Quark and Gluon Tagging at the LHC

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Being able to distinguish light-quark jets from gluon jets on an event-by-event basis could significantly enhance the reach for many new physics searches at the Large Hadron Collider. Through an exhaustive search of existing and novel jet substructure observables, we find that a multivariate approach can filter out over 95% of the gluon jets while keeping more than half of the light-quark jets. Moreover, a combination of two simple variables, the charge track multiplicity and the $p_T$-weighted linear radial moment (girth), can achieve similar results. While this pair appears very promising, our study is only Monte Carlo based, and other discriminants may work better with real data in a realistic experimental environment. To that end, we explore many other observables constructed using different jet sizes and parameters, and highlight those that deserve further theoretical and experimental scrutiny. Additional information, including distributions of around 10,000 variables, can be found on the website [http://jets.physics.harvard.edu/qvg](http://jets.physics.harvard.edu/qvg).

The Large Hadron Collider at CERN produces billions of jets a second. Understanding these jets may be the key to unraveling physics beyond the standard model. The jets at the LHC can be coarsely partitioned into quark or gluon jets. Almost always, the jets we would like most to study are quark jets and so removing the gluon jet background, if possible, could greatly enhance the search reach for many new physics scenarios. In this letter, we show that through a comprehensive study of light quark (uds) vs gluon discriminants, many of which take advantage of the improved resolution of the LHC detectors, gluon-tagging with good efficiency is achievable on an event-by-event basis.

There are many situations in which quark/gluon tagging would be useful. For example, the jets produced in supersymmetric decay chains are usually entirely quark jets, while their backgrounds are mostly gluon jets. Quark tagging would be especially helpful in cases like R-parity violating SUSY where there are no additional handles like leptons, photons or missing energy. Even when there are leptonic decay modes, quark tagging could help measure the hadronic branching ratio of new physics, which will be needed to verify the model. Interesting standard model physics also tends to be quark-heavy. For example, vector-boson fusion requires the tagging of two forward jets which are always quarks, while background jets in this forward region tend to be dominated by gluons. Alternatively, there are some scenarios where the new physics is in gluon jets (e.g. coloron models [1], which produce 4 or more gluon jets), for which gluon tagging would also help.

Quark and gluon jets are an extremely useful abstraction, discussed in hundreds of papers and many experimental studies, despite their not having a precise theoretical or experimental definition [2]. Any flavor tagging is only meaningful to the extent that there is a correspondence between hard partons and jets, which is the standard starting point for almost every collider search involving hadronic final states. The correspondence is affected by things like the jet algorithm, the event’s topology, and the distance between energetic deposits. In this paper, quark or gluon jets refer to the parton which is produced in the hard process at leading order in perturbation theory and initiates the parton shower. At this level there is no ambiguity in what is meant by the jet flavor since there is no interference between different final states. In fact, the flavor is well-defined to to all orders in perturbation theory up to the same power corrections that affect any collinear and IR safe jet algorithm’s parton correspondence [3]. These power corrections involve the jet size $R$ (equivalently the jet’s mass-to-energy ratio $m/E$) and $\Lambda_{QCD}/E$.

In this paper, we study pure quark or gluon samples and only consider observable properties of these jets. The expectation is that properties of jets coming from, say, SUSY decays will be more similar to the properties of the quark jets in our simulation, despite the fact that jet properties are not expected to be completely universal – jet substructure is affected by things like adjacent jets and differing color-connections. The result of this paper is a rough ranking of jet observables that can be used to distinguish light flavor and help find new physics signals. Once promising observables are found and their distributions measured, it might be possible to construct an even more powerful tagger.

Gluon tagging at the LHC is both more useful and achievable than at the Tevatron. The LHC’s proton-proton initial state, higher energy, and higher luminosity increase the number of jets produced and make gluon jet backgrounds more common. CMS’s particle flow [4] and ATLAS’s individually calibrated TopoClusters [5], along with each detector’s improved calorimeter resolution, allow unprecedented measurement of the energy and track distribution within jets. As we will see, the better the resolution on the jet constituents, the better the tagger will be. We also find that the higher $p_T$ jets of the LHC are more reliably tagged than lower $p_T$ jets of previous colliders, as long as the tracking remains reliable.

Much is known experimentally about quark and gluon jets (see [6] for a summary). One important LEP result
was that $b$-jets were more similar to gluon jets than to light-quark jets \cite{7,8}: due to the longer decay chain of $B$-hadrons, the number of particles and angular spread is larger for a $b$-jet than a light-quark jet. The similarity of $b$-jets to gluon jets should be lessened in the LHC’s higher $p_T$ jets because the QCD shower produces more particles, whereas the particle multiplicity is relatively fixed in the $B$-hadron decay. There are already sophisticated and very detector-specific methods for $b$-tagging. Current $b$-taggers rely mostly on impact parameters or a secondary vertex, so they are independent of the observables we consider. Therefore, we restrict our study to discriminating light quarks (uds) from gluons.

The accumulated knowledge from decades of experiments and perturbative QCD calculations have been incorporated into Monte Carlo event generators, in particular Pythia \cite{9} and Herwig \cite{10}. These programs also include sophisticated hadronization and underlying event models which have also been tuned to data. Small differences still exist between these tools (and between the tools and data), but they provide an excellent starting point to characterize which observables might be useful in gluon-tagging. The approach to gluon-tagging discussed here is to find observables which appear promising and then can be measured and calibrated on samples of mixed or pure quark or gluon jets at the LHC \cite{3}.

To understand the structure of a jet, it is important to distinguish observables which average over all events from observables which are useful on an event-by-event basis. One example of an averaged observable is the classic integrated jet shape, $\Psi(r)$, which has already been measured at the LHC \cite{11}. This jet shape is defined as the fraction of a jet’s $p_T$ within a cone of radius $r$. Traditionally, jet shapes are presented as an average over all jets in a particular $p_T$ or $\eta$ range. For any $r$, the averaged jet shape becomes a single number, which is generally larger for quarks than for gluons because a greater fraction of a typical quark jet’s $p_T$ is at the center of the jet. On traditional jet shape plots, error bars for each $r$ are proportional to the standard deviation of the underlying distribution, but that distribution is not a narrow Gaussian around the average. For example, the event-by-event distributions for $\Psi(r=0.1)$ are shown in Figure 1 for quarks and gluons. Jet shapes averaged over these distributions throw out useful information about the location and $p_T$’s of particles within the jet, along with their correlations. For event-by-event discrimination, it is crucial to have distributions, whereas most public data only describes averages. In this study we consider $\Psi(r)$ and many other variables to see which are best suited to quark/gluon tagging.

To generate samples of quark and gluon jets we considered samples of dijet events and $\gamma$+jet events. These were generated with Madgraph v4.1.26 \cite{12} and showered through both Pythia v8.140 \cite{9} and Herwig++ v2.4.2 \cite{10} with the default tunes. Jets, reconstructed using FastJet v2.4.2 \cite{13}, were required to have $|\eta| < 1$.

We needed to isolate samples of quark and gluon jets with the similar jet $p_T$’s. Unfortunately, we cannot get similar jet $p_T$’s by having similar $p_T$’s at the hard parton level, since the showering changes the $p_T$ significantly, and differently for quarks and gluons. This is an unphysical difference, since the parton $p_T$ is set artificially, and we have to avoid our tagger picking up on it. The solution we chose was to generate and shower a wide spectrum of dijet and $\gamma$+jet events, and require the resulting Anti-$k_T$ R=0.5 jets to lie within 10% of the central value for each of six $p_T$ windows, centered around 50, 100, 200, 400, 800, and 1600 GeV. (The underlying hard partons spanned a range from half to twice the central value.) The $p_T$ spectrum within each window matches the falling spectrum of the underlying dijet or $\gamma$+jet samples, which are nearly identical for quarks and gluons in narrow windows chosen. When the entire event is reclustered with a different jet size, as was done when examining how the observables change with $R$, the resulting jet $p_T$ no longer necessarily lies within the narrow $\pm 10\%$ window. In fact, how the jet $p_T$ changes with $R$ forms a quark/gluon discriminant similar to integrated jet shape.

With each sample of similar-$p_T$ jets, there are two main types of observables useful in separating quarks from gluons: discrete ones, which try to distinguish individual particles/tracks/subjets, and continuous ones that can treat the energy or $p_T$ within the jet as a smooth function of $(\delta \eta, \delta \phi)$ away from the jet axis in order to form combinations like geometric moments.

The discrete category includes the number of distinguishable tracks, small subjets, or reconstructed parti-
cles. Functions of this information, such as the average or the spread (standard deviation) of their $p_T$’s were considered. We find that this class of observables provide the best discrimination at high quark efficiency (mild cuts) and high jet $p_T$.

The average multiplicity of any type of particle, along with its variance, are sensitive to the QCD charges of the underlying gluon ($C_A = 3$) or quark ($C_F = 4/3$). To leading order,

$$\frac{\langle N_p \rangle}{\langle N_g \rangle} = \frac{C_A}{C_F} \quad \text{and} \quad \frac{\sigma_p^2}{\sigma_g^2} = \frac{C_A}{C_F}. \quad (1)$$

The OPAL collaboration, among others, studied the charged particle multiplicity in light-quark and gluon jets of energy around 40 GeV to 45 GeV [13] and found distributions that agree well with the Monte Carlo event generators and with analytic predictions.

We find that the strongest discrete observable is the number of charged particles within the jet, where charged particles were required to have $p_T > 500$ MeV. Lower cutoffs actually lead to better discrimination power, so how well the LHC detectors will be able to resolve particle $p_T$ will have important consequences for gluon tagging.

Another discrete observable is the subjet multiplicity, which was also studied at LEP [13, 16]. Extremely small subjets approach the limit of particles and are sensitive to hadronization, but larger subjets probe the better modeled, perturbative physics and give the largest ratio between quark and gluon subjet multiplicities. For the higher-energy jets of the LHC, the optimal jet size is far smaller than the calorimeter resolution. We found that counting $R_{mb}=0.1$ anti-$k_T$ jets was more powerful than other subjet algorithms and larger sizes, but not as powerful as counting the number of charged tracks. Subjet multiplicity can serve as a reasonable substitute if charged track multiplicity proves less reliable in some circumstances (perhaps at very high $\eta$). Counting all hadrons works even better than charged tracks.

Other observables in the discrete category that show reasonable discrimination power include the average distance to jet axis $\langle r \rangle$, the $p_T$ fraction of the $N$th hardest track or subjet, and the subjet splitting scale (when the jet is reclustered with the $k_T$ algorithm). Finally, there are observables that take advantage of the electrical charge that quarks carry. Since the hardest hadrons produced at the end of the shower have charges correlated with the initiating quark, adding up the charges of all tracks weighted by their $p_T$ gives some small discrimination.

The second, more continuous, category of observables includes jet mass, jet broadening [17], and the family of radial moments like girth [18], angularities [19], and the optimal moment which are described below. These tend to perform better at lower jet $p_T$ and for achieving high quark purity through harsh cuts. Other observables that try to capture the 2D shape or color connections of the jet, like pull [20], eccentricity, or planar flow [21], are less powerful in this application.

We find the best single observable in the continuous category is the linear radial moment – a measure of the ‘width’ or ‘girth’ of the jet – constructed by adding up the $p_T$ deposits within the jet, weighted by distance from jet axis. It is defined as

**Linear Radial Moment (Girth):**

$$g = \sum_{i \in \text{jet}} \frac{p_{Ti}}{p_T} |r_i| \quad (2)$$

where $r_i = \sqrt{\Delta y_i^2 + \Delta \phi_i^2}$ and where the true boost-invariant rapidity $y$ should be used for the (massive) jet axis instead of the geometric pseudorapidity $\eta$. Under the assumption of central jets with massless constituents at small angles, this linear moment is identical to jet broadening, defined as the sum of momenta transverse to the jet axis normalized by the sum of momenta. While jet broadening is natural at an $e^+e^-$ collider, the linear radial moment is more natural and works a bit better at the LHC. Other geometric moments involving different powers of $r$ were not as powerful at discriminating quarks from gluons, including the jet mass, which is equal to the $r^2$ geometric moment in the same limit.

By weighting the $p_T$ by other functions of $r$, whole families of radial-kernel observables can be constructed:

**Kernel Moment :**

$$K = \sum_{i \in \text{jet}} \frac{p_{Ti}}{p_T} K(r_i) \quad (3)$$

Angularities [19, 21] are one such example, where the $p_T$ and $r$ are usually replaced by energy and angle. Angularities are often normalized by the jet mass rather than the jet $p_T$, and we considered both normalizations. Both angularities and kernels which are powers of $r$ suffer from sensitivity to the edge of the jet where their kernels are greatest. This becomes problematic in crowded environments with adjacent jets.

Rather than try to guess a useful kernel, we attempted to optimize its shape numerically. By parameterizing the kernel as a spline with 5 to 10 points, a genetic algorithm was used to maximize gluon rejection for several different quark efficiencies. In all cases, the optimal kernels rose linearly from the axis of the jet out to $r \sim 0.3$, then turned over and decreased smoothly to zero at the edge of the jet, but the gluon rejection in call cases was rather insensitive to the region away from the center. These optimal kernels performed slightly better than the linear radial moments, but not enough to justify additional focus here.

By looking at combinations of observables, additional quark/gluon discrimination is achieved. The 2D histograms for the best discrete and continuous observables, charged particle count and the linear radial moment, are shown in Figure 4. While the two observables are correlated, it is still helpful to use both. In the third panel
of this figure, we show the 2D bin-by-bin likelihood distribution. Given these variables, the discriminant that achieves optimal gluon rejection for a fixed quark efficiency is a simple cut on the appropriate likelihood contour. Cutting out the top-right corner, for example, eliminates the most egregiously gluey jets. In practice, this can be pre-computed or measured in each jet $p_T$ window. As part of jet energy scale calibrations, ATLAS [22] has measured these two variables in dijet, $\gamma$-jet, and multijet samples and used them individually to determine the flavor composition to 10% precision.

The same method can be applied for more than 2 observables, but then the exact likelihood becomes impossible to map efficiently with limited training samples. A multivariate technique like Boosted Decision Trees can be employed to approximate this multidimensional likelihood distribution, as explained in [18].

In summary, quite a number of single variables do comparably well, while some (like pull or planar flow) do quite poorly at gluon tagging. We examined many combinations of observables, and found significant improvement by looking at pairs, but only marginal gains beyond that. The results for the gluon rejection as a function of quark efficiency are shown for a number of the more interesting observables and combinations in Figure 3 for 200 GeV jets. The relative performance of variables changed little with $p_T$ even though the optimal cuts do. Definitions and distributions of these variables, and thousands of others, can be found on [http://jets.physics.harvard.edu/qvg](http://jets.physics.harvard.edu/qvg) Good pairs of variables included one from the discrete category described above, such as particle count, and one more continuous shape variable, like the linear radial moment (girth).

As an example using these curves to estimate the improvement in a search’s reach, consider $X \rightarrow WW \rightarrow q\bar{q}q\bar{q}$ whose background is mostly 4-jets from QCD, each of which is a gluon 80% of the time [3]. By operating at 60% quark efficiency, only $1/10^{13}$ of gluons pass the tagger, which means $(20\%)^4$ of the total QCD background passes. One measure of statistical significance in a counting experiment is $S/\sqrt{B}$, perhaps within a particular invariant mass window. Any starting significance can be improved by a factor of 3.2 using these cuts. The 60% operating point was chosen to maximize this significance improvement for this particular background composition, which highlights the need to characterize background rejection for all signal efficiencies.

Measurements of these variables are underway, but it would be very interesting to see distributions of and correlations between as many of the variables in Figure 3 as possible. To this end, it has recently been observed that 99% pure samples of quark jets can be obtained in $\gamma+2$jet events, and 95% pure samples of gluon jets can be
obtained in 3-jet events [3]. These samples could provide a direct evaluation of the tagging technique at all jet $p_T$, verify and help improve the Monte Carlo generators, and provide a test of perturbative QCD.

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