



# Estimating Incumbency Advantage Without Bias

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# *Estimating Incumbency Advantage without Bias\**

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In this paper we prove theoretically and demonstrate empirically that all existing measures of incumbency advantage in the congressional elections literature are biased or inconsistent. We then provide an unbiased estimator based on a very simple linear regression model. We apply this new method to congressional elections since 1900, providing the first evidence of a positive incumbency advantage in the first half of the century.

## **Introduction**

Incumbency advantage is the most frequently studied factor in the last 15 years of congressional elections research. Discovery of the increase in incumbency advantage has sparked controversy over its origins, historical pattern, current magnitude, and electoral, democratic, and policy consequences.<sup>1</sup> Even the most basic descriptive information about incumbency advantage is flawed because every previous measure based on aggregate data in this literature is plagued by selection bias, inconsistencies, and inefficiencies. We prove this result and also propose a new unbiased and statistically efficient measure that is very easy to calculate. Congressional elections from 1900 to the present fit our theoretical model. Our empirical reanalyses do not challenge the existing scholarly consensus that incumbency advantage is much larger than it once was. However, we do challenge the prevailing belief that incumbency advantage was nonexistent, and the evidence that it was negative, in House elections during the first half of this century (Alford and Brady 1988). We show instead that the average advantage of incumbency in these elections was about two percentage points.

In section 1 of this paper, we define incumbency advantage, and section 2 gives an intuitive sense of the problems identifying it even in ideal electoral systems. Section 3 formally proves that sophomore surge and retirement slump are biased estimates of incumbency advantage, and section 4 briefly reveals the flaws

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<sup>1</sup>See Alford and Brady (1988); Alford and Hibbing (1981); Ansolabehere, Brady, and Fiorina (1988); Born (1979); Cain, Ferejohn, and Fiorina (1987); Collie (1981); Cook (1983); Cover and Mayhew (1977); Erikson (1971, 1972); Ferejohn (1977); Fiorina (1977); Garand and Gross (1984); Hinckley (1981); Jacobson (1987); King and Gelman (in press); Kostroski (1973); Krebbiel and Wright (1983); and Mayhew (1974).

in other existing measures. We offer an improved measure in section 5 that is based on a simple regression model with three explanatory variables and is therefore very easy to implement (see equation 6, below). This familiar regression framework also enables one to see potential problems more clearly. Empirical analyses in section 6 demonstrate the biases in sophomore surge and retirement slump and the advantages of our measure. Finally, in section 7, we analyze the problem of uncontested seats in estimates of incumbency advantage.

### 1. Definition of Incumbency Advantage

We define the theoretical incumbency advantage for a single legislative district election as

$$\psi = w^{(I)} - w^{(O)} \quad (1)$$

where

$w^{(I)}$  = proportion of the vote received by the incumbent *legislator* in his or her district, if he or she runs against major party opposition (thus,  $w^{(I)}$  is unobserved in an open seat election), and

$w^{(O)}$  = proportion of the vote received by the incumbent *party* in that district, if the incumbent legislator does *not* run and all major parties compete for this open seat; ( $w^{(O)}$  is unobserved if the incumbent runs for reelection).

We define the aggregate incumbency advantage for an entire legislature as the average of the incumbency advantages for all districts in a general election. This theoretical definition applies within a single election year and allows incumbency advantage to differ across districts. Our results hold in this general case, but below we often assume for simplicity that it varies over time but not across districts, and that the incumbency advantage accruing to Democratic and Republican legislators is the same.

Note that the definition in equation 1 does *not* assume that the candidates in  $w^{(I)}$  and  $w^{(O)}$  are identical in all respects except for incumbency status. Our theoretical definition of incumbency advantage properly includes the electoral advantages of all the perquisites of office: constituency service, fund-raising, name recognition, visibility, and others. We also do not make the counterfactual assumption that candidate quality is the same for incumbents and challengers, allowing quality also to be included in our definition.

### 2. Bias in Intuitive Measures

The incumbency advantage in a district depends on both  $w^{(I)}$  and  $w^{(O)}$ ; unfortunately, a real election in a single district will reveal only one of these (or none, if the election is uncontested). This problem of measuring incumbency

**Table 1. Election Outcomes in a Hypothetical Congressional District with No Incumbency Advantage**

Democratic Vote in Election 1	Incumbent Party in Election 2	Democratic Vote in Election 2	Change in Vote for the Incumbent from 1 to 2
0.45	R	0.45	0
0.45	R	0.55	-0.1
0.55	D	0.45	-0.1
0.55	D	0.55	0

advantage can therefore be thought of as causal estimation with missing data (Rubin 1974). An inappropriate analysis of these data can too easily lead to selection bias (Achen 1986).

In this brief section, we explore our definition of incumbency advantage by applying it to a theoretical electoral system in which incumbency has no effect on votes; that is,  $\psi = 0$  for all district elections. This analysis provides intuition about the central problem in analyzing these data.

Consider a congressional district that, on average, supports the two political parties equally and gives no advantage to incumbency. Assume the Democratic proportion of the two-party vote in two consecutive elections in this district,  $v_1$  and  $v_2$ , differ from 0.5 only due to independent random factors. Also assume that the winner in an open seat election at time 1 runs again as an incumbent in election 2. Then, four typical and equally likely outcomes are displayed in Table 1.

In two of the four cases in this table, the incumbent party in election 2 loses votes; in the other two cases, nothing changes. Looking just at changes between elections, it appears that incumbency either has no effect (the first and last case) or reduces the vote (the middle two cases). One might be tempted to conclude that incumbency actually hurts a candidacy. In fact, we set up these hypothetical district elections so that incumbency has no effect at all. The vote decline from time 1 to time 2 is merely a statistical selection effect, owing to the fact that incumbent candidates in the second election always received at least half of the vote in the first. This selection problem is reflected in most existing measures, a subject to which we now turn.

### 3. Sophomore Surge and Retirement Slump

In this section we formally analyze the two most popular measures of incumbency advantage: sophomore surge and retirement slump. Sophomore surge

is the average vote gain for freshman winners in election 1 who run again in election 2. Retirement slump is the average vote loss for the parties whose candidates won election 1 and did not run in election 2. Note that sophomore surge and retirement slump are each based on only a small fraction of legislative races (see, e.g., Cover and Mayhew 1977). In two separate subsections, we demonstrate the direction and then the size of the bias in these measures.

We begin with several definitions:

$v_t^{(i)}$  is the Democratic proportion of the two-party vote in a contested district election at time  $t$  ( $t = 1, 2$ ) with an incumbent running. If the district is an open seat election,  $v_t^{(i)}$  is unobserved.

$v_t^{(o)}$  is the Democratic proportion of the vote in an open seat contested district election at time  $t$  ( $t = 1, 2$ ). If the incumbent runs,  $v_t^{(o)}$  is unobserved.

$I_t$  equals 1 if a Democratic incumbent runs for reelection, 0 if no incumbent runs, and  $-1$  if a Republican incumbent is seeking reelection.

$\psi_t$  is the incumbency advantage in election  $t$ , the proportion of the vote an incumbent running in a district would receive solely because he or she had been elected before and served as the incumbent representative.

$\delta$  is the nationwide partisan swing, the average change in the Democratic proportion of district votes between elections 1 and 2.

$u_2^{(o)}$  is defined for convenience as  $v_2^{(o)} - \delta$ , the district Democratic vote proportion in an open seat after correcting for nationwide partisan swing.

$f(v_t^{(o)})$  is the theoretical probability distribution that generates  $v_t^{(o)}$  in election  $t$ .

All these variables and parameters refer to a single electoral district and exist in theory, although are not all observed in every election in practice. This section thus retains our theoretical focus; we consider data and empirical estimators only after sorting out the theoretical issues.

#### *Proof of Bias in Sophomore Surge and Retirement Slump*

We assume that the incumbency effect  $\psi$  is equal for each pair of elections 1 and 2, so  $v_t^{(i)} = v_t^{(o)} + \psi I_t$ . We also assume that the marginal density  $f$  is symmetric between elections 1 and 2, except for the nationwide vote swing  $\delta$ . Write the joint density function of partisan preferences  $f(v_1^{(o)}, v_2^{(o)})$ . Then the symmetry condition states:

$$f(v_1, v_2 + \delta) = f(v_2, v_1 + \delta)$$

This assumption allows the vote in any district to differ from election 1 to election 2, but constrains the shape of the probability density to remain constant.

The mean of this density is shifted by the nationwide partisan swing  $\delta$  and is thus allowed to vary between election years.<sup>2</sup>

For a district with a Democratic sophomore incumbent at time 2, the sophomore surge is defined as the proportion of the vote this candidate receives running for reelection at time 2 minus the proportion he or she received in his or her first election at time 1:

$$\begin{aligned} SS_D &= (v_2^{(f)} - v_1^{(o)} \mid I_1 = 0, I_2 = 1) \\ &= (\psi + v_2^{(o)} - v_1^{(o)} \mid I_1 = 0, I_2 = 1) \\ &= (\psi + \delta + u_2^{(o)} - v_1^{(o)} \mid I_1 = 0, I_2 = 1) \end{aligned} \quad (2)$$

The condition ( $I_1 = 0, I_2 = 1$ ) indicates that the district was an open seat in time 1 and that the Democrat won election 1 and ran as an incumbent in election 2. (Our mathematical treatment ignores the rare occasions that a freshman victory is obtained by beating an incumbent.)

How good a summary of incumbency advantage is the sophomore surge? We analyze this question by calculating the expected value of  $SS_D$ . If sophomore surge were an unbiased estimate of the incumbent advantage, then its expected value would be  $\psi$ . However,

$$\begin{aligned} E(SS_D) &= \psi + \delta + [E(u_2^{(o)} \mid I_1 = 0, I_2 = 1) \\ &\quad - E(v_1^{(o)} \mid I_1 = 0, I_2 = 1)] \\ &= \psi + \delta + [E(u_2^{(o)} \mid v_1^{(o)} > 0.5) - E(v_1^{(o)} \mid v_1^{(o)} > 0.5)] \\ &\leq \psi + \delta \end{aligned} \quad (3)$$

This result is proved rigorously in the appendix. Essentially, the inequality comes from the expression  $SS_D = v_2^{(f)} - v_1^{(o)}$  in equation 2. The positive term  $v_2^{(f)}$  can take on any value from 0 to 1, while the negative term  $-v_1^{(o)}$  is constrained to exceed 0.5. This works to diminish the sophomore surge to below the incumbency advantage.

Similarly, we define the Republican sophomore surge as:

$$\begin{aligned} SS_R &= \{(1 - v_2^{(f)}) - (1 - v_1^{(o)}) \mid I_1 = 0, I_2 = -1\} \\ &= \{-(v_2^{(f)} - v_1^{(o)}) \mid I_1 = 0, I_2 = -1\} \end{aligned} \quad (4)$$

and similar analysis shows:

$$E(SS_R) \leq \psi - \delta \quad (5)$$

<sup>2</sup>This assumption is thus considerably more realistic than uniform partisan swing (Butler 1951), is more flexible than that made by King (1990), and is essentially equivalent to that made by King and Gelman (in press), except that we do not restrict  $f$  to take any particular mathematical form.

Inequalities (3) and (5) show that *sophomore surge underestimates the true incumbency effect, even after correcting for nationwide swing.*

The expected retirement slump for the Democrats from election 1 to election 2 is:

$$\begin{aligned} E(RS_D) &= E(v_1^{(I)} - v_2^{(O)} \mid I_1 = 1, I_2 = 0) \\ &= E(v_1^{(I)} - v_2^{(O)} \mid v_1^{(I)} > 0.5) \geq \psi - \delta \end{aligned}$$

using virtually the same analysis as above. Similarly,

$$E(RS_R) = E[-(v_1^{(I)} - v_2^{(O)}) \mid I_1 = -1, I_2 = 0] \geq \psi + \delta$$

This analysis indicates that *retirement slump is higher on average than the incumbency effect, even after correcting for nationwide swing.*

### Extent of the Discrepancy

In the last subsection, we proved that sophomore surge and retirement slump differ systematically from incumbency advantage. In this subsection, we calculate the size of this theoretical discrepancy by adopting a simple parametric model.

We calculate the expected values of sophomore surges and retirement slumps, assuming that district votes (without incumbency) follow a bivariate normal distribution:

$$(v_1^{(O)}, v_2^{(O)}) \sim N\left(\mu, \mu + \delta, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tau^2\right)$$

In this case, the conditional expectation of a district vote change is just a linear regression:

$$E(v_2^{(O)} - v_1^{(O)} \mid v_1^{(O)}) = \delta - (1 - \rho)(v_1^{(O)} - \mu)$$

The expected sophomore surge for Democrats becomes

$$\begin{aligned} E(SS_D) &= \psi + \delta \\ &\quad - (1 - \rho) \int_{0.5}^{\infty} (v_1 - \mu) \frac{1}{\tau} \phi\left(\frac{v_1 - \mu}{\tau}\right) dv_1 / P(v_1^{(O)} > 0.5) \end{aligned}$$

Using the standard result on the expectation of a truncated normal variable (Johnson and Kotz 1970, 81),

$$E(SS_D) = \psi + \delta - (1 - \rho)\tau \frac{\phi[(\mu - 0.5)/\tau]}{\Phi[(\mu - 0.5)/\tau]}$$

where  $\phi$  is the normal probability density function and  $\Phi$  is the normal cumulative distribution function.

Similarly,

$$E(SS_R) = \psi - \delta - (1 - \rho)\tau \frac{\phi[(0.5 - \mu)/\tau]}{\Phi[(0.5 - \mu)/\tau]}$$

$$E(RS_D) = \psi - \delta + (1 - \rho)\tau \frac{\phi[(\psi + \mu - 0.5)/\tau]}{\Phi[(\psi + \mu - 0.5)/\tau]}$$

$$E(RS_R) = \psi + \delta + (1 - \rho)\tau \frac{\phi[(\psi + 0.5 - \mu)/\tau]}{\Phi[(\psi + 0.5 - \mu)/\tau]}$$

To find the practical meanings of these calculations, we must choose reasonable values for the parameters  $\mu$  (the expected vote at time 1 if no incumbent is running,  $E(v_1^{(0)})$ ,  $\rho$  (the correlation between  $v_1^{(0)}$  and  $v_2^{(0)}$ ; Democratic votes with no incumbent running at times 1 and 2),  $\tau^2$  (the variance of the district vote at time 1 and at time 2), and  $\psi$  (the incumbency advantage). In fact, the data from contested U.S. congressional elections fit this bivariate normal model fairly well, so we chose empirically reasonable parameters. Table 2 displays the following theoretical discrepancies

$$\frac{1}{2} [E(SS_D + SS_R)] - \psi$$

and

$$\frac{1}{2} [E(RS_D + RS_R)] - \psi$$

for the six sets of parameter values specified by:  $\mu = (0.5 \text{ and } 0.6)$ ,  $\rho = 0.8$ ,  $\tau = 0.1$ , and  $\psi = (0, 0.05, \text{ and } 0.10)$ . If these discrepancies were close to zero for all reasonable parameter values, then sophomore surge and retirement slump would be good measures.

Unfortunately, the discrepancies reported in Table 2 illustrate that *sophomore surge* is typically almost two percentage points lower than the true incumbency effect, no matter how large the latter is. *Retirement slump* is almost two

**Table 2. Discrepancies of Sophomore Surge and Retirement Slump, Relative to the Incumbency Advantage**

	Sophomore Surge		Retirement Slump	
	$\mu = 0.5$	$\mu = 0.6$	$\mu = 0.5$	$\mu = 0.6$
$\psi = 0$	-0.016	-0.018	0.016	0.018
$\psi = 0.05$	-0.016	-0.018	0.010	0.013
$\psi = 0.10$	-0.016	-0.018	0.006	0.009



*percentage points larger than the incumbency effect when  $\psi$  is small, but the bias diminishes with a larger incumbency effect.* Biases of two percentage points on the scale of Democratic votes represent statistical biases of at least one-fifth of the true incumbency advantage; this bias can even change the sign of the estimated incumbency effect.

#### 4. Other Previous Measures of Incumbency Advantage

Existing measures of incumbency advantage are relatively ad hoc but intuitive constructs. None were created within any standard theory of inference, and until now, none have been evaluated to see whether they conform to any desirable statistical criteria. In this section we briefly and more informally point out the biases and inconsistencies in five measures of incumbency advantage that are used less frequently than sophomore surge and retirement slump.

Erikson (1971) was the first to study the incumbency advantage systematically and is perhaps most sensitive to the tricky problems involved in estimation. He considers regression to the mean (selection bias), reciprocal causation, nationwide partisan swing, and other factors. He also produces estimates of the incumbency advantage that, according to our empirical analyses, often produce relatively small biases. However, he uses a "regression on residuals" procedure that is quite biased in general (see King 1986, Appendix 1).

Second, Alford and Brady (1988) propose using the average of sophomore surge and retirement slump as a measure of incumbency advantage. They call this average "slurge." Since sophomore surge is an underestimate and retirement slump is an overestimate, one might hope that an average of the two would yield a measure that is about right. Indeed, this average is a better estimate than either of the two quantities alone. However, Table 2 indicates that the biases in sophomore surge and retirement slump cancel only when the true incumbency advantage is zero. Otherwise, sophomore surge is more of an underestimate than retirement slump is an overestimate; thus, "slurge" will generally underestimate the true incumbency advantage.

A third measure was proposed by Garand and Gross (1984), who use the difference in the vote margin between incumbent winners and nonincumbent winners. This is an intuitive measure of how incumbent districts differ from nonincumbent districts. However, as Jacobson (1987, 128) and Alford and Brady (1988, 6) note, selection bias is a serious problem, since incumbent losers are excluded entirely. This measure in general attributes party strength in a district to incumbency and therefore overestimates the incumbency advantage.

Collie (1981) proposed a fourth measure by tracking districts moving between marginal and safe categories as different cohorts of open seat winners are reelected in subsequent elections. This is an excellent descriptive procedure and provides a good feel for individual patterns of House reelection and defeat. Unfortunately, it is also a flawed way of measuring incumbency advantage because

of regression to the mean and because Collie only tracks districts with incumbents who continue to win.

Finally, several variants of sophomore surge and retirement slump also exist. For example, many correct for nationwide partisan swing, a wise procedure when comparing changes over time. We showed above that these measures are still biased. Another correction is to calculate scores separately for the Democrats and Republicans and then to average them. Payne (1980) does this, so his measure will not track with votes. This procedure causes no additional biases beyond the problems described in section 3, but it is statistically inefficient if incumbency advantage is the same for both parties. Alford and Hibbing (1981) calculate sophomore surge and retirement slump for the second and third reelections instead of only the first. This provides important information about electoral career paths, but it is biased for the same reasons as the standard sophomore surge and retirement slump measures are biased.<sup>3</sup>

The problems with these and other measures suggest that scholars should take inference more seriously. The best approach to inference is *not* to invent some measure and immediately proceed to empirical analyses. Instead, we need to derive estimators with known statistical properties. This can be done either by formally evaluating intuitively created measures after the fact or through the formal procedures associated with a theory of statistical inference (King 1989).

### 5. An Improved Estimator of Incumbency Advantage

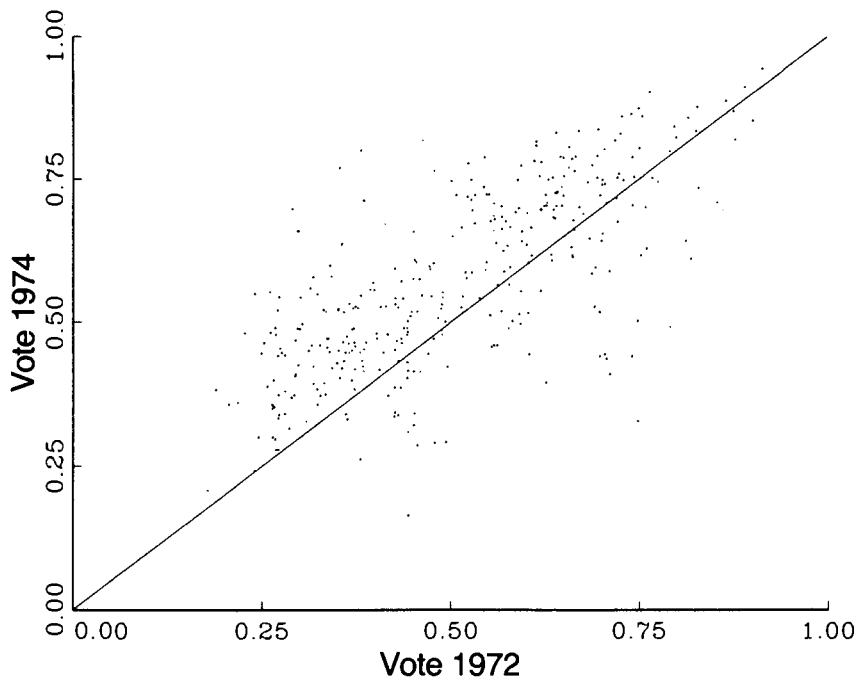
We create our estimator in a familiar regression framework. This way, the problems we must avoid in constructing an estimator with desirable statistical properties are very easily recognized, since they are listed in most textbooks (see Hanushek and Jackson 1977). We first present this estimator and then discuss possible problems.

To begin, denote  $v_1$  and  $v_2$  as the Democratic proportions of the two-party vote in elections 1 and 2, respectively. As an illustration, Figure 1 shows these variables for contested districts in 1972 and 1974. As above, we let  $I_t$  equal 1 if a Democratic incumbent runs for reelection, 0 if no incumbent runs, and  $-1$  if a Republican incumbent is seeking reelection, in election  $t$ . In addition,  $P_2$  is 1 if the Democrat wins election 1, and  $-1$  if the Republican wins. In this section and the next, we follow the uniform practice in the literature of discarding uncontested seats; since this may cause selection bias, we discuss the problem and a solution in section 7.

For a pair of election years, we base our measure of the incumbency effect

<sup>3</sup>Other measures give an indication of the incumbent's advantage but are not intended to provide a specific numerical estimate of the vote proportion due to incumbency. For example, Cover and Mayhew (1977) calculate, among other things, the percentage of incumbents winning with at least 60% of the vote. Jacobson (1987), Ansolabehere, Brady, and Fiorina (1988), and others have also calculated a variety of statistics measuring individual incumbents' vulnerability.

Figure 1. Partisan Swing



$\psi$  on a linear regression of votes on incumbency status, controlling for previous votes and partisan swing:

$$E(v_2) = \beta_0 + \beta_1 v_1 + \beta_2 P_2 + \psi I_2 \quad (6)$$

where the least squares estimate of  $\psi$  is our measure of incumbency advantage. Including  $v_1$  considerably reduces the variance of the estimate of  $\psi$  and eliminates a large source of possible bias. Including the party of the winner,  $P_2$ , allows the regression line of  $v_2$  on  $v_1$  and  $I_2$  to be at different levels for Democrats and Republicans, with the slope of the line held constant. If  $P_2$  were not included,  $\psi$  would probably be underestimated. Our estimator is not the same as retirement slump or sophomore surge; we maximize statistical efficiency by basing our measure on all contested seats rather than on only the approximately 10% contested by sophomores or left open by retirees.<sup>4</sup>

We could add a control variable  $I_1$  to this equation for incumbency status at time 1, but we find in practice that this has no appreciable effect on our estimate of  $\psi$  and therefore omit it. (If including  $I_1$  had an effect on the estimates in an

<sup>4</sup>Some comments we received on an earlier version of this paper show confusion over the definition of retirement slump. See section 2 of this paper for a precise definition.

application of this estimator to a different legislature, it should be included.) In the remainder of this section, we consider several other possible problems with our measure.

### *Exogeneity of the Decision to Run*

One possible objection to this estimator is that it assumes that decisions to run for reelection,  $I_2$ , are exogenous to votes in the second election,  $v_2$ . For example, if incumbents frequently decided not to run for reelection because they knew they would be likely to lose, this estimator would be inconsistent. The primary reason for an incumbent (who obviously won the previous election) to believe he or she will be defeated is probably alleged corruption of some kind. Thus, the study by Peters and Welch (1980) should provide sufficient reason to think that  $I_2$  is essentially exogenous to  $v_2$ . In the six election years from 1968 to 1978, Peters and Welch find only 80 cases of alleged corruption among incumbents (0.03 of all races). Among these House members, a slightly smaller fraction did run for reelection (0.813) than among all incumbents (0.879), but in two of the six elections those who were accused actually ran *more* frequently. Moreover, they break down their 80 cases into several categories because some seem quite benign (like abuse of the franking privilege). In the two categories where they found any significant electoral effect (morals and bribery charges), only 18 incumbents were implicated. This small number is a trivial fraction (0.007) of all races in this period. Other periods with more widespread corruption might be more problematic, but the complications entailed in making a correction would be more trouble than it is likely to be worth.

We can learn more about this estimator by noting that  $I_2$  may be written as the product of two factors:  $I_2 = R_2 P_2$ . The variable  $P_2$  is defined above, and  $R_2$  refers to the decision of the incumbent to seek reelection, coded as 1 if the incumbent runs and 0 otherwise. Using this relationship, we can reexpress equation 6 as follows:

$$E(v_2) = \beta_0 + \beta_1 v_1 + \beta_2 P_2 + \psi R_2 P_2 \quad (7)$$

The variable  $I_2 \equiv R_2 P_2$  can then be thought of as an "interaction" effect. However, only one of the two possible main effects,  $P_2$ , is included separately in this equation. This nonhierarchical model is appropriate because we seek to estimate the incumbency effect averaged over incumbents of both parties.<sup>5</sup>

<sup>5</sup>The estimate of  $\psi$  from running equation 6 or 7 is equivalent to running the following regression:

$$E(w_2) = \gamma_0 + \gamma_1 w_1 + \gamma_2 P_2 + \psi R_2$$

where  $w_1$  is the proportion of the vote for the winning candidate in election 1,  $w_2$  is the incumbent party's vote proportion in election 2 (i.e., the vote proportion in election 2 for the winning party in election 1), and  $P_2$  and  $R_2$  are defined as above. (The other parameters are of secondary interest, but  $\gamma_0$ ,  $\gamma_1$ , and  $\gamma_2$  are linear functions of  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ .)

### *Control Variables*

In House elections, we find that our key explanatory variable,  $I_2$ , is independent of the control variable  $v_1$ , after conditioning on the other control variable  $P_2$ .<sup>6</sup> In other words, incumbents do not base their decision of whether to seek reelection on their vote total in the previous election. This fortunately makes our results fairly insensitive to the assumption that the modeled relationship is linear.

We could improve this estimator by adding other control variables to equation 6, but one must be very careful to add only variables that occur before the winner's decision to seek reelection ( $R_2$ ). For example, including a measure of the quality of the opposition candidate would be tempting but inappropriate because the quality of the opposition candidate is largely dependent on the incumbent's decision about whether to run for reelection,  $I_2$ . Since the ability to scare away challengers is an important power of incumbency, we want this to be included as part of our estimate of incumbency advantage.<sup>7</sup>

### *Personal Incumbency Advantage*

We defined incumbency advantage as the vote proportion gained by a party due to running an incumbent candidate in a district election. This is the sum of two effects: (1) personal advantage gained by a candidate due to his or her incumbency, and (2) advantage gained by a party because its incumbent candidate is of higher quality than the typical open seat candidate. This higher candidate quality presumably manifests itself in greater vote-getting ability separate from the personal incumbency advantage.<sup>8</sup> We estimate the relative size of effects (1) and (2) on our incumbency measure as follows. Sophomore surge is a biased estimate of the personal incumbency advantage effect (1). If effect (2) were sub-

<sup>6</sup>For each pair of elections, we calculated the proportion of incumbents who received vote totals in the previous election of between 0.50 and 0.55, 0.55 and 0.60, 0.60 and 0.65, and so on. These proportions were all approximately equal, and a chi-square test could not reject the hypothesis of equality of proportions.

<sup>7</sup>An appropriate control variable to add might be quality of *potential* opposition candidates. This variable would be appropriate because it is prior to the incumbent's decision to run,  $R_2$ . In one extreme, if a district contained no potential high-quality opposition candidates, our measure (which does not include this control variable) would overestimate the incumbency advantage; the reason is that the effect of an incumbent's decision to run for reelection on the potential opposition party candidates' decision would add little to the incumbent's vote total. In districts with many high-quality potential opposition candidates, the incumbent's decision to run for reelection could have a larger effect on the vote total by scaring away some of these candidates. Of course, data on potential high-quality opposition candidates would be very difficult to collect even if one were to conduct a detailed analysis of each congressional district. Alternatively, one could include a measure of incumbent candidate quality, whether or not the incumbent decides to run for reelection, or perhaps the quality of the opposition party candidate in the last election. To further increase statistical efficiency, one might even include district-level economic variables as controls.

<sup>8</sup>We thank Robert Erikson for pointing this out to us.

Table 3. Hypothetical Data with Incumbency Advantage of 0.10

$E(V^0) = 0.55$										$E(V^0) = 0.45$											
$P_1$	$R_1$	$\varepsilon_1$	$V_1$	$R_2$	$\varepsilon_2$	$V_2$	$P_1$	$R_1$	$\varepsilon_1$	$V_1$	$R_2$	$\varepsilon_2$	$V_2$	$P_1$	$R_1$	$\varepsilon_1$	$V_1$	$R_2$	$\varepsilon_2$	$V_2$	
1	1	0.06	0.71	1	0.06	0.71	1	1	0.06	0.61	1	0.06	0.61	1	1	0.06	0.61	1	0.06	0.61	0.61
1	1	0.06	0.71	1	-0.06	0.59	1	1	0.06	0.61	1	0.06	0.61	1	1	0.06	0.61	1	-0.06	0.49	0.49
1	1	0.06	0.71	0	0.06	0.61	1	1	0.06	0.61	0	0.06	0.61	0	1	0.06	0.61	0	0.06	0.51	0.51
1	1	0.06	0.71	0	-0.06	0.49	1	1	0.06	0.61	0	0.06	0.61	0	1	0.06	0.61	0	-0.06	0.39	0.39
1	1	-0.06	0.59	1	0.06	0.71	1	1	-0.06	0.49	1	-0.06	0.41	1	1	-0.06	0.49	1	0.06	0.41	0.41
1	1	-0.06	0.59	1	-0.06	0.59	1	1	-0.06	0.49	1	-0.06	0.29	1	1	-0.06	0.49	1	-0.06	0.29	0.29
1	1	-0.06	0.59	0	0.06	0.61	1	1	-0.06	0.49	0	-0.06	0.51	1	1	-0.06	0.49	0	0.06	0.51	0.51
1	1	-0.06	0.59	0	-0.06	0.49	1	1	-0.06	0.49	0	-0.06	0.39	1	1	-0.06	0.49	0	-0.06	0.39	0.39
1	0	0.06	0.61	1	0.06	0.71	1	0	0.06	0.51	1	0.06	0.61	1	0	0.06	0.51	1	0.06	0.61	0.61
1	0	0.06	0.61	1	-0.06	0.59	1	0	0.06	0.51	1	-0.06	0.49	1	0	0.06	0.51	1	-0.06	0.49	0.49
1	0	0.06	0.61	0	0.06	0.61	1	0	0.06	0.51	0	0.06	0.51	1	0	0.06	0.51	0	0.06	0.51	0.51
1	0	0.06	0.61	0	-0.06	0.49	1	0	0.06	0.51	0	-0.06	0.39	1	0	0.06	0.51	0	-0.06	0.39	0.39
1	0	-0.06	0.49	1	0.06	0.51	1	0	-0.06	0.39	1	0.06	0.41	1	0	-0.06	0.39	1	0.06	0.41	0.41
1	0	-0.06	0.49	1	-0.06	0.39	1	0	-0.06	0.39	1	-0.06	0.29	1	0	-0.06	0.39	1	-0.06	0.29	0.29

1	0	-0.06	0.49	0	0.06	0.61	1	0	-0.06	0.39	0	0.06	0.51
1	0	-0.06	0.49	0	-0.06	0.49	1	0	-0.06	0.39	0	-0.06	0.39
-1	1	0.06	0.51	1	0.06	0.71	-1	1	0.06	0.41	1	0.06	0.41
-1	1	0.06	0.51	1	-0.06	0.59	-1	1	0.06	0.41	1	-0.06	0.29
-1	1	0.06	0.51	0	0.06	0.61	-1	1	0.06	0.41	0	0.06	0.51
-1	1	0.06	0.51	0	-0.06	0.49	-1	1	0.06	0.41	0	-0.06	0.39
-1	1	-0.06	0.39	1	0.06	0.51	-1	1	-0.06	0.29	1	0.06	0.41
-1	1	-0.06	0.39	1	-0.06	0.39	-1	1	-0.06	0.29	1	-0.06	0.29
-1	1	-0.06	0.39	0	0.06	0.61	-1	1	-0.06	0.29	0	0.06	0.51
-1	1	-0.06	0.39	0	-0.06	0.49	-1	1	-0.06	0.29	0	-0.06	0.39
-1	0	0.06	0.61	1	0.06	0.71	-1	0	0.06	0.51	1	0.06	0.61
-1	0	0.06	0.61	1	-0.06	0.59	-1	0	0.06	0.51	1	-0.06	0.49
-1	0	0.06	0.61	0	0.06	0.61	-1	0	0.06	0.51	0	0.06	0.51
-1	0	0.06	0.61	0	-0.06	0.49	-1	0	0.06	0.51	0	-0.06	0.39
-1	0	-0.06	0.49	1	0.06	0.51	-1	0	-0.06	0.39	1	0.06	0.41
-1	0	-0.06	0.49	1	-0.06	0.39	-1	0	-0.06	0.39	1	-0.06	0.29
-1	0	-0.06	0.49	0	0.06	0.61	-1	0	-0.06	0.39	0	0.06	0.51
-1	0	-0.06	0.49	0	-0.06	0.49	-1	0	-0.06	0.39	0	-0.06	0.39

stantial, then our measure would be correspondingly larger than the personal advantage (1). In this case, our regression estimate of incumbency advantage would be significantly greater than sophomore surge, beyond the difference predicted by our analysis of the theoretical bias in sophomore surge in section 2. Our empirical analysis in section 6 shows that this does not happen: the average difference between the two estimates is 2.4 percentage points, which is less than one percentage point more than the theoretical bias of sophomore surge shown in Table 2. Thus, we conclude that there is little difference between personal incumbency advantage and our measure. This conclusion—that open seat candidates are not significantly weaker than incumbent candidates, after correcting for personal incumbency advantage—is plausible because the weakest open seat candidates are eliminated in primary challenges.

The assumptions of regression analysis can be violated in many ways. We have already considered endogeneity, omitted variable bias, selection bias, and nonlinearity. We also discuss a different kind of selection bias that might occur due to dropping uncontested districts in section 7. The remaining possible violations, such as heteroscedasticity and (spatial) autocorrelation, are much less likely in our model. We therefore conclude that our estimator is consistent, unbiased, and statistically efficient.

#### *A Simple Check on New Measures*

Finally, we provide a relatively simple and intuitive tool with which scholars can judge new estimators, that captures the ideas behind many formal statistical criteria. Table 3 presents a set of 64 hypothetical district election results. We created these theoretical data with a true incumbency effect of  $\psi = 0.10$  by taking all possible combinations of the following five variables:  $R_1$  (with values 0 and 1), expected votes with no incumbent running  $E(V^{(o)})$  (0.45 and 0.55),  $R_2$  (0 and 1),  $V_1 \equiv E(V^{(o)}) + \psi P_1 R_1 + \varepsilon_1$  (where  $\varepsilon_1$  takes on the values 0.06 and  $-0.06$ ), and  $V_2 \equiv E(V^{(o)}) + \psi P_2 R_2 + \varepsilon_2$  (where  $\varepsilon_2$  takes on values 0.06 and  $-0.06$ ). This example assumes that expected votes are constant over time within districts and that incumbency is the only candidate-specific effect.

Table 3 is useful for evaluating any potential estimator in the hypothetical situation where only these five variables are relevant: the decision to run in each of two elections, expected votes, and random error in each election. In order to introduce other variables (or other values of these variables) in this hypothetical system, one must make sure to use all possible combinations of the values.

Although the true incumbency effect in these hypothetical data is 0.10, sophomore surge is 0.04, retirement slump is 0.12, and “slurge” is 0.08. Our estimator is based on a regression of the Democratic proportion of the two-party vote at time 2 ( $v_2$ ) on the Democratic proportion of the two-party vote at time 1 ( $v_1$ ), the party of the winner ( $P_2$ ), and incumbency status ( $I_2$ ):  $E(v_2) = \beta_0 + \beta_1 v_1 + \beta_2 P_2 + \psi I_2$ . It correctly estimates  $\hat{\psi} = 0.10$ .



## 6. Empirical Analyses

In order to examine our theoretical predictions of bias in sophomore surge and retirement slump and to apply our improved measure of incumbency advantage, we have collected electoral data from every election to the House of Representatives held in this century.<sup>9</sup>

As is standard practice in the literature, we excluded districts with third-party victories. When third parties endorsed a major-party candidate, as is allowed in New York, for example, we combine these votes and allocate them to the major-party candidate. Votes for other third parties are omitted. For each biennial general election, we estimate the incumbency advantage using district election results in that election and the previous election. We do not estimate incumbency advantage for election years ending in "2" because these immediately follow redistricting, and tracking votes across these periods is not feasible at the district level.

Figure 2 plots our estimate of incumbency advantage (the solid line), along with sophomore surge (the dashed line) and retirement slump (the dotted line). These empirical results precisely support our theoretical analysis. Sophomore surge was lower than our estimate by an average of 2.4 percentage points for the entire period. For most of the years, using sophomore surge as a measure of incumbency advantage would (incorrectly) indicate that incumbents are electorally disadvantaged. Retirement slump was about a percentage point above our measure until about 1960, when incumbency advantage rose, decreasing the bias in retirement slump. Our measure is also less variable than the two alternatives. This reflects the fact that our measure is more reliable and statistically efficient because it is based on all the districts contested in two successive election years.

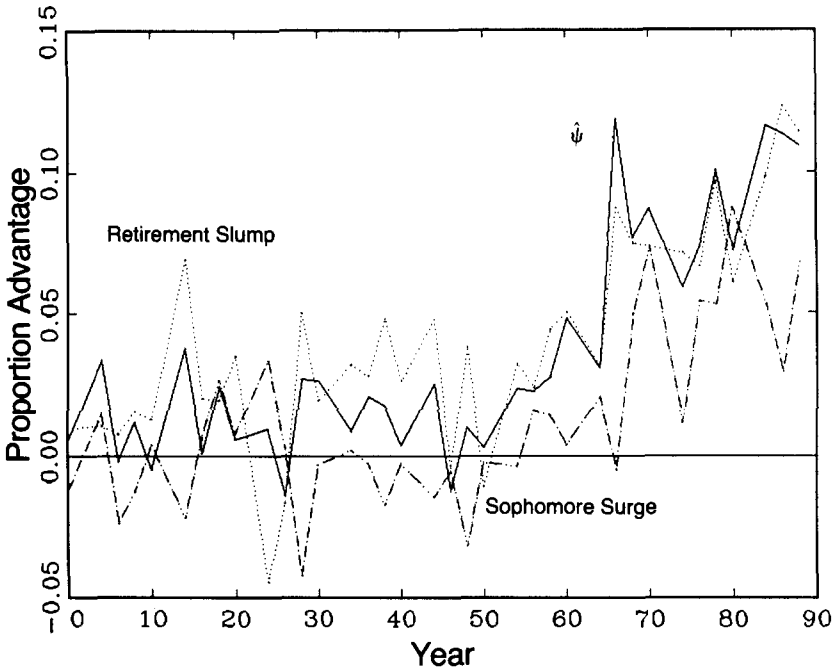
The results in Figure 2 are consistent with the congressional elections literature which shows that incumbency advantage is much larger now than it has been in the past. However, the figure also contradicts the most complete analysis of

<sup>9</sup>Data collection was surprisingly difficult. We compared data from Congressional Quarterly, the ICPSR, and from a data set collected from these sources by Professor Gary Jacobson. Because of the various coding rules, and coding errors, there were numerous inconsistencies. We resolved as many as we could by extensive cross-checking.

The ICPSR collection does not include a variable for incumbency, and it is stored in a particularly inconvenient form, but it does include the proper names of the candidates. We were able to code incumbency by comparing the names of the winning candidates in successive elections. Unfortunately, this collection includes literally hundreds of problems. Most of the problems involve the alphabetically coded names. These appear in numerous inconsistent formats: (1) FIRST LAST; (2) LAST, FIRST; (3) F. LAST; (4) LAST, F.; (5) LAST; and (5) FRITS LATS; among other combinations and misspellings. We wrote a simple pattern recognition program that helped find many of these errors. We then had research assistants look through each of the 50,000+ names to find additional errors.

The Congressional Quarterly data collection is better, but still has many errors, some identical to the ICPSR but some entirely new.

Figure 2. Estimates of House Incumbency Advantage



incumbency advantage in the first half of this century. Alford and Brady (1988, 15) conclude that “all of the numerous advantages that accompany incumbency yielded no *electoral* advantage to the incumbent until sometime after 1950.” Our empirical results provide evidence of at least a 1% or 2% incumbency advantage as far back as 1900. Indeed, our theoretical results also explain precisely why Alford and Brady concluded as they did: their measure (“slurge”) is an underestimate of the true incumbency advantage.<sup>10</sup>

### 7. Uncontested Seats

The ability of some incumbents to scare away all challengers is probably the biggest advantage of incumbency, and yet no measure—including ours—completely captures this phenomenon. We begin this section with an intuitive discussion of a possible selection bias caused by dropping uncontested seats,

<sup>10</sup>To test for separate Democratic and Republican incumbency advantages, one can let  $\psi = \gamma_0 + \gamma_1 P_2$ . Then, Democratic incumbency advantage is  $\gamma_0 + \gamma_1$  and Republican incumbency advantage is  $\gamma_0 - \gamma_1$ . We estimated these for our data from the House of Representatives and found no significant difference between the two.

then discuss a model that in principle could be used to generate estimates without bias. Our conclusion from this analysis is that if our measure is biased at all it only slightly underestimates the true incumbency advantage.

### *Intuition*

To understand the problem that uncontested seats cause in measures of incumbency advantage, we define  $Y_i$  as the *effective vote*: the unobserved proportion of the two-party vote that the Democratic candidate would have won in district  $i$  had election 2 been contested. (The vote in election 1,  $v_1$  in equation 6, is also censored in this way, but this causes fewer problems and is much more difficult to correct. We focus here only on censoring in the dependent variable,  $Y_i$ .) For contested elections, the effective vote is merely the observed Democratic proportion of the two-party vote ( $Y_i = V_{2i}$ ). If  $Y_i$  were observed in every district (i.e., if every election were contested), the regression in equation 6 would give the correct estimate of incumbency advantage.

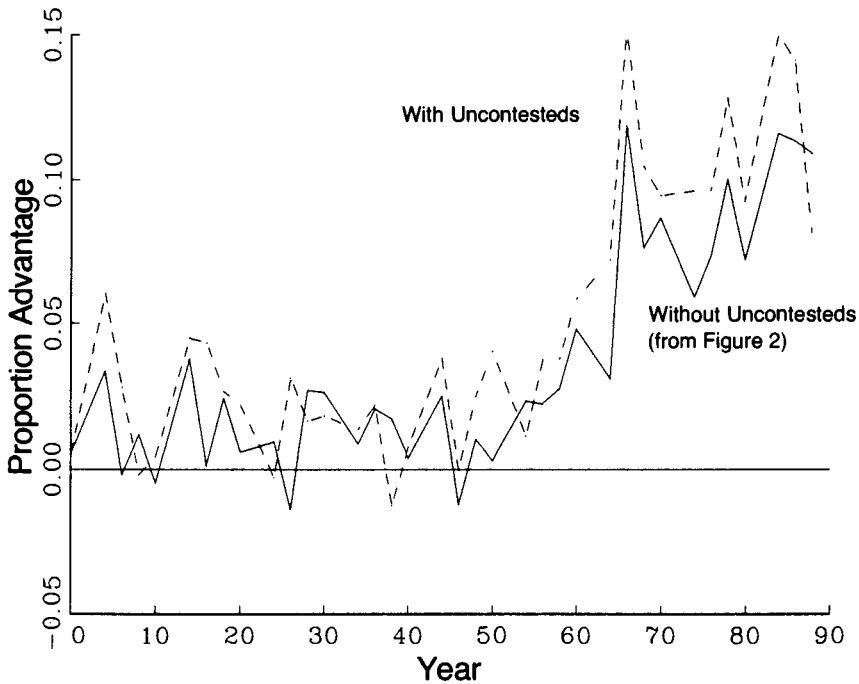
Unfortunately, the selection of districts into our sample of contested elections is neither exhaustive nor random. Instead, districts with very high or very low expected values of  $Y_i$  are much less likely to be contested. In general, bias will occur only if, conditional on the explanatory variables, the sample selection rule is correlated with the (unobserved) dependent variable (see Achen 1986)—precisely the case with all incumbency advantage estimators.<sup>11</sup>

Selection bias of this sort will generally cause one to *underestimate* the true incumbency advantage. The reason can be seen intuitively by studying Figure 1. Imagine, for simplicity, that  $Y_i$  were observed for every district and represented as a point in this figure. Now suppose that every district with a value of  $Y_i$  (on the vertical axis) above 0.7 were uncontested. A regression line fit to all the points would be steeper than one fit to only those below 0.7. If we also assumed that contested districts were the only ones where  $0.3 < Y_i < 0.7$ , the regression line would be even flatter. A proper statistical procedure should be based on the line with all the unobserved and observed values of  $Y_i$ , instead of only the observed ones. Of course, the real problem is even more difficult, since district elections do not become uncontested, and thus censored from our sample, by any simple deterministic rule.

In assessing the bias due to selection effects, one must be careful to distinguish selection due to explanatory variables, which causes no bias, and selection on the dependent variable, which does. For example, our estimator cannot be biased when potential opponents are scared off by variables included in our model: vote, incumbency status, and party of the winner at time 1. Our estimator might be biased if potential opponents decide not to contest an election due to factors we do not measure, such as a political scandal. It might also be biased if,

<sup>11</sup>In other words, selection bias occurs when the selection rule and the error term are correlated.

**Figure 3. Estimates of Incumbency Advantage with Uncontested**



for example, a potential opponent were scared off because he or she knew that the incumbent could raise considerable money, beyond that of the typical incumbent and beyond that reflected in the prior vote total. Because the explanatory variables in equation 6 incorporate most of what explains whether congressional elections are contested, we believe that these selection effects induce very little bias in  $\psi$ .

We can provide a rough estimate of an upper bound on  $\psi$ . Since the unobserved values of  $Y_i$  are presumably extreme, we set them to their most extreme possible values (0 or 1) and apply equation 6. This does not solve the problem, but it should give an upper bound on the true incumbency advantage. Figure 3 plots this new estimate, along with our original estimates from Figure 2. The figure shows that an appropriate selection bias correction would increase the estimate of incumbency advantage by a very small amount: the broken line in Figure 3 is only greater than the solid line by only 0.01 on average and 0.04 at the maximum. Since the broken line is an overestimate, the true incumbency effect therefore lies somewhere between these two lines, and due to the reasoning above we believe that it is much closer to our original estimate.

### A Stochastic Model

This problem can also be formalized with a somewhat more elaborate stochastic model. This model could produce consistent estimates of incumbency advantage, but it is substantially more complicated, is more difficult to implement, and relies on several untestable assumptions.

Recall that if the explanatory variables contain all the information in the selection rule (the rule by which districts become uncontested), then  $\hat{\psi}$  is not biased. To analyze this problem, denote  $U$  as a vector of relevant unobserved variables. The variables  $U$  do not include our explanatory variables ( $v_1$ ,  $P_2$ , and  $I_2$ ) or other variables uncorrelated with them. They might include scandals, pending retirement decisions, the lack of good challengers, the strength of the party organization not reflected in the prior vote total, or other variables that would help predict  $v_2$ .

If one were able to collect data for  $U$ , then an easy correction to our measure is to include it as an additional explanatory variable in our regression equation 6. Since this information is not available, we can theoretically correct this bias by assuming a probability distribution for  $U$  and a model for the probability of an uncontested district.

To show this, we construct a likelihood function with factors for contested and uncontested districts. Define a variable  $C_2$  that equals zero if at time 2 a district is uncontested and one if it is contested. Then the contested part of the likelihood function is the joint distribution of  $Y_2$ ,  $C_2$ , and  $U$ , integrated over the unknown variable  $U$ . The uncontested part of the likelihood function is the distribution of  $C$  and  $U$ , integrated over  $U$ . Taking the product of the two groups of observations, the likelihood function is as follows:

$$L(\beta | Y_2) \propto \prod_{C_2=1} \int f(Y_2, C_2, U | \beta) dU \quad (8)$$

$$\times \prod_{C_2=0} \int P(C_2 = 0, U | \beta) dU \quad (9)$$

To understand this function better, we factor each part into separate conditional probability distributions:

$$L(\beta | Y_2) \propto \prod_{C_2=1} \int f(Y_2 | C_2 = 1, U, \beta) \quad (10)$$

$$\times P(C_2 = 1 | U, \beta) f(U | \beta) dU$$

$$\times \prod_{C_2=0} \int P(C_2 = 0 | U, \beta) f(U | \beta) dU \quad (11)$$

where  $f(Y_2 | C_2 = 1, U, \beta)$  is just the normal distribution implied in our regression estimator (as before for contested districts only,  $C_2 = 1$ ). The factors  $P(C_2 = 1 | U, \beta)$  and  $P(C_2 = 0 | U, \beta)$  are conditional probability models for a district being contested or uncontested, respectively. Finally,  $f(U | \beta)$  is a prior distribution for the unobserved variable  $U$ .

Whereas deriving a reasonable model for  $P(C_2 = 1 | U, \beta)$  seems conceivable, making reasonable assumptions about  $f(U | \beta)$  is much more problematic. Determining the probability distribution for variables we observe is difficult; assuming them for particular unobserved variables is quite hazardous. However, deriving a reasonable distribution for a vector of variables  $U$ , even the contents of which we do not know, seems impossible. One could make assumptions for these distributions, and then maximize the likelihood function in equation 10, but the results would depend heavily on these untestable assumptions. Our analyses earlier in this section also suggest that the results would not be much different from our simpler regression estimator in equation 6.

### 8. Concluding Remarks

This paper provides theoretical and empirical evidence that every measure of incumbency advantage in the congressional literature is either biased or inconsistent. We offer a simple unbiased measure that solves most existing problems. One remaining problem is uncontested seats, for which we provide a theoretical model and bounds on the true coefficient. We also provide the first direct evidence that being an incumbent was a net electoral benefit in the House of Representatives in the first half of the twentieth century.

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### APPENDIX

This appendix provides a rigorous proof of the bias for Democratic sophomore surge stated in equation 3. Essentially the same procedures can be used to prove bias for Republican sophomore surge and for retirement slump.

The expected value of Democratic sophomore surge is as follows:

$$\begin{aligned} E(SS_D) &= E(v_2^{(D)} - v_1^{(D)} | v_1^{(D)} > 0.5) \\ &= E(\psi + v_2^{(D)} - v_1^{(D)} | v_1^{(D)} > 0.5) \\ &= \psi + \int_{0.5}^{\infty} \int_{-\infty}^{\infty} (v_2 - v_1) \frac{f(v_1, v_2)}{P(v_1^{(D)} > 0.5)} dv_2 dv_1 \end{aligned}$$

Relabeling  $u_2 = v_2 - \delta$ :

$$E(SS_D) = \psi + \delta + \frac{1}{P(v_1^{(D)} > 0.5)} \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{\infty} du_2 (u_2 - v_1) f(v_1, u_2 + \delta)$$

Using the symmetry of  $f$  in the second term below in the brackets:

$$E(SS_D) = \psi + \delta + \frac{1}{P(v_1^{(O)} > 0.5)} \\ \times \left[ \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{\infty} du_2 u_2 f(v_1, u_2 + \delta) - \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{\infty} du_2 v_1 f(u_2, v_1 + \delta) \right]$$

Switching the labels of  $v_1$  and  $u_2$  in the second term in the brackets:

$$E(SS_D) = \psi + \delta + \frac{1}{P(v_1^{(O)} > 0.5)} \\ \times \left[ \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{\infty} du_2 u_2 f(v_1, u_2 + \delta) - \int_{-\infty}^{\infty} dv_1 \int_{0.5}^{\infty} du_2 v_1 f(u_2, v_1 + \delta) \right]$$

Canceling the overlap in the two-dimensional regions of integration yields:

$$E(SS_D) = \psi + \delta + \frac{1}{P(v_1^{(O)} > 0.5)} \\ \times \left[ \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{0.5} du_2 u_2 f(v_1, u_2 + \delta) - \int_{-\infty}^{0.5} dv_1 \int_{0.5}^{\infty} du_2 u_2 f(v_1, u_2 + \delta) \right] \\ \leq \psi + \delta + \frac{1}{P(v_1^{(O)} > 0.5)} \\ \times \left[ \int_{0.5}^{\infty} dv_1 \int_{-\infty}^{0.5} du_2 (0.5) f(v_1, u_2 + \delta) - \int_{-\infty}^{0.5} dv_1 \int_{0.5}^{\infty} du_2 (0.5) f(v_1, u_2 + \delta) \right]$$

Switching back the labels of  $v_1$  and  $u_2$  and again using symmetry of  $f$  on the second term in the brackets reveals it to equal the first term.

$$E(SS_D) \leq \psi + \delta \quad (12)$$

QED

## REFERENCES

- Achen, Christopher. 1986. *The Statistical Analysis of Quasi-Experiments*. Berkeley: University of California Press.
- Alford, John, and David W. Brady. 1988. "Partisan and Incumbent Advantage in U.S. House Elections, 1846–1986." Center for the Study of Institution and Values, Rice University.
- Alford, John R., and John R. Hibbing. 1981. "Increased Incumbency Advantage in the House." *Journal of Politics* 43: 1042–61.
- Ansolabehere, Stephen, David Brady, and Morris Fiorina. 1988. "The Marginals Never Vanished?" Technical report, Graduate School of Business, Stanford University.
- Born, Richard. 1979. "Generational Replacement and the Growth of Incumbent Reelection in the U.S. House." *American Political Science Review*. 73:811–17.
- Butler, David E. 1951. Appendix. *The British General Election of 1950*, ed. H. G. Nicholas. London: Macmillan.
- Cain, Bruce, John Ferejohn, and Morris Fiorina. 1987. *The Personal Vote: Constituency Service and Electoral Independence*. Cambridge: Harvard University Press.

- Collie, Melissa P. 1981. "Incumbency, Electoral Safety, and Turnover in the House of Representatives, 1952-1976." *American Political Science Review* 75:119-31.
- Cook, Timothy E. 1983. Review Essay, *American Political Science Review* 77:1018.
- Cover, Albert D., and David R. Mayhew. 1977. "Congressional Dynamics and the Decline of Competitive Congressional Elections." In *Congress Reconsidered*, ed. Lawrence C. Dodd and Bruce I. Oppenheimer. New York: Praeger.
- Erikson, Robert S. 1971. "The Advantage of Incumbency in Congressional Elections." *Polity* 3:395-405.
- . 1972. "Malapportionment, Gerrymandering, and Party Fortunes in Congressional Elections." *American Political Science Review* 66:1234-55.
- Ferejohn, John A. 1977. "On the Decline of Competition in Congressional Elections." *American Political Science Review* 28:127-46.
- Fiorina, Morris. 1977. *Congress: Keystone of the Washington Establishment*. New Haven: Yale University Press.
- Garand, James C., and Donald A. Gross. 1984. "Change in the Vote Margins for Congressional Candidates: A Specification of the Historical Trends." *American Political Science Review* 78:17-30.
- Hanushek, Eric, and John Jackson. 1977. *Statistical Methods for Social Scientists*. New York: Academic Press.
- Hinckley, Barbara. 1981. *Congressional Elections*. Washington, DC: Congressional Quarterly Press.
- Jacobson, Gary C. 1987. "The Marginals Never Vanished: Incumbency and Competition in Elections to the U.S. House of Representatives." *American Journal of Political Science* 31:126-41.
- Johnson, Norman L., and Samuel Kotz. 1970. *Distributions in Statistics: Continuous Univariate Distributions-1*. New York: Wiley.
- King, Gary. 1986. "How Not to Lie with Statistics: Avoiding Common Mistakes in Quantitative Political Science." *American Journal of Political Science* 30:666-87.
- . 1989. *Unifying Political Methodology: The Likelihood Theory of Statistical Inference*. New York: Cambridge University Press.
- . 1990. "Representation through Legislative Redistricting: A Stochastic Model." *American Journal of Political Science* 33:787-824.
- King, Gary, and Andrew Gelman. In press. "Systemic Consequences of Incumbency Advantage in Congressional Elections." *American Journal of Political Science*.
- Kostroski, Warren. 1973. "Party and Incumbency in Postwar Senate Elections." *American Journal of Political Science* 67:1213-34.
- Krehbiel, Keith, and John R. Wright. 1983. "The Incumbency Effect in Congressional Elections: A Test of Two Explanations." *American Journal of Political Science* 27:140-57.
- Mayhew, David R. 1974. "Congressional Elections: The Case of the Vanishing Marginals." *Polity* 6:295-317.
- Payne, James L. 1980. "The Personal Electoral Advantage of House Incumbents." *American Politics Quarterly* 8:375-98.
- Peters, John G., and Susan Welch. 1980. "The Effects of Charges of Corruption on Voting Behavior in Congressional Elections." *American Political Science Review* 74:697-708.
- Rubin, Donald. 1974. "Estimating Causal Effects of Treatments in Randomized and Non-Randomized Studies." *Journal of Educational Statistics* 66:688-701.