \text{ cm}$ , the end point of our calculation. The kinetic temperature  $T$  starts from a value of about 60 K, after a sharp drop from 200 K at the outer edge of the high-density shell, and decreases monotonically to about 10 K at the end point. Figure 2 shows fractional abundances of

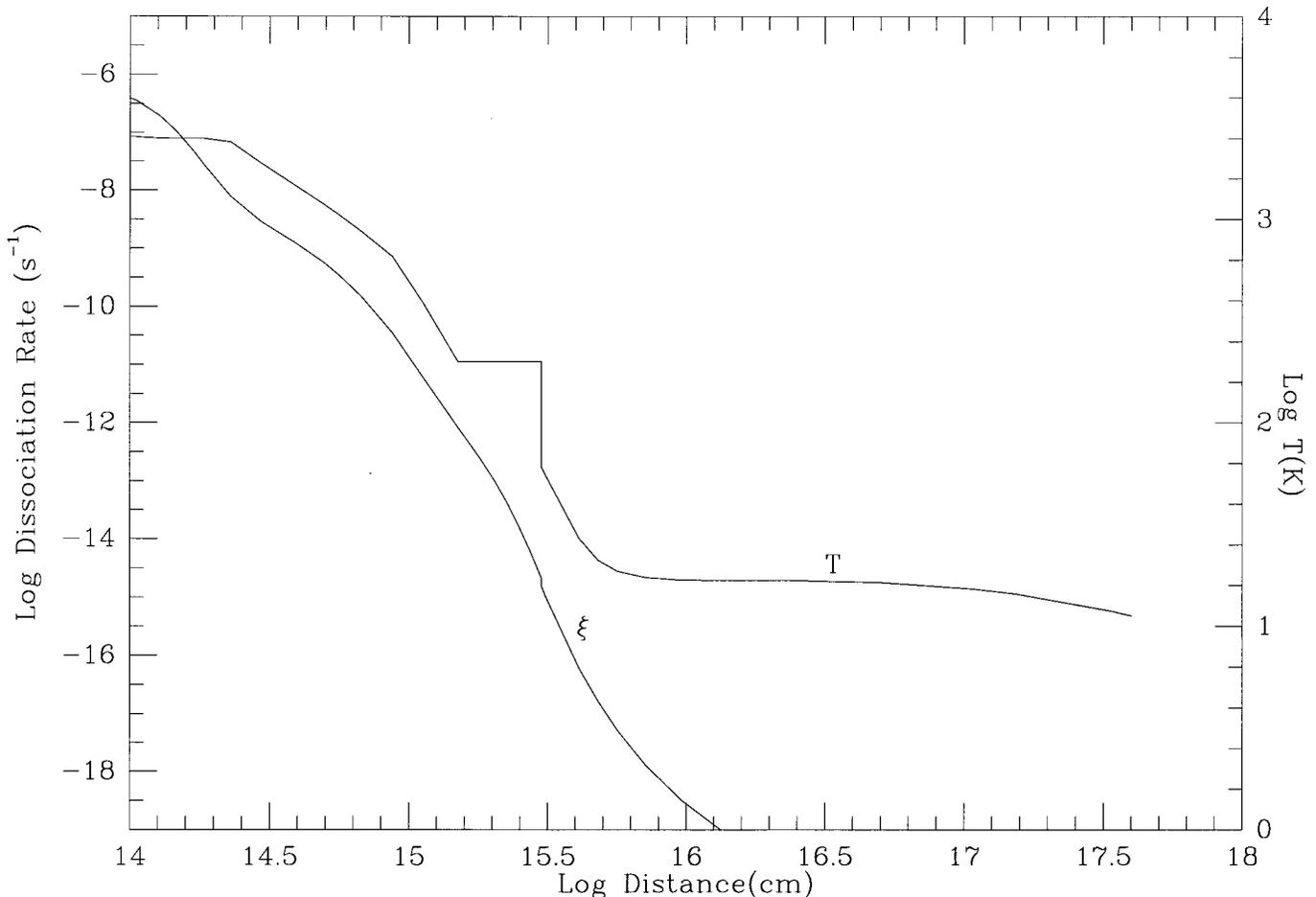


FIG. 1.— $\text{H}_2$  dissociation rate and kinetic temperature as functions of the radial distance from the inner surface of the high-density shell. The high-density shell is on the left, before the sharp jump at  $3 \times 10^{15} \text{ cm}$  in temperature; the stellar wind region is on the right, after the jump.

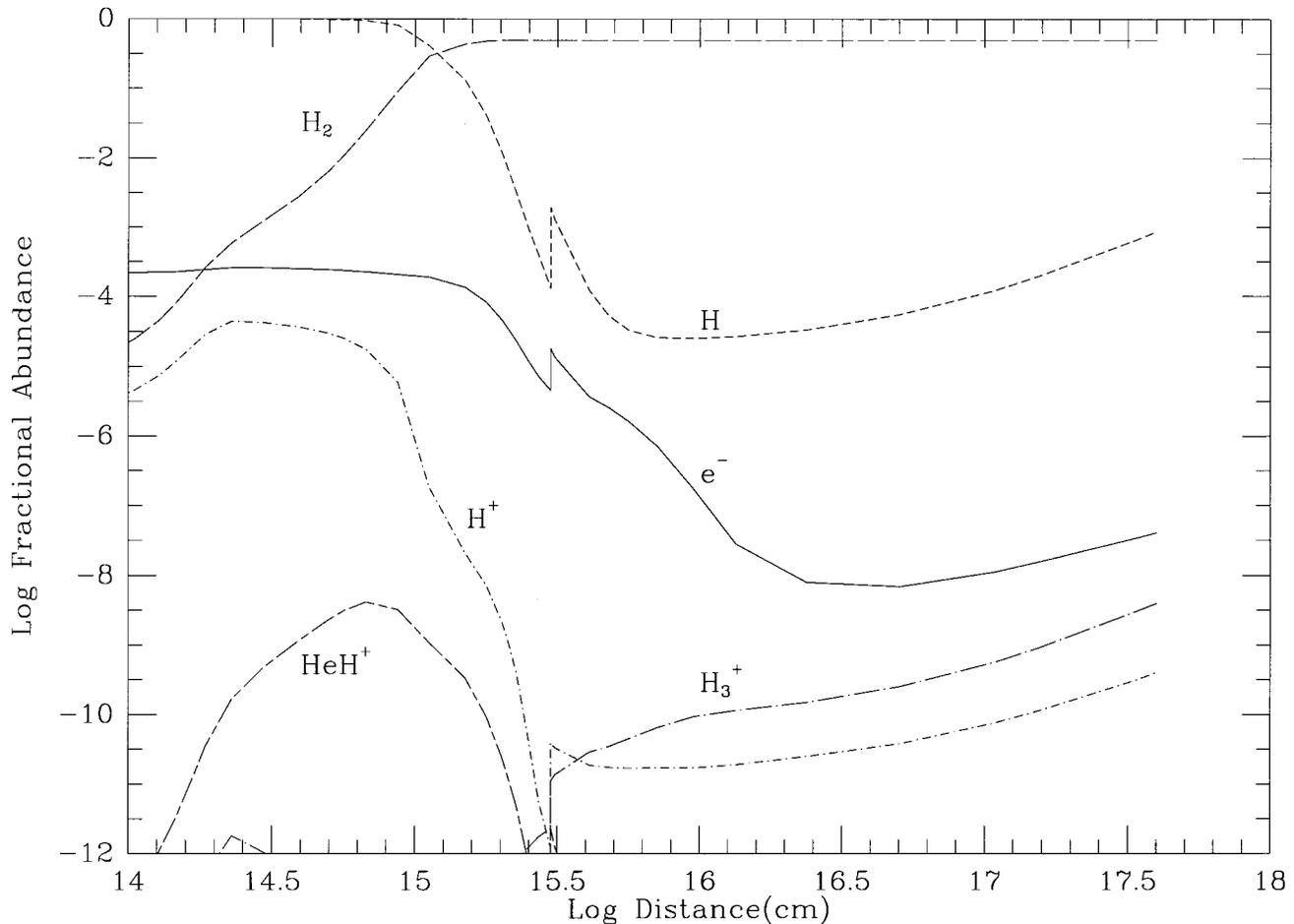


FIG. 2.—Fractional abundances of  $e$ ,  $H$ ,  $H_2$ ,  $H^+$ ,  $H_3^+$ , and  $HeH^+$  as functions of the radial distance from the inner surface of the high-density shell. The high-density shell is on the left, before the sharp jumps at  $3 \times 10^{15}$  cm in fractional abundances of  $H$ ,  $H_3^+$ , and so on; the stellar wind region is on the right, after the jumps.

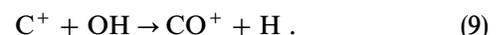
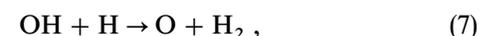
$H$ ,  $H_2$ ,  $H^+$ ,  $H_3^+$ ,  $HeH^+$ , and electrons. The fractional abundances of  $H^+$ ,  $H_3^+$ , and  $HeH^+$  are always very small and of negligible observational consequence. Within the high-density shell, the gas is atomic in the inner shell and molecular at the outer boundary. The  $H_2$  abundance increases monotonically from about  $10^{-5}$  to 0.5. The  $H$  and electron abundances are nearly constant at 1 and  $2 \times 10^{-4}$  for the inner half shell, but they decrease monotonically to  $10^{-4}$  and  $3 \times 10^{-6}$  at the outer edge. In the stellar wind region, the  $H_2$  and  $H$  abundances are almost constant at 0.5 and  $10^{-4}$ , while the electron abundance decreases from about  $10^{-5}$  to about  $10^{-8}$  before it starts to increase and reaches a value of  $5 \times 10^{-8}$  at the end point of our calculation. The increase in  $H$  abundance at large radial distances is due to the increase of  $\zeta/n_H$ , where  $\zeta = 1.3 \times 10^{-17} \text{ s}^{-1}$  is the cosmic-ray ionization rate and the density  $n_H$  goes with the inverse square of the radial distance. In Figure 3 we present the fractional abundances of  $C$ ,  $O$ ,  $C^+$ ,  $CO$ ,  $OH$ ,  $CH$ ,  $H_2O$ , and  $CH^+$  as functions of radial distance. We concentrate on  $CO$ ,  $OH$ ,  $CH$ ,  $H_2O$ , and  $CH^+$ , since these species were detected by *ISO* and they exist only in the neutral envelope. In the high-density shell the fractional abundances of  $OH$ ,  $CH^+$ , and  $H_2O$  reach their peak values of  $10^{-6}$ ,  $3 \times 10^{-8}$ , and  $3 \times 10^{-8}$ , respectively, at about the place where the temperature is about 1000 K. The  $CH$  abundance peaks farther outside at a value of  $2 \times 10^{-7}$ , whereas that of  $CO$  peaks at the outer edge with almost all carbon locked up in

$CO$ . In the stellar wind region,  $CO$  contains almost all the carbon. The carbon and oxygen that are not in  $CO$  exist almost entirely in atomic form. The abundances of  $CH$ ,  $OH$ , and  $H_2O$  increase outward, but the densities of these species are small, since the total density goes with the inverse square of the radial distance.

Since the density and temperature of the stellar wind region are low, the detected *ISO* lines and  $H_2$  infrared lines are predominantly produced in the high-density shell. Thus we only discuss the distribution and major formation and destruction processes for the observed species in the high-density shell.  $OH$  and  $CH^+$  exist in regions where the temperature is around 1000 K or higher.  $OH$  is produced by the endothermic reaction



and is removed by



The endothermic reaction



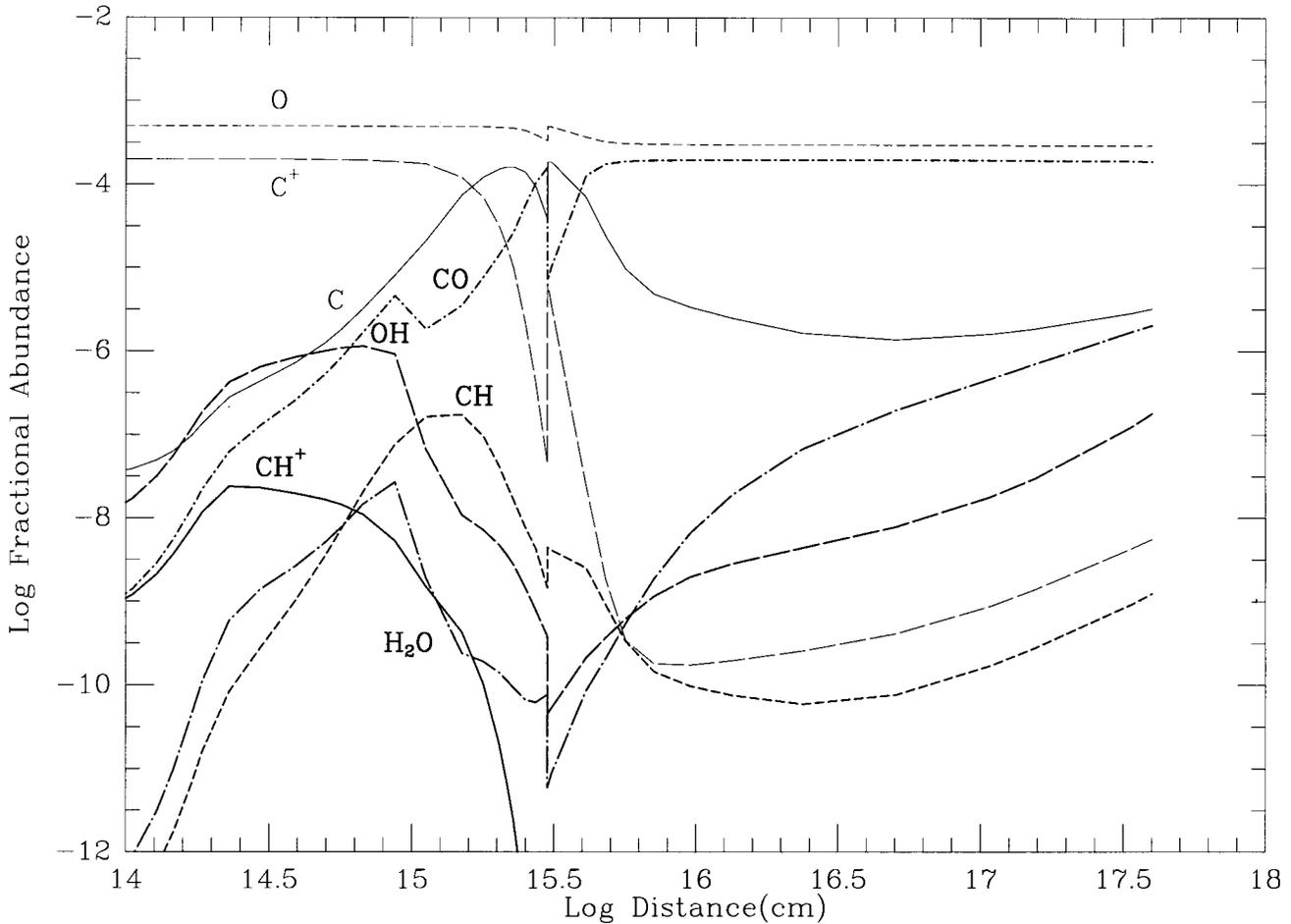
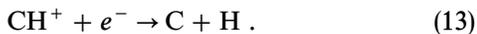
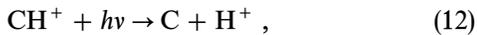
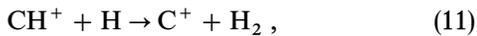


FIG. 3.—Fractional abundances of C, O, C<sup>+</sup>, CO, OH, CH, H<sub>2</sub>O, and CH<sup>+</sup> as functions of the radial distance from the inner surface of the high-density shell. The high-density shell is on the left, before the sharp jumps at  $3 \times 10^{15}$  cm in fractional abundances of OH, CH, CO, and so on; the stellar wind region is on the right, after the jumps.

leads to CH<sup>+</sup>, and CH<sup>+</sup> is removed by



The molecule CO is abundant in the outer boundary layer of the high-density shell. At the outer edge of this layer where the temperature is 200 K, almost all carbon is locked up in CO. At the inner part of this layer where the temperature ranges from 200 to about 1000 K, the CO abundance is smaller. This warm high-density boundary layer is the region where high-*J* CO rotational lines originate. In the region, CO is formed through reaction (8) and through reaction (9) followed by

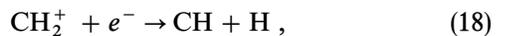
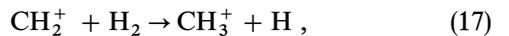
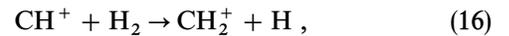


and removed through

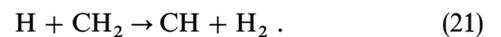
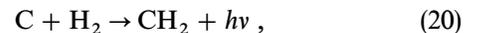


The boundary layer of the high-density shell is also where CH resides. In the inner region of the boundary, where the temperature ranges from several 100 K to about 1000 K,

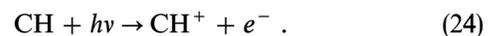
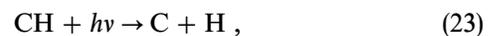
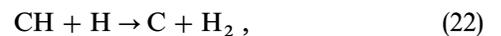
CH is produced by



At the edge of the high-density shell where the temperature is 200 K, CH is produced by



CH is always removed by



We also made a calculation including the influence of the diffuse interstellar radiation field. The thermal, chemical, and ionizational structures of the high-density shell are unaltered. Those of the outer stellar wind region show a modest change, but the corresponding temperature and

abundances are still too small to have observational consequences on *ISO* data and H<sub>2</sub> infrared data.

### 3. EMISSION-LINE INTENSITIES

We computed the intensities of CO, OH, CH, and CH<sup>+</sup> rotational lines based on the fractional abundances presented in the last section. Far-infrared emission from the low-density stellar wind region is negligible, because the temperature and density of the region are too low. Thus only the high-density shell was included in our calculation. We employed a modified escape probability formalism. For each molecular species, we first assumed all emission lines to be optically thin, and we calculated the fractional population of rotational levels at each depth step by solving the statistical equilibrium equations (Yan & Dalgarno 1997). Then we computed the opacity and escape probability (de Jong, Dalgarno, & Boland 1980) of each rotational line traversing the whole depth of the high-density shell. At this point, the fractional abundances of rotational levels at each depth step were recomputed by solving the statistical equilibrium equations where now Einstein *A* values were replaced by the effective *A* values, which are products of the *A* values and the corresponding escape probabilities. We iterated the above procedure until the fractional abundances of rotational levels converged. The intensity of each rotational line was computed by integrating the effective emissivity through the whole depth of the high-density shell. The effect of stimulated absorption and emission from dust-emitted infrared continuum radiation is included in our calculation.

In solving the statistical equilibrium equations for molecular rotational levels, Einstein *A* values and collisional excitation rate coefficients were needed. Molecular lines in the high-density shell are excited by collisions with H and H<sub>2</sub>. Electron impact excitation is negligible, since the fractional abundance of electrons is very small. For CO, we included the lowest 40 rotational levels of the ground vibrational level in our calculation. We adopted the energy levels, Einstein *A* values, and H<sub>2</sub> impact rate coefficients of McKee et al. (1982) and H impact rate coefficients of Chu & Dalgarno (1975). For OH, we included the lowest 10 rotational levels. The energy levels and Einstein *A* values were taken from Offer & van Dishoeck (1992) and Destombes et al. (1977), respectively. When the temperature is high, the H<sub>2</sub> impact quenching rate coefficients are not sensitive to the temperature (Dewangan, Flower, & Alexander 1987; Offer & van Dishoeck 1992). We assumed that they are independent of the temperature, and we used the high-temperature (600 K) H<sub>2</sub> impact-quenching rate coefficients of Offer & van Dishoeck (1992). The excitation rate coefficients were calculated from detailed balance. There are not any available theoretical or experimental data on the H impact rate coefficients. The H impact rate coefficients are typically within 1–10 times the corresponding H<sub>2</sub> impact values, and we assumed them to be the same as the corresponding H<sub>2</sub> impact values to fit the observed OH line fluxes. For CH, we also included the lowest 10 rotational levels. The energy levels and Einstein *A* values were taken from Brown & Evenson (1983). There are no data available on the H and H<sub>2</sub> impact rate coefficients for CH. We assumed that they are the same and equal to 1.3 times the corresponding H<sub>2</sub>-OH impact rate coefficients to fit the observations. For CH<sup>+</sup>, we included the lowest 20 rotational levels. Neither

theoretical nor experimental data are available on the H and H<sub>2</sub> impact rate coefficients. We assumed the H and H<sub>2</sub> impact rate coefficients to be the same. To fit observations, we chose H<sub>2</sub> collisional quenching rate coefficients that are 2.5 times the corresponding HCO<sup>+</sup> values (Turner 1994). The upward rate coefficients were derived from detailed balance. The molecular line width was needed in calculating the opacities and the effective *A* values; we adopted a value of 2 km s<sup>-1</sup> (Jaminet et al. 1991).

The observed and computed intensities for CO, OH, CH and CH<sup>+</sup> rotational lines are listed in Tables 1–4. The theoretical CO line fluxes are in very good agreement with the observed fluxes except for *J* = 21–20 up to *J* = 24–23, where the theoretical values are 2–3 times the observed values. The discrepancies for these four CO lines may be attributed to *ISO* calibration or data reduction uncertainty, since the spectra are weak and noisy (Liu et al. 1996). Except for the 119.34 μm line, the calculated OH line fluxes agree with the observed values with the maximum deviation being less than 20% (see Table 2). The 119.34 μm line emis-

TABLE 1  
OBSERVED AND COMPUTED CO LINE FLUXES OF NGC 7027

$\lambda_{\text{vac}}(\mu\text{m})$	Identification	Flux (Observed) <sup>a</sup>	Flux (Theoretical) <sup>a</sup>
186.00.....	<i>J</i> = 14–13	22.5	23.1
173.63.....	<i>J</i> = 15–14	21.2	19.7
162.81.....	<i>J</i> = 16–15	17.8	15.8
153.27.....	<i>J</i> = 17–16	10.0	12.4
144.78.....	<i>J</i> = 18–17	9.04 <sup>b</sup>	10.0
137.20.....	<i>J</i> = 19–18	12.1 <sup>b</sup>	8.61
130.37.....	<i>J</i> = 20–19	8.44 <sup>b</sup>	7.84
124.19.....	<i>J</i> = 21–20	4.95 <sup>b</sup>	7.44
118.58.....	<i>J</i> = 22–21	5.13 <sup>b</sup>	7.21
113.46.....	<i>J</i> = 23–22	3.03 <sup>b</sup>	7.00
108.76.....	<i>J</i> = 24–23	4.80 <sup>b</sup>	6.75
104.44.....	<i>J</i> = 25–24	9.56 <sup>b</sup>	6.43
100.46.....	<i>J</i> = 26–25	~ 10	6.05
96.77.....	<i>J</i> = 27–26	~ 10	5.61
90.16.....	<i>J</i> = 29–28	< 10	4.64

<sup>a</sup> In 10<sup>-20</sup> W cm<sup>-2</sup>.

<sup>b</sup> Weak and noisy.

TABLE 2  
OBSERVED AND COMPUTED OH LINE FLUXES OF NGC 7027

$\lambda_{\text{vac}}(\mu\text{m})$	Identification	Flux (Observed) <sup>a</sup>	Flux (Theoretical) <sup>a</sup>
119.34.....	<sup>2</sup> Π <sub>3/2</sub> <i>J</i> = $\frac{5}{2} - \frac{3}{2}$	15.3 ± 1.1	29.5 <sup>b</sup>
84.51.....	<sup>2</sup> Π <sub>3/2</sub> <i>J</i> = $\frac{7}{2} - \frac{5}{2}$	19.8 ± 1.5	22.3
65.20.....	<sup>2</sup> Π <sub>3/2</sub> <i>J</i> = $\frac{9}{2} - \frac{7}{2}$	< 24 (3 σ)	3.9
163.26.....	<sup>2</sup> Π <sub>1/2</sub> <i>J</i> = $\frac{3}{2} - \frac{1}{2}$	6.2 ± 0.7	4.9
98.73.....	<sup>2</sup> Π <sub>1/2</sub> <i>J</i> = $\frac{5}{2} - \frac{3}{2}$	6.9 ± 1.2	7.0
79.15.....	<sup>2</sup> Π <sub>1/2-3/2</sub> <i>J</i> = $\frac{1}{2} - \frac{3}{2}$ <sup>c</sup>	17.6 ± 2.3	17.8

<sup>a</sup> In 10<sup>-20</sup> W cm<sup>-2</sup>.

<sup>b</sup> The flux may be reduced to about 15 if we include the effect of absorption by interstellar OH.

<sup>c</sup> From <sup>2</sup>Π<sub>1/2</sub> *J* =  $\frac{1}{2}$  to <sup>2</sup>Π<sub>3/2</sub> *J* =  $\frac{3}{2}$ .

TABLE 3  
OBSERVED AND COMPUTED CH LINE FLUXES OF NGC 7027

$\lambda_{\text{vac}}(\mu\text{m})$	Identification	Flux (Observed) <sup>a</sup>	Flux (Theoretical) <sup>a</sup>
149.2.....	${}^2\Pi_{3/2}(F_2)J = \frac{3}{2} - \frac{1}{2}$	4.2	4.1 <sup>b</sup>
115.7.....	${}^2\Pi_{3/2}(F_2)J = \frac{3}{2} - \frac{3}{2}$	2.0	1.6
180.6.....	${}^2\Pi_{1/2}(F_1)J = \frac{5}{2} - \frac{3}{2}$	3.3	3.9
118.7.....	${}^2\Pi_{1/2}(F_1)J = \frac{7}{2} - \frac{5}{2}$	~2.0	1.8

<sup>a</sup> In  $10^{-20}$  W cm<sup>-2</sup>.

<sup>b</sup> The flux may be reduced to about 2.7 if we include the effect of absorption by interstellar CH.

sion can be absorbed by interstellar OH molecules, since the line ends in the rotational ground state. Indeed, there is an interstellar cloud in the sight line with strong Na absorption lines, and the velocity of the cloud coincides with the central velocity of the approaching high-density half-shell (Dinerstein, Sneden, & Uglum 1995; Jaminet et al. 1991). Interstellar clouds with strong absorption lines typically have OH column densities of about  $10^{14}$  cm<sup>-2</sup> (van Dishoeck & Black 1986), which corresponds to an opacity of about 2 for the line assuming a velocity width of 4 km s<sup>-1</sup>. Therefore, up to half of the theoretical 119.34  $\mu\text{m}$  line flux may be absorbed by interstellar OH, and the resultant flux of about  $15 \times 10^{-20}$  W cm<sup>-2</sup> is in good agreement with the observed value of  $15.3 \times 10^{-20}$  W cm<sup>-2</sup>. Interstellar absorption of the OH 79.15  $\mu\text{m}$  line that also ends in the rotational ground state may reduce the calculated flux by as much as 20%. After including this effect, the deviation of the theoretical flux from the observed value is still less than 20%. Measurements of OH absorption would yield the column density of interstellar OH and provide additional constraints on our model. The derived CH line fluxes (Table 3) are in good agreement with the observed values. The calculated fluxes change somewhat with the adopted collisional rate coefficients, adding some degree of uncertainty into our result.

Liu et al. (1997) argued against identifying the 149.2  $\mu\text{m}$  line with HeH<sup>+</sup> and favored a CH origin. Our Table 3 seems to support their assertion. However, as in the case of the OH line at 119.34  $\mu\text{m}$ , the CH 149.2  $\mu\text{m}$  line emission in the approaching high-density half-shell may be absorbed by interstellar CH molecules. The typical CH column density of interstellar clouds with strong absorption lines is about  $4 \times 10^{13}$  cm<sup>-2</sup> (van Dishoeck & Black 1986), which corresponds to an opacity of about 0.5 for this line assuming a velocity width of 4 km s<sup>-1</sup>. Therefore, about  $\frac{1}{3}$  of the

TABLE 4

OBSERVED AND COMPUTED CH<sup>+</sup> LINE FLUXES OF NGC 7027

$\lambda_{\text{vac}}(\mu\text{m})$	Identification	Flux (Observed) <sup>a</sup>	Flux (Theoretical) <sup>a</sup>
179.61.....	$J = 2-1$	$15.1 \pm 0.5$	11.0
119.86.....	$J = 3-2$	$21.8 \pm 1.7$	19.7
90.02.....	$J = 4-3$	$23.2 \pm 2.5$	23.9
72.15.....	$J = 5-4$	$25.0 \pm 4.1$	24.9
60.26.....	$J = 6-5$	$24.1 \pm 3.3$	23.6

<sup>a</sup> In  $10^{-20}$  W cm<sup>-2</sup>.

observed 149.2  $\mu\text{m}$  line flux may be from HeH<sup>+</sup> at the edge of the ionized nebula (Cecchi-Pestellini & Dalgarno 1993). Future optical measurements of CH absorption are needed to determine the column density of interstellar CH and provide a better constraint on the HeH<sup>+</sup> flux.

The computed CH<sup>+</sup> line fluxes are in excellent agreement with observed values (see Table 4) except for the 179.6  $\mu\text{m}$  line, which also coincides with an ortho-H<sub>2</sub>O line. Thus we estimate the water contribution to the 179.6  $\mu\text{m}$  line flux to be  $4 \times 10^{-20}$  W cm<sup>-2</sup>. Further discussion on the possible H<sub>2</sub>O emission is presented in the next section.

We have also used other line widths (1 and 5 km s<sup>-1</sup>) and other escape probability formulas (Neufeld & Kaufman 1993; Osterbrock 1989) in our calculation. The relative line strengths of the species are essentially the same. We conclude that the theoretical and observational fluxes of CO, OH, CH, and CH<sup>+</sup> lines are in good agreement. The agreement provides strong support for our model.

We computed the H<sub>2</sub> infrared spectrum generally following the approach of Sternberg & Dalgarno (1989), but we also included the effect of formation of H<sub>2</sub> in rovibrationally excited levels through chemical reactions. We included the extinction effect of the grains in the envelope by assuming a  $1/\lambda$  extinction law and a visual extinction of 3 (Middlemass 1990). Our results indicate that H<sub>2</sub> is pumped predominantly by ultraviolet fluorescence and formation on grains, but chemical reactions that lead to the formation of H<sub>2</sub> in excited levels can contribute up to 5%. Observational results exist for a number of prominent lines of H<sub>2</sub>. The observed and computed fluxes for these lines are listed in Table 5, and they agree quite well. Our calculation is also in good agreement with the result of a recent mapping of NGC 7027 in the H<sub>2</sub>  $v = 1-0$  S(1) line by Graham et al. (1993). We derived an intensity of  $4.6 \times 10^{-4}$  ergs s<sup>-1</sup> cm<sup>-2</sup> sr<sup>-1</sup> for the line of sight through the center of the neutral envelope. After subtracting the He I 2.11  $\mu\text{m}$  contribution, the average intensity for the central region from Graham et al. (1993) is  $5 \times 10^{-4}$  ergs s<sup>-1</sup> cm<sup>-2</sup> sr<sup>-1</sup>. We also show predicted fluxes for the 30 strongest H<sub>2</sub> lines in Table 6 and predicted fluxes for the 15 strongest H<sub>2</sub> lines in the *K* band in Table 7. We note that the predicted fluxes for the H<sub>2</sub> pure rotational lines in Table 6 are rather uncertain (with an accuracy of a factor of a few), since these fluxes depend on the rates of reactions of heavy elements with H<sub>2</sub> in each pure rotational level and reverse reactions that form H<sub>2</sub> in each pure rotational level. Considerable uncertainties exist in the rate coefficients for these reactions and in the elemental abundances. The transitions in Table 7 are mainly those

TABLE 5

OBSERVED AND COMPUTED H<sub>2</sub> LINE FLUXES OF NGC 7027

Identification	$\lambda_{\text{vac}}(\mu\text{m})$	Flux (Observed) <sup>a</sup>	Flux (Theoretical) <sup>a</sup>
$v = 1-0\text{S}(1)$ .....	2.122	$14 \pm 1^b$	13.21
$v = 1-0\text{Q}(1)$ .....	2.407	~10 <sup>c</sup>	13.81
$v = 1-0\text{Q}(3)$ .....	2.424	~10 <sup>c</sup>	10.05
$v = 1-0\text{S}(2)$ .....	2.034	<6 <sup>b</sup>	4.18
$v = 1-0\text{Q}(2)$ .....	2.413	<5 <sup>c</sup>	4.20
$v = 1-0\text{S}(0)$ .....	2.223	$4.6 \pm 0.7^b$	3.62
$v = 2-1\text{S}(1)$ .....	2.248	$1 \pm 0.5^b$	0.87

<sup>a</sup> In  $10^{-20}$  W cm<sup>-2</sup>.

<sup>b</sup> From Beckwith et al. 1980.

<sup>c</sup> From Treffers et al. 1976.

TABLE 6  
PREDICTED FLUXES OF THE 30 STRONGEST H<sub>2</sub> LINES  
FROM NGC 7027

Identification	$\lambda_{\text{vac}}(\mu\text{m})$	Flux (Theoretical) <sup>a</sup>
<i>v</i> = 1-0S(7) .....	1.748	1.166
<i>v</i> = 1-0S(5) .....	1.836	3.834
<i>v</i> = 1-0S(4) .....	1.892	2.183
<i>v</i> = 1-0S(3) .....	1.958	9.103
<i>v</i> = 1-0S(2) .....	2.034	4.175
<i>v</i> = 2-1S(3) .....	2.074	0.898
<i>v</i> = 1-0S(1) .....	2.122	13.205
<i>v</i> = 1-0S(0) .....	2.223	3.621
<i>v</i> = 1-0Q(1).....	2.407	13.805
<i>v</i> = 1-0Q(2).....	2.413	4.199
<i>v</i> = 1-0Q(3).....	2.424	10.054
<i>v</i> = 1-0Q(4).....	2.437	2.600
<i>v</i> = 1-0Q(5).....	2.455	5.083
<i>v</i> = 1-0Q(6).....	2.476	1.161
<i>v</i> = 1-0Q(7).....	2.500	2.037
<i>v</i> = 1-0O(2).....	2.627	3.901
<i>v</i> = 1-0O(3).....	2.803	12.690
<i>v</i> = 1-0O(4).....	3.004	3.619
<i>v</i> = 1-0O(5).....	3.235	6.546
<i>v</i> = 1-0O(6).....	3.501	1.220
<i>v</i> = 1-0O(7).....	3.808	1.668
<i>v</i> = 0-0S(8) .....	5.053	13.125
<i>v</i> = 0-0S(7) .....	5.511	76.965
<i>v</i> = 0-0S(6) .....	6.109	71.762
<i>v</i> = 0-0S(5) .....	6.909	220.388
<i>v</i> = 0-0S(4) .....	8.026	124.266
<i>v</i> = 0-0S(3) .....	9.665	247.500
<i>v</i> = 0-0S(2) .....	12.279	64.614
<i>v</i> = 0-0S(1) .....	17.035	47.283
<i>v</i> = 0-0S(0) .....	28.221	2.387

<sup>a</sup> in  $10^{-20}$  W cm<sup>-2</sup>.

observed in the planetary nebula BD +30°3639 (Shupe et al. 1998), although there are differences in the relative fluxes.

#### 4. DISCUSSION AND CONCLUSIONS

So far we concentrated on the high-density shell, although we presented the thermal and chemical structure of the stellar wind region as well. Jaminet et al. (1991)

TABLE 7  
PREDICTED FLUXES OF THE 15 STRONGEST K-BAND H<sub>2</sub>  
LINES FROM NGC 7027

Identification	$\lambda_{\text{vac}}(\mu\text{m})$	Flux (Theoretical) <sup>a</sup>
<i>v</i> = 1-0S(2) .....	2.034	4.175
<i>v</i> = 3-2S(5) .....	2.066	0.152
<i>v</i> = 2-1S(3) .....	2.074	0.898
<i>v</i> = 1-0S(1) .....	2.122	13.205
<i>v</i> = 3-2S(4) .....	2.128	0.072
<i>v</i> = 2-1S(2) .....	2.154	0.372
<i>v</i> = 3-2S(3) .....	2.201	0.212
<i>v</i> = 1-0S(0) .....	2.223	3.621
<i>v</i> = 2-1S(1) .....	2.248	0.866
<i>v</i> = 3-2S(2) .....	2.287	0.085
<i>v</i> = 4-3S(3) .....	2.344	0.070
<i>v</i> = 2-1S(0) .....	2.356	0.248
<i>v</i> = 3-2S(1) .....	2.386	0.165
<i>v</i> = 1-0Q(1).....	2.407	13.805
<i>v</i> = 1-0Q(2).....	2.413	4.199

<sup>a</sup> in  $10^{-20}$  W cm<sup>-2</sup>.

mapped NGC 7027 in the  $J = 3-2$  and  $2-1$  lines of CO. They showed that the kinetic temperature is  $30 \pm 3$  K at a radius of  $8''$  ( $r = 1.2 \times 10^{17}$  cm) and varies with radius approximately like  $r^{-0.85}$ , while the CO:H<sub>2</sub> abundance ratio is  $2_{-1}^{+2} \times 10^{-4}$ . Figures 1 and 3 show that the computed kinetic temperature and CO:H<sub>2</sub> abundance ratio are largely in agreement with observation. [Note that the distance ( $D$ ) in these figures is measured from the inner radius of the high-density shell so that  $D = (r - 6.8 \times 10^{16})$  cm.] Thus our model supports the assertion of Jaminet et al. (1991) that there is a considerable increase in heavy element abundances, particularly that of carbon (the carbon abundance in the ionized region is much higher than the ISM value), during the last 1000 yr of mass loss from the central star. We also made a calculation of the thermal and chemical structure of the high-density shell by assuming the heavy element abundances to be the same as those of the ionized region. We then computed the CH<sup>+</sup> and CH fluxes based on the model. We must assume unreasonably low collisional excitation rate coefficients to force the calculated fluxes to be in agreement with observations. The logical conclusion is that the heavy element abundances in the neutral envelope are close to the ISM values. Future submillimeter observations of C I 609  $\mu\text{m}$  emission from NGC 7027 may provide further confirmation.

We removed the assumption of a constant temperature of 200 K in the outer boundary layer of the high-density shell to see the effect of the shock. The computed temperature decreased rapidly to less than 20 K toward the outer boundary. The resulting intensities of the CO  $J = 14-13$ ,  $15-14$ , and  $16-15$  lines fall far short of the observed values, while those of the CO high- $J$  lines such as  $27-26$  and  $26-25$  remain unchanged. Our model results suggest, therefore, that the far-infrared emission lines of NGC 7027 originate from the high-density shell, whose inner part is a photodissociation region (PDR) and outer part is a shocked region.

We also varied the thickness of the high-density shell in the range of  $(2-4) \times 10^{15}$  cm and the temperature of the shocked region in the range of 100–400 K. The influence on the predicted intensities is negligible for CH<sup>+</sup> and OH and small for CH, because these species are most abundant in the high-temperature ( $\sim 1000$  K) PDR. The influence on the CO low- $J$  ( $J < 20$ ) line intensities, however, is significant. About half of the CO low- $J$  line intensities are produced in the shocked region. The higher the temperature of the shocked region, the higher the  $J$  at which the observed CO intensity peaks. Agreement with observations is best with a shell thickness of  $3 \times 10^{15}$  cm and a shocked gas temperature of 200 K, which are the adopted values in our model.

A number of water far-infrared lines have been tentatively identified (Cernicharo et al. 1997; Liu et al. 1997). They are ortho-H<sub>2</sub>O 179.6, 174.6, 108.05, and 75.39  $\mu\text{m}$  lines with fluxes of  $(4, 1.5, 5.5, \text{ and } 6.6) \times 10^{-20}$  W cm<sup>-2</sup>, respectively. Our model cannot reproduce these line fluxes. The lines may originate from a thin layer of shocked gas in the high-density shell just inside the stellar wind region where the temperature ranges from about 300 to about 1000 K. Shock models (Draine, Roberge, & Dalgarno 1983) showed that at such temperatures a significant fraction of oxygen exists in H<sub>2</sub>O. The required H<sub>2</sub>O column density to fit the observation (Volk & Kwok 1997) can be reached in a shell that is a small fraction of the high-density shell. Furthermore, the

observed relative strengths of the water lines are similar to the theoretical result for shocked gas (Neufeld & Melnick 1987). Therefore the temperature of 200 K we adopted for the shocked region can be reinterpreted as the average temperature of the shocked region, and water emission can be accounted for, while the results of our calculation remain unaltered. Detailed modeling of the shock resulting from interacting stellar winds, which is beyond the scope of this paper, is needed for a comprehensive study of the H<sub>2</sub>O far-infrared emission. If the detection of water lines is confirmed, it further supports our assertion that the neutral envelope of NGC 7027 is under the combined influence of photodissociation and a shock.

The CH<sup>+</sup> emission-line intensities from our model are in good agreement with the observed values. Our model suggests that the high-CH<sup>+</sup> column density (Liu et al. 1997) can be attributed to the high temperature of the CH<sup>+</sup> formation region, which may be an important step in solving the long standing puzzle of the high CH<sup>+</sup> fractional abundance in the general ISM.

Based on our study in this paper, we draw the following conclusions:

1. The neutral envelope of NGC 7027 is under the combined influence of photodissociation and a shock.
2. It is highly likely that the carbon abundance in the neutral envelope is much lower than that in the ionized region.
3. The CH<sup>+</sup>, OH and CH lines originate from the dense, high-temperature ( $\sim 1000$  K) PDR.
4. High- $J$  (from  $J = 29-28$  down to  $J = 17-16$ ) CO lines originate from the dense, high-temperature PDR, whereas about half the intensities of low- $J$  ( $16-15$ ,  $15-14$ , and  $14-13$ ) CO lines originate from the dense, high-temperature PDR; the rest come from the outer dense shocked region.

We thank the referee, Sun Kwok, for suggestions that led to significant improvements of this paper. S. R. F. and M. Y. gratefully acknowledge support from NASA grant NAG5-4957 under the LTSA program. A. D. acknowledges support from the National Science Foundation, Division of Astronomy.

#### REFERENCES

- Beckwith, S., et al. 1980, *AJ*, 85, 886  
 Brown, J. M., & Evenson, K. M. 1983, *ApJ*, 268, L51  
 Burton, M. G., Hollenbach, D. J., & Tielens, A. G. G. M. 1990, *ApJ*, 365, 620  
 Cecchi-Pestellini, C., & Dalgarno, A. 1993, *ApJ*, 413, 611  
 Cernicharo, J., et al. 1997, *ApJ*, 483, L65  
 Chu, S. I., & Dalgarno, A. 1975, *Proc. R. Soc. London A*, 342, 191  
 de Jong, T., Dalgarno, A., & Boland, A. 1980, *A&A*, 91, 68  
 Destombes, J. L., Marliere, C., Baudry, A., & Brillet, J. 1977, *A&A*, 60, 55  
 Dewardan, D. P., Flower, D. R., & Alexander, M. H. 1987, *MNRAS*, 226, 505  
 Dinerstein, H. L., Sneden, C., & Uglum, J. 1995, *ApJ*, 447, 262  
 Draine, B. T. 1978, *ApJS*, 36, 595  
 Draine, B. T., Roberge, W. G., & Dalgarno, A. 1983, *ApJ*, 264, 485  
 Ellis, H. B., & Werner, M. W. 1984, *BAAS*, 16, 462  
 Federman, S. R., Glassgold, A. E., & Kwan, J. 1979, *ApJ*, 227, 446  
 Federman, S. R., & Yan, M. 1998, in preparation  
 Gatley, I., Depoy, D. L., & Fowler, A. M. 1988, *Science*, 242, 1264  
 Graham, J. R., et al. 1993, *AJ*, 105, 250  
 Hoare, M. G., Roche, P. F., & Clegg, R. E. S. 1992, *MNRAS*, 258, 257  
 Hollenbach, D., & McKee, C. F. 1989, *ApJ*, 342, 306  
 Jaminet, P. A., et al. 1991, *ApJ*, 380, 461  
 Kwok, S. 1994, *PASP*, 106, 344  
 Kwok, S., Purton, C. R., & FitzGerald, M. P. 1978, *ApJ*, 219, L125  
 Liu, X.-W., et al. 1996, *A&A*, 315, L257  
 ———. 1997, *MNRAS*, 290, L71  
 McKee, C. F., Storey, J. W., Watson, D. M., & Green, S. 1982, *ApJ*, 259, 647  
 Middlemass, D. 1990, *MNRAS*, 244, 294  
 Millar, T. J., Farquhar, P. R. A., & Willacy, K. 1997, *A&AS*, 121, 139  
 Mufson, S. L., Lyon, J., & Marioni, P. A. 1975, *ApJ*, 201, L85  
 Neufeld, D. A., & Kaufman, M. J. 1993, *ApJ*, 418, 263  
 Neufeld, D. A., & Melnick, G. J. 1987, *ApJ*, 322, 266  
 Offer, A. R., & van Dishoeck, E. F. 1992, *MNRAS*, 257, 377  
 Osterbrock, D. E. 1989, *Astrophysics of Gaseous Nebulae and Active Galactic Nuclei* (Mill Valley: University Science Books)  
 Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, in *Numerical Recipes* (Cambridge: Cambridge Univ. Press), 718  
 Shupe, D. L., et al. 1998, *ApJ*, 498, 267  
 Stephens, T. L., & Dalgarno, A. 1972, *J. Quant. Spectrosc. Radiat. Transfer*, 12, 569  
 Sternberg, A., & Dalgarno, A. 1989, *ApJ*, 338, 197  
 ———. 1995, *ApJS*, 99, 565  
 Tielens, A. G. G. M., & Hollenbach, D. 1985, *ApJ*, 291, 722  
 Tine, S., Lepp, S., Gredel, R., & Dalgarno, A. 1997, *ApJ*, 481, 282  
 Treffers, R. R., Fink, V., Larson, H. P., & Gautier, T. N. 1976, *ApJ*, 209, 793  
 Turner, B. E. 1994, *ApJ*, 430, 727  
 van Dishoeck, E. F., & Black, J. H. 1986, *ApJS*, 62, 109  
 ———. 1988, *ApJ*, 334, 771  
 Volk, K., & Kwok, S. 1997, *ApJ*, 477, 722  
 Yan, M. 1996, Ph.D. thesis, Harvard University  
 Yan, M., & Dalgarno, A. 1997, *ApJ*, 481, 296