Five Problems in Quantum Gravity

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Abstract
We present five open problems in quantum gravity which one might reasonably hope to solve in the next decade. Hints appearing in the literature are summarized for each one.

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1. Introduction

Reconciling quantum mechanics and general relativity is one of the great scientific challenges of our time. A definitive resolution will undoubtedly require both experimental and theoretical advances. At present, hopes for relevant physical experiments are distant. On the other hand, we are rich in puzzles and gedanken experiments which can help us understand the severe constraints imposed by theoretical consistency. The results of these gedanken experiments have led to dramatic theoretical advances over the last several decades, with a notable role played by string theory. Along the way, the problem of quantum gravity has revealed an unanticipated depth and richness, reaching into, tying together and sometimes solving problems in disparate areas of math and physics. Compelling new paradigms have been suggested for the structure of the universe around us. At the same time, it seems clear that what we have learned so far is only the tip of the iceberg and there is much more to come.

In this lecture, I will describe five problems of varying difficulty in quantum gravity that strike me as ripe for attack and might conceivably be solved in the next decade.\footnote{Each of the five problems discussed here has hundreds or thousands of relevant references, so it is impossible to give a comprehensive list. Those given are simply meant to be representative and provide the reader with an entry to the literature.}
problems here are “sharp” in the sense that there is a definite number or function involved. A successful computation of the number/function from some theoretical starting point is then a good indicator we are on the right track. Perhaps consideration of these problems will help us find the rest of the iceberg.

2. Universality of the Bekenstein-Hawking area-entropy law

2.1. The problem

The Bekenstein-Hawking area law \[ S_{BH} = \frac{\text{Area}}{4\hbar G} \] (2.1) applies universally to all horizons - cosmic, black hole and observer - in general relativity. Consistency with the generalized laws of thermodynamics requires that this entropy can be accounted for by counting some kind of quantum microstates. This accounting was achieved, including the \( \frac{1}{4} \) prefactor, for certain five-dimensional supersymmetric black holes in string theory, and then generalized to a wide variety of contexts, including many which do not involve string theory. However in this construction and its generalizations, the fact that the entropy is proportional to the area comes out only at the last step of a long computation. It is not obvious why this should always turn out to be so. A simple universal relation like (2.1) demands a simple universal explanation. The problem is to find it.

2.2. Some hints

(i) If we tile the horizon with Planck-sized cells, and assign one degree of freedom to each cell, then the entropy, which is extensive, will go like the area. This suggests that the microstates can be described as living on the horizon itself. The hard part is to naturally get the \( \frac{1}{4} \) from such a picture.

(ii) The entropy has a one-loop correction, proportional to \( \hbar^0 \), which is finite when expressed in terms of the one-loop corrected Newton’s constant, modulo some subtleties. This correction can be viewed as the entanglement entropy of the quantum states of the quantum fields inside and outside the horizon. It is dominated by short-wavelength modes.

Should the reader wish to suggest additional short hints or sharp problems, a communication to the author for use in a revised or expanded version of this note would be appreciated.
and is therefore naturally proportional to the area \( A \). If gravity is induced \( G \), which means that Newton’s constant is zero at tree level and arises as a one loop correction, then the entanglement entropy is responsible for all of the entropy, and reproduces the area law with the correct coefficient \( \beta \). This might in fact be the case in string theory, where the Einstein action is induced at one loop from open strings, but this notion has yet to be made precise. Recent progress \( [11] \) has revealed a rich holographic relation between entanglement entropy and minimal surfaces including horizons. Related observations appear in \( [12] \).

(iii) A universal relation in general relativity could be the image of a universal relation in statistical mechanics. One such candidate is the Cardy formula \( [13] \) which relates the asymptotic growth of states of a 2D CFT to its central charge. This formula was the basis of the original stringy computation in \( [3] \). It was later shown \( [14] \) that the Cardy/Area relation transcends its stringy origins, and follows in many cases simply from properties of the diffeomorphism group \( [15] \). Could the area law always be the Cardy formula? One suggestion along these lines \( [16] \) is that there might be some kind of universal 2D CFT, where the conformal transformations act in the \((r, t)\) plane, for all horizons. Another hint is that the general formula for 4D Kerr-Newman entropy can be written in the Cardy form

\[
S_{BH} = 2\pi (\sqrt{c_L L_0 / 6} + \sqrt{c_R L_0 / 6}) \text{ for } c_L = c_R = 6, \quad L_0 = (M^4 - Q^2 M^2 - J^2), \quad \bar{L}_0 = (M^2 - \frac{1}{2} Q^2)^2.
\]

3. de Sitter entropy

3.1. The problem

de Sitter space has an event horizon with thermodynamic properties described by the area law \( (2.1) \). The problem is to reproduce the de Sitter entropy by microstate counting. The location of the horizon is observer dependent, like the horizon in Rindler space which also obeys an area law. Since the de Sitter area law is numerically the same as the black hole area law, they must have a common explanation. It is very hard to imagine how this can work. For the black hole, we can at least say approximately “where” the microstates are located; there is an object with states to be counted. The object whose states we are supposed to count in de Sitter space is more elusive.
3.2. Some hints

(i) One way to proceed is to try to find de Sitter space as a solution of string theory and then, as in the black hole problem \[3\], find a duality transformation which maps it to a quantum system whose microstates can be counted. de Sitter solutions are difficult, but not impossible, to describe because they are never supersymmetric. Some recent rather simple constructions can be found in \[17\].

(ii) de Sitter space has an asymptotic boundary which is similar in many respects to that of anti-de Sitter space, but differs in that the boundaries are at timelike rather than spacelike infinity. The similarity suggests the possibility of a holographic dS/CFT correspondence \[18\] along the lines of the AdS/CFT correspondence \[19,15\]. The microstates might then be counted in the dual CFT. Some tantalizing numerological evidence for this was found in \[20\].

4. Partition function of 3D AdS-Einstein gravity

4.1. The problem

In three dimensions, all solutions of Einstein gravity with a negative cosmological constant are locally, but not necessarily globally, AdS$_3$. There are therefore no local degrees of freedom, and one might think the theory is too trivial to be interesting. On the other hand, it contains black hole solutions \[21\], so the quantum version, if it indeed exists, must at least be rich enough to account for the black hole microstates. A sharp question is to compute the quantum partition function of pure 3D AdS-Einstein gravity as a function of Newton’s constant $G$ and the cosmological constant $-\ell^{-2}$. It is surprising that pure 3D gravity has been studied for decades by now and it is still not known if there is a consistent quantum version.

4.2. Some hints

(i) In \[22\], it was proposed, based on local equivalence of the equations of motion, that pure AdS$_3$ gravity can be solved by rewriting it as a Chern-Simons gauge theory and then using the holographic duality to a (reduced) boundary WZW model. However this proposal has run into problems \[23\]. One way of stating the problem is that the euclidean partition function $Z$ constructed this way fails to be invariant under modular transformations. Since these are large diffeomorphisms this is a quantum anomaly. A closely related statement is
that the WZW microstates are not numerous enough to account for the black hole entropy (although see [24]). So it seems that in fact the Chern-Simons gauge theory is not quantum equivalent to pure gravity - perhaps because it includes singular field configurations with vanishing metric. Nevertheless, the connection to Chern-Simons gauge theory smells like an important hint.

(ii) On very general grounds [15], we expect that 3D AdS gravity should be dual to a 2D CFT with central charge \( c = \frac{3\ell}{2G} \). Solving the theory is equivalent to specifying this CFT. It was suggested in [23] that, rather than directly quantizing the Einstein-Hilbert action, this CFT might simply be deduced by various consistency requirements. Namely, the central charge must be \( c = \frac{3\ell}{2G} \), \( Z \) must be modular invariant (since these are large diffeomorphisms) and its pole structure must reflect the fact that there are no perturbative excitations. Adding the additional assumption of holomorphic factorization (i.e. decoupling of the left and right movers in the CFT), it was shown [23] that \( Z \) is uniquely determined to be a certain modular form \( Z_{ext} \). Unfortunately \( Z_{ext} \) does not agree with the Euclidean sum-over-geometries [25] which indicates that the assumption is not valid for pure gravity. Modular invariance and the restriction on the pole structure are still strong, if not uniquely determining, hints on the form of \( Z \) for pure gravity. Determining \( Z \) for pure 3D quantum Einstein gravity - if it exists - is an important open problem.

5. Extreme Kerr-Newman

5.1. The problem

The easiest kind of black holes to understand are the stable, charged, supersymmetric ones. Perhaps the hardest are the neutral Schwarzschild black holes. There are no useful parameters to expand in, and they represent an excited quantum system which is decaying. An intermediate step between the easy supersymmetric and the difficult Schwarzschild black holes is the two parameter family of extreme Kerr-Newman black holes. These have zero Hawking temperature as well as variable parameters and so are perhaps simpler to understand than Schwarzschild. The problem is to give a statistical accounting of the entropy \( S_{EKN} = \pi \sqrt{Q^4 + 4J^2} \) of an extreme charge \( Q \) spin \( J \) Kerr-Newman black hole.

\( \text{Interestingly, the assumption becomes a consistency requirement for chiral gravity [24] whose action contains an additional gravitational Chern-Simons term. Therefore if quantum chiral gravity exists, the argument of [23] can be applied and its partition function must be } Z_{ext}. \text{ This conclusion does agree with the sum-over-geometries [27].} \)
5.2. Some hints

(i) It has been understood for some time that the near horizon region of supersymmetric black holes is governed by an attractor mechanism, in which the geometry is determined by the charges independently of the asymptotic data of the spacetime \[28\]. This mechanism is essential in order for the area-law entropy to be an intrinsic property of the black hole. Recently it has been understood that the attractor mechanism operates for all extreme black holes supersymmetric or not \[29\], implying that the number of quantum microstates is invariant under at least some changes in the external parameters.

(ii) Interestingly, the attractor geometry always contains an enhanced \( SL(2, R) \) isometry, which becomes a (warped) AdS\(_3\) when the fibration of the \( U(1) \) of angular momentum or electric charge is included. This suggests that extreme black holes may always be related to 2D CFTs. Indeed, since these lectures were given, progress has been made along these lines \[30\].

6. Information release

6.1. The problem

In the seventies \[31\] Hawking gave a very simple and beautiful argument that information is destroyed by black holes. As our understanding has progressed, despite the simplicity of the argument, Hawking’s original scenario has seemed less and less plausible \[32\]. A variety of papers have appeared pointing to possible loopholes, but there does not appear to be a general consensus on how, where and why Hawking’s argument fails. The problem is to explain how the information is released from a decaying black hole.

A sharp question, which may serve to clarify the discussion, is: What is the rate of information return to \( I^+ \) of an evaporating black hole? To be more precise, a mixed state \( \rho(t^-) \) on \( I^+ \) at retarded time \( t^- \) can be defined by tracing over the portion of the full state on \( I^+ \) with support after \( t^- \). One can then compute the entanglement entropy \( S_{ent}(t^-) \) of \( \rho \) from the usual expression \(-\text{tr} \rho \ln \rho\). If the information is all returned, then \( S_{ent}(\pm \infty) = 0 \). The problem is to compute the function \( S_{ent}(t^-) \). If we truly understand black hole dynamics, we should be able to compute this function.

While there are many possibilities, several candidates stand out:

(1) **Bad question:** The question/answer is for some reason ill-defined.
(2) **Information destruction:** $S_{\text{ent}}(t^-)$ grows monotonically as expected from Hawking’s analysis for a time of order $M^3$ (for 4D Schwarzschild), and then stays constant when the black hole evaporates to zero radius and disappears. Information is destroyed.

(3) **Long-lived remnant:** $S_{\text{ent}}(t^-)$ grows monotonically as expected from Hawking’s analysis for a time of order $M^3$, and then slowly decreases back to zero over a time of order $M^4$ until the black hole becomes Planckian. The information is slowly released in infrared quanta with energy of order $M^{-4}$ by a conventional long-lived remnant.

(4) **Non-local remnant:** $S_{\text{ent}}(t^-)$ grows monotonically as expected from Hawking’s analysis for a time of order $M^3$, and then decreases back to zero over a time of order $M^{8/3}$. This is possible if, when the black hole becomes sub-Planckian, the information is stored not in a local Planck-sized region around the origin, but rather non-locally in a region of radius $M^{8/3}$: (using $S \sim (E R)^{3/4}$ and $E \sim 1$ this accommodates an entropy $S \sim M^2$). Of course, according to semiclassical gravity macroscopic locality would seem, by the standard argument, to have to be violated in order for the information to get to such a large region. This can be called a “non-local remnant”.

(5) **Maximal information return:** Late time radiation is correlated with early time radiation, and $S_{\text{ent}}$ is nearly zero at the (retarded) time when the black hole becomes Planckian. The profile of $S_{\text{ent}}$ is the same as it would be for any blackbody burning down to its ground state, as detailed in [33]. This of course also seems to violate macroscopic locality.

6.2. Some hints

(i) An important hint here comes from thinking about what happens in string theory. Formation/evaporation of a small (relative to the AdS$_5$ radius) black hole in AdS$_5$ can be described as a process in the dual Yang-Mills gauge theory living on the $S^3$ boundary [19,34]. We know that the gauge theory is unitary and entropy is well-defined, so this rules out (1) and (2) above. We can also rule out (3). Since the long-lived remnant lifetime is parametrically longer than the evaporation time, the existence of such remnants implies that the gauge theory must contain of order $e^{S_{\text{BH}}} = e^{4\pi M^2}$ states with masses less than the remnant mass. Since the remnant mass is of order the Planck mass and $M$ can be

4 In a time $M^4$ there will be an energy of order one in radially propagating infrared modes spread over a region of size $M^4$. This is the size needed to accommodate an entropy of order $M^2$. 
arbitrarily large, this says there are infinitely many states below any fixed energy. But we
know this is not the case for gauge theory on a sphere so (3) is ruled out. (4) on the other
hand cannot be so easily ruled out because the non-local remnant lifetime is shorter than
the evaporation time and our understanding of the relation of gauge theory and gravity
states is imprecise. Indeed we do not even know what states correspond to a small black
hole, so we certainly can not analyze the evaporation profile. However in principle these
questions seem answerable.

(ii) More hints come from thinking about the behavior of field theory two point functions
at infinity in the presence of an eternal black hole \[35\]. Semiclassically, these correlators
have a thermal exponential decay at large time separations. However such behavior is in-
compatible with unitarity if the black hole is in a pure state: there are always exponentially
rare spikes at late times corresponding to Poincare recurrences. To compute such expo-
nentially rare processes we must include nonperturbative effects. It would be extremely
interesting to understand in some exact theory like string theory how one sees such spikes
in the nonperturbative semiclassical expansion.

The problem of computing spikes in correlators seems simpler and more well-posed
than the information release problem. An understanding of the former would undoubtedly
shed significant light on and give important hints about the latter. However they are not
exactly the same problem. For one thing, the information release problem already appears
in perturbation theory, while the spikes are a nonperturbative phenomenon. Interesting
recent work in this direction appears in \[36\].

7. Conclusion

We have our work cut out for us.

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References


