



Developments in Control Authority for Flapping-Wing Micro-Aerial Vehicles

Citation

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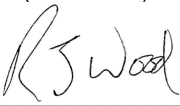
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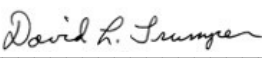
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March 25, 2022

Developments in Control Authority for Flapping-Wing Micro-Aerial Vehicles

A dissertation presented

by

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to

The John A. Paulson School of Engineering and Applied Sciences

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

Engineering Sciences

Harvard University

Cambridge, Massachusetts

March 2022

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Control Authority and Autonomy for the RoboBee

Abstract

Bio-inspired flight at the insect scale encourages aspirations of unmatched speed and agility. However, flapping-wing micro-aerial vehicles are subject to unique challenges in control, including constraints in computation, sensing, and actuation; extreme disturbances relative to the vehicles' small size; and modeling complexity of the underactuated flapping-wing system. I introduce control and modeling strategies tailored to the unique qualities of flapping-wing flight, applying these methods in flight with the Harvard RoboBee, an 86 mg flapping-wing micro-aerial vehicle, with dramatic improvements to vehicle characterization and performance. Through this work, the RoboBee achieves full control authority for the first time, opening the door to aggressive maneuvers, full sensor utility, and robust flight; anticipating applications beyond the lab and within unconstrained environments.

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Acknowledgments

First, I would like to thank my advisor, Professor Robert Wood, for his tremendous support over the past years. It has been an absolute pleasure to work with you, and I have truly enjoyed being a member of your laboratory. Additionally, I would like to thank my committee, including Professors David Brooks and David Trumper, as well as previous committee members Professor Radhika Nagpal and Dr. Scott Kuindersma, for their kind support and feedback, as well as for helpful conversations regarding new ideas and research directions.

I would also like to thank the members of the Harvard Microrobotics Laboratory for their collaboration and friendship. I would particularly like to thank Dr. Patrick Hyun, with whom it has been a joy to collaborate, as well as the other past and present members of the RoboBee team, especially Professors Farrell Helbling, Kevin Chen, and Noah Jafferis, and Dr. Avik De. This laboratory has been a truly wonderful community, and I'm thankful to have been able to engage, work, and laugh with you all over these years.

I am deeply grateful to my family for all of their love and support, throughout my degree and during my engineering education. My husband has brought endless fun and joy to my graduate school years, and our children have been absolute rays of sunshine and motivation. I am thankful, as well, to my parents, for their encouragement, excitement, and support throughout my education and forays into robotics, and to my siblings, for their companionship. I am also deeply grateful to my wonderful grandparents and in-laws for all of their encouragement.

Finally, I would like to acknowledge funding from the National Science Foundation Graduate Research Fellowship Program and the Office of Naval Research, which has made this work possible.

Chapter 1

Introduction

Decades of multidisciplinary research in aerial vehicles have allowed humans to take to the skies in dramatic fashion: the Apollo 11 spaceflight took the astronauts to the moon, the U.S. Air Force Thunderbirds and U.S. Navy Blue Angels achieve feats of high-speed coordination and agility in fixed-wing flight, and commercial passenger jets regularly bring collections of hundreds of passengers across continents and oceans – so reliably that the achievement is barely given a thought.

However, despite generations of research and discovery in aviation, a relatively unexplored frontier remains: the pursuit of flight at the smallest scales seen in nature. Flapping-wing propulsion, demonstrated in the flight of insects and birds, provides an avenue for achieving small-scale flight, begetting a growing area of research in flapping-wing micro-aerial vehicles (FWMAVs), which show promise in overcoming propulsion and scaling challenges seen by rotor-based and fixed-wing vehicles [1]. Additionally, by designing FWMAVs, we may learn more about the propulsion, maneuverability, and behavior of insects themselves, by imitating them through biomimicry in design.

However, the advantages of FWMAVs come with a vast array of new challenges.

As size decreases, hardware options for power, sensing, actuation, and computation are greatly reduced, requiring custom options and limiting autonomy [2]. Additionally, as aerial robots achieve flight at smaller scales, ambient conditions (especially air currents) present increasingly dramatic relative disturbances.

A particular set of challenges emerges when considering control of these vehicles. At the insect scale, passive flexures are commonly incorporated to define wingstroke trajectories [3]. Due to these passive components, vehicle dynamics are both (1) difficult to observe and (2) significantly variable with flapping frequency and vehicle manufacturing anomalies [4]. Also, as wing frequency increases and approaches resonance (which is the typical operating regime for FWMAVs with oscillating actuators, including piezoelectric actuator-driven systems [5]), the morphology of these passive elements results in significant passive filtering of control signals, which dramatically limits control authority, especially in directing vehicle heading [6]. Increasing the challenges for control, the complexity of flapping-wing flight renders it difficult to model [7], complicating the understanding of the specific forces and torques produced during flight.

Despite these challenges, FWMAVs show promise in expanding the available scales of engineered flight towards the centimeter length, and smaller. One of the most significant milestones towards making this vision a reality is the achievement of *full control authority* of these vehicles: the ability to produce controllable, reliable, and flightworthy thrust, roll torque, pitch torque, and yaw torque. The work presented herein describes the pursuit and achievement of this goal, opening the door to exciting opportunities in insect-scale aerial mobility.

1.1 Motivation for FWMAs: Why Wings?

As we pursue the advancement of insect-scale flight, the primary consideration in vehicle design is the method of propulsion used to take flight. This decision significantly affects the design of actuation and control strategies, and in the case of flapping wings, motivates research in modeling and control of flapping systems.

Propulsion strategies for micro-aerial vehicles may be generally divided into rotor-based, fixed-wing, and flapping-wing [8]. Fixed-wing flight has been responsible for a variety of aerodynamic achievements, from the first human flight [9] to modern commercial air travel at incredible scales. Rotor-based flight has recently expanded from the development of helicopters and large rotorcraft to a large field of research especially focused on quadrotors and multi-agent aerial behavior. Flapping wings, seen in the hopes of early aerial vehicle design, including Leonardo da Vinci’s proposed flying machines [10], have recently grown into an expanding field of bird- and insect-inspired flapping-wing robots.

Given these options for aerial propulsion, why should researchers favor wings in small-scale flight, as opposed to methods more common in engineering? First, flapping wings show promise at small scales simply due to the observation that nature’s own agile fliers apply the same technique to achieve small-scale flight, with examples of aerial winged maneuvers in fliers as small as 1 mm in length [11]. In addition to the bio-inspired motivation for flapping wings, however, we observe limitations of fixed-wing and rotor-based systems as vehicle size is scaled down. For example, as wing size decreases, the Reynolds number of the associated fluid flow also decreases (designating a shift towards laminar flow regimes). As the Reynolds number decreases (especially below 200,000 [12]), the relative performance of fixed-wing vehicles decreases [13]. Rotor-based vehicles, too, exhibit scaling-related challenges: these vehicles rely on independently-controlled motors, which are limited

in efficiency at decreasing sizes [14]. Flapping wings, however, are extremely common in nature at these scales, suggesting utility for small-scale aerial agility, and developments in actuator design, including development of piezoelectric and dielectric elastomer (DEA) actuators [15, 16], have allowed FWMAVs to take flight with wingspans as short as 3 cm and vehicle mass as low as 60 mg [17]. While limitations in hardware for power, computation, and sensing remain among all propulsion methods at this scale, flapping wings provide a promising avenue to overcome scaling challenges and achieve flight at the centimeter scale and smaller.

1.2 FWMAV Development Background

Given the potential for advancement in small-scale flight using bio-inspired flapping-wing propulsion, it is no surprise to observe that the field of FWMAVs is rapidly growing, with numerous examples of current and recent vehicle development. To examine the existing field, we first define a scale of interest (including mass and wingspan) for these vehicles. Kumar et al. defines micro-aerial vehicles as within 10 – 50 cm and 100 – 500 g [2]. However, with a particular focus on insect-scale flight, defined by Chen et al. as < 10 cm and < 5 g, we choose a smaller range of interest. We choose, then, to consider FWMAVs at 2 – 50 cm wingspan and 50 mg – 100 g, to proceed in capturing the scope of current FWMAV development.

We will examine both the morphology of existing vehicles, and their strategies for actuation and control, to demonstrate challenges and opportunities in control authority and autonomy at the insect scale.

1.2.1 Morphology and Vehicles

We first consider morphology and vehicle design across the scale of FWMAVs. At the larger sizes (determined by mass and wingspan) lie vehicles largely actuated via motors, with autonomy available in many systems via onboard power and sensing. Of vehicles with mass 10 – 100 g, the AeroVironment Nano Hummingbird (19 g, 16.5 cm wingspan) [18], Purdue autonomous hummingbird (20.4 g, 17.0 cm wingspan) [19] each rely on two wings actuated by independent motors. The TU Delft DelFly (16 g, 28 cm wingspan) [20] also uses two motors; however, each motor controls a pair of wings. The larger UIUC BatBot (B2) (93 g, 46.9 cm wingspan) also applies motor-driven actuation, though its propulsion differs in that it utilizes complex, bio-inspired limb-like wings with joints, inspired by the particular morphology and biomechanics of bats [21].

A slightly smaller class of FWMAVs, with mass 1 – 10 g, includes the CMU flapping-wing vehicle (2.7 g, 7 cm per wing) [22] and the Harvard Robot Moth (3.22 g, 16 cm wingspan) [23]. Both vehicles use motors to drive two wings; however, the CMU flyer includes motors for each wing, while the Robot Moth has a single motor for both wings, and also applies passive tail stabilization. Hines et al. demonstrate a tethered version of their vehicle in [22], whereas the Robot Moth is untethered, with power supplied by an onboard battery.

At the smallest end of the scale (in mass and wingspan) lie vehicles with mass ≤ 1 g. These vehicles rely largely on custom actuation (as opposed to motors). The two-wing Harvard RoboBee (80 mg) [24] and its four-wing counterpart, the RoboBee X-Wing (90 mg baseline, 259 mg with onboard solar power hardware) [25] each rely on two piezoelectric actuators (to independently control each wing and wing pair, respectively). The two vehicles are similar in construction; however, the X-Wing has been equipped with onboard solar power to achieve untethered flight with a light source of approximately three

suns [25]. The University of Tokyo swallowtail-inspired FWMAV (400 mg, 14 cm wingspan) achieves flight with two wings powered via a rubberband. Finally, the soft actuator flapping-wing vehicle by Chen et al. (100 mg per actuator) [26] relies on soft actuators to power two, four, or eight wings in tethered flight.

Like their biological counterparts, these vehicles exhibit a strong relationship between size and required flapping frequency [27]. For example, the UIUC Bat Bot and DelFly, among the larger vehicles, achieve flight with flapping frequencies of approximately 10 Hz [21, 19]; the mid-size Harvard Robot Moth flaps at 25 Hz [23], and the RoboBee flaps at about 160 Hz [24]. While this provides an interesting connection with biology, it also serves to motivate actuator design at smaller scales: for example, the piezoelectric actuators commonly used for the smallest vehicles exhibit both fast actuation and high deflection to accommodate these high frequencies [28].

A key distinguishing feature between various FWMAV morphologies is the use of oscillatory and non-oscillatory actuators. Oscillatory actuators (e.g., piezoelectric actuators) allow independent control of flapping frequency and amplitude; whereas non-oscillatory actuators are limited to frequency control. (We note that while motors are typical for non-oscillatory actuation, it is possible to control both frequency and amplitude with oscillating motors [29].) Insects rely on both frequency and amplitude modulation [30]; therefore, vehicles with oscillatory actuators hold particular promise in bio-inspired, insect-like flight.

1.2.2 Control Strategies

The class of FWMAVs differs not only in morphology and actuation, but also in control, from the perspective both of control authority (the ability to produce necessary thrust and torques for maneuvers) and control strategies.

As expected due to the mitigation of micro-scale hardware constraints, the larger

FWMAVs are as a whole, more commonly autonomous and controllable, often with onboard power and sensor-based feedback control. One of the most dramatic examples of controlled flight at this scale comes from recent work with the Purdue autonomous hummingbird: the vehicle uses reinforcement learning-based control with Vicon motion-capture feedback to perform a backflip in flight [31]. The similarly-inspired AeroVironment Nano Hummingbird is noteworthy in control for its several minute hover time [18].

Of vehicles with onboard control, partial feedback is common, as achieving full state feedback (position and orientation, with associated velocities) requires a large sensor load. The DelFly achieves autonomous flight (in part via using a bio-inspired clap-and-fling wingstroke to enhance lift) and applies optic flow to avoid obstacles in midair; however, [20] describes partial control feedback, specific to obstacle avoidance and height control. Similarly, the Bat Bot also relies on partial feedback for autonomous flight, using PD feedback to control attitude, without directly controlling position [21].

Many insect-scale FWMAV rely entirely on external feedback (due largely to the prohibitive mass of sensing packages), or fly entirely in open-loop. The RoboBee and soft actuator FWMAV, for example, each rely on Vicon motion-capture feedback for control [24, 26]. The Robot Moth, CMU flapping-wing vehicle, and University of Tokyo swallowtail-inspired FWMAV all rely on open-loop flight [23, 22, 32]. We also note that the Robot Moth, while open-loop, does have a tail used for passive stability which may be adjusted between flights [23].

Computational methods have also proven useful in evaluating modeling and control strategies for FWMAVs, including simulation of FWMAV control [33]. While simulated control methods provide insight into possible strategies, applying such methods in hardware would be challenging due to unmodeled vehicle dynamics and extreme sensing restrictions. Additionally, comparison of modeling strategies evaluated various flapping-wing flight mod-

els, compared to a computational fluid dynamics study reference, for use in describing vehicle stability (particularly useful for control design), determining that quasi-steady models with rotational and translational components could be sufficient to describe FWMAV stability [34]. Computational and mathematical analysis, in combination with control experiments, encourages full utility of flapping-wing systems.

While FWMAV design and control have made great strides in achieving flapping-wing flight across a variety of platforms, substantial work remains in achieving sufficient control to truly mimic the agility and maneuverability of insects.

1.2.3 The Harvard RoboBee

The Harvard RoboBee, among the smallest existing FWMAVs, is the featured platform within this work, used for the control experiments presented in the following chapters. The RoboBee (shown in Fig. 1.1) is a two-wing FWMAV, with each wing flapping independently via a piezoelectric bimorph actuator [35]. The vehicle achieves flight with flapping frequencies of approximately 150 Hz (near resonance [36]). Historically, control has been achieved with reliable thrust, roll torque, and pitch torque [37], including disturbance rejection [38] and aggressive maneuvers [39]. Yaw torque has previously been attempted via physical vehicle enhancements and modifications [40, 41]; however, recent control efforts (highlighted in this thesis) have allowed the introduction of reliable and controllable yaw torque via novel flapping strategies and control space expansion [6, 24]. The vehicle is currently powered and controlled by signals sent via tether from an offboard computer and power amplifiers; however, recent groundwork has been laid for onboard sensing [42, 43, 44, 45], control [46], and power [47, 48, 25].

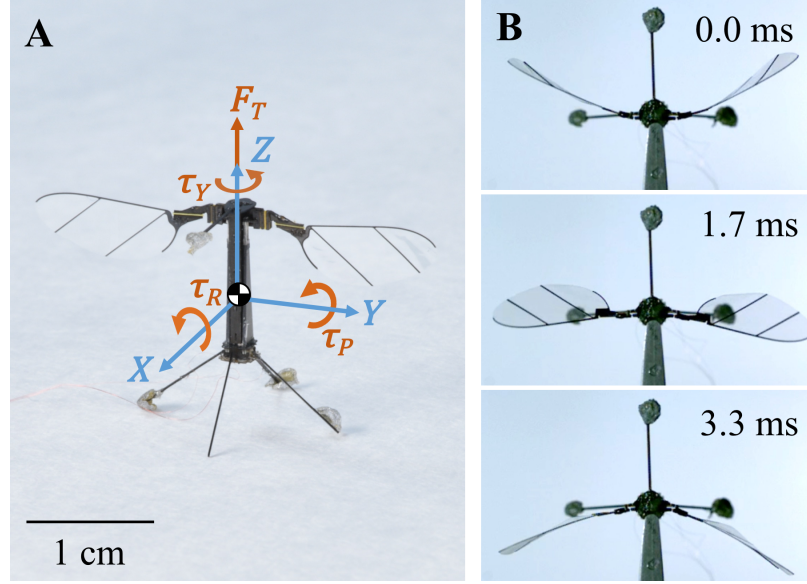


Figure 1.1: The Harvard RoboBee, showing (a) full stationary vehicle, with body coordinate frame in X , Y , Z , along with thrust F_T , roll torque τ_R , pitch torque τ_P , and yaw torque τ_Y ; and (b) an example of a flapping wingstroke at 150Hz.

1.3 Approaching Full Control Authority

We define *full control authority* as the ability to control a vehicle's thrust, roll torque, pitch torque, and yaw torque. By this definition, a quadrotor is a quintessential example of an aerial system with full control authority; the vehicle can control its body position and orientation in three-dimensional space via control of its thrust and three body torques, produced by modulating the velocities of four rotors. Effectively, then, when we ask whether a vehicle exhibits full control authority, we ask if it can match a quadrotor in controllability.

Achieving full control authority is a substantial challenge for FWMAV, especially at the smallest scales. A commonly challenging control axis for these vehicles is yaw torque [49, 50, 51, 52] (see Fig. 1.1), which therefore limits the use of sensors (i.e., a camera heading cannot be directly controlled) in addition to stability and maneuverability. Since

FWMAV often produce significantly smaller yaw torque than roll and pitch [53, 54, 55], existing research has sought to overcome this limitation, either via yaw-specific vehicle modification [40, 56, 41] or simply through removal of yaw authority requirements within control [57, 58, 59].

Full control authority, including reliable and controllable thrust and body torques, will allow FWMAVs to use sensors more effectively, in addition to expanding their ability to achieve stable and agile flight. These advantages in maneuverability and sensor utility will also assist in migration from the lab into unconstrained environments, as vehicles will improve in reliability, stability, and autonomy as a direct result of advances in control authority.

1.4 Thesis Overview

This thesis describes the achievement of full control authority of the RoboBee, focusing particularly on advancement of yaw torque via design of flight conditions and operating points based on vehicle dynamics, expansion of the control space, and improvements to modeling and characterization of FWMAVs. After the review of FWMAVs and control authority presented above (*1: Introduction*), we proceed by discussing a method to achieve reliable heading control for FWMAVs (*2: Yaw Torque Authority for a Flapping-Wing Micro-Aerial Vehicle*), then discussing advances in modeling to describe vehicle behavior and inform control strategies, as well as demonstrating full control authority of the RoboBee in flight (*3: Modeling and Control of FWMAVs with Harmonic Sinusoids*). We then proceed to describe the expansion of the control space for piezoelectric actuator-driven FWMAVs through frequency modulation (*4: Frequency-Modulated Control for FWMAVs*). The work presented herein anticipates the transition to control-autonomous flight with upcoming mi-

crocontroller and system integration projects. The final section (*5: Conclusions and Future Work*) will anticipate these advances in autonomy, focusing on relevant prior and current work in sensing, control hardware, and appropriate control strategy, while highlighting insights into performance and limitations for a control-autonomous vehicle, and proposing appropriate metrics for control-autonomous flight.

Chapter 2

Yaw Torque Authority for a Flapping-Wing Micro-Aerial Vehicle

Chapter 2 is adapted from Steinmeyer et al. 2019 [6], ©2019 IEEE.

2.1 Introduction

Flapping-wing micro-aerial vehicles rely on subtle changes in the kinematics of high-frequency wing flapping to produce roll, pitch, and yaw torques. To generate yaw torque, the Harvard RoboBee changes the ratio of upstroke to downstroke speed (“split-cycling”) by applying a second harmonic to the fundamental flapping signal for each wing. However, since flapping typically occurs near resonance (for efficiency), these higher harmonics are filtered out by the transmission and actuator dynamics. Therefore, reliable yaw control authority has proven elusive. We propose a method to generate yaw torque sufficient for in-flight control by using split-cycle flapping in an “iso-lift” regime, to mitigate reso-

nant filtering by decreasing the flapping frequency and increasing the drive voltage, which produces lift identical to typical flight conditions. We model the expected torque at iso-lift conditions and apply this method to the physical RoboBee, achieving reliable, controllable yaw torque. Finally, we demonstrate yaw control with a simple heading controller, achieving a step response with a time constant an order of magnitude faster than previous attempts.

2.2 Motivation

The recent advent of micro-aerial vehicles (MAVs) has allowed controlled flight at historically prohibitive scales, while also presenting a new realm of challenges for power, sensing, and control [27], [60]. To achieve the necessary force and torque authority for flight at this scale, researchers seek inspiration from biological systems in the development of insect-inspired robots [61], [62], [18]. For example, the Harvard RoboBee [63] is an 80 mg MAV which achieves flapping-wing flight by applying sinusoidal signals to two piezoelectric bimorph actuators [35].

The RoboBee has achieved controlled flight [64] and has demonstrated torque production in roll, pitch, and yaw [65] (see Fig. 2.1 for axes definitions), but yaw torque is historically the weakest [40]. As such, roll and pitch torques are applied to perform lateral maneuvers, and direct yaw control is largely ignored [64]. While lateral control may be achieved in this manner, the lack of yaw control limits the capabilities of these vehicles; for example, yaw control is necessary for heading stabilization.

One promising method for yaw torque production, particularly for under-actuated vehicles such as the RoboBee, is called “split-cycling”; that is, varying the relative speeds of the wing upstroke and downstroke to produce a net drag [66], as shown in Fig. 2.2. The wingstroke is determined by a sinusoidal signal sent to the actuators, with stroke kinemat-

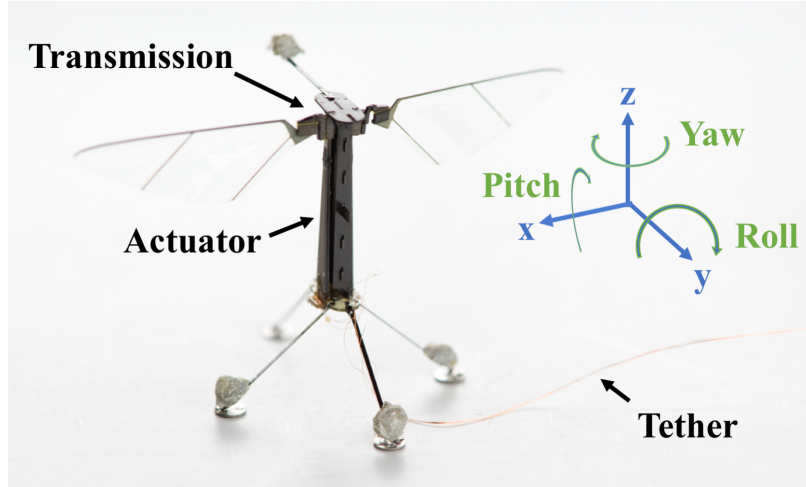


Figure 2.1: The Harvard RoboBee, with defined rotational axes.

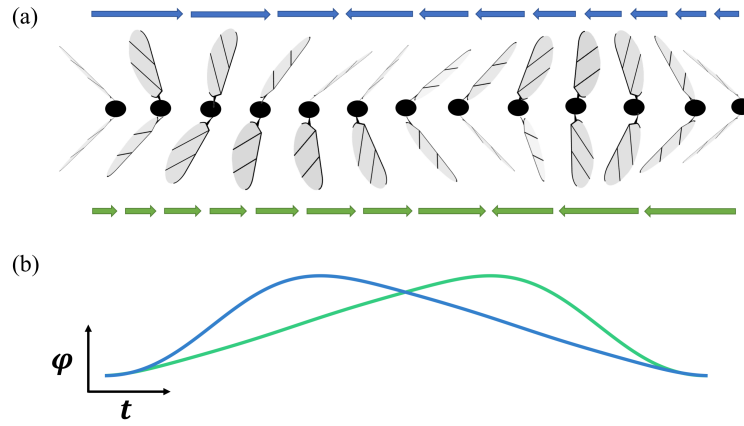


Figure 2.2: Split-cycle wing-flapping is used to induce a net drag to produce yaw torque. (a) A full split-cycle wing stroke; blue and green arrows indicate the transition from upstroke to downstroke for each wing. (b) An example of paired split-cycle wingstroke motions; blue and green lines correspond to stroke angles of the wings shown above.

ics passively determined by the transmission, hinge, and wing. Although this method has been applied in a basic demonstration of yaw control for the dual-actuator RoboBee [45], it required almost a full second for a 90-degree turn, and performance was heavily affected by local airflow in the indoor test arena. As such, RoboBee yaw authority is not sufficient for robust control and aggressive maneuvers. While strategies have been proposed to improve yaw control authority (including active control of the wing angle-of-attack using an additional actuator [41] and sleeve-stop hinges to vary the wing stiffness profile [40]), a solution which does not substantially complicate manufacturing and which produces controllable torque at lift conditions has not yet been found.

Since the RoboBee transmission (Fig. 2.1) acts as a low-pass filter at frequencies higher than the flapping resonant frequency ([65], [36], [53]), and the split-cycle method produces a difference between upstroke and downstroke speed by applying higher-order harmonics to the wingstroke sinusoid [58], we may infer that poor yaw control authority is the result of filtering of the higher harmonics in the split-cycle command. As such, at the operating frequency and voltage applied for efficient flight, yaw torques become negligible and effectively useless.

In this work, we propose a method to produce yaw torque sufficient for aggressive maneuvering via split-cycling combined with simultaneous frequency and voltage modulation. Yaw torque is produced by shifting the operating conditions to a state of lower frequency and higher voltage (such that lift remains consistent with normal flight: “iso-lift” conditions) during split-cycling to reduce the attenuation of the higher-order harmonics of the split-cycle sinusoid. Applying this method, we achieved yaw torques sufficient for aggressive maneuvering under conditions suitable for flight (i.e., reliable yaw torques which allow full revolutions within tenths of a second), and implemented this technique in simple heading control, demonstrating a step response an order of magnitude faster than previously

achieved on the dual-actuator RoboBee.

2.3 Yaw Torque Modeling

We developed a model to explore the stroke angle frequency response. In addition to predicting resonance and regions conducive to efficient flight, the model also predicts the filtering of higher-order sinusoids (i.e., filtering of the split-cycle effect through the transmissions and actuators). We also modeled thrust forces and axial torques to predict the ideal (unfiltered) control authority at given operating conditions. We then combined the torque and force models with the transmission-actuator transfer function to predict the forces and torques produced by the RoboBee to demonstrate the filtering of the split-cycle effect for yaw torque generation.

2.3.1 Split-Cycle Filtering

We first derived a second-order transfer function to describe the frequency-dependent relationship between the actuator drive voltage and the resulting flapping motion, following the approach applied in [53] and [67]. Note that the most accurate model for the current RoboBee is found in [36]; however, we will instead use the simpler second-order model, which provides sufficient fidelity and matches the key features of the model in [36], including the resonant frequency and high frequency roll-off. Physical parameters from [36], [53], and [67] (actuator mass m_a , actuator and transmission stiffnesses k_a and k_t , damping b , transmission ratio T , shoulder length r_{cp} , and moment of inertia J_ϕ) are incorporated into the model and appear in Table 2.1.

We modeled the transmission and actuator combination as a spring-mass-damper system with m_{eq} , b_{eq} , and k_{eq} (equivalent mass, damping, and stiffness, respectively) with respect to the flapping frequency ω . The transfer function $H(\omega)$ represents the relationship

between the voltage input signal, $V(\omega)$, and the wing stroke angle, $\Phi(\omega)$, in the frequency domain, which is scaled with a gain A_H such that $H(\omega) = \Phi(\omega)/V(\omega)$:

$$H(\omega) = \frac{A_H}{m_{eq}(\omega i)^2 + b_{eq}(\omega i) + k_{eq}} \quad (2.1)$$

The constants m_{eq} , b_{eq} , and k_{eq} , calculated as described in [67], are given in Table 2.1, as well as all parameters required for their calculation. The resulting frequency response is shown in Fig. 2.3, also indicating the typical flapping frequency range used for flight (chosen for its proximity to resonance, resulting in high-amplitude flapping), and the proposed operating regime. The resulting split-cycle filtering effect is shown in Fig. 2.4.

A harmonic voltage input signal is considered and given as $V(t) = V_{amp}\psi(t)$ where $2V_{amp}$ is the peak-to-peak voltage and ψ represents the shape of the harmonics with a fundamental frequency ω and coefficients a_1 , a_2 , and a_3 corresponding to the first three harmonics:

$$\psi(t) = a_1 \sin(\omega t) \pm a_2 \sin(2\omega t) + a_3 \sin(3\omega t). \quad (2.2)$$

The coefficients, a_1 , a_2 , and a_3 , are chosen such that

$$\max_{t \in (-\pi/\omega, \pi/\omega)} |\psi(t)| = 1 \quad (2.3)$$

holds. The stroke angle ϕ may then be described by transforming V to $H(\omega)$ using (2.1):

$$\phi(t) = V_{amp} \sum_{n=1}^3 a_n |H(n\omega)| \sin(n\omega t - \angle H(n\omega)) \quad (2.4)$$

where $\angle H(n\omega)$ is the phase of the system evaluated at frequency $n\omega$. For our purposes, the fundamental frequency is the primary wingbeat frequency of the MAV, the second harmonic is responsible for the split-cycle effect (the variation in the ratio of upstroke to

Table 2.1: Transmission and Actuator Modeling Parameters

Actuator mass	m_a	25 mg
Wing moment of inertia	J_ϕ	51.1 mg · mm ²
Transmission ratio	T	2666
Shoulder length	r_{cp}	1.42 × 9.56 mm
Aerodynamic damping	b	2.03 × 10 ⁻⁶ μNs/m
Actuator stiffness	k_a	300 N/m
Transmission stiffness	k_t	28.2 μNm/rad
Equivalent mass	m_{eq}	0.388 g
Equivalent damping	b_{eq}	0.196 Ns/m
Equivalent stiffness	k_{eq}	500.4 N/m

downstroke speed), and the third harmonic serves to flatten the signal (to allow higher-amplitude fundamental flapping frequencies, and therefore higher-amplitude wingstrokes, within set voltage limits), and to smooth the signal after adding a second harmonic.

2.3.2 Force and Torque Modeling

The lift force generated by a wing flapping with stroke angle $\phi(t)$ may be calculated as follows [65], for a wing flapping in air with density ρ , with wingshape parameter β (as described in [68]) and lift coefficient $C_L(\alpha)$ (where α is the wing angle-of-attack) [69]:

$$F_L = \frac{1}{2}\rho\beta C_L(\alpha)\dot{\phi}^2. \quad (2.5)$$

Similarly, the drag force is expressed as

$$F_D = \frac{1}{2}\rho\beta C_D(\alpha)\dot{\phi}|\dot{\phi}| \quad (2.6)$$

where $C_D(\alpha)$ is the drag coefficient.

With shoulder length r_{cp} (see Fig. 2.1 and Table 2.1), the yaw torque for a single

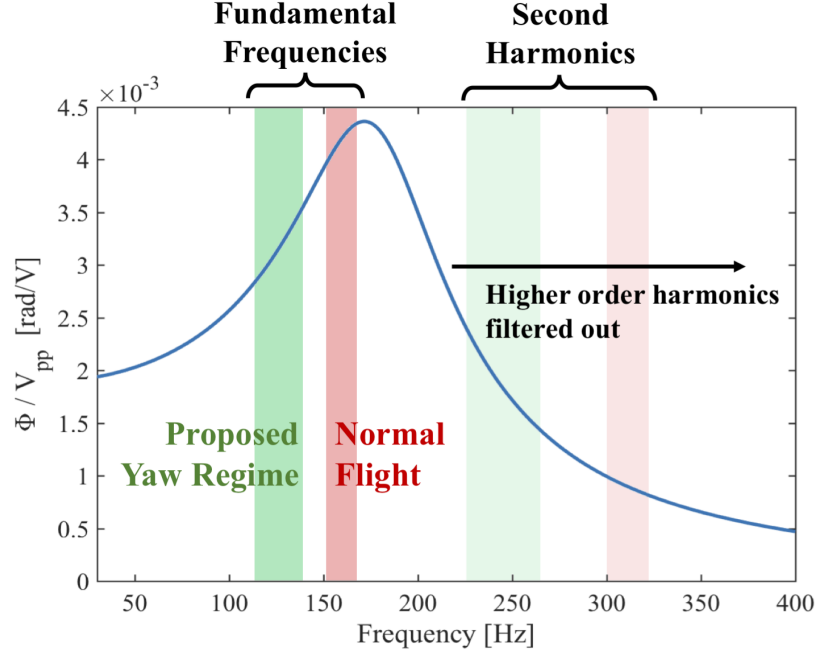


Figure 2.3: A frequency response demonstrating wingstroke angle amplitude at different operating conditions, predicting significant filtering (via the actuator and transmission) of higher-order harmonics. The proposed iso-lift split-cycle method requires a lower fundamental frequency than normal flight, resulting in lower filtering of the second harmonic, and therefore an increase in yaw torque.

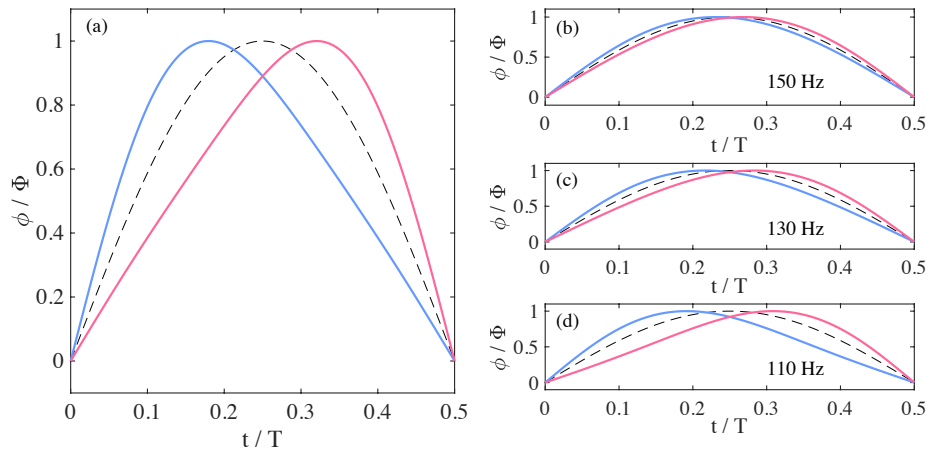


Figure 2.4: Filtering of yaw-generating wingstroke signals through the actuator and transmission: An unfiltered signal is shown in (a), with corresponding filtered signals at 150Hz (b), 130Hz (c), and 110Hz (d) according to the transfer function presented in Fig. 2.3.

wing may be calculated as follows:

$$\tau_{yaw} = \frac{1}{2} r_{cp} \rho \beta C_D(\alpha) \dot{\phi} |\dot{\phi}|. \quad (2.7)$$

We may calculate the time-averaged lift force for a single wing, F_L^{avg} , using the wingstroke period $T = \frac{\omega}{2\pi}$:

$$F_L^{\text{avg}} = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} F_L dt \quad (2.8)$$

$$= \frac{\omega}{4\pi} \rho \beta C_L \int_0^{\frac{2\pi}{\omega}} \dot{\phi}^2 dt \quad (2.9)$$

$$= \frac{1}{4} \rho \beta C_L (\omega V_{amp})^2 \sum_{n=1}^3 n^2 (a_n |H(n\omega)|)^2. \quad (2.10)$$

Similarly, the time-averaged yaw torque τ_{yaw}^{avg} for a single wing may be calculated as:

$$\tau_{yaw}^{\text{avg}} = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \tau_{yaw} dt \quad (2.11)$$

$$= \frac{\omega}{4\pi} r_{cp} \rho \beta C_D \int_0^{\frac{2\pi}{\omega}} \dot{\phi} |\dot{\phi}| dt. \quad (2.12)$$

Note that total thrust equates to the sum of lift forces from each wing, and total yaw torque is the sum of yaw torques from each wing.

Considering the expressions for F_L^{avg} and τ_{yaw}^{avg} , we note that apart from the wingstroke $\phi(t)$, both expressions have the same relationship with non-constant parameters (that is, V_{amp} and ω). For our purposes, this means that if the split-cycle effect (due to wingstroke shape $\psi(t)$ within $\phi(t)$) is filtered by the transmission and actuator (modeled according to $H(\omega)$), yaw torque cannot be increased without simultaneously increasing the lift force, though the two are intended to be independent.

2.3.3 Iso-lift Conditions

We now define “iso-lift conditions” as the set of operating voltage, frequency, and shape parameters which result in equivalent time-averaged lift force. We propose that, during yaw maneuvers, the RoboBee shift to a lower-frequency iso-lift condition, thereby reducing split-cycle filtering due to the post-resonance roll-off and allowing independence between yaw and lift. Effectively, we predict that a lower operating frequency and higher applied voltage will permit yaw torque while maintaining a desired lift.

Let $p_A := (V_A, \omega_A, a_1, a_2, a_3)$ and $p_B := (V_B, \omega_B, b_1, b_2, b_3)$ be two sets of parameters used to generate time-averaged lift forces $F_L^{avg}(p_A)$ and $F_L^{avg}(p_B)$, respectively, by calculating lift using (2.10), and let $h : \mathbb{R}^5 \rightarrow \mathbb{R}$ be defined as

$$h(p_A, p_B) = F_L^{avg}(p_A) - F_L^{avg}(p_B). \quad (2.13)$$

Then p_A and p_B satisfy the iso-lift condition if and only if $h(p_A, p_B) = 0$ holds, or equivalently, if and only if

$$\sum_{n=1}^3 n^2 ((\omega_A V_A a_n |H(n\omega_A)|)^2 - (\omega_B V_B b_n |H(n\omega_B)|)^2) = 0 \quad (2.14)$$

holds. The above definition gives an equivalence relation, namely $p_A \sim p_B$, and the set of parameters can be partitioned by its equivalence class. An example of sets of parameters in the same equivalence class are shown in Table 2.2; each set generates the same time-averaged lift force.

To model the effect of operating at iso-lift conditions on the resulting yaw torque, we first set a baseline lift condition: an efficient (i.e., close to resonance [63]) frequency, voltage, and shape combination with no second harmonic (and therefore no split-cycle

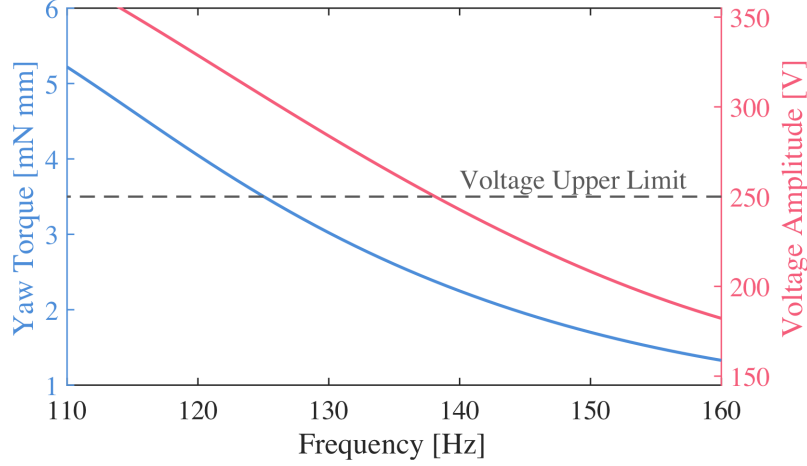


Figure 2.5: Yaw torque (blue) increase at lower operating frequencies at iso-lift conditions, with required voltage shown in red. We impose a voltage limit for RoboBee wing-flapping to promote system longevity; higher voltages can cause damage to the actuators.

effect, and no yaw torque). For the MAV used in these experiments, this baseline condition is a 150 Hz flapping frequency, 180 V drive voltage, and shape defined by sinusoid harmonic coefficients $\begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} = \begin{pmatrix} 1.09 & 0 & 0.10 \end{pmatrix}$; this achieves sufficient lift for takeoff of the RoboBee. (Note that flight conditions vary slightly between MAVs due to manufacturing imperfections and for an MAV over its lifespan as it experiences flexure wear [70]).

Based on these operating conditions and the previously-defined model for calculating lift and torque with split-cycle filtering, a plot of iso-lift conditions is shown in Fig. 2.5, with a defined acceptable operating voltage cutoff. The actuators for this RoboBee were tested up to 300 V, and are typically operated under 200 V; therefore, 250 V was set as a maximum allowable voltage during these experiments.

2.4 Torque Generation Experiments

We conducted open-loop yaw velocity measurements at a set of iso-lift conditions equivalent in thrust to RoboBee hovering conditions, in line with the described model.

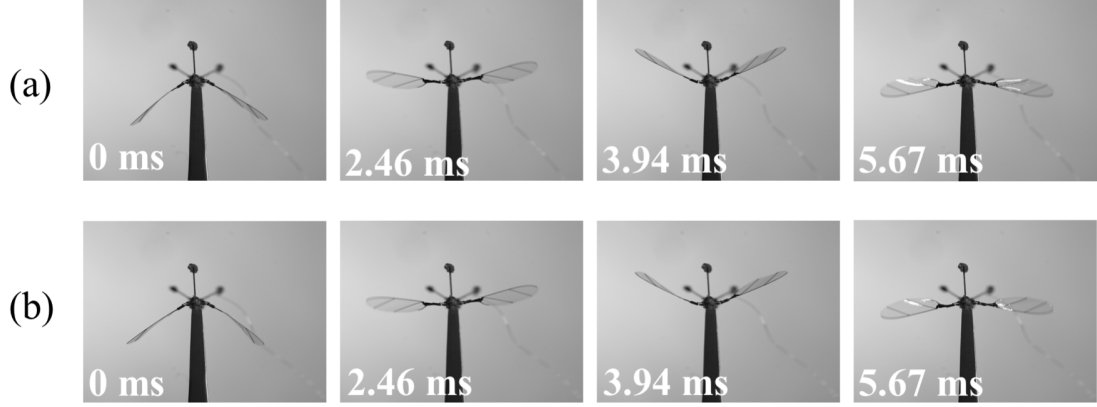


Figure 2.6: Wing kinematics are maintained for iso-lift conditions at 140 Hz with a RoboBee operating in typical flight conditions (a) and iso-lift yaw conditions (b).

We applied positive and negative yaw torque conditions, along with a baseline (no yaw) flight condition, at a set of decreasing frequencies, and calculated a flapping signal sinusoid for each wing with the following structure, such that $\phi_N(t)$ corresponds to baseline conditions, and $\phi_Y(t)$ corresponds to yaw conditions. V_A , ω_A , and $\begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix}$ correspond to the baseline conditions for voltage, flapping frequency, and the shape parameters, respectively; similarly, V_B , ω_B , and $\begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix}$ correspond to the yaw conditions for voltage, flapping frequency, and the shape parameters (experimental parameters are provided in Table 2.2).

$$\phi_N(t) = V_A [a_1 \sin(\omega_A t) + a_2 \sin(2\omega_A t) + a_3 \sin(3\omega_A t)] \quad (2.15)$$

$$\phi_Y(t) = V_B [b_1 \sin(\omega_B t) \pm b_2 \sin(2\omega_B t) + b_3 \sin(3\omega_B t)] \quad (2.16)$$

The positive and negative coefficients for the second harmonic for yaw, b_2 , will produce positive and negative torques, respectively.

Before conducting yaw torque experiments, we observed wingstroke kinematics for the RoboBee at iso-lift conditions to confirm that iso-lift yaw parameters would be

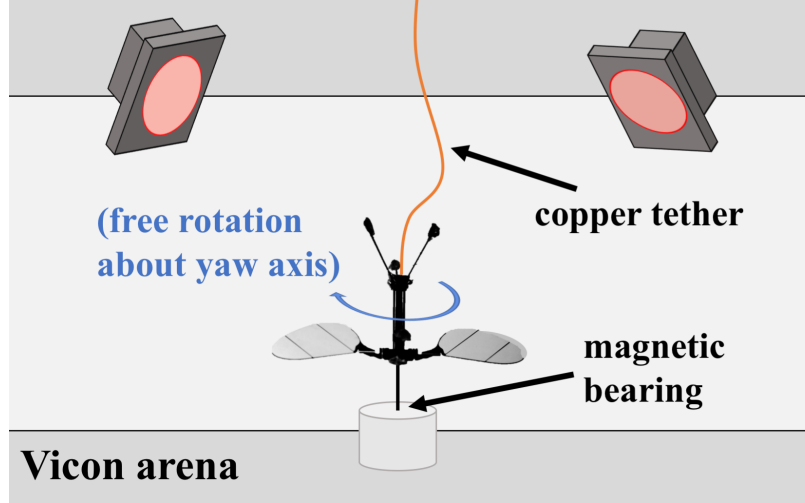


Figure 2.7: Experimental setup for yaw torque measurements: The RoboBee is fixed upside-down in a Vicon arena via a magnetic bearing, which allows full rotation about its yaw axis. Note that a copper tether is attached to the MAV to provide power and control signals; the RoboBee must overcome torque from the tether and friction from the bearing in order to rotate.

flightworthy (to supplement the lift calculations in Section 3.4), applying normal flight signals and iso-lift yaw signals at 140 Hz, with corresponding parameters found in Table 2.2). As shown in Fig. 2.6, sufficient similarity exists in stroke kinematics between the iso-lift baseline flight and yaw torque operating conditions to confirm similar flight conditions between the two (thereby affirming validity of the iso-lift condition model).

During our experiments, we constrained the RoboBee to a single degree of freedom

Table 2.2: Torque Experimental Parameters

Sinusoid Shape (Coefficients)	a_1	a_2	a_3	b_1	b_2	b_3
	1.09	0	0.099	0.934	0.216	0.047

$\omega_{A,B}$ [rad/s]	V_A [V]	V_B [V]
$140 \cdot 2\pi$	210.9	242.8
$145 \cdot 2\pi$	194.5	224.6
$150 \cdot 2\pi$	180.0	208.3
$155 \cdot 2\pi$	167.5	194.1

(i.e., rotation about the yaw axis) such that it could rotate freely; the experimental setup is shown in Fig. 2.7. We measured angular displacement using Vicon motion-capture cameras (Vicon T040 System, Oxford UK) operating at 500 Hz. We applied a voltage-ramped signal with a duration of 0.4 s (0.1 s of linear ramping up and down; 0.2 s at full voltage) for each experiment; this corresponds to between 56 and 62 periods (at 140 Hz and 155 Hz, respectively).

We quantified the increase in yaw torque by measuring the time-varying angular velocity about the yaw axis. (We provide the average velocity in lieu of a direct measure of torque because this quantity is the clearest yaw torque metric provided directly by our motion capture methods.) At the maximum torque condition (with split-cycle shape defined in Table 2.2), the MAV achieves an average of 17.28 rad/s and -18.82 rad/s, with corresponding standard deviations of 0.97 rad/s and 2.26 rad/s (indicating strong consistency across trials), for positive and negative yaw angular velocities over the full ramped (peak voltage) signal, respectively. This corresponds to angular displacements of 4.56 rad and -4.29 rad (with standard deviations of 0.59 rad and 0.74 rad) over 0.2 s at full voltage. We note that yaw torque magnitudes may be calculated from the motion capture data by considering the observed dynamics of the MAV in conjunction with ambient effects, including drag, tether effects, and bearing friction. While a full model of these effects is outside of the scope of this work, we note that the linear increase in velocity observed with increased applied yaw torque during experiments corresponds directly with the modeled yaw torque increase shown in Fig. 2.5.

Applying the iso-lift split cycle method for yaw torque generation, the RoboBee can accomplish a full rotation in a fraction of a second at flight conditions. This degree of yaw torque authority is sufficient for aggressive maneuvering, and therefore we proceed in implementation of an associated yaw controller.

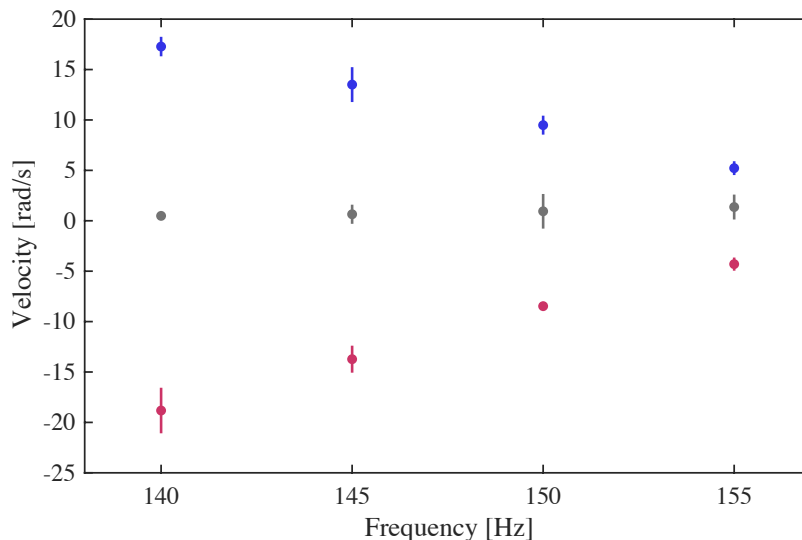


Figure 2.8: Measured open-loop yaw velocities across a selection of iso-lift operating conditions. Velocities from positive and negative yaw torque are shown in blue and red, respectively; the baseline condition (no yaw torque applied) is shown in gray. Each data point represents an average measurement, with standard deviations shown by error bars. Four samples were taken for all yaw conditions and 145Hz-155Hz baseline conditions; two samples were taken for the 140Hz baseline condition.

2.5 Yaw Control Demonstration

After demonstrating sufficient yaw torque authority for aggressive control, we demonstrate basic implementation of iso-lift split-cycle yaw torque in a simple heading controller.

We simplified the controller design by reducing the five-parameter set of signal sinusoid properties, as in (2.15) and (2.16), to a single parameter. This parameter μ , visualized in Fig. 2.9, scales from -1 to 1 such that the extremes correspond to maximum negative and positive yaw torque, respectively, and the zero-point is normal, efficient flight. Iso-lift conditions are maintained across the full span of μ .

The parameters used in these experiments at $\mu = 0$ and $\mu = \pm 1$ are provided in Table 2.3, which are at sub-hovering iso-lift conditions compared to the baseline conditions in

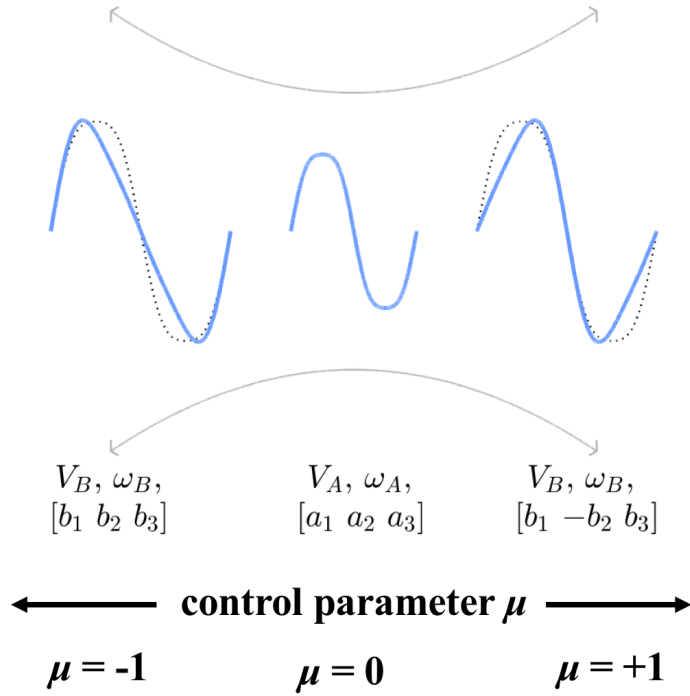


Figure 2.9: A smooth multi-modal control parameter for yaw control. The parameter μ interpolates between iso-lift conditions: peak-to-peak signal voltage (V_A , V_B); flapping frequency (ω_A , ω_B); and sinusoid shape (a_1 , a_2 , a_3 ; b_1 , b_2 , b_3 , corresponding to amplitudes of the first, second, and third harmonics of a sinusoid at the fundamental operating frequency), centered on standard efficient flight (A) and extending to conditions of maximal positive and negative yaw torque (B).

Table 2.2. To generate a continuous transition between yaw torques while preserving the iso-lift condition, a convex combination between (V_A, a_1, a_2, a_3) and (V_B, b_1, b_2, b_3) is considered by varying the parameter μ from $[0, 1]$, and a convex combination between $(V_B, b_1, -b_2, b_3)$ and (V_A, a_1, a_2, a_3) with μ varying from $[-1, 0]$. If we denote $(V_\mu, c_{\mu 1}, c_{\mu 2}, c_{\mu 3})$ as the new parameter set for a given μ , then the corresponding flapping frequency, ω_μ , which satisfies the iso-lift condition, is found by solving the following optimization problem,

$$\omega_\mu = \underset{\omega}{\operatorname{argmin}} \quad h(p_A, p_\mu(\omega))^2, \quad (2.17)$$

where $p_\mu(\omega) = (V_\mu, \omega, c_{\mu 1}, c_{\mu 2}, c_{\mu 3})$ and $h(p_A, p_\mu)$ is defined in (2.13). The above optimization may have multiple solutions; we choose the one below ω_A . Finally, a smooth parameter fit on the numerically-generated list of p_μ is used in the following yaw control experiment.

We implemented a PID heading controller with manually-tuned gains using the control parameter μ , with the RoboBee constrained to axial yaw rotation as described in Section 2.4. The reference signal was a heading angle step function, with an initial value of 0 rad and a final value of $\frac{\pi}{2}$ rad. The RoboBee started at an arbitrary angle, and the control signal was ramped as in the yaw torque measurement experiments.

The measured step response is shown in Fig. 2.10 (with associated images of

Table 2.3: Torque Experimental Parameters

Flapping Frequency [rad/s]	ω_A	$150.0 \cdot 2\pi$
	ω_B	$139.2 \cdot 2\pi$
Operating Voltage [V]	V_A	160.0
	V_B	220.0
Sinusoid Shape (Harmonic Coefficients)	a_1	1.09
	a_2	0
	a_3	0.099
	b_1	0.934
	b_2	0.216
	b_3	0.047

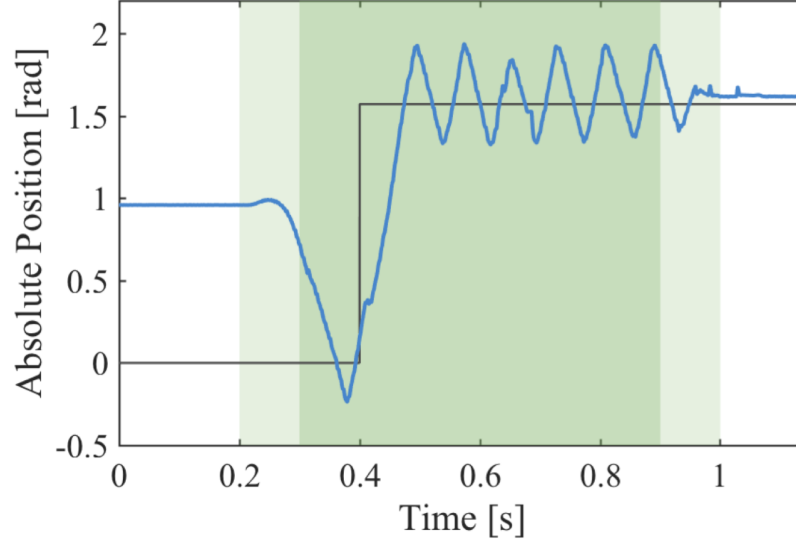


Figure 2.10: Step response of the RoboBee using PID control with smooth multimodal control parameter μ . The control signals ramp up and down linearly during testing (light green), but within the section of maximum control authority (dark green), the RoboBee follows the step function with a time constant of 0.1 s.

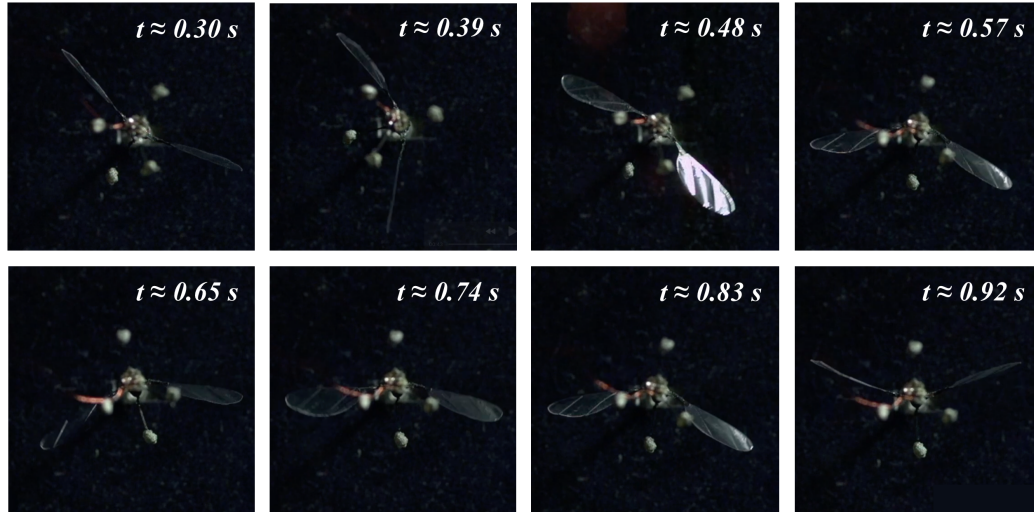


Figure 2.11: RoboBee during PID step response for heading control with a constrained vehicle, associated with the control experiment described in Section 2.5.

the vehicle during the control test shown in Fig. 2.11). At full control authority, the RoboBee demonstrated a time constant on the order of 0.1 s, an order of magnitude faster than that demonstrated in [45]. Observing oscillations about the reference signal, we infer that the RoboBee exhibited comparable control authority in both positive and negative yaw torque. While controller optimization will improve the response (e.g., by mitigating the oscillation about the reference signal), we conclude that iso-lift split-cycle yaw torque generation successfully demonstrates substantial yaw control authority.

2.6 Discussion and Future Work

We can demonstrate the significance of reliable yaw torque for aerial vehicle control by considering the case of a quadrotor position controller (specifically, a geometric tracking controller by Lee et al. [71]), which in its default case tracks position and heading. As shown in Fig. 2.12, when we remove the heading requirement for this controller, the position tracking becomes dramatically unstable; when we apply no yaw torque, position tracking succeeds, albeit with notably decreased performance; and finally, a reliable damping torque about the yaw axis allows position control to succeed. We observe, then, that the inclusion of reliable yaw torque within a control strategy has ramifications for stability and position control, in addition to allowing full attitude control of the vehicle.

The yaw torque authority achieved by the iso-lift split-cycle method demonstrates sufficient yaw torque for aggressive controlled maneuvers, while maintaining flight-worthy lift. This result was verified by the demonstration of PID heading control, achieving significant performance improvements over previous yaw control attempts. Nevertheless, we must ensure that this method is a viable long-term and robust solution for yaw torque control authority on the RoboBee; this requires further investigation into effects from operating at

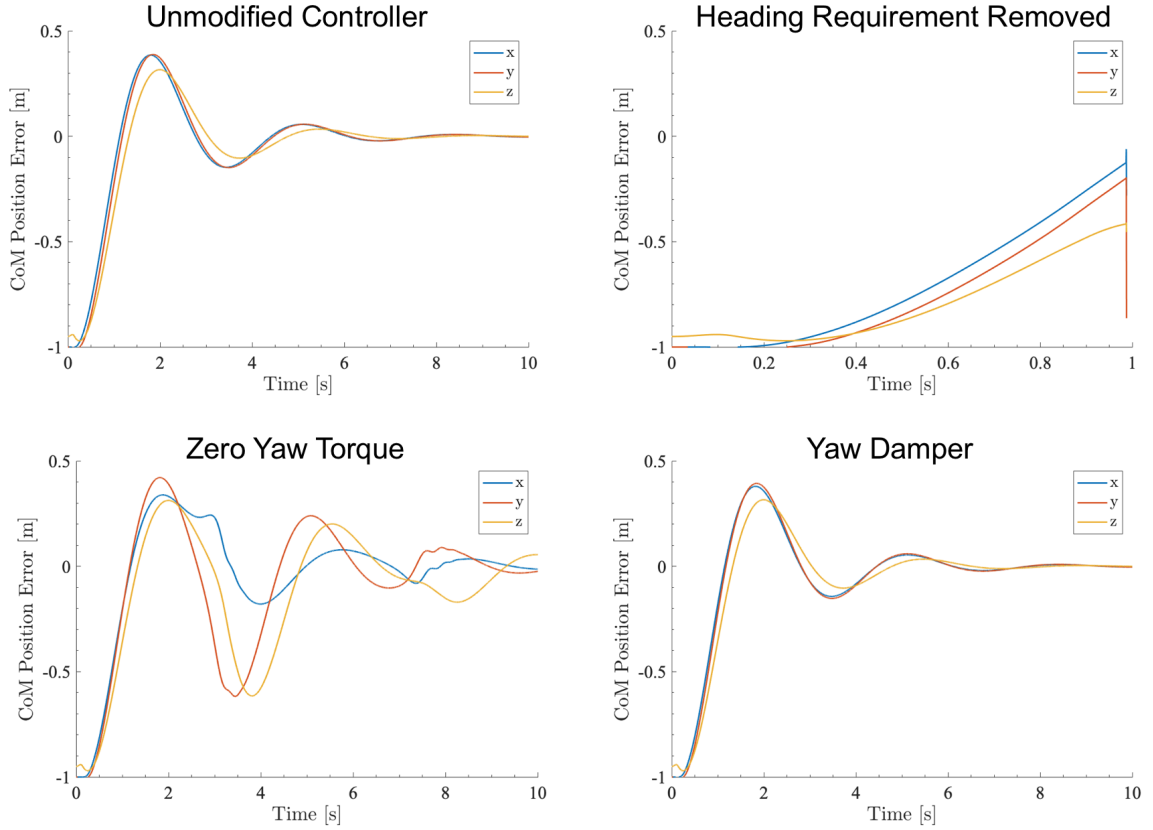


Figure 2.12: Modification of quadrotor position controller with heading requirement, showing performance with heading requirement removed, and with yaw control effort replaced with zero applied yaw torque and a simple damper. We observe that removal of the heading requirement (upper right) results in significantly reduced position control performance (compared to the baseline case, in the upper left), indicating that for this controller, the heading requirement is critical. With zero applied yaw torque (bottom left), the position controller is effective, though with lower performance than the original. When even a yaw damper is applied (bottom right), as opposed to full yaw control, position control performance is nearly identical to position control with active yaw control. This progression demonstrates that yaw authority, even regardless of the direct control method, has a strong effect on position control.

high voltage conditions and to couple the proposed yaw control strategy with roll and pitch control.

This method of yaw torque generation is particularly favorable because it requires no modification to the existing RoboBee design, which has evolved over the past decade [63]. Therefore, flight performance will not decrease for an MAV with yaw capabilities, nor is any significant effort required to transition between normal flight and yaw domains. We do note, however, that the yaw domain corresponds to lower power efficiency (where efficiency is considered to be the ratio of lift to power exerted by the actuators; power is directly proportional to $V^2\omega$, for applied voltage V and flapping frequency ω [36]). It follows that iso-lift yaw conditions should be used purposefully during RoboBee flight.

During experiments, the RoboBee flexures wore down to such a degree that the resonant frequency of the MAV dropped approximately 15 Hz. Therefore, operating frequencies which were originally low enough to reduce split-cycle filtering sufficiently to generate yaw torque became too close to the resonant point, and could no longer produce yaw torque. While in practice this can be reversed by replacing worn-down hinges with new, identical copies (a repair which requires only a matter of minutes), this observation indicates that the iso-lift split-cycle method is particularly susceptible to shifts in resonant frequency, and therefore to the state of wear of an individual MAV. It follows that, to achieve robust yaw control, a degree of adaptivity should be integrated into the controller to account for any change in resonance. Additionally, experiments should be conducted to measure any increase in flexure wear due to operation in the lower-efficiency yaw torque domain to anticipate and mitigate deterioration due to operation in yaw conditions.

While simple PID control was successfully demonstrated using iso-lift split-cycle yaw torque generation, a rich area of exploration remains in designing an optimal controller. Performance will be improved by insights into transitions between sinusoid shapes, ampli-

tudes, and frequencies while transitioning from normal flight to yaw conditions, whether independently or as a lumped parameter (as applied herein). Optimization of mode transitions may also decrease flexure deterioration during operation.

Finally, the proposed methods for generating yaw torque sufficient for control authority will be implemented in flight to complement existing control authority in thrust, roll torque, and pitch torque. Not only will this implementation allow a full demonstration of simultaneous yaw torque and lift, but it will open up the expanded set of control parameters used in iso-lift conditions to pave the way for exploration of aggressive control to fully exploit the maneuverability of the RoboBee.

Chapter 3

Modeling and Control of FWMAVs with Harmonic Sinusoids

Chapter 3 is adapted from McGill et al. 2021 [24], ©2021 IEEE.

3.1 Introduction

Flapping-wing micro-aerial vehicles (FWMAVs) provide unique control challenges both in control authority and in the complexity of modeling flapping-wing aeromechanics. We provide a control-critical, closed-form model for FWMAV flight, particularly tailored to augmenting yaw torque authority of the vehicle (a common challenge for FWMAVs). The model provides invertible nonlinear closed-form expressions for thrust and moments in terms of trigonometric polynomial input voltages and the wingstroke shape, and respects asymmetries inherent to the vehicle. We apply the model to characterize the Harvard RoboBee, providing an optimization-based system identification process. Using the inverted mapping between wingstroke shape and control axes, we achieve lateral and heading control of the vehicle. Our model and asymmetric system identification, in collaboration with the

chosen control scheme, allow full control authority of the position and the heading of the vehicle.

3.2 Motivation

A primary challenge in control of flapping-wing micro-aerial vehicles (FWMAVs) lies in understanding the time-varying aerodynamic forces generated in flight. While fixed-wing vehicles can have an invertible linear mapping between input and output (e.g., quadrotors use four rotor speeds to generate thrust and three moments), the control space of FWMAVs is typically larger, including harmonic flapping trajectory shape (which may be decomposed into a Fourier series representation.)

We may compute aerodynamic forces within a wingstroke using a quasi-steady aerodynamic model, based on the blade-element method, using known kinematics and the wing shape [72]. Quasi-steady aerodynamic forces rely critically on wing pitch angles, which modulate lift and drag. However, the small scale of most FWMAVs limits observability and controllability of wing pitch (particularly for under-actuated transmission mechanisms, as in the RoboBee [53]), and vehicle weight limitations prohibit the use of encoders on the wing joint [19, 73]. For example, many FWMAVs operating at high flapping frequencies (as in [19, 58, 56]) rely on passive dynamics to achieve a desired pitch rotation. Being unobserved, passive wing pitching precludes instantaneous understanding of wing forces, so FWMAVs often use a cycle-averaged estimation of aerodynamic forces and moments for control [53, 19, 56]. (Offline, these forces may be directly measured [53, 54] or computed via the observation of wing kinematics [56].) A model with simple analytic expressions for FWMAV forces and torques based on the input wingstroke actuation would be particularly useful for control.

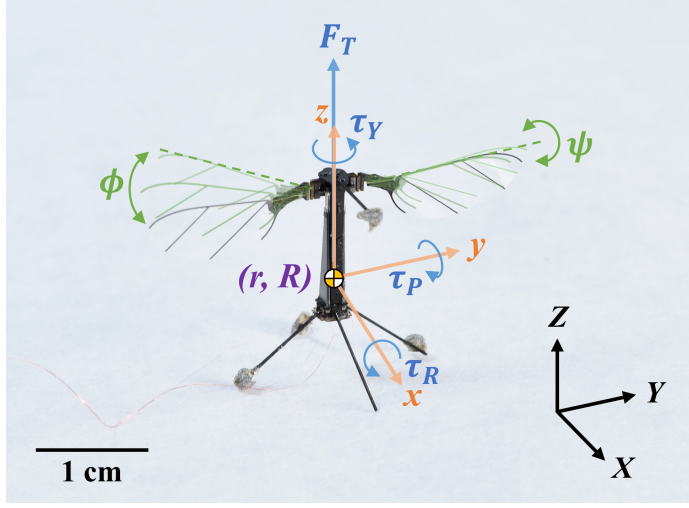


Figure 3.1: The Harvard RoboBee; state described via the CoM at r with rotation matrix R (observed in flight) and wing stroke ϕ and pitch ψ angles (not directly observed), with control authority of the thrust force F_T and roll (τ_R), pitch (τ_P), and yaw (τ_Y) moments.

An additional challenge in FWMAV flight is control authority; especially for yaw torque, which is commonly the weakest control axis [49, 40, 55]. While two-wing FWMAVs can generate thrust, roll, and pitch with a symmetric wingstroke shape, yaw torque authority requires bilateral asymmetry to generate a non-zero cycle-averaged moment. Two-wing FWMAVs can use a “split-cycle” method to generate yaw torques, modulating the ratio of upstroke to downstroke speeds [53, 19, 74], as shown in Fig. 3.2. However, yaw torque authority via this method produces significantly less torque compared to the other torques (roll and pitch) [53, 54, 55], resulting in substantial obstacles for active heading control [58, 75]. Therefore, previous FWMAV controllers have required yaw-amplifying vehicle design modifications [56, 40], or have simply left heading free [58, 57].

Alternately, yaw may be generated by varying the second harmonic of the flapping signal, combined with a sub-resonant fundamental frequency (to reduce passive filtering of higher-order harmonics) [6]. This second harmonic injection method achieves a similar wingstroke shape to the split-cycle method, without piece-wise sinusoid concatenation (see

Fig. 3.2). While this method has the potential to dramatically improve FWMAV control authority, it begets challenges in modeling. While a closed-form model for drag force (and by extension, yaw torque) exists for split-cycle flapping [53], deriving a closed-form, analytic model for the second harmonic injection method effectively requires finding the roots of a high-degree (≥ 6) trigonometric polynomial, which is impossible per the Abel-Ruffini theorem [76]. Recent work (described in Chapter 2) has verified the second harmonic injection method for yaw torque in flight [56], but its corresponding lift and drag forces were found numerically, using data from video of the flapping of a constrained (static) vehicle.

A further challenge in FWMAV control is the presence of asymmetries introduced into the vehicle during manufacturing [54], which result in sizeable control offsets in flight. Whereas FWMAV models do not commonly account for imperfections inherent to a vehicle, assuming symmetry despite the likelihood of manufacturing anomalies [53, 77], control design strategies attempt to mitigate dynamic effects resulting from asymmetries via force and torque offsets within the controller, whether constant or adaptive [54, 37]. However, these strategies serve to mask the characteristics of a vehicle, rather than to understand or even exploit them in control.

In this chapter, we derive an analytic closed-form, invertible model describing cycle-average thrust and torques for a FWMAV driven by multiple sinusoidal control signals, allowing full vehicle control. Our method allows yaw control via the second harmonic injection method, and we include the effects of this method on thrust and the three body torques. We also include asymmetries due to physical discrepancies between opposite actuator-wing pairs. We provide an optimization-based system identification method to characterize a FWMAV, including its asymmetries, using the nonlinear model, applying open-loop data from 40 unique flights. Our mapping allows general controllers (e.g., those

designed for quadrotors) to directly control a FWMAV. We apply a geometric, coordinate-free controller from [71] to the Harvard RoboBee; a two-wing, 86 mg FWMAV. We apply active heading control both while hovering in place and while tracking a trajectory, to demonstrate in-flight heading control for the RoboBee, and achieve full control authority for the first time.

3.3 FWMAV Modeling with Cycle-Averaged Aerodynamics

The propulsion of FWMAVs relies on aerodynamic lift and drag forces generated during flapping. The blade element based quasi-steady aerodynamic model is commonly used to compute the instantaneous quasi-steady aerodynamic force within a cycle, though it requires knowledge of wing configurations (the wing stroke angle ϕ , and the wing pitch angle ψ), which is not available during controlled flight [58, 56, 54]. Therefore, FWMAV models commonly rely on simplifying assumptions to identify a mapping between actuator inputs (i.e., harmonic voltage signals) and the harmonic wing trajectory.

Assumption 1 *The body dynamics and the wing dynamics are decoupled.*

Assumption 2 *Body dynamics depend on cycle-averaged (time-independent) aerodynamic forces.*

Assumption 1 applies to FWMAVs whose wing mass is markedly lower than the body mass (e.g., the wing mass of the Harvard RoboBee contributes to less than 1% of its total mass). Further analysis of the contribution of wing inertia to whole body dynamics may be found in [78] and [79]. Assumption 2 applies to FWMAVs characterized by high-frequency flapping, relative to slower body dynamics. (The Harvard RoboBee typically flaps at 160 Hz, the RoboFly at 170Hz, [56], and Bee+ at 100 Hz [49]). This assumption

allows identification of time-independent estimations for lift and drag forces, for practical use in controlled flight [19, 73, 56, 49].

3.3.1 Cycle-Averaged Quasi-Steady Aerodynamic Forces

Consider harmonic wing trajectories, with stroke angle $\phi_i(t)$ and a harmonic wing pitch angle $\psi_i(t)$ given for each i -th wing, where $i \in \{L, R\}$ indicates the left and right wings, respectively. Let T be the period of the fundamental of the harmonic wing stroke motion, $\phi(t)$, and $\omega := 2\pi/T$ is the flapping frequency. Then, the blade-element method can be used to compute the cycle-averaged lift and drag force ($\bar{F}_L^i$ and $\bar{F}_D^i$) produced by harmonic wing flapping [72, 53],

$$\bar{F}_L^i := \frac{\omega \rho \beta}{4\pi} \int_0^{\frac{2\pi}{\omega}} C_L(\alpha_i) \dot{\phi}_i^2 dt \quad (3.1)$$

$$\bar{F}_D^i := \frac{\omega \rho \beta}{4\pi} \int_0^{\frac{2\pi}{\omega}} C_D(\alpha_i) \dot{\phi}_i |\dot{\phi}_i| dt. \quad (3.2)$$

with angle of attack $\alpha_i(t)$ (varying with wing pitch ψ). The wing stroke velocity is $\dot{\phi}_i$, the ambient air density is given by ρ , and the wing geometry parameter is β . We use experimentally-identified lift and drag coefficients, $C_L(\alpha_i)$ and $C_D(\alpha_i)$, from a *Drosophila* wing [72, 69].

3.3.2 Cycle-Averaged FWMAV Dynamics

Suppose that $r = (x, y, z)$ is the center of mass (CoM) of the vehicle, and the origin of the body-attached coordinate frame is located at the CoM as shown in Fig. 3.1. We align the body frame with the principal axis of the vehicle, and model the vehicle body as a rigid body with (r, R) (for rotation matrix R) as a homogeneous coordinate in $SE(3)$, the Special Euclidean Group, which is a semi-direct product between $\mathbb{R}^3$ and $SO(3)$, the

Special Orthogonal Group.

$$\dot{r} = v \quad (3.3)$$

$$\dot{R} = R\hat{\Omega} \quad (3.4)$$

$$m\dot{v} = -mge_3 + Re_3F_T \quad (3.5)$$

$$I\dot{\Omega} = -\Omega \times I\Omega + \tau, \quad (3.6)$$

The vector $e_3 = (0, 0, 1)^\top$ is a unit vector in $\mathbb{R}^3$, m is the vehicle mass, g is gravitational acceleration, F_T is the thrust force, and $\tau = (\tau_r, \tau_p, \tau_y)$ is a vector of torques along the roll, pitch, and yaw axes. The principal moment of inertia, $I \in \mathbb{R}^{3 \times 3}$, is a diagonal matrix with components (I_{xx}, I_{yy}, I_{zz}) , and $\hat{\Omega}$ is a skew-symmetric matrix such that, $\hat{\Omega}y = \Omega \times y$ (here, $\times$ indicates a cross product). For the RoboBee, we use $m = 86$ mg (measured), $I_{xx} = 1.42$ g·mm², $I_{yy} = 1.34$ g·mm², and $I_{zz} = 0.45$ g·mm² (provided in [58] and [64]). We assume that the direction of the thrust F_T is aligned with the body z axis.

The thrust and the torques are generated by the aerodynamic forces acting on each wing. Following Assumption 2, we may estimate the generated thrust and torques using the cycle-averaged lift and drag forces:

$$F_T = \bar{F}_L^L + \bar{F}_L^R \quad (3.7)$$

$$\tau_r = r_{cp}(\cos(\phi_{mean}^L)\bar{F}_L^L - \cos(\phi_{mean}^R)\bar{F}_L^R) \quad (3.8)$$

$$\tau_p = r_{cp}(\bar{F}_L^L \sin(\phi_{mean}^L) + \bar{F}_L^R \sin(\phi_{mean}^R)) \quad (3.9)$$

$$\tau_y = r_{cp}(\bar{F}_D^L - \bar{F}_D^R). \quad (3.10)$$

Distance from the body z -axis to the wing center of pressure is r_{cp} , and average stroke angle is $\phi_{mean}^{L,R}$.

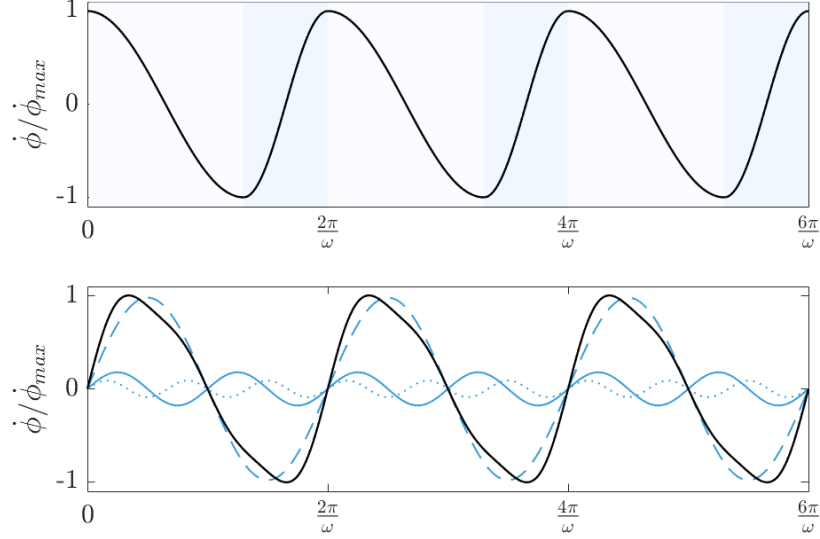


Figure 3.2: Flapping trajectories (wing velocity $\dot{\phi}$) used to generate yaw torque for FWMAVs; the split-cycle method (top) and second harmonic injection method (bottom), with dashed fundamental frequency, solid second harmonic, and dotted third harmonic.

3.4 Analytic Mapping from Input Voltage to Thrust and Torque with Multiple Sinusoids

In designing our model for FWMAVs, we consider sinusoidal flapping signals composed of three harmonics and one DC component, per Eq. (3.12). Also, we give special consideration to inherent vehicle asymmetries often introduced during manufacturing, which are nearly impossible to avoid.

3.4.1 Input Voltage to Wing Trajectory Approximation

We apply the second-order linear system approximation describing actuator and transmission dynamics in [67], which models the transfer function (from the harmonic input voltage signal to the harmonic trajectories of the wing stroke) using an equivalent mass, m_{eq} , damping coefficient, b_{eq} , and stiffness k_{eq} . This transfer function H characterizes the

actuator and transmission dynamics at a flapping frequency ω as follows:

$$H(\omega) = \frac{A_H}{m_{eq}(\omega j)^2 + b_{eq}(\omega j) + k_{eq}}, \quad (3.11)$$

where $\Phi(\omega) = H(\omega)V(\omega)$ for Φ and V , representing the Fourier transform of the wing stroke angle ϕ and the input voltage signal $\mathcal{V}$, respectively. We actuate each wing by applying a harmonic voltage signal

$$\mathcal{V}^i(t) = V_{off}^i + V_{amp}^i \sum_{n=1}^3 a_n \sin(n\omega t) \quad (3.12)$$

for each $i \in \{L, R\}$, where a_n are Fourier coefficients which determine the shape of the harmonic signals, V_{amp} is the signal amplitude, and V_{off} is the DC offset. The voltage signals to each wing are composed of superimposed sinusoids at the fundamental flapping frequency and its second and third harmonics, as shown in Fig. 3.2, with corresponding coefficients a_i , $i \in [1, 3]$. We select each a_i such that a_1 primarily influences wingstroke amplitude, a_2 biases the relative speeds of the upstroke to downstroke and induces a net yaw torque, and a_3 flattens and smooths the sinusoid, decreasing voltage requirements for a given a_1 [25]. In practice, we choose constant a_1 and a_3 , with signal shape controlled by a_2 . We choose a_1 and a_3 such that the maximum amplitude of the base shape function $\sum_{n=1}^3 a_n \sin(n\omega t)$ is one, for some nominal $a_2 = 0$ (zero yaw condition).

The instantaneous wingstroke angle, ϕ , at time t is

$$\phi(t) = (V_{off} + \nu)|H(0)|\text{sgn}\angle H(0) + V_{amp} \sum_{n=1}^3 a_n |H(n\omega)| \sin(n\omega t - \angle H(n\omega)), \quad (3.13)$$

where $|\cdot|$ is the absolute value. In practice, the mean-stroke angle, ϕ_{mean} (DC component of Eq. (3.13)), has a bias introduced during actuator manufacturing, where ν is the required

offset voltage to compensate this bias.

3.4.2 Closed-Form Cycle-Averaged Lift Force

We compute the cycle-averaged aerodynamic lift and drag forces in Eqs. (3.1-3.2), with the second-order expression for wing stroke angle ϕ^i from Eq. (3.13), as a function of tunable parameters such as the amplitude V_{amp} , the offset V_{off} , and harmonic shape parameters $\{a_n\}_{n=1}^3$. However, acquiring closed-form expressions is a substantial challenge, in part due to the lack of observability of the wing pitch. To proceed in deriving a mapping from the input voltage to the cycle-averaged thrust and torques, we use the following assumption:

Assumption 3 ([53]) *We use a constant angle-of-attack (AoA) in the cycle-averaged aerodynamic force and drag calculations; $\dot{\alpha}^i = 0$.*

Using this assumption, we find estimated lift and drag forces (which hold for any constant AoA), enabling us to provide analytical closed-form expressions for time-averaged thrust and torques. We learn the lift and drag gains through the optimization-based system identification process in Section 3.5.

Solving Eq. (3.1) for a closed-form $\bar{F}_L$, applying Assumption 3 with wingstroke angle $\phi(t)$ per Eq. (3.13), yields

$$F_L = \frac{1}{4}\rho\beta C_L\omega^2 V_{amp}^2 \sum_{n=1}^3 n^2 (a_n |H(n\omega)|)^2, \quad (3.14)$$

allowing analytic expressions for thrust, roll torque, and pitch torque. However, finding a closed-form solution for the drag force (and by extension, yaw torque) remains a challenge even with a constant AoA (per Assumption 3) due to the sign changes in the integrand of Eq. (3.2).

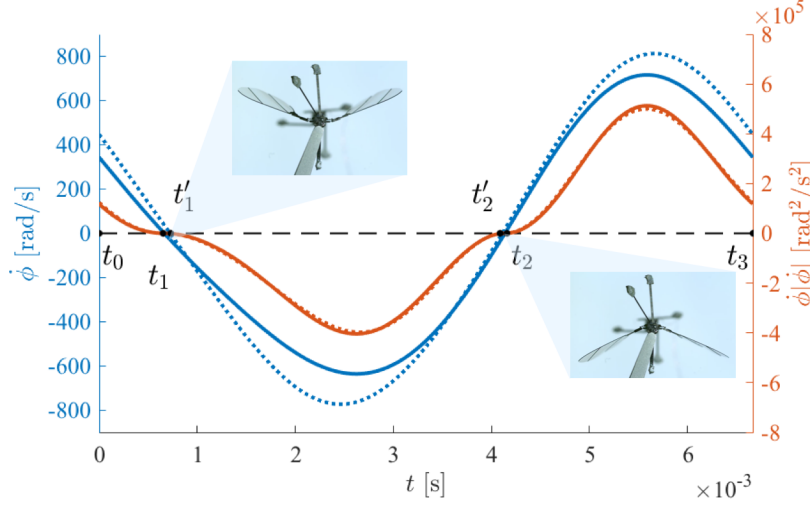


Figure 3.3: Visualization of model assumptions: (blue) wingstroke speed $\dot{\phi}$, with full $\dot{\phi}$ shown with a solid line, and fundamental frequency approximation shown with a dotted line; (red) full $\dot{\phi}|\dot{\phi}|$ shown using a solid line, and simplification (with third harmonic term removed) shown with a dotted line. (For $\dot{\phi}$ in this figure, we use $\omega = 150$ Hz, $V_{amp} = 180$ V, $a_1 = 1.1$, $a_2 = 0.1$, and $a_3 = 0.1$.)

3.4.3 Closed-Form Cycle-Averaged Drag Force

While the second-harmonic drag and yaw torque method provides substantial modeling challenges, we maintain that it provides distinct advantages over the split-cycle method in that it is fully analytic and allows instantaneous, continuous yaw commands.

We first find an appropriate approximation for the closed-form drag force, $\overline{F}_D$, according to the integral in Eq. (3.2) and using Assumption 3. A representative example of $\dot{\phi}|\dot{\phi}|$ of the RoboBee flap in the fixed platform is illustrated in Fig. 3.3. During each wingstroke cycle, there always exist two zero crossings within the interval $[t_0, t_0 + 2\pi/\omega]$. Without loss of generality, we assign $t_0 = 0$, $t_3 = \frac{2\pi}{\omega}$, and zero-crossing points of $\dot{\phi}|\dot{\phi}|$ to be t_1 and t_2 (see Fig. 3.3). Suppose $\dot{\phi}(t) \geq 0$ for all $t \in [t_0, t_1] \cup [t_2, t_3]$ and $\dot{\phi}(t) \leq 0$ for all $t \in [t_1, t_2]$. Let Q be the anti-derivative of $\dot{\phi}^2$, which can be explicitly computed using Eq. (3.13) ($Q := \int \dot{\phi}^2$), then

$$\int_0^{\frac{2\pi}{\omega}} \dot{\phi}|\dot{\phi}|dt = Q(t_3) - Q(t_0) + 2(Q(t_1) - Q(t_2)). \quad (3.15)$$

Thus, the closed form for $\overline{F}_D$ depends on explicit solutions at zero-crossings t_1 and t_2 of $\dot{\phi}(t_i) = 0$. However, finding direct solutions for t_1 and t_2 given $\dot{\phi}$ requires a closed-form solution for this six-degree trigonometric polynomial: Given its arbitrary phases, this is not possible (per the Abel-Ruffini theorem [76]).

Therefore, we approximate t_1 and t_2 , using zero-crossings of the fundamental, $V_{amp}[A \cos(\omega t - a)]$. This yields approximate roots of $\dot{\phi}(t) = 0$, with $t'_1 = \frac{2a+\pi}{2\omega}$ and $t'_2 = \frac{2a+3\pi}{2\omega}$. Then, we evaluate the antiderivative, $Q(t)$ at each t'_i . Applying these within Eq. (3.15) provides an analytical approximation for the cycle-averaged drag force, per Eq. (3.2):

$$\begin{aligned} \overline{F}_D \approx \frac{\rho\beta V_{amp}^2 C_D}{60\pi} \quad [& 40AB \cos(2a - b) - AC \sin(3a - c) \\ & + 12BC (\cos(5a - b - c) + 5 \cos(a + b - c)) \quad], \end{aligned} \quad (3.16)$$

where $A = a_1\omega|H(\omega)|$, $a = \angle H(\omega)$, $B = 2a_2\omega|H(2\omega)|$, $b = \angle H(2\omega)$, $C = 3a_3\omega|H(3\omega)|$, and $c = \angle H(3\omega)$, allowing a closed-form expression for yaw torque per Eq. (3.10).

3.4.4 Full Closed-Form Mapping of Input Voltage to Force/Torques Mapping, Including Yaw Torque

Before providing a full closed-form expression for drag force, we make one final simplification: the magnitude of the transfer function, $H(\omega)$ in Eq. (3.11), for the RoboBee at the third harmonic, $|H(3\omega)|$, is significantly smaller than that of the lower harmonics [53, 6]. The third harmonic term has only a very small contribution to lift and drag; instead, it serves to flatten the input voltage signal [6]. (Coefficient a_3 is very small compared to a_1 ;

10% in [6].) It follows that the contribution to the cycle-averaged drag in Eq. (3.16) from $C = 3a_3\omega|H(3\omega)|$ is relatively small, which allows us to assume $C = 0$ in Eq. (3.16). Finally, since the mean stroke angle ϕ_{mean} (in Eqs. (3.8-3.9)) is very small (less than ± 10 deg), we may apply small angle approximations $\sin(\phi_{mean}) \approx \phi_{mean}$ and $\cos(\phi_{mean}) \approx 1$ in the final expressions as in [53].

Suppose that the shape parameters for the left and right input voltage signals in Eq. (3.12), $\{a_i^L\}_{i=1}^3$ and $\{a_i^R\}_{i=1}^3$, are symmetrical, such that $a_1^L = a_1^R$, $a_2^L = -a_2^R$, and $a_3^L = a_3^R$. Then, we may write closed-form expressions for the thrust, F_T , and three moments, (τ_r, τ_p, τ_y) as a function of five input parameters: two amplitudes of the first order harmonic, $V_L := V_{amp}^L$ and $V_R := V_{amp}^R$, two offset voltages, V_{off}^L and V_{off}^R , and the shape parameter for the second order harmonic, a_2 . Defining $\zeta = a_1^2|H(\omega)|^2 + 4a_2^2|H(2\omega)|^2$, we have:

$$F_T \approx \frac{1}{4}\rho\beta C_L\omega^2(\eta V_L^2 + V_R^2)\zeta \quad (3.17)$$

$$\tau_r \approx r_{cp}\rho\beta C_L\omega^2(\eta V_L^2 - V_R^2)\zeta \quad (3.18)$$

$$\tau_p \approx r_{cp}\rho\beta C_L|H(0)|\text{sgn}\angle H(0)\omega^2 \quad (3.19)$$

$$\begin{aligned} & \times (\eta V_L^2(V_{off}^L + \nu) + V_R^2(V_{off}^R + \nu))\zeta \\ \tau_y \approx & \frac{4}{3\pi}r_{cp}\rho\beta C_D\omega^2 a_1(a_2 + \mu)|H(\omega)||H(2\omega)| \\ & \times \cos(2\angle H(\omega) - \angle H(2\omega))(\eta V_L^2 + V_R^2). \end{aligned} \quad (3.20)$$

In addition to the demonstration of modeling assumptions provided in Figure 3.3, we also provide a visual comparison of calculated forces and torques produced with typical flapping conditions (across a range of frequencies) in Fig. 3.4. We find that force and torque calculations with the new analytic expressions above match well with numerically-computed forces and torques (without the defined modeling assumptions), using MATLAB

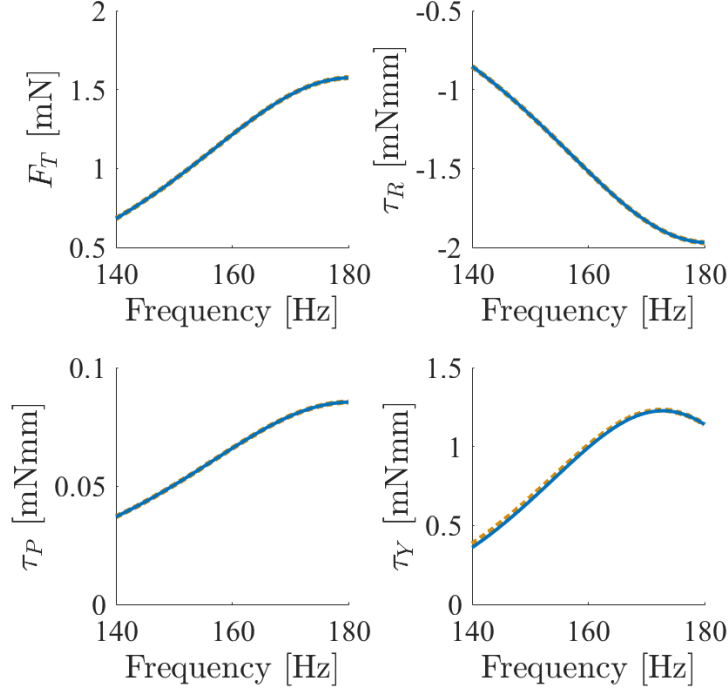


Figure 3.4: Agreement between numerical and analytical (closed-form) solutions for thrust and torques given typical flapping conditions across a range of frequencies. Numerical solutions are provided with red dashed lines; analytical solutions are provided with blue solid lines.

to find numerical solutions to the integrals in Eqs. (3.1) and (3.2).

We also introduce a yaw offset term, μ (similar to ν , which accounts for mechanical bias in the pitch offsets), to describe the effective change in a_2 due to wing asymmetries and imperfections introduced during manufacturing. In addition, we include the parameter η to account for discrepancies in force production between wing-actuator pairs on the left and right sides. (It is practically infeasible, when manufacturing, for the two sides to be identical.) Figure 3.5 provides an example of expected variation in observed forces and torques, given representative flapping signal parameters, with manufacturing parameters varied randomly within 10% of the intended component design. We observe significant variation in forces and torques with these simulated manufacturing discrepancies, which

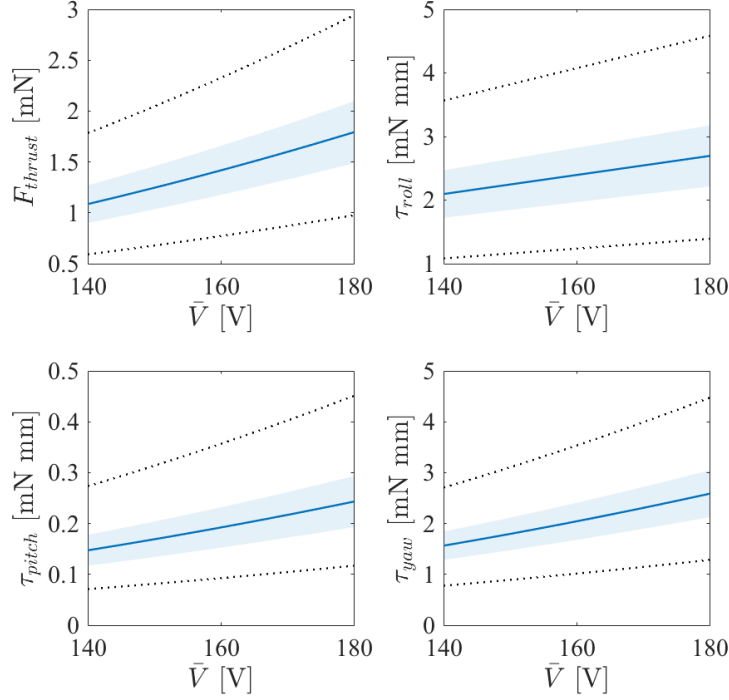


Figure 3.5: Effect of manufacturing variability on RoboBee forces and torques. Given typical flapping parameters with varying frequency, force and torque control outputs are provided for the ideal case, shown by the solid blue line. With manufacturing variability within 10% for key physical parameters, the shaded blue region encompasses ± 1 standard deviation from the mean (anticipated) value, with maximum and minimum values over 10000 trials shown by the dotted black lines [80].

can result in control inputs on the order of a multiple of two different from the intended control (i.e., control with a perfect vehicle) [80]. Asymmetry parameters η , ν , and μ are found via the following optimization process.

3.5 Optimization-Based System Identification

To characterize a given FWMAV according to the model described in Section 3.4, we apply an optimization-based system identification process using open-loop data. The closed-form mapping from the input voltage to the thrust and moments in Eqs. (3.17-3.20)

requires knowledge of physical parameters including air density, ρ , wing geometry β , lift and drag coefficients, C_L and C_D , and the transfer function $H(\omega)$. We use a motion capture system (Vicon T040 System, Oxford UK) to collect open-loop flight data to numerically estimate cycle-averaged thrust and moments.

In anticipation of control applications, and to simplify the model, we first rephrase vehicle forces and torques in terms of direct wing-flapping signal inputs and grouped physical parameters. We choose control inputs u_i , $i \in [1, 5]$, as follows: $u_1 = V_L^2$, $u_2 = V_R^2$, $u_3 = a_2$, $u_4 = V_{off}^L$, and $u_5 = V_{off}^R$. These parameters allow independent modification of thrust and body torques. Next, we choose grouped parameters γ_i , $i \in [1, 3]$ and δ_i , $i \in [1, 3]$ (grouped such that δ_i parameters are easier to directly measure, for use in system identification and control). We assign $\gamma_1 = \frac{1}{4}\rho\beta C_L\omega^2$, $\gamma_2 = r_{cp}$, and $\gamma_3 = \frac{C_D}{C_L} \cdot \frac{8}{3\pi} \cos(2\angle H(\omega) - \angle H(2\omega))$; and $\delta_1 = a_1\omega|H(\omega)|$, $\delta_2 = 2\omega|H(2\omega)|$, and $\delta_3 = |H(0)|\text{sgn}\angle H(0)$. With the constant lift and drag coefficients assumption, along with Assumption 3, our model becomes

$$F_T = \gamma_1(\delta_1^2 + \delta_2^2 u_3^2)(\eta u_1 + u_2) \quad (3.21)$$

$$\tau_r = \gamma_1\gamma_2(\delta_1^2 + \delta_2^2 u_3^2)(\eta u_1 - u_2) \quad (3.22)$$

$$\tau_p = \gamma_1\gamma_2\delta_3(\delta_1^2 + \delta_2^2 u_3^2)(\eta u_1(u_4 + \nu) + u_2(u_5 + \nu)) \quad (3.23)$$

$$\tau_y = \gamma_1\gamma_2\gamma_3\delta_1\delta_2(u_3 + \mu)(\eta u_1 + u_2). \quad (3.24)$$

In preparation for the optimization-based system identification process that follows, we separate the parameters from Eqs. (3.21-3.24) into four groups, as follows:

$$\bar{p}_1 := (\gamma_1, \gamma_2, \eta), \bar{p}_2 := (\nu, \delta_3), \bar{p}_3 := (\mu, \gamma_3) \quad (3.25)$$

$$\bar{q} := (\delta_1, \delta_2) \quad (3.26)$$

We assume $\{\bar{p}_i\}_{i=1}^3$ are unknown parameters, related to the lift and drag coefficients and vehicle asymmetry, and $\bar{q}$ are known parameters, found using nominal actuator and transmission physical parameters (provided in [6]).

3.5.1 Cycle-Averaged Thrust and Moment Measurement

Applying the closed-form model to characterize the RoboBee, we measure vehicle forces and torques from dynamic tests with a variety of wingstroke control conditions, using this data to find an optimal fitting between our model and the data. Since the cycle-averaged dynamics described in Eqs. (3.3-3.6) are markedly slower than the flapping frequency (150 Hz or higher), and the Vicon motion-capture system, at 500 Hz, is fast enough to sample the relatively slow-varying body dynamics, we can estimate the cycle-averaged thrust and moments using the motion-capture data recorded during flight.

First, we gathered a comprehensive set of 40 open-loop flights (0.35 seconds), varying control parameters to achieve measurable thrust and body torque combinations, with a 150 Hz flapping frequency (approximately 15 Hz below resonance for our vehicle). Flights (open-loop, and later controlled) were contained within a motion-capture arena with position and attitude captured using the Vicon system, and power and control lines supplied from off-board computers and amplifiers.

We calculate acceleration as the vehicle lifts off (25 ms of flight; approximately four cycles of flapping data at full voltage). We find thrust and body torques using the low-pass filtered derivatives,

$$F_T^e = \langle \bar{R}e_3, m\dot{\bar{v}} + mge_3 \rangle \quad (3.27)$$

$$\tau^e = I\dot{\bar{\Omega}} + I\bar{\Omega} \times \bar{\Omega} \quad (3.28)$$

where $\langle \cdot, \cdot \rangle$ is the inner-product in $\mathbb{R}^2$; and $\bar{R}$, $\bar{v}$, and $\bar{\Omega}$ are Vicon data (cycle-averaged, with filtered derivatives).

3.5.2 Optimization-Based System Identification

We design an optimization-based mapping process between FWMAV data and the model in Eqs. (3.21-3.24), which is characterized by nonlinear relationships between unknown parameters $\{\bar{p}_i\}_{i=1}^3$ (grouped in Eq. (3.25)). We observe that the closed-form model has a nearly triangular structure, as the number of unknown parameters in the closed form increases from F_T to τ_y . Therefore, we formulate an optimization sequence, identifying the optimal parameters for $\bar{p}_1^* := (\gamma_1^*, \gamma_2^*, \eta^*)$ first, then using the optimal $\bar{p}_1^*$ to find optimal parameters for $\bar{p}_2$ and $\bar{p}_3$. To find $\bar{p}_1^*$, we formulate a nonlinear optimization problem, minimizing the weighted sum of estimation errors for F_T and τ_r . Suppose that $\chi = \gamma_1 * \eta$ and $\bar{F}_T^e, \bar{\tau}_r^e \in \mathbb{R}^N$ are vectorized measured thrust and roll torque from N open-loop flights. Then, our problem is

$$(\gamma_1^*, \chi^*, \eta) := \arg \min_{\bar{p}_1 \in \mathbb{R}^3} J(\bar{p}_1)$$

with cost function

$$J(\bar{p}_1) := \|\bar{F}_T^e - \bar{F}_T(\bar{p}_1)\|_2^2 + \lambda \|\bar{\tau}_r^e - \bar{\tau}_r(\bar{p}_1)\|_2^2,$$

using tunable weight $\lambda > 0$, where $\|\cdot\|_2$ is the 2-norm. $\bar{F}_T$ and $\bar{\tau}_r$ are vectorized estimates (according to the closed-form model) in $\mathbb{R}^N$ using the commanded control inputs $\{u_i\}_{i=1}^5$.

The change of variables using χ allows us to reformulate the expression for F_T to

have a linear dependence on γ_1 and χ , maintaining bilinearity only in $\bar{\tau}_r$:

$$\bar{F}_T(\bar{p}_1) = \Delta_1 \begin{pmatrix} \gamma_1 \\ \chi \end{pmatrix}, \bar{\tau}_r(\bar{p}_1) = \gamma_2 \Delta_2 \begin{pmatrix} \gamma_1 \\ \chi \end{pmatrix}$$

for known $\Delta_1, \Delta_2 \in \mathbb{R}^{N \times 2}$. Let $\bar{p}_{11}^* = (\gamma_1^*, \chi^*)$ be the partial state of the optimal solution $\bar{p}_1^*$. Then, we have a nominal solution of $\gamma_2^* = (\Delta_2 \bar{p}_{11}^*)^\dagger \bar{\tau}_r^e$ and

$$((\bar{F}_T^e)^\top \Delta_1 + \lambda (\bar{\tau}_r^e)^\top \Delta_2 \gamma_2^*) = \bar{p}_{11}^{*\top} (\Delta_1 \Delta_1^\top + \lambda \Delta_2 \Delta_2^\top \gamma_2^2)$$

where $\dagger$ is the pseudo-inverse. By numerically solving the above necessary condition, we obtain the optimal $\bar{p}_1^*$.

Next, we solve two independent optimization problems to find the optimal $\bar{p}_2^* = (\nu^*, \delta_3)$ and $\bar{p}_3^* = (\mu, \gamma_3)$:

$$\begin{aligned} (\nu^*, \delta_3^*) &:= \arg \min_{\bar{p}_2 \in \mathbb{R}^2} \|\bar{\tau}_p^e - \bar{\tau}_p(\bar{p}_2, \bar{p}_1^*)\|_2^2 \\ (\mu^*, \gamma_3^*) &:= \arg \min_{\bar{p}_3 \in \mathbb{R}^2} \|\bar{\tau}_y^e - \bar{\tau}_y(\bar{p}_3, \bar{p}_1^*)\|_2^2. \end{aligned}$$

We provide the model fitting results in Fig. 3.6. The model predicts the trends in thrust and torques with varied control inputs, and we find that the root-mean-square error across all trials between the model-predicted and measured values are 0.0220 mN for thrust force (18.1% of the range of measured values), 0.0540 mNmm for roll torque (10.8%), 0.0328 mNmm for pitch torque (10.8%), and 0.0116 mNmm for yaw torque (7.8%).

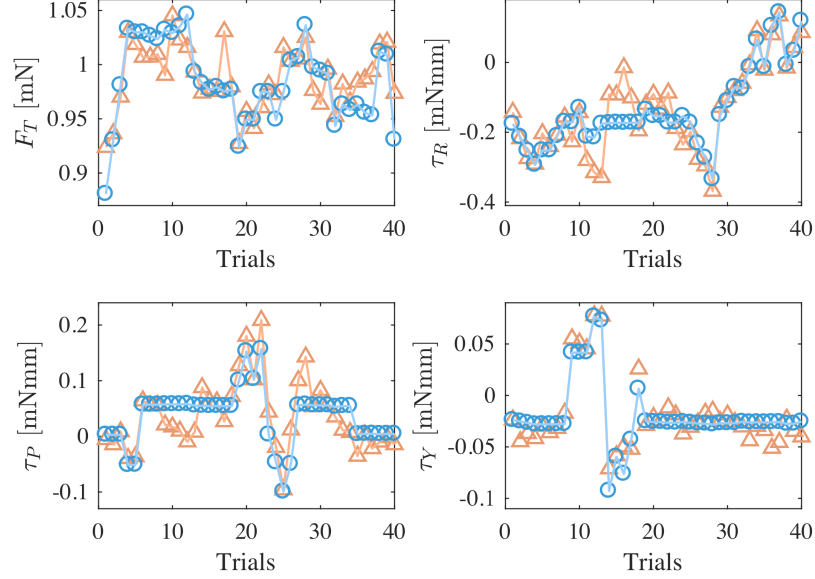


Figure 3.6: Measured (orange) and model-estimated (blue) forces and torques over 40 open-loop flights, using optimal system identification.

3.6 Geometric Control with Inverse Mapping

With this closed-form model for FWMAV flight, fit to our vehicle in Section 3.5, we perform a lateral and heading control task. Previous RoboBee controllers in [58, 75, 57] only tracked reference positions, with a free heading angle (simply minimizing yaw velocity).

3.6.1 Inverse Mapping for Feedback Control

We produce an analytical inverse of the model, to allow direct calculation of control parameters required for desired forces and torques (respecting vehicle asymmetries, and thus not requiring unmodeled offsets). We have five control parameters, $\{u_i\}_{i=1}^5$ to yield four desired outputs (one thrust and three moments) per Eqs. (3.21-3.24). This indicates that the mapping is surjective, but not injective. Thus, there exists an extra degree of freedom to generate the same desired output. We simplify our inverse mapping by choosing symmetrical voltage offsets, $V_{off}^L = V_{off}^R$ (i.e., $u_4 = u_5$). For simplicity, we neglect the

contributions of μ to u_1 and u_2 . We compute the following nonlinear mapping using the MATLAB symbolic solver to directly find control inputs for desired thrust and torque set $(F_T, \tau_r, \tau_p, \tau_y) \in \mathbb{R}^+ \times \mathbb{R}^3$:

$$u_1 = \frac{(\gamma_2 \gamma_3 F_T + (\gamma_2^2 \gamma_3^2 F_T^2 - 4\tau_y^2)^{1/2})(\gamma_2 F_T + \tau_r)}{4\eta \gamma_1 \gamma_2^2 \gamma_3 \delta_1^2 F_T} \quad (3.29)$$

$$u_2 = \frac{(\gamma_2 \gamma_3 F_T + (\gamma_2^2 \gamma_3^2 F_T^2 - 4\tau_y^2)^{1/2})(\gamma_2 F_T - \tau_r)}{4\gamma_1 \gamma_2^2 \gamma_3 \delta_1^2 F_T} \quad (3.30)$$

$$u_3 = \frac{\delta_1(\gamma_2 \gamma_3 F_T - (\gamma_2^2 \gamma_3^2 F_T^2 - 4\tau_y^2)^{1/2})}{2\delta_2 \tau_y} - \mu \quad (3.31)$$

$$u_4 = \frac{\tau_p}{\gamma_2 \delta_3 F_T} - \nu, \quad (3.32)$$

with a valid domain for $F_T > 0$. If $\tau_y = 0$, applying L'Hôpital's rule for the limit as $\tau_y \rightarrow 0$ yields $u_3 = -\mu$.

To demonstrate the utility of the inverse model, we provide a demonstration of direct calculation of flapping signal parameters, given a set of desired time-varying forces and torques, in Fig. 3.7. For control inputs (forces and torques) within the expected typical operating regime for the RoboBee, resulting voltage and harmonic coefficient signal parameters calculated from the inverse model fall within acceptable and normal boundaries for flight-worthy flapping conditions.

3.6.2 Geometric Position and Heading Control

With the control authority observed in Section 3.5, we anticipate that our vehicle is equipped with the ability to operate according to quadrotor-like control methods on $SE(3)$. We apply a geometric tracking controller on $SE(3)$ following [71], which provides coordinate-free asymptotic control of CoM position and heading.

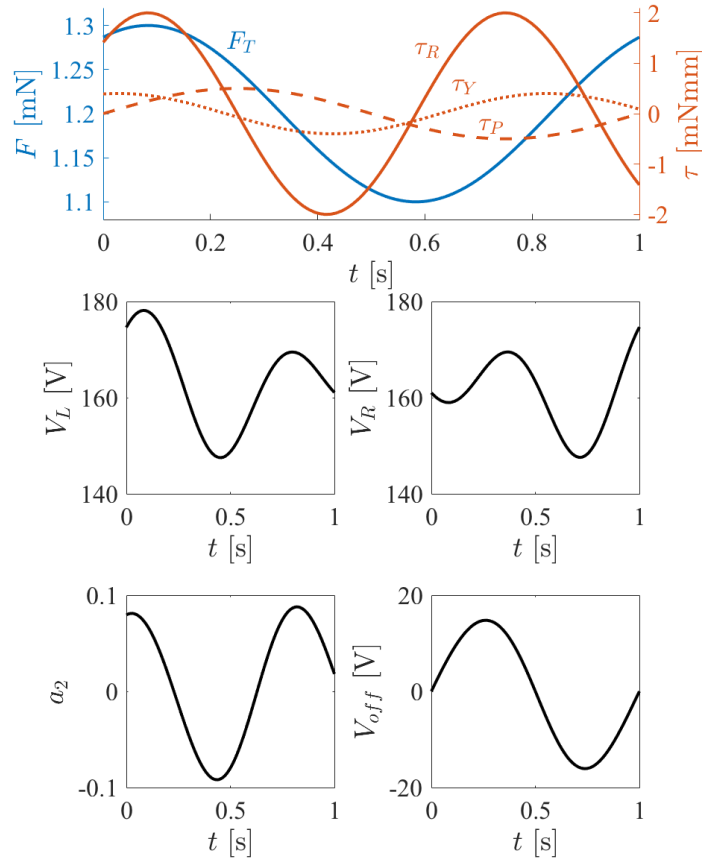


Figure 3.7: Control parameters to yield varying thrust and torques (constant flapping frequency). Desired forces and torques are shown in blue and red, respectively (top); corresponding flapping signal parameters are shown in black (bottom).

Suppose that $r_d(t)$ is the desired CoM trajectory, and $b_1^d(t) \in \mathbb{R}^3$ is the desired heading orientation in the inertial frame (defined by the x -axis in Fig. 3.1) at time t . Then, the feedback control law is defined as

$$\begin{aligned} F_T &= \langle -k_x e_x - k_v e_v + m g e_3 + m \ddot{r}_d, R e_3 \rangle \\ \tau &= -k_R e_R - k_\Omega e_\Omega + \Omega \times I \Omega \\ &\quad - I(\dot{\Omega} R^\top R_d \Omega_d - R^\top R_d \dot{\Omega}_d), \end{aligned}$$

where $e_x := r - r_d$, $e_v := v - \dot{r}_d$, R_d is the desired orientation, and Ω_d is the desired angular velocity. R_d and Ω_d are uniquely defined with the desired thrust vector and the desired heading b_1^d (see [71] for more details). Orientation error is captured by $e_R := 1/2(R_d^\top R - R^\top R_d)^\vee$ (where $\vee$ is the inverse of the skew-symmetric hat map of a matrix), which represents the distance metric on $SO(3)$; Angular velocity error is captured by $e_\Omega := \Omega - R^\top R_d \Omega_d$, related to the derivative of $R_d^\top R$; and $k_x, k_v, k_R, K_\Omega \in \mathbb{R}^{3 \times 3}$ are diagonal matrices with positive entities representing controller gains.

3.6.3 Hovering Flight with Variable Heading Angle

We apply this controller during a hovering flight, commanding a fixed position, $r_d = (-0.012, -0.034, 0.14)$ mm, and tracking a variable heading vector. We command heading vectors $b_1^d = (1/\sqrt{2}, 0, 0)$ and $(-1/\sqrt{2}, 0, 0)$, representing heading angles of $\pm 45^\circ$ while upright, with a linear transition between the two in a step-like function (shown in Fig. 3.8). We define the heading angle θ as the angle between the heading vector b_1 (aligned with the x -axis in Fig. 3.1) and the global y -axis, such that $\theta := \arctan \frac{b_1(2)}{b_1(1)}$, with desired heading θ_d . The vehicle stabilized at θ_d set-points, maintaining hovering stability and converging to θ_d with a maximum heading error of 14.2° after convergence. Additionally, during the

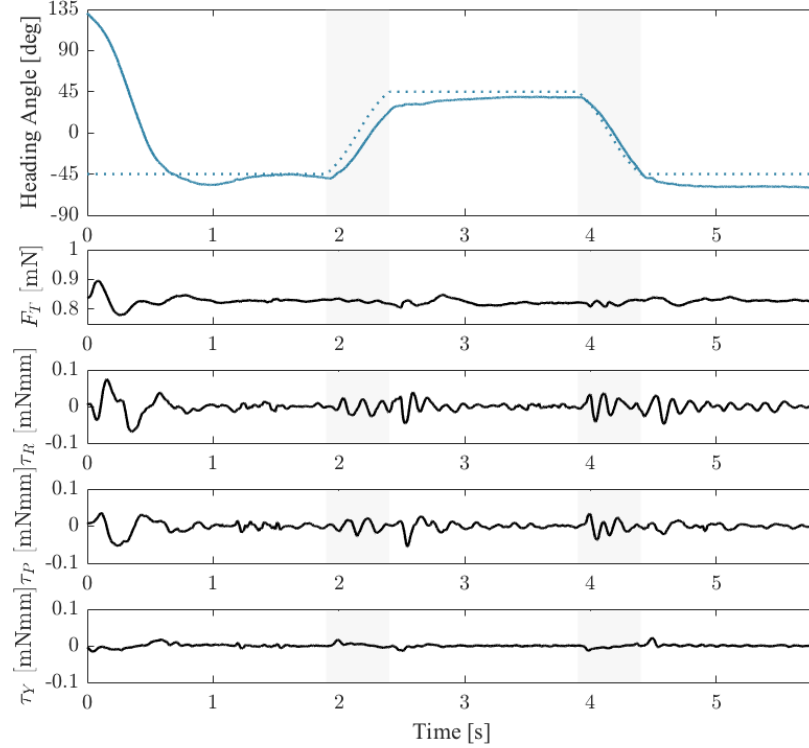


Figure 3.8: Heading response for tracking a desired heading angle during hovering flight, with estimated forces and torques (from motion-capture data) during the control response.

transitions in θ_d between set-points (shown in gray), the vehicle tracked the π rad/s angular velocity of θ_d . In Fig. 3.8, we also report forces and torques produced by the vehicle during its response (estimated according to Section 3.5.1). We observe that the RoboBee exhibits controllable yaw torque in flight; the first such demonstration for this vehicle.

Observing the measured torques for the maneuver (shown in Fig. 3.8), we observe that (1) yaw torque is substantially smaller in magnitude than roll and pitch torques; and (2) the highest yaw torques occurs during the beginning and end of a heading angle change, without substantial yaw torque required throughout the maneuver. This is consistent with observed behavior in fruit flies (*Drosophila melanogaster*), in both cases suggesting that dynamics in heading maneuvers are largely governed by inertia, rather than friction [81].

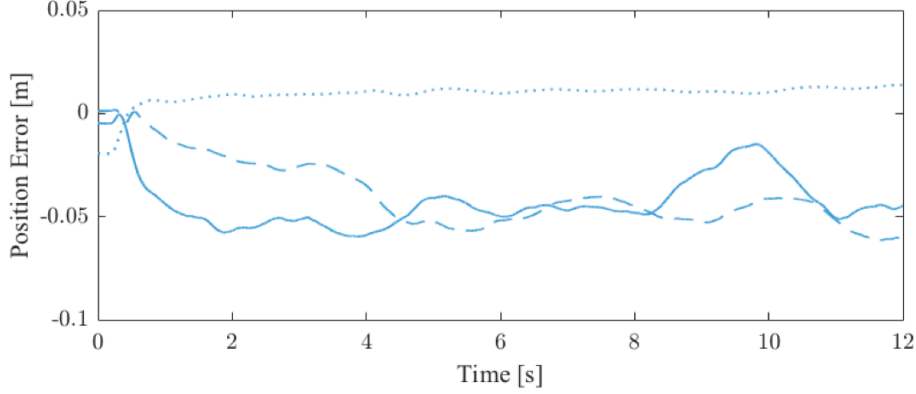


Figure 3.9: Position error in the x (solid line), y (dashed line), and z (dotted line) directions over a 12-second hovering flight.

3.6.4 Long-Duration Hovering Flight

Applying the same control method for a long-duration hovering flight with no heading angle change, we perform a 12-second flight, reporting the position error in the x , y , and z directions in Fig. 3.9. We note that the flight ended at a predetermined time, as opposed to a vehicle stability issue or other circumstance; therefore, with this control strategy, we anticipate that FWMAV would be able to fly for a longer duration. Additionally, this demonstrated flight is among the longest for this vehicle; typical demonstrated flight durations are within 7 seconds (and often much shorter). We conclude, then, that the new analytic model in combination with the chosen control strategy provide an avenue for stability over increasing flight duration.

3.6.5 Lateral Control with Active Heading Tracking

We now demonstrate that the RoboBee can achieve full control authority (controllable thrust and body torques, with attitude and lateral control) using these methods. We validate full control authority of the vehicle with a demonstration of a maneuver wherein the RoboBee circles (laterally) about a center point, with its gaze fixed on that point;

$r_d := (\cos(2\pi/5t) + x_d, \sin(2\pi/5t) + y_d, z_d)$ which creates a circle with a period of five seconds, centered at $c := (x_d, y_d, z_d)$, and the heading vector is defined as $b_d^1 := c - r_d$. Results of this maneuver are shown in Fig. 3.10. We observe a position error (reported as mean $\pm$ standard deviation) of -0.68 ± 2.40 cm in the x direction, 1.78 ± 2.23 cm in the y direction, and 0.167 ± 0.561 cm in the z direction (corresponding to an effective center-point shift of 2.04 cm, with a circle radius expanded by 2.96 cm). This performance is comparable to the previous state-of-the-art for RoboBee control [58], which is notable given that the applied controller was not designed for FWMAVs, nor does it include adaptive elements. Additionally, [58] does not achieve active heading control; in this demonstration, the vehicle tracks its desired heading with an error of $-4.13 \pm 2.70^\circ$.

3.7 Discussion and Future Work

FWMAV modeling may complement control strategies both by capturing vehicle asymmetries and giving an analytic, physically intuitive understanding of its dynamics. We present a FWMAV dynamics model which allows analytical wingstroke and vehicle dynamics descriptions and closed-form nonlinear expressions describing force and torque generation via quasi-steady analysis, while respecting inherent vehicle asymmetries. We validate our model with experimental data used to characterize the vehicle, and demonstrate the utility of this system identification process in hovering and heading tracking via a geometric controller. In applying these methods, we demonstrate in-flight heading control of the RoboBee for the first time, and achieve full control authority with simultaneous attitude and lateral control. These modeling, system identification, and control processes provide marked advantages in control authority for this vehicle, and may be applied on a wider scale to other FWMAV designs. We anticipate that with the inclusion of an adaptive

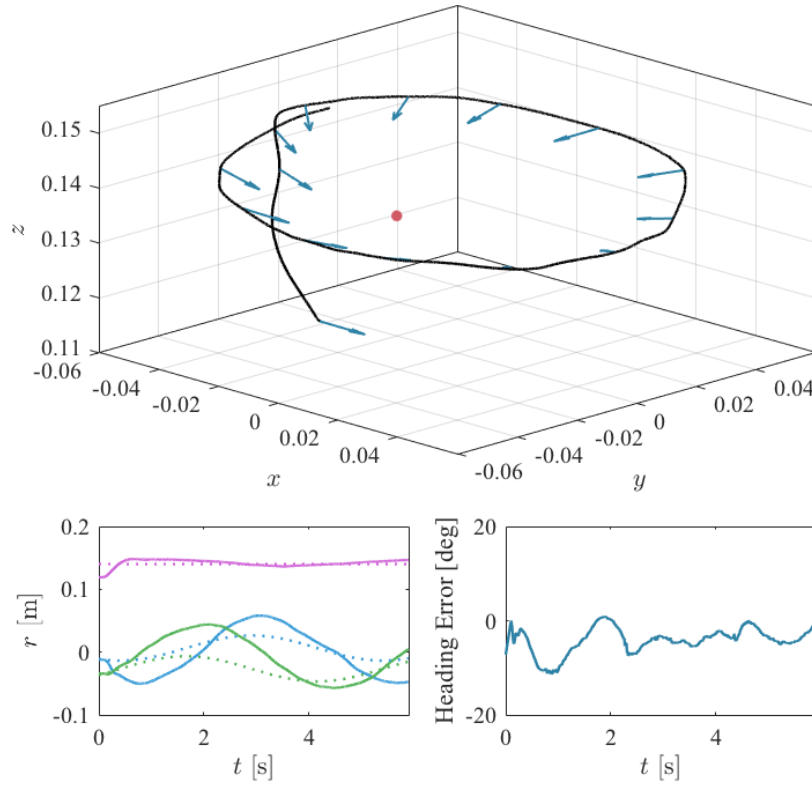


Figure 3.10: Controlled flight of the RoboBee circling and facing a center point in midair; (top) Three-dimensional vehicle trajectory in black, with the observed heading in blue and the desired center point in red; (bottom left) Desired (dotted) and observed position, with x in blue, y in green, and z in magenta; and (bottom right) heading angle error.

law, the controller in conjunction with our model and system identification process would further improve the tracking performance.

Chapter 4

Frequency-Modulated Control for Insect-Scale Flapping-Wing Vehicles

4.1 Introduction

Insects exhibit both amplitude and frequency modulation during wingstrokes in flight, a combination with potential advantages in FWMAV flight as we attempt to mimic the agility of biological fliers. However, frequency-modulated control is uncommon in insect-scale FWMAVs, and these vehicles have not yet demonstrated graceful, mid-wingstroke frequency transitions in flight. We propose a method to allow active frequency control as a primary control variable, and describe a method to achieve smooth frequency transitions in flight. We demonstrate that frequency, in combination with other flapping parameters, may be used to generate arbitrary, time-varying forces and torques, in a manner directly applicable to control. Additionally, we explore the advantages of frequency modulation in

FWMAV flight, applying a frequency-based method to control the Harvard RoboBee during hovering flight.

4.2 Motivation and Background

Considering the family of FWMAVs wherein each wing is independently controlled by a single actuator (as opposed to more complex systems [21], or vehicles whose wings are collectively controlled by one actuator [23, 32]), we may observe that oscillating actuators in FWMAVs present a unique opportunity to mimic insect flight, as they allow active control of both wing stroke amplitude and flapping frequency (a feature of insect flight [30]), in addition to wing stroke offset and phase. This opportunity is primarily found in piezoelectric actuators; however, Zhang et al. present a motor-based vehicle with the capacity to achieve frequency and amplitude modulation, with a single actuator for each wing [29]. (When motors are used without oscillating motion, instead limited to control of motor velocity, the combination of active frequency and amplitude control with a single actuator becomes infeasible.)

Frequency-modulated control for FWMAVs has largely been considered independently from amplitude and bias control. For example, He et al. proposes a model for forces and torques in FWMAV flight based on flapping frequency, but not amplitude [82]. However, incorporating active frequency and amplitude control may allow improvement towards the goal of insect-like agility, as biological insects use both frequency and amplitude control in flight.

The potential utility of independent frequency and amplitude modulation in FWMAV control is the subject of a small but growing body of research. Anderson et al. proposed a control strategy for piezoelectric-actuated FWMAVs, controlling bias, phase

shift, and harmonic flapping coefficients of the flapping signal [83], also positing that direct frequency-modulated control would be possible, albeit only near resonance, and with symmetric frequency between the vehicle’s wings. Simultaneous harmonic coefficient modulation and voltage control in FWMAV flight was also explored in [24], which provided a corresponding analytic model for resulting forces and torques. Oppenheimer et al. provide a controller for piezoelectric-actuated FWMAVs using frequency control (without active amplitude control, although pitch control is achieved by incorporating a wing bias), with wingstroke frequency changed once per wingbeat (as opposed to more continuous modulation) [84].

4.2.1 Frequency Modulation in Insect Flight

In exploring the potential advantages of incorporating active frequency modulation as a control method within flapping-wing flight, it is useful to consider how the combination of wingbeat frequency and amplitude modulation is used in biological insect flight.

Insects may increase either wingstroke amplitude or flapping frequency to augment lift, and typically vary the two parameters independently. Faruque et al. claim that since flapping-wing aerial vehicles (both biological and mechanical) are designed for optimal performance within a specific frequency range, we may consider frequency as a supplemental control parameter instead of as a primary control method [30]. (We note that Anderson et al. came to a similar conclusion when considering frequency modulation in control for FWMAVs [83], as discussed in Section 4.2).

The modulation of flapping frequency in flight is not necessarily subtle: Fruit flies (*Drosophila*) demonstrate frequency variation in closed-loop flight on the order of 15% [85], and hawkmoths (*Manduca sexta*) modulate their flapping frequency during maneuvers via synchronous muscles with significant modulation of frequency, on the order of 30% (and up

to 50%) [86].

The methods (both passive and active) of frequency modulation are also of interest in understanding frequency as a control variable. In practice, insects modulate flapping frequency by modulating nerve impulse frequencies [87]. In *Drosophila melanogaster*, asynchronous muscle firing rate corresponds directly to frequency changes [88]. Flapping frequency may also be heavily influenced by the ambient environment. Vogel et al. found that insect flapping frequency can vary inversely with air density, which may correspond to an effective variation in wing inertia with increasing air density [89]. Additionally, while it is commonly accepted that insects flap at resonance for efficiency, Ha et al. suggest that flapping frequency is independent of the insect's natural frequency, and is instead related to body mass and the area of the forewing [90].

Additionally, a variety of explanations have been observed and proposed for the use of frequency modulation in insect flight. Mosquitos (*Aedes aegypti*), for example, modulate fundamental flapping frequency and harmonics as a communication method, including in courtship [91], and hawkmoths (*Manduca sexta*) appear to use frequency modulation to aid in control in the presence of perturbations [86]. Pons et al. proposed that insects may not necessarily have one optimal resonant state, but instead, a set of optimal flight states and resonant points (corresponding, for example, to maximal energy efficiency or maximal flight performance). In this case, frequency changes in flight could be used to move between these optimal states [92]. Finally, in considering the mechanism for large frequency changes during insect flight, Nachtigall et al. posit that dramatic changes are only possible with changes in the resonant properties of the insect thorax [93].

4.3 Frequency-Modulated Control Methods

First, we will discuss the advantages of high- and low-frequency flight regimes in efficiency and control, and experimentally determine voltage requirements for maintaining lift at varying flapping frequencies. Next, we will present a method to apply continuous frequency transitions in oscillating, flapping-wing flight (based on [94], which applies graceful transitions to periodic walking), such that frequency transitions may occur throughout the wingstroke. Finally, we will create an inverse model (based on the FWMAV model in [24]) expressing frequency as a control input (along with actuator voltage offset, ratio of voltage amplitude at each wing, and the coefficient of the second harmonic of the flapping frequency signal) to predict flapping control parameters required to generate desired thrust and body torques for closed-loop flight.

4.3.1 Iso-lift Contours with Varying Flight Conditions

In typical flight for the smallest FWMAV (including [37] and [29]), wingstroke amplitude (along with bias and flapping sinusoid shape) are the primary control parameters, coupled with a distinct pre-determined flapping frequency based on characterization of the particular vehicle.

However, studies in both robotics [6, 24, 56] and biology [92, 86] suggest that active frequency modulation may pose distinct benefits for controlled flight. Specifically, while high-frequency flight (with frequencies close to the resonant point of the vehicle or insect) is advantageous in power efficiency, lower-frequency regimes have the potential to dramatically increase the controllability of the system. In the case of FWMAVs, for example, reducing the flapping frequency in closed-loop flight allows active heading control, whereas yaw torque is substantially filtered out at higher frequencies [6].

We therefore are interested in the existence of *iso-lift manifolds* [6]; that is,

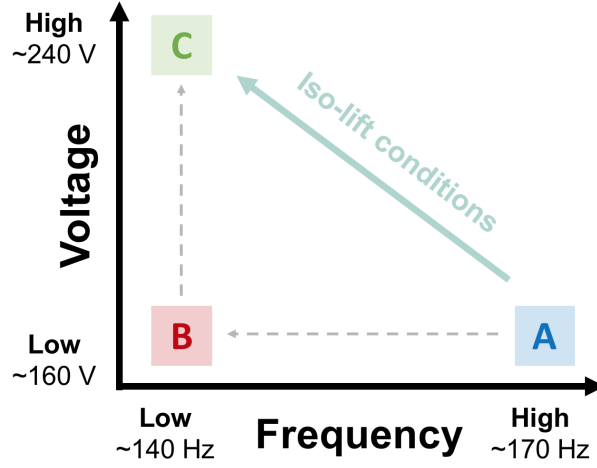


Figure 4.1: Visual explanation for iso-lift conditions within flapping-wing flight: (A) Typical flight occurs near resonance to maximize power efficiency, which comes at the expense of control authority; (B) reducing flapping frequency increases control authority, but without simultaneously increasing actuator voltage, a vehicle will exhibit substantially lower lift; so (C) when frequency is decreased, actuator voltage must increase to allow lift to be maintained. This leads to iso-lift conditions between (A) and (C), where (A) maximizes power efficiency, and (C) maximizes control authority.

flapping-wing flight conditions wherein lift can be maintained with different combinations of voltage and frequency. We visualize this concept in Fig. 4.1, showing roughly the expectation that as flapping frequency decreases (assuming that all frequencies are at or below resonance) and applied actuator voltage increases, lift may be maintained in flapping-wing flight. We note that in non-piezoelectric actuated FWMAVs capable of simultaneous frequency and amplitude modulation (e.g., [29]), we may assume wingstroke amplitude to be analogous to applied actuator voltage.

A primary limiting factor for flight along these iso-lift manifolds is the required voltage increase at the piezoelectric actuator in order to maintain lift at lower frequencies: for example, the bimorph piezoelectric actuators used in RoboBee flight are tested only up to 300 V [36], above which the performance and reliability are unknown. Therefore, we set a “soft” upper limit of approximately 240 V for controlled flight using these actuators, which

is substantially higher than voltage required for typical flight near the resonant frequency (less than 200 V) for a vehicle not carrying an additional payload.

While modeling in [6] suggested that low-frequency iso-lift conditions sufficient for heading control in the RoboBee (e.g., with frequency reduced sufficiently to mitigate passive yaw torque filtering) can maintain lift in typical flight conditions at higher frequencies without excessive actuator voltage, this has not yet been verified in free flight. We therefore conducted a series of pseudo-open-loop flights, with active control of roll and pitch torques, to maintain an upright vehicle with free heading, and thrust determined by the combination of actuator voltage amplitude and flapping frequency.

We define a single effective actuator voltage V_{eff} for FWMAV flight to simplify the definition of the iso-lift manifold, since the left and right wings are actuated independently, and roll torque is applied by modulating the relative voltage of the left and right wings, therefore impacting lift force [53]. (Flapping frequency is equal for both wings.) For time-varying left and right voltage amplitudes V_{amp}^L and V_{amp}^R , we define

$$V_{eff}^2 = (V_{amp}^L - V_{amp}^R)^2 + \left(\frac{V_{amp}^L + V_{amp}^R}{2} \right)^2. \quad (4.1)$$

Choosing a representative set of open-loop voltage (V_{eff}) and flapping frequency combinations, we conducted 36 flights with the chosen conditions (with active roll and pitch control, but no heading or position control). The state of the vehicle was recorded over the duration of each flight using a Vicon motion-capture arena (Vicon T040 System, Oxford UK), and thrust estimates were calculated based on the movement of the vehicle while in flight. A contour map of the resulting thrust for each frequency and voltage combination is shown in Fig. 4.2, with thrust reported as a multiple of vehicle body weight. (We also note that for an upright vehicle, thrust and lift forces are equivalent.)

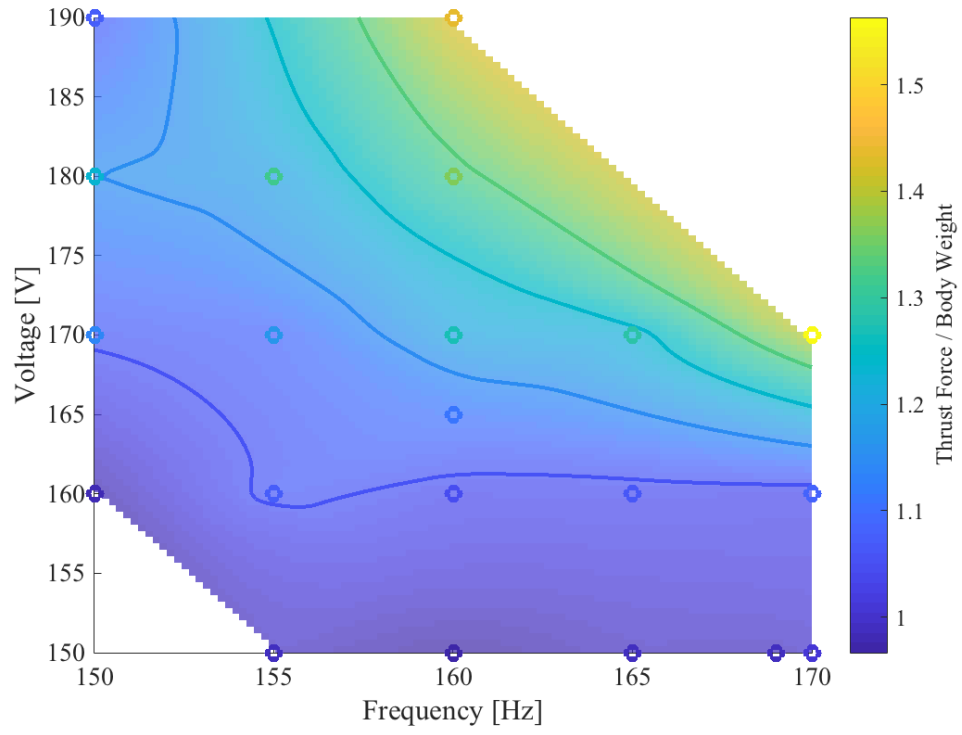


Figure 4.2: Iso-lift contour map depicting observed thrust for a set of frequency and actuator combinations for RoboBee free flight, with active roll and pitch control, and open-loop heading and thrust.

In Fig. 4.2, we observe a set of contours along the iso-lift manifold between 150 Hz and 170 Hz, demonstrating that for this range of flapping frequencies, thrust sufficient to lift the vehicle and allow room for variation within a control loop is possible at decreasing frequencies, with the lower bound at 150 Hz sufficient for observable and controllable yaw torque.

4.3.2 Smooth Frequency Transitions for Piezoelectric Actuators

While some of the smallest FWMAVs have the physical ability to mimic insects in the use of independent amplitude and frequency modulation [84, 29, 53], as of yet, amplitude modulation (with a set frequency, based on vehicle characteristics) is the most common strategy, as seen in [24, 29]. While Oppenheimer et al. present a frequency modulation-based control strategy [84], their method does not allow frequency modulation mid-wingstroke (instead commanding one frequency shift per wingbeat). However, observation of *Manduca sexta* has demonstrated that, in the face of perturbations, insects can adjust wingbeat frequency even during a single wingstroke [86]. Therefore, to gain the largest control advantages from frequency-modulated control and to allow continuity of the control signal, we seek a method for smooth transitions between frequencies, regardless of position within the wingstroke.

Based on the strategies presented in [94], we present a method for graceful frequency transitions for active frequency control in FWMAV flight with oscillating actuators. To allow these graceful transitions, instead of directly controlling shifts between commanded frequency setpoints, we instead focus on modulating the derivative of the commanded frequency, in order to maintain smoothness throughout the transition.

For example, consider the case of a frequency shift between arbitrary frequency commands: If commanded wingstroke angle is defined, arbitrarily, as $\Psi(t) = A \sin(\omega t)$ (for

frequency ω , time t , and amplitude A), an abrupt command between consecutive time steps t_1 and t_2 such that $\Psi(t_1) = A \sin(\omega_1 t_1)$ and $\Psi(t_2) = A \sin(\omega_2 t_2)$ will lead to a discontinuity in the resulting flapping sinusoid command $\Psi(t)$. Considering the dynamics of FWMAVs, this discontinuity may be amplified by any phase differences in the actuator and transmission frequency response (shown in Fig. 4.3) between ω_1 and ω_2 . Therefore, we choose to define graceful transitions between commanded frequencies, as follows.

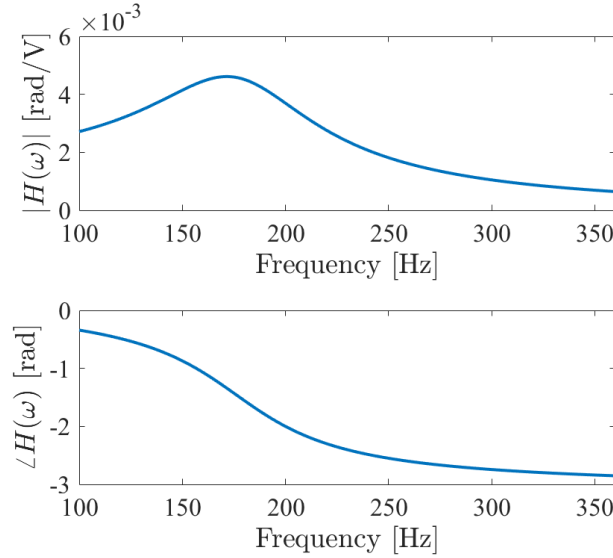


Figure 4.3: Frequency response in magnitude and phase, resulting from actuator and transmission dynamics of the RoboBee, based on the transfer function defined in [6].

We consider a frequency shift of time duration dt (where dt can be near-instantaneous, or slower than the period of the oscillating signal). We command a frequency shift centered at time $t = t_{shift}$. We define the function $\sigma(t)$ to describe the transition between frequencies ω_1 and ω_0 (effectively, a function which begins at 0, and after duration dt has reached 1). While $\sigma(t)$ can be defined arbitrarily, we choose

$$\sigma(t) = \frac{\tanh(a(t - t_{shift})) + 1}{2}, \quad (4.2)$$

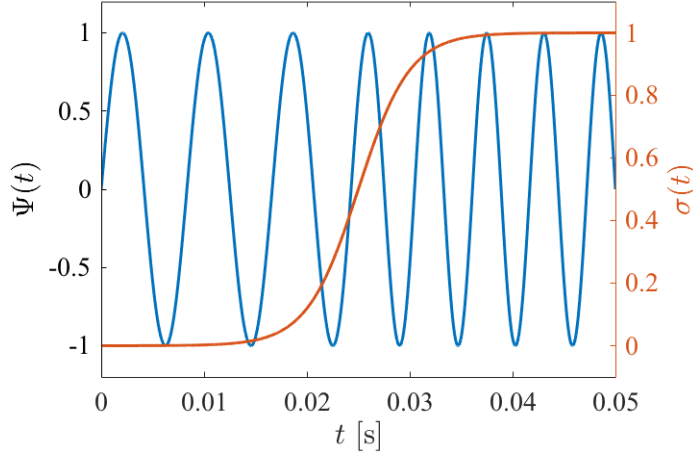


Figure 4.4: Demonstration of a graceful transition between arbitrary frequencies, 120 Hz at $t = 0$ s and 180 Hz at $t = 0.05$ s, with resulting flapping sinusoid command $\Psi(t)$ shown in blue, and corresponding smooth transition function $\sigma(t)$ shown in red.

where a defines the speed of the frequency shift. Defining $\Omega(t) = \omega t$, we set

$$\dot{\Omega} = \omega_1 \sigma(t) + \omega_0 (1 - \sigma(t)). \quad (4.3)$$

Then, our sinusoid command at time t becomes

$$\Psi(t) = A \sin(2\pi\Omega(t)). \quad (4.4)$$

We demonstrate this graceful frequency transition strategy with a set of arbitrary commanded frequencies between 110 Hz and 190 Hz over a sinusoidal signal of duration 0.1 s, shown in Fig. 4.5. We observe that the flapping sinusoid maintains smoothness throughout all commanded frequency changes (with a chosen to result in near-instantaneous step commands).

In order to ensure that the graceful frequency transition method proposed herein is suitable for use in FWMVs with piezoelectric actuators, verifying that (1) the commanded

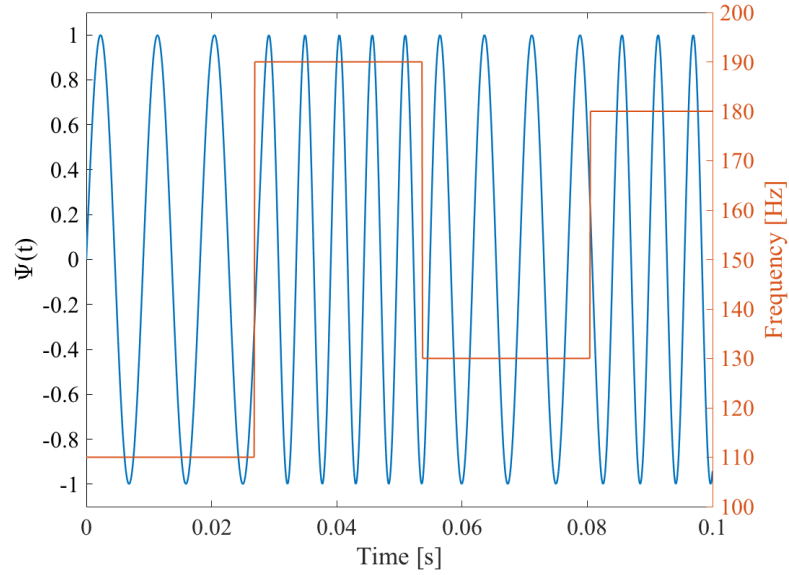


Figure 4.5: Demonstration of frequency modulation of an oscillating signal with graceful transitions. Frequency commands are shown in red, and the resulting continuous actuator drive signal is shown in blue. We note that the frequency shift commands occur at arbitrary points during the signal (i.e., not at the extrema or middle), and that the frequency shifts are very abrupt relative to those anticipated within controlled flight, to demonstrate smooth transitions even in the presence of rapid frequency shifts.

frequency shift may be observed as anticipated during flapping, and (2) rapid frequency transitions do not damage the piezoelectric actuators, we applied frequency transitions on a static vehicle to observe the resulting wing motions. The results of a representative experimental frequency shift are shown in Fig. 4.6

We applied a transition between 150 Hz and 170 Hz (within the range of the iso-lift contour tests shown in Fig. 4.2). Flapping was recorded via high-speed camera, with wing angle data taken from the resulting video. We then fit a sinusoidal signal of matching frequency (including the frequency transition) to the measured wing angle trajectory.

Based on the fitted trajectory, we observed that the initial and final flapping frequencies of the observed signal matched the frequency commands sent to the vehicle; however, the frequency transition occurred over a longer duration than commanded. Recall that parameter a in $\sigma(t)$ influences frequency transition speed: while the commanded frequency transition applied $a = 500$, the observed transition on the physical vehicle corresponded to $a \approx 57.3$ (indicating a substantially slower rate of frequency change). Upon examination of the commanded signal at different points within the control loop and commands to hardware, we posit that the majority of the transition speed reduction is due to software-based filtering of the control signal (bringing a from 500 to approximately 96.2), while the remaining transition speed reduction is hardware-based (whether due to attenuation from actuator dynamics or amplifier slew rate limiters).

Additionally, we observe a substantial phase shift between beginning and end of the flapping signal (before and after the transition), on the order of 45° . Finally, we observe that while applied actuator voltage was constant throughout the signal, the frequency increase resulted in a substantial wingstroke amplitude increase, as predicted by the actuator and transmission dynamics shown in Fig. 4.3.

In a control loop, we may implement this graceful frequency transition by choosing

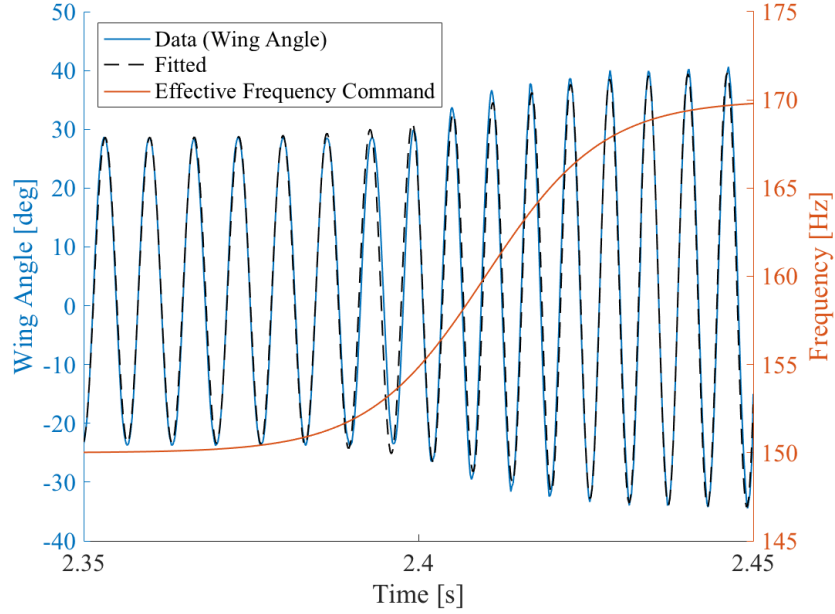


Figure 4.6: Static demonstration of graceful flapping frequency transition between 150 Hz and 170 Hz, with observed wing angle response shown in blue, corresponding fitted signal based on angle data shown in dashed black, and frequency shown in red.

transition speed parameter a (according to a preference between fast or slow frequency changes; insects demonstrate frequency changes over long and short timescales depending on flight conditions and perturbations [86]), and calculating the corresponding shift duration t_d (which may be lengthened to include room for error in effective a). Then, when a frequency shift is initiated by the controller at time t , we set $t_{shift} = t + \frac{t_d}{2}$, and update the frequency at each control loop iteration accordingly.

4.3.3 Inverse Model for Frequency-Modulated Control

In order to use frequency modulation during controlled flight, we present an “inverse” model to predict control inputs, including flapping frequency, which will result in desired thrust force and roll, pitch, and yaw torques. The thrust and torques can then be directly commanded using any number of flight controllers, as demonstrated in [24] which

applied this method with amplitude-based flapping signals to control a FWMAV using a geometric controller designed for quadrotors.

Specifically, we aim to determine thrust and torques in terms of flapping signal parameters u , as follows:

$$u = \begin{bmatrix} \Lambda & V_{off} & \omega & a_2 \end{bmatrix}. \quad (4.5)$$

Given unique voltage amplitudes V_L and V_R at the left and right wings, respectively, we define $\Lambda = \frac{V_R}{V_L}$. We have two options in using Λ to define V_L and V_R : First, we may hold a set center amplitude $\bar{V}$, and define $V_L = \bar{V} + \tilde{V}$ and $V_R = \bar{V} - \tilde{V}$. Then, $\Lambda = \frac{V_R}{V_L} = \frac{\bar{V} - \tilde{V}}{\bar{V} + \tilde{V}}$, and similarly, $\tilde{V} = \bar{V} \left(\frac{1+\Lambda}{1-\Lambda} \right)$. Second, we may hold constant V_L and simply define $V_R = \Lambda V_L$. While the latter option is mathematically simpler, we anticipate that we can reduce coupling between roll and thrust by the former. V_{off} denotes voltage offset of the sinusoidal flapping signal (symmetric between the left and right wings), and ω denotes flapping frequency. As in [24], we will define the flapping sinusoid using a combination of sinusoids at the fundamental flapping frequency ω and its second and third harmonics, whose amplitude coefficients are denoted by a_1 , a_2 , and a_3 , respectively; only a_2 is directly controlled, whereas a_1 and a_3 are held constant.

We note that $\Lambda = \frac{V_R}{V_L}$ such that when $V = V_R$ we have

$$\eta V_L^2 \pm V_R^2 = V^2(\eta \pm \Lambda^2). \quad (4.6)$$

Then, modifying the model presented in [24], assuming symmetric V_{off} between the left and right wings, we define the thrust force F_T , roll torque τ_R , pitch torque τ_P , and

yaw torque τ_Y as

$$F_T = \frac{1}{4}\rho\beta C_L\omega^2 V^2(\eta + \Lambda^2)\zeta \quad (4.7)$$

$$\tau_R = r_{cp}\rho\beta C_L\omega^2 V^2(\eta - \Lambda^2)\zeta \quad (4.8)$$

$$\tau_P = r_{cp}\rho\beta C_L |H(0)| \text{sgn} \angle H(0) \omega^2 (V_{off} + \nu) V^2(\eta + \Lambda^2)\zeta \quad (4.9)$$

$$\begin{aligned} \tau_Y = \frac{4}{3\pi}\rho\beta C_D\omega^2 a_1(a_2 + \mu) |H(\omega)| |H(2\omega)| \\ \times \cos(2\angle H(\omega) - \angle H(2\omega)) V^2(\eta + \Lambda^2), \end{aligned} \quad (4.10)$$

where

$$\zeta = a_1^2 |H(\omega)|^2 + 4a_2^2 |H(2\omega)|^2. \quad (4.11)$$

Here, ρ is ambient air density; β corresponds to the wing shape; C_L and C_D are lift and drag coefficients, respectively (held constant in this model, though in reality they vary with angle of attack throughout the wing stroke); r_{cp} is the shoulder width of the vehicle; and $H(\omega)$ is the transfer function corresponding to actuator and transmission dynamics, defined in [6]. Additionally, parameters η , ν , and μ correspond to physical anomalies and imperfections in vehicle design, determined during vehicle characterization [24].

Based on this force and torque model, we seek analytic solutions for flapping parameters u in terms of desired forces and torques. However, this solution is nontrivial and intractably complicated. Therefore, we seek reasonable assumptions about the model to allow a fully analytic solution for u in terms of F_T , τ_R , τ_P , and τ_Y .

First, we examine the term ζ : Recall that the amplitude of the fundamental frequency (ω) component of the flapping sinusoid, is a_1 , and the amplitude of the second harmonic is (at 2ω) is a_2 . (Fundamental frequency ω is typically slightly below the resonant frequency of the FWMAV; the second harmonic at 2ω is far above the vehicle's resonant point.)

In typical flight conditions, $a_1 > a_2$, and $|H(\omega)| > |H(2\omega)|$ (which we can also observe in Fig. 4.3) [6]. Since these terms are squared, we presume that $a_1^2 |H(\omega)|^2 \gg 4a_2^2 |H(2\omega)|^2$. If we neglect $4a_2^2 |H(2\omega)|^2$ within ζ , resulting value for ζ is nearly identical to when the term is included. Therefore, we redefine ζ as $\zeta = a_1^2 |H(\omega)|^2$, and will maintain this throughout the rest of the model inversion.

Next, we observe that the complexity in inverting the model to find an analytic solution for u comes largely from the frequency response to the transmission and actuator dynamics, according to the transfer function $H(\omega) = \frac{A_H}{m_{eq}(\omega i)^2 + b_{eq}(\omega i) + k_{eq}}$ [6] (for effective mass, damping, and spring coefficients m_{eq} , b_{eq} , and k_{eq} , and amplitude A_H). We can directly find the magnitude and phase of $H(\omega)$; that is,

$$|H(\omega)| = \frac{A_H}{\sqrt{(k_{eq} + \omega^2 m_{eq})^2 + (b_{eq} \omega)^2}} \quad (4.12)$$

$$\angle H(\omega) = -\arctan\left(\frac{b_{eq} \omega}{k_{eq} - \omega^2 m_{eq}}\right). \quad (4.13)$$

Using these expressions, we define

$$P(\omega) = |H(\omega)| |H(2\omega)| \cos(2\angle H(\omega) - \angle H(2\omega)) \quad (4.14)$$

such that

$$\tau_Y = \frac{4}{3\pi} \rho \beta C_D \omega^2 a_1 (a_2 + \mu) P(\omega) (\Lambda^2 + 1) \quad (4.15)$$

Since using the exact transfer function leads to an intractable solution for u , we use linear approximations for the frequency response $H(\omega)$ near flapping conditions, directly for magnitude $|H(\omega)|$ and indirectly for $P(\omega)$.

We define approximations for $|H(\omega)|$ and $|H(2\omega)|$, respectively:

$$\hat{H}_1(\omega) = m_H^{\omega^*}(\omega - \omega^*) + |H(\omega^*)| \quad (4.16)$$

$$\hat{H}_2(\omega) = 2m_H^{2\omega^*}(\omega - \omega^*) + |H(2\omega^*)|, \quad (4.17)$$

where the magnitudes of the frequency responses for ω and 2ω are linearized about a central frequency ω^* , where $\frac{d}{d\omega}|H(\omega^*)| = m_H^{\omega^*}$ and $\frac{d}{d\omega}|H(2\omega^*)| = m_H^{2\omega^*}$.

Then, we define

$$\hat{\zeta}(\omega) = a_1^2 \hat{H}_1^2(\omega). \quad (4.18)$$

Similarly, we consider a lumped estimate for $P(\omega)$ as follows:

$$\hat{P}(\omega) = m_P^{\omega^*}(\omega - \omega^*) + P(\omega^*). \quad (4.19)$$

Again, $P(\omega)$ is linearized about a central frequency ω^* , where $\frac{d}{d\omega}P(\omega^*) = m_P^{\omega^*}$.

Then, we can rephrase our force and torque model as

$$F_T = \frac{1}{4}\rho\beta C_L\omega^2(V^2(\eta + \Lambda^2))\hat{\zeta}(\omega) \quad (4.20)$$

$$\tau_R = r_{cp}\rho\beta C_L\omega^2(V^2(\eta - \Lambda^2))\hat{\zeta}(\omega) \quad (4.21)$$

$$\tau_P = r_{cp}\rho\beta C_L|H(0)|\text{sgn}\angle H(0)\omega^2(V_{off} + \nu)(V^2(\eta + \Lambda^2))\hat{\zeta}(\omega) \quad (4.22)$$

$$\tau_Y = \frac{4}{3\pi}\rho\beta C_D\omega^2 a_1(a_2 + \mu)\hat{P}(\omega)(V^2(\eta + \Lambda^2)). \quad (4.23)$$

We note that the coefficient a_2 appears only in the expression for yaw torque, τ_Y . Therefore, we can solve the system of three equations, Eq. (4.20-4.22) for the other

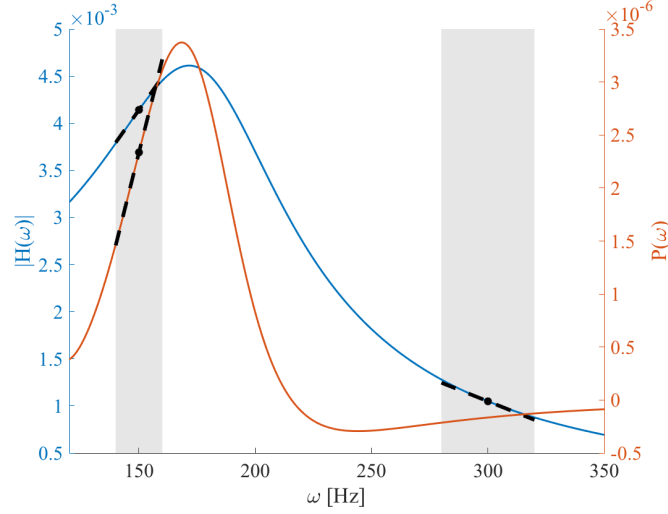


Figure 4.7: Linearization of the frequency response (in black dashed lines, centered at ω^* and $2\omega^*$) at for $P(\omega)$ at ω^* (red), and for $|H(\omega)|$ at ω^* and $2\omega^*$ (blue). Typical flapping flight operating frequencies are shown in the left gray shaded region; the width of the right shaded region is double the range of typical frequencies, due to the second harmonic response at 2ω .

components of u (Λ , V_{off} , and ω) as follows:

$$\Lambda = \sqrt{\frac{4r_{cp}F_T + \tau_R}{4r_{cp}F_T - \tau_R}} \quad (4.24)$$

$$V_{off} = \frac{\tau_P}{4r_{cp}|H(0)|\text{sgn}(|H(0)|)F_T} - \nu \quad (4.25)$$

$$\omega = \frac{m_H^{\omega^*}\omega^* - |H(\omega^*)| - \sqrt{(|H(\omega^*)| - m_H^{\omega^*})^2 + \frac{2\sqrt{2}m_H^{\omega^*}}{a_1} \sqrt{\frac{4r_{cp}F_T - \tau_R}{r_{cp}\rho\beta C_L\eta V^2}}}}{2m_H^{\omega^*}}. \quad (4.26)$$

We note that solving this system of equations actually yields three solutions for Λ , V_{off} , and ω ; however, we choose the only solution of the three whose ω corresponds to flightworthy flapping conditions. (Other solutions provided solutions for ω with flapping frequency too slow to fly; on the order of 10 Hz, or negative ω .)

Now, with known Λ , V_{off} , and ω , we can complete our solution for u by using Eq.

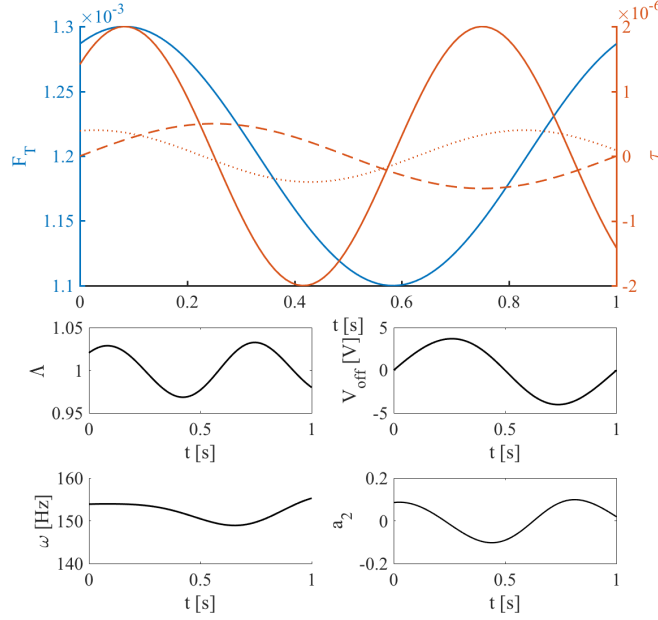


Figure 4.8: Flapping parameters $u = [\Lambda \ V_{off} \ \omega \ a_2]$ (bottom) to yield desired varying thrust (F_T in blue) and torques (solid τ_R , dashed τ_P , dotted τ_Y (top); in red). Desired forces and torques are shown in blue and red, respectively (top).

(4.23) to directly solve for a_2 :

$$a_2 = \frac{3\pi\tau_Y}{4\rho\beta C_D \omega^2 a_1 \hat{P}(\omega)(V^2(\eta + \Lambda^2))} - \mu. \quad (4.27)$$

Given the expressions for $u = [\Lambda \ V_{off} \ \omega \ a_2]$ now in terms of desired forces and torques, we provide an example of direct calculation of flapping parameters required to generate arbitrary forces and torques, shown in Fig. 4.8.

4.3.4 Control Implementation

With a model inversion to provide flapping parameters u to generate desired forces and torques, assuming that the commanded forces and torques are within the operating range of the vehicle, we may apply an arbitrary controller to control the FWMAV in flight.

Non-control parameters (including ambient conditions such as ρ , or vehicle parameters such as β and the effective mass, spring, and damping coefficients of the transfer function $H(\omega)$) may be estimated using an open-loop flight characterization process, as in [24].

4.4 Experiments

We applied the frequency-based model and control strategy described above in hovering flight with the RoboBee. Per Section 4.3.3, we chose to define V_L and V_R with Λ such that V_L and V_R varied around a set $\bar{V}$ (in this case, $\bar{V} = 200$ V). The results of this implementation and flight are shown in Fig. 4.9.

We observe that frequency changes (primarily to regulate thrust) within the flight-worthy regime of about 145-162 Hz. Given that $\bar{V}$ is fixed, frequency is primarily responsible for regulating z position error, while frequency reduction also allows greater heading control authority. We observe that the simultaneous decrease in ω and increase in a_2 work together to reduce heading error significantly over the course of the flight.

We also observe that during the flight reported in Fig. 4.9, left-right voltage ratio parameter $\Lambda = \frac{V_R}{V_L}$ lies consistently in the range $\Lambda > 1$, indicating that the left wing produces greater lift than the right wing (given the same voltage amplitude and frequency), so a higher voltage is applied to the right wing to reduce asymmetry and mitigate roll torque bias.

Additionally, we may observe the different timescales of control reactions in ω and a_2 : a_2 increases steadily before varying about a value of approximately 0.115; ω decreases over the flight, though demonstrates a bounce-like decrease as opposed to a smooth transition. We anticipate that decrease in ω largely serves to temper altitude (as this variable is primarily responsible, in this case, for lift); however, as the magnitude a_2 increases, it

results in an increase to the overall voltage sinusoid amplitude sent to each wing (given constant a_1). Therefore, it is likely here that ω is responding to the challenge of simultaneously mitigating altitude error and the lift increase due to the a_2 yaw torque response.

While running additional experimental flights, we observed that there is a strong dependence between the magnitude of the yaw-inducing a_2 coefficient and the stability of the flight. We hypothesize that this is due to the fact that changes in a_2 can significantly affect the vehicle's thrust, which is mitigated during amplitude-based control strategies by the voltage amplitude at each wing. However, since the mean amplitude $\bar{V}$ is constant in this case, this mitigation is not present. Therefore, greater attention should be paid to normalization of the sinusoid shape, such that changes in a_2 do not significantly affect the thrust. (We also note that the vehicle used for these particular experiment was subject to actuator damage; while this damage was accounted for during the vehicle characterization process, it may have exacerbated any dependencies between various control parameters.)

4.5 Discussion and Future Work

We present a strategy for frequency-modulated control with graceful transitions, via designing direct expressions for desired control forces and torques in terms of flapping control parameters, including frequency. We also note that the particular implementation suggested herein applies partial amplitude modulation, in addition to frequency modulation: the parameter α modulates the relative voltage amplitude of the left and right wings (leaving one side constant, or varying both around a set center amplitude). We validate this method in hovering flight, demonstrating stability using the frequency-based control strategy.

By implementing smooth frequency modulation within controlled flight, we are able to observe stability in position and heading, in addition to a partnership between

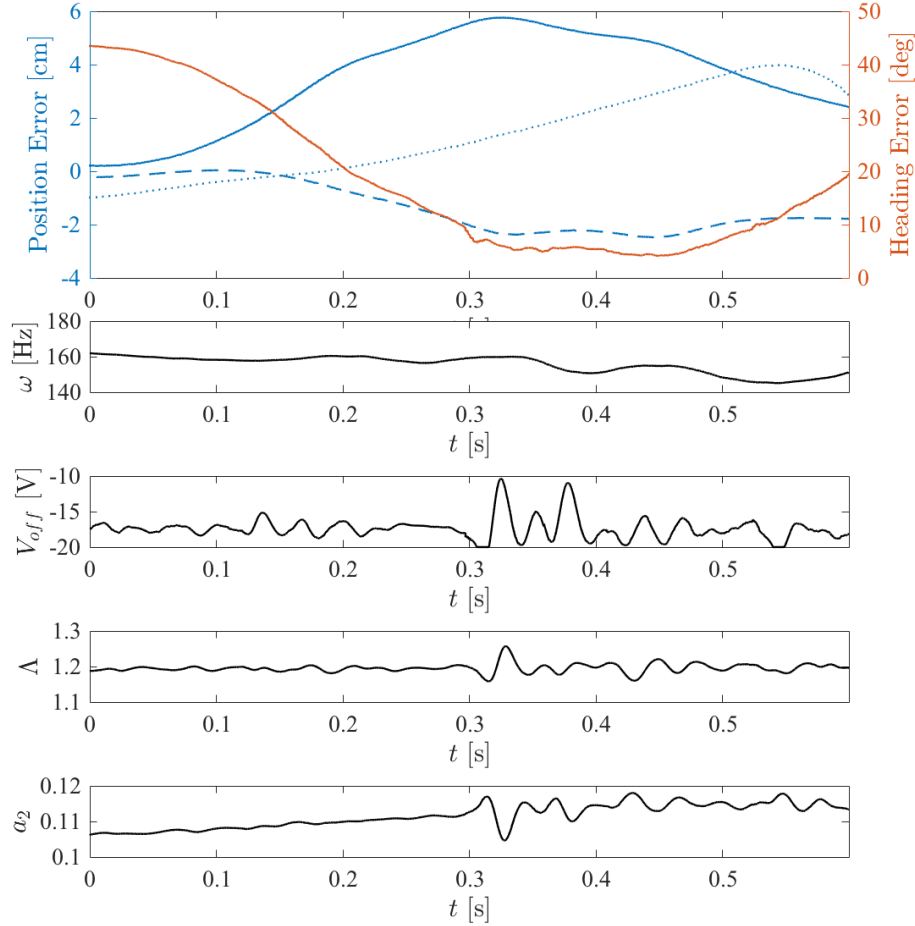


Figure 4.9: Position and heading error for a hovering flight using frequency modulation. (top) Blue solid, dashed, and dotted lines report position error in x , y , and z , and red solid line reports heading error. (bottom) Four plots showing the flapping sinusoid inputs ω , V_{off} , Λ , and a_2 to generate desired control torques, with constant $V_{amp} = 200$ V.

frequency and flapping sinusoid shape (governed by a_2) to maintain altitude and heading control. We do, however, notice that without full amplitude control, this coupling results in thrust variation with changes in yaw torque; this affects the vehicle’s ability to generate arbitrary thrust and torque combinations. During future flights, we anticipate observing certain distinct advantages of higher and lower frequency regimes in flight (such as increased efficiency at higher frequencies, or increased performance in maneuvers at lower frequencies), as suggested by Pons et al. in studying insect flight [92].

Introducing graceful frequency modulation in flight, in collaboration with existing amplitude modulation strategies, allows advancement in bio-inspired flight control strategies. Fields of research in insect flight dynamics (especially related to frequency modulation) and FWMAV control are progressing simultaneously, allowing exciting connections between biology and robotics as both provide insights and inspiration in tandem.

Chapter 5

Conclusions and Future Work

5.1 Introduction

With a fully controllable vehicle coupled with advances in modeling and expansion of the control space, we may anticipate a major next step in control for the RoboBee: full control autonomy.

In the context of FWMAVs, we define *control autonomy* as the ability to execute a control strategy using onboard hardware, within the extreme payload and power restrictions for the vehicle. A close partner to this challenge is the pursuit of *sensor autonomy*, or the vehicle's ability to achieve sufficient proprioceptive and exteroceptive information for flight via an onboard sensing package (as opposed to external motion capture cameras, or other offboard information).

Therefore, as we conclude discussion of recent advances in modeling and control of the RoboBee, we shift focus to look ahead towards a control-autonomous and sensor-autonomous vehicle.

5.2 Approaching Control Autonomy

As the RoboBee approaches control autonomy (complementing current research in power and sensor autonomy), controller design must encourage efficiency such that the resulting solutions are suitable for control-autonomous flight. As a preemptive measure, a set of metrics should be defined to guide controller design and evaluate candidate controllers for suitability for autonomy.

Controller efficiency will be considered both in terms of power (mechanical and electrical) and computation. Within power efficiency, we will consider both efficiency of wing kinematics and of vehicle trajectories; within computational efficiency, we will consider the suitability of a controller for implementation on a microcontroller for control autonomy.

5.2.1 Computational Efficiency

In designing control strategies appropriate for control-autonomous flight with on-board computation, we must heavily consider the computational efficiency of the method, in addition to speed and latency requirements to maintain stability. De et al. achieved control of the RoboBee via a computationally-efficient modular controller, demonstrating expansion of control considerations from strictly stability and flight maneuver performance to include feasibility of use with onboard computational hardware in flight [80].

Additionally, in considering the interplay between hardware requirements and control strategy within severe computational restrictions (i.e., using hardware within the payload restrictions of the smallest FWMVs), strategies must consider speed and latency, both in defining minimum requirements for control, and in computational requirements to allow sufficient performance with flight-worthy hardware. Three specific variables of note in this case are latency (especially related to sensor feedback), speed of the control loop, and feedback rate for proprioception and exteroception. Future work should examine the

efficacy of various control strategies in the face of speed and latency restrictions, in addition to the effect of control strategy choice on each factor.

Implementation of onboard sensing (proprioceptive and exteroceptive) will likely require the largest computational expense (apart from the controllers themselves). This includes both state estimation and external sensing of the environment. Exteroceptive sensing will vary significantly in computational expense based on the chosen sensor package; for example, proximity sensors would require significantly lower computational requirements than vision-based sensors, though vision-based sensors would provide greater information about the environment, and may assist heavily in proprioception as well. Sensors will vary in filtering requirements, which will be an additional computational expense. Sensor expectations must therefore be considered in tandem with controller design, as not only does the sensor choice affect state estimation used by the controller, but the controller and sensors must compete for computational and power resources.

Finally, designing controllers with respect to efficiency will not only simply allow these controllers to operate onboard the RoboBee, but will increase the ability of the vehicle to carry the hardware required for control. If controllers are consistently designed to maximize lift production with respect to input power (in addition to maintaining stable hovering and trajectory following), the RoboBee is able to maximize the use of its own payload. Therefore, by maintaining consistent focus on controller efficiency, we create a symbiotic relationship between the power, sensing, and control elements of the RoboBee, encouraging the system to perform at its full potential.

5.2.2 Power Efficiency

Controller design for an autonomous system must also consider the matter of power efficiency. In defining power efficiency metrics for the RoboBee, we may constrain

the scope of the problem to consider efficiency with respect to lift force production. That is, a controller is considered more efficient if it achieves greater thrust from the same input power.

The input power (or the power supplied to the actuators) is a function of actuator capacitance C_{act} , as well as the voltage amplitude and fundamental frequency of the flapping signal to the actuators V_{pp} and f , respectively; Equation 5.1 follows the model used by Jafferis et al. [36]:

$$P_{act} = \frac{1}{2} C_{act} V_{pp}^2 f \quad (5.1)$$

Output power is here considered to be the rate of work in the direction of thrust. For simplification of the analysis, we will not consider power expended towards body torques (though this could be incorporated into a future model), and will use the time-averaged model for lift force initially presented in [53] and expanded in [6].

The time-averaged lift model from [6] (modified to consider individual voltage amplitudes for the left- and right-wing flapping signals) is provided in Equations 5.2-5.4.

$$\bar{F}_T = \bar{F}_{L,left} + \bar{F}_{L,right} \quad (5.2)$$

$$= \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} F_{L,left} dt + \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} F_{L,right} dt \quad (5.3)$$

$$= \left[\frac{1}{4} \rho \beta C_L \omega^2 \sum_{n=1}^3 n^2 (a_n |H(n\omega)|)^2 \right] (V_{amp,L}^2 + V_{amp,R}^2) \quad (5.4)$$

The time-averaged (per wingstroke) work in the direction of thrust is $W_T = \int \bar{F}_T \cdot dz'$ (where z' is in the direction of the thrust axis, as shown in Figure 5.1), or $W_T = \bar{F}_T \Delta z'$, where $\Delta z'$ is the displacement in the direction of thrust (though we note that the direction of thrust changes with the vehicle orientation during flight). Then, to find power $P_T = \frac{dW_T}{dt}$,

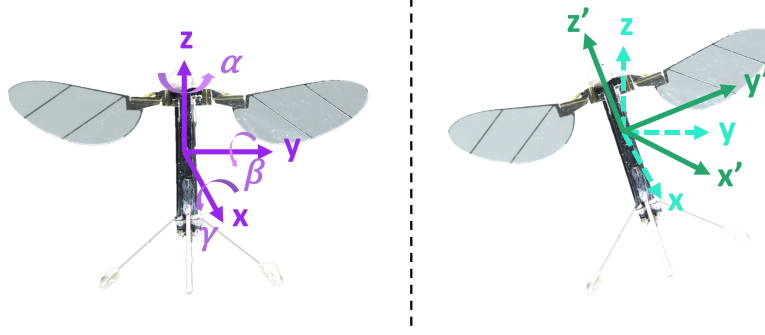


Figure 5.1: RoboBee axes representation; (a) position (x, y, z) and orientation (α, β, γ) in roll, pitch, and yaw, respectively; (b) global reference basis (x, y, z) and RoboBee frame basis (x', y', z') .

we note that $\bar{F}_T$ is independent of time, so we have

$$P_T = \bar{F}_T \cdot v_{z'}, \quad (5.5)$$

where $v_{z'}$ is the velocity in the direction of thrust.

Using P_{act} (input power) and P_T (output power) from Equations 5.1 and 5.5, respectively, we may define power efficiency η as

$$\eta = \frac{P_T}{P_{act}}. \quad (5.6)$$

We largely analyze hovering when considering the performance of the RoboBee; in this case, we will consider efficiency as

$$\eta^* = \frac{\bar{F}_T}{P_{act}}. \quad (5.7)$$

This representation is more applicable to hovering and specific analysis of flapping kinematics, while the former (Equation 5.6) is more relevant to maneuvers.

5.2.3 Flapping Kinematics

For any task, the RoboBee should produce necessary control authority (thrust force and body torques) to achieve a given maneuver, while maximizing efficiency (η^* , in Equation 5.7) of its flapping kinematics; that is, applying flapping signal parameters – flapping frequency, voltages, and sinusoid shape (i.e., harmonic coefficients) – for minimal power expenditure.

We note also that any set of control parameters should also lie within an acceptable range to maximize actuator longevity. This requires fundamental flapping frequency at or below resonance, and voltage amplitudes below 250 V (though ideally below 200 V).

Recent work in yaw torque authority generation [6] provides an example of this efficiency optimization problem: yaw torque flapping signals are filtered by high (near-resonant) flapping frequencies; flapping at a lower frequency increases yaw torque authority, but reduces lift. For example, a flapping sinusoid with parameters $\omega = 150 \cdot 2\pi$ rad/s, $V_{amp} = 180$ V, and harmonic coefficients – to govern the sinusoid shape – (1.09, 0, 0.099) produces the same thrust as a flapping sinusoid with $\omega = 140 \cdot 2\pi$ rad/s, $V_{amp} = 242.8$ V, and harmonic coefficients (0.934, 0.216, 0.047); both produce 1.2 mN of thrust. The former requires 24.5 mW of power, and the latter requires 41.7 mW; however, yaw torque is almost entirely attenuated at the lower-power conditions.

It follows that the effective hovering efficiency (η^*) required by these sinusoidal signals are 0.049 N/W and 0.029 N/W, respectively. We expect that during a maneuver – especially an aggressive maneuver, characterized by high force and torque requirements – the RoboBee will require periods of low-efficiency flight, though it should operate in these domains as infrequently as possible. We will therefore set two metrics for power-efficient wing-flapping: a time-averaged and an instantaneous minimum efficiency. Since thus far, the low-frequency flapping required for yaw torque is the least efficient regime for RoboBee

flight, this will set our efficiency limit. We set the instantaneous flapping power efficiency recommendation, then, at $\eta^* \geq 0.3$ N/W. We also set a time-averaged recommendation for efficiency η^* at $\bar{\eta}^* \geq 0.4$ N/W over the course of a maneuver.

Finally, we note that while Jafferis et al. suggested the power model in Equation 5.1, the authors also suggested that actual power requirements may be substantially higher than those suggested by the model. Therefore, we recommend using this metric as a guideline for comparison of power requirements between control strategies, rather than as an absolute expectation for power requirements during autonomous flight.

5.2.4 Flight Path

As the RoboBee approaches aggressive maneuvers, trajectory optimization (for rapid and precise execution of a maneuver, in addition to energy efficiency) will be an ongoing effort. From a high-level perspective, this problem may be approached from the efficiency perspective by analyzing the power efficiency (using η in Equation 5.6) of distinct guiding strategies (e.g., bio-inspired path planning, or choosing the most direct path).

An example is provided in Figure 5.2: If a RoboBee attempts to navigate through a set of suspended rings, general strategies include selection of the smoothest path, a bio-inspired path, or a parabolic path. While the smooth path may be more direct, it would require the RoboBee to maintain a significant orientation deviation (from the upright position) throughout nearly the entire maneuver; bio-inspired or parabolic paths may allow more power-efficient navigation.

Efficient path-planning for the RoboBee is an open research effort which will be explored in greater detail as the RoboBee approaches increasingly aggressive maneuvers.

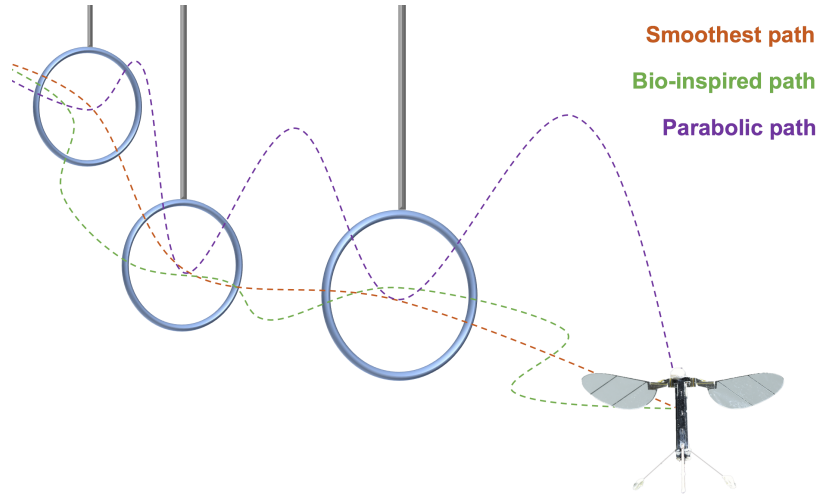


Figure 5.2: A selection of high-level paths possible for a given task (navigation through rings), including the smoothest path, a theoretical bio-inspired path, and a parabolic path; each considered for comparison of flight path efficiency. We note that these cases are chosen to provide examples of the wide range of possible flight paths, though their individual characteristics are arbitrary.

5.3 Approaching Sensor Autonomy

The area of sensor autonomy for FWMAVs is made increasingly difficult by the small payloads of these systems. FWMAVs, especially towards the insect scale, are subject to extreme payload constraints; even very small sensors can be significant loads for the vehicle to carry.

For example, we consider the InvenSense MPU9150 IMU [95], previously used for attitude stabilization with the RoboBee [42]. Without considering pull-up resistors or any required attachments, the sensor package has dimensions $4 \text{ mm} \times 4 \text{ mm} \times 1 \text{ mm}$. If we estimate the mass of the IMU in this package using the density of silicon (approximately 2.33 g/cm^3), this sensor has an approximate mass of 37.3 mg. Given that the RoboBee has a payload of approximately 170 mg [96], this is a significant load for the vehicle to carry: this single sensor requires 21.9% of the full vehicle payload. When we consider that, in a fully autonomous vehicle, this payload would also be divided among a power source and

computational hardware, it is clear that sensors must be chosen intelligently to balance the challenges of (1) providing sufficient proprioception and exteroception for control, and (2) allow for the inclusion of other hardware for autonomy.

An important consideration, then, in designing sensing packages for FWMAVs, is to choose sensors which yield sufficient information for control, while not weighing down the vehicle with overwhelming additional mass. We may consider strategies used for other existing FWMAVs (including those with much larger payloads than the RoboBee) in finding potential solutions in sensor package design: For example, the Bat Bot and DelFly both rely on partial feedback, as opposed to full state feedback (full position and attitude estimation, along with velocities for each; effectively, a 12×1 vector of state information). The Bat Bot achieves autonomous flight with attitude control and no direct position control [21], while the DelFly uses optic flow for obstacle avoidance, while additionally including height control [20].

One additional consideration in small-scale flapping-wing flight is the extreme sensor disturbances caused directly by a vehicle's flapping wings. In previous RoboBee work, Fuller et al. found that the accelerometer within an IMU could not provide useful information in flight, as the vehicle's wings produced overwhelming noise [42]. (Similar concerns were addressed by Tu et al. in [97].) Indeed, as vehicles approach the insect scale, we expect that the low body inertia will result in stronger vehicle body motion from flapping. Additionally, practically speaking, since the vehicles are so small relative to the attached sensors (e.g., highest sensor dimension on the order of 10% or more of the vehicle body length, for the smallest FWMAVs), the sensor simply cannot be displaced from the wings by a large margin. Therefore, the flapping motion effect is amplified at the sensor due to proximity between the sensor and vehicle propulsion elements.

We may conclude, then, that the not only must sensors be chosen to provide

sufficient state estimation and environmental awareness for autonomous flight, but also must be chosen considering the extreme relative dynamics of the flapping-wing vehicle.

5.3.1 Previous RoboBee Sensor Progress

As one of the smallest insect-scale FWMAVs, the RoboBee is subject to some of the most restrictive payload limitations among vehicles of its kind. Despite this challenge, previous research has allowed the RoboBee to achieve partial state feedback and exteroception using commercial and custom sensors.

Exploration of commercial sensors onboard the vehicle has largely focused on inertial measurement units (IMUs) and their subcomponents (i.e., accelerometers, gyroscopes, and magnetometers) time-of-flight (ToF) sensors, which provide depth and proximity information, and cameras. A ToF sensor (VL6180x, STMicroelectronics) was applied for altitude estimation in closed-loop flight, maintaining altitude error within half of the vehicle’s body length [43]. The gyroscope component of an IMU (MPU9150, Invensense) was used for attitude stabilization in flight [42]; while the accelerometer data from the IMU was also available, the authors found that the flapping wings generated overwhelming noise for the accelerometer data, such that it was not useful in flight. Additionally, a magnetometer (HMC1043, Honeywell) was used to track heading angle in constrained flight (with the vehicle attached to a vertical guide wire), demonstrating feedback control [45]. Finally, Mange et al. presented a gaze stabilization mechanism using optic flow with a small-scale camera, particularly tailored to surgical devices (NanEye 2D, AWAIBA) [98].

Previous research has also applied custom biomimetic sensing towards FWMAV control. Common to vision sensing for biological insects are the ocelli, which are particularly suited for detecting rapid changes in an environment’s light [99]. Fuller et al. presented an ocelli-inspired, photoresistor-based vision sensor for FWMAVs, applying the sensor in flight

to stabilize the orientation of the RoboBee [44].

5.3.2 Expansion of Onboard Sensing

As sensor autonomy is a key element in achieving a fully-autonomous system, we look forward to expanding the use of sensors for onboard control of the RoboBee in the immediate future. Particular steps towards achieving this goal include developing strategies to isolate sensor data from wing-flapping noise (especially accelerometer data, which would be valuable in proprioception); determining the minimum state estimation requirements for autonomous flight (e.g., the required portion of the full 12×1 state vector, which varies depending on control strategy, as shown with examples from various FWMAVs in Section 1.2.2; and sufficient exteroceptive information); and selecting an appropriate sensor package (including associated sensor placement and orientation) to achieve these sensing information requirements. As in [44], custom sensor designs may be considered; however, immediate efforts will focus on incorporation of commercially-available sensing options.

5.4 Discussion

The advances in control strategies for FWMAVs discussed in this work will not only improve the maneuverability and stability of flapping-wing vehicles at the smallest scales, but will also allow efficient use of onboard sensing packages as we approach full autonomy. With full control authority of the RoboBee, through strategies generalizable to the wider field of FWMAVs, we look forward to the opportunity to approach new questions in computationally-efficient control strategies; aggressive maneuvers; and robust methods for control of vehicles with differing morphologies, and whose characteristics change during use.

Additionally, as we improve the agility and maneuverability of these systems, we

open the door to exciting connections to biology. Already, recent work has applied the work presented herein to expand yaw torque authority through reduced-frequency flapping to motivate the study of the use of frequency modulation in hawkmoth flight [86]. As we further approach the goal of mimicking the impressive biological feats displayed in insect flight, we may continue to learn about biological mechanisms for aerial maneuvers and acrobatics by attempting to recreate them with flapping-wing robots.

Exciting possibilities lie ahead for flapping-wing vehicles. As these vehicles move closer to full autonomy, we look ahead to opportunities in swarm behavior and operation in natural environments, opening the doors to channeling the full potential of small-scale flight.

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