A dipolar quantum gas microscope

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A dipolar quantum gas microscope

a dissertation presented
by
Gregory Alan Phelps
to
The Department of Physics

in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
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Harvard University
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A dipolar quantum gas microscope

Abstract

Quantum gas microscopy provides an exciting platform for the study of in situ atom-atom interactions. Recent advances in quantum gas microscopy have allowed the probing of the Bose-Hubbard and Fermi-Hubbard Hamiltonian with a variety of atomic species. In this thesis, we are currently extending these platforms with the introduction of an Erbium Dipolar Quantum Gas Microscope which will allow us to study dipole-dipole interactions in a lattice.

Erbium has several exciting properties, which increase the control and flexibility of these systems. These include stable bosonic and fermionic isotopes, a large magnetic dipole moment (7uB), a large spin value (J=6 and F=19/2 for bosonic and fermionic isotopes), a rich Feshbach spectrum, and several useful broad and narrow atomic transitions. A broad 401 nm transition can be used for imaging, which in combination with a 266 nm lattice spacing will provide fast time scales and access to a dipole-dipole interaction of approximately 30 Hz.

This thesis will document the progress toward an Erbium Quantum Gas Microscope and include a variety of systems which have been developed. It will also show the first ever 841 nm, red-detuned Magneto-Optical Trap with temperatures as low as 400 nK and a phase space density up to 0.05. This, in combination with other improvements, will allow for faster cycle times and more control over the system. These developments will allow us to benefit from the long-range interaction of Erbium and probe the Extended Bose-Hubbard and Extended Fermi-Hubbard Hamiltonian to an unprecedented degree.
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To all people who encouraged this path in life.
Acknowledgments

Before I discuss my time at Harvard, I think it is necessary to mention one important person and the main reason I took this direction in life. I grew up in an environment which did not value education and the majority of people never made it to college - a substantial percentage of students didn’t even graduate high school. I was extremely lucky in finding one individual who made the biggest difference in my life. Hector Ayala (a.k.a. Mr. Ayala), a High school English teacher, encouraged me to pursue a direction in life which led me here. Without this person, I am 100% certain my life would have taken a very different direction (most likely not involving college). Hector, you taught me integrity, honor and work ethic. I am forever indebted to you.

My path at Harvard University has been odd. I was accepted to Harvard as a theorist in Atomic Molecular and Optical (AMO) Physics, having never worked in an experimental lab. I worked for some time as a theorist and finally ended up an experimentalist in a physics lab reputed to be heavily engineering oriented. Along my path I have learned many things, beyond what I thought I would learn during my PhD, and I am grateful to those people at Harvard who helped along the way.

Of course, I want to thank my advisor Markus Greiner. It was difficult to convince you that a theorist would make a good experimentalist, but you were open to trying. I am grateful that you gave us freedom in designing the experiment and the direction we pursue. Your perspective on the design of the experiment has been invaluable.

I would also like to thank my other committee members, Mikhail Lukin and John Doyle, for taking the time to advise me. I would like to thank Misha for initially accepting me into his group for theory work - my theoretical knowledge of physics was deepened by this experience. Buffer gas sources were a common topic of conversation with John Doyle - I would be curious to see how it increases our loading rates.

My transition from theorist to experimentalist was facilitated by a few outstanding members of the Harvard physics community. Rob Hart is an amazing person to work with. Rob is a wonderful teacher and tinkerer. It was great to teach Physics 15C with Rob so many times and I truly appreciate him allowing me to contribute to the How To Make Anything course.
Joe Peidle has always been interested in the things I am doing and has a contagious passion for teaching experimental physics.

Stan Cotreau, the resident machinist and shop instructor, gets his own paragraph. It has truly been a pleasure working with Stan. I especially enjoyed all the discussions, stories and banter. Stan taught me countless techniques about building and machining. After working for 7 years in the machine shop I still look forward to coming in and chatting with Stan - I will miss that when I finally leave Harvard. I will definitely use the things I learned from Stan for the rest of my life.

I joined the Erbium lab shortly after it was conceived, but before it was decided what was really happening. None of the experiment existed and very little had been thought deeply about. The post-doc on the experiment was Susannah Dickerson. Susannah’s enthusiasm and thoroughness was critical to the start of the experiment. Fellow graduate students Anne Hébert and Aaron Krahn were a pleasure to work with. I will always remember husky hat, “would you do it for 10 dollars?” , and stomach ache Fridays. I am certain they will bring the experiment to the next level and pursue fruitful careers after. I wish the new student Furkan Öztürk a successful PhD. I also want to thank all the other members of the Greiner lab for great lunch conversations and physics discussions.

Of course, I would be nowhere without my friends and family. I would like to thank both my parents and my brothers, Michael and Jonathan. I must thank my extended, but in my mind immediate family: Felipe, Shannon, Little Felipe, Fabian, and now Lily. They didn’t always understand all the “quantum physics” I was talking about, but they listened and showed interest. I can’t imagine my life without them. Dr. Shaggy soon! My good friend Erik Bauch is not without mention. He has been a great friend from day one at Harvard. I look forward to more gym days after this thesis is submitted.

Finally, I would like to thank my wife, Alex. She was and is a constant source of encouragement, support, and advice.
Introduction

Does a basic theory describing a few particles explain the behavior of a large collection of particles? A single, slowly moving, electron in a periodic potential is well described by the Schrödinger equation, but a large collection of electrons in a periodic potential exhibits emergent phenomena, such as superconductivity and magnetism - both of which are not easily described by the basic underlying theory. A wonderful quote from P.W. Anderson captures this concept:

“At each stage entirely new laws, concepts, and generalizations are necessary, requiring inspiration and creativity to just as great a degree as in the previous one. Psychology is not applied biology, nor is biology applied chemistry.”

While the behavior of individual particles is fairly well understood, even a few particles together exhibit already a more complex behavior.

Perhaps the complexity of emergent phenomena relates to the complexity of simulating a large number of particles from a fundamental, bottom-up approach. Indeed, the inability to simulate leads to a disconnect between the physical scales. In this case it is necessary to write
a theory describing the macroscopic collection which neglects microscopic details. However, a deep understanding of this connection is still necessary to truly understand a phenomenon. The complexity of a large quantum mechanical system can be seen in the scaling of the dimension of the Hilbert space for a collection of two level systems. A single two level system has the following state:

$$|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle.$$ 

Only one coefficient is necessary to describe this system. If there are two particles, the state is described by 3 coefficients,

$$|\psi\rangle = c_{11} |1\rangle |1\rangle + c_{12} |1\rangle |2\rangle + c_{21} |2\rangle |1\rangle + c_{22} |2\rangle |2\rangle.$$ 

Extending this to $N$ two level systems, the number of coefficients grows exponentially as $2^N - 1$. This scaling emerges from the entanglement in the system. If it were a classical system, however, then the number of coefficients would scale as $N$. The exponential growth in information means it is impossible to simulate on a classical computer a large collection quantum objects composed of simple building blocks.

Quantum gas microscopy of neutral atoms in a lattice provides one possible route to quantum simulation. These systems provide a natural platform for the study of condensed matter systems. The single-site resolution of the objective in these systems allows for unprecedented control and measurement of the quantum system. A variety of Quantum Gas Microscopes exist based on alkali and rare-earth atoms: Rubidium, Lithium, Potassium, and Ytterbium. These systems rely on on-site interactions to realize an interacting system. A few atomic species move away from on-site interaction to include a long range magnetic dipole interaction. This extends these platforms to a Hamiltonian with nearest neigh-
Since our understanding of quantum systems would be limited without numerical simulation, a variety of institutions and companies have pushed the limits of simulating quantum systems on classical computers [35, 88, 51, 26]. The limitations in classical simulations, however, necessitates the implementation of real quantum simulators. A variety of commercial and experimental systems are also being developed for quantum simulation: Rydberg atoms [14, 69, 65, 112, 77], ions [113, 110, 40] and superconducting qubits. (Intel, Google, IBM, Rigetti [85], D-Wave)

Since the majority of this work is on the ongoing development of a Dipolar Quantum Gas Microscope, the work here will not include a completed system. It will, however, include the majority of components and logic behind the construction of a third generation Quantum Gas Microscope.

As a third generation machine, several factors are important when deciding to build a new experiment. Most importantly, a new Quantum Gas Microscope must bring features not available in current systems. Erbium, a Lanthanide atom, is an atom which definitely fulfills this requirement. First, Erbium features multiple stable isotopes, one of which is a Fermion. Switching between isotopes is as easy as re-locking lasers to a different frequency. Second, Erbium has a magnetic dipole interaction of approximately $7\mu_B$, which gives it an interaction strength approximately 50 times larger than alkali atoms. This will allow for the investigation of the physics of real materials including long range interaction and the exploration of the Extended Hubbard Model. Finally, Erbium possesses a zoo of optical transitions, many of which are easily accessible by commercial laser systems - this should increase controllability of the system [64].

Recent Rydberg experiments have shown faster than Hz repetition rates, which is useful for adjusting the experiment in real time, state tomography, lower uncertainties, and poten-
tially machine learning techniques. Utilizing the features of Erbium discussed above, it should be possible to increase the experimental repetition rate for a Quantum Gas Microscope. The current benchmarks within the Greiner group is the Rubidium experiment \cite{11}, at approximately a 40 second repetition time and the Lithium experiment \cite{87} at approximately 20 seconds. While both of these times are not unreasonable for a machine producing a Bose-Einstein condensate (BEC) or Fermi gas, the future of this type of science will benefit greatly from increased repetition rates. Currently, in the Erbium experiment it is possible to reach a phase space density of 0.05 in 200 ms. Following this with a fast evaporation will increase the repetition rate of this experiment substantially. This should allow the experiment to collect in 1 day the same amount of data that it took the Rubidium lab 2 weeks to produce. Not only is there an improvement in signal to noise due to the increased number of sample points, the data is less prone to variations and complications due to drift. In addition, a fast repetition rate will allow Bose-Hubbard quantum simulators \cite{28,67,93} to compete with other quantum computing platforms.

A few other exciting features of Erbium allow for more control and ease in implementation of the experiment. For one, Erbium features a rich Feshbach spectrum \cite{43}, which will be useful for controlling the scattering properties of Erbium. Additionally, Erbium has a fast transition at 401 nm, which should allow for scattering up to $94 \times 10^6$ photons/second. In combination with the large mass of Erbium, it will be possible to implement high fidelity imaging of atoms in a short spacing lattice without the need for cooling techniques. Next, there is J=6 to J=7 transition at 1299 nm in Erbium with a transition rate of $\Gamma = (2\pi) \times 2.3$ Hz \cite{12}. This transition may prove useful for shelving atoms in a meta-stable state for imaging or storing the quantum state. Finally, Erbium has a narrow transition at 841 nm with a transition rate of $\Gamma = (2\pi) \times 8$ kHz \cite{12}. This transition will allow for the generation of low heating artificial gauge fields, which will allow the exploration of physics beyond the reach of real condensed
matters. Additionally, this transition is broad enough, as is shown in Strontium [60], to operate a red-detuned narrow-line magneto-optical trap. In this configuration it is possible to load dipole traps with high phase space densities - making BEC production significantly faster [98], a primary goal of this experiment.

This thesis is organized as follows:

- **Chapter 1** presents a theoretical overview of necessary concepts when building a Quantum Gas Microscope. It covers basic atom-light interaction, Bose-Einstein condensation, physics of optical lattices, Bose-Hubbard Hamiltonian, and the dipole interaction.

- **Chapter 2** provides an overview of useful properties of Erbium. It also provides spectroscopic data for 3 of the major optical cooling transitions.

- **Chapter 3** discusses unwanted disorder in optical lattices. More specifically, it covers the source of disorder and discusses best practices for designing a low disorder optical lattice.

- **Chapter 4** explores the idea of fast imaging in an optical lattice. It presents simulations of imaging heavy atoms with a fast transition.

- **Chapter 5** covers the experimental setup in detail. It covers all laser systems, the vacuum chamber, magnetic coils, the control system, an accordion lattice, Second Harmonic Generation of 532 nm light, and briefly discusses the custom objective designed within the lab.

- **Chapter 6** details the realization of several benchmarks towards the Quantum Gas Microscope. It discusses 583nm MOT in some detail, the generation of a Bose-Einstein
condensate, and the first ever red-detuned narrow line MOT on the 841 nm transition of Erbium.

- **Chapter 7** examines several directions currently under study. Specifically, phase transitions of the Extended Hubbard model, sub-wavelength lattices, artificial gauge fields, and cooperative resonances.
Neutral atoms and their interactions with light and each other is a deeply studied topic. A solid foundation in this these topics is necessary for understanding and building a Quantum Gas Microscope. Some of these topics considered in this chapter have not been used in the system to date. However, many of the preliminary calculations and considerations required a deep understanding of basic atomic and optical physics.

1.1 Atom-Light interaction

Arguably, the most important interaction in a Quantum Gas Microscope is the atom-light interaction. In this case the atom interacts with an oscillating electric field. Note, in all cases considered in this thesis, the electric can be treated classically. The light interaction typically couples through the stronger electric-dipole interaction, but it can also couple through higher order electric field interactions leading to extremely narrow optical transitions.
1.1.1 Two-level atom

If an atom interacts with a near resonant light field, the resulting interaction can be approximated by a quantum system with two levels coupled to each other. The Hamiltonian governing this system (assuming dipole coupling only) is given by

\[ \hat{H} = \hbar \omega |2\rangle \langle 2| - d \cdot E(r, t) \]  

(1.1)

where \( \omega \) is the excited state transition frequency, \( d \) is the dipole operator and \( E(r, t) \) is the oscillating electric field. If the electric field is assumed to be a running wave, then the electric field has the form

\[ E(r, t) = \frac{1}{2} \left( E_0 e^{-i(k \cdot r - \omega_L t)} + c.c. \right) \]  

(1.2)

where \( k \) is the wavevector of light and \( \omega_L \) is the laser frequency. The dipole operator can be expanded in the \( |1\rangle, |2\rangle \) basis to give

\[ d = d_{12} |1\rangle \langle 2| + d_{21} |2\rangle \langle 1| \]  

(1.3)

Here the dipole operator does not contain terms that couple the same state due to wavefunction symmetry considerations. It is clear that the Schrödinger equation can be difficult to solve with two high frequency counter-rotating terms. To get some intuition in one experimentally relevant parameter regime, it is worth considering the case where \( |\omega - \omega_L| \ll \omega \). In this regime the rotating wave approximation (RWA) can be applied. It is convenient to go to the rotating frame of the laser by using the unitary transformation matrix

\[ \hat{U} = e^{-i\omega_L |2\rangle \langle 2|} \]  

(1.4)
The corresponding Hamiltonian in the rotating frame is given by

\[ \hat{H} \rightarrow \hat{U} \hat{H} \hat{U}^\dagger - i\hbar \frac{d\hat{U}^\dagger}{dt}. \]  

(1.5)

The Hamiltonian has the following form

\[
\begin{align*}
\dot{H} &= \hbar (\omega - \omega_l)|2\rangle \langle 2| - (d_{12}e^{i\omega_l t}|1\rangle \langle 2| + d_{21}e^{-i\omega_l t}|2\rangle \langle 1|) \cdot E(r, t) \\
&= -\hbar \delta|2\rangle \langle 2| - \frac{1}{2} \left( \langle 1| e^{-i\vec{k} \cdot \vec{r}} \frac{d_{12} \cdot E_0}{\hbar} |2\rangle |2\rangle \langle 1| \right) \cdot E(r, t) \\
&- \frac{1}{2} \left( \langle 2| e^{i\vec{k} \cdot \vec{r}} |1\rangle \langle 1| \right) - \frac{1}{2} \left( \langle 1| e^{-i\vec{k} \cdot \vec{r}} \frac{d_{21} \cdot E_0^*}{\hbar} |2\rangle |2\rangle \langle 1| \right) \\
\end{align*}
\]

(1.6)

where \( \delta = \omega_l - \omega \) is the detuning. Since the RWA is being used, fast rotating terms in the Hamiltonian can be neglected. Intuitively, these terms do not effect the dynamics on longer time scales since the dynamics driven by the terms will average out. Therefore, the Hamiltonian can be written in the RWA as

\[
\begin{align*}
\dot{H} &= -\hbar \delta|2\rangle \langle 2| - \frac{\hbar}{2} \left( \Omega e^{-i\vec{k} \cdot \vec{r}} |1\rangle \langle 2| + \Omega^* e^{i\vec{k} \cdot \vec{r}} |2\rangle \langle 1| \right) \\
\end{align*}
\]

(1.7)

where \( \Omega = \frac{d_{12} \cdot E_0}{\hbar} \) is the Rabi frequency.

1.1.2 The Master Equation

The previous Hamiltonian does not include effects from coupling of the two level atom to the vacuum. The presence of the vacuum causes atom decay from the excited state by effectively coupling the closed system to an infinitely large Markovian reservoir. The derivation of this coupling is relatively simple, but tedious, and reveals the following master equation for the
evolution of the density matrix \[^80\]

\[
\dot{\rho} = \frac{1}{\hbar} [\hat{H}, \rho] - \frac{\gamma}{2} (\{\sigma_{22}, \rho\} - 2\sigma_{12}\rho\sigma_{21}) 
\]  

(1.8)

where \(\sigma_{nm} = |n\rangle\langle m|\). If the density matrix takes the form \(\rho = \rho_{11}\sigma_{11} + \rho_{12}\sigma_{12} + \rho_{21}\sigma_{21} + \rho_{22}\sigma_{22}\), then the evolution of the coefficients according to the master equation (Eq. 1.8) are

\[
\begin{align*}
\dot{\rho}_{11} &= \gamma\rho_{22} + \frac{i}{2}(\Omega\rho_{21} - \Omega^*\rho_{12}) \\
\dot{\rho}_{12} &= (-i\delta - \frac{\gamma}{2})\rho_{12} + \frac{i}{2}\Omega(\rho_{22} - \rho_{11}) \\
\dot{\rho}_{22} &= -\gamma\rho_{22} - \frac{i}{2}(\Omega\rho_{21} - \Omega^*\rho_{12})
\end{align*}
\]  

(1.9)

It is worth solving for the steady state of the master equation, which yields

\[
\begin{align*}
\rho_{22} &= \frac{|\Omega|^2/4}{(\xi)^2 + \delta^2 + |\Omega|^2/2} \\
\rho_{12} &= \frac{i\Omega (2\rho_{22} - 1)}{\xi + i\delta}.
\end{align*}
\]  

(1.10)

If the saturation parameter, which is just a metric of the strength of the Rabi frequency to the natural linewidth, is defined to be \(s = 2|\Omega|/\gamma\), the excited state fraction becomes

\[
\rho_{22} = \frac{1}{2} \frac{s}{1 + s + 4(\delta/\gamma)^2}.
\]  

(1.11)

The saturation parameter, \(s\), can be written as \(s = I/I_s\), where \(I\) is the intensity of light and \(I_s\) is known as the saturation intensity. To find the saturation intensity, the relationship between \(\gamma\) and the square of the dipole moment, \(|\mathbf{d}_{12}|^2\), can be used:

\[
\gamma = \frac{\omega^3|\mathbf{d}_{12}|^2}{3\pi\varepsilon_0\hbar c^3},
\]  

(1.12)
where $\varepsilon_0$ is the permittivity of free space and $c$ is the speed of light. This is known as the Einstein A coefficient and can be derived from Wigner-Weisskopf theory [94]. The intensity of light goes as $I = \varepsilon_0 c |E|^2 / 2$. Using the definition of $s$ and the relationship between the Rabi frequency and electric-field, the saturation intensity becomes

$$I_s = \frac{c\varepsilon_0 h^2 \gamma^2}{4|d_{12} \cdot \hat{e}|^2}$$  \hspace{1cm} (1.13)

where $E_0 = E_0 \hat{e}$. If $|d_{12} \cdot \hat{e}| = |d_{12}|$, then the saturation intensity can be further reduced to

$$I_s = \frac{\pi hc\gamma^2}{3\lambda^3}.$$  \hspace{1cm} (1.14)

This relates experimentally known parameters to the excited state fraction.

1.1.3 The Lorentz Oscillator

When working primarily with far-detuned fields, the atom-light interaction can be modeled as a classical system [49]. The force between the electron-proton behaves like a spring and the incident light field acts as a driving force

$$m \frac{d^2 x}{dt^2} = -m\omega_0^2 x - m\gamma_{\omega_0} \frac{dx}{dt} - qE(t)$$ \hspace{1cm} (1.15)

where $m$ is approximately the mass of the electron, $\omega_0$ is the transition frequency, $\gamma_{\omega_0}$ is the damping rate, and $q$ is the charge of the electron. The driving electric field can be written as $E(t) = E_0 \cos(\omega_0 t)$. The damping rate comes from Larmor’s formula

$$\gamma_{\omega_0} \approx \frac{q^2 \omega_0^2}{6\pi\varepsilon_0 mc^3}.$$ \hspace{1cm} (1.16)
The asymptotic behavior of Eq. (1.15) is

\[ x(t) = a \exp(i\omega_l t) + b \exp(-i\omega_l t), \]  

(1.17)

where \( a \) and \( b \) take the form

\[ a = b^* = \frac{q}{2m} \frac{1}{\omega_0^2 - \omega_l^2 - i\omega_l \gamma} E_0. \]  

(1.18)

The corresponding dipole moment goes as

\[ p(t) = qx(t) = qa e^{i\omega_l t} + c.c. \]  

(1.19)

Therefore, the polarizability becomes

\[ \alpha = \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega_l^2 + i\omega_l \gamma}. \]  

(1.20)

From this it is a simple matter to find the dipole energy and the scattering rate. More specifically, it is easy enough to see that dipole energy scales as

\[ U_{\text{dip}} = -\langle p(t)E(t) \rangle_t, \]  

(1.21)

and the scattering rate is

\[ P_{\text{scat}} = \hbar \omega_l \Gamma_{\text{scat}} = -\langle \dot{p}(t)E(t) \rangle_t. \]  

(1.22)
Here $\langle \rangle_t$ denotes the time average. Putting it all together gives

\[
U_{\text{dip}} = -\frac{3\pi c^2}{2\omega_0^3} \left( \frac{\gamma}{\omega_0 - \omega_l} + \frac{\gamma}{\omega_0 + \omega_l} \right) I
\]

\[
\Gamma_{\text{scat}} = \frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\omega_l}{\omega_0} \right)^3 \left( \frac{\gamma}{\omega_0 - \omega_l} + \frac{\gamma}{\omega_0 + \omega_l} \right)^2 I,
\]

where the on-resonant damping rate has been introduced $\gamma = (\omega_0/\omega_l)^2 \gamma_{_\omega_l}$. For far off-resonant fields, the correction to the trap depth and scattering can be substantial. In the near-detuned regime, $\omega_l \approx \omega_0$, the trapping and scattering go as

\[
U_{\text{dip}} = -\frac{3\pi c^2 \gamma}{2\omega_0^3} I
\]

\[
\Gamma_{\text{scat}} = \frac{3\pi c^2}{2\hbar\omega_0^3} \left( \frac{\gamma}{\delta} \right)^2 I.
\]

The ratio of the scattering rate to trap depth goes as

\[
\frac{\hbar \Gamma_{\text{scat}}}{U_{\text{dip}}} = \frac{\gamma}{\delta}.
\]

The main point to take away from this is it is better to be far detuned with a high intensity laser to minimize scattering rate.

1.1.4 Semi-classical approach

This can also be found directly from the full Hamiltonian in the non-RWA. If we assume $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$, then the evolution of the excited state coefficient according to the full Hamiltonian is

\[
\dot{c}_2 = i\omega c_2 - \frac{\gamma}{2} c_2 + ic_1 \frac{d_{21}}{2\hbar} \cdot (E_0 e^{-i\omega_l t} + E_0^* e^{i\omega_l t})
\]

(1.26)
where it has been assumed the atom is located at \( r = 0 \) and have added the phenomenological decay term. It is a reasonable assumption that \( c_1 = 1 \) and \( |c_2| \ll 1 \), which is a good approximation to the far detuned case. The solution to this equation (ignoring transients) yields

\[
c_2(t) = -\frac{d_{21}}{2} \cdot \left( \frac{E_0}{i\gamma/2 + \omega + \omega_1} e^{-i\omega_1 t} + \frac{E_0^*}{i\gamma/2 + \omega - \omega_1} e^{i\omega_1 t} \right). \tag{1.27}
\]

The corresponding trapping energy is

\[
U = -\langle \langle \mathbf{d} \cdot \mathbf{E}(r, t) \rangle \rangle_t \approx -\langle \langle d_{12} c_2(t) + c_2^*(t) d_{21} \rangle \cdot \mathbf{E}(t) \rangle_t \tag{1.28}
\]

where \( \langle \rangle_t \) denotes the time average. Calculating this and taking the time average gives the following

\[
U = -\frac{1}{4} \left( \frac{|d_{12} \cdot E_0|^2}{\omega + \omega_I} + \frac{|d_{12} \cdot E_0^*|^2}{\omega - \omega_I} \right). \tag{1.29}
\]

It is obvious from this form that the counter-rotating term couples differently from co-rotating term. This is of particular importance for circularly polarized light in a multi-level atom. For a two level atom independent of polarization, \( |d_{12} \cdot E_0|^2 = |d_{12} \cdot E_0^*|^2 = |d_{12}|^2 |E_0|^2 \). This means the trapping energy \( U \) takes the form,

\[
U = -\frac{3\pi c^2}{2\omega^3} \left( \frac{\gamma}{\omega + \omega_I} + \frac{\gamma}{\omega - \omega_I} \right) I. \tag{1.30}
\]

In the near-detuned regime, where \( |\omega_I - \omega| \ll \omega \), the standard trapping and scattering rate are recovered:

\[
U = -\frac{3\pi c^2 \gamma}{2\omega^3} \frac{I}{\delta I} \tag{1.31}
\]
where $\delta = \omega_l - \omega$ is the detuning from resonance.

1.2 Atom-Magnetic Field Interaction

Similar to the dipole interaction, the Hamiltonian of the interaction of an atom with a magnetic field takes the form

$$\hat{H}_{\text{int}} = -\mu \cdot \mathbf{B}. \quad (1.32)$$

where $\mu$ is the magnetic moment and $\mathbf{B}$ is the magnetic field. In principle, light contains a magnetic part, but the interaction at optical frequencies is significantly weaker than the electric dipole interaction [21]. So, in most cases involving optical frequencies the magnetic interaction can be neglected. This is not the case for DC or RF fields and gives rise to the Zeeman effect. For a weak magnetic field in the z-direction $\mathbf{B} = B_z \hat{z}$, the Zeeman effect takes the form

$$E_Z \approx \mu_B g_j m_j B_z \quad (1.33)$$

where $\mu_B$ is the Bohr magneton, $g_j$ is the Landé g-factor, and $m_j$ is the magnetic quantum number. This breaks down at stronger fields where terms containing various magnetic couplings need to be considered: spin-orbit coupling, coupling to the nuclear spin, etc...

1.3 Optical Lattices

Optical lattices are typically formed by interfering two far-detuned light beams; however, more beams can interfere to give more complicated lattice structures. These structures are used for several reasons. First, it allows the study crystalline like materials on an experimentally accessible time-scale. The atom in the lattice is a simulation of electrons in some crystal structure (the optical lattice). Second, it allows the formation of strongly interacting systems
by strongly confining atoms and increasing the interaction strength. Of course, this allows scientists to explore physics previously inaccessible to dilute gas experiments.

To better understand the physics of atoms in a lattice, a single atom in a conservative trap needs to be considered, which is governed by the Hamiltonian

$$\hat{H}_0 = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r}).$$

(1.34)

Here no assumptions have been made about the potential. Typically in an optical lattice the potential has some periodic structure. Specifically, the potential an atom experiences has translational invariance governed by the following

$$U(\mathbf{r}) = U(\mathbf{r} + \mathbf{R}),$$

(1.35)

where $\mathbf{R}$ is a Bravais translation lattice vector. From this point it is possible to setup a formalism for handling periodic potentials before making assumptions about the exact form of the potential.

1.3.1 Bloch Waves

The wave-function of a single atom in the optical lattice, due to the periodic structure, takes the following form

$$\psi_{n,k}(\mathbf{r}) = e^{ik \cdot \mathbf{r}} u_{n,k}(\mathbf{r}),$$

(1.36)

where $\mathbf{k}$ is some wave-vector, $n$ is the band, and $u_{n,k}(\mathbf{r})$ has the same period as the lattice. This is known as Bloch’s theorem and the wave-functions are called Bloch waves. Due to symmetries of the lattice, the function $u_{n,k}(\mathbf{r})$ must be invariant to translations by a Bravais
lattice vector, $\mathbf{R}$. That is to say,

$$u_{n,k}(\mathbf{r} + \mathbf{R}) = u_{n,k}(\mathbf{r}). \quad (1.37)$$

### 1.3.2 Band Structure

Using the form of the wave-function, the Hamiltonian governing $u(\mathbf{r})$ can be written as

$$\hat{H} = \frac{(\mathbf{p} + \mathbf{q})^2}{2m} + U(\mathbf{r}), \quad (1.38)$$

where $\mathbf{q} = \hbar \mathbf{k}$ is the lattice momentum and naturally $\hat{H}u_{n,q}(\mathbf{r}) = E_{n,q}u_{n,q}(\mathbf{r})$. Here the subscripts $k$ and $q$ have been exchanged. This eigenvalue problem can be solved by writing the potential and wave-function in terms of reciprocal lattice vectors, $\mathbf{g}_1$ and $\mathbf{g}_2$

$$U(\mathbf{r}) = \sum_{n_1,m_1} U_{n_1,m_1} e^{-i(n_1\mathbf{g}_1 + m_1\mathbf{g}_2) \cdot \mathbf{r}} \quad (1.39)$$

$$u_{n,q}(\mathbf{r}) = \sum_{n_1,m_1} \phi_{n_1,m_1}^{(n)} e^{-i(n_1\mathbf{g}_1 + m_1\mathbf{g}_2) \cdot \mathbf{r}} \quad (1.40)$$

where 2-dimensions are assumed. The following relation will be helpful later

$$\int \int_{\text{unit cell}} e^{-i(n\mathbf{g}_1 + m\mathbf{g}_2) \cdot \mathbf{r}} d\mathbf{r} = |\mathbf{r}_1 \times \mathbf{r}_2| \delta_{n,0} \delta_{m,0}. \quad (1.41)$$

Here $\mathbf{r}_1$ and $\mathbf{r}_2$ represent Bravais lattice vectors. Using Eq. 1.38, the momentum and potential parts of the Hamiltonian take the following form

$$\frac{(\mathbf{p} + \mathbf{q})^2}{2m} u(\mathbf{r}) = \frac{\hbar^2}{2m} \sum_{n_1,m_1} \phi_{n_1,m_1}^{(n)} ((n_1\mathbf{g}_1 + m_1\mathbf{g}_2) + \mathbf{q}/\hbar)^2 e^{-i(n_1\mathbf{g}_1 + m_1\mathbf{g}_2) \cdot \mathbf{r}}, \quad (1.42)$$
Putting these together and using the orthogonality relation in Eq. (1.41) produces

\[
\frac{\hbar^2}{2m} c_{n_0,m_0}^{(n)} (n_0 \mathbf{g}_1 + m_0 \mathbf{g}_2) + \mathbf{q}/\hbar)^2 + \sum_{n_1,m_1,n_2,m_2} U_{n_1,m_1} c_{n_2,m_2}^{(n)} \delta_{n_1+n_2,n_0} \delta_{m_1+m_2,m_0} = E_{n,q} c_{n_0,m_0}^{(n)} \tag{1.44}
\]

It is worth non-dimensionalizing Eq. (1.44) by scaling energy to the recoil energy and momentum to the lattice momentum vector $|\mathbf{k}|$. That is to say, define $\mathbf{q} = \hbar|\mathbf{k}| \tilde{\mathbf{q}}_1, \mathbf{g}_1, 2 = \hbar|\mathbf{k}| \tilde{\mathbf{g}}_1, 2$, $U_{n,m} = E_r \tilde{U}_{n,m}$, and $E_{n,q} = E_r \tilde{E}_{n,q}$. Here $E_r = \frac{\hbar^2 |\mathbf{k}|^2}{2m}$. This gives the following equations which solves the Hamiltonian,

\[
c_{n_0,m_0}^{(n)} (n_0 \tilde{\mathbf{g}}_1 + m_0 \tilde{\mathbf{g}}_2) + \tilde{\mathbf{q}})^2 + \sum_{n_1,m_1,n_2,m_2} \tilde{U}_{n_1,m_1} c_{n_2,m_2}^{(n)} \delta_{n_1+n_2,n_0} \delta_{m_1+m_2,m_0} = \tilde{E}_{n,q} c_{n_0,m_0}^{(n)} \tag{1.45}
\]

Without specifying the lattice structure, it is difficult to go any further. This can be used to calculate the band structure and Bloch waves for an arbitrary 2D periodic potential, but care should be taken to reduce the parameter space if possible.

1D Lattice

Assume the following periodic potential

\[
U(x) = U_0 \cos^2(k_0 x) - \frac{U_0}{2} = \frac{U_0}{2} \cos(2k_0 x) \tag{1.46}
\]
The offset here is chosen for simplicity and does not affect the final result. Using Eq. 1.44 and correcting for the fact that the system is only in 1 dimension, the equations become

\[ c_n (2n + \tilde{q})^2 + \sum_{n_1,n_2} \tilde{U}_{n_2} c_{n_1} \delta_{n_1+n_2,n} = \tilde{E}_q c_n. \] (1.47)

\( U(x) \) can be written in the following way

\[ \tilde{U}(x) = \frac{\tilde{U}_0}{4} \left( e^{2i\tilde{k}_0 x} + e^{-2i\tilde{k}_0 x} \right), \] (1.48)

which helps to identify \( \tilde{U}_n \),

\[ U_n = \begin{cases} \frac{\tilde{U}_0}{4}, & n = 1 \\ \frac{\tilde{U}_0}{4}, & n = -1 \\ 0, & \text{otherwise}. \end{cases} \] (1.49)

This results in the following equations

\[ c_n (2n + \tilde{q})^2 + \frac{\tilde{U}_0}{4} (c_{n-1} + c_{n+1}) = \tilde{E}_q c_n \] (1.50)

This equation can be solved numerically as an eigenvalue problem. Since this is an infinite dimensional problem, it is necessary to truncate the \( n \) at some value. If \( N \) bands are under consideration, then \(-5N \leq n \leq 5N\) is a good range to start with. Indeed the number of bands under consideration should be small compared to the value of \( n \) the calculation is truncated at. The coefficients \( c_N \) will become exceedingly small for lower bands as \( N \) is increased. The Bloch bands for various lattice depths can be seen Figure 1.1. Similarly, the energy of the Bands can be found as a function of lattice depth can be found in Figure 1.2.
Figure 1.1: Band structure of a 1D lattice. The lower energy band has been adjusted to have an energy of $0E_r$.

1.3.3 Wannier Functions

In the tight-binding regime, it makes sense to write the wave-function in a basis where atoms are localized on lattice sites rather than filling the whole lattice as with a Bloch wave. One choice of Wannier function contains all quasi-momentum states and has the form

$$w_n(r - R) = \frac{V}{(2\pi)^3} \int_{BZ} dk \ e^{-ik \cdot R} \psi_{n,k}(r)$$

(1.51)

where $\psi_{n,k}(r)$ is given in Eq. 1.36, and $R = n_1 r_1 + n_2 r_2 + n_3 r_3$ is a multiple of a Bravais lattice vectors. This is done in 3-dimensions for completeness, but it is a simple matter to go to lower dimensions. These wave-functions are not unique - there is nothing preventing the choice of a different phase of the Bloch waves. More specifically, there is a freedom of choice in the function $\phi_n(k)$ such that

$$w_n(r - R) = \frac{V}{(2\pi)^3} \int_{BZ} dk \ e^{-ik \cdot R} e^{-i\phi_n(k)} \psi_{n,k}(r).$$

(1.52)

Significant work has gone into finding the maximally localized Wannier functions for more
Figure 1.2: Bloch bands as a function of lattice depth. Blue filled regions denote Bloch bands and red line is the lattice depth for reference.

complicated structures [78]. Wannier functions are normalized such that

\[
\int dr \ w_n^*(r - \mathbf{R}_1)w_m(r - \mathbf{R}_2) = \delta_{n,m}\delta_{\mathbf{R}_1,\mathbf{R}_2}. \tag{1.53}
\]

This normalization is ensured by the relation

\[
\int dr \ \psi_n^*_{\mathbf{k}_1}(r)\psi_m_{\mathbf{k}_2}(r) = \frac{(2\pi)^3}{V} \delta_{n,m}\delta^3(\mathbf{k}_1 - \mathbf{k}_2). \tag{1.54}
\]

The inverse transform for producing the Bloch wave-function from Wannier functions is

\[
\psi_{n,\mathbf{k}}(r) = \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}}w_n(r - \mathbf{R}). \tag{1.55}
\]

Due to the discrete nature of the inverse transform, it is convenient to work in a discrete basis
Figure 1.3: Plots showing the increase of the Wannier function (in black) as the lattice depth is increased. The lattice is shown (blue) for reference. As the lattice depth is increased the Wannier function becomes more localized and quickly starts to resemble the ground state wave-function of the quantum harmonic oscillator.

in \( \mathbf{k} \)-space. In this case, the Wannier function takes the form

\[
w_n(r - R) = \frac{1}{N} \sum_{k} e^{-i \mathbf{k} \cdot R} \psi_{n,k}(\mathbf{r})
\]

(1.56)

where \( N \) is the number of \( \mathbf{k} \)-space points. For \( N \gg 1 \), the discrete basis approaches the continuous case.

A few of the Wannier functions for the 1D case can be found in Figure 1.3. It can be seen from this that the Wannier functions delocalize as the lattice depth is decreased and approach the harmonic oscillator ground state as the depth is increased. The higher Bloch band Wannier functions are also delocalized and have significant overlap with neighboring sites as can be seen in Figure 1.4.
1.3.4 Blue vs Red-detuned Lattice

It is worth commenting on a red versus blue-detuned lattice. As can be seen in Eq. [1.31] the dipole force will attract (repel) the atom to areas of high intensity if the laser is red-detuned (blue-detuned) relative to the atomic resonance. Therefore, if a lattice is generated with a red-detuned (blue-detuned) laser, the atoms will tend to sit at regions of high (low) intensity. If scattering is neglected, and the atom sits in an infinite lattice, then the difference between red and blue-detuned lattices is non-existent. However, there are several useful properties of red and blue-detuned lattices in real systems.

The scattering rate will be substantially lower in a blue detuned lattice due to the lower intensity at the atom location. In the tight binding regime, the scattering rate of the lowest band Wannier function in the red versus blue-detuned lattice has the following form

\[
\eta_{\text{scat}} = \frac{\Gamma_{\text{scat, blue}}}{\Gamma_{\text{scat, red}}} = \frac{\int |I_{\text{blue}}(r)|^2 |w_i(r)|^2 dr}{\int |I_{\text{red}}(r)|^2 |w_i(r)|^2 dr} \quad (1.57)
\]

Here \(I_{\text{red(blue)}}(r)\) are the intensity (or potential) the atoms experience in the red(blue)-detuned
Figure 1.5: Scattering ratio for blue vs red detuned lattices. This is computed using Wannier functions.

case. Evaluating this for various lattice depths produces the results in Figure 1.5. As can be seen, the differences are not as substantial at low lattice depths, but at $40E_r$, the scattering rate can be 10 times lower in the blue-detuned case. This suggests it is possible to work at closer detunings to an atomic resonance in the blue-detuned case, which can be useful if optical power is limited. Interestingly, the overall heating rate in a red vs blue detuned lattice is identical. This is due to the fact that excitation to higher bands has a higher probability in the blue detuned case, which yields the same overall heating rate. The main problem is scattering a single photon should localize the atom and partially collapse the wave-function regardless if it heats to an excited band.

One feature in the blue-detuned case is there is no overall harmonic confinement. (See Figure 1.6) In many experiments, this can be problematic since local chemical potential is defined by the overall harmonic confinement. This requires the use of additional red-detuned beams or donut beams (in the blue detuned case) to produce confinement.

The ground state energy of in the two cases also behaves differently. Suppose the lattice potential in the red detuned case is of the form $U_{\text{red}}(x) = -NE_r \cos^2(kx)$, where $N > 0$. In the blue-detuned case the form is $U_{\text{blue}}(x) = NE_r \sin^2(kx)$. Then in the Lamb-Dicke regime the
potential can be approximated as a harmonic confinement giving $U_{\text{red}}(x) \approx -NE_r(1 - k^2x^2)$ and $U_{\text{blue}}(x) \approx NE_rk^2x^2$. From the quantum harmonic oscillator, the ground state energy will take the form

$$E_{\text{red}} = (-N + \sqrt{N})E_r$$

$$E_{\text{blue}} = \sqrt{NE_r}.$$ (1.58)

Following the same logic, there is also a difference in the strength of disorder (see Chapter 3) in the optical lattice.

1.4 Atoms in Optical Lattices

1.4.1 Second Quantized Hamiltonian

The second quantized many-body Hamiltonian describing Bosonic interacting particles in a potential is [70]

$$\hat{H} = \int dr \hat{\Psi}^\dagger(r, t)\hat{H}_0\hat{\Psi}(r, t)$$

$$+ \frac{1}{2} \int dr dr' \hat{\Psi}^\dagger(r, t)\hat{\Psi}^\dagger(r', t)V(r - r')\hat{\Psi}(r, t)\hat{\Psi}(r', t)$$

$$- \int dr \hat{\Psi}^\dagger(r, t)\mu(r)\hat{\Psi}(r, t)$$
where $\hat{H}_0$ is the single particle Hamiltonian given in Eq. [1.34] and $\mu(r)$ is the local chemical potential. The two particle interaction term is described by $V(r - r')$. In principle the interaction term consists of a contact interaction term as well as long-range interaction terms

$$V(r - r') = \frac{4\pi \hbar^2 a}{m} \delta^{(3)}(r - r') + V_{\text{long range}}(r - r')$$  \hspace{1cm} (1.60)

Here $a$ is the s-wave scattering length and $m$ is the mass of the particle. In the limit of $n^{1/3}a \ll 1$, where $n$ is the density of the gas (which equates to the inter-particle spacing being much larger than the interaction length scale), the solution to Eq. [1.59] can be approximated by a mean-field approach

$$\dot{\Psi}(r, t) = \Psi(r, t) + \delta \dot{\Psi}(r, t),$$  \hspace{1cm} (1.61)

where $\Psi(r, t) = \langle \dot{\Psi}(r, t) \rangle$ is the mean-field wave-function and $\delta \dot{\Psi}(r, t)$ is the fluctuating field operator. Using the Heisenberg equation, the time-dependent, mean-field part of the wave-function is described by the following

$$i\hbar \frac{d}{dt} \Psi(r, t) = \hat{H}_0 \Psi(r, t) + \frac{4\pi \hbar^2 a}{m} |\Psi(r, t)|^2 \Psi(r, t) - \mu \Psi(r, t).$$  \hspace{1cm} (1.62)

This assumes no long-range interaction. While this mean-field approach describes weakly interacting systems, it fails to capture the majority of physics in a more strongly interacting system. Of course, the main reason for putting atoms into an optical lattice is to bring the atoms into a strongly interacting regime. In this case, a new framework is necessary to describe the system.
1.4.2 Bose-Hubbard Hamiltonian

The Hamiltonian in Eq. [1.59] still describes the strongly interacting system. In order to approach the problem, it is necessary to write the Bosonic creation and annihilation operators in a useful basis. More specifically, if the atoms are confined to the lowest Bloch-band, a natural choice for the Bosonic operators is to write them in the Wannier basis

\[ \hat{\Psi}(\mathbf{r}) = \sum_{i} \hat{b}_i \psi_1(\mathbf{r} - \mathbf{r}_i). \]  

(1.63)

Here \( \hat{b}_i^\dagger, \hat{b}_i \) are the bosonic creation and annihilation operators for an atom in site \( i \). Indeed, these operators obey the canonical commutation relations \( [\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij} \). Inserting Eq. [1.63] into Eq. [1.59] yields

\[
\hat{H}_{BH} = \sum_{i,j} \left[ \int d\mathbf{r} \, \psi_1^*(\mathbf{r} - \mathbf{r}_i) \hat{H}_0 \psi_1(\mathbf{r} - \mathbf{r}_j) \right] \hat{b}_i^\dagger \hat{b}_j

+ \frac{1}{2} g \sum_{i,j,k,l} \left[ \int d\mathbf{r} \, \psi_1^*(\mathbf{r} - \mathbf{r}_i) \psi_1^*(\mathbf{r} - \mathbf{r}_j) \psi_1(\mathbf{r} - \mathbf{r}_k) \psi_1(\mathbf{r} - \mathbf{r}_l) \right] \hat{b}_i^\dagger \hat{b}_j^\dagger \hat{b}_k \hat{b}_l

+ \frac{1}{2} \sum_{i,j,k,l} \left[ \int d\mathbf{r} d\mathbf{r}' \, \psi_1^*(\mathbf{r} - \mathbf{r}_i) \psi_1^*(\mathbf{r}' - \mathbf{r}_j) V_{\text{long range}}(\mathbf{r} - \mathbf{r}') \psi_1(\mathbf{r} - \mathbf{r}_k) \psi_1(\mathbf{r}' - \mathbf{r}_l) \right] \hat{b}_i^\dagger \hat{b}_j^\dagger \hat{b}_k \hat{b}_l
\]

(1.64)

The first term in the Hamiltonian contains the tunneling energy as well as the single particle on-site energy. Terms which contain large separation between particles can be neglected since the Wannier function has little overlap a few sites away. This equates to any terms with large differences between \( i, j \) or \( i, j, k, l \). Therefore, the first term in the Hamiltonian can be rewritten
as

\[
\sum_{i,j} \left[ \int dr \, w_i^*(r - r_i) \hat{H}_0 w_1(r - r_j) \right] \hat{b}^\dagger_i \hat{b}_j = \sum_i \left[ \int dr \, w_i^*(r - r_i) \hat{H}_0 w_1(r - r_i) \right] \hat{n}_i + \\
\sum_{\langle i,j \rangle} \left[ \int dr \, w_i^*(r - r_i) \hat{H}_0 w_1(r - r_j) \right] \hat{b}^\dagger_i \hat{b}_j + \\
\sum_{\langle\langle i,j \rangle\rangle} \left[ \int dr \, w_i^*(r - r_i) \hat{H}_0 w_1(r - r_j) \right] \hat{b}^\dagger_i \hat{b}_j + \\
+ \text{Next-Next-Nearest-Neighbor Terms} + \ldots
\]

(1.65)

where \( \langle i,j \rangle \) represents the sum over nearest neighbors and \( \langle\langle i,j \rangle\rangle \) over next-nearest neighbors. The first term in Eq. (1.65) is the single particle on-site energy, the next term is nearest neighbor tunneling, and so on. The contact interaction term in Eq. (1.64) can also be expanded to yield

\[
\frac{1}{2} g \sum_{i,j,k,l} \left[ \int dr \, w_i^*(r - r_i) w_i^*(r - r_j) w_1(r - r_k) w_1(r - r_l) \right] \hat{b}^\dagger_i \hat{b}^\dagger_j \hat{b}_k \hat{b}_l = \\
= \frac{1}{2} g \sum_i a_{iiii} \hat{n}_i (\hat{n}_i - 1) + \\
\frac{1}{4} g \sum_{\langle i,j \rangle} \left\{ 4a_{ijij} \hat{n}_i \hat{n}_j + 2 \hat{a}_{ijij} \hat{n}_i \hat{n}_j + a_{ijij} \hat{b}^\dagger_i \hat{b}^\dagger_j \hat{b}_i \hat{b}_j + h.c. \right\} + \\
+ \text{Next-Nearest-Neighbor processes} + \ldots
\]

(1.66)

Here \( a_{ijkl} \) is given by

\[
a_{ijkl} = \int dr \, w_i^*(r - r_i) w_i^*(r - r_j) w_1(r - r_k) w_1(r - r_l).
\]

(1.67)
Long-range interaction terms can be derived similarly and simply require exchanging

\[ g \alpha_{ijkl} \to \int d\mathbf{r} d\mathbf{r}' w_1^*(\mathbf{r} - \mathbf{r}_i) w_1^*(\mathbf{r} - \mathbf{r}_j) V_{\text{long range}}(\mathbf{r} - \mathbf{r}') w_1(\mathbf{r} - \mathbf{r}_k) w_1(\mathbf{r} - \mathbf{r}_l) \]  \hspace{1cm} (1.68)

in Eq. 1.66. Plenty of physics can be seen from the terms in Eq. 1.66. The first term is simply the on-site interaction energy shift. The terms containing \( \hat{n}_i \hat{n}_j \) are density dependent tunneling terms. The term containing \( \hat{n}_i \hat{n}_j \) is an energy shift due to neighboring sites having atoms. Finally, the term containing \( \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_j \) is a two-particle hopping term, where two particles on site \( j \) are moved to site \( i \).

1.4.3 Tunneling

The tunneling term in the Bose-Hubbard model has the following form

\[ J_{i,j}^m = J_{i,j}^m(\mathbf{r}_i, \mathbf{r}_j) = \int d\mathbf{r} w_m^*(\mathbf{r} - \mathbf{r}_j) \hat{H}_0 w_n(\mathbf{r} - \mathbf{r}_i), \]  \hspace{1cm} (1.69)

where \( i, j \) denote some indexing of sites, \( \mathbf{r}_i \) is the position of that site, and \( n, m \) represent bands. This tunneling term can be related to the Fourier transform of the band structure. To see this, evaluation of the term

\[ \hat{H}_0 w_n(\mathbf{r} - \mathbf{r}_i) = \frac{V}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} e^{-i\mathbf{k} \cdot \mathbf{R}} \hat{H}_0 \psi_{n,\mathbf{k}}(\mathbf{r}) \]
\[ = \frac{V}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} e^{-i\mathbf{k} \cdot \mathbf{R}} \psi_{n,\mathbf{k}}(\mathbf{r}) E_{n,\mathbf{k}} \]  \hspace{1cm} (1.70)

From this and using the relation in Eq. 1.54, the tunneling takes the form

\[ J_{i,j}^m = \frac{V}{(2\pi)^3} \delta_{n,m} \int_{\text{BZ}} d\mathbf{k} e^{-i\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_i)} E_{n,\mathbf{k}} = \delta_{n,m} \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot (\mathbf{r}_j - \mathbf{r}_i)} E_{n,\mathbf{k}} \]  \hspace{1cm} (1.71)

29
Due to the higher energy of excited bands, the tunneling rates can be substantially higher. In Figure 1.7 is a plot of the tunneling energy in $E_r$ for various lattice depths in the 1-D lattice given in Eq. 1.46. Additionally, tunneling to other sites such as next-nearest-neighbor can be seen in Figure 1.8. It is obvious from this plot that at even low lattice depth tunnelings other than nearest neighbor can be neglected.

Indeed, tunneling between bands is not permitted from the bare lattice Hamiltonian. Introducing tilt or disorder in the lattice sites will cause tunneling into higher bands. Rewriting the Hamiltonian in the presence of the perturbing potential as $\hat{H}_0 \to \hat{H}_0 + \delta U(r)$, suggests the potential-induced inter-band tunneling takes the form

$$\delta J_{i,j}^{n,m} = \int dr \, w_n^*(r - r_j) \delta U(r) w_n(r - r_i). \quad (1.72)$$

Here the tunneling has been modified to $J_{i,j}^{n,m} \to J_{i,j}^{n,m} + \delta J_{i,j}^{n,m}$.

Band Structure From Bloch Waves in a 1D Lattice

Another useful relation can be derived by considering the band structure energy for the 1D lattice given in Eq. 1.46. The energy is given by

$$E_{n,k} = \int_{-\lambda/4}^{\lambda/4} dr \, \psi_n^* \hat{H} \psi_n(r)$$

$$= \sum_{r_1,r_2} e^{ik(r_2-r_1)} \int_{-\lambda/4}^{\lambda/4} dr \, w_{n}^*(r-r_1) \hat{H}w_{n}(r-r_2) \quad (1.73)$$

$$= \sum_{m_0,n_0} e^{ik \frac{\lambda}{4}(m_0-n_0)} \int_{-\lambda/4}^{\lambda/4} dr \, w_n^*(r - \frac{\lambda}{2}m_0) \hat{H}w_n \left(r - \frac{\lambda}{2}n_0 \right)$$

where $\hat{H}$ is given in Eq. 1.34. It is relatively easy to see that the integral and sum can be writ-
\[
E_{n,k} = \int_{-\infty}^{\infty} dr \, w_n^*(r) \, \hat{H} w_n(r) + \left( e^{ikr_1} + e^{-ikr_2} \right) \int_{-\infty}^{\infty} dr \, w_n^* \left( r - \frac{\lambda}{2} \right) \hat{H} w_n(r) + \ldots \\
= \int_{-\infty}^{\infty} dr \, w_n^*(r) \, \hat{H} w_n(r) + \sum_{m=1}^{\infty} \cos \left( \pi m \frac{q}{\hbar k_0} \right) J_{m,0}^{n,n}
\]

(1.74)

The first term in Eq. 1.74 represents the on-site energy due to the lattice potential. This term in a deep lattice can be approximated by Eq. 1.58. The result in Eq. 1.74 is interesting since it relates the tunneling rates to the band structure. It can be seen that in the deep in the tight binding limit, where next nearest neighbor tunneling is significantly weaker than nearest neighbor tunneling, the band structure is well approximated by

\[
E_{n,q} \approx \int_{-\infty}^{\infty} dr \, w_n^*(r) \, \hat{H} w_n(r) + 2 \cos \left( \pi \frac{q}{\hbar k_0} \right) J_{1,0}^{n,n}.
\]

(1.75)

It can be seen from this equation that the nearest neighbor tunneling energy is approximated by

\[
J_{1,0}^{n,n} \approx \frac{\Delta E_{n,q}}{4}.
\]

(1.76)

where \( \Delta E_{n,q} = \max (E_{n,q}) - \min (E_{n,q}) \). For lattice depths greater than 15\(E_r\), the lowest-band tunneling can be approximated by the following equation 16

\[
J_{1,0}^{1,1}/E_r \approx \frac{4}{\sqrt{\pi}} \left( \frac{U_0}{E_r} \right)^{3/4} e^{-2\sqrt{U_0/E_r}}.
\]

(1.77)
1.4.4 On-site interaction

The on-site interaction term, as shown in Eq. [1.66], takes the form

$$\hat{H}_{\text{contact}} = \frac{1}{2} g \sum_i \left( \int d\mathbf{r} |w_1(\mathbf{r} - \mathbf{r}_i)|^4 \right) \hat{n}_i (\hat{n}_i - 1).$$

(1.78)

If the lattice is deep enough that the harmonic approximation is valid, the on-site interaction can be calculated. The Wannier function for a cubic lattice, in the Harmonic approximation, takes the form

$$w_1(\mathbf{r} - \mathbf{r}_i) \approx \left( \frac{m \bar{\omega}}{\pi \hbar} \right)^{3/4} e^{-\frac{m \bar{\omega}}{2h} (x-x_i)^2} e^{-\frac{m \bar{\omega}}{2h} (y-y_i)^2} e^{-\frac{m \bar{\omega}}{2h} (z-z_i)^2},$$

(1.79)

where $\bar{\omega}^3 = \omega_x \omega_y \omega_z$, and $\omega_a = \frac{2}{\hbar} E_r^{(\beta)} \sqrt{\frac{U_\beta}{E_r^{(\beta)}}}$ for $\beta = x, y, z$. Here $U_\beta$ is the lattice depth and $E_r^{(\beta)} = \hbar^2 k_r^2 / (2m)$ is the recoil energy in the $\beta$ direction. Integrating the contact interaction

Figure 1.7: Tunneling rate as a function of lattice depth for different Bloch bands. Here $n = 1$ is the lowest Bloch band.
Figure 1.8: Tunneling for Nearest Neighbor (NN) compared to Next-Nearest Neighbor (N-NN) and Next-Next-Nearest Neighbor (N-N-NN).

The on-site interaction term makes tunneling to a neighboring site containing an atom an off-resonant process. In fact, it is similar to a two level atom with a detuning. In this case, this process will scale as $U^2 / J$, where $J$ is the nearest neighbor tunneling energy.

In most Quantum Gas Microscope experiments the atoms live in a 2D layer with a square lattice, which gives $k_x = k_y$. In this case, the on-site interaction becomes proportional to the recoil energy of the square lattice

$$
U / E_r^a = \frac{8 \pi}{(2 \pi)^{3/2}} k_z a \left( \frac{U_y}{E_r^{(y)}} \frac{U_z}{E_r^{(z)}} \right)^{1/4}. \quad (1.81)
$$

Since the entire Bose-Hubbard Hamiltonian can be normalized to the recoil energy, the only ways to effectively increase $U$ relative to other parameters is to increase $k_z$ perpendicular to the 2D layer or to change the scattering length. Increasing trap depth has little effect on $U$.

The on-site interaction term gives

$$
U = g \int d\mathbf{r} |w_1(\mathbf{r} - \mathbf{r}_i)|^4 = \frac{4 \pi \hbar^2 a}{(2 \pi)^{3/2} m} k_x k_y k_z \left( \frac{U_x}{E_r^{(x)}} \frac{U_y}{E_r^{(y)}} \frac{U_z}{E_r^{(z)}} \right)^{1/4}. \quad (1.80)
$$

In most Quantum Gas Microscope experiments the atoms live in a 2D layer with a square lattice, which gives $k_x = k_y$. In this case, the on-site interaction becomes proportional to the recoil energy of the square lattice
1.5 Phase Transitions in the Bose-Hubbard Model

Considering the most dominant terms in the Bose-Hubbard Hamiltonian gives the following Hamiltonian

$$\hat{H}_{BH} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i \varepsilon_i \hat{n}_i$$  \hspace{1cm} (1.82)

where $\langle \rangle$ is the sum over nearest neighbors and $\varepsilon_i$ is the local energy of the atom on site $i$. Usually, $\varepsilon_i$ is defined by the overall harmonic confinement. Assuming $\varepsilon_i$ is fixed for all positions, the ground state of this Hamiltonian in the limit of $J/U \ll 1$ is given by

$$|\Psi_{\text{Mott}}\rangle \propto \prod_{i=1}^{N} (\hat{b}_i^\dagger)^n |0\rangle,$$  \hspace{1cm} (1.83)

where $|0\rangle$ is the vacuum state and $n$ is the occupation number of the sites. This is known as the Mott insulating state which amounts to $n$ atoms on every lattice site. This state is incompressible and has a gapped spectrum due to the energy cost to move one atom from one site to another. In the opposite limit of $J/U \gg 1$, the Hamiltonian has a super-fluid ground state with the wave-function

$$|\Psi_{\text{SF}}\rangle \propto \left( \sum_{i=1}^{M} \hat{b}_i^\dagger \right)^N |0\rangle$$  \hspace{1cm} (1.84)

This is similar to a coherent state and approaches one in the limit of $M \to \infty$ and $M/N$ fixed. Indeed, like a coherent state it has a well defined phase and fluctuating atom numbers on every site. The phase diagram was originally derived by Fisher in Ref. [39] and an example of the phase diagram in 2D can be seen in Figure 1.9. Note the Mott insulating state persists at non-zero tunneling due to the gapped spectrum preventing tunneling events.
Figure 1.9: Phase diagram with Mott insulator to super-fluid transition.

1.6 Dipole Interaction

The first long range interaction to consider is the Dipole-Dipole interaction. This interaction not only has long-range nature, but is also anisotropic. The general form for two magnetic dipoles separated by a vector \( r \) and with magnetic moments given by \( \mathbf{m}_1, \mathbf{m}_2 \) is

\[
V_{m,dd} = -\frac{\mu_0}{4\pi|r|^3} \left( 3(\mathbf{m}_1 \cdot \hat{r})(\mathbf{m}_2 \cdot \hat{r}) - \mathbf{m}_1 \cdot \mathbf{m}_2 \right). \tag{1.85}
\]

It is worth comparing magnetic dipole interaction to an induced electric dipole interaction. In the electric case, the form of the dipole-dipole interaction is nearly identical

\[
V_{e,dd} = -\frac{1}{4\pi\varepsilon_0|r|^3} \left( 3(\mathbf{e}_1 \cdot \hat{r})(\mathbf{e}_2 \cdot \hat{r}) - \mathbf{e}_1 \cdot \mathbf{e}_2 \right). \tag{1.86}
\]

where \( \mathbf{e}_1, \mathbf{e}_2 \) are the induced dipole moments. Thus, the scaling factors for the magnetic dipole
interaction is \( \mu_0 \mu^2/|r|^3 \) and \( d^2/(\varepsilon_0 |r|^3) \), where \( d \) is the electric dipole moment. The ratio of these two scaling factors is a metric of the interaction strength difference, \( \mu_0 \mu^2 \varepsilon_0 / d^2 \). The typical electric dipole moment is approximately 1 Debye and for Erbium, the magnetic dipole is approximately \( 7 \mu_B \), where \( \mu_B \) is the Bohr magneton. Inserting this into the ratio yields approximately 0.004. Therefore, the energy of the dipole interaction is around 1000 times weaker for the magnetic dipole interaction.

In most experiments an external magnetic field is applied which forces the dipoles to align along that field. In this case the magnetic dipole interaction takes the form

\[
V_{dd}(r) = \frac{\mu_0 \mu^2}{4\pi |r|^3} (1 - 3 \cos^2 \theta)
\]

where \( \theta \) is the angle between the dipole moments and the vector \( r \).

1.7 Extended Bose-Hubbard Hamiltonian

One of the goals of this experiment is to explore physics of the Extended Bose-Hubbard Hamiltonian. This Hamiltonian comes from including the long-range dipole dipole interaction in the Hamiltonian in Eq. [1.64]. The dipole interaction term contains the following components

\[
V_{ijkl} = \int \int drdr' \, w_i^*(r - r_i)w_j^*(r' - r_j)V_{dd}(r - r')w_k(r - r_k)w_l(r' - r_l).
\]

The most relevant terms for this experiment are the on-site interaction energy shift and the nearest neighbor interaction. The on-site energy term takes the form

\[
V_{iii} = \int \int drdr' \, |w_i(r - r_i)|^2 |w_i(r' - r_i)|^2 V_{dd}(r - r')
\]
Note this term can be written in terms of momentum space, which greatly simplifies its calculation [70]. The neighboring terms have the form

\[
V_{ijij} = \int \int d\mathbf{r} d\mathbf{r}' |w_1(\mathbf{r} - \mathbf{r}_i)|^2 |w_1(\mathbf{r}' - \mathbf{r}_j)|^2 V_{dd}(\mathbf{r} - \mathbf{r}').
\] (1.90)

Due to the long-range character of this interaction and the fairly localized extent of the Wannier-function, it is a fair approximation to assume the dipole interaction does not vary over the extent of the Wannier function

\[
V_{ijij} \approx V_{dd}(\mathbf{r}_i - \mathbf{r}_j) \int \int d\mathbf{r} d\mathbf{r}' |w_1(\mathbf{r} - \mathbf{r}_i)|^2 |w_1(\mathbf{r}' - \mathbf{r}_j)|^2 = V_{dd}(\mathbf{r}_i - \mathbf{r}_j).
\] (1.91)

It is rather complicated to compute the true 6-dimensional integral [10] in Eq. 1.90. This approximation is best in deep lattices and can be slightly different in shallow lattices. The Hamiltonian then takes the familiar form

\[
\hat{H} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \frac{1}{2} \sum_{i,j} V_{ijij} \hat{n}_i \hat{n}_j + \sum_i \epsilon_i \hat{n}_i
\] (1.92)

where \(U\) now contains terms from the dipole interaction and s-wave scattering.
CHAPTER 2

Characteristics of Erbium

Erbium is an atom belonging to the Lanthanide series (rare earth) - the section of the periodic table that sits off to the side, since if it was included in-line, the periodic table would not fit in most textbooks. Erbium is a metallic material which reacts relatively strongly with oxygen to produce Erbium(III) Oxide. We know this experimentally due to the flames which leaped from our chamber after removing the oven section for repairs. Arguably, Erbium’s most important use is in the telecommunications industry. In this usage, silica optical fibers are doped with erbium to produce Erbium-Doped Fiber Amplifier (EDFA) \([36, 79]\), which have transmission in the C-band 1530-1565 nm and the L-band 1565-1610 nm. As an aside, it is amazing that the wavelengths produced by EDFAs are located at the minimum of absorption for silica fibers. Beyond uses in the telecommunications industry, Erbium has a number of properties which make it an interesting atom for use in quantum gas microscopy. Most notably, Erbium has a long range magnetic dipole-dipole interaction. Arguably, the most important feature for this experiment is the rich electronic structure which gives rise to many atomic

\[^{1}\text{An interaction is defined to be long range if the scaling } V(r) \propto r^{-D} \text{ if } D \text{ is smaller than or equal to the dimensionality of the system. Therefore, technically Erbium has a long range magnetic dipole-dipole interaction in 3-dimensions, but not in 2-dimensions.}\]
transitions. The linewidths of these transitions range from a few $\mu$Hz to a few tens of MHz. Finally, there is a rich Feshbach spectrum with many narrow and broad resonances, which allows tunability over the interaction strength in the system.

2.1 Atomic Structure

The atomic structure of Erbium is quite complicated compared to that of the alkali atoms. It is this structure that gives rise to the large dipole moment found in Erbium. The ground state of Erbium consists of a Xenon core with an additional 14 electrons filling first the 6s state followed by the 4f state - given by Hund’s rule. The ground state can be written as

\[
\text{[Xe]}^4f^{12}6s^2(3H_6).
\]

Here $^3H_6$ is the electron configuration for the 14 additional electrons around the Xenon core.

2.2 Isotopes

There are 6 stable isotopes of Erbium. The isotopes having abundances ranging from 0.14% to 33.6%. (See Table 2.1) The $^{167}$Er isotope is fermionic with nuclear spin $I = 7/2$ and the remaining isotopes are bosonic with nuclear spin $I = 0$. Luckily, the fermionic isotope has large enough abundance for trapping and cooling.

2.3 Atomic Levels and Transitions

Erbium, due to its complicated electronic structure, has many atomic transitions. All atomic levels from the NIST database are included in Figure 2.1. There are approximately 670

\[1\text{With LS coupling the term symbol may be written in the form }^{2S+1}L_J, \text{ where } S \text{ is the total spin, } L \text{ is the orbital angular momentum and } J \text{ is the total orbital angular momentum.}\]
<table>
<thead>
<tr>
<th>Isotope</th>
<th>Mass</th>
<th>Abundance</th>
<th>Nuclear Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{162}$Er</td>
<td>161.928775</td>
<td>0.14%</td>
<td>0</td>
</tr>
<tr>
<td>$^{164}$Er</td>
<td>163.929198</td>
<td>1.61%</td>
<td>0</td>
</tr>
<tr>
<td>$^{166}$Er</td>
<td>165.930290</td>
<td>33.6%</td>
<td>0</td>
</tr>
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<td>$^{167}$Er</td>
<td>166.932046</td>
<td>22.95%</td>
<td>$\frac{7}{2}$</td>
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<tr>
<td>$^{168}$Er</td>
<td>167.932368</td>
<td>26.8%</td>
<td>0</td>
</tr>
<tr>
<td>$^{170}$Er</td>
<td>169.935461</td>
<td>14.9%</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.1: Isotopes of Erbium from Ref [64].

states included in the database - including transitions up to around 50000 cm$^{-1}$. The transitions used in this experiment are also included in the Figure 2.1 for reference. Erbium also has an extremely narrow quadrupole transition ($J = 6 \rightarrow J = 4$) at 1986 nm with transition rate $\Gamma = (2\pi) \times 7.3\mu$Hz. This transition could be used for implementation of an atomic clock [63]. For completeness, information on the laser cooling transitions [12] is compiled in Table 2.2.

<table>
<thead>
<tr>
<th>Transition wavelength</th>
<th>Electron configuration</th>
<th>Transition rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>401 nm</td>
<td>$4f^{12}(^{3}H_{6})6s6p(^{1}P_{1}^{o})(6, 1)^{2}_{j}$</td>
<td>$(2\pi)30$ MHz</td>
</tr>
<tr>
<td>583 nm</td>
<td>$4f^{12}(^{3}H_{6})6s6p(^{3}P_{1}^{o})(6, 1)^{2}_{j}$</td>
<td>$(2\pi)190$ kHz</td>
</tr>
<tr>
<td>631 nm</td>
<td>$4f^{11}(^{4}P_{5/2}^{o})(5d_{3/2}6s^{2}(13/2, 5/2)^{2}_{j}$</td>
<td>$(2\pi)28$ kHz</td>
</tr>
<tr>
<td>841 nm</td>
<td>$4f^{11}(^{4}P_{3/2}^{o})(5d_{3/2}6s^{2}(15/2, 5/2)^{2}_{j}$</td>
<td>$(2\pi)8$ kHz</td>
</tr>
<tr>
<td>1299 nm</td>
<td>$4f^{12}(^{3}H_{6})6s6p(^{3}P_{1}^{o})(6, 1)^{2}_{j}$</td>
<td>$(2\pi)2.3$ Hz</td>
</tr>
</tbody>
</table>

Table 2.2: Laser cooling transitions from Ref [12].
Figure 2.1: Atomic level structure up to approximately 50000 cm$^{-1}$ as a function of total angular momentum $J$. Black levels denote even parity (ungerade) and black odd parity (gerade). Dipole allowed transitions can occur between levels of differing parity. Three transitions used in the system are shown. Blue denotes the 401 nm transition with a transition strength of $\Gamma_{401} \approx (2\pi) \times 29.7$ MHz. Yellow denotes the 583 nm with a transition strength of $\Gamma_{583} \approx (2\pi) \times 190$ kHz. This transition is used to initially capture of the atoms in a Magneto Optical Trap (MOT). Red denotes the 841 nm transition with a transition strength of $\Gamma_{841} \approx (2\pi) \times 8$ kHz, which is used for further cooling/compression after the initial MOT.

2.4 Isotope Shifts

We have accurately measured the isotope shifts for all stable isotopes of Erbium. (see Figure 2.2) Since all near resonant wavelengths used in our experiment are locked to our ultra-low-expansion (ULE) cavity, it is a simple matter to accurately measure the frequency splitting of the isotopes when used in combination with a wavemeter. The $F = 19/2 - 21/2$ is reported for the fermionic isotope of Erbium, since this is the only frequency important for MOT operation. Measurement of all isotope shifts were performed with the Magneto Optical Trap (MOT). The MOT was compressed and then all light and magnetic fields were turned off. The
401 nm transition was measured using absorption imaging and varying the detuning. Counting atom number without compensation for detuning reveals the transition. The 583 nm and 841 nm were found using a short (4 ms), low intensity pulse, which caused a slight displacement of the atomic cloud horizontally during time-of-flight. Fitting the displacement allowed extraction of the transition frequency. Note if the intensity is not low enough, the displacement does not follow the shape of the linewidth. Doppler shifts will cause the line to shift slightly to the blue-detuned side. Using this method the transition frequency can be estimated to less than the linewidth. This measurement is slightly more challenging with the 841 nm transition since the displacement is small. This can be remedied by taking many measurements and averaging, or by increasing the intensity and fitting the appropriate function to the measured displacement.

**Figure 2.2:** Isotope shifts for the 401 nm, 583 nm, and 841 nm transitions. The linear fits are include only the $^{164}$Er, $^{166}$Er, $^{168}$Er and $^{170}$Er isotopes - this is intended to show the shift from a linear trend for the $^{162}$Er isotope.
<table>
<thead>
<tr>
<th>Transition wavelength</th>
<th>Isotope</th>
<th>Frequency shift (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>401 nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>162</td>
<td>1874</td>
</tr>
<tr>
<td></td>
<td>164</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>166</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>167 (F = 19/2 - 21/2)</td>
<td>142</td>
</tr>
<tr>
<td></td>
<td>168</td>
<td>-851</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>-1715</td>
</tr>
<tr>
<td>583 nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>162</td>
<td>2161.6</td>
</tr>
<tr>
<td></td>
<td>164</td>
<td>974.9</td>
</tr>
<tr>
<td></td>
<td>166</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>167 (F = 19/2 - 21/2)</td>
<td>-2008.1</td>
</tr>
<tr>
<td></td>
<td>168</td>
<td>-974.3</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>-1965.6</td>
</tr>
<tr>
<td>841 nm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>162</td>
<td>-3253.31</td>
</tr>
<tr>
<td></td>
<td>164</td>
<td>-1423.43</td>
</tr>
<tr>
<td></td>
<td>166</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>167 (F = 19/2 - 21/2)</td>
<td>238.73</td>
</tr>
<tr>
<td></td>
<td>168</td>
<td>1435.36</td>
</tr>
<tr>
<td></td>
<td>170</td>
<td>2914.05</td>
</tr>
</tbody>
</table>

**Table 2.3:** Table of measured isotope shifts relative to \(^{166}\text{Er}\) isotope.
2.5 Dipole Moment

Due to the large total angular momentum, \( J = 6 \), of the ground state configuration of Erbium, there exists a modest magnetic dipole moment of approximately \( 7 \mu_b \). To be more concrete, the effective dipole interaction can be calculated by

\[
\mu_{dd} = m_j g_j \mu_b,
\]  

(2.1)

where \( m_j \) is the angular momentum projection, \( g_j \) is the Landé g-factor and \( \mu_b \) is the Bohr magneton. In the stretched states, \( m_j = \pm 6 \), the dipole interaction strength is 49 times stronger than in the alkali atoms.

2.6 Feshbach Resonances

Erbium has a rich Feshbach spectrum with every isotope having a different spectrum. These resonances can be used for tuning the on-site interaction energy of the Bose-Hubbard Hamiltonian. In addition, tuning the scattering length in a bulk system can show dipolar collapse \([66, 8]\). This spectrum has been measured carefully for \(^{166}\text{Er} \) and \(^{168}\text{Er} \) in Ref \([43]\).
Disorder in optical lattices

Unwanted disorder in an optical lattice can be a limiting factor when exploring new physics. These variations may require additional techniques to mitigate the undesired effects. There are numerous sources of disorder in an optical lattice: dust on optics may be the culprit, the scratch-dig on optics, or even small inclusions in the glass. Dust may be manageable if the optics setup is in a clean room, but this may not be enough. The scratch-dig on optics can be solved by purchasing super-polished optics; however, these optics are delicate, cost-prohibitive, and even cleaning them can lead to scratches. Inclusions can somewhat be limited by selecting optics with extremely high quality glass, but they cannot be eliminated entirely. The main problem of going this route is most companies do not offer optics with the quality necessary to nearly eliminate disorder, nor is it cost-effective. A more pragmatic approach is to understand the origin of disorder in optical lattices and understand how to manage it by smart design.

In the case of Erbium, disorder can be problematic while exploring dipolar physics. For example, if one is exploring the fractional filling phases of the Extended Bose-Hubbard Hamiltonian, disorder can prevent these phases from forming. (See section 7.2) The characteristic energy scale for the dipole-dipole interaction in our system will be approximately $\omega_{dd}$.
$(2\pi) \times 30$ Hz and the recoil energy of our lattice will be of order $\omega_{\text{recoil}} \approx (2\pi) \times 4.2$ kHz. In order to form fractional filling phases, the lattice depth needs to be around $22E_{\text{recoil}}$, where $E_{\text{recoil}} = h\omega_{\text{recoil}}$. This means site-to-site variations need to be less than the ratio of the dipole energy to the lattice depth $\frac{\Delta E_{\text{dip}}}{\Delta \omega_{\text{recoil}}}$ = 0.0003 for fractional filling phases to form. Indeed, small areas may form in the potential which satisfy this condition, even in the presence of disorder, but most likely the areas will be smaller than intended and change over time.

With additional work disorder can be mitigated by use of a beam shaping device. If the disorder can be accurately measured, then an additional potential can be generated using a digital micro-mirror device [114]. The main disadvantage to this technique is added complexity. The disorder may also change over time as dust accumulates on the optics, necessitating recalibration of the system. It seems best to use this method as a last resort and design the lattice to already minimize disorder.

The term disorder used here can be made more concrete by defining it as unwanted fluctuations in the intended potential on length scales smaller than the characteristic length scale of the potential. For a Gaussian beam, the characteristic length scale of the potential is the beam waist. The parabolic potential at the potential minimum of a Gaussian can be normalized to the trap waist:

$$U(x) = -U_0 e^{-\frac{2x^2}{w^2}} \approx -U_0 + 2U_0 \left(\frac{x}{w}\right)^2,$$

where $U_0$ is the trap depth and $w$ is the waist of the Gaussian beam forming the trap. Other potentials could also be considered, but most experiments use Gaussian beams for the formation of dipole traps and lattices; therefore, for the remainder of this chapter Gaussian beams will be the only type considered.
3.1 Mott Insulator as probe for disorder

A Mott insulator can be used as a probe for disorder. The shape of the Mott insulator transitions ($n = 0$ to $n = 1$, etc...) is dictated primarily by the potential shape and the on-site interaction. So, small fluctuations in potential depth can lead to oddly shaped Mott insulators [107]. The sensitivity of the Mott insulator to disorder can be understood from the Mott insulator to super-fluid phase diagram [39]. At $J/U = 0$, the transition between the Mott insulating lobes occurs at chemical potential $\mu = NU$, where $J$ is the tunneling energy, $U$ is the on-site interaction energy, $N$ is a positive integer, and $\mu$ is the chemical potential. In the presence of a potential, $\mu$ is dictated by the potential as well as the atom number. At low tunneling the region separating the Mott insulating lobes is smallest, which makes it more sensitive to disorder.

In order to quantify the sensitivity of the Mott insulator to disorder, consider an $n = 1$ Mott insulator in a 2-d parabolic confining potential. Further assume that the trapping frequencies are identical in the two directions. In this case, the harmonic confining potential may be written (up to a constant offset) as

$$U(x, y) \approx \frac{2U_0}{w^2}(x^2 + y^2) = \frac{2U_0}{w^2}r^2,$$

(3.1)

where $U_0$ is the trap depth, $w$ is the waist of the Gaussian beam, and $r = \sqrt{x^2 + y^2}$ is the radial distance. This equation can also be written in terms of trap frequencies $U(x, y) = \frac{1}{2}m\Omega^2 r^2$.

The question to answer is how sensitive is the edge of the Mott insulator to fluctuations in trap depth? To quantify this, the change in energy at the edge of the Mott insulator needs to be calculated. Assuming the lattice wavelength is $\lambda_{\text{lat}}$ and a radial distance $r_N$, the number of
atoms filling the potential in the $n = 1$ Mott insulating shell is $N_{\text{atoms}} \approx \pi (2r_{\text{atoms}}/\lambda_{\text{lat}})^2$. The atom number is limited to some upper bound (depending on on-site interaction and trap parameters). If not, then the $n = 2, 3, \ldots$ Mott shells will start to be populated. This gives a relation between the radial distance of the edge of the Mott insulator and the atom number.

This implies an energy change per lattice site in the radial direction of

$$
\Delta U = \frac{\partial U}{\partial r} \bigg|_{r=r_{\text{atoms}}} \cdot \frac{\lambda_{\text{lat}}}{2} = \frac{U_0}{w^2} \frac{\lambda_{\text{lat}}^2}{w} \sqrt{\frac{N_{\text{atoms}}}{\pi}}
$$

$$
= \frac{\hbar^2}{2E_p} \frac{\Omega^2}{w^2} \sqrt{\pi^3 N_{\text{atoms}}}
$$

at the edge of the Mott insulator. Here the energy change has also been written in terms of trap frequency, $\Omega$. Indeed, the change is smallest when the atom number is small; however, having a larger waist with a smaller lattice spacing leads to the most sensitivity. Equation 3.2 states that increasing in the energy on a given site at the edge of the Mott insulator by $\Delta U$ will prevent that site from being populated by an atom.

3.2 Disorder in a red and blue detuned lattice

Without doing a direct calculation of the disorder, it is possible to see how disorder will scale in a red and blue detuned lattice. Given two counter-propagating electric fields and a perturbing electric field, the total electric field along the $\vec{k} = k\hat{x}$ direction (ignoring polarization) can be written as

$$
E(x) = E_0 e^{ikx} + E_0 e^{-ikx} + \delta E_0 e^{-ik'x-i\phi}
$$

Here $E_0$ is the electric field of the desired lattice, $\delta E_0$ is the perturbing electric field (disorder), $\phi$ is the phase of the perturbing field, and $k' = \vec{k}' \cdot \hat{x}$ is the wavevector for the disorder. For simplicity, the electric field $E_0$ and perturbing field $\delta E = e E_0$ are taken to be real and positive.
- in this case, \( \varepsilon \ll 1 \). Ignoring factors of \( O(\varepsilon^2) \), the corresponding intensity of this electric field is of the form

\[
I(x) \propto 4E_0^2 \cos^2(kx) + 4\varepsilon E_0^2 \cos(kx) \cos(k'x + \varphi).
\] (3.3)

Before going any further it is important to note that \( k' \approx k \). This statement simply means that disorder is at a longer length scale than the distance between sites. This approximation will not always hold true, but the maximum intensity change occurs when \( k = k' \). In order to see the maximum possible effect of the additional perturbing beam, the phase \( \varphi \) has to be chosen to minimize or maximize intensity around a potential minimum. As was discussed in Section 1.3.4, in a red-detuned lattice the atom is attracted to the maximum intensity of the light field and to the minimum in a blue-detuned lattice. This means the potential can be written as

\[
U_{\pm}(x) \approx \pm NE_r \cos^2(kx) \pm \varepsilon NE_r \cos(kx) \cos(kx + \varphi),
\] (3.4)

where \( U_+ \) is the blue-detuned potential and \( U_- \) is the red-detuned potential. Here the potential depth has been set to \( NE_r \), where \( N \in \mathbb{R}_{\geq 0} \) and \( E_r \) is the recoil energy. \( U_-(x) \) needs to be expanded around \( x = 0 \) and \( U_+(x) \) around \( x = \frac{\pi}{2k} \). Again, ignoring terms of \( O(\varepsilon^2) \) yields,

\[
U_-(x) \approx -NE_r(1 + \varepsilon \cos(\varphi)) + NE_r k^2 (1 + \varepsilon \cos(\varphi)) \left( x + \frac{\sin(\varphi)}{2k} \varepsilon \right)^2
\]

\[
U_+(x) \approx NE_r k^2 (1 + \varepsilon \cos(\varphi)) \left( x - \frac{\pi}{2k} + \frac{\sin(\varphi)}{2k} \varepsilon \right)^2
\] (3.5)

The corresponding energy of the ground state is

\[
|E_-| \approx -NE_r(1 + \varepsilon \cos(\varphi)) + \sqrt{NE_r} + \frac{\varepsilon}{2} \cos(\varphi) \sqrt{NE_r}
\]

\[
|E_+| \approx \sqrt{NE_r} + \frac{\varepsilon}{2} \cos(\varphi) \sqrt{NE_r}.
\] (3.6)
The maximum possible deviation of the state energy to the unperturbed energy is given by

\[
\Delta E_\pm \approx \left( N - \sqrt{N} \right) E_r \varepsilon
\]

\[
\Delta E_\pm \approx \frac{\sqrt{N}}{2} E_r \varepsilon.
\]

Since the calculation of the energy required the tight-binding approximation, the potential depth satisfies \( N \gg 1 \). This guarantees \( \Delta E_- \) is strictly positive for \( \varepsilon > 0 \). The ratio of the two energy energy differences in Eq. (3.7) scales as

\[
\frac{\Delta E_-}{\Delta E_+} = \frac{N - \sqrt{N}/2}{\sqrt{N}/2} \approx 2\sqrt{N}.
\]

Of course, this implies that at a lattice depth of \( 22E_r \) the difference in ground state energies due to disorder is approximately 8.5 times greater in a red-detuned lattice compared to blue-detuned.

Note that all other possible \( k \)-vector components contributing to the disorder will mix predominantly with the carrier and mixing between the various beams will contribute \( \mathcal{O}(\varepsilon^2) \). This treatment has considered the electric-field linear in \( \varepsilon \). If instead the intensity is linear in \( \varepsilon \), then the electric-field would scale as \( \delta E_0 = \sqrt{\varepsilon}E_0 \). To include this all previous equations in the section should have \( \varepsilon \to \sqrt{\varepsilon} \). It is evident from this scaling that an extremely small amount of light can mix strongly with the carrier causing significant fluctuations in intensity.

3.3 Disorder in other experiments

It is instructive to look at the level of disorder in another experiment to see how much better the disorder needs to be in this experiment. The easiest experiment to directly compare to is the Rubidium quantum gas microscope in the Greiner Lab. In this experiment the lat-
tice wavelength is $\lambda_{\text{lat}} \approx 1360$ nm. According to Ref. [107] the lattice depth is $U_0 = 22E_r$ and the transverse confinement is 45 Hz. It is easy to verify that the recoil energy is $E_r/\hbar \approx (2\pi) \times 1270$ Hz given these parameters. According to Eq. 3.2, the energy change per lattice site at the edge of the Mott insulator is $\Delta U/\hbar = (2\pi)49$ Hz. The Mott insulator in Figure 2(a) of Ref. [107] looks to have fluctuations of at least a few sites in some areas, which would approximately double the amount of energy change due to disorder. In the Rubidium experiment the lattices are also blue-detuned, so the fractional shift due to disorder has already been reduced by a factor approximately 8.5 to the red-detuned case (according to Eq. 3.8). Therefore, if the experiment used a red-detuned beam, the amount of uncorrected disorder in that experiment would be approximately 420 Hz per lattice site at $22E_r$ lattice depth! This is a factor of 50 times greater than the requirements for the fractional filling phases to emerge! Even in the blue-detuned case this is a factor of 6 times worse than is required.

3.4 Scaling factors for disorder

Optical lattices can be generated in several ways. In Figure 3.1 three possible ways of generating a lattice are shown. First, as is done in the Rubidium QGM, the lattices can be projected through the objective. In this regime a high NA objective is used to achieve a lattice spacing of $\lambda_{\text{lat}} = \frac{\lambda}{2N\lambda}$. Second, the lattice can be focused through a lens and retro-reflected by a mirror or a cat-eye. Finally, the lattice can be generated using an optical cavity. Other methods exist, but will not be considered here.

To explore these configurations, consider a Gaussian beam propagating through a lens as shown in Figure 3.2. The focus of the Gaussian beam is at a distance $f$ from the lens, where $f$ is the focal length. On the same side as the focus is a scatterer located at $z = z_0$, described by the mask function $D(x,y)$. An appropriate choice for $D(x,y) = 1 - G(x,y)$ is a circular
obstruction, where

\[ G(x, y) = \begin{cases} 
1 & x^2 + y^2 \leq 1 \\
0 & \text{otherwise.} 
\end{cases} \]  
(3.9)

It is also possible to include phase and amplitude variations due to the scatterer, but the results should be similar to the case of an opaque object. The electric field for the unperturbed Gaussian beam is given by

\[ E_0(x, y, z_0) = E_0 \frac{w_0}{w(z_0 + f)} e^{-\frac{r^2}{w^2(z_0 + f)}} e^{-i(\kappa(z_0 + f) + k \frac{z^2}{2R(z_0 + f)} - \psi(z_0 + f))}, \]  
(3.10)

where \( w(z) = w_0 \sqrt{1 + (z/z_R)^2} \) is the \( 1/e^2 \) waist of the beam, \( z_R = \pi w_0^2/\lambda \) is the Rayleigh range, \( R(z) = z (1 + (z_R/z)^2) \) is the radius of curvature, and \( \psi(z) = \arctan(z/z_r) \) is the Gouy
Figure 3.2: Diagram for calculation. Scatters are located at $z = z_0$ given by mask $D(x, y)$. A lens of focal length $f$ is located at $z = 0$ and the lattice beams are focused at $z = f$.

The electric field at the location of the mask is then given by

$$E(x, y, z_0) = E_0(x, y, z_0) \left[ 1 - G \left( \frac{x - x_0}{\delta r_0}, \frac{y - y_0}{\delta r_0} \right) \right], \quad (3.11)$$

where $x_0, y_0$ is the center location of the scatterer, and $\delta r_0$ is its radius. Now, the electric field after the lens at $z = f$ will be the Fourier transform of $E(x, y, z_0)$ with an additional phase factor (see Appendix A),

$$E_F(x', y') = \mathcal{F} \left\{ E_0(x, y, z_0) \right\} (k_x, k_y) e^{-i\phi(x', y')} \bigg|_{k_x = k_x', k_y = k_y'},$$

where

$$\mathcal{F} \left\{ E_0(x, y, z_0) \right\} (k_x, k_y) = \frac{k}{f} \left\{ E_0(x, y, z_0) G \left( \frac{x - x_0}{\delta r_0}, \frac{y - y_0}{\delta r_0} \right) \right\} (k_x, k_y) \cdot e^{-i\phi(x', y')} \bigg|_{k_x = k_x', k_y = k_y'}. \quad (3.13)$$

Here the phase $\phi(x', y') = k(f - z_0) + k^2 (f + z_0)(x'^2 + y'^2)$ and $k = 2\pi/\lambda$.

Before taking Fourier transforms it is useful to approximate the electric field of the term containing $G(x, y)$ in Eq. 3.11. First assume that $\delta r_0 \ll w(z_0 + f)$, then it is safe to replace
\[
\exp\left\{-\frac{r^2}{w^2(z_0+f)}\right\} \rightarrow \exp\left\{-\frac{r_0^2}{w^2(z_0+f)}\right\}, \text{ where } r_0^2 = x_0^2 + y_0^2. \text{ To include the direction of the local wave-vector, complex exponential term is expanded to first order around the center of the scatterer,}
\]
\[
e^{-ik\frac{r^2}{2w(z_0+f)}} \rightarrow e^{ik\frac{r_0^2}{2w(z_0+f)}} - ik\frac{(x_0+y_0)}{w(z_0+f)}. \]

Combining these approximations allows the term containing \(G(x,y)\) to be written as
\[
E_0(x,y,z_0)G\left(\frac{x-x_0}{\delta r_0}, \frac{y-y_0}{\delta r_0}\right) \approx \tilde{E}_0 e^{-ik\frac{(x_0+y_0)}{w(z_0+f)}} G\left(\frac{x-x_0}{\delta r_0}, \frac{y-y_0}{\delta r_0}\right). \tag{3.14}
\]

Here \(\tilde{E}_0\) contains all phase and amplitude terms,
\[
\tilde{E}_0 = |E_0| \frac{w_0}{w(z_0+f)} e^{-\frac{\delta^2}{w^2(z_0+f)}} e^{i\Phi} \tag{3.15}
\]
where \(\Phi\) is the complex phase.

It is now a simple matter to compute the field after the lens. Because the unperturbed field has its focus located a distance \(f\) away from the lens, it is particularly easy to compute the its field contribution after the lens. Without going it to too much unnecessary detail, the field becomes
\[
\tilde{E}_0(x',y') = \frac{E_0 k}{2f} w_0^2 e^{-i\frac{k_0}{w_0}(k_0^2+k_z^2)} e^{-i2\beta_f} \bigg|_{k_z = k_f, k_y = k_f'} \tag{3.16}
\]
Notice the defocus term is no longer present due to the focus of the Gaussian being located a distance \(f\) from the lens. It is easy to see the effective waist after the lens is \(w_f = \frac{2f}{k w_0}\). The
perturbing field can be calculated to be

\[
\overline{E}_G(x', y') = E_0 \frac{k}{f} \delta r_0^2 \tilde{G} \left( \delta r_0 \left( k x' - k \frac{x_0}{R(z_0 + f)} \right), \delta r_0 \left( k y' - k \frac{y_0}{R(z_0 + f)} \right) \right) \\
\times e^{i\frac{k x_0 x'}{f}} e^{i\frac{k y_0 y'}{f}} e^{-i k (f-z_0) - i \frac{k f}{2}(x'^2+y'^2)}
\]

(3.17)

where

\[
\tilde{G}(k_x, k_y) = \frac{J_1 \left( \sqrt{k_x^2 + k_y^2} \right)}{\sqrt{k_x^2 + k_y^2}}. \tag{3.18}
\]

Here \( J_1(x) \) is a Bessel function of the first kind. Now that an approximate electric field has been obtained, analysis of each term reveals scaling factors for disorder. The function \( \tilde{G}(k_x, k_y) \) in Eq. 3.17 is the Airy disk when light passes through a circular aperture. This function gives a few scaling factors, which, when satisfied, helps to minimize disorder. First, the radius (smallest \( r \) which has zero electric-field) of the Airy disk needs to be larger than the waist after the lens:

\[
3.83 \frac{f}{k\delta r_0} > w_{\text{final}} \rightarrow 3.83 \frac{w_0}{2} > \delta r_0 \tag{3.19}
\]

Here \( x = 3.83 \) is the first zero of the Bessel function, \( J_1(x = 3.83) \approx 0 \). Second, the center of the Airy disk should be closer to the axis than the width of the Airy disk

\[
\frac{fr_0}{R(z_0 + f)} < 3.83 \frac{f}{k\delta r_0}.
\]

Since \( R^{-1}(z) \) is bounded by \(-z_R^{-1} \leq 2R^{-1}(z) \leq z_R^{-1} \) for all \( z \), the inequality can be further
If the size of the scatterer is small compared to the waist, then the distance \( r_0 \) can be quite substantial. However, these two inequalities do not guarantee no disorder. The remaining defocus term can also be responsible for significant fringing. Examining the defocus term, the same logic can be applied to it as was applied to the Airy disk. The interference of the carrier and the defocus term produces ring structures. It is important that the width of these rings (defined by the distance to oscillate by half a wave from the center of the ring pattern),

\[
\frac{k}{2f^2}|f + z_0|r'^2 = \pi \rightarrow r' = f\sqrt{\frac{\lambda}{|f + z_0|}}
\]

be larger than the final waist size

\[
f\sqrt{\frac{\lambda}{|f + z_0|}} > \frac{2f}{k\omega_0} \rightarrow \pi z_R > |f + z_0|.
\]  

This puts a restriction on the \( z \) location of the scatterers. Finally, the interference pattern on axis due to the rings should have a length scale larger than the final waist size. This will guarantee the phase does not change too quickly on axis due to the quadratic term. The effective wave-vector for this interference is given by \( k_{\text{eff}} = kr_0/f \), which yields

\[
\frac{2\pi}{k_{\text{eff}}} > \frac{2f}{k\omega_0} \rightarrow r_0 < \pi \omega_0.
\]  

Equations 3.19 and 3.22 can be combined to

\[
r_0 < \min \left( \pi \omega_0, \frac{3.83}{2} \frac{\omega^2_0}{\delta r_0} \right)
\]
These restrictions will guarantee in most cases length scales larger than the waist size after the lens. The inequalities can be made more restrictive by replacing $<$ with $\ll$, etc... Doing this will guarantee absolutely minimal disorder. There is a small gray zone given by Eq. 3.23 in which the scatterer with $r_0 > w_0$ causes some disorder. This can be mitigated by requiring the scatterer be much smaller than the initial waist $\delta r_0 \ll w_0$, which is not unreasonable. In reality, it is impossible to limit scatters to a given distance from the optical axis. The reason scatterers can exist outside the specified region is that the perturbing electric-field, and therefore the strength of the disorder, exponentially decreases with distance from the axis (see Eq. 3.15). At a distance of $r_0 = 3w_0$, the perturbing electric field strength is approximately 8000 times weaker than if the scatterer was on axis. However, this is not true if the derived conditions are not satisfied. For example, if the scatterer is many Rayleigh ranges away from the lens, then it is possible to have substantial amplitude at large $r_0$. Finally, it is worth stating that an aperture at the Fourier plane of the lens (even the lens itself) naturally provides a restriction on length scales in the system. If the diameter of the aperture is $d$, then the shortest disorder length scale is limited to

$$\lambda_{\text{aperture}} \approx 4\frac{\lambda}{d}. \quad (3.24)$$

In a perfect world, this says an aperture can be placed at the Fourier plane of the system and disorder should be removed entirely - this is not entirely true. If the lattice is projected through an objective, there are many optical surfaces after the aperture which could potentially scatter light (even experience a double reflection). It is possible to run into a situation which still has disorder.
3.4.1 Scatterers after the focusing lens

Typically the focusing lens will exist outside the vacuum chamber. In between there might be a few mirrors and a vacuum window (or a glass cell). Since the effect of the focusing lens has been explored, a relevant question to ask is how the subsequent optical elements effect the beam quality? The answer to this can be found by analyzing the scaling factors for propagation of a single slit.

The max angle for interference from point source (see Figure 3.3) can be found by using beam waist:

\[
\sin(\theta) = \frac{w(z)}{\sqrt{z^2 + w^2(z)}}
\]  
(3.25)

This gives a shortest length scale at the origin for interference of

\[
l_{\text{int}} = \frac{\lambda}{\sin(\theta)} = \frac{\lambda}{\sqrt{z^2 + w^2(z)}}
\]  
(3.26)

That is to say, all disorder in the system is limited to length scales \(l_{\text{int}} \leq l_{\text{disorder}} < \infty\). At large distances \(z \gg z_R\), the minimum length scale becomes \(l_{\text{int}, z \rightarrow \infty} = \pi w_0\). This is not equal to the waist, \(w_0\), since higher NA components are required to generate the actual waist size. Smaller
Figure 3.4: Propagation of disorder from defects after the final imaging lens. Disorder is reduced significantly in one Rayleigh range.

length scales can be achieved, however the strength of the disorder will drop off exponentially at distances, from the center axis of the beam, further than the waist.

If a single slit of width $2\delta r_0$ is considered, then according to standard Fraunhofer diffraction theory, the width of the central peak in the distribution will be $w_{\text{slit}} \approx \lambda z / \delta r_0$. This approximation will be valid for $z \gg \delta r_0^2 / \lambda$, which can also be thought of as approximately the Rayleigh range of a beam with a diameter equal to the slit width. The width of the central peak follows the same form for the width of a Gaussian beam as a function of distance from the minimum waist. Ideally, the width of the central peak should be much larger than the waist to guarantee no disorder - this implies the $z \gg w_0 \delta r_0 / \lambda$. This is easily satisfied in most cases, even at a single Rayleigh range distance $z \approx \pi w_0^2 / \lambda$, since the size of the scatter should be smaller than the waist. In practice this distance can be even smaller since the strength of the disorder drops off as $1/z$. To be safe, however, it is best to keep the distance between the final waist and other optical surfaces to a maximum. A good starting point is approximately $z > z_R/3$ away from the waist. If this can be achieved, then disorder will be minimized. Surfaces closer than this need to be extremely high quality with no surface defects.
3.5 Disorder with temporally incoherent light

One way to improve disorder is to use a temporally incoherent light source. In the retro-reflected lattice or the cavity lattice this will not work since the path lengths will most likely be much larger than the coherence length of the light. However, in a projected lattice, the path lengths of the interfering beams are nearly identical. In this case it is also important to understand that temporally incoherent light will minimally improve disorder much from an individual scatterer - this is due to the path-lengths of the main beam and the scattered portion of the beam being nearly identical. If one examines Figure 3.5, the portion of the scattered beam that interferes most strongly will be the one that crosses the main beam axis at a distance $f$ from the lens. It can easily be shown that the optical path length for this portion of the beam (for a thin lens) will be approximately $z_0 + f$, which is independent of the distance of the scatterer from the beam axis. The primary difference will come from the curvature of the Gaussian beam. The scatterer will acquire a path length difference (relative to the beam axis) of $\delta z \approx \frac{x_0^2}{2R(\lambda_0 + \Delta \lambda)} \leq \frac{1}{2\pi} \frac{x_0^2}{w_0^2}$. Indeed the lengths scales of the disorder from this scatterer will be approximately $\lambda f/x_0$. It is worth examining the strength of the interference as a function of scatterer location. As is shown in Ref. [19], the strength of the interference (assuming a Gaussian spectrum) will scale as

$$\text{Re}[y(\tau)] \propto e^{-\left(\frac{\pi c \Delta \lambda}{\lambda_0^2 \sqrt{\ln 2}} \right)^2},$$

(3.27)

where $\tau$ is the time delay of the scattered beam, $c$ is the speed of light, $\lambda_0$ is the central wavelength of the spectrum, and $\Delta \lambda$ is the -3dB width of the spectrum. Setting $\tau = \delta z/c$ yields the
Figure 3.5: Imaging of a point source which reveals contributions to disorder from off-axis scatterers.

The positional dependence of the interference.

\[
\text{Re} \left[ \gamma(z_0, x_0) \right] \propto e^{-\frac{1}{4} \left( \frac{\pi \Delta \lambda}{\lambda_0 \sqrt{\ln 2}} \right)^2 \left( \frac{\lambda_0 f}{d} \right)^4 \left( \frac{(v_0)^2}{(v_0)^2 + (2\beta)^2} \right)^2} 
\approx e^{-\frac{1}{4} \left( \frac{\pi \Delta \lambda}{\lambda_0 \sqrt{\ln 2}} \right)^2 \left( \frac{\lambda_0 f}{d} \right)^4 \left( \frac{(v_0)^2}{(v_0)^2 + (2\beta)^2} \right)^2},
\]

where in the second part of the equation the position has been replaced by the equivalent length scale \( d \) produced by the scatterer. Note that this assumes \( \Delta \lambda \ll \lambda_0 \). Equation (3.28) reveals that shorter length scales are strongly suppressed. Of course, increasing the spectral width helps to reduce interference. At places where the wavefront curvature of the Gaussian is small, the benefit of temporally incoherent light is reduced. This suggests that incoherent light reduces disorder as long as all scatterers are within a few Rayleigh ranges from the focus of the Gaussian beam.

The major improvement comes from double reflections from optical surfaces. In this case, the light has traveled sufficient distance to be outside the coherence length and the beams no longer interfere. This is also useful for suppressing disorder from scatter off of the vacuum chamber. Outside the coherence length, the interference fringes quickly average out. Just to be
Figure 3.6: Two possible lens positions for creating the same final waist size at the atom location. One has a longer focal length lens and the other shorter. The longer focal length lens is less sensitive to disorder.

pedantic, the electric field of a lattice and the scatterer will take the approximate form

$$E(x) = E_0 e^{ikx} + E_0 e^{-ikx} + \delta E_0 e^{-ik'x+ip}$$

(3.29)

where the $\varphi$ term is time dependent and averages terms to 0. The resulting intensity will scale as

$$\langle I(x) \rangle \propto 4E_0^2 \cos^2(kx) + \delta E_0^2 + 2 \left( \langle E_0 e^{ikx} + E_0 e^{-ikx} \rangle \delta E_0 e^{-ik'x+ip} \right)$$

(3.30)

where $\langle \rangle$ denotes time averaging and $\langle e^{i\varphi} \rangle = 0$. Again, relating the perturbation to the primary electric field, $\delta E_0 = \varepsilon E_0$, shows that the perturbation only changes the local energy by $\mathcal{O}(\varepsilon^2)$. 

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3.6 Minimizing disorder

All of the relations in Section 3.4 suggest three inequalities, which when satisfied should produce a system with minimal disorder:

\[ f \ll z_R \]
\[ |z_0| < z_R \] (3.31)
\[ \delta r_0 \ll w_0. \]

If the focal length is much less than the Rayleigh range of the incident beam, all potential scatterers/optics are within a Rayleigh range of the focus of the incident beam (before the focusing lens), and the scatterers are sufficiently small (smaller than the incident beam waist), then disorder should not exist. In addition, it is required to have all optical surfaces after the final lens be as far away from the final focus as possible - preferably more than a Rayleigh range. Numerical simulations verify minimal disorder when these relations are satisfied and one example of those simulations can be found in Figure 3.7. In this figure two configurations are shown: one satisfying the inequalities in Eq. 3.31 and the other not. This is achieved by choosing the focal length and beam size to produce the same final waist, but with different configurations as is shown in Figure 3.6. In addition, the scatterers are identical and are placed at the location where the two beam sizes are identical. Significant differences in the level of disorder can be seen. It is a good idea to put an iris or aperture in the Fourier plane to help remove stray light that could cause disorder.

A cavity lattice will also show very little disorder since the cavity should mode filter disorder from before the cavity and the remaining scattering from the mirrors should be the dominant source; this is in the regime of section 3.4.1, which is less sensitive to scatterers. Two possible configurations for generating the same final waist are shown in Figure 3.6. One
Figure 3.7: Two possible configurations producing very different levels of disorder. In one case the inequalities in Eq. 3.31 are satisfied (long focal length) and very little disorder is present. In the other case the inequalities are not satisfied.

where the Rayleigh range before the final lens is longer than the focal length of the lens, and one shorter. Scatterers located at the same position on the far left of the figure will produce significantly more disorder for the shorter focal length lens. In addition, the shorter focal length lens will no longer be far from the final focus, so scatterers on the lens itself become important. A final comment, projecting a lattice will almost inevitably be in the worst case where disorder is maximal. This method of lattice generation should be avoided unless absolutely necessary or the system parameters happen to satisfy the relations in Eq. 3.31.
Fast Imaging

A variety of methods for imaging in 2D array of atoms in a lattice already exist. These methods range from Raman sideband cooling [27, 87], imaging the atoms in a deep lattice [92], and optical molasses [11]. While these methods are in principle applicable to Erbium, the properties of Erbium suggest a more simple approach. Using the broad 401 nm transition in Erbium with a line-width of approximately 30 MHz, it is possible to scatter around $94 \times 10^6$ photons/s. In a few microseconds it is possible to collect a small, but significant number of photons from the atoms. Erbium also benefits from a large atomic mass, which reduces the recoil velocity from a photon, and thereby, reduces the random walk distance the atom will experience. $^6$Li, for example, would experience a 30 times increase in distance walked for the same number of scattered photons at 401 nm. In the case of two orthogonal counter-propagating beams in the plane of the atoms, which completely saturate the transition, the Erbium atoms will experience the random walk in Figure 4.1. It is easy to see that an atom will leave the confines of the lattice site of a 532 nm lattice in a little more than 2 $\mu$s of imaging. At this point, it can become nearly impossible to distinguish where an atom was originally located, thus reducing the fidelity of the imaging scheme. However, as will be shown in Section 4.3, collecting a few
Figure 4.1: Random walk distance in the 2D plane of the imaging system and vertically as a function of average number of scattered photons. The widths of lattice sites (relative to the initial atom location) generated by a 532 nm and 1064 nm laser are added for reference.

photons is sufficient for high fidelity imaging.

4.1 Effective Quantum Efficiency

There are a variety of imaging camera options available, all with their own strengths and weaknesses. The most widely used is the Electron Multiplication Charge-Coupled Device (EMCCD), which has high quantum efficiency and low readout noise, but suffers from low frame rates, multiplicative noise\(^1\), low pixel count. Another option is a Scientific CMOS (sCMOS) camera. These typically have lower quantum efficiency than the EMCCD, but faster frame rates and higher dynamic range. Another option, is the Intensified CCD (iCCD). The quantum efficiency of these cameras is substantially lower, but allows for fast gating.

In the case of the low photon count expected for fast imaging, the EMCCD can be beneficial due to the low effective readout noise. The typical quantum efficiency for 401nm is around 72 percent.\(^1\) This quantum efficiency is effectively reduced by the multiplicative noise

\(^1\)Oxford Instruments Andor iXon897 EXF
of \( f = 1.41 \) from the amplification process in EMCCD cameras.

\[
Q_{\text{EMCCD}} \approx \frac{Q_E}{f} \tag{4.1}
\]

The effective quantum efficiency can be increased by using the EMCCD in a photon counting mode. The idea here is to remove multiplicative noise by having a low photon density and asking the question: is there a photon on the pixel or not? By not trying to distinguish the number of photons, the multiplicative noise is reduced to \( f = 1 \) in the case of low photon density. In this case, the effective quantum efficiency approaches the bare value, \( Q_E \).

A subtly to photon counting is the quantum efficiency now depends on how a photon is counted. In order to more consistently distinguish photon counts from noise, the threshold for electron counts is increased. This causes the counting to have fewer false positives, but at the expense of true photons not being detected, which causes a reduction in effective quantum efficiency.

At the low photon densities used in photon counting, Clock Induced Charge (CIC)\(^2\) becomes another noise source and can effectively decrease the quantum efficiency. In order to compensate any decrease in quantum efficiency it is necessary to scatter more photons to recover the original signal to noise - clearly this is damaging when trying to combat the random walk from scattering.

Other losses in the imaging system effectively reduce the quantum efficiency of the camera. The primary source of lost photons is the collection of only a small fraction of scattered photons from the objective numerical aperture (NA). The percentage of collected photons can be

\(^2\)Specified CIC in EMCCDs can be a misleading quantity, since it depends strongly on the company’s choice for threshold for counting a photon. Depending on the design of the system it can be advantageous to increase the effect of CIC to increase the effective quantum efficiency. For example, an Andor iXon 897 EMCCD detects a maximum of 90% of photons-electrons at the expense of a CIC of 0.016 events/pixel - the specified CIC is 0.0018 events/pixel, which corresponds to a detection probability of 40%.
found by integrating over the correct region on a sphere:

\[ \alpha_{\text{collected}} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\arcsin NA} \sin \phi d\phi d\theta = \frac{1}{2} \left( 1 - \sqrt{1 - NA^2} \right). \]  

(4.2)

As one would expect, the collected photon number is reduced by at least a factor of 2. In fact, even at the large NA of 0.85, only 23.7% of the scattered photons are collected by the objective. Another loss which leads to an effective reduction in quantum efficiency is other imaging system losses. It is reasonable to lose 20% of photons through the imaging system - every surface is likely to lose around 1 percent. This does not include losses from commercially available achromatic lenses with glasses that tend to absorb near UV wavelengths. If the imaging system loss is characterized by the parameter \( T_{\text{img}} \), the final effective quantum efficiency is reduced to:

\[ QE_{\text{effective}} \approx QE \frac{\alpha_{\text{collected}} T_{\text{img}}}{T_{\text{img}}}. \]  

(4.3)

Using the bare quantum efficiency of 72%, 80% imaging system transmission, 0.85 NA, and no photon counting, the effective quantum efficiency is reduced to approximately 10%. This estimate has neglected other noise sources in the camera which reduce the effective quantum efficiency. This number can be used to estimate the average number of detected photo-electrons for imaging around 2 \( \mu \)s of approximately 19 photo-electrons.

4.2 Fidelity

Imaging fidelity can be difficult to define in a general way for all systems, since it depends on the goal of the experiment. For the purpose of this thesis, the probability/fidelity for correctly identifying two cases will be shown - identifying an atom (or empty site) on the central
Figure 4.2: Example configuration of atoms.

site for the same configuration of surrounding atoms. An example of these configurations can be seen in Figure 4.2. The reason this is chosen, rather than a single number, is it captures the differences between the cases and shows the effects of false negatives or positives on imaging fidelity. In addition, there can be a misleading enhancement in imaging fidelity by considering filling fraction. Consider the case where an atom is missing in a background of filled sites. The algorithm can be tuned to correctly identify all filled sites correctly, but can miss the empty site without effecting fidelity too much. This might be misleading as detecting empty sites might be more important.

4.3 Simulation

To simulate the imaging system, most sources of loss and noise in the system were considered. The simulation works by following the atoms in momentum and position space as they stochastically scatter photons. Atom loss to meta-stable states is also added. The loss from the NA and imaging system are also stochastic. The point spread function (PSF) is simulated by generating a distribution consistent with the aberration of the imaging system, which al-
Figure 4.3: SIMULATED image of a Mott insulating state using the described imaging technique. There are approximately 400 atoms with 1.7 μs imaging time, 0.85 NA, a quantum efficiency of 72%, CIC of 0.016 events/pixel, 80% imaging system transmission, a lattice spacing of 266 nm, a magnification of 500, and a camera pixel size of 16 microns. Green lines representing lattice sites are included for visual reference.

To increase the speed of the simulation, $5 \times 10^6$ photon positions were pre-sampled with the Metropolis-Hastings algorithm. Those photon positions were randomly sampled from to generate the images. The simulation was run 800 times with a specified configuration. Note that the lattice spacing in all simulations presented here is 266 nm - increased lattice spacing yields significantly higher fidelity. The collected photo-electron numbers on each lattice site were evaluated. A deconvolution algorithm was also tried, and showed some improvement,
but the results here will not include deconvolution. A threshold was set and the number of sites the algorithm identified having an atom (no atom) were counted and compared to the number of sites that actually have an atom (no atom).

In photon counting mode having too few pixels for a lattice site results in the reduction of measured counts - the photon density per pixel needs to be much less than 1. A simulation of counted photo-electrons as a function of magnification is shown in Figure 4.4. The theoretical maximum shown considers all sources of photon reduction on the lattice site. The results in Figure 4.4 suggest a magnification of 500 is suitable for imaging these low photon numbers.

The results of a few simulations are shown in Figures 4.5, 4.6, 4.7. In Figure 4.5 the case of mostly empty sites is considered. What should be evident is the ability to identify correctly an empty site from an occupied site depends on the configuration of sites around the atom.

The peak fidelity (assuming a filled site is as important as an empty site) in this case is approximately 99%. However, choosing the best threshold for identifying atoms in the top figure bar incorrectly identifies empty sites in the bottom configuration. By choosing an algorithm with a fixed threshold for identifying sites, the maximum possible fidelity for a given filling
Figure 4.5: Simulation of mostly empty sites with a 266 nm lattice spacing. The case of a fully empty background is considered in the top panel and the case of one filled site next to the center is shown. The configurations are shown on the left, the binned photo-electron counts are in the center, and the fidelity as a function of threshold is on the right. Black dashed lines are added to the fidelity plots to show the point at which the fidelity of identifying an empty site is the same as a filled site. Grey dashed lines are extended to both the upper and lower plot as a reference.

fraction is reduced. Simulations suggest the peak fidelity for a fixed threshold is around 96% for a filling fraction of 1/9. Similar, but more dramatic differences can be seen in Figure 4.6.

Finally, the case of nearly full filling can be seen in Figure 4.7.

The results of the simulation suggest it is useful to adapt the threshold to the specific configuration of atoms surrounding the site. Initial results from the adaptive threshold method showed similar improvements to the deep learning method [90], but with a more clear understanding of what is happening behind the scenes. Further investigation into these methods were put aside, since the experiment is implementing an accordion lattice for imaging, which allows imaging of spin states and also increases the imaging fidelity to above 99.9 percent in
Figure 4.6: Simulation of approximately half filled sites with a 266 nm lattice spacing. The case of a fully empty background is considered in the top panel and the case of one filled site next to the center is shown. The configurations are shown on the left, the binned photo-electron counts are in the center, and the fidelity as a function of threshold is on the right. Black dashed lines are added to the fidelity plots to show the point at which the fidelity of identifying an empty site is the same as a filled site. Grey dashed lines are extended to both the upper and lower plot as a reference.
Figure 4.7: Simulation of mostly filled sites with a 266 nm lattice spacing. The case of a fully empty background is considered in the top panel and the case of one filled site next to the center is shown. The configurations are shown on the left, the binned photo-electron counts are in the center, and the fidelity as a function of threshold is on the right. Black dashed lines are added to the fidelity plots to show the point at which the fidelity of identifying and empty site is the same as a filled site. Grey dashed lines are extended to both the upper and lower plot as a reference.
all configurations. However, these algorithmic methods may prove useful for initial experiments.
Experimental setup

The experimental setup has been quite involved in development. A significant amount of time went into developing systems which are not used on the experiment yet, but will be in the near future.

One important aspect to the experiment is turn-key operation. In general, when systems become so complicated, it is necessary to have reliability in the system behavior. Poor design or bad luck can lead to daily (sometimes even hourly) system maintenance to keep the machine running. When a large number of systems work together, the probability of failure increases and “cutting corners” in the design/construction can lead to an unreliable system. This philosophy has guided the development of the systems on the experiment and the experiment has benefited from it.

The room housing the experiment was still under construction while the experimental direction was being developed. The lab space itself is extremely advanced. It has a separate control area where the control computers are located, which allows for a safer work space and a more stable experiment. The experiment room has room for two 5’x10’ optical tables. Above the optical tables are clouds constructed of 80-20 and have temperature and humidity regulated
Figure 5.1: Optical tables when first installed in lab with 80-20 clouds above each table. The experiment is on the table further back and most laser setups exist on the closer table.

laminar flow through multiple HEPA filters. There is room for storage of devices above the optical tables. There is plenty of counter and cabinet space for storage of equipment. A back room exists for housing high heat load and noisy electronics. The ceiling has lower pressure than the room, which draws air from the room into the ceiling. The backroom is cooled by air-flow into the ceiling. By careful placement of vents the heat from a variety of devices can be removed.

5.1 Vacuum chamber

The vacuum chamber currently consists of an oven section, a 2D molasses section, a Zeeman slower, a Magneto Optical Trap (MOT) and Science chamber, and a Ti-Sublimation Pump. There are plans to build another section on this chamber to transport atoms to another Science
Figure 5.2: Image of vacuum chamber with a few optics. On the left is the 2D Molasses section followed to the right by the Zeeman slower, the MOT Chamber, and the Ti-Sublimation section.

chamber where there is more optical access.

5.1.1 Oven

Due to Erbium’s low vapor pressure at room temperature, it is necessary to heat the Erbium to approximately 1100°C to get a flux of atoms. The oven we chose was based on Professor Francesca Ferlaino’s Group’s recommendation. The oven consists of a Tantalum crucible inside a vacuum housing. The crucible is surrounded by heating filaments and water cooling lines. The crucible consists of two sections - a so called Effusion Cell and a “Hot-lips” section. The majority of the Erbium metal sits in the Effusion cell section, which is separated from the “Hot-lips” by a tube with a small aperture. The tube is intended to help collimate the atom beam. The effusion cell section is operated at 1100°C and the “Hot-lips” at 1200°C. The temperature difference is intended to prevent build-up of Erbium on output of the crucible.

1 CreaTec Fischer Co. GmbH. Dual-Filament Effusion Cell DFC-40-10-WK-2B
2 CreaTec Ta-10-2B
One problem operating at such high temperatures is degradation of the filaments. The oven was initially operated at 1200C (1300C) for the Effusion cell (“Hot-lips”). This was useful in finding the initial signals for the transitions and trapping of the atoms. Eventually the oven temperature was increased to 1300C (1400C) for the Effusion cell (“Hot-lips”) due to reduced atom flux. After operating at this temperature for a couple of months, but not every day, the power requirements to the filaments started to increase. What was happening was the heating filaments were reacting with the Erbium and started to crystallize. Eventually, the PID loop controlling the oven became unstable and started to oscillate wildly. This prompted the return of the oven for repair.

After opening the oven section it was obvious that the reason the oven temperature needed to be increased was a build-up of Erbium on the crucible aperture. A second oven and multiple crucibles were purchased in case this happens again. Luckily, under the current operating conditions of the experiment, it is possible to run the oven at 1000C (1100C) for the Effusion cell (“Hot-lips”). This will help to extend the lifetime of the oven and reduce the rate of refilling the oven.

The oven controller has an Uninterruptible Power Supply (UPS) to prevent sudden cooling in the event of a power failure. The oven is also water-cooled using chemically treated water from the lab water system. (See Section 5.2) There is an interlock system (Fig. 5.5 and Section 5.2.1) in place which is intended to shut down the oven in a controlled manner in the event of a power failure or flow reduction. The flow rates of the water are monitored and if the flow rate drops below a set threshold, the oven controller is set to ramp the oven temperature down at a reasonable rate. Of course, this means if water stops flowing, the water in the lines near the oven will evaporate. This has happened before and caused no serious problems.
**Figure 5.3**: CAD rendering of Molasses Section. Oven to the left followed by the octagonal chamber and multiple Ion pumps.

5.1.2 2D Molasses Section

The 2D Molasses section (see Fig. 5.3) consists of an octagonal vacuum chamber with 6 of the 8 viewports with windows. The bottom viewport has a blank on it and the top connects to an Ion pump. A second Ion pump is installed to help maintain good vacuum. The vacuum in this section readily reaches $10^{-11}$ Torr. This section is separated from the MOT chamber by the Zeeman slower which consists of a long tube (approximately 400mm long, 8mm in diameter). This tube acts as a differential pump preventing pressure increases in the MOT chamber if the pressure were to go up in the Molasses section. This section is also separated from the MOT chamber by a VAT all-metal gate valve. This valve can be closed while the Erbium reservoir in the oven is being replenished. An additional Viton sealed gate valve can be closed during oven exchange.
Figure 5.4: CAD rendering of MOT Chamber. This chamber is made entirely of 316LN Stainless Steel. The steel was annealed after to reduce magnetism caused by welding and cold-working the steel.

5.1.3 MOT Chamber

The MOT chamber is made entirely of 316LN stainless steel and can be seen in Figure 5.4. The chamber has four large CF63 view-ports at an angle to prevent issues from double reflections. The top and the bottom have CF100 flanges for mounting larger view-ports. Two additional CF40 flanges for mounting additional windows or an additional chamber. The MOT chamber is connected to a Ti-Sublimation chamber for further reducing pressure.

5.1.4 Ti-Sublimation Pump

The titanium sublimation pump is connected directly to the MOT chamber to have a larger conductance. When used in conjunction with the Gamma Vacuum Titan Ion Pump connected
to the system, pressures of $10^{-11}$ Torr are achievable. There is a shutter to cover the entrance window for the Zeeman slowing beam during firing of the Ti-Sub. It is extremely important to close this shutter before firing since the Titanium will coat the window. Additionally, a shield is installed in the Ti-Sub chamber to prevent Titanium from coating things inside the MOT chamber.

5.2 Water Cooling System

The water cooling system implemented in this experiment is quite advanced for an experiment of this size. A 150 gallon reservoir feeds an OptiTemp chiller. A series of filters and a mixing tank provides extremely stable temperature and water flow. The water temperature is set to 70F to avoid being near the dew point in the lab. A series of three distribution manifolds are installed in the experiment.

A variety of devices need to be water cooled in the experiment. All RF amplifiers in the experiment are water-cooled. Two distribution boxes monitor the temperature and flow rate going to critical components of the experiment: all coils and RF amplifiers. In the event of too large of a temperature rise or reduction in flow below a set value, the boxes will trip the main interlock of our system, which will shut down power to all water-cooled devices.

5.2.1 Oven Water Interlock

An interlock is installed on the oven to prevent overheating in the event of a water-cooling failure. The interlock is composed of an Arduino Mega\textsuperscript{3} which measures the flow rate of water into the oven (see Figure 5.5) with two flow meters.\textsuperscript{4} The oven controller is instructed to

\textsuperscript{3}An Arduino was chosen to reduce the learning curve with programming and replacing the microcontroller. The Arduino Mega has been running reliably, without being shut down, for more than two years.

\textsuperscript{4}Digmesa 938-1556/01
Figure 5.5: Water system for oven interlock. The flows are monitored by two flow meters (FM) on each line. A specially designed circuit monitors the flow and shuts down the oven in a controlled manner if the flow rate drops below a specific value.

turn off by two TTLs. The ramp-down is controlled at 25 Celcius/min. The Arduino also monitors the state of two switches which instruct the oven to shut-down or go to a standby mode where the temperatures are reduced to 500C for the effusion cell and 800C for the "hot-lips.” The oven is set to standby mode overnight, and is very rarely ever shut completely down.

5.3 Magnetic Coils

5.3.1 Earth Compensation Coils

Compensation coils for earths field are built around the entire experiment. They are installed on the 80-20 frame surrounding the experiment. The coils for the atom-beam direction have 45 turns each, the coils for the other two directions have 25 turns each. These coils with active stabilization should be able to cancel the field to below mG.

5.3.2 Zeeman Slower Coils

The Zeeman slower is a duplicate of the Zeeman slower used in Francesca Ferlaino’s Erbium/Dysprosium experiment - the coils were wound in Innsbruck. The original design of the Zeeman slower can be found in Albert Frisch’s thesis [42].

Since the experiment is designed to have the MOT function as a science chamber, heat load is extremely important as it causes thermal gradients across the chamber and changes in the
chamber size. The coil closest to the chamber is capable of raising the flange on the chamber to 30-40 Celsius above room temperature. To combat these effects, a series of refrigerator coils were installed (see Figure 5.6) to remove the need for coils near the chamber. The coils were wrapped around the already installed Zeeman slower. They are insulated from the chamber with custom PTFE mounts and are also water-cooled. The MOT loading was re-optimized with these coils and showed no change in loading rate.

5.3.3 MOT and Feshbach Coils

The MOT coils and Feshbach coils are integrated into one water-cooled mount. (See Figure 5.7) The coils are approximately in a Helmholtz configuration. There are three coil pairs on the mount. One is for generating the MOT gradient. It generates 0.493 G/cm/A at the center of the chamber. There is the “Big Feshbach” coils produce 3.21 G/A and are capable of generating 500G fields. Finally, a smaller coil pair, “Small Feshbach”, generate 0.748 G/A. The “Small Feshbach” is used for fast, precise, control of the field, which can prove useful when
turning the coils on quickly, jumping through Feshbach resonances or fine control of the field.

5.3.4 Refrigerator Coils

A series of refrigerator coils are installed around the chamber. The refrigerator coils consist of hollow copper tubing with water flowing in the tubing. There are two coil pairs for producing fields along the atom beam direction and orthogonal to it. The coils have 8 turns each and are connected permanently to the lab water cooling system. The low turn count requires larger currents, but the coils do not heat the chamber. The coils along the beam axis have been used to compensate the magnetic field of the Zeeman slower at the MOT location. The coils are connected with an IGBT for switching between running the same current through each coil or driving each coil separately. Most of the time the coils are run separately. The coils orthogonal to the atom beam axis are run from the same supply and are used for imaging. During absorption imaging the coils are quickly turned on to put the quantization axis along the imag-
ing beam.

It would be a simple matter to control the refrigerator coils and Feshbach coils to create an arbitrary field rotation. This could be useful for defining the direction of the dipoles in the optical lattice or fine tuning earths field cancellation.

5.3.5 Stern-Gerlach Coils

In vacuum coils (Fig. 5.8) were designed and built for generating fast magnetic field gradients. The idea here is to use an accordion lattice in conjunction with a large magnetic field gradient to separate out all spin states for imaging. The requirements for this to occur are magnetic fields of approximately 100 G/cm pulsing on in a few hundred microseconds. To achieve fast rise times, the coils were designed with a single turn. The leads are designed to cancel any stray fields from the leads at the atom location. In practice, the coils, when combined with a driver consisting of capacitors, is able to pulse 4000A in a few hundred microseconds - enough to see spin separation within the lattice. The coils were placed in vacuum to avoid the eddy currents induced in the copper gaskets sealing the chamber. These coils are currently used for Stern-Gerlach imaging of the BEC and the MOT and all 13 spin states have been resolved in a 15uK MOT. These coils could also be used for bringing an RF field into the chamber. The polarization of this RF field will be along the vertical direction, but with rotation of the quantization axis, the RF can couple different magnetic sub-levels.

5.4 Control System

The control system used in the experiment is the same as was originally developed for the Lithium experiment in the Greiner Lab \[73, 95\]. It is an FPGA based system, which allows for control and measurement of the experiment with a time resolution of 100 ns. The system uses
Figure 5.8: In vacuum coil system for generating large magnetic field gradients in a short time.
two main boards - a DDS board and a general output board.

The DDS board is called the Symphonia board. It has 4 DDS outputs with output up to 7 dBm and a frequency range of 10 MHz to 400 MHz. There are 6 Analog to Digital Converters (ADC) and 2 Digital to Analog Converters (DAC). Four of the 6 ADCs are used for PI control of the outputs of the DDS. This is useful for intensity stabilization of most laser systems. The other ADCs and DACs are not used in the experiment.

The general board is called the Master Spy board. This is the most versatile board and allows measurement and control of a variety of devices. The devices (known as daughter boards) are connected to the Master Spy board through ethernet cables. The communication is a Serial Peripheral Interface (SPI) communication over 3 Low-Voltage Differential Signal (LVDS) pairs. The power to the boards is carried through other wires in the ethernet cables. The outputs/inputs are galvanically isolated from the Master Spy boards by dedicated chips, which prevents ground loops from forming when connecting systems together. A variety of daughter boards exist, but the most widely used devices are TTLs, DACs and ADCs.

One of the main benefits of this system is it is not a state machine. Many aspects of the system are run in a state machine configuration, but it is possible to build feedback systems using information measured from the experiment. Several PID loops have been implemented with the FPGA system and it is possible to measure the values on an ADC and save them in the experimental results.

In addition, the control software known as Blastia has been developed to be object-oriented, which allows for ease in implementing new systems. Although, this flexibility comes at the expense of a steep learning curve.

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5 Analog Devices ADUM****

6 A finite state machine in this context is a series of pre-programmed states which are switched between upon a trigger or clock signal.
Trigger and Clock Generation

The trigger for the entire experiment is generated using an FPGA. The FPGA in the experiment is software triggered, which is then converted to a TTL output from the FPGA. The TTL triggers a function generator to generate a single pulse. This pulse is then sent to all FPGAs in the system along equal length BNC cables, which guarantees timing. All FPGAs are synced to better than 10 ns, which is acceptable for this experiment. If more triggers are needed, it may be necessary to amplify the output of the function generator. The clock for the experiment is generated using a Stanford Research Systems Rubidium frequency standard. This provides stable 10MHz for all FPGAs to run on.

5.5 Lasers

5.5.1 401 nm Laser

The 401 nm laser system consists of an M2 Solstis Ti-Sapphire laser. The Solstis is pumped with a Sprout G-18. Pumping with the full 18W we were able to achieve approximately 8W at 802 nm, which is near the peak in the gain spectrum of the Ti-Sapphire laser. From this approximately 250mW is picked off for monitoring the wavelength and locking to the main ULE cavity. The remainder is then frequency doubled in the M2 ECDX doubling cavity. The system originally produced 3W at 401 nm, but after some time it was only able to maintain 1.8-2W output at 401 nm. This is sufficient power for the system.

This laser system has proven to be the most difficult in the experiment. The main problem is degradation of components due to exposure to near-UV wavelengths. It is worth describing in some detail our experience with this system. A diagram of the laser setup can be found in

7SRS FS725
• **Pump Laser:** The pump laser for this system has been extremely problematic. When the M2 was first purchased it was pumped with an older Verdi V18. The problem with this was the laser had aged to the point where there were occasionally mode hops. In principle this should not affect the output of the Ti-Sapphire laser, but the accompanying intensity fluctuation perturbed the Solstis etalon lock. This would cause the Solstis to jump frequency (usually several GHz) at unpredictable times. Due to the cost of replacing the Verdi, a Sprout G-18 laser was purchased. At first this laser solved the problem and behaved quite well for around one year. After one year the Sprout started experiencing some form of degradation. The main sign for this was the inability for the Sprout to reach 18W - it would reach around 15W. This would be fine for our system, other than a reduced power at 401 nm, if the drop in power and degradation didn’t result in the Sprout becoming more noisy. It started causing the same etalon locking issues that we experienced before with the Verdi. This was fixed temporarily by Lighthouse Photonics re-optimizing the system remotely, but it only lasted a few weeks. The laser finally had to be sent in for repairs. These repairs lasted for around another year. The Sprout is currently degrading again and will have to be sent in again for repairs. It seems that this will occur regularly with the Sprout G-18.

• **M2 Solstis:** Overall, other than pump issues, the Solstis behaves as expected. The main annoyance with this laser is the long warm-up times. The Solstis takes approximately 2 hours to settle after the pump has been turned on. If the laser is locked two hours after startup, the ULE locking system and etalon locking range will keep the system locked for approximately 24 hours. At this point the system will drift too far and the laser will need to be re-locked. If the system is locked within the first few hours, it will need to
be re-locked often until the system has settled.

- **M2 ECDX:** The M2 ECDX is a fairly well behaved doubling system. It is sensitive to perturbations and can lose lock if there is a small rap on the table, but it stays locked as long as the frequency of the Solstis is stable. Occasionally (usually every 6 months) there will be a power drop which is often due to a dust particle landing on the doubling crystal face. This can be remedied by carefully cleaning the optical surfaces of the crystal, as well as the mirrors, with HPLC grade methanol. The power is usually completely recovered with this process. The cavity has needed realignment a few times, but this is a relatively simple process.

- **Beamsplitters:** 401 nm light is strongly absorbed by many standard optics which might be used at near-IR wavelengths. The beamsplitters originally purchased were from Red Optronics and were epoxied. The 401 nm light would burn a spot on the epoxy every few weeks and the setup had to be realigned. Optically contacted, fused silica cubes were purchased from Spectral Optics. The cubes were replaced and have no degradation has been seen even in high intensity locations. In some locations with particularly high intensity, $\alpha$-BBO Glan Taylor polarizers.\(^8\) Again, there have been no issues with these polarizers.

- **Optical Fibers:** We had originally planed to avoid optical fibers altogether with the 401 nm light, but due to some issues having the apparatus and M2 system on the same table, the laser was moved to the laser table. In some ways this was a blessing in disguise and has freed up a lot of space for the apparatus. Originally all silica fibers\(^9\) were used but effects from Stimulated Brillouin Scattering (SBS) when sending higher pow-

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\(^{8}\)Thorlabs GLB10-405  
\(^{9}\)Thorlabs PM-S405-XP
ers through the fibers (>100mW) were observed. Not only this, but the fibers would degrade rather quickly, which could be attributed to the high intensity at the entrance face of the fiber. This was remedied by purchasing all silica fibers from Oz Optics Ltd. These fibers have an AR coated end-cap contacted to the fiber face which reduces the intensity on the glass-air interface. In addition, the fiber uses a triplet fiber collimator and an $f = -100$mm fused silica lens for coupling into the fiber. The focus position is adjustable through the fiber connector. With this combination we are able to achieve approximately 80 percent coupling efficiency when accounting for losses through all eight optical surfaces before the fiber. The output of the fibers are also sealed from dust. The collimators are sealed and assembled using Thorlabs components. The components of the collimators are cleaned for vacuum use before the fiber is inserted. With this it is possible to deliver more than 100mW of power through each fiber with no signs of degradation with over two years of use. The limits of the fiber are not pushed too much since many of the processes leading to degradation are exponential in intensity.

- **Tellurium Dioxide AOM:** Tellurium Dioxide AOMs were originally used in the 401nm setup, but had problems when putting relatively low power through the AOMs. With around 100mW of optical power focused to 100 microns in diameter through the AOM there was severe beam distortions due to absorption of the 401 nm light. Similar effects were seen when using 1mm diameter beams with 500mW of power. Typically the AOM absorbed around 5-10% of the light in each pass, which equated to a significant power lost. The Tellurium Dioxide would also degrade over time and the absorption would go up to 20-30 percent on each pass over time. This has been attributed to color.

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10Oz Optics QPMJ-A3HPCA3HPC-400-3/125-3A-10-1-AR2
11Thorlabs TC06APC-405
center formation in the glass. Two TeO2 AOMs are used in the 401nm setup, both on the spectroscopy channels. These require relatively low optical power, so smaller optical absorption effects are seen. A TeO2 AOM was originally used on the Zeeman slower channel, but beam pointing issues due to RF thermal effects proved to be too much. Fiber coupling efficiency after the AOM would start out significantly lower and settle after a few seconds at the aligned efficiency as the RF was turned on. (See Appendix [A])

- **Quartz AOM:** To fix this problem with absorption and beam pointing on the higher power channels, quartz AOMs from Brimrose\(^\text{12}\) were purchased. This remedied the absorption problems from TeO2, but introduced another. Quartz AOMs are more polarization sensitive and require a specific polarization to operate with high diffraction efficiency. This means making a double pass AOM (which we needed for the molasses channels) is not as easy as it is with TeO2 AOMs. A Faraday rotator before the AOM was tried first, but the Faraday rotator would also degrade. That being said, the intensity was above the damage threshold of the Faraday rotator. Finally, two AOMs are used in series to get the appropriate frequency shift. They are arranged in such a way as to cancel beam displacements when scanning the frequency. 700mW passes through the first AOM with a beam diameter of 0.5mm and there has been no damage over the last two years. The RF thermal effects in the Zeeman slower channel are no longer present.

- **Other optics:** All other optics in the system have been replaced with either quartz or fused silica. All mirrors are laser-line mirrors and wave-plates are quartz.

\(^{12}\)Brimrose CQMF-80-5-401, CQMF-230-100-401
Figure 5.9: Diagram of 401nm optical setup.
5.5.2 583 nm Laser

The 583 nm laser system consists of an 1166 nm Toptica DL Pro with narrow line-width option. This laser acts as a seed for an MPB Raman amplifier, which is then frequency doubled in a single pass to 583 nm. The Raman amplifier is seeded with approximately 50 mW of 1166 nm. The whole system can deliver up to 2W of 583 nm light, but we typically operate at around 800 mW. A part of the 1166 nm light is used for locking the DL pro to the Ultra-Low-Expansion (ULE) cavity. The fundamental is locked since Electo-Optic-Modulators (EOM) in the visible are known to degrade over time.

The 800 mW of 583 nm is divided up between four channels. This allows independent control of the optical power for each of the three Magneto Optical Trap (MOT) axes. In fact, due to the choice of control system, intensity control loops are implemented for all three axes as well as have full frequency control over approximately 60 MHz, or approximately \( 315 \Gamma_{583} \). A diagram of the setup can be found in Figure 5.10.

5.5.3 841 nm Laser

The 841 nm light is derived from a Toptica DL Pro with narrow line-width option. A 60 dB isolator is installed in the laser housing. This laser delivers around 75 mW of light after the isolator, but much less than this is needed since the saturation intensity of the 841 nm transition is small. Control of the laser light is similar to the 583 nm setup. There is a channel for each MOT axis and an additional for spectroscopy. A few hundred microWatts of light is taken for measuring the wavelength and around 250 \( \mu \)W of light is used for locking the laser to its own ULE cavity. A diagram of the 841 nm setup can be found in Figure 5.11.
Figure 5.10: Diagram of 583 nm optical setup including the 1166 nm seed for the MPB Raman amplifier.
Figure 5.11: 841 nm laser schematic.
5.5.4 Laser Frequency Locking

Laser frequency locking is achieved by locking the lasers to a ULE cavity using Pound-Drever-Hall (PDH) locking. In principle it is possible to lock the 401 nm laser directly to Erbium using modulation transfer spectroscopy, but due to Erbium’s low vapor pressure at room temperature it needs an oven to achieve enough atoms for locking. It is also possible to lock to a Hollow Cathode Lamp (HCL), but the only company that produced a HCL with the correct geometry to have an optical beam go through the gas stopped production. The other two transitions used need a cavity since the line-widths are significantly narrower and a good signal to noise would be nearly impossible. So, all lasers are locked directly to ULE cavities. Note the 401 nm and 583 nm transition share the same ULE cavity, and another cavity specifically for the 841 nm transition.

401 nm laser lock

PDH locking of 802 nm is achieved using an EO-Space fiber EOM and an ULE cavity. The EOM is driven with two summed frequencies, the offset frequency for shifting the laser frequency to the cavity resonance and the PDH frequency of 38MHz. The offset frequency is generated by a Rigol DSG815 and the PDH frequency by a Kiethley 3390. The error signal achieved is approximately 1 V peak-peak. Residual Amplitude Modulation (RAM) has been minimized by proper alignment of the polarization into the fiber EOM.

PID locking of the 401 nm laser is achieved using a spare Toptica FALC 110, but it may be exchanged at some point for a different system. Note the locking is done on the fundamental at 802 nm - this avoids issues with EOMs at near-UV wavelengths as well as unlocking of the doubling cavity. The fast path is used for controlling the fast piezo in the Solstis and gives us approximately 100kHz of bandwidth at 40 MHz of range. The unlimited integrator is used on
Figure 5.12: Schematic of the original ULE cavity optical setup including feedback for the main wavelengths being locked to the cavity. Four paths to the ULE cavity allow locking and frequency interrogation of various lasers in the lab. The 401 nm and 583 nm laser light are locked to the cavity by their fundamental wavelengths. Additionally, 841 nm can be put into the cavity as well as wavelengths below 800 nm. The 802 nm and 1166 nm wavelengths have sidebands imprinted by fiber EOMs.
the slow path and gives us approximately 25 GHz of range. The line-width of the ULE cavity at 802 nm is approximately 10 MHz. The line-width of the locked 802 nm light is approximately 100 kHz without much effort on the locking, which equates to 200 kHz at 401 nm. A diagram of the setup can be found in Figure 5.12.

583 nm laser lock

The fundamental of the 583 nm light is also locked. EO-Space fiber EOM is used for imprinting the frequencies on the beam. RAM is controlled by careful alignment of the polarization into the fiber EOM and does not present a limitation at this time. The same frequency generators are used as is used for the 401 nm laser. In this system 1.6 V peak-peak on the error signal is achieved. The PDH sidebands have a frequency of 4.9 MHz which is much larger than the ULE cavity line-width of 200 kHz. A Toptica FALC 110 is also used for the PID. The fast path output is used to directly drive the DC port of the DL Pro and the unlimited integrator is used to feed back on the piezo from the Toptica DLC. Using this setup a line-width of 10 kHz is easily achieved and line-widths of 1 kHz can be achieved with some work, which is much smaller than the atomic transition line-width.

After careful measurement the cavity drift appears to be approximately 5.6 kHz/day at this time, but it will change over time. This could easily be fed-forward on to keep the frequency stable over much longer periods of time.

841 nm laser lock

The 841 nm laser is locked to its own cavity. A separate cavity was purchased for this laser since the cavity line-width of the other ULE cavity at 841 nm is approximately 9 MHz and the line-width of the atomic transition is 7 kHz. Ideally the laser line-width should be much
smaller than 7 kHz. This makes the system more sensitive to RAM and other sources of drift. The cavity line-width of the new cavity is 30 kHz at 841 nm. The locking scheme is slightly different compared to the 583 nm laser. In this case a fiber EOM is not used for generating the PDH sidebands, but only for the cavity frequency offset. Instead, a free space EOM with a resonance frequency near 10 MHz. This EOM has an angle cut on the face which allows us to filter the orthogonal polarization to reduce RAM. In addition, there is less additional RAM from the slight polarization mismatch of the optical fibers and the crystal that is found in fiber EOMs. This EOM can easily be temperature stabilized to further reduce RAM. Finally, it has a DC port for further reduction of RAM if necessary. A setup of the 841 nm lock can be found in Figure 5.13.

The error signal is around 1.6 V peak-peak. The PID is also delivered by a Toptica FALC 110. By careful consideration of sources of phase delay and careful tuning, the bandwidth
of the system has been optimized to around 2-3 MHz. Since we do not have a second laser to beat this one against, we used the in loop noise spectrum to estimate the line-width of the laser. It appears to be on the order of a few hundred Hertz, but better estimates would need more concrete measurement of the line-width.

With a little more work this cavity could be used as a frequency reference for all experiments in the Greiner lab. In principle it should be possible to narrow the line-width to Hertz level. With feedforward techniques and repeated measurements, it should be possible to resolve the transition frequency of the 7 kHz line down to a few Hertz. This would provide the labs with a Hertz level optical frequency source for locking other optical cavities to or possibly a frequency comb.

Since the cavity is relatively new, it appears to be drifting around 31.5 kHz/day. This is not unusual for a new cavity as the ULE glass is still flowing into its steady state. The other ULE cavity is approximately 3 years old, so its drift has reduced significantly.

5.5.5 1064 nm Fiber Laser

We currently use an Azur Light Systems (ALS) 1064 nm fiber laser for evaporation. The system delivers 45 W after the built in internal isolator. The system is operated in a constant cur-
rent mode, since the internal power control PID adds noise at low frequency and has a substantial servo bump around 10 kHz. One strange thing was occasional increases in the intensity noise spectrum of the laser, which could not be explained. It would correspond to 10-20 dB increases in noise at random times if monitoring for longer periods. This does not effect the experiment because this laser is used solely for our dipole trap, which is relatively insensitive to the levels of noise from the laser. In addition, the laser power in the dipole trap has a PID which helps to lower the noise at relevant frequencies.

One serious drawback of this system is the beam pointing when the delivery fiber to the laser head is moved. Watching on a camera a few meters away large beam displacements were observed when the delivery fiber was touched - much more than ALS specified. To remedy this the ALS was mounted on a rack next to the table. The fiber was then surrounded by an aluminum tube and filled with foam insulation. The last 12 inches of the fiber is mounted securely to an ABS block, which was specially machined to secure it. (See Figure 5.14) The beam pointing is no longer sensitive to movement of the fiber, since only the first few inches of the fiber out of the laser rack can move.

As is shown in Figure 5.16, the output of the laser is then fiber coupled again. This was done to remove the problem of realignment of the Optical Dipole Trap (ODT) in case the fiber laser failed (this happened once already). A fiber coupling mount was designed (see Fig. 5.15)
to mount a photonic crystal fiber\textsuperscript{4}, which has allowed 26 W of optical power through the fiber, without any degradation, and a 90 percent coupling efficiency. An optical fiber of 2 m was chosen to avoid SBS at high optical powers. More power could, in principle, be delivered, but this has been limited by the available power from the laser. The fiber coupler consists of a copper block with a custom SMA fiber connector mount made from stainless steel. This fiber mount screws directly to the copper block. Copper was chosen for its mass and thermal conductivity, which together keep the fiber more stable. The copper block has an SM1 thread on the inside to accommodate a fused silica triplet lens.\textsuperscript{5} The lens position is adjusted by three screws in the back of the fiber coupling block and is locked in place by an SM1 retaining ring. It can be time consuming to get the initial position of the lens correct, but once it is properly placed the high coupling efficiency is permanently maintained. The temperature of the mode stripper is maintained by a peltier element. In addition, the cooling block for the mode stripper acts to stabilize the position of the fiber tip and remove stress from the SMA connector.

Additionally, a \(\lambda/2\) waveplate mounted to a galvo is used for fast control of power distribution as shown in Figure 5.16. A compensation plate is added after on a galvo to produce the exact opposite motion and reduce displacement of the beam. Fiber couplings are unaffected by this setup and power is able to hand-off between paths in 5-10 ms. The power control allows for reduction of the power to 1 part in 10000.

5.5.6 1550 nm Fiber Laser

An Erbium Doped Fiber amplifier will be used for trapping of atoms.\textsuperscript{13} This fiber laser delivers around 32 W of single frequency 1550 nm. The laser is seeded by an NKT Adjustik X15 laser and delivers low noise, narrow linewidth 1550 nm light. This laser has been tested on

\textsuperscript{14}NKT LMA-PM-15, 2-5 m long
\textsuperscript{15}OptoSigma HFTLSQ-15-20PF1
\textsuperscript{16}IPG EAR-30K-C-LP-SF
Figure 5.16: 1064 nm laser setup.

the atoms and shows significantly longer trap lifetimes than the 1064 nm laser - upwards of 40 seconds in some cases. This laser will be used for evaporation of the fermionic species of Erbium $^{5}$ as well as possibly an optical transport beam.

5.6 2D Molasses

The 2D molasses section is extremely important for increasing loading rate into the MOT. This section serves to cool the transverse velocity of the atom beam coming from the oven - effectively acting as a collimating lens for the atoms. Using the 401 nm transition, it is possible to quickly cool the transverse velocity distribution of the atom beam moving with approximately 400 m/s.

The molasses is derived from two high power optical fibers. (See section 5.5.1) A schematic of the setup can be found in Figure 5.17. After the collimating lens the beam size is 1.5 mm waist. The remaining optical power after the wave-plate, and polarizing beamsplitter is ap-
Figure 5.17: Diagram of 2D Molasses section including absorption spectroscopy setup.

approximately 80 mW on each line. It is necessary to put around 130 mW before the optical fiber to reach this power. A cylindrical telescope is used to expand the beam to 15 mm x 1.5 mm waists (30 mm x 3 mm diameters). The 15 mm direction of the beam is along the atom beam and the 1.5 mm direction is orthogonal to the beam. The purpose of this is to increase the interaction length of the atom beam and the optical beam. The 3mm waist is chosen to match the approximate atom beam diameter at the oven location. The maximum achievable saturation intensity with 80 mW of power is $3.6I_s$ per beam. A single quarter wave-plate is used after the chamber for the retro-reflection to prevent light going back through the fiber. In optimizing there were no effects from adding the standard quarter-waveplate before the chamber.

Careful alignment of these beams and the oven are critical for enhancing the loading rate into the MOT. Using a fluorescence signal in the MOT chamber, the bellows on the oven was
adjusted to maximize fluorescence intensity. After this, the detuning of the molasses light was set to $\Delta = -\Gamma/2$ and the mirrors and wave-plates were adjusted to increase the fluorescence. There was some coupling between the two molasses beams. This process was repeated several times until the atom-flux was maximized. After this process was finished, the final loading rate increase into the MOT due to the molasses section was 34.

5.7 Zeeman Slower

The Zeeman slower beam was optimized to operate at a total power of 50 mW before the quarter wave-plate. (See Figure 5.18) This beam is derived from a high power optical fiber. (See Figure 5.9) The beam is 8mm in diameter and slightly focused at the oven location.

Alignment of this beam was fairly difficult since there is no reference to look at. The beam was aligned as well as possible without an atom number signal. A probe beam was used to measure the velocity distribution of the atom beam in the MOT chamber. The probe beam was angled approximately 30 degrees from the atom beam. Calculated currents were used as a starting point for the Zeeman slower coils. The currents were adjusted to maximize the fluorescence of the probe beam, which was set at a frequency to measure around 50 m/s atoms. After the proper settings were found, the fluorescence was measured for various velocities by scanning the detuning of the probe beam - this effectively measured the velocity distribution of the atoms along the beam direction. Scans were taken while trying to bring the peak of the velocity distribution down to around 5-10 m/s. This allowed the initial settings for the MOT to be found. After the MOT was operating to the maximum achievable loading rates, the final beam alignment could be performed. This involved monitoring the loading rate of the MOT and adjusting the beam alignment and polarization. After repeated alignment and re-optimization of MOT parameters, the atom loading rate was increased by a factor of 4.
5.8 Yellow MOT

A schematic of the yellow MOT on the experiment can be found in Figure 5.18. The yellow MOT was originally setup as a non-scanning MOT. The beams were made to be approximately 2 inches in diameter. This MOT yielded a loading rate of $200 \times 10^6$ atoms/s. It was operated at an intensity of approximately $12I_s$ and a detuning of approximately -5 MHz.

Later the MOT was upgraded to include scanning [60, 76]. The scanning was implemented using an HP ESG-D3000A frequency generator at center frequency of 80 MHz, mixed with a 30 MHz signal from the DDS to produce the 110 MHz necessary to drive the AOMs. The HP was frequency modulated by a triangle wave produced external function generator at a modulation frequency of 190 kHz - this sets the spacing between the peaks in the spectrum. The frequency spectrum is broadened to by approximately 4 MHz. The intensity necessary to operate this MOT is around $50I_s$. The loading rate compared to the non-scanning MOT is approximately 2-3 times larger.

The intensity of the MOT is controlled by feedback. This was necessary to better stabilize the compressed MOT location and make the sequence more repeatable.

5.9 841 nm MOT

The 841 nm MOT is overlapped with the 583 nm MOT beams using dichroic mirrors. A schematic of the 841 nm MOT on the experiment can be found in Figure 5.18. The beams were made to be approximately 1 inch in diameter at the atom location - they don’t need to be larger since this MOT operates in a more compressed regime. Intensities of approximately $400I_s$ are achievable in this setup. The 583 nm and 841 nm MOT share custom waveplates\textsuperscript{[17]} made for $\lambda/4$ retardation at both wavelengths.

\textsuperscript{17}Knight Optical 2" $\lambda/4$ waveplates - designed for both 583 nm and 841 nm.

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5.10 Dipole trap

A schematic of the dipole trap used for BEC generation can be found in Figure 5.18. This dipole trap is no longer installed in the system, since the experiment is trying for a faster evaporation. On the experiment table an additional 110 MHz AOM was used to scan the dipole trap [91]. The frequency for this AOM was generated using an HP ESG-D3000A signal generator. By modulating the FM port of the signal generator with a Keithley function generator programmed for the correct ramp scanning was achieved. The amount of scanning was controlled by a DAC on the modulation port of the Keithley, which controlled the amplitude. A few groups have used the scanning method to achieve fast evaporation to BEC. The beam generated by the NKT FIBER 1 line (see Figure 5.18) had a final waist of around 18 microns in both directions. With scanning, the dipole trap could be increased to a waist of 108 microns by 18 microns. Tuning of the waist allows for better control of the trapping frequencies and the trap depth at the same time - a necessity for achieving runaway evaporation [54]. Up to 26 Watts could be delivered after the fiber, but with scanning, the optical power available at the atoms was reduced to approximately 14 Watts. The crossed direction had a trap waist of 80 microns in both directions. Up to 12 Watts of power could be delivered to the atoms with this setup.

5.11 Lattice

The lattice will be generated using a 532 nm laser. (See Section 5.13) A lot of work has gone into the design and implementation of the lattice, but it is not ready for public documentation. The lattice has been designed in such a way to prevent effects from disorder - an important aspect to exploring some of the physics with the dipole interaction. A super lattice with phase locked 1064 nm and 532 nm will also eventually be installed. This will allow efficient gen-
Figure 5.18: Diagram of experimental setup around the vacuum chamber.
eration of double wells [75, 83, 56, 17] for entropy re-distribution techniques [29] or other physics. The main science lattice will be retro-reflected from mirrors mounted directly to the chamber, which will help to prevent fluctuations due to differential movements. An additional two accordion lattices will be added for imaging and manipulation of atoms in the science lattice.

5.12 Accordion Lattice

In order to be able to compress the atoms into a 2D system and for loading into the vertical standing wave, a novel accordion lattice has been designed. Several groups have designed and implemented other designs [59, 106, 7, 73]. In addition, the lattice can be used as a primary lattice, which should allow for the high fidelity imaging of close spacing lattices. A concept drawing of the accordion lattice can be found in Figure 5.19. The centerpiece of this accordion lattice consists of a beamsplitter designed to take one input beam and separate into two parallel output beams. By adjusting the position of the input beam the output beams can be separated. A lens then takes the Fourier transform of this electric field and creates a fringe pattern with variable spacing.

A significant development effort went into designing the beamsplitter for this operation. It consists of two Thorlabs dove prisms.\textsuperscript{18} The doveprisms were then specially coated with 45

\textsuperscript{18}Thorlabs PS993
degree AR coats on the entrance faces and a coating designed for approximately 72 degrees angle of incidence. The beamsplitter is designed to be 50:50 with p-polarized light between 400 nm - 700 nm. The coating was specially designed to work with Norland Optics NOA61 UV cure adhesive. One critical point for this beamsplitter is it needs to be properly aligned. If the beamsplitter is not properly aligned, then the output beams will not intersect after the focusing lens. Additionally, the overlap of the two beams can change as the input beam is scanned, which can lead to heating or reduced trap depths.

Note that standard beamsplitters from Thorlabs or other optics companies are rarely aligned well enough to function in this accordion lattice, nor do the companies do not have the capabilities to align the beamsplitters better. To approach this, an alignment procedure for this beamsplitter was devised. The initial alignment consisted of a long focal length, 2 inch achromatic lens, \( f = 1000 \) mm. A large collimated 532 nm beam was used to find the exact focus of the lens. An aperture was then used to reduce the beam size to around 5 mm in diameter. A custom mount to hold the beamsplitter during alignment was made. A 5 axis stage allowed for angle alignment as well as positional alignment of the two prisms relative to each other. NOA61 epoxy was place between the prisms and the prisms were put together. The prims were assembled in a location such that the 5 mm beam entered one face of the prism and would be split into two beams. The beams then passed through the lens and were focused.
onto a camera at the location of the lens focus. Any misalignment of the beamsplitter is revealed by the relative location of the two beams to the exact focus position. By adjusting the angular alignment of the prisms, the two beams could be made to overlap. This produced interference fringes which made it slightly difficult to see the exact center. The beamsplitter is now approximately aligned, but it is possible to do much better. To do the final fine alignment a large mirror was placed in the beam path after the prism, but before the lens as shown in Figure 5.20. The green light was then turned off, since it was not needed for this part. As can be seen in Figure 5.20, the beamsplitter plus mirror looks like an interferometer. The mirror angle is aligned by carefully adjusting the angle while looking through the beamsplitter. Note the horizontal angle of the beamsplitter is not important for the alignment. By looking at a pattern through the beamsplitter, adjustment of the vertical position can be achieved by overlapping the images of the pattern from the two paths in the interferometer. Once this is aligned as well as possible, positional and angular adjustment of the beamsplitter can be made. Fine adjustments are made until white light fringes from the room light are seen in the interferometer as is shown in Figure 5.20. This is simply the instance where the path lengths are nearly perfectly aligned. Now, by repeatedly adjusting the position of the beamsplitter and the vertical angle of the mirror, the fringes are made as large as possible while maintaining the fringes parallel to the reflection plane. After acceptable alignment is achieved, high intensity UV is used to initially cure the epoxy. Once cured enough for removal from the mount, the prism undergoes a final cure in high intensity UV for 1 hour.

The coherence length of room light is quite short (a few microns) and requires extremely precise alignment of the beamsplitter. The quality of alignment is evident in almost no positional change of the two intersecting beams as the fringe spacing is changed. Images showing the accordion lattice in a test setup can be found in Figure 5.21. The phase of the lattice is tracked during expansion and contraction of the lattice and the relative movement of the
Figure 5.21: Example images of accordion lattice and corresponding phase of fringes as the fringe spacing is changed.

fringes in waves can be seen in Figure 5.21. The lattice while expanding over a factor of 20 in lattice spacing only shifts by 0.3 waves peak to peak. In principle, this fringe shift is entirely due to the surface quality of the beamsplitter and can be improved significantly by using custom higher quality dove prisms. The fringe stability at large lattice spacing was also sensitive to air fluctuations as can be seen in Figure 5.22, where a large reduction in fluctuation can be seen by reducing air currents. The phase of the lattice can be further stabilized with the introduction of a compensation plate mounted to a galvo in one of the beam paths. By adjusting the angle of the compensation plate, the path length can be modified to compensate out any phase shift. Using feedforward can remove shifts due to surface imperfections of the beamsplitter and feedback could be used for compensation of dynamically changing phase.

The design of this beamsplitter marks a significant step forward in the generation of optical accordion lattices. Absolute stability of this accordion is guaranteed by the monolithic design. In addition, this system would work for generation of an accordion lattice from highly incoherent light, which may prove useful for future experiments.
Figure 5.22: Fluctuations of fringe position with large lattice spacing with and without covers to reduce air currents.

5.13 Green Light Generation

The lattice system will require green light in order to achieve close spacing in the lattice. The requirements for such a lattice system are high power, frequency stable, low noise 532 nm light. Several commercial systems exist, but come at significant cost while not meeting all requirements for this system. Additionally, the green light will need to be phase locked to a 1064 nm source for super-lattice geometries.

Several reasons exist to develop a doubling system. For example, one upside to commercial systems is the ready-to-use nature of the system - it requires less development. One downside to a commercial system is having to keep at least two lasers on hand to replace one in case of failure. It is not uncommon for lasers to fail and have to go through lengthy repairs, which can lead to significant downtime of an experiment. Since the experiment already requires a 1064 nm laser for evaporation, as well as a backup laser, it makes sense to develop a doubling system around this laser. Additionally, most single frequency, high power, 532 nm lasers come with a large natural linewidth.\(^9\) The frequency fluctuations, which result in larger linewidths,

\(^9\)Coherent Verdi V18 at 5 MHz linewidth and Light House Photonics Solo-10W at 1 MHz
cause fluctuations of the trap center [15]. The scaling of this heating goes as square of the trapping frequency $\nu^2$, which equates to $U_0$ where $U_0$ is the trap depth. The heating rate can, thus, be substantially higher for deep lattices. The heating rate is also related to the length, $L$, of the retro path, since the position change of the fringe position is given by $\delta x/\lambda = L\delta f/c$, where $\delta f$ is the frequency change of the laser. This indicates that the heating rate will scale as $L^2$. There is also an unfavorable scaling proportional to mass due to the ground state wavefunction size. In this case Erbium will heat approximately 30 times faster than Lithium for the same laser setup.

Several methods of green light generation have been attempted in this lab, predominately single pass generation. The first method was single pass generation using an MgO:PPLN crystal. The oven and crystal were purchased from the same company. A single-frequency 50W 1064nm fiber amplifier was used to pump the crystal. The 1064nm light was focused slightly larger than the Boyd-Kleinman focusing criterion [18]. Additionally, the light was focused closer to the entrance face of the crystal to avoid high intensity green at the exit face of the crystal. In this implementation of the system the crystal cracked (see Figure 5.23) when generating around 3W of green while pumping with approximately 10W of 1064nm light. This power level did not reach the minimum 10W requirement of this system. Additionally, the crystal became extremely sensitive to temperature. The main problem was local heating of the crystal due to absorbed light.

The second method that was tried was generation using PPMgSLT crystal. This crystal

\[\text{linewidth.}\]
\[\text{Covesion MSHG1064-1.0-40}\]
\[\text{Azur Light Systems}\]
\[\text{Focusing 5-10um above is advantageous due to thermal effects when generating high power green light.}\]
\[\text{High intensity green leads to Green Induced Infrared Absorption (GIIRA) which causes additional thermal effects. In principle, MgO doping helps to prevent GIIRA, but it does not completely eliminate it.}\]
\[\text{Oxide Corporation PPMgSLT 30mm long crystal with no wedges and 0.5mm height}\]
Figure 5.23: Fractured crystal from Covesion.

has higher damage threshold than MgO doped PPLN. Similar techniques were used with this crystal as with the Covesion crystal, such as larger focus size and focusing towards the entrance face of the crystal. The crystal was pumped with 35W of 1064nm, limited by losses of the optical system. The doubled power was around 12W, which is approximately 34 percent conversion efficiency. Pumping with the full 45W available from the laser is possible, in principle, and should yield powers of 15W or greater. Doubling caused an increase in intensity noise of approximately 10dB relative to the raw 1064 nm light, which is not uncommon in doubling systems. The line-width of the green light is twice that of the 1064 nm light. When seeded with narrow line-width sources, laser line-widths of 10kHz and below are easily achieved.

The crystal was mounted in a home-built crystal oven. (See Figure 5.24) Good thermal contact between the crystal and the oven walls was ensured by small pieces of Indium foil. The temperature was controlled to 0.002 Celcius using a Peltier element and a thermistor. Temperature feedback was controlled by a Thorlabs TED4015 TEC controller. A picture of the doubled light test setup can be found in Figure 5.25.

The Residual Intensity Noise (RIN) of the green light can be further reduced using a zeroth order AOM. By picking off a small amount of light (2-5 percent), the RIN up to 150kHz can be reduced to $-150\text{dBc/Hz}$, which is far better than the requirements of the system, without

\footnote{Thorlabs TH10K}
Figure 5.24: Custom oven and crystal for frequency doubling. The brass parts house the crystal and can be stabilized to around 0.002 C. The aluminum cooling plate underneath is water cooled and the translation stage allows for 3 axes of motion.

losing much light.

5.14 Imaging

There are two imaging systems installed to date. These imaging systems are for absorption imaging along the horizontal and vertical directions. They are 4f systems with magnifications of approximately 1. The camera used is a Point Grey GS3-U3-15S5M-C. Additionally, an Andor iXon 897 EXF has been purchased for imaging atoms in the lattice.

5.14.1 Reflective Objective

A custom objective has been designed within the group for use in the microscope. The objective is a reflective design consisting of a spherical mirror\(^{26}\) and an aspheric correction plate\(^{27}\) Both optics are made from fused silica. The two optics are separated by a spacer also made

\(^{26}\)Manufactured by Optimax Systems, Inc. and coated by Laser Optik GmbH.

\(^{27}\)Asphericon GmbH.
Figure 5.25: Doubled 1064 nm light using the custom doubling crystal oven. Several dichroic mirrors filter residual 1064 nm light to produce up to 12 W of 532 nm. The box was placed around the oven to better help stabilize the temperature - in the final setup it will be replaced with a more fire-resistant housing.

from fused silica to avoid misalignment during baking. The objective is designed specifically for 401 nm and will be placed inside the vacuum chamber. A rendering of the objective and a toy model can be seen in Figure 5.26. The working distance of the objective is around 25mm, and the atoms will be located around 18mm from any optical surface. The objective has been aligned and has a diffraction limited NA of 0.85. The objective has non-diffraction limited optical access to an NA of 0.95 which allows accordion lattice beams to generate a lattice spacing of $\frac{\lambda}{2NA}$. 

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28Manufactured by Insaco Inc.
Figure 5.26: Custom reflective objective designed in the lab. The lavender highlighting on the left figure shows the path of the light. The right figure is a rendering of the objective, which has been assembled and tested.
Experimental realization

6.1 583 nm MOT

Various narrow-line Magneto-Optical Traps (MOT) have been implemented for a variety of atomic species \[60, 13, 76, 89, 50, 37\]. The first MOT implemented in this experiment was inspired by the work in Francesca Ferlaino’s group \[41\]. A multi-species MOT of Erbium and Dysprosium has recently been implemented, again in Francesca Ferlaino’s group \[55\]. Due to the narrow-line width a few methods for increasing loading rate of these narrow-line MOTs have been explored \[76, 68\].

The 583 nm transition in Erbium has a 190 kHz line-width. The narrowness of a narrow-line MOT is related to the maximum force of the MOT relative to gravity:

\[
\eta = \frac{\hbar k \Gamma}{2mg}. \tag{6.1}
\]

Here \(\Gamma\) is the transition line-width, \(k = \frac{2\pi}{\lambda_{583}}\), \(m\) is the mass of the atom, and \(g\) is gravity. A MOT is considered narrow if \(\eta \approx 1\), just barely supporting against gravity. It is a stretch to
Figure 6.1: The amount of compression of the yellow MOT is controlled by detuning as shown qualitatively in figure (a). A quantitative plot (b) shows the potential experienced by the atoms, in the vertical direction, for two different configurations with large and small detunings. In (c) is an image of the atoms in the MOT, without scanning. This trap contains approximately $10^9$ atoms at approximately 100 uK.

truly call this a narrow-line MOT, since $\eta_{583} = 247$, but it is indeed narrow compared to the alkali MOTs.

In order to quantify MOT operation in the narrow-line case, consider the force experienced on an atom with some position $z$ and a velocity $v$:

$$F_z(v, z) \approx \frac{\hbar k \Gamma}{2} \left( \frac{s}{1 + 2s + 4(\Delta - kv - \delta \mu \partial B_z/h \frac{\partial z}{\Gamma})^2} \right) - mg, \quad (6.2)$$

where $s = I/I_s$ is the saturation parameter (see Section 1.1.2) and $\partial_z B$ is the magnetic field gradient. $\delta \mu/\mu_B = m_e g_e + m_g g_g$ is the differential magnetic moment between the ground state (excited state) with magnetic quantum numbers $m_g$ ($m_e$) and Landé g-factors $g_g$ ($g_e$).

The force is predominately from the bottom beam since the atoms tend to sag to the bottom of the trap where force is large enough to support against gravity - in this location, only pho-
tons from the upwards propagating beam are scattered. The preferential scattering from the upwards beam causes the cloud to become polarized in the $m_j = -6$ state. The $2s$ is the denominator of Eq. 6.2 comes from bottom beam and the $1/4$ of the intensity in the correct polarization of the side beams. This is a good approximation due to the narrow-line transition and the polarizing nature of a narrow-line MOT. The potential of this force can be seen in Figure 6.1. In this figure the full force, including the downwards beam, has been included.

The stable location of an atom can be found by solving for when the force, $F_z(v = 0, z_c) = 0$,

$$\Delta - \delta \mu \partial B_z z_c / \hbar = - \frac{1}{2} \Gamma \sqrt{(\eta - 2)s} - 1.$$  \hspace{1cm} (6.3)

The location of the atom is dependent on detuning, magnetic field, and intensity. The damping rate $F_z(v, z_c) \approx \alpha v$ can also be calculated

$$\alpha = -2 \hbar k^2 \sqrt{(\eta - 2)s} - 1 / \eta^2 s.$$  \hspace{1cm} (6.4)
This damping rate can be used to estimate equilibration time of the MOT. It must be noted that this damping coefficient is only true in the vertical direction for atoms already trapped. To consider the capture velocity of the MOT the force from the other beams must be considered. The force for this atom will go as

\[ F(v_x, x) \approx \frac{\hbar k \Gamma}{2} \left( 1 + 2s + 4 \frac{(\Delta - kv_x - \frac{1}{2} \delta \mu \partial B_x / \hbar)^2}{\Gamma^2} \right) - \frac{\hbar k \Gamma}{2} \left( 1 + 2s + 4 \frac{(\Delta + kv_x - \frac{1}{2} \delta \mu \partial B_x / \hbar)^2}{\Gamma^2} \right) \]  

(6.5)

The damping coefficient near \( x = 0 \) goes as

\[ \alpha_x = \frac{8\hbar k^2 \Delta s}{\Gamma \left( 1 + 2s + 4 \frac{\Delta^2}{\Gamma^2} \right)^2} \]  

(6.6)

The capture velocity of the MOT can be estimated by finding the velocity as a function of position,

\[ v(x) = v_0 + \frac{\alpha_x m}{x}. \]  

(6.7)

Therefore, the capture velocity given a MOT beam of diameter \( d \) is approximately

\[ v_{\text{capture}} \approx -\frac{\alpha_x m}{d}. \]  

(6.8)

The capture velocity from this should be a lower bound. The damping force at larger velocities is substantially higher than near \( v = 0 \), therefore, the actual capture velocity should be larger than this simple estimation. A more substantive estimation can be found by numerically integrating the force equation.

The final temperature of the MOT can be found using the damping coefficient in Eq. 6.4.
Figure 6.3: Theoretical temperature of the 583 nm MOT.

To find a formula for the temperature, the diffusion coefficient $D_p$ needs to be calculated

$$D_p = \hbar^2 k^2 \langle \rho_{22} \rangle \approx \frac{\hbar^2 k^2 \Gamma}{2\eta}$$  \hspace{1cm} (6.9)

where $\langle \rho_{22} \rangle$ is the excited state fraction. The temperature is then given by

$$T = \frac{D_p}{k_B|\alpha|} = \frac{\hbar \Gamma s}{4 k_B \sqrt{(\eta - 2)s - 1}}.$$  \hspace{1cm} (6.10)

A similar derivation can be found in Ref. [74]. Interestingly, the temperature of the MOT is set solely by the intensity. A theoretical plot of the temperature of the 583 nm MOT is shown in Figure 6.3. The plot also includes the doppler limit and the recoil limit.

6.1.1 Discussion of 583 nm MOT

The implementation of the 583 nm MOT in the experiment started as a non-scanning MOT. After careful alignment of the 2D Molasses section and the Zeeman slower, the loading rate into the MOT was approximately $200 \times 10^6$ atoms/s. Peak atom numbers were approximately $800 \times 10^6$. The loading rate was substantially increased by the 2D Molasses section, yielding
a 34 times increase in loading rate. In this implementation, the setup was power limited to approximately $12I_s$ per beam. A picture of the actual MOT can be seen in Figure 6.1. The sagging of the MOT in the trap is useful for preventing scattering from the Zeeman slowing beam.

Later the 583 nm MOT was upgraded to a scanning MOT (see Section 6.1 for details on the implementation). This MOT required more optical power, approximately 4 times more power per channel. The dynamics of this MOT were slightly more complicated, but the net effect was a 3 times increase in loading rate. With careful tuning, the loading rates for $^{166}Er$ can be increased to around $710 \times 10^6$ atoms/s with peak atom numbers around $1.6 \times 10^9$ as shown in Figure 6.2. This increase in loading rate was partly attributed to the higher intensity, however, operating the old MOT with larger intensities would not increase the loading rate indefinitely due to a limit in the maximum detuning set by the MOT beam size - scanning helped to overcome this limitation. A more easily attainable and stable loading rate is around $350 \times 10^6$ atoms/s. These large loading rates allowed for the trapping of all stable isotopes of Erbium, opening the possibility of evaporation for all stable isotopes. In the scanning configuration, the MOT no longer maintains the shape as in Figure 6.1. The MOT is pushed further up and interacts more strongly with the Zeeman slowing beam. An untested possible improvement might be to removing scanning from the vertical direction. It is not clear that scanning from this beam is entirely necessary since the speed of the atoms in the vertical direction is substantially lower.

The beams used in both setups were approximately 50 mm in diameter. If the beam size is reduced to 25 mm diameter, the loading rates drop by a factor of 2. In both setups the MOT operates with all 6 beams, but it is possible to get similar, but slightly lower, loading rates without the downward vertical beam.

All quoted loading rates were with compressing the MOT to -800 kHz detuning at an in-
tensity of approximately $0.3I_s$. The compression started with turning off scanning and immediately starting to ramp the detuning, magnetic fields, and intensity to the final compressed value. It was important to allow the system to equilibrate at the final compressed MOT settings for approximately 50-100 ms. If allowed to equilibrate, the final temperature the MOT could reach was approximately $10\,\mu$K at around $0.1I_s$. Further reducing the intensity did not bring the temperature lower, it only heated the cloud and caused atom loss. In most cases, even for loading the ODT, the MOT intensity was limited to $0.1 - 0.3I_s$.

6.2 BEC

The original BEC production in this lab was inspired by the evaporation sequence and dipole trap in Ref. [42]. The traps were composed of $18\,\mu$m x $18\,\mu$m waist with the ability to scan the beam waist to $108\,\mu$m in the horizontal direction. (See Section 5.10) BECs with final atom numbers between 30-50k atoms were produced with the $^{168}$Er isotope. The sequence for initial evaporation can be found in Figure 6.5. The evaporation sequence took around 7 seconds initially, but tuning of the system reduced time to just below 5 seconds. The onset of degeneracy can be seen in Figure 6.4. The thermal background is fitted for visual reference and the corresponding temperature is noted on each plot. This sequence was taken from the last 1 second of evaporation.

The efficiency of evaporative cooling is defined by

$$\eta = -\frac{d\ln \text{PSD}}{d\ln N},$$

(6.11)

The PSD of can be found by the following formula

$$\text{PSD} = N\hat{\Omega}^3 \left( \frac{\hbar}{k_b T} \right)^3,$$

(6.12)
where $\bar{\Omega} = (\Omega_x, \Omega_y, \Omega_z)^{1/3}$ is the geometric trap frequency. This is simply a measure of the number of atoms in a de Broglie volume, defined by the cube of the de Broglie wavelength, $\lambda_{dB}^3$.

The evaporation efficiency in this experiment was $\eta \approx 2$. The evaporation efficiency was low due to some atom loss process. When the dipole traps were initially loaded with trap depths near 200 $\mu$K and atom cloud temperatures of around 30 $\mu$K, there was some unexplained heating which caused a faster than expected atom loss, possibly due to the fiber laser spectrum.

The number of trapped atoms was initially around $10^7$ with MOT loading times of around 1 second. The initial lifetime of the atoms at 30 $\mu$K was less than 1 second. In order to reach degeneracy, it was simply a matter of loading the trap with more atoms than should be necessary and just dealing with the atom loss. Note, for evaporation it was important to choose the magnetic field carefully. The field was chosen to be around 0.38 Gauss, below the first Feshbach resonance in $^{168}$Er.

Trap frequency measurements were made at several parts of the evaporation sequence. They were measured by turning the ODT off for a short amount of time and measuring the oscillations that resulted in momentum space. The trap frequencies at the end of the sequence were chosen to form a pancake shaped trap, which is known to prevent dipolar collapse \[61\].

This dipole trap is no longer in the system. The evaporation sequence has taken a path which could lead to significantly faster evaporation. See section \[7.1\] for comments.

### 6.3 841nm MOT

In order to reach degeneracy faster, we decided to implement a second stage red-detuned MOT using the 8kHz, 841nm transition in Erbium. The idea here is to increase loading efficiency into the ODT and at the same time increase the PSD initially loaded in the ODT.
PSD of 0.1 was loaded into the ODT, the old evaporation scheme would be much closer to
degeneracy as shown in Figure 6.5. In fact, better evaporation schemes might lead to a sub-
stantially faster BEC production. Many of the equations derived in Section 6.1 still apply to
the 841 nm MOT.

The maximum MOT force compared to gravity (Eq. 6.1) for the 841 nm transition is ap-
proximately $\eta_{841} \approx 7.2$. This MOT is approaching the limit to support against gravity. A
variety of groups have worked with narrow-line MOTs using transitions of comparable (or
narrower) widths. A similar MOT was implemented with the 841 nm transition in Erbium us-
ing blue detuned light [13]. The MOT by Berglund et. al. could only reach temperatures of 2
$\mu$K and could not operate red detuned of resonance - most likely due to the hand-off from the
blue laser and stronger response to magnetic fields for Erbium. Probably the most well known
narrow-line MOT is the 7.6 kHz, 689 nm transition of Strontium [60]. The Strontium MOT
can reach an extremely low temperature and a high phase space density.

As for the MOT implemented in this experiment, the temperature estimated from Eq. 6.10
for the 841 nm transition is shown in Figure 6.8. Theoretically the MOT should reach a final
temperature near the Doppler cooling limit, and just a few times larger than the recoil limit.
Figure 6.5: Original sequence for BEC production.
To get this MOT to work, the atoms were first loaded into the 583 nm MOT where they were compressed and cooled. The 583 nm MOT was switched off and the 841 nm MOT was turned on. The atom transfer efficiency between the 583 nm MOT and the 841 nm MOT was nearly 100%. The best transfer efficiency was found when the 583 nm MOT was let go slightly above the equilibrium position of the 841 nm MOT - it was not extremely sensitive to this, but it was important that it was above. The 841 nm MOT would capture the 20 \(\mu\)K atoms and almost immediately cool them to around 5 \(\mu\)K. Before compressing, the 841 nm MOT had a relatively long lifetime, around 500 ms.

Going through a similar compression sequence to the 583 nm MOT, the 841 nm MOT was further compressed and cooled. The sequence of compression can be found in Figure 6.7. It is important to note that the optimal compression occurred when the offset magnetic field was ramped in such a way as to maintain the position of the MOT during compression.

Final temperature of the compressed 841 nm MOT versus intensity can be found in Figure 6.7. The lowest temperature reached was around 400 nK, but the atom numbers were substantially lower at this temperature. Most atoms were lost by an intensity of \(2I_o\), so temperatures below 400 nK were undetectable. Interestingly, no effects from Feshbach resonances were observed. Absorption imaging pictures of the compressed 841 nm MOT can be found in Figure 6.6. The two pictures are for -105 kHz detuning and -72 kHz detuning. Further compression resulted in significant atom loss. A scale bar is included for reference. The atom number saturates at around \(10 \times 10^6\) atoms depending on how aggressive the compression is - larger atom numbers can be achieved, but the MOT is quite large since the detunings are \(|\Delta| > 100 \text{ kHz}\).

To get a better estimate of the MOT dynamics and PSD, it is worth computing the trapping frequencies for the MOT. Note, a similar derivation can be found in Ref [37]. To derive the
trapping frequencies first consider the magnitude of the magnetic field

$$|B(x)| = \left| \frac{\partial B_z}{\partial z} \right|_{x=0} \sqrt{z^2 + \left( \frac{x}{2} \right)^2 + \left( \frac{y}{2} \right)^2}$$  \hspace{1cm} (6.13)$$

where the strongest gradient from the quadrapole field points along the vertical z direction.

The force at zero velocity in the presence of this magnetic field, neglecting the force from the gradient itself, is given by

$$F_x = F_y = \frac{\hbar k \Gamma}{2} \frac{s/4}{1 + 2s + 4\Delta^2(x - x_0)/\Gamma^2} - \frac{\hbar k \Gamma}{2} \frac{s/4}{1 + 2s + 4\Delta^2(x - x_0)/\Gamma^2} = 0$$  \hspace{1cm} (6.14)$$

$$F_z = \frac{\hbar k \Gamma}{2} \frac{s}{1 + 2s + 4\Delta^2(x)/\Gamma^2} - mg$$  \hspace{1cm} (6.15)$$

The force in the x and y directions cancels out due to the opposing beams. However, the force in the z direction is exactly what was found in Eq. 6.2. Expanding Eq. 6.15 around the equi-

Figure 6.6: Compression of the 841nm MOT. The temperature of these clouds is around 1uK. The detuning is -105kHz (a) and -72kHz (b). The atom number is approximately $5 \times 10^6$ (a) and $2 \times 10^6$ (b). The point in the center of the images marks the zero of the magnetic field and the ellipse marks the position of resonance for the atoms. Gravity points downwards.
librium position \( x = y = 0 \) and \( z = z_c \), yields the following

\[
F_z(x) = \frac{\hbar k \Gamma}{2} \frac{s}{1 + 2s + 4\Delta^2(x)/\Gamma^2} - mg \approx \left( \frac{\partial F_z}{\partial z} \right)_{x,z_c} (z - z_c) \quad (6.16)
\]

From here it is easy to show that the derivative in force goes as

\[
\left( \frac{\partial F_z}{\partial z} \right)_{x=z_c} = 8 \frac{mg(\partial B_z)\partial \Delta(z_c)}{\hbar \Gamma^2 \eta s}.
\]

Here the effective detuning at the equilibrium position is defined by

\[
\Delta(z_c) = \frac{1}{2 \Gamma} \sqrt{\eta - 2} s - 1.
\]

The trapping in the horizontal direction is slightly more subtle. The predominant source of trapping in this direction is not the light force from the horizontal direction. Because atoms are resonant on an ellipsoid and tend to sag toward the bottom of the trap, there still exists a horizontal restoring force from the interplay of the vertical light beam, the magnetic field, and gravity. If the atoms are located at \( z_c \) in the vertical direction, then the effective potential they experience (neglecting force from the magnetic field) is given by

\[
U_{\text{gravity}}(x) = -mg \sqrt{z_c^2 + \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2} \approx -mg|z_c| + mg \frac{x^2}{8|z_c|} + mg \frac{y^2}{8|z_c|}
\]

This force relates to the magnetic field gradient given in Eq. 6.13, since the atoms are forced to follow a constant magnetic field. From Eq. 6.19, it is easy to derive the force experienced
Figure 6.7: Sequence for 583 nm MOT loading and compression as well as 841 nm hand-off and compression.
Figure 6.8: 841 nm MOT temperature as a function of intensity. Included are measured values of the MOT temperature.

by the atoms in the \( \hat{x} \) and \( \hat{y} \) directions

\[
F_{\text{gravity}} = -mg \frac{x}{4|z_c|} \ddot{x} - mg \frac{y}{4|z_c|} \ddot{y} \tag{6.20}
\]

Using Eq. [6.20] and Eq. [6.17] it is a simple matter to find the horizontal trapping frequencies,

\[
\Omega_x = \Omega_y = \sqrt{\frac{g}{4|z_c|}}, \tag{6.21}
\]

and the vertical frequency,

\[
\Omega_z = \sqrt{\frac{8g(\partial B_z)\delta \mu \Delta(z_c)}{\hbar \Gamma^2 \eta_s}}. \tag{6.22}
\]

By measuring the temperature of the cloud and the atom number, it is easy to make an estimate of the phase space density in the MOT. Similarly the PSD of the MOT can be estimated by measuring the size of the MOT using absorption imaging and measuring the atom number
and temperature with time-of-flight. The corresponding equation is given by

\[
\text{PSD} = \frac{N}{V} \lambda_{\text{dB}}^3,
\]

(6.23)

where \( V = \frac{\pi^{3/2}\sigma_x\sigma_y\sigma_z}{2} \) is the volume of the compressed MOT and \( \lambda_{\text{dB}} = \frac{\hbar}{\sqrt{2\pi mk_bT}} \) is the thermal de Broglie wavelength. Estimations of the MOT volume and atom number put the density near \( N/V \approx 2 \times 10^{12} \text{ cm}^{-3} \) with a corresponding PSD around 0.02 – 0.05. Similar estimates from the 583 nm MOT suggest a PSD of \( \text{PSD}_{583} \approx 4 \times 10^{-4} \). The MOT therefore delivers a two order of magnitude increase in PSD. Techniques to further enhance the PSD loaded into an ODT from the 841 nm MOT are currently under investigation. Simulations suggest a PSD of 0.1 with a few million atoms is not unreasonable.
Outlook

7.1 Fast-BEC Production

A variety of methods exist for fast BEC production: compression and Raman sideband cooling [32], laser cooling [99], fast evaporative cooling [97, 98], and dynamically shaped traps [91]. The method currently being explored is a mixture of laser cooling on the 841 nm narrow-line transition and fast evaporative cooling in dynamically shaped traps. In other words, if it is possible to achieve an initially large PSD and large atom number in a dipole trap, then a fast evaporation sequence should produce BEC in around 500 ms.

One difficulty with Erbium is the excited state polarizability in the 841 nm excited state for a 1064 nm dipole trap is around 20% of the ground state. This is in contrast to Strontium, where by tuning the polarization, the differential stark shift for on the 689 nm transition is around 10 times smaller. This prevents Doppler cooling techniques with 841 nm in the dipole trap from working and can make achieving high PSD in the ODT difficult.

Several techniques to overcome this are currently under investigation. Another interesting method for Fast-BEC production may come from a slightly modified Sisyphus cooling mecha-
**Figure 7.1**: Phase diagram of the Extended Bose Hubbard model computed in mean-field in (a) including fractional filling phases. In (b) is the potential of the lattice being implemented in this experiment with a reference line for $\mu = 2U_{nn}$, which indicates the maximum number of sites with the checkerboard phase. In (c) is a figure showing the approximate locations of the super-solid (SS) phase. Note that this is not from calculation and is only there to qualitatively show the SS location.

nism originally discussed in Ref. [57] and implemented in Refs. [31, 30].

### 7.2 Phase Transitions of the Extended Hubbard Model

The addition of the long-range dipole interaction to the Bose-Hubbard model modifies the standard Mott insulator to super-fluid phase diagram. [104, 103] Effects from the dipole interaction in a lattice have been observed in Ref. [10]. The presence of the dipole interaction leads to a spontaneous breaking of translational symmetry and gives rise to non-uniform fractional filling phases. In principle, these phases can have any filling fraction, but timescales of the experiment should limit it to nearest neighbor interaction with half-filling phases - heating.
and decoherence in the system should limit the time over which the fractional filling phases can evolve. Observation of any of these phases would be a smoking gun for dipolar physics in the lattice.

These fractional filling phases are found between the Mott insulating lobes and can give rise to a number of atom configurations as shown in Figure 7.1. The phase diagram is computed from a mean-field approach assuming fourth nearest neighbor interaction and is detailed in Ref [104]. The lowest energy configuration is the checkerboard phase which is indicated by the lobe extending furthest out in tunneling, $J/U_{nn}$. This is clearly the most stable configuration since movement of any atom/hole in the lattice costs energy on the order of the nearest neighbor dipole interaction strength. The phases are also meta-stable and will tunnel between each other.

Since the lattice will be red-detuned in this experiment, the lattice beams not only produce lattice sites, but also provide overall harmonic confinement. In Figure 7.1 (b) the energy of the lattice sites is plotted in 1D for the planned waist sizes in the experiment at the correct lattice depth to produce the necessary tunneling of $J/U_{nn} < 0.3$. A black line is added for reference at $\mu = 2U_{nn}$, which is approximately the maximum chemical potential change supported by the checkerboard phase. The lattice in its current design would produce around 150 lattice sites with the fractional filling phases.

It has been theorized that there is a super-solid phase [22] on the lobes of the fractional filling phases as shown in Figure 7.1 (c). These may be experimentally difficult to observe due the required control over chemical potential. The addition of potential shaping optics, such as an SLM or DMD [114] or a blue-detuned anti-confining beam generated by 841 nm light may make it possible to observe these phases.
7.3 Low Heating Artificial Gauge Fields

A charged particle moving through a magnetic field can experience a phase accumulation known as the Aharonov-Bohm phase. Since the movement of electrons mostly governs the behavior of materials, the introduction of a magnetic field can produce topological systems such as quantum Hall states [105, 46, 9], chiral spin liquids [25], or realize Hamiltonians such as the interaction Harper-Hofstadter model [53, 102].

Ultra-cold atomic systems provide a novel platform for studying the microscopic dynamics of systems, but lack the charged nature of electrons in a crystal. In order to emulate magnetic fields, several techniques have been pioneered to produce artificial gauge fields, which show similar physics to an electron in a magnetic field. The majority of methods rely on using Raman transitions to produce the gauge field [96]. Artificial gauge fields have been produced using clock transitions [108], narrow transitions in Dysprosium [20, 33], Raman transitions with synthetic dimensions in alkali atoms [23, 72], or lattice modulation techniques [81, 101, 102, 2, 4, 3]. Several reviews have covered gauge fields with ultra-cold atoms [47, 34].

As is investigated in Ref [33], the narrow transitions in Dy and Er provide a perfect platform for generating Raman transitions with reduced heating. It can be shown that the heating rate for the alkali atoms is dictated by the broader transitions and the $^2P_{3/2}$ and $^2P_{1/2}$ states having a canceling effect on the Raman coupling strength due to opposite signed Clebsch-Gordon coefficients [53]. This is in contrast to using a single narrow transition, which does not have the same canceling effect and exhibits stronger Raman coupling for the same field strength. Indeed, this results in lower scattering rates for the same coupling strength.

The 841 nm transition provides a similar transition rate and structure as the 741 nm transition of Dy. Using the correct Raman beam polarization it is possible to generate a synthetic
dimension with up to 13 states. (See Figure 7.2) The number of states may be limited by using a strong magnetic field to quadratically split the energies to prevent Raman transitions to other magnetic sublevels.

### 7.4 Sub-wavelength Lattices

Standard lattices are limited to a spacing greater than half the lattice wavelength. By utilizing the internal structure of the atom it is possible to create sub-wavelength trapping features, which is due to the non-linear atomic response. Sub-wavelength lattices offer the potential for increasing interaction and tunneling energy scales in the lattice [58], increasing dipole interaction, position dependent interactions, and the study of molecular collisions [115].

These potentials have been theoretically investigated in a number of contexts and several techniques exist for generating sub-wavelength features. As pointed out before, it is necessary to utilize some non-linear response to produce these features. The most discussed methods utilize non-adiabatic corrections to the Born-Oppenheimer potentials from dark states [38, 115, 58]. Another method uses spin dependent optical lattices mixed with stroboscopically changing the lattice phase to produce periodic potentials of sub-wavelength spacing [84].
Figure 7.3: Sub-wavelength lattice production using the $\Lambda$ configuration and two driving lasers. In (a) is a proposed level scheme in Erbium consisting of a 585.17 nm and 451.99 nm transition. In (b) is a diagram of the $\Lambda$ configuration used for producing sub-wavelength features. In (c) are example potentials exhibiting sub-wavelength features.

Finally, it is possible to use multiple dressing fields, which, when properly tuned can produce averaged potentials with sub-wavelength features [111]. These techniques can also be used for sub-wavelength addressing of atoms [48]. The $\Lambda$ dark state technique was recently used in Ytterbium to produce sub-wavelength features [109].

An interesting $\Lambda$ transition in Erbium (see Figure 7.3) exists, which could enable implementation of the dark state technique. A diagram of the $\Lambda$ 3 level atom can be seen in Figure 7.3 (b). The Erbium transition utilizes the ground state, $J = 6$, state of Erbium ($|1\rangle = |J = 6, m_j = -6\rangle$), the $J = 4$ clock state ($|2\rangle = |J = 4, m_j = -4\rangle$) coupled by two circularly polarized Raman lasers with wavelengths 585.17 nm and 451.99 nm. There are other transitions here which utilize the lowest $J = 5$ state for $|2\rangle$ and may be easier to implement 2D lattices with due to the linear polarization of one of the photons. The $J = 4$ clock state in the proposed scheme has a decay rate of around $\Gamma = (2\pi) \times 7 \mu$Hz [63], which is effectively zero for all possible experiments. The other two transitions are narrow with transition rates of around
\[ \Gamma = (2\pi) \times 42 \text{ kHz} \] (\[ \Gamma = (2\pi) \times 120 \text{ kHz} \]) for the 451.99 nm (585.17 nm) transition \[^{[100]}\].

The transition appears mostly closed with a decay from the excited state to the \( J = 5 \) lower state of rate \( \Gamma = (2\pi) \times 600 \text{ Hz} \). The similarity, however, of the \( J = 4 \) and \( J = 6 \) state may be advantageous since similar imaging wavelengths exist for both states. It may also be possible to use this Raman transition for STIRAP to shelve the atoms for imaging spin states or storage of a quantum state.

Generation of sub-wavelength lattices with multiple sites next to each other and spaced by a few tens of nano-meters may be possible by stroboscopically adjusting the phase of the lattice potential, dictated by \( \Omega_{23}(x) \). Doing this should create several closely spaced sharp features. Imaging of these atoms may be possible by increasing the separation of the atoms also with stroboscopic techniques.
A.1 Gaussian Beams

Gaussian beams are fundamental to most AMO experiments. The Gaussian beam is the fundamental mode of optical resonators and fiber optics. In addition, the propagation of a Gaussian beam through a lens produces a different Gaussian beam - it is preserved through many optics. In physical systems the Gaussian beam is used since it closely resembles a harmonic trap.

The Gaussian beam is one solution of the paraxial wave equation

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2ik \frac{\partial u}{\partial z}
\]  

(A.1)

where \( E(x) = u(x)e^{-ikz+iot} \). The electric field is polarization independent due to the paraxial approximation. Outside the paraxial approximation, a focused beam can be calculated, but is more challenging to compute \[1\]. Solving the paraxial wave equation yields the Gaussian
electric field

\[ E(x, y, z) = E_0 \sqrt{ \frac{w_x w_y}{w_x(z-z_x)w_y(z-z_y)}} e^{-\frac{z^2}{w_x^2(z-z_x)}} - \frac{z^2}{w_y^2(z-z_y)} e^{-i\left(kz + k\frac{z^2}{2w_x(z-z_x)} + k\frac{z^2}{2w_y(z-z_y)} - \frac{w_x(z-z_x)}{2} - \frac{w_y(z-z_y)}{2}\right)}, \quad (A.2) \]

where \( w_a \) is the minimum beam waist, \( w_a(z) = w_a \sqrt{1 + \left(\frac{z}{z_{a,R}}\right)^2} \) \( (A.3) \)
is the position dependent waist,

\[ R_a(z) = z \left(1 + \frac{z_{a,R}^2}{z^2}\right) \quad (A.4) \]
is the radius of curvature of the beam, and the so-called Gouy phase takes the form

\[ \psi_a(z) = \arctan \left(\frac{z_{a,R}}{z}\right). \quad (A.5) \]

Here \( z_{a,R} = \pi w_a^2/\lambda \) is the Rayleigh range of the beam, and \( a \) can be either \( x, y \). The resulting intensity goes as

\[ I(x, y, z) = I_0 \frac{w_x w_y}{w_x(z-z_x)w_y(z-z_y)} e^{-\frac{2z^2}{w_x^2(z-z_x)}} - \frac{2z^2}{w_y^2(z-z_y)} \quad (A.6) \]

The intensity can be written in terms of power by integrating the intensity distribution to give

\[ I_0 = \frac{2P}{\pi w_x w_y}. \]
Gaussian Beam Divergence

The divergence of a Gaussian beam is given by

\[ \theta = \lim_{z \to \infty} \frac{w_0(z)}{z} = \frac{w_0}{z_{a,R}}. \]  

(A.7)

This is effectively the NA required for this beam, but it is good practice to use \( 2\theta \) as the required NA. It is this quantity that is related to aberrations induced by optics. If the required NA is too large for a given optic, then aberration will effect beam shape. For spherical optics maintaining \( 2\theta < 0.03 \) will prevent significant aberration on the beam. Of course, if \( 2\theta \) were to be doubled, then there would be 16 times more spherical aberration.

A.2 Fourier Transforming Property of Lens

One of the most important concepts to an optics-heavy lab is the fact that lenses produce Fourier transforms of the electric field. It is this property which maintains Gaussian beams when using lenses to change the beam size. The concept is fairly easy to understand physically - a plane wave is effectively mapped to a point. Of course, this point is convolved with the point spread function (PSF) of the imaging system, but is of little consequence as long as
**Figure A.2:** Fourier transforming property of a lens. Plane waves are focused to a point after the lens.

the PSF size is much smaller than the waist size.

To be more accurate, consider the diagram in Figure A.2. An electric field is located at a distance \(|z_0|\) from the lens of focal length \(f\). The electric field at \(z = z_0\) can be decomposed into a series of plane waves by a Fourier transform

\[
E_0(x) \rightarrow \tilde{E}_0(k_x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E_0(x) e^{-ikx} dx
\]

The electric field after the lens is proportional to

\[
E(x') \propto \tilde{E}_0 \left( \frac{kx'}{f} \right) e^{-ikz_0 \cos \theta - i\frac{k}{\cos \theta}}
\]

The terms in the exponent containing \(\theta\) can be expanded in the paraxial approximation and yields

\[
z_0 \cos \theta + \frac{f}{\cos \theta} \approx (z_0 + f) + \frac{1}{2} (f - z_0) \theta^2
\]

\[
\approx (z_0 + f) + \frac{1}{2} (f - z_0) \left( \frac{x'}{f} \right)^2.
\]
Figure A.3: Beam profile at infinity for different types of aberration.

Putting it all together yields the final electric field

\[
E(x') \propto \tilde{E}_0 \left( \frac{k x'}{f} \right) e^{-i k (z_0 + f) - i L_z f (f - z_0) \left( \frac{x'}{f} \right)^2}.
\]  
(A.11)

Notice the spherical wave-front term, which is entirely due to the electric field being outside the Fourier plane of the lens. Extending this to two dimensions is relatively simple and yields the following

\[
E(x', y') = \frac{k}{f} \tilde{E}_0 \left( \frac{k x'}{f}, \frac{k y'}{f} \right) e^{-i k (z_0 + f) - i L_z f (f - z_0) \frac{x'^2 + y'^2}{f^2}}.
\]  
(A.12)

The factor \( k/f \) has been added to preserve the total power in the Fourier transformed field.

A.3 Aberration

Optical aberration can be decomposed into ortho-normal functions over the unit disk. This aberration is in phase space and takes the form \( E \propto \exp \left( i \phi(x, y) \right) \). The beam profile at infinity will resemble those in Figure A.3. These polynomials are denoted by \( Z_n^m(\rho, \phi) \) and have the orthogonality relation
<table>
<thead>
<tr>
<th>Zernike Polynomial</th>
<th>Noll Index</th>
<th>Equation</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0^0$</td>
<td>1</td>
<td>1</td>
<td>Piston</td>
</tr>
<tr>
<td>$Z_1^{-1}$</td>
<td>3</td>
<td>$2\rho \sin(\varphi)$</td>
<td>Vertical Tilt</td>
</tr>
<tr>
<td>$Z_1^{-1}$</td>
<td>2</td>
<td>$2\rho \cos(\varphi)$</td>
<td>Horizontal Tilt</td>
</tr>
<tr>
<td>$Z_2^{-2}$</td>
<td>5</td>
<td>$\sqrt{6}\rho^2 \sin(2\varphi)$</td>
<td>Oblique Astigmatism</td>
</tr>
<tr>
<td>$Z_2^0$</td>
<td>4</td>
<td>$\sqrt{3}(2\rho^2 - 1)$</td>
<td>Defocus</td>
</tr>
<tr>
<td>$Z_2^2$</td>
<td>6</td>
<td>$\sqrt{6}\rho^2 \cos(2\varphi)$</td>
<td>Vertical Astigmatism</td>
</tr>
<tr>
<td>$Z_3^{-1}$</td>
<td>7</td>
<td>$\sqrt{8}(3\rho^3 - 2\rho) \sin(\varphi)$</td>
<td>Vertical Coma</td>
</tr>
<tr>
<td>$Z_3^1$</td>
<td>8</td>
<td>$\sqrt{8}(3\rho^3 - 2\rho) \cos(\varphi)$</td>
<td>Horizontal Coma</td>
</tr>
<tr>
<td>$Z_4^0$</td>
<td>11</td>
<td>$\sqrt{5}(6\rho^4 - 6\rho^2 + 1)$</td>
<td>Spherical</td>
</tr>
</tbody>
</table>

Table A.1: Table of most common Zernike polynomials.

\[ \int_0^{2\pi} \int_0^1 Z_{m_1}^{n_1}(\rho, \varphi)Z_{m_2}^{n_2}(\rho, \varphi) \rho d\rho d\varphi = \pi \delta_{n_1, n_2} \delta_{m_1, m_2}. \quad (A.13) \]

The electric field takes the form

\[ E \propto e^{i\varphi(x,y)} = e^{i \sum a_{n,m}Z_{m}^{n}(\rho, \varphi)}. \quad (A.14) \]

The most well known polynomials are listed in Table A.1. The Noll index [86] is referred to here since it is a common notation used in optics design software, such as Zemax.
Acousto-optic modulators - not so much fun

Acousto-optic modulators (AOM) are an essential ingredient to any AMO lab. The essentially black box operation of these devices, RF in, diffraction out, is complicated by several material properties. In most cases an AMO lab will not run into issues with AOMs, but there have been several note-worthy technical issues that have come up while building this experiment.

- **Thermal Lensing** Thermal lensing is induced by the laser beam and optical absorption of the crystal. Since several high power lasers were used on this experiment, this effect was fairly prevalent. Typically, the high power laser beam is focused to a predetermined spot size after the AOM. For example, for generation of lattices, dipole traps, or even focusing onto an optical fiber. The main mechanism for thermal lensing is the temperature dependent change in the index of refraction \( \text{[62]} \). In this case the focal length of the thermal lens is given by

\[
f_{th} = \frac{2\pi w_0^2 \kappa}{P_{in} \eta} \left( \frac{dn}{dT} \right)^{-1}
\]

(B.1)

where \( \kappa \) is the thermal conductivity, \( P_{in} \) is the optical power, \( w_0 \) is the beam waist, \( \eta \) is
the fraction of absorbed optical power, and \( dn/dT \) is the change of index of refraction due to temperature. Increasing the beam size increases the focal length of the thermal lens, since the intensity goes down - causing local curvature to change. However, this naive approach to thermal lensing from the AOM will not decrease the amount of thermal lensing in the focused spot. To illustrate, suppose a beam with waist \( w_0 \) and power \( P_{in} \) is incident on the AOM. The AOM is a distance \( f \) away from a lens, which has a focal length \( f \). Using the ray transfer matrix method for Gaussian beam propagation, the displacement of the final waist from the case without thermal lensing is

\[
\delta z = -\frac{f^2}{f_{th}}. \tag{B.2}
\]

Now, it is easy enough to show that the final Rayleigh range after the focusing lens will be \( z_f = f^2/z_R \), where \( z_R \) is Rayleigh range of the incident beam. So, the fractional change relative to the Rayleigh range will be given by

\[
\frac{\delta z}{z_f} = -\frac{z_R}{f_{th}}. \tag{B.3}
\]

Since both \( f_{th} \) and \( z_R \) depend quadratically on the original beam waist, the thermal lens always produces the same positional focus change relative to the Rayleigh range. In essence, this means that no choice of initial beam waist or focusing lens \( f \) will change the amount of the thermal lensing in Rayleigh ranges. This is remarkable in some ways, as well as problematic. The only way to reduce the effect of thermal lensing is to use less optical power, reduce the absorption, or increase the thermal conductivity. These results are also true for any optic in the beam path - this is why it is so important to choose the best glass type when designing a high power optical system.
• **Astigmatism**

Astigmatism from an AOM comes from the finite sized aperture of the acoustic wave. The acoustic wave strength will drop off in some polynomial way and change the local index of refraction of the glass. This may be thought of as a thin piece of glass introducing some aberration on the beam. This effect seems to also be RF power dependent, which can make its compensation difficult. The best practice here is to keep the beam much smaller than the active aperture of the AOM - a factor of two (or more) smaller seems to work well. Reducing the beam waist can have significant effects in reducing the amount of aberration introduced by the AOM, especially for higher order aberrations.

• **RF Thermal Effects**

One of the most difficult effects to deal with is thermal effects from RF. Not only are they difficult to compensate, but they are highly power dependent. Thermal RF effects appear to be most strong in Tellurium Dioxide AOMs. These effects were explored in quartz AOMs and no RF thermal effects were seen. This is interesting considering Tellurium Dioxide has a much higher figure of merit \[24\] compared to quartz, which means Tellurium Dioxide requires less optical power for the same diffraction efficiency. The last phenomena that could cause RF thermal effects is absorption of the acoustic wave. If the absorption of the acoustic wave were significantly higher in TeO$_2$, the absorbed energy could locally heat the crystal.

The largest beam pointing appears to be in the vertical direction (perpendicular to the acoustic and optical wave). This can be attributed to a thermal gradient in the crystal due to poor thermal contact on one side of the crystal. This displacement can result in fiber coupling changes of approximately 50 percent over a few seconds.
Several techniques to handle thermal effects exist. For example, leaving the AOM on all the time and using an optical shutter mixed with pulsing the AOM on and off for a short time - this is used in this experiment for getting consistent imaging pulses. A more sophisticated method, which is of particular use when optical shutters cannot be used, consists of using a second frequency in the AOM. The second frequency diffracts the unwanted light to another position where it is dumped. By careful power hand-off between the desired frequency and the second frequency, RF thermal effects can be mostly removed [44]. This technique was implemented in this lab using Direct Digital Synthesis (DDS) and an FPGA. Double pass AOMs also reduce thermal effects since the beam pointing is compensated on the return pass through the AOM.

Interestingly, thermal effects seem to be reduced in some TeO₂ AOMs, which suggests better thermal management is important. The strongest thermal effects were seen with Gooch and Housego TeO₂ AOMs as well as Isomet Corporation. The smallest thermal effects were seen in IntraAction Corporation AOMs.
References


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