Essays in Financial Economics

Citation

Permanent link
http://nrs.harvard.edu/urn-3:HUL.InstRepos:42029651

Terms of Use
This article was downloaded from Harvard University’s DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA

Share Your Story
The Harvard community has made this article openly available. Please share how this access benefits you. Submit a story.

Accessibility
Essays in Financial Economics

Abstract

The first chapter studies how dealers affect the liquidity of the corporate bonds market. Using corporate bond transaction data with dealer identifiers, I find that a large component of the bid-ask spread is dealer-dependent. Customers incur different trading costs depending on which dealer handles their transactions. The dealer-specific component of trading costs is related to several characteristics of dealers, including their connectedness, credit risk, and portfolio risk. These findings are consistent with an inventory risk model of the bid-ask spread, whereby shocks to a dealer’s inventory cost affects the equilibrium bid-ask spread.

The second chapter, joint with Ali Ozdagli, studies how interest rates influence the investment behavior of insurance companies. Life insurance companies, the largest institutional holders of corporate bonds, tilt their portfolios towards higher-yield bonds when interest rates decline. This tilt seems to be primarily driven by an increase in duration rather than credit risk and insurers do not seem to increase the credit risk of their bonds as interest rates decline. Moreover, the duration gap between their assets and liabilities deviates from zero for extended periods of time both in negative and positive directions. We propose a new model of duration-matching under adjustment costs that conforms with these patterns and test other implications of this model.

The third chapter, joint with Luis Viceira, documents that the short-run correlations of returns across countries have increased substantially from 1986 to 2016, both for equities and bonds. We identify increased correlations of discount rate shocks, a transitory component of returns, as the main driver of the upward shift in stock return correlations. We
conclude that the increase in short-run correlations does not imply decreased long-horizon benefit for diversification in global equities market. In addition, we investigate the optimal intertemporal global portfolio choice problem for long horizon investors in the presence of permanent shocks and transitory shocks to asset values.
## Contents

Abstract .................................................................................................................. iii  
Acknowledgments .................................................................................................... xii  

1 Corporate Bond Liquidity and Dealer Inventory Cost ........................................ 1  
1.1 Introduction ........................................................................................................ 1  
1.2 Data and Summary Statistics ........................................................................... 7  
1.3 Estimating the Dealer-Specific Trading Cost ..................................................... 9  
  1.3.1 Measuring the Trading Cost through Markup ............................................. 9  
  1.3.2 Estimating the Dealer-Specific Trading Cost ........................................... 11  
1.4 Analyzing the Dealer Specific Trading Cost ...................................................... 18  
1.5 Model and Testable Implications ..................................................................... 27  
1.6 Testing the Model ............................................................................................. 39  
  1.6.1 Bid-Ask Spread and Dealer’s Connectedness ............................................ 40  
  1.6.2 Bid-Ask Spread and Dealer’s Credit Risk ................................................ 44  
  1.6.3 Bid-Ask Spread and Portfolio Risk ........................................................... 47  
1.7 Conclusion ......................................................................................................... 49  

2 Interest Rates and Insurance Company Investment Behavior ............................ 50  
2.1 Introduction ....................................................................................................... 50  
2.2 Data and Stylized Facts .................................................................................... 55  
  2.2.1 Data Construction ...................................................................................... 55  
  2.2.2 Measuring Life Insurers’ Tilt for Higher-Yield Bonds .............................. 56  
  2.2.3 Stylized Facts ............................................................................................. 57  
2.3 Duration Matching by Life Insurance Companies ............................................ 66  
  2.3.1 Duration Gap ............................................................................................. 66  
  2.3.2 Do Insurers Always Maintain Zero Duration Gap? ................................ 67  
2.4 Duration Matching with Adjustment Costs: The Target Duration Hypothesis 70  
  2.4.1 The Model .................................................................................................. 70  
  2.4.2 Testing the Model ....................................................................................... 73  
2.5 Results ............................................................................................................... 75  
  2.5.1 The Interest Rate and the Option to Surrender and Lapse ....................... 75
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5.2</td>
<td>Estimating Parameters in the Partial Adjustment Model</td>
<td>76</td>
</tr>
<tr>
<td>2.5.3</td>
<td>Adjustment Cost and the Speed of Adjustment</td>
<td>79</td>
</tr>
<tr>
<td>2.5.4</td>
<td>Active Duration Adjustment and the Option to Surrender and Lapse</td>
<td>84</td>
</tr>
<tr>
<td>2.6</td>
<td>Interest Rate Sensitivity of the Return on Equity: Model vs. Data</td>
<td>85</td>
</tr>
<tr>
<td>2.7</td>
<td>Discussion of Alternative Explanations</td>
<td>90</td>
</tr>
<tr>
<td>2.8</td>
<td>Conclusion</td>
<td>93</td>
</tr>
<tr>
<td>3</td>
<td>Global Portfolio Diversification for Long-Horizon Investors</td>
<td>95</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>95</td>
</tr>
<tr>
<td>3.2</td>
<td>Asset Return Decomposition</td>
<td>102</td>
</tr>
<tr>
<td>3.3</td>
<td>Global Portfolio Diversification with Time-Varying Discount Rates</td>
<td>104</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Model</td>
<td>105</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Correlated Return News and Portfolio Risk Across Investment Horizons</td>
<td>107</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Calibrated Example</td>
<td>110</td>
</tr>
<tr>
<td>3.4</td>
<td>Empirical Investigation of the Sources of Return Correlations in Global Capital Markets</td>
<td>113</td>
</tr>
<tr>
<td>3.4.1</td>
<td>VAR Specification and Estimation of Return Decomposition</td>
<td>113</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Summary Statistics and VAR Estimates</td>
<td>115</td>
</tr>
<tr>
<td>3.4.3</td>
<td>News Decomposition of Cross-Country Correlations of Stock and Bond Returns</td>
<td>118</td>
</tr>
<tr>
<td>3.5</td>
<td>Robustness Checks</td>
<td>125</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Alternative Measure of Market Integration</td>
<td>125</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Direct Measures of Cash Flow Correlations</td>
<td>127</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Correlated Stock Market Volatility News</td>
<td>130</td>
</tr>
<tr>
<td>3.6</td>
<td>The Impact of Real and Financial Integration on Long-Run Global Portfolio Diversification</td>
<td>132</td>
</tr>
<tr>
<td>3.6.1</td>
<td>The Risk of Globally Diversified Stock and Bond Portfolios Across Investment Horizons</td>
<td>132</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Optimal Global Equity Portfolio Diversification at Long Horizons</td>
<td>136</td>
</tr>
<tr>
<td>3.7</td>
<td>Conclusions</td>
<td>140</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>144</td>
</tr>
</tbody>
</table>

**Appendix A** Appendix to Chapter 1

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Supplementary Figures and Tables</td>
<td>152</td>
</tr>
<tr>
<td>A.2</td>
<td>Estimating Dealer-Specific Markup Using the Kalman Filter</td>
<td>154</td>
</tr>
<tr>
<td>A.3</td>
<td>Decomposing the Dealer-Level Markup</td>
<td>155</td>
</tr>
<tr>
<td>A.4</td>
<td>Supplementary Derivations of the Model</td>
<td>158</td>
</tr>
</tbody>
</table>
Appendix B  Appendix to Chapter 2  165
  B.1  Supplementary Figures and Tables ........................................ 165

Appendix C  Appendix to Chapter 3  167
  C.1  Asset Return Decomposition ............................................. 167
  C.2  Derivation of Results in Section 3.2 .................................. 174
  C.3  Symmetrical Model for Asset Returns .................................. 182
  C.4  Data Description .......................................................... 188
  C.5  VAR Model Estimation ...................................................... 193
  C.6  Fisher Transformation and Correlation Contribution ............... 204
  C.7  Semidefinite Programming Method ..................................... 205
  C.8  VAR Model with Stochastic Volatility ................................. 206
  C.9  Complementary Results of the Paper .................................. 215
### List of Tables

1.1 Summary Statistics ................................................. 8
1.2 Estimated Parameters for the Dealer Specific Markup Equation ....... 16
1.3 Comparing $R^2$ of Different Regression Specification on Bid-Ask Spread ... 17
1.4 Estimated Mean Reversion Speed in Relative Markup .................... 20
1.5 Relationship Between Markups from Dealer Buy and Dealer Sell .......... 26
1.6 Bid-Ask Spread and Dealer’s Connectedness .......................... 43
1.7 Bid-Ask Spread and Dealer’s Credit Risk ............................ 45
1.8 Bid-Ask Spread and Bond Volatility ............................... 48

2.1 Estimated Equity Duration and Convexity of the Life Insurance Sector .... 68
2.2 Estimating Parameters in Partial Adjustment Model ..................... 78
2.3 The Adjustment Cost and the Speed of Adjustment (Book Leverage) ....... 81
2.4 The Adjustment Cost and the Speed of Adjustment (Market Leverage) .... 82
2.5 Active Duration Adjustment and Option to Surrender and Lapse (Quarterly) 86
2.6 Active Duration Adjustment and Option to Surrender and Lapse (Annual) .. 87
2.7 Equity Duration: Model vs Data .................................. 89

3.1 Summary Statistics ................................................. 116
3.2 Correlation Summary Statistics ..................................... 117
3.3 Cross-Country Return Correlation Decomposition ......................... 120
3.4 Average $R^2$ Using Principal Components as Global Factors ............. 126
3.5 Direct Measure of Cash Flow Correlations ............................ 128
3.6 Optimal Global Equity Portfolio Allocations and Expected Utility ........ 138

A.1 Estimated Mean Reversion Speed in Relative Markup (from dealer buys) ... 152
A.2 Estimated Parameters of the State-Space Model ........................ 155

C.1 Correlation Summary Statistics ..................................... 191
C.2 Pooled VAR(1) Model Estimates .................................... 193
C.3 VAR(1) Model Estimates [Australia] .................................. 197
C.4 VAR(1) Model Estimates [Canada] .................................... 198
C.5 VAR(1) Model Estimates [France] .................................... 199
C.6 VAR(1) Model Estimates [Germany] .............................................. 200
C.7 VAR(1) Model Estimates [Japan] ................................................. 201
C.8 VAR(1) Model Estimates [United Kingdom] ................................. 202
C.9 VAR(1) Model Estimates [United States] ...................................... 203
C.10 Return Correlation Decomposition (Bonds vs. Stocks Within Countries and Across Countries) ................................................. 215
C.11 Optimal Global Equity Portfolio Allocations and Expected Utility (value weighted myopic portfolio) ........................................ 216
## List of Figures

1.1 Corporate Bond Inventory Level of Prime Dealers ................................. 3  
1.2 Transaction Price of a Sample Corporate Bond ................................... 10  
1.3 Estimated Dealer-Level Markup .......................................................... 14  
1.4 Cumulative Relative Cost from Strategic Trading Problem ....................... 23  
1.5 Markup and Dealer-Customer Trade Quantity ....................................... 34  
1.6 Interdealer-Network of Corporate Bond Trading .................................... 41  

2.1 Interest Rate and Excess Yield on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4) .................................................. 59  
2.2 Interest Rate and Duration Tilt on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4) .................................................. 62  
2.3 Interest Rate and Credit Risk Tilt on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4) .................................................. 64  
2.4 Time Varying Duration and Convexity of the Life Insurance Sector’s Equity 69  
2.5 Interest Rate and the Option to Surrender and Lapse ........................... 77  
2.6 Time Series of Duration Slack and Yield Slack ................................. 92  

3.1 Stock and Bond Correlations Across Countries .................................... 97  
3.2 Coefficient on $\sigma^{\text{EC}}_{|\text{DR}, DR}$ as a Function of Investment Horizon $k$ .......................... 109  
3.3 Annualized Portfolio Risk and Optimal Allocation to Risky Assets as a Function of Investment Horizon ........................................ 112  
3.4 Contributions of News Components to Overall Cross-Country Unexpected Return Covariance ................................................. 119  
3.5 Average Cross-Country Correlations of VAR News (Stocks) .................. 123  
3.6 Average Cross-Country Correlations of VAR News (Bonds) ................. 124  
3.7 Cross-Country Correlations of Proxies for Equity Cash Flow Fundamentals 129  
3.8 Cross Country Correlations of Stock Volatility News ........................... 131  
3.9 Equal Weighted Portfolio Risk as a Function of Investment Horizon (Equities and Bonds) .................................................. 134  

A.1 Estimated Dealer Level Markup (from dealer buys) ............................ 153
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.2</td>
<td>Kalman Filter Estimate of Dealer-Level Markup</td>
<td>156</td>
</tr>
<tr>
<td>B.1</td>
<td>CDS Spread Matched Excess Yield and 10 year Treasury Yield (2002Q1-2016Q4)</td>
<td>165</td>
</tr>
<tr>
<td>B.2</td>
<td>Time Series of Maturity Slack and Yield Slack</td>
<td>166</td>
</tr>
<tr>
<td>C.1</td>
<td>International credit spreads</td>
<td>207</td>
</tr>
<tr>
<td>C.2</td>
<td>International realized variance (RVAR) and expected variance (EVAR)</td>
<td>211</td>
</tr>
<tr>
<td>C.3</td>
<td>Cross country correlation of heteroscedastic VAR news (stocks)</td>
<td>212</td>
</tr>
<tr>
<td>C.4</td>
<td>Impact of stochastic volatility news on equity portfolio risk</td>
<td>213</td>
</tr>
<tr>
<td>C.5</td>
<td>Stock-bond correlations across and within countries</td>
<td>218</td>
</tr>
<tr>
<td>C.6</td>
<td>Annualized Portfolio Risk and Optimal Allocation to Risky Assets as a Function of Investment Horizon (2 symmetric countries)</td>
<td>219</td>
</tr>
<tr>
<td>C.7</td>
<td>Relative Contribution of Covariances of Return Components to Overall Return Covariance</td>
<td>220</td>
</tr>
<tr>
<td>C.8</td>
<td>Value Weighted Portfolio Risk as a Function of Investment Horizon (Equities and Bonds)</td>
<td>221</td>
</tr>
</tbody>
</table>
Acknowledgments

First, I am immeasurably grateful to my thesis committee members John Campbell, Luis Viceira, and Emil Siriwardane. As a group they provided invaluable advice, coaching and encouragement throughout the program. Luis first taught me how to do research and encouraged me to pursue a Ph.D. in finance. He has been my close mentor and collaborator over the years. John has been a role model for me, and I continue to learn from him on how to be better academically, personally, and professionally. Emil has taught me so much on the nuts and bolts of research, from framing a question, designing empirical methods, to strategies of writing a paper.

Second, I would like to thank Ali Ozdagli as a wonderful friend and close collaborator, and colleagues John Zhou, Yixin Chen, and Randy Cohen for the great discussions throughout my graduate career.

I would also like to thank the economics and finance faculty members Neil Shephard, James Stock, Jeremy Stein, Victoria Ivashina, Adi Sunderam, Robin Greenwood, and Lauren Cohen for their advice over the years.

In addition, I received invaluable feedback from Oliver Randall on chapter 1; Malcolm Baker, Bo Becker (discussant), Domenico Giannone (discussant), Ralph Koijen, Richard Rosen (discussant), and Stephane Verani on chapter 2; Christopher Polk, Christian Lundblad (discussant), and Gonzalo Rubio (discussant) on chapter 3.

Finally, I’m extremely grateful to my family. I could not have pursued and achieved this accomplishment without the support from my parents Zhouyuan Wang and Zhen Liu, and my wife Tsz Mui Kwan.
Chapter 1

Corporate Bond Liquidity and Dealer Inventory Cost

1.1 Introduction

Corporate bonds have become an increasingly important asset class in the United States. As of 2017Q4, the amount of US corporate debt outstanding was $9.0 trillion. This is a size comparable to the stock market ($32.1 trillion) and the treasury market ($14.5 trillion).\footnote{Corporate bond and treasury outstanding is from https://www.sifma.org/resources/research/bond-chart/ and equity outstanding is from the World Bank.} Unlike stocks, which are exchange traded, corporate bonds are traded over-the-counter with dealers acting as market-makers by quoting prices at which they are willing to buy or sell. Therefore, corporate bonds are generally less liquid and less transparent when compared to equities (Edwards, Harris, and Piwowar 2007, Bao, Pan, and Wang 2011). Given the important role that bond financing plays in our economy, it is of central importance to understand how dealers affect the liquidity of the corporate bonds market. This question is of interest to investors (e.g. hedge funds, asset management companies, insurance companies), who care about reducing the market impact of trade. It is also important to regulators and policy makers, who want to design policies to improve market liquidity and
financial stability.

In this paper, I show that dealers’ inventory risks are important drivers of market illiquidity. In the market microstructure literature, there are numerous papers on the asymmetric information theory of market liquidity, which describe the bid-ask spread as compensation for adverse selection costs (Glosten-Milgrom 1985, Kyle 1985). However, this literature ignores the inventory costs undertaken by dealers, and assumes that market-makers are risk neutral. Stoll (1978) was the first to propose an inventory risk model for the bid-ask spread. However, early research using intermediary inventory data in the stock market found little evidence of price pressures (Madhavan and Smidt 1991, 1993, Hasbrouck and Sofianos 1993). Even though more recent research has found price pressures of inventory in the stock market, this effect was not persistent. This may be partially explained by the fact that the market-makers in the stock market do not keep inventory positions on their portfolio for extended periods, generally limiting them to a half-life of less than a day (Hendershott and Menkveld 2014).

Do dealers’ inventory costs matter for market liquidity? The corporate bond market provides an ideal laboratory to test the importance of inventory risk. There are two main reasons for this. First, corporate bonds (OTC) are much less liquid than stocks (electronic), and the bid-ask spreads are significantly larger in magnitude. Second, unlike market-makers in stocks, who generally do not hold inventories overnight, dealers in the corporate bond market hold large inventories on their portfolio for extended periods of time. Figure 1.1 shows the corporate bond inventory level of prime dealers from the Federal Reserve Bank of New York, which demonstrates that dealers in the corporate bond market carry large inventories on their portfolios over time. The prime dealer inventory reached its peak of over 250 billion dollars at the end of 2007, and dropped to about 100 billion dollars post-crisis.

This paper uses dealer-level transaction data from the Financial Industry Regulatory Authority (FINRA) to understand the role dealers’ inventory costs play in corporate bond liquidity. This data has made it possible to link each transaction to the dealer who facilitates the trade, and allows me to test hypotheses related to dealer inventory at a granular level.
Figure 1.1: Corporate Bond Inventory Level of Prime Dealers

This figure plots the corporate bond inventory levels of prime dealers from 2001 to 2013. The data is from the Federal Reserve Bank of New York website.

https://www.newyorkfed.org/markets/gsds/search.html
The first empirical question I investigate is whether customers trading with different dealers (with different inventory costs) face different trading costs (bid-ask spread). To answer this question, I propose a methodology to estimate the dealer-specific component in trading cost and compare this component across different dealers at the same point in time. This allows me to empirically separate the inventory cost channel of the bid-ask spread from the asymmetric information channel. The asymmetric information channel predicts that bonds that are more informationally sensitive would have a higher bid-ask spread, but does not explain the systematic price deviations found at the dealer-level. To focus on the dealer-level variations, bond-by-time fixed effects are added to the panel data regressions. This specification allows me to tease out the adverse selection effect, which is likely to depend on bond and time, but not on dealers.

It turns out there are dealer-level deviations in trading costs which last for extended periods. To further explore dealer-level price deviation, I study a hypothetical trading problem. This problem allows customers to strategically trade with dealers who have low predicted trading costs. Hypothetically, I evaluate how much customers could potentially reduce trading costs by using this strategy over time.

The empirical findings support the hypothesis that inventory risk has an important effect on dealers’ pricing behavior. The results show that the dealer-specific trading costs (markup) vary significantly across dealers. In addition, the dealer-specific trading costs are highly persistent and predictable. When a specific dealer has a higher or lower trading cost than others, this pattern often persists for an extended period of time, with a two-week half-life of the converge time. This suggests that in addition to an asymmetric information effect, each dealer also has an effect on market liquidity, which might be related to his inventory cost. A hypothetical strategy designed to exploit this persistent dealer-level price deviation reduces annual trading costs by 11.3%, assuming customers can strategically trade with two dealers and rebalance their portfolio daily.

After documenting the above facts related to dealer-specific trading costs, I investigate the possible explanations for the variations in these costs. Using the theoretical framework
proposed by Randall (2015a), I derive several predictions linking dealer-level trading costs with dealer characteristics in an inventory cost model. The model predicts that three factors could contribute to the differences in dealer-level bid-ask spreads (trading cost) of the same bond.

The first factor is related to the funding constraints of dealers. Dealers often have large balance sheets of risky inventory assets. To fund these, dealers may rely heavily on short term collateralized loans such as repurchase agreements (Brunnermeier and Pedersen 2008). This means that it will be more difficult for dealers with tighter funding constraints to hold large inventories, and therefore they need to be compensated with a higher bid-ask spread.

The connectedness of a dealer in the inter-dealer trading network is the second factor affecting the dealer-level bid-ask spread. Inter-dealer trading gives dealers large risk-sharing benefits, since it is easier for them to manage inventory risks by trading immediately among themselves instead of waiting to trade with the next incoming customer (Ho and Stoll 1983). Dealers with better connections in the inter-dealer trading network are able to clear large inventory positions in a shorter period of time, thus facing lower inventory risks and requiring a lower bid-ask spread.

The third factor is associated with dealers’ portfolio risks. As dealers want to keep the risk of their aggregate inventory portfolio low, they care about how much marginal risk a given bond position adds to their existing bond portfolio. The dealer-level bid-ask spread is generally higher on a bond that adds more risk to the dealer’s current inventory.

As predicted by the model, I find that all three of these factors have significant effects on the corporate bond bid-ask spread. Different specifications, including time fixed effect, bond fixed effect, and bond-by-time fixed effect, are used to examine the quantitative magnitudes of these effects. The bond-by-time fixed effect allows for the absorption of each individual bond’s exposure to any systematic factors, which gives a clean evaluation of the net effect coming from individual dealer’s inventory cost. Section 1.6 presents the detailed results. First, I evaluate the association between a dealer’s network centrality and the dealer-specific bid-ask spread. By comparing the dealer-level trading costs of the same bond within the
same month across dealers, I find that dealers with one standard deviation higher centrality than others charge 4.7 bps lower. Second, I evaluate the effect of dealers’ credit risk on dealer-specific bid-ask spreads. It is estimated that a one percentage point increase in a dealer’s CDS spread, capturing higher dealer-level credit risk, would be associated with a 13.8 bps increase in the bid-ask spread of a given corporate bond. When controlled for the bond-by-time fixed effect, this magnitude becomes 3.3 bps. This means that for the same bond at the same point in time, a dealer with a higher credit risk (one percentage point higher CDS spread) charges 3.3 bps higher in bid-ask spread. Finally, I examine the effect of portfolio risk on the bid-ask spread. In the cross section, when monthly bond volatility goes up by 1%, the bid-ask spread goes up by 11.9 bps.

This paper is also related to a recent literature examining how dealers’ inventory risks influence the pricing and liquidity in OTC markets. Randall (2015a) proposes a modeling framework that links dealers’ inventory costs to the prices and quantities of inter-dealer trades and customer-dealer trades. In section 1.5, I extend Randall’s model by providing microfoundations of dealers’ inventory costs, and give predictions that link the bid-ask spread or markup to dealer characteristics. Randall (2015b) finds that as aggregate inventory cost increases, the dealer sector as a whole adds more investment grade bonds to their inventories, and lowers the percentage of high yield bonds, suggesting a flight to quality. However, due to data limitations, Randall (2015b) does not track the inventories at the dealer level or explore variations of pricing behavior in the cross section of dealers. Acquiring transaction data that includes dealer identifiers allows this paper to fully examine these dealer-level variations. Friewald and Nagler (2016) study the implications of inventory models on the cross section of bond returns. They find that bonds with high average inventories across dealers are underpriced due to selling pressure. A high-minus-low inventory-sorted portfolio delivers a risk-adjusted return of 21 bps per week.

The finding that inventory risk is an important determinant of corporate bond liquidity has implications for regulators and policy makers. Consider, for instance, the liquidity management problem for an economy in a liquidity crisis in which trading costs are
extremely high and in which the market freezes. In such an environment, it is crucial to alleviate dealers’ inventory risks and reduce their costs of liquidity provision. The current framework suggests two possible approaches to this problem. The first one is to loosen dealers’ margin/funding constraints through capital injections, which lowers their borrowing costs. The second approach is to directly purchase risky assets from dealers’ inventory portfolios. This method is consistent with several actions taken by the Federal Reserve following the 2008 crisis, such as quantitative easing and the Troubled Asset Relief Program (TARP), which aimed to reduce the cost of capital and remove troubled assets from dealers’ portfolios.

The remainder of the paper is organized as follows. Section 1.2 introduces the data and methodology. Section 1.3 discusses how I estimate the dealer-specific trading cost. Section 1.4 conducts a detailed analysis of the estimated dealer-specific trading costs. Section 1.5 introduces the model and testable implications derived from it. Section 1.6 presents empirical results from testing the model. Section 1.7 concludes the paper.

1.2 Data and Summary Statistics

The primary source of data for this paper is the Academic Corporate Bond TRACE (academic TRACE) Dataset. This data has transaction reports for all corporate bonds dating back to July 2002. In addition, it has anonymous identifiers of the counterparties in each transaction. For dealer-to-dealer transactions, anonymous identifiers for both buyers and sellers are reported. For customer-to-dealer or dealer-to-customer transactions, only dealer identifiers are disclosed. The data is cleaned using the filtering algorithm from Dick-Nielsen (2009). The algorithm filters cancellations, corrections and inter-dealer transactions, which avoids the double-counting of trading volume.

The information on issuance and amount outstanding of bonds come from Mergent FISD. I also use the NAIC bond transaction data from FISD, which has historical transactions of corporate bonds by insurance companies, including the names of dealers in each transaction. The daily CDS spreads come from Markit.
Table 1.1: Summary Statistics

The table reports the summary statistics of the sample of corporate bonds and the characteristic of dealers. The sample starts in July 2002 and ends in March 2015. The dealer’s CDS spread $CDS_{d,t}$ is the average CDS spread of dealer $d$ in month $t$. The bond bid-ask spread $S_{i,d,t}$ is the bid-ask spread of bond $i$ by dealer $d$ in month $t$. The markup from sell $MU^S_{i,d,t}$ is the difference between the price at which the dealer sell to customers and the inter-dealer price. And the markup from buy $MU^B_{i,d,t}$ is the difference between the inter-dealer price and the price at which the dealer buy from customers.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>dealer’s CDS spread $CDS_{d,t}$ (%)</td>
<td>1.12</td>
<td>2.24</td>
<td>0.32</td>
<td>0.80</td>
<td>1.37</td>
</tr>
<tr>
<td>bond bid-ask spread $S_{i,d,t}$ (bps)</td>
<td>138.20</td>
<td>159.11</td>
<td>34.17</td>
<td>84.49</td>
<td>185.55</td>
</tr>
<tr>
<td>bond markup from sell $MU^S_{i,d,t}$ (bps)</td>
<td>67.24</td>
<td>145.60</td>
<td>5.52</td>
<td>31.89</td>
<td>103.63</td>
</tr>
<tr>
<td>bond markup from buy $MU^B_{i,d,t}$ (bps)</td>
<td>-59.50</td>
<td>161.26</td>
<td>-89.13</td>
<td>-30.66</td>
<td>-3.16</td>
</tr>
<tr>
<td>bond monthly volatility $r^\text{Bond}_{i,t}$ (%)</td>
<td>4.89</td>
<td>7.20</td>
<td>1.61</td>
<td>2.95</td>
<td>5.56</td>
</tr>
</tbody>
</table>

I use an algorithm to assign daily CDS spreads to each anonymous dealer in academic TRACE data. The algorithm does not recover the actual identity of each dealer, and the dealers in my final dataset remain anonymous. I first manually create a link between the dealers in the NAIC bond transaction data and those in the Markit CDS data. After this link is created, I replace the names of dealers in both datasets with randomly generated anonymous identifiers. Then I match the transactions in academic TRACE and NAIC bond transactions by CUSIP, price, volume and date. This allows me to construct a link between the anonymous dealer identifier in academic TRACE and my self-created anonymous dealer identifier in NAIC bond transactions. Finally, I assign the daily Markit CDS spread to academic TRACE data using the self-created anonymous identifier. The final output of this algorithm adds an additional variable, namely dealers’ daily CDS spreads, to the academic TRACE data. About 44.1% of the transactions in academic TRACE are assigned with dealers’ CDS spreads.

Table 1.1 reports the summary statistics of the main variables used in the empirical part of this paper. The details on constructing these variables are provided in Section 1.6.
1.3 Estimating the Dealer-Specific Trading Cost

This section describes the methodology used to estimate the dealer-specific trading costs. Using this methodology, I am able to compare the transaction cost of trading with each individual dealer versus that of trading with other dealers, which allows for further study of the factors driving the dealer-specific trading costs.

1.3.1 Measuring the Trading Cost through Markup

The empirical literature has proposed many measures of corporate bond illiquidity (trading cost). The most commonly used one is the Amihud (2002) illiquidity measure, which was originally designed for the stock market. It is calculated as the ratio of daily return to daily dollar volume. This measure captures the price impact $\lambda$, as shown in Kyle (1985). Bao, Pan and Wang (2011) proposed a method for measuring the bid-ask spread in corporate bonds by using the negative autocovariance of price difference. This measure captures the bid-ask bounce $\gamma$ as in Roll (1984).

However, neither method is able to capture the features of a dealership market, where all customers only trade with dealers. In fact, all the corporate bond transactions can be placed in one of the following three categories: dealers buying from customers, dealers selling to customers, and inter-dealer trade.

One salient empirical feature is that these three types of trades are carried out at different prices. In general, dealers sell to customers at a price higher than the inter-dealer price, and buy from customers at a price lower than inter-dealer price. More explicitly, if one considers a setting where dealer $d$ is trading bond $i$ at time $t$, the feature from the data can be expressed as:

$$p_{i,d,t}^S > p_{i,d,t}^{\text{interdealer}} > p_{i,d,t}^B$$  \hspace{1cm} (1.1)

Figure 1.2 shows the prices at which dealers sell to (red circle) and buy from (green circle) customers, as well as the inter-dealer prices (blue plus) of a single corporate bond on a typical trading day. The average inter-dealer price on this day was about $100, while dealers sold to customers at an average price of $101 and bought from customers at
The figure plots the transaction price of a sample corporate bond (issued by FORD) over a day (Mar 02, 2005). The blue plus sign indicates the prices of inter-dealer transactions. The red circle indicates the prices at which dealers sell bonds to customers, and the green circle indicates the prices at which dealers buy from customers.

Figure 1.2: Transaction Price of a Sample Corporate Bond
an average price of about $99. One measure of trading cost that captures this type of feature is the “markup”, which is defined as the difference between the dealer-customer price and the inter-dealer price (Randall 2015a). Specifically, the markup of dealer \( d \) selling bond \( i \) is \( \text{MU}_{i,d,t}^S = P_{i,d,t}^S - P_{i,d,t}^{\text{Interdealer}} \), and the markup of dealer \( d \) buying bond \( i \) is \( \text{MU}_{i,d,t}^B = P_{i,d,t}^B - P_{i,d,t}^{\text{Interdealer}} \). The difference between the two, \( S_{i,d,t} = \text{MU}_{i,d,t}^S - \text{MU}_{i,d,t}^B \), is the bid-ask spread earned by dealer \( d \) on bond \( i \).

1.3.2 Estimating the Dealer-Specific Trading Cost

In this section, I describe the methodology used to estimate the dealer-specific trading cost, which is customers’ estimated cost of buying and selling bonds through a specific dealer. We also call it “estimated dealer-level markup.” Investors will pay a higher costs if they buy bonds from dealers with a higher estimated markup of selling. This section then looks at why the costs investors face can be different across dealers at the same point in time, and empirically examines several drivers of dealer-specific trading costs.

Recall that the “markup” is the difference between the dealer-customer price and the inter-dealer price, and that there are two types of dealer-customer price: the price at which dealers sell to customers, and the price at which dealers buy from customers. To avoid redundancy, I use the former to illustrate the methodology of estimating the markup (i.e. sell markup). The markup calculated from the the latter (i.e. buy markup) is similar. I assume that dealer \( d \) sells bond \( i \) to customers at time \( t_n \) (day \( t \), timestamp \( n \)) at a price:

\[
P_{i,d,t_n}^S = \text{MU}_{i,d,t_n}^S + P_{l,t_n}^I
\]

where \( P_{i,t_n}^I \) is the inter-dealer price, and \( \text{MU}_{i,d,t_n}^S \) is the difference between the dealer’s sell price and the inter-dealer price. The inter-dealer market is assumed to be competitive, and \( P_{i,t_n}^I \) is a common price that does not depend on a specific dealer \( d \). I further assume that the difference between the dealer-customer price and the inter-dealer price has a bond-specific
component and a dealer specific component:

\[
MU_{i,d,t}^S = C_{i,t}^S + M_{d,t}^S + F_{i,d,t}^S + \epsilon_{i,d,t}^S
\]  

(1.3)

The error term \(\epsilon_{i,d,t}^S\) is assumed to be i.i.d. across bonds with a zero mean and a bounded variance. The term \(M_{d,t}^S\) captures the idea that each dealer might price bonds differently due to differential exposure to inventory risks. At each point in time, a dealer facing higher inventory cost might demand a higher markup as compensation for risk. Thus inferring how much \(M_{d,t}^S\) differs in the cross section helps us understand how much inventory cost matters for a dealer’s pricing policy. To be more general, we include the term \(F_{i,d,t}^S\) which can depend on both the dealer and the bond. This term captures the idea that a particular dealer might have higher/lower markup for one bond, based on his cost of providing liquidity in that bond.

Equation 1.3 can be averaged across all transactions in day \(t\), to get a daily estimate:

\[
P_{i,d,t}^S = MU_{i,d,t}^S + P_{i,t}^I
\]

\[
= C_{i,t}^S + M_{d,t}^S + F_{i,d,t}^S + \epsilon_{i,d,t}^S + P_{i,t}^I
\]

(1.4)

The dealer-level markup is defined as:

\[
MU_{d,t}^S \triangleq \frac{1}{I} \sum_{i=1}^{I} \left( p_{i,d,t}^S - p_{i,t}^I \right)
\]

(1.5)

which measures the average cost of buying bond \(i\) from dealer \(d\) on day \(t\).

In the Appendix, we show that under reasonable assumptions, the dealer-level markup can be decomposed into the following two components:

\[
MU_{d,t}^S = C_{i}^S + M_{d,t}^S
\]

(1.6)

We are interested in understanding the differences in \(M_{d,t}^S\) across dealers. Equation 1.6 suggests that for two dealers \(d_1\) and \(d_2\) we have:

\[
MU_{d_1,t}^S - MU_{d_2,t}^S = M_{d_1,t}^S - M_{d_2,t}^S
\]

(1.7)
In order to compare $M_{d,t}$ across dealers, one can instead estimate the dealer-level markup $MU_{d,t}$ and compare the estimated markup across dealers. By definition, $MU_{d,t} = \frac{1}{T} \sum_{i=1}^{T} (p_{i,d,t}^S - p_{i,t}^I)$ can be calculated directly from the data by first estimating daily volume weighted average prices (VWAP) for dealer $d$ from dealer-customer trades and inter-dealer trades, and then averaging across bonds.

However, when the dealer-level markups $MU_{d,t}$ are calculated directly from the data the results are very noisy, as plotted in Figure 1.3 (Panel A) in blue. To obtain a smoother estimate of dealer-level markups, I use the following simple linear filtering algorithm. One can also use a standard filtering method such as the Kalman filter (results provided in the Appendix). The benefit of this simple algorithm is that the parameters have intuitive meanings. The linear filtering model assumes that $MU_{d,t+1} = \mu_{d,t}^S + \nu_{t+1}$ where $\mu_{d,t}^S = E \left( MU_{d,t+1}^S | \mathcal{F}_t \right)$ is the smoothed estimate of markup, $MU_{d,t+1}^S$ is calculated from data, and $\nu_{t+1} \sim N(0, \sigma_d^2)$. The conditional mean of markup is assumed to follow the process:

$$\mu_{d,t}^S = \omega_d + \alpha_d \mu_{d,t-1}^S + \beta_d MU_{d,t}^S$$  \hspace{1cm} (1.8)

where $\alpha_d$ is the persistence of trading cost, and $\beta_d$ captures the update of new information each period.

I estimate the parameters of the model $(\omega_d, \alpha_d, \beta_d, \sigma_d)$ for each dealer ($d = 1, ..., D$) using MLE. The point estimates are reported in Table 1.2 for the top four dealers by trading volume over our whole sample. These estimates are quite consistent across dealers, with $\alpha_d \approx 0.9$ and $\beta_d \approx 0.1$. This suggests that the markup is a highly persistent process. The standard deviation $\sigma_d$ ranges from 29.8 to 36.3 bps across the four dealers. The conditional mean $\mu_{d,t}^S = E \left( MU_{d,t+1}^S | \mathcal{F}_t \right)$ estimated from the model is called the “estimated dealer-level markup.”

Panel A of Figure 1.3 plots the time series of both the dealer level markup $MU_{d,t}^S$ and the estimated dealer-level markup $\mu_{d,t}^S$ for a single dealer. After the filtering, the estimated dealer level markup (orange) is much smoother than the original markup calculated from the data (blue).
Figure 1.3: Estimated Dealer-Level Markup

The figure plots the time series of dealer-level markup. Panel A plots the time series of both the dealer-level markup $MU^S_{d,t}$ (blue) and estimated dealer-level markup $\mu^S_{d,t}$ (orange) for a single dealer. Panel B plots the estimated dealer-level markup $\mu^S_{d,t}$ for the top four dealers in our data. Panel C plots the relative markup $RM^S_{d,t} = \mu^S_{d,t} - \frac{1}{D-1} \sum_{i \neq d} \mu^S_{i,t}$ of the top four dealers.
Figure 1.3 (Continued) Panel C
Table 1.2: Estimated Parameters for the Dealer Specific Markup Equation

The table reports point estimates of parameters \((\omega_d, \alpha_d, \beta_d, \sigma_d)\) of the filtering model for each dealer \((d = 1, \ldots, D)\) using maximum likelihood. The point estimates are reported for the top four dealers by trading volume over our whole sample. The units of \(\omega_d\) and \(\sigma_d\) are converted into bps.

<table>
<thead>
<tr>
<th>Dealer 1</th>
<th>Dealer 3</th>
<th>Dealer 2</th>
<th>Dealer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_d)</td>
<td>0.325</td>
<td>0.328</td>
<td>0.466</td>
</tr>
<tr>
<td>(\alpha_d)</td>
<td>0.899</td>
<td>0.909</td>
<td>0.874</td>
</tr>
<tr>
<td>(\beta_d)</td>
<td>0.094</td>
<td>0.085</td>
<td>0.117</td>
</tr>
<tr>
<td>(\sigma_d)</td>
<td>31.525</td>
<td>36.344</td>
<td>33.410</td>
</tr>
</tbody>
</table>

Panel B of Figure 1.3 compares the estimated markups of the top four dealers. There are two main observations from this plot. First, the estimated markup does not differ much from one dealer to another. As \(MU_d^{S} = CS_t + MS_d^{S}\), this suggests that the common component \(CS_t\) drives most of the variations in markup. In fact, the components \(CS_t\) and \(MS_d^{S}\) can be separately identified by imposing a constraint that \(1/D \sum_{d=1}^{D} MS_d^{S}, t = 0\) (i.e. the cross sectional mean of dealer specific components in the markups is zero). Second, although the estimated markup does not differ much from one dealer to another, the deviation of a single dealer’s estimated markup from the average markup of the other dealers (i.e. the relative markup) can last for an extended period of time, usually weeks or sometimes even months. For example, the estimated markup of Dealer 2 was higher than others during the peak of the crisis, and this wedge did not disappear until a few weeks later. This suggests that the dealer-specific component of the markup \(MS_d^{S}\) has an economically meaningful magnitude and differs from one dealer to another, potentially due to idiosyncratic shocks to each dealer’s inventory cost.

Similarly, the dealer-level markup can be estimated using dealer buys. As previously stated, in general, dealers buy at a price lower than the inter-dealer price, which results in
The table compares the $R^2$ of different regression specifications and the contributions of different components in explaining the bid-ask spread, constructed at bond-dealer-month level. The right hand side of the regressions have dummy variables representing different fixed-effects. We add time, bond, dealer, bond-by-time, and dealer-by-time fixed effects respectively. The contribution of each component is defined as the variance of each estimated fixed-effect divided by the variance of the bid-ask spread.

<table>
<thead>
<tr>
<th>Regressions</th>
<th>Specification 1</th>
<th>Adjusted $R^2$</th>
<th>40.18%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{i,d,t} = D_t + D_i + D_d + \varepsilon_{i,d,t}$</td>
<td>Specification 2</td>
<td>Adjusted $R^2$</td>
<td>64.84%</td>
</tr>
<tr>
<td>$S_{i,d,t} = D_{i,t} + D_{d,t} + \varepsilon_{i,d,t}$</td>
<td>Component Contributions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2(D_t) / \sigma^2(S_{i,d,t})$</td>
<td>15.4%</td>
<td>$\sigma^2(D_i) / \sigma^2(S_{i,d,t})$</td>
<td>28.4%</td>
</tr>
<tr>
<td>$\sigma^2(D_d) / \sigma^2(S_{i,d,t})$</td>
<td>6.2%</td>
<td>$\sigma^2(\varepsilon_{i,d,t}) / \sigma^2(S_{i,d,t})$</td>
<td>51.1%</td>
</tr>
<tr>
<td>$\sigma^2(D_{i,t}) / \sigma^2(S_{i,d,t})$</td>
<td>55.5%</td>
<td>$\sigma^2(D_{d,t}) / \sigma^2(S_{i,d,t})$</td>
<td>7.7%</td>
</tr>
<tr>
<td>$\sigma^2(\varepsilon_{i,d,t}) / \sigma^2(S_{i,d,t})$</td>
<td>29.7%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As previously explained, the bid-ask spread earned by dealer $d$ on bond $i$ in month $t$ is $S_{i,d,t} = M_{i,d,t}^S - M_{i,d,t}^B$. One can check how much of this bid-ask spread is explained by the dealer-level component versus the bond-level component by adding different fixed effects in a regression framework. Table 1.3 compares the $R^2$ of different specifications, and the contribution of each components in the regression. The left hand side of the regression is $S_{i,d,t}$, the bid-ask spread constructed at bond-dealer-month level. We define the contribution of each component as the variance of each estimated fixed-effect divided by the variance of the bid-ask spread. For example, in the specification $S_{i,d,t} = D_t + D_i + D_d + \varepsilon_{i,d,t}$, the variance of the bid-ask spread could be decomposed into the following:

a negative markup. Figure A.1 in the Appendix plots the estimated dealer-level markup using dealer buys from the top eight dealers in our data. The stylized patterns here are similar to Figure 1.3 — a common component drives most of the variations in markup, but there is also a dealer-level idiosyncratic component, which generally persists for weeks.
\[ \sigma^2(S_{i,t}) = \sigma^2(D_i) + \sigma^2(D_t) + \sigma^2(D_d) + \sigma^2(\varepsilon_{i,d,t}) + \text{covariances} \]

where the “covariances” include twelve covariance terms. The contribution of time fixed effect is \( \frac{\sigma^2(D_t)}{\sigma^2(S_{i,t})} = 15.4\% \). While bond fixed effect and dealer fixed effect respectively contribute 28.4\% and 6.2\% to the total variations in bid-ask spread. The unexplained part \( \sigma^2(\varepsilon_{i,d,t}) \) accounts for half of the variations in the bid-ask spread. Unsurprisingly, a large fraction of the bid-ask spread is determined by the nature of the bond \( (D_i) \), such as its credit risk. However, the dealer-level component \( (D_d) \) also determines a significant portion of the bid-ask spread, which is about 22\% \((= 6.2/28.4)\) of the magnitude of the bond-level component. The “covariances” only accounts for about 1\% of the variations in bid-ask spread.

The second specification include both the bond-by-time fixed effect and the dealer-by-time fixed effect. The former explains 55.5\% and the latter explains 7.7\% of the variations in the bid-ask spread. This again suggests that a significant part of the bid-ask spread is dealer-dependent, and different dealers could require very different trading costs (bid-ask spreads) on the same bond as compensation for liquidity provision.

### 1.4 Analyzing the Dealer Specific Trading Cost

This section analyzes the estimated dealer-specific markup \( \mu_{d,t} \). I show that there are substantial variations in the estimated markup across dealers, and estimate the half-life of the relative markup. In addition, the future realized markup is predictable using the current period’s estimated markup. This predictability is tested using a strategic trading decision problem. The results again corroborate my finding that the markup of a dealer can deviate from other dealers for nontrivial periods of time, suggesting that the effect of inventory risks on dealer-level trading costs might be large.
Mean Reversion Speed in Relative Markup

Recall that our estimated markup is $\mu_{d,t}^S = E \left( MU_{d,t+1}^S | \mathcal{F}_t \right)$. I define dealer $d$’s markup relative to other dealers (relative markup) as

$$
RM_{d,t}^S = \mu_{d,t}^S - \frac{1}{D-1} \sum_{i \neq d} \mu_{i,t}^S
$$

(1.9)

This variable captures how much the estimated markup of dealer $d$ deviates from other dealers.

Figure 1.3 Panel C plots the relative markup for each of the top four dealers. The relative markup for each dealer is a mean-reverting process and generally fluctuate between 60bps and -60bps. Table 1.4 Panel A reports the summary statistics of the relative markups. Dealer 1 has a negative mean of -8.75bps, while Dealers 2 and 3 have slightly positive means. The relative markups have standard deviations of roughly 10bps, and they exhibit excess kurtosis.

The mean-reverting speed of relative markup can be estimated from the following regression:

$$
RM_{d,t+1}^S - RM_{d,t}^S = \theta_0 + \theta_1 (RM_{d,t} - \bar{RM}_d) + \epsilon_{t+1}
$$

(1.10)

where $\bar{RM}_d$ is the average value of the relative markup. Table 1.4 reports the estimated speed of mean reversion $\theta_1$ and its 95% confidence interval. To get a better sense of the magnitude of these estimates, one can compute the half-life of this mean reversion process $\frac{-\ln 2}{\ln (1+\theta_1)}$. The average half-life of the relative markup is 2.3 weeks. These estimates suggest that the dealer-level adjustment speed of the markup is slow, partially reflecting the fact that the inventory adjustment in the corporate bond market is slow. This means that dealers can bear large inventory risks. The estimated magnitude of a half-life using the dealer-level markup is comparable to the half-life estimated by previous research using the quantity of inventory level. By directly analyzing the inventory level, Friewald and Nagler (2016) find that the average half-life of individual bond inventories is about five to six weeks, and Dick-Nielsen and Rossi (2018) find that the half-life of an inventory adjustment process following a bond downgrade is about five to seven days. These estimates of half-life are in
Table 1.4: Estimated Mean Reversion Speed in Relative Markup

Panel A reports the summary statistics of the relative markup $RM_{d,t}^S$. The units are in basis points. Panel B reports the estimated mean reversion speed of the relative markup $RM_{d,t}^S$ from the following regression

$$RM_{d,t+1}^S - RM_{d,t}^S = \theta_0 + \theta_1 (RM_{d,t} - \bar{RM}_d) + \epsilon_{t+1}$$

We estimate the regression separately for each dealer. The half-life of this mean reversion process is $\frac{-\ln 2}{\ln (1 + \theta_1)}$, and we report the units in weeks. For all four dealers, the estimates for $\theta_0$ are close to zero and statistically insignificant. To avoid redundancy, we do not report $\hat{\theta}_0$ here.

<table>
<thead>
<tr>
<th>Panel A</th>
<th>Dealer 1</th>
<th>Dealer 2</th>
<th>Dealer 3</th>
<th>Dealer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>p25</td>
<td>-13.36</td>
<td>-3.97</td>
<td>-1.41</td>
<td>-5.61</td>
</tr>
<tr>
<td>p50</td>
<td>-7.78</td>
<td>1.07</td>
<td>3.86</td>
<td>0.02</td>
</tr>
<tr>
<td>p75</td>
<td>-3.18</td>
<td>7.25</td>
<td>11.19</td>
<td>5.44</td>
</tr>
<tr>
<td>mean</td>
<td>-8.75</td>
<td>2.58</td>
<td>5.03</td>
<td>-0.11</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>8.78</td>
<td>10.91</td>
<td>10.34</td>
<td>9.77</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.74</td>
<td>2.14</td>
<td>-0.04</td>
<td>-0.28</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.54</td>
<td>12.64</td>
<td>5.88</td>
<td>5.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>Dealer 1</th>
<th>Dealer 2</th>
<th>Dealer 3</th>
<th>Dealer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>-0.070</td>
<td>-0.053</td>
<td>-0.050</td>
<td>-0.062</td>
</tr>
<tr>
<td>95% CI of $\theta_1$</td>
<td>[-0.092, -0.049]</td>
<td>[-0.071, -0.034]</td>
<td>[-0.065, -0.034]</td>
<td>[-0.077, -0.048]</td>
</tr>
<tr>
<td>Half-Life (weeks)</td>
<td>1.901</td>
<td>2.570</td>
<td>2.727</td>
<td>2.151</td>
</tr>
<tr>
<td>95% CI of Half-Life</td>
<td>[1.438, 2.774]</td>
<td>[1.894, 3.954]</td>
<td>[2.073, 3.954]</td>
<td>[1.725, 2.841]</td>
</tr>
</tbody>
</table>
enormous contrast to the stock market inventory, which has a half-life of less than a day (Hendershott and Menkveld 2014).

Table A1 in the Appendix shows the estimated mean reversion speed in the relative markup from dealer-buys. These results are consistent with that of dealer-sells, with an average half-life of about two weeks.

A Hypothetical Strategic Trading Problem

It has been documented in the earlier sections that the estimated relative markup for one dealer can deviate from its mean for an extended period of time. This suggests that if the estimated cost of buying from dealer \( d \) (i.e. \( \mu_{S_d}^{d_i} \)) is high today, it is most likely to remain high tomorrow as well. This naturally leads to a hypothetical question: could investors reduce trading costs tomorrow, by strategically trading with dealers who have low estimated markups today? The answer is not obvious, because even if the predicted markup \( \mu_{S_d}^{d_i} \) is low today, the realized markup tomorrow \( MU_{S_d}^{d_i} = \mu_{S_d}^{d_i} + v_{t+1} \) might not necessarily be low, due to the uncertainty in the realization of \( v_{t+1} \). As I will show in this section, strategically trading with dealers with low predicted markup will significantly reduce the cumulative trading costs. The goal of this section is not to find an implementable strategy to reduce trading cost, but to use this hypothetical trading problem to help us understand how much pricing behavior can differ across dealers. This section begins by setting up the problem as follows.

Setup of the Strategic Trading Problem

1. On day \( t \), find the dealer who has the highest and lowest predicted markup for the following day:

   \[
   \begin{align*}
   d_{\text{min}} &= \arg\min_d \left[ \mu_{S_d}^{d_i} \right] = \arg\min_d \left[ \omega_d + \alpha_d \mu_{d,t-1}^S + \beta_d MU_{d,t}^S \right] \\
   d_{\text{max}} &= \arg\max_d \left[ \mu_{S_d}^{d_i} \right] = \arg\max_d \left[ \omega_d + \alpha_d \mu_{d,t-1}^S + \beta_d MU_{d,t}^S \right]
   \end{align*}
   \]

2. On day \( t + 1 \), strategically trade with dealer \( d_{\text{min}} \) and avoid trading with dealer \( d_{\text{max}} \).
thereby reducing realized cost by $MU_{d_{\text{max}},t+1} - MU_{d_{\text{min}},t+1}$

3. Repeat the above procedures over time, assuming a daily rebalance.

The reduced cumulative relative cost of using this strategy is:

$$\pi_{0\rightarrow T} = \prod_{t=0}^{T} \left(1 + MU_{d_{\text{max}},t+1} - MU_{d_{\text{min}},t+1}\right)$$

(1.11)

If the realized markups are not predictable over the next day, the cumulative relative cost $\pi_{0\rightarrow T}$ are expected to be constant over time. If there are predictable components which are captured by the model, then $\pi_{0\rightarrow T}$ will increase over time.

Figure 1.4 plots the time series of $\pi_{0\rightarrow T}$, starting from one at the beginning of our sample. The cumulative relative costs increase over time, confirming that the markups can be different across dealers for extended periods of time. In the context of the model to be discussed in section 1.5, this suggests that differences in dealers’ inventory costs have large effects on their pricing behavior. When investors have relationships with two dealers, this strategic trading problem yields a cumulative cost difference of 11.3% per year. The cost differences are measured by comparing the best-case scenarios (i.e. trading with Dealer $d_{\text{min}}$) and the worst-case scenarios (i.e. trading with Dealer $d_{\text{max}}$), assuming a daily rebalance. This cumulative cost differences increase as investors have relationships with more dealers with whom they can strategically choose to trade.

However, in reality, investors do not always switch among multiple dealers for their trades. Hendershott et al. (2017) find that on an annual basis, roughly 30% of insurers only trade with a single dealer. One possible reason for this is that although investors may be able to reduce their transaction costs by trading strategically, there are also higher costs on informational leakage about their orders when they start to trade with more dealers. In addition, there might be a relationship component in customer-dealer trading. Customers may value their relationships with the current dealers, so they do not always switch to other dealers even if this might reduce their costs in the short term, in the hope that current dealers will provide them with better services in the future.

Finally, note that the cumulative relative costs are growing at a faster rate in the post-
The figure plots cumulative relative cost $\pi_{0\rightarrow T} = \prod_{t=0}^{T} \left( 1 + MU_{d_{\text{max}},t+1} - MU_{d_{\text{min}},t+1} \right)$ over time. The plot compares strategic trading decisions, assuming investors could trade with two, three, or four dealers. For the case where investors could trade with only 2 dealers, we compute the daily relative cost for dealer pairs (1,2), (1,3), (1,4), (2,3), (2,4), (3,4), and calculate a pairwise average. Similar calculation is made for the case with 3 dealers. Panel A plots the cumulative relative cost for markup estimated from dealer sell orders, and Panel B plots cumulative relative cost for markup estimated from dealer buy orders.
crisis period. This finding is consistent with mounting evidence that post-crisis regulations have impaired corporate bond liquidity. The literature has documented that dealers are less willing to take and hold bond inventory from customers in the post-crisis period (Adrian et al. 2017, Bessembinder et al. 2018). In addition, the Volcker Rule in particular has reduced dealers’ ability for market-making, which has led to a decrease in corporate bond market liquidity (Bao, O’Hara and Zhou 2018). The fact that cumulative relative costs are growing faster post-crisis suggests that dealer level markups can deviate from the cross-sectional mean for longer periods of time. This supports the view that the tightening of regulations might have increased the inventory risks borne by dealers.

**Relationship Between Markups from Dealer Buy and Dealer Sell**

In section 1.3.1 I defined the markup of dealer \( d \) selling bond \( i \) is \( MU^S_{i,d,t} = P^S_{i,d,t} - P^\text{Interdealer}_{i,d,t} \), and markup of dealer \( d \) buying bond \( i \) is \( MU^B_{i,d,t} = P^B_{i,d,t} - P^\text{Interdealer}_{i,d,t} \). One salient feature in the data that \( MU^S_{i,d,t} > 0 \) and \( MU^B_{i,d,t} < 0 \), which means dealers generally sell bonds at a price higher than the inter-dealer price, and buy bonds at a price lower than the inter-dealer price. A related empirical question is that how the dealer’s buy markup \( MU^B_{i,d,t} \) and sell markup \( MU^S_{i,d,t} \) correlate with each other. In other words, when a dealer’s sell price relative to the inter-dealer price (i.e. sell markup) increases, whether the buy price relative to the inter-dealer price (i.e. buy markup) tends to increase or decrease at the same time. As it will be clear from the model in section 1.5, the correlation between buy markup and sell markup helps us understand the driving forces of a dealer’s pricing behavior. Consider a shock that increases a dealer’s cost of providing liquidity. The dealer will then require higher compensation for liquidity provision (i.e. higher bid-ask spread). To achieve a wider bid-ask spread, the dealer will have the incentive to both increase the sell price and reduce the buy price relative to the inter-dealer price (i.e. higher \( MU^S_{i,d,t} \) and lower \( MU^B_{i,d,t} \)). This tends to make the correlation between sell markup and buy markup negative.

However, another economic driver related dealer’s inventory management is also at play. As mentioned earlier, the dealer’s inventory is a mean reverting process (Friewald
and Nagler 2016, Dick-Nielsen and Rossi 2018) consistent with the idea that he tries to keep inventories at an optimal level. Consider a shock that increases a dealer’s inventory positions to an unusually high level. In order to clear this extra inventory, the dealer has more willingness to sell and less willingness to buy additional bonds. The former means reducing the sell price relative to the inter-dealer price (i.e. reducing sell markup) and the latter means reducing the buy price relative to the inter-dealer price (i.e. reducing buy markup). This tends to make the correlation between sell markup and buy markup positive.

To examine this empirically, we run a panel regression of buy markup $MU^B_{i,d,t}$ on sell markup $MU^S_{i,d,t}$. Table 1.5 Panel A reports the regression results. Column (1) suggests that the buy markup and sell markup are strongly positively correlated, with a highly significant coefficient of 0.385. Column (2) adds bond-by-month fixed effect and results in a significant coefficient of 0.25. This specification compares the dealers’ pricing behavior at the same point in time on the same bond, and suggests that the dealers who have high buy markup on the bond also tend to have high sell markup. Column (3) adds bond-by-dealer fixed effect and results in a significant coefficient of 0.33. This means if we examine a dealer’s pricing behavior on a given bond, the buy markup and the sell markup tend to move up and down at the same time. Column (4) added a time fixed effect in addition to column (3) and suggests the results is robust.

In addition, I also examine how the deviations by a dealer from the average dealer markup correlate across the buys and sells. To do this, I first construct the relative buy markup $(RM^B_{i,d,t})$, which is the deviation of the buy markup of dealer $d$ $(MU^B_{i,d,t})$ from the average dealer markup $(\frac{1}{D} \sum_{d=1}^{D} MU^B_{i,d,t})$. The relative sell markup is constructed similarly. Panel B of Table 1.5 reports the panel regression result of the relative buy markup on the relative sell markup. The results are similar to Panel A. Note that the regression coefficient in Panel B column (2) is exactly the same as Panel A column (2), because the average dealer markup we subtract in relative buy/sell markup is absorbed by the bond-by-time fixed effect.

The empirical finding that buy markup and sell markup are positively correlated is
Table 1.5: Relationship Between Markups from Dealer Buy and Dealer Sell

Panel A reports the regression result of buy markup on sell markup. Columns (1) through (4) include different fixed effects in the regression. Panel B reports the regression result relative buy markup on relative sell markup. Robust t-statistics are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.

### Panel A
Correlation Between Buy Markup and Sell Markup

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
<th>Column (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MU</td>
<td>0.385***</td>
<td>0.252***</td>
<td>0.333***</td>
<td>0.360***</td>
</tr>
<tr>
<td></td>
<td>(10.34)</td>
<td>(9.97)</td>
<td>(40.87)</td>
<td>(54.86)</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Month FE</td>
<td>YES</td>
<td>–</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Dealer FE</td>
<td>YES</td>
<td>NO</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Bond × Month FE</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Bond × Dealer FE</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>3585577</td>
<td>3309202</td>
<td>3295058</td>
<td>3295058</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.275</td>
<td>0.41</td>
<td>0.388</td>
<td>0.414</td>
</tr>
</tbody>
</table>

### Panel B
Correlation Between Buy Relative Markup and Sell Relative Markup

<table>
<thead>
<tr>
<th></th>
<th>Column (1)</th>
<th>Column (2)</th>
<th>Column (3)</th>
<th>Column (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM</td>
<td>0.313***</td>
<td>0.252***</td>
<td>0.298***</td>
<td>0.299***</td>
</tr>
<tr>
<td></td>
<td>(12.02)</td>
<td>(9.97)</td>
<td>(60.26)</td>
<td>(60.33)</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Month FE</td>
<td>YES</td>
<td>–</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Dealer FE</td>
<td>YES</td>
<td>NO</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Bond × Month FE</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Bond × Dealer FE</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>3585577</td>
<td>3309202</td>
<td>3295058</td>
<td>3295058</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.164</td>
<td>0.203</td>
<td>0.330</td>
<td>0.332</td>
</tr>
</tbody>
</table>
further discussed in section 1.5.

1.5 Model and Testable Implications

This section briefly introduces the model of inventory costs and dealers’ pricing behavior, which is a simplified version of Randall (2015a). I also provide a micro-foundation for the inventory costs in the model, generating several testable implications that link the bid-ask spread to dealers’ characteristics.

Model

The economy has a single risky asset and a risk-free asset (cash).\(^2\) There is a continuum of dealers and customers. Customers are risk neutral and get utilities from consuming their wealth. They trade only once - when they are born. All of their wealth is consumed one period later. Dealers live for an infinite time and bear a per period cost of holding an inventory of risky assets.

Dealers find counterparties through an over-the-counter (OTC) search market (Duffie, Garleanu, and Pedersen 2005, 2007). The trading of the risky asset happens in both a competitive inter-dealer market and a dealer-customer market where both parties could bargain on price and quantity. A dealer has probability \(\pi_i\) meeting with another dealer in the inter-dealer market each period, probability \(\pi_j\) meeting with customer \(j\) in the dealer-customer market. Thus, there is a probability of \(1 - \pi_i - \sum_j \pi_j\) that the dealer could not find a counterparty to trade with. Note that a dealer cannot trade with both a customer and another dealer in the same period by assumption. In this setup, one of the following three situations happens for a given dealer every period:

1. Trading \(q_i\) units of risky asset with another dealer in the inter-dealer market at price \(p_i\) (with probability \(\pi_i\))

\(^2\)In the model, I use a single risky asset to illustrate how a dealer’s inventory risk can influence his pricing behavior. However, in reality a dealer holds a portfolio of bonds, and could have different inventory levels and different markups on different bonds.
2. Trading \( q_t^j \) units of risky asset with customer \( j \) at price \( p_t^j \) (with probability \( \pi_t^j \))

3. Not trading at all (with probability \( 1 - \pi_t - \sum_j \pi_t^j \))

I formalize the sign of trading volume \( q_t \) such that \( q_t > 0 \) when a dealer is buying and \( q_t < 0 \) when a dealer is selling.

Dealers are averse to holding the risky asset, thus bear a per period inventory cost \( f_t \times (a_t^d)^2 \). The coefficient \( f_t \) is a cost parameter, and \( a_t^d \) is the inventory level of dealer \( d \). The assumption of quadratic inventory cost function is also used in Stoll (1978), and Mildenstein and Schleef (1983). This allows for a closed-form solution of the bid-ask spread and gives the tractable property of “linear price impact”. Assuming the economy has a random state \( s_t \) at \( t \), then \( f_t = f(s_t) \) is assumed to be dependent on this state. In a later part of this section, I will provide the microfoundation of \( f(s_t) \) that allows it to be linked to data.

I use the notations \( d_t \) to denote the amount of cash possessed by dealer \( d \) in period \( t \), \( q_t \) to denote the amount of risky asset the dealer trades in period \( t \), and \( p_t \) to denote the prices at which these trades happen. Thus we have \( q_t = q_t^j \) and \( p_t = p_t^j \) if case 1 happens; \( q_t = q_t^c \) and \( p_t = p_t^c \) if case 2 happens; and \( q_t = 0 \) if case 3 happens.

Each period, the dealer pays an inventory cost \( f_t \times (a_t^d)^2 \), and incurs a cash transfer \( -p_t q_t \) through buying/selling the risky asset. For simplicity, the interest rate on dealers’ cash holding is assumed to be zero. It follows that the dealer’s per period cash flow is \( -p_t q_t - f_t \times (a_t^d)^2 \). A dealer with time preference \( \delta \) who maximizes the total expected cash flow will have the following value function:

\[
V_t^d(d_t, a_t^d, s_t) = \max_{\{q_t, p_t\}_{t=1}^\infty} E_t \left[ \sum_{u=0}^\infty \delta^u \left( -p_{t+u} q_{t+u} - f_{t+u} \times (a_{t+u}^d)^2 \right) \right] \tag{1.12}
\]

The dealer is subject to the following case-dependent evolution processes on cash (i.e. budget constraints) and risky inventory:

1. If an inter-dealer trade happens in period \( \tau \), then the dealer’s cash in period \( \tau + 1 \) becomes \( d_{\tau+1}^d = d_t^d - f_t \times (a_t^d)^2 - p_t^d q_t^d \), and the inventory positions become \( a_{\tau+1}^d = a_t^d + q_t^d \)

2. If a dealer-customer trade happens in period \( \tau \), then the dealer’s cash in period \( \tau + 1 \) becomes \( d_{\tau+1}^d = d_t^d - f_t \times (a_t^d)^2 - p_t^c q_t^c \), and the inventory positions become \( a_{\tau+1}^d = a_t^d + q_t^c \)
3. If no trade happens, then the dealer’s cash in period $\tau + 1$ becomes $\$d_{\tau+1} = \$d_\tau - f_\tau \times (a_t^d)^2$, and inventory positions remain $a_{\tau+1}^d = a_t^d$.

One can conjecture and verify that the dealer’s value function has the following functional form:

$$V_t^d (\$d_t^d, a_t^d, s_t) = \gamma_t^S \$d_t^d + \gamma_t^a a_t^d + \gamma_t^{aa} (a_t^d)^2 + \gamma_t$$

which is linear in the amount of cash he possesses $\$d_t^d$, and quadratic in his inventory level $a_t^d$. In the Appendix, I follow the method in Randall (2015a) to verify that the value function has this functional form, and derive the expressions for the parameters $\gamma_t^S, \gamma_t^a, \gamma_t^{aa}$ and $\gamma_t$ in my simplified version of Randall’s original model.

**Dealer-customer trades**

There are a continuum of customers with different types. The type of customer $j$ is defined by his valuation of the risky asset $V_t^{cj} = E_j^t (V_{t+1})$, and bargaining power $\eta_j$. Every customer could have a different valuation about the risky asset, depending on his expectation. With probability $\pi_j^c$, the dealer meets with customer $j$ to negotiate a trade. Each customer trades at most once during his lifetime. Given the structure of the OTC market, customers can only trade with dealers, but not with other customers. It is assumed that customers and dealers both have bargaining power, and the quantity $q_t^c$ and the price $p_t^c$ are determined by Nash Bargaining. A customer’s gain from trading is higher when he sells the risky asset at a higher price relative to his valuation. If customer $j$ sells $q_t^c$ units of the bond to dealer $d$, the customer’s gain from trading versus not trading is:

$$\text{Gain}_t^c = \delta q_t^c (p_t^c - V_t^c)$$

The customer will have positive gains from trading as long as he could buy at a lower price than his valuation or sell at a higher price than his valuation. A utility-maximizing customer might be tempted to trade an infinite amount. However, this is infeasible because it increases dealer’s inventory cost and consequently changes the equilibrium price $p_t^c$.

To discuss dealer’s gain from trading, we first define the value function of a dealer as a

---

3For notation simplicity, the subscript $j$ is omitted for the customer in this section.
function of amount traded \( q^c_t \). In this case, dealer’s period \( t \) flow payoff is \(-p^c_t q^c_t - f_t \times (a^d_t)^2\), and his continuation value is \( \delta E_t [V^d_{t+1}] \). His starting wealth at period \( t + 1 \) is \( $^d_{t+1} = $^d_t - p^c_t q^c_t - f_t \times (a^d_t)^2 \). Thus we can define the dealer’s period \( t \) value function conditional on trading \( q^c_t \) as:

\[
\tilde{V}^d_t (s^d_t, a^d_t, s_t, q^c_t) = -p^c_t q^c_t - f_t \times (a^d_t)^2 + \delta E_t \left[ V^d_{t+1} (s^d_t - p^c_t q^c_t - f_t \times (a^d_t)^2, a^d_t + q^c_t, s_{t+1}) \right]
\]

(1.15)

Then the dealer’s gain from trading \( q^c_t \) is his value function conditional on trading \( q^c_t \) versus not trading at all:

\[
\text{Gain}^d_t = \tilde{V}^d_t (s^d_t, a^d_t, s_t, q^c_t) - \tilde{V}^d_t (s^d_t, a^d_t, s_t, 0)
\]

(1.16)

The equilibrium price and quantity \( p^c_t \) and \( q^c_t \) are determined through Nash Bargaining. Specifically, the dealer and the customer negotiate to maximize the Nash product:

\[
\max_{q^c_t, p^c_t} (\text{Gain}^d_t)^{\eta} (\text{Gain}^c_t)^{1-\eta}
\]

(1.17)

where \( 0 < \eta < 1 \) is the bargaining power parameter of the customer, and \( 1 - \eta \) is the bargaining power parameter of the dealer. A higher \( \eta \) means a higher bargaining power of the customer. By plugging equations 1.15 and 1.16 into equation 1.16, and taking the first order condition with respect to \( p^c_t \) and \( q^c_t \), one can solve the equilibrium price and quantity.

In the Appendix, I show that \( p^c_t \) and \( q^c_t \) satisfy the following equations:

\[
P^c_t = \frac{\eta}{2} \frac{\delta E_t (\gamma^a_{t+1} + 2a^d_t \gamma^aa_{t+1})}{1 + \delta E_t (\gamma^aa_{t+1})} + \left( 1 - \frac{\eta}{2} \right) V^c_t
\]

\[
= -\eta \times \delta q^c_t \times E_t (\gamma^aa_{t+1}) + V^c_t
\]

(1.18)

and

\[
q^c_t = -a^d_t + \frac{V^c_t \left( 1 + \delta E_t (\gamma^a_{t+1}) \right) - \delta E_t (\gamma^aa_{t+1})}{2\delta E_t (\gamma^aa_{t+1})}
\]

(1.19)

It is also shown in the Appendix that the coefficient \( E_t (\gamma^aa_{t+1}) < 0 \), so equation 1.18 suggests that customers sell to dealers (i.e. \( q^c_t > 0 \)) when the negotiated price is higher than customers’ valuations of the risky asset \( V^c_t \). When the customer sells \( q^c_t \) units of bond to the dealer,
the transaction price increases with the customer’s bargaining power $\eta$. Equation 1.18 is also quite intuitive: when the dealer already has high inventory level (higher $a^d_t$), he is less willing to buy more risky assets from the customer (lower $q^c_t$).

Trades only happen when both a dealer and a customer have positive gains from trading.

**Inter-dealer trades**

In each period, dealer $d$ enters the inter-dealer market with probability $\tau_t$. Dealers trade in the inter-dealer market for risk-sharing. A nice feature of the Randall (2015a) framework is that dealers have identical preferences and choose to perfectly share their risks, thus they have equal inventory positions after inter-dealer trading.

The inter-dealer market is assumed to be competitive, thus dealers take the inter-dealer price as given and optimize over the quantity they trade. If dealer $d$ enters the inter-dealer market at period $t$, his optimization problem is equivalent to solving the following Bellman equation, in which the dealer chooses to trade optimal quantity $q^i_t$ given price $p^i_t$:

$$V^d_t (s^d_t, a^d_t, s_t) = \max_{q} p^i_t q^i_t - f_t \times \left( a^d_t \right)^2 + \delta E_t \left[ V^d_{t+1} (s^d_{t+1}, a^d_{t+1}, s_{t+1}) \right]$$  \hspace{1cm} (1.20)

It is shown in the Appendix that the first order condition of this Bellman equation gives the following result:

$$a^d_t + q^i_t = \frac{p^i_t \left( 1 + \delta E_t \left( \gamma^{s}_{t+1} \right) \right) - \delta E_t \left( \gamma^{a}_{t+1} \right)}{2 \delta E_t \left( \gamma^{aa}_{t+1} \right)}$$  \hspace{1cm} (1.21)

The left hand side of equation 1.18 is the dealer’s inventory at period $t$ after inter-dealer trading. This equation suggests an intertemporal trade-off of inventory management that depends on the current price of the risky asset. As shown in the Appendix, the coefficient $E_t \left( \gamma^{aa}_{t+1} \right) < 0$ and $E_t \left( \gamma^{s}_{t+1} \right)$. When the current inter-dealer price $p^i_t$ is low, the dealer will build up his current inventory by buying more risky assets (i.e. increase $q^i_t$), in the hope of selling them to either customers or other dealers at higher prices in the future.

**The markup and bid-ask spread**

This section derives an expression for the markup — the price difference between dealer-customer trades and inter-dealer trades. To distinguish dealers trading in the inter-dealer market and dealers trading in the dealer-customer market, I use $a^d_{t,j}$ to denote the average
inventory of the dealers in the inter-dealer market, and \( a_t^{d,c} \) to denote the inventory of a dealer in the dealer-customer market. Using this new notation, and combining equations 1.19 and 1.21, it follows that the price of inter-dealer trade is:

\[
p_t = 2d \left( \left( a_t^{d,c} + q_t^c \right) - \left( a_t^{d,i} + q_t^i \right) \right) (-E_t(\gamma_{t+1}^{aa})) + V_t^c
\]

(1.22)

The inter-dealer price \( p_t \) is decreasing in the inventory level of dealers in the inter-dealer market \( a_t^{d,i} + q_t^i \). This is because when the dealers have higher inventories, they are willing to lower their prices in order to clear the inventories in the inter-dealer market.

By taking the difference of equations 1.18 and 1.22, one can obtain:

\[
p_t^c - p_t^i = \delta \left( (2 - \eta) q_t^c + 2 \left( a_t^{d,i} + q_t^i - a_t^{d,c} \right) \right) (-E_t(\gamma_{t+1}^{aa}))
\]

(1.23)

In the Appendix, I derive that the coefficient \( \gamma_{t+1}^{aa} \) has the following expression:

\[
\gamma_{t+1}^{aa} = - \sum_{u=0}^{\infty} \left[ (\delta \tilde{\pi})^u \times E_{t+1} (f_{t+u+1}) \right]
\]

(1.24)

where \( \tilde{\pi} = 1 - \pi_t - \pi_c + \sum_j \pi_j^c \eta_j \). Combing equations 1.23 and 1.24, we can obtain the final expression for the markup as follows:

\[
p_t^c - p_t^i = \delta \left( (2 - \eta) q_t^c + 2 \left( a_t^{d,i} + q_t^i - a_t^{d,c} \right) \right) \sum_{u=0}^{\infty} \left[ (\delta \tilde{\pi})^u \times E_t (f_{t+u+1}) \right]
\]

(1.25)

The markup is a function of (1). quantity traded \( q_t^c \) (2). the inventory cost parameter \( E_t (f_{t+u+1}) \) (3). customer’s bargaining power \( \eta \) (4). the inventory of the dealer trading with a customer \( a_t^{d,c} \) relative the average inventory of dealers in the inter-dealer market \( a_t^{d,i} + q_t^i \) and (5). a term \( \tilde{\pi} \) containing probabilities \( \pi_t \) and \( \pi_c \). The discussion below elaborates the intuition why these terms can influence the markup.

The markup depends on \( q_t^c \), the dealer-customer quantity traded, because of the price impact of trading. Similar to Kyle (1985), this model has the tractable property of “linear price impact”, in the sense that the markup is linear in \( q_t^c \). With this expression for markup, one can simply measure the illiquidity using the price impact of dealer-customer trade.
I formally define the bid-ask spread as the difference between the markup of a dealer selling one unit to a customer vs. buying one unit from a customer:

\[
S_t = (p_i^c - p_i^f) \mid q_i^c = 1 - (p_i^c - p_i^f) \mid q_i^f = 1 = 2\delta(2 - \eta) \sum_{u=0}^{\infty} [(\delta \pi)^u \times E_t(f_{t+u+1})]
\]

which measures the "round trip trading cost".

Figure 1.5 plots the markup \(p_i^c - p_i^f\) against the dealer-customer quantity traded \(q_i^c\), and illustrates how the bid-ask spread relates to the buy-markup and the sell-markup. The downward sloping line in this figure is the markup at which the dealer is willing to trade with a customer, for a given quantity \(q_i^c\). The slope of this line \(\frac{\partial (p_i^c - p_i^f)}{\partial q_i^c}\) measures the illiquidity of the bond, in the sense that a steeper slope represents a larger price impact of dealer-customer trade.

If the markup of the dealer selling one unit is denoted as \(MU_i^S = (p_i^c - p_i^f) \mid q_i^c = 1\) and buying one unit as \(MU_i^B = (p_i^c - p_i^f) \mid q_i^f = 1\), then the spread is equal to:

\[
S_t = MU_i^S - MU_i^B
\]
as shown in Figure 1.5.

From equation 1.25, we have:

\[
\frac{\partial MU_i^S}{\partial a_i^{d,c}} = -2\delta \sum_{u=0}^{\infty} [(\delta \pi)^u \times E_t(f_{t+u+1})] = \frac{\partial MU_i^B}{\partial a_i^{d,c}}
\]

(1.27)

This means a change in the dealer’s inventory level \(a_i^{d,c}\) has an identical effect on the buy markup \(MU_i^B\) and the sell markup \(MU_i^S\). The intuition is as follows. The intercept of the line with the vertical axis in Figure 1.5 is proportional to \(2(a_i^{d,i} + q_i^c - a_i^{d,c})\), measuring the dealer’s inventory level \((a_i^{d,c})\) relative to the average inventories of dealers in the inter-dealer market \((a_i^{d,i} + q_i^c)\). If the dealer happens to have a high inventory level \(a_i^{d,c}\) relative to the average dealers in the inter-dealer market, he would be anxious to unload his inventories. Thus this dealer would be less willing to buy, and more willing to sell. Therefore, the prices at which the dealer is willing to buy from or sell to customers will both decrease. In other
Figure 1.5: Markup and Dealer-Customer Trade Quantity

The figure plots the markup $p_i^c - p_i^l$ against the dealer-customer quantity traded $q_i^c$, and illustrates how the bid-ask spread is related to the buy markup and the sell markup and . $q_i^c > 0$ indicates a dealer buying from a customer and $q_i^c < 0$ indicates a dealer selling to a customer. The bid-ask spread is equal to the difference in the sell markup and the buy markup: $S_i = MU_i^S - MU_i^B$. 

$\begin{align*}
p_i^c - p_i^l \\
q_i^c
\end{align*}$
words, an increase in the dealer’s inventory \((a^i_t \text{ goes up relative to } a^i_t + q^i_t)\) will decrease both buy markup and sell markup by the same amount, which means the line in Figure 1.5 shifts downwards. Since buy markup and sell markup moves by the same amount, the slope of this line is left unchanged. Thus changes in the the dealer’s inventory level have no direct effect on the bid-ask spread.

A change in the dealer’s inventory level moves buy-markup and sell-markup in the same direction is an important feature of the model, as discussed in Randall (2015a). If the changes in the inventory level has large influence on a dealer’s pricing behavior, then one should predict that the buy-markup and sell-markup have a positive correlation. The empirical facts reported in Table 1.5 is consistent with this feature of the model.

The expression of the bid-ask spread in equation 1.26 has an intuitive explanation. The spread (market illiquidity) could be higher due to any one of the follow three factors:

1. A higher inventory cost parameter \(E_t (f_{t+u+1})\), resulting in dealers being less willing to provide liquidity.

2. Lower customer bargaining power \(\eta\), meaning that dealers could get compensated with a higher spread from the dealer-customer negotiation process.

3. Less likelihood for dealers to meet with other dealers or customers every period (lower \(\pi_t, \pi_c\)), resulting in dealers’ inability to quickly clear inventory.

**Microfoundations for Inventory Cost**

In Randall (2015a), the inventory cost parameter \(f_t\) is not directly microfounded. This section provides a microfoundation for the inventory cost, which allows me to link the model to the data. First, I assume dealers face margin/funding constraints:

\[
\left( a^d_t \right)^2 m^d_t + \mu_t \leq 0 \tag{1.28}
\]

and the margin requirement \(m^d_t\) is an increasing function of the riskiness of dealer \(d\). This constraint captures the idea that when dealer \(d\) has a large risk of default, the margin
constraint he faces is tighter, and it is costlier for the dealer to provide liquidity.

In the margin-based asset pricing literature (Garleanu and Pedersen 2011), the margin constraint is a hard constraint, which cannot be violated. Here it is being modeled as a soft constraint, which can be violated at a cost, and $-\mu_t \geq 0$ measures the extent to which this constraint is violated. Intuitively, $-\mu_t$ can be viewed as an additional inventory cost to each dealer resulting from the margin requirement. As indicated above, the margin requirement $m^d_t = g(CDS_t)$ is an increasing function of dealers’ default risk (measured by CDS spread). This function is normalized, so that when a dealer has zero default risk (i.e. zero CDS spread), the margin requirement is zero (i.e. $g(0) = 0$), and when a dealer has a very large default risk, the margin requirement approaches 1.

Thus, when dealers have no default risk, $m^d_t = 0$, the constraint can be satisfied with zero violation (i.e. $-\mu_t = 0$). When dealers have a positive default risk, $m^d_t > 0$, and in order for the constraint to be satisfied, there must be a positive violation (i.e. $-\mu_t \geq (a^d_t)^2 m^d_t > 0$). The violation of the constraint $-\mu_t$ will enter into dealers’ utility function as a cost. Another way to interpret this constraint is that in order to provide liquidity, dealers have to raise capital from financiers via collateralized borrowing. When dealers have higher default risk, the cost of borrowing is higher.

The second constraint is a value at risk (VaR) constraint:

$$\left( a^d_t \right)^2 \eta_t^{\text{RISK}} + \eta_t \leq 0$$

(1.29)

where $\eta_t^{\text{RISK}}$ is a function of dealers’ bond portfolio risk. This constraint captures the idea that the inventory cost is larger if a dealer holds a portfolio of riskier bonds. In general, financial intermediaries are faced with very strict capital regulations. Dealers holding risky bonds will have higher risk-weighted assets, and therefore are required to hold more liquid capital. Consequently, a dealer holding riskier bonds will face a much tighter VaR constraint. Again, this is modeled as a soft constraint, with $-\eta_t \geq 0$ measuring the extent to which this constraint is violated. The constraint violation $-\eta_t$ will also enter dealers’ utility function as an additional cost.
The dealer’s problem could thus be rewritten as:

$$\max_{q_t, p_t, m_t, h_t} E_t \left[ \sum_{u=0}^{\infty} \delta^u \left( -p_{t+u}q_{t+u} + \mu_{t+u} + \eta_{t+u} \right) \right]$$

such that

$$\left( a_t^d \right)^2 m_t^d + \mu_t \leq 0$$

and

$$\left( a_t^d \right)^2 f_t^{d, \text{RISK}} + \eta_t \leq 0$$

This constrained maximization problem is equivalent to the initial problem:

$$\max_{\{q_t, p_t\}_{t=1}^{\infty}} E_t \left[ \sum_{u=0}^{\infty} \delta^u \left( -p_{t+u}q_{t+u} - f_t^d \times \left( a_t^d \right)^2 \right) \right]$$

where

$$f_t^d = m_t^d + f_t^{d, \text{RISK}}$$

Thus, introducing the margin/funding constraints and the value at risk constraints provides a microfoundation for the inventory cost parameter $f_t$. Within this framework, several factors can drive the markup differences from one dealer to another. First, if a dealer has a higher/lower inventory level than the average, the inventory imbalance will make the dealer more willing to sell/buy the bond, which is then reflected in his pricing behavior. Second, some dealers may face higher inventory costs relative to other dealers. This can be due to either the tighter margin/funding constraints or tighter VaR constraints that dealers are facing. However, although inventory levels can have large effects on the markup, the spread itself is not directly linked to inventory level. This is because any changes in inventory levels will shift dealers’ selling prices and buying prices in the same direction, leaving the spread unchanged.
Testable Implications

According to the model, the bid-ask spread:

\[ S_t = 2\delta (2 - \eta) \sum_{u=0}^{\infty} (\delta \tilde{\pi})^u \times E_t(f_{t+u+1}) \]

with \( \tilde{\pi} = 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \) is related to several factors which can be tested with the data on dealer characteristics. First, the bid-ask spread is related to the connectedness of dealers. Dealers who have a higher probability of meeting with other dealers (higher \( \pi_i \)) or customers (higher \( \pi_c \)) during each period will have a lower spread. In the OTC market, there are often delays in finding a counterparty due to search frictions (Duffie, Garleanu and Pedersen 2005, 2007), and the equilibrium spread should reflect these costs. Intuitively, dealers who have a higher probability of meeting with counterparties in a short period of time will incur lower search costs. Empirically, the likelihood of a dealer meeting counterparties in every period is proxied using his centrality in the inter-dealer network.

Second, the spread is related to the inventory cost. As mentioned previously in this section, the inventory cost parameter is \( f_t = m_t + h^{RISK}_t \), where \( m_t = g(CDS_t) \) is an increasing function of dealers’ CDS spreads. The first term is related to dealers’ default risk, which is a measure of dealers’ funding costs. In other words, dealers with a higher credit risk will face tighter funding/margin constraints, and thus incur higher costs of holding inventory. In the equilibrium, it is expected that dealers with a higher default risk should have higher dealer-specific trading costs. The second term is related to dealers’ portfolio risk, which suggests that dealers holding a riskier inventory portfolio will face a tighter VaR constraint and will thus be less willing to acquire additional risky assets. This suggests a positive association between the dealer-specific bid-ask spreads and the riskiness of dealers’ bond portfolios. In addition, the marginal risk a new bond position adds to a dealer’s existing portfolio is also directly related to the dealer’s willingness of acquiring this new inventory.

In the next section, I provide the empirical results for these testable implications, allowing quantitative evaluations of how much each channel affects the dealer-level spread.
1.6 Testing the Model

This section presents empirical results from testing the predictions of the model, which associates the dealer-specific spread with several determinants. The dataset has variations at bond, dealer, and time levels, which are fully explored in the regressions. One advantage of this data is that bond-by-time fixed effects could be added to the regressions to absorb the parts that are not directly relevant to my study. Remember that the dealer specific spread of bond \( i \) for dealer \( d \) is 4:

\[
S_{i,d,t} = MU_{i,d,t}^S - MU_{i,d,t}^B \\
= \left( C_{i,t}^S + M_{d,t}^S + \varepsilon_{i,d,t}^S \right) - \left( C_{i,t}^B + M_{d,t}^B + \varepsilon_{i,d,t}^B \right) \\
= \left( C_{i,t}^S - C_{i,t}^B \right) + \left( M_{d,t}^S - M_{d,t}^B \right) + \left( \varepsilon_{i,d,t}^S - \varepsilon_{i,d,t}^B \right)
\]

(1.30)

My objective is to understand what drives the variations in the dealer-level spread \( M_{d,t}^S - M_{d,t}^B \). Therefore, adding bond-by-time fixed effect takes out the common component of spread \( C_{i,t}^S - C_{i,t}^B \) and allows me to focus instead on the dealer specific part of the spread.

First, the main variable of interest, bid-ask spread, is constructed at a bond-dealer-monthly level. The bid-ask spread of bond \( i \) charged by dealer \( d \) in month \( t \), \( S_{i,d,t} \), is constructed using the following procedure. On each trading day \( \tau \), the daily bid-ask spread of bond \( i \) by dealer \( d \) is constructed as

\[
S_{i,d,\tau} = \frac{P_{i,d,\tau}^S - P_{i,d,\tau}^B}{(P_{i,d,\tau}^S + P_{i,d,\tau}^B)/2},
\]

where \( P_{i,d,\tau}^S \) (\( P_{i,d,\tau}^B \)) is the average price of all sell (buy) transactions made by dealer \( d \) for bond \( i \) on day \( \tau \). To make sure that \( S_{i,d,\tau} > 0 \), I only include observations where dealer’s daily sell prices are higher than buy prices 5. I also remove outliers where \( S_{i,d,\tau} > 15\% \), accounting for less than 1% of the data.

Finally, I aggregate the daily bid-ask spread to a monthly frequency using an arithmetic average over all the trading days in month \( t \):

\[
S_{i,d,t} = \frac{1}{\text{Days}_t} \sum_{\tau \in t} S_{i,d,\tau}.
\]

The final trading cost variable has a 25th percentile of 0.34%, a 50th percentile of 0.84% and a 75th percentile of

---

4Empirically, since dealers sell above the inter-dealer price, and buy below the inter-dealer price, generally we have \( MU_{i,d,t}^S > 0 \) and \( MU_{i,d,t}^B < 0 \). In other words, one can think of \( S_{i,d,t} \) as \( |MU_{i,d,t}^S| - |MU_{i,d,t}^B| \).

5In the data, about 12.5% of the observations has \( P_{i,d,\tau}^S < P_{i,d,\tau}^B \). This is probably due to the fact that buy and sell transactions happen at different times of the day and new information arrives over the course of a trading day.
1.8%.

1.6.1 Bid-Ask Spread and Dealer’s Connectedness

This section examines the relationship between the bid-ask spread and dealers’ connectedness. The model predicts that dealers with a higher probability of meeting with other dealers/customers ($\pi^d / \pi^c$) each period will have a lower spread. In the trading network, a higher probability of meeting with counterparties means that dealers are better connected. Better connectedness gives dealers a comparative advantage in managing their inventory risk, because it allows them to constantly trade with counterparties and clear their extra inventories relatively quickly. The network centrality, a measure of dealers’ connectedness, is used as a proxy of $\pi^d$. The data does not distinguish one customer from another. Thus the likelihood of a dealer meeting with customer $c$, $\pi^c$, cannot be measured directly. However, a reasonable conjecture is that if a dealer has better connections with other dealers, he might also have better connections with customers.

**The Inter-dealer Network**

It is important to understand some features of the inter-dealer network before starting the regression analysis about bid-ask spread and dealer connectedness. The network is defined by a graph $(N, G)$, consisting of a set of nodes $N = 1, ..., D$ representing the $D$ dealers, and a real-valued $D \times D$ adjacency matrix $G$ where $G_{ij}$ represents the relationship between dealers $i$ and $j$. The monthly relationship between two dealers is defined by whether there has been any inter-dealer trading between them. To eliminate the effect of very weak relationships, I define direct relationships as those having inter-dealer trades of at least a par-value of $\$1$ million in a given month.

Figure 1.6 contains a network plot of this inter-dealer relationship. Panel A plots the inter-dealer network for Jan 2012, where each node (black) represents a dealer, and each edge (red) represents a trading relationship. The network exhibits a strong core-periphery structure, in which a set of interconnected core dealers trade with each other, accounting for most of the trading volume, and many sparsely connected peripheral dealers trade less
The figure plots the inter-dealer network of corporate bond trading. Panel A plots the inter-dealer network for Jan 2012, where each node (black) represents a dealer in our data, and each edge (red) represent a trading relationship. Panel B plots this network structure for the top 50 dealers by total volume in Jan 2012. The thickness of each edge is proportional to the volume between each pair of dealers. Panel C plots the cumulative fraction of volume against the number of dealers in the network.
frequently. This core-periphery structure is pervasive in over-the-counter (OTC) markets and could arise endogenously as an equilibrium balance between trade competition and inventory efficiency (Wang 2016, Di Maggio, Kermani, and Song 2017, Eisfeldt, Herskovic, Rajan and Siriwandane 2018).

Panel B plots this network structure for the top 50 dealers, as defined by total volume traded in January 2012. The thickness of each edge is proportional to the amount of trading carried out between each pair of dealers. The plot suggests that even among the top 50 dealers, there is still a large dispersion in pairwise volume, as the top 10 dealers trade with each other much more frequently. This pattern is even more obvious in Panel C, which plots the cumulative fraction of volume against the number of dealers in the network. Although there are more than 400 dealers, the top 11 dealers account for about 70% of the total volume.

The patterns from figure 1.6 suggest a large cross-sectional dispersion in the connectedness of dealers. As shown in the model, better-connected dealers have a lower inventory risk because they are able to clear large inventories faster. Empirically, the social network literature uses centrality to measure connectedness. I follow Di Maggio, Kermani, and Song (2017) and use eigenvector centrality to obtain a monthly measure of dealer connectedness. The eigenvector centrality $C^e$ of all dealers is obtained by solving the following equation (Bonacich 1986):

$$GC^e = \lambda C^e$$

where $C^E$ is the eigenvector of $G$ and $\lambda$ is the associated eigenvalue. The eigenvector centrality is defined as the eigenvector associated with the largest eigenvalue. Eigenvector centrality captures the idea that dealers are more central if their neighbors are also more central.

To test this hypothesis, I run the following regression:

$$S_{i,d,t} = \gamma \times C^e_{d,t} + D_{i,t} + \eta_{i,d,t}$$

where $S_{i,d,t}$ is the average trading cost of bond $i$ charged by dealer $d$ in month $t$, and $D_{i,t}$ is
The table reports the regression of bond-dealer level trading cost on dealer centrality. Panel A uses unweighted centrality, where every inter-dealer relationship is treated equally. Panel B uses weighted centrality where the strength of relationship is weighted by the pairwise trading volume between two dealers. Robust t-statistics are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.

<table>
<thead>
<tr>
<th>Dealer Connectedness and Bid-Ask Spread</th>
<th>Panel A: Unweighted Centrality</th>
<th>Panel B: Weighted Centrality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Standardized $C_{d,t}^e$</td>
<td>(1) Standardized $C_{d,t}^w$</td>
</tr>
<tr>
<td></td>
<td>-3.14*** (-5.43)</td>
<td>-12.90*** (-18.06)</td>
</tr>
<tr>
<td></td>
<td>-4.28*** (-12.02)</td>
<td>-14.42*** (-26.37)</td>
</tr>
<tr>
<td></td>
<td>-4.74*** (-17.82)</td>
<td>-13.60*** (-22.12)</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Month FE</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Bond × Month FE</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>6917131</td>
<td>6917131</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.364</td>
<td>0.369</td>
</tr>
</tbody>
</table>

Table 1.6 reports the regression of bond-dealer level trading cost on dealer centrality. Panel A uses unweighted centrality, whereby every inter-dealer relationship is treated equally. To make the coefficients more interpretable, the centrality variable in the regression is standardized. The results in column (1) of panel A suggest that when dealer centrality is higher by one standard deviation, the spread is 3.14 bps lower. Column (2) adds both bond fixed effect and time fixed effect, and the estimate becomes -4.28 bps. Column (3) adds bond-by-time fixed effect to the regression. As suggested in equation 1.30, adding a bond-by-time fixed effect allows me to eliminate the common component of spread and focus on the dealer-specific part, which is the main object of interest. Column (3) suggests that a one standard deviation increase in dealer centrality is associated with a 4.74 bps decrease in the bid-ask spread. The magnitude in column (3) is close to that in column (1) and (2), suggesting that adding a bond-by-time fixed effect does not notably change the estimate. This is because the centrality is simply a cross-sectional measure, and it does not include a time-varying common component. Panel B uses weighted centrality, whereby the
strength of the relationship is weighted by the pairwise trading volume between two dealers. The results are similar to those in Panel A, but the magnitudes are about 3 times greater.

The regression results suggest that dealers with higher centrality charge a lower bid-ask spread in equilibrium. This is consistent with the prediction that better-connected dealers are able to clear inventory faster, thus facing lower costs of providing liquidity.

1.6.2 Bid-Ask Spread and Dealer’s Credit Risk

This section tests the relationship between dealers’ spread and credit risk. The model shows that dealers with a higher credit risk will have a higher bid-ask spread. The intuition is that dealers with a higher credit risk face tighter funding constraints, and a higher cost of collateralized funding means more costly liquidity provisions. Empirically, dealers’ credit risk can be measured by their CDS spread. Recall that the inventory cost parameter \( f_t = m_{d,t} + h_t^{RISK} \) and the dealers’ margin requirements \( m_{d,t} = g(CDS_{d,t}) \) are increasing functions of the riskiness of dealer \( d \).

To test this prediction I run the following regression:

\[
S_{i,d,t} = \beta \times CDS_{d,t} + D_{i,t} + \zeta_{i,d,t}
\]

where \( S_{i,d,t} \) is the average trading cost of bond \( i \) charged by dealer \( d \) in month \( t \). The model predicts that the coefficient \( \beta > 0 \).

Table 1.7 Panel A reports the regression results of bond-dealer level bid-ask spreads on dealers’ CDS spreads. I get positive and statistically significant estimates for \( \beta \) in all three specifications, consistent with the prediction of the model. The point estimate in column (1) suggests that for each one percentage point rise in dealers’ CDS spreads, the bid-ask spread charged by the dealer goes up by 0.138 percentage point. To put this magnitude in context, note that the median trading cost is 0.84 percentage point. This suggests that a one percentage point increase in the CDS spread is related to a 16% increase in average trading cost (0.138/0.84=16%).

Column (2) includes both bond and time fixed effects and gives a point estimate of 5.01
Table 1.7: Bid-Ask Spread and Dealer’s Credit Risk

Panel A of the table reports the regression results of bond-dealer level bid-ask spread on dealers’ credit risk (measured using CDS spread). Panel B compares the effect of dealers’ credit risk and connectedness on the bid-ask spread. Both variables are standardized in this regression, and connectedness is measured using the weighted centrality. The Robust t-statistics are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.

### Panel A

<table>
<thead>
<tr>
<th>Dealer Credit Risk and Bid-Ask Spread</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_{i,d,t})</td>
<td>S_{i,d,t}</td>
<td>S_{i,d,t}</td>
<td>S_{i,d,t}</td>
</tr>
<tr>
<td>CDS_{d,t}</td>
<td>0.138***</td>
<td>0.0501***</td>
<td>0.0326***</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>YES</td>
<td>–</td>
</tr>
<tr>
<td>Month FE</td>
<td>NO</td>
<td>YES</td>
<td>–</td>
</tr>
<tr>
<td>Bond × Month FE</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>3199877</td>
<td>3199877</td>
<td>2847348</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.381</td>
<td>0.459</td>
<td>0.701</td>
</tr>
</tbody>
</table>

### Panel B

Comparing the Magnitude of Connectedness Effect and Credit Risk Effect

<table>
<thead>
<tr>
<th>Dealer Connectedness, Credit Risk and Bid-Ask Spread</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_{i,d,t})</td>
<td>S_{i,d,t}</td>
<td>S_{i,d,t}</td>
<td>S_{i,d,t}</td>
</tr>
<tr>
<td>Standardized (C_{d,t}^c)</td>
<td>-9.29***</td>
<td>-11.46***</td>
<td>-11.10***</td>
</tr>
<tr>
<td>(\text{Standardized CDS}_{d,t})</td>
<td>30.20***</td>
<td>10.02***</td>
<td>6.12***</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>YES</td>
<td>–</td>
</tr>
<tr>
<td>Month FE</td>
<td>NO</td>
<td>YES</td>
<td>–</td>
</tr>
<tr>
<td>Bond × Month FE</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>3199513</td>
<td>3199513</td>
<td>2847056</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.385</td>
<td>0.465</td>
<td>0.707</td>
</tr>
</tbody>
</table>
bps. After adding a bond-by-month fixed effect, this estimate remains a similar magnitude
(3.26 bps) and is highly statistically significant. The time-varying trading costs might be
correlated with dealers’ credit risks, for example, due to responses to macroeconomic shocks
or the market risk. In other words, there might be a common component in addition to
dealers’ credit risks that drives variations in both the bond-dealer level bid-ask spread
and the dealer-level CDS spread. As suggested by equation 1.30, adding a bond-by-time
fixed effect allows me to remove the effect of this common component and focus on the
dealer-specific part of the bid-ask spread. The point estimate decreases from 13.8 bps to 3.26
bps, suggesting that this common component is dominant in the bid-ask spreads. However,
column (3) provides strong evidence that the dealer-specific part of the bid-ask spread
related to dealer-specific credit risk is still large and significant.

The results suggest that dealers with a higher credit risk have a higher bid-ask spread
in the equilibrium, consistent with the prediction that riskier dealers face tighter funding
constraints, and thus bear higher inventory costs. Consequently, they require higher
compensation for liquidity provision.

To compare the relative importance of the effects of connectedness and credit risk, I
combine these two effects into one regression to evaluate their contributions in explaining
the bid-ask spread:

$$S_{i,d,t} = \gamma \times C_d + \beta \times CDS_{d,t} + D_{i,t} + \xi_{i,d,t}$$

Table 1.7 Panel B reports the regression results. Both centrality and CDS are standardized
to zero mean and unit variance, so that the magnitudes are easily comparable. The results
suggest that a one standard deviation increase in dealers’ centrality is associated with a
9.29bps reduction in the bid-ask spread when we include bond fixed effect (column 1) and a
11.46 bps reduction in the bid-ask spread when we further include time fixed effect (column
2). The result is robust when we add bond-by-time fixed effect and the magnitude stays
large (11.1 bps).

A one standard deviation increase in dealers’ CDS spread is associated with a 30.2 bps
increase in the bid-ask spread when we include bond fixed effect (column 1) and 10.02 bps
increase in the bid-ask spread when we further include time fixed effect. The magnitude slightly reduces to 6.12 bps when we include bond-by-time fixed effect and remains highly significant.

1.6.3 Bid-Ask Spread and Portfolio Risk

This section examines how bid-ask spreads are related to dealers’ portfolio risks. As discussed earlier, the inventory cost parameter in the model has two parts $f_t^d = m_t^d + h_t^{d,RISK}$. The first part is the funding constraint related to dealers’ credit risks, and the second part is the value-at-risk constraint related to dealers’ bond portfolio risks. Intuitively, dealers with higher portfolio risks face greater uncertainty about the value of their inventory portfolios, thus assuming a higher inventory cost and requiring a higher bid-ask spread as compensation.

To illustrate the intuition of the point mentioned above, consider the following simple example. Suppose a dealer holds a portfolio of bonds with return $R_{dt}$. He is considering adding an additional $\theta$ units of inventory of bond $i$ to his current portfolio. The variance of his new portfolio becomes:

$$Var\left((1 - \theta)R_{dt} + \theta R_{it}\right) = (1 - \theta)^2 Var(R_{dt}) + \theta^2 Var(R_{it}) + 2(1 - \theta)\theta Cov(R_{dt}, R_{it})$$

The first term $Var(R_{dt})$ is the dealer’s current inventory portfolio risk. The second term $Var(R_{it})$ is the risk of bond $i$, and the third term $Cov(R_{dt}, R_{it})$ is the covariance of bond $i$ with the dealer’s current inventory portfolio. The second and third terms reflect how much bond $i$ contributes to dealer $d$’s portfolio risk, and should thus be related to the bid-ask spread of bond $i$ charged by dealer $d$. This suggests that the following predictions relate bid-ask spread to portfolio risk:

1. The bid-ask spread of bond $i$ is positively correlated with the risk of bond $i$, $Var(R_{it})$
2. The bid-ask spread of bond $i$ charged by dealer $d$ is positively correlated with the covariance of bond $i$ and dealer’s $d$’s existing inventory portfolio, $Cov(R_{dt}, R_{it})$

To test the first prediction, the monthly bond volatility $\sigma_{it}^{Bond} = \sqrt{Var(R_{it})}$ is constructed.
The table reports the regression results of bond-dealer level (bid-ask) spread on bond volatility. Robust t-statistics are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.

<table>
<thead>
<tr>
<th>Bond Volatility and Bid-Ask Spread</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{i,t}^{Bond}$</td>
<td>$s_{i,d,t}$</td>
<td>$s_{i,d,t}$</td>
<td>$s_{i,d,t}$</td>
<td>$s_{i,d,t}$</td>
</tr>
<tr>
<td></td>
<td>9.48***</td>
<td>11.90***</td>
<td>6.82***</td>
<td>11.3***</td>
</tr>
<tr>
<td></td>
<td>(26.19)</td>
<td>(23.60)</td>
<td>(26.11)</td>
<td>(25.38)</td>
</tr>
<tr>
<td>Bond FE</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Month FE</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Dealer×Month FE</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>5044929</td>
<td>5045985</td>
<td>5044929</td>
<td>5038707</td>
</tr>
<tr>
<td>R-sqr</td>
<td>0.455</td>
<td>0.313</td>
<td>0.493</td>
<td>0.448</td>
</tr>
</tbody>
</table>

The daily bond return is calculated as the daily log difference of the volume weighted average price (VWAP). I then calculate the standard deviation of daily bond returns over monthly samples, and multiply this number by $\sqrt{21}$ to get the monthly volatility $\sigma_{i,t}^{Bond}$ (assuming 21 trading days in a month). In our sample, the monthly bond volatility $\sigma_{i,t}^{Bond}$ has a 25th percentile of 1.6%, a median of 2.9%, and a 75th percentile of 5.6%, which are smaller than the volatility of single stocks.

I run the following regression to test the first hypothesis:

$$S_{i,d,t} = \theta \sigma_{i,t}^{Bond} + D_i + D_t + D_{d,t} + \nu_{i,d,t}$$

The right hand side variable $\sigma_{i,t}^{Bond}$ does not contain any dealer-level variation, and we are only evaluating the quantitative relationship between bond volatility and the bid-ask spread charged by an average dealer. Table 1.8 reports the result of this regression.

Column (2) suggests that in the cross section, when monthly bond volatility increases by 1%, the bid-ask spread goes up by 11.9 bps. As discussed above, the third quartile of bond volatility is 4% (=5.6%-1.6%) higher than its first quartile. This suggests a higher bid-ask spread of 47.6 bps, which is sizable. Column (1) includes a bond fixed effect in the
regression and the magnitudes slightly decrease. Column (3) adds both bond and month fixed effects, and (4) includes the dealer-by-month fixed effect. The results are robust to different specifications. Overall, these results suggest that the risk (volatility) of individual bonds are important determinants of the bid-ask spread.

1.7 Conclusion

I have used dealer-level transaction data to understand the role of dealers’ inventory costs in corporate bond liquidity (trading cost). The corporate bond market provides an ideal laboratory to test the importance of inventory risk, because dealers in corporate bonds hold large inventories for weeks, thus creating a much greater inventory cost compared to the stock market. I find that at a given point in time, different dealers may set different markups on the same bond, suggesting the existence of a dealer-specific factor driving their pricing behavior. Moreover, the cross sectional discrepancies in dealer-level markup may last for an extended period of time with a half-life period of two weeks, close to the half-life of dealers’ inventory position adjustment periods. It is further shown that the dealer-level trading costs are related to several characteristics of dealers’ inventory costs, such as dealers’ funding costs, inventory clearing speeds, and inventory portfolio risk. This suggests that inventory cost is an important driver of the corporate bond market illiquidity (i.e. the bid-ask spread).

A large body of literature on asymmetric information explains the bid-ask spread as compensation for adverse selection cost. This literature ignores the effect of inventory cost, and assumes dealers to be risk-neutral. This paper suggests that, at least in the U.S. corporate bond market, inventory risk is an important determinant of the bid-ask spread, and dealers’ inventory costs have a substantial influence on their pricing behavior.

Overall, the empirical evidence is consistent with the “inventory theory” of bid-ask spread—as in Garman (1976), Stoll (1978), Amihud and Mendelson (1980)—which suggests that dealers’ inventory risks have large effects on trading costs. In times when liquidity "dries up", regulators and policy makers could improve market liquidity by reducing inventory risks of dealers, either through capital injections or asset purchases.
2.1 Introduction

The effect of interest rates on financial institutions’ investment behavior has been the center of attention of academics, policymakers, and the media. A particular financial stability concern has been that the low-interest-rate environment prevailing since the 2008 financial crisis may heighten incentives of financial institutions to invest in riskier assets (Bernanke 2013; Stein 2013; Rajan 2013; Yellen 2014). We study how changes in interest rates affect the investment and risk-taking behavior of life insurance companies, the largest institutional holders of corporate bonds, using a new regulatory database that includes a long time series starting in 1994 and covers the whole universe of life insurance companies.

We show that insurance companies are tilting their portfolios towards higher-yield bonds when interest rates decline. At first, this seems to be consistent with “reaching for yield” behavior in a low-interest-rate environment (Becker and Ivashina 2015; Choi and Kronlund 2017). However, we find that the tilt towards higher-yield bonds seems to be primarily driven by an increase in duration rather than an increase in credit risk, and insurers do not

---

1Co-authored with Ali Ozdagli
seem to increase their credit risk as interest rates decline.\footnote{Becker and Ivashina (2015) first documented reaching-for-yield behavior: insurance companies systematically bias their portfolios towards higher-yield bonds within the same rating category. Choi and Kronlund (2017) study how the reaching-for-yield behavior of bond mutual funds is affected by interest rates.}

An alternative hypothesis that can explain this phenomenon is that insurers hedge their risk through duration matching of assets and liabilities (Domanski, Shin, and Sushko 2017). The insurance company wants to adjust its portfolio to keep the duration gap between assets and liabilities close to zero in an effort to reduce its interest rate risk because of regulations that tie risk-based capital surcharges to interest rate risk (Lombardi 2006) and because the demand for their products depends on their health and riskiness (Koijen and Yogo 2015). The duration of liabilities reacts to changes in interest rates because of the behavior of policyholders. Many insurance products offer policyholders the option to contribute additional funds at their discretion or to close out (surrender) a contract in return for a predetermined payment. When interest rates change, it is more likely that policyholders will act on these options (Berends, McMenamin, Plestis, and Rosen 2013; NAIC 2014). In particular, lower interest rates increase liability duration by decreasing the likelihood of surrender and increasing the likelihood of paid-up additions. Therefore, the duration gap decreases when interest rates fall to which the insurance company reacts by increasing the duration of its assets in order to pull the duration gap back to zero.

Under continuous duration matching, the equity duration of insurance companies should be close to zero. In contrast with this implication, we find that the duration of equity deviates from zero both in positive and negative directions for extended periods of time. Therefore, we propose a stylized model of duration matching with adjustment costs, in the spirit of capital adjustment costs that have been popular in the literature studying firms’ investment decisions since Abel and Eberly (1994). In the context of insurance companies, these adjustment costs may stem from the fact that selling and purchasing assets in large quantities may have greater marginal cost due to market frictions like price pressure or due to greater cost of effort by investment managers. This intuitive idea of frictions to portfolio adjustment is also confirmed in our discussions with regulators and conforms with the fact
that the insurers engage in bond acquisitions and disposals intermittently.\footnote{Based on the quarterly observations, a typical insurance firm trades bonds about 2/3 of the quarters. One concern is that this observation may imply that life insurers rely on derivatives to manage interest rate risk. However, derivatives have traditionally not played a large role in risk management in the life insurance industry (Berends, McMenamin, Plestis, and Rosen 2013).}

In our model, the duration of an insurer’s assets varies over time in response to interest rate changes both because of nonzero convexity of bonds and because of the insurer’s active adjustment to its duration through acquisitions and disposals. Our interest in the investment behavior of insurance companies requires us to isolate the second effect. To capture this effect, our model allows us to create a novel definition of \textit{active duration adjustment}, measured as the difference between the duration of the insurer’s total holdings at the end of a given period and the duration of its legacy assets (the hypothetical duration of the holdings if the insurer were not to make any changes to its portfolio) under the new interest rates.

Our model shows that an insurer’s active duration adjustment is driven by the difference between the duration of its legacy assets and the insurer’s \textit{target duration}. Therefore, our model suggests a target duration hypothesis which is analogous to the target leverage hypothesis in corporate finance. In the absence of adjustment costs, the insurer would always invest to set the duration of assets equal to the target duration. In the presence of adjustment costs, however, the insurer closes the gap gradually rather than immediately at every period. The speed of this adjustment depends positively on the cost of carrying interest rate risk due to deviations from a zero duration gap and negatively on the cost of adjustment. We show that the solution of our model leads to a reduced-form econometric model that can be directly estimated with the data using a standard regression approach, akin to econometric models used to test the target leverage hypothesis in empirical corporate finance (for example, DeAngelo and Roll 2015).

Our model, while stylized, has several powerful implications which can be tested with our long and comprehensive cross-sectional data of insurance companies. Consistent with the implication that adjustment towards the target duration happens gradually, we find that it takes an insurance firm about 11 quarters to close half the duration gap, leading to
extended periods of exposure to interest rate risk. Moreover, consistent with the predictions of the model regarding the relationship between target duration, leverage, and interest rates, we find that the active duration adjustment is positively related to leverage and negatively related to the product of leverage and the interest rate, and the interest rate does not have an additional effect on active duration adjustment beyond its interaction with leverage.

Our model also predicts that the speed of adjustment should be slower for firms that face larger costs of rebalancing their portfolio. Accordingly, we find that firms with larger holdings have a slower speed of adjustment consistent with the argument that they need larger trades for the same amount of duration adjustment and thereby face larger costs due to the price pressure generated by their trades. Similarly, we find that firms with less liquid portfolios have a slower speed of adjustment consistent with the argument that they face larger trading costs when they want to adjust their portfolios.

The premise of our model lies in the argument that policyholders’ behavior reacts to interest rate changes. When interest rates are lower, policyholders have greater incentives to hold on to their insurance contracts due to lack of other high-yield investment opportunities. This implies that policy surrenders and lapses become less likely as interest rates decline, which increases target duration of the insurer. Consistent with this argument, we find a positive relationship between interest rates and surrender/lapse rates, and this association generates a negative relationship between surrender/lapse rates and active duration adjustment.

Finally, we use our estimated model to calculate the duration of equity predicted by our model. We show that this predicted duration of equity matches with the empirical interest rate sensitivity of equity returns of insurance companies.

Overall, our results suggest that insurance companies tilt their portfolio towards higher duration assets in an effort to minimize their interest rate risk subject to adjustment costs. This poses challenges to financial stability that are separate from reaching-for-yield behavior.

---

4 A lower surrender rate lengthens the duration of the payments insurance companies have to make as the underlying risk will materialize in the future. A lower lapse rate increases target duration primarily by increasing the liabilities, and hence leverage, of the insurance company. See Section 2.5.4 for details.
In particular, reaching for yield in a low-interest-rate environment suggests that central banks should raise interest rates to prevent financial institutions’ excessive risk taking that can generate additional negative effects if the economy experiences adverse shocks. In comparison, duration matching under adjustment costs suggests that the insurance companies are exposed to interest rate risk for an extended period of time even if their goal is to minimize risk. In this framework, the central bank should take into account the sign of the duration gap when deciding to raise interest rates. If the duration gap is positive, then an increase in interest rates can reduce the target duration and hence increase the duration gap further, thereby increasing the interest rate risk of the insurance companies rather than reducing it. In the current environment, however, the equity duration (and hence duration gap) of U.S. insurance companies is negative, thereby giving an additional incentive for the Federal Reserve to raise rates to reduce the duration mismatch faced by insurance companies due to adjustment costs.

Our results are also related to the previous literature on insurance company investment behavior. The seminal paper in this literature, Becker and Ivashina (2015), studies how the reaching-for-yield behavior of insurance companies changes before and during the financial crisis and finds that insurance companies reach for credit risk before, but not during, the financial crisis. Our focus is on understanding the drivers of the changes in excess yield on insurance companies’ portfolios as the interest rate changes. We find similar results as Becker and Ivashina (2015) for their time window 2004:Q3-2010:Q4; however, we find that the bulk of excess yield in insurance companies’ bond holdings relative to the market can be attributed to changes in duration risk rather than credit risk over the 1996–2016 sample period.
2.2 Data and Stylized Facts

2.2.1 Data Construction

We construct our dataset by combining data from several sources. The data for life insurance companies’ corporate bond holdings comes from NAIC statutory filings. Schedule D of insurance filings has detailed information on investment by life, health, and property and casualty (P&C) insurance companies, including corporate bonds, stocks, and municipal bonds. We obtain our data of insurance company holdings directly from NAIC through a special agreement with the Federal Reserve. The data have a complete coverage of all the NAIC-reporting insurance companies from 1994Q1 to 2016Q4. Schedule D has both annual files with year-end portfolio holdings information, and quarterly files which contain asset acquisition and disposal information within each quarter. The exact date and amount of each insurance company’s acquisition/disposal transactions are documented. Thus, we know their portfolio rebalancing behavior at a very granular level.

The corporate bond pricing information comes from Mergent FISD bond transactions (1994–2002) and TRACE (2002–2016). The Mergent FISD consists of all transactions of publicly traded corporate bonds by life insurance companies, property and casualty insurance companies, and health insurance companies beginning in January 1994. Previous research has shown the FISD data are representative of corporate bond transactions (Warga 2000; Campbell and Taksler 2003). The TRACE data have transaction reports for all corporate bonds back to July 2002. The data are cleaned using the filtering algorithm in Dick-Nielsen (2009). We obtain the bond issuance information from Mergent FISD, which provides coupon, maturity, offering amount, and rating. We measure credit risk using both the distance to default and the CDS spread. The distance to default (DD) is publicly available from the Credit Research Initiative at National University of Singapore, which is available for our entire sample (1994–2016). The CDS spread comes from Markit and starts from 2002.

Our sample covers a relatively high interest rate period from 1994 to 2000 and the post-recession low interest environment from 2010 to 2016. As far as we know, our sample
has a longer time span compared to other papers that investigate investment behavior of financial institutions in the bond market.\(^5\) With a long sample of 23 years, we are able to study how insurance companies’ investment behavior differs as interest rates change.

### 2.2.2 Measuring Life Insurers’ Tilt for Higher-Yield Bonds

Insurance companies are the largest institutional holders of corporate and foreign bonds. According to the U.S. Flow of Funds Accounts, in 2015Q4, life insurers held $2.36 trillion of corporate and foreign bonds, quantitatively similar to mutual and pension funds taken together.\(^6\) Insurance regulations require insurance companies to maintain minimum levels of capital on a risk-adjusted basis, called risk-based capital (RBC). To determine the capital requirement for credit risk, corporate bonds are sorted into six broad categories (National Association of Insurance Commissioners (NAIC) risk categories 1 through 6) based on their credit ratings, with higher numbered categories subject to higher capital requirements.\(^7\)

As discussed in Becker and Ivashina (2015), due to the regulations and the presence of government guarantees, insurance companies may attempt to increase the yield in their bond portfolio by taking on extra priced risk, while leaving capital requirements unaffected. Therefore, we focus on corporate bond holdings of insurance companies, conditional on NAIC risk categories. Our main hypothesis is that the incentives of insurance companies to invest in higher-yield bonds within a given NAIC rating category is related to the level of interest rates. To measure this empirically, we compare the average yield of insurance company corporate bond holdings with the average yield of the aggregate corporate bond portfolio (Choi-Kronlund 2016), within each NAIC rating category.

\(^5\) For example, the sample period in Becker and Ivashina (2015) is from 2004 to 2010, and the sample period in Choi and Kronlund (2017) is from 2002 to 2012; both are less than half of the length of our sample.

\(^6\) Mutual funds and pension funds are the second and third largest institutional holders in the US corporate bond market, with holdings of $1.74 and $0.7 trillion, respectively, for the same time period.

\(^7\) The NAIC categories map into S&P ratings in the following way: $\textit{NAIC1} = \{\textit{AAA, AA, A}\}$, $\textit{NAIC2} = \textit{BBB}$, $\textit{NAIC3} = \textit{BB}$, $\textit{NAIC4} = \textit{B}$, $\textit{NAIC5} = \textit{CCC}$, $\textit{NAIC6} = \{\textit{CC, C, D}\}$
We define the excess yield \( \text{ExYld}_{i,t} \) of insurance company \( i \) at date \( t \) within the NAIC1 category as the average yield of insurance company \( i \)'s NAIC1 bond portfolio relative to the average yield of all outstanding NAIC1 bonds in the market:

\[
\text{ExYld}_{i,t} = \frac{\sum_j H_{i,j,t} y_{j,t}}{\sum_j H_{i,j,t}} - \frac{\sum_k A_{k,t} y_{k,t}}{\sum_k A_{k,t}}
\]

where \( y_{j,t} \) is the yield on bond \( j \), \( H_{i,j,t} \) is the amount of bond \( j \) held by insurance company \( i \) and \( A_{k,t} \) is the amount of bond \( k \) outstanding, both measured as face value at the end of quarter \( t \).

This measure also gives the excess yield in the aggregate insurance sector when we let \( i \) be the universe of all insurance companies. Comparing the relative yield of an insurance company's portfolio to the market within an NAIC designation allows us to control for the unobservable factors that drive variation in the market yield. Similarly, we could also define the \( \text{ExYld}_{i,t} \) in NAIC2 designation. The main results we present in the paper are based on NAIC1 designation.\(^8\)

2.2.3 Stylized Facts

We document three stylized facts by examining how this excess yield is related to interest rates, and the underlying risk quantities insurance companies are loading on. We use the 10-year Treasury Constant Maturity Rate as the interest rate variable because it has duration very comparable to both the assets and liabilities of typical insurance companies, and therefore should be the most relevant discount rate insurers use while making investment decisions (Domanski-Shin-Sushko 2017; Hartley-Paulson-Rosen 2016). In later parts of the paper, we present a partial adjustment model of duration matching to rationalize these facts.

\(^8\)Over 60% of corporate bond holdings of insurance companies is in the NAIC1 category, with over an additional 30% in the NAIC2 category. The robustness of our results using NAIC2 category bond holdings are available upon request. Since the holdings in the remaining NAIC categories are less than 10% of their total corporate bond holdings, we do not study other categories.
Stylized Fact 1: The excess yield of life insurance sector’s corporate bond portfolio increases as interest rates decline.

Figure 2.1 plots the excess yield of the bond portfolio for life insurance companies and the level of the 10-year treasury yield (1994q1–2016q4). As the interest rate declines, life insurance companies tend to hold portfolios with a higher yield relative to the rest of the market, within the same rating category. Insurance companies on average hold higher-yield bonds than the market in the NAIC1 category, and hold bonds with similar yield to the market in the NAIC2 category. However, the negative relationship between excess yield and the interest rate holds in both rating categories. In the NAIC1 (NAIC2) category, a 1 percentage point decrease in the 10-year treasury yield is associated with a 10.9 (3.6) basis point increase in excess yield on insurance companies’ bond portfolio.

One possible explanation of this pattern is “reaching for yield.” The literature has argued that a financial institution’s risk-taking appetite is stronger when interest rates are low (Choi and Kronlund 2017; Chodorow-Reich 2014; Barbu, Fricke and Moench 2016; Ma, Lian and Wang 2017; Di Maggio and Kacperczyk 2017). According to this view, insurance companies might have a tendency to take excess risk to generate additional returns because the lower interest rate has reduced the expected return on their existing portfolio.

There are two major sources of risk in the corporate bond market that insurers could load on in order to generate higher expected returns. The first source is credit risk. As argued in Becker and Ivashina (2015), one way for insurance companies to reach for yield is to increase their holdings of bonds with greater credit risk within the same NAIC rating category. The second source is duration risk. Lengthening the bond portfolio’s duration is an alternative way for insurers to increase the portfolio’s expected return. In fact, if the excess yield on insurers’ portfolio is driven by a reaching-for-yield incentive, they will strategically choose between loading on credit or duration risk based on the optimal risk-return trade-off.

In order to unpack the risk quantities of insurance companies’ portfolio into these two sources of risk, we come up with a matching algorithm. In particular, for every NAIC1
Figure 2.1: Interest Rate and Excess Yield on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4)

The figure plots the excess yield on life insurance sector’s corporate bond portfolio against the 10-year yield on US treasury. The excess yield is first constructed at bond-quarter level, for NAIC category 1 and 2 separately. For every bond held by the life insurance sector, we compute the excess yield by subtracting the market average yield (i.e. average yield on all bonds outstanding for a given NAIC category) from the bond yield. Then we aggregate the bond level excess yield to insurance sector level weighting by amount held by the insurance sector. Panel A reports for bonds in NAIC1 category, and Panel B for NAIC2 category.
bond that insurance companies hold on their balance sheet in a given quarter, we find 10 bonds among all the NAIC1 bonds outstanding (excluding the bond itself) with the closest duration to the bond we want to match with. Then we subtract the average yield of the 10 duration-matched bonds from the yield of the bond that insurers hold. We call this excess yield “duration-matched excess yield” of the bond. Particularly, we are interested in understanding whether duration is a “sufficient statistic” that explains the excess yield on the insurance companies’ portfolio. The 10 bonds in each control group are chosen among the universe of bonds in the same NAIC category, which represent insurance company’s possible investment space in that category. If the insurance companies are tilting on characteristics other than duration that makes the yields different, then this control group will not pick up the other characteristics, and this effect will show up in the “duration-matched excess yield”. But if insurance companies are only tilting on duration, or the net effect of the other characteristics they tilt on is zero, then the “duration-matched excess yield” should be close to zero.

In our empirical design, we are always using market holdings of corporate bonds as the benchmark (control) group, following Becker-Ivashina (2015) and Choi-Kronlund (2017). Similarly, we use only corporate bonds in duration matching, in line with Choi and Kronlund (2017) who use maturity buckets within the corporate bond universe. We aggregate this duration-matched excess yield to the whole insurance sector by value weighting the bond-level metric by the total par amount held by the insurance sector. We repeat this exercise for the NAIC2 category.

The duration matching procedure has an important advantage over a linear regression framework where duration is used as a regressor, which is a commonly used method in the literature. Since we repeat the matching exercise every quarter, our approach takes into account the time-varying price of duration risk. This guarantees that the changes in excess yield are driven by changes in the quantity of duration risk, rather than mechanical changes due to the fluctuations in the price of duration risk, which would affect the excess yield.
even if there’s no change in the investment behavior of the insurance companies.  

There is another advantage of our approach compared to linear regression. This duration-matched excess yield captures how much of the excess yield remains on the insurance sector’s portfolio after we control for duration. By comparing the yield of the insurance sector’s bond holdings with a control group from all the bonds outstanding with similar duration in the same NAIC category, we can properly take care of any nonlinear relationship between duration and yield, which could not be fully controlled in a linear regression framework.

**Stylized Fact 2:** After controlling for duration, the “duration-matched excess yield” does not react to interest rate changes.

Figure 2.2 plots the duration-matched excess yield against the 10-year Treasury yield. Unlike the excess yield, the duration-matched excess yield no longer increases when the interest rate declines. In the NAIC1 category, the “duration matched excess yield” is insensitive to changes in the interest rate (Panel A), and in the NAIC2 category, it even slightly declines in the low-interest-rate-environment (Panel B). When interest rates are high, the excess yield (scattered in orange) and duration matched excess yield (scattered in blue) are indistinguishable from each other, whereas their difference widens when the interest rate declines. This pattern tells us that the negative association between insurers’ excess yield and the interest rate we see in Stylized Fact 1 can be attributed to the difference in the duration profile of their portfolios relative to the market, suggesting that insurers may be increasing their asset portfolio duration as the interest rate declines.

Indeed, this hypothesis is verified in Figure 2.2, Panel C. We calculate for the NAIC1 category the “excess duration” of the insurance sector’s bond holdings (holding-weighted average duration of insurance company portfolio minus the average duration of the market). We see that, on average, insurance companies hold higher duration bonds. Moreover, the

---

9We also look at the quantity of duration risk directly in our analysis.
Figure 2.2: Interest Rate and Duration Tilt on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4)

Panel A and Panel B plots the duration matched excess yield (in percentage points) against the 10-year yield (in percentage points) on US treasury for NAIC category 1 and 2 respectively (blue scatters). As a comparison, we also plot the excess yield shown in Figure 2.1 (orange scatters). The duration matched excess yield is the difference between bond yield and average yield of 10 bonds with close duration from the market. Panel C and Panel D report the excess duration (duration tilt) of the insurance sector’s corporate bond portfolio, which is the difference between the insurers’ bond portfolio duration and the average duration of all bonds in the market. We do this for bonds with NAIC category 1 (Panel C) and 2 (Panel D) separately.
excess duration varies a lot with the interest rate. When interest rates are around 7%, the excess duration is almost zero, and it then increases monotonically to around 2.5 when interest rates decline to 1.5%. We observe a similar qualitative pattern in Panel D for NAIC2 category.

One might also wonder how much of the variation of the excess yield on the insurance company’s bond portfolio is driven by changes in the credit risk profile of the portfolio. To shed light on this question, we adopt a similar matching algorithm for credit risk. For every bond held by insurance companies on their balance sheet in a given quarter, we find 10 bonds among all the bonds outstanding that have the closest distance-to-default (DD) to it. This allows us to compare, after controlling for credit risk characteristics, how much of the excess yield is left on insurance companies’ bond portfolio. This approach also addresses the potential concern whether some of the patterns in Panels A and B in Figure 2.2 are driven by the correlation of duration and credit risk (i.e., by controlling for duration we also controlled for credit risk).

**Stylized Fact 3:** *After controlling for credit risk, the “credit-risk-matched excess yield” still increases as the interest rate decreases.*

As shown in Figure 2.3 (Panels A & B), the “DD-matched excess yield” still has a very negative correlation with the level of the interest rate. In fact, the “DD-matched excess yield” is almost indistinguishable from the excess yield. This means the credit risk alone does not explain much of the changes in excess yield on insurers’ bond portfolio in response to changes in interest rates. This is verified in Panels C & D of Figure 2.3 which examine the excess credit risk on insurance companies’ bond portfolio.

The distance to default measure is publicly available, covers a large cross-section of firms, and goes back to the beginning of our sample (1994Q1), thus we use it as our benchmark.

---

10 We use modified duration, which is a price sensitivity measure. It is defined as the percentage change in the price of the bond when the yield increases by 1 percentage point.
Figure 2.3: Interest Rate and Credit Risk Tilt on Insurance Sector’s Bond Portfolio (1994Q1-2016Q4)

Panel A and Panel B plots the distance-to-default (DD) matched excess yield against the 10-year yield on US treasury (blue scatters) for NAIC category 1 and 2 respectively. As a comparison, we also plot the excess yield shown in Figure 2.1 (orange scatters). The duration matched excess yield is the difference between bond yield and average yield of 10 bonds with close duration from the market. Panel C and Panel D reports the excess duration (duration tilt) of insurance sector’s corporate bond portfolio, which is the different between insurer’s bond portfolio duration and average duration of all bonds in the market. We do this for bonds with NAIC category 1 (Panel C) and 2 (Panel D) separately.
measure of credit risk. However, we also use the CDS spread as an alternative measure of credit risk to corroborate our findings, following Becker and Ivashina (2015). This addresses the concern that the default probability computed using the Merton model sometimes does not turn out to be the best measure of default risk when evaluated using out-of-sample forecasting ability (Bharath and Shumway 2008). The CDS spread captures the risk-neutral default probabilities, which adjust for investors’ risk aversion, while the distance to default does not. In our study, we care about insurance company investment behavior, thus using the CDS spread as a robustness check may better capture their risk-taking behavior. Figure B.1 in the appendix reports the plot of CDS-matched excess yield against the 10-year treasury yield. The CDS spread data are available from 2002Q1 to 2016Q4. The results suggest that the “CDS-matched excess yield” still has a very negative correlation with the level of the interest rate, putting these concerns to rest.

To summarize, we show that insurance companies are tilting their portfolios toward higher-yield bonds when interest rates decline. At first, this seems to be consistent with “reaching for yield” in a low-interest-rate environment. However, we find that the tilt toward higher-yield bonds seems to be primarily driven by an increase in duration rather than credit risk, and insurers do not seem to increase their credit risk as interest rates decline.

These patterns cannot be squared with a rational model in which insurance companies take on excessive risk to reach for yield when interest rates are low because (i) exposure to credit risk does not react to changes in interest rates, and (ii) life insurance companies in general have longer liability duration than asset duration, and increasing asset duration in response to interest rate declines would actually reduce their risk, rather than causing them to take on additional risk. In the next section, we propose an alternative explanation for these stylized facts: insurance companies are gradually adjusting their asset portfolio duration to meet a duration target that minimizes their interest rate risk subject to adjustment costs.
2.3 Duration Matching by Life Insurance Companies

2.3.1 Duration Gap

Understanding the concept of duration matching starts with understanding the concept of duration gap. We adopt the definition from Mishkin and Eakins (2012) and define the duration gap as $G = D_A - \frac{L}{A}D_L$, where $D_A$ is the duration of assets, $D_L$ is the duration of liabilities, and $L/A$ is leverage (liabilities/assets). The definition is motivated by the fact that an insurance company with zero duration gap will have an equity value immune to interest rate changes. To see this, note that $-rac{AE}{Ar} = -\frac{\partial(A-L)}{Ar} = -A\frac{\partial \ln A}{Ar} + L\frac{\partial \ln L}{Ar} = A(D_A - \frac{L}{A}D_L) = A \times G$. Dividing both sides by the equity value of an insurance company and noting that $D_E = -\frac{\partial \ln E}{Ar}$, we get the identity $G = \frac{E}{A}D_E$. The insurance company wants to adjust its portfolio to keep the duration gap between assets and liabilities close to zero in an effort to reduce the interest rate risk because of regulations that tie risk-based capital surcharges to interest rate risk (Lombardi 2006) and because the demand for their products depends on their health and riskiness (Koijen and Yogo 2015).

Duration matching can also rationalize the fact that insurance companies have a shorter duration of assets relative to their liabilities (EIOPA 2014a, b, Graph 78), a fact hard to rationalize in a framework where insurance companies acquire high duration bonds to “reach for yield”. In a framework where insurers reach for yield by acquiring higher duration assets, asset duration should exceed liability duration and a lower interest rate environment would exacerbate this difference. Given that the life insurance sector has an average modified asset duration less than 9 even in the highest quarter, and many policies (liabilities) have time spans of 10-30 years, this implication of reaching for yield is difficult to reconcile with the data. However, duration matching readily explains why $D_A < D_L$. Since the insurer’s goal is to attain a zero duration gap, i.e., $D_A = \frac{L}{A}D_L$, we have $D_A < D_L$ because leverage $L/A < 1$.

In principle, life insurers could also use derivatives to manage their interest rate risk, in addition to adjusting asset duration. However, since using derivatives are expensive,
it has traditionally played little role in risk management of the life insurance industry (Berends, McMenamin, Plestis, and Rosen 2013). Therefore, we do not consider interest rate derivatives in the analysis and assume insurers have to rely on duration matching to manage their interest rate risk.

2.3.2 Do Insurers Always Maintain Zero Duration Gap?

We start our analysis with the simplest duration matching framework: insurers continuously rebalance to attain a zero duration gap, so that equity is always immune to interest rate fluctuations. This is similar to the stylized example of duration matching as in Domanski, Shin and Sushko (2017). A few testable implications come out directly from this framework.

Consider an insurance company aiming to always keep the duration gap equal to zero, $G = 0$. As the interest rate changes, the duration gap can deviate from zero, and hence the insurer needs to engage in dynamic hedging. How the insurance company rebalances its portfolio depends on $dG/dr$, the sensitivity of the current duration gap to the interest rate. Our stylized fact implies that insurance companies increase the duration of assets, $D_A$, after an interest rate decrease, which is consistent with a scenario that the duration gap falls below zero and insurance companies have to lengthen asset duration to close the gap. This implies $dG/dr > 0$.

Therefore, the simplest duration-matching framework suggests $G = 0$ and $dG/dr > 0$. These predictions are testable using the duration and convexity of insurance companies’ equity. As discussed in the previous section, the sensitivity of equity to the interest rate is directly linked to the duration gap: $-\frac{\partial E}{\partial r} = A \times G$. So if the duration gap is equal to zero, the equity value must be perfectly immune to interest rate fluctuations. In other words, the duration of equity $D_E \equiv -\frac{1}{E} \frac{dE}{dr} = 0$. Note that we have the mathematical result $\frac{dG}{dr} = \frac{d}{dr} \left( -\frac{1}{A} \frac{dE}{dr} \right) = -\frac{1}{A} \frac{d^2 E}{dr^2} + \frac{dE}{dr} \frac{1}{A} \frac{dA}{dr}$. And when duration is perfectly matched, we have $\frac{dE}{dr} = 0$, which means $\frac{dG}{dr} = -\frac{1}{A} \frac{d^2 E}{dr^2}$. So $dG/dr > 0$ implies $-\frac{dE}{dr^2} > 0$. This means the convexity of equity $C_E = \frac{1}{E} \frac{d^2 E}{dr^2} < 0$. In sum, perfectly matched duration leads to the following predictions:
Table 2.1: Estimated Equity Duration and Convexity of the Life Insurance Sector

The table reports the estimated coefficient from the regression (1994 to 2016, weekly data)
\[ \text{Ret}_{E,t} = Q - D_E \Delta y_{10,t} + \frac{C_E}{2} (\Delta y_{10,t})^2 \]

<table>
<thead>
<tr>
<th>Coeff</th>
<th>$-D_E$</th>
<th>$\frac{C_E}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>0.057***</td>
<td>-0.092</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.51)</td>
<td>(3.42)</td>
</tr>
</tbody>
</table>

1. On average, insurers maintain a zero duration gap, thus $D_E = 0$

2. Insurance companies actively increase $D_A$ after an interest rate decrease, implying $C_E < 0$

The duration and convexity of equity could be directly estimated using the following regression (Campbell, Lo, and MacKinlay 1997), where $\text{Ret}_{E,t}$ is the equity returns and $\Delta y_{10,t}$ is the change in 10-year Treasury yield:

\[ \text{Ret}_{E,t} = Q - D_E \Delta y_{10,t} + \frac{C_E}{2} (\Delta y_{10,t})^2 \] (2.2)

We construct weekly equity returns using the SNL U.S. Insurance Life & Health Equity Index (1994-2017) and regress them on weekly changes in the 10-year treasury yield. Table 2.1 reports the point estimates of the regression coefficients in the whole sample, which suggests that on average $D_E = -0.057$ and is statistically significant. The convexity is not significantly different from zero. This rejects the predictions from the simplistic duration matching model, and hence the notion that insurers always maintain a zero duration gap.

To better understand the reason behind the failure of the simplest duration matching model, we run the same regression over two-year rolling windows and study the evolution of $D_E$ and $C_E$. From Figure 2.4, we see that the lower interest rates in recent years may have pushed the insurers toward $D_E < 0$, consistent with Hartley, Paulson, and Rosen (2016). As we will discuss in later sections of the paper, one reason might be that the increase in the duration of liabilities due to the implicit options in some insurance contracts, for example the pre-determined withdrawal/surrender value of life insurance products and annuities,
Figure 2.4: Time Varying Duration and Convexity of the Life Insurance Sector’s Equity

The figures plots the 2-year rolling estimation of coefficients in the regression \( \text{Ret}_{E,t} = Q - D_E \Delta y_{10,t} + \frac{C_E}{2} (\Delta y_{10,t})^2 \) from 1994 to 2016. The shadow area indicates the 95% confidence interval of the estimated coefficient. Panel A plots the rolling estimation of the coefficient \(-D_E\) and Panel B plots the rolling estimation of coefficient \(\frac{C_E}{2}\).
which are out of the money in the low interest rate environment. This effect is similar to the effect of the prepayment option in the context of mortgages and banks.

The results also suggest that insurance companies do not fully adjust their asset duration to perfectly match with liability duration every period. More realistically, insurance companies’ duration matching behavior is better described by a “partial adjustment” framework. This framework introduces market frictions: adjusting a large fraction of the portfolio in the corporate bond market in a short period of time is costly, due to price pressures and illiquidity of the market. When liability duration increases, insurance companies try to rebalance their portfolio to increase $D_A$, but can only do so gradually over time. Moreover, when the interest rate continues to decline and further widens this gap, insurance companies will adjust their asset portfolio duration to chase a “time-varying target”.

2.4 Duration Matching with Adjustment Costs: The Target Duration Hypothesis

In this section, we provide the theoretical foundations for dynamic duration matching under adjustment costs. We show how the solution of a simple theoretical model leads to a reduced-form model that can be directly estimated in the data using a standard regression approach. Since this reduced-form model turns out to be analogous to econometric models of the target leverage hypothesis in corporate finance (for example, DeAngelo and Roll 2015), we call our framework the target duration hypothesis.

2.4.1 The Model

The insurance company wants to adjust its portfolio to keep the duration gap close to zero in an effort to reduce its interest rate risk because of regulations that tie risk-based capital surcharges to interest rate risk (Lombardi 2006). In addition, the demand for insurance
products depend on their health and riskiness (Koijen and Yogo 2015).\footnote{Although retail investors might not fully understand the relationship between the duration gap and interest rate risk when they purchase an insurance policy, the risk might affect the ratings of insurers, and hence affect agents’ recommendations to retail investors.} However, there are costs of rebalancing the firm’s asset portfolio to make large adjustments in asset duration. For example, selling and purchasing assets in large quantities may have greater marginal cost due to market frictions like price pressures or due to greater cost of effort by investment managers. This intuitive idea of frictions to portfolio adjustment is also confirmed in our discussions with regulators and conforms with the fact that the insurers do not engage in bond acquisitions and disposals in every period.\footnote{Based on the quarterly observations, a typical insurance firm trades bonds about 2/3 of the quarters. One concern is that this observation may imply that life insurers rely on derivatives to manage interest rate risk. However, derivatives have traditionally not played a large role in risk management in the life insurance industry due to large costs of hedging (Berends, McMenamin, Plestis, and Rosen 2013).} It is a widely held view that duration matching can be costly, as restructuring the balance sheet is time consuming, costly, and generally not desirable (Saunders and Cornett 2001).

The firm is trying to balance between its desire to minimize the cost of having a duration gap different from zero and its desire to minimize the cost of adjustments to its portfolio duration.\footnote{Ideally, the insurance company would buy assets of which cash flows perfectly match the cash flows of the liabilities, thereby achieving duration-matching (zero gap) and convexity-matching simultaneously. However, uncertainty about the timing of the payoffs (such as changes in mortality rates) and difficulty to find assets that perfectly match the cash flows is a hurdle against this goal; as a result, convexity of insurance companies is not regulated and do not appear in our cost function. In a multi-period setting, the convexity choice can still become important as it will affect the future expected duration gap, depending on expected changes in interest rates, which complicates the model without providing additional intuition regarding our framework of duration-matching under adjustment costs.} As a result, the firm’s objective function at date $t$ is given by

$$
\text{max} \left[ -\frac{\phi}{2} (G_t)^2 + \frac{\psi}{2} (\Delta D_{A,t})^2 \right],
$$

where $G$ is the duration gap and $\Delta D_A$ is how much the firm adjusts its asset duration. The first term in this objective function captures the cost of deviating from a zero duration gap, a deviation that increases interest rate risk regardless of the direction of the deviation. The second term captures the increasing marginal cost of adjusting duration regardless of the...
direction of the duration adjustment.\textsuperscript{14}

The duration gap is defined as in Mishkin and Eakins (2017) as $G \equiv D_A - \frac{1}{A} D_L$, where $D_A$ is the duration of assets, $D_L$ is the duration of liabilities, and $L/A$ is leverage (liabilities/assets). The new asset duration after the firm’s portfolio adjustment is given by $D_{A,t} = D_{A,t}^0 + \Delta D_{A,t}$, where $D_{A,t}^0$ is the duration of legacy assets at time $t$; that is, $D_{A,t}^0$ is what would be the asset duration if the firm were not to make any portfolio adjustment ($\Delta D_{A,t} = 0$) but rather keep the same portfolio as at the beginning of the period. Moreover, since keeping the duration gap close to zero means that the firm should keep the duration of assets ($D_A$) close to $\frac{1}{A} D_L$, we define $D_A^* \equiv \frac{1}{A} D_L$ as the target asset duration. Putting this information together, the objective function of the firm becomes to choose the optimal adjustment to its portfolio duration

$$\max_{\Delta D_{A,t}} \left[ \frac{\phi}{2} (D_{A,t}^0 + \Delta D_{A,t} - D_A^*)^2 + \frac{\psi}{2} (\Delta D_{A,t})^2 \right].$$ \hspace{1cm} (2.4)

The duration of liabilities reacts to changes in interest rates because of the behavior of the policyholders. Many insurance products offer policyholders the option to contribute additional funds at their discretion or to close out (surrender) a contract in return for a predetermined payment. When interest rates change, it is more likely that policyholders will act on these options (Berends, McMenamin, Plestis, and Rosen 2013). In particular, lower interest rates increase liability duration by decreasing the likelihood of surrender and increasing the likelihood of paid-up additions. We can capture this relationship by allowing the duration target to depend on interest rates, $D_A^* (r)$. Furthermore, as a result of the non-zero convexity of the bonds, the duration of legacy assets is affected by changes in interest rates. We can express this dependence as $D_{A,t}^0 = D_A^0 (r_t)$. As a result, the objective function of the firm becomes

$$\max_{\Delta D_{A,t}} \left[ \frac{\phi}{2} (D_A^0 (r_t) + \Delta D_{A,t} - D_A^* (r_t))^2 + \frac{\psi}{2} (\Delta D_{A,t})^2 \right].$$ \hspace{1cm} (2.5)

\textsuperscript{14}We have chosen quadratic cost functions for ease of derivation of first-order conditions. Alternatively, the first-order condition of this model can be seen as a linear approximation to the first-order condition of a model with more general cost functions.
The FOC of this problem is given by

\[ \Delta D_{A,t} = -\frac{\phi}{\psi + \phi} \left[ D^0_A (r_t) - D^*_A (r_t) \right] . \] (2.6)

This expression is familiar to empirical researchers working with the target leverage hypothesis in corporate finance, e.g., DeAngelo and Roll (2015). In particular, this expression captures how fast the firm reacts to imbalances in the duration of its legacy assets in relation to its target duration, \( D^0_A (r_t) - D^*_A (r_t) \). In analogy with the target leverage hypothesis, \( 0 < \frac{\phi}{\psi + \phi} < 1 \) is the speed of adjustment to the target duration. The speed of adjustment is positively related to the cost of missing the duration target, \( \phi \), and negatively related to the adjustment cost, \( \psi \). The parameters \( \phi \) and \( \psi \) are not separately identified in the model, thus they could be collapsed into one parameter in principle. However, we use two parameters to provide a more intuitive explanation of the model.

### 2.4.2 Testing the Model

Our data provide comprehensive information regarding the holdings of every insurance company, which we aggregate at the firm-quarter level. We can calculate the duration of legacy assets at the end of a given quarter \( t \), \( D^0_A (r_t) \), from the data directly because we observe the holdings of the insurer at the end of the last quarter. Similarly, we can calculate the active adjustment to duration as the difference between the duration of the holdings and the duration of the legacy assets at the end of quarter \( t \), \( \Delta D_{A,t} = D_{A,t} - D^0_{A,t} \).

The duration of liabilities is hard to measure because the liabilities of insurance companies do not have the same level of detail as its assets. Therefore, we model the dependence of the liabilities’ duration on the interest rate as a linear function so that the target duration is given by \( D^*_A (r) = \frac{1}{\lambda} (a + b \times r) \), where \( a > 0 \) and \( b < 0 \) because the duration of liabilities is a positive and decreasing function of interest rates. Plugging this expression into the first order condition of the model,

\[ \Delta D_{A,t} = -\frac{\phi}{\psi + \phi} \left[ D^0_A (r_t) - D^*_A (r_t) \right] , \] (2.7)
we obtain the following expression that can be estimated using a linear regression,

\[
ActiveDurationAdjustment = -\frac{\phi}{\psi + \phi} \times LegacyDuration + \frac{\phi}{\psi + \phi} \times Leverage \times (a + b \times r) \\
= -\frac{\phi}{\psi + \phi} \times LegacyDuration + \frac{\phi}{\psi + \phi} a \times Leverage \\
+ \frac{\phi}{\psi + \phi} b \times Leverage \times r.
\]

It is customary in empirical work to put the uninteracted terms into a regression when interacted terms are present. Therefore, our final regression also includes the uninteracted (stand-alone) interest rate, \(r\), as follows

\[
ActiveDurationAdjustment_{i,t} = \text{const}_i + a \times LegacyDuration_{i,t} + \beta \times Leverage_{i,t-1} \\
+ \gamma \times Leverage_{i,t-1} \times r_t + \delta \times r_t + \text{error}_{i,t},
\]

where each observation is at the level of firm \(i\) and quarter \(t\). Since the dependent variable is the adjustment in duration, the firm fixed effect, \(\text{const}_i\), controls for any trend in the duration of the holdings that may be correlated with interest rates.

This regression allows us to test the following predictions of our model:

1. The coefficient of \(LegacyDuration\) \((a = -\frac{\phi}{\psi + \phi})\) satisfies \(-1 < a < 0\).
2. The coefficient of \(Leverage\) \((\beta = \frac{\phi}{\psi + \phi} a)\) is positive.
3. The coefficient of \(Leverage \times r\) \((\gamma = \frac{\phi}{\psi + \phi} b)\) is negative.
4. The coefficient of stand-alone interest rate \(r\) \((\delta)\) is zero.

As an additional test of our model, we study the speed of the duration adjustment, \(\frac{\phi}{\psi + \phi}\), for different groups of firms. In particular, we note that the speed of adjustment should be slower for firms that face larger costs of rebalancing their portfolio, \(\psi\). Accordingly, we predict that firms with larger holdings should have a slower speed of adjustment because they need larger trades for the same amount of duration adjustment and thereby face larger costs due to the price pressures generated by their trades. Similarly, we predict that firms
with less liquid portfolios should have a slower speed of adjustment because they face larger trading costs when they want to adjust their portfolios.

As another test, we check if the surrender behavior of the policyholders is consistent with our results. Since the positive link between the policy surrender and interest rate generates a link between the liability duration and interest rates, we test the following two-stage regression

\[
\text{SurrenderRatio}_{i,t} = \theta_i + \eta \times r_t + \varphi \times \text{LegacyDuration}_{i,t} + \epsilon_{i,t}
\]

\[
\text{ActiveDurationAdjustment}_{i,t} = \text{const}_i + \alpha \times \text{LegacyDuration}_{i,t} + \beta \times \text{SurrenderRatio}_{i,t} + \text{error}_{i,t}
\]

where we expect \( \eta > 0, \alpha < 0, \) and \( \beta < 0. \) We estimate similar regressions where the surrender ratio is replaced by lapse ratio.

Finally, we use our estimates in order to calculate the predicted duration of equity by our model and compare it with the empirical interest rate sensitivity of equity returns.

In the next section, we show that these predictions of the model are confirmed in the data, suggesting that the target duration hypothesis is a good representation of the investment decisions of insurance companies.

2.5 Results

2.5.1 The Interest Rate and the Option to Surrender and Lapse

We collect data for the amount of policies surrendered and lapsed by each insurance company every year. By dividing these variables by the total amount of policies in force, we obtain a ratio of policies surrendered and lapsed. These ratios are capturing the tendency of policyholders to surrender or lapse their policies. Figure 2.5 plots the surrender rate and lapse rate against the 10-year treasury yield. We see a strong positive association. In higher interest rate environments there is also a high tendency for policyholders to surrender or lapse their policy. As the interest rate declined from 6.5% to 2%, the surrender rate
decreased from 1.8% to 0.8%, and the lapse rate decreased from 7.7% to 4.7%. This is because when interest rates are high, there are better alternative investment opportunities that policyholders can substitute into. On the other hand, since many life insurance and annuity products have embedded guarantees, policyholders would prefer to receive the minimum guaranteed rate on these products in a low-interest-rate environment.

An increase in the surrender rate and the lapse rate will influence the target duration of an insurance company. An increase in the surrender rate will reduce the liability duration, $D_L$, because the future liabilities become current liabilities (cash liability). How lapses affect the target duration is less straightforward. If a policyholder stops paying the premium, life insurance policies (whole life, variable universal life, and universal life insurance policies) with existing cash values will use its account value to pay for the unpaid premium. If the account value is insufficient to pay for the policyholder’s premium, then the policy will be considered lapsed. Recall that the target duration is equal to $D^*_A = \frac{L}{A} D_L$. Suppose an insurance company has two policies with payouts $L_1$ and $L_2$ (so that total liability $L = L_1 + L_2$), and duration of the two payouts are $D_{L1}$ and $D_{L2}$. The target duration of the insurer is $D^*_A = \frac{1}{A} \frac{L_1 D_{L1} + L_2 D_{L2}}{L_1 + L_2} = \frac{L_1 D_{L1} + L_2 D_{L2}}{A}$. Suppose, without loss of generality, policy 1 is lapsed; the target duration becomes $D^*_{A, \text{Lapse}} = \frac{L_2}{A} D_{L2} < D^*_A$. This means more lapses will also reduce the target duration of an insurance company.

When interest rates are low, the surrender rate and lapse rate are also low because investors will prefer to hold their policy with its guaranteed payment. This mechanism will increase the target duration of insurance companies. As a result, insurance companies will actively increase their asset duration in order to reduce the duration gap.

### 2.5.2 Estimating Parameters in the Partial Adjustment Model

As illustrated in the previous section, we run the following regression:

$$
ActiveDurationAdjustment_{i,t} = \text{const}_i + a \times LegacyDuration_{i,t} + \beta \times Leverage_{i,t-1} \\
+ \gamma \times Leverage_{i,t-1} \times r_t + \delta \times r_t + \text{error}_{i,t},
$$
Figure 2.5: Interest Rate and the Option to Surrender and Lapse

The figures plots the surrender rate (Panel A) and lapse rate (Panel B) against the 10-year treasury yield. The surrender rate is the amount of insurance policy surrendered each year as a fraction of total insurance contract in force. We aggregate the data from company level to the whole life insurance sector. The lapse rate is constructed similarly.
Table 2.2: Estimating Parameters in Partial Adjustment Model

The table reports the estimated coefficients in the partial adjustment model from the regression
\[ ActiveDurAdj_{i,t} = con_i + \alpha \times LegacyDur_{i,t} + \beta \times Leverage_{i,t} + \gamma \times Leverage_{i,t} \times r_{t}^{10} + \delta \times r_{t}^{10} + \epsilon_{i,t}. \]
The standard errors are double clustered by firm and quarter. Panel A runs the regression using book leverage constructed using insurance company’s quarterly filings. Panel B runs the regression using the subset of insurance companies whose parents are publicly listed firms. The book leverage is the book value of liability divided by total asset \( \frac{L}{A} \). The market leverage is constructed using \( 1 - \frac{E}{A} \) where \( E \) is the market capitalization at every quarter end. All regressions include firm FE.

<table>
<thead>
<tr>
<th>Panel A. Book Leverage</th>
<th>Variables</th>
<th>ActiveDurAdj_{i,t}</th>
<th>Panel B. Market Leverage</th>
<th>Variables</th>
<th>ActiveDurAdj_{i,t}</th>
</tr>
</thead>
<tbody>
<tr>
<td>LegacyDur_{i,t}</td>
<td>-0.0625***</td>
<td>(0.00458)</td>
<td>LegacyDur_{i,t}</td>
<td>-0.0596***</td>
<td>(0.00972)</td>
</tr>
<tr>
<td>BkLeverage_{i,t-1}</td>
<td>0.340***</td>
<td>(0.0560)</td>
<td>MktLeverage_{i,t-1}</td>
<td>0.242**</td>
<td>(0.104)</td>
</tr>
<tr>
<td>( r_{t}^{10} )</td>
<td>0.452</td>
<td>(0.766)</td>
<td>( r_{t}^{10} )</td>
<td>2.104</td>
<td>(1.775)</td>
</tr>
<tr>
<td>BkLeverage_{i,t-1} * ( r_{t}^{10} )</td>
<td>-2.981***</td>
<td>(0.961)</td>
<td>MktLeverage_{i,t-1} * ( r_{t}^{10} )</td>
<td>-4.876**</td>
<td>(2.260)</td>
</tr>
<tr>
<td>Observations</td>
<td>59,559</td>
<td>0.053</td>
<td>Observations</td>
<td>12,001</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis
*** \( p < 0.01 \), ** \( p < .05 \), * \( p < 0.1 \)

The regression allows us to test the predictions from the model by estimating the parameters. For all insurance companies, we have book leverage measured using liability divided by total assets, from insurance companies’ quarterly filings. And for public insurance companies, we can get market leverage using \( 1 - \frac{E}{A} \), where \( E \) is the market capitalization of stocks, and \( A \) is the total assets. Panels A and B in Table 2.2 report the estimation results using book and market leverage, respectively.

The estimation supports all four predictions from the model:

1. The coefficient of legacy duration \( \alpha = -0.0625 \) and is significant, satisfying \(-1 < \alpha < 0\). Note that \( -\alpha = \frac{\phi}{\psi+\phi} = 0.0625 \) is the speed of adjustment. The point estimate implies the
time to close half of the duration gap is about 11 quarters (Half Life = \( \frac{\ln(1/2)}{\ln(1-0.0625)} \)), assuming the duration target does not change. This suggests that the duration adjustment is gradual, and there are barriers to adjusting the asset portfolio immediately. In section 2.5.3, we show that insurers with different adjustment costs can have very different speeds of adjustment.

2. The coefficient of leverage \( \beta = 0.340 \) is positive and significant. The point estimate implies the coefficient \( a = \frac{\beta}{\phi/(\psi+\phi)} = \frac{0.340}{0.0625} = 5.44 \).

3. The coefficient of leverage interacted with the interest rate is \( \gamma = -2.981 \) is negative and significant. This implies a negative coefficient of \( b = \frac{\gamma}{\phi/(\psi+\phi)} = \frac{-2.981}{0.0625} = -47.70 \). Remember that \( D^*(r) = \frac{1}{\lambda}(a + b \times r) \); this means there is a negative long-run relationship between the interest rate and the duration target. If the 10-year treasury yield goes down by 1 percentage point, the target duration goes up by \( 0.477 \times 0.9 \) for the average firm with leverage of 0.9. This is economically meaningful.

4. The coefficient of the stand-alone interest rate \( r \) is \( \delta = 0.452 \) and is statistically indistinguishable from zero.

### 2.5.3 Adjustment Cost and the Speed of Adjustment

Our framework is based on the premise that insurance companies face costs when they want to rebalance their portfolios. Such costs depend on the liquidity of an insurer’s portfolio, because more illiquid assets are more costly to trade, as well as the size of the holdings of the insurer, because firms with larger holdings need larger trades and thereby face larger costs due to price pressures for the same amount of duration adjustment. Since a lower cost of rebalancing the portfolios, \( \psi \), allows an insurer to adjust the duration of its assets faster, our model suggests that insurers with more illiquid assets and larger holdings adjust their portfolio at a slower speed, i.e., have lower \( \frac{\phi}{\psi+\phi} \).

To test these implications about the relation of the speed of adjustment to holdings size and liquidity precisely, we focus on those insurance companies for which we have detailed trading and volume information on the bonds that constitute more than 90% of their holdings. This approach ensures that the duration and liquidity of the insurers’ holdings
are not systematically missing (and hence potentially biased) in a way that is correlated with the size and liquidity of an insurer’s holdings. To first establish that this approach does not introduce a sample selection bias to our previous results, columns (1) of Table 2.3 and Table 2.4 show that the results in this restricted sample are similar to the unrestricted sample in Table 2.2.

To test the first hypothesis, we sort the insurance companies into two groups based on whether the size of their holdings are above or below the sample median in a given quarter. The regression

\[
ActiveDurationAdjustment_{i,t} = \text{const}_i + \alpha \times \text{LegacyDuration}_{i,t} + \beta \times \text{Leverage}_{i,t-1} + \gamma \times \text{Leverage}_{i,t-1} \times r_t + \delta \times r_t + \text{error}_{i,t}
\]

is estimated separately for large insurers and small insurers to identify the coefficients \( \alpha = -\frac{\phi}{\psi + \phi} \), of which magnitude gives the speed of adjustment.

Columns (2) and (3) in Table 2.3 report the coefficients when the regression uses book leverage. For large companies, we have an adjustment speed of \( \frac{\phi}{\psi + \phi} = 0.0519 \), implying a half-life of about 13 quarters. For small companies, the adjustment speed increases to 0.0820, implying a half-life of about 8 quarters. The difference between these two groups (0.0302) is statistically significant. The other coefficients in the regressions, \( \beta = \frac{\phi}{\psi + \phi} \alpha \) and \( \gamma = \frac{\psi + \phi}{\psi + \phi} \beta \), become insignificant in this subsample, except the coefficient of lagged leverage in column (3) for small firms. These coefficients lose significance because we have fewer observations in this restricted sample, and the companies are further split into large and small ones. However, the point estimates for \( \beta = \frac{\phi}{\psi + \phi} \alpha \) and \( \gamma = \frac{\psi + \phi}{\psi + \phi} \beta \) are also greater in magnitude in column (3) relative to column (2), which is consistent with the conclusion from the estimates of \( \alpha = -\frac{\phi}{\psi + \phi} \) that smaller insurance companies have a greater speed of adjustment.

To test the second hypothesis, we first measure the liquidity of insurance companies’ holdings. The quarterly liquidity of each corporate bond \( i \) is measured using turnover, a commonly used proxy for liquidity (Datar et al. 1998; Avramov and Chordia 2006;
The table compares the speed of adjustment between insurance companies with high v.s. low adjustment costs. We divide the insurance companies into large v.s. small groups (columns 2 and 3), and into illiquid v.s. liquid groups (columns 4 v.s 5). The liquidity is measured using quarterly turnover of corporate bonds, then aggregated to insurer level.

For each group, we estimate the adjustment speed from the regression $ActiveDurAdj_{i,t} = \text{con}_i + \alpha \times LegacyDur_{i,t} + \beta \times Leverage_{i,t} + \gamma \times Leverage_{i,t} \times r_{i,t}^{10} + \delta \times r_{i,t}^{10} + \epsilon_{i,t}$. The standard errors are double clustered by firm and quarter. Columns (1) reports the estimation results for all companies in the restricted sample. Column (2) and (3) report the regression results for large v.s. small companies. And columns (4) and (5) report the regression results for illiquid v.s. liquid companies. The book leverage is the book value of liability divided by total asset $L/A$. All regressions include firm FE.

<table>
<thead>
<tr>
<th>Portfolio Size</th>
<th>Portfolio Liquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
</tr>
<tr>
<td>LegacyDur_{i,t}</td>
<td>-0.0546*** (0.0055)</td>
</tr>
<tr>
<td></td>
<td>-0.0820*** (0.0082)</td>
</tr>
<tr>
<td>BkLeverage_{i,t-1}</td>
<td>0.388*** (0.0689)</td>
</tr>
<tr>
<td></td>
<td>0.342*** (0.116)</td>
</tr>
<tr>
<td>r_{i,t}^{10}</td>
<td>0.183 (1.136)</td>
</tr>
<tr>
<td></td>
<td>-0.254 (1.571)</td>
</tr>
<tr>
<td>BkLeverage_{i,t-1} * r_{i,t}^{10}</td>
<td>-4.840*** (1.621)</td>
</tr>
<tr>
<td></td>
<td>-4.274 (2.680)</td>
</tr>
<tr>
<td>Difference between groups</td>
<td>-0.0302*** (0.0114)</td>
</tr>
<tr>
<td>Observations</td>
<td>27,138 13,521 13,575</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.078 0.088 0.091</td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis
*** $p < 0.01$, ** $p < .05$, * $p < 0.1$
Table 2.4: The Adjustment Cost and the Speed of Adjustment (Market Leverage)

The table compares the speed of adjustment between insurance companies with high v.s. low adjustment costs. We divide the insurance companies into large v.s. small groups (columns 2 and 3), and into illiquid v.s. liquid groups (columns 4 v.s 5). The liquidity is measured using quarterly turnover of corporate bonds, then aggregated to insurer level.

For each group, we estimate the adjustment speed from the regression \( \text{ActiveDurAdj}_{i,t} = c_{i,t} + \alpha \times \text{LegacyDur}_{i,t} + \beta \times \text{Leverage}_{i,t} + \gamma \times \text{MktLeverage}_{i,t} \times r^{10}_t + \delta \times r^{10}_t + \varepsilon_{i,t} \). The standard errors are double clustered by firm and quarter. Columns (1) reports the estimation results for all companies in the restricted sample. Column (2) and (3) report the regression results for large v.s. small companies. And columns (4) and (5) report the regression results for illiquid v.s. liquid companies. The market leverage is constructed using \( 1 - \frac{E}{A} \) where \( E \) is the market capitalization at every quarter end. All regressions include firm FE.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Large</td>
<td>Small</td>
<td>Illiquid</td>
<td>Liquid</td>
</tr>
<tr>
<td>LegacyDur_{i,t}</td>
<td>-0.0684***</td>
<td>-0.0487***</td>
<td>-0.0924***</td>
<td>-0.0423*</td>
<td>-0.104***</td>
</tr>
<tr>
<td></td>
<td>(0.0106)</td>
<td>(0.00976)</td>
<td>(0.0174)</td>
<td>(0.0225)</td>
<td>(0.0120)</td>
</tr>
<tr>
<td>MktLeverage_{i,t}</td>
<td>0.349</td>
<td>0.262</td>
<td>0.791</td>
<td>0.247</td>
<td>0.547</td>
</tr>
<tr>
<td></td>
<td>(0.291)</td>
<td>(0.212)</td>
<td>(0.765)</td>
<td>(0.373)</td>
<td>(0.379)</td>
</tr>
<tr>
<td>r^{10}_t</td>
<td>-0.0206</td>
<td>2.973</td>
<td>-0.438</td>
<td>-2.806</td>
<td>2.193</td>
</tr>
<tr>
<td></td>
<td>(4.124)</td>
<td>(3.343)</td>
<td>(9.804)</td>
<td>(5.599)</td>
<td>(5.381)</td>
</tr>
<tr>
<td>MktLeverage_{i,t-1} * r^{10}_t</td>
<td>-5.973</td>
<td>-6.241</td>
<td>-13.44</td>
<td>-0.677</td>
<td>-9.178</td>
</tr>
<tr>
<td></td>
<td>(6.231)</td>
<td>(4.237)</td>
<td>(17.13)</td>
<td>(6.467)</td>
<td>(7.251)</td>
</tr>
<tr>
<td>Difference between groups</td>
<td>-0.0436**</td>
<td>-0.0613**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0210)</td>
<td>(0.0277)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>6,069</td>
<td>4,063</td>
<td>1,996</td>
<td>2,926</td>
<td>2,920</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.080</td>
<td>0.104</td>
<td>0.092</td>
<td>0.100</td>
<td>0.146</td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis

*** \( p < 0.01 \), ** \( p < .05 \), * \( p < 0.1 \)
Rouwenhorst 1999). It is defined as the ratio of quarterly total volume traded and amount outstanding, i.e., \( \text{Liq}_{i,t} = \frac{\text{Volume}_{i,t}}{\text{AmtOut}_{i,j,t}} \). Then the liquidity of an insurance company \( j \)'s portfolio is measured as the weighted average liquidity of bonds held by the company, \( \text{PortfLiq}_{j,t} = \frac{\sum \text{Liq}_{i,t} \times \text{AmtHeld}_{i,j,t}}{\sum \text{AmtHeld}_{i,j,t}} \), where \( \text{AmtHeld}_{i,j,t} \) is the amount of bond \( i \) held by insurer \( j \) by the end of quarter \( t \). We sort the insurance companies into two groups based on whether their holdings' liquidity is above or below the sample median in a given quarter.

Columns (4) and (5) in Table 2.3 report the coefficients when the regression uses book leverage. For insurance companies with less liquid portfolios, we have an adjustment speed of \( \frac{\phi}{\phi + \psi} = 0.0529 \), implying a half-life of about 13 quarters. For insurance companies with more liquid portfolios, we have a must faster adjustment speed of 0.0871, which implies a half-life of about 8 quarters. The difference between these two groups (more liquid v.s. less liquid portfolios) is statistically significant. Again, the point estimates in column (2) for \( \beta = \frac{\phi}{\phi + \psi} a \) and \( \gamma = \frac{\phi}{\phi + \psi} b \) are much greater in magnitude than in column (1).

Finally, Table 2.4 reports the results for the same analysis using market leverage instead of book leverage. This further reduces the sample size by restricting the sample to public insurance companies. The results are qualitatively similar. Columns (2) and (3) suggest a half-life of 13 quarters for large insurers and 7 quarters for smaller insurers, and the difference between the two groups is statistically significant. Columns (4) and (5) in Table 2.4 suggest a half-life of 16 quarters for insurers with illiquid portfolios and 6.3 quarters for insurers with liquid portfolios.

Overall, Tables 2.3 and 2.4 support the prediction from our model that insurance companies with higher adjustment costs have lower adjustment speeds. Specifically, companies with more illiquid portfolios rebalance their bond assets more slowly to reduce the liquidity cost, and companies that hold more assets also adjust more gradually to avoid large price impacts of trades.
2.5.4 Active Duration Adjustment and the Option to Surrender and Lapse

The mechanism in our model is that interest rate changes affect policy holders’ surrender and lapse behavior, thus affecting the target duration of insurance companies, which then transmits into active duration adjustment on the asset side. For the mechanism to work, any surrender or lapse caused by interest rate fluctuations should lead to active duration adjustment. The mechanism (for the case of surrenders) is testable using the following two-stage instrumental variable regression:

\[
\text{SurrenderRatio}_{i,t} = \theta_i + \eta \times r_t + \varphi \times \text{LegacyDuration}_{i,t} + \epsilon_{i,t}
\]

\[
\text{ActiveDurationAdjustment}_{i,t} = \text{const}_i + \alpha \times \text{LegacyDuration}_{i,t} + \beta \times \text{SurrenderRatio}_{i,t} + \text{error}_{i,t}
\]

where \( \text{SurrenderRatio}_{i,t} \) is the first stage regression estimate, which we use in the second stage regression. The \( \text{LegacyDuration}_{i,t} \) is also included in the first stage as is standard in the implementation of an instrumental variable approach in two-stage least squares regression. Our results are robust if we exclude \( \text{LegacyDuration}_{i,t} \) in this regression.

This mechanism creates two predictions:

1. In the first stage regression, we have \( \eta > 0 \). When the interest rate is higher, there will be more policy surrenders.

2. In the second stage regression, we have \( \beta < 0 \). Companies with a higher (lower) surrender ratio will have to actively decrease (increase) their asset duration.

Similarly, we could test the effect of lapses on active duration adjustment by replacing the surrender ratio with the lapse ratio in the regression.

Table 2.5 reports the estimation for the two-stage regressions using quarterly data. For both the surrender ratio and the lapse ratio, we find strong support for \( \eta > 0 \) and \( \beta < 0 \). In the first stage regression, a 1 percentage point decrease in the interest rate is associated with a 0.30 percentage point decrease in the surrender ratio and a 0.68 percentage point decrease in the lapse ratio. In the second stage, a 1 percentage point decrease in the predicted
surrender ratio is associated with a positive quarterly active duration adjustment of 0.0795. And a 1 percentage point decrease in the lapse ratio is associated with a quarterly active duration adjustment of 0.035. A one standard deviation change in the surrender ratio (0.07) corresponds to a change in the active duration adjustment of 0.56 (=7.95*0.07), and a one standard deviation change in the lapse ratio (0.10) corresponds to a quarterly change in the active duration adjustment of 0.345 (=3.45*0.10). The magnitudes are economically meaningful and comparable to a one standard deviation change in the active duration adjustment (0.55).

Table 2.6 reports the results for the same two-stage regressions using annual data. The effect of the surrender ratio and the lapse ratio on the active change in duration is about four times compared to the quarterly data.

2.6 Interest Rate Sensitivity of the Return on Equity: Model vs. Data

As a final test of the model, we compare the interest rate sensitivity of equity returns implied by the model vs. the data. In particular, remember from Section 2.3.2 that the duration of equity can be calculated by running a regression of stock returns on changes in yields:

\[ Ret_{E,t} = Q - D_E \Delta y_{10,t} + \varepsilon, \]  
(2.8)

where \( D_E \) is the duration of equity and \( \Delta y_{10,t} \) is the change in the 10-year Treasury yield.

Also note that the duration of equity is related to the duration of assets, \( D_A \), and the target duration of equity, \( D^*_A \), via

\[ D_E = G \frac{A}{E} = \left( D_A - \frac{L}{A} D_L \right) \frac{A}{E} = (D_A - D^*_A) \frac{A}{E}. \]  
(2.9)

Using the relationship implied by the model, \( \Delta D_A = -\frac{q}{\delta + \phi} [D^0_A - D^*_A] \), the definition of active duration adjustment, \( \Delta D_A = D_A - D^0_A \), and the relationship between assets, equity,
Table 2.5: Active Duration Adjustment and Option to Surrender and Lapse (Quarterly)

The table reports the estimation result of the two stage regressions, which studies how interest rate changes transmits to the active duration adjustments on the asset side of insurance companies. The first stage regresses the surrender ratio on interest rate and legacy duration. The second stage regresses the quarterly active duration adjustment on predicted surrender ratio and legacy duration. All regressions include firm FE.

### Panel A. Active Duration Adjustment and Surrender Ratio

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>1st Stage</th>
<th>2nd Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>LegacyDur(_{i,t})</td>
<td>4.3 × 10(^{-5})</td>
<td>LegacyDur(_{i,t})</td>
</tr>
<tr>
<td>(r_t^{10})</td>
<td>0.295***</td>
<td>(SurrenderRatio_{i,t})</td>
</tr>
<tr>
<td>Observations</td>
<td>51,520</td>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.576</td>
<td>R-squared</td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis

*** p < 0.01, ** p < .05, * p < 0.1

### Panel B. Active Duration Adjustment and Lapse Ratio

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>1st Stage</th>
<th>2nd Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>LegacyDur(_{i,t})</td>
<td>-4.82 × 10(^{-4})</td>
<td>LegacyDur(_{i,t})</td>
</tr>
<tr>
<td>(r_t^{10})</td>
<td>0.679***</td>
<td>(LapseRatio_{i,t})</td>
</tr>
<tr>
<td>Observations</td>
<td>51,520</td>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.488</td>
<td>R-squared</td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis

*** p < 0.01, ** p < .05, * p < 0.1
Table 2.6: Active Duration Adjustment and Option to Surrender and Lapse (Annual)

The table reports the estimation result of the two stage regressions, which studies how interest rate changes transmits to the active duration adjustments on the asset side of insurance companies. The first stage regresses the surrender ratio on interest rate and legacy duration. The second stage regresses the annual active duration adjustment on predicted surrender ratio and legacy duration. All regressions include firm FE.

<table>
<thead>
<tr>
<th>Panel A. Active Duration Adjustment and Surrender Ratio</th>
<th>1st Stage</th>
<th>2nd Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>VARIABLES</td>
<td>$\text{SurrenderRatio}_{i,t}$</td>
<td>$\text{ActiveDurAdj}_{i,t}$</td>
</tr>
<tr>
<td>LegacyDur$_{i,t}$</td>
<td>$2.8 \times 10^{-5}$</td>
<td>$-0.253^{***}$</td>
</tr>
<tr>
<td></td>
<td>(2.86 $\times 10^{-4}$)</td>
<td>(0.0229)</td>
</tr>
<tr>
<td>$r_{10}^{i}$</td>
<td>$0.304^{***}$</td>
<td>$-32.94^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.0692)</td>
<td>(10.16)</td>
</tr>
<tr>
<td>Observations</td>
<td>12,909</td>
<td>12,889</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.586</td>
<td></td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis

$^{***} p < 0.01, ^{**} p < .05, ^{*} p < 0.1$

<table>
<thead>
<tr>
<th>Panel B. Active Duration Adjustment and Lapse Ratio</th>
<th>1st Stage</th>
<th>2nd Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>VARIABLES</td>
<td>$\text{LapseRatio}_{i,t}$</td>
<td>$\text{ActiveDurAdj}_{i,t}$</td>
</tr>
<tr>
<td>LegacyDur$_{i,t}$</td>
<td>$-4.74 \times 10^{-4}$</td>
<td>$-0.261^{***}$</td>
</tr>
<tr>
<td></td>
<td>(6.09 $\times 10^{-4}$)</td>
<td>(0.0236)</td>
</tr>
<tr>
<td>$r_{10}^{i}$</td>
<td>$0.698^{***}$</td>
<td>$-14.31^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.116)</td>
<td>(3.669)</td>
</tr>
<tr>
<td>Observations</td>
<td>12,909</td>
<td>12,889</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.482</td>
<td></td>
</tr>
</tbody>
</table>

Robust Standard errors in parenthesis

$^{***} p < 0.01, ^{**} p < .05, ^{*} p < 0.1$
and leverage, \( \frac{A}{E} = 1/(1 - \text{Leverage}) \), we obtain

\[
D_E = (D_A - D_A^*) \frac{A}{E} = \left(1 - \frac{\psi + \phi}{\phi} \right) \Delta D_A \left( \frac{1}{1 - \text{Leverage}} \right).
\]

(2.10)

Thus, we can define the model-implied duration of equity as:

\[
D_{E, \text{Model}} = \left(1 - \frac{\psi + \phi}{\phi} \right) \Delta D_A \left( \frac{1}{1 - \text{Leverage}} \right).
\]

(2.11)

This model-implied duration of equity, \( D_{E, \text{Model}} \), can be directly computed from the data using this expression and our regression results in Section 2.5.2. In particular, our regression results directly provide the estimate for \( \frac{\psi + \phi}{\phi} \), which is the inverse of the speed of adjustment, and we can directly observe \( \Delta D_A \), the active duration adjustment, and leverage from the data. We allow the adjustment speed \( \frac{\psi + \phi}{\phi} \) to be time-varying when computing model-implied duration \( D_{E, \text{Model}} \). Specifically, the regression in Section 2.5.2 is run over a three-year rolling window (the past 12 quarters) to estimate the adjustment speed.

A natural way to test the validity of the model-implied duration of equity is running the following regression in a panel setting:

\[
Ret_{E,i,t} = Q_i - \beta D_{E,i,t-1}^{\text{Model}} \Delta y_{10,t} + \epsilon_{i,t}.
\]

(2.12)

where \( Ret_{E,i,t} \) is the equity return of insurance company \( i \) in quarter \( t \), and \( D_{E,i,t}^{\text{Model}} \) is the model-implied duration of insurance company \( i \) in quarter \( t \), which we compute in our data. Comparing equations 2.8 and 2.12, we should observe that \( \beta > 0 \), and more specifically \( \beta \approx 1 \). However, since \( D_{E, \text{Model}}^{\text{Model}} \) is just an estimate, it can suffer from measurement error leading to estimates of \( \beta \) biased toward zero.

Table 2.7 presents the estimates of this regression. The results are quantitatively similar regardless of whether we use the model-implied duration based on book leverage (Panel A) or the one based on market leverage (Panel B). The first column gives the result from a standard panel regression. The reassuring result is that the estimate for the coefficient \( \beta \) is positive, approximately equal to 0.4. However, it is significantly different from 1, which would be the value of \( \beta \) if the model were perfect. This result can at least partially be
Table 2.7: Equity Duration: Model vs Data

The table reports the estimated coefficients from the regression
\[ \text{Ret}_{i,t} = \text{con}_t - \beta \times D_{\text{Model},i,t} \times \Delta y_{10,t} + \varepsilon_{i,t} \]
The standard errors are double clustered by firm and quarter. Panel A runs the regression where \( D_{\text{Model},i,t} \) is calculated using book leverage from insurance company’s quarterly filings. The book leverage is the book value of liability divided by total asset \( \frac{L}{A} \). Panel B runs the same regression using market leverage. The market leverage is constructed using \( 1 - \frac{E}{A} \) where \( E \) is the market capitalization at every quarter end. All the right hand side variables are winsorized at 5% to remove the effect of outliers. All regressions include firm FE. *** \( p < 0.01 \), ** \( p < .05 \), * \( p < 0.1 \)

### Panel A. Model-Implied Equity Duration based on Book Leverage

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-D_{\text{Model}} \times \Delta y_{10,t})</td>
<td>0.442***</td>
<td>1.120***</td>
<td>1.132***</td>
<td>1.136***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.335)</td>
<td>(0.412)</td>
<td>(0.372)</td>
</tr>
<tr>
<td>Observations</td>
<td>11,687</td>
<td>11,481</td>
<td>11,571</td>
<td>11,409</td>
</tr>
</tbody>
</table>

### Panel B. Model-Implied Equity Duration based on Market Leverage

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-D_{\text{Model}} \times \Delta y_{10,t})</td>
<td>0.408***</td>
<td>1.149**</td>
<td>1.283***</td>
<td>1.266***</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.468)</td>
<td>(0.290)</td>
<td>(0.323)</td>
</tr>
<tr>
<td>Observations</td>
<td>11,734</td>
<td>11,493</td>
<td>11,428</td>
<td>11,263</td>
</tr>
</tbody>
</table>

attributed to the attenuation bias due to measurement error as discussed above. In order to address the attenuation bias, we also run the same regression using an instrumental variable approach as described below.

The first instrument for \( D_{\text{Model}} \) (IV1) is lagged \( D_{\text{Model}} \), where the identification assumption is that while \( D_{\text{Model}} \) is persistent, the errors in the adjustment cost model regressions are not. The second instrument (IV2) for \( D_{\text{Model}} \) is

\[
D_{\text{Model},i,t} = \left( 1 - \frac{\psi + \phi}{\phi} \right) \Delta D_A \left( \frac{1}{1 - \text{Leverage}} \right),
\]

(2.13)
where $\hat{\Delta D_A}$ is the predicted value of active duration adjustment that comes from the regressions in Section 2.5.2. This instrument is valid because while $\hat{\Delta D_A}$ is highly correlated with the actual value of $\Delta D_A$, it is not correlated with the error term in the regression that introduces the measurement error. Similar to the adjustment speed, the instrument $\hat{\Delta D_A}$ is also calculated using a rolling window of three-years (the past 12 quarters). Columns 2-4 of Table 2.7 show that using IV1 and IV2 produce very similar results, both when used separately and together, and significantly increase the estimated coefficient from a value of roughly 0.4 to a value of 1.12 to 1.13. These empirical facts support the prediction of the model that $\beta \approx 1$, and shows that the implied-duration of equity from our partial adjustment model is comparable with the equity duration estimated from stock returns.

2.7 Discussion of Alternative Explanations

Our model of duration matching under adjustment cost can explain why insurance companies deviate from a zero duration gap for extended periods of time and hence conforms with the pattern in Figure 2.4. This pattern cannot readily be explained by a “reaching for duration” hypothesis because Figure 2.4 suggests that the duration gap has been negative in the last decade (post-2009) and the insurance companies reaching for yield could close this gap by buying longer duration bonds and simultaneously earning higher yields. Still, one may naturally ask whether there can be alternative explanations for the large negative duration gap in the last decade. In this section, we discuss two explanations alternative to the adjustment cost model.\(^{15}\)

One explanation for the recent period can be that insurance companies might be already holding the highest duration corporate bonds in the market but these bonds are still not enough to give them perfect duration matching. In other words, there are simply not enough high duration corporate bonds outstanding. Another explanation can be insurance companies might not want to increase their asset duration if higher duration corporate

\(^{15}\)We thank John Campbell and Ralph Koijen for discussions motivating these alternative explanations.
bonds in the market are issued by less risky firms and hence provide lower yields. We investigate these explanations by answering two questions.

First, what would the duration of insurance companies be if they would hold highest duration bonds in the same NAIC category? If insurance companies could significantly increase the duration of their assets by holding highest duration bonds without changing their risk-based capital, it would be hard to argue that there are simply not enough high duration bonds. Second, would the insurance companies earn lower yields if they actually held these highest duration bonds? If they would not earn lower yields by holding higher duration bonds, it would be hard to argue that insurance companies avoid longer duration assets because these assets would provide them with lower investment income.

To answer the first question, we take the total par amount of bonds held by insurance companies and calculate what the duration of their holdings would be if they invested this amount in highest duration bonds. Suppose, without loss of generality, the insurance company holdings at quarter $t$ is given by $H_t$ and the bonds in the market are ranked from high to low duration, $D_{1,t}, D_{2,t}, D_{3,t}, \ldots$, with outstanding amounts $A_{1,t}, A_{2,t}, A_{3,t}, \ldots$ respectively. Since the insurance companies do not hold the whole market there is a value of $K$ for which $\sum_{i=1}^{K} A_{i,t} = H_t$. Then we can calculate the maximum possible duration that the insurance company assets can have as $D_{A,t}^{\text{max}} = (\sum_{i=1}^{K} D_{i,t} A_{i,t}) / \sum_{i=1}^{K} A_{i,t}$. The difference between $D_{A,t}^{\text{max}}$ and the actual asset duration of the insurance companies, $D_{A,t}$ gives us the “duration slack”, $D_{A,t}^{\text{slack}} = D_{A,t}^{\text{max}} - D_{A,t}$, i.e. how much more insurance companies could increase their duration.\footnote{For robustness, we use maturity as an alternative measure because duration can only be calculated when the yield of a bond can be calculated. The “maturity slack” can be introduced in a similar way. The results for maturity slack are similar to duration slack, and are available in the appendix.}

Figure 2.6 panels A and B plot the duration slack over time for both NAIC1 and NAIC2 categories, i.e. how much extra duration insurance companies could possibly attain within each NAIC rating category. We see a gradual increase in the slack duration in the recent decade in both rating categories. The duration slack is about 7 for both NAIC1 and NAIC2 bonds by 2016. This result suggests insurance companies are still quite far from the
Figure 2.6: Time Series of Duration Slack and Yield Slack

The figure plots the time series of duration slack and yield slack for both NAIC1 and NAIC2 bonds. Duration slack is defined as the difference between maximum possible duration (that the insurance company assets can have) and the actual asset duration of the insurance companies. The yield slack is defined as the difference in yield between the maximum duration portfolio and the insurance companies’ actual portfolio.
maximum asset duration they could achieve, going against the argument that there are not enough long duration bonds in the market.

To answer the second question, we use a similar approach to calculate the “yield slack”, i.e. how much more yield the insurance companies could earn by investing in highest duration bonds in the same NAIC rating category. Using the same ordering of the bonds as before, we can calculate yield slack as

$$y_{i,t}^{\text{slack}} = (\sum_{i=1}^{K} y_{i,t} A_{i,t}) / \sum_{i=1}^{K} A_{i,t} - y_{A,t}$$

where $y_{i,t}$ is the yield on bond $i$ and $y_{A,t}$ is the yield on assets held by the insurance companies.

Figure 2.6 panels C and D plot the yield slack over time for both NAIC1 and NAIC2 categories. The yield slack has almost always been positive and has been, on average, above one percentage point in the recent decade. This suggest that by holding higher duration bonds, insurance companies could have achieved more than 1 percentage point higher yields than their current portfolio. Therefore, insurance companies would not earn lower yields if they were to include higher duration assets in their portfolio, suggesting that potential loss of investment income is an unlikely reason for not minimizing the duration gap by increasing the asset duration.

In sum, in a frictionless world without adjustment cost, holding a higher-duration portfolio would allow insurance companies to both close the duration gap and obtain higher yield. Therefore, the additional evidence in this section is consistent with our adjustment cost framework.

2.8 Conclusion

In this paper, we document three stylized facts about the interest rate and life insurance companies’ investment behavior. As the interest rate declines, life insurance companies increase the excess yield on their corporate bond portfolio (relative to the market). Using a matching algorithm to unpack the risk quantities of insurance companies’ portfolio, we find that most of the excess yield is driven by “duration tilt” rather than “credit risk tilt”. We propose a “target duration hypothesis” to explain insurance companies’ investment behavior which is consistent with the stylized facts. According to the hypothesis, insurance companies
adjust asset duration to match a time-varying “duration target”. When the interest rates increase, policyholders strategically close out a contract in return for a predetermined payment. This changes insurers’ “duration target”, which then transmits into active duration adjustment. Finally, we test several predictions from the target duration model.
Chapter 3

Global Portfolio Diversification for Long-Horizon Investors

3.1 Introduction

A large body of empirical research in Finance has documented the existence of significant benefits from global portfolio diversification stemming from the historically low level of correlation of global equity market returns. Given these correlations, investors would need implausibly large and mutually inconsistent return expectations on their own stock markets to justify holding a domestically biased equity portfolio (French and Poterba, 1991). The cross-country correlations of long-term government bond returns are also historically low, suggesting that the case for holding a globally diversified bond portfolio is also strong (Campbell, Serfaty-de-Medeiros, and Viceira, 2010).

However, in recent decades global equity markets have experienced a significant increase in return correlations resulting from trade and financial globalization (Goetzmann, Li, and Rouwenhorst 2005, Quinn and Voth, 2008, Solnik and McLeavey 2009 and Bekaert and

---

Hodrick 2017). Figure 3.1 documents this empirical phenomenon. It plots the cross-country average 3-year moving correlations of monthly equity and bond excess returns across seven major markets that account for the bulk of global stock and bond market capitalization: Australia, Canada, France, Germany, Japan, United Kingdom, and the United States. The figure plots these correlations for the 1970-2016 period. The figure shows a secular increase in the cross-country correlations of stock and bond returns since 1970, and a further temporary significant increase in global stock return correlations during the global financial crisis of 2008-2009. Goetzmann, Li, and Rouwenhorst (2005) reports a similar figure for stock returns dating back to the second half of the 19th century, and shows that historical episodes of trade and capital flow liberalizations appear to be associated with increased correlations of global equity markets.

The significant secular increase in global return correlations raises the question of whether the gains from international portfolio diversification have also declined correspondingly. In an environment with constant investment opportunities, an increase in cross-country return correlations unambiguously reduces the benefits of international portfolio diversification for all investors, unless there is a compensating increase in expected returns. However, there is considerable academic research documenting time variation in investment opportunities in the form of predictable variation in discount rates, volatility, and risk. If investment opportunities are not constant, is it still true that an increase in

---

2 The gains from global portfolio diversification have also been questioned on other fronts. It has been argued that global stock returns become more correlated in falling markets and exhibit negative co-skewness. However, empirically this effect is not enough to eliminate the gains from global equity portfolio diversification; moreover, there is no evidence of negative co-skewness at longer horizons (De Santis and Gerard 1997, Ang and Bekaert 2002, Longin and Solnik 2001, Hartmann et al. 2004, Chua et al. 2009, Leibovitz and Bova 2009, Asness et al. 2011). A second argument is that domestic portfolios focused on global companies could potentially produce the same diversification gains as a global portfolio, but the empirical evidence suggests the two are not substitutes, especially when including medium and small capitalization stocks (Errunza et al. 1999, Cheol et al. 2010). A third argument relies not so much on questioning that there are gains from global diversification but on attributing them to sector diversification (Carrieri, Errunza, and Sarkissian, 2012). However, the empirical evidence suggests that the diversification benefits of global equities come from both country factors and industry factors (Heston and Rouwenhorst 1994, Campa and Fernandes 2006).


4 There appears to be predictable variation in discount rates, both real interest rates and risk premia, at the asset class level and at the individual stock level (Campbell 1991, Cochrane, 2008 and 2011, Vuolteenaho 2002).
Figure 3.1: Stock and Bond Correlations Across Countries

This figure plots average correlations of stock returns across countries and bond returns across countries. Monthly averages are computed using pairwise return correlations across seven different countries over 3-year rolling windows (Australia, Canada, France, Germany, Japan, United Kingdom, and United States). Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate. The sample is from Jan 1986 to Dec 2016.
This paper explores this question both theoretically and empirically. We examine global portfolio diversification in the presence of time variation in discount rates, both real interest rates and risk premia, and in market volatility. We show that in such environment, both the risk of globally diversified portfolios and optimal international portfolio diversification are a function of investment horizon, and an increase in return correlations does not necessarily imply a reduction in the benefits of global portfolio diversification for long horizon investors.

Our argument builds on the distinction between shocks to cash flows or fundamentals (“cash flow news”) and shocks to discount rates (“discount rate news”) that arises when discount rates are time varying. This distinction implies that asset returns can be correlated because either cash flows are correlated, or discount rates are correlated. Empirically, cash flow shocks appear to be highly persistent, while discount rate shocks appear to be transitory (Campbell and Shiller, 1988, Campbell 1991, Campbell and Vuolteenaho 2004).

We show that the impact on portfolio risk of correlated persistent cash flow shocks on portfolio risk is independent of investment horizon, while the impact of correlated transitory discount rate shocks is a decreasing function of investment horizon. Therefore if cash flows become more correlated across markets, the scope for global portfolio diversification declines for all investors regardless of their investment horizon. By contrast, if discount rates become more correlated, the scope for global portfolio diversification declines for short-term investors, but less so for long-term investors, since discount rates have only a temporary impact on valuations and returns. Correlated persistent shocks to market volatility increase portfolio risk at all horizons.

We next conduct an empirical investigation of global portfolio diversification in equities...
and sovereign bonds in the period 1986-2016. Using the return decomposition and news estimation framework of Campbell (1991), we estimate the sources of cross-country return correlations for stocks and bonds in the entire sample period as well as in two subperiods, 1986-1999 and 2000-2016. Although we do not account explicitly for estimation uncertainty in our analysis, we use simultaneously the whole cross-section of countries in our estimation to increase power; we also provide ample auxiliary evidence supporting the main conclusions derived from our main news estimation approach.

Our empirical analysis reveals an economically and statistically significant increase in the average cross-country correlation of discount rate news, both real rate news and risk premia news, from the 1986-1999 period to the 2000-2016 period for both stocks and bonds, although the increase in the correlation of risk premia news has been more pronounced for equities than for bonds. We also find a significant increase in the average cross-country correlation of nominal bond cash flow news, or inflation news. However, we do not find a significant increase in the correlation of cash flow news for stocks. Multiple direct measures of equity cash flows corroborate this finding.

We also estimate market volatility news for the cross-section of stock markets included in our sample following the methodology of Campbell, Giglio, Polk, and Turley (2017). We find that the average cross-country correlation of persistent shocks to market volatility has remained fairly stable and low over the entire sample period, with the exception of a temporary but significant increase during the financial crisis of 2008-2009.

Our results add to the extensive empirical literature measuring financial integration, particularly to a nascent but growing body of research that explores the sources and effects of globalization on capital markets. Following Ammer and Mei (1996), we interpret the

---

6Our start date is constrained by data availability for the seven countries included in Figure 3.1.

7There is disagreement in the literature about how precisely one can estimate time variation in expected returns: See Campbell and Yogo (2006), Campbell and Thompson (2008), Goyal and Welch (2008), and Pastor and Stambaugh (2009 and 2012).

increase in the cross-country correlations of discount rate news across subperiods as an indicator of increased financial market integration. Accordingly our estimates suggest that a stronger degree of financial integration of global markets in the most recent period is the main driver of the increment in one-period stock return correlations shown in Figure 3.1. We show that our results are robust to considering the alternative measure of capital market integration proposed by Pukthuanthong and Roll (2009), which we extend to accommodate the distinction between cash flow news and discount rate news.

Arguably today the marginal investor in developed equity markets is more likely to be a global investor, and more so for equity markets than for bond markets, for which regulatory capture or “financial repression” (Reinhart and Rogoff, 2014) might induce a lesser degree of integration. Davis and van Wincoop (2017) documents a large increase in the global correlation between capital inflows and outflows from 1970-1990 to 1990-2011, which they attribute to an increase in financial globalization. Lustig, Stathopoulus, and Verdelhan (2016) estimates stochastic discount factors (SDF) for G10 countries using bond data, and show that permanent shocks to each SDF are highly correlated and exhibit very similar volatility in the 1985-2012 period.

For nominal bonds, the increase in the cross-country correlations of cash flow news reflects an increase in the cross-country correlation of inflation news, since their real cash flows vary inversely with inflation. Our results suggest that increased correlation of inflation news across monetary areas has also been an important contributor to the increase in one-period bond return correlations in the most recent period. These findings add to research that documents a large increase in the average cross-country correlation of inflation, suggesting the presence of a global factor in inflation (Wang and Wen 2007, Mumtaz, Simonelli and Surico 2011, Neely and Rapach 2011, and Henriksen, Kydland and Sustek 2013). This increased correlation in inflation could be the result of successful inflation targeting by central banks, which has operated as an implicit mechanism of coordination in monetary policy and has reduced country-specific variation in inflation expectations others.
The final section in this paper investigates the implications of our empirical findings about global return news correlations for global portfolio diversification in two related ways. First, we compute the risk of global portfolios of stocks and bonds as function of investment horizon for each subperiod (Campbell and Viceira 2005). Second, we compute optimal intertemporal global equity portfolio allocations and expected utility implied by our estimates across periods for investors with different degrees of relative risk aversion and investment horizons (Campbell, Chan, and Viceira 2003, Jurek and Viceira 2011).

Our portfolio risk analysis shows that, consistent with our theoretical findings, the significant increase in one-period return correlations in the 2000-2016 period relative to the 1986-1999 has increased the short-run risk of global equity portfolios but not their long-run risk. In fact, we estimate a decline in long-run global equity portfolio risk in the second subperiod as the result of both a correlation effect and a volatility effect. The correlation effect is that increased correlation of transitory discount rate news accounts for most of the increase in return correlations, and we have shown that correlated discount rate shocks have a minimal effect on long-run return correlations. The volatility effect is an estimated increase in the degree of stock return predictability in the second subperiod which in turn implies a reduction in stock return volatility at long horizons. It is well known that the persistent run up in global stock market valuations in the late 1980’s and the 1990’s weakened the evidence of stock return predictability, which has been restored in the most recent period.

By contrast, we estimate that the risk of global bond portfolios has increased at all horizons in the second subperiod, as a result of the increase in the correlations of bond cash flow news. This upward shift in the risk of global bond portfolios is detrimental to long-only bond investors, but beneficial to investors with long-term liabilities such as pension funds. For such investors, increased bond return correlations expand the universe of bonds they can use to hedge their local pension liabilities. These benefits can be especially large to investors whose liabilities are large relative to the size of their domestic bond markets and are exposed to adverse price pressure when they try to hedge their liabilities in their local

Our analysis of the optimal intertemporal global equity portfolio allocations and expected utility implied by our news estimates shows that the increase in the cross-country correlations of stock returns has not led to reduction in the benefits of global equity portfolio diversification at long horizons. Because the increase in return correlations results from correlated discount rate news, long-horizon investors still find that holding global equity portfolios helps diversify cash flow risk.

The paper is organized as follows. Section 2 introduces the basic asset return decomposition into cash flow news and discount rate news. Section 3 explores long-run portfolio risk and optimal intertemporal global portfolio diversification in a stylized symmetrical model of global markets. This section provides insights into the differential effects of each type of return news on long-run global portfolio risk and portfolio choice. Section 4 conducts an empirical analysis of the changes in cross-country stock and bond return correlations over time and the sources of these changes. Section 5 introduces auxiliary evidence of the empirical results on Section 4 and investigates correlated persistent shocks to market risk. Section 6 examines the implications of our estimates of cash flow news and discount rate news for the risk of globally diversified portfolios of stocks and bonds across investment horizons, and for optimal intertemporal portfolio choice. Finally, Section 7 concludes. An Online Appendix provides full details on all the derivations of the results in the paper and all supplementary empirical results not reported in the main body of the paper.\footnote{This Appendix is available at http://www.people.hbs.edu/lviceira/publications.html.}

3.2 Asset Return Decomposition

The starting point of our analysis is the log-linear approximation to present value relations of Campbell and Shiller (1988) and the return decomposition of Campbell (1991). A log-linearization of the return on an asset around the unconditional mean of its dividend-price ratio—where dividend is a proxy for cash flow—implies the following decomposition of
realized returns:

\[ r_{s,t+1} - \mathbb{E}_t [r_{s,t+1}] = (\mathbb{E}_t - \mathbb{E}_t) \sum_{j=0}^{\infty} \rho_s^j \Delta d_{t+1+j} - (\mathbb{E}_t - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho_s^j r_{t+1+j} \]

\[ \equiv N_{CF,s,t+1} - N_{DR,s,t+1}. \]

where \( r_{s,t} \) denotes the natural log of the gross total return on the asset and \( \Delta d_{t+1} \) the change in its log dividend (or cash flow). The constant \( \rho_s \equiv 1/ (1 + \exp (\overline{d} - p)) \) is a log-linearization parameter, where \( \overline{d} - p \) denotes the unconditional mean of the log dividend-price ratio.

Equation (C.1) shows that the unexpected log return on an asset reflects changes in either its expected future cash flows or in its expected future returns (or discount rates). Following standard terminology in this literature, we will refer to the former as cash flow shocks or cash flow news \((N_{CF,s,t+1})\), and to the latter as discount rate shocks or discount rate news \((N_{DR,s,t+1})\). We can further decompose \( N_{DR,s,t+1} \) into the sum of news about excess log returns or risk premia \((N_{RP,s,t+1})\) and news about the return on the reference asset used to measure excess returns \((N_{RR,s,t+1})\). In our empirical analysis we follow standard practice and use cash (i.e., a short-term nominal bond like a T-bill in the US) as the reference asset.

We also consider nominal bonds with fixed maturities in addition to equities. Nominal bond cash flows (i.e., coupons) are fixed in nominal terms and thus vary inversely with the price level in real terms. Therefore in logs bond cash flow news are the negative of inflation news. The Appendix provides detailed expressions of the news components of stock and bond returns.

Asset return news components are not directly observable, but we can infer them from a return generating model. We follow Campbell (1991) and assume that the asset return generating process follows a first-order vector autoregressive (VAR) model. It is important to note three observations about this VAR specification.

First, a VAR(1) specification is not restrictive in the sense that it can easily accommodate higher order lags through a straightforward change in the state vector. Second, it is also well know that return decompositions are sensitive to the particular specification of the
components of the state vector (Chen and Zhao 2009). Our empirical specification of the state vector includes variables for which there is wide consensus that they capture time variation in risk premia, inflation, and real interest rates. We also conduct an additional analysis in Section 5 that corroborates that our results are robust to our specification of the VAR. Third, our main empirical analysis is based on a homoskedastic version of the VAR, but we also consider a heteroskedastic specification along the lines of Campbell, Giglio, Polk, and Turley (2017) in Section 5.

### 3.3 Global Portfolio Diversification with Time-Varying Discount Rates

The return decomposition (C.1) implies that there are two potential sources of correlation in asset returns when discount rates are time varying: correlated cash flows and correlated discount rates. This section develops a symmetrical model of investment opportunities with N asset markets (or “countries”) to analyze the contribution of each source of correlation to portfolio risk across investment horizons, portfolio choice, and the benefits of portfolio diversification at long horizons.

Our return generating model is a direct extension to a multi-market setting of the canonical model of time-varying investment opportunities of Campbell and Viceira (1999), Barberis (2000), and Pastor and Stambaugh (2009, 2012). This stylized model is particularly helpful to interpret the results of our subsequent empirical analysis of global portfolio risk and portfolio choice at long horizons. Please refer to the Appendix for derivations of all results in this section.
3.3.1 Model

There are $N$ ex-ante identical markets with identical return generating process described by the following single-state variable VAR(1) model:

\begin{align*}
    r_{i,t+1} &= \mu_1 + \beta s_{i,t} + u_{i,t+1} \\
    s_{i,t+1} &= \mu_2 + \phi s_{i,t} + u_{s_{i,t+1}},
\end{align*}

(3.3)

where $r_{i,t+1}$ denotes the log return on country $i$, and $s_{i,t+1}$ denotes the single state variable that drives the time variation in the conditional expected return on country $i$: $E_t[r_{i,t+1}] = \mu_1 + \beta s_{i,t}$. The parameters $\mu_1$, $\mu_2$, $\beta$, and $\phi$ are identical across countries. Without loss of generality we normalize $\beta > 0$. To preserve stationarity, we must have $|\phi| < 1$.

The conditional within-country variance-covariance matrix of the innovations to the VAR is also identical across countries and constant over time:

\[
    V_t[u_{i,t+1}] = \begin{bmatrix}
        \sigma_{uu}^{wc} & \sigma_{us}^{wc} \\
        \sigma_{us}^{wc} & \sigma_{ss}^{wc}
    \end{bmatrix}.
\]

(3.5)

where $u_{i,t+1} = (u_{i,t+1}, u_{s_{i,t+1}})'$ and the superscript $wc$ denotes within-country quantities.

Finally, the conditional cross-country covariance matrix of VAR innovations between any pair of countries is also identical across country pairs and constant over time:

\[
    C_t[u_{i,t+1}, u_{j,t+1}] = \begin{bmatrix}
        \sigma_{uu}^{xc} & \sigma_{us}^{xc} \\
        \sigma_{us}^{xc} & \sigma_{ss}^{xc}
    \end{bmatrix}
\]

(3.6)

for all $i$ and $j$. The superscript $xc$ denotes cross-country quantities.

The stylized model of country returns defined by equations (3.3)-(3.6) implies that countries are identical and symmetrical with respect to the structure of their return dynamics and the cross-country correlation structure of returns and state variables. Of course the realized paths of returns and the state variable in each country will vary across countries. For example, in this model the expected excess return on country $i$ is given by $\mu_1 + \beta s_{i,t}$, whose realizations depend on the realizations of the country-specific state variable $s_{i,t}$.
A straightforward application of the return decomposition (C.1) to the VAR(1) model (3.3)-(3.6) shows that the shocks to the model (3.3)-(3.4) are related to structural cash flow and discount rate shocks as follows:

\[ N_{DR,i,t+1} = \lambda u_{si,t+1}, \quad (3.7) \]
\[ N_{CF,i,t+1} = u_{i,t+1} + \lambda u_{si,t+1}, \quad (3.8) \]

with

\[ \lambda = \frac{\rho \beta}{1 - \rho \phi}. \]

Therefore discount rate news are proportional to innovations to the state variable driving expected returns, with proportionality constant \( \lambda \). This constant is increasing in the persistence (\( \phi \)) of the state variable or expected returns, the loading of expected returns on the state variable (\( \beta \)), and the log-linearization parameter \( \rho \). Note that \( \lambda = 0 \) when expected returns are constant, i.e., when \( \beta = 0 \). In that case all variation in realized returns is driven exclusively by cash flow news: \( u_{i,t+1} = N_{CF,i,t+1} \).

Our assumptions about the conditional covariance structure of the innovations to the VAR (3.5)-(3.6), together with equations (3.7) and (3.8), imply that the conditional variances and covariances of news are constant over time and identical both within country and across countries. To fix notation, we write

\[ C_t[N_{CF,i,t+1}, N_{CF,j,t+1}] \equiv \sigma_{CF,CF}^m, \quad (3.9) \]
\[ C_t[N_{CF,i,t+1}, N_{DR,j,t+1}] \equiv \sigma_{CF,DR}^m, \quad (3.10) \]
\[ C_t[N_{DR,i,t+1}, N_{DR,j,t+1}] \equiv \sigma_{DR,DR}^m, \quad (3.11) \]

where \( m \equiv wc \) when \( i = j \), and \( m \equiv xc \) when \( i \neq j \). For example, \( \sigma_{CF,CF}^{xc} \) denotes both the conditional cross-country covariance of cash flows news.
3.3.2 Correlated Return News and Portfolio Risk Across Investment Horizons

The symmetrical model of Section 3.1 provides a convenient framework to explore the impact of the cross-country correlation of each type of return news on portfolio risk and portfolio choice across investment horizons.

Consider the equally-weighted portfolio of the N identical and symmetrical markets, which is also the mean-variance optimal portfolio. The risk of this portfolio at horizon \( k \), defined as the conditional variance of the \( k \)-horizon log portfolio return normalized by the investment horizon, is a weighted average of the normalized within-country conditional variance and the cross-country covariance of \( k \)-horizon returns:

\[
\frac{1}{k} \mathbb{V}_t[r_{p,t+k}^{(k)}] = \frac{1}{N} \frac{1}{k} \mathbb{V}_t[r_{i,t+k}^{(k)}] + (1 - \frac{1}{N}) \frac{1}{k} \mathbb{C}_t[r_{i,t+k}^{(k)} r_{j,t+k}^{(k)}].
\]  

(3.12)

where \( r_{i,t+k}^{(k)} = \sum_{l=1}^{k} r_{i,t+l} \) is the log return at horizon \( k \), and

\[
\mathbb{C}_t[r_{i,t+k}^{(k)} r_{j,t+k}^{(k)}] = \sum_{l=1}^{k} \mathbb{C}_t[r_{i,t+l}, r_{j,t+l}] + 2 \sum_{l=1}^{k-1} \sum_{m=1}^{k-l} \mathbb{C}_t[r_{i,t+l}, r_{j,t+l+m}].
\]  

(3.13)

The expression for the within-country conditional return variance \( \mathbb{V}_t[r_{i,t+k}^{(k)}] \) follows immediately from (3.13) by noting that \( \mathbb{V}_t[r_{i,t+k}^{(k)}] = \mathbb{C}_t[r_{i,t+k}^{(k)}, r_{i,t+k}^{(k)}]. \)

We are interested in expressing the conditional within-country and cross-country moments of \( k \)-period returns as a function of the conditional moments of return news. A forward recursion of the dynamic equations of the VAR(1) model (3.3)-(3.4) shows that future one-period realized returns are given by

\[
r_{i,t+l} - \mathbb{E}_t[r_{i,t+l}] = N_{CF,i,t+l} - N_{DR,i,t+l} + \frac{\beta}{\lambda} \sum_{m=1}^{l-1} \phi^{m-1} N_{DR,i,t+l-m},
\]  

(3.14)

where we have replaced the reduced-form shocks \( u_{i,t+l} \) and \( u_{s,t+l} \) with the structural shocks \( N_{CF,i,t+l} \) and \( N_{DR,i,t+l} \) using (3.7) and (3.8). Note that \( \beta/\lambda > 0 \).

Equation (3.14) illustrates the permanent and transitory nature of cash flow news and return predictability. To see that, note from the definition of \( k \)-horizon log return that the moments on the right hand side of (3.12) are all proportional to \( k \) when returns are unpredictable.
discount rate news respectively. It shows that, conditional on information at time \( t \), the realized return on an asset \( l \) periods ahead is a function only of the contemporaneous cash flow shock. But it depends on the entire history of discount rate shocks between \( t + 1 \) and \( t + l \), such that a positive discount rate shock drives realized returns down contemporaneously, but this effect reverses over time at a speed determined by the autoregressive coefficient \( \phi \).

Using the forward recursion (3.14) it is straightforward to show that the cross-country component (3.13) of portfolio risk at horizon \( k \) is given by:

\[
\frac{1}{k} C_t(r_{i,t+k}^{(k)}, r_{j,t+k}^{(k)}) = \sigma_{CF,CF}^{xc} + \left[ a(k)^2 + b(k) \right] \times \sigma_{DR,DR}^{xc} - 2 \times a(k) \times \sigma_{CF,DR}^{xc}. \tag{3.15}
\]

The coefficients \( a(k) \equiv a(k; \beta, \phi, \rho) \) and \( b(k) \equiv b(k; \beta, \phi, \rho) \) are given in the Appendix.

For \( k = 1 \), equation (3.15) reduces to

\[
C_t(r_{i,t+1}, r_{j,t+1}) = \sigma_{CF,CF}^{xc} + \sigma_{DR,DR}^{xc} - 2 \sigma_{CF,DR}^{xc}. \tag{3.16}
\]

Equations (3.15) and (3.16) show how correlated cash flow news and correlated discount rate news impact portfolio risk across investment horizons. At a one-period horizon, the cross-country covariance of each type of news has identical impact on the cross-country covariance of returns and portfolio risk per period. However, at horizons \( (k > 1) \) equation (3.15) shows that each type of return news cross-country covariance has a different effect on portfolio risk.

Specifically, the unit coefficient on \( \sigma_{CF,CF}^{xc} \) implies that its effect on portfolio risk remains the same at all horizons, while the horizon-dependent coefficient on \( \sigma_{DR,DR}^{xc} \)—and on \( \sigma_{CF,DR}^{xc} \)—implies that its effect changes with investment horizon. In the limit, the cross-country component of portfolio risk per period (3.15) converges to

\[
\lim_{k \to +\infty} \frac{1}{k} C_t(r_{i,t+k}^{(k)}, r_{j,t+k}^{(k)}) = \sigma_{CF,CF}^{xc} + \left( 1 - \frac{1 - \rho \phi}{\rho - \rho \phi} \right)^2 \times \sigma_{DR,DR}^{xc} - 2 \times \left( 1 - \frac{1 - \rho \phi}{\rho - \rho \phi} \right)^2 \times \sigma_{CF,DR}^{xc}, \tag{3.17}
\]

where the coefficient on \( \sigma_{DR,DR}^{xc} \) is smaller than one whenever \( \rho > \phi \) and sufficiently close
The figures plots the coefficient on $\sigma_{DR,DR}^{xc} = a(k; \beta, \phi, \rho)^2 + b(k; \beta, \phi, \rho)$ as a function of investment horizon $k$. We use parameters estimated from U.S. data for calibration: $\beta = 0.0121$, $\phi = 0.9864$, $\rho = 0.9982$. The expressions for $a(k; \beta, \phi, \rho)$ and $b(k; \beta, \phi, \rho)$ are given in the Appendix.

to one, and zero when $\rho = 1$.\footnote{Note that $\rho$ measures the importance of distant cash flow news and discount rate for valuations and returns (see equation [C.1]), while $\phi$ determines the persistence of discount rate news. Therefore, the conditions $\rho > \phi$ and $\rho \rightarrow 1$ essentially say that in the limit correlated discount rate news do not matter for long-run portfolio risk if they are not sufficiently persistent.} These conditions hold in all the cases we consider in our empirical analysis.

Figure 3.2 plots the coefficient on $\sigma_{DR,DR}^{xc}$ for values of $\beta$, $\phi$, and $\rho$ calibrated to U.S. data in our sample. The figure shows that, for this empirically relevant calibration, the coefficient on $\sigma_{DR,DR}^{xc}$ declines monotonically as $k$ increases and rapidly approaches values well under 0.3 at horizons of 10 years or more, consistent with the intuition that correlated discount rate news matter less for portfolio risk than correlated cash flow news at long horizons.

Equivalently, the long-horizon covariation of asset returns is primarily determined by the covariance structure of cash flow innovations. The covariance of discount rate innovations...
matters for long-run return correlations only if discount rate news are extremely persistent.

A similar logic applies to the variation of the within-country component of portfolio risk. Since \( \mathbf{V}_t[r_{i,t+k}] = \mathbf{C}_t[r_{i,t+k}, r_{i,t+k}] \), it follows that:

\[
\frac{1}{k} \mathbf{V}_t[r_{i,t+k}] = \sigma_{\text{CF,CF}}^\text{wC} + \left[a(k)^2 + b(k)\right] \times \sigma_{\text{DR,DR}}^\text{wC} - 2 \times a(k) \times \sigma_{\text{CF,DR}}^\text{wC}. \tag{3.18}
\]

Of course, the within-country \( k \)-return portfolio variance (3.18) is also the \( k \)-horizon risk of a single-country portfolio.

Campbell and Viceira (2005), Pastor and Stambaugh (2012), and others have studied the properties of \( \mathbf{V}_t[r_{i,t+k}] / k \) as a function of the moments of the shocks to the VAR(1). Equation (3.18) writes single-country portfolio risk as a function of the moments of cash flow news and discount rate news. This derivation helps us gain intuition into why empirically portfolio risk per unit of time appears to decline at long horizons when asset returns are predictable: Discount rate shocks are transitory shocks whose impact on long-run portfolio return variability is smaller than the impact of permanent cash flow shocks.

When returns are not predictable (i.e., \( \beta = 0 \)) and all return variation comes from cash flow news, equations (3.15) and (3.18) reduce to \( \sigma_{\text{CF,CF}}^\text{wC} \) and \( \sigma_{\text{CF,CF}}^\text{wC} \) respectively. Portfolio risk per period is the same across all investment horizons and equals

\[
\frac{1}{k} \mathbf{V}_t[r_{p,t+k}] = \frac{1}{N} \sigma_{\text{CF,CF}}^\text{wC} + (1 - \frac{1}{N}) \sigma_{\text{CF,CF}}^\text{wC}.
\]

### 3.3.3 Calibrated Example

We now illustrate how the cross-country covariance of each return news component affects portfolio risk across investment horizons within the context of this symmetrical model. We calibrate the VAR(1) return dynamics (3.3)-(3.4) to US excess stock returns, with the log dividend-price ratio as the state variable.

We use these estimates to compute portfolio risk per period \( \sqrt{\mathbf{V}_t[r_{p,t+k}] / k} \) on an equally-weighted portfolio of seven U.S. stock market clones and the optimal intertemporal allocation to global equities and cash of an investor who maximizes expected utility of terminal wealth at a finite horizon (Campbell and Viceira (2005, Jurek and Viceira 2011). We set the coefficient
of relative risk aversion of this investor to 5.

We consider three different scenarios for the cross-country correlations of return news. The first scenario and baseline case, sets all cross-country news correlations to zero. The second scenario and the third scenario both set the cross-country correlation of one-period returns to the same positive value, but this correlation comes from a different type of cross-country news correlation: The second scenario ("CF integration") generates positive cross-country return correlations exclusively from correlated cash flow news;\textsuperscript{12} the third scenario ("DR integration") generates positive cross-country return correlations exclusively from correlated discount news.\textsuperscript{13}

Figure 3.3 plots annualized portfolio risk (Panel A) and the mean optimal equity portfolio allocation (Panel B) as a function of investment horizon for each of the three scenario. The intercepts in Panel B reflect the one-period or instantaneously mean-variance efficient allocation to risky assets, while the deviations from the intercepts reflect intertemporal hedging demands. To facilitate interpretation, we set the unconditional expected returns and the risk-free rate such that the mean-variance allocation to cash is zero in the baseline scenario, which in turn implies a positive allocation to cash in the other two scenarios with correlated returns.

Consistent with our results in Section 3.3, Panel A shows that portfolio risk per period declines as investment horizon increases as a result of return predictability, with a magnitude that depends on the source of cross-country return correlations. Uncorrelated news generate a more pronounced decline than correlated news. Most interestingly, correlated discount rate news generate a much larger decline in long-horizon portfolio risk than correlated cash

\textsuperscript{12}In U.S. data, $\sigma_{DR,DR}^{uc}/\sigma_{CF,CF}^{uc} = 2.6$, that is, discount rate news are 2.6 times more volatile than cash flow news. Holding this ratio to 2.6 for all countries and setting all other cross-country news correlations to zero, the maximum admissible value of the cross-country correlation of cash flow news that ensures a positive semidefinite variance-covariance matrix of shocks across all markets is 0.60. This in turn implies a cross-country correlation of returns of 0.062.

\textsuperscript{13}The value of the cross-country correlation of discount rate news that generates the same value of the cross-country correlation of one-period returns as in the second scenario is 0.10. It is much smaller than the cross-correlation of cash flow news because the volatility of discount rate news is much larger than the volatility of cash flows news.
Figure 3.3: Annualized Portfolio Risk and Optimal Allocation to Risky Assets as a Function of Investment Horizon

The figure plots annualized portfolio risk $\sqrt{V_t(\sigma_{p,t+k})/k}$ (panel A) and optimal allocation to risky assets (panel B) as a function of investment horizon $k$ (months). We compare the term structure of portfolio risk and optimal allocation for 3 scenarios: (1) Baseline case with zero cross-country return news correlations, both for CF news and DR news. (2) CF news integration case, where cross-country return correlations come from positive cross-country CF news correlations; cross-country correlations of DR news are zero. (3) DR integration case, where cross-country return correlations come from positive cross-country DR news correlation; cross-country correlations of CF news are zero. To make Scenarios 2 and 3 comparable, we set the cross-country correlation of one-period returns at the same value (0.07). Panel A plots portfolio risk in each scenario for a portfolio of seven symmetric countries. Panel B plots optimal allocation to risky assets (for a portfolio of seven countries) as a function of time remaining to terminal date. The total optimal allocation is the sum of two parts: myopic allocation (equals the intercept at $\tau = 1$) and hedging allocation. The investor has horizon of $K = 360$ (30 years) and rebalance his allocation each period. The $x$-axis $\tau$ is the time remaining to the terminal date. We compare the term structure of optimal allocation to risky assets across the same 3 scenarios described above. We set the expected excess returns so that in the benchmark case, the myopic investor ($\tau = 1$) allocate $1/N$ to each risky asset ($14.3\%$ for $N = 7$) and zero to cash. The expected excess returns are kept the same across the three cases to make them comparable.
flow news, even though both imply the same level of portfolio risk at short horizons.

Panel B shows that total portfolio demand for stocks is increasing in investment horizon in all three scenarios because shocks to the state variable—or equivalently expected excess returns—are negatively correlated with realized stock excess returns, implying that a long position in the risky assets helps hedge against a fall in expected returns. However, the increase in optimal portfolio demand also depends critically on the source of cross-country return correlations. Intertemporal hedging demands are significantly smaller when correlated cash flow news is the driver of cross-country return correlation.

Figure 3.3 illustrates the main point of our argument. Investors can achieve a significantly larger reduction in long-run portfolio risk through global portfolio diversification when the driver of cross-country return correlations is correlated discount rate news than when the driver is correlated cash flow news, even if both result in the same level of short-run portfolio risk and short-run or myopic portfolio allocations. Equivalently, if global return correlations increase, the risk of a globally diversified portfolio increases at short horizons regardless of the source of the increase in cross-country return correlations. But it increases much less at long horizons when the source of the increase in return correlations is capital market integration (or correlated discount rates) than when it is real markets integration (or correlated cash flows).

3.4 Empirical Investigation of the Sources of Return Correlations in Global Capital Markets

3.4.1 VAR Specification and Estimation of Return Decomposition

The stylized symmetrical model presented in Section 3 highlights the importance of understanding the sources of cross-country correlations of returns to evaluate the benefits of international portfolio diversification at long horizons. We now present an empirical analysis of the return news decomposition presented in Section 2 for stocks and government bond returns of seven major developed economies that account for at least 80% of total
global stock market capitalization throughout our sample period: Australia, Canada, France, Germany, Japan, the U.K., and the U.S. The sample period expands January 1986 through December 2016, the longest period for which we have complete data on returns and state variables for all these countries.

We estimate a homoskedastic pooled VAR(1) model for the seven countries in our sample with a country-specific vector of intercepts and a common matrix of slope coefficients. Our specification of the state vector for the VAR(1) model includes the log return on equities and bonds in excess of the return on their domestic T-bill to ensure that the return decomposition is currency independent (Campbell, Sefarty de Medeiros, and Viceira, 2010). It also includes state variables known to predict excess returns on stocks and bonds—log dividend-price ratios and yield spreads—, and variables that help capture the dynamics of real interest rates and inflation—log nominal short-term interest rates and log inflation (Campbell, Chan, and Viceira, 2003, Campbell and Viceira, 2005). We obtain monthly data for the state variables in all seven countries from a variety of sources. The Appendix provides a detailed description of the data and its sources. We consider a heteroskedastic version of this VAR in the later sections of the paper.

We estimate a pooled VAR(1) model for the entire sample in an attempt to use as much cross-country and time-series information as possible to estimate the process for expected returns, because our sample is relatively short in the time series dimension and we also want to analyze changes in the cross-country correlation of news components in our sample period. We extract estimates of the news components of stock and bond excess returns for each country from the estimates of this VAR(1) system using the return decomposition described in Section 2. We estimate news components for both the entire sample period and two subperiods, 1986-1999 and 2000-2016. We obtain subperiod estimates by splitting the vector of innovations while holding the coefficients at their full sample estimates.

We hold the slope coefficients constant across subperiods for two reasons. First, the state variables that capture expected excess returns, inflation, and the nominal short-rate follow highly persistent processes that require long samples to be precisely estimated. Second, we
don’t have strong priors as to why the slopes of the VAR system might have changed across periods, while we do have strong priors as to why the correlation structure of the shocks, particularly across countries, might have changed.

Of course, if the expected return processes for stocks and bonds differ across markets and change over time, a full-sample pooled estimation can introduce biases in the estimation of news components and their volatilities and correlations. However, estimates based on individual country VAR’s do not appear to fundamentally change our results, and the analysis of cash flow correlations presented in Section 5 suggests that our results do not depend on changes over time in the structure of return predictability. Accordingly, we use our entire sample period to estimate the slope coefficients. In practice, this procedure tempers the evidence of return predictability for those markets for which there is more in-sample evidence of return predictability, such as the U.K. and the U.S.

For stocks, our specification of the state vector allows us to explicitly identify unexpected stock excess returns and the discount rate news components of stock returns—real rate news and risk premium news—from equations in the VAR, and obtain cash flow news as the sum of unexpected excess returns and discount rate news. Section 5 provides evidence that our results are robust to this identification strategy for equity cash flow news. For bonds, our specification allows us to explicitly identify bond cash flow news from the inflation equation in the VAR, and obtain the risk premium news component of bond returns as the residual. The Appendix provides details of the return decomposition for stocks and bonds.

### 3.4.2 Summary Statistics and VAR Estimates

Table 3.1 and Table 3.2 present summary statistics for stock and bond excess returns over the entire sample period and the subperiods 1986.01-1999.12 and 2000.01-2016.12. This partition of the sample is motivated by our interest in exploring the sources of the changes in cross-country stock and bond return correlations that have occurred during our sample period, illustrated in Figure 3.1, and their impact on international portfolio diversification across investment horizons.
Table 3.1: Summary Statistics

This table reports summary statistics of monthly bond and stock returns for the whole sample (January 1986 to December 2016), early sample (January 1986 to December 1999) and late sample (January 2000 to December 2016). Estimates of means, volatilities, and Sharpe Ratios are all scaled to annualized units. Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate.

<table>
<thead>
<tr>
<th>Whole Sample: January 1986 to December 2016</th>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>2.8%</td>
<td>3.2%</td>
<td>3.8%</td>
<td>3.0%</td>
<td>1.2%</td>
<td>3.4%</td>
</tr>
<tr>
<td>Volatility</td>
<td>17.3%</td>
<td>15.3%</td>
<td>19.4%</td>
<td>21.7%</td>
<td>20.1%</td>
<td>15.6%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.16</td>
<td>0.21</td>
<td>0.19</td>
<td>0.14</td>
<td>0.06</td>
<td>0.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Whole Sample: January 1986 to December 2016</th>
<th>Bonds</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>2.8%</td>
<td>3.2%</td>
<td>3.7%</td>
<td>3.1%</td>
<td>3.3%</td>
<td>2.8%</td>
</tr>
<tr>
<td>Volatility</td>
<td>6.5%</td>
<td>6.0%</td>
<td>5.2%</td>
<td>4.9%</td>
<td>5.2%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.44</td>
<td>0.53</td>
<td>0.70</td>
<td>0.64</td>
<td>0.64</td>
<td>0.38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Early Sample: January 1986 to December 1999</th>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>2.5%</td>
<td>2.8%</td>
<td>7.8%</td>
<td>4.8%</td>
<td>1.5%</td>
<td>5.8%</td>
</tr>
<tr>
<td>Volatility</td>
<td>21.2%</td>
<td>15.6%</td>
<td>21.1%</td>
<td>21.4%</td>
<td>22.0%</td>
<td>17.4%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.12</td>
<td>0.18</td>
<td>0.37</td>
<td>0.23</td>
<td>0.07</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Early Sample: January 1986 to December 1999</th>
<th>Bonds</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>3.6%</td>
<td>2.9%</td>
<td>3.1%</td>
<td>2.1%</td>
<td>4.4%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Volatility</td>
<td>7.6%</td>
<td>7.1%</td>
<td>5.7%</td>
<td>5.0%</td>
<td>6.9%</td>
<td>9.4%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.47</td>
<td>0.40</td>
<td>0.55</td>
<td>0.42</td>
<td>0.65</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Late Sample: January 2000 to December 2016</th>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>3.0%</td>
<td>3.6%</td>
<td>0.5%</td>
<td>1.5%</td>
<td>0.9%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Volatility</td>
<td>13.5%</td>
<td>15.0%</td>
<td>18.0%</td>
<td>22.0%</td>
<td>18.4%</td>
<td>14.1%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.22</td>
<td>0.24</td>
<td>0.03</td>
<td>0.07</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Late Sample: January 2000 to December 2016</th>
<th>Bonds</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
</tr>
<tr>
<td>Mean</td>
<td>2.3%</td>
<td>3.5%</td>
<td>4.1%</td>
<td>3.9%</td>
<td>2.4%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Volatility</td>
<td>5.5%</td>
<td>5.0%</td>
<td>4.9%</td>
<td>4.8%</td>
<td>3.2%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.41</td>
<td>0.70</td>
<td>0.85</td>
<td>0.82</td>
<td>0.74</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Table 3.2: Correlation Summary Statistics

The table reports the overall average correlations within and across countries for the full period (January 1986 to December 2016) as well as for each subperiod (January 1986 to December 1999, January 2000 to December 2016), based on individual country-pair stock and bond return correlations. Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate. The individual country-pair correlations are reported in the Appendix.

<table>
<thead>
<tr>
<th></th>
<th>Within Countries</th>
<th>Across Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bonds  Stocks</td>
<td>Bonds  Stocks</td>
</tr>
<tr>
<td>Full Period</td>
<td>Bonds  1.00  Stocks  0.06</td>
<td>Bonds  0.49  Stocks  0.62</td>
</tr>
<tr>
<td>Early Sample</td>
<td>Bonds  1.00  Stocks  0.30</td>
<td>Bonds  0.40  Stocks  0.54</td>
</tr>
<tr>
<td>Late Sample</td>
<td>Bonds  1.00  Stocks  -0.25</td>
<td>Bonds  0.64  Stocks  -0.23</td>
</tr>
<tr>
<td>Difference</td>
<td>Bonds  0.00  Stocks  -0.54</td>
<td>Bonds  0.25  Stocks  -0.37</td>
</tr>
</tbody>
</table>

Table 3.1 shows that the sample Sharpe Ratio of bonds in every country is significantly larger than the Sharpe Ratio of equities, both in the whole sample and in each subperiod, with the sole exception of the U.K. and the U.S. during the 1986-1999 period. The superior sample performance of bonds reflects that the cross-country average bond excess return has remained stable at about 3.2% per annum, while the average stock excess return has declined from 5.1% to 1.9% p.a. between the first and the second half of the sample period. Excess return volatility in each market has experienced a small decline between the first and second subperiod and in each country, but on average it has been around 6% p.a. for bonds and 18% p.a. for stocks.

Table 3.2 reports average within-country and cross-country correlations of bond and stock excess returns over the entire sample period and the two subperiods. Complementing Figure 3.1, it shows that cross-country return correlations have increased significantly from the early to the late subperiod for both stocks and bonds.

---

14The Appendix reports the full correlation matrices.
The Appendix reports the estimates of the pooled VAR(1) model and for each individual country. The top panel in each table reports coefficient estimates with \( t \)-statistics in parentheses and the \( R^2 \) statistic for each equation in the model. The bottom panel reports the correlation matrix of residuals, with the diagonal elements showing annualized standard deviations multiplied by 100 and the off-diagonal elements showing correlations.

We summarize here the estimation results. Our estimates reproduce the well-known results that the dividend-price ratio forecasts positively stock excess returns and that the yield spread forecasts positively bond excess returns. The estimates for the equations corresponding to the log dividend-price ratio, the nominal short-term interest rate, and the log yield spread show that each variable is generally well-described by a persistent univariate AR(1) process. Log inflation follows a less persistent process. As we will see, this has important implications for the benefits of global diversification of bond portfolios.

The correlation matrix of residuals shows a large negative cross-country average correlation between unexpected stock excess returns and shocks to the dividend-price ratio, both in the full sample and in each subperiod. We also estimate a negative average correlation between unexpected bond excess returns and shocks to the yield spread, although its magnitude is much smaller.\(^{15}\) Because the dividend-price ratio and the yield spread are the main predictors of stock and bond excess returns, respectively, these negative correlations imply that shocks to expected excess returns are negatively correlated with realized excess returns. That is, stocks and bonds tend to do well when expected excess returns fall, thus providing investors with a hedge against a deterioration in investment opportunities.

3.4.3 News Decomposition of Cross-Country Correlations of Stock and Bond Returns

Our VAR estimates allow us to extract estimates of the news components of stock and bond returns to explore the sources of cross-country return correlations and their changes.

\(^{15}\)By contrast, Campbell, Chan, and Viceira (2003) and Campbell and Viceira (2005) report a positive estimate of this correlation for the U.S. in the postwar period up to the early 2000’s.
Figure 3.4: Contributions of News Components to Overall Cross-Country Unexpected Return Covariance

The figure plots breakdown of contributions of different news components to unexpected stock return correlations across countries (Panel A) and unexpected bond return correlations across countries (Panel B). In Panel A (stocks across countries), the cash flow news component contribution is calculated as $\frac{1}{N(N-1)/2} \sum_i \sum_{j \neq i} \frac{\text{Cov}(N_{CF,i}, N_{CF,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)}$, the real rate news component contribution is calculated as $\frac{1}{N(N-1)/2} \sum_i \sum_{j \neq i} \frac{\text{Cov}(N_{RR,i}, N_{RR,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)}$, the risk premium news component contribution is calculated as $\frac{1}{N(N-1)/2} \sum_i \sum_{j \neq i} \frac{\text{Cov}(N_{RP,i}, N_{RP,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)}$, and the cross components is calculated as $\frac{1}{N(N-1)/2} \sum_i \sum_{j \neq i} \left( \frac{\text{Cov}(N_{CF,i}, -N_{RR,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)} + \frac{\text{Cov}(N_{CF,i}, -N_{RP,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)} + \frac{\text{Cov}(-N_{RR,i}, -N_{RP,j})}{\text{Cov}(\tilde{x}_i, \tilde{x}_j)} \right)$.

The component contributions in panel B are calculated similarly. Note that by definition, values in the component contributions sum up to 1.

between the 1986-1999 subperiod and the 2000-2016 subperiod. Table 3.3 reports the average cross-country correlations of the news components of excess stock and excess bond returns for each subperiod as well as p-values of the differences between subperiods based on bootstrap and Fisher transformation methods. Figure 3.4 plots the proportional contribution of each component to the average cross-country return covariance for stocks (left panel) and bonds (right panel). (The Appendix describes the statistical tests and the calculation of the contribution of each component to total correlation.)
Table 3.3: Cross-Country Return Correlation Decomposition

This table decomposes the sources of global stock return correlations and bond return correlations. Correlations among individual stock/bond return components (i.e., cash-flow, real-rate, and risk premium news) across countries are shown in the table. Estimates are reported for each subperiod as well as the difference between the two subperiods. Tests for significant correlation differences between subperiods are based on bootstrap and Fisher r-to-z methods for calculating p-values.

<table>
<thead>
<tr>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Bonds</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF (s)</td>
<td>RR (s)</td>
<td>RP (s)</td>
<td>CF (b)</td>
<td>RR (b)</td>
<td>RP (b)</td>
<td>CF (s)</td>
<td>RR (s)</td>
<td>RP (s)</td>
<td>CF (b)</td>
<td>RR (b)</td>
<td>RP (b)</td>
<td></td>
</tr>
<tr>
<td>Subperiod 1</td>
<td>CF (s)</td>
<td>0.41</td>
<td>0.03</td>
<td>0.39</td>
<td>0.47</td>
<td>0.28</td>
<td>0.63</td>
<td>0.34</td>
<td>0.35</td>
<td>0.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RR (s)</td>
<td>-0.30</td>
<td>-0.33</td>
<td>0.49</td>
<td>-0.39</td>
<td>-0.59</td>
<td>0.63</td>
<td>0.10</td>
<td>0.08</td>
<td>0.22</td>
<td>0.24</td>
<td>0.07</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>RP (s)</td>
<td>0.06</td>
<td>0.25</td>
<td>0.25</td>
<td>0.08</td>
<td>0.07</td>
<td>0.22</td>
<td>0.22</td>
<td>0.25</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td>CF (s)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>RR (s)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.20</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>RP (s)</td>
<td>0.18</td>
<td>0.00</td>
<td>0.03</td>
<td>0.22</td>
<td>0.25</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

p-values (bootstrap)

<table>
<thead>
<tr>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Bonds</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF (s)</td>
<td>RR (s)</td>
<td>RP (s)</td>
<td>CF (b)</td>
<td>RR (b)</td>
<td>RP (b)</td>
<td>CF (s)</td>
<td>RR (s)</td>
<td>RP (s)</td>
<td>CF (b)</td>
<td>RR (b)</td>
<td>RP (b)</td>
<td></td>
</tr>
<tr>
<td>Subperiod 1</td>
<td>CF (s)</td>
<td>0.18</td>
<td>0.00</td>
<td>0.00</td>
<td>0.18</td>
<td>0.00</td>
<td>0.02</td>
<td>0.20</td>
<td>0.24</td>
<td>0.00</td>
<td>0.22</td>
<td>0.25</td>
<td>0.01</td>
</tr>
<tr>
<td>RR (s)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
Following Ammer and Mei (1996), we refer to cross-country discount rate news correlations as a measure of capital markets integration. To understand this terminology, consider a world in which capital markets are perfectly integrated, so there is a unique marginal investor pricing all assets. Since discount rates are determined by investors, we would expect discount rates to move synchronously. Alternatively, we can also think of a world with integrated capital markets as a world in which shocks to investor risk aversion or investor sentiment propagate rapidly across markets. In either case, we expect discount rate news to be highly correlated across markets. By contrast, cash flows need not be perfectly correlated in such world, just like we don’t expect the cash flows on two individual stocks in the same stock market to be perfectly correlated, as they can be subject to idiosyncratic shocks in addition to common aggregate shocks.

Table 3.3 shows that capital market integration is the main source of the significant increase in global cross-country correlations of stock excess returns from the early subperiod to the late subperiod: The cross-country correlations of both the real rate news component and the risk premium component of discount rate news have experienced increases which are economically and statistically significant, from 0.39 to 0.63 and from 0.49 to 0.63 respectively. By contrast, the cross-country correlations of cash flow news have experienced a much smaller increase, from 0.41 to 0.47 from the early to the late subperiod.

Figure 3.4 shows that the the risk-premium component of stock returns is the most important contributor to cross-country return covariance in each subperiod, and that its contribution has become even more important in the late subperiod at 84% from 54%. The cash flow news covariance is the second largest contributor, but its contribution is much smaller at about 20% in both subperiods. This figure also shows that cross-news covariances explain a very small fraction of the total cross-country stock return covariance.

Figure 3.5 plots the time series of the 3-year moving average of average cross-country correlations of shocks to stock excess returns (Panel A), cash flow news (Panel B), real rate news (Panel C), and risk premium news (Panel D), both including the outlier October 1987 observation and excluding it. Panel A shows an upward trend in the average cross-country
correlation of realized stock excess returns, with the exception of a temporary decline in 2014-2015.\textsuperscript{16} Panel B shows that the average cross-country correlation of cash flow news exhibits no time series trend, while the cross-country correlations of both real interest rate news and risk premium news exhibit a clear upward trend.

Table 3.3 shows interesting contrasting results for bonds relative to equities. First, all components of bond news have experienced a significant increase in their average cross-country correlations from the early period to the late period, a point that Figure 3.6 confirms visually. In particular, the average correlation of bond cash flow news, i.e., inflation news, has almost doubled from 0.34 to 0.64, similar to the real rate news component. Second, the cross-country correlation of risk premium news is considerably smaller for bonds than for stocks in each subperiod, suggesting that bond markets are less globally integrated than stock markets.

Figure 3.4 shows that real rate news covariance is the most important contributor to cross-country bond return covariance and risk premium news covariance comes second. The cash flow or inflation news covariance explains only a small fraction of total bond return cross-country covariance.

Our estimates of the increase in the cross-country correlations of bond cash flow news add to a body of research in Economics that documents a large increase in the average cross-country correlation of inflation and suggests the presence of a global factor in inflation (Wang and Wen 2007, Mumtaz, Simonelli and Surico 2011, Neely and Rapach 2011, and Henriksen, Kydland and Sustek 2013). This increased correlation in inflation could be the result of successful inflation targeting by central banks, which has operated as an implicit mechanism of coordination in monetary policy and it has reduced country-specific variation in inflation expectations (Cecchetti and Schoenholtz, 2014, 2015).

Overall, our empirical results present strong evidence that financial integration has been a powerful driver of the increase in the cross-country correlation of stock and bond returns between 1986-1999 and 2000-2016. There is a growing literature exploring global

\textsuperscript{16}This decline is not attributable to a specific time observation or country pair.
Figure 3.5: Average Cross-Country Correlations of VAR News (Stocks)

This figure plots the three year 3-year moving average of average cross-country correlations of shocks to stock excess returns (Panel A), cash flow news (Panel B), real rate news (Panel C), and risk premium news (Panel D), both including the October 1987 observation and excluding it.
Figure 3.6: Average Cross-Country Correlations of VAR News (Bonds)

This figure plots the three year 3-year moving average of average cross-country correlations of shocks to bond excess returns (Panel A), cash flow news (Panel B), real rate news (Panel C), and risk premium news (Panel D).
financial integration. Davis and van Wincoop (2017) document a large increase in the global correlation between capital inflows and outflows from 1970-1990 to 1990-2011, which they attribute to an increase in financial globalization. Lustig, Stathopoulos, and Verdelhan (2016) estimate stochastic discount factors (SDF) for G10 countries using bond data, and show that permanent shocks to each SDF are highly correlated and exhibit very similar volatility in the 1985-2012 period. Our results highlight the importance of accounting for time variation in discount rates to understand financial globalization.

3.5 Robustness Checks

3.5.1 Alternative Measure of Market Integration

Thus far we have used only cross-country correlations of returns and their news components as our metric for financial integration. Pukthuanthong and Roll (2009) propose using as an alternative metric of integration the $R^2$ from regressing returns on global factors estimated from a principal component analysis. This methodology is particularly helpful to determine if a relatively low degree of cross-country correlations could be the result of a multifactor structure instead of evidence of lack of integration.

We apply the Pukthuanthong-Roll methodology to realized returns and the news components of returns. For each return and news component series and for each subsample, we find the first three principal components every year and the $R^2$ from a simple least squares regression.

Table 3.4 reports average $R^2$ over the two subperiods for each series. The results from this exercise confirm the conclusions from the correlation analysis: We find a substantial increase in $R^2$ in each case except for stocks cash flow news, for which the increase in $R^2$ is negligible.
Table 3.4: Average \( R^2 \) Using Principal Components as Global Factors

This table applies the Pukthuanthong-Roll methodology to realized returns, unexpected returns and the three news components of returns. For a given return or news component series, we find the first three principal components every year and obtain the \( R^2 \) from a simple least squares regression using PCs as global factors. The table reports average \( R^2 \). Panel A corresponds to stocks, and Panel B corresponds to bonds.

<table>
<thead>
<tr>
<th>Panel A: Stocks</th>
<th>All</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency Hedged Stock Returns</td>
<td>0.68</td>
<td>0.60</td>
<td>0.73</td>
<td>0.13</td>
</tr>
<tr>
<td>Unexpected Stock Returns</td>
<td>0.67</td>
<td>0.59</td>
<td>0.74</td>
<td>0.15</td>
</tr>
<tr>
<td>CF News (Stocks)</td>
<td>0.53</td>
<td>0.51</td>
<td>0.54</td>
<td>0.03</td>
</tr>
<tr>
<td>RR News (Stocks)</td>
<td>0.58</td>
<td>0.42</td>
<td>0.71</td>
<td>0.28</td>
</tr>
<tr>
<td>RP News (Stocks)</td>
<td>0.59</td>
<td>0.48</td>
<td>0.68</td>
<td>0.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Bonds</th>
<th>All</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency Hedged Bond Returns</td>
<td>0.68</td>
<td>0.59</td>
<td>0.74</td>
<td>0.15</td>
</tr>
<tr>
<td>Unexpected Bond Returns</td>
<td>0.64</td>
<td>0.55</td>
<td>0.71</td>
<td>0.17</td>
</tr>
<tr>
<td>CF News (Bonds)</td>
<td>0.56</td>
<td>0.39</td>
<td>0.70</td>
<td>0.31</td>
</tr>
<tr>
<td>RR News (Bonds)</td>
<td>0.57</td>
<td>0.40</td>
<td>0.71</td>
<td>0.31</td>
</tr>
<tr>
<td>RP News (Bonds)</td>
<td>0.46</td>
<td>0.38</td>
<td>0.52</td>
<td>0.13</td>
</tr>
</tbody>
</table>
3.5.2 Direct Measures of Cash Flow Correlations

The empirical stock return news decomposition performed in Section 5 identifies stock cash flow news as the difference between realized excess stock returns and discount rate news. If discount rate news are not well identified, then estimates of cash flow news will inherit the misspecification of the return prediction model (Chen and Zhao 2009). Estimation error in the discount rate news component of returns could potentially lead us to underestimate the contribution of cash flow news to the secular increase in the correlation of global stock returns. Moreover, the use of full-sample, pooled estimates of the slope coefficients of the VAR model to estimate news components could also introduce bias our results.\(^{17}\)

To attenuate these concerns, we follow Chen and Zhao (2009) and model cash flow news directly from five proxies of aggregate equity cash flows: real dividend growth, real corporate earnings growth, real GDP growth, real consumption growth, and real industrial production growth. (The Appendix for full details of the data source for these variables). We estimate univariate models for each one of them and compute rolling cross-country correlations of the innovations.

Table 3.5 and Figure 3.7 report the results of this analysis. Table 3.5 shows a small but statistically not significant increase in the cross-country correlations of all these measures except real industrial production growth. Figure 3.7 confirms visually these results. None of the variables under consideration exhibit any upward trend in cross-country correlations. Correlations exhibit variation over time but overall they oscillate around a constant average around 15%-30%. The average magnitude of the correlations is somewhat lower than the average correlation level exhibited by our estimates of cash flow news, suggesting that, if anything, our approach overestimates the correlation of cash flow news.

These empirical results strongly suggest that the observed large increase in the correlation of global stock returns is not caused by increased correlations of cash flow fundamentals.

\(^{17}\)For example, suppose that in the late sample cash flow news gets more volatile and more correlated across countries; and assume the process for discount rate news is the exact same as in the early period. This means that the dividend-price ratio will be more correlated across countries because growth has become more correlated across countries. If we re-estimated the VAR in the late period, we would properly recover that the increased return correlation is due to cash flow news.
Table 3.5: Direct Measure of Cash Flow Correlations

The table reports cross-country correlation of real GDP growth, real consumption growth, real industrial production growth, real dividend growth and real corporate earnings growth over early sample and late samples. The correlations are computed using the AR(1) residual of each variable. Specifically, we first run a AR(1) regression for growth in each macro variable $\Delta X_{t+1} = a + \beta X_t + \varepsilon_{t+1}$, and then compute the average pairwise cross-country correlations of the residuals. The GDP, consumption and corporate earnings correlations are constructed using quarterly observations, and industrial production growth and dividend growth correlations are constructed using monthly observations. We also report correlation in late sample excluding crisis (2007Q4-2009Q4 for quarterly data and 2007.12-2009.12 for monthly data), and the difference in correlation between late sample and early sample. Data for corporate earnings are only available starting from 1994, thus we redefine early sample as 1994-2005 and late sample as 2006-2016. p-values calculated using Fisher’s transformation are reported.

<table>
<thead>
<tr>
<th></th>
<th>Early Sample 1986-1999</th>
<th>Late Sample 2000-2016</th>
<th>Late Sample (excluding crisis) 2006-2016</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real GDP Growth (quarterly)</strong></td>
<td>0.120</td>
<td>0.365</td>
<td>0.136</td>
</tr>
<tr>
<td>Difference (late - early)</td>
<td>0.246</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>[0.078]</td>
<td>[0.466]</td>
<td></td>
</tr>
<tr>
<td><strong>Real Consumption Growth (quarterly)</strong></td>
<td>0.016</td>
<td>0.134</td>
<td>0.092</td>
</tr>
<tr>
<td>Difference (late - early)</td>
<td>0.119</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>[0.259]</td>
<td>[0.345]</td>
<td></td>
</tr>
<tr>
<td><strong>Real Industrial Production Growth (monthly)</strong></td>
<td>0.171</td>
<td>0.131</td>
<td>0.079</td>
</tr>
<tr>
<td>Difference (late - early)</td>
<td>-0.040</td>
<td>-0.092</td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>[0.348]</td>
<td>[0.193]</td>
<td></td>
</tr>
<tr>
<td><strong>Real Dividend Growth (monthly)</strong></td>
<td>0.075</td>
<td>0.145</td>
<td>0.061</td>
</tr>
<tr>
<td>Difference (late - early)</td>
<td>0.070</td>
<td>-0.014</td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>[0.249]</td>
<td>[0.448]</td>
<td></td>
</tr>
<tr>
<td><strong>Real Corporate Earnings Growth (quarterly)</strong></td>
<td>0.028</td>
<td>0.082</td>
<td>-0.058</td>
</tr>
<tr>
<td>Difference (late - early)</td>
<td>0.054</td>
<td>-0.085</td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>[0.402]</td>
<td>[0.357]</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.7: Cross-Country Correlations of Proxies for Equity Cash Flow Fundamentals

This figure plots the three year 3-year moving average of average cross-country correlations of shocks to GDP growth (Panel A), industrial production growth (Panel B), consumption growth (Panel C) and corporate earnings (Panel D). Quarterly GDP, monthly industrial production index and quarterly consumption are in real terms and in local currency. Quarterly corporate sector earnings are in nominal terms and in local currency, and we adjust for inflation to convert them into real terms. We run a AR(1) regression $\Delta X_{t+1} = \alpha + \beta X_t + \epsilon_{t+1}$ for the log growth of real GDP, real industrial production, real consumption and real corporate earnings at country level. And then we compute the average pairwise correlation of residual from the AR(1) regression over a 3 year rolling window (36 months for IP, 12 quarters for GDP, consumption and earnings). Corporate earnings in our sample starts from 1994 due to data availability. GDP, IP is plotted starting from 1970 and consumption from 1980.
Therefore, if the increase in the correlation of global stock returns is not the result of increased correlation of fundamentals, it must be the result of increased correlation of discount rates.

### 3.5.3 Correlated Stock Market Volatility News

Our analysis so far has not considered the well-known empirical regularity that stock market volatility—and return volatility more generally—is time varying and subject to persistent shocks (Campbell, Giglio, Polk, and Turley 2017, CGPT henceforth). If persistent volatility shocks are a feature of capital markets, it is important to understand whether they are correlated across markets and what this means for global portfolio diversification.\(^{18}\)

We estimate volatility news for our cross-section of country stock markets following CGPT two-stage heteroskedastic-VAR methodology. This requires expanding our baseline specification of the state vector to include the default spread and stock market return variance for each country. We construct our international sample of default spreads building on the work of Kang and Pflueger (2015). Monthly realized stock market variance is based on within-month daily stock market returns denominated in U.S. dollars. The Appendix provides full detail of our data sources, data construction procedures, and shows the estimation results.

Figure 3.8 plots the time series of the 3-year moving average of average cross-country correlations of volatility news to expected variance (Panel A) and to realized stock return variance from the first stage regression (Panel B).\(^{19}\) Panel A in the figure shows that the cross-country correlation of volatility news has been very low on average and fairly stable over time except for short-lived sharp increases during the crash of October 1987 and during the

\(^{18}\)Chacko and Viceira (2005) shows that it is optimal for investors to time market volatility, and for long-term investors to tilt their portfolios away from stocks when volatility shocks are persistent and negatively correlated with realized stock returns. See also Liu (2007). Moreira and Muir (2017a and 2017b) show the profitability of market volatility timing.

\(^{19}\)The Appendix reports similar plots for stock excess returns, stock cash flow news, real rate news, and risk premium news. We omit those in the main text as they are very similar to the ones we obtain in the homoskedastic case.
Figure 3.8: Cross Country Correlations of Stock Volatility News
This figure plots the time series of the 3-year moving average of average cross-country correlations of volatility news for expected variance (EVAR, Panel A) and the average cross-country correlations of innovations to realized stock return variance from the first stage regression (RVAR, Panel B).
2008-2009 financial crisis. (Because we plot rolling 3-year correlations, correlation appears to be high during the subsequent period. In reality, only a few observations in late 2008 and early 2009 are responsible for this increase). Panel B shows that the cross-country correlation of shocks to realized volatility is a noisy version of the the cross-country correlation of volatility news shown in Panel A.

To understand the implications of the results in Figure 3.8 for portfolio risk at long horizons, we have extended the symmetric model of Section 3 to allow for time-varying return and expected return volatility as in CGPT. This analysis shows that persistent stochastic volatility shocks increase portfolio risk at all horizons, but that cross-country correlation of volatility shocks has only a small added impact on portfolio risk. (The Appendix describes in detail how we have extended the model and the results from the calibration of the model.)

### 3.6 The Impact of Real and Financial Integration on Long-Run Global Portfolio Diversification

Section 3 shows that the impact of an increase in cross-country return correlations on portfolio risk and portfolio choice at long horizons depends on the source of the increase—correlated cash flow news or correlated discount rate news. Section 4 has documented empirically an economically and statistically significant increase in the cross-country correlations of stock and bond excess returns between 1986-1999 and 2000-2016. We now explore the implications of our empirical results for portfolio risk and optimal global portfolio diversification at long horizons.

#### 3.6.1 The Risk of Globally Diversified Stock and Bond Portfolios Across Investment Horizons

We start with an analysis of portfolio risk of all-equity and all-bond portfolios across investment horizons and across subperiods. We consider both equal-weighted (EW) and
value-weighted (VW) portfolios of the seven markets in our sample and present results for the EW portfolios, which are more representative of the average country experience, as the VW portfolios are largely dominated by the U.S. market experience. (The Appendix presents results for the VW portfolios, which are very similar to those for the EW portfolios.)

Figure 3.9 presents our main portfolio risk results. The upper plots in Panel A and Panel B plots the percent annualized standard deviation of portfolio excess returns implied by our VAR estimates for each subperiod as a function of investment horizon $k$. The panel shows a declining pattern in the risk of globally diversified portfolios of both equities and bonds as a function of investment horizon in both subperiods. This pattern is much more pronounced for global equity portfolios than global bond portfolios.

The upper plot in Panel A also shows a reduction in the risk of global equity portfolios in the second subperiod, particularly at long horizons. At a 25-year horizon (300 months), portfolio risk is about 10% p.a. in the early sample and 8% p.a. in the late sample. This is an economically significant difference, especially when compounded over 25 years. By contrast, the risk of global bond portfolios has grown from the early to the late subperiod.

The portfolio risk decomposition (3.12) is helpful to understand the drivers of this change in long-run portfolio risk across subperiods: changes in country return volatilities or changes in cross-country return correlations (or both). The middle and lower plots in Panel A and Panel B report the results from performing this decomposition.

The middle plots in Panel A and Panel B of Figure 3.9 show the cross-country average of conditional $k$-horizon excess return volatility implied by our VAR estimates, in percent annual terms. The plots show a declining pattern for both equities and bonds that reflects the well-known dampening effect of return predictability on long-horizon return volatility at the individual market level. The plots also show that the average excess return volatility of both equities and bonds is lower in the late subperiod than in the early subperiod at all

---

20 That is, $100 \times \sqrt{(12/k) \mathbf{V}_t[x_{r.t+k}^{(k)}]}$.

21 That is, $100 \times \sum_{i=1}^{N} \left( w_i / \sum_{i=1}^{N} w_i \right) \sqrt{(12/k) \mathbf{V}_t[x_{r.t+k}^{(k)}]}$, where $w_i$ equals either market $i$ capitalization weight (VW portfolio) or $1/7$ (EW portfolio).
**Figure 3.9: Equal Weighted Portfolio Risk as a Function of Investment Horizon (Equities and Bonds)**

The figure compares the early sample (1986.01-1999.12) and late sample (2000.01-2016.12) equal weighted portfolio risk across investment horizons for equities (Panel A) and bonds (Panel B). For each panel, we plot the annualized conditional standard deviation of portfolio excess returns, annualized average conditional volatility (across N countries) of excess returns, and pairwise average conditional correlation of cross-country excess returns. Portfolios are equal-weighted.
investment horizons. Therefore changes in country return volatility help explain the decline in global equity portfolio risk in the late subperiod, but cannot explain the increase in bond portfolio risk.

The reduction in stock return volatility at all horizons in the late period reflects the restoration of stock return predictability after the run up in valuations relative to fundamentals of the second half of the 1990’s that weakened the empirical evidence on return predictability (Cochrane, 2008).

The lower plots in Panel A and Panel B of Figure 3.9 show the percent cross-country average of pairwise conditional correlations of \( k \)-horizon excess returns. This plots show that cross-country return correlations for both stocks and bonds are a declining function of investment horizon in both periods, although the pattern is much more pronounced for equities than for bonds. This pattern reflects the presence of correlated transitory components in returns as shown in Section 3: in the case of equities, Section 4 shows that the main contributor to return cross-country covariances is transitory equity premia news; in the case of bonds, it is real rate news, which is also transitory although less so than risk premia news.

The panel also shows an increase in cross-country return correlations for both equities and bonds at all horizons in the late period relative to the early period. In the case of equities, this increase is very small at long horizons reflecting that the main driver of the increase in stock return correlations is a large increase in the correlations of the transitory components of returns, particularly risk premia news. Therefore, the estimated significant increase in short-run cross-country return correlation has not contributed to an increase in global portfolio risk at long horizons in the late subperiod.

\[ \text{Corr}_i[x_{t+k}^{(k)}, x_{t+k}^{(k)}], \text{ where } w_i = \begin{cases} \text{either market capitalization weight (VW portfolio) or } 1/7 \text{ (EW portfolio).} 
\end{cases} \]

22That is, \( 100 \times \sum_{i=1}^{N} \sum_{j=1}^{N} \left( w_i w_j / \sum_{i=1}^{N} w_i w_j \right) \text{Corr}_i[x_{t+k}^{(k)}, x_{t+k}^{(k)}] \), where \( w_i \) equals either market \( i \) capitalization weight (VW portfolio) or \( 1/7 \) (EW portfolio).

23We use semidefinite programming methods to re-estimate the overall variance-covariance matrix of VAR innovations across all countries in the late subperiod subject to the constraint that the elements of the within-country variance-covariance matrix of innovations for each country remain at the same values as in the early subperiod. This exercise shows that indeed global equity portfolio risk would have not increased in the late subperiod despite a substantive increase in short-run cross-country return correlations. See the Appendix.
For bonds the increase is highly significant at all horizons, which explains the increase in the risk of global bond portfolios in the late period. Section 4 shows that the main drivers of the increase in bond correlations are large increases in the cross-country correlations of real rate news and cash flow or inflation news. Both of them are the most persistent components of bond returns.

### 3.6.2 Optimal Global Equity Portfolio Diversification at Long Horizons

An alternative way of evaluating the importance of the secular increase in the correlations of global equity and bond returns for long-term investors is to compute optimal intertemporal portfolio allocations and expected utility implied by our estimates of return dynamics in each subperiod. We consider two types of equity long-horizon investors.

The first one is the investor we consider in Section 3.2, that is, an investor with power utility preferences over terminal wealth at a finite horizon (Jurek and Viceira, 2011). We refer to this investor as the “JV investor.” The second investor is an infinitely lived investor with Epstein-Zin utility over intermediate consumption (Campbell and Viceira 1999, Campbell, Chan, and Viceira (2003). We refer to this investor as the “CCV investor.” For calibration purposes, we set the time discount factor to 0.92 and the coefficient of relative risk aversion to 5 for both investors. We also set the elasticity of intertemporal substitution of consumption of the CCV investor to one.\(^{24}\)

In order to compute optimal portfolio allocations we need to take a stand on unconditional expected returns and the risk free rate. In the spirit of the approach pioneered by Black and Litterman (1992), we set the vector of unconditional expected excess returns and the risk free rate such that the myopic or one-period mean-variance optimal portfolio allocation in the early sample equals either the EW global equity portfolio or the VW global equity portfolio, given the estimated variance-covariance of one-period returns. We report

\(^{24}\)We solve for the optimal intertemporal portfolio allocation of this investor building on the approximate solution methods of Campbell and Viceira (1999) and Campbell, Chan, and Viceira (2003). They show that the optimal intertemporal portfolio policy for this investor is an affine function of the vector of state variables similar to the solution in Jurek and Viceira (2011) that has two components, a myopic or one-period component and an intertemporal hedging component.
allocations in the EW case. (The Appendix reports results for the EW case). This assumption allows us to understand how optimal portfolio allocations change across investment horizons within each period, and across periods, for reasons related exclusively to changes in risk across investment horizons.

Table 3.6 report optimal global equity portfolio allocations (Panel A) and expected utility (Panel B) for the two investors in each subperiod. The first numerical column Panel A reports the mean optimal one-period (or mean-variance) allocation to stocks, which is the same for both investors. The second column and the third column report the vector of mean intertemporal hedging demands for a JV investor with a 20-year horizon and the CCV investor, respectively. Panel B reports expected utility expressed as a certainty equivalent of wealth for JV investors with horizons of 5, 10, 15, and 20 years, and expected utility per unit of wealth for the CCV investor. We compute expected utility for two cases: when the investor can invest only in U.S. equities, and when the investor has access to global markets.

Panel A in Table 3.6 reports portfolio allocation results for the early sample. By construction, the myopic allocation is 100% invested in the EW equity portfolio. The total intertemporal hedging demand for stocks is positive and large for both investors, at about 110% for the JV investor and 70% for the CCV investor. The intertemporal hedging demand of the CCV investor is smaller than that for the JV investor because, although the CCV investor is infinitely lived, his investment horizon is effectively shorter. To see this, note that the CCV investor consumes every period, while the JV investor delays consumption till the end of his long horizon of 20 years. Given our parametric assumptions, the duration of the consumption liabilities the CCV investor is funding out of his wealth is about 13.5 years, significantly shorter than that of the JV investor, which is 20 years.
Table 3.6: Optimal Global Equity Portfolio Allocations and Expected Utility

Panel A reports optimal global equity portfolio allocations by “JV” investor and “CCV” investor. The CCV investor has Epstein-Zin preference and the expected value function defined as $E[V_t] = \left(1 - \delta\right)^{-\psi/(1-\psi)} \left(\frac{C_t W_t}{1 - \delta}\right)^{1/(1-\psi)}$. The JV investor’s utility is power utility defined on terminal wealth $E_t[1^{g} W_t + K]$. The myopic demand is the allocation of those two investors at investment horizon 1. An investor’s allocation is the sum of myopic demand and hedging demand. We report the JV hedging demand for an investor at horizon of 20 years (240 months). We compare across 3 scenarios: optimal allocation in early sample, late sample and late sample with hypothetical covariance matrix that controls for within-country correlation. To make it comparable, we fix the monthly implied excess returns across these 3 scenarios. We set implied excess returns for equal weight portfolio such that investor hold the myopic demand equal to $1/N$ in each country. “Total” allocation is the sum of the allocations to each country. Panel B reports the expected utility by “JV” investor (with $RRA_{\gamma} = 5$) and “CCV” investor (with $RRA_{\gamma} = 5$), assuming they allocate optimally to the 7 countries investment space as reported in Panel A. We also report investor’s expected utility by constraining the investment space to USA only. We assume investor has initial wealth of one dollar and look at investment horizons $K$ of 5 years (60 months), 10 years (120 months), 15 years (180 months) and 20 years (240 months). We report the certainty equivalent for the JV investor (with $RRA_{\gamma} = 5$). The results are obtained by Monte Carlo simulation using 2,000 VAR paths sampled using the method of antithetic variates. The certainty equivalent of wealth is computed by evaluating the mean utility realized across the simulated paths and computing $W_{CE} = u^{-1}\left(E[u(W_{t+K})]\right)$.

<table>
<thead>
<tr>
<th></th>
<th>Myopic JV demand</th>
<th>JV hedging demand at 20 yr</th>
<th>CCV hedging demand</th>
<th>Panel B: Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>K=60</td>
<td>K=120</td>
<td>K=180</td>
</tr>
<tr>
<td><strong>Early Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>14.29%</td>
<td>23.12%</td>
<td>12.99%</td>
<td>7 countries</td>
</tr>
<tr>
<td>CAN</td>
<td>14.29%</td>
<td>16.63%</td>
<td>10.96%</td>
<td>USA only</td>
</tr>
<tr>
<td>FRA</td>
<td>14.29%</td>
<td>-7.05%</td>
<td>-4.29%</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>14.29%</td>
<td>31.66%</td>
<td>19.42%</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>14.29%</td>
<td>18.84%</td>
<td>12.04%</td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>14.29%</td>
<td>-1.31%</td>
<td>0.60%</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>14.29%</td>
<td>28.80%</td>
<td>18.26%</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100.00%</td>
<td>110.69%</td>
<td>69.98%</td>
<td></td>
</tr>
<tr>
<td><strong>Late Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>82.76%</td>
<td>78.37%</td>
<td>55.07%</td>
<td>7 countries</td>
</tr>
<tr>
<td>CAN</td>
<td>23.59%</td>
<td>55.21%</td>
<td>35.26%</td>
<td>USA only</td>
</tr>
<tr>
<td>FRA</td>
<td>29.67%</td>
<td>29.95%</td>
<td>22.34%</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>-12.73%</td>
<td>-2.71%</td>
<td>-3.17%</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>-1.82%</td>
<td>18.81%</td>
<td>9.09%</td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>52.44%</td>
<td>14.73%</td>
<td>8.80%</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>-41.32%</td>
<td>-8.29%</td>
<td>-8.72%</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>132.58%</td>
<td>106.07%</td>
<td>118.67%</td>
<td></td>
</tr>
<tr>
<td><strong>Late Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>11.52%</td>
<td>5.87%</td>
<td>3.46%</td>
<td>7 countries</td>
</tr>
<tr>
<td>CAN</td>
<td>28.37%</td>
<td>3.66%</td>
<td>1.50%</td>
<td>USA only</td>
</tr>
<tr>
<td>FRA</td>
<td>14.67%</td>
<td>15.23%</td>
<td>8.50%</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>7.28%</td>
<td>23.30%</td>
<td>12.84%</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>-0.35%</td>
<td>8.99%</td>
<td>6.11%</td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>16.98%</td>
<td>0.44%</td>
<td>1.43%</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>6.18%</td>
<td>59.10%</td>
<td>44.63%</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>84.64%</td>
<td>116.59%</td>
<td>78.47%</td>
<td></td>
</tr>
</tbody>
</table>
The relative composition of the intertemporal hedging allocation across markets is qualitatively similar for both investors. Their optimal intertemporal portfolio demands tilt total portfolio demand toward U.S. equities, and to a smaller but still significant degree, toward German and Japanese equities.

Panel B shows very large gains in expected utility for long-horizon investors from the ability to invest globally in all subperiods. The certainty equivalent of wealth for the JV investor and the expected utility of consumption per unit for the CCV investor are both an order of magnitude larger for a long-horizon investor with access to global equity markets than for a similar investor able to invest only in the U.S. stock market. Moreover, for the JV investor, welfare gains increase exponentially with investment horizon: The gain from having access to seven markets is proportionally much larger at a 20-year horizon than at a 10-year horizon. These large benefits of portfolio diversification are consistent with those reported in Jarek and Viceira (2011, Tables VI and VIII) and Campbell, Chan, and Viceira (2003, Table 5) for U.S. investors who gain access to bonds when they can invest only in U.S. equities and cash.

Panel A also reports optimal equity portfolio allocations and expected utility implied by our estimates of the return generating process in the late sample, holding unconditional expected excess returns and the risk free rate at the same values as in the early sample. The increase in the cross-country correlations of one-period returns generates a one-period myopic allocation with long and short positions and an overall levered position in equities, illustrating the fact that increased correlations do not necessarily imply less willingness to hold risky assets in a portfolio in the absence of borrowing and short-selling constraints. The panel also shows a significant increase in intertemporal hedging demands for stocks in the late sample, at 186% for the JV investor and 119% for the CCV investor. Expected utility from investing globally also increases significantly for the JV investor at long horizons and for the the CCV investor.

As we have noted in Section 6.1, the changes in intertemporal hedging demand and the welfare gains in the late period with respect to the early period could be the result of either
changes in country return volatility, or changes in cross-country correlations. To isolate the effect of the second, the lowest section of each panel in Table 3.5 reports optimal equity portfolio allocations and expected utility in the late sample holding within-country stock return predictability constant across samples (see the Appendix).

Table 3.6 shows that, under this constrained estimation, total equity optimal myopic portfolio demand is somewhat smaller in the late sample than in the early sample, and total intertemporal hedging demand somewhat larger. This hides the fact that the investor still holds optimally long and short positions. The utility gains from investing globally relative to investing in the US are still very large despite the increase in short-run return correlations. Moreover, there are still significant utility gains from investing globally relative to the early period even though we are holding within-country mean-reversion constant across periods. Therefore this welfare gain is a result of the change in the correlation structure of return news.

Overall, the results from this section suggest again that the increase in short-term correlations of stock excess returns resulting from financial globalization have not diminished the benefits of international portfolio diversification for long-horizon investors. This is so because the most relevant risk to these investors is cash flow risk, and cash flow shocks have not become significantly more correlated across countries in the late sample. Therefore, long-horizon investors still have ample room to diversify cash flow risk through global diversification. Moreover, if anything, the benefits have increased for unconstrained investors, who can take advantage of the increase in short-term correlations to build long-short myopic portfolios with lower overall risk.

3.7 Conclusions

We have documented a significant secular increase in the cross-country return correlation of global stock and bond markets since the turn of the 21st century, and explored its implications for portfolio risk, optimal intertemporal global portfolio choice, and the benefits of global portfolio diversification as a function of investment horizon. Our analysis builds
from a framework with time-varying, mean-reverting discount rates—real interest rates and risk premia—in which asset valuations vary over time in response to cash flow news and to discount rate news, both of which can be correlated across markets.

We show empirically that the main source of the increase in global return correlations has been financial globalization, which has made discount rate shocks significantly more correlated across markets. By contrast, we don’t find empirical evidence that the globalization of trade has resulted in an increase of the cross-country correlations of the second component of realized returns, cash flow shocks. We also find no evidence of an increase in the cross-country correlation of long-term volatility and risk shocks in the period of globalization. We estimate the average cross-country correlation of volatility or risk shocks to be close to zero throughout the 1986-2016 period, except for brief but significant spikes in late 1987 and during the financial crisis of 2008-2009 and its immediate aftermath.

We find that, although the increase in global stock return correlations has reduced the benefits of global portfolio diversification for short-horizon investors, long-horizon equity investors still benefit from holding internationally diversified equity portfolios in the 2000-2016 period as much as they did in the preceding 1986-1999 period, when global return correlations were much lower. Long-run global equity portfolio risk has not increased, optimal long-horizon portfolios are as globally diversified and invest in equities as much as in the preceding period, and the expected utility of long-horizon investors from holding global equity portfolios has increased.

We have shown that these results on global portfolio diversification follow from the differential impact that correlated discount rate news and correlated cash flow news have on long-run return correlations, global portfolio risk, and optimal global intertemporal portfolio choice. We show that an increase in the cross-country correlation of cash flow news leads to a one-to-one increase in cross-country return correlations at all horizons, while the impact of an increase in the cross-country correlation of discount rate news on return correlations declines as investment horizon increases.

This differential impact derives from the persistence of each type of shock. Cash-flow
news correlations have a much larger impact on long-horizon return correlations than discount rate news correlations because cash flow shocks are highly persistent and affect valuations and returns at all horizons, while discount rate shocks are transitory shocks whose impact on valuations and returns dissipates at long horizons. Therefore, cash flow news are more relevant to long-term investors than discount rate news. Because empirically cash flow news exhibit low cross-country correlations and these correlations do not appear to have increased over time, long-horizon investors still have ample margin to reduce equity cash flow risk through international equity portfolio diversification. By contrast, short-horizon investors care equally about both discount rate risk and cash flow risk, and discount rate risk has become strongly more correlated across markets over time.

We have also documented that the empirical evidence of return predictability appears to have strengthened in the 2000-2016 period relative to the 1986-1999 period, resulting in a decline in stock return volatility at all horizons. This country return volatility effect has also contributed to reduce the risk of globally diversified equity portfolios at long horizons and to increase the utility benefits of holding globally diversified portfolios for long-horizon investors.

With respect to bond markets, we find that the significant increase in the cross-country correlation of bond returns has been driven by both increased correlation of discount rate news resulting from global capital markets integration, and increased correlation of nominal bond cash flow news resulting from increased correlation of inflation across monetary areas. Long-run cross-country bond return correlations have increased as much as short-run correlations, implying that the benefits of international bond portfolio diversification have declined as much for long-horizon long-only bond investors as for short-horizon investors. However, the increased correlation of global bond markets at short and long horizons is beneficial to investors with long-dated liabilities such as pension funds. The scope for hedging liabilities using global bonds has increased. This can be particularly beneficial to investors with large long-dated liabilities whose own domestic bond markets are small.

Our research could expand in different directions. First, it would be interesting to
document why trade globalization does not appear to have led to an increase in the global correlation of cash flow news identified from equity returns. Although our results about stock and bond cash flow news correlations are consistent with a body of literature in empirical macroeconomics documenting a large increase in cross-country correlations of inflation but no increase cross-country correlations of real output, it would be interesting to explore this phenomenon more systematically at a more granular level. Second, although we have shown that persistent volatility shocks do not appear to have become more correlated over time, their correlation appears to increase significantly at times of sharp market declines. These are times in which expected returns also increase, suggesting that the increase in risk is compensated by a corresponding increase in expected returns. It would be interesting to explore the implications for intertemporal portfolio choice and for global portfolio diversification at long horizons of the joint comovement of discount rate news and risk news. Finally, we have documented but not explored in detail that the negative stock-bond correlation is a persistent global phenomenon. Understanding the economic drivers of this phenomenon at a global scale is another potential venue of future research.
References


Appendix A

Appendix to Chapter 1

A.1 Supplementary Figures and Tables

Table A.1: Estimated Mean Reversion Speed in Relative Markup (from dealer buys)

The table reports the estimated mean reversion speed of relative markup (from dealer buys) \( RM_{d,t}^B \) from the following regression

\[
RM_{d,t+1}^B - RM_{d,t}^B = \theta_0 + \theta_1 (RM_{d,t} - \bar{RM}) + \epsilon_{t+1}
\]

We estimate the regression separately for each dealer. The half-life of his mean reversion process is \( \frac{-\ln 2}{\ln(1 + \theta_1)} \), and we report the unit in weeks.

<table>
<thead>
<tr>
<th></th>
<th>Dealer 1</th>
<th>Dealer 2</th>
<th>Dealer 3</th>
<th>Dealer 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>-0.064</td>
<td>-0.073</td>
<td>-0.041</td>
<td>-0.086</td>
</tr>
<tr>
<td>95% CI of ( \theta_1 )</td>
<td>[-0.084, -0.044]</td>
<td>[-0.089, -0.056]</td>
<td>[-0.053, -0.028]</td>
<td>[-0.105, -0.066]</td>
</tr>
<tr>
<td>Half-Life (weeks)</td>
<td>2.103</td>
<td>1.836</td>
<td>3.319</td>
<td>1.544</td>
</tr>
<tr>
<td>95% CI of Half-Life</td>
<td>[1.585, 3.093]</td>
<td>[1.480, 2.394]</td>
<td>[2.529, 4.799]</td>
<td>[1.245, 2.020]</td>
</tr>
</tbody>
</table>
Figure A.1: Estimated Dealer Level Markup (from dealer buys)

The figure plots the time series of dealer-level markup calculated from dealer buy orders. Panel A and B plots the estimated dealer level markup $\mu_{d,t}^B$ (from dealer buys) for the top 8 dealers in our data.
A.2 Estimating Dealer-Specific Markup Using the Kalman Filter

This section reports the estimated dealer-specific markup using the Kalman Filter. The results are similar to the linear filter used in the main text of the paper.

It is assumed that the markup of each dealer follows a state-space model:

\[ y_t = \omega + s_t + u_t \]

\[ s_t = \phi s_{t-1} + v_t \]

where \( y_t \overset{\Delta}{=} MU_{d,t} \) is the markup calculated from data, \( s_t \) is a latent factor, and the disturbances are distributed as:

\[ u_t \sim i.i.d. \ N(0, \sigma_u^2) \]

\[ v_t \sim i.i.d. \ N(0, \sigma_v^2) \]

The Kalman filter for this state-space model has the following standard iterative steps:

Prediction:

\[ s_{t|t-1} \overset{\Delta}{=} E[s_t|\mathcal{F}_{t-1}] = \phi s_{t-1|t-1} \]

\[ P_{t|t-1} \overset{\Delta}{=} E[(s_t - s_{t|t-1})^2 | \mathcal{F}_{t-1}] = \phi^2 P_{t-1|t-1} + \sigma_v^2 \]

Observation:

\[ \mu_{t|t-1} \overset{\Delta}{=} E[y_t|\mathcal{F}_{t-1}] = \omega + s_{t|t-1} \]

\[ V_{t|t-1} \overset{\Delta}{=} E[(y_t - \mu_{t|t-1})^2 | \mathcal{F}_{t-1}] = P_{t|t-1} + \sigma_u^2 \]

Updating:

\[ s_{t|t} \overset{\Delta}{=} E[s_t|\mathcal{F}_t] = s_{t|t-1} + \frac{P_{t|t-1}}{V_{t|t-1}} (y_t - \mu_{t|t-1}) \]

\[ P_{t|t} \overset{\Delta}{=} E[(s_t - s_{t|t-1})^2 | \mathcal{F}_t] = P_{t|t-1} - \frac{p_{t|t-1}^2}{V_{t|t-1}} \]
Table A.2: Estimated Parameters of the State-Space Model

The table reports the estimated parameters \((\phi, \sigma_u, \sigma_v, \omega)\) for each dealer using maximum likelihood. The units of \(\sigma_u, \sigma_v\) and \(\omega\) are reported in bps.

<table>
<thead>
<tr>
<th>Dealer</th>
<th>(\phi)</th>
<th>(\sigma_u)</th>
<th>(\sigma_v)</th>
<th>(\omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dealer 1</td>
<td>0.992</td>
<td>29.967</td>
<td>3.262</td>
<td>46.46</td>
</tr>
<tr>
<td>Dealer 2</td>
<td>0.994</td>
<td>31.717</td>
<td>3.266</td>
<td>52.29</td>
</tr>
<tr>
<td>Dealer 3</td>
<td>0.994</td>
<td>34.863</td>
<td>2.966</td>
<td>54.43</td>
</tr>
<tr>
<td>Dealer 4</td>
<td>0.991</td>
<td>28.160</td>
<td>3.311</td>
<td>52.46</td>
</tr>
</tbody>
</table>

The parameters for each dealer in the model \(\theta = (\phi, \sigma_u, \sigma_v, \omega)\) are estimated using maximum likelihood by assuming \(y_t \sim N(\mu_{t|t-1}, V_{t|t-1})\). The log-likelihood function is given by:

\[
\ln L_T(\theta) = \frac{1}{T} \sum_{t=1}^T \ln l_t
= \frac{1}{T} \sum_{t=1}^T \left( -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln V_{t|t-1} - \frac{1}{2} \frac{(y_t - \mu_{t|t-1})^2}{V_{t|t-1}} \right)
\]

Table A.2 reports the estimated parameters in the state space model using MLE. The parameter \(\phi\) is close to 1, which means the latent factor in the markup is persistent. The units of \(\sigma_u, \sigma_v\) and \(\omega\) are reported in bps. Figure A.2 reports the estimated dealer-level markup \(\mu_{t|t-1}^d = E [MU_{d,t} | F_{t-1}]\) for the top four dealers in our sample, which is similar to the results in Figure 1.3.

### A.3 Decomposing the Dealer-Level Markup

The dealer-level markup is defined as:

\[
MU_{d,t}^S \triangleq \frac{1}{I} \sum_{i=1}^I \left( p_{i,d,t}^S - p_{i,t}^I \right)
\]  

(A.1)
The figure plots the time series of dealer-level markup estimated using Kalman filter. Panel A plots the time series of both the dealer-level markup $\mu_{d,t}^S$ (blue) and estimated dealer-level markup $\mu_{t\mid t-1}^d$ (orange) for a single dealer. Panel B plots the estimated dealer-level markup $\mu_{t\mid t-1}^d$ for the top 4 dealers in our data.
which measures the average cost of buying bond \( i \) from dealer \( d \) on day \( t \).

Plugging in equation 1.4 to replace \( P_{i,d,t}^S - P_{i,t}^I \) results in:

\[
MU_{d,t}^S = \frac{1}{L} \sum_{l=1}^{L} C_{i,t}^S + \frac{1}{L} \sum_{l=1}^{L} M_{d,t}^S + \frac{1}{L} \sum_{l=1}^{L} F_{i,d,t}^S + \frac{1}{L} \sum_{l=1}^{L} \epsilon_{i,d,t}^S
\]

(A.2)

where \( \frac{1}{L} \sum_{l=1}^{L} C_{i,t}^S \) and \( \frac{1}{L} \sum_{l=1}^{L} F_{i,d,t}^S \) are denoted as \( C_t^S \) and \( F_{d,t}^S \) respectively. We impose the following two assumptions to further simplify equation A.2:

1. When \( L \) is large enough, one can assume the average of i.i.d error term \( \frac{1}{L} \sum_{l=1}^{L} \epsilon_{i,d,t}^S \to 0 \).

2. Since \( M_{d,t}^S \) and \( F_{d,t}^S \) are both terms that depend on dealer and time, they cannot be separately identified. One can impose a restriction that \( F_{d,t}^S = \frac{1}{L} \sum_{l=1}^{L} F_{i,d,t}^S = 0 \). This means a dealer might have relatively higher markup on one bond and lower markup on another, but the average terms \( \frac{1}{L} \sum_{l=1}^{L} F_{i,d,t}^S \) can be normalized to zero. An alternative way to think about this normalization is that \( M_{d,t}^S \) and \( F_{d,t}^S \) can be collapsed into a new term, and we simply rename this new term \( M_{d,t}^S \).

Thus we have \( MU_{d,t}^S = C_t^S + M_{d,t}^S \), which means dealer-level markup can be decomposed into a bond-level component \( C_t^S \) and a dealer-level component \( M_{d,t}^S \).
A.4 Supplementary Derivations of the Model

This part of the appendix derives several results for the model in section 1.5, which is a simplified version of Randall (2015a). Without loss of generality, the model assumes that the cash held by dealers generate zero interest rate. In addition, we assume the single risky asset has infinite horizons to simplify the derivations. Randall (2015a) provides a more general setup where the asset can mature in finite periods, followed by the issuance of a new issuance. Although the general setup might be closer to reality, the simplified model is enough to deliver the intuitions and predictions behind the inventory risk channel.

A dealer who maximizes the total expected cash flow will have the following value function:

\[ V_d(t, a_d, s_t) = \max_{f, p_t} E_t \left[ \sum_{n=0}^{\infty} \delta^n \left( -p_{t+u}q_{t+u} - f_{t+u} \times \left( a_{t+u}^d \right)^2 \right) \right] \]

In equation 1.13, I conjectured the value function has the following functional form:

\[ V_d(t, a_d, s_t) = g_t^d \ s_t + g_t^a a_d^t + g_t^{aa} \left( a_d^t \right)^2 + \gamma_t \]

To verify this conjecture, we solve the maximization problem in equation 1.12 for each of the following three scenarios: inter-dealer trading, dealer-customer trading, and no trading.

(a). Inter-dealer trading

We assume the inter-dealer market is competitive, where each dealer take prices as given and optimize over the quantity \( q_i^d \). The Bellman equation in this scenario is:

\[ V_{d,i}^d(s_i^d, a_i^d, s_t) = \max_{q_i^d} - f_i \times \left( a_i^d \right)^2 - p_i^d q_i^d + \delta E_t \left[ V_{d,i+1}^d(s_{i+1}^d, a_{i+1}^d, s_{t+1}) \right] \]  

(A.3)

where

\[ s_{i+1}^d = s_i^d - f_i \times \left( a_i^d \right)^2 - p_i^d q_i^d \]  

(A.4)

and

\[ a_{i+1}^d = a_i^d + q_i^d \]  

(A.5)

Taking the first order condition of the Bellman equation with respect to \( q_i^d \) gives:
where
\[ E \]

This verifies that the value function has the form in equation 1.13 when inter-dealer trading happens, and gives the recursive expression for the coefficients in this equation.

(b). Customer-dealer trading

To simplify the notation, we leave out the subscript for variables associated with the
customer \( j \). For example, customer \( j \)'s valuation of risky asset \( V_t^i \) is written as \( V_t^j \), bargaining power \( \eta_j \) is written as \( \eta \).

The prices \( p_i^c \) and traded quantities \( q_i^c \) between a dealer and a customer are determined by Nash-Bargaining. From equation 1.15, equation 1.16 and the conjectured functional form of value function, we have the dealer’s gain from trading \( q_i^c \) vs. not trading given by:

\[
\text{Gain}_i^d = \tilde{V}_t^d(s_{i1}, a_{i1}^d, s_t, q_i^c) - \tilde{V}_t^d(s_{i1}, a_{i1}^d, s_t, 0)
\]

\[
= -p_i^c q_i^c + \delta E_t \left[ V_{t+1}^d(s_{i1} - p_i^c q_i^c - f_t \times (a_{i1}^d, a_{i1}^d + q_i^c, s_{t+1}) - V_{t+1}^d(s_{i1} - f_t \times (a_{i1}^d, a_{i1}^d, s_{t+1}) \right]
\]

\[
= -p_i^c q_i^c + \delta E_t \left[ -\gamma_{t+1}^d p_i^c q_i^c + \gamma_{t+1}^d q_i^c + \gamma_{t+1}^d \left( q_i^c \right)^2 + 2a_{i1}^d q_i^c \right]
\]

\[
= q_i^c E_t \left[ \delta \gamma_{t+1}^d + \delta \left( 2a_{i1}^d + q_i^c \right) \gamma_{t+1}^d - \left( 1 + \delta \gamma_{t+1}^d \right) p_i^c \right]
\]

The customer’s gain from trading trading \( q_i^c \) vs. not trading is given by:

\[
\text{Gain}_i^c = \delta q_i^c (p_i^c - V_i^c)
\]

Both parties need to have positive gains from trade for the transaction to happen. Thus the necessary conditions for trading are \( \text{Gain}_i^d > 0 \) and \( \text{Gain}_i^c > 0 \).

The optimal prices \( p_i^c \) and traded quantities \( q_i^c \) are determined by solving the Nash-Bargaining problem:

\[
\max_{q_i^c, p_i^c} (\text{Gain}_i^d)^{\eta} (\text{Gain}_i^d)^{1-\eta}
\]

where \( 0 < \eta < 1 \) denotes the bargaining power of the customer. Taking the first derivative with respect to \( q_i^c \) and \( p_i^c \) on the log of this objective function, we get the following:

\[
\left\{ \frac{1}{q_i^c} + (1 - \eta) \frac{\delta E_t (\gamma_{t+1}^d)}{E_t [\delta \gamma_{t+1}^d + \delta (2a_{i1}^d + q_i^c) \gamma_{t+1}^d - (1 + \delta \gamma_{t+1}^d) p_i^c]} - 1 \right\} \frac{1}{p_i^c - V_i^c} + (1 - \eta) \frac{\delta E_t (\gamma_{t+1}^d)}{E_t [\delta \gamma_{t+1}^d + \delta (2a_{i1}^d + q_i^c) \gamma_{t+1}^d - (1 + \delta \gamma_{t+1}^d) p_i^c]} = 0
\]

We can solve \( q_i^c \) and \( p_i^c \) from these two equations to get:
\begin{align*}
\begin{cases}
p^c_t = \frac{\eta}{2} \delta E_t \left( \gamma^d_t + \frac{2\eta \gamma^a_t}{\gamma^d_t} \right) + \left( 1 - \frac{\eta}{2} \right) V^c_t \\
q^c_t = -a^d_t + \frac{V^c_t \left( 1 + \delta E_t \left( \gamma^d_t \right) - \delta E_t \left( \gamma^a_t \right) \right)}{2\delta E_t \left( \gamma^a_t \right)}
\end{cases}
\end{align*}
(A.10)

Using equation A.10 and the conjecture, we have:

\[V^{d,c}_t (s^d_t, a^d_t, s_t) = -f_t \times \left( a^d_t \right)^2 - p^c_t q^c_t + \delta E_t \left[ \gamma^{d}_t \left( s^d_t - f_t \times \left( a^d_t \right)^2 - p^c_t q^c_t \right) + \gamma^a_t \left( a^d_t + q^c_t \right) \right] + \gamma^{a}_t \left( a^d_t + q^c_t \right)^2 + \gamma^{t+1}_t \]

\[= s^d_t \delta E_t \left( \gamma^{d}_t \right) + p^c_t q^c_t \left[ -1 - \delta E_t \left( \gamma^{d}_t \right) - f_t \times \left( a^d_t \right)^2 \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) \right] + \delta E_t \left( \gamma^{t+1}_t \right)

+ \delta \left( a^d_t + q^c_t \right) \left[ E_t \left( \gamma^{a}_t \right) + \left( a^d_t + q^c_t \right) E_t \left( \gamma^{a}_t \right) \right]

= \delta E_t \left( \gamma^{t+1}_t \right) \times s^d_t + \left[ \eta \delta E_t \left( \gamma^{a}_t \right) + \left( 1 - \eta \right) V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) \right] \times a^d_t + \left[ \eta \delta E_t \left( \gamma^{a}_t \right) + \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) f_t \right] \times \left( a^d_t \right)^2

+ \left\{ \begin{array}{l}
\delta E_t \left( \gamma^{d}_t \right) + \frac{V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) - \delta E_t \left( \gamma^{a}_t \right)}{2\delta E_t \left( \gamma^{a}_t \right)} \left[ \frac{\delta}{2} \left( 1 - \eta \right) E_t \left( \gamma^{a}_t \right) \\
- \frac{1 - \eta}{2} V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) \right] \end{array} \right\}

= \gamma^{d,c}_t \times s^d_t + \gamma^{a,c}_t \times a^d_t + \gamma^{a,c}_t \times \left( a^d_t \right)^2 + \gamma^c_t
\end{align*}
(A.11)

where

\[\begin{align*}
\gamma^{d,c}_t &= \delta E_t \left( \gamma^{d}_t \right) \\
\gamma^{a,c}_t &= \eta \delta E_t \left( \gamma^{a}_t \right) + \left( 1 - \eta \right) V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) \\
\gamma^{a,a,c}_t &= \eta \delta E_t \left( \gamma^{a}_t \right) - \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) f_t \\
\gamma^c_t &= \delta E_t \left( \gamma^{d}_t \right) + \frac{V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) - \delta E_t \left( \gamma^{a}_t \right)}{2\delta E_t \left( \gamma^{a}_t \right)} \left[ \frac{\delta}{2} \left( 1 - \eta \right) E_t \left( \gamma^{a}_t \right) - \frac{1 - \eta}{2} V^c_t \left( 1 + \delta E_t \left( \gamma^{d}_t \right) \right) \right]
\end{align*}\]
(A.12)

This verifies that the value function has the form in equation 1.13 when dealer-customer trading happens, and gives the recursive expression for the coefficients in this equation.
(c). No trading

When there is no trading, the dealer incurs a per-period inventory cost, and then enters into next period. There is no optimization involved in this scenario. The value function of a dealer when there is no trading is:

\[
V^{\text{d,nt}}(d_t, a_t^d, s_t) = -f_t \times \left( a_t^d \right)^2 + \delta E_t \left[ V^{\text{d,nt}}(d_{t+1}, a_{t+1}^d, s_{t+1}) \right]
\]

\[
= -f_t \times \left( a_t^d \right)^2 + \delta E_t \left[ \gamma_{t+1}^s \left( d_t^d - f_t \times \left( a_t^d \right)^2 \right) + \gamma_{t+1}^a a_t^d + \gamma_{t+1}^{aa} \left( a_t^d \right)^2 + \gamma_t \right]
\]

\[
= \delta E_t \left( \gamma_{t+1}^s \right) \times d_t^d + \delta E_t \left( \gamma_{t+1}^a \right) \times a_t^d + \left[ - \left( 1 + \delta E_t \left( \gamma_{t+1}^s \right) \right) f_t + \delta E_t \left( \gamma_{t+1}^{aa} \right) \right] \times \left( a_t^d \right)^2 + \delta E_t \left( \gamma_{t+1} \right)
\]  
(A.13)

Value function with uncertain trade type

We can combine the results in (a), (b) and (c) to get the (expected) value function before the realization of trade type. We denote \( \pi_i \) as the probability that inter-dealer trade happens, \( \pi_j^c \) as the probability that the dealer trade with customer \( j \), and \( \pi_c \) as \( \sum_j \pi_j^c \). Obviously, the probability of no trading happening is \( 1 - \pi_i - \pi_c \). Using equations A.8, A.11 and A.13, we have:

\[
V^d_t(d_t^d, a_t^d, s_t) = \pi_i V^{d,i}_t + \sum_j \pi_j^c V^{d,j}_t + (1 - \pi_i - \pi_c) V^d_t
\]

\[
= \delta E_t \left( \gamma_{t+1}^s \right) \times d_t^d + \left[ \left( 1 + \delta E_t \left( \gamma_{t+1}^s \right) \right) \left( \pi_i p_t^i + \sum_j \pi_j^c (1 - \eta_j) \right) V^c_t \right] + \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) \delta E_t \left( \gamma_{t+1}^{aa} \right) \times a_t^d
\]

\[
+ \left[ - \left( 1 + \delta E_t \left( \gamma_{t+1}^s \right) \right) f_t + \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) E_t \left( \gamma_{t+1}^{aa} \right) \right] \times \left( a_t^d \right)^2
\]

\[
+ \pi_i \gamma_i^i + \sum_j \pi_j^c \gamma_j^i + (1 - \pi_i - \pi_c) \delta E_t \left( \gamma_{t+1} \right)
\]

\[
= \gamma_t^s \times d_t^d + \gamma_t^a \times a_t^d + \gamma_t^{aa} \times \left( a_t^d \right)^2 + \gamma_t
\]
where

\[
\begin{align*}
\gamma_t^S &= \delta E_t \left( \gamma_{t+1}^S \right) \\
\gamma_t^a &= \left( 1 + \delta E_t \left( \gamma_{t+1}^a \right) \right) \left( \pi_i p_i^1 + \sum_j \pi_j^c \left( 1 - \eta_j \right) V_t^c \right) + \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) E_t \left( \gamma_{t+1}^a \right) \\
\gamma_t^{aa} &= -\left( 1 + \delta E_t \left( \gamma_{t+1}^a \right) \right) f_t + \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) E_t \left( \gamma_{t+1}^{aa} \right) \\
\gamma_t &= \pi_i \gamma_i^1 + \sum_j \pi_j^c \gamma_j^c + \delta \left( 1 - \pi_i - \pi_c \right) E_t \left( \gamma_{t+1} \right)
\end{align*}
\]

(E.14)

Equations A.14 provide recursive relationships of the coefficients. Solving these recursive relationships gives the following:

\[
\begin{align*}
\gamma_t^S &= 0 \\
\gamma_t^a &= \sum_{u=0}^{\infty} \left[ \left( \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) \right)^u \left( \pi_i E_t \left( p_i^1 + u \right) + \sum_j \pi_j^c \left( 1 - \eta_j \right) E_t \left( V_{t+1}^c \right) \right) \right] \\
\gamma_t^{aa} &= -\sum_{u=0}^{\infty} \left[ \left( \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j^c \eta_j \right) \right)^u E_t \left( f_{t+1} \right) \right]
\end{align*}
\]

Deriving the markup

From equation A.10 we have

\[
p_i^c = \frac{\eta}{2} \left[ \frac{\delta E_t \left( \gamma_{t+1}^a \right) + 2\delta a_i^{dc} E_t \left( \gamma_{t+1}^{aa} \right)}{1 + \delta E_t \left( \gamma_{t+1}^S \right)} + \left( 1 - \frac{\eta}{2} \right) V_t^c \right]
\]

and

\[
\delta E_t \left( \gamma_{t+1}^a \right) = -\left( q_i^c + a_i^{dc} \right) 2\delta E_t \left( \gamma_{t+1}^{aa} \right) + V_t^c \left( 1 + \delta E_t \left( \gamma_{t+1}^S \right) \right)
\]

Combining equations A.15 and A.16, and use the fact that $E_t \left( \gamma_{t+1}^S \right) = 0$, we have:

\[
p_i^c = \frac{\eta}{2} \left[ -\left( q_i^c + a_i^{dc} \right) 2\delta E_t \left( \gamma_{t+1}^{aa} \right) + V_t^c \left( 1 + \delta E_t \left( \gamma_{t+1}^S \right) \right) + 2\delta a_i^{dc} E_t \left( \gamma_{t+1}^{aa} \right) \right] + \left( 1 - \frac{\eta}{2} \right) V_t^c
\]

\[
= -\eta \delta q_i^c E_t \left( \gamma_{t+1}^{aa} \right) + V_t^c
\]

(A.17)

Using equation A.6 and the fact that $E_t \left( \gamma_{t+1}^S \right) = 0$, we have the following:

\[
p_i^c = \delta E_t \left( \gamma_{t+1}^a \right) + 2\delta \left( a_i^{dc} + q_i^c \right) E_t \left( \gamma_{t+1}^{aa} \right)
\]
Again, we could replace $\delta E_i (\gamma_t^{a+1})$ in this equation using equation A.16, which then gives:

$$p_i^c = -\left( q_i^c + a_i^{d,c} \right) 2\delta E_i (\gamma_t^{a+1}) + V_i^c + 2\delta \left( a_i^{d,i} + q_i^i \right) E_i (\gamma_t^{a+1})$$

$$= V_i^c + 2\delta \left( \left( a_i^{d,i} + q_i^i \right) - \left( a_i^{d,c} + q_i^c \right) \right) E_i (\gamma_t^{a+1})$$

(A.18)

We could get the expression of the markup by combining equations A.17 and A.18:

$$p_i^c - p_i^d = \delta \left[ \eta q_i^c + 2 \left( \left( a_i^{d,i} + q_i^i \right) - \left( a_i^{d,c} + q_i^c \right) \right) \right] \left( -E_i (\gamma_t^{a+1}) \right)$$

$$= \delta \left[ \eta q_i^c + 2 \left( \left( a_i^{d,i} + q_i^i \right) - \left( a_i^{d,c} + q_i^c \right) \right) \right] \times \sum_{\mu=0}^{\infty} \left[ \delta \left( 1 - \pi_i - \pi_c + \sum_j \pi_j \eta_j \right) \right] u \left( f_{1-u} \right)$$

(A.19)
Appendix B

Appendix to Chapter 2

B.1 Supplementary Figures and Tables

Figure B.1: CDS Spread Matched Excess Yield and 10 year Treasury Yield (2002Q1-2016Q4)

Panel A and Panel B plots the CDS Spread matched excess yield against the 10-year yield on US treasury (blue scatters) for NAIC category 1 and 2 respectively. As a comparison, we also plot the excess yield shown in Figure 3.1 (orange scatters). The CDS Spread matched excess yield is the difference between bond yield and average yield of 10 bonds with close duration from the market.
Figure B.2: Time Series of Maturity Slack and Yield Slack

The figure plots the time series of maturity slack and yield slack for both NAIC1 and NAIC2 bonds. Maturity slack is defined as the difference between maximum possible maturity (that the insurance company assets can have) and the actual asset maturity of the insurance companies. The yield slack is defined as the difference in yield between the maximum maturity portfolio and the insurance companies’ actual portfolio.
Appendix C

Appendix to Chapter 3

C.1 Asset Return Decomposition

A log-linearization of the return on an asset around the unconditional mean of its dividend-price ratio—where dividend is a proxy for cash flow—implies the following decomposition of realized returns:

$$ r_{s,t+1} - \mathbb{E}_t [r_{s,t+1}] = (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}. $$

(C.1)

where $r_{s,t}$ denotes the natural log of the gross total return on the asset and $\Delta d_{t+1}$ the change in its log dividend (or cash flow). The constant $\rho_s \equiv 1/ (1 + \exp (\bar{d} - \bar{p}) )$ is a log-linearization parameter, where $\bar{d} - \bar{p}$ denotes the unconditional mean of the log dividend-price ratio.

Equation (C.1) shows that the unexpected log return on an asset reflects changes in either its expected future cash flows or in its expected future returns (or discount rates). Following standard terminology in this literature, we will refer to the former as cash flow shocks or cash flow news, and to the latter as discount rate shocks or discount rate news, and write more succinctly

$$ r_{s,t+1} - \mathbb{E}_t [r_{s,t+1}] \equiv N_{CF,s,t+1} - N_{DR,s,t+1}. $$

(C.2)
We can further decompose \( N_{DR,s,t+1} \) into news about excess log returns—or risk premia—, and news about the return on the reference asset used to compute excess returns:

\[
N_{DR,s,t+1} = N_{RR,s,t+1} + N_{RP,s,t+1},
\]

(C.3)

with

\[
N_{RR,s,t+1} \equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{\infty} \rho_j r_{f,t+1+j} \right],
\]

\[
N_{RP,s,t+1} \equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{\infty} \rho_j x_{s,t+1+j} \right],
\]

where \( x_{s,t+1+j} = r_{s,t+1+j} - r_{f,t+1+j} \) denotes log excess returns with respect to the log return on the benchmark asset \( r_{f,t+1+j} \). In our empirical analysis we follow standard practice and use cash (i.e., a short-term nominal bond like a T-bill in the US) as the reference asset, and measure returns in real terms. For example, \( r_{f,t+1} = y_{1,t} - \pi_{t+1} \), where \( y_{1,t} \) denotes the yield on a one-period nominal bond at \( t \), which is also its nominal return at \( t+1 \), and \( \pi_{t+1} \) denotes log inflation.

The preceding expressions assume the asset is a perpetual claim on cash flows such as equities. In our empirical analysis we also consider nominal bonds with fixed maturities and whose cash flows (i.e., coupons) are fixed in nominal terms and thus vary inversely with the price level in real terms. Section A.1 below shows that for a $1-coupon nominal bond with maturity \( n \),

\[
r_{n,t+1} - \mathbb{E}_t [r_{n,t+1}] = N_{CF,n,t+1} - N_{RR,n,t+1} - N_{RP,n,t+1},
\]

(C.4)
with

\[ N_{CF,n,t+1} = -N_{INFL,n,t+1} \equiv - (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j r_{t+1+j} \right], \]

\[ N_{RR,n,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j r_{f,t+1+j} \right], \]

\[ N_{RP,n,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j x_{n-j,t+1+j} \right], \]

and \( \rho_b = 1/(1 + \exp(-p_n)) \).

The news components defined above are not directly observable, but we can infer them from a return generating model. We follow Campbell (1991) and assume that the asset return generating process follows a first-order vector autoregressive (VAR) model:

\[ \tilde{z}_{t+1} = a + A \tilde{z}_t + u_{t+1}, \]

where \( \tilde{z}_{t+1} \) is a state vector that includes the excess log return on the assets under consideration, variables that predict excess returns, and variables that capture the dynamics of inflation and the short-term interest rate. The vector of innovations \( u_{t+1} \) is uncorrelated over time with conditional variance-covariance matrix \( V_t [u_{t+1}] \). Given a specification for \( \tilde{z}_{t+1} \), it is straightforward to derive the components of the return decomposition as a function of the vector \( u_{t+1} \) of innovations to \( \tilde{z}_{t+1} \) and the parameters of the VAR(1).

**Excess Bond Returns Decomposition (3 News Components)**

Define the log one-period nominal return on a nominal \( n \)-period coupon bond as

\[ r_{n,t+1}^S = \log \left( 1 + R_{n,t+1}^S \right) = \log (P_{n-1,t+1} + C) - \log (P_{n,t}) \]

\[ = p_{n-1,t+1} - p_{n,t} + \log (1 + \exp (c - p_{n-1,t+1})) \]

\[ \approx k + \rho_b p_{n-1,t+1} + (1 - \rho_b) c - p_{n,t}, \]

(C.6)
where \( \rho_b = \frac{1}{1 + \exp(c - p)} \) and \( k = -\log(\rho_b) - (1 - \rho_b) \log\left(\frac{1}{\rho_b} - 1\right) \). Solving forward and imposing the terminal condition that \( p_{n-j,t+j}|_{j=n} = 0 \), we get that

\[
p_{n,t} = (k + (1 - \rho_b) c) \left( \sum_{j=0}^{n-1} \rho_b^j \right) - \sum_{j=0}^{n-1} r_{n-j,t+1+j}^s \rho_b^j.
\]

Plugging this expression in to the unexpected bond return from Eq. (C.6), we get that

\[
(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ r_{n,t+1}^s \right] = (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \rho_b p_{n-1,t+1} \right] - (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ p_{n,t} \right]
\]

\[
= (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \rho_b p_{n-1,t+1} \right]
\]

\[
= - (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} r_{n-j,t+1+j}^s \rho_b^j \right].
\]

We can write \( r_{n,t+1}^s = x r_{n,t+1} + r_{f,t+1}^s \) where \( x r_{n,t+1} \) is the excess log 1-period return on a nominal \( n \)-period coupon bond and \( r_{f,t+1}^s \) is the realized nominal return of the 1-period nominal bond, which is the same as the yield of the 1-period nominal bond \( y_{1,t}^N \).

Decomposing the surprise bond return in Eq. (C.7) gives

\[
(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ x r_{n,t+1} + r_{f,t+1}^s \right] = - (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} \rho_b^j x r_{n-j,t+1+j} \right] - (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} \rho_b^j r_{f,t+1+j}^s \right].
\]

The LHS can be simplified by noting that

\[
(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ r_{f,t+1}^s \right] = (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ y_{1,t}^N \right] = 0.
\]

To simplify the RHS, we simply note that the realized nominal return of the 1-period nominal bond is the realized real return of the 1-period nominal bond plus realized inflation: \( r_{f,t+1}^s = r_{f,t+1} + \pi_{t+1} \). The second term on the RHS is then

\[
(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} \rho_b^j r_{f,t+1+j}^s \right] = (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} \rho_b^j r_{f,t+1+j} \right] + (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{n-1} \rho_b^j \pi_{t+1+j} \right].
\]

(C.8)

Putting together the simplified LHS and RHS, we have the following 3 news component decomposition for unexpected excess bond returns:
(\(E_{t+1} - E_t\)) \[x_{n,t+1}\] = \(N_{CF,n,t+1} - N_{RR,n,t+1} - N_{RP,n,t+1}\)

where

\(N_{CF,n,t+1} = -N_{INFL,n,t+1} \equiv - (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j^p \tau_{t+1+j} \right] \),

\(N_{RR,n,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j^f \tau_{f,t+1+j} \right] \), and

\(N_{RP,n,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=1}^{n-1} \rho_j^r \tau_{r,t+1+j} \right] \). (C.9)

To extract the news components from the VAR, consider the vector of state variables

\[\tilde{z}_{t+1} = \left[ x_{r,t+1}, x_{n,t+1}, d_{t+1} - p_{t+1}, \tau_{t+1}, y_{1,t+1}, y_{10,t+1} - y_{11,t+1} \right]. \quad \text{(C.10)}\]

The main VAR equation is \(\tilde{z}_{t+1} = a + A \tilde{z}_t + u_{t+1}\), which leads to \(E_t [\tilde{z}_{t+j}] = A^j \tilde{z}_t\) and \((E_{t+1} - E_t) [\tilde{z}_{t+j}] = A^{j-1} u_{t+1}\). It is then straightforward to see how the decomposition can be written in VAR notation:

\[(E_{t+1} - E_t) [x_{n,t+1}] = e_2' u_{t+1},\]

\[N_{CF,n,t+1} = -e_4' \left( \sum_{j=1}^{n-1} \rho_j^p A^j \right) u_{t+1},\]

\[N_{RR,n,t+1} = e_5' \left( \sum_{j=1}^{n-1} \rho_j^f A^{j-1} \right) u_{t+1} - e_4' \left( \sum_{j=1}^{n-1} \rho_j^r A^j \right) u_{t+1},\]

and

\[N_{RP,n,t+1} = N_{CF,n,t+1} - N_{RR,n,t+1} - (E_{t+1} - E_t) [x_{n,t+1}].\]

We get \(N_{RR,n,t+1}\) by using Eq. (C.8) to express real rate news in terms of nominal rate news and inflation news. Finally, we back out \(N_{RP,n,t+1}\) as the residual.
Excess Stock Returns Decomposition (3 News Components)

We start with Campbell-Shiller decomposition which decompose the news on real stock return into news on growth of log real dividend and news on log real interest rate

\[(E_{t+1} - E_t) [r_{s,t+1}] = NCF_{s,t+1} - NDR_{s,t+1},\]

where

\[NCF_{s,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_j \Delta d_{t+1+j} \right] \text{ and} \]

\[NDR_{s,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho_j r_{s,t+1+j} \right]. \tag{C.11} \]

We can relate the 2 news component decomposition to the 3 news component decomposition as follows. Note that the excess return could be written as \(xr_{s,t+1+j} = r_{s,t+1+j} - r_{f,t+1+j}\), we have

\[NDR_{s,t+1} = (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho_j xr_{s,t+1+j} \right] + (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_j rf_{s,t+1+j} \right] - (E_{t+1} - E_t) [rf_{s,t+1+j}].\]

Combining this with the decomposition we have

\[(E_{t+1} - E_t) [xr_{s,t+1}] + (E_{t+1} - E_t) [rf_{s,t+1}] = NCF_{s,t+1} - NDR_{s,t+1}\]

\[= NCF_{s,t+1} - (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho_j xr_{s,t+1+j} \right]\]

\[- (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_j rf_{s,t+1+j} \right] + (E_{t+1} - E_t) [rf_{s,t+1+j}].\]

Thus we have

\[(E_{t+1} - E_t) [xr_{s,t+1}] = NCF_{s,t+1} - NRR_{s,t+1} - NRP_{s,t+1}\]
where

\[ N_{CF,s,t+1} \equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=0}^{\infty} \rho_j^s \Delta d_{t+1+j} \right], \]

\[ N_{RR,s,t+1} \equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=0}^{\infty} \rho_j^s r_{f,t+1+j} \right], \]

\[ N_{RP,s,t+1} \equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[ \sum_{j=1}^{\infty} \rho_j^s x_{r_{s,t+1+j}} \right]. \]

(C.12)

With the same vector of state variables \( z_{t+1} \) as in Eq. (C.10), we write the decomposition in VAR notation:

\[(\mathbb{E}_{t+1} - \mathbb{E}_t) [x_{r,s,t+1}] = e_{1}^t u_{t+1},\]

\[ N_{CF,s,t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t) [x_{r,s,t+1}] + N_{RR,s,t+1} + N_{RP,s,t+1}, \]

\[ N_{RR,s,t+1} = e_{5}^t \left( \sum_{j=1}^{\infty} \rho_j A \right) u_{t+1} - e_{4}^t \left( \sum_{j=0}^{\infty} \rho_j A \right) u_{t+1}, \]

and

\[ N_{RP,s,t+1} = e_{1}^t \left( \sum_{j=1}^{\infty} \rho_j A \right) u_{t+1}. \]

Similar to the case with bonds, we get \( N_{RR,n,t+1} \) by using an infinite-sum version of Eq. (C.8) to express real rate news in terms of nominal rate news and inflation news. Note that the first term in \( N_{RR,s,t+1} \) starts from \( j = 1 \) instead of \( j = 0 \) because \((\mathbb{E}_{t+1} - \mathbb{E}_t) [y_{1,t}^N] = 0\). Finally, we back out \( N_{CF,s,t+1} \) as the residual.
C.2 Derivation of Results in Section 3.2

We want to derive the general formula for \( k \) period portfolio return variance, where the portfolio is constructed by holding equal weight on \( N \) identical markets. The starting point is from our stylized symmetrical model of asset returns of Section 3

\[
\begin{align*}
    r_{i,t+1} &= \mu_1 + \beta s_{i,t} + u_{i,t+1} \\
    s_{i,t+1} &= \mu_2 + \phi s_{i,t} + u_{si,t+1}
\end{align*}
\]  

(C.13)

and we could also write the VAR residual in terms of news terms \( u_{i,t+1} = N_{CF,i,t+1} - N_{DR,i,t+1} \) and \( u_{si,t+1} = \frac{1}{\lambda} N_{DR,i,t+1} \), where \( \lambda = \frac{\rho \beta}{1 - \rho \beta} \). The log portfolio return over \( k \) period horizon (from \( t \) to \( t+k \)) is \(^1\)

\[
    r_{p,t+k}^{(k)} = r_{0}^{(k)} + \alpha_t'(r_{i,t+k}^{(k)} - r_{0}^{(k)}) I + \frac{1}{2} \alpha_t(k)^2 \sigma_t(k)^2 - \frac{1}{2} \alpha_t(k) \Sigma_t(k) \alpha_t(k)
\]  

(C.14)

and the variance of \( k \) period portfolio return is

\[
    \text{Var}[r_{p,t+k}^{(k)}] = \frac{1}{N} \text{Var}[r_{i,t+k}^{(k)}] + (1 - \frac{1}{N}) C_v[r_{i,t+k}^{(k)}, r_{j,t+k}^{(k)}]
\]  

(C.15)

where \( r_{i,t+k}^{(k)} = \sum_{l=1}^{k} r_{i,t+l} \) is the \( k \) period log return of market \( i \).

The term of interest in the expression is the cross-country covariance. Let’s now derive the general expression for the covariance term. Note that the 1 period return at \( t+l \) could be written as

\[
    r_{i,t+l} = \mu_1 + \beta s_{i,t+l-1} + u_{i,t+l} \\
    = \mu_1 + \beta (\phi s_{i,t+l-2} + u_{si,t+l-1}) + u_{i,t+l} \\
    \ldots
\]

\[
    = \mu_1 + \beta \phi^{l-1} s_{i,t} + \beta \sum_{m=1}^{l-1} \phi^{m-1} u_{si,t+l-m} + u_{i,t+l}
\]  

(C.16)

\(^1\)The formula for portfolio return below is derived in the appendix of Campbell and Viceira (2002) “Strategic Asset Allocation: Portfolio Choice for Long-Term Investors”
and

\[ C_t[r_{i,t+l}, r_{j,t+l}] = C_t[\beta \sum_{m=1}^{l-1} \phi_m u_{s_{i,t+l-m}} + u_{i,t+l}, \beta \sum_{m=1}^{l-1} \phi_m u_{s_{j,t+l-m}} + u_{j,t+l}] \]

\[ = C_t[\frac{\beta}{\lambda} \sum_{m=1}^{l-1} \phi_m N_{DF,i,t+l-m} + N_{CF,i,t+l} - N_{DF,j,t+l}, \frac{\beta}{\lambda} \sum_{m=1}^{l-1} \phi_m N_{DF,j,t+l-m} + N_{CF,j,t+l} - N_{DF,j,t+l}] \]

(C.17)

We make the assumption that \((for \ \forall l \geq 1, i \neq j)\)

\[ C_t[N_{CF,i,t+l}, N_{CF,j,t+l}] = \sigma_{CF,CF}^x \]

\[ C_t[N_{CF,i,t+l}, N_{DR,j,t+l}] = \sigma_{CF,DR}^x \]

\[ C_t[N_{DR,i,t+l}, N_{DR,j,t+l}] = \sigma_{DR,DR}^x \]

Thus we have

\[ C_t[r_{i,t+l}, r_{j,t+l}] = \left[ \frac{\beta^2}{\lambda} \frac{(1 - (\phi^2)^{l-1})}{1 - \phi^2} + 1 \right] \sigma_{DR,DR}^x + \sigma_{CF,CF}^x - 2 \sigma_{CF,DR}^x \]

(C.18)

For the cross-period & cross-country covariance, we have

\[ C_t[r_{i,t+l}, r_{j,t+l+p}] = C_t[\beta \sum_{m=1}^{l-1} \phi_m u_{s_{i,t+l-m}} + u_{i,t+l}, \beta \sum_{m=1}^{l+p-1} \phi_m u_{s_{j,t+l+p-m}} + u_{j,t+l+p}] \]

\[ = C_t[u_{i,t+l} + \beta u_{s_{i,t+l-1}} + \beta \phi u_{s_{i,t+l-2}} + \cdots + \beta \phi^{l-2} u_{s_{i,t+1}}, \]

\[ \beta \phi^{p-1} u_{s_{j,t+l}} + \beta \phi^p u_{s_{j,t+l-1}} + \beta \phi^{p+1} u_{s_{j,t+l-2}} + \cdots + \beta \phi^{l+p-2} u_{s_{j,t+1}] \]

\[ = \beta \phi^{p-1} C_t[u_{s_{i,t+l-1}}, u_{s_{j,t+l}}] + \beta^2 \phi p C_t[u_{s_{i,t+l-1}}, u_{s_{j,t+l-1}}] \]

\[ + \beta^2 \phi^{p+2} C_t[u_{s_{i,t+l-2}}, u_{s_{j,t+l-2}}] + \cdots + \beta^2 \phi^{p+2(l-2)} C_t[u_{s_{i,t+l}}, u_{s_{j,t+l-1}]} \]

\[ = \frac{\beta \phi^{p-1}}{\lambda} (\sigma_{CF,DR}^x - \sigma_{DR,DR}^x) + \frac{\beta^2 \phi^p}{\lambda^2} \frac{1 - (\phi^2)^{l-1}}{1 - \phi^2} \sigma_{DR,DR}^x \]

(C.19)

with \(p \geq 1\). Using the results above, we could get the \(k\) period cross-country return
covariance

\[ C_r^{(k)} = \sum_{i=1}^{k} C_i[r_{t+1}, r_{t+i}] + 2 \sum_{i=1}^{k-1} \sum_{p=1}^{k-i} C_i[r_{t+1}, r_{t+i+p}] \]

\[ = \sum_{i=1}^{k} \left( \left[ \frac{\beta^2}{\lambda^2} \frac{(1 - (\phi^2)^{i-1})}{1 - \phi^2} + 1 \right] \sigma_{DR,DR}^{xc} + \sigma_{CF,CF}^{xc} - 2 \sigma_{CF,DR}^{xc} \right) \]

\[ + 2 \sum_{i=1}^{k-1} \sum_{p=1}^{k-i} \left( \frac{\beta \phi^{i-1}}{\lambda} \left( \sigma_{CF,DR}^{xc} - \sigma_{DR,DR}^{xc} \right) + \frac{\beta^2 \phi^{i-1}}{\lambda^2} \frac{1 - (\phi^2)^{i-1}}{1 - \phi^2} \sigma_{DR,DR}^{xc} \right) \]

\[ = \left( \left[ \frac{\beta^2}{\lambda^2} \frac{k - 1 - (\phi^2)^k}{1 - \phi^2} + k \right] \sigma_{DR,DR}^{xc} + k \sigma_{CF,CF}^{xc} - 2k \sigma_{CF,DR}^{xc} \right) \]

\[ + 2 \sum_{i=1}^{k-1} \left( \frac{\beta}{\lambda(1 - \phi)} \frac{1 - (\phi^2)^{k-1}}{1 - \phi} \right) \sigma_{CF,DR}^{xc} + \frac{\beta^2 \phi}{\lambda^2(1 - \phi)^2} \frac{(k - 1 - (\phi^2)^{k-1})^2}{(1 - \phi)^2} \sigma_{DR,DR}^{xc} \]

\[ + 2 \left( \frac{\beta}{\lambda(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{1 - \phi} \right) \sigma_{CF,DR}^{xc} + \left( \frac{\beta}{\lambda(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{1 - \phi} \right) \sigma_{CF,CF}^{xc} \]

\[ + 2 \left( \frac{\beta}{\lambda(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{1 - \phi} \right) \sigma_{DR,DR}^{xc} \]

We further simplify the coefficient on \( \sigma_{DR,DR}^{xc} \) as

\[ \frac{\beta^2}{\lambda^2} \frac{k - 1 - (\phi^2)^k}{1 - \phi^2} + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)^2} \frac{(k - 1 - (\phi^2)^{k-1})^2}{(1 - \phi)^2} \frac{1 - (\phi^2)^{k-1}}{1 - \phi^2} \]

\[ - 2 \frac{\beta}{\lambda(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{1 - \phi} + k \]

\[ = k \left( \frac{\beta^2}{\lambda^2} \frac{1 - (\phi^2)^k}{k(1 - \phi^2)} + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{k(1 - \phi)^2} \frac{1 - (\phi^2)^{k-1}}{k(1 - \phi^2)} \right) \]

\[ + 2 \sum_{i=1}^{k-1} \sum_{p=1}^{k-i} \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} \frac{(k - 1 - (\phi^2)^{k-1})^2}{(1 - \phi)^2} \frac{1 - (\phi^2)^{k-1}}{1 - \phi^2} \]

\[ = k \left( \frac{\beta^2}{\lambda^2} \frac{1 - (\phi^2)^k}{k(1 - \phi^2)} + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} \frac{k - 1 - (\phi^2)^{k-1}}{k(1 - \phi)^2} \frac{1 - (\phi^2)^{k-1}}{k(1 - \phi^2)} \right) \]
\[
-2 \frac{\beta}{\lambda(1-\phi)} \left( k - 1 \frac{1 - \phi^{k-1}}{k} - \phi \frac{1 - \phi^{k-1}}{k(1-\phi)} + 1 \right)
\]

\[
k \left\{ \frac{\beta^2}{\lambda^2(1-\phi)(1+\phi)} \left( 1 - \frac{1 - (\phi^2)^k}{k(1+\phi)(1+\phi)} + 2 \frac{\phi}{(1+\phi)} \left( \frac{k - 1}{k} + \frac{(\phi^{k-1} - 1)(\phi - \phi^{k-1})}{k(1-\phi)} - \frac{1 - (\phi^2)^{k-1}}{k(1-\phi)(1+\phi)} \right) \right) - 2 \frac{\beta}{\lambda(1-\phi)} \left( \frac{k - 1}{k} - \phi \frac{1 - \phi^{k-1}}{k(1-\phi)} + 1 \right) \right\}
\]

\[
= k \left\{ \left( \frac{\beta}{\lambda(1-\phi)} \right)^2 \left( \frac{1 - \phi}{1+\phi} - \frac{1 - (\phi^2)^k}{k(1+\phi)(1+\phi)} + 2 \frac{\phi}{(1+\phi)} \left( \frac{k - 1}{k} + \frac{(\phi^{k-1} - 1)(\phi - \phi^{k-1})}{k(1-\phi)} - \frac{1 - (\phi^2)^{k-1}}{k(1-\phi)(1+\phi)} \right) \right) - 2 \frac{\beta}{\lambda(1-\phi)} \left( \frac{k - 1}{k} - \phi \frac{1 - \phi^{k-1}}{k(1-\phi)} + 1 \right) \right\}
\]

If we define \(a(k; \beta, \phi, \lambda) \equiv 1 - \left( \frac{\beta}{\lambda(1-\phi)} \right) \left( \frac{k - 1}{k} - \phi \frac{1 - \phi^{k-1}}{k(1-\phi)} \right)\) then the cross country return covariance could be written as

\[
\frac{1}{k} C_t[r_{it+k}^{(k)}, r_{jt+k}^{(k)}] = \sigma_{CF,CF}^{xc} + \left[ a(k; \beta, \phi, \lambda)^2 + b(k; \beta, \phi, \lambda) \right] \sigma_{DR,DR}^{xc} - 2a(k; \beta, \phi, \lambda)\sigma_{CF,DR}^{xc}
\]

(C.20)

\[
b(k; \beta, \phi, \lambda) \equiv \left( \frac{\beta}{\lambda(1-\phi)} \right)^2 \left( \frac{1 - \phi}{1+\phi} - \frac{1 - (\phi^2)^k}{k(1+\phi)(1+\phi)} + 2 \frac{\phi}{(1+\phi)} \left( \frac{k - 1}{k} + \frac{(\phi^{k-1} - 1)(\phi - \phi^{k-1})}{k(1-\phi)} - \frac{1 - (\phi^2)^{k-1}}{k(1-\phi)(1+\phi)} \right) \right)
\]

(C.21)

we could show that \(\lim_{k \to +\infty} b(k; \beta, \phi, \lambda) = 0\).

Finally we have the asymptotic result

\[
\lim_{k \to +\infty} \frac{C_t[r_{it+k}^{(k)}, r_{jt+k}^{(k)}]}{k} = \sigma_{CF,CF}^{xc} + 2 \left( \frac{\beta}{\lambda(1-\phi)} - 1 \right) \sigma_{CF,DR}^{xc}
\]

\[
+ \left( \frac{\beta^2}{\lambda^2(1-\phi^2)} + \frac{2\beta^2 \phi}{\lambda^2(1-\phi^2)(1-\phi)} - \frac{2\beta}{\lambda(1-\phi^2)} + 1 \right) \sigma_{CF,DR}^{xc}
\]

(C.22)

Now we derive the range of the coefficients for variance-covariance terms in Eq (12),
note that $\lambda = \frac{\rho \beta}{1 - \rho \phi}$

\[
\frac{\beta}{\lambda(1 - \phi)} - 1 = \frac{1 - \rho \phi}{\rho} \frac{1}{(1 - \phi)} - 1 > \frac{1}{\rho} - 1 > 0
\]

and

\[
\frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} - \frac{2\beta}{\lambda(1 - \phi)} + 1
\]

\[
= \left( \frac{\beta}{\lambda(1 - \phi)} \right)^2 - \frac{2\beta}{\lambda(1 - \phi)} + 1
\]

\[
= \left( \frac{\beta}{\lambda(1 - \phi)} - 1 \right)^2
\]

\[
= \left( \frac{1 - \rho \phi}{\rho - \rho \phi} - 1 \right)^2
\]

we know that $\rho$ and $\phi$ are close to but smaller than 1, and if we assume that $\rho > \frac{1}{1 - \phi}$, we have $\left( \frac{1 - \rho \phi}{\rho - \rho \phi} - 1 \right)^2 < 1$. Thus we could have

\[
0 < \frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} - \frac{2\beta}{\lambda(1 - \phi)} + 1 < 1
\]

under the assumption.

**Numerical Calibration:**

We try to use the formula to explain the positive gap between the portfolio variance of the benchmark case and the case in which integration is purely driven by increased DR news correlation. In our benchmark case, we set $\sigma_{CF,CF}^{xc} = \sigma_{CF,DR}^{xc} = \sigma_{DR,DR}^{xc} = 0$, therefore

\[
\lim_{k \to +\infty} \sqrt{V_i[r_{p,t+k}^{(k)}]/k} = \lim_{k \to +\infty} \sqrt{\frac{1}{N}V_i[r_{i,t+k}^{(k)}]/k}
\]  \hspace{1cm} \text{(C.23)}

And for the integrated case purely driven by increased DR news correlation, we have

\[
= \lim_{k \to +\infty} \sqrt{\frac{1}{N}V_i[r_{i,t+k}^{(k)}]/k} + (1 - \frac{1}{N}) \left( \frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} - \frac{2\beta}{\lambda(1 - \phi)} + 1 \right)\sigma_{DR,DR}^{xc}
\]  \hspace{1cm} \text{(C.24)}
and we have

\[
\frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2\phi}{\lambda^2(1 - \phi^2)(1 - \phi)} - \frac{2\beta}{\lambda(1 - \phi)} + 1 = 0.0175 \tag{C.25}
\]

therefore explains the positive gap between the two variance plot in our 2 country symmetrical experiment.

The coefficient of the term \(\sigma_{DR,DR}^{xc}\) in Eq (11) standardized by \(k\)

\[
\frac{1}{k} \left( \frac{\beta^2 (k^{1-(\phi^2)^k})}{\lambda^2 1 - \phi^2} + 2 \frac{\beta^2\phi}{\lambda^2(1 - \phi^2)(1 - \phi)} (k - 1 + \frac{(\phi^{k-1} - 1)(\phi - \phi^{k-1})}{1 - \phi} - \frac{1 - (\phi^2)^{k-1}}{1 - \phi^2}) - 2 \frac{\beta}{\lambda(1 - \phi)} (k - 1 - \phi^{-1}) + k \right)
\]

is a function of investment horizon \(k\), and the coefficient annualized by \(k\) should converge to the value in Eq (15). The coefficient as a function of \(k\) is plotted in Figure 3.2.

In the next step, we calibrate the variance under the two cases (integration purely driven by increased cross country CF-CF/ DR-DR correlation). Under the limit case where \(k \to +\infty\) we have

\[
\left( \frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2\phi}{\lambda^2(1 - \phi^2)^2} - \frac{2\beta}{\lambda(1 - \phi)} + 1 \right)\sigma_{DR,DR}^{xc} = 0.000010
\]

where \(\sigma_{DR,DR}^{xc} = p_{DR,DR}^{xc}\sigma_{DR}\sigma_{DR}\) and cross country DR correlation \(p_{DR,DR}^{xc} = 0.25\). Similarly we get

\[
\sigma_{CF,CF}^{xc} = p_{CF,CF}^{xc}\sigma_{CF}\sigma_{CF} = 0.0012
\]

where \(p_{CF,CF}^{xc} = 0.335\). In the calibration, we see that when integration purely driven by increased cross country CF-CF correlation, the impact on portfolio variance is permanent. When the integration is purely driven by increased cross country DR-DR correlation, the impact on portfolio variance is temporary, and dies out at long horizons. This matches with our intuition perfectly, and we see from the calibration that \(\left( \frac{\beta^2}{\lambda^2(1 - \phi^2)} + \frac{2\beta^2\phi}{\lambda^2(1 - \phi^2)^2} - \frac{2\beta}{\lambda(1 - \phi)} + 1 \right)\sigma_{DR,DR}^{xc} \ll \sigma_{CF,CF}^{xc}\).

**Lemma:** Assuming

179
(1) $0.5 < \rho < 1$ and $0.5 < \phi < 1$ (trivially satisfied for time preference factor $\rho$ and persistence of state variable $\phi$).

(2) $\rho > \frac{2\phi^2 + 3\phi + 1}{\phi^2 + 3\phi + 2}$

We can conclude that the coefficient $\frac{1}{k} \left[ a(k; \beta, \phi, \lambda)^2 + b(k; \beta, \phi, \lambda) \right]$ is positive and decreasing in $k$ (these are sufficient but not necessary conditions). The impact of covariance term $\sigma_{DR,DR}^c$ on per-period portfolio variance decreases as investment horizon $k$ increases.

**Proof:**

$$ f(k) = \frac{1}{k} \left[ a(k; \beta, \phi, \lambda)^2 + b(k; \beta, \phi, \lambda) \right] $$

$$ = \frac{1}{k} \left( \frac{\beta^2}{\lambda^2} \left( k - \frac{1 - (\phi^2)^k}{1 - \phi^2} \right) + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} \left( k - 1 + \frac{(\phi^k - 1)(\phi - \phi^{-1})}{1 - \phi} \right) - \frac{2}{\phi^2} \frac{\beta}{\lambda(1 - \phi)} \left( k - 1 - \frac{1 - \phi^k}{1 - \phi} \right) + k \right) $$

$$ = \left( \frac{\beta^2}{\lambda^2} \left( 1 - \frac{1 - (\phi^2)^k}{1 - \phi^2} \right) + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} \left( 1 - 1 + \frac{(\phi^k - 1)(\phi - \phi^{-1})}{k(1 - \phi)} \right) - \frac{1 - (\phi^2)^k}{k(1 - \phi^2)} - 2 \frac{\beta}{\lambda(1 - \phi)} \left( 1 - \frac{1}{k} - \frac{\phi^k}{k - 1 - \phi^k} \right) \right) $$

$$ = \text{Const} + \frac{1}{k} \left( -\frac{\beta^2}{\lambda^2} \frac{(1 - \phi^k)(1 + \phi^k)}{(1 - \phi^2)^2} + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)} \frac{(2 + \phi - \phi^k + 1)(\phi^k - 1)}{(1 - \phi^2)^2} + 2 \frac{\beta}{\lambda(1 - \phi)} \frac{1 - \phi^k}{1 - \phi^k} \right) $$

$$ = \text{Const} + \frac{1}{k} \frac{\beta}{\lambda(1 - \phi)^2} \left( \frac{\beta}{\lambda} \frac{(2\phi^2 + \phi - 1) - 2\phi^2 - 3\phi - 1}{(1 + \phi)^2(1 - \phi)} + 2 \right) $$

where

$$ \text{Const} = \frac{\beta^2}{\lambda^2} \frac{1}{1 - \phi^2} + 2 \frac{\beta^2 \phi}{\lambda^2(1 - \phi^2)(1 - \phi)} - 2 \frac{\beta}{\lambda(1 - \phi)} + 1 $$

$$ = \frac{\beta^2(1 - \phi) + 2\beta^2 \phi - 2\beta \lambda(1 - \phi^2) + \lambda^2(1 - \phi^2)(1 - \phi)}{\lambda^2(1 - \phi^2)(1 - \phi)} $$

$$ = \frac{(\beta - \lambda(1 - \phi))^2}{\lambda^2(1 - \phi^2)^2} > 0 $$

Note that $\rho$ and $\phi$ are close to but smaller than 1, and $\frac{\beta}{\lambda} = \frac{1 - \rho \phi}{\rho}$. We want to find sufficient conditions so that $f(k)$ is decreasing in $k$. Since $f(k) = g(k)h(k)$ and $f'(k) = g'(k)h(k) + g(k)h'(k)$, $f'(k) < 0 \iff g(k)h'(k) < -g'(k)h(k)$. Since $g(k) > 0$, it will be sufficient if we could show that $g'(k) < 0$, $h'(k) < 0$ and $h(k) > 0$. 

180
We first show that $g(k) \equiv \frac{1}{k} \frac{1-\phi^k}{(1-\phi)^2}$ decrease in $k$ for $\phi \in (0,1)$. Take the first order derivative we get $g'(k) = \frac{\beta}{\lambda} \frac{1}{(1-\phi)^2} \frac{\phi^{k-k\ln(\phi)}-1}{k^2}$. To show $g'(k) < 0$, we need to show that $m(\phi) = \phi^k(1-k\ln(\phi)) - 1 < 0$ for $\phi \in (0,1)$ and $\forall k$. This could be easily proved since $m'(\phi) = -k^2\phi^k\ln(\phi) > 0$ for $\phi \in (0,1)$ and $m(1) = 0$. Thus $g(k)$ is positive and decrease in $k$. Then we want to know the property of $h(k) = \frac{1}{\lambda} \frac{\phi^{(2\phi^2+\phi-1)-2\phi^2-3\phi-1}}{(1+\phi)^2(1-\phi)} + 2$. We also notice given that $2\phi^2 + \phi - 1 > 0$ (which hold as long as $\phi > 0.5$), $h(k)$ is decreasing in $k$. Thus it would be sufficient to prove the lemma if we know $h(k) > 0$ for $\forall k$. Since $h(k)$ is decreasing in $k$, we only need $\lim_{k \to \infty} h(k) = -\frac{\beta}{\lambda} \frac{2\phi^2+3\phi+1}{(1+\phi)^2(1-\phi)} + 2 = -\frac{1-\rho}{\rho(1-\phi)} \frac{2\phi^2+3\phi+1}{(1+\phi)^2} + 2 > 0$ to hold. This is equivalent to $\rho > \frac{2\phi^2+3\phi+1}{\phi^2+3\phi+2}$. Under this condition, we know both $g(k)$ and $h(k)$ are positive and decreasing, therefore $f(k) = g(k)h(k)$ is positive and decreasing in $k$. 

181
C.3 Symmetrical Model for Asset Returns

We introduce a two-state-variable symmetrical model for stocks, which includes excess stock return and dividend price ratio as state variables. In particular, the dynamics of the variables are given by:

\[ x_{r,t+1} = \mu_1 + \beta(d_t - p_t) + u_{x_{r},t+1} \]  
\[ d_{t+1} - p_{t+1} = \mu_2 + \phi(d_t - p_t) + u_{d_{p,t+1}} \]  

We denote \( u_t = [u_{x_{r,t}}, u_{d_{p,t}}]' \) and assume the VAR shocks are covariance stationary \( E(u_t) = 0 \), \( E(u_t u_s) = \begin{cases} \Sigma^{wc} & (t = s) \\ 0 & (t \neq s) \end{cases} \). The superscript \( wc \) stands for within-country, and we use \( xc \) to represent cross-country in later part of the paper.

Connect VAR shocks to structural shocks

We decompose stock excess returns into two structural shocks: cash flow news and discount rate news. In the symmetrical model (VAR) with two state variables, there’s actually a one-to-one mapping from the structural shocks to VAR shocks. Recall from the decomposition

\[ N_{RR,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_s^j r_{f,t+1+j} \right] = (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_s^j \left( y_{1,t+j} - \pi_{t+1+j} \right) \right] = 0 \]

This is because the short nominal rate and inflation are assumed to be zero in our symmetrical model.

\[ N_{RP,t+1} \equiv (E_{t+1} - E_t) \left[ \sum_{j=0}^{\infty} \rho_s^j x_{r,t+1+j} \right] = \frac{\rho_s \beta}{1 - \rho_s \phi} u_{d_{p,t+1}} \]

Therefore we have the discount rate news

\[ N_{DR,t+1} = N_{RR,t+1} + N_{RP,t+1} = \frac{\rho_s \beta}{1 - \rho_s \phi} u_{d_{p,t+1}} \]
and the cash flow news is calculated from the identity

\[ N_{CF,t+1} = (E_{t+1} - E_t) [x_{r,t+1}] + N_{DR,t+1} = u_{xr,t+1} + \frac{\rho_s \beta}{1 - \rho_s \phi} u_{dp,t+1} \]

To summarize, we have

\[
\begin{bmatrix}
N_{CF,t+1} \\
N_{DR,t+1}
\end{bmatrix} = 
\begin{bmatrix}
1 & \frac{\rho_s \beta}{1 - \rho_s \phi} \\
0 & \frac{\rho_s \beta}{1 - \rho_s \phi}
\end{bmatrix} 
\begin{bmatrix}
u_{xr,t+1} \\
u_{dp,t+1}
\end{bmatrix}
\]

which connects the VAR shocks to structural shocks. Or in matrix notation \( \epsilon_{t+1} = Pu_{t+1} \), where \( \epsilon_{t+1} \) is the structural shock, \( u_{t+1} \) the VAR shocks and \( P \) the transformation matrix.

**From single country to a world with N identical countries**

To further explore the benefit of international diversification, we design an experiment in a world with \( N \) clones (\( N \)-replica world composed of \( N \) identical countries, and we use the US data to get empirical results). To explain the experiment in detail, we first introduce some notations. Let \( \Sigma^{wc} \equiv Var(u_{t+1}) \) be the within country VAR covariance matrix, and \( \Sigma^{xc} \equiv Cov(u_{i,t+1}, u_{j,t+1}) \) \((i \neq j)\) is defined as the cross-country VAR covariance matrix (between country \( i \) and \( j \)). Since all covariance matrix \( \Sigma \) could be decomposed into volatility component \( G \equiv diag(\Sigma)^{1/2} \) and correlation component \( \Gamma \equiv diag(\Sigma)^{-1/2} \Sigma diag(\Sigma)^{-1/2} \), we have the following decomposition for within-country and cross-country VAR covariance matrix

\[ \Sigma^{wc} \equiv G \Sigma^{wc} G' \]  
(C.30)

\[ \Sigma^{xc} \equiv G \Sigma^{xc} G' \]  
(C.31)

By using this notation we have implicitly assumed all countries are identical, i.e. \( \Sigma^{wc}_i = \Sigma^{wc}_j \) and \( \Sigma^{xc}_{ij} = \Sigma^{xc}_{im} \ (i \neq j, l \neq m) \), which also implies \( G_{\Sigma,i} = G_{\Sigma,j}, \Gamma_{\Sigma,i}^{wc} = \Gamma_{\Sigma,j}^{wc}, \Gamma_{\Sigma,i}^{xc} = \Gamma_{\Sigma,j}^{xc} \).
Then the covariance matrix for the global VAR shock in the N-replica economy is

$$
\Sigma_{\text{glo}} = \begin{bmatrix}
\Sigma^{wc} & \Sigma^{xc} & \ldots & \Sigma^{xc} \\
\Sigma^{xc} & \Sigma^{wc} & \ldots & \Sigma^{xc} \\
\vdots & \vdots & \ldots & \vdots \\
\Sigma^{xc} & \Sigma^{xc} & \ldots & \Sigma^{wc}
\end{bmatrix}
$$

with $\Sigma^{wc}$ as diagonal blocks and $\Sigma^{xc}$ as off diagonal blocks. Later we use $\Sigma_{\text{glo}}$ international portfolio allocation analysis.

**Connect the VAR covariance matrix to structural covariance matrix in a world with N identical countries**

Let $\Omega^{wc} \equiv Var(\epsilon_{t+1})$ be the within country structural covariance matrix, and $\Omega^{xc} \equiv Cov(\epsilon_{i,t+1}, \epsilon_{j,t+1}) (i \neq j)$ is defined as the cross-country structural covariance matrix (between country $i$ and $j$). Analogous to the decomposition above, we have

$$
\Omega^{xc} \equiv G_\Omega \Gamma^{xc}_\Omega G'_\Omega \tag{C.32}
$$

$$
\Omega^{wc} \equiv G_\Omega \Gamma^{wc}_\Omega G'_\Omega \tag{C.33}
$$

From the relation $\epsilon_{t+1} = Pu_{t+1}$, we can take cross-country covariance $Cov(\epsilon_{i,t+1}, \epsilon_{j,t+1}) = PCov(u_{i,t+1}, u_{j,t+1})P'$ and get an identity $\Omega^{xc} = P\Sigma^{xc}P'$. Of course, $\Omega^{wc} = P\Sigma^{wc}P'$ also holds.

The identity could be rewritten as

$$
G_\Omega \Gamma^{xc}_\Omega G'_\Omega = PG_\Sigma \Gamma^{xc}_\Sigma G'_\Sigma P' \tag{C.34}
$$

Applying the vec operator to both sides and using the trick that $\text{vec}(ABC) = (C' \otimes A) \cdot \text{vec}(B)$ (see Hamilton 1994 Proposition 10.4) we have

$$
(G_\Omega \otimes G_\Omega) \cdot \text{vec}(\Gamma^{xc}_\Omega) = ((PG_\Sigma \otimes PG_\Sigma)) \cdot \text{vec}(\Gamma^{xc}_\Sigma) \tag{C.35}
$$

Now we’ve got a mapping from cross-country structural shock correlation matrix to cross-
country VAR shock correlation matrix. If \((PG_\Sigma \otimes DG_\Sigma)\) is nonsingular, we could rewrite the relationship as

\[
vec (\Gamma^\Sigma_{xc}) = ((PG_\Sigma \otimes PG_\Sigma)^{-1} (G_\Omega \otimes G_\Omega)) \cdot vec (\Gamma^\Sigma_{xc}) 
\] (C.36)

And similarly, we have

\[
(G_\Omega \otimes G_\Omega) \cdot vec (\Gamma^\Sigma_{wxc}) = ((PG_\Sigma \otimes PG_\Sigma)) \cdot vec (\Gamma^\Sigma_{wxc}) 
\] (C.37)

We could also analogously define the covariance matrix for the global structural shock

\[
\Omega_{glo} = \begin{bmatrix}
\Omega^{wxc} & \Omega^{xc} & \cdots & \Omega^{xc} \\
\Omega^{xc} & \Omega^{wxc} & \cdots & \Omega^{xc} \\
\vdots & \vdots & \ddots & \vdots \\
\Omega^{xc} & \Omega^{xc} & \cdots & \Omega^{wxc}
\end{bmatrix}
\]

And equations (33) and (34) give us the connection between \(\Omega_{glo}\) and \(\Sigma_{glo}\).

**Illustrative example using the symmetrical model**

From the analysis above, we know there’s a connection between the global structural shocks and global VAR shocks. And we could design some experiments using this connection to study the effect of international integration on portfolio allocation. Empirically, we follow the steps below:

1. Estimate a single country symmetrical model using the US historical data. From this we could get a estimate for the covariance matrix \(\Sigma^{wxc}\) (or equivalently \(G_\Sigma\) and \(\Gamma^{wxc}_{\Sigma}\)). \(P\) matrix could also be calculated from the reduced form VAR coefficients.

2. Using the identity \(\Omega^{wxc} = P\Sigma^{wxc}P'\), we have an estimate of \(\Omega^{wxc}\) (or equivalently \(G_\Omega\) and \(\Gamma^{wxc}_{\Omega}\)).

3. Manually set values for the cross-country structural shock correlation matrix \(\Gamma^{xc}_{\Omega}\). From equation (?) we will be able to get the implied cross-country VAR shock correlation matrix \(\Gamma^{xc}_{\Sigma}\).
4. Construct the implied global VAR covariance matrix $\Sigma_{glo}$, based on our input $\Gamma_{\Omega}^{xc}$ in step 3. Given $\Sigma_{glo}$, we could study the implications of international integration on global portfolio allocation.

Specifically, we assign 3 set of values to $\Gamma_{\Omega}^{xc}$ in step 3 above, each corresponds a scenario below:

**1st Scenario:** $\Gamma_{\Omega}^{xc} = 0$

This is a benchmark case without international integration, where all cross-country structural shocks are uncorrelated.

**2nd Scenario:** $\Gamma_{\Omega}^{xc} = \begin{bmatrix} \Gamma_{\Omega,11}^{xc} & 0 \\ 0 & 0 \end{bmatrix}$

where $\Gamma_{\Omega,11}^{xc}$ denote the cross-country CF news correlation.

This is a case with international integration, and the integration is purely driven by increased CF news correlation:

**3rd Scenario:** $\Gamma_{\Omega}^{xc} = \begin{bmatrix} 0 & 0 \\ 0 & \Gamma_{\Omega,22}^{xc} \end{bmatrix}$

where $\Gamma_{\Omega,22}^{xc}$ denote the cross-country DR news correlation.

This is a case with international integration, and the integration is purely driven by increased DR news correlation.

**Implied Correlation Structure of VAR in Section 3.3**

<table>
<thead>
<tr>
<th>Corr</th>
<th>First Scenario</th>
<th>Second Scenario</th>
<th>Third Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_{xr,s}$</td>
<td>$u_{xr,s}$</td>
<td>$u_{xr,s}$</td>
</tr>
<tr>
<td>$u_{xr,s}$</td>
<td>0</td>
<td>0.070</td>
<td>0.070</td>
</tr>
<tr>
<td>$u_{dp}$</td>
<td>0</td>
<td>0</td>
<td>-0.087</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corr</th>
<th>First Scenario</th>
<th>Second Scenario</th>
<th>Third Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_{dp}$</td>
<td>$u_{dp}$</td>
<td>$u_{dp}$</td>
</tr>
<tr>
<td>$u_{xr,s}$</td>
<td>0</td>
<td>0</td>
<td>-0.087</td>
</tr>
<tr>
<td>$u_{dp}$</td>
<td>0</td>
<td>0</td>
<td>0.109</td>
</tr>
</tbody>
</table>
From 2 state variables (symmetrical model) to 6 state variables (general model)

It’s very easy to incorporate the symmetrical model in a more general framework. Recall that our general model for a single country is a VAR with 6 state variables

\[ \tilde{z}_{t+1} = a + A\tilde{z}_t + u_{t+1} \]

where \( \tilde{z}_{t+1} = [x_{r,t+1}, x_{n,t+1}, d_{t+1} - p_{t+1}, \pi_{t+1}, y_{1,t+1}^N, y_{10,t+1}^N - y_{1,t+1}^N] \). Our symmetrical model is a special case of the general model with

\[
a = \begin{bmatrix}
\mu_1 \\
0 \\
\mu_2 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
0 & 0 & \beta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \phi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

and

\[
u_{t+1} = \begin{bmatrix}
u_{x_{r,t+1}} \\
0 \\
u_{d_{p,t+1}} \\
0 \\
0 \\
0
\end{bmatrix}
\]
C.4 Data Description

We consider a number of time series from 7 major OECD countries, which accounts for 62% of total world market shares by end of 2014. The full sample period is 1986:01 to 2016:12, yielding 372 monthly observations. We split the full sample to two sub-periods, with the sub-period 1 from 1986:01 to 1999:12 and the sub-period 2 from 2000:01 to 2016:12. Returns are in U.S. dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate.

Currency-hedged Return

Before further explaining our data in details, we first introduce the concept of currency hedged excess return. Consider a home investor from US buying assets in a foreign country (for example in Japan), we are interested in his excess returns from this investment denominated in home currency. We use a superscript * to denote a foreign variable. \( S_t \) denotes the spot foreign exchange rate, and an increase in \( S_t \) means home currency is weakening relative to foreign currency. To conduct this trade, the investor at time \( t \) has to exchange 1 US dollar into \( \frac{1}{S_t} \) Japanese yen and invest in Japanese capital market, then converts the money back to USD at time \( t + 1 \) when the investment is liquidated. Thus the (unhedged) 1-period return in Japanese market (measured in dollars) is

\[
1 + R_{JPN,t+1}^h \equiv (1 + R_{JPN,t+1}^*) \frac{S_{t+1}}{S_t}
\]

where \( R_{JPN,t+1}^* \) is return in Japanese asset denominated in Japanese yen (local return).

However, due to the uncertainty in future exchange rate \( S_{t+1} \), the investor will want to lock down the future exchange rate using a currency forward at forward rate \( F_t \). So the currency hedged return of a US investor investing in Japan is defined as

\[
1 + R_{JPN,t+1}^h \equiv (1 + R_{JPN,t+1}^*) \frac{F_t}{S_t}
\]

Recall from the covered interest rate parity (CIP), we also have

\[
1 + i_{US,t+1} = (1 + i_{JPN,t+1}^*) \frac{F_t}{S_t}
\]
where \( i_{US,t+1} \) is the nominal interest rate for the US, while \( i_{JPN,t+1} \) is the nominal interest rate for Japan. The intuition for this equation is that the investor should not have arbitrage opportunities, or alternatively, should be indifferent to invest locally or abroad if the currency risk of investing in foreign country is hedged. This equation holds pretty well unless there’s counter-party risk or barriers to financial integration (transaction costs, taxes, capital controls, et cetera).

Combining the two equations above, we know that the excess currency hedged return of a US investor investing in Japan is

\[
\frac{1 + R^h_{JPN,t+1}}{1 + i_{US,t+1}} = \frac{1 + R^*_{JPN,t+1}}{1 + i^*_{JPN,t+1}}
\]

or in log terms

\[
r^h_{JPN,t+1} - r_{f,US,t+1} = r^*_{JPN,t+1} - r^*_{JPN,t+1}
\]

where \( r_{f,US,t+1} = \ln(1 + i_{US,t+1}) \) and \( r^*_{JPN,t+1} = \ln(1 + i^*_{JPN,t+1}) \) are the risk free rates in US and Japan. Thus, we have shown that the excess currency-hedged return of US investors investing in Japan is the same as the excess return of Japanese investors investing in home country (local excess return).

**Main Variables**

Now we introduce our main variables briefly.

**Returns, Dividend Yield and Inflation**

The international portfolio we consider are constructed from country level index in equity and bonds. The country level stock returns are measured as dollar returns on MSCI net total return indices, which reinvest dividends after the deduction of withholding taxes. We use Merill Lynch total return indices (7yr-10yr) to get bond returns. The dividend yield is measured as the log of MSCI dividend yield (MSDY), which is calculated using the trailing 12-month cash earnings per share figure. All the data on stock and bond returns as well as dividend yields are from Datastream. Table reports sample correlations of monthly
bond and stock returns for the period January 1986 to December 2016 as well as the two 
sub-samples. Returns are in U.S. Dollar currency-hedged terms in excess of the three-month 
U.S. Treasury bill rate.

For the inflation, we get data from both Datastream and Global Financial Data (GFD). We 
first get annualized inflation rates from Datastream. But for France and UK, the data 
does not go back far enough because data comes from newer HICP that started in 1990’s; 
thus, we compute inflation manually using CPI for France and RPI for UK from GSD.

**Foreign Exchange Rates**

We get spot currency levels and one-month forward currency levels from Datastream. The 
currency levels are all in terms of 1 US dollar except for British Pound (GBP), so we invert 
GBP to get correct reference frame. The (unhedged) currency returns are calculated as 
\[ \ln \left( \frac{S_{t+1}}{S_t} \right) \] for spot currency levels for 1 USD, and the currency-hedged returns are calculated 
as \[ \ln \left( \frac{F_t}{S_t} \right) \] for forward and spot currency levels for 1 USD. Note that French and German data 
switch to Euros at the beginning of 1999.

**Short Term and Long Term Nominal Interest Rate**

We use 1 month T-bill rate for US short term nominal interest rate, and for other countries 
we use different rates on short term financial instruments including 1 month Euribor rates, 
bank loan rates or overnight money market interest rates. The data are from GFD and 
central bank websites. Long term nominal interest rate are represented using 10 year yields. 
The US series is from CRSP Fixed Term Indices and other countries from GFD.

**Data Source**

The dataset has country-level time series mainly collected from Datastream and Global 
Financial Data (GFD). For the concrete tickers, refer to our online appendix: http://www. 
## Table C.1: Correlation Summary Statistics

This table reports sample correlations of monthly bond and stock returns for the whole sample (January 1986 to December 2016), early sample (January 1986 to December 1999) and late sample (January 2000 to December 2016). Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate.

<table>
<thead>
<tr>
<th></th>
<th>Bonds</th>
<th>Stocks</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
<td>AUS</td>
<td>CAN</td>
<td>FRA</td>
<td>GER</td>
<td>JPN</td>
<td>UKI</td>
<td>USA</td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.55</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>0.46</td>
<td>0.52</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>0.49</td>
<td>0.58</td>
<td>0.86</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>0.22</td>
<td>0.33</td>
<td>0.30</td>
<td>0.39</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>0.53</td>
<td>0.44</td>
<td>0.57</td>
<td>0.59</td>
<td>0.27</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.55</td>
<td>0.71</td>
<td>0.60</td>
<td>0.64</td>
<td>0.31</td>
<td>0.39</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>0.21</td>
<td>-0.04</td>
<td>-0.06</td>
<td>-0.11</td>
<td>-0.11</td>
<td>0.13</td>
<td>-0.16</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.07</td>
<td>0.10</td>
<td>-0.07</td>
<td>-0.11</td>
<td>-0.04</td>
<td>0.03</td>
<td>-0.09</td>
<td>0.63</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>-0.03</td>
<td>-0.02</td>
<td>0.09</td>
<td>-0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>-0.14</td>
<td>0.57</td>
<td>0.63</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>-0.03</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.10</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.19</td>
<td>0.56</td>
<td>0.60</td>
<td>0.84</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>-0.10</td>
<td>0.00</td>
<td>-0.03</td>
<td>-0.08</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.16</td>
<td>0.44</td>
<td>0.46</td>
<td>0.51</td>
<td>0.46</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>0.12</td>
<td>0.07</td>
<td>0.03</td>
<td>-0.03</td>
<td>0.01</td>
<td>0.15</td>
<td>-0.06</td>
<td>0.66</td>
<td>0.68</td>
<td>0.73</td>
<td>0.68</td>
<td>0.45</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.04</td>
<td>0.08</td>
<td>-0.02</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.03</td>
<td>-0.05</td>
<td>0.63</td>
<td>0.78</td>
<td>0.71</td>
<td>0.69</td>
<td>0.49</td>
<td>0.79</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>
### Table C.1 Correlation Summary Statistics (Continued)

#### Correlations (Jan. 1986 - Dec. 1999)

<table>
<thead>
<tr>
<th></th>
<th>Bonds</th>
<th>Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AUS CAN FRA GER JPN UKI USA</td>
<td>AUS CAN FRA GER JPN UKI USA</td>
</tr>
<tr>
<td>Bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.44 1.00</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>0.31 0.39 1.00</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>0.31 0.46 0.78 1.00</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>0.18 0.34 0.30 0.43 1.00</td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>0.44 0.29 0.45 0.46 0.24 1.00</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.40 0.64 0.48 0.51 0.31 0.17 1.00</td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>0.44 0.01 0.01 -0.01 -0.12 0.29 -0.10</td>
<td>1.00</td>
</tr>
<tr>
<td>CAN</td>
<td>0.39 0.30 0.08 0.06 0.04 0.21 0.08</td>
<td>0.64 1.00</td>
</tr>
<tr>
<td>FRA</td>
<td>0.18 0.12 0.40 0.31 0.09 0.22 0.08</td>
<td>0.48 0.55 1.00</td>
</tr>
<tr>
<td>GER</td>
<td>0.25 0.13 0.24 0.23 -0.02 0.12 0.06</td>
<td>0.51 0.54 0.76 1.00</td>
</tr>
<tr>
<td>JPN</td>
<td>0.08 0.17 0.13 0.12 0.14 0.15 0.00</td>
<td>0.34 0.39 0.42 0.32 1.00</td>
</tr>
<tr>
<td>UKI</td>
<td>0.37 0.19 0.20 0.17 0.04 0.33 0.09</td>
<td>0.64 0.66 0.62 0.58 0.37 1.00</td>
</tr>
<tr>
<td>USA</td>
<td>0.35 0.34 0.19 0.12 0.05 0.22 0.24</td>
<td>0.58 0.78 0.59 0.55 0.36 0.74 1.00</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Bonds</th>
<th>Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AUS CAN FRA GER JPN UKI USA</td>
<td>AUS CAN FRA GER JPN UKI USA</td>
</tr>
<tr>
<td>Bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.73 1.00</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>0.66 0.70 1.00</td>
<td></td>
</tr>
<tr>
<td>GER</td>
<td>0.71 0.73 0.94 1.00</td>
<td></td>
</tr>
<tr>
<td>JPN</td>
<td>0.35 0.33 0.36 0.39 1.00</td>
<td></td>
</tr>
<tr>
<td>UKI</td>
<td>0.72 0.76 0.78 0.84 0.37 1.00</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.74 0.83 0.72 0.76 0.36 0.76 1.00</td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>-0.21 -0.12 -0.17 -0.24 -0.08 -0.22 -0.24</td>
<td>1.00</td>
</tr>
<tr>
<td>CAN</td>
<td>-0.29 -0.13 -0.21 -0.26 -0.17 -0.22 -0.24</td>
<td>0.66 1.00</td>
</tr>
<tr>
<td>FRA</td>
<td>-0.31 -0.20 -0.25 -0.34 -0.13 -0.27 -0.36</td>
<td>0.71 0.72 1.00</td>
</tr>
<tr>
<td>GER</td>
<td>-0.34 -0.24 -0.29 -0.36 -0.12 -0.28 -0.38</td>
<td>0.66 0.65 0.92 1.00</td>
</tr>
<tr>
<td>JPN</td>
<td>-0.34 -0.24 -0.20 -0.28 -0.31 -0.31 -0.33</td>
<td>0.61 0.55 0.62 0.59 1.00</td>
</tr>
<tr>
<td>UKI</td>
<td>-0.22 -0.09 -0.16 -0.24 -0.06 -0.15 -0.22</td>
<td>0.71 0.72 0.86 0.79 0.56 1.00</td>
</tr>
<tr>
<td>USA</td>
<td>-0.30 -0.21 -0.23 -0.30 -0.11 -0.25 -0.30</td>
<td>0.73 0.78 0.83 0.81 0.62 0.84 1.00</td>
</tr>
</tbody>
</table>
### C.5 VAR Model Estimation

**Table C.2: Pooled VAR(1) Model Estimates**

This table reports the results for pooled VAR(1) estimates.

**Panel A**

<table>
<thead>
<tr>
<th>Model estimates</th>
<th>Coefficients on lagged variables</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>(1) log stock excess returns</td>
<td>0.081</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>(2.249)</td>
<td>(1.151)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>-0.050</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(-4.786)</td>
<td>(1.939)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.078</td>
<td>-0.141</td>
</tr>
<tr>
<td></td>
<td>(-2.057)</td>
<td>(-1.390)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.004</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(2.580)</td>
<td>(-1.674)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(1.282)</td>
<td>(-4.188)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(1.952)</td>
<td>(1.070)</td>
</tr>
</tbody>
</table>
Panel B

**Within-country Residual Correlation Matrix (1986.01-2016.12)**

Averaged over 7 countries

Average annualized volatility*100 in diagonal

<table>
<thead>
<tr>
<th>(1) log stock excess returns</th>
<th>(2) log bond excess returns</th>
<th>(3) log dividend yield</th>
<th>(4) log inflation</th>
<th>(5) log short rate</th>
<th>(6) log yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.702</td>
<td>0.062</td>
<td>-0.897</td>
<td>0.024</td>
<td>-0.018</td>
<td>-0.031</td>
</tr>
<tr>
<td>0.062</td>
<td>5.829</td>
<td>-0.055</td>
<td>-0.076</td>
<td>-0.183</td>
<td>-0.461</td>
</tr>
<tr>
<td>-0.897</td>
<td>-0.055</td>
<td>19.684</td>
<td>0.025</td>
<td>0.033</td>
<td>0.023</td>
</tr>
<tr>
<td>0.024</td>
<td>-0.076</td>
<td>0.025</td>
<td>1.115</td>
<td>0.055</td>
<td>0.013</td>
</tr>
<tr>
<td>-0.018</td>
<td>-0.183</td>
<td>0.033</td>
<td>0.055</td>
<td>0.102</td>
<td>-0.711</td>
</tr>
<tr>
<td>-0.031</td>
<td>-0.461</td>
<td>0.023</td>
<td>0.013</td>
<td>-0.711</td>
<td>0.119</td>
</tr>
</tbody>
</table>

**Cross-country Residual Correlation Matrix (1986.01-2016.12)**

Averaged over 7 countries

Diagonal terms are average cross-country correlation of the same state variable

<table>
<thead>
<tr>
<th>(1) log stock excess returns</th>
<th>(2) log bond excess returns</th>
<th>(3) log dividend yield</th>
<th>(4) log inflation</th>
<th>(5) log short rate</th>
<th>(6) log yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.610</td>
<td>-0.050</td>
<td>-0.571</td>
<td>0.006</td>
<td>0.003</td>
<td>0.030</td>
</tr>
<tr>
<td>0.000</td>
<td>0.458</td>
<td>0.002</td>
<td>-0.072</td>
<td>-0.051</td>
<td>-0.288</td>
</tr>
<tr>
<td>-0.546</td>
<td>0.044</td>
<td>0.531</td>
<td>0.017</td>
<td>0.010</td>
<td>-0.039</td>
</tr>
<tr>
<td>0.013</td>
<td>-0.036</td>
<td>0.014</td>
<td>0.186</td>
<td>0.032</td>
<td>0.001</td>
</tr>
<tr>
<td>0.007</td>
<td>-0.045</td>
<td>0.009</td>
<td>0.049</td>
<td>0.128</td>
<td>-0.062</td>
</tr>
<tr>
<td>-0.010</td>
<td>-0.257</td>
<td>0.000</td>
<td>0.015</td>
<td>-0.087</td>
<td>0.259</td>
</tr>
</tbody>
</table>
### Table C.2 (Continued)

**Panel C**

#### Within-country Residual Correlation Matrix (1986.01-1999.12)

Averaged over 7 countries. Diagonal terms are annualized average volatility*100.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>19.213</td>
<td>0.293</td>
<td>-0.926</td>
<td>-0.026</td>
<td>-0.088</td>
<td>-0.109</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.293</td>
<td>6.743</td>
<td>-0.290</td>
<td>-0.071</td>
<td>-0.209</td>
<td>-0.400</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.926</td>
<td>-0.290</td>
<td>20.863</td>
<td>0.058</td>
<td>0.083</td>
<td>0.115</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>-0.026</td>
<td>-0.071</td>
<td>0.058</td>
<td>1.058</td>
<td>0.041</td>
<td>0.021</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.088</td>
<td>-0.209</td>
<td>0.083</td>
<td>0.041</td>
<td>0.136</td>
<td>-0.721</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.109</td>
<td>-0.400</td>
<td>0.115</td>
<td>0.021</td>
<td>-0.721</td>
<td>0.153</td>
</tr>
</tbody>
</table>

#### Cross-country Residual Correlation Matrix (1986.01-1999.12)

Averaged over 7 countries. Diagonal terms are average cross-country correlation of the same state variable.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.538</td>
<td>0.080</td>
<td>-0.527</td>
<td>-0.060</td>
<td>-0.047</td>
<td>-0.012</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.183</td>
<td>0.370</td>
<td>-0.177</td>
<td>-0.060</td>
<td>-0.072</td>
<td>-0.213</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.508</td>
<td>-0.084</td>
<td>0.509</td>
<td>0.069</td>
<td>0.045</td>
<td>0.015</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>-0.016</td>
<td>-0.020</td>
<td>0.027</td>
<td>0.093</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.035</td>
<td>-0.054</td>
<td>0.030</td>
<td>0.034</td>
<td>0.097</td>
<td>-0.032</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.074</td>
<td>-0.196</td>
<td>0.078</td>
<td>0.033</td>
<td>-0.050</td>
<td>0.188</td>
</tr>
</tbody>
</table>
### Panel D

**Within-country Residual Correlation Matrix (2000.01-2016.12)**

Averaged over 7 countries.

Diagonal terms are annualized average volatility*100.

<table>
<thead>
<tr>
<th></th>
<th>(1) log stock excess returns</th>
<th>(2) log bond excess returns</th>
<th>(3) log dividend yield</th>
<th>(4) log inflation</th>
<th>(5) log short rate</th>
<th>(6) log yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>16.244</td>
<td>-0.239</td>
<td>-0.871</td>
<td>0.071</td>
<td>0.129</td>
<td>0.095</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>-0.239</td>
<td>4.863</td>
<td>0.247</td>
<td>-0.086</td>
<td>-0.125</td>
<td>-0.643</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.871</td>
<td>0.247</td>
<td>18.416</td>
<td>-0.008</td>
<td>-0.080</td>
<td>-0.120</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.071</td>
<td>-0.086</td>
<td>-0.008</td>
<td>1.135</td>
<td>0.091</td>
<td>0.017</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.129</td>
<td>-0.125</td>
<td>-0.080</td>
<td>0.091</td>
<td>0.053</td>
<td>-0.625</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.095</td>
<td>-0.643</td>
<td>-0.120</td>
<td>0.017</td>
<td>-0.625</td>
<td>0.074</td>
</tr>
</tbody>
</table>

**Cross-country Residual Correlation Matrix (2000.01-2016.12)**

Averaged over 7 countries.

Diagonal terms are average cross-country correlation of the same state variable.

<table>
<thead>
<tr>
<th></th>
<th>(1) log stock excess returns</th>
<th>(2) log bond excess returns</th>
<th>(3) log dividend yield</th>
<th>(4) log inflation</th>
<th>(5) log short rate</th>
<th>(6) log yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.700</td>
<td>-0.220</td>
<td>-0.633</td>
<td>0.068</td>
<td>0.110</td>
<td>0.083</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>-0.225</td>
<td>0.605</td>
<td>0.216</td>
<td>-0.101</td>
<td>-0.008</td>
<td>-0.442</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.600</td>
<td>0.198</td>
<td>0.573</td>
<td>-0.030</td>
<td>-0.061</td>
<td>-0.104</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.046</td>
<td>-0.070</td>
<td>-0.006</td>
<td>0.249</td>
<td>0.057</td>
<td>0.014</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.115</td>
<td>-0.035</td>
<td>-0.040</td>
<td>0.107</td>
<td>0.271</td>
<td>-0.171</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.100</td>
<td>-0.439</td>
<td>-0.140</td>
<td>0.004</td>
<td>-0.206</td>
<td>0.486</td>
</tr>
</tbody>
</table>

196
Table C.3: VAR(1) Model Estimates [Australia]

Panel A. Model estimates

<table>
<thead>
<tr>
<th></th>
<th>Coefficients on lagged variables</th>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.080 0.024 -0.273 0.023 0.118 -0.774</td>
<td>0.714</td>
<td>0.018</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.081) (0.566) (-1.540) (1.049) (0.105) (-0.553)</td>
<td>(0.298)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.027 -0.041 0.110 0.009 -0.389 0.698</td>
<td>2.597</td>
<td>0.047</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.322) (-2.144) (1.889) (1.433) (-0.862) (1.643)</td>
<td>(2.586)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.164 -0.051 0.291 0.950 1.044 -0.158</td>
<td>-4.370</td>
<td>0.923</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.058) (-0.948) (1.486) (40.451) (0.774) (-0.103)</td>
<td>(-1.519)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>-0.001 0.001 -0.003 0.000 0.737 0.117</td>
<td>0.001</td>
<td>0.709</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.558) (0.751) (-0.974) (-0.598) (10.216) (2.553)</td>
<td>(0.021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.000 0.000 0.002 0.000 0.044 0.985</td>
<td>0.179</td>
<td>0.956</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.305) (0.605) (0.880) (0.474) (2.262) (44.080)</td>
<td>(3.591)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.001 0.000 -0.003 0.000 -0.043 0.004</td>
<td>0.786</td>
<td>0.702</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.797) (0.255) (-1.821) (-1.052) (-2.238) (0.180)</td>
<td>(15.170)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>17.152</td>
<td>0.210</td>
<td>-0.918</td>
<td>0.001</td>
<td>-0.041</td>
<td>-0.032</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.210</td>
<td>6.349</td>
<td>-0.177</td>
<td>-0.058</td>
<td>-0.061</td>
<td>-0.288</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.918</td>
<td>-0.177</td>
<td>18.997</td>
<td>0.004</td>
<td>0.027</td>
<td>0.040</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.001</td>
<td>-0.058</td>
<td>0.004</td>
<td>0.437</td>
<td>0.091</td>
<td>-0.068</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.041</td>
<td>-0.061</td>
<td>0.027</td>
<td>0.091</td>
<td>0.215</td>
<td>-0.933</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.032</td>
<td>-0.288</td>
<td>0.040</td>
<td>-0.068</td>
<td>-0.933</td>
<td>0.229</td>
</tr>
</tbody>
</table>
Table C.4: VAR(1) Model Estimates [Canada]

Panel A. Model estimates

<table>
<thead>
<tr>
<th>Coefficients on lagged variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.033</td>
<td>0.116</td>
<td>0.155</td>
<td>0.008</td>
<td>0.509</td>
<td>-0.983</td>
<td>1.908</td>
</tr>
<tr>
<td></td>
<td>(0.843)</td>
<td>(1.915)</td>
<td>(1.238)</td>
<td>(0.757)</td>
<td>(0.787)</td>
<td>(-0.902)</td>
<td>(0.755)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.007</td>
<td>-0.078</td>
<td>0.044</td>
<td>0.002</td>
<td>-0.065</td>
<td>0.429</td>
<td>2.555</td>
</tr>
<tr>
<td></td>
<td>(0.654)</td>
<td>(-2.810)</td>
<td>(0.652)</td>
<td>(0.741)</td>
<td>(-0.194)</td>
<td>(0.969)</td>
<td>(2.210)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.076</td>
<td>-0.128</td>
<td>-0.217</td>
<td>0.978</td>
<td>-0.518</td>
<td>-0.450</td>
<td>-5.245</td>
</tr>
<tr>
<td></td>
<td>(-1.529)</td>
<td>(-2.016)</td>
<td>(-1.648)</td>
<td>(73.081)</td>
<td>(-0.655)</td>
<td>(-0.378)</td>
<td>(-1.867)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.000</td>
<td>0.008</td>
<td>-0.008</td>
<td>0.000</td>
<td>0.109</td>
<td>0.247</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>(0.110)</td>
<td>(1.726)</td>
<td>(-0.774)</td>
<td>(-0.228)</td>
<td>(1.632)</td>
<td>(2.717)</td>
<td>(-0.675)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(-1.635)</td>
<td>(-0.172)</td>
<td>(-3.091)</td>
<td>(-1.499)</td>
<td>(-0.025)</td>
<td>(136.467)</td>
<td>(1.451)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.000</td>
<td>0.001</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.006</td>
<td>0.952</td>
</tr>
<tr>
<td></td>
<td>(1.156)</td>
<td>(2.073)</td>
<td>(2.259)</td>
<td>(0.893)</td>
<td>(-0.014)</td>
<td>(-0.849)</td>
<td>(49.350)</td>
</tr>
</tbody>
</table>

Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>15.048</td>
<td>0.119</td>
<td>-0.911</td>
<td>0.090</td>
<td>-0.016</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.119</td>
<td>5.837</td>
<td>-0.113</td>
<td>0.009</td>
<td>-0.309</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.911</td>
<td>-0.113</td>
<td>16.963</td>
<td>-0.045</td>
<td>0.035</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.090</td>
<td>0.009</td>
<td>-0.045</td>
<td>1.170</td>
<td>0.027</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.016</td>
<td>-0.309</td>
<td>0.035</td>
<td>0.027</td>
<td>0.095</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.041</td>
<td>-0.367</td>
<td>0.036</td>
<td>-0.014</td>
<td>-0.724</td>
</tr>
</tbody>
</table>
Table C.5: VAR(1) Model Estimates [France]

Panel A. Model estimates

<table>
<thead>
<tr>
<th>Coefficients on lagged variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.024</td>
<td>0.100</td>
<td>0.467</td>
<td>0.007</td>
<td>1.062</td>
<td>-0.310</td>
<td>3.542</td>
</tr>
<tr>
<td></td>
<td>(0.559)</td>
<td>(1.440)</td>
<td>(2.129)</td>
<td>(0.625)</td>
<td>(1.009)</td>
<td>(-0.170)</td>
<td>(0.753)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.027</td>
<td>-0.030</td>
<td>0.079</td>
<td>0.008</td>
<td>-0.650</td>
<td>0.568</td>
<td>2.187</td>
</tr>
<tr>
<td></td>
<td>(2.457)</td>
<td>(-2.003)</td>
<td>(1.338)</td>
<td>(2.601)</td>
<td>(-2.693)</td>
<td>(1.290)</td>
<td>(1.976)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.100</td>
<td>-0.081</td>
<td>-0.581</td>
<td>0.968</td>
<td>-0.688</td>
<td>-0.781</td>
<td>-6.171</td>
</tr>
<tr>
<td></td>
<td>(-1.893)</td>
<td>(-1.118)</td>
<td>(-2.523)</td>
<td>(66.628)</td>
<td>(-0.575)</td>
<td>(-0.414)</td>
<td>(-1.222)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.000</td>
<td>0.005</td>
<td>-0.002</td>
<td>0.000</td>
<td>-0.028</td>
<td>0.264</td>
<td>0.192</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(2.072)</td>
<td>(-0.187)</td>
<td>(-0.124)</td>
<td>(-0.502)</td>
<td>(3.149)</td>
<td>(0.973)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.003</td>
<td>0.000</td>
<td>-0.002</td>
<td>1.099</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(-3.076)</td>
<td>(-0.381)</td>
<td>(-3.523)</td>
<td>(-2.961)</td>
<td>(-0.406)</td>
<td>(144.631)</td>
<td>(1.756)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.008</td>
<td>-0.018</td>
<td>0.927</td>
</tr>
<tr>
<td></td>
<td>(1.282)</td>
<td>(1.786)</td>
<td>(1.701)</td>
<td>(0.866)</td>
<td>(1.586)</td>
<td>(-2.023)</td>
<td>(28.193)</td>
</tr>
</tbody>
</table>

Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>19.104</td>
<td>0.090</td>
<td>-0.858</td>
<td>-0.029</td>
<td>-0.011</td>
<td>-0.072</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.090</td>
<td>5.076</td>
<td>-0.022</td>
<td>-0.146</td>
<td>-0.150</td>
<td>-0.500</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.858</td>
<td>-0.022</td>
<td>21.924</td>
<td>0.127</td>
<td>-0.015</td>
<td>0.060</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>-0.029</td>
<td>-0.146</td>
<td>0.127</td>
<td>0.912</td>
<td>0.095</td>
<td>0.033</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.011</td>
<td>-0.150</td>
<td>-0.015</td>
<td>0.095</td>
<td>0.080</td>
<td>-0.747</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.072</td>
<td>-0.500</td>
<td>0.060</td>
<td>0.033</td>
<td>-0.747</td>
<td>0.097</td>
</tr>
</tbody>
</table>
Table C.6: VAR(1) Model Estimates [Germany]

Panel A. Model estimates

<table>
<thead>
<tr>
<th>Coefficients on lagged variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.085</td>
<td>0.097</td>
<td>0.020</td>
<td>0.022</td>
<td>-0.548</td>
<td>-2.237</td>
<td>3.821</td>
</tr>
<tr>
<td></td>
<td>(1.544)</td>
<td>(1.503)</td>
<td>(0.082)</td>
<td>(1.497)</td>
<td>(-0.579)</td>
<td>(-1.380)</td>
<td>(0.802)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.007</td>
<td>-0.040</td>
<td>0.055</td>
<td>0.001</td>
<td>-0.350</td>
<td>-0.185</td>
<td>1.017</td>
</tr>
<tr>
<td></td>
<td>(0.662)</td>
<td>(-3.110)</td>
<td>(1.014)</td>
<td>(0.445)</td>
<td>(-1.559)</td>
<td>(-0.484)</td>
<td>(1.077)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.170</td>
<td>-0.112</td>
<td>-0.069</td>
<td>0.951</td>
<td>0.328</td>
<td>1.066</td>
<td>-7.180</td>
</tr>
<tr>
<td></td>
<td>(-2.900)</td>
<td>(-1.675)</td>
<td>(-0.269)</td>
<td>(59.279)</td>
<td>(0.321)</td>
<td>(0.647)</td>
<td>(-1.494)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.003</td>
<td>0.005</td>
<td>-0.010</td>
<td>0.000</td>
<td>-0.125</td>
<td>0.236</td>
<td>-0.416</td>
</tr>
<tr>
<td></td>
<td>(1.202)</td>
<td>(1.912)</td>
<td>(-0.968)</td>
<td>(0.620)</td>
<td>(-2.217)</td>
<td>(2.026)</td>
<td>(-1.874)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.003</td>
<td>0.000</td>
<td>0.002</td>
<td>1.003</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(-1.749)</td>
<td>(1.028)</td>
<td>(-4.439)</td>
<td>(-1.613)</td>
<td>(0.603)</td>
<td>(272.038)</td>
<td>(1.903)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.002</td>
<td>-0.004</td>
<td>0.969</td>
</tr>
<tr>
<td></td>
<td>(1.423)</td>
<td>(1.877)</td>
<td>(2.609)</td>
<td>(1.277)</td>
<td>(0.735)</td>
<td>(-0.657)</td>
<td>(58.023)</td>
</tr>
</tbody>
</table>

Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>21.496</td>
<td>-0.095</td>
<td>-0.875</td>
<td>0.085</td>
<td>0.083</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>-0.095</td>
<td>4.789</td>
<td>0.081</td>
<td>-0.127</td>
<td>-0.310</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.875</td>
<td>0.081</td>
<td>23.371</td>
<td>-0.046</td>
<td>-0.042</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.085</td>
<td>-0.127</td>
<td>-0.046</td>
<td>1.140</td>
<td>0.033</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.083</td>
<td>-0.310</td>
<td>-0.042</td>
<td>0.033</td>
<td>0.054</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>-0.034</td>
<td>-0.529</td>
<td>0.017</td>
<td>0.096</td>
<td>-0.581</td>
</tr>
</tbody>
</table>
Table C.7: VAR(1) Model Estimates [Japan]

Panel A. Model estimates

<table>
<thead>
<tr>
<th>Coefficients on lagged variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.048</td>
<td>0.119</td>
<td>0.211</td>
<td>0.011</td>
<td>-0.535</td>
<td>-0.291</td>
<td>4.355</td>
</tr>
<tr>
<td></td>
<td>(1.128)</td>
<td>(1.795)</td>
<td>(0.885)</td>
<td>(1.077)</td>
<td>(-0.815)</td>
<td>(-0.095)</td>
<td>(0.646)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.035</td>
<td>-0.034</td>
<td>0.136</td>
<td>0.009</td>
<td>-0.018</td>
<td>0.888</td>
<td>7.955</td>
</tr>
<tr>
<td></td>
<td>(3.297)</td>
<td>(-2.619)</td>
<td>(2.095)</td>
<td>(3.311)</td>
<td>(-0.112)</td>
<td>(1.172)</td>
<td>(4.093)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.096</td>
<td>-0.128</td>
<td>-0.225</td>
<td>0.975</td>
<td>0.631</td>
<td>-2.666</td>
<td>-13.637</td>
</tr>
<tr>
<td></td>
<td>(-1.609)</td>
<td>(-1.592)</td>
<td>(-0.783)</td>
<td>(65.865)</td>
<td>(0.846)</td>
<td>(-0.722)</td>
<td>(-1.537)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.181</td>
<td>0.374</td>
<td>-0.138</td>
</tr>
<tr>
<td></td>
<td>(-0.083)</td>
<td>(0.383)</td>
<td>(0.034)</td>
<td>(-0.098)</td>
<td>(4.731)</td>
<td>(1.692)</td>
<td>(-0.279)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>0.002</td>
<td>0.984</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(-2.430)</td>
<td>(-0.166)</td>
<td>(-2.350)</td>
<td>(-2.439)</td>
<td>(1.027)</td>
<td>(121.339)</td>
<td>(-0.253)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.004</td>
<td>0.924</td>
</tr>
<tr>
<td></td>
<td>(-2.229)</td>
<td>(2.376)</td>
<td>(-0.921)</td>
<td>(-2.424)</td>
<td>(-1.194)</td>
<td>(0.421)</td>
<td>(41.469)</td>
</tr>
</tbody>
</table>

Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>19.861</td>
<td>0.002</td>
<td>-0.860</td>
<td>0.041</td>
<td>-0.043</td>
<td>0.001</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.002</td>
<td>4.927</td>
<td>-0.015</td>
<td>0.016</td>
<td>-0.187</td>
<td>-0.742</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.860</td>
<td>-0.015</td>
<td>22.954</td>
<td>0.004</td>
<td>0.066</td>
<td>0.005</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.041</td>
<td>0.016</td>
<td>0.004</td>
<td>1.475</td>
<td>0.032</td>
<td>-0.016</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.043</td>
<td>-0.187</td>
<td>0.066</td>
<td>0.032</td>
<td>0.038</td>
<td>-0.407</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.001</td>
<td>-0.742</td>
<td>0.005</td>
<td>-0.016</td>
<td>-0.407</td>
<td>0.059</td>
</tr>
</tbody>
</table>
Table C.8: VAR(1) Model Estimates [United Kingdom]

### Panel A. Model estimates

<table>
<thead>
<tr>
<th>Coefficients on lagged variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>Rsq</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns</td>
<td>0.093</td>
<td>0.029</td>
<td>0.248</td>
<td>0.026</td>
<td>0.031</td>
<td>-0.930</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>(1.833)</td>
<td>(0.518)</td>
<td>(2.419)</td>
<td>(1.917)</td>
<td>(0.048)</td>
<td>(-0.627)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>(2) log bond excess returns</td>
<td>0.023</td>
<td>-0.069</td>
<td>-0.010</td>
<td>0.006</td>
<td>0.034</td>
<td>-0.006</td>
<td>1.086</td>
</tr>
<tr>
<td></td>
<td>(1.249)</td>
<td>(-1.452)</td>
<td>(-0.128)</td>
<td>(1.273)</td>
<td>(0.119)</td>
<td>(-0.010)</td>
<td>(0.843)</td>
</tr>
<tr>
<td>(3) log dividend yield</td>
<td>-0.083</td>
<td>-0.025</td>
<td>-0.264</td>
<td>0.975</td>
<td>-0.147</td>
<td>0.287</td>
<td>-1.345</td>
</tr>
<tr>
<td></td>
<td>(-1.487)</td>
<td>(-0.413)</td>
<td>(-2.374)</td>
<td>(63.898)</td>
<td>(-0.220)</td>
<td>(0.183)</td>
<td>(-0.404)</td>
</tr>
<tr>
<td>(4) log inflation</td>
<td>0.003</td>
<td>0.004</td>
<td>-0.012</td>
<td>0.000</td>
<td>0.133</td>
<td>0.243</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(0.584)</td>
<td>(0.976)</td>
<td>(-0.891)</td>
<td>(0.336)</td>
<td>(2.046)</td>
<td>(2.197)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>(5) log short rate</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.000</td>
<td>0.005</td>
<td>1.010</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(-2.122)</td>
<td>(0.928)</td>
<td>(-3.303)</td>
<td>(-2.033)</td>
<td>(1.492)</td>
<td>(135.818)</td>
<td>(2.432)</td>
</tr>
<tr>
<td>(6) log yield spread</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>-0.006</td>
<td>-0.017</td>
<td>0.942</td>
</tr>
<tr>
<td></td>
<td>(1.949)</td>
<td>(0.069)</td>
<td>(1.735)</td>
<td>(1.683)</td>
<td>(-1.190)</td>
<td>(-2.033)</td>
<td>(48.304)</td>
</tr>
</tbody>
</table>

### Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th>(1) log stock excess returns</th>
<th>(2) log bond excess returns</th>
<th>(3) log dividend yield</th>
<th>(4) log inflation</th>
<th>(5) log short rate</th>
<th>(6) log yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.385</td>
<td>0.153</td>
<td>-0.907</td>
<td>-0.011</td>
<td>-0.053</td>
<td>-0.099</td>
</tr>
<tr>
<td>0.153</td>
<td>7.345</td>
<td>-0.144</td>
<td>-0.102</td>
<td>-0.304</td>
<td>-0.424</td>
</tr>
<tr>
<td>-0.907</td>
<td>-0.144</td>
<td>17.152</td>
<td>0.066</td>
<td>0.058</td>
<td>0.091</td>
</tr>
<tr>
<td>-0.011</td>
<td>-0.102</td>
<td>0.066</td>
<td>1.412</td>
<td>0.117</td>
<td>0.033</td>
</tr>
<tr>
<td>-0.053</td>
<td>-0.304</td>
<td>0.058</td>
<td>0.117</td>
<td>0.075</td>
<td>-0.578</td>
</tr>
<tr>
<td>-0.099</td>
<td>-0.424</td>
<td>0.091</td>
<td>0.033</td>
<td>-0.578</td>
<td>0.096</td>
</tr>
</tbody>
</table>
### Table C.9: VAR(1) Model Estimates [United States]

#### Panel A. Model estimates

<table>
<thead>
<tr>
<th>( )</th>
<th>Coefficients on lagged variables</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
<th>( )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns &amp; 0.111 &amp; 0.062 &amp; 0.030 &amp; 0.023 &amp; 0.212 &amp; 3.259 &amp; 6.290 &amp; 0.024</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (2.633) &amp; (0.876) &amp; (0.219) &amp; (2.505) &amp; (0.290) &amp; (-1.799) &amp; (-1.693)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) log bond excess returns &amp; -0.015 &amp; -0.079 &amp; 0.034 &amp; -0.003 &amp; -0.831 &amp; 1.422 &amp; 3.670 &amp; 0.075</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (-0.979) &amp; (-3.138) &amp; (0.584) &amp; (-0.854) &amp; (-2.323) &amp; (2.263) &amp; (2.760)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) log dividend yield &amp; -0.092 &amp; -0.044 &amp; -0.001 &amp; 0.979 &amp; 0.369 &amp; 1.421 &amp; 4.942 &amp; 0.981</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (-2.117) &amp; (-0.643) &amp; (-0.006) &amp; (101.402) &amp; (0.556) &amp; (0.749) &amp; (1.350)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) log inflation &amp; 0.002 &amp; 0.009 &amp; -0.012 &amp; 0.000 &amp; 0.448 &amp; 0.158 &amp; -0.052 &amp; 0.263</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (0.593) &amp; (1.635) &amp; (-1.394) &amp; (0.346) &amp; (5.998) &amp; (1.506) &amp; (-0.246)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) log short rate &amp; -0.001 &amp; 0.001 &amp; 0.000 &amp; 0.000 &amp; 0.006 &amp; 1.029 &amp; 0.154 &amp; 0.964</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (-3.505) &amp; (0.742) &amp; (-0.321) &amp; (-2.639) &amp; (1.063) &amp; (99.190) &amp; (5.568)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) log yield spread &amp; 0.001 &amp; 0.000 &amp; 0.000 &amp; 0.000 &amp; 0.003 &amp; -0.050 &amp; 0.803 &amp; 0.777</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&amp; (3.487) &amp; (0.412) &amp; (-0.092) &amp; (2.650) &amp; (0.342) &amp; (-3.578) &amp; (25.018)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Panel B. Residual correlation matrix

<table>
<thead>
<tr>
<th>( )</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) log stock excess returns &amp; 15.127 &amp; -0.034 &amp; -0.959 &amp; -0.024 &amp; 0.031 &amp; -0.024</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) log bond excess returns &amp; -0.034 &amp; 6.147 &amp; 0.015 &amp; -0.133 &amp; 0.013 &amp; -0.452</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) log dividend yield &amp; -0.959 &amp; 0.015 &amp; 15.323 &amp; 0.038 &amp; 0.005 &amp; 0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) log inflation &amp; -0.024 &amp; -0.133 &amp; 0.038 &amp; 0.971 &amp; -0.025 &amp; 0.076</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) log short rate &amp; 0.031 &amp; 0.013 &amp; 0.005 &amp; -0.025 &amp; 0.133 &amp; -0.883</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) log yield spread &amp; -0.024 &amp; -0.452 &amp; 0.002 &amp; 0.076 &amp; -0.883 &amp; 0.160</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C.6 Fisher Transformation and Correlation Contribution

Fisher Transformation

We use Fisher transformation to test the hypothesis that cross-country correlations of the news components of excess stock returns are different between 1986-1999 subperiod and the 2000-2016 subperiod. Define \( z = \frac{1}{2} \ln \left( \frac{1+r}{1-r} \right) \). If \((X, Y)\) is bivariate normal, and if \((X_i, Y_i)\) used to form \( r \) are independent, then \( z \sim \mathcal{N} \left( \frac{1}{2} \ln \left( \frac{1+r}{1-r} \right), \frac{1}{N-3} \right) \), where \( N \) is the sample size. For two samples of data, the early subperiod \((1)\) and the late subperiod \((2)\), define \( z_1 = \frac{1}{2} \ln \left( \frac{1+r_1}{1-r_1} \right) \) and \( z_2 = \frac{1}{2} \ln \left( \frac{1+r_2}{1-r_2} \right) \). The difference is \( z_1 - z_2 \sim \mathcal{N} \left( \frac{1}{2} \ln \left( \frac{1+r_1}{1-r_1} \right) - \frac{1}{2} \ln \left( \frac{1+r_2}{1-r_2} \right), \frac{1}{N_1-3} + \frac{1}{N_2-3} \right) \). p-values can then be obtained in the normal way.

Correlation Contribution

For stocks, we can decompose the excess return news \( \tilde{x}_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t) [x_{s,t+1}] = N_{CF,t+1} - N_{RR,t+1} - N_{RP,t+1} \). For bonds we can decompose its excess return news as \( \tilde{x}_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t) [x_{r,t+1}] = N_{CF,r,t+1} - N_{RR,r,t+1} - N_{RP,r,t+1} \). (an increase in \( N_{CF,r,t+1} \) for bonds is interpreted as negative inflation news).

The reported “Component Contributions” in Figure 3.4 look at how much of the average covariance in excess returns is being explained by covariances of news components. E.g., in Figure 3.4, the stocks cash flow/stocks real rate across countries component contribution is calculated as \( \frac{1}{N(N-1)/2} \sum_{i \neq j} \frac{\text{Cov}(N_{CF,i}, N_{RR,j})}{\text{Cov}(\tilde{x}_{s,i}, \tilde{x}_{s,j})} \). For a given \((i,j)\) pair, the denominator \( \text{Cov}(\tilde{x}_{s,i}, \tilde{x}_{s,j}) = \text{Cov}(N_{CF,i} - N_{RR,i} - N_{RP,i}, N_{CF,j} - N_{RR,j} - N_{RP,j}) \) can be broken into 9 covariances of news components. Therefore, the 9 terms in the “Component Contributions” table always sum up to 1.
C.7 Semidefinite Programming Method

We do a constrained minimization problem to estimate the covariance matrices which satisfy two constraints: A). volatility matrix and within-country correlation are the same across two sample period. B). covariance matrix is positive semi-definite. First we decompose a covariance matrix into volatility matrix and correlation matrix

\[
\Sigma = D\Gamma D = \begin{pmatrix}
\sigma_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sigma_m \\
\end{pmatrix} \begin{pmatrix}
1 & \cdots & \rho_{1m} \\
\vdots & \ddots & \vdots \\
\rho_{1m} & \cdots & 1 \\
\end{pmatrix} \begin{pmatrix}
\sigma_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sigma_m \\
\end{pmatrix}
\]

Where the \( \sigma_i \) and \( \rho_{ij} \) (\( i, j = 1, \ldots, m \)) are the coefficients to be estimated. Suppose \( \hat{\Sigma}_1 \) and \( \hat{\Sigma}_2 \) are the sample covariance matrices for early period and late period (known), then we need to estimate two covariance matrix \( \Sigma_1 = D_1 \Gamma_1 D_1 \) and \( \Sigma_2 = D_2 \Gamma_2 D_2 \) with the constraint \( D_1 = D_2 = D \) and \( \Gamma_{1\text{within}} = \Gamma_{2\text{within}} \). We use the minimum distance estimation, and this is a well defined constrained optimization problem

\[
\min_{\Sigma_1, \Sigma_2} \left\{ \| \hat{\Sigma}_1 - \Sigma_1 \|_2 + \| \hat{\Sigma}_2 - \Sigma_2 \|_2 \right\}
\]

\[
\iff \min_{D, \Gamma_1, \Gamma_2} \left\{ \| \hat{\Sigma}_1 - D\Gamma_1 D \|_2 + \| \hat{\Sigma}_2 - D\Gamma_2 D \|_2 \right\}
\]

\[
\text{s.t. } \Gamma_i \succeq 0 \ (i = 1, 2)
\]

\[
\Gamma_{2\text{within}} = \Gamma_{1\text{within}}
\]

where \( \| \cdot \|_2 \) represents the norm in \( L^2 \) space (\( \| A - B \|_2 = \sum_{i,j}(a_{ij} - b_{ij})^2 \)), the notation \( \Gamma \succeq 0 \) means the matrix \( \Gamma \) is positive semi-definite, and \( \Gamma_{\text{within}} \) denotes the within-country correlation. To solve the Semidefinite programming (SDP) problem, we use the MATLAB package CVX by Stephen Boyd. [http://cvxr.com/cvx/doc/sdp.html](http://cvxr.com/cvx/doc/sdp.html)
C.8  VAR Model with Stochastic Volatility

Estimating VAR with Stochastic Volatility

We follow the methodology in Campbell, Giglio, Polk and Turley (CGPT 2017) in estimating VAR with stochastic volatility. Our VAR includes 8 state variables: stock excess returns, bond excess returns, dividend yield, inflation, short rate, yield spread, credit spread and EVAR. This adds two additional variables to our baseline VAR (credit spread and EVAR). The credit spread is constructed following the methodology in Kang and Pflueger (2013). It’s constructed as the log yields of investment grade corporate bond index subtracted by log yields of nominal government bond\(^2\). For U.S. credit spread, we use Moody’s Baa log yield minus Aaa log yield. Figure C.1 plots the country level credit spread in our sample. As argued in CGPT 2017, shocks to credit spread to some degree reflect news about aggregate default probabilities, which in turn should reflect news about the market’s future cash flows and volatility.

We use daily MSCI price index (MSPI) denominated in USD to constructed monthly realized variance (RVAR). The daily return is constructed by taking the daily difference of the price index \(r_{t+1} = \ln\left(\frac{P_{t+1}}{P_t}\right)\). The monthly realized variance is the sum of daily squared return. In estimation of the VAR, we use a two stage method (as in CGPT 2017). In the first stage, we construct period \(t+1\) expected market variance (EVAR\(_t\)) based on information available at period \(t\) (i.e. all state variables at period \(t\): \(x_t\)). Following CGPT, we fit the regression using weighted Least Squares (WLS). Specifically, we weight each observation \((RVAR_{t+1}, x_t)\) by previous period’s realized variance \(RVAR_{t-1}\). And we use a shrinkage factor as indicated in CGPT to ensure the ratio of weights across observations is not too extreme. In the second stage, we estimate a VAR with the first stage fitted value EVAR as a state variable. The second stage VAR is also estimated using WLS except that now the weight becomes \(EVAR_{t-1}\). We continue to apply the shrinkage factor in the second stage estimation. The results of the first stage regressions and

\(^2\)We selected government bonds with appropriate maturity so that the duration of it roughly match the duration of corporate bond indexes.
This figure shows the monthly credit spreads for Australia, Canada, France, Germany, Japan, the UK, and the US. It’s constructed as the log yields of investment grade corporate bond index subtracted by log yields of duration matched nominal government bond. For U.S. credit spread, we use Moody’s Baa log yield minus Aaa log yield.
second stage VAR estimations for 7 countries are reported in the online appendix: http://www.people.hbs.edu/lviceira/VW-IntlDiv-2018-07-07-Appendix.pdf

Simulating Symmetrical Model with Stochastic Volatility

To understand the impact of stochastic volatility on portfolio risk, we add volatility shock into our stylized symmetrical model of asset returns of Section 3 and simulate the symmetrical model with stochastic volatility. The new model has the following data generating process

\[
\begin{align*}
    r_{t+1} &= \mu_r + \beta s_t + \sigma_t u_{r,t+1} \\
    s_{t+1} &= \mu_s + \phi s_t + \sigma_t u_{s,t+1} \\
    \sigma_{t+1} &= (1 - \psi) + \psi \sigma_t + \nu_{\sigma,t+1}
\end{align*}
\]

The only difference from our previous symmetrical model is that here we added add a volatility, which follows a AR(1) process with persistence \( \psi \). Now the innovations to other variables (\( s_t \) and \( r_t \)) become heteroskedastic. In the simulation, we assume a symmetrical model for 7 countries, and the shocks to the 7 country VAR follow a multivariate normal process. In the simulation, we set \( \phi = 0.9857 \) and \( \beta = 0.0123 \), which are estimated from US data. For the volatility persistence, we compared two values in simulation: \( \psi = 0.9 \) and \( \psi = 0.99 \).

As a robustness check, we first reproduced the results in Figure 3.3 Panel A by simulating the 7 country symmetrical model of 2 state variables (excess stock return, dividend price ratio) over a horizon of 800 periods. We simulate 20000 paths. Then we simulate our symmetrical model with stochastic volatility specified above. We set the within-country correlation of volatility news and excess stock return news \( \text{corr}(\nu_{\sigma,i}, u_{r,i}) \) to be -0.625 and the within-country correlation of volatility news and dividend yield news \( \text{corr}(\nu_{\sigma,i}, u_{d,i}) \) to be 0.595. The numbers come from our VAR estimation results.

We focus on two exercises in the simulation. In the first exercise, the volatility news are
not correlated across countries (i.e. \( corr(v_{i,j}, v_{i,j}) = 0 \) for \( \forall i \neq j \)). Compare this with the symmetrical model of 2 state variables, we could see the impact of stochastic volatility on portfolio risk. In the second exercise, we make volatility news correlated across countries (\( corr(v_{i,j}, v_{i,j}) = 0.3 \) for \( \forall i \neq j \)) and everything else the same as in the first exercise. This exercise studies how volatility integration impacts portfolio risk. In both exercises, we tried two specifications for the volatility persistence (\( \psi = 0.9 \) and \( \psi = 0.99 \)). We see that when volatility is more persistent, the impact on portfolio risk is greater.

Figure C.4 plots the annualized global portfolio risk generated by the model as a function of investment horizon for different degrees of persistence in volatility (0.90 in Panel A and 0.99 in Panel B) and different cross-country correlations (zero on left plots and positive on right plots).³

The left column of each panel in Figure C.4 shows the impact on portfolio risk of adding stochastic volatility to a model with constant volatility in a scenario in which volatility shocks are uncorrelated across countries. The three solid lines in the plots correspond to the scenarios we have considered for the model with constant volatility of Section 3. These are the lines shown on Panel A of Figure C.4. This column shows that stochastic volatility increases portfolio risk at all horizons, especially at short horizons. The increase in market risk is more pronounced as volatility becomes more persistent.

The right column of each panel in Figure C.4 shows the impact of correlated stochastic volatility shocks. The three solid lines in the plots correspond to the case with stochastic volatility with uncorrelated volatility shocks—i.e., the dashed lines on the left column. These plots show that correlated volatility further increases portfolio risk, especially at long horizons. However, this increase is significant only when volatility shocks are highly persistent and correlated cash flow news is the source of correlated returns across countries. In that case, correlated volatility shocks amplify the effect of cash flow news correlation on portfolio risk at long horizons.

These results suggest that stochastic volatility shocks increase portfolio risk at all horizons

³Since there is no analytical expression \( \sqrt{\sum_{l}^{(k)} p_{i,l+k} / k} \), we evaluate it through simulation.
when they are highly persistent. However, allowing for correlated volatility shocks has only a small added impact on portfolio risk, except if returns are also correlated across countries, and the source of this correlation is correlated cash flow news. This scenario is not empirically plausible, because the main source of correlation in returns is correlated discount rate news, not correlated cash flow news. Therefore, these results suggest that while stochastic volatility increases portfolio risk at all horizons, this risk doesn’t necessarily increase more during periods in which risk becomes more correlated across markets. These results suggest that stochastic volatility shocks increase portfolio risk at all horizons when they are highly persistent. However, allowing for correlated volatility shocks has only a small added impact on portfolio risk, except if returns are also correlated across countries, and the source of this correlation is correlated cash flow news. This scenario is not empirically plausible, because the main source of correlation in returns is correlated discount rate news, not correlated cash flow news. Therefore, these results suggest that while stochastic volatility increases portfolio risk at all horizons, this risk doesn’t necessarily increase more during periods in which risk becomes more correlated across markets. In light of this last consideration, the empirical analysis in our paper assumes away time variation in volatility. That is, we present results based on a homoskedastic VAR model.
Figure C.2: International realized variance (RVAR) and expected variance (EVAR).

This figure shows the monthly realized variance (RVAR) and expected variance (EVAR) for Australia, Canada, France, Germany, Japan, the UK, and the US. The monthly realized variance is constructed from daily MSCI price index (MSPI) denominated in USD.
Figure C.3: Cross country correlation of heteroscedastic VAR news (stocks).

This figure plots the three year 3-year moving average of average cross-country correlations of shocks to stock excess returns, cash flow news, real rate news, and risk premium news, both including the October 1987 observation and excluding it. The news components are extracted from heteroscedastic VAR.
Panel A: Impact of stochastic volatility news on equity portfolio risk

(volatility persistence $\psi = 0.9$)

Figure C.4: Impact of stochastic volatility news on equity portfolio risk

This figure plots the equity portfolio risk $\sqrt{V_t\left[\frac{\mu_t^{(k)}_{\psi, t+k}}{\bar{\mu}_t}\right]} / k$ as a function of investment horizon $k$. As there’s no analytical expression, we evaluate it by simulating our symmetrical model with stochastic volatility. The left column of each panel plots the portfolio risk in a homoskedastic symmetrical model (solid line) and in a heteroskedastic version of the symmetrical model with stochastic volatility news uncorrelated across countries (dashed line). In each version of the model, we compare the term structure of portfolio risk across 3 scenarios (as described in Figure 3.3). The right column of each panel plots the portfolio risk in a heteroskedastic version of the symmetrical model of Section 3 with stochastic volatility news uncorrelated across countries (solid line) and with volatility news correlated across countries (dashed line). In this version of the model, volatility follows an AR(1) process with persistence parameter $\psi$. Panel A is simulated with volatility persistence $\psi = 0.9$ and Panel B is simulated with $\psi = 0.99$. 
Figure C.4 (Continued)

Panel B: Impact of stochastic volatility news on equity portfolio risk

(volatility persistence $\psi = 0.99$)
C.9 Complementary Results of the Paper

Complementary Results of Table 3.3

Table C.10: Return Correlation Decomposition (Bonds vs. Stocks Within Countries and Across Countries)

The left panel (right panel) of this table decomposes the sources of global bond v.s. stock return correlations within countries (across countries). Correlations among individual return components (i.e., cash-flow, real-rate, and risk premium news) within countries are shown in the table. Estimates are reported for each subperiod as well as the difference between the two subperiods. Tests for significant correlation differences between subperiods are based on bootstrap and Fisher r-to-z methods for calculating p-values.

<table>
<thead>
<tr>
<th></th>
<th>Bonds vs. Stocks Within Countries</th>
<th>Bonds vs. Stocks Across Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CF(s)</td>
<td>RR(s)</td>
</tr>
<tr>
<td>Subperiod 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (b)</td>
<td>0.10</td>
<td>-0.13</td>
</tr>
<tr>
<td>RR (b)</td>
<td>-0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>RP (b)</td>
<td>0.66</td>
<td>-0.65</td>
</tr>
<tr>
<td>Subperiod 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (b)</td>
<td>-0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>RR (b)</td>
<td>-0.91</td>
<td>0.98</td>
</tr>
<tr>
<td>RP (b)</td>
<td>0.84</td>
<td>-0.86</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF (b)</td>
<td>-0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>RR (b)</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>RP (b)</td>
<td>0.18</td>
<td>-0.21</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-values</td>
<td>CF (b)</td>
<td>0.00</td>
</tr>
<tr>
<td>(bootstrap)</td>
<td>RR (b)</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>RP (b)</td>
<td>0.00</td>
</tr>
<tr>
<td>p-values</td>
<td>CF (b)</td>
<td>0.00</td>
</tr>
<tr>
<td>(Fisher r-to-z)</td>
<td>RR (b)</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>RP (b)</td>
<td>0.00</td>
</tr>
</tbody>
</table>
**Complementary Results of Table 3.6**

**Table C.11: Optimal Global Equity Portfolio Allocations and Expected Utility (value weighted myopic portfolio)**

Panel A reports optimal global equity portfolio allocations by “JV” investor and “CCV” investor. The CCV investor has Epstein-Zin preference and the expected value function defined as $\bar{V}_t = (1 - \delta)^{-\psi/(1-\psi)} \left( \frac{C_t}{M_t} \right)^{1/(1-\psi)}$. The JV investor’s utility is power utility defined on terminal wealth $E_t[\frac{1}{1-\gamma} W_{t+K}]$. The myopic demand is the allocation of those two investors at investment horizon 1. An investor’s allocation is the sum of myopic demand and hedging demand. We report the JV hedging demand for an investor at horizon of 20 years (240 months). We compare across 3 scenarios: optimal allocation in early sample, late sample and late sample with hypothetical covariance matrix that controls for within-country correlation. To make it comparable, we fix the monthly implied excess returns across these 3 scenarios. We set implied excess returns for value weight portfolio such that investor hold the myopic demand equal to market cap weight. “Total” allocation is the sum of the allocations to each country. Panel B reports the expected utility by “JV” investor (with RRA $\gamma = 5$) and “CCV” investor (with EIS $\psi \rightarrow 1$ and RRA $\gamma = 5$), assuming they allocate optimally to the 7 countries investment space as reported in Panel A. We also report investor’s expected utility by constraining the investment space to USA only. We assume investor has initial wealth of one dollar and look at investment horizons $K$ of 5 years (60 months), 10 years (120 months), 15 years (180 months) and 20 years (240 months). We report the certainty equivalent for the JV investor (with RRA $\gamma = 5$). The results are obtained by Monte Carlo simulation using 2,000 VAR paths sampled using the method of antithetic variates. The certainty equivalent of wealth is computed by evaluating the mean utility realized across the simulated paths and computing, $W_{CE} = u^{-1} \left( E[u(\hat{W}_{t+K})] \right)$.

<table>
<thead>
<tr>
<th>Panel A: Optimal Global Equity Portfolio Allocations</th>
<th>Panel B: Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country</td>
<td>Myopic demand</td>
</tr>
<tr>
<td>--------</td>
<td>---------------</td>
</tr>
<tr>
<td>Early Sample</td>
<td></td>
</tr>
<tr>
<td>AUS</td>
<td>1.51%</td>
</tr>
<tr>
<td>CAN</td>
<td>2.83%</td>
</tr>
<tr>
<td>FRA</td>
<td>5.22%</td>
</tr>
<tr>
<td>GER</td>
<td>5.07%</td>
</tr>
<tr>
<td>JPN</td>
<td>16.09%</td>
</tr>
<tr>
<td>UKI</td>
<td>10.38%</td>
</tr>
<tr>
<td>USA</td>
<td>58.88%</td>
</tr>
<tr>
<td>Total</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
Table C.11 (Continued)

<table>
<thead>
<tr>
<th>Country</th>
<th>Panel A: Optimal Global Equity Portfolio Allocations</th>
<th>Panel B: Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Myopic demand</td>
<td>JV hedging demand at 20 yr</td>
</tr>
<tr>
<td>AUS</td>
<td>23.09%</td>
<td>25.04%</td>
</tr>
<tr>
<td>CAN</td>
<td>10.04%</td>
<td>34.31%</td>
</tr>
<tr>
<td>FRA</td>
<td>12.06%</td>
<td>27.13%</td>
</tr>
<tr>
<td>GER</td>
<td>-39.79%</td>
<td>-20.41%</td>
</tr>
<tr>
<td>JPN</td>
<td>5.17%</td>
<td>22.73%</td>
</tr>
<tr>
<td>UKI</td>
<td>50.51%</td>
<td>4.61%</td>
</tr>
<tr>
<td>USA</td>
<td>62.40%</td>
<td>72.43%</td>
</tr>
<tr>
<td>Total</td>
<td>123.47%</td>
<td>165.85%</td>
</tr>
</tbody>
</table>

Late Sample

<table>
<thead>
<tr>
<th>Country</th>
<th>Panel A: Optimal Global Equity Portfolio Allocations</th>
<th>Panel B: Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-9.05%</td>
<td>6.92%</td>
</tr>
<tr>
<td>CAN</td>
<td>25.97%</td>
<td>2.42%</td>
</tr>
<tr>
<td>FRA</td>
<td>-2.32%</td>
<td>17.46%</td>
</tr>
<tr>
<td>GER</td>
<td>-18.95%</td>
<td>22.21%</td>
</tr>
<tr>
<td>JPN</td>
<td>-1.51%</td>
<td>12.20%</td>
</tr>
<tr>
<td>UKI</td>
<td>19.72%</td>
<td>3.45%</td>
</tr>
<tr>
<td>USA</td>
<td>82.91%</td>
<td>50.24%</td>
</tr>
<tr>
<td>Total</td>
<td>96.77%</td>
<td>114.89%</td>
</tr>
</tbody>
</table>
Complementary Results of Figure 3.1

Figure C.5: Stock-bond correlations across and within countries

This figure plots average stock-bond correlations across countries and within countries. Monthly averages are computed using pairwise return correlations within and across seven different countries over 3-year rolling windows (Australia, Canada, France, Germany, Japan, United Kingdom, and United States). Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate. The sample is from Jan 1986 to Dec 2016.
Complementary Results of Figure 3.3

The figure plots annualized portfolio risk $\sqrt{\frac{V_t(\mathcal{R}(t, k))}{k}}$ (panel A) and optimal allocation to risky assets (panel B) as a function of investment horizon $k$ (months) for an asset space of 2 symmetrical countries, which complements to Figure 3.3 in the main paper (7 symmetric countries). We compare the term structure of portfolio risk and optimal allocation for 3 scenarios: (1) Baseline case with zero cross-country return news correlations, both for CF news and DR news. (2) CF news integration case, where cross-country return correlations come from positive cross-country CF news correlations; cross-country correlations of DR news are zero. (3) DR integration case, where cross-country return correlations come from positive cross-country DR news correlation; cross-country correlations of CF news are zero. To make Scenarios 2 and 3 comparable, we set the cross-country correlation of one-period returns at the same value (0.07). Panel A plots portfolio risk in each scenario for a portfolio of seven symmetric countries. Panel B plots optimal allocation to risky assets (for a portfolio of seven countries) as a function of time remaining to terminal date. The total optimal allocation is the sum of two parts: myopic allocation (equals the intercept at $\tau = 1$) and hedging allocation. The investor has horizon of $K = 360$ (30 years) and rebalance his allocation each period. The x-axis $\tau$ is the time remaining to the terminal date. We compare the term structure of optimal allocation to risky assets across the same 3 scenarios described above. We set the expected excess returns so that in the benchmark case, the myopic investor ($\tau = 1$) allocate $1/N$ to each risky asset ($50\%$ for $N = 2$) and zero to cash. The expected excess returns are kept the same across the three cases to make them comparable.
Contributions of news components to unexpected bond v.s. stock return correlations within countries (Panel A) and bond v.s. stock return correlations across countries (Panel B) are broken down in the columns. In Panel A (bond v.s. stock return correlations within countries), the cash flow component contribution is calculated as $\frac{1}{N} \sum_i \frac{\text{Cov}(N_b, CF, i, N_s, CF, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)}$, the real rate component contribution is calculated as $\frac{1}{N} \sum_i \frac{\text{Cov}(N_b, RR, i, N_s, RR, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)}$, the risk premium component contribution is calculated as $\frac{1}{N} \sum_i \frac{\text{Cov}(N_b, RP, i, N_s, RP, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)}$, and the cross components is calculated as

$$\frac{1}{N} \sum_i \left( \frac{\text{Cov}(N_b, CF, i, N_s, RR, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)} + \frac{\text{Cov}(N_b, CF, i, N_s, RP, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)} + \frac{\text{Cov}(N_b, RR, i, N_s, CF, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)} + \frac{\text{Cov}(N_b, RR, i, N_s, RP, i)}{\text{Cov}(\tilde{x}_i, \tilde{x}_i)} \right).$$

The component contributions in the panel B is calculated similarly (but with pairwise average across countries). Note that by definition, values in the component contributions sum up to 1.
Complementary Results of Figure 3.9

**Figure C.8: Value Weighted Portfolio Risk as a Function of Investment Horizon (Equities and Bonds)**

The figure compares the early sample (1986.01-1999.12) and late sample (2000.01-2016.12) value weighted portfolio risk across investment horizons for equities (Panel A) and bonds (Panel B). For each panel, we plot the annualized conditional standard deviation of portfolio excess returns, annualized average conditional volatility (across N countries) of excess returns, and pairwise average conditional correlation of cross-country excess returns. Portfolios are value-weighted.