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Photon sorters and QND detectors using single photon emitters

D. Witthaut,1, 2 M. D. Lukin,3 and A. S. Sørensen1
1QUANTOP, Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark
2Max-Planck-Institute for Dynamics and Self-Organization, D-37073 Göttingen, Germany
3Department of Physics, Harvard University, Cambridge MA 02138
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We discuss a new method for realizing number-resolving and non-demolition photo detectors by strong coupling of light to individual single photon emitters, which act as strong optical non-linearities. As a specific application we show how these elements can be integrated into an error-proof Bell state analyzer, whose efficiency exceeds the best possible performance with linear optics even for a modest atom-field coupling. The methods are error-proof in the sense that every detection event unambiguously projects the photon state onto a Fock or Bell state.

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Experimental realization of number-resolving, non-demolition photo (QND) detectors is a long-standing challenge in quantum optics and quantum information science. Conventional photodetectors measure only the intensity or the energy of an incoming light pulse, and are not capable of measuring photon states in a QND fashion. More advanced measurements schemes can be constructed using optical non-linearities, but these are typically very weak since photons rarely interact with each other. In this Rapid Communication, we show how to overcome this problem by strong coupling of light to individual single photon emitters. This provides a strong optical non-linearity, which enables the realization of number-resolving photon sorters and quantum non-demolition photo detectors. A common approach to realizing strong coupling between photons and atoms relies on cavity QED, where the light field is confined to a high–Q resonator [1–3].

In order to reach the strong coupling regime in small integrated devices, great advances have been made using photonic crystals [4, 5], tapered optical fibers coupled to a single atom [6, 7], microwave transmission lines coupled to a flux qubit [8], or surface plasmons modes coupled to a single photon emitter in free space [16, 17]. In the present paper we will explore possible applications of emitters coupled to such a one-dimensional photonic continuum for photo detection, but the ideas and formalism we use can also be applied to cavity QED as well as other methods of achieving strong optical non-linearities [18–20].

First, we consider passive devices based on simple two-level emitters. The interaction with the emitter naturally leads to a photon turnstile effect, which can be used to implement a number resolving photon sorter. Secondly, we consider a waveguide coupled to a three-level emitter controlled by a classical laser field. This setup offers significantly more opportunities at the expense of a more complex optical setup. In particular we discuss QND photo detectors. As a possible applications of these devices we will show how to construct optical Bell-state analyzers. A Bell measurement is an essential ingredient in quantum information, as it enables efficient quantum repeaters [21] as well as universal quantum optical computers [22]. Unfortunately such a measurement cannot be realized with linear optics [23], but requires a strong nonlinearity. We focus on realistic systems with losses and discuss how to make devices error-proof. Even in the case of an error, the measurement shall at most give an inconclusive but never a wrong result.

One of the conceptually simplest extensions of linear optics is a device capable of non-destructively distinguishing single and two photon pulses. Such a photon sorter can be realized with only passive optical elements and simple two level emitters using the setup sketched in Fig. 1. The procedure is most easily explained if the emitters are coupled to semi-infinite waveguides extending only in one direction, but can also be realized with infinite waveguides if more optical elements are used [11–13].

We assume that the incoming pulse enters the interferometer in the upper arm labelled by $\hat{a}_{in}$. A single photon is split at the beam splitter and brought to interact with the emitters, where it experiences a phase shift $\hat{t}_k$:

$$t_k = \frac{ck - \hbar \omega_0 + i\hbar(\gamma - \Gamma)/2}{ck - \hbar \omega_0 + i\hbar(\gamma + \Gamma)/2},$$

where $k$ is the photon wavenumber. Similar to the quantum jump approach [24] we have added an imaginary part to the resonance frequency of the emitter $\omega_0 - i\gamma/2$ describing a coupling of the emitter to other modes than the waveguide with a rate $\gamma$. The coupling strength is given by the rate of spontaneous emission into the waveguide $\Gamma$. The phase shift $\hat{t}_k$ depends on the wavenumber $k$ but is the same in both arms of the interferometer. The interferometer can therefore be balanced such that a single photon always leaves the setup in mode $\hat{a}_{out}$. This is
different if two photons enter the setup and interact in-
the frequency width $\sigma$ of a single emitter. A single photon will always be emitted in 
mode $\hat{a}_{\text{out}}$ while two photons are likely to be emitted in mode 
$\hat{b}_{\text{out}}$ but never split into one photon in each arm. (b) The 
success probability of the photon sorter, i.e. the probabil-
ity that two photons are scattered to the mode $\hat{b}_{\text{out}}$, as a function of the 
frequency width $\sigma$ of a Gaussian input pulse.

The beam splitter mixes the modes $\hat{a}$ and $\hat{b}$ such that 
$\hat{a}^+_k \hat{a}^+_p \rightarrow \hat{a}^+_k \hat{a}^+_p + 2 \hat{a}^+_k \hat{b}^+_p + \hat{b}^+_k \hat{b}^+_p$. When the photons then interact with the same emitter, this introduces a strongly correlated 'bound state' contribution

$$f_1(k) f_1(p) \rightarrow t_k t_p f_1(k) f_1(p) + f_B(k, p),$$

whose precise form can be found in Refs. [13 14]. Finally, after interacting with the beam splitter once again, the two-photon input state [2] is transformed to

$$|\Psi_{\text{out}}\rangle = \frac{1}{2\sqrt{2}} \int dk dp f_B(k, p) \hat{b}^+_k \hat{b}^+_p |0\rangle$$

Note that there are no mixed terms (e.g. $\hat{a}^+_k \hat{b}^+_p$), so that the two photons always leave the interferometer in the same arm. There is a significant probability that the two photons leave the setup in mode $\hat{b}_{\text{out}}$, whereas a single photon always leaves the interferometer in mode $\hat{a}_{\text{out}}$, such that the interferometer acts as a photon sorter.

The success probability of the photon sorter, i.e. the probability that two photons are scattered to the mode $\hat{b}_{\text{out}}$ is given by

$$p_s = \frac{1}{4} \int dk dp \| f_B(k, p) \|^2.$$
The photon detection schemes discussed above may be used in a variety of contexts where the measurement of more involved properties of light is required. A particularly important application is the design of an optical Bell state analyzer, which distinguishes the four Bell states

\[ |\phi^\pm\rangle = \frac{1}{2} \int dk \int dp f_1(k)p \left( \hat{a}_{1k} \hat{a}_{3p} \pm \hat{a}_{2k} \hat{a}_{4p} \right) |0\rangle \]

\[ |\psi^\pm\rangle = \frac{1}{2} \int dk \int dp f_1(k)p \left( \hat{a}_{1k} \hat{a}_{4p} \pm \hat{a}_{2k} \hat{a}_{3p} \right) |0\rangle, \]

where the subscripts 1–4 refer to four different photonic modes. In principle, a BSA can be achieved directly from the scheme for photonic quantum gates in cavity QED [3]. Such setups, however, often require rapid switching of the optical path and delay lines for photons, which is experimentally unfavorable. Here, we consider a modified version of Ref. [3], which avoids these elements and integrate it into a BSA. This setup shall be efficient and error-proof even for an imperfect coupling, i.e. \( \gamma \neq 0 \), so that it cannot give a wrong measurement result.

For resonant input photons, the photo QND detector introduced above is sufficient to realize a simple error-proof BSA using the setup shown in Fig. 2 (b). We assume that the logical state of both control and target photon are encoded into two spatial modes. The control photon (modes \( \hat{a}_{1,2} \)) passes the setup well before the target photon (modes \( \hat{a}_{3,4} \)). For the Bell states \( |\phi^\pm\rangle \), both photons pass the same arm of the setup subsequently. The emitter coupled to this arm is transferred from state \( |g\rangle \) to \( |s\rangle \) and back to state \( |g\rangle \) after interacting with the control and target photon, respectively. The other emitter always remains in the internal state \( |g\rangle \). On the contrary, the two photons pass through different arms of the interferometer for the Bell states \( |\psi^\pm\rangle \), so that both emitters are transferred to the state \( |s\rangle \). A measurement of the internal state of the emitters thus allows to distinguish between the subspaces spanned by \( |\phi^\pm\rangle \) (atomic state \( |gg\rangle \)) on the one hand and \( |\psi^\pm\rangle \) (atomic state \( |ss\rangle \)) on the other hand. Whether it is the plus or the minus sign is revealed by the coincidence pattern of detectors placed after a beamsplitter mixing the modes 1 and 2 as well as 3 and 4. Taking into account photon loss, the success probability of this BSA is given by

\[ p_{\text{success}} = \eta^2 |t_0|^2 \int dk dp |f_2(k,p)(1-t_k)(1-t_p)|^2 \]  

regardless of which of the Bell states is incident. This result (with \( \eta = 1 \)) is plotted in Fig. 2 (c) as a function of the loss rate \( \gamma/\Gamma \) for Gaussian input pulses. One finds that a Purcell factor of \( \Gamma/\gamma \approx 5.8 \) is sufficient to exceed the \( \eta^2 \times 50 \% \) limit of linear optics.

The present setup is, however, not strictly error-proof if the input photons are not completely resonant. While the measurement result \( |ss\rangle \) leads to an unambiguous Bell state measurement, the result \( |gg\rangle \) does not. It can be almost certainly attributed to the subspace spanned by \( |\phi^\pm\rangle \), but there is a small probability that it has been triggered by the states \( |\psi^\pm\rangle \). This error can, however, be suppressed to a large extent by choosing a rotation angle of \( \alpha = 1/(1-t_0) \). In this case the residual probability to obtain an erroneous measurement result for the input state \( |\psi^\pm\rangle \) is given by

\[ p_{\text{error}} = \eta^2 |t_0|^2 \int dk dp |f_2(k,p)(1-t_k)(1-t_p)|^2. \]  

For a Gaussian wavepacket, \( p_{\text{error}} \) vanishes as \( \sigma^4/(\gamma+\Gamma)^4 \) for \( \sigma/(\gamma+\Gamma) \to 0 \). It thus remains small also for a non-monochromatic input photon as shown in Fig. 2 (d).

In order to realize a fully error-proof BSA, one needs another measurement stage, which unambiguously detects \( |\phi^\pm\rangle \). In principle this can be realized by exchanging the modes \( \hat{a}_1 \) and \( \hat{a}_2 \) and then repeating the above scheme [23]. This would, however, require rapid switching of the optical path between control and target photon.

As we will now show, a fully passive, error-proof BSA may in fact be constructed using the photon sorters introduced above. The setup to perform these operations is summarized in Fig. 3 (a). Assume that the four optical modes containing the Bell state in Eqn. 8 are incident on a beamsplitter array mixing the modes 1 and 4 as well as 2 and 3 (denoted by BS1 in Fig. 3). The states \( |\psi^\pm\rangle \) are mapped onto \( \left( \hat{a}_1^\dagger \hat{a}_4^\dagger \pm \hat{a}_2^\dagger \hat{a}_3^\dagger \right) |0\rangle \), suppressing the pulse shape for simplicity. The two photons are always located in the same mode for the states \( |\psi^\pm\rangle \), whereas they are always located in two different modes for the
states $|\phi^{\pm}\rangle$. If each of the modes is now incident on the photon sorter introduced above, the states $|\psi^{\pm}\rangle$ are separated to the modes $b_{1,...,4}$ with a significant probability. It is then possible to distinguish between $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$ with linear optics and conventional photodetectors only, giving rise to an unambiguous Bell state measurement. If no photon is detected we can simply go on with the two photons in the modes $a_{1,...,4}$. In order to detect also the Bell states $|\phi^{\pm}\rangle$, one undoes the effect of the first beam splitter array BS$_1$ and then mixes the modes 1 and 3 as $c_{1,...,4}$ by a linear optical BSA and subsequently detected. The photons are either measured in the two detector arrays projecting unambiguously onto one of the Bell states or leave the BSA in the modes $a_{1,...,4}$. In the latter case another measurement can be attempted.

The proposed BSA works probabilistically — with a non-vanishing probability the photons are not detected but just transmitted through the complete setup. The success probability is given by the product of two photons to be scattered to the modes $b_{1,...,4}$ or $c_{1,...,4}$, respectively, and thus given by the success probability of the photon sorter, which is given in Eqn. (5) and plotted in Fig. 3(b). If the detection fails and the photons are transmitted, one can just repeat all operations. However, in actual experiments photon losses are inevitable and the coupling of the emitter to the one-dimensional waveguide is not perfect. Photon loss only leads to an inconclusive measurement result and Fig. 3(b) shows the resulting success probability of an array of 1 (dashed line) and 5 (solid line) concatenated BSAs, as a function of the ratio $\gamma/\Gamma$. After passing this array, the remaining modes are detected with a linear optical BSA. As shown in the figure already a modest Purcell factor of $\Gamma/\gamma \approx 3.3$ is sufficient to exceed the 50% limit of linear optics.

In conclusion we have shown that the coupling of single emitters to one-dimensional waveguides opens up new possibilities for number resolving, non-demolition photo detection. We have explicitly shown how to construct photon sorters and QND detectors, and that these systems can be used for efficient Bell state analysis. Most importantly the devices are error proof in the sense that imperfect coupling only leads inconclusive and not wrong results. As a consequence the devices work with modest coupling efficiencies, which are well within reach of current experiments.

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