



Numerical Studies in the Formation of Planetary Systems: What We Can Learn from Inclinations and Angular Momenta

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Numerical Studies in the Formation of Planetary Systems: What We Can Learn from Inclinations and Angular Momenta

A dissertation presented

by

Matthew Zachary Heising

to

The Department of Astronomy

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

Astronomy & Astrophysics

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Numerical Studies in the Formation of Planetary Systems: What We Can Learn from Inclinations and Angular Momenta

Abstract

This thesis presents numerical work performed to answer a few targeted questions in planet formation theory, applied both to specific systems and more generally. To put these questions in context, it begins with a broad overview of the current state of the field of planet formation, as motivated by observations both within and outside the Solar System.

From there, in Chapters 2 and 3, I turn to the study of two particular systems of interest, the TRAPPIST-1 and Kepler-90 systems. The inclination dispersions of these systems are quantified, and it is shown that their remarkable flatness, particularly that of the TRAPPIST-1 system, makes them outliers among the known planetary systems. Motivated by this, a numerical experiment is performed for each of these systems in which we use hydrodynamic simulations of differently massed disks, together with the planets, to show that the systems' flatness places upper bounds on the masses of their respective protoplanetary disks. Implications of this upper bound are discussed with attention to specific formation models, and some avenues for the formation of these systems are ruled out.

In Chapter 4, I present a more general discussion of angular momentum deficit (AMD). In particular, I show that breaking it up into components depending solely on inclinations or solely on eccentricities can reveal underlying dynamics in certain contexts in which AMD is not conserved over time. As an illustrative example, N-body simulations of a Nice model scenario for the late-stage evolution of the Solar System are performed, which show how one such component of AMD can at times dominate over the other, and how this can evolve with time. Some consequences for planet formation from this analysis are qualitatively explored.

The thesis concludes with an overview of the aforementioned work and a discussion of possible next steps to which this work paves the way.

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1

Introduction

To introduce the work in my doctoral thesis, I will start by giving an overview of planet formation theory and relevant observations, with the goal of motivating the need for highly precise three-dimensional (3D) hydrodynamic simulations of protoplanetary disks as presented in this work.

1.1 Classical Planet Formation

Before the discovery of the first exoplanet around a Sun-like star in 1995 (Mayor & Queloz 1995), the Solar System provided the only source of reference for those trying to unravel how planets were formed. This is not to say that the problem was in any way simple. The Solar System alone provided a rich data set, and even when the scope of the problem was constrained to only understanding its formation, there were several key physical features that presented some difficulty in explaining.

The most basic element of the Solar System that must be explained by any suitable theory of its formation is that the planets all orbit nearly in the same plane, and in the same direction. Of the eight planets in the Solar System, the one that is most inclined relative to the invariable plane - that is, the fixed plane that is perpendicular to the total angular momentum of the Solar System - is Mercury, with an inclination of 6.34° . Of the giant planets, which contain the vast majority of the angular momentum of the system, the orbits are even more coplanar, with the largest deviation being Uranus' inclination of 1.02° . Furthermore, the invariable plane is closely aligned with the Solar equator - that is, the Sun rotates on the same axis and in the same direction on which the planets orbit it.

The long-accepted explanation of this apparent coincidence in the orbits of the Solar System planets is what was described as the “nebular hypothesis,” of which the first serious quantitative scientific description dates back to Laplace (1796). In

this framework, to understand planet formation one needs to first go back to the formation of stars. The environments in which stars form are molecular clouds, which are large clouds consisting of mainly gas and some dust. When a region of such a cloud exceeds a critical density, it will begin to collapse under its own gravity - in the context of the nebular hypothesis, this collapsing material is known as the nebula. As molecular clouds are non-uniform, the nebula will inevitably contain some nonzero net angular momentum. Due to conservation of angular momentum, as the nebula contracts into what would become a star, the rate of rotation drastically increases. Eventually, this rotation is fast enough that gravity will not be able to hold all of the matter in a sphere, causing much of the material to flatten out into a rotating disk around the protostar. In the nebular hypothesis, it is from this disk, now referred to as a “protoplanetary disk,” that planets eventually form, which naturally explains the observable fact that the orbits of the Solar System planets and the rotation of the Sun all occur on similar axes.

There was however a problem with early versions of the nebular hypothesis that plagued theorists for much of the 19th and 20th centuries (Woolfson 1993). In the initial model, planets formed from the direct collapse of annular rings that broke off from the protostar due to its fast rotation. If the Solar System planets and the Sun formed from the same material in this way, it wasn’t understood why the Sun, which contained 99.86% of the total mass of the system, would contribute less than 1% of the total angular momentum, with the vast majority contained in the orbits of the planets.

A breakthrough came from exploring in more detail exactly how planets would form in a protoplanetary disk. As proposed by Lyttleton (1961), the dust in the protoplanetary disk that has been inherited from the initial molecular cloud would naturally flatten out into the midplane of the disk due to drag from the surrounding gas. Confined to the midplane, collisions between dust particles would occur much more frequently, leading to coagulation. Subsequent collisions would allow for the growth of solid bodies up to roughly 1 km in size, referred to as “planetesimals.”

Safronov (1972) further explored the evolution of these planetesimals. Due to their gravitational influence on each other, these planetesimals would excite eccentricities in their orbits. These more eccentric orbits would lead to collisions between planetesimals, leading to either fragmentation or aggregation, depending on the rel-

ative masses and velocities of the two bodies. Through this process, including the effect of gravitational focusing, much larger bodies could be grown, up to the size of terrestrial planets.

The giant planets of the Solar System required some further explanation. A core accretion model for their formation was developed by Perri & Cameron (1974) and Mizuno (1980). In this model, a solid core is initially formed, under the same processes as described by Safronov (1972). If this core grows large enough, it can create a hydrodynamic instability in the mostly gaseous protoplanetary disk, leading to very efficient gas accretion. Eventually, accretion is terminated by the planet opening a gap in the disk due to its mass. With this model, the formation of gas giants became quite feasible.

There was another fundamental feature of the Solar System that still needed to be explained, however. Why was it that the four outer planets were so much more massive than the four inner planets, which evidently were not able to efficiently accrete gas? The solution came in the form of what is known as the “ice line.” As temperature would decrease with radius in the protoplanetary disk, there is a certain radius beyond which (treating the disk as roughly two-dimensional) a particular compound, such as H_2O or CO , would freeze out and exist in the solid state. These compounds, known as “volatiles,” contributed a large amount to the amount of solid matter available past their respective ice lines, allowing for the faster growth of planetary cores, and thereby explaining the discrepancy in the mass and composition of the inner vs. outer planets.

Hayashi (1981), building off of the work of Weidenschilling (1977), employed the concept of the ice line to work backwards from the composition of the Solar System planets to create an estimate of how much mass, as a function of radius, would have been necessary for the initial protoplanetary disk to contain in order to form the planets. This was coined the “Minimum Mass Solar Nebula” (MMSN), and represented the first meaningful constraint on the mass of the Solar protoplanetary disk. From simple facts about the Solar System planets’ orbits, masses, and compositions, a theory had been created that was not only largely successful in describing their formation, but also implied the presence of a mostly gaseous disk that had since dissipated, and yielded important predictions for its surface density profile.

1.2 Lessons From Exoplanets

It is an interesting historical coincidence that the classical theory of planet formation did not have that much time to mature before it was dramatically challenged by the discovery of the first exoplanets. The first exoplanet around a Sun-like star, 51 Pegasi b (Mayor & Queloz 1995), was discovered to have a minimum mass of 0.5 Jupiter masses but a semimajor axis of only 0.05 AU, significantly closer-in to its host star than even Mercury is to the Sun. Clearly this did not fit in the paradigm of giant planets existing beyond the ice line. Several more similar planets, dubbed “hot Jupiters,” were discovered shortly thereafter (Udry et al. 2000), with some “warm Jupiters,” further out but still within their theoretical snow lines, following suit (Marcy & Butler 2000). While these hot Jupiters were the easiest types of planets to detect with the available methods and were indeed later shown to be outliers, their existence nevertheless had to be explained.

As it turns out, the theory to explain the existence of these hot/warm Jupiters was already mainly in place. Lin & Papaloizou (1979) and Papaloizou & Lin (1984) showed that a protoplanet with a mass similar to that of Jupiter will exert a torque on a protoplanetary disk large enough to open a gap. Once this gap is opened, the planet becomes tied to the viscous evolution of the disk. As angular momentum in the disk is lost due to viscosity and the gas moves inward, so too will the planet be forced to migrate inward. This form of planetary migration became known as Type II migration. Because the planet becomes tied to the viscous evolution of the disk, the migration speed v depends on the disk viscosity as

$$v = -\frac{3\nu}{2r} \quad (1)$$

where ν is the disk’s kinematic viscosity and r is cylindrical radius (Armitage 2010).

Type II migration had not previously been thought to be an important part of traditional planet formation models. However, the discovery of hot Jupiters brought it much greater attention as a means of creating Jupiter-mass planets close to their stars. Moreover, more detailed calculations of gas accretion of giant planets showed that they could sometimes reach close to their final mass significantly before their protoplanetary disk had been dispersed (Pollack et al. 1996). With this in mind, Lin et al. (1996) was successfully able to explain the unusual orbit of 51 Pegasi b through traditional planet formation theory with the help of Type II migration. This became

the default explanation for several other hot and warm Jupiters. With this new theoretical framework, the existence of these planets, combined with observationally deduced typical lifetimes of protoplanetary disks, gave meaningful constraints on the viscosity of protoplanetary disks.

Migration models of planet formation continued to evolve over subsequent years. As some smaller bodies were discovered, small enough to not open gaps in their disks but too large to have formed in situ under conventional models, Type I migration (Goldreich & Tremaine 1979, 1980), which depends not on the disk’s viscosity but instead its surface density profile and the planet’s mass, was invoked to explain their migration (Terquem & Papaloizou 2007). In addition, the theory was later aided by the development of a “pebble accretion” model (Lambrechts & Johansen 2012) that implied a faster, more efficient core accretion for giant planets, allowing them more time to migrate. These developments allowed for yet more information about protoplanetary disks to be inferred from observations of exoplanets, under the assumptions of their formation models.

As exoplanet detection techniques matured, statistically relevant surveys of exoplanets could be conducted. These included radial-velocity surveys, which detected exoplanets through the Doppler shift of the host star’s orbit induced by the planet (Howard et al. 2010; Mayor et al. 2011), as well as the *Kepler* mission, which photometrically detected transits of exoplanets in front of their host stars (Borucki et al. 2010). With the surveys, theories of planet formation could be further tested.

One apparent discrepancy that developed from this influx of data, as pointed out by Hansen & Murray (2012), was that close-in planets were statistically less massive on average than would be predicted by population synthesis models (e.g. Howard et al. 2012) that assume giant planets form beyond the ice line and then migrate in. Furthermore, in this regime, the mass upper limit under which planets were observed to fall increased with semimajor axis. This led Hansen & Murray (2012) to develop an alternate model for the formation of close-in giant planets and super-Earths in which smaller bodies first migrate inward, giving the inner protoplanetary disk an enhanced supply of rocky material. This excess material would allow for the formation of larger planets through core accretion inside the ice line, making possible in situ formation of close-in giant planets. The observational basis for this model was that it would more naturally reproduce the distribution in close-in exoplanets of

mass vs. semimajor axis.

While this model solved some of the apparent problems of conventional migration models, it was unclear if the physical properties of typical protoplanetary disks would allow for it. Employing the model of in situ formation, Chiang & Laughlin (2013) used a similar technique to what had been used to construct the MMSN to calculate a “Minimum Mass Extrasolar Nebula” (MMEN) that would be required to form a typical exoplanetary system with close-in Super-Earths. If this MMEN, with relatively larger surface densities than in the MMSN, did indeed better reflect the typical protoplanetary disk that planets formed in, a number of other important consequences for planet formation would follow. Notably, if planetesimals were formed from dust in situ, it would be expected that close-in planets around sufficiently hot stars would not exist, as the local temperatures would cause dust to sublimate before it could form planetesimals. In addition, if the larger surface densities implied by the MMEN extended to larger radii than could be observationally constrained, that would also allow for the easier formation of gas and ice giants beyond the ice line.

In the last decade, no clear consensus between these two competing models has been reached. It is clear that this remains an important question for planet formation, not least because the implications for protoplanetary disks between the two models are significant.

As the number of detected exoplanet systems has drastically increased over the last decade, some more outlying cases have also become apparent. In some cases, these outliers have spurred more “exotic” formation models to explain their existence. One such system is the TRAPPIST-1 system (Gillon et al. 2017), which consists of seven Earth-size planets orbiting an M dwarf with a mass of $0.09 M_{\odot}$ (Van Grootel et al. 2018). Besides the large number of planets, there are several other features that make the system distinct from other known exoplanetary systems. All seven planets are closely-packed in a near-resonant chain. They are all of similar size and density, suggesting a composition that is similar to Earth’s but somewhat depleted in iron (Agol et al. 2021). Furthermore, the system is remarkably flat, with mutual inclinations significantly lower than what has typically been observed for other systems (Luger et al. 2017a - see also Chapter 2.2 of this work). As there is still a severe lack of data for planetary systems around M-dwarfs, it is impossible to say whether the TRAPPIST-1 system is an outlier among such systems, but it is

certainly an outlier when compared to the planetary systems that have been observed to date.

To explain such an unusual system, Ormel et al. (2017) proposed an equally unusual model for its formation. In their model, the TRAPPIST-1 planets form one by one, all initially at the H_2O ice line, due to streaming instabilities leading to the accumulation of pebbles in that region, enhancing the available solid material. The planets are able to grow quickly through pebble accretion, and are then transported to the inner disk via Type I migration. A planet’s inward migration will cease when it enters a mean motion resonance with the planet interior to it. Later, when the disk disperses and the magnetospheric cavity of the star expands, the planets are nudged slightly out of resonance. Importantly, due to the planets all forming in one pebble-enhanced region, the model does not require very high surface densities in the inner protoplanetary disk as a whole.

While this model is unconventional and might seem unnecessarily prescriptive, it is successful in explaining some of the characteristic features of the TRAPPIST-1 system. Notably, it naturally produces planets of similar composition that are close to but not quite in resonances with each other. The low mutual inclinations of the TRAPPIST-1 system are not directly addressed by the model of Ormel et al. (2017). However, as is shown in this thesis, they also suggest a relatively low-mass disk indicative of such a formation scenario. What such a model implies not only for the formation of the TRAPPIST-1 system but also about protoplanetary disks around M dwarfs is significant, especially if such planetary systems are found to be common around M dwarfs.

1.3 Lessons From Disks

As models of planet formation could reveal information about protoplanetary disks, so too was the inverse true. A major breakthrough came in the form of ALMA (ALMA Partnership et al. 2015b), a millimeter and submillimeter array of radio telescopes that could perform unprecedented high resolution observations of circumstellar disks. For the first time, detailed structure of circumstellar disks could be directly observed, enabling further calculation of disk properties and possible evidence of planet-disk interactions.

The first major success of ALMA was delivered very soon after its inception with observations of the young star HL Tau and its surrounding disk. The observations, at wavelengths of 2.9, 1.3, and 0.87 mm were sensitive to the thermal emission of dust particles in HL Tau’s disk. This dust emission revealed a detailed structure in the disk consisting of seven pairs of bright and dark rings. As such dark rings could apparently be explained by giant planets creating gaps in the disk, this discovery was putatively suggested to be the first direct observational evidence of planet formation in process. Further evidence for this theory came from the fact that the rings appeared to be elliptical in nature, implying that they were formed by planets on eccentric orbits, and that the radial locations of the rings seemed to indicate the presence of orbital resonances.

However, alternate theories to explain the observed rings in HL Tau’s disk were also proposed. Zhang et al. (2015) described a model in which the three main dark rings were formed not by planets, but rather by pebble growth near the condensation fronts of three different volatiles: H_2O , CO , and N_2 , each corresponding to a different radius in the disk. Alternatively, Takahashi & Inutsuka (2016) proposed a model in which the rings are instead formed by secular gravitational instabilities in the disk. Furthermore, even if the model of gap formation via dynamical planetary interactions were assumed, complex hydrodynamic and radiative transfer modeling of the gas and dust was required in order to infer the masses of the potential planets (Dipierro et al. 2015; Pinte et al. 2016; Jin et al. 2016). These competing models demonstrate the importance of detailed numerical modeling of disks that is required in order to meaningfully interpret the available observational evidence.

The imaging of HL Tau’s disk represented only the first of many successes of ALMA. The Disk Substructures at High Angular Resolution Project (DSHARP) presented dust continuum observations of 20 more circumstellar disks (Andrews et al. 2018). In the morphologies of each of these disks, a wide variety of substructure was revealed. These substructures include not only concentric rings similar to those observed in the disk of HL Tau (Huang et al. 2018a), but also non-axisymmetric structures such as crescents (Isella et al. 2018), spiral arms (Huang et al. 2018b), and misaligned inner disks Pérez et al. (2018). The evolving picture of the morphologies of these disks became quite more complex.

These new structures required new numerical modeling to explain their origin.

For example, while some spiral arm structures appeared to be induced by perturbations to the disk from an orbital companion (Wagner et al. 2015), while others are more indicative of formation by gravitational instability (GI) in the disk (Huang et al. 2020). To distinguish between these two models, detailed hydrodynamic simulations have been performed (Dong et al. 2016; Baehr & Zhu 2021). To adequately test these models, fully 3D hydrodynamic simulations are a necessity; in the case of spiral arms induced by an orbital companion, the companion will in general have some inclination relative to the disk, and the vertical temperature profile of the disk will also affect the morphology of the arms (Rosotti et al. 2020), while in the case of GI-induced spiral arms, global effects depending on the disk’s thickness contribute to the instability (Lodato & Rice 2004).

In combining what we can learn from our own Solar System, observations of extrasolar planets, and observations of circumstellar disks, the need for precise numerical modeling of protoplanetary disks is apparent. The moving-mesh hydrodynamic code AREPO (Springel 2010) is one such tool that is well-suited for such simulations, as its quasi-Lagrangian nature allows for 3D simulations of disks, simultaneously with planets, with low numerical viscosity and adaptable geometry.

The rest of this thesis is organized as follows. In chapter 2, I present work on the TRAPPIST-1 system, in which we use hydrodynamic simulations of its protoplanetary disk, performed with AREPO, to use the extreme flatness of the system as a constraint on its formation. In chapter 3, a follow-up paper to this work is presented, in which we investigate the Kepler-90 system with the same manner of simulations in order to understand how the dynamics uncovered by our work on the TRAPPIST-1 simulation are affected when comparing to another system that has some key similarities but also some major differences. In chapter 4, I present a more general theoretical discussion of angular momentum deficit, a dynamical quantity that proved to be a useful metric in understanding the results from our work in chapters 2-3, and use N-body simulations to explore what can be learned from separating it into inclination and eccentricity components in certain contexts. In chapter 5, I conclude by giving an overview of the work presented in this thesis and discussing potential next steps.

2

How Flat Can a Planetary System Get? I. The case of TRAPPIST-1

Heising, M. Z., Sasselov, D. D., Hernquist, L., et al. 2021, ApJ, 913, 126. doi:10.3847/1538-4357/abf8a8

Abstract

The seven planets orbiting TRAPPIST-1 in a compact near-resonant chain offer a unique case to study in planet formation theory. We demonstrate in this paper that the remarkable flatness of the system, exceeding that of any other known planetary system, is an important constraint on the mass of the gaseous disk in which it formed and attained its current configuration. We use three-dimensional (3D) hydrodynamic simulations of the gas and planets to study specific formation models. In particular, we report simulations motivated by the model proposed by Ormel et al. (2017) - in this model, the dispersal of the gas disk pushes the planets from an initial resonant chain into their present configuration. We find that a disk with the mass used in this model is consistent with the flatness of the TRAPPIST-1 system, but a more massive disk is not, with the transition occurring between 15 and 50 times the mass of the Ormel et al. disk. This mass upper limit rules out certain models of the formation of the system, namely in situ formation and disk migration on long timescales.

2.1 Introduction

The TRAPPIST-1 system, consisting of seven Earth-size planets (Gillon et al. 2017) orbiting a $0.09 M_{\odot}$ star (Van Grootel et al. 2018), offers for what the time being is a unique case to test our understanding of various aspects of planet formation. The system was first detected with the TRAPPIST telescopes and has since been

followed up with observations taken by Spitzer and K2, among others. These follow-up observations have been important in better constraining the planets’ sizes, orbits, and most importantly, their masses, determined by transit-timing variation (TTV) analysis. These revised masses, with uncertainties ranging from 5% to 12% (Grimm et al. 2018), are crucial to understanding the stability of the system as well as the composition of the planets, which has important implications for the formation of the system.

One defining feature of the system is that the planets are all closely packed in a near-resonant chain (Luger et al. 2017b). This aspect of the system is very important to understanding its formation history. The compactness of the system indicates that there are no unseen planets between planet b and planet h, since any such system would be dynamically unstable. More importantly, however, it implies that the planets migrated to attain these resonances (Goldreich & Schlichting 2014). While it is not immediately apparent exactly where the planets formed, their moderate water budgets of $\lesssim 30\%$ implies that they did not form completely beyond the H₂O iceline (Dorn et al. 2018). Ormel et al. (2017) propose a scenario in which the TRAPPIST-1 planets originated as planetary embryos that formed at the H₂O iceline and then migrate inward and grow in the disk through pebble accretion. The implications of this scenario are further explored in this paper.

One aspect of the TRAPPIST-1 system that we do not believe has received as much attention as it deserves is its flatness. The standard deviation of the reported inclinations of the system (Gillon et al. 2017; Luger et al. 2017b) is 0.072° . By this metric, it is the flattest of all systems with three or more planets on the NASA Exoplanet Archive. This flatness is potentially a very important constraint on its formation, and is something that can only be tested with 3D simulations.

Interacting planets, like in the closely packed system of TRAPPIST-1, tend to perturb and increase each others orbital eccentricities in unison with their mutual inclinations. The presence of a gas disk damps both, the damping of small inclinations being particularly efficient (Lubow & Ogilvie 2001; Tanaka & Ward 2004; Arzamasskiy et al. 2018). The question we address in this paper is what happens when the gas is gone. The initial placement of the planets in a gas disk effectively gives the system a perturbation, as the gas causes the locations of resonances in the system to change, among other effects, with a larger disk mass causing a larger

perturbation. When the gas is removed, whatever change it induced on the planets’ inclinations will be “locked-in,” as it is not expected that the mutual inclinations will be able to be reduced purely through planet-planet interactions. Petrovich et al. (2020) describes an in-depth model for inclination excitation through disk dispersal. The goal of this paper is to find the minimum disk mass for which the perturbation to the orbits of the TRAPPIST-1 planets is inconsistent with the very small mutual inclinations they are observed with today.

2.2 Inclination Dispersion of the TRAPPIST-1 System

To quantify the inclination dispersion of the TRAPPIST-1 system, we adopt a method used by Fabrycky et al. (2014) and Zhu et al. (2018). For each planet pair in the system, we define a quantity

$$\xi = \frac{T_{dur,in}/P_{in}^{1/3}}{T_{dur,out}/P_{out}^{1/3}} \quad (2)$$

where T_{dur} is the transit duration and P is the orbital period. This quantity can be rewritten as

$$\xi = \sqrt{\frac{(1 + r_{in})^2 - b_{in}^2}{(1 + r_{out})^2 - b_{out}^2}} \quad (3)$$

where r is the planet-to-star radius ratio and b is the impact parameter. Since $r \ll 1$ for all planets in the TRAPPIST-1 system, ξ depends almost entirely on the planets’ impact parameters, and thus probes their mutual inclination. For a perfectly flat system, $b_{out}^2 \geq b_{in}^2$, resulting in a value of ξ somewhat larger than unity, depending exactly on the inclination of the system as a whole relative to the viewer. The distribution of ξ for all 21 planet pairs in the TRAPPIST-1 system is shown as the blue histogram in Figure 1.

To constrain the inclination dispersion, we then model the system as follows. We assume the system has inclinations that are distributed according to a Rayleigh distribution with dispersion σ . We further assume that the eccentricity of each planet is sufficiently low that T_{dur} depends only on the inclination – this assumption is justified based on both TTV and stability analyses (Grimm et al. 2018).

For each simulated planet pair, we randomly draw an impact parameter for the outer planet, $b_{out,sim}$ from a uniform distribution from 0 to b_{max} , where b_{max} is the largest impact parameter the planet could have had while still having been detected, related to the observed transit signal-to-noise ratio (S/N_{obs}) as

$$b_{max} = \sqrt{(1+r)^2 - \left(\frac{S/N_{thresh}}{S/N_{obs}}\right)^4 \times ((1+r)^2 - b^2)} \quad (4)$$

where S/N_{thresh} is the signal-to-noise threshold below which a planet would not be detected. Although S/N_{obs} was not publicly available to us, we note from the strong dependence on it in Equation 4 that any planet with $S/N_{obs} > S/N_{thresh}$ by any significant margin would have $b_{max} \approx 1 + r$. As such, and by inspection of the K2 transits in Grimm et al. (2018), we use this approximation for b_{max} for planets b-g and use $b_{max} = 0.75$ for planet h – our results are not very sensitive to exactly what value is used here.

We then randomly draw an impact parameter for the inner, $b_{in,sim}$ planet from a normal distribution centered on $b_{out,sim}(P_{in}/P_{out})^{2/3}$ and standard deviation $\sigma/(d_{in} + r_{in})$, where d is the ratio of the stellar radius to the semimajor axis. This normal distribution corresponds to the desired Rayleigh distribution of inclinations for the system. A detectable planet requires $|b_{in,sim}| \leq b_{in,max}$, and if this condition is not met, we resample $b_{in,sim}$ until it is.

For a given value of σ , for each of the 21 planet pairs in the TRAPPIST-1 system, we use this method to generate 6000 simulated planet pairs, and calculate ξ_{sim} for each. The resulting distribution of ξ_{sim} for $\sigma = 0.05^\circ$ is shown as the orange histogram in Figure 1. While the simulated values do not perfectly line up with the observed values, this should not be a big surprise, since 21 planet pairs is not a very large sample size.

To determine the true value of σ for the TRAPPIST-1 system, we repeat this analysis for 50 values of σ equally spaced from 0° to 0.3° . For each value of σ , we calculate the likelihood, $\mathcal{L}$, of the observed distribution to be drawn from the simulated distribution as

$$\mathcal{L} = \prod_{j=1}^{21} \int P(\ln \xi | \sigma) \exp \left(-\frac{(\ln \xi_j - \ln \xi)^2}{2\sigma_{\ln \xi, j}^2} \right) d \ln \xi \quad (5)$$

where the product is taken over each of the planet pairs, $P(\ln \xi | \sigma)$ is the probability distribution of the simulated $\ln \xi$ given a certain value of σ , ξ_j is the observed value of ξ for the j th planet pair, and $\sigma_{\ln \xi, j}$ is the fractional uncertainty of ξ_j based on the measured uncertainties of T_{dur} . Using a spline function, we produce a smooth plot of $\ln(\mathcal{L}/\mathcal{L}_{max})$ versus σ to find the value of σ that maximizes the likelihood. We also calculate the 1σ confidence interval as the region for which $\ln(\mathcal{L}/\mathcal{L}_{max}) > -1/2$. Our result is that $\sigma = 0.05^{+0.20}_{-0.05}$ degrees. Notably, we cannot place a meaningful lower bound on σ . This is similar to a result that Luger et al. (2017a) attained, that the orbital planes of the planets are aligned to within 0.3° at 90% confidence.

To put this result in context, we plot in Figure 2 the inclination dispersion of the TRAPPIST-1 system on top of the distribution of inclination dispersions for the *Kepler* planets as a function of the number of planets in each system, reproduced from Zhu et al. (2018). This plot shows that as the number of planets in a system increases, the inclination dispersion of the system decreases, though this trend is not well understood from a physical perspective. Even when comparing it to other seven-planet systems, TRAPPIST-1 appears to be an outlier with a much lower than typical inclination dispersion. Comparing TRAPPIST-1 to all the *Kepler* systems, of which there are many more, it is an even larger outlier. Of course, it should be noted that the planets on which the analysis of Zhu et al. (2018) was based orbited Sun-like stars, of which TRAPPIST-1 is not. Perhaps it is not surprising then that TRAPPIST-1 is an outlier in this context, if typical planetary systems around M-dwarfs were to have lower inclination dispersions than those around Sun-like stars, though the data does not yet exist to support this claim, nor is it clear exactly what the physical mechanism for this would be.

To further put TRAPPIST-1 in context, we repeat the above analysis for the *Kepler*-90 system, a similarly populous (though not as compact) system with eight planets orbiting a $1.12 M_\odot$ star (Shallue & Vanderburg 2018). However, this system was not as well modeled by a Rayleigh distribution of inclinations, due to planet i being a significant outlier. As a result, we could only achieve a lower bound for the inclination dispersion of the system of 0.84° , also shown in Figure 2. Because of this, we repeat this analysis for the system with planet i removed. As can be seen, the resulting inclination dispersion is much closer to that of the TRAPPIST-1 system, though still likely somewhat higher.

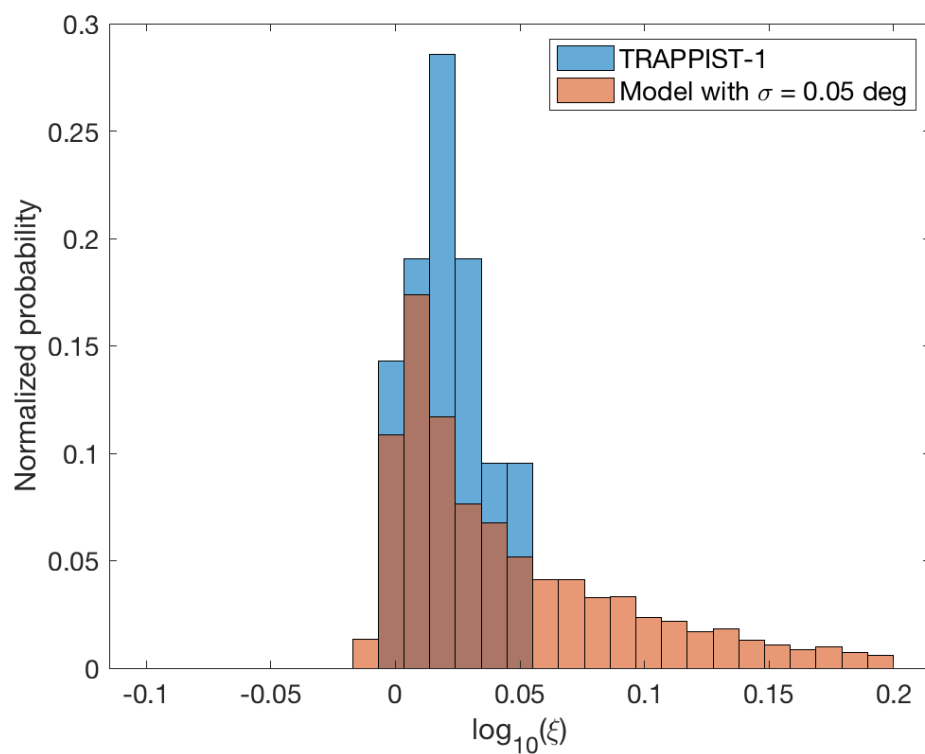


Fig. 1.— Observed (blue) and simulated (orange) distributions of ξ for the TRAPPIST-1 system.

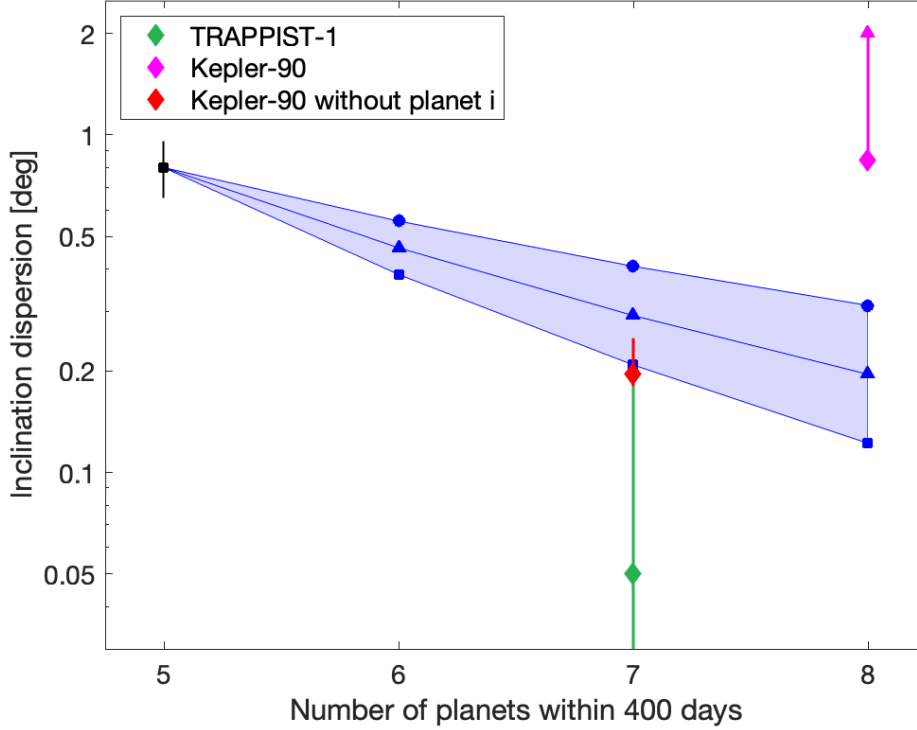


Fig. 2.— Inclination dispersions of the TRAPPIST-1 and Kepler-90 systems, both with and without planet i, compared to the inclination dispersion distribution as a function of number of planets (multiplicity) for the *Kepler* systems, reproduced from Zhu et al. (2018). The dispersion distribution was normalized to a multiplicity of 5 in Zhu et al. (2018) because it was the largest multiplicity for which there were enough Kepler systems to well constrain the inclination dispersion, and because for systems with smaller observed multiplicities, there is greater chance of additional unknown planets. The blue shaded region shows the expected 2σ range of inclination dispersions. Note that the lower error bar for the TRAPPIST-1 system extends to 0, and that the inclination dispersion for the Kepler-90 system represents a lower bound. The lower error bar for the Kepler-90 system without planet i is quite small and just barely extends past the marker. The upper error bar for the TRAPPIST-1 system and that of the Kepler-90 system without planet i both coincide at 0.25 degrees.

2.3 Disk Simulations - Methods & Initial Conditions

Our goal for the rest of the paper is to understand what constraints such an extreme flatness places on formation models of the TRAPPIST-1 system. The inclinations of planets is influenced not only by planet-planet interactions but also by gravitational interactions with the gaseous disk in which they form. To explore this, we simulate the planets' orbits during the phase in which they have attained their current masses and semimajor axes (or close to them), but the gas disk is still present. The question we want to answer is how different disk masses affect the dynamics of their planets, especially with regards to their inclinations.

For a complete description of the dynamics of the system, we must self-consistently model the gravitational interactions between the planets and the disk with the hydrodynamic evolution of the disk in three dimensions. To do this, we use the hydrodynamic code AREPO (Springel 2010; Pakmor et al. 2016) for our simulations. AREPO solves the equations of hydrodynamics (either the Euler equations or the Navier-Stokes equations, depending on whether viscosity is included) on a Voronoi mesh that moves with the fluid, giving the code a quasi-Lagrangian nature. It uses a finite volume approach to evolve the hydrodynamic variables of each cell forward in time. A source term is added to the hydrodynamic equations to include the effects of gravity. This hydrodynamic evolution is coupled to an N-body solver for collisionless particles, which represent planets in our simulations. The gravitational potential felt by both the gas and planets includes the self-gravity of the gas, the gravity from the planets, and a softened central potential due to the host star. AREPO has primarily been used for cosmological simulations (see Vogelsberger et al. (2014), for example), though it is well-suited for a wide array of problems. In particular, Muñoz et al. (2013) implemented an approach to account for viscosity and subsequently applied the code to simulations of circumstellar disks (Muñoz et al. 2014). This implementation of viscosity is also used in our simulations.

As a fiducial model for the disk in which the TRAPPIST-1 system formed, we consider the scenario proposed by Ormel et al. (2017) (and later expanded upon in Schoonenberg et al. (2019)). In this model, the seven planets form sequentially, each initially at the water iceline, before migrating in, continuing to accrete pebbles along the way. The migration of each planet stops when it reaches a resonance with its interior planet (or when it reaches the magnetospheric cavity of the disk, in the case of

planet b). This results in a resonant chain with planet b at a semimajor axis of 0.0102 AU and period ratios (starting from c:b) of 3:2, 3:2, 3:2, 4:3, and 3:2. The idea the authors explore is that as the gas is dispersed due to magnetospheric rebound, planets will for a time couple to the edge of the inner cavity via a corotation torque (Liu et al. 2017), disturbing them from their original resonant chain and allowing them to attain the periods we observe them with today. Of particular interest to our work is that this model does not require a very massive disk, making it a useful starting point for our simulations. This is in contrast to a more classical disk migration model of TRAPPIST-1’s formation, described by Tamayo et al. (2017), for example, which invokes a more massive disk.

The seven TRAPPIST-1 planets are initialized with semimajor axes to give the period ratios as described above - note that these are not the semimajor axes with which the system is observed today, and in general are somewhat smaller, but they are of similar magnitude. The planets’ masses are given by Grimm et al. (2018)¹, and they are given eccentricities of be zero. The inclinations are calculated by first setting them equal to the observed inclinations reported in Gillon et al. (2017) and Luger et al. (2017b), with longitudes of ascending node independently randomly sampled from a uniform distribution from $[0, 2\pi)$; the system is then given a rotation such that the invariable plane of the system would lie in the x and y dimensions. While it is impossible to know the true orbits of the TRAPPIST-1 planets given degeneracies between their inclinations, their longitudes of ascending node, and their transit impact parameters, this approach gives our simulated planets inclinations that are consistent with observing inclinations. The star is forced to maintain a fixed position in our simulations, and is allowed to accrete mass within a radius that begins at 0 at the start of the simulation and increases linearly to reach a maximum of 0.001 AU at a time of 7.7 orbits of planet b. Over the duration of

¹Since the simulations in this work were performed, more precise mass measurements for the TRAPPIST-1 system were obtained in Agol et al. (2021). To fully understand what impact these revised masses would have on this work, of course, the simulations would need to be performed again completely. However, the changes in mass are relatively small - the largest relative change in mass is that of planet b, which increased by a factor of 1.35. Furthermore, the overall planetary architecture remains relatively similar, with most planets increasing in mass from the values reported in Grimm et al. (2018). Given that, we suspect these revisions in mass would not largely change the findings of this work.

our simulation of the TRAPPIST-1 system with the disk described by Ormel et al. (2017), the star accretes a total of 1.59×10^{-5} its initial mass.

The gaseous disk in our simulations begin with a surface density Σ given by a softened Lynden-Bell-Pringle density profile (Lynden-Bell & Pringle 1974):

$$\Sigma(R) = \Sigma_0 \left(\frac{f_{soft}(R, R_{soft})}{R_c} \right)^{-p} \exp \left[- \left(\frac{R}{R_c} \right)^{2-p} \right] \quad (6)$$

where R is the cylindrical radius, Σ_0 is a constant given by the total disk mass, p is a surface density power law index, R_c is the disk's characteristic radius, R_{soft} is a softening parameter, and f_{soft} is a smooth softening function that keeps the surface density profile from diverging at $R = 0$. In particular, $f_{soft}(R, R_{soft}) = R_{soft}/2.8$ for $R = 0$ and $f_{soft}(R, R_{soft}) = R$ for $R \geq R_{soft}$, giving a smooth transition to a Newtonian potential; the exact form of this spline softening function is adopted from the smoothed-particle hydrodynamics code GADGET (Springel et al. 2001). In our simulations, $p = 0.5$ and $\Sigma_0 = 8.26 \times 10^{-5} M_*/\text{AU}^2$, where $M_* = 0.08 M_\odot$ is the mass of TRAPPIST-1. These values correspond to the inner disk described by Ormel et al. (2017) in Section 2.1 of their paper. This surface density profile was chosen by Ormel et al. (2017) due to consideration of accretion rates typical for M-stars and an assumed α -viscosity of 0.001, which is also used in our simulations. Importantly, this surface density profile provides enough gas to produce the desired corotation torques mentioned above. We use a characteristic radius $R_c = 0.2 \text{ AU}$ and $R_{soft} = 0.006 \text{ AU}$ so that the disk density profile in the vicinity of the planets is well-described by a power law. The resulting gas disk has a total mass of $0.83 M_\oplus$, of which $0.09 M_\oplus$ is interior to planet h. This disk is much smaller than the complete disk described by Ormel et al. (2017), the surface density of which falls off as R^{-1} for $R \gg 1 \text{ AU}$. We assume that the orbits of the TRAPPIST-1 planets are not significantly affected by gas at much larger radii than the planets' own semimajor axes. A plot of the initial surface density profile is shown in Figure .

It is important to note here that this is a relatively low-mass disk from the perspective of planet formation. In particular, the ratio of the mass of the disk interior to the outermost planet in our simulation (at 0.048 AU) to the sum of the masses of the planets themselves is only 0.018. In contrast, the Solar System's gas disk is believed to have been at least as massive as the Minimum Mass Solar Nebula (Hayashi 1981), the mass of which, interior to Mars, is a factor of 1.1 times greater

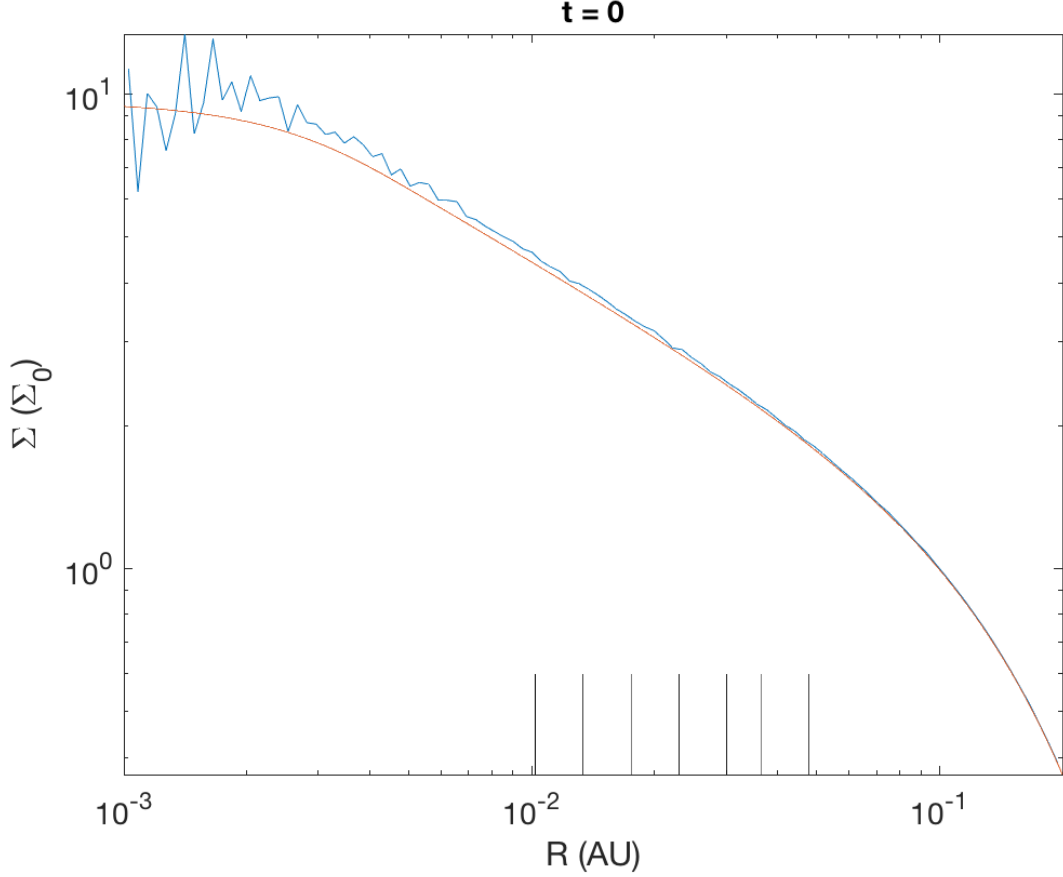


Fig. 3.— Initial surface density profile of the disk described by Ormel et al. (2017), plotted out to $R = R_c$. The red curve shows the analytic surface density profile as described by Equation 6, while the blue curve shows the measured surface density profile after the density of each cell assigned in the initial conditions has been converted to a mass at the start of the simulation. The black lines indicate the semi-major axes of the TRAPPIST-1 planets. In the vicinity of the planets, the surface density profile follows a power law as $\Sigma(R) \propto R^{-0.5}$.

than the sum of the masses of our four terrestrial planets.

Ormel et al. (2017) use a disk with a constant axis ratio of 0.03. However, we found that it was too difficult to resolve such a thin disk. Instead, we used a constant axis ratio of 0.1, so that the sound speed, c_s , was characterized by

$$c_s(R) = 0.1 \sqrt{\frac{GM_*}{f_{soft}(R, R_{soft})}} \quad (7)$$

where G is the gravitational constant. Again, the softening function is used to avoid a divergence at $R = 0$. We force this functional form of the sound speed to be maintained at all times during the simulations using a locally isothermal equation of state.

Our approach to generate the initial conditions of the gas is taken from Muñoz et al. (2015). Our aim is to divide up the computational domain into N cells of roughly equal mass. To do this, we need to specify positions of all N cells - for our simulations described here, $N = 1.5 \times 10^6$. In cylindrical coordinates, we draw ϕ from a uniform distribution from $[0, 2\pi)$. We use Equation 6 to Monte Carlo sample values of R .

To sample z , however, we must first solve for the vertical density profile, which is non-trivial when self-gravity is important. To do this, we divide up the disk into many radial bins. At each radial bin, for a given R , we must have

$$\frac{c_s^2(R)}{\rho} \frac{\partial \rho}{\partial z} = -\frac{\partial \Phi_{0,z}}{\partial z} - \frac{\partial \Phi_{g,z}}{\partial z} \quad (8)$$

where $\Phi_{0,z} \equiv \Phi_0(R, z) - \Phi_0(R, 0)$ is the z -dependent part of the Keplerian potential Φ_0 of the central star and $\Phi_{g,z} \equiv \Phi_g(R, z) - \Phi_g(R, 0)$ is the z -dependent part of the potential Φ_g due to self-gravity. $\Phi_{g,z}$ is related to ρ through the Poisson equation as

$$\frac{\partial^2 \Phi_g(R, z)}{\partial z^2} = \frac{\partial^2 \Phi_{g,z}(R, z)}{\partial z^2} = 4\pi G \rho(R, z). \quad (9)$$

Using an iterative method, described in detail in Muñoz et al. (2015), the vertical density profile is solved for such that Equation 8 is met to within a tolerance parameter. With an appropriate guess, this method generally converges within ~ 5 iterations.

Due to the nature of the Voronoi mesh used in AREPO, the computational domain, a $(1.6 \text{ AU})^3$ box for our simulations, must be sampled throughout. This includes regions away from the disk that would have very low density and therefore would not be well sampled by the Monte Carlo method described above. To accommodate this, a “background grid” of cells is created. This background grid, consisting of roughly 75,000 cells for our simulations, must completely sample the computational domain, and must transition smoothly into the Monte Carlo-sampled points in the disk.

However, due to the different methods in which the disk cells and the background cells are sampled, it is extremely difficult to ensure a smooth transition. As such, many cells in this transition region will be irregular. To solve this, the following regularization method, known as Lloyd’s algorithm (Lloyd 1982), is performed. From the initial locations of the gas cells, a Voronoi mesh is constructed. For each cell in the Voronoi mesh, a center of mass is calculated. Each mesh-generating point is then moved to the center of mass of its cell, and a new Voronoi mesh is then calculated. This process is then repeated two times, for a total of three iterations. Once the new set of positions is established, the vertical density profile is again calculated as described above, and each gas cell is initialized with its proper density. Density slices through the initial disk are shown in Figure 4.

To calculate the velocity structure of the gas, we must take into account both the pressure gradient of the disk and self-gravity. For a locally isothermal disk, the azimuthal velocity, v_ϕ , takes the form (Wang et al. 2010)

$$\frac{v_\phi}{R} = \frac{v_c}{R} + \frac{1}{\rho} \frac{\partial p}{\partial R} \bigg|_{R=0} - \frac{\partial c_s^2}{\partial R} \log \left(\frac{\rho(R, z)}{\rho_0(R)} \right) \quad (10)$$

where $\rho_0(R)$ is the midplane density and $v_c^2 = v_K^2(z=0) + v_{c,d}^2$ is the circular speed due to gravity, both from the central star and the disk. $v_{c,d}^2$ can be calculated as (Mestel 1963)

$$v_{c,d}^2 = G \frac{M_d(< R)}{R} + 2G \sum_{k=1}^{\infty} \alpha_k \left(\frac{2k+1}{R^{2k+1}} \int_0^R dR' \Sigma(R') R'^{2k+1} - 2kR^{2k} \int_R^{\infty} dR' \frac{\Sigma(R')}{R'^{2k}} \right). \quad (11)$$

In practice, this summation converges in roughly 10 terms.

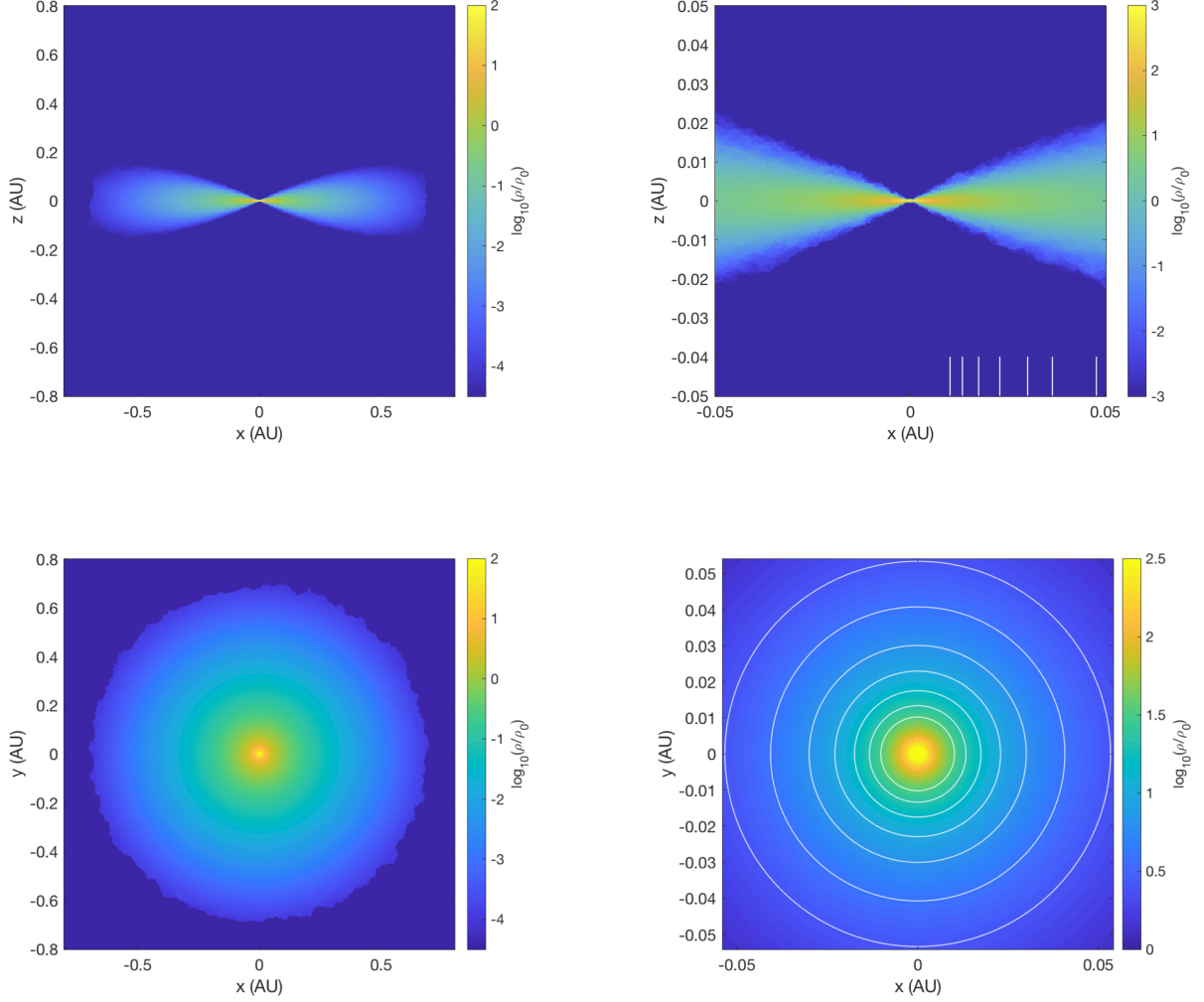


Fig. 4.— Density slices of the disk at the start of the simulation. The upper panels show a slice through $y = 0$, while the lower panels show a slice through $z = 0$. The left panels show the full computational domain, while the right panels show a zoomed-in view, with white lines indicating the semimajor axes of the TRAPPIST-1 planets. The normalization density, ρ_0 , is defined as $\rho_0 \equiv \Sigma_0/(hR_c)$, where $h = 0.1$ is the axis ratio of the disk.

2.4 Results

To understand the effect the gas disk has on the TRAPPIST-1 planets in the scenario proposed by Ormel et al. (2017), we must first understand the effects the planets have on each other. To this end, we use AREPO to perform a simulation of the system as described above except without any gas for 77,000 orbits of planet b. The evolution of the planets’ eccentricities and inclinations are shown in the upper four panels of Figure 5.

The eccentricities display variation on relatively short timescales. The planets attain moderate eccentricities, particularly planet d, which has an eccentricity exceeding 0.1 at times. This is in contrast to some other N-body simulations of the TRAPPIST-1 system. Eccentricities in Papaloizou et al. (2018) generally do not exceed 0.01 on long timescales, though Tamayo et al. (2017) use migration to generate stable configurations of the TRAPPIST-1 system with eccentricities ranging from $\sim 0.01 - 0.1$. However, this difference is not surprising, primarily for two reasons: first, because our simulations begin with the planets in exact commensurabilities corresponding to lower-order resonances than they are in today; and second, because our simulations do not include an eccentricity-damping term common to both Papaloizou et al. (2018) and Tamayo et al. (2017). We confirmed that an N-body simulation initialized with the observed semimajor axes yields smaller eccentricities that better match other such studies of the system.

The inclinations display variation on longer timescales. In particular, the inclinations of planets b and c and planets f and g show a clear anti-correlation, a sign of secular behavior. We have verified this by performing a secular analysis of the system. The inclinations from this secular analysis are shown in the bottom panels of Figure 5, and largely reproduce the inclinations of the system as simulated with full N-body dynamics. Overall, the inclinations do not deviate by orders of magnitude on this timescale and the inclination dispersion of the system remains small.

To determine the effect the gaseous disk has on the orbits of the TRAPPIST-1 planets, we perform the following numerical experiment. We set up a simulation with both planets and gaseous disk initialized as described above. The system is evolved for 200 orbits of planet b. At $t = 200P_b$, where P_b is the orbital period of planet b, we begin a new simulation in which all the gas is instantly removed. This

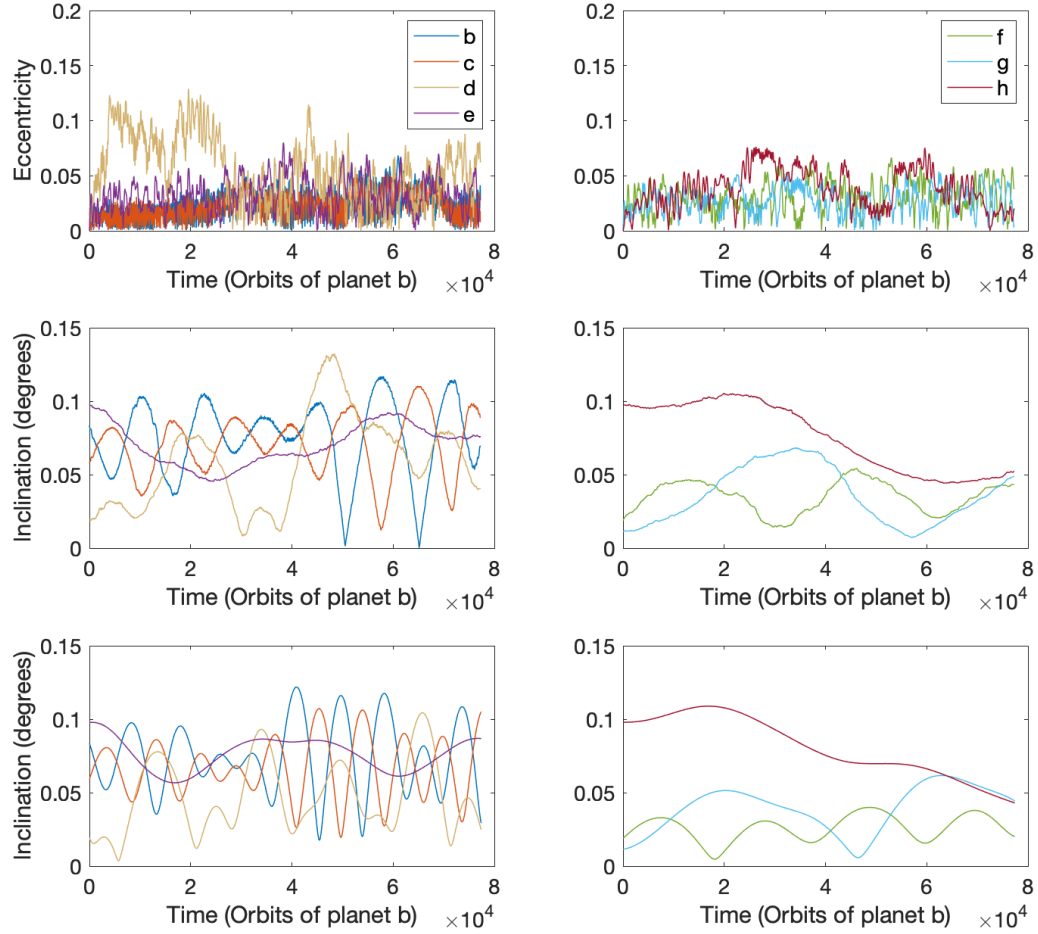


Fig. 5.— Top four panels: eccentricities and inclinations of TRAPPIST-1 planets for the gas-free run. For clarity, planets b-e and planets f-h are shown in separate panels. Bottom panels: inclination of the TRAPPIST-1 planets as calculated via secular analysis.

instant removal of the gas is not meant to represent a physically realistic process, but rather allows us to determine what effect the gas has on the orbits of the planets by comparison between the two runs. In performing this experiment, our goal is not to model the evolution of the planets' orbits for the entire lifespan of the disk, but rather to study the planets in the last stages of planetary formation, when the planets had already reached close to what is their current orbital configuration, and when much of the disk mass might have already been lost.

The eccentricities and inclinations from these runs are shown in Figure 6. At $t = 200P_b$, where the gas-free run begins, the eccentricities display a range from 0.005 to 0.030, while the inclinations range from 0.019° to 0.098° . While the two runs do diverge slightly, there does not seem to be an important difference between the two runs, particularly with regards to the planets' inclinations. An obvious exception is planet d, the inclination of which does diverge significantly over the course of the two simulations. However, the two inclinations remain very similar immediately following the removal of the gas at $t = 200P_b$, and it is only at $t = 576P_b$ that the two inclinations begin to diverge. Because of this, we do not believe this divergence to be caused by gas dynamics, and is instead simply an example of a small perturbation in a dynamically complex N-body system leading to large differences in a later state. Besides planet d, qualitatively, the behavior of the planets is largely unchanged between the two runs.

Figure 7 shows density slices through the disk at $t = 100P_b$. The vertical structure of the disk does not change much over the course of the simulation, as can be seen by comparison with the initial disk. Changes in the radial structure are due to the central sink particle and the gravitational effects of the planets, though individual spiral density waves are not easily distinguishable due to the tightly packed nature of the system.

To further explore how the gaseous disk affects the evolution of the TRAPPIST-1 system, we repeat the above experiment for three more disks with Σ_0 increased by factors of 4, 15, and 50. For each of these differently massed disks, the system was similarly evolved with the gas for 200 orbits of planet b, at which point a gas-free simulation branches off. The results of these more massive disk runs are shown in Figure 8. As noted in the introduction, the goal of this experiment is to determine the least massive disk capable of causing a significantly large perturbation to the orbits

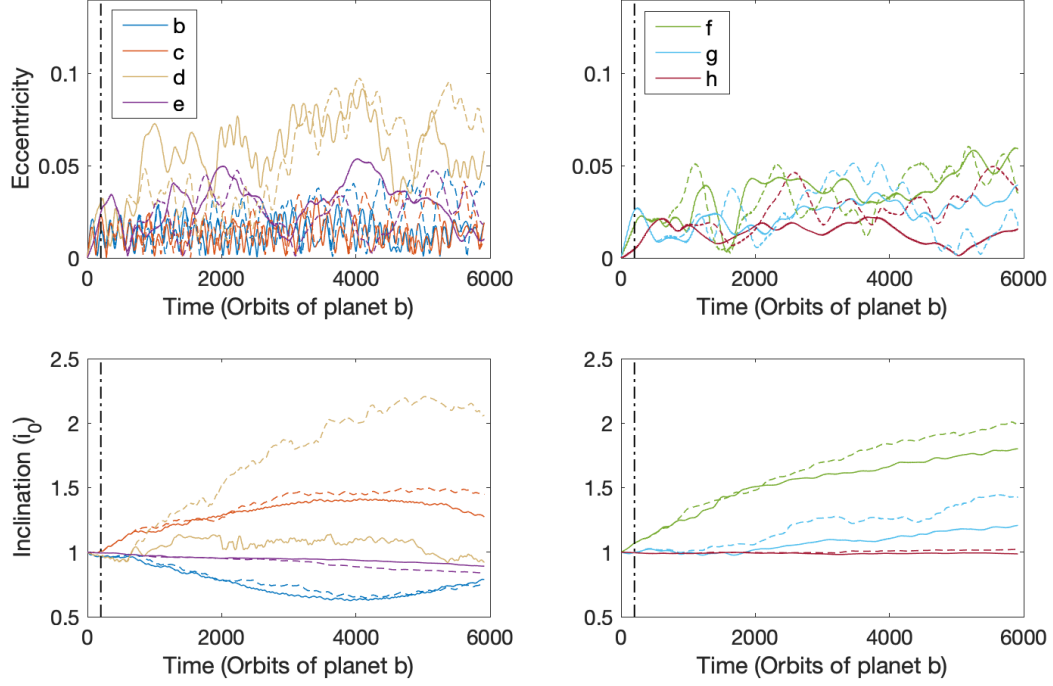


Fig. 6.— Eccentricities and inclinations of TRAPPIST-1 planets simulated with the disk described by Ormel et al. (2017). The solid lines show the run with gas, and the dashed lines beginning at $t = 200P_b$ (demarcated with the black dash-dot line) show the run in which the gas is removed. To more easily see the difference in inclinations between the gas run and the gas-free run, the inclinations are normalized to their initial values.

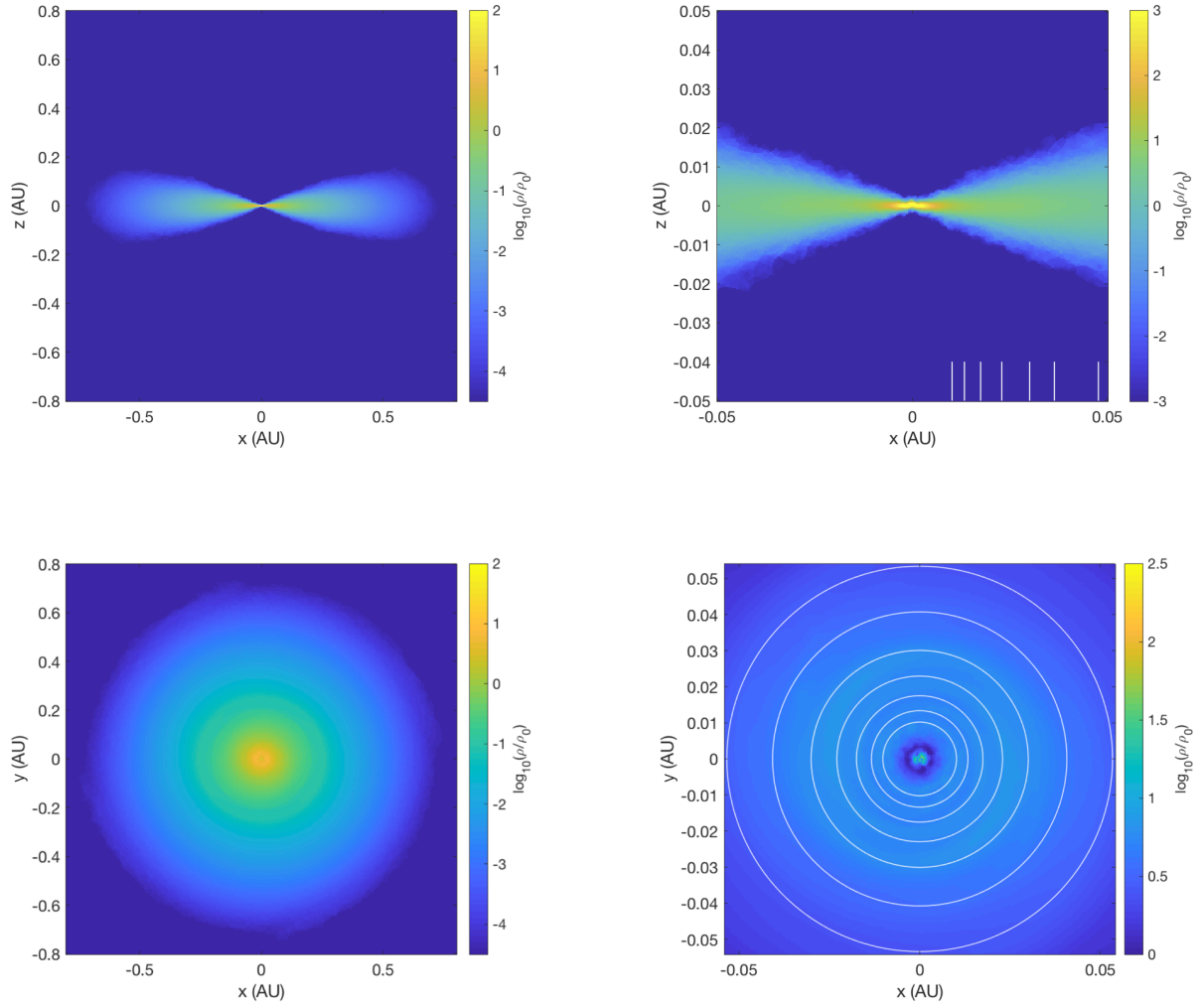


Fig. 7.— Density slices after 100 orbits of planet b. The panels are the same as in Figure 4.

of the planets, as measured by their mutual inclinations relative to their observed values.

As the disk mass increases, the inclinations with gas diverge from the inclinations without gas more quickly. This effect is most apparent in the outer planets, particularly f and g. The Ormel et al. disk and the $4 \times \Sigma_0$ disk are apparently in one dynamical regime, while the $50 \times \Sigma_0$ disk is in another (the $15 \times \Sigma_0$ disk appears to be in somewhat of a transition region, with most of the planets behaving as if it were in the less massive regime, but planet g in particular behaving as if it were in the more massive regime). In the more massive disk regime, it is the gas, not the planets, that is driving the evolution of the inclinations. As such, the planets in this regime are no longer constrained by the energy budget of their initial orbits. While the exact long-term behavior of the planets cannot be predicted from a run of this length, we argue that the system, being driven by the gas, will not retain its low inclination dispersion. This suggests an upper bound on the mass of the TRAPPIST-1 disk of roughly $15\text{--}50 \times \Sigma_0$. This upper limit represents a bound on how much of the gas disk still remained when the planets had reached what is close to their present day configuration. The history of what happened earlier in the disk’s lifetime, when it might have been more massive, is lost.

It is again important to note here that although the instant removal of the gas does not reflect a realistic model of how the gas would dissipate, it nonetheless allows us to differentiate between dynamical regimes dominated by planet-planet interactions and those dominated by gas-planet interactions. Indeed, if a more realistic treatment of the removal of the gas were used, in which the gas dissipated over much longer timescales, we would expect to see this same change of dynamical regime.

To better understand the evolution of the system, we consider the angular momentum deficit (AMD), which can serve as a useful metric for understanding the dynamical temperature of a planetary system (Laskar & Petit 2017). AMD can be calculated as

$$\text{AMD} = \sum_{k=1}^n \Lambda_k \left(1 - \sqrt{1 - e_k^2} \cos i_k \right), \quad (12)$$

where the summation is carried out over all n planets, e_k is eccentricity, i_k is inclination, and $\Lambda_k = m_k \sqrt{G m_* a_k}$, where a_k is semimajor axis, m_k is planetary mass, m_* is stellar mass, and G is the constant of gravitation. Of note, when this expression is

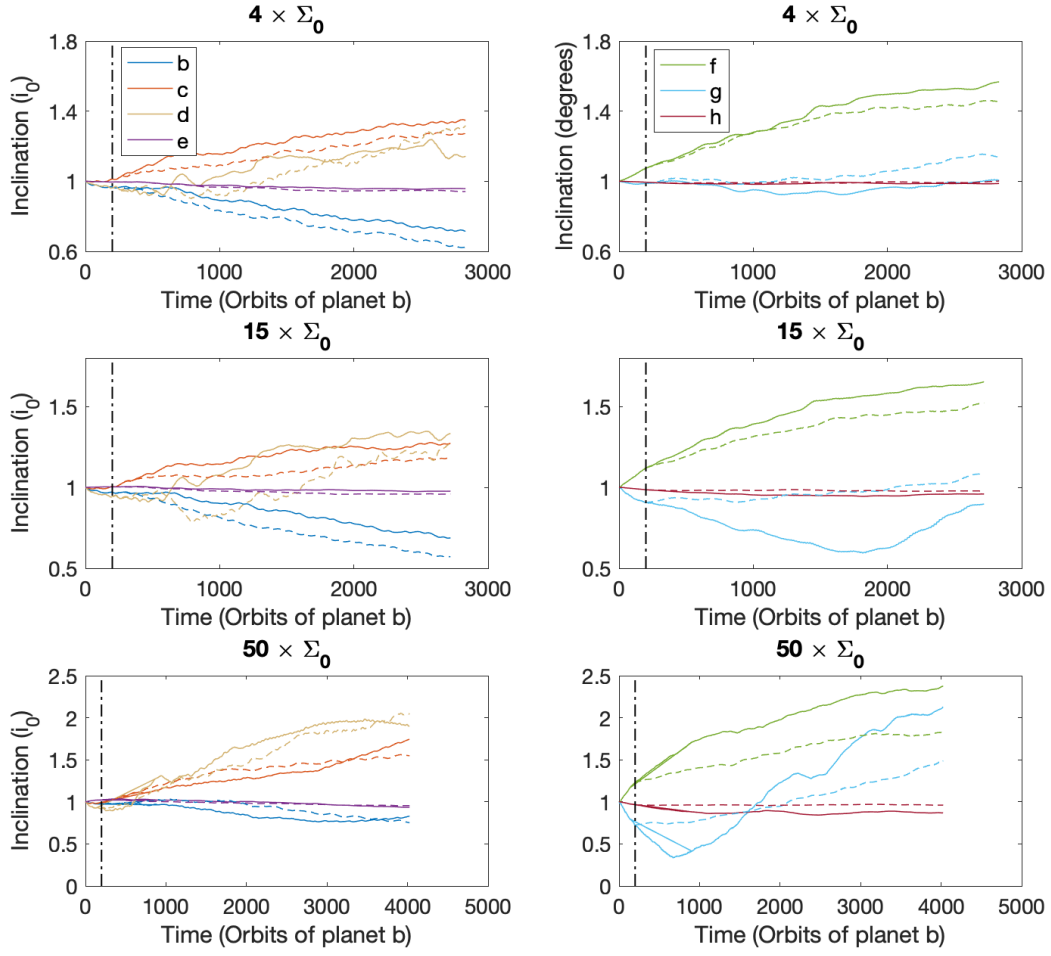


Fig. 8.— Inclinations of the TRAPPIST-1 planets for the more massive disks - the lines are defined as in Figure 6. Note that the y-axis scale changes for different disk masses.

Taylor expanded for small values of e and i , e and i are treated identically. However, in the simulations performed in this work, e maintains values that are typically 1-2 orders of magnitude larger than i , making AMD an ill-suited metric for capturing the evolution of inclinations. As such, we define a new parameter, the inclination angular momentum deficit (IAMD), as

$$\text{IAMD} \equiv \sum_{k=1}^n \Lambda_k (1 - \cos i_k), \quad (13)$$

which can be interpreted as the angular momentum deficit due to inclination alone. While this quantity does not convey any particular dynamical significance, it is useful for this work as a way to characterize the inclinations in the system as a whole. Furthermore, in keeping with Turrini et al. (2020), we normalize this quantity so that it can be better compared with other systems by dividing by the circular angular momentum; that is,

$$\text{NIAMD} \equiv \frac{\text{IAMD}}{\sum_{k=1}^n \Lambda_k}. \quad (14)$$

This quantity is calculated over time for each disk and for both the gas and gas-free runs, and is shown in Figure 9. It can be seen that for all except the most massive disk, NIAMD for either the gas or gas-free runs does not change much on these timescales, and in general NIAMD does not increase much from its initial value if at all. However, for the $50 \times \Sigma_0$ disk, we do see an increase in NIAMD. In particular, we see a relatively sharp increase in NIAMD in the gas run that overtakes the NIAMD of the gas-free run. This behavior further supports the idea that for gas disks above a certain limit in mass, the gas drives the inclinations of the planets to values that are inconsistent with those with which they are observed today. The fact that we do not see this behavior in the $15 \times \Sigma_0$ disk leads us to revise our estimate of the upper limit to $50 \times \Sigma_0$ disk.

2.5 Conclusions

We have presented 3D hydrodynamic simulations of the disk and planets of the TRAPPIST-1 system to study its unusual flatness. In particular, we used the low

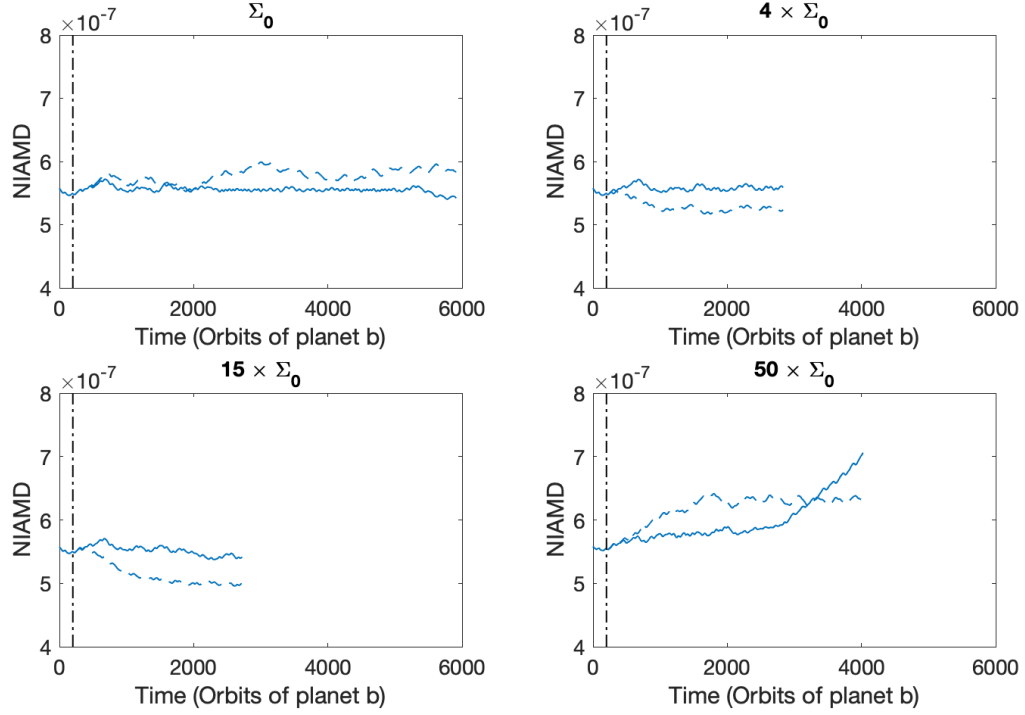


Fig. 9.— Normalized inclination angular momentum deficit (NIAMD) as a function of time for each of the simulated disks. Solid lines show the gas runs, and the dashed lines beginning at 200 orbits show the gas-free runs.

inclination dispersion of the system as a constraint on its formation. We determined that the gas disk described by Ormel et al. (2017) is consistent with the low inclination dispersion we observe the planets with today. It is conceivable, then, that the dispersal of such a disk could have pushed the planets from an initial resonant chain into the configuration with which we see them today, as Ormel et al. (2017) propose. However, when we increase the mass in the disk by a factor of 50 or more, we find that the gas has a strong effect on the evolution of the planets’ inclinations. We argue that such a massive disk is inconsistent with the low inclination dispersion of the TRAPPIST-1 planets.

This result can inform our understanding of the TRAPPIST-1 system beyond just the specific formation scenario proposed by Ormel et al. (2017). For example, Tamayo et al. (2017) perform simulations of the TRAPPIST-1 system in which the resonant chain is formed by migrating planet h inward with a timescale $\tau_a = 6 \times 10^5$ orbital periods. Equating this to the migration timescale for Type 1 migration (Tanaka et al. 2002), we find that this corresponds to a surface density of $0.20M_*/\text{AU}^2$ at the observed semimajor axis of planet h (Grimm et al. 2018). However, our $15 \times \Sigma_0$ disk model, which serves as an upper bound for the true mass of the TRAPPIST-1 gas disk at the last stages of planet formation, has a surface density of only $0.013M_*/\text{AU}^2$ at this radius. We therefore contend that such a migration rate for planet h is too fast. Similarly, a specific scenario explored in Section 5 of Papaloizou et al. (2018) uses a model for Type 1 migration that implies a surface density of $0.056M_*/\text{AU}^2$ at this radius – such a model can also be ruled out.

While the TRAPPIST-1 system appears at first to be an outlier among planetary systems, both in terms of the compact resonant chain that the planets fall into as well as the extreme flatness of their orbits, it is important to remember that the vast majority of known exoplanet systems are hosted by Sun-like stars, and what the “typical” M-star planetary system looks like is still very much an unknown. Furthermore, whether or not TRAPPIST-1 -like systems are common, the dynamics explored in this paper may be relevant across a wide range of planetary systems.

We would like to thank Diego Muñoz for his assistance in setting up simulations of protoplanetary disks with AREPO and for helpful discussions with him on interpreting the results of this work. We would also like to thank the anonymous referees for this work for the improvements they suggested. This work used

the Extreme Science and Engineering Discovery Environment (XSEDE; Towns et al. 2014) Stampede2 at the Texas Advanced Computing Center through allocations TG-AST190012 and TG-AST190034.

Appendix A High Resolution Run

To ensure that our simulations have sufficient resolution to accurately capture the gas dynamics for the purposes of our simulations, we perform a test in which we perform a comparatively high resolution run. For this run, the disk is initialized as before, except with 1×10^7 gas cells as opposed to 1.5×10^6 . We perform this test for the $50 \times \Sigma_0$ disk, as this is the disk for which effects due to the gas are most strongly felt by the planets. This high-resolution run is carried out for 461 orbits of planet b; the gas is not removed at any point during this run. We compare this run to our lower simulation gas run in Figure 10. As can be seen, the behavior of the planets is largely unchanged between the two simulations, indicating that a resolution corresponding to 1.5×10^6 gas cells is sufficient to capture the gas dynamics for our purposes.

Appendix B Comparison of N-body Dynamics to REBOUND

While AREPO has proven to be a successful code to tackle a large array of hydrodynamic problems, it hasn't yet been used as much to explore N-body planetary dynamics. To validate AREPO's performance as an N-body solver, and to verify that the timesteps used in this work were sufficiently small to accurately capture the dynamics of the system, we performed simulations using the N-body code REBOUND (Rein & Liu 2012) to which we could compare our AREPO simulation runs. All REBOUND simulations were performed with the LEAPFROG integrator (a similar integration scheme to the one used by AREPO) and with a timestep equal to 0.001 times the orbital period of the innermost planet.

A first comparison was performed with a simple two-planet system. The masses of the two planets in this system were set to those of TRAPPIST-1b and TRAPPIST-1c, respectively. The initial semimajor axis of planet b was set to be the same as that of TRAPPIST-1b, while planet c was initialized with a period of 1.422 times

that of planet b, far from any major commensurability. Both planets were initialized with zero eccentricity and with small inclinations characteristic of the TRAPPIST-1 system. The system was evolved for 80000 orbits of planet b.

A comparison of the planets' eccentricities between the AREPO run and the REBOUND run is shown in Figure 11, while a comparison of the planets' inclinations is shown in Figure 12. As can be seen, the two codes yield very similar results. In particular, the same long-term trends in eccentricity are reproduced by both codes. While there are no notable long-term trends apparent in the inclinations for this system, the inclinations cover a very similar range of values between both codes. Part of the difference in appearance in the results from the two codes is due to the lower output rate of the AREPO run, though this does not reflect the timestep used in the simulation.

A second comparison test was also performed using the TRAPPIST-1 system itself. The planets were given the same initial conditions as they were for the AREPO simulations in this work, and the system was also evolved for 80000 orbits of planet b. The eccentricities and inclinations from this REBOUND simulation are shown in Figure 13, and they can be compared to those shown in Figure 5 for the corresponding AREPO simulation. Due to the chaotic nature of the TRAPPIST-1 system (see Tamayo et al. (2017), for example), it is not expected that two such simulations would fully agree out to large timescales. Indeed, we do see some divergence between the two simulations. However, the eccentricities and inclinations in both cases span a similar range of values and vary over similar timescales. Furthermore, there are some trends that agree well between the two simulations over their entire duration, particularly those in the inclinations of the outer planets.

Because of the similar behavior observed between the two codes both for the simple two-body system and for the TRAPPIST-1 system, we are confident that the N-body performance of AREPO used for all simulations in this work accurately captures the true planetary dynamics.

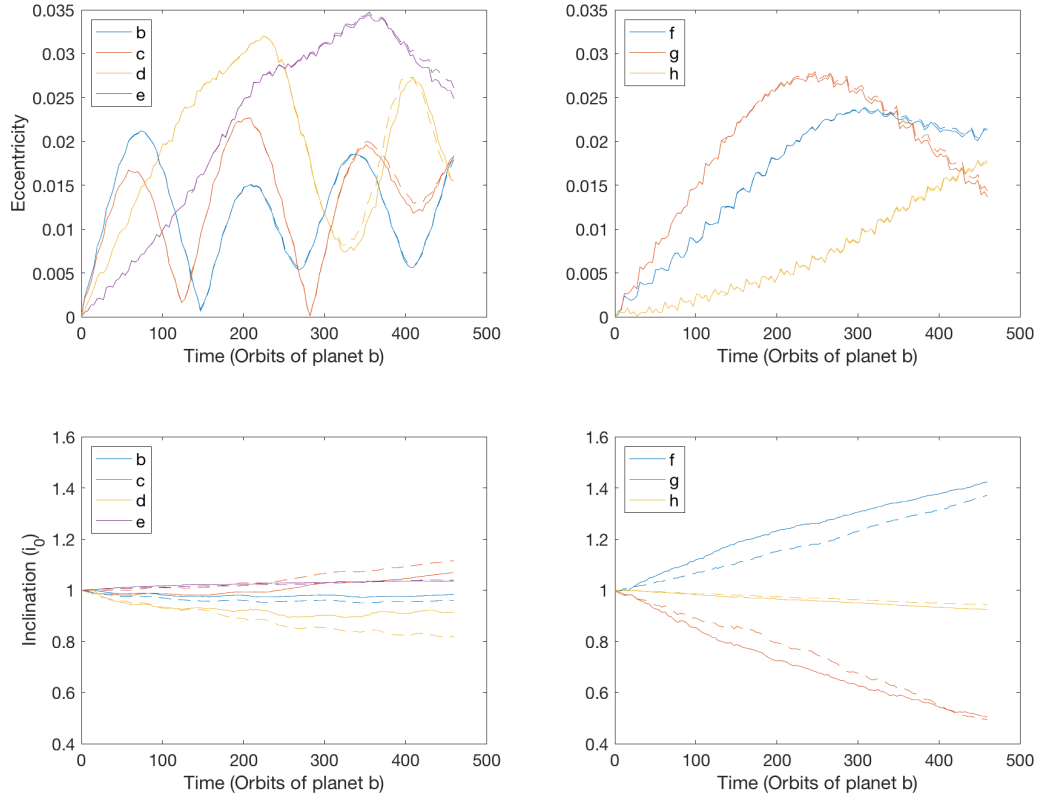


Fig. 10.— Eccentricities and inclinations of TRAPPIST-1 planets simulated with the $50 \times \Sigma_0$ disk. The solid lines show the run with 1.5×10^6 gas cells, and the dashed lines show the run with 1×10^7 gas cells.

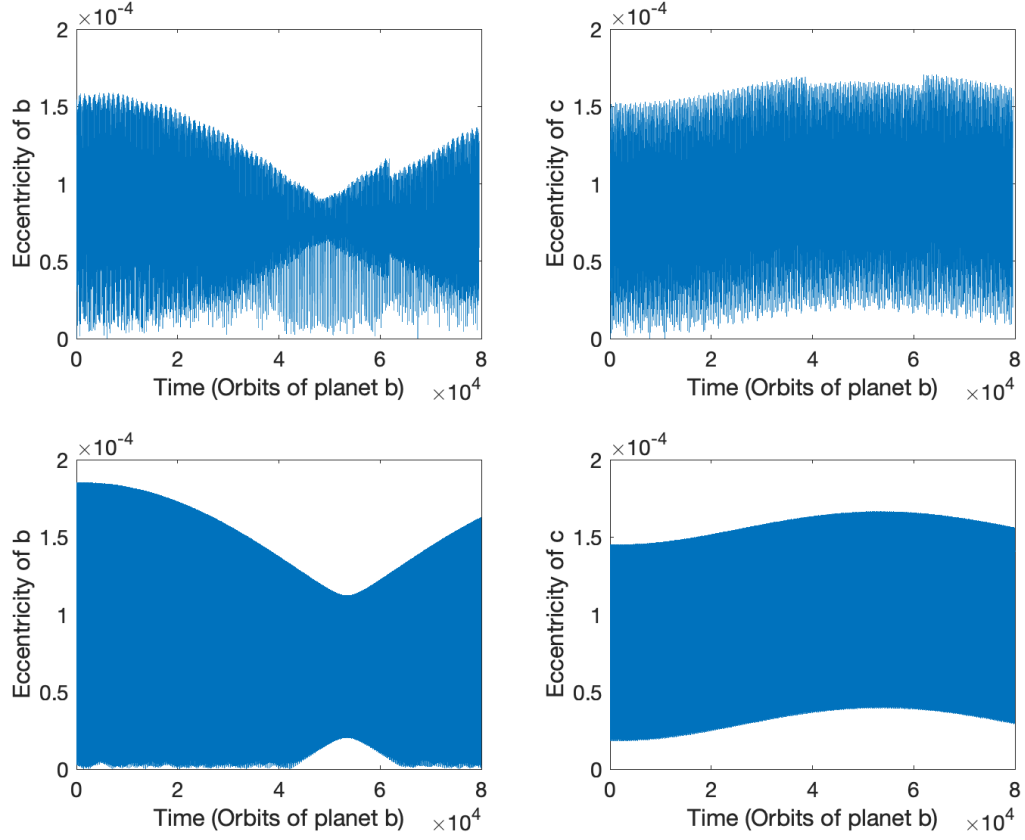


Fig. 11.— Eccentricities of the planets in the simple two-body system used for comparison of AREPO to REBOUND. The upper panels show the results from the AREPO simulation, while the lower panels show the results from the REBOUND simulation. Note that the REBOUND plots reflect a larger number of data points than were used for the AREPO plots.

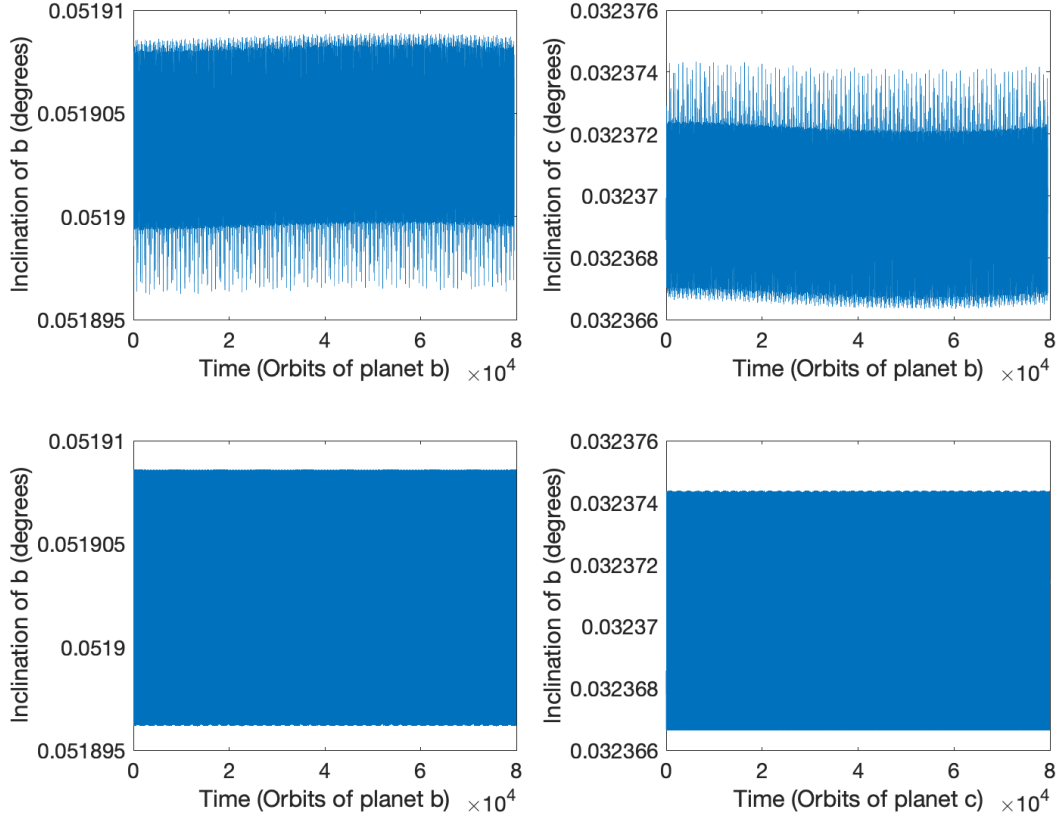


Fig. 12.— Inclinations of the planets in the simple two-body system used for comparison of AREPO to REBOUND. The upper panels show the results from the AREPO simulation, while the lower panels show the results from the REBOUND simulation. Note that the REBOUND plots reflect a larger number of data points than were used for the AREPO plots.

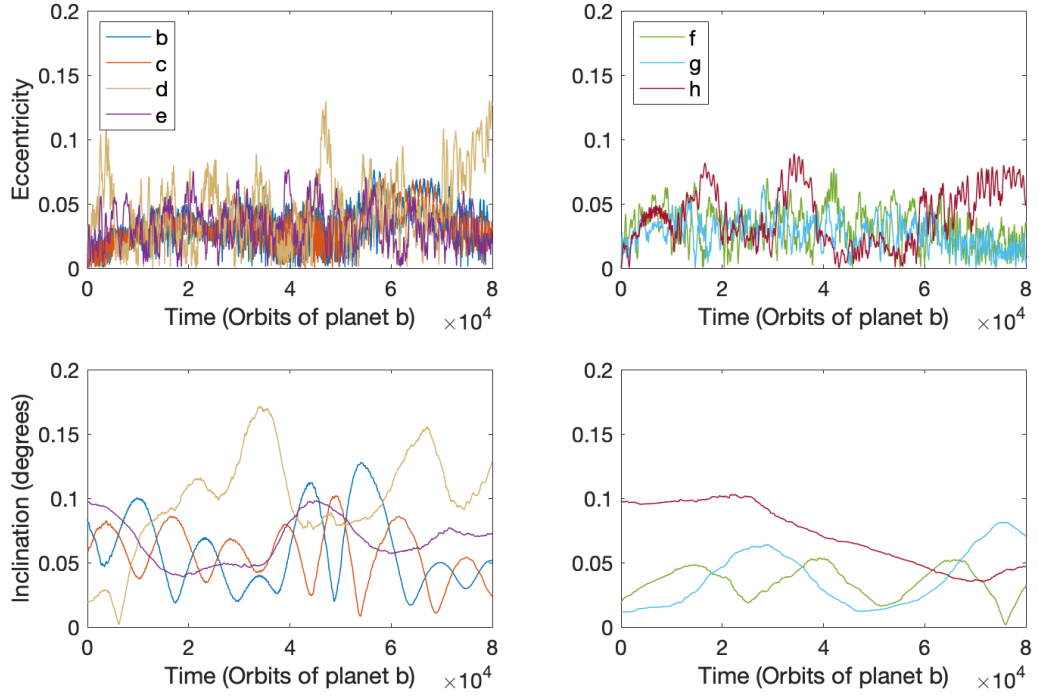


Fig. 13.— Eccentricities and inclinations of the TRAPPIST-1 system as simulated by REBOUND. For comparison, the results from the corresponding AREPO simulation are shown in Figure 5.

3

How Flat Can a Planetary System Get? II. The case of Kepler-90

Heising, M. Z., Sasselov, D. D., & Hernquist, L. 2021, ApJ, submitted

Abstract

We present in this paper a follow-up work to Heising et al. (2021), in which the dynamics of the TRAPPIST-1 system was explored, with special attention paid to the extraordinary flatness of the system and what it could tell us about its formation. Here, we turn to the Kepler-90 system, another many-planet system with very low mutual orbital inclinations. Notably, the Kepler-90 system displays some key differences to the TRAPPIST-1 system - most significant among them is the mass of the host star which is more than 10 times higher. Therefore, the comparison between the two systems offers an opportunity to study scaling with central mass. We use three-dimensional hydrodynamic simulations of the gas disk and the embedded planets to explore the effect of gas disk mass (and its removal) on mutual inclinations. From these simulations we found an upper limit for the mass of the gas disk during the last stages of the system's formation. In comparing this result to Heising et al.(2021), we find this upper limit in disk mass - relative to the stellar mass, to be lower than the upper limit we derived for the TRAPPIST-1 system, indicating that the effect of the gas on the mutual inclinations of the planets is felt even more strongly for the Kepler-90 system.

3.1 Introduction

With eight planets orbiting a $1.2 M_{\odot}$ star (Cabrera et al. 2014), the Kepler-90 system currently contains the most confirmed exoplanets of any system know, equal

only to our own Solar System. Using the transit method, seven planets were initially detected from *Kepler* photometry (Cabrera et al. 2014; Lissauer et al. 2014; Schmitt et al. 2014). An eighth planet, planet i, sitting between planets c and d in semimajor axis, was later identified with a deep learning technique that was applied to detecting transits in *Kepler* photometry (Shallue & Vanderburg 2018).

In this paper, we follow on from our previous work on the TRAPPIST-1 system (Heising et al. 2021, i.e. Chapter 2), applying the same techniques to the Kepler-90 system. In that work, we determined that the low mutual inclinations of the TRAPPIST-1 system provided a constraint on the mass of the gas disk they formed in during the last stages of planet formation. Here, we turn to the Kepler-90 system to better understand if we see the same dynamics at play in a different system, and if we can draw similar conclusions about the system, or whether the fundamental dynamics at play were due to particularities of the TRAPPIST-1 system.

The Kepler-90 system offers an insightful point of comparison to the TRAPPIST-1 system. They contain a similarly large number of planets (there are seven known exoplanets orbiting TRAPPIST-1). In addition, they both have relatively low mutual inclinations - this point is further addressed in Section 3.2. However, there are also some key differences between the systems. Kepler-90 has a stellar mass that is 13 times larger than that of TRAPPIST-1. In addition, while the planets of the TRAPPIST-1 system are tightly packed in a near-resonant chain, there is more variation in how the Kepler-90 planets are distributed in period - planets d, e, and f lie in a (2:3:4) mean motion resonant chain, and planets b and c are close to a 4:5 resonance (Cabrera et al. 2014), but others adjacent planet pairs are far apart, with the period ratio between planets d and i being 4.13. The semimajor axes of the planets in the two systems are shown schematically in Figure 14. Furthermore, there is also a difference in how the planets are distributed in size. The TRAPPIST-1 planets are all quite similar in size, with radii ranging from 0.76 to 1.13 Earth radii. The Kepler-90 planets, on the other hand, extend all the way from 1.19 to 11.3 Earth radii. There is also a clear trend with size and semimajor axis, with the two most outer planets being the largest by a significant margin (see Figure 16), whereas there are no obvious trends with size and semimajor axis in the TRAPPIST-1 system. It is these differences that make Kepler-90 an interesting system to study for this follow-up work.

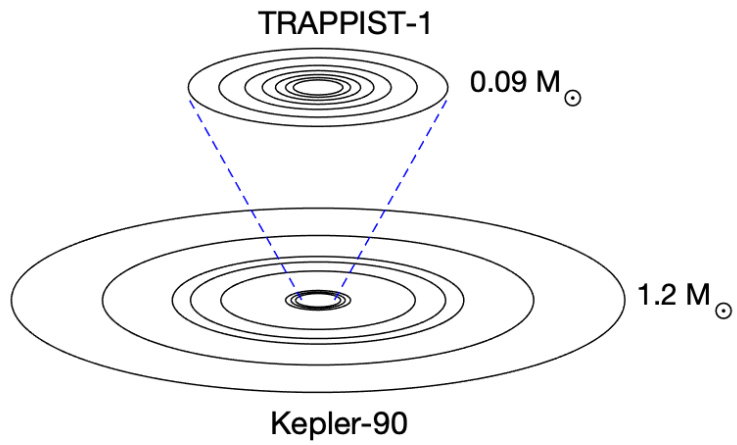


Fig. 14.— Schematic representation of the orbits of the Kepler-90 system and the version of the TRAPPIST-1 described in Heising et al. (2021), with eccentricities and inclinations assumed zero for simplicity. The TRAPPIST-1 system is shown at eight times scale, with a true extent relative to the Kepler-90 system indicated by the blue dashed lines. The stellar mass of each system is also indicated.

The underlying question that we are trying to answer is the same as the question we posed for our study of the TRAPPIST-1 system. We are interested in what happens to the planets in the system as the gas is dispersed, particularly as it relates to their inclinations. As the presence of the gas changes the locations of resonances in the system, it effectively acts as a perturbation. When the gas is removed, the effect of this perturbation will be “locked-in,” as there is no readily available way for the mutual inclinations of the system to be reduced purely through planet-planet interactions. As in Heising et al. (2021), our goal is to determine the minimum disk mass for which the perturbation to the orbits of the Kepler-90 planets is inconsistent with the mutual inclinations they are observed with today.

3.2 Inclination Dispersion of the Kepler-90 System

In Heising et al. (2021), we described a method for calculating the inclination dispersions of the TRAPPIST-1 and Kepler-90 systems, using the same procedure used by Fabrycky et al. (2014) and Zhu et al. (2018). This method entails Monte Carlo sampling impact parameters for each planet assuming that the inclinations follow a Rayleigh distribution with a given dispersion. For each pair of adjacent planets, given a certain dispersion, 6000 pairs of simulated impact parameters were generated. From these simulated impact parameters, a quantity ξ was then calculated, defined as

$$\xi = \frac{T_{dur,in}/P_{in}^{1/3}}{T_{dur,out}/P_{out}^{1/3}} \quad (15)$$

where T_{dur} is the transit duration and P is the orbital period. The simulated values of ξ were then compared to the observed values of the system to quantify how well a given dispersion fit the system, and to find a best fit. The results of this fit are shown in Figure 15, which also shows for context the inclination dispersion distribution as a function of number of planets for the *Kepler* systems, as calculated by Zhu et al. (2018).

As can be seen, the inclination dispersion of the Kepler-90 system is significantly larger than that of the TRAPPIST-1 system. In fact, only a lower bound could be measured, as the system is not well-fit by a Rayleigh distribution of inclinations. By inspection, we found that the reason for this was the inclination of planet i, which

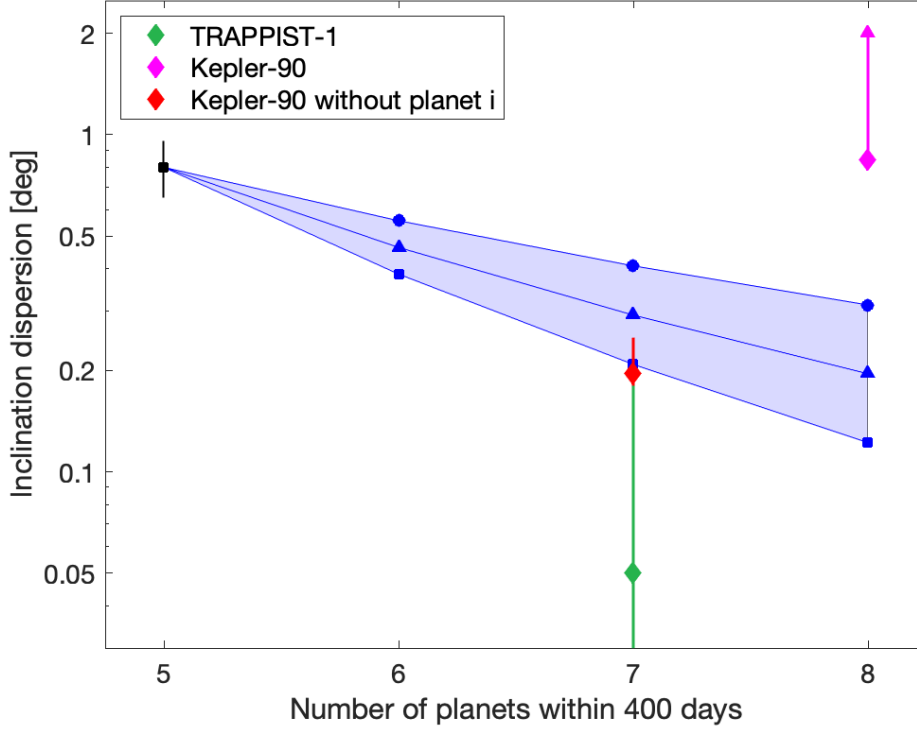


Fig. 15.— Inclination dispersions of the TRAPPIST-1 and Kepler-90 systems, both with and without planet i, compared to the inclination dispersion distribution as a function of number of planets (multiplicity) for the *Kepler* systems, reproduced from Zhu et al. (2018). The blue shaded region shows the expected 2σ range of inclination dispersions. Note that the lower error bar for the TRAPPIST-1 system extends to 0, and that the inclination dispersion for the Kepler-90 system represents a lower bound. The lower error bar for the Kepler-90 system without planet i is quite small and just barely extends past the marker. The upper error bar for the TRAPPIST-1 system and that of the Kepler-90 system without planet i both coincide at 0.25 degrees.

was an apparent outlier. The same analysis was then performed to measure the inclination dispersion of the system with planet i removed. The resulting measurement, also shown in Figure 15, was much more precise and consistent with the range of values measured for the TRAPPIST-1 system. Of note, planet i is one of the smallest planets in the system, and by our mass estimates for the planets in the system (described in Section 3.3), contains only 1.4% of the total planetary mass. As such, from a dynamical perspective, the TRAPPIST-1 system and the Kepler-90 system are closer in mutual inclinations than it would at first appear.

3.3 Disk Simulations - Methods & Initial Conditions

As was the case for our work in the TRAPPIST-1 system in Heising et al. (2021), we now turn to disk simulations to understand what the flatness of the system can tell us about its formation. In particular, we are interested in placing a constraint on the mass of the gas disk during the last stages of planet formation, when the planets had nearly attained their current orbits. We assume that after the gas was dispersed, no dynamical process could significantly alter the overall flatness of the planets' orbits. As described in Heising et al. (2021), we use the hydrodynamic code AREPO (Springel 2010; Pakmor et al. 2016), which solves the equations of hydrodynamics on a Voronoi mesh that moves with the fluid, to perform these simulations. In Heising et al. (2021), we compared our N-body simulations of the TRAPPIST-1 system performed with AREPO to simulations performed with the N-body code REBOUND (Rein & Liu 2012), and found good agreement, validating the N-body performance of AREPO for this work.

Before describing how such simulations were performed, though, we must address an important caveat, which is that at the time the work described in this paper was carried out, no masses had been measured for any of the Kepler-90 planets - only radii. For any dynamical study, of course, masses are a necessity, and so we had to estimate the planets' masses from their radii, as reported by Cabrera et al. (2014) and Shallue & Vanderburg (2018). To do this, we used the model described by Chen & Kipping (2017), which describes an empirical relation between mass and radius over several orders of magnitude, with transitions between different classes of objects at certain masses. The resulting masses are shown in Figure 16. Planet

c was found to be in the Terrian regime of Chen & Kipping (2017), while all other planets lie in the Neptunian regime.

Since the completion of the simulations performed in this work, which took upwards of two months to run, an analysis of transit timing variations performed by Liang et al. (2020) yielded measurements for the masses of planets g and h. These mass measurements, $15.0^{+0.9}_{-0.8}$ Earth masses for planet g and 203^{+5}_{-5} Earth masses for planet h, differ significantly from our mass estimates for these two planets of 50.4 and 88.4 Earth masses, respectively. As these two planets are the largest in the system, we expect this change in mass would have a meaningful impact on the dynamics of the system as a whole. However, we would not expect such a change in mass to alter the qualitative findings of this work. Furthermore, since planet h, which has a dominant dynamical effect, was found to be more massive than we had estimated, we would expect this to strengthen our results, if anything. This point is further addressed in Section 3.5.

The initial conditions for the planets were calculated as they were in Heising et al. (2021). Values for the planets' periods were taken from Cabrera et al. (2014) and Shallue & Vanderburg (2018). Eccentricities were assumed to be initially zero. Inclinations were taken from values reported by Cabrera et al. (2014) and Shallue & Vanderburg (2018), before a rotation was applied such that the xy-plane would coincide with the invariable plane of the system. The star is again forced to maintain a fixed position, and is allowed to accrete gas within a radius that, compared to the value used in Heising et al. (2021), is scaled to the semimajor axis of Kepler-90b relative to that of TRAPPIST-1b.

The initial conditions of the gas disk were also calculated using the same methods described in Heising et al. (2021). We adopt a surface density Σ given by a softened Lynden-Bell-Pringle density profile (Lynden-Bell & Pringle 1974):

$$\Sigma(R) = \Sigma_0 \left(\frac{f_{soft}(R, R_{soft})}{R_c} \right)^{-p} \exp \left[- \left(\frac{R}{R_c} \right)^{2-p} \right] \quad (16)$$

where R is the cylindrical radius, Σ_0 is a constant given by the total disk mass, p is a surface density power law index, R_c is the disk's characteristic radius, R_{soft} is a softening parameter, and f_{soft} is a smooth softening function that keeps the surface density profile from diverging at $R = 0$. In particular, $f_{soft}(R, R_{soft}) = R$ for

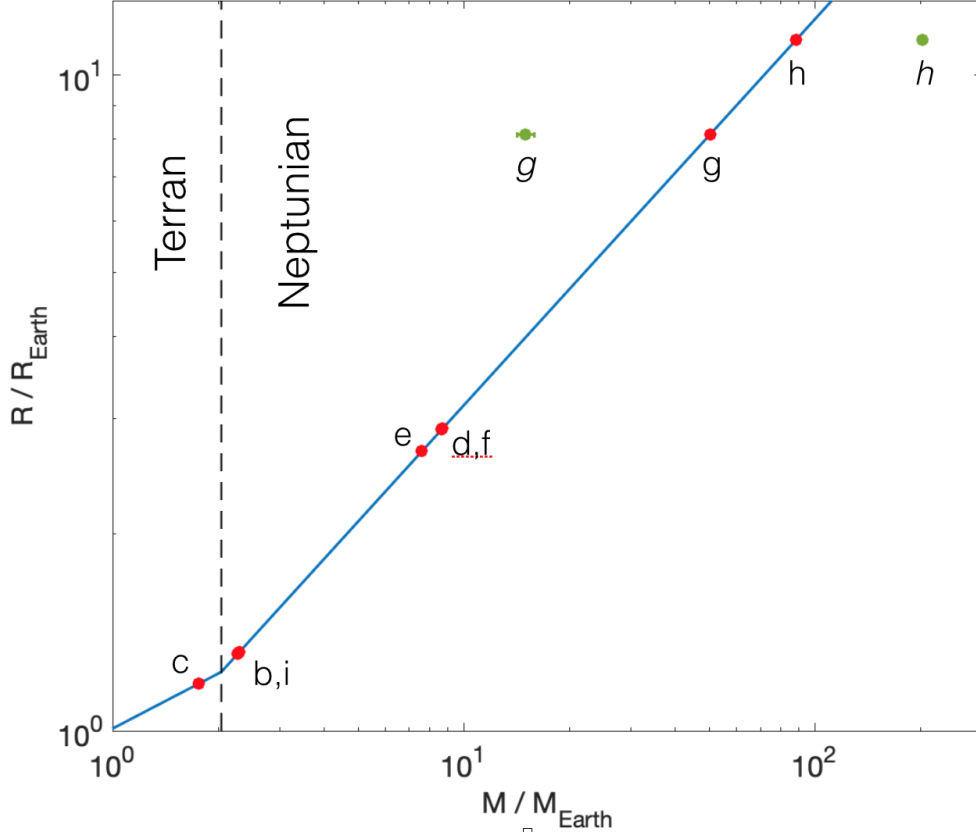


Fig. 16.— Masses of the Kepler-90 planets as predicted from their radii using the relation described in Chen & Kipping (2017). Note that as planets b and i, as well as planets d and f, are very similar in size, they show up as overlapping dots. The green dots and error bars show the masses of planets g and h as measured by Liang et al. (2020). Note that the error bars on the mass of planet h are smaller than the marker size.

$R \geq R_{soft}$ and $f_{soft}(R, R_{soft}) = R_{soft}/2.8$ for $R = 0$. As before, we set $p = 0.5$. The rest of the values are determined by scaling the disk to match the disks described in Heising et al. (2021). The mass of the disk scales with the stellar mass, R_c scales with the semimajor axis of the outermost planet of each system, and R_{soft} scales with the semimajor axis of the innermost planet of each system. This scaling implies a relatively low-mass disk, as was the case for the least massive disk explored in Heising et al. (2021); it contains a total of $3.6M_{\oplus}$ interior to planet h at 1.01 AU, whereas the Minimum Mass Solar Nebular (Hayashi 1981) contains $331M_{\oplus}$ interior to the same radius. As before, the resulting disk is well-described by a power law in the vicinity of the planets, and we assume that the planets' orbits would not be significantly altered by gas at much larger radii than their own semimajor axes. A constant axis ratio of 0.1 was similarly adopted for the disk, with a sound speed c_s characterized by

$$c_s(R) = 0.1 \sqrt{\frac{GM_*}{f_{soft}(R, R_{soft})}} \quad (17)$$

where G is the gravitational constant.

We use the method described in Muñoz et al. (2015) to create a Monte Carlo sample of points in the disk, so that when the Voronoi mesh Arepo uses is created from these points, the cells will have roughly equal mass. Points are initially sampled in ϕ and R using the surface density profile described in Equation 16. To sample points in z , a vertical density profile, which depends on the self-gravity of the gas, must first be calculated at each radius to satisfy the condition

$$\frac{c_s^2(R)}{\rho} \frac{\partial \rho}{\partial z} = -\frac{\partial \Phi_{0,z}}{\partial z} - \frac{\partial \Phi_{g,z}}{\partial z} \quad (18)$$

where $\Phi_{0,z} \equiv \Phi_0(R, z) - \Phi_0(R, 0)$ is the z -dependent part of the Keplerian potential Φ_0 of the central star and $\Phi_{g,z} \equiv \Phi_g(R, z) - \Phi_g(R, 0)$ is the z -dependent part of the potential Φ_g due to self-gravity. $\Phi_{g,z}$ is related to ρ through the Poisson equation as

$$\frac{\partial^2 \Phi_g(R, z)}{\partial z^2} = \frac{\partial^2 \Phi_{g,z}(R, z)}{\partial z^2} = 4\pi G \rho(R, z). \quad (19)$$

An iterative process is then used to calculate the vertical density profile that satisfies Equation 18. The z -coordinates of the mesh-generating points are then sampled from this profile at a given radius.

As in Heising et al. (2021), a background grid of cells is created so that the computational domain is well-sampled throughout, including regions where there is little mass. To help this background grid smoothly transition to the Monte Carlo-sampled cells, Lloyd’s algorithm (Lloyd 1982) is used to regularize the mesh, as described in Heising et al. (2021). From this new set of points, each gas cell is initialized with its proper density. Density slices through the initial disk are shown in Figure 17. Initial velocities are then calculated for each cell, taking into account both the pressure gradient of the disk and self-gravity, as was described in Heising et al. (2021).

3.4 Results

To compare the dynamics of the Kepler-90 system to that of the TRAPPIST-1 system, as studied in Heising et al. (2021), we perform the same numerical experiment. A simulation with gas and planets is initialized as described above and is evolved for 200 orbits of planet b. At that point in time, the simulation branches into two different runs - a gas run, in which things “proceed as normal”, with gas and planets continuing to evolve together; and a gas-free run, in which the gas is instantly removed. As before, we note that the purpose of instantly removing the gas is not to realistically model the disk’s dispersal, but rather to determine the effect of the gas on the orbits of the planets by comparison between the two runs.

The eccentricities and inclinations from these runs are shown in Figure 18. At $t = 200P_b$, where the gas-free run begins, the eccentricities display a range from 1.1×10^{-4} to 5.0×10^{-3} , while the inclinations range from 0.043° to 0.38° . While some planets do eventually diverge in eccentricity, it takes some time for this to happen, likely as a stochastic result of small changes, and there are no obvious trends. In inclination, there is some systematic divergence between the two runs, which begins over the period of roughly $400P_b$ to $1000P_b$, where P_b is the orbital period of planet b. However, these divergences are fairly small, and the magnitude of the mutual inclinations in the system is largely unchanged. Such behavior indicates that the system is not dominated by gas dynamics, and that the planets could retain their relatively low mutual inclinations.

Density slices through the disk at $t = 100P_b$ are shown in Figure 19, which

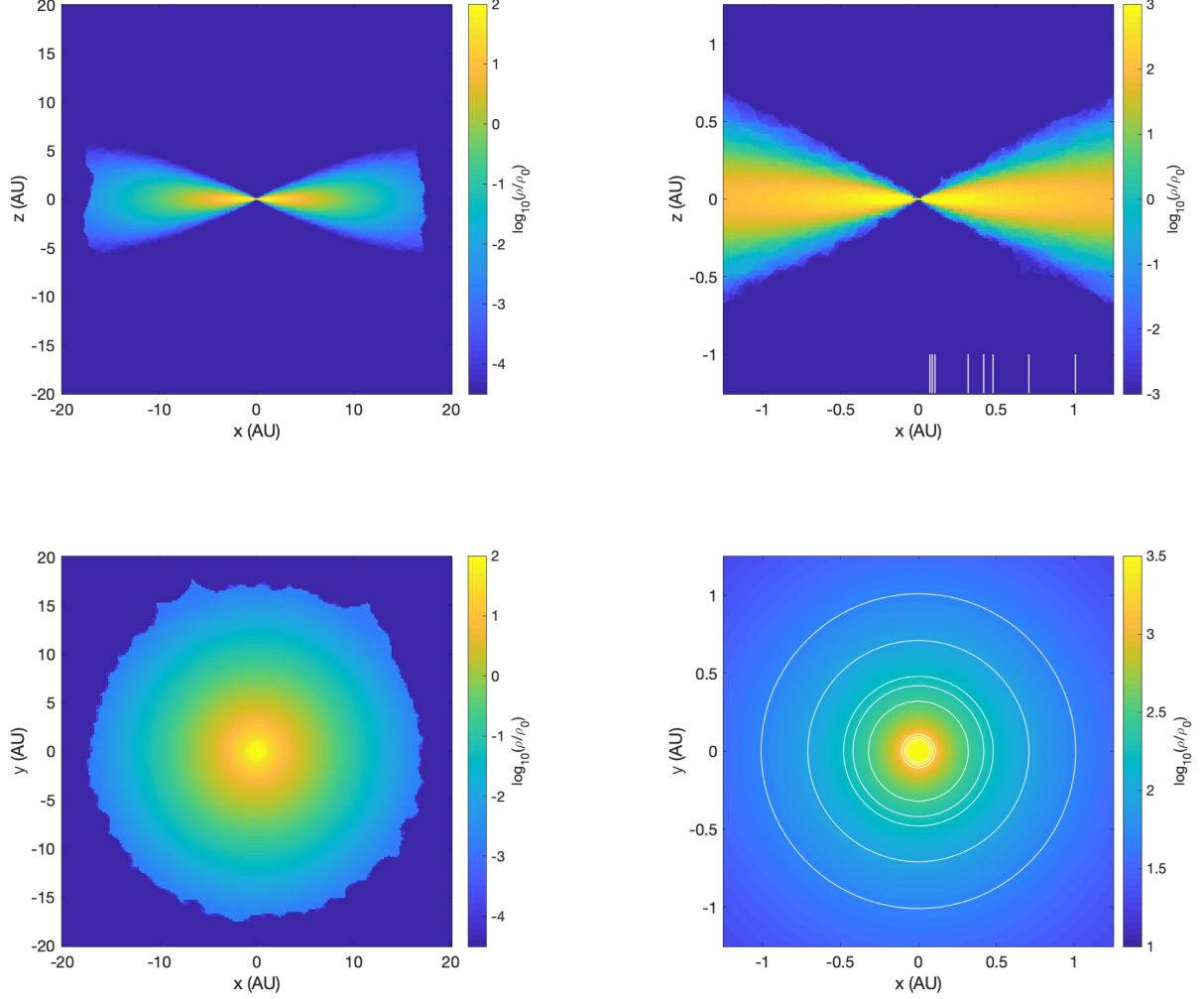


Fig. 17.— Density slices of the disk at the start of the simulation. The upper panels show a slice through $y = 0$, while the lower panels show a slice through $z = 0$. The left panels show the full computational domain, while the right panels show a zoomed-in view, with white lines indicating the semimajor axes of the TRAPPIST-1 planets. The normalization density, ρ_0 , is defined as $\rho_0 \equiv \Sigma_0/(hR_c)$, where $h = 0.1$ is the axis ratio of the disk.

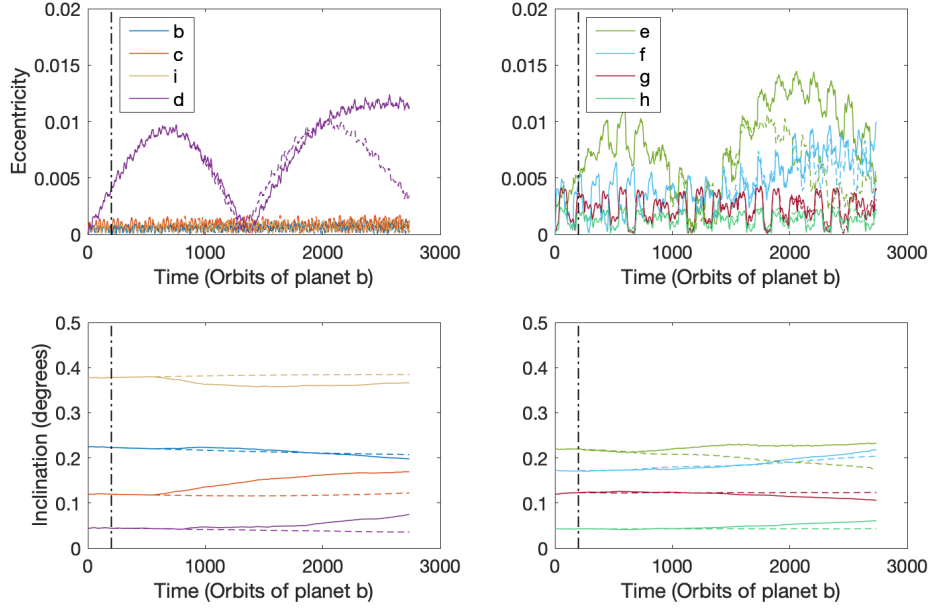


Fig. 18.— Eccentricities and inclinations of the Kepler-90 planets simulated with the Σ_0 disk. The solid lines show the run with gas, and the dashed lines beginning at $t = 200P_b$ (demarcated with the black dash-dot line) show the run in which the gas is removed.

can be compared with the initial disk shown in Figure 17. The vertical density structure is not significantly altered on this timescale. In the radial structure, the main development is that spiral density waves can be seen to propagate from the two outermost planets, which contain the bulk of the planetary mass. These waves, which were not observed in the simulations of the TRAPPIST-1 system performed in Heising et al. (2021), highlight that the planets in this system strongly influence the gas dynamics.

To test how the dynamics of the system depend on disk mass, we repeat the above experiment for disks with Σ_0 increased by factors of 4, 15, and 50, as was done in Heising et al. (2021). The resulting inclinations from these simulations are shown in Figure 20. As can be seen, for each of these disk masses, there is a much starker divergence between the gas and gas-free runs as was present for the Σ_0 disk. Furthermore, most planets diverge promptly at $t = 200P_b$, indicating that the evolution of their inclinations is driven almost entirely by gas dynamics. As disk mass is increased, the effect is only felt more strongly, with inclinations diverging more rapidly and reaching larger values. As this behavior would be inconsistent with the relatively low mutual inclinations the Kepler-90 system is observed with today, we infer an upper limit of $4 \times \Sigma_0$ for how massive the Kepler-90 disk could have been during the last stages of planet formation.

To better quantify the inclinations of the system as a whole, we turn to the normalized inclination angular momentum deficit, or NIAMD, as was defined in Heising et al. (2021). NIAMD indicates the loss in angular momentum, as compared to a system with the same semimajor axes and masses but circular orbits, due solely to the planets' inclinations. Mathematically, it can be expressed as

$$\text{NIAMD} \equiv \frac{\sum_{k=1}^n \Lambda_k (1 - \cos i_k)}{\sum_{k=1}^n \Lambda_k}, \quad (20)$$

where the summation is carried out over all n planets, i_k is inclination, and $\Lambda_k = m_k \sqrt{G m_* a_k}$, where a_k is semimajor axis, m_k is planetary mass, m_* is stellar mass, and G is the constant of gravitation.

The NIAMD for each disk mass and for both the gas and gas-free runs is shown in Figure 21. Here, a similar story is told as could be seen when looking only at the individual inclinations. For the Σ_0 disk, there is minimal divergence between the gas and gas-free runs. However, for the more massive disks, NIAMD diverges

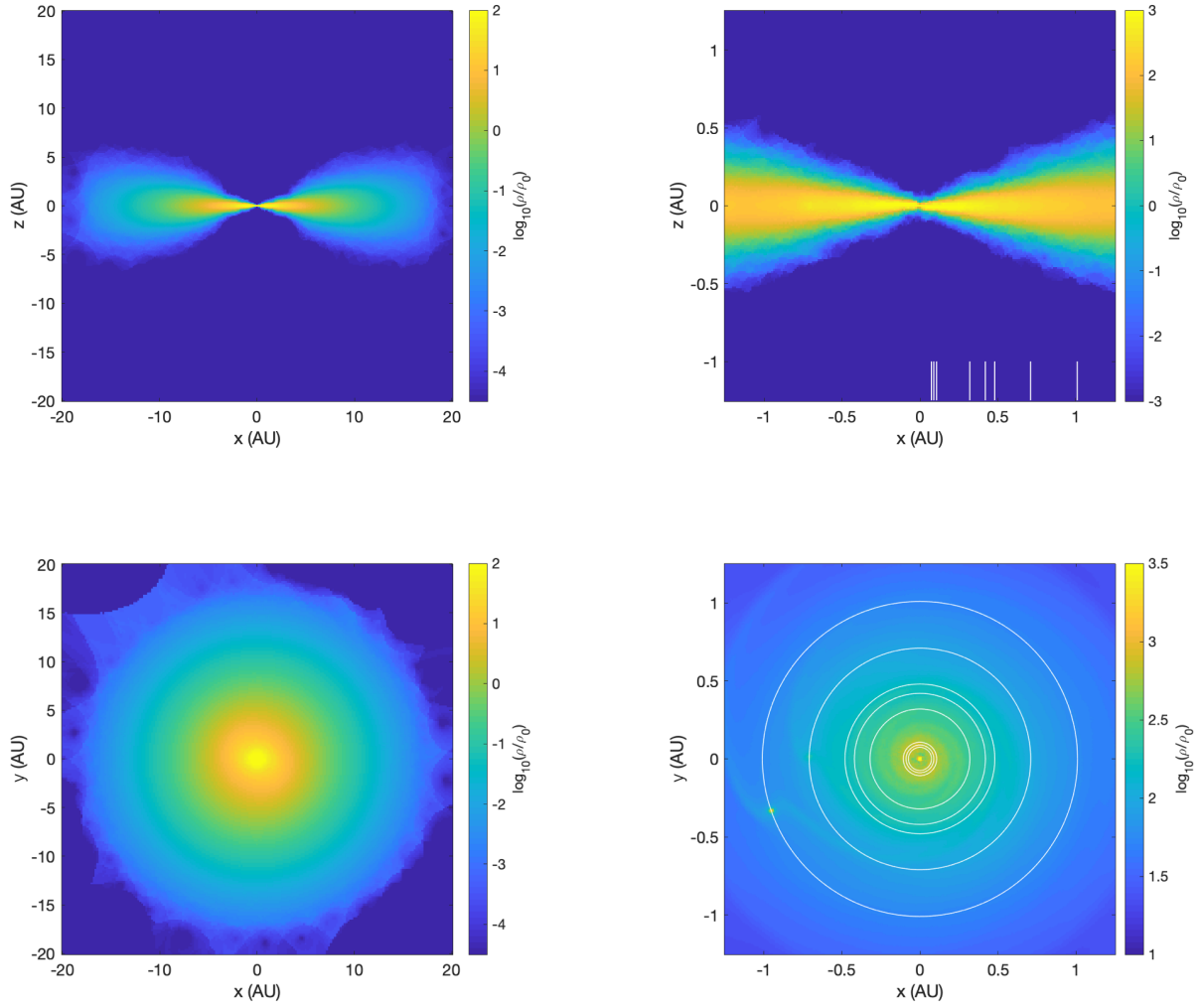


Fig. 19.— Density slices after 100 orbits of planet b. The panels are the same as in Figure 17.

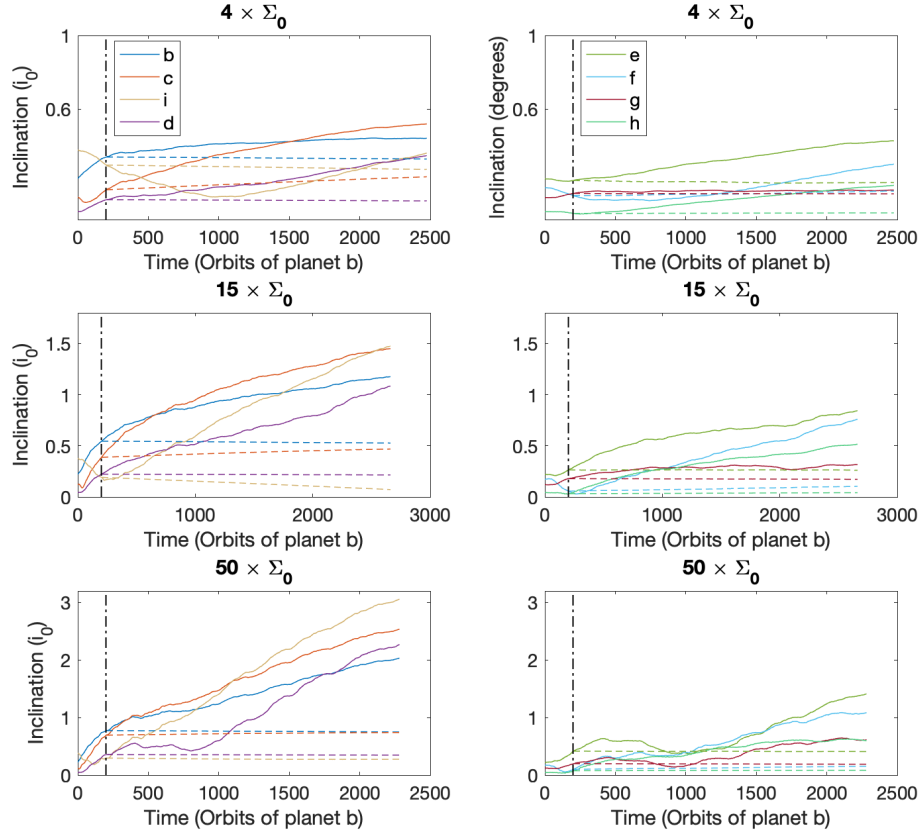


Fig. 20.— Inclinations of the Kepler-90 planets for the more massive disks - the lines are defined as in Figure 18.

significantly, and this divergence is apparent immediately after $t = 200P_b$. This behavior strengthens our upper limit of $4 \times \Sigma_0$ proposed above. We note also that the NIAMD reaches much larger values than it did in our simulations of the TRAPPIST-1 system, as can be seen by comparison to Figure 9 in Heising et al. (2021).

3.5 Conclusions

In this work, we have presented hydrodynamic simulations of the Kepler-90 system to probe its dynamics during the last stages of planet formation, at the end of its disk’s lifetime. Particular attention was paid to the relatively low mutual inclinations of the system, and how they were affected by varying disk mass. This work was motivated by and directly compared to similar simulations of the TRAPPIST-1 system performed in Heising et al. (2021).

In comparing the Kepler-90 system to the TRAPPIST-1 system, we have observed qualitatively similar dynamics. As disk mass is increased, inclinations eventually began to diverge between runs where gas is present and runs where gas is not. The key difference that we have uncovered is that this change occurs at lower mass for the Kepler-90 system, implying an upper limit of $4 \times \Sigma_0$ as opposed to the upper limit of $15 - 50 \times \Sigma_0$ found in Heising et al. (2021). It is likely that this change is due to the difference in distribution of mass of the system - there is overall more planetary mass in the Kepler-90 system than there is in the TRAPPIST-1 system, and this mass is contained mostly in the outer two planets, leading to the development of significant spiral density waves, providing a source of feedback between the gas and the planets. If this is indeed the case, it would imply that the stronger dependence on disk mass observed for the Kepler-90 system, as compared to the TRAPPIST-1 system, would be strengthened by the larger mass for Kepler-90h reported by Liang et al. (2020).

To date, there has been little work done on models of the formation of the Kepler-90 system specifically to which our upper mass limit could be compared. More generally, though, the similar dynamics uncovered by this work and Heising et al. (2021) implies that such behavior can be found in multi-planet systems of widely varying scale and architecture. Furthermore, it shows that mutual inclinations of a system can be an important constraint on its formation, a constraint that is perhaps

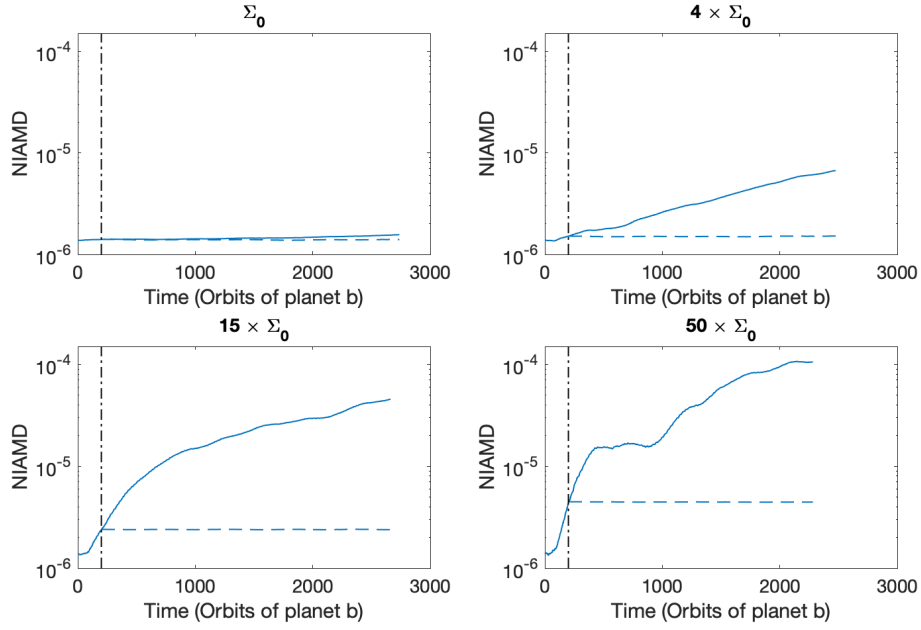


Fig. 21.— Normalized inclination angular momentum deficit (NIAMD) as a function of time for each of the simulated disks. Solid lines show the gas runs, and the dashed lines beginning at 200 orbits show the gas-free runs. Note that the y-axis is on a log scale.

overlooked today.

This work used the Extreme Science and Engineering Discovery Environment (XSEDE; Towns et al. 2014) Stampede2 at the Texas Advanced Computing Center through allocation TG-AST190034.

Breaking Angular Momentum Deficit Down to its Parts

Abstract

Angular momentum deficit (AMD) has proven to be a useful tool to characterize the dynamics and formation of planetary systems in a variety of contexts. Here, we present work that explores aspects of AMD that are not often discussed, namely its separation into inclination and eccentricity components that can evolve separately as a function of time under certain conditions. As a demonstrative example, numerical simulations from the early stages of the Nice model are presented. We find that in this context, AMD can at times grow to be almost entirely dominated by inclinations, giving valuable insight into the underlying dynamics of such an evolving system. This treatment of AMD can be a valuable tool for work on exoplanet systems, and has some possible implications for planet formation theory modeling.

4.1 Introduction

With more and more multi-exoplanet systems being discovered, there has been an increased need for a metric to characterize and compare the orbital dynamics of these systems. Focusing on individual orbital parameters, namely semimajor axis, eccentricity, and inclination (Winn & Fabrycky 2015), can be a useful way to classify systems, but can potentially be misleading in certain contexts; a given eccentricity of a certain planet, for example, will mean very different things for the dynamics of the system depending on the planet’s mass and semimajor axis in relation to the rest of the system. What is often more desirable is a way to classify the system as a whole.

In Laskar (1997, 2000), angular momentum deficit (AMD) was introduced as a useful metric for this purpose of assessing the degree of orbital excitation. AMD represents the absolute difference between the angular momentum in a planetary system and the angular momentum that system would have if the orbits were all circular and coplanar (this representing the maximum angular momentum a system could have given a set of masses and semimajor axes). Qualitatively, this gives an indication of the dynamical temperature of a system, as larger eccentricities and inclinations correspond to larger AMD, with more weight being given to more massive and longer-period planets in a system. Quantitatively, AMD derives from the normal component with respect to a reference plane (the invariable plane, observed in the Solar System, but not in exoplanet systems). Mathematically, AMD can be expressed as (Laskar (1997, 2000)):

$$\text{AMD} = \sum_{k=1}^n \Lambda_k \left(1 - \sqrt{1 - e_k^2} \cos i_k \right), \quad (21)$$

where the summation is carried out over all n planets in a system, e_k is eccentricity, i_k is inclination, and $\Lambda_k = m_k \sqrt{G m_* a_k}$, where a_k is semimajor axis, m_k is planetary mass, m_* is stellar mass, and G is the constant of gravitation. In the absence of resonances, Λ_k are constants in the averaged equations of motion, a fact used first by P.-S. Laplace at the end of the 18th century to point out that changes in orbital eccentricities and inclinations are bounded in the secular equations to first order (Laskar (2000)).

Laskar & Petit (2017) showed that AMD can be used to classify the stability of planetary systems in an analysis considering what value of AMD would be required to allow collisions between planets in a particular system (in collisional growth models). Chambers (2001) used AMD to evaluate stable solutions in his Solar System simulations. And Turrini et al. (2020) introduced the concept of normalized angular momentum deficit so that different exoplanetary systems could be directly compared with a single metric.

As was shown in Laskar & Petit (2017), AMD is a conserved quantity for a system that can be approximated by secular dynamics. As semimajor axis is conserved in this approximation, this is simply a statement of conservation of angular momentum. For this reason, AMD is typically taken to be a fixed quantity. However, while the secular approximation is valid in many contexts, it does not always hold.

For examples, when mean motion resonances are present, an AMD-stability analysis may not be appropriate. For some systems, especially chaotic ones, semimajor axis may vary greatly. In such cases, a typical AMD calculation will not capture the dynamics at play. However, looking at AMD not as a fixed quantity, but instead as a function of time, may yield additional insights into the system. As an example, we employed an analysis of a time-dependent AMD in a study of the dynamics of the TRAPPIST-1 system of planets during the dissipation of its gaseous protoplanetary disk (Heising et al. 2021, i.e. Chapter 2). Monitoring the changes of the system’s AMD in our 3D hydrodynamic simulations allowed a quantitative evaluation of the effect of the disk mass on the mutual inclinations of the TRAPPIST-1 planets.

Another notable aspect of typical AMD analysis is that eccentricity and inclination is treated approximately symmetrically. This can be seen by applying to Equation 21 a Taylor expansion to lowest order in e and i , which yields

$$\text{AMD} \approx \sum_{k=1}^n \Lambda_k \left(\frac{e_k^2 + i_k^2}{2} \right). \quad (22)$$

In the context of AMD, we can see that eccentricity and inclination contribute equally. In practice, this means that the AMD value of many exoplanet systems will be dominated by one quantity over the other. In particular, the transit method for exoplanet detection selects for systems with low mutual inclinations, particular for higher-multiplicity systems (Zhu et al. 2018), as planets with larger inclinations would not transit. However, eccentricities can still be moderately large in such systems, due to planet-planet interactions, resulting in an AMD that is determined almost entirely by the eccentricities. It is true that in such cases, eccentricities do play the dominant role in dynamics, and so it is appropriate that the AMD reflects that. In certain contexts, though, there might be reason to pay particular attention to the inclinations, making the AMD a poor metric for this purpose. In (Heising et al. 2021), we were faced with exactly this situation, prompting us to introduce a related quantity, the inclination-AMD (IAMD), defined as

$$\text{IAMD} \equiv \sum_{k=1}^n \Lambda_k (1 - \cos i_k). \quad (23)$$

Simply put, the IAMD represents the angular momentum deficit due to inclination alone, or equivalently, AMD if all eccentricities in the system were set to zero. Simi-

larly, and for completeness, we can define the eccentricity-AMD (EAMD) as

$$\text{EAMD} \equiv \sum_{k=1}^n \Lambda_k \left(1 - \sqrt{1 - e_k^2} \right). \quad (24)$$

We note that to lowest order, $\text{AMD} \approx \text{IAMD} + \text{EAMD}$.

The separation of AMD into IAMD and EAMD as a function of time is further explored in this work with numerical simulations, and possible implications are discussed. We suggest that AMD, used in time and in its parts, could be valuable as a measure of instability triggers in models of the formation and evolution of multi-planetary systems, as for example, during the dissipation of the gaseous disk, or when two massive planets are reaching a mean-motion resonance (see the Nice model in Sect. 2), etc.

4.2 Nice Model Simulations

For an example of a system in which AMD is subject to change over time, we turn to the Nice model description of the Solar System (Tsiganis et al. 2005; Morbidelli et al. 2005; Gomes et al. 2005). In the original formulation of this model, the four outer planets, which initially do not span as far out in the Solar System, interact with an exterior disk of planetesimals. The three outermost planets each in turn scatter these planetesimals inward, which by conservation of angular momentum causes them to drift somewhat outward. When the planetesimals reach Jupiter, however, due to its large mass, they are driven into much more eccentric orbits, and many are ejected from the Solar System. This in turn results in Jupiter drifting inward. As Saturn migrates outward and Jupiter inward, the two eventually reach a 1:2 mean motion resonance, destabilizing the system, and causing the orbits of the outer planets to evolve dramatically, allowing them to reach their current configuration.

Much work has been done on refining (Levison et al. 2011), or in some cases refuting (Boehnke & Harrison 2016), aspects of the Nice model. Our goal in this paper is not to validate or expand upon the Nice model in any way. Instead, we use it as a dynamical test case to explore AMD in the context as discussed above. In particular, we are interested in characterizing the system as it approaches instability with the resonance of Jupiter and Saturn.

The exact setup of our simulations was very similar to that described in Tsiganis et al. (2005). We begin with Jupiter, Saturn, Uranus, and Neptune at semimajor axes of 5.45 AU, 8.3 AU, 12 AU, and 15.25 AU respectively. Of note, this puts Saturn interior to the 1:2 mean motion resonance with Jupiter by 0.35 AU. A disk of 5,000 planetesimals extends from 15.25 AU to 32.5 AU, with a total mass of $40M_{\oplus}$ and a surface density profile that falls off with radius as $\Sigma \sim R^{-1}$. To save computational time, the self-gravity of the disk was ignored, as was done in Tsiganis et al. (2005). A total of seven such simulations were performed. For each, the longitudes of each body were initialized randomly between 0 and 2π , and the eccentricities and inclinations of each body were initialized randomly between 0 and 10^{-3} . Each simulation was evolved with a timestep of 0.5 years and was integrated out to 5.3×10^6 years, a timescale that is similar to how long it typically takes Jupiter and Saturn to reach resonance in the simulations of Tsiganis et al. (2005).

To carry out the simulations described above, we used the open source N-body code REBOUND (Rein & Liu 2012). The simulations were integrated with WHFAST (Rein & Tamayo 2015), a symplectic Wisdom-Holman integrator (Wisdom & Holman 1991; Kinoshita et al. 1991).

The results from three illustrative simulations are shown in Figure 25 (the remainder are shown in Appendix A). For each simulation, the semimajor axes of the four planets as a function of time are shown in the left panels. For clarity, since these changes are mostly relatively small on this timescale, all the semimajor axes have been normalized to their initial values. The right panels show the AMD of the planets (not including planetesimals) broken into the IAMD and EAMD components. Here, we have normalized each AMD value by dividing by the total circular angular momentum of the system, that is, by $\sum_{k=1}^n \Lambda_k$, as was done by Turrini et al. (2020), for instance.

Simulation 4 represents the typical outcome that we observed. Neptune, exchanging angular momentum with the disk of planetesimals and scattering many of them inward, quickly begins migrating outward. Uranus and Saturn in turn experience some outward migration as well. The AMD of the system gives us an insight into the underlying dynamics that are at play in this process. Because e and i are initialized to similar values, EAMD and IAMD are roughly equal at the beginning of the simulation. However, almost immediately, the planets attain larger eccen-

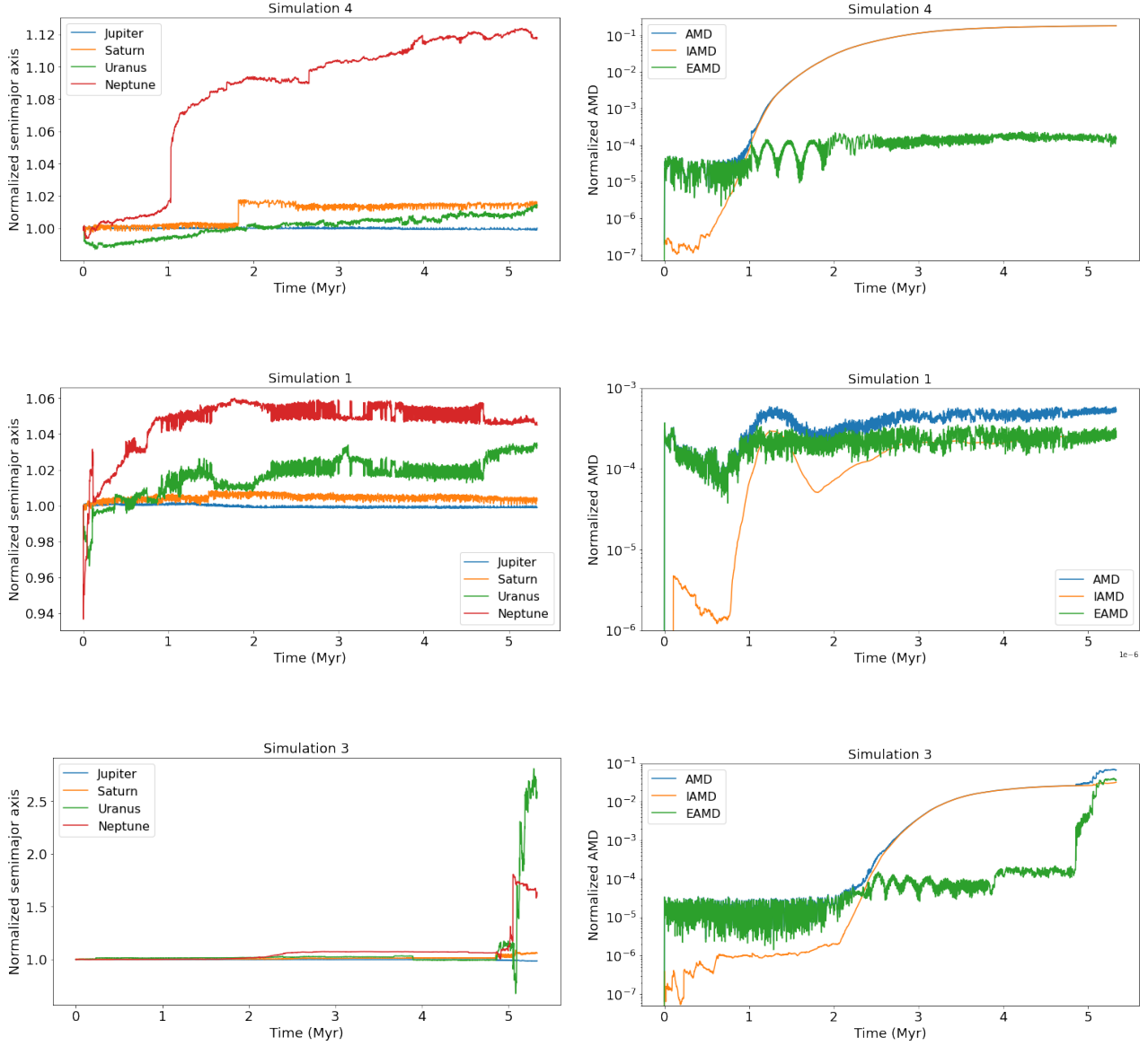


Fig. 22.— On the left panels, the semimajor axes of the Solar System giant planets for different Nice Model simulations are shown as a function of time, normalized to their initial values for clarity. On the right panels, the angular momentum deficit (AMD) of those planets from the same simulations is presented, normalized to the circular angular momentum of the planetary system, and separated into its IAMD and EAMD components.

tricies due to planet-planet interactions, causing the EAMD to exceed IAMD by two orders of magnitude. In this regime, the AMD is almost completely dominated by the eccentricities. As the system evolves, though, we see a dramatic growth in IAMD of several orders of magnitude, while EAMD stays much more constant, not changing by more than roughly an order of magnitude. As a result, the AMD shifts to being extremely inclination-dominated. It is the planets' inclinations, not their eccentricities, that underlie the dynamics of this early Nice model scenario.

In Simulation 1, we see largely similar behavior in the evolution of the semimajor axes of the planets (with some differences at the very beginning of the simulation). However, when we look at AMD, the picture is somewhat different. Again, EAMD immediately grows and greatly exceeds IAMD for the first $\sim 10^6$ years. While IAMD does eventually grow, though, it doesn't surpass EAMD. Indeed, this is the only simulation we observed for which it appears an equipartition between EAMD and IAMD has been reached for this process. Still, it is important to note that again it seems to be the IAMD that is most indicative of long-term changes to AMD; this is apparent in the growth, dip, and regrowth of both IAMD and AMD from roughly $1 - 3 \times 10^6$ years. In contrast, EAMD does not change much on a long term scale after its initial burst.

Simulation 3 is a case in which the system destabilizes due to Jupiter and Saturn reaching their mean motion resonance on this timescale. Until this happens, the evolution of the planets' semimajor axes and AMD proceeds similarly to that of Simulation 4 (and most others). When destabilization occurs, Uranus and Neptune quickly are pushed out to much wider orbits, and (at least temporarily) switch places, as was seen in many of the simulations of Tsiganis et al. (2005). The destabilization of the system is felt much more by the planets' eccentricities than their inclinations, as we can see by the comparison of EAMD to IAMD during this time. Post-destabilization, it appears possible that an equipartition of EAMD and IAMD has been reached. A true understanding of the post-destabilization dynamics would require significantly longer simulations and is beyond the scope of this paper, but it seems likely that the framework of separating EAMD and IAMD as a function of time would be helpful for this.

As noted above, the plots of AMD in Figure 25 represents only that from the planets, not the planetesimals. As angular momentum is not conserved in only the

planets, it is of course not surprising that AMD is also not conserved, even at times when the semimajor axes of the planets are relatively constant. To better understand the AMD of the system, it is important that we also consider the contribution from the disk of planetesimals.

In Figure 23, the AMD of the planetesimal disk is shown for Simulation 4. As many planetesimals are ejected over the course of the simulation (by the end, only 1,554 of the original 5,000 planetesimals remain bound to the Sun), only the planetesimals that have not been ejected are included in the calculation of AMD. The upper panel shows the AMD of the planetesimals only, broken up into EAMD and IAMD. As can be seen, there is a steep increase in AMD over the first 2×10^6 years, initially driven by eccentricities, which are excited by planet-planetesimal interactions. Eventually, IAMD of the planetesimals catches up, and near equipartition is reached. Because of the very large eccentricities and inclinations attained, the normalized AMD reaches up to 0.92 - that is, the angular momentum of the planetesimals is only 0.08 of what the circular angular momentum of the planetesimals would be.

In the lower panel, we show the AMD of the whole system, here divided into the component from the planets and the component from the planetesimals. As can be seen, the AMD of the system in the beginning is heavily dominated by the planetesimals, despite the less total mass in the planetesimals, due to the energetic orbits the planetesimals are quickly excited to. The planets, on the other hand, take more time to be significantly excited, and as a result the AMD contribution from the planets does not play a large role until later. What's surprising is that the AMD from the planets eventually becomes very close to equal that of the planetesimals, and the two values remain extremely close. We don't believe there to be any underlying dynamical reason to explain this behavior, and indeed, it is not seen in every simulation.

4.3 Discussion

The motivation for exploring the aspects of AMD investigated by this paper came from two earlier works, Chapters 2 and 3 of this thesis. In these works, we performed hydrodynamic + N-body simulations to explore the dynamics of two plan-

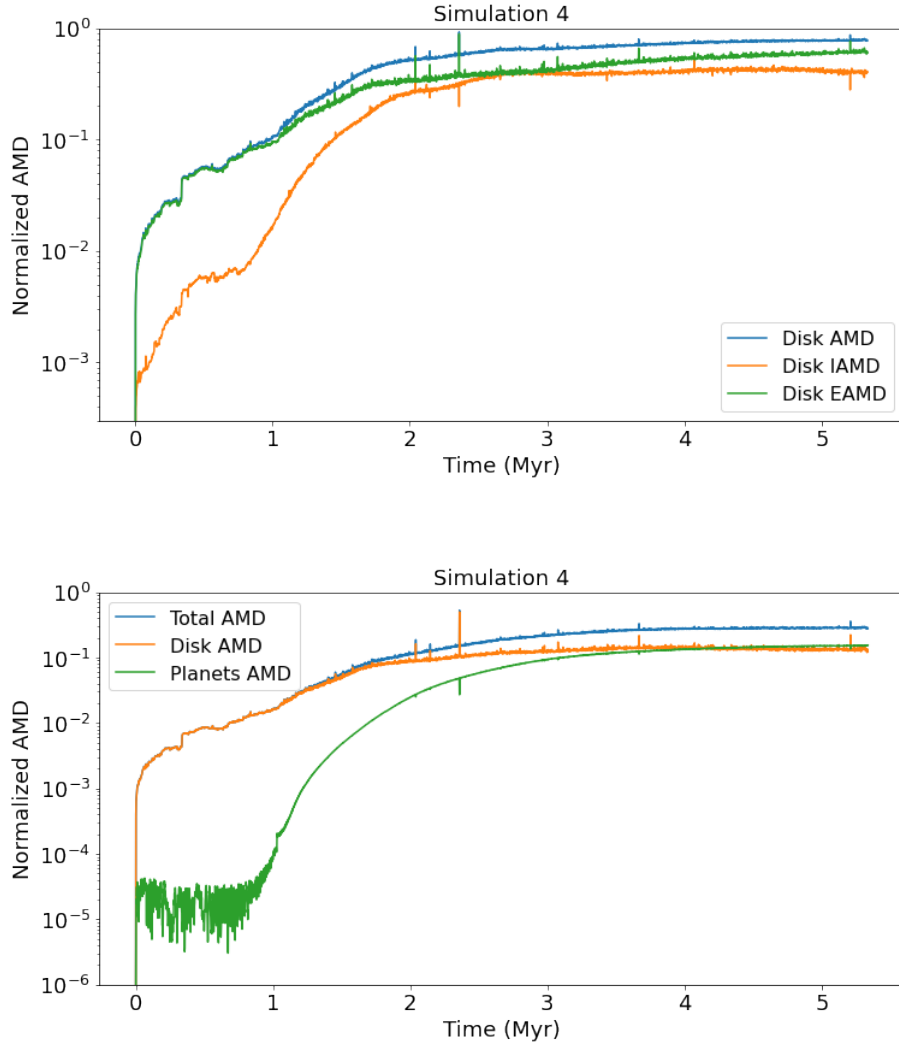


Fig. 23.— In the upper panel, the AMD from only the disk of planetesimals is shown, separated into IAMD and EAMD, while in the lower panel, the total AMD of the system is presented, here separated into the component due to the disk and the component due to the planets. Note that the value of the disk AMD in the lower panel is lower than that of the upper panel; this is because in the upper panel, the AMD has been normalized to the circular angular momentum of only the planetesimals, while in the lower panel, it has been normalized to the circular angular momentum of the planets and the planetesimals together.

etary systems, TRAPPIST-1 and Kepler-90, respectively, in the late stages of planet formation, when the protoplanetary disk was being dispersed. In particular, we were interested in using the low mutual inclinations of these two systems as constraints on the mass of the disks in which they formed. To understand the response of these systems to different mass disks and the subsequent removal thereof, we required a dynamical metric for the overall flatness of the system. Initially we tried using AMD for this purpose. However, we found (particularly for the TRAPPIST-1 system) that due to the initially flat orbits and significant planet-planet interactions, eccentricities heavily outweighed inclinations over the course of our simulations, causing AMD to give almost no information on the flatness of the system. It was because of this that we introduced the concept of IAMD as a metric better-suited for our purposes. Indeed, in comparing the different simulations we performed, we found a signal in the IAMD that was not present in the AMD, and revealed important dynamical information.

In the simulations performed in this work, we again see that IAMD and EAMD can contain distinct information, although the nature of it changed. Here, AMD was often dominated by IAMD, and it was in the inclinations that the dynamical effects associated with the early stages of the Nice model were most strongly felt. The fact that we find the separation of IAMD and EAMD as a function of time useful in both of these two cases suggests it may have use in a wide variety of dynamical contexts.

It is important and interesting to note that in his original formulation of AMD, Laskar (1997) also separated out the terms associated with eccentricities and inclinations of the Solar System planets. In looking at the individual terms that make up the AMD of the Solar System, it can be seen that there is not a clear equipartition, with the eccentricities of the outer planets outweighing their inclinations overall. However, to our knowledge, this work represents the first instance of separating EAMD and IAMD as a function of time, which is of course only possible in the context of simulations. It is only by looking at how these quantities change with time that we can see certain signals that reveal information about a system’s dynamics. Of course, there is no information that is contained in IAMD or EAMD that is not contained in the individual inclinations, eccentricities, and semimajor axes of a system. But by analyzing systems in this framework, which has the advantage of encapsulating the dynamics of a multi-planet system into a single quantity that takes into account how much each body contributes to the angular momentum of the system, it is possible

that new insights will be gained.

The simulations presented in this work also suggest some implications for planet formation. In Laskar & Petit (2017), a formation model for planetary systems is presented that uses AMD to constrain when collisions are possible. A key assumption of their model is that AMD is reduced by collisions, and is constant between collisions. Because of this, they argue that AMD will only ever go down, eventually reaching a value for which collisions are no longer possible, at which point the system will be AMD-stable. The assumption of constant AMD between collisions is valid under a secular approximation, but in certain contexts that cannot be adequately approximated as such, such as the Nice model scenario explored in this work, this assumption does not hold, and indeed we see that AMD can grow significantly. If a system were forming under such conditions, it would not necessarily reach a final AMD-stable point, and further collisions would be possible.

Another argument in the formation model of Laskar & Petit (2017) is that there is a negative feedback loop between a planet’s mass and its capacity for collisions, placing a limit on how many collisions it can experience before growing too massive. The argument stems from the assumption that for an individual body, $\text{AMD} \propto me^2$, where m is its mass and e its eccentricity (see Equation 22). Because of this, as a body’s mass is increased due to collisions, its eccentricity is not free to be as large for the same AMD. As such, the more collisions a body undergoes, the more circular its orbit will become, making future collisions more difficult. However, as was seen in this work, it is not always true that $\text{AMD} \propto me^2$, as certain dynamical processes can cause a system to be in a state where AMD is almost completely dominated by inclinations (or eccentricities). If a system is in such an inclination-dominated state, AMD is nearly independent of eccentricity, so that collisions would not necessarily lead to circular orbits.

It is interesting to note that either of these assumptions breaking down allows for more collisions, and for completely different reasons, potentially leading to systems with fewer and more massive planets in certain contexts. We want to stress that none of this is to challenge the work of Laskar & Petit (2017), but to argue that different behavior can be possible under certain conditions. There is much more to explore in the separation of EAMD and IAMD and what causes a system to become dominated by one or the other, and we are hopeful that this will be a useful idea for

future endeavors.

Appendix A Complete Nice Model Simulations

For completeness, the remainder of the Nice Model simulations not shown in Figures 1 and 2 are presented here.

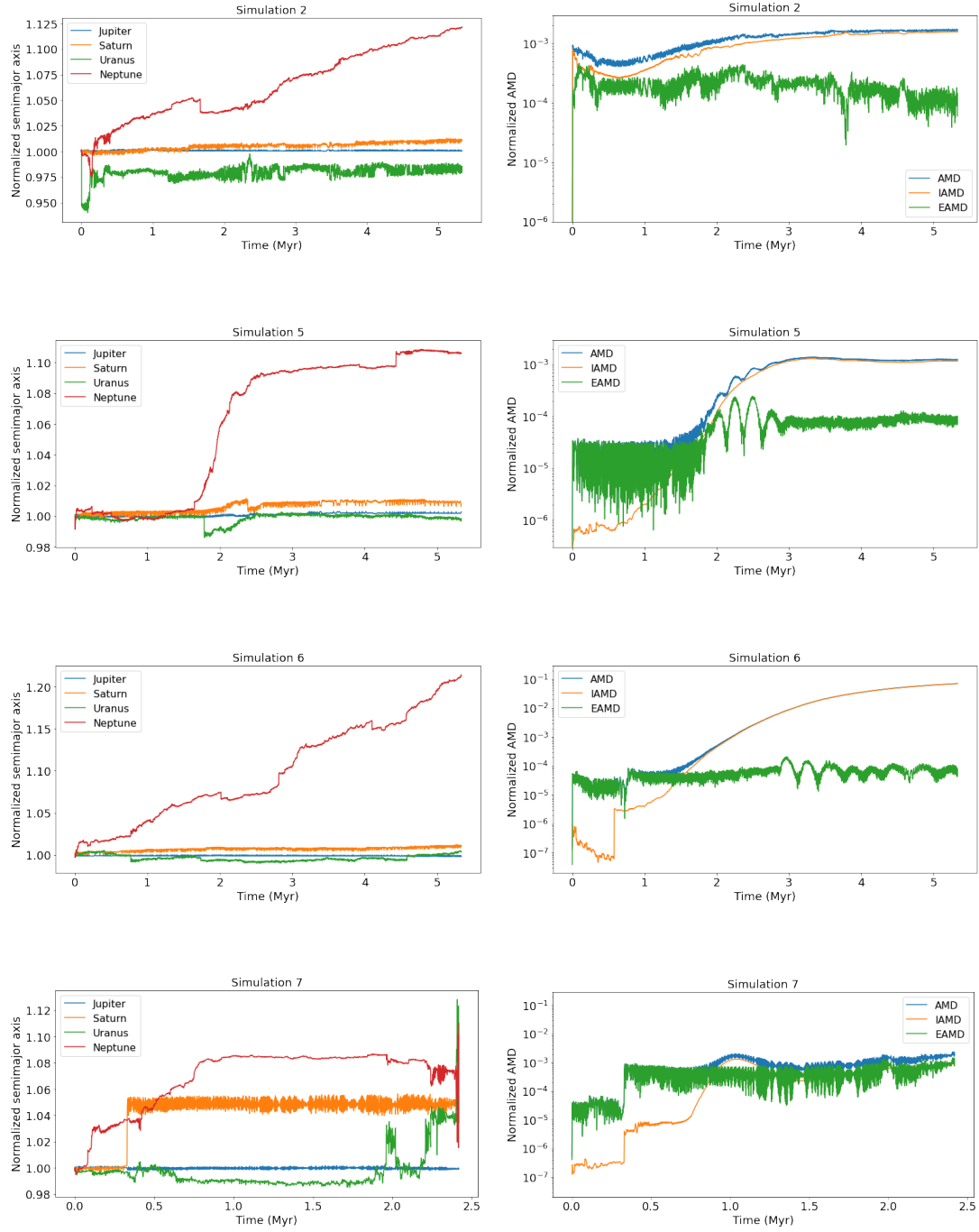


Fig. 24.— Semimajor axes and AMD of the planets as a function of time for the remaining Nice Model simulations, as in Figure 1. Note that the timescale for Simulation 7 is smaller due to the simulation terminating at that point.

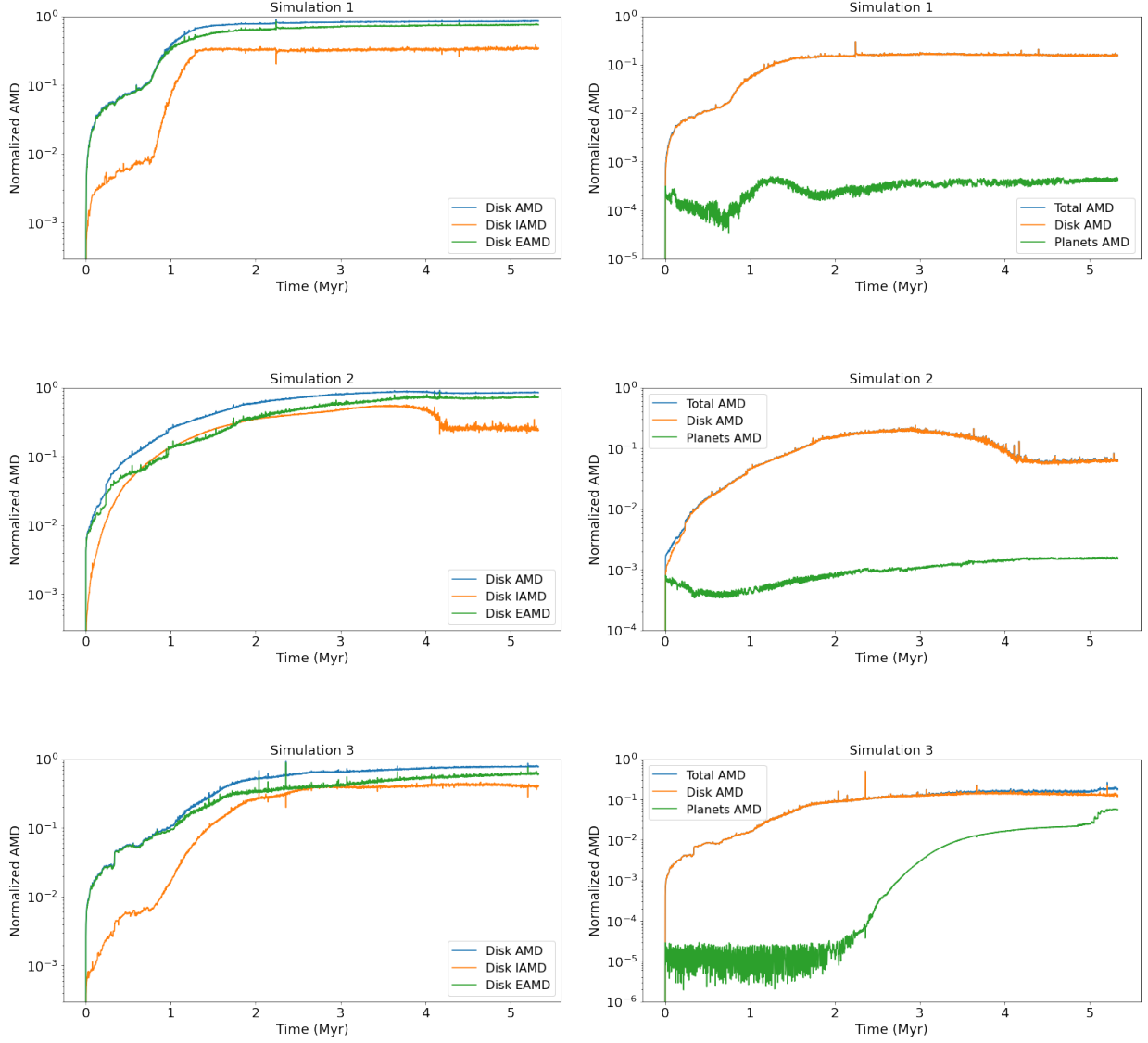


Fig. 25.— (Continued on next page) AMD of the planetesimal disk and total AMD of the remaining Nice Model simulations, as in Figure 2.

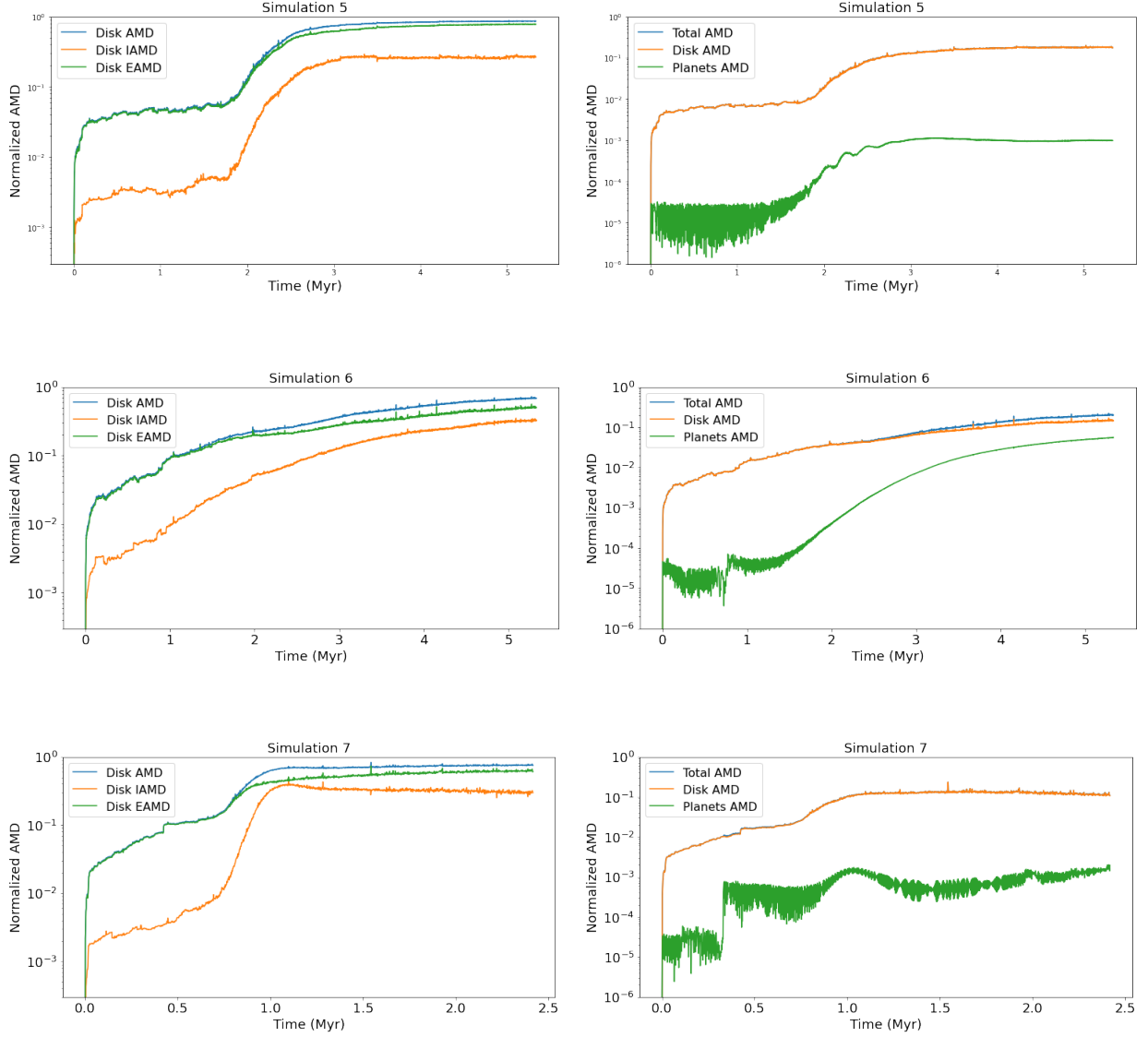


Fig. 25.— (Continued from previous page) Note that the timescale for Simulation 7 is smaller due to the simulation terminating at that point.

Conclusions and Future Steps

As observations of exoplanets and circumstellar disks continue to improve, our picture of the Solar System’s place in the Milky Way becomes more clear. However, our understanding of the processes that lead to the formation of such planetary systems, while growing more complete, also becomes more complicated. With such diversity of planetary architectures and disk structures, the goal of a unified theory of planet formation becomes increasingly difficult.

In this thesis, I have presented work that adds a small piece to this puzzle. Our work on planet-disk interactions in low-inclination systems, while concerning only two systems directly, highlights the important role that mutual inclinations play in understanding the formation of such systems, a role that has perhaps been overlooked by the field, and that we believe may be a useful tool to constrain other specific systems down the line. Similar, our work on IAMD and EAMD may help us better understand AMD in certain non-secular dynamical contexts, which might have meaningful implications for planet formation.

The most important findings of this thesis can be summarized as thus:

1. The TRAPPIST-1 and Kepler-90 systems both have remarkably low mutual inclinations, particularly when discounting the low-mass outlier Kepler-90i. While such low mutual inclinations might arguably be expected from the observed decreasing trend between inclination dispersion and multiplicity in Kepler systems, this nevertheless distinguishes these systems from the vast majority of observed exoplanet systems, and it is likely that the TRAPPIST-1 system is even flatter than this trend would predict. It remains unknown whether such flatness is typical of planetary systems around M dwarfs.
2. If the disk that the TRAPPIST-1 system formed in were to exceed a certain mass, the perturbation induced by the dispersing disk would excite the mutual inclinations of the planets to values inconsistently high compared to

their present-day observed values, with no readily available way for them to be damped without the disk. This dynamical behavior is made apparent by considering IAMD as a function of time as a metric for the system. The upper mass limit of the TRAPPIST-1 disk implies a formation model that requires a low-mass disk, such as that described by Ormel et al. (2017).

3. When a scaled-up version of the same numerical experiment was applied to the Kepler-90 system, we similarly found evidence for an upper mass limit, with larger disk masses leading to inconsistently high mutual inclinations. However, it was found that the effect observed in the TRAPPIST-1 system was felt even more strongly here, with higher mutual inclinations being excited, and the transition occurring at a relatively lower disk mass. We postulate that this difference is due to the different planetary architectures of the two systems, with the vast majority of planetary mass and angular momentum of the Kepler-90 system being contained in its two outermost planets, leading to the development of pronounced spiral density waves in the disk and changing the overall dynamics of planet-disk interactions.
4. While AMD is often treated as a fixed quantity, which is valid under secular dynamics, there are certain contexts in which AMD as a function of time can reveal information about the dynamics of a system. In particular, changes to AMD are sometimes felt separately by the inclinations and eccentricities of the system, and one can dominate over the other, as we demonstrate with N-body simulations of a simple Nice model scenario. This “decoupling” of IAMD and EAMD, together with the capacity for AMD to grow with time in such systems, might allow for more impacts in the late stages of planet formation, though this analysis is only qualitative.

Far from closing the door on any area in planet formation, these findings open the way for many next steps that could be explored.

The TRAPPIST-1 and Kepler-90 systems were the most obvious choices of study for the work presented in this thesis, primarily because of their low mutual inclinations, but also because of their high multiplicities and architectures that allowed for complex planet-planet interactions. However, there are of course other systems that this numerical experiment could be applied to. It would be interesting, for example,

to test systems with somewhat larger mutual inclinations, to determine at what point the behavior we observed breaks down. In addition, as more systems continue to be discovered, doubtlessly other interesting targets will present themselves. In particular, as more systems around M dwarfs are observed (Nowak et al. 2020), it will become more clear exactly how common TRAPPIST-1-like systems are in nature. Furthermore, a better understanding of the dynamics at play in these two cases might be made possible by studying artificial systems of some sort. While this would be difficult to do in any systematic way, due to the significant computational resources required, some important insights might be gained by allowing for the variation of certain physical parameters.

There are also natural future directions to take the study of AMD presented in this work. The N-body simulations we performed were of only a very particular scenario. However, there are many other contexts in which AMD is not necessarily conserved that might make for just as interesting cases to study. One such case, related to our work on the TRAPPIST-1 and Kepler-90 systems, is how AMD changes when a gas disk is slowly removed. Torques from disk-planet interactions, which depend on the overall surface density profile of the disk, will affect the overall AMD, effectively causing the planets to feel a different central force. How the AMD responds to the gradual removal of this torque might be another interesting application of the separation into IAMD and EAMD components. In addition, a more quantitative description of the implications to planet formation discussed in Section 4.3 of this work might also be a fruitful, if more ambitious, avenue to explore.

As the diversity of known planetary systems continues to increase, forcing theories of planet formation to evolve and expand, understanding the interplay between planetary and disk dynamics will become ever more important. In order to do so, numerical studies presented in this work and those like them will be critical to make sense of such complex systems.

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