Microscopic Entropy of the Black Ring

hep-th/0411187

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Abstract

A surprising new seven-parameter supersymmetric black ring solution of five-dimensional supergravity has recently been discovered. In this paper, M-theory is used to give an exact microscopic accounting of its entropy.

A supersymmetric black ring has been recently discovered [1]. This is an asymptotically flat black hole solution of five-dimensional supergravity whose event horizon has toroidal topology $S^1 \times S^2$. A more general seven-parameter supersymmetric black ring solution was found in [2, 3]. Previous studies of non-BPS or singular black rings include [4]. Concentric black ring solutions have been found in [3, 5].

In M-theory the solutions of [1, 2] correspond to supertubes [6], namely to configurations with M5-branes wrapped around four-cycles of a six-torus and the fifth brane dimension forming a circle, or ring, stabilized by angular momentum in five-dimensional space-time. There is also M2-brane charge density distributed around the ring. The brane wrappings around the different cycles are summarized by the array

q_1	M2:	1	2	-	-	-	-	-
q_2	M2:	-	-	3	4	-	-	-
q_3	M2:	-	-	-	-	5	6	-
p^1	M5:	-	-	3	4	5	6	ψ
p^2	M5:	1	2	-	-	5	6	ψ
p^3	M5:	1	2	3	4	-	-	ψ

where q_A and p^A, $A = \{1, 2, 3\}$, are the numbers of M2-branes and M5-branes, respectively, wrapping each cycle, and ψ is the five-dimensional angular coordinate around the ring. The solution of the five-dimensional effective supergravity theory is reproduced in the appendix of this paper. It is characterized by seven independent parameters, which we choose to be the set of six brane numbers listed above, together with the angular momentum J_ψ around the ring.

The Bekenstein-Hawking entropy, $S = \mathcal{A}_{horizon}/(4G)$, of this supergravity solution is given in [2] in the form

$$S_{BR} = 2\pi\sqrt{q_1 q_2 q_3 - k_1 k_2 k_3 - J_\phi^2 - D(J_\psi - J_\phi)}\,, \tag{1}$$

where $k_1 \equiv q_1 - p^2 p^3$ (and similarly for permutations of 1, 2 and 3), $D \equiv p^1 p^2 p^3$ and J_ϕ is the angular momentum in the plane orthogonal to the ring. Replacing J_ϕ by its expression (13) in terms of q_A, p^A, we get

$$S_{BR} = 2\pi\sqrt{\frac{D^2}{4} - DJ_\psi - \frac{1}{4}\Big((p^1 q_1)^2 + (p^2 q_2)^2 + (p^3 q_3)^2\Big) + \frac{1}{2}D\left(\frac{q_1 q_2}{p^3} + \frac{q_2 q_3}{p^1} + \frac{q_1 q_3}{p^2}\right)} \tag{2}$$

where we have written everything in terms of the seven independent integers q_A, p^A, J_ψ. We wish to compare this expression with the formula we get from the microscopic computation. [1]

Our starting point is reference [8], where a derivation of the entropy of a four-dimensional black hole in a Calabi-Yau$\times S^1$ compactification of M-theory was given.[2] The construction

[1] A previous discussion of this problem can be found in [7].

[2] Although proper $SU(3)$ holonomy was assumed in [8], the results quoted here are valid for a six-torus.

involves an M5-brane wrapped about $\sum_A p^A \Sigma_A$, where Σ_A is an integral basis of four cycles in the Calabi-Yau. The resulting string in five dimensions is a chiral (0,4) CFT, whose left-moving central charge is given to leading order by $c_L = 6D$, where

$$D = D_{ABC} p^A p^B p^C \tag{3}$$

and D_{ABC} are the triple intersection numbers of the Σ_A. For the case of a six-torus, relevant to the solutions of [1, 2], D_{ABC} is equal to 1/6 if (ABC) form a permutation of (123) and zero otherwise. Therefore we can write

$$c_L = 6D = 6p^1 p^2 p^3. \tag{4}$$

Membrane charge arises as the momentum zero modes q_A of a Narain lattice of scalars in the CFT. The left-moving oscillator number, denoted $\hat{q}_0$ in [8], was shown to be related to the left-moving momentum q_0 by[3]

$$\hat{q}_0 = q_0 + \frac{1}{12} D^{AB} q_A q_B + \frac{c_L}{24}, \tag{5}$$

where D^{AB} is the inverse of the matrix D_{AB}, defined as

$$D_{AB} = D_{ABC} p^C = \frac{1}{6} \begin{pmatrix} 0 & p^3 & p^2 \\ p^3 & 0 & p^1 \\ p^2 & p^1 & 0 \end{pmatrix}. \tag{6}$$

The second term in $\hat{q}_0$ comes from the momentum carried by the Narain scalar zero modes, while the last term is the usual oscillator zero-point contribution to the momentum. The entropy of excited states with left oscillator number $\hat{q}_0$ is then given by the Cardy formula

$$S = 2\pi \sqrt{c_L \hat{q}_0 / 6}, \tag{7}$$

where $\hat{q}_0$ is the left moving momentum available to be distributed among the oscillators.

In [8], a further S^1 compactification from five to four dimensions was performed, and four-dimensional black holes were made by wrapping the string around this S^1. The resulting macroscopic entropy agreed with the microscopic result following from equation (7). In the present context we do not wish to compactify to four dimensions. However we can still put the same (0,4) CFT on a circle in five dimensions by simply tying up the ends of the string, rather than winding it around the S^1. The string is now stabilized dynamically by the angular momentum J_ψ. Hence we are proposing that the microscopic description of the black ring is simply as a ring of the same CFT encountered in [8].

This can be checked by computing the microscopic entropy. The left-moving momentum q_0 should equal J_ψ up to a sign, since this is the momentum around the ring, and both are integrally quantized. Taking the sign such that $q_0 = -J_\psi$ we find

$$\hat{q}_0 = -J_\psi + \frac{D}{4} + \frac{1}{2}\left(\frac{q_1 q_2}{p^3} + \frac{q_2 q_3}{p^1} + \frac{q_1 q_3}{p^2}\right) - \frac{1}{4D}\Big((p^1 q_1)^2 + (p^2 q_2)^2 + (p^3 q_3)^2\Big). \tag{8}$$

[3]The last zero-point term was omitted in [8] because it was subleading in the parameter range considered therein.

Substituting this expression into the Cardy formula (7) for the entropy and comparing with the supergravity result (2) we find perfect agreement.

In conclusion, the microscopic description of the black ring solution of [1, 2] is that of a ring of the same Calabi-Yau-wrapped M5-brane (0,4) CFT encountered in [8].

Appendix

The five-dimensional metric has the form

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1}\frac{R^2}{(x-y)^2}\left[\frac{dy^2}{y^2-1} + (y^2-1)d\psi^2 + \frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right] \quad (9)$$

where $f^{-1} = (H_1H_2H_3)^{1/3}$,

$$H_1 = 1 + \left(\frac{4G}{\pi}\right)^{2/3}\frac{q_1 - p_2p_3}{2R^2}(x-y) - \left(\frac{4G}{\pi}\right)^{2/3}\frac{p_2p_3}{4R^2}(x^2-y^2) \quad (10)$$

and H_2 and H_3 are given by obvious permutations.

The coordinates have ranges $x \in [-1,1], y \in [-\infty,-1], \phi,\psi \in [0,2\pi]$ and asymptotic infinity lies at $x \to y \to -1$. Further, for the black ring solution we have $\omega = \omega_\phi d\phi + \omega_\psi d\psi$, where

$$\begin{aligned}\omega_\phi &= -\frac{G}{2\pi R^2}(1-x^2)[p^1q_1 + p^2q_2 + p^3q_3 - D(3+x+y)]\,, \quad (11)\\ \omega_\psi &= \frac{1}{2}\left(\frac{4G}{\pi}\right)^{\frac{1}{3}}(1+y) - \frac{G}{2\pi R^2}(y^2-1)[p^1q_1 + p^2q_2 + p^3q_3 - D(3+x+y)]\,,\end{aligned}$$

with $D = p^1p^2p^3$. The solution possesses two angular momenta as measured at infinity. In terms of the brane numbers and the radius R they are given by

$$J_\phi = \frac{1}{2}(p^1q_1 + p^2q_2 + p^3q_3 - D)\,, \quad (12)$$

$$J_\psi = \left(\frac{\pi}{4G}\right)^{\frac{2}{3}}R^2(p^1+p^2+p^3) + \frac{1}{2}(p^1q_1 + p^2q_2 + p^3q_3 - D)\,. \quad (13)$$

Note that in terms of the charges used in the first reference in [2] we have

$$q_i = \left(\frac{\pi}{4G}\right)^{\frac{2}{3}}Q_i^{EEMR}\,, \qquad p^i = \left(\frac{\pi}{4G}\right)^{\frac{1}{3}}q_i^{EEMR}\,. \quad (14)$$

Acknowledgments

We thank H. Elvang, R. Emparan, D. Jafferis, J. Maldacena, D. Marolf, H. Reall and H. Verlinde for valuable discussions. This work was supported in part by DOE grant DE-FG02-91ER40654.

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