# Cross sections for gluon + gluon → heavy quarkonium + gluon

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# CROSS-SECTIONS FOR GLUON + GLUON → HEAVY QUARKONIUM + GLUON

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## ABSTRACT

Using the helicity amplitude formalism, we derive simple cross-section formulae for the production of heavy quarkonia through the process $g + g \rightarrow ({}^{2S+1}L_J) + g$ when L = S or P. Within the framework of perturbative QCD, these subprocesses are important for the hadroproduction of heavy quarkonia.

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Hadroproduction of the $J/\Psi$ has recently been observed at the CERN $p\bar{p}$ collider [1]. It is to be expected that the hadroproduction of heavy quarkonia, in general, will be an important source of information on various aspects of high-energy physics: it allows one to study $B-\bar{B}$ mixing [1 - 3], to search for extra b quarks [4], and to test perturbative QCD [5].

Within the framework of perturbative QCD, the production of heavy quarkonia is described by several subprocesses, such as $gg \to {}^{2S+1}L_J$, $q\bar{q} \to {}^{2S+1}L_J$, $g\,q \to {}^{2S+1}L_J\,q$, $g\,\bar{q} \to {}^{2S+1}L_J\,\bar{q}$, $q\,\bar{q} \to {}^{2S+1}L_J\,g$, $g\,g \to {}^{2S+1}L_J\,g$, etc. These processes are easily evaluated using the standard Feynman diagram techniques [5], except for the case $g\,g \to {}^{2S+1}L_J\,g$, described by the Feynman diagrams of Fig. 1. In this case, the number of Feynman diagrams is rather large and the covariant summation over the polarization degrees of freedom leads to long and cumbersome expressions which, for the $^3P$ channels, were handled by symbolic manipulation programs.

In this letter, we present our cross section formulae for the process

$$g(k_1) + g(k_2) \to {}^{2S+1}L_J + g(k_3), \qquad (1)$$

where $k_1$, $k_2$ and $k_3$ represent the momenta of the gluons. Our results are limited to the lowest order, i.e., $\alpha_s^3$, not including the much more complicated higher-order QCD corrections. The formulae were obtained using the helicity formalism [6] developed earlier, where one introduces an explicit representation of the three gluon polarization vectors. E.g.,

$$\not{\epsilon}_1^{\pm} = N\left[\not{k}_1\not{k}_2\not{k}_3(1 \mp \gamma_5) + \not{k}_3\not{k}_2\not{k}_1(1 \pm \gamma_5)\right],$$
$$N = (2stu)^{-1/2}, \qquad (2)$$

where $s = (k_1 + k_2)^2$, $t = (k_2 - k_3)^2$, and $u = (k_1 - k_3)^2$.

In this way, we obtain simple expressions for the two independent helicity amplitudes of $g\,g \to g\,q\,\bar{q}$. With the method of Guberina, Kühn, Peccei, and Rückl [7], it then becomes a straightforward task to obtain the cross section formulae for process (1). Further details of this procedure will be presented in a forthcoming paper.

We simply list our results:

$$^1S_0 : \quad \frac{d\sigma}{dt} = \frac{\pi \alpha_s^3 R_0^2}{M s^2} \; \frac{P^2(M^8 - 2M^4P + P^2 + 2M^2Q)}{Q(Q - M^2P)^2} \, ,$$

$$^3S_1 : \quad \frac{d\sigma}{dt} = \frac{10\pi \alpha_s^3 R_0^2}{9 s^2} \; \frac{M(P^2 - M^2Q)}{(Q - M^2P)^2} \, ,$$

$$^1P_1 : \quad \frac{d\sigma}{dt} = \frac{40\pi \alpha_s^3 R_1'^2}{3 M s^2} \; \frac{-M^{10}P + M^6P^2 + Q(5M^8 - 7M^4P + 2P^2) + 4M^2Q^2}{(Q - M^2P)^3} \, ,$$

$$^3P_0 : \quad \frac{d\sigma}{dt} = \frac{4\pi \alpha_s^3 R_1'^2}{M^3 s^2} \; \frac{1}{Q(Q - M^2P)^4} \Big[ 9M^4P^4(M^8 - 2M^4P + P^2) - 6M^2P^3Q(2M^8 - 5M^4P + P^2) - P^2Q^2(M^8 + 2M^4P - P^2) + 2M^2PQ^3(M^4 - P) + 6M^4Q^4 \Big] \, ,$$

$$^3P_1 : \quad \frac{d\sigma}{dt} = \frac{12\pi \alpha_s^3 R_1'^2}{M^3 s^2} \; \frac{P^2[M^2P^2(M^4 - 4P) - 2Q(M^8 - 5M^4P - P^2) - 15M^2Q^2]}{(Q - M^2P)^4}$$

$$^3P_2 : \quad \frac{d\sigma}{dt} = \frac{4\pi \alpha_s^3 R_1'^2}{M^3 s^2} \; \frac{1}{Q(Q - M^2P)^4} \Big[ 12M^4P^4(M^8 - 2M^4P + P^2) - 3M^2P^3Q(8M^8 - M^4P + 4P^2) + 2P^2Q^2(-7M^8 + 43M^4P + P^2) + M^2PQ^3(16M^4 - 61P) + 12M^4Q^4 \Big] \, , \qquad (3)$$

with $P = st + tu + us$, $q = stu$, and $M^2 = s + t + u$, i.e., the $(\text{mass})^2$ of the produced resonance. In these formulae, $R_o$ is related to the $(q\bar{q})$ wave function at the origin and is given by

$$R_o^2 = M^2\,\Gamma(^3S_1 \to e^+e^-) / 4\alpha^2 e_q^2 , \tag{4}$$

while $R_1'$ is related to its derivative at the origin. Using the quarkonium potential of ref.[8] with $\Lambda = 0.2$ GeV, one finds for the $(c\bar{c})$ system

$$R_1'^2 / M_\chi^2 = 0.006\ (\text{GeV})^3 , \tag{5}$$

where $M_\chi$ is the mass of a $^3P$ state.

The cross section formulae (3) can be written in many different ways. An alternative set of formulae is:

$$^1S_0 : \quad \frac{d\sigma}{dt} = \frac{\pi\alpha_s^3 R_0^2}{8Ms^2}\left[\frac{M^4 - s^2 - t^2 - u^2}{(s-M^2)(t-M^2)(u-M^2)}\right]^2 \frac{M^8 + s^4 + t^4 + u^4}{stu} ,$$

$$^3S_1 : \quad \frac{d\sigma}{dt} = \frac{5\pi\alpha_s^3 R_0^2 M}{9s^2}\left[\frac{s^2}{(t-M^2)^2(u-M^2)^2} + \frac{t^2}{(u-M^2)^2(s-M^2)^2} + \frac{u^2}{(s-M^2)^2(t-M^2)^2}\right] ,$$

$$^1P_1 : \quad \frac{d\sigma}{dt} = \frac{20\pi\alpha_s^3 R_1'^2}{3Ms^2}\ \frac{1}{[(s-M^2)(t-M^2)(u-M^2)]^2}$$

$$\left\{ M^4(s^2 + t^2 + u^2 + M^4) + \frac{2stu[s^4 + t^4 + u^4 + M^4(s^2 + t^2 + u^2) + 2M^8]}{(s-M^2)(t-M^2)(u-M^2)}\right\} ,$$

$$^3P_0: \quad \frac{d\sigma}{dt} = \frac{2\pi\alpha_s^3 R_1'^2}{M^3 s^2} \quad \frac{1}{[(s-M^2)(t-M^2)(u-M^2)]^2}$$

$$\Big\{ 8M^2 \Big[ \frac{tu(t^4-t^2u^2+u^4)}{(s-M^2)^2} + \frac{us(u^4-u^2s^2+s^4)}{(t-M^2)^2} + \frac{st(s^4-s^2t^2+t^4)}{(u-M^2)^2} \Big]$$

$$+ 4M^4 \big[ M^2(st+tu+us) - 5stu \big]$$

$$+ (st+tu+us)^2 \Big[ \frac{9M^8}{stu} + \frac{1}{(s-M^2)(t-M^2)(u-M^2)} \Big( 8M^4(s^2+t^2+u^2)$$

$$- 16M^2 stu + \Big( 1 - 9M^2 \Big( \frac{1}{s} + \frac{1}{t} + \frac{1}{u} \Big) \Big) (s^4+t^4+u^4) \Big) \Big] \Big\} ,$$

$$^3P_1: \quad \frac{d\sigma}{dt} = \frac{12\pi\alpha_s^3 R_1'^2}{M^3 s^2} \quad \frac{1}{[(s-M^2)(t-M^2)(u-M^2)]^2}$$

$$\Big\{ M^2 \Big[ \frac{t^2u^2(t^2+u^2)}{(s-M^2)^2} + \frac{u^2s^2(u^2+s^2)}{(t-M^2)^2} + \frac{s^2t^2(s^2+t^2)}{(u-M^2)^2} \Big]$$

$$+ \frac{2(s^2t^2+t^2u^2+u^2s^2)(s^2t^2+t^2u^2+u^2s^2+M^2stu)}{(s-M^2)(t-M^2)(u-M^2)} \Big\} ,$$

$$^3P_2: \quad \frac{d\sigma}{dt} = \frac{4\pi\alpha_s^3 R_1'^2}{M^3 s^2} \quad \frac{1}{[(s-M^2)(t-M^2)(u-M^2)]^2}$$

$$\Big\{ M^2 \Big[ \frac{t^2u^2(t^2+4tu+u^2)}{(s-M^2)^2} + \frac{u^2s^2(u^2+4us+s^2)}{(t-M^2)^2} + \frac{s^2t^2(s^2+4st+t^2)}{(u-M^2)^2} \Big]$$

$$+ 12M^2 \big[ 3(s^3t+t^3u+u^3s+st^3+tu^3+us^3) + 4M^2 stu \big]$$

$$+ \frac{2(st+tu+us-M^4)(st+tu+us)^2}{(s-M^2)(t-M^2)(u-M^2)} \Big[ st+tu+us$$

$$- 24M^4 - 6M^2(st+tu+us-M^4)\Big(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\Big) \Big] \Big\} . \tag{6}$$

It is seen that the helicity formalism [6] is capable of yielding simple and useful formulae for the process $g\ g \rightarrow {}^{2S+1}L_J\ g$. Such formulae are expected to have many applications. An especially outstanding example is related to the very recent study of Glover, Halzen and Martin [9] on the physics from $J/\psi$-tags in $p\bar{p}$ collisions. The present formulae can lead to a significant saving of computer time, perhaps by a factor of 5, to obtain the cross section for $gg \rightarrow \chi\, g$, offering the possibility of more refined studies.

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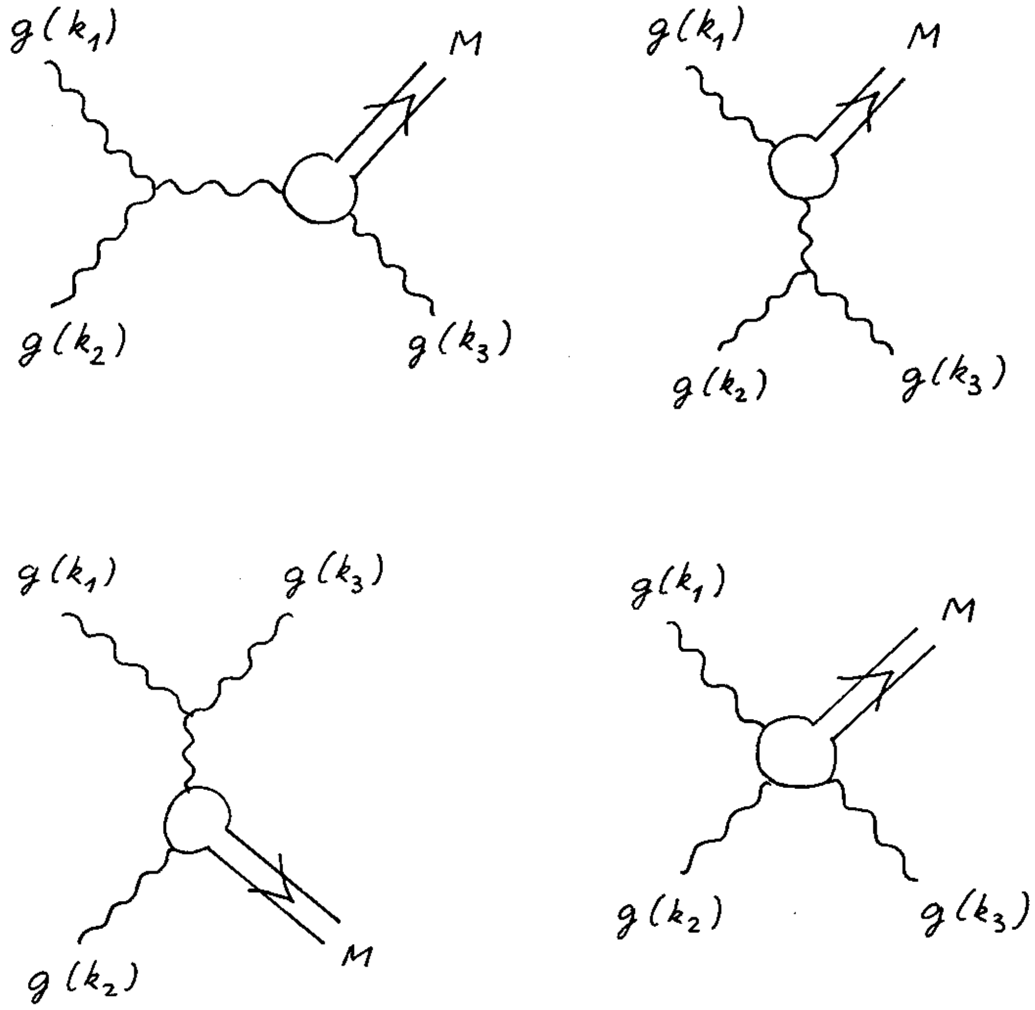
## FIGURE CAPTION

Fig. 1: Feynman diagrams for $g + g \rightarrow {}^{2S+1}L_J + g$.




+ permutations