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Spin liquids and the cuprate superconductors

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Spin liquids and the cuprate superconductors

128th Statistical Mechanics Conference
Rutgers University
May 19, 2025

Subir Sachdev

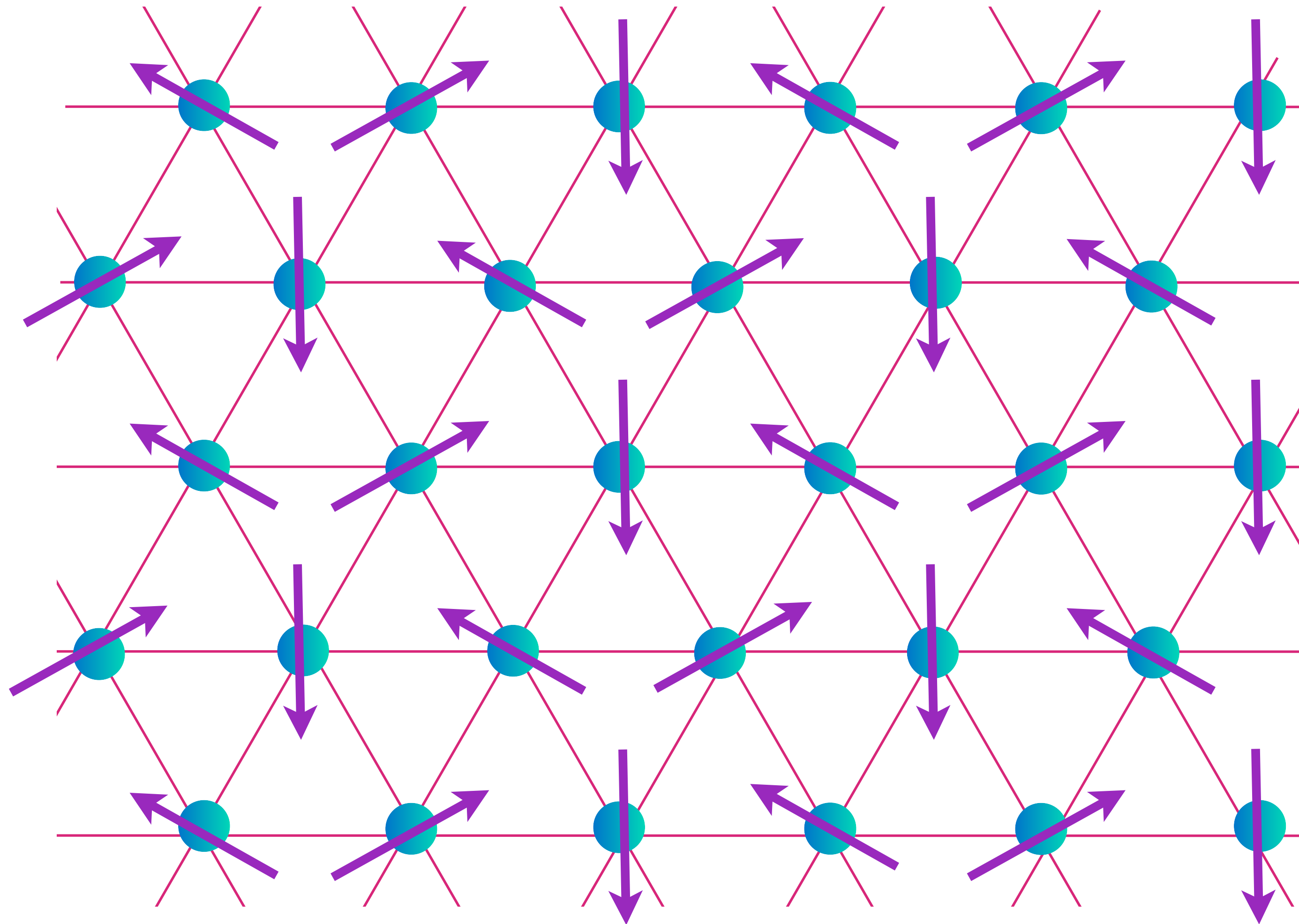


Spin liquids
on the
triangular and
square lattices

Triangular lattice antiferromagnet

Spin model with $S=1/2$ per unit cell

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



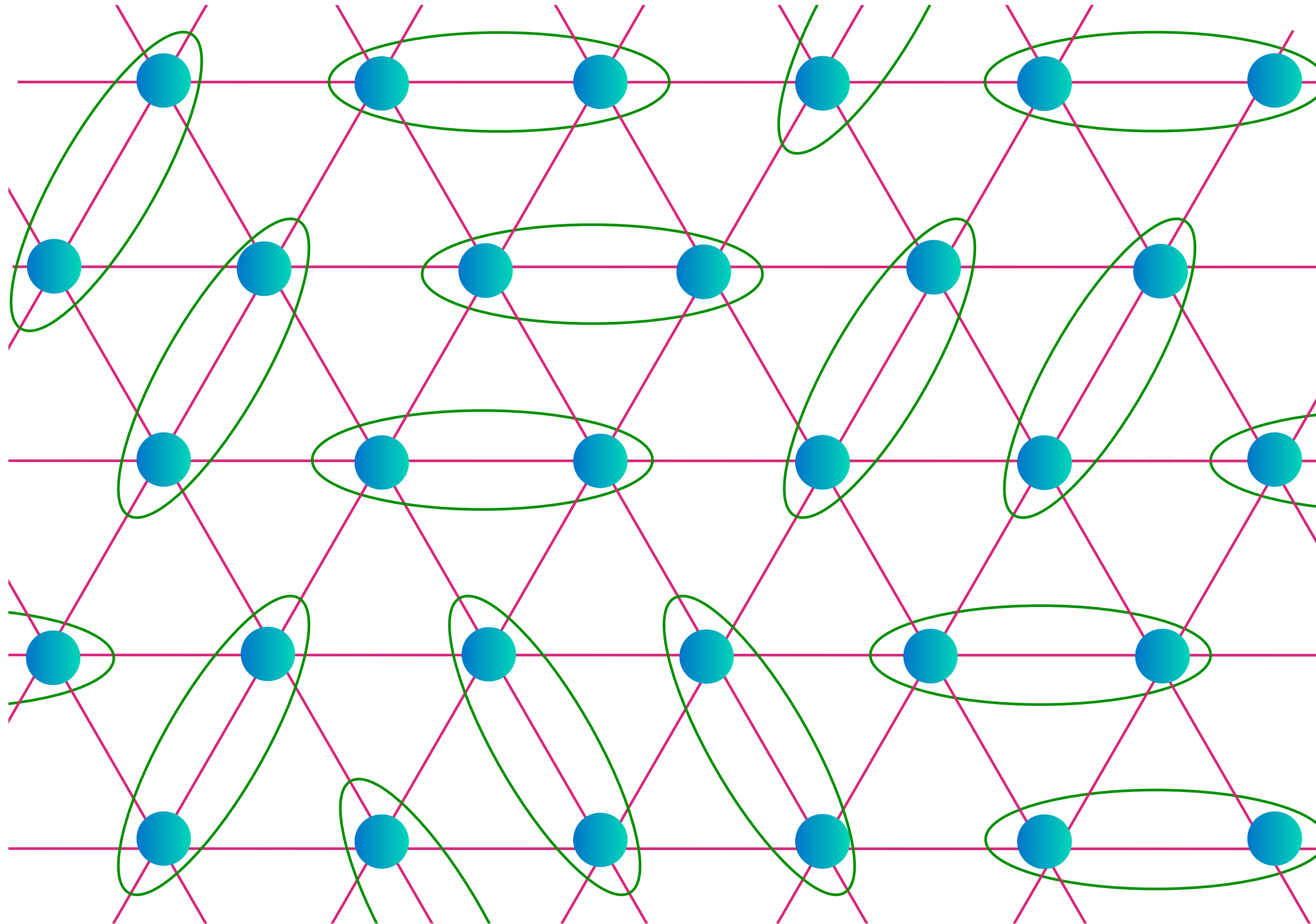
$$\begin{aligned} [S_\alpha, S_\beta] &= i\epsilon_{\alpha\beta\gamma} S_\gamma \\ S_\alpha^2 &= S(S+1); \\ S &= 1/2 \\ S_z |\uparrow\rangle &= (1/2) |\uparrow\rangle \\ S_z |\downarrow\rangle &= -(1/2) |\downarrow\rangle \\ &\text{on each site } i \end{aligned}$$

Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

Spin model with $S=1/2$ per unit cell

$$\text{green oval with two blue dots} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



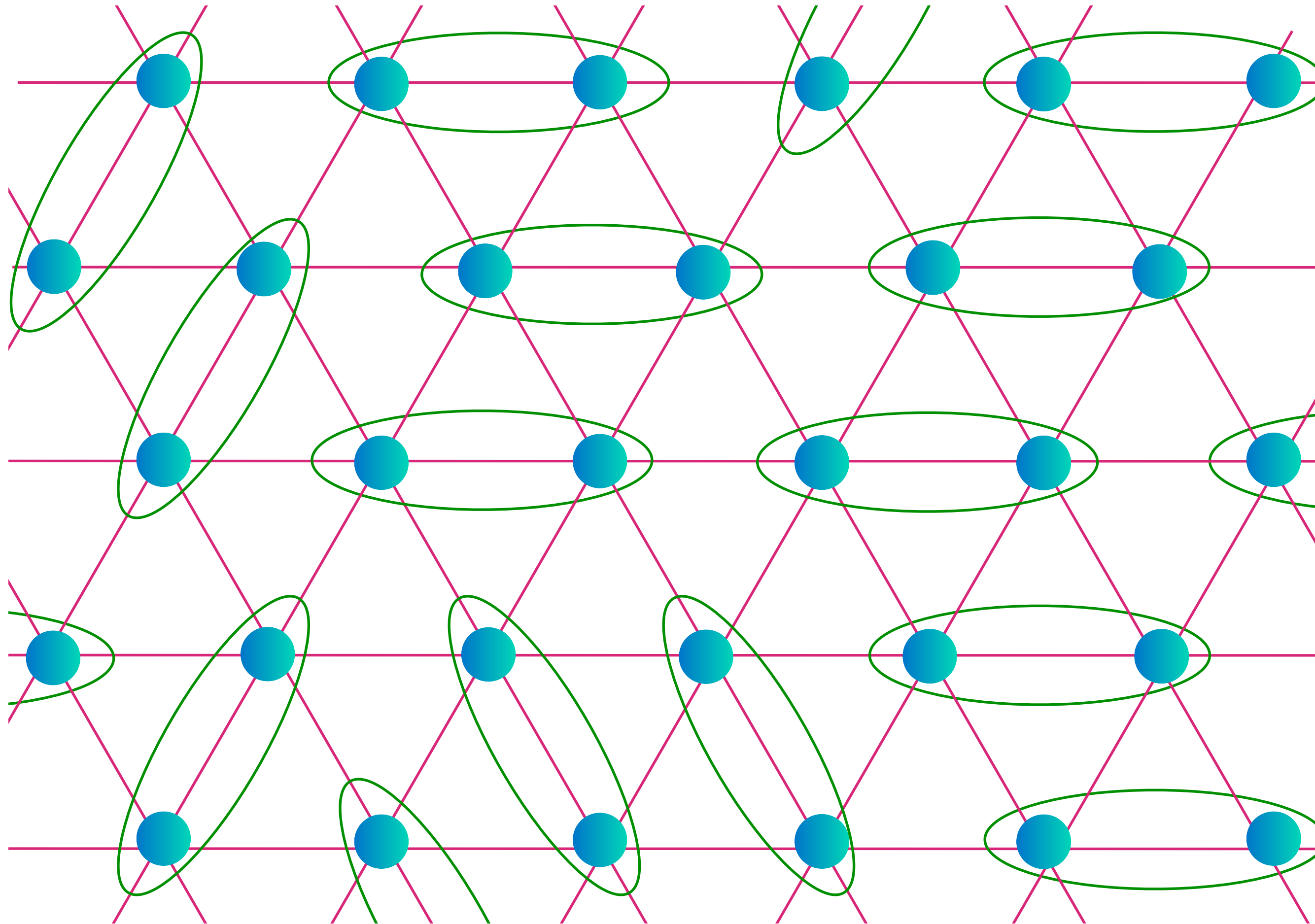
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering
of lattice

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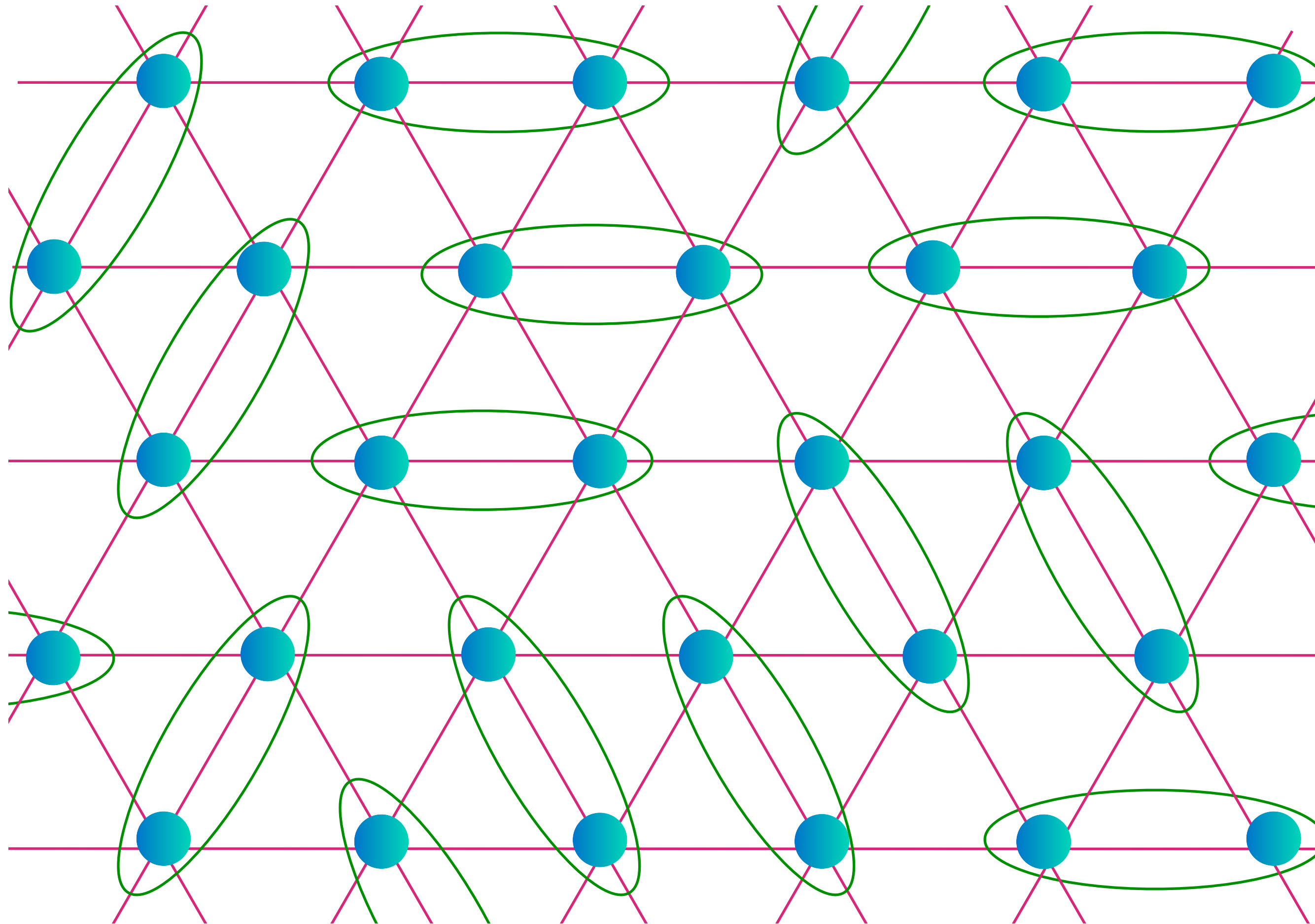
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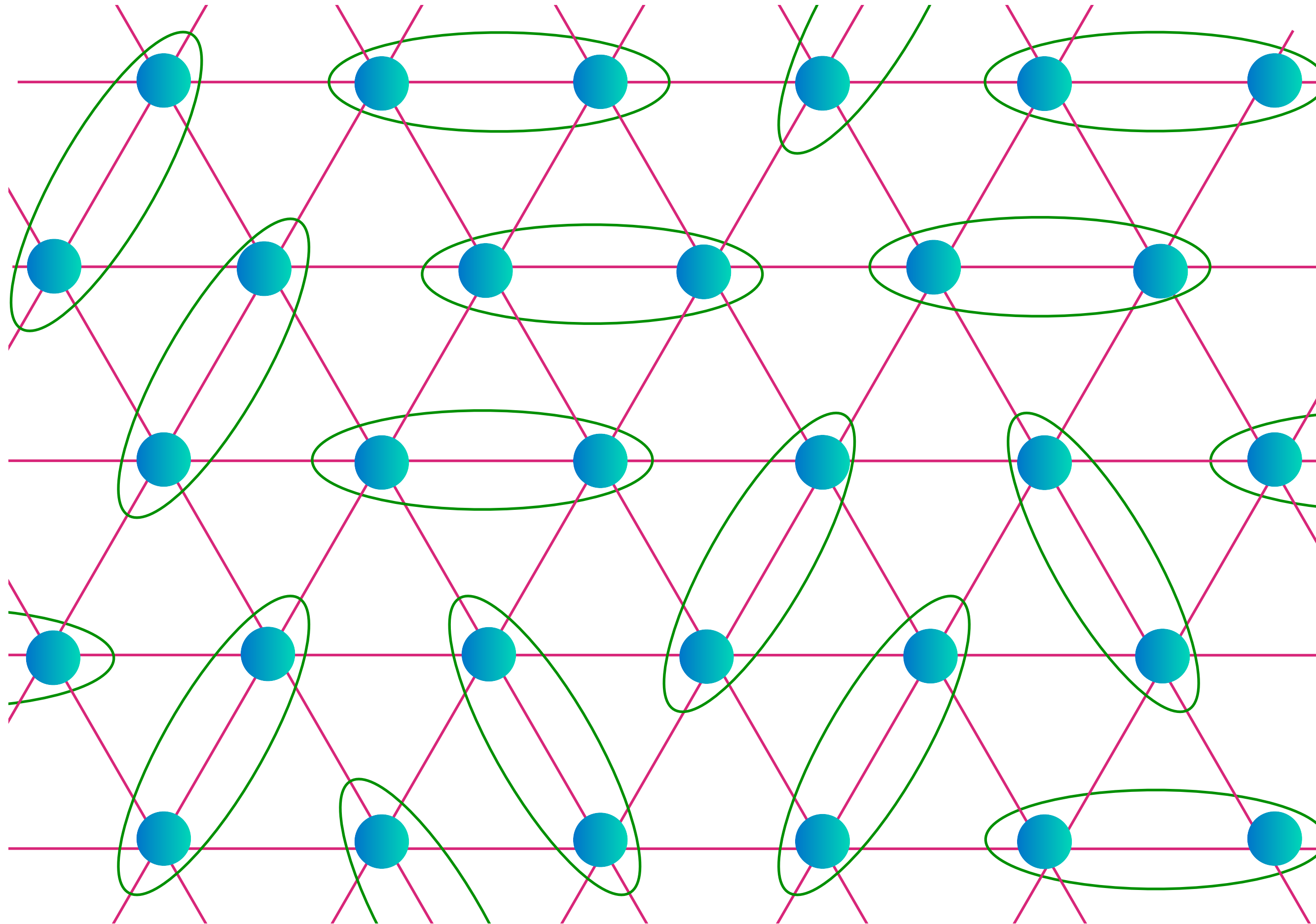
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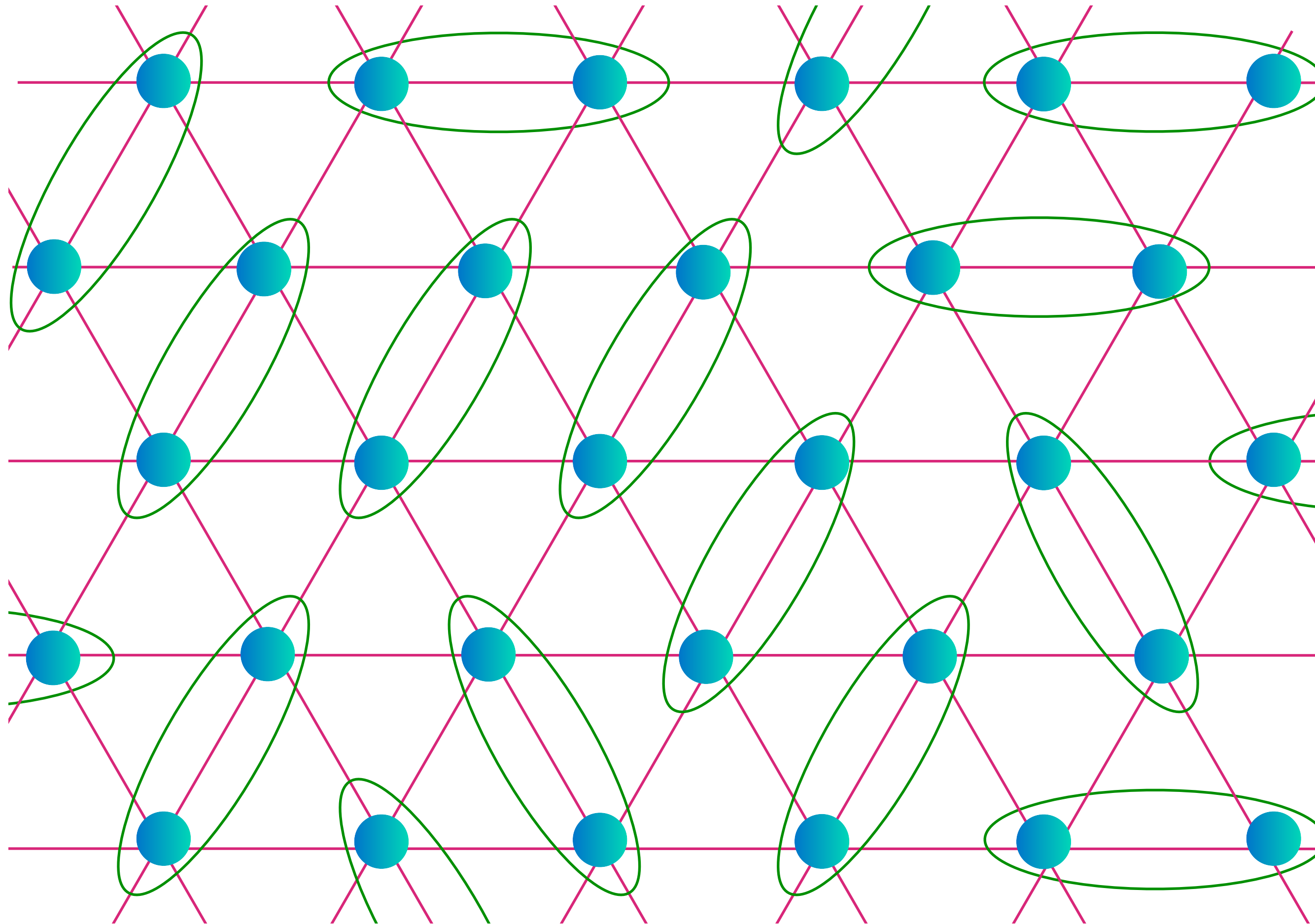
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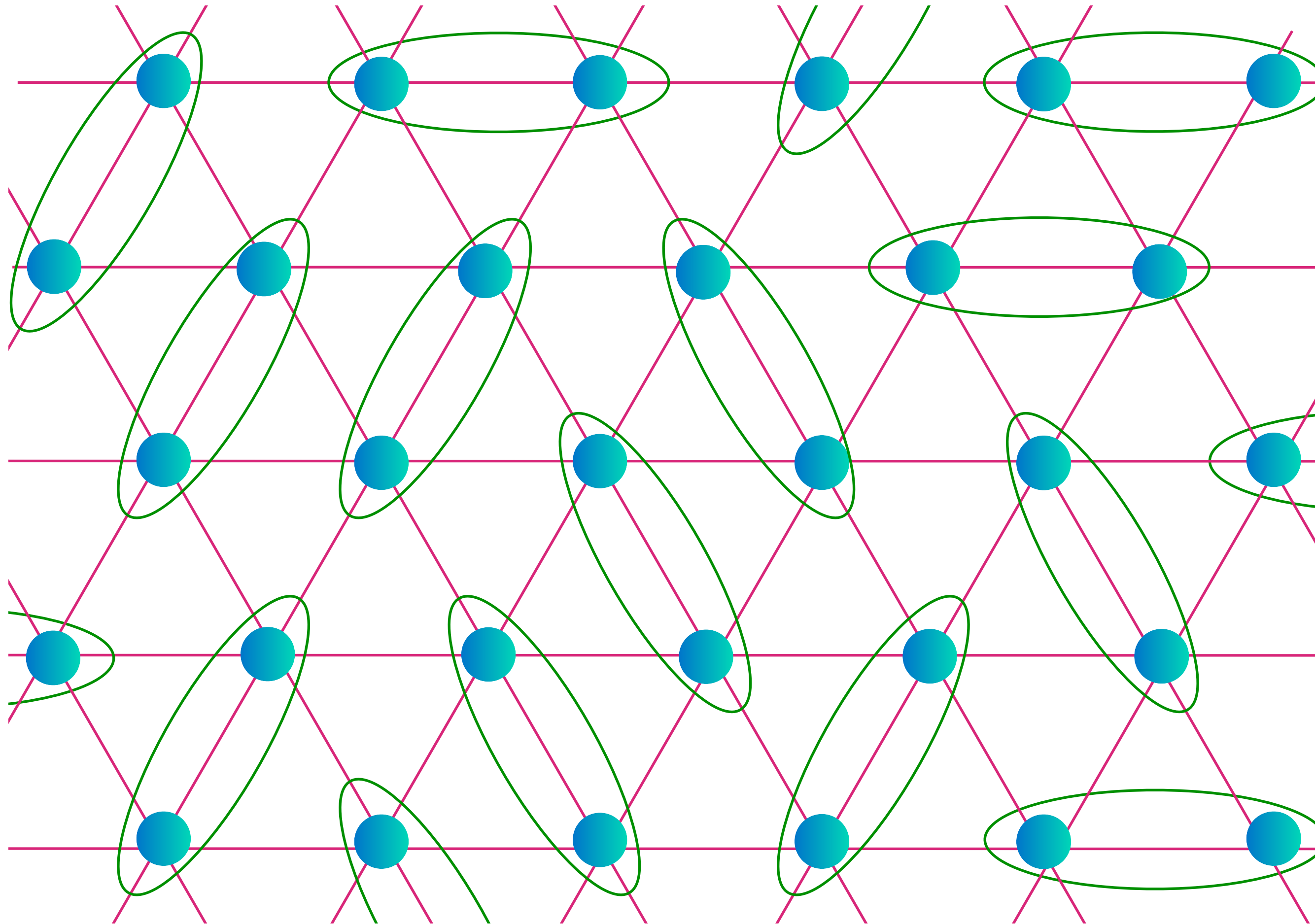
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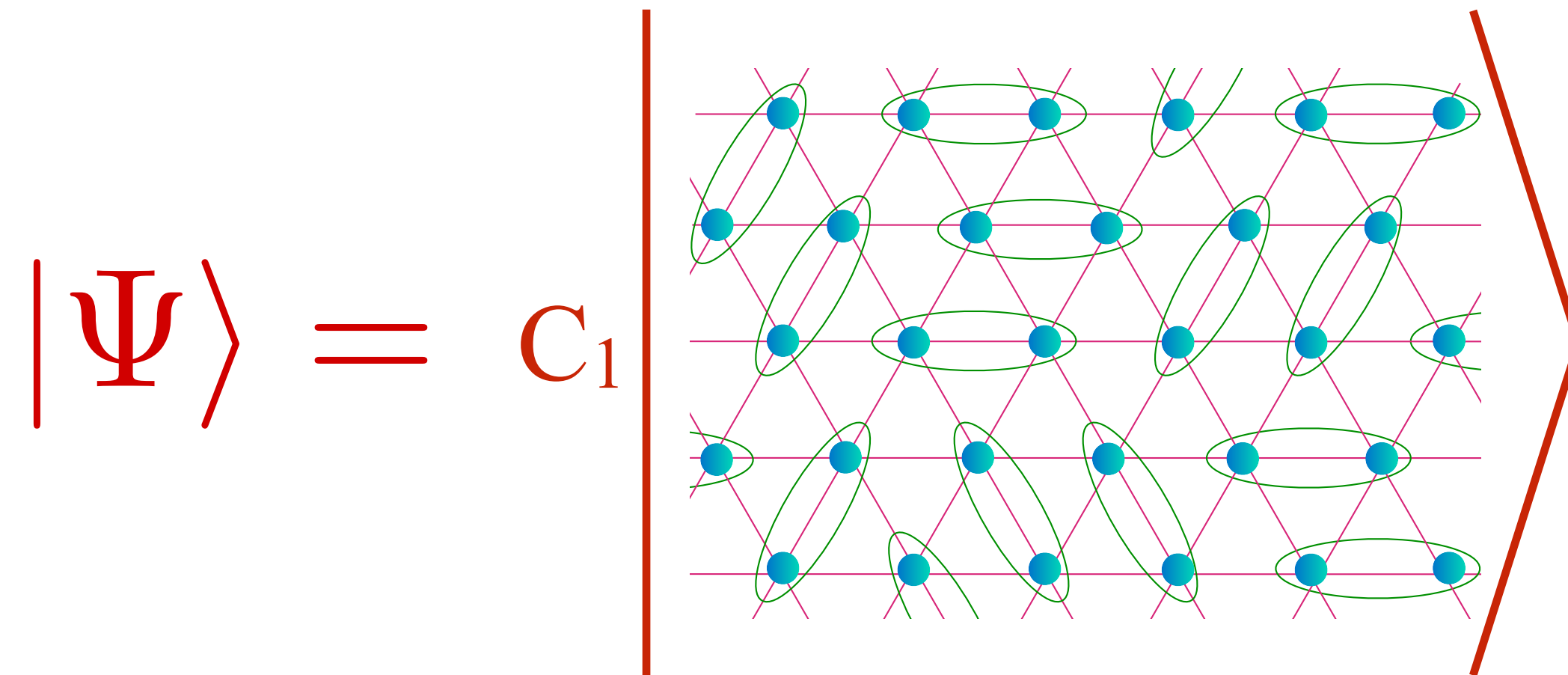
$$\text{green oval with two blue dots} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|\Psi\rangle = C_1 \left[\text{diagram 1} \right] + C_2 \left[\text{diagram 2} \right] + C_3 \left[\text{diagram 3} \right] + C_4 \left[\text{diagram 4} \right] + C_5 \left[\text{diagram 5} \right] + \dots$$

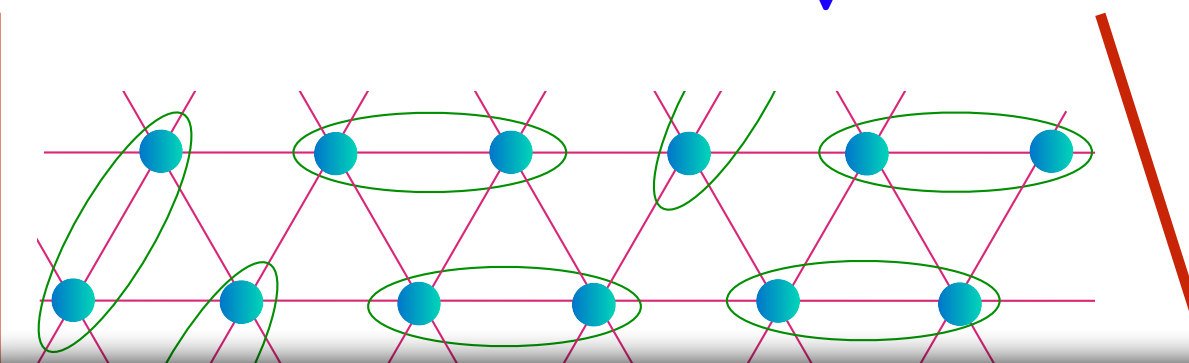
The diagrams show a 4x6 grid of blue dots (spins) on a triangular lattice. Green ovals represent valence bonds between pairs of spins. The orientation of the ovals varies between the five diagrams, illustrating different configurations of resonating valence bonds. Each diagram is enclosed in large red square brackets with a red chevron pointing to the right.

Spin liquid: resonating valence bonds

$$\text{green oval with two blue dots} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



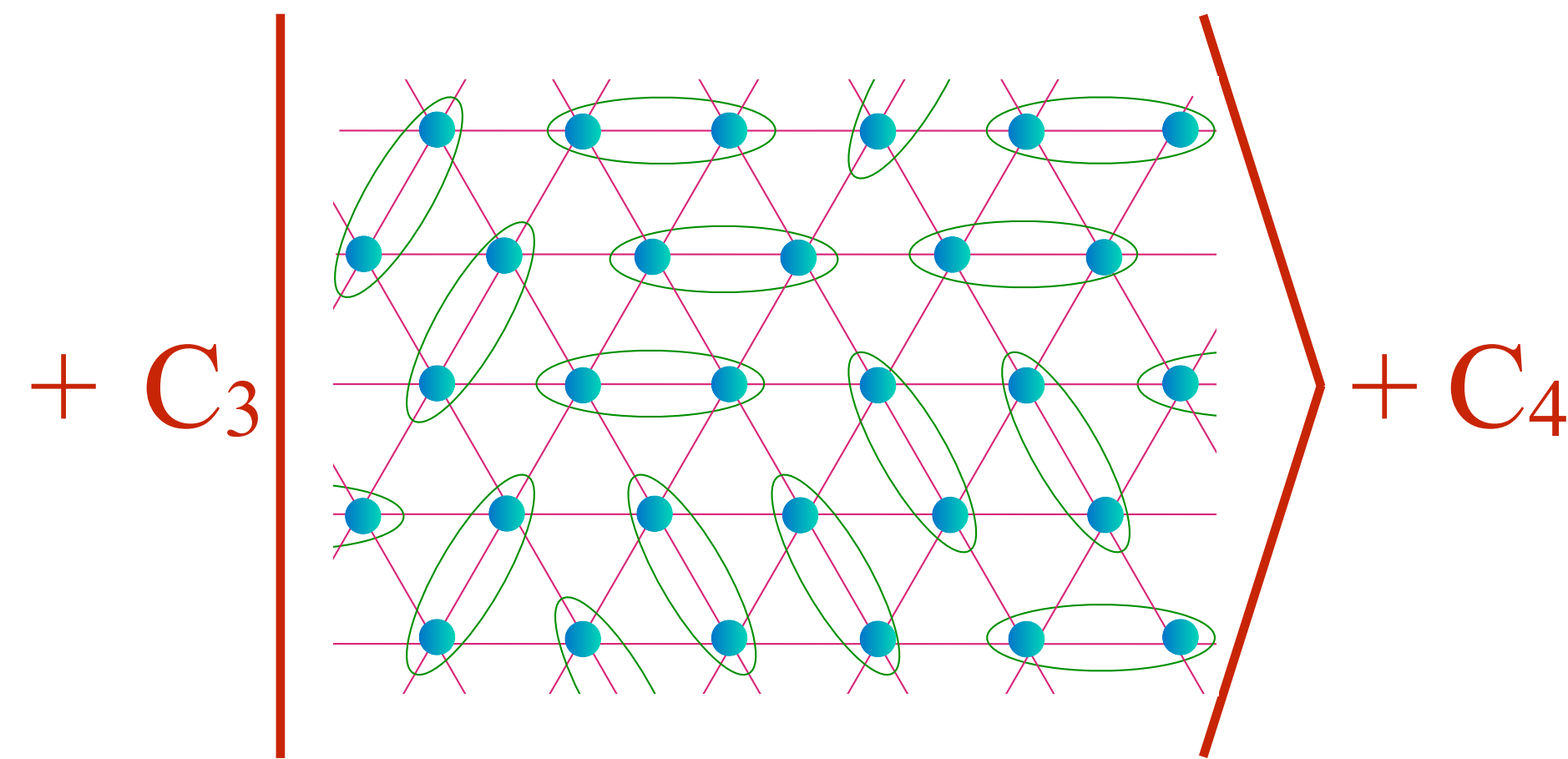
+



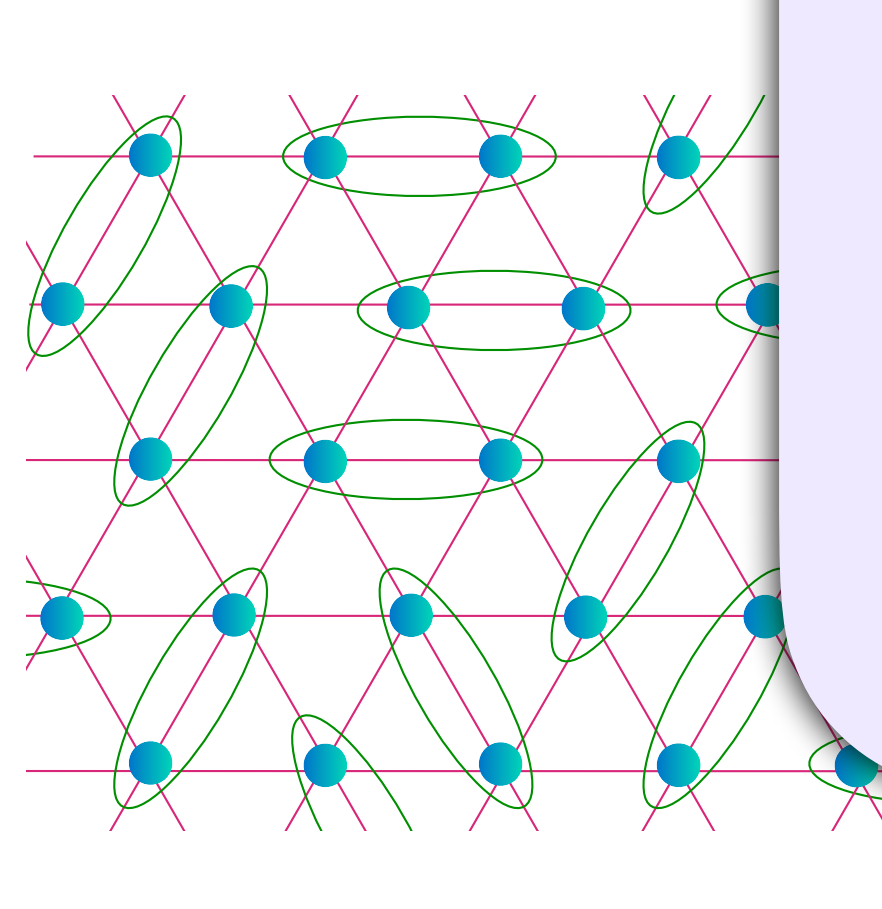
Key feature: fractionalization.

Excitations are particle-like,
but cannot be created
by local operators.

The excitations are classified
under distinct
superselection/anyon sectors.

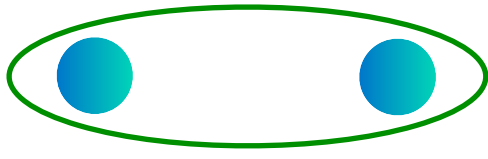


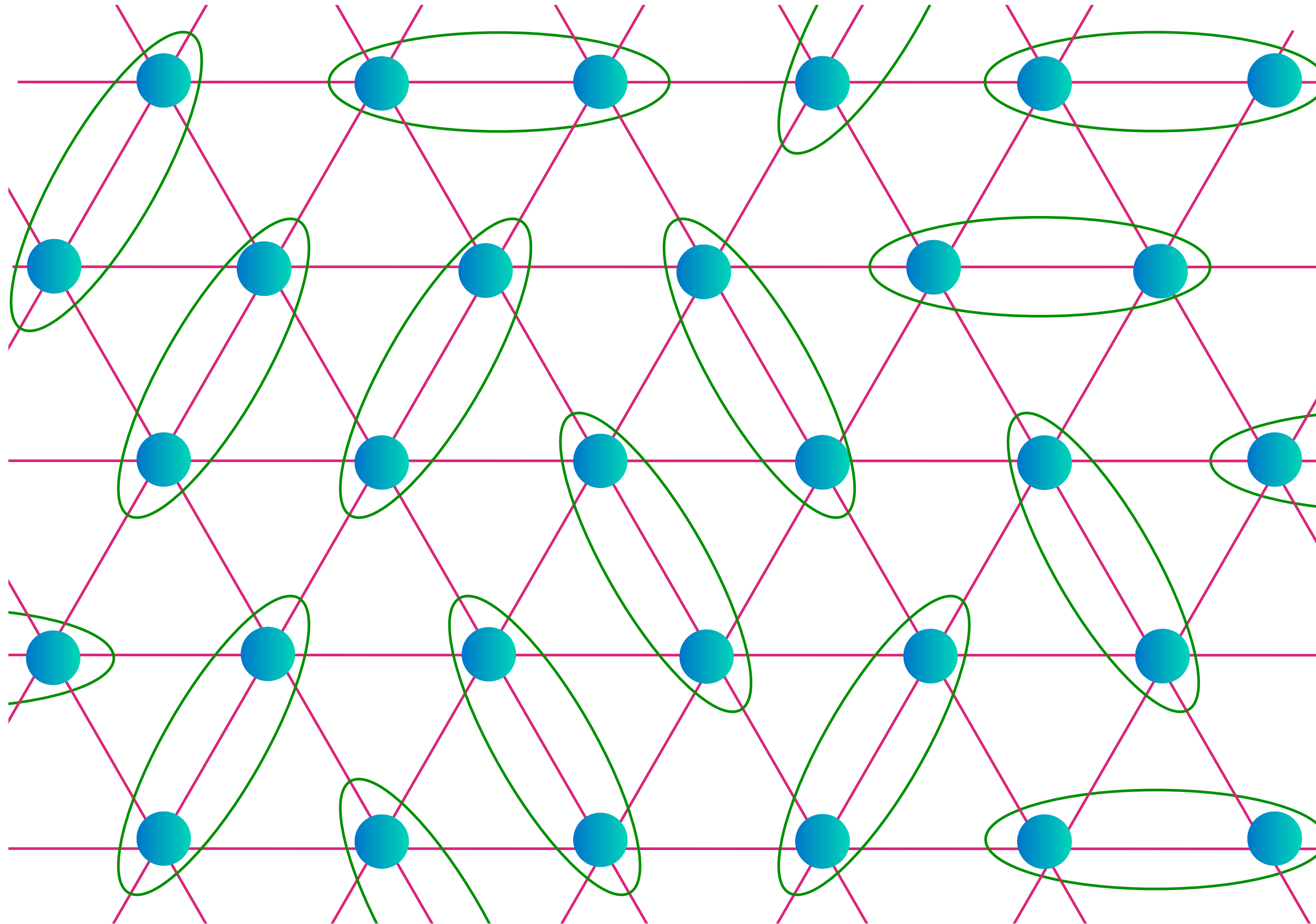
+



RVB: Z_2 spin liquid

Fractionalized excitations: a “spinon”
with spin $S=1/2$

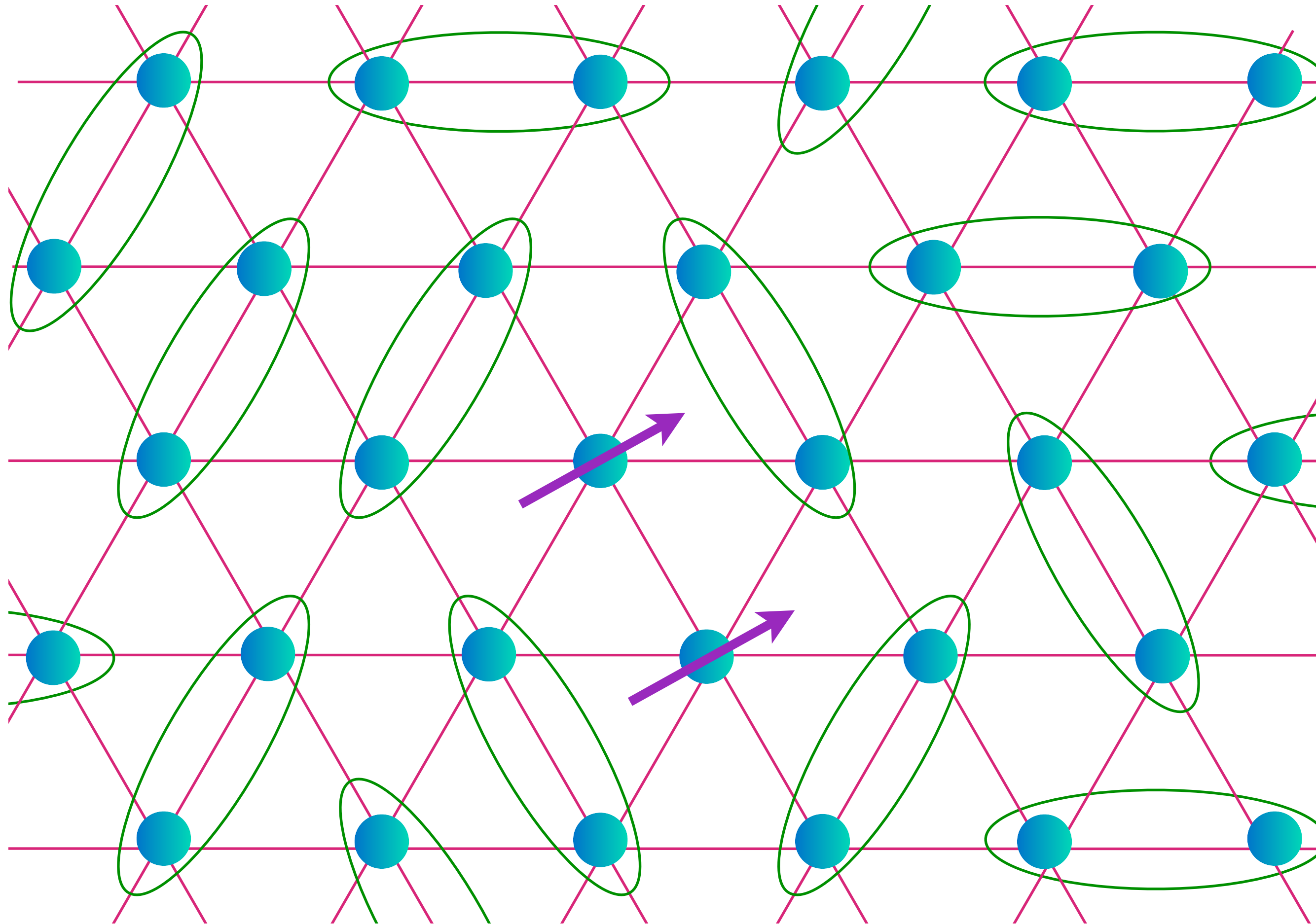

$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



RVB: Z_2 spin liquid

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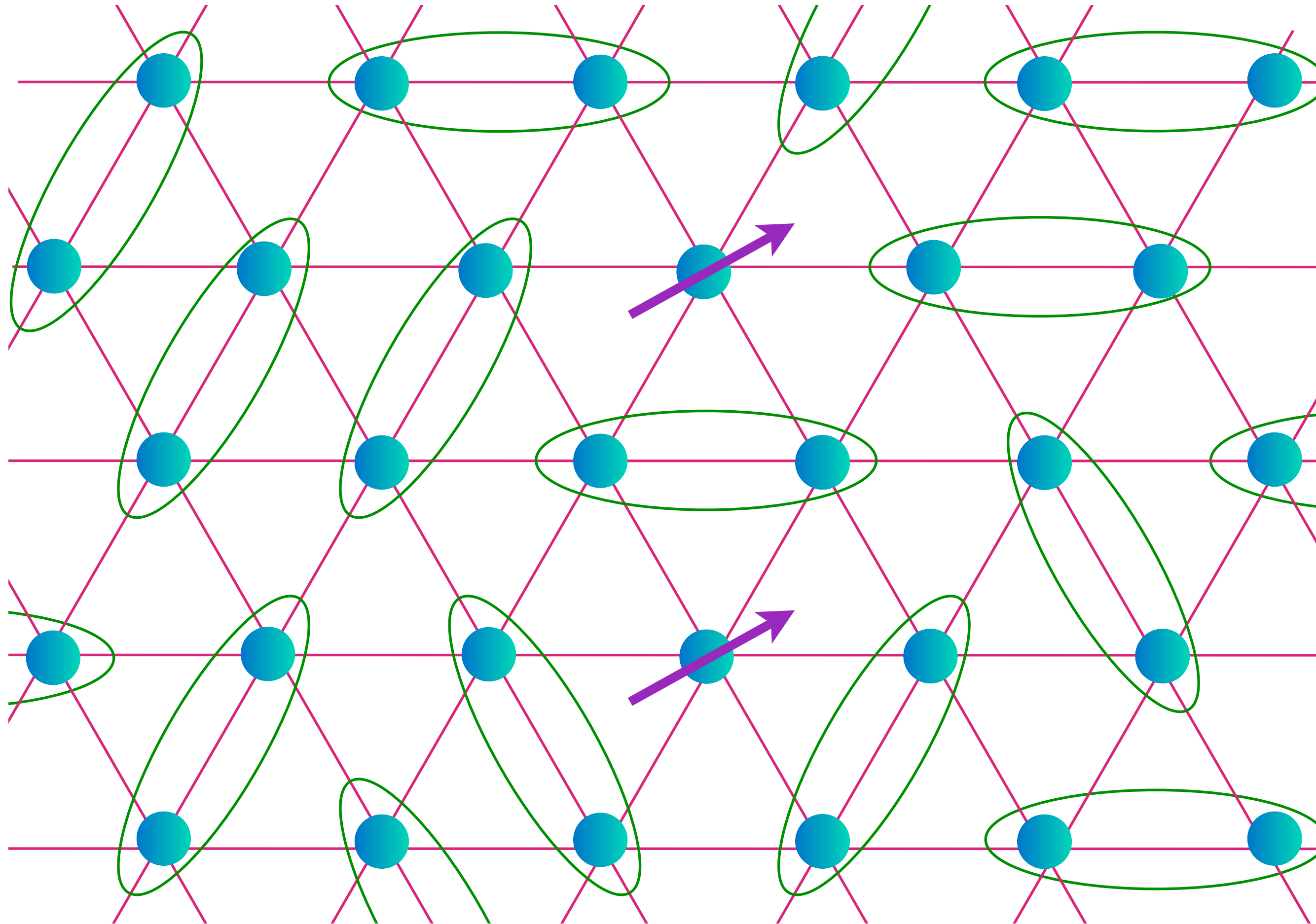
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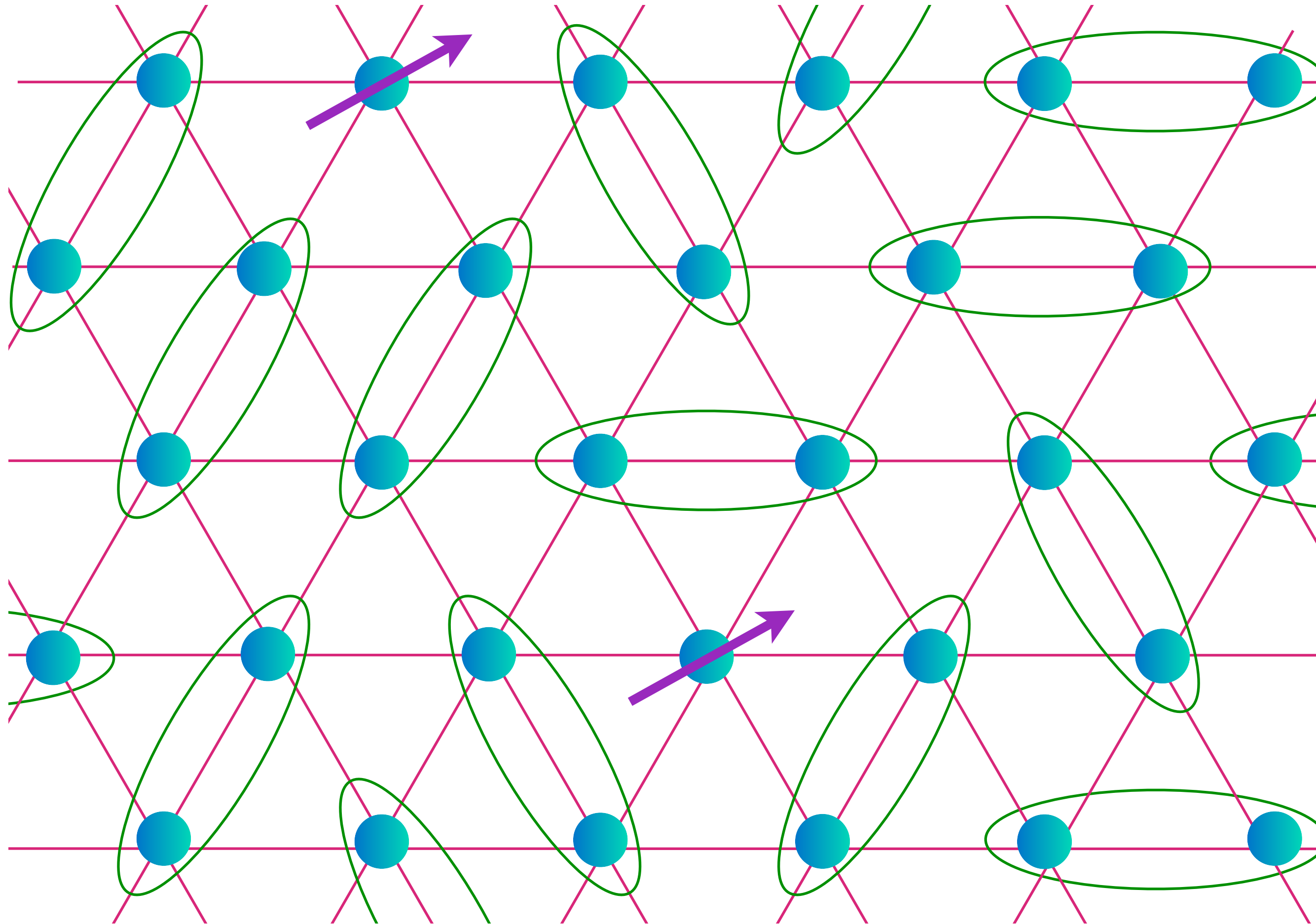
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RVB: Z_2 spin liquid

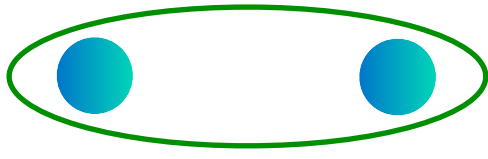
Fractionalized excitations: a “spinon”
with spin $S=1/2$

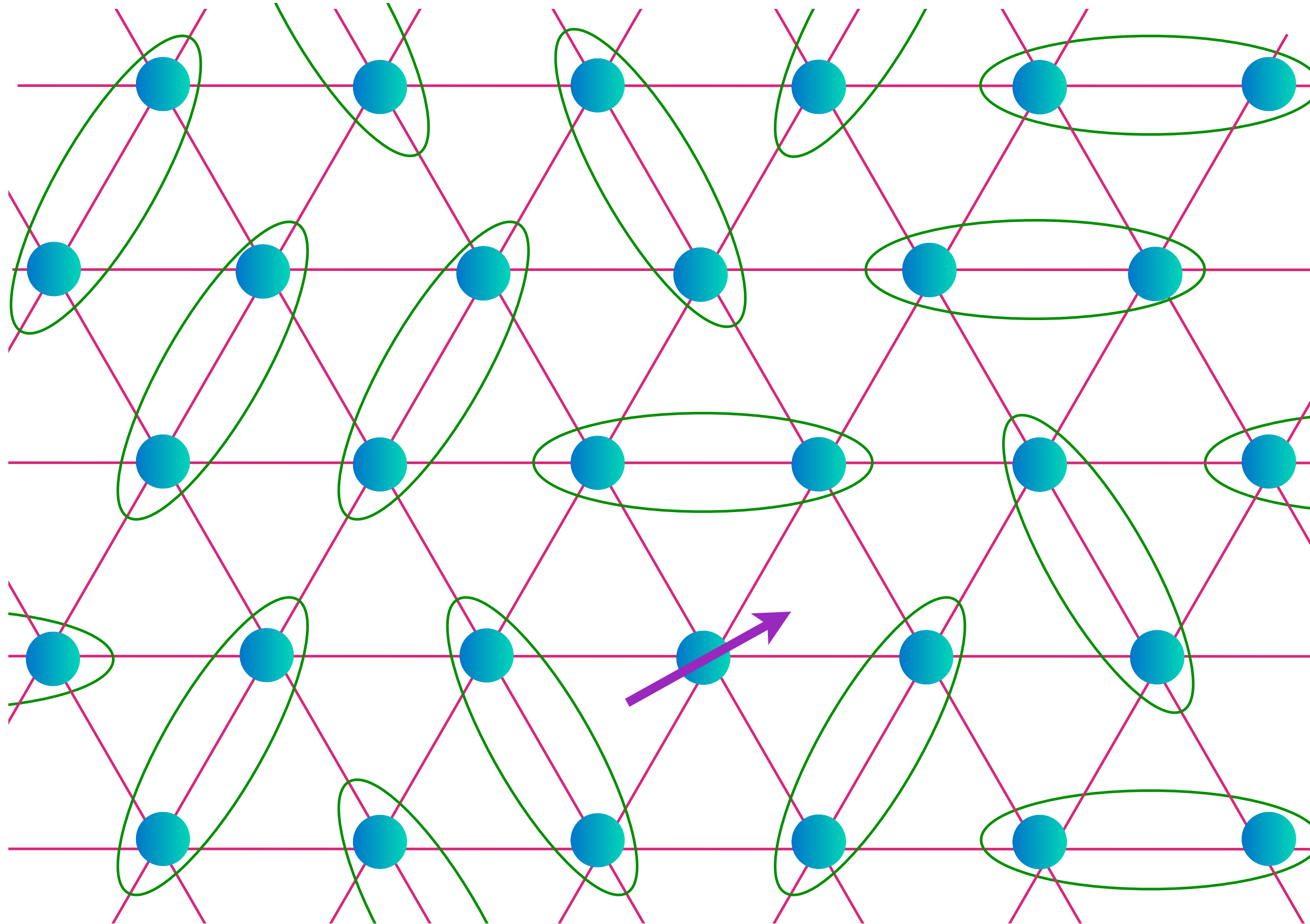
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RVB: Z_2 spin liquid

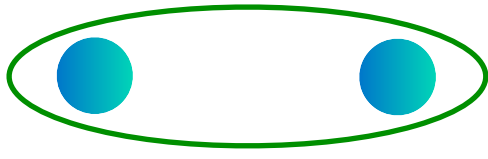
Fractionalized excitations: a “spinon”
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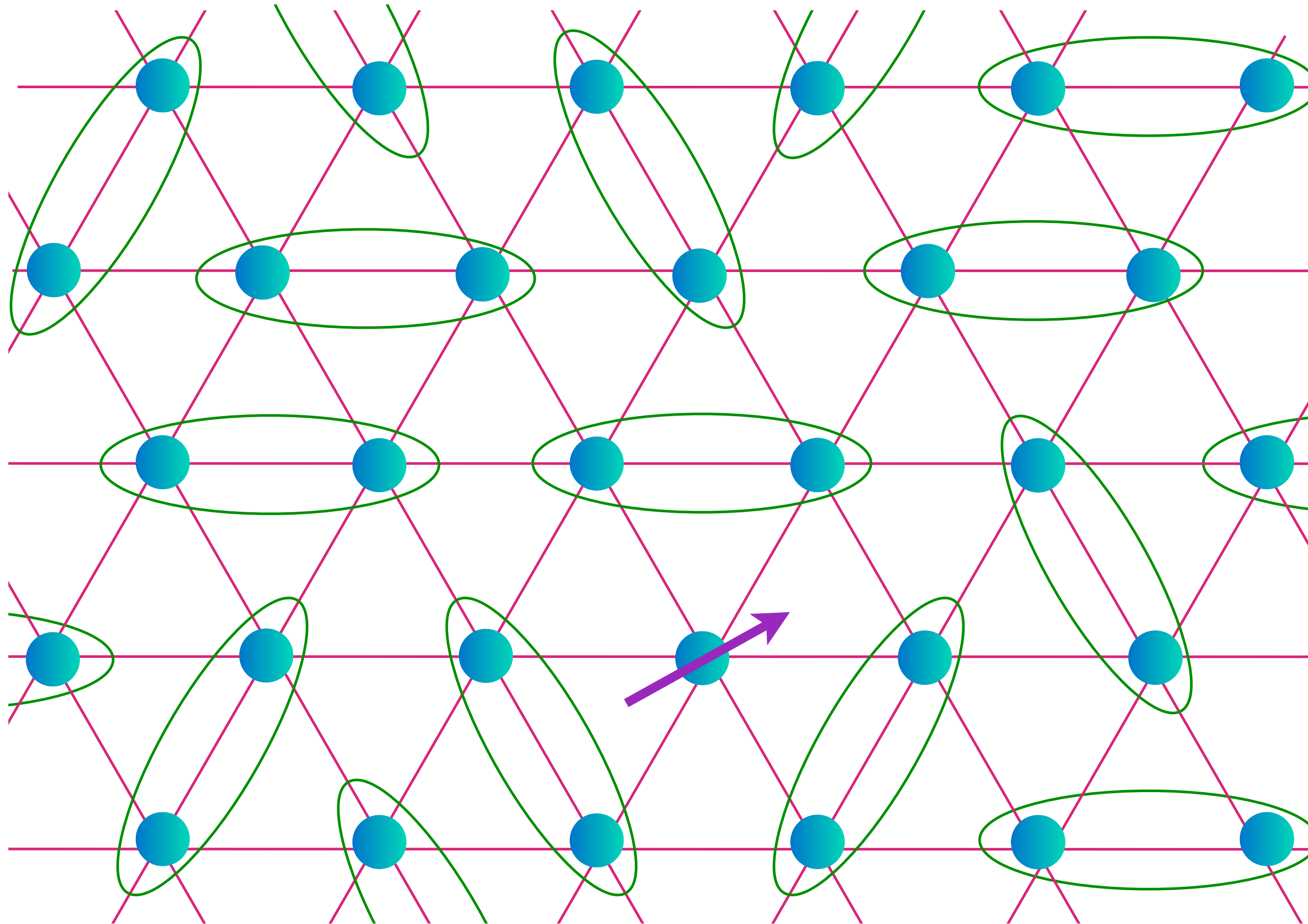

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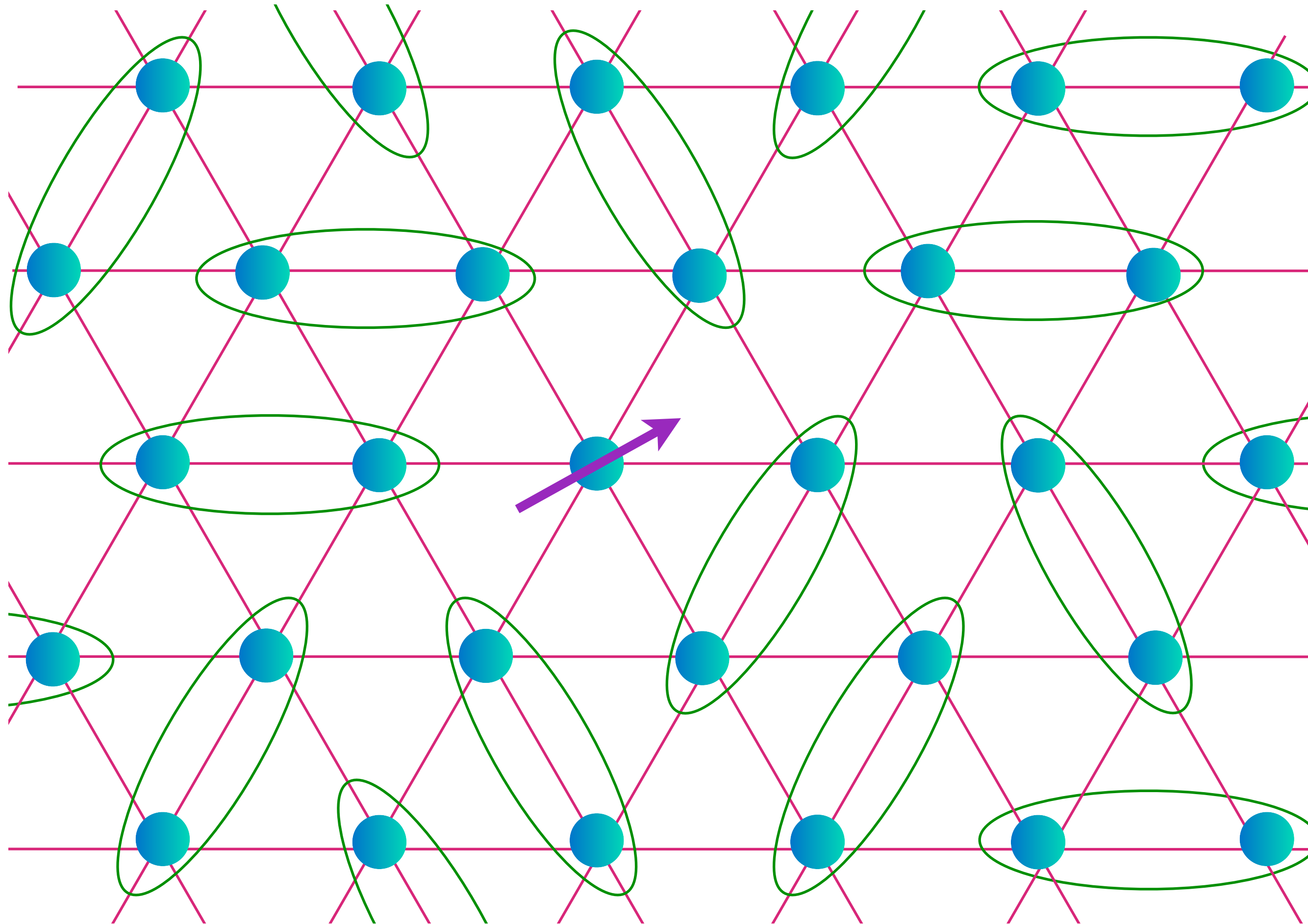

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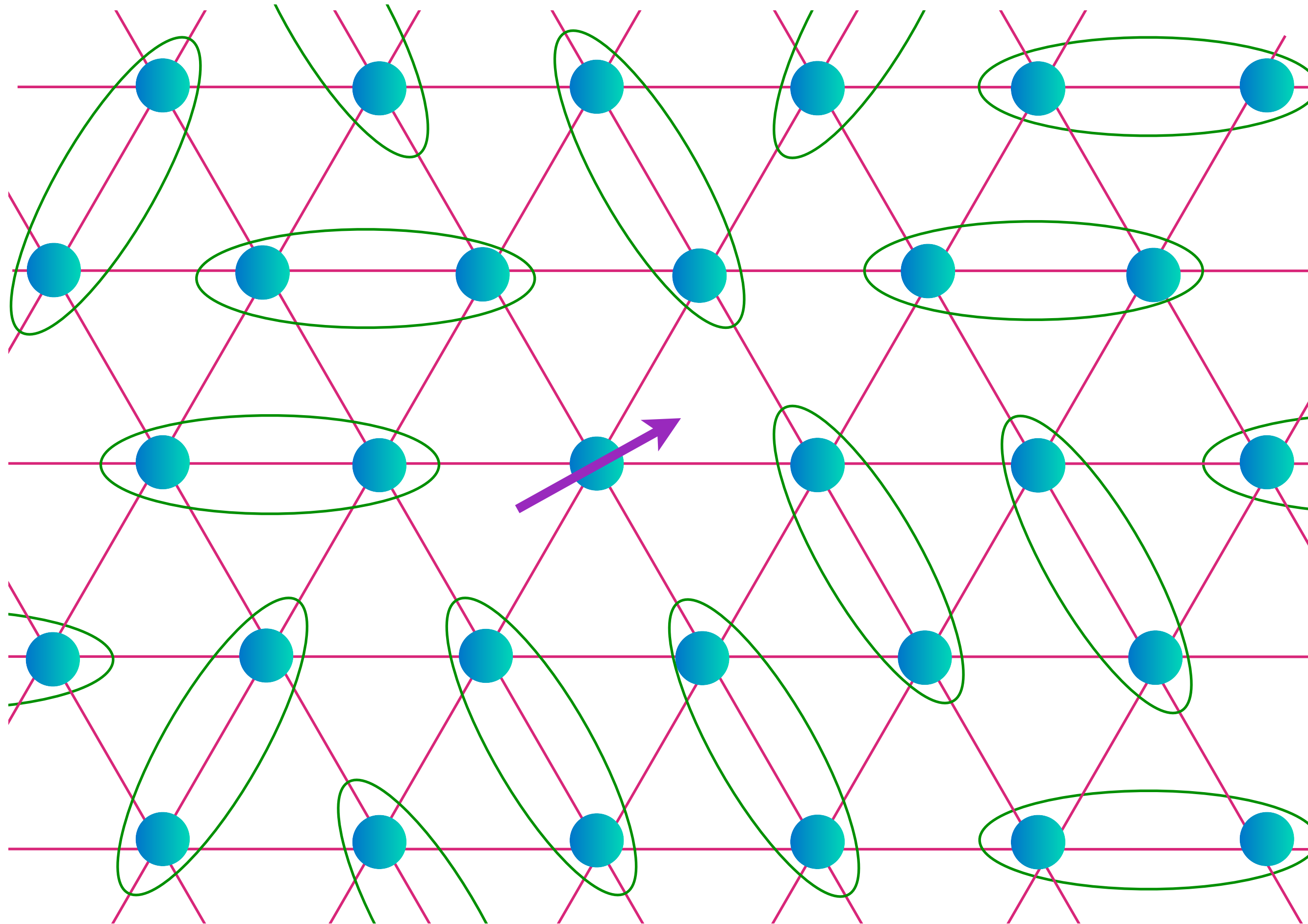
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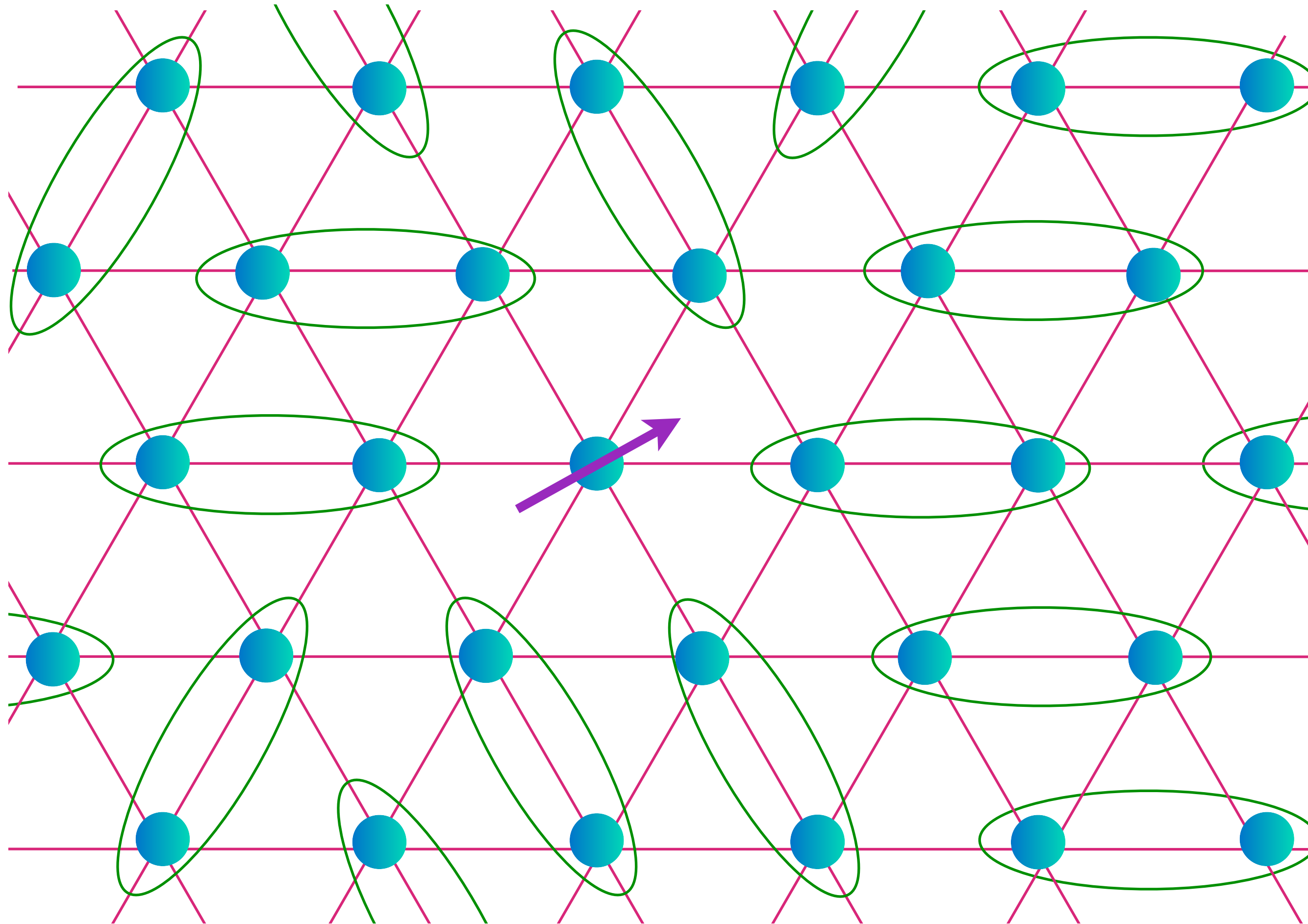
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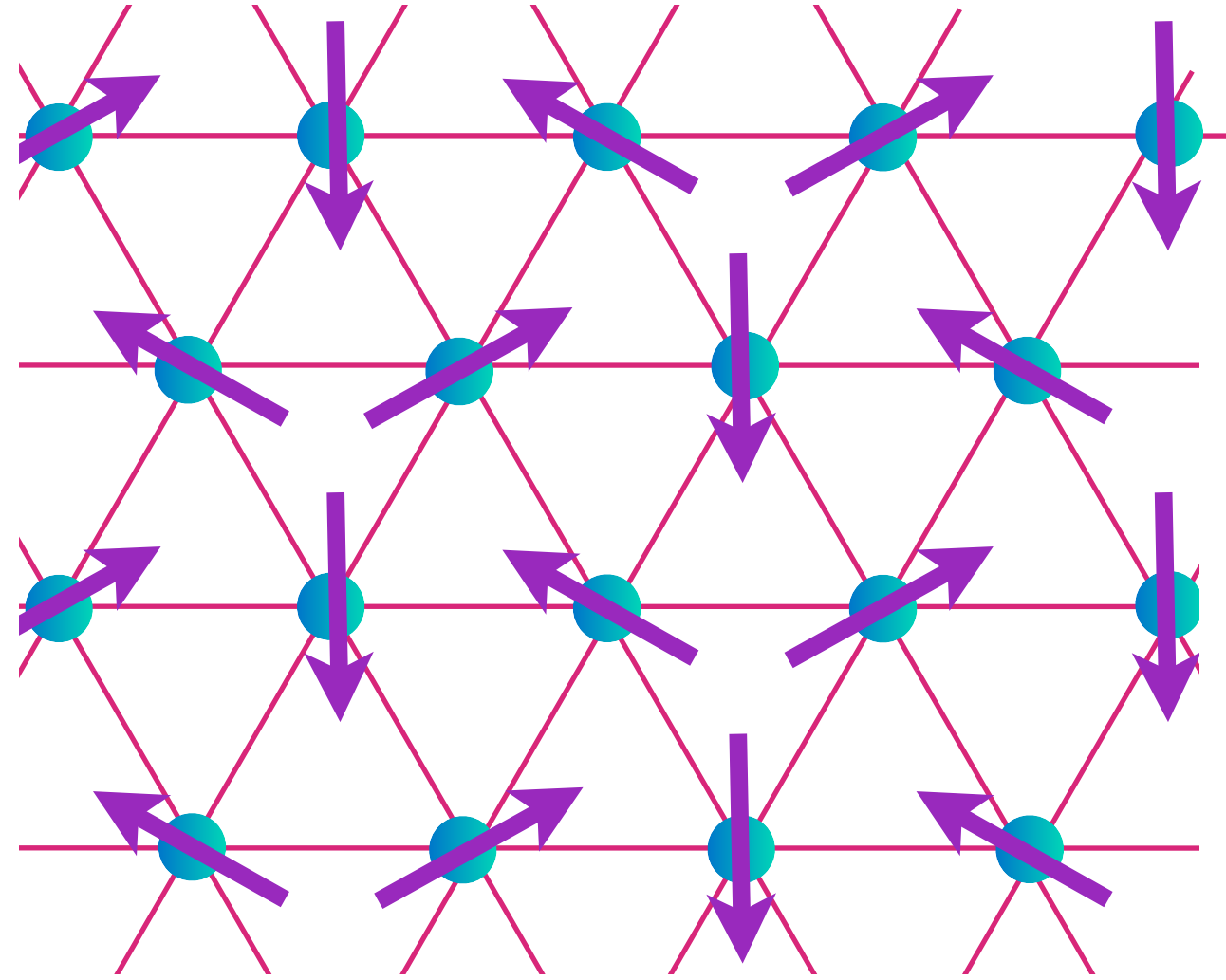
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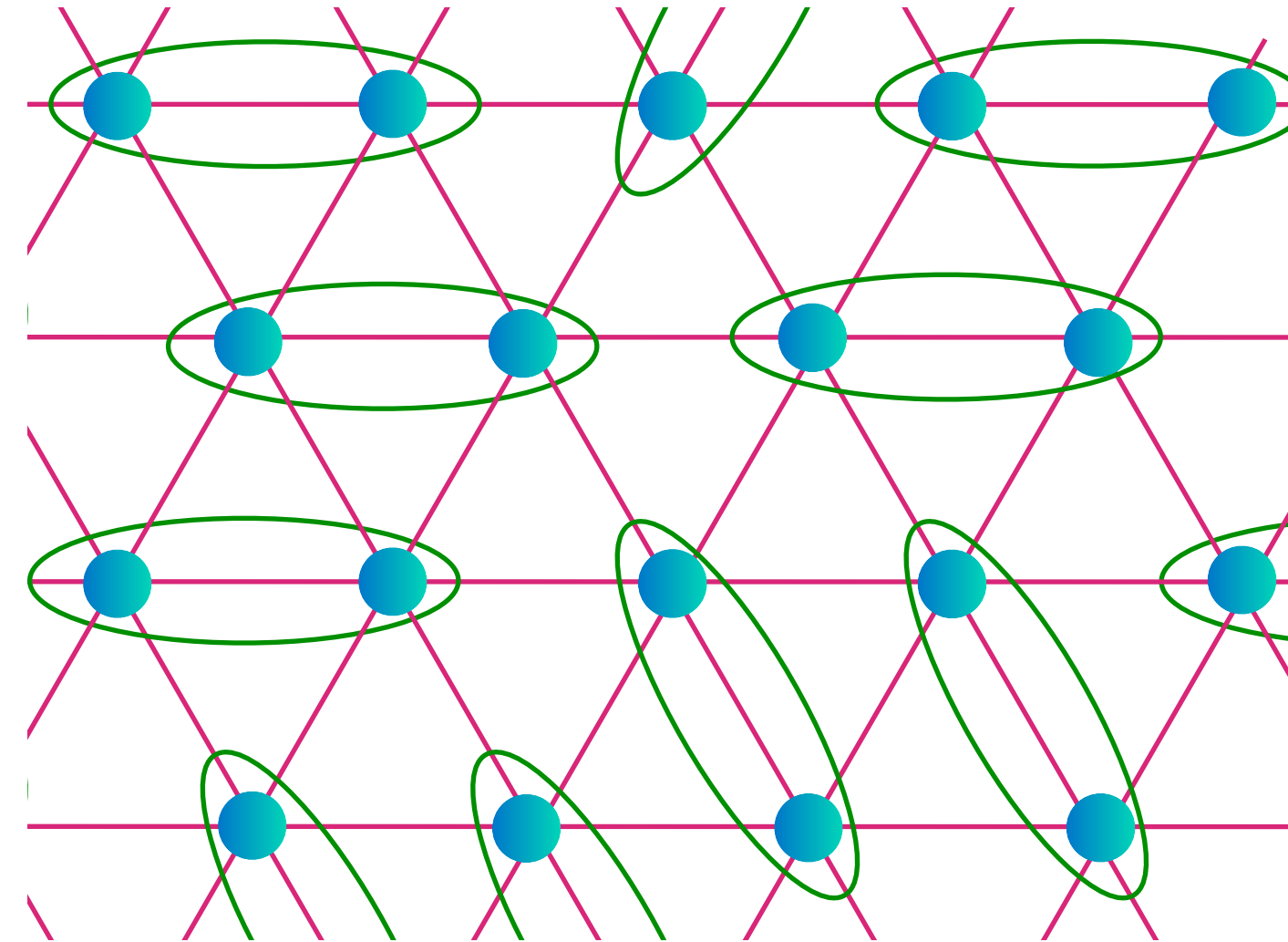
$$\text{[Diagram: Two blue dots in a green oval]} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$S=1/2$ triangular lattice



$\mathbb{Z}_2$ spin liquid



Second neighbor
exchange J_2

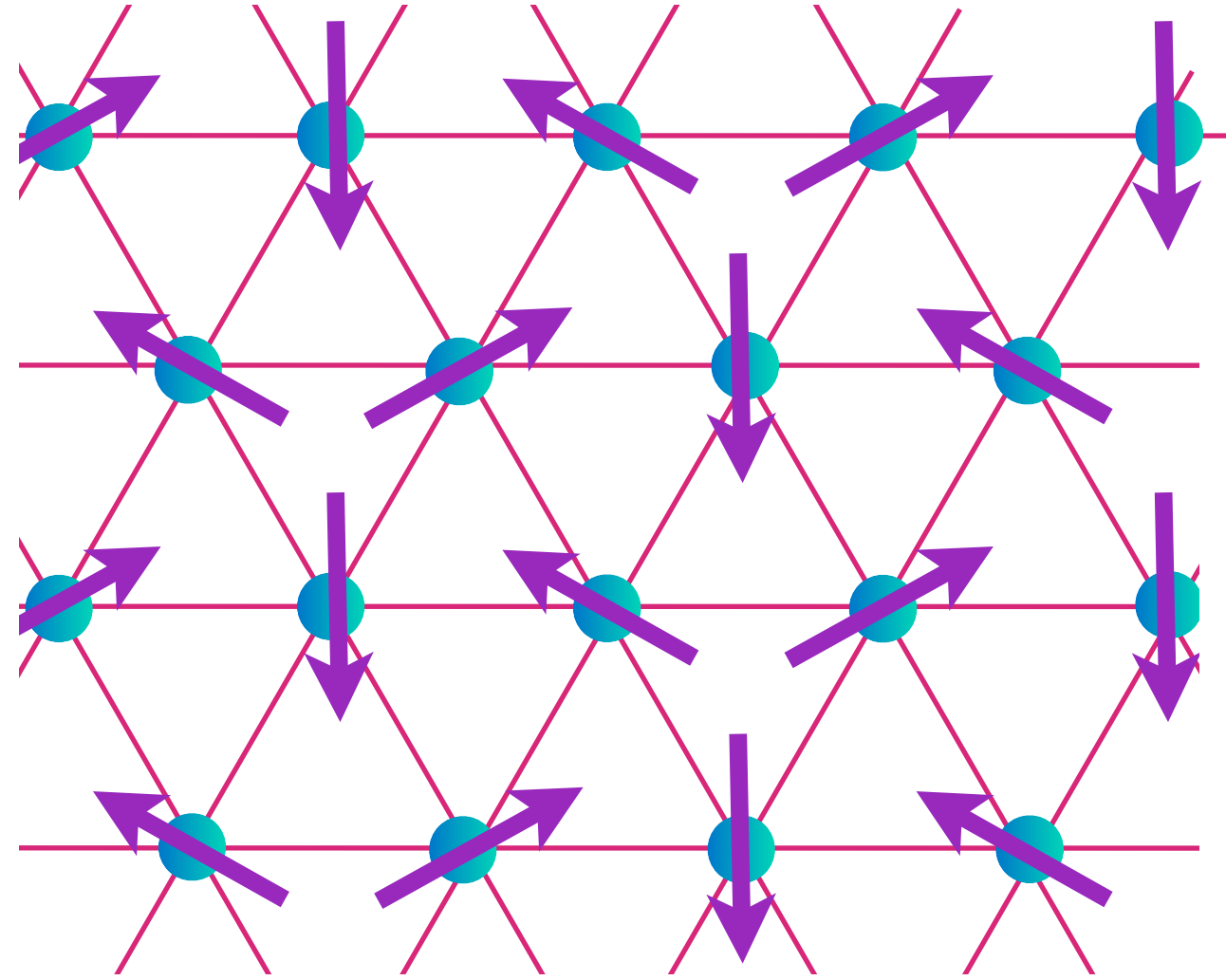
Read and Sachdev (1990):

The RVB spin liquid has an emergent $\mathbb{Z}_2$ gauge field.

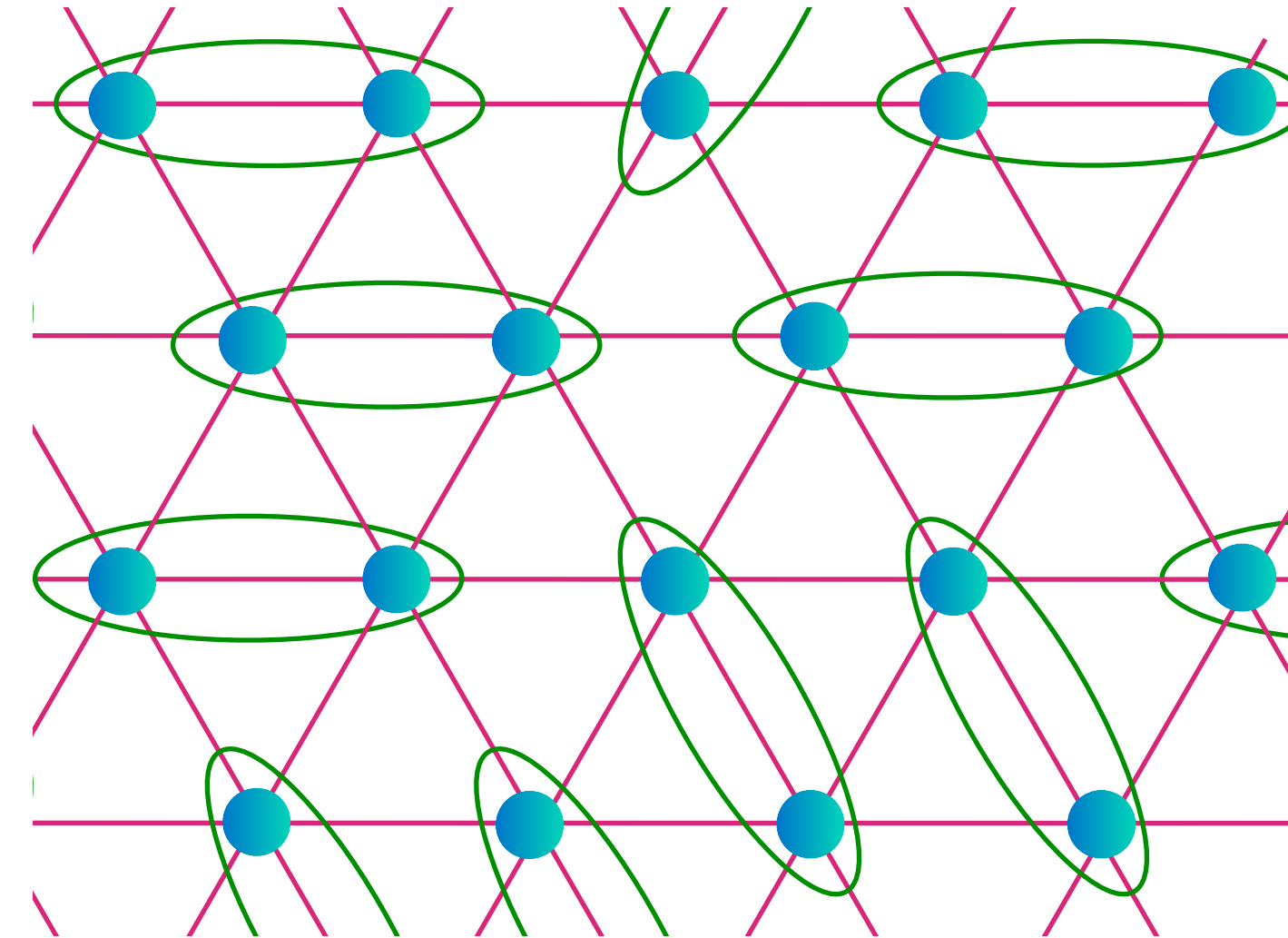
The spinons carry electric $\mathbb{Z}_2$ gauge charges.

There are also emergent particles ('visons' or ' m ') which carry magnetic $\mathbb{Z}_2$ gauge charges.

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$\mathbb{Z}_2$ spin liquid



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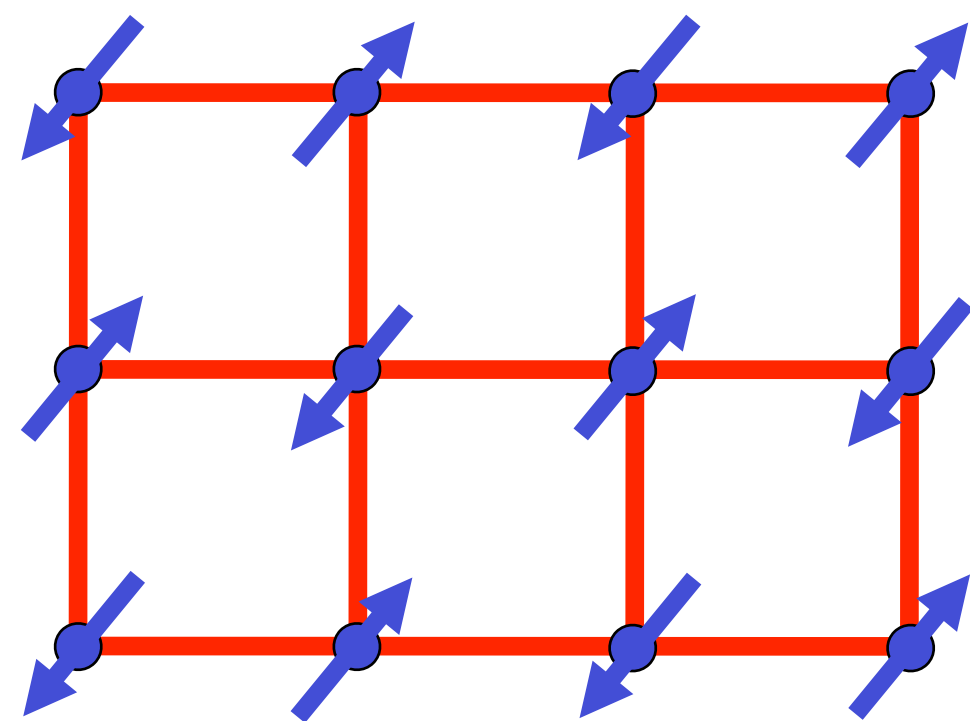
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There are also emergent particles (‘visons’ or ‘ m ’) which carry magnetic $\mathbb{Z}_2$ gauge charges.

The superselection structure of these excitations (“unitary modular tensor category”) is the same as that found in Kitaev’s toric code (1997).

$S=1/2$ square lattice



$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Gapped spin liquid

N/n_b

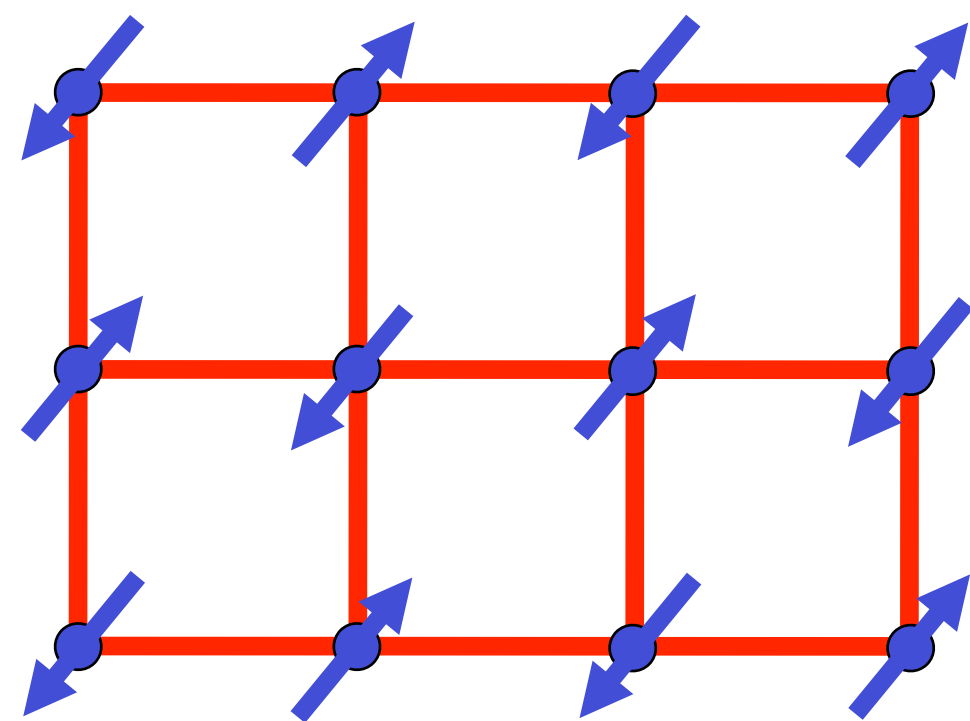
Bosonic partons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$

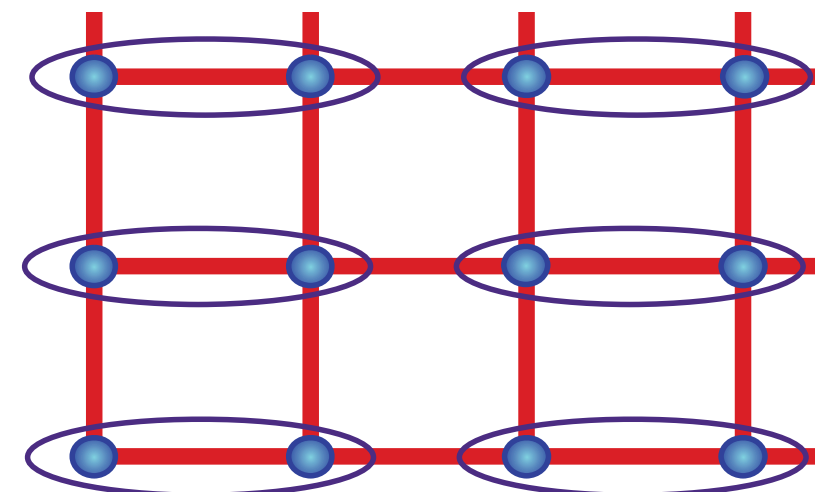


D. Arovas and A. Auerbach, PRB **38**, 316 (1988)

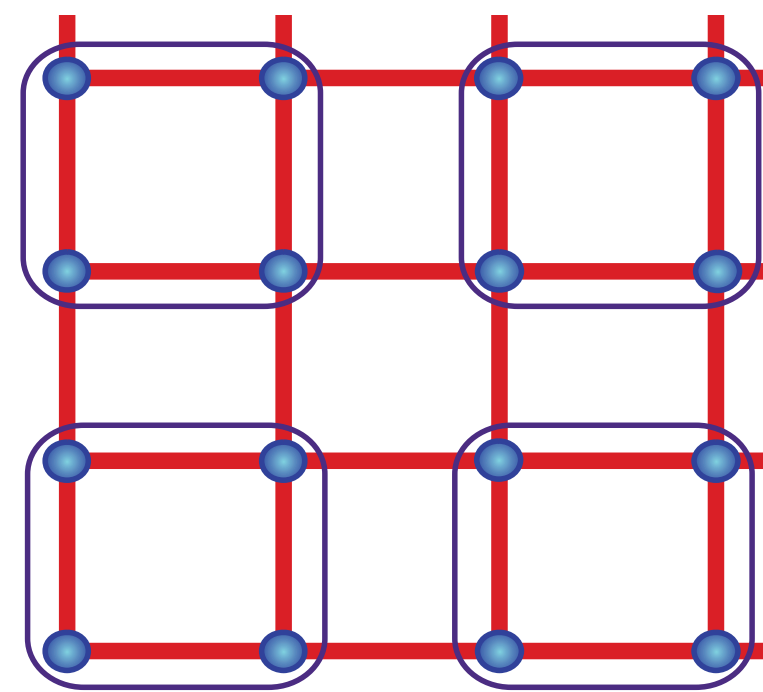
$S=1/2$ square lattice



$\langle b_\alpha \rangle \neq 0$:
Néel order



or



$\langle b_\alpha \rangle = 0$:
Valence bond solid (VBS)

J_2/J_1

Bosonic partons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$

U(1) gauge symmetry: $b \rightarrow be^{i\theta}$

$\mathbb{CP}^1$ U(1) gauge theory.

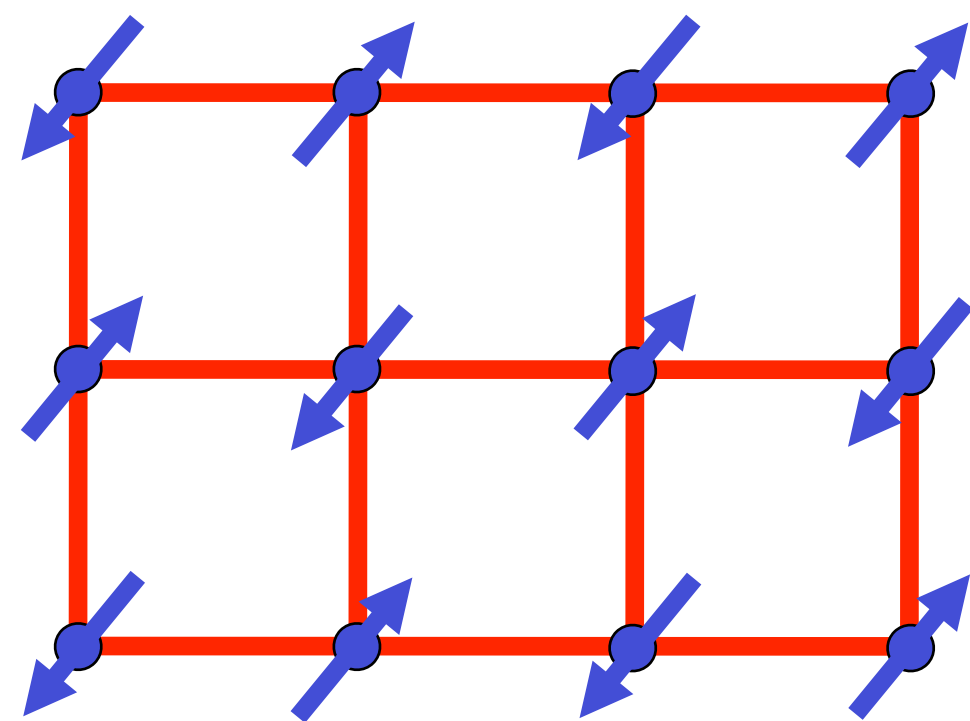
Monopole Berry phases
implement
Lieb-Schultz-Mattis (LSM)
'anomalies'.

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}^\dagger; \quad \mathcal{L}_z = |(\partial_\mu - ia_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

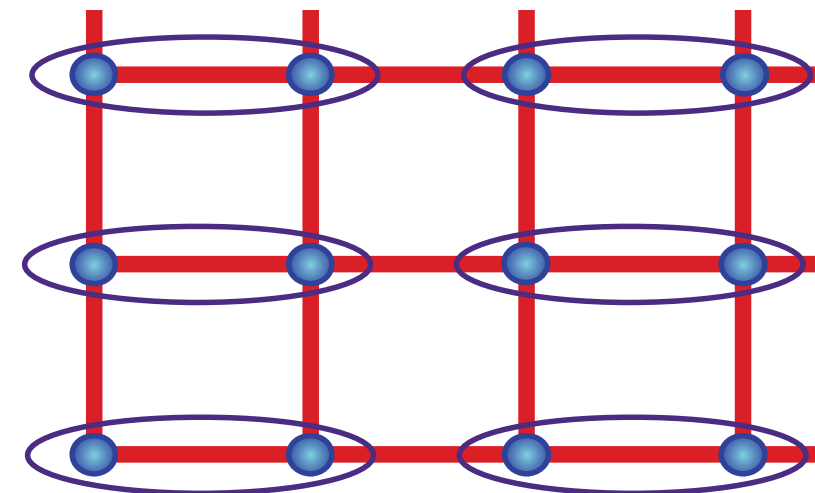
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)

N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

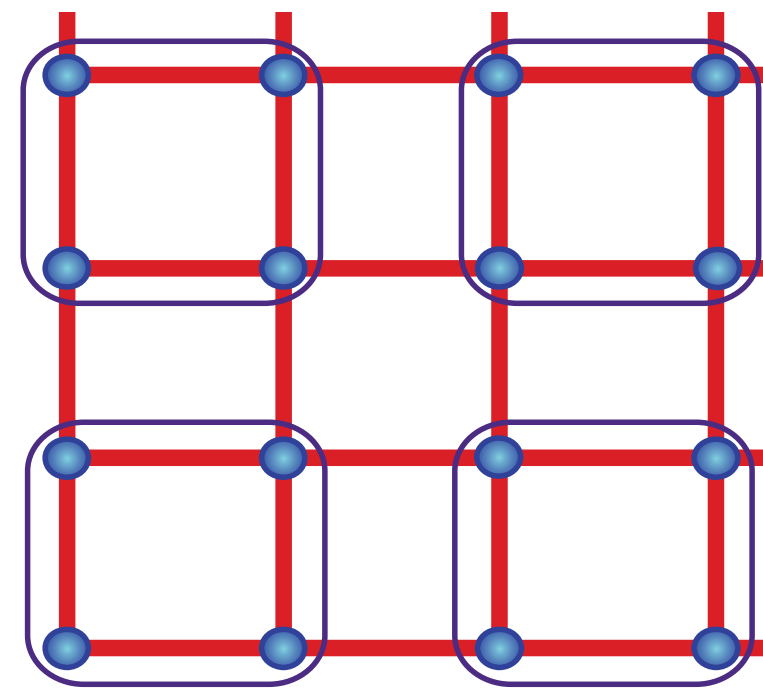
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$\langle b_\alpha \rangle = 0$:
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J_2/J_1

Critical, gapless
spin liquid?

Bosonic partons

$$\mathbf{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$

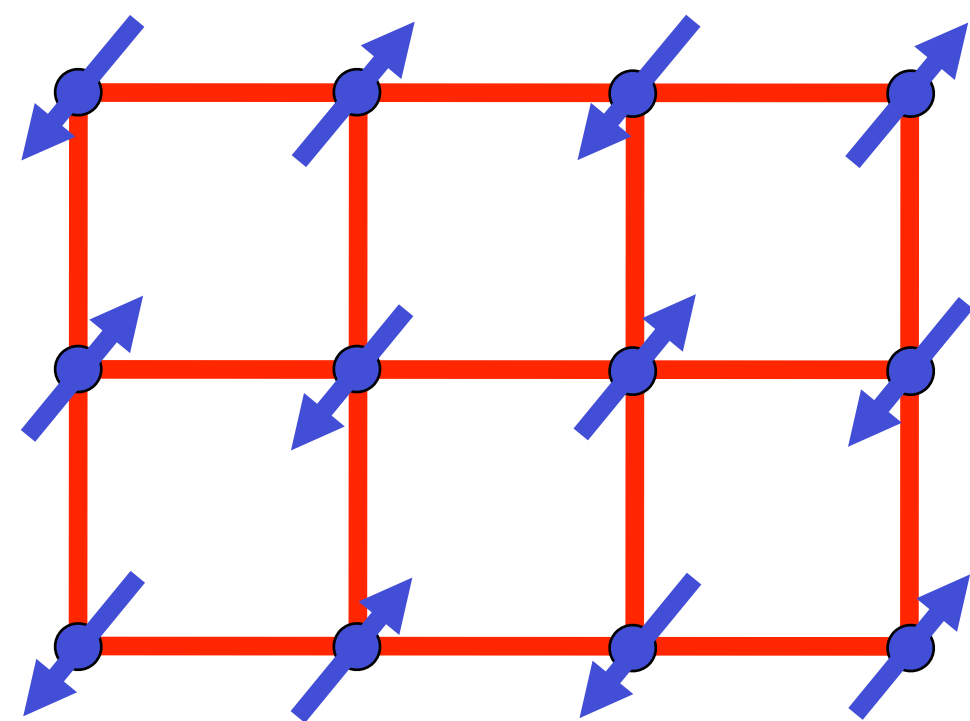
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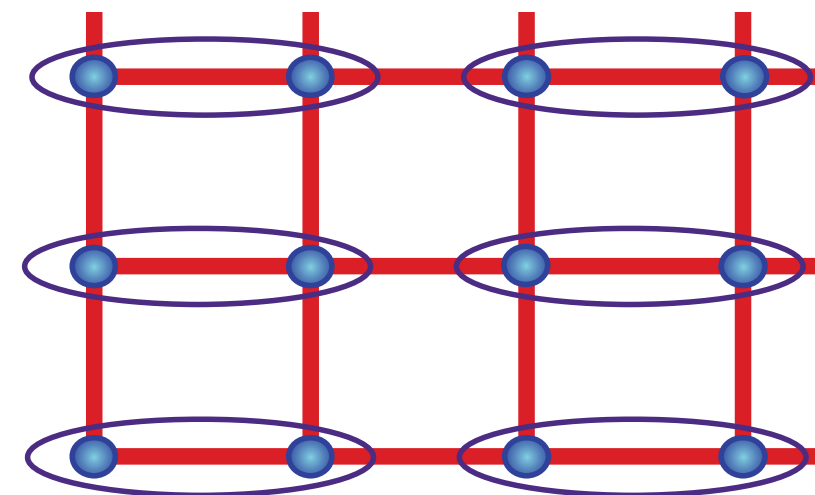
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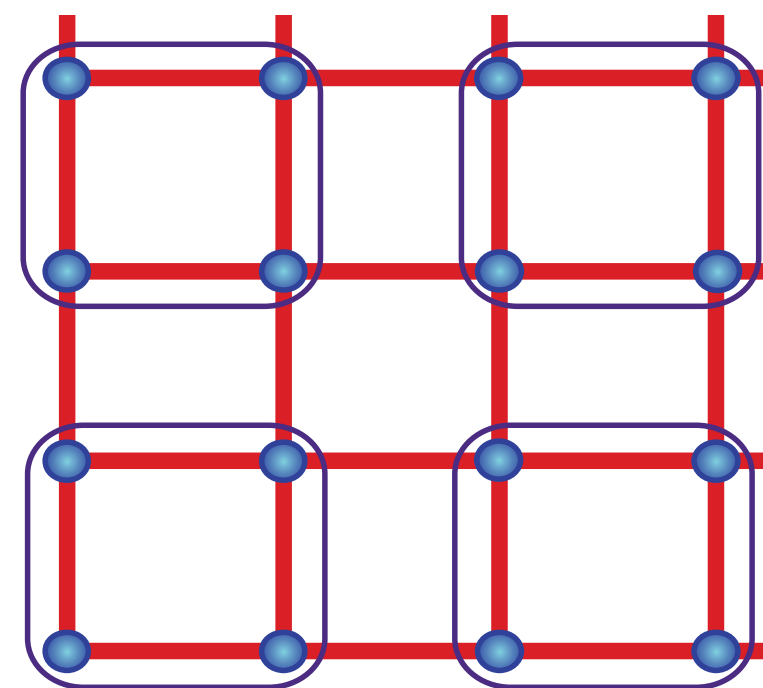
$S=1/2$ square lattice



Néel order



or



Valence bond solid (VBS)

J_2/J_1

Critical, gapless spin liquid?

$$\mathcal{L} = i\bar{\Psi}_s \gamma_\mu D_\mu \Psi_s.$$

Fermionic partons

$$\mathbf{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$



I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

Ying Ran and X.-G. Wen, cond-mat/0609620

Critical gapless spin liquid:
 $SU(2)$ QCD with
 $N_f = 2$ massless fermions.
 Confining instability to precisely
 the Néel and VBS orders
 of $\mathbb{CP}^1$ theory.

$S=1/2$ square lattice

Bosonic spinons:
 $\mathbb{CP}^1$ U(1) gauge theory

SU(2) gauge theory of $N_f = 2$
fundamental, massless, Dirac fermions.

Obtained from a saddle-point of
fermionic spinons moving in π -flux.

SO(5) non-linear σ -model
of Néel/VBS orders
with $k = 1$ WZW term

$S=1/2$ square lattice

Bosonic spinons:
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Nearly-
critical
 $S=1/2$
square
lattice anti-
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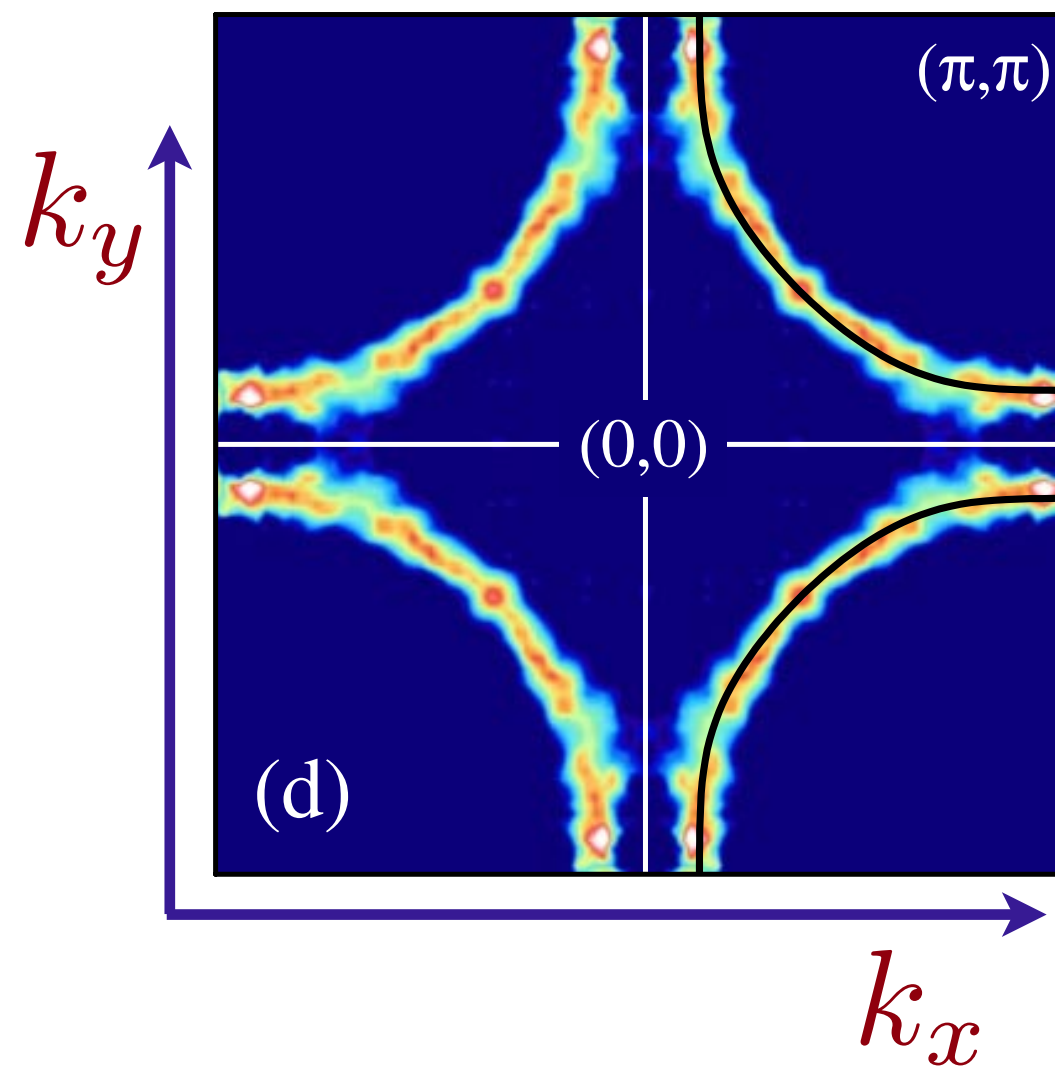
SO(5) non-linear σ -model
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Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Fractionalization
in a metal:
Fractionalized
Fermi liquids (FL*)

Ordinary metals

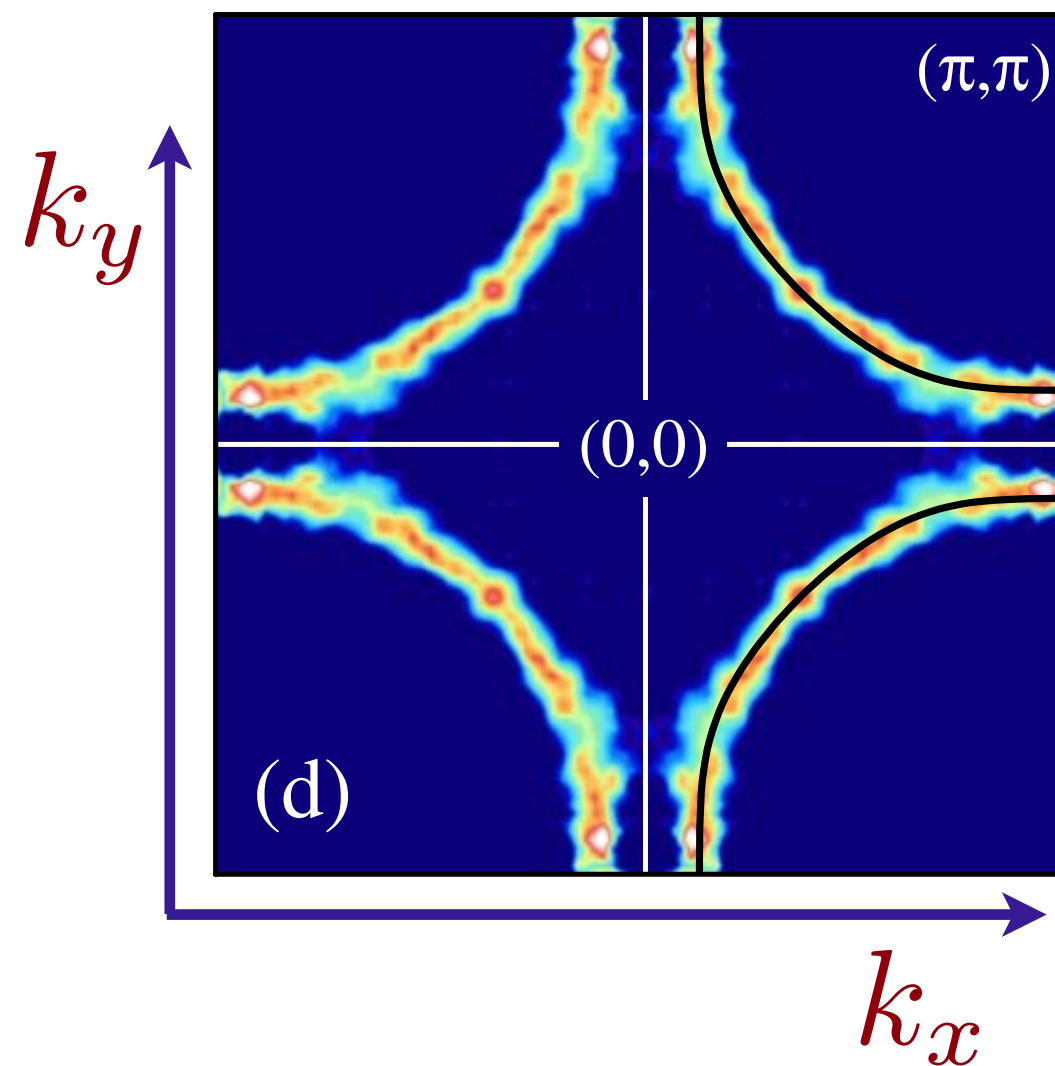
Nearly-free gas of fermions, with a Fermi surface between empty and occupied states (Sommerfeld, 1927).



Ordinary metals

Nearly-free gas of fermions, with a Fermi surface between empty and occupied states (Sommerfeld, 1927).

Area enclosed by the Fermi surface is the same as that for free fermions with the same symmetry
Luttinger, 1960 - perturbative



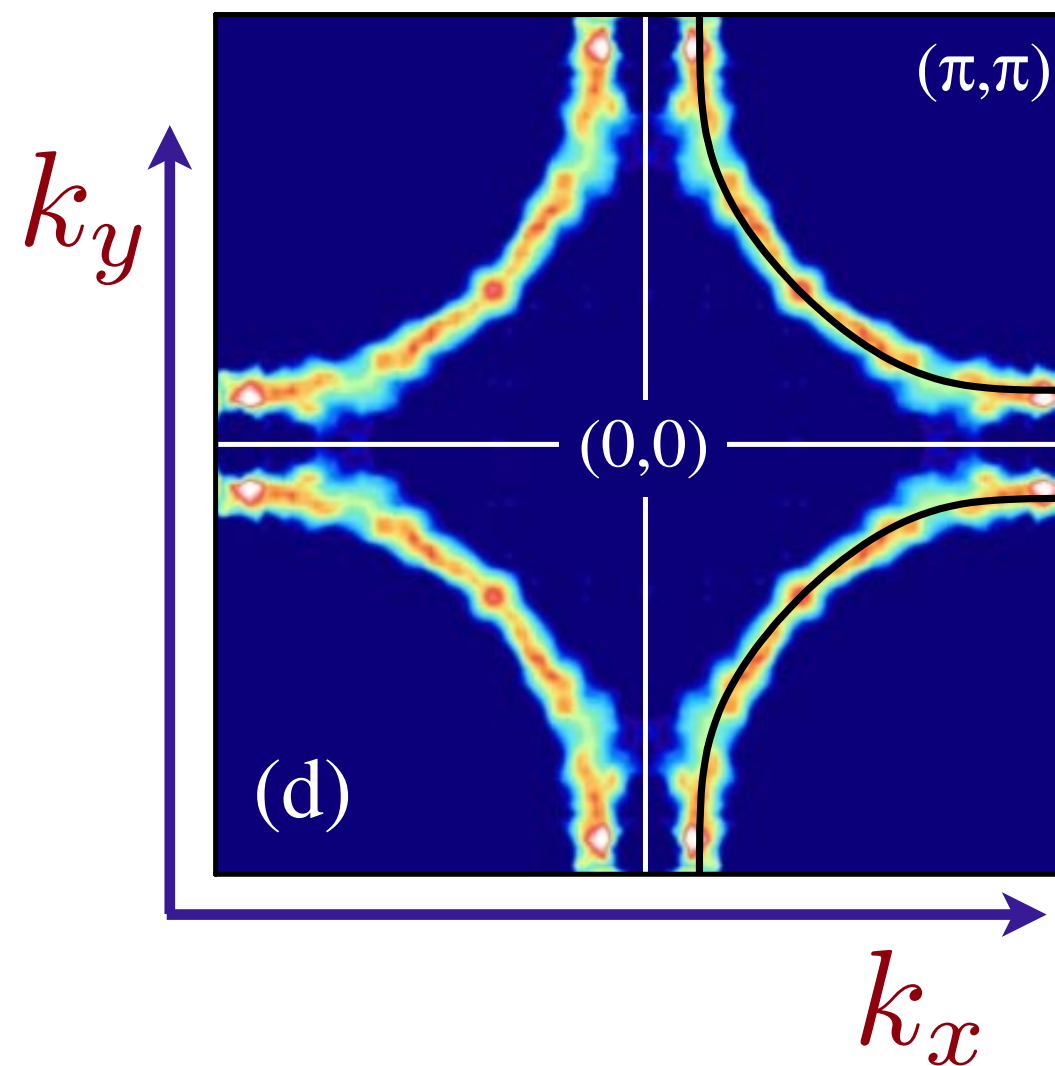
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Luttinger, 1960 - perturbative;

Oshikawa, 2000 - non-perturbative using LSM anomaly



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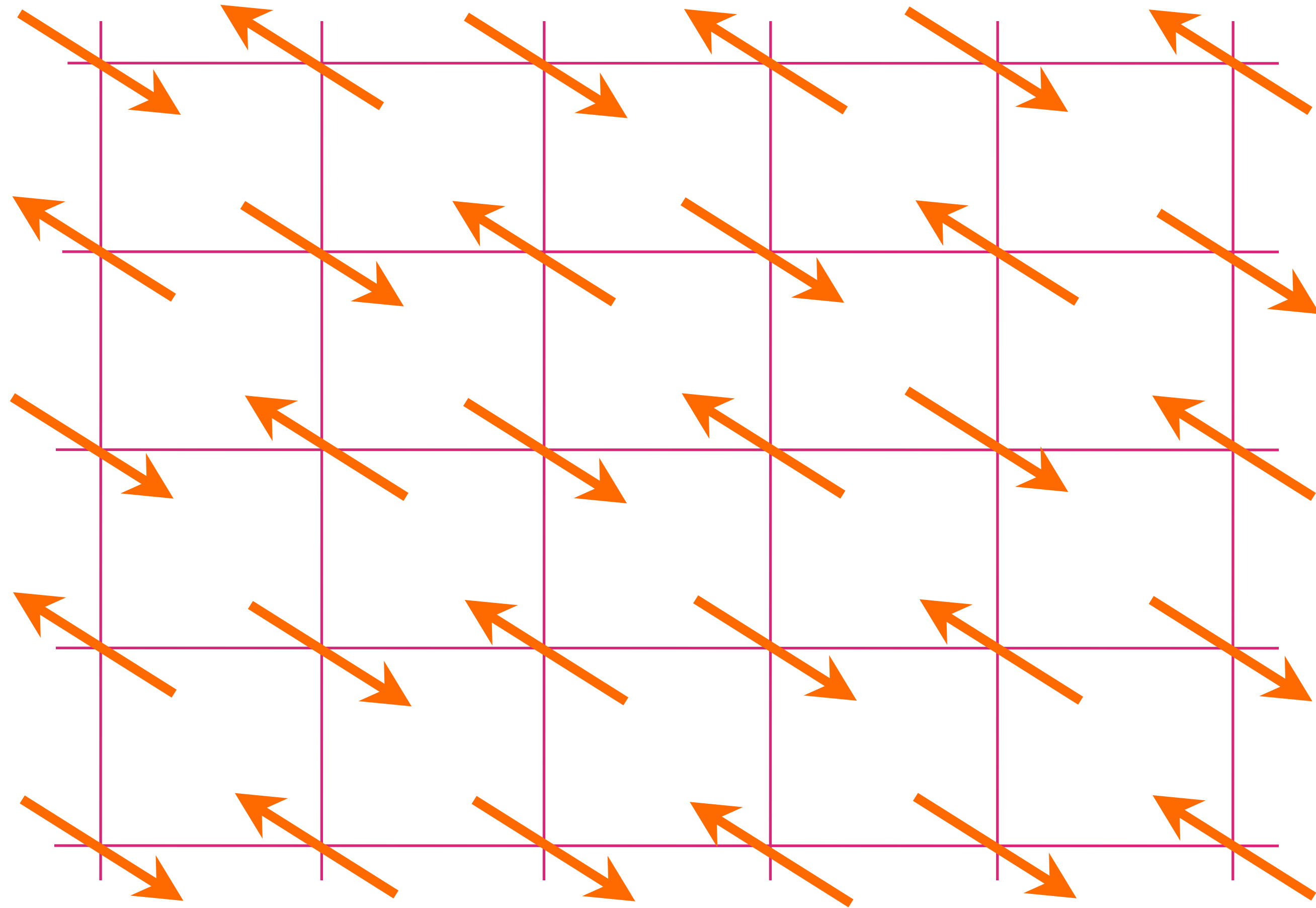
Luttinger, 1960 - perturbative;

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Fractionalized Fermi liquids (FL*)

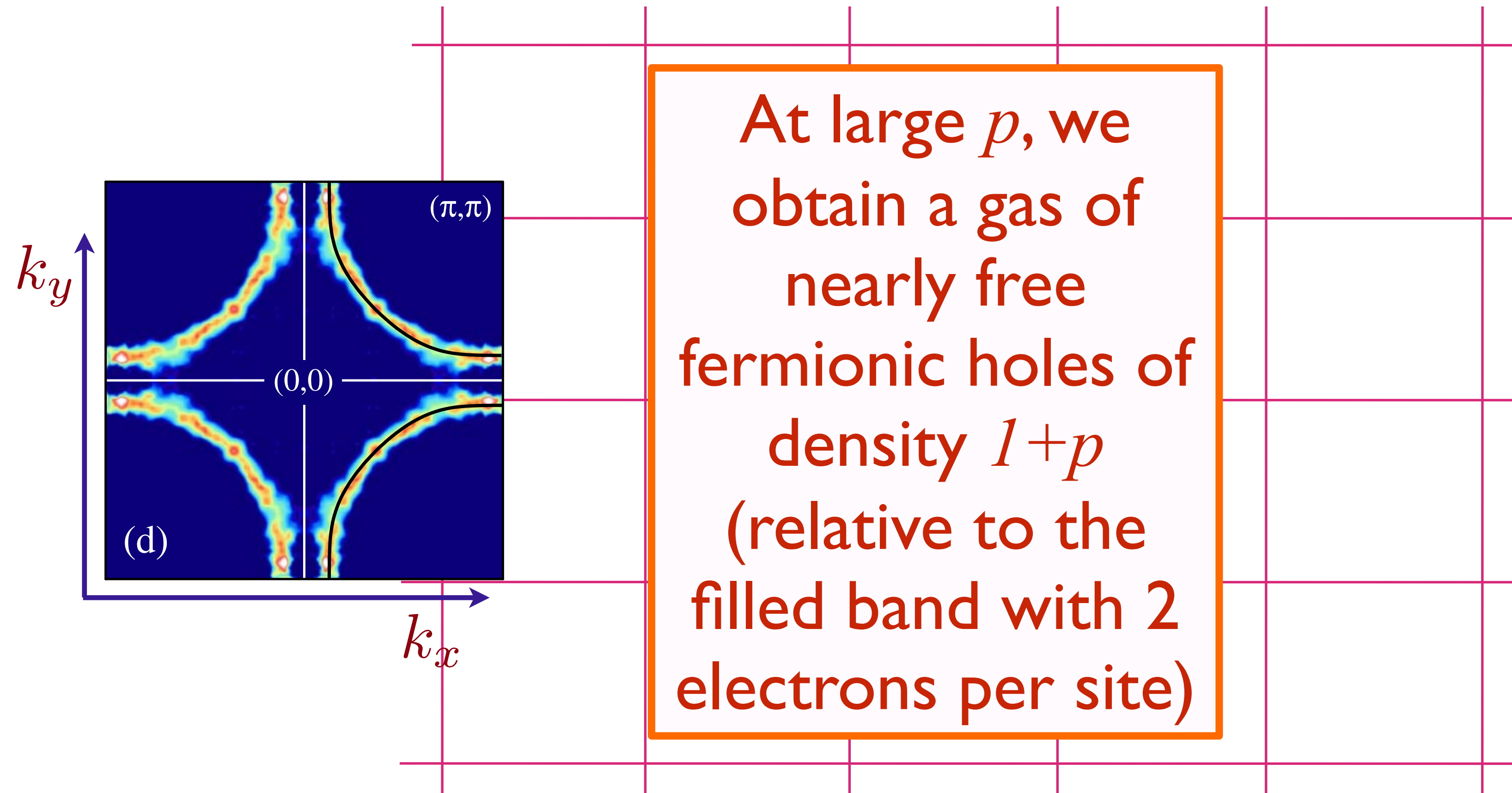
There can be metals with Fermi surface area not equal to the Luttinger-Oshikawa value provided the fractionalized excitations of a spin liquid are also present.

Insulating antiferromagnet

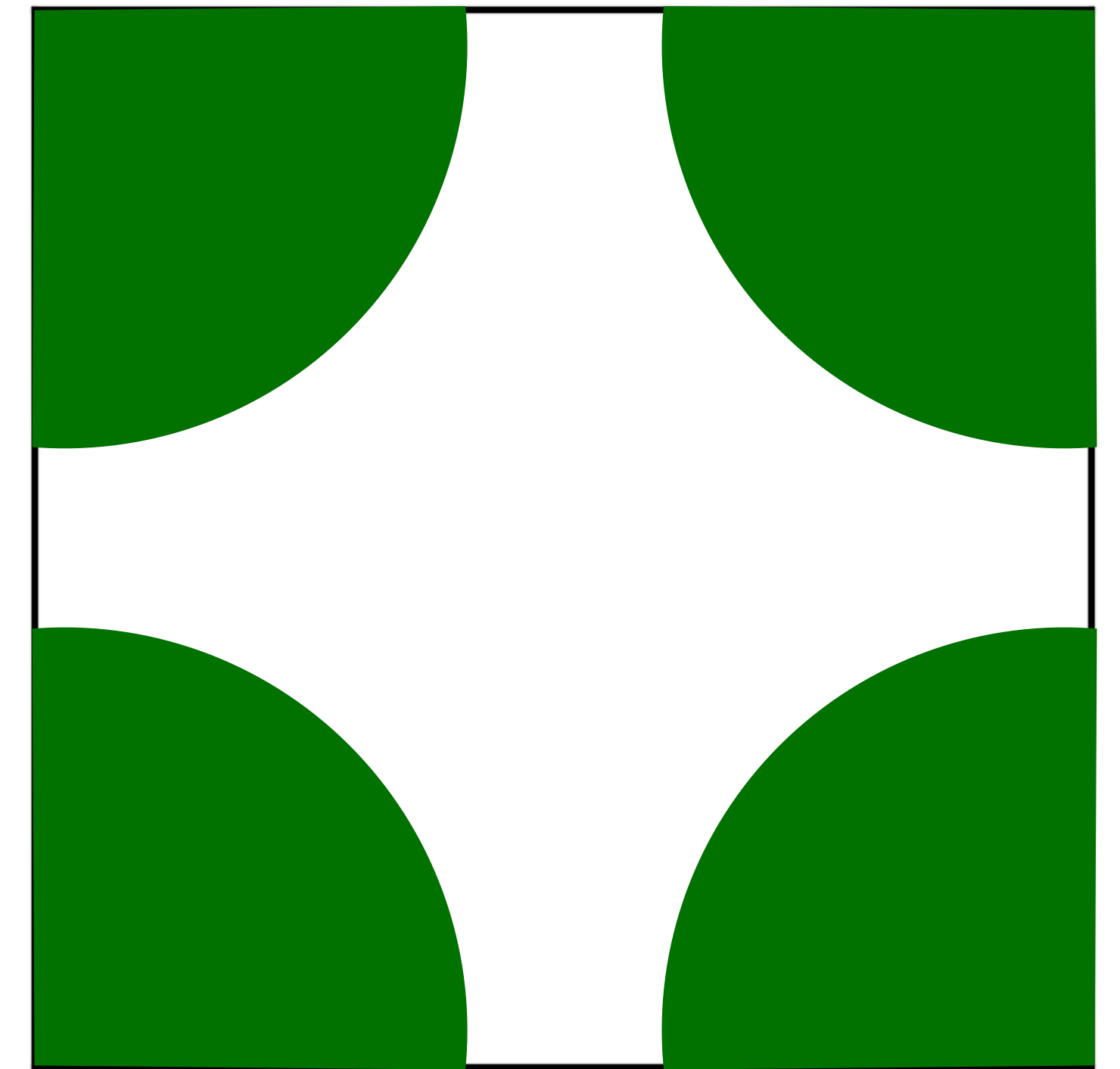


Doping an insulating antiferromagnet with holes of density p

Ordinary metal



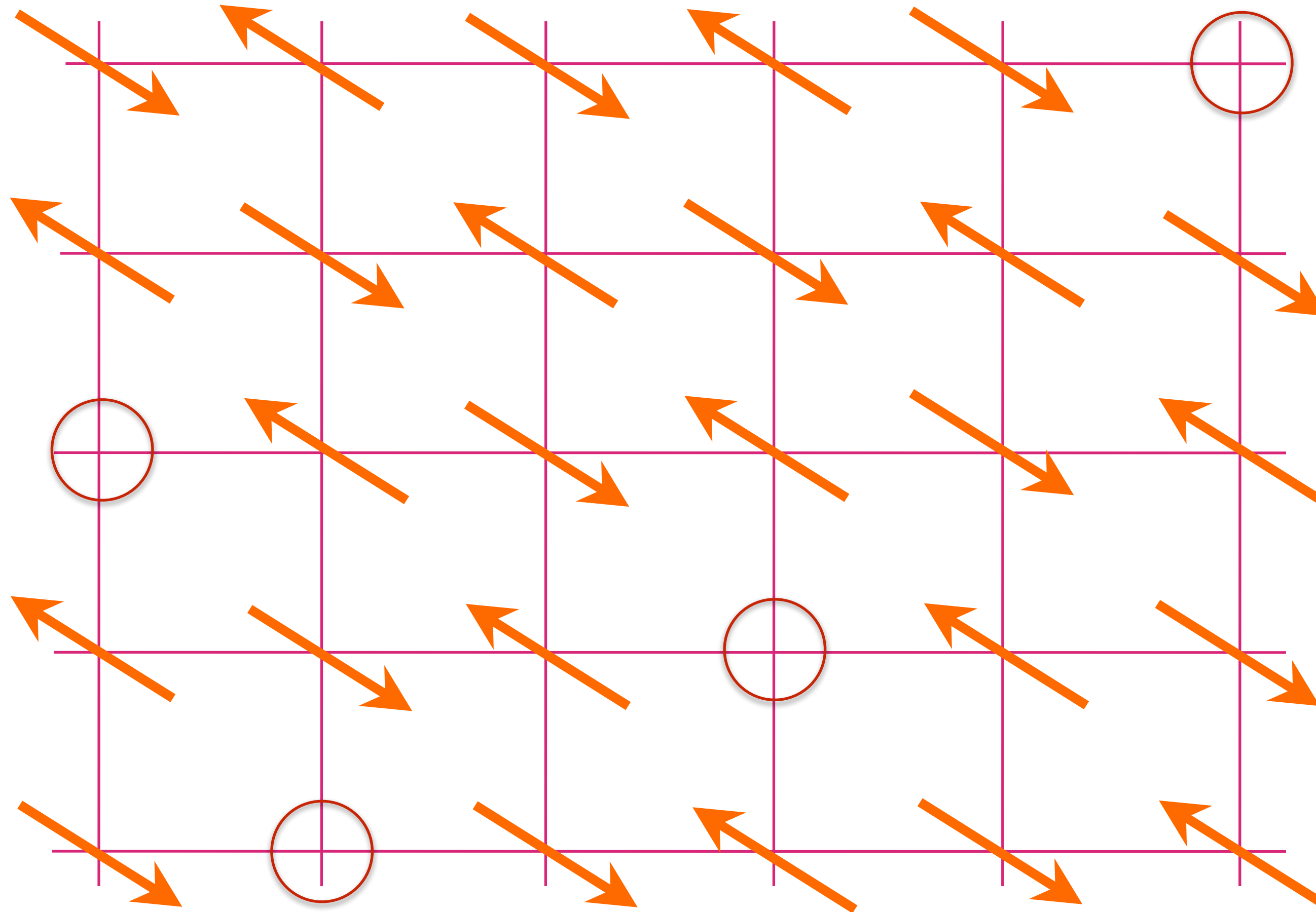
Luttinger area.
No broken symmetry



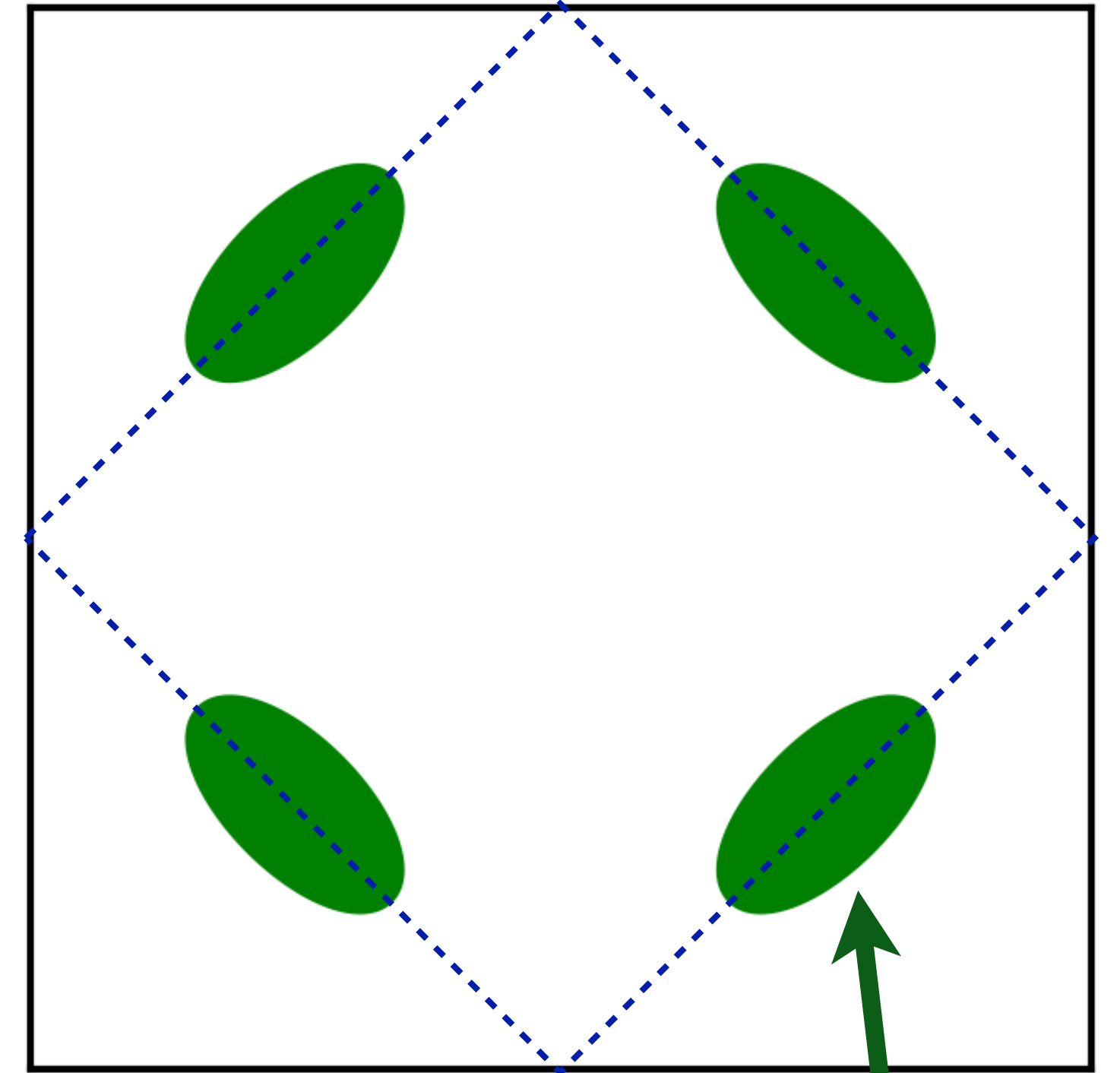
Area $(1+p)/2$

Doping an insulating antiferromagnet with holes of density p

AF metal



Luttinger area.
Broken symmetry

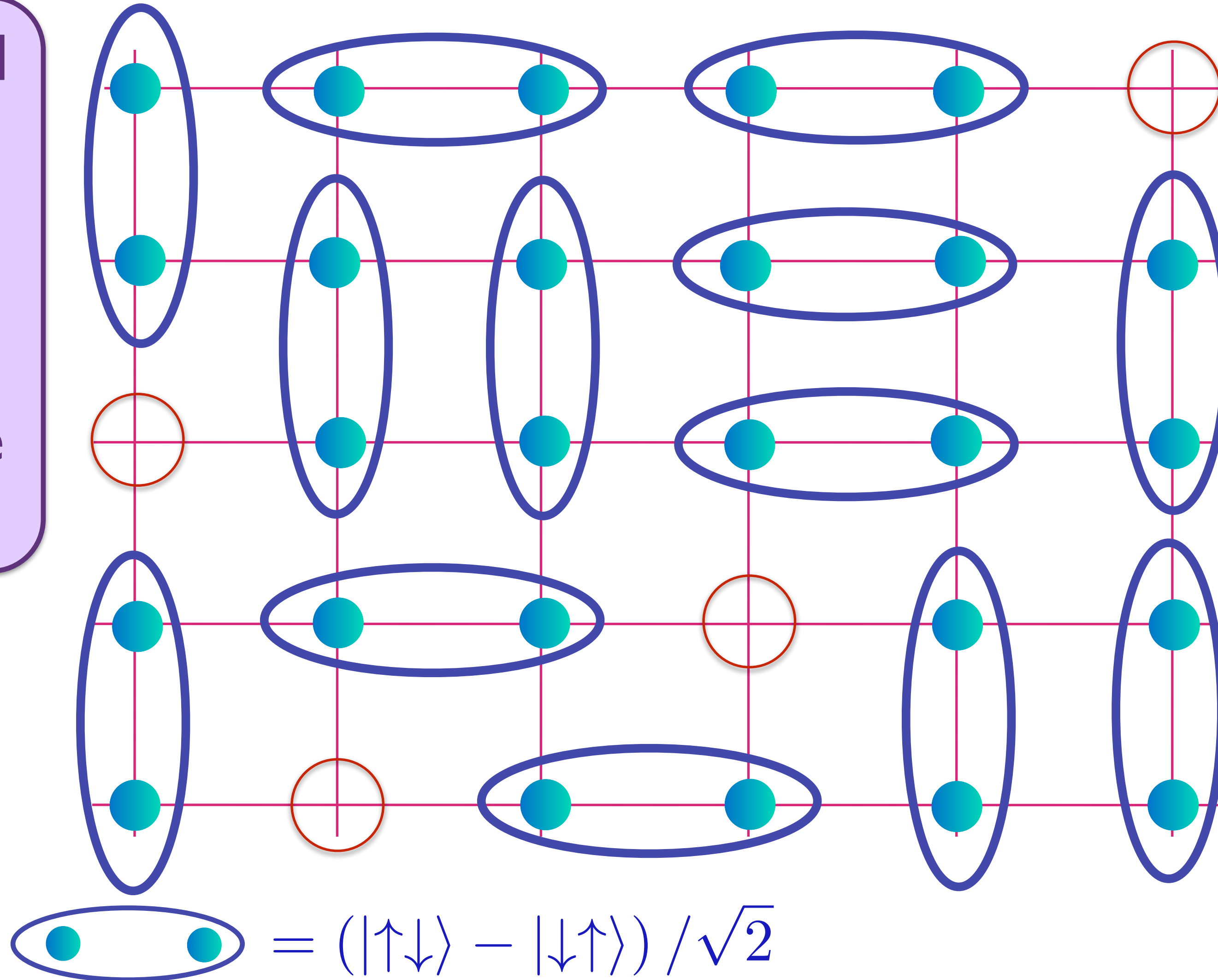


Area $p/4$

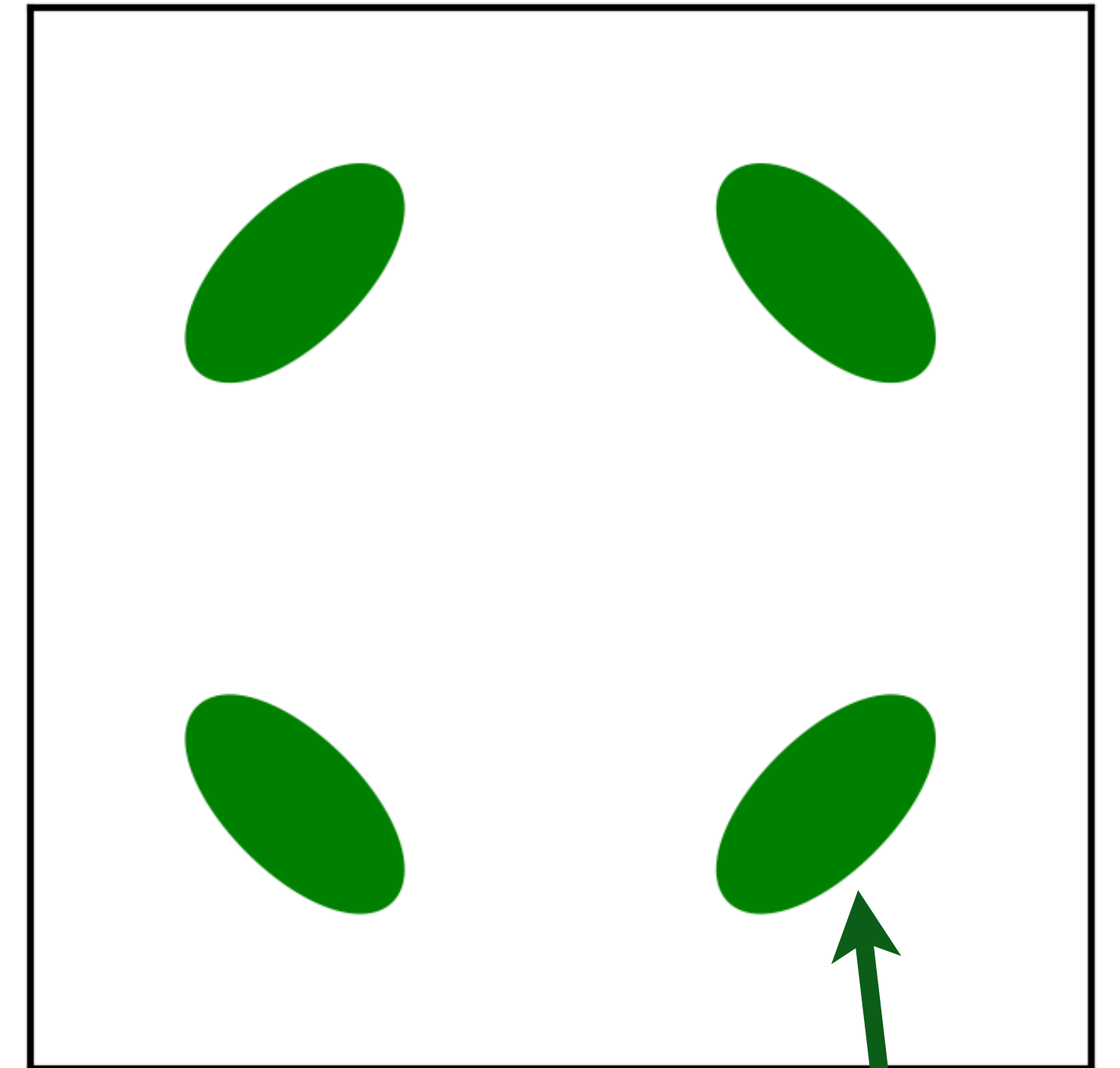
Doping an insulating antiferromagnet with holes of density p

Holon metal

Spin liquid
with
density p
of
spinless,
charge $+e$
“holons”.



non-Luttinger area.
Spin liquid

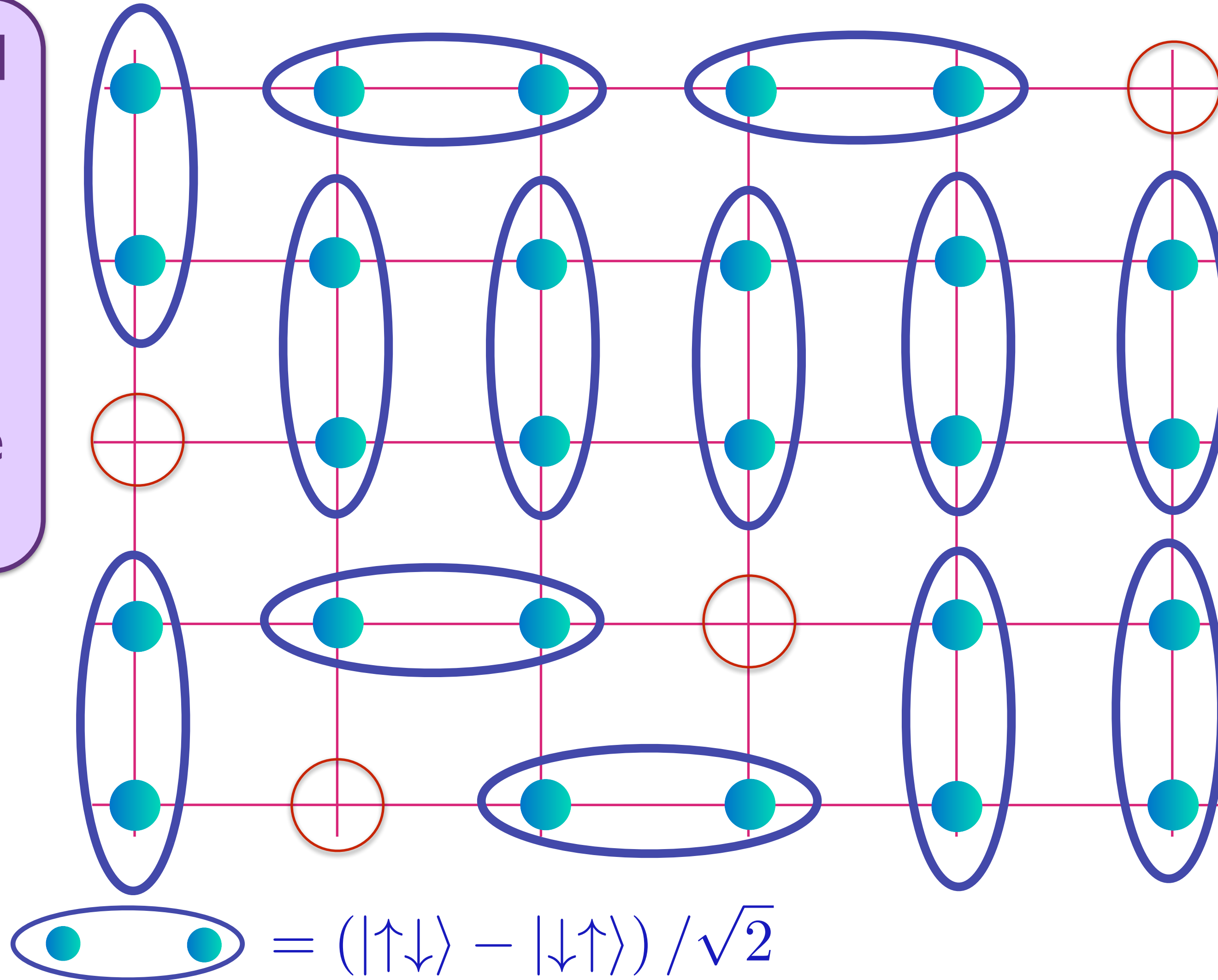


Area $p/4$

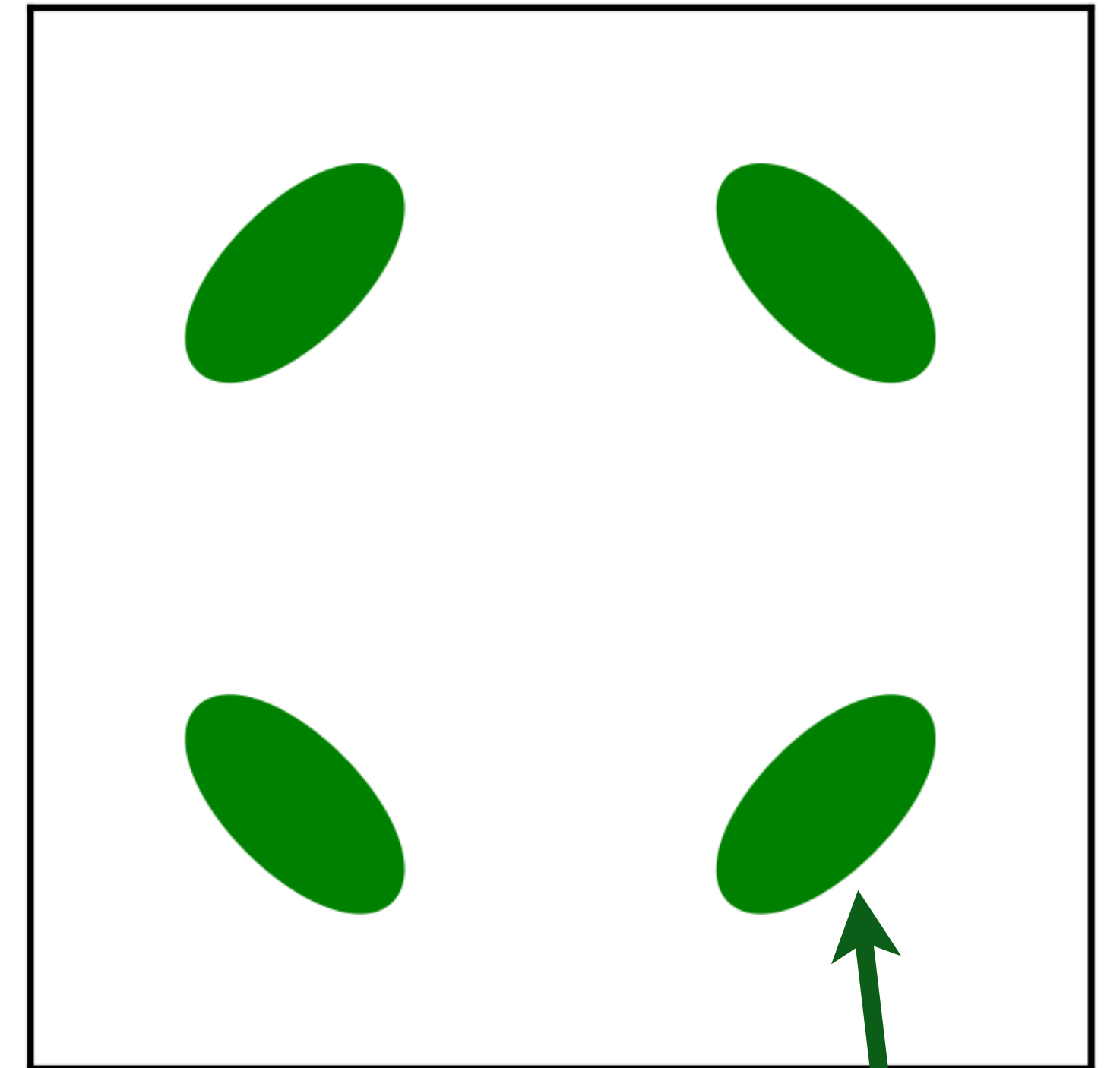
Doping an insulating antiferromagnet with holes of density p

Holon metal

Spin liquid
with
density p
of
spinless,
charge $+e$
“holons”.



non-Luttinger area.
Spin liquid

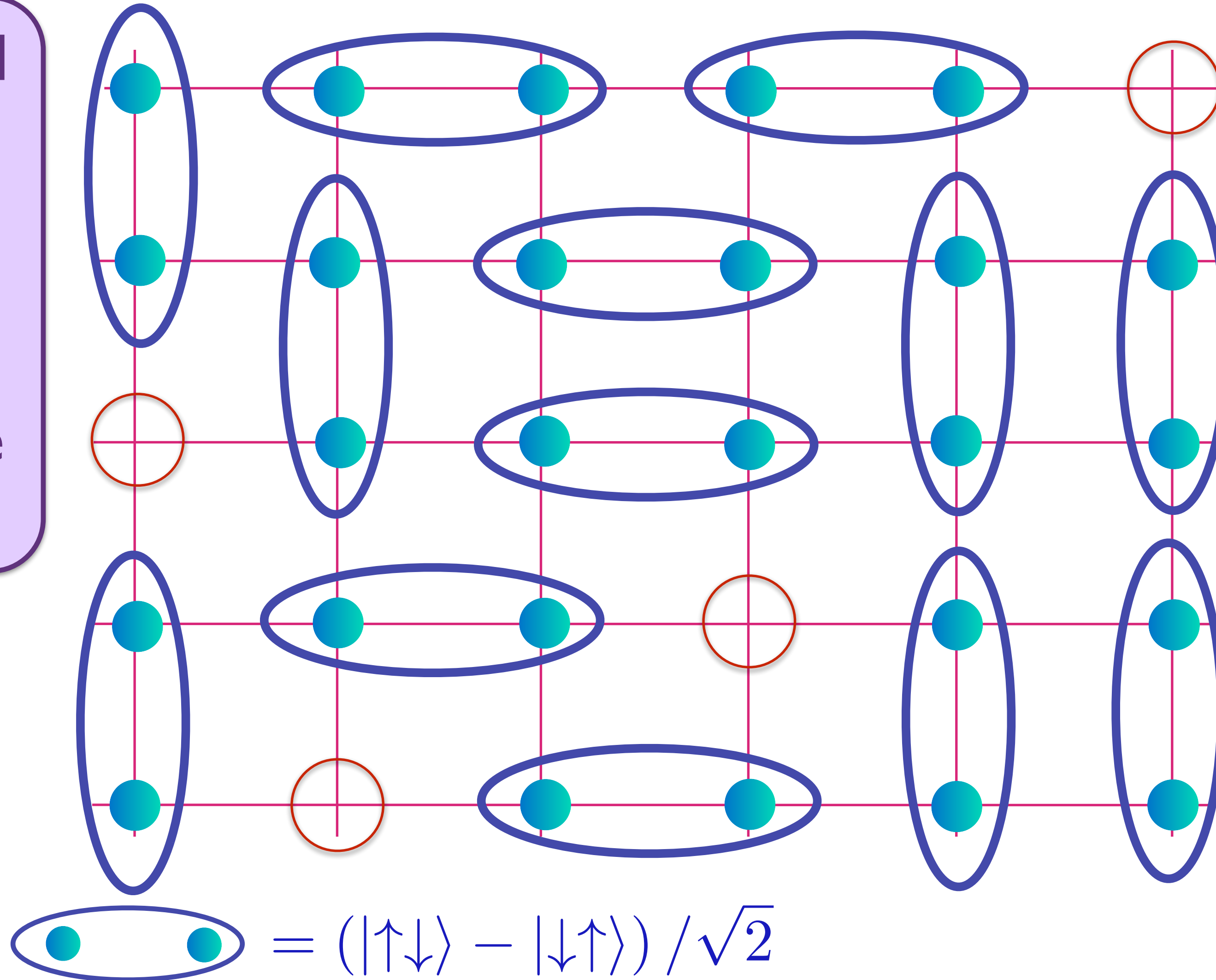


Area $p/4$

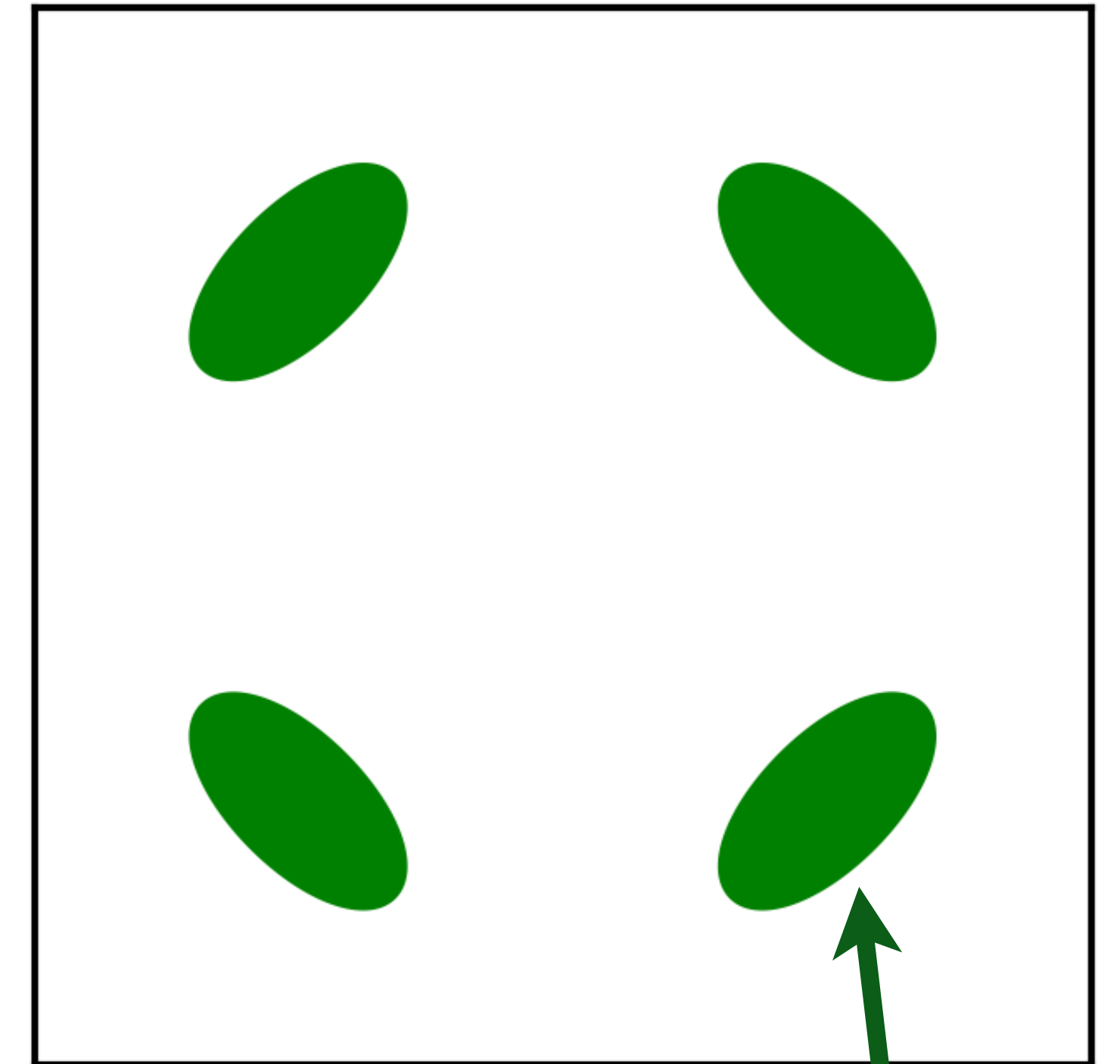
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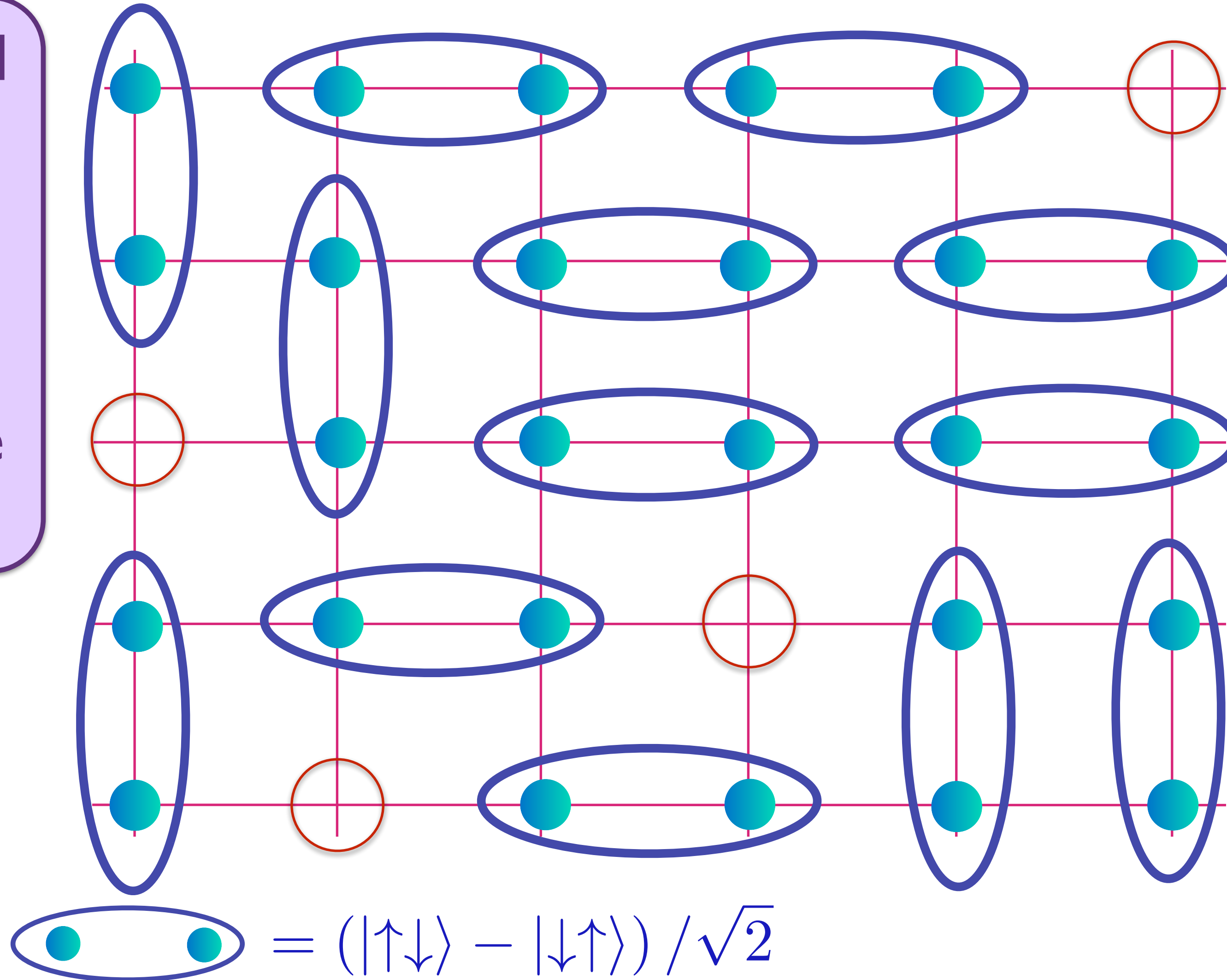


Area $p/4$

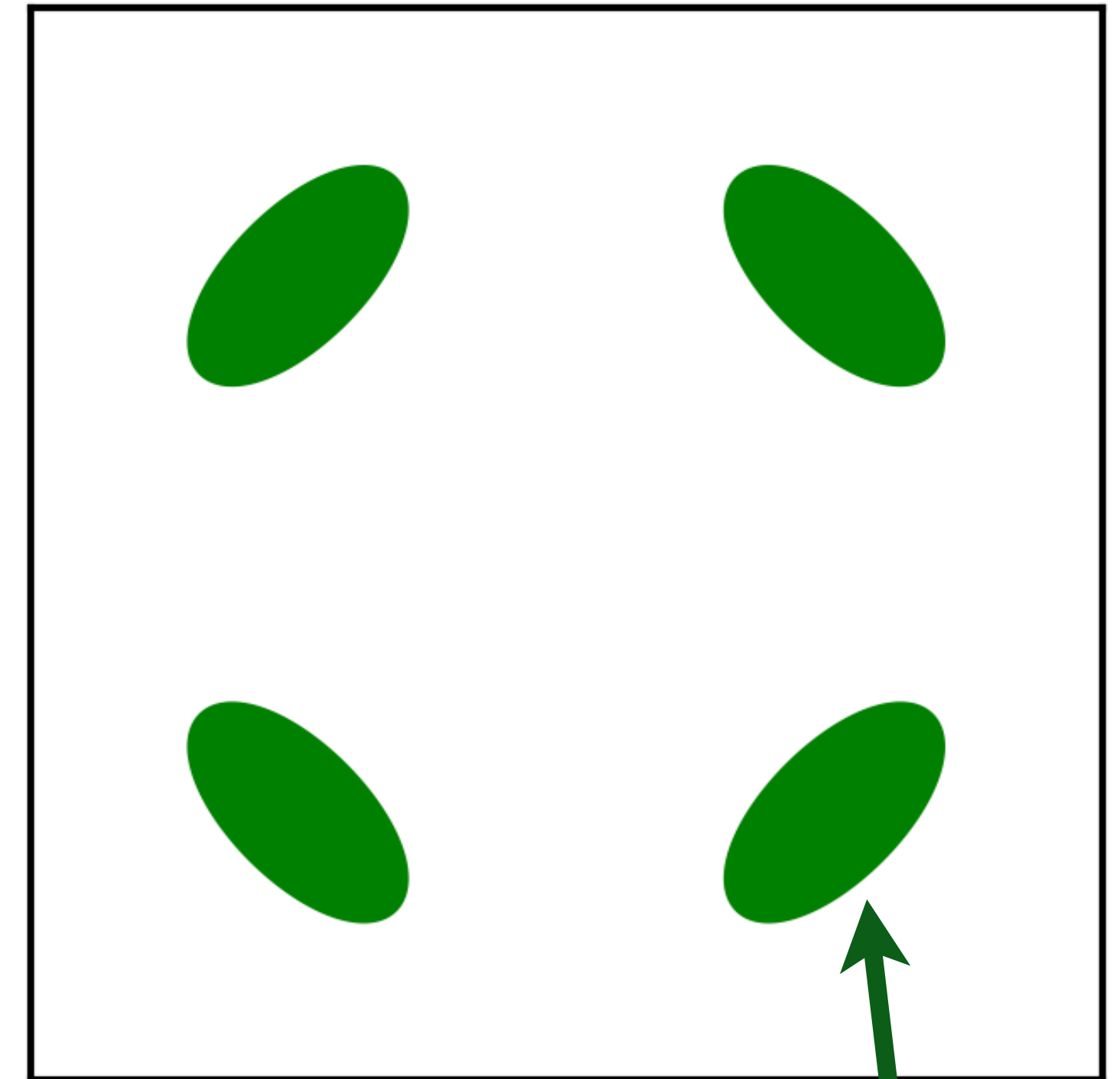
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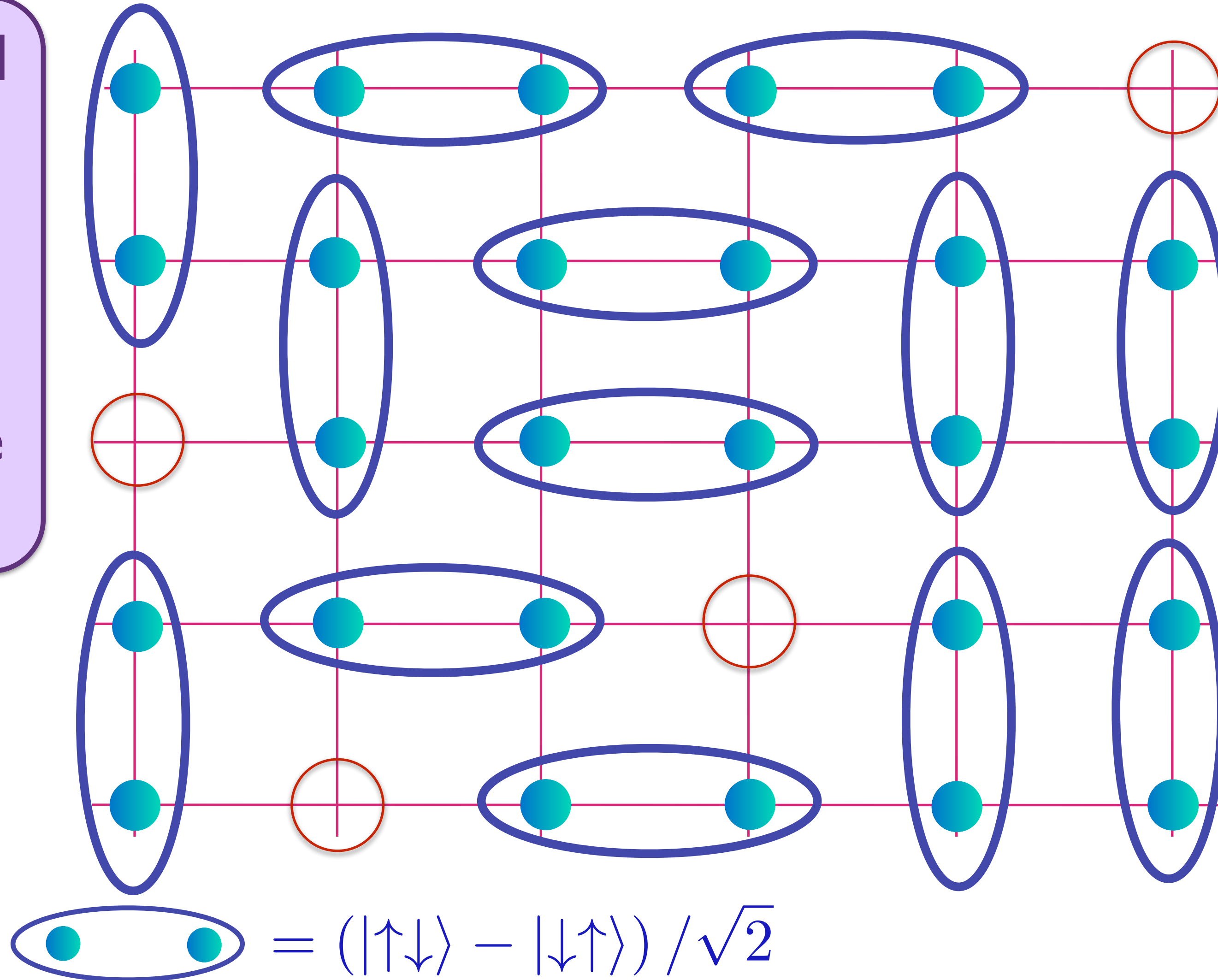


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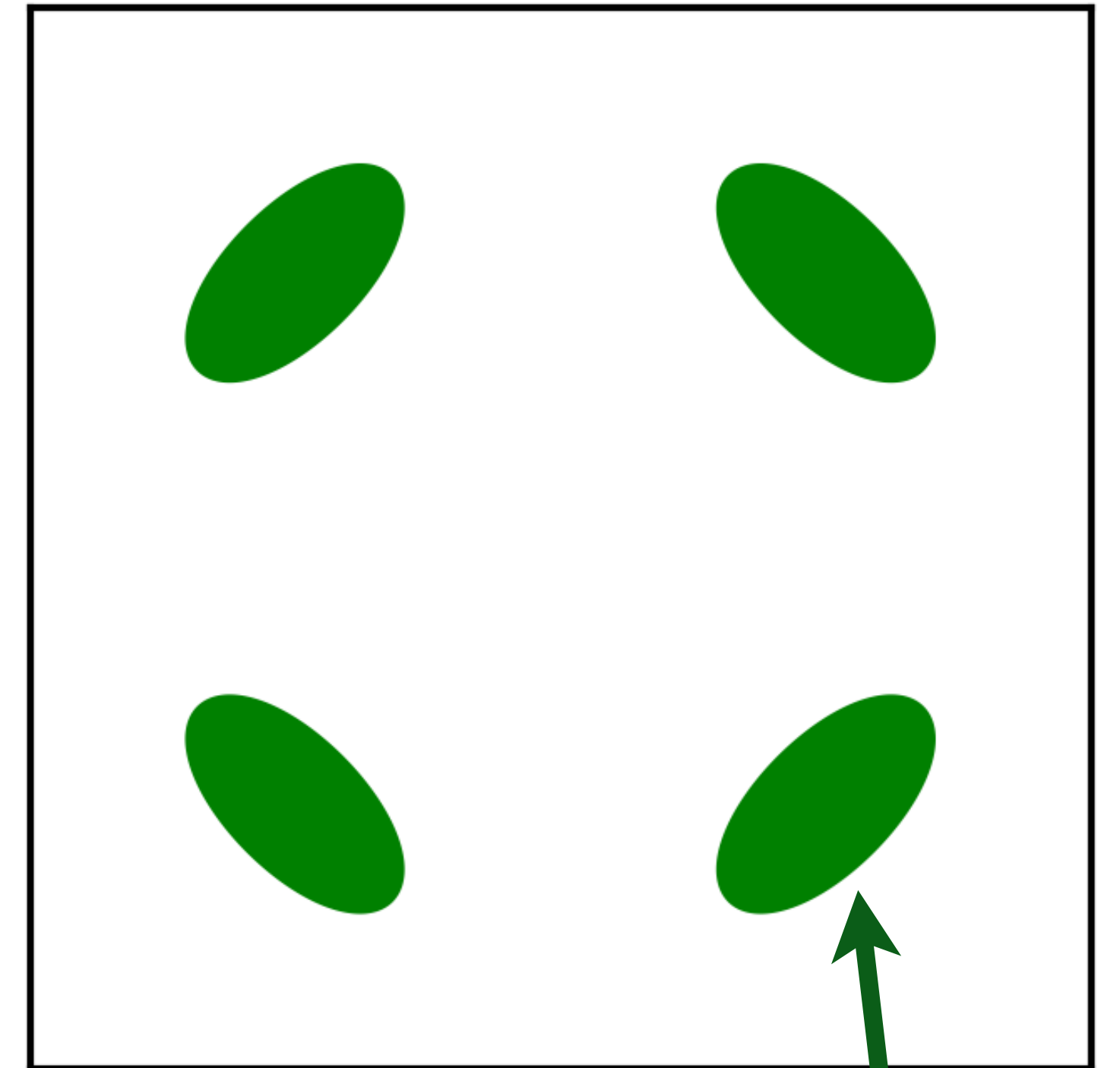
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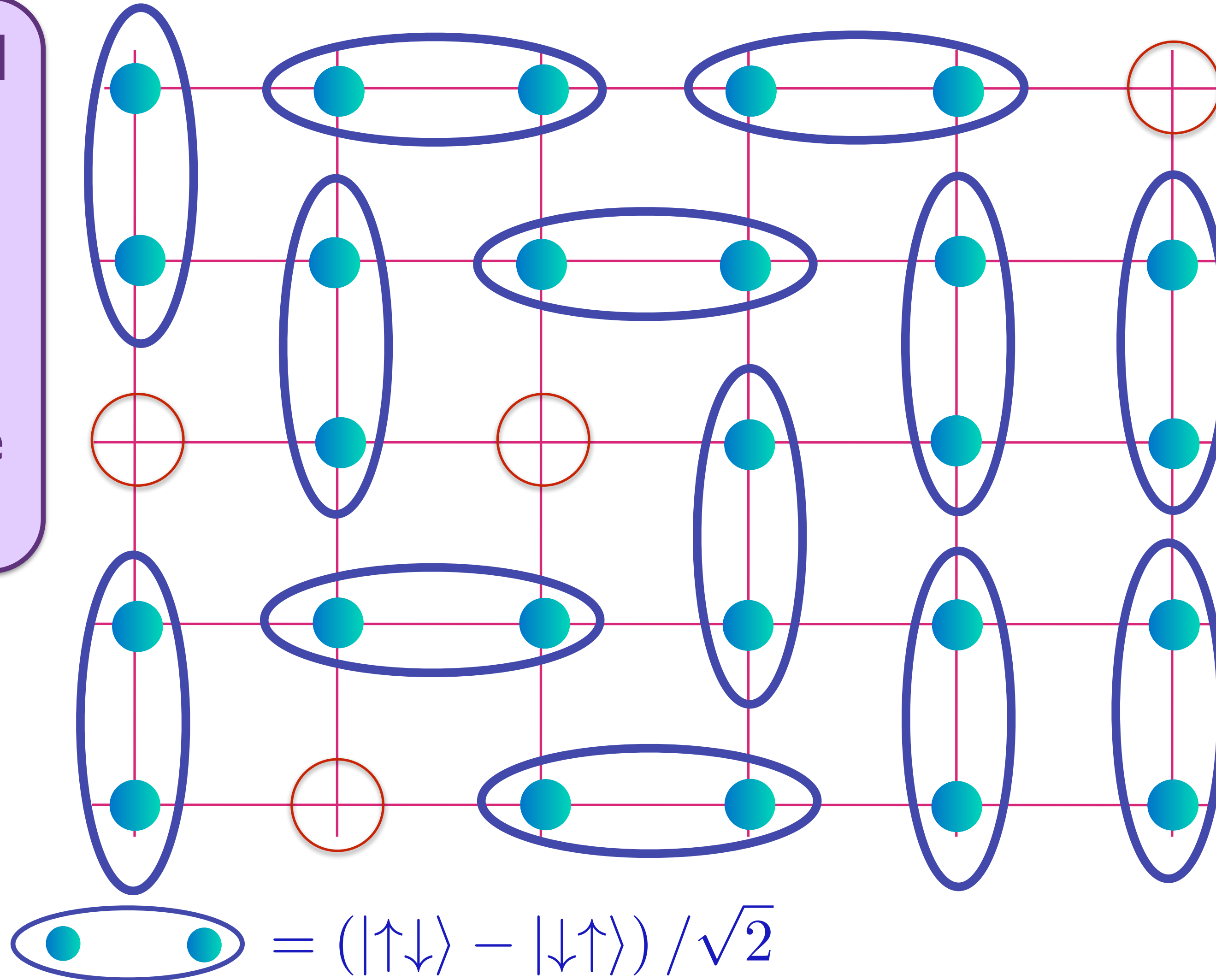


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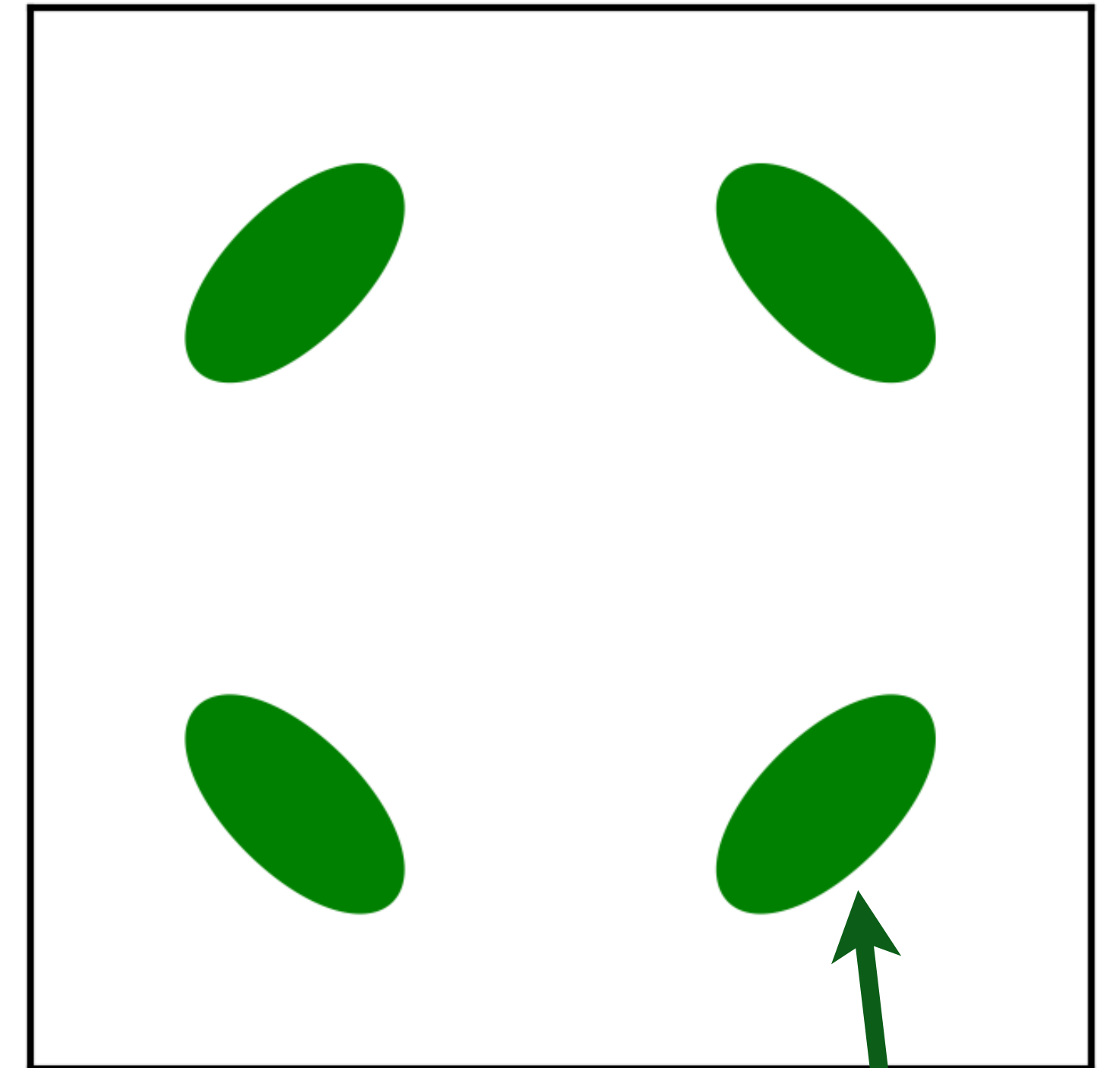
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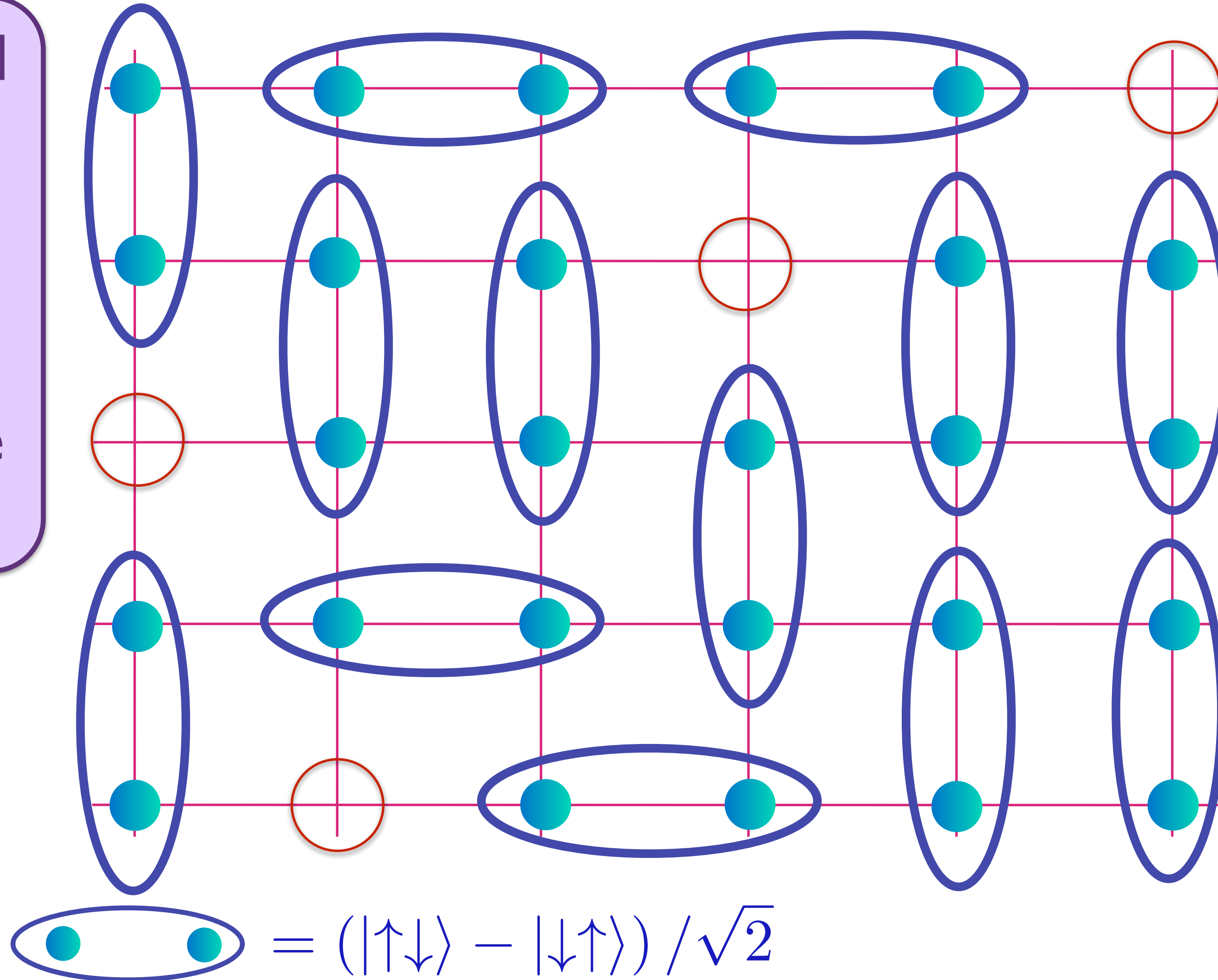


Area $p/4$

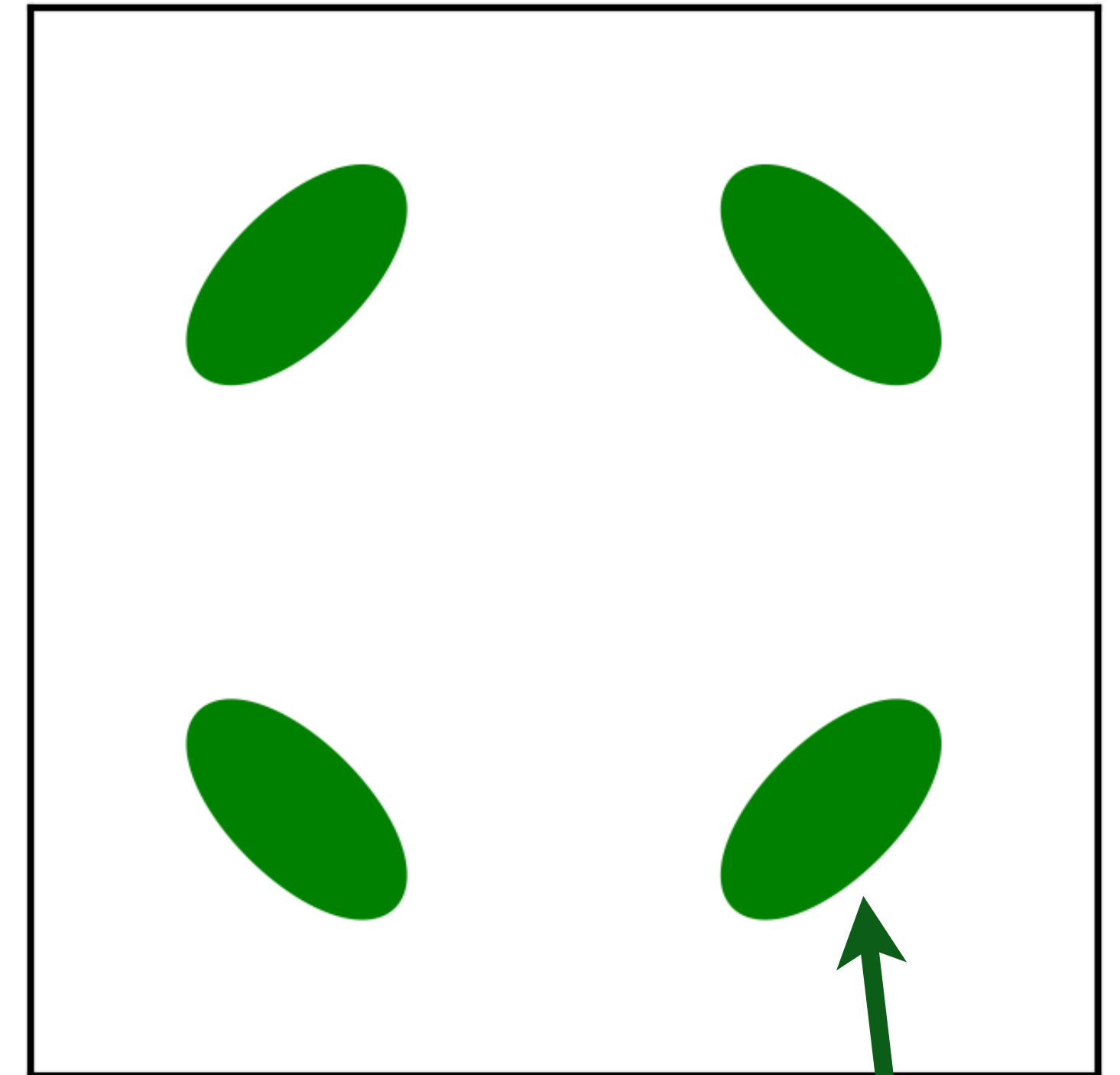
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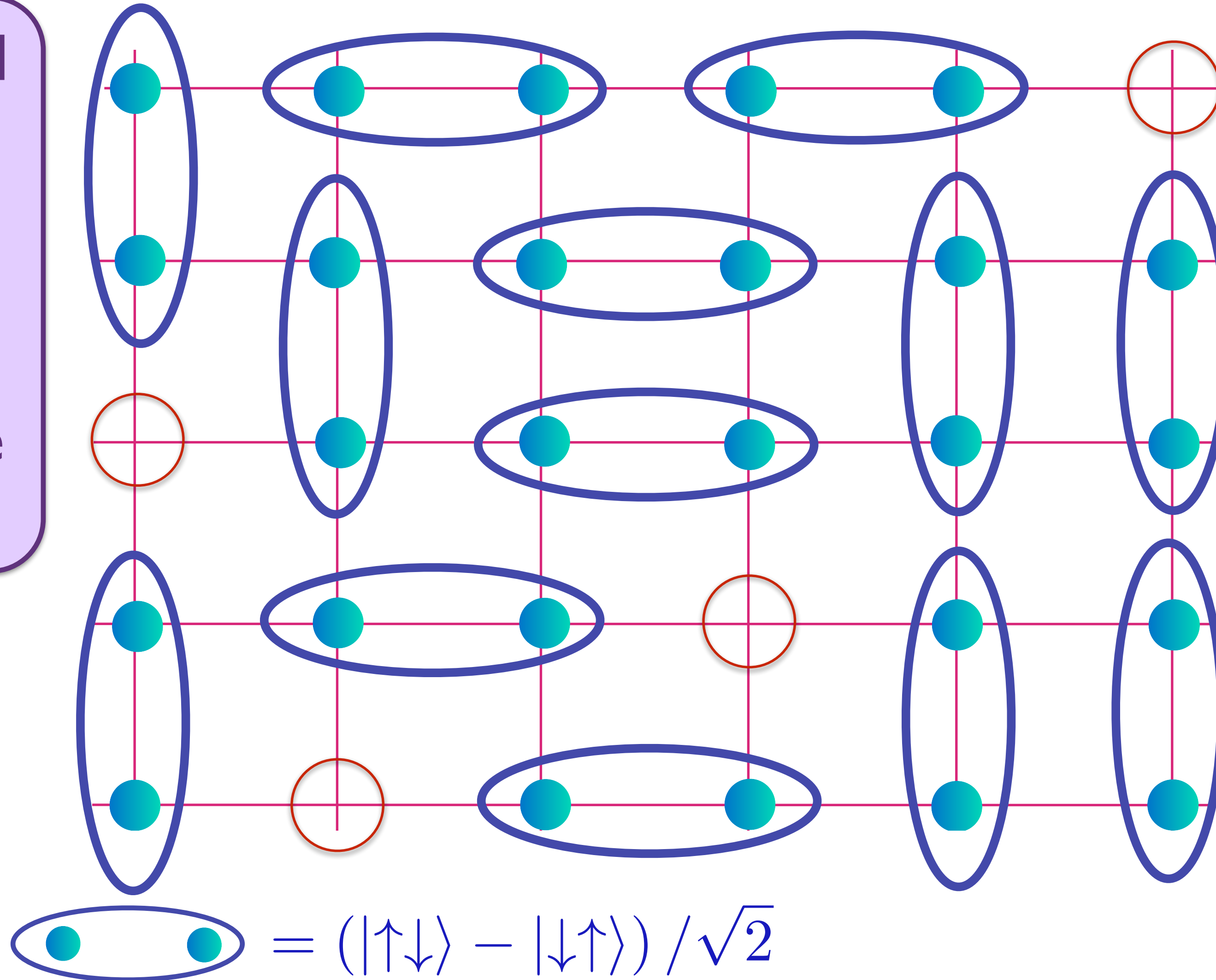


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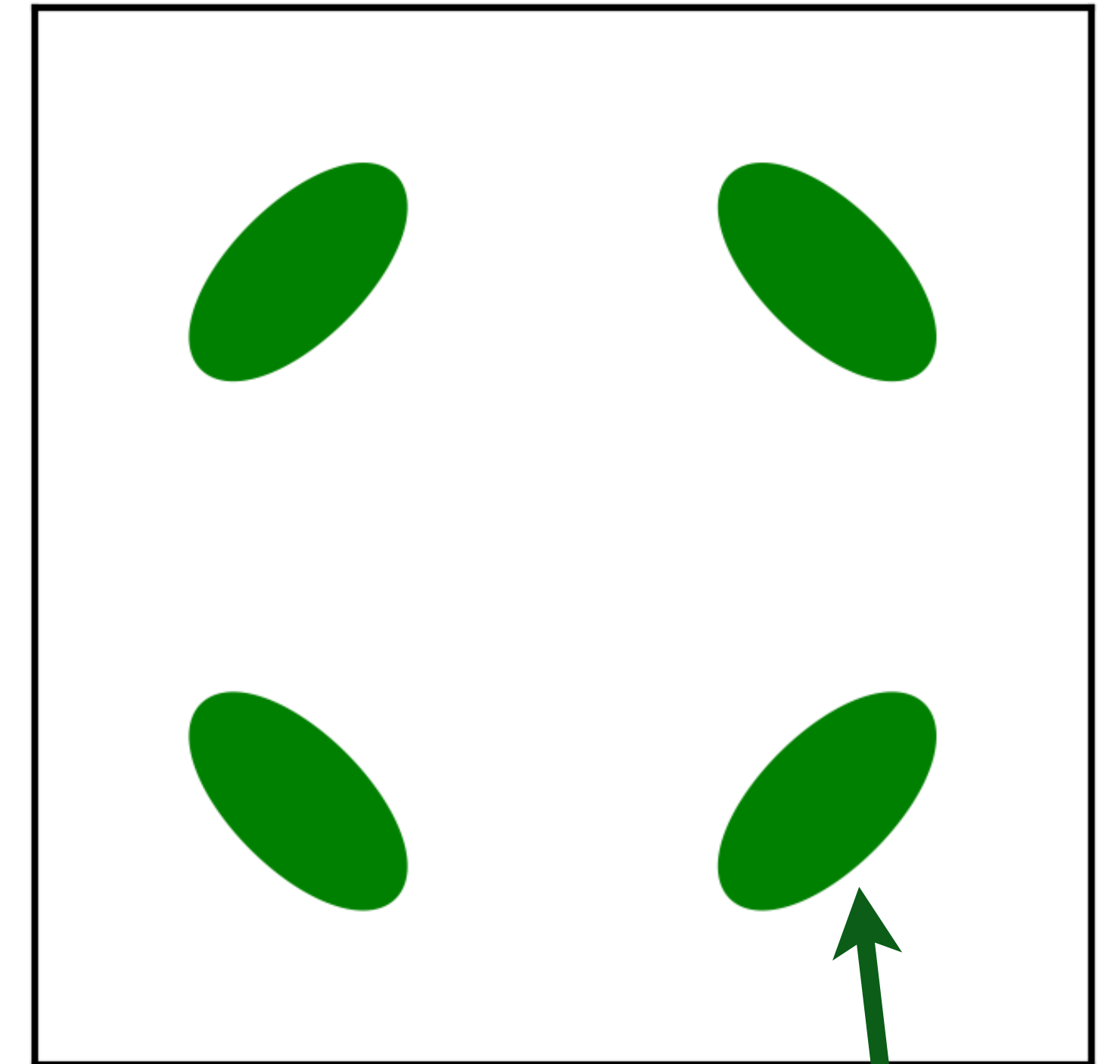
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Spin liquid

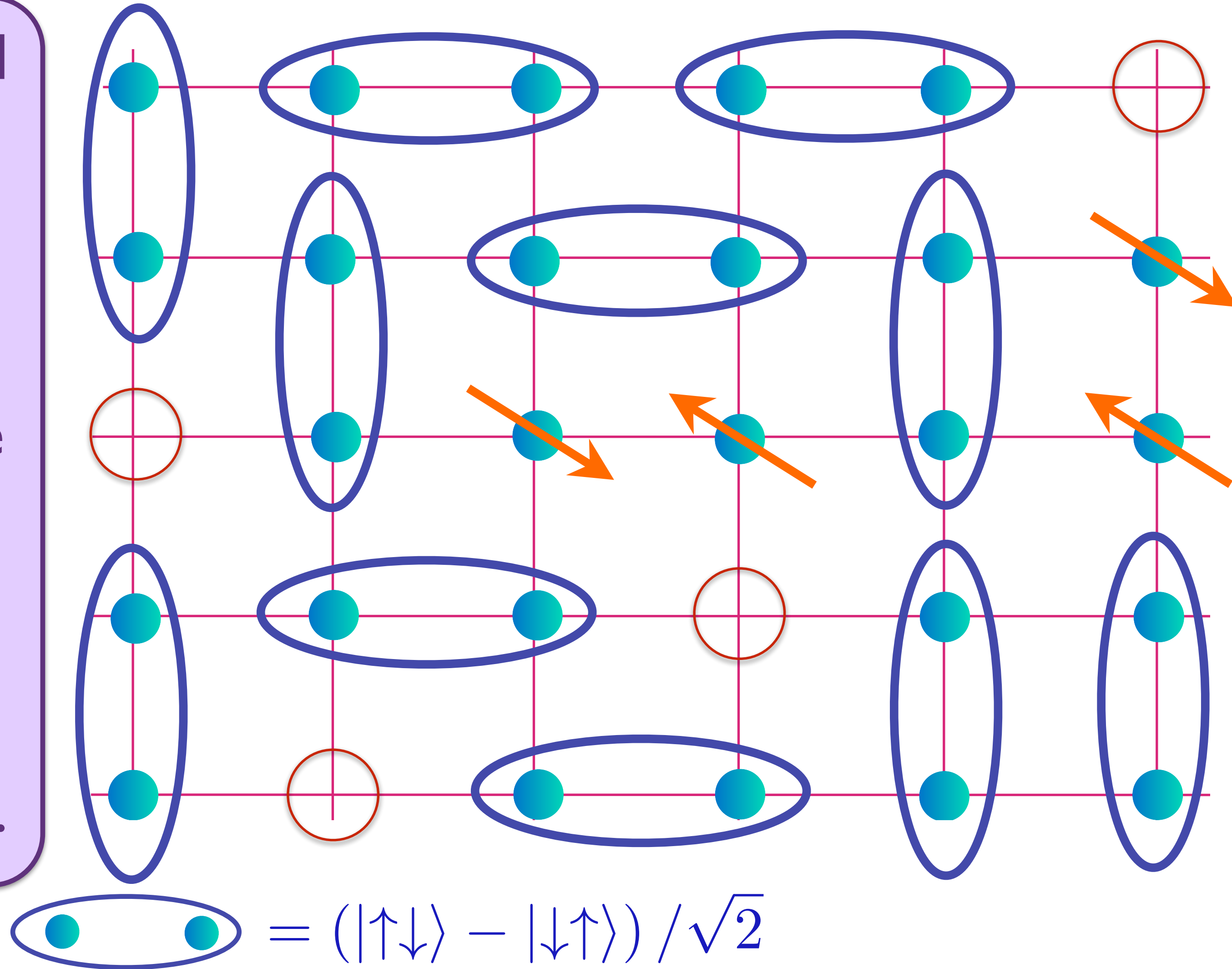


Area $p/4$

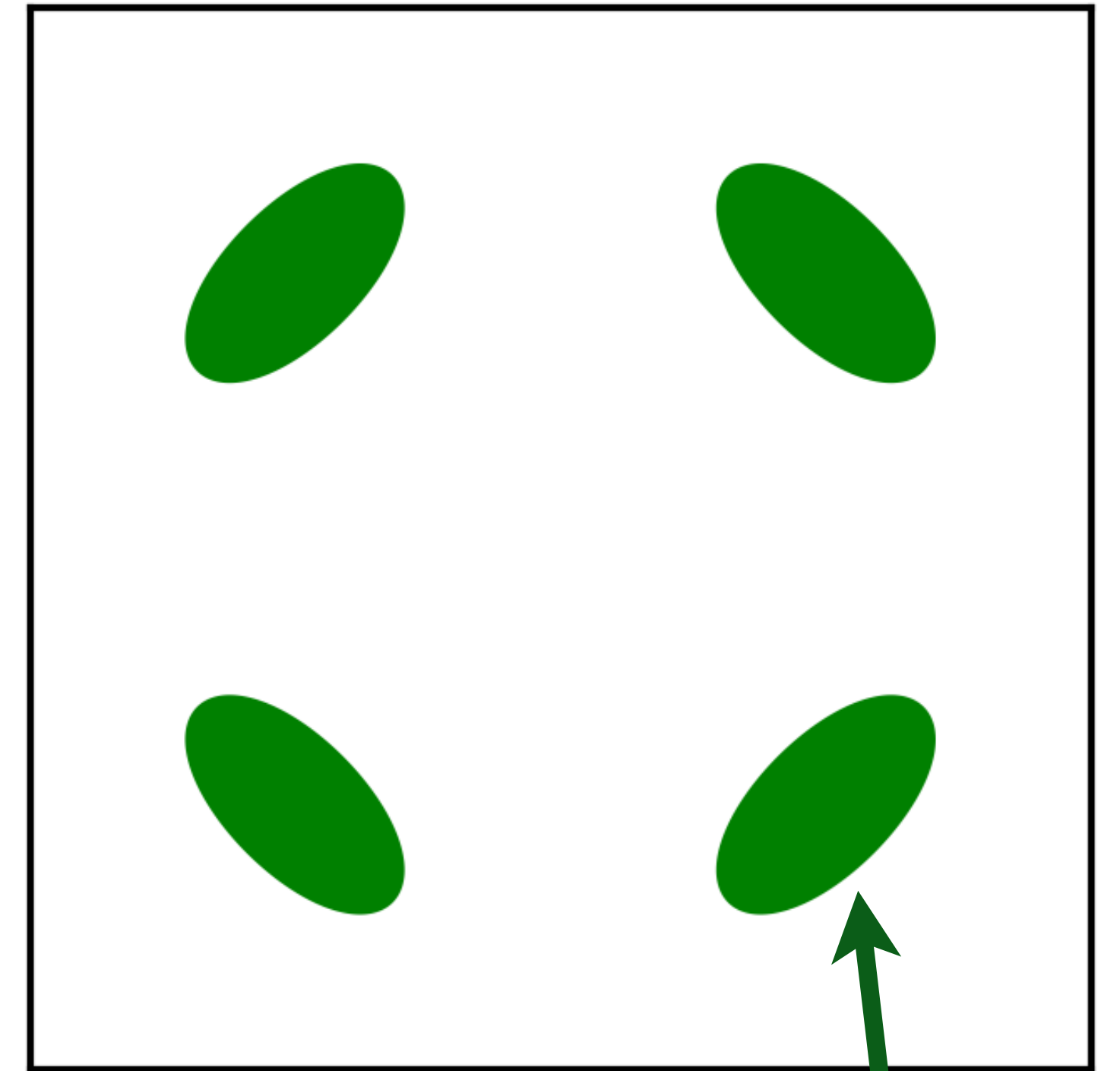
Doping an insulating antiferromagnet with holes of density p

Holon metal

Spin liquid with density p of spinless, charge $+e$ “holons” and charge 0 spin-1/2 “spinons”.



non-Luttinger area.
Spin liquid

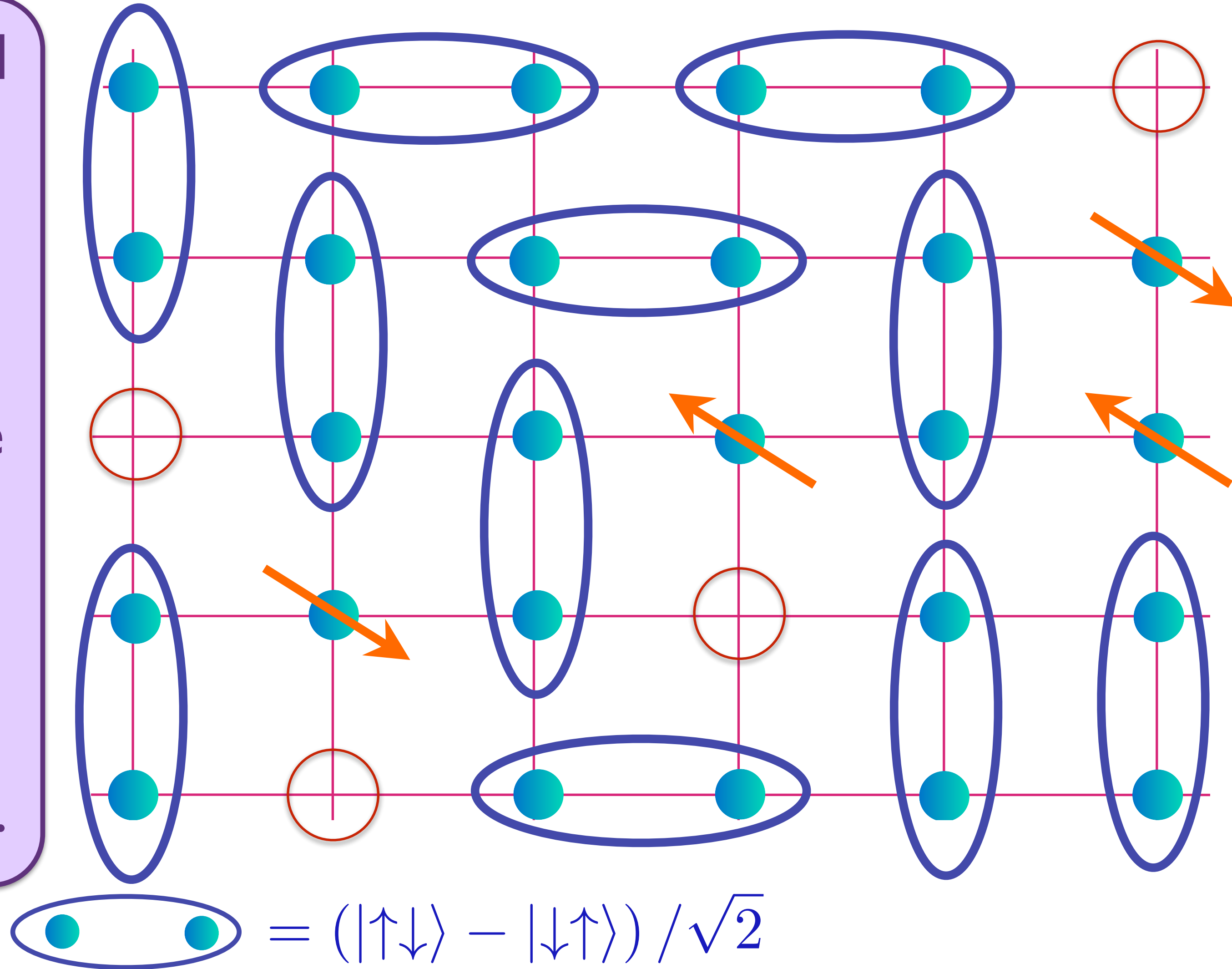


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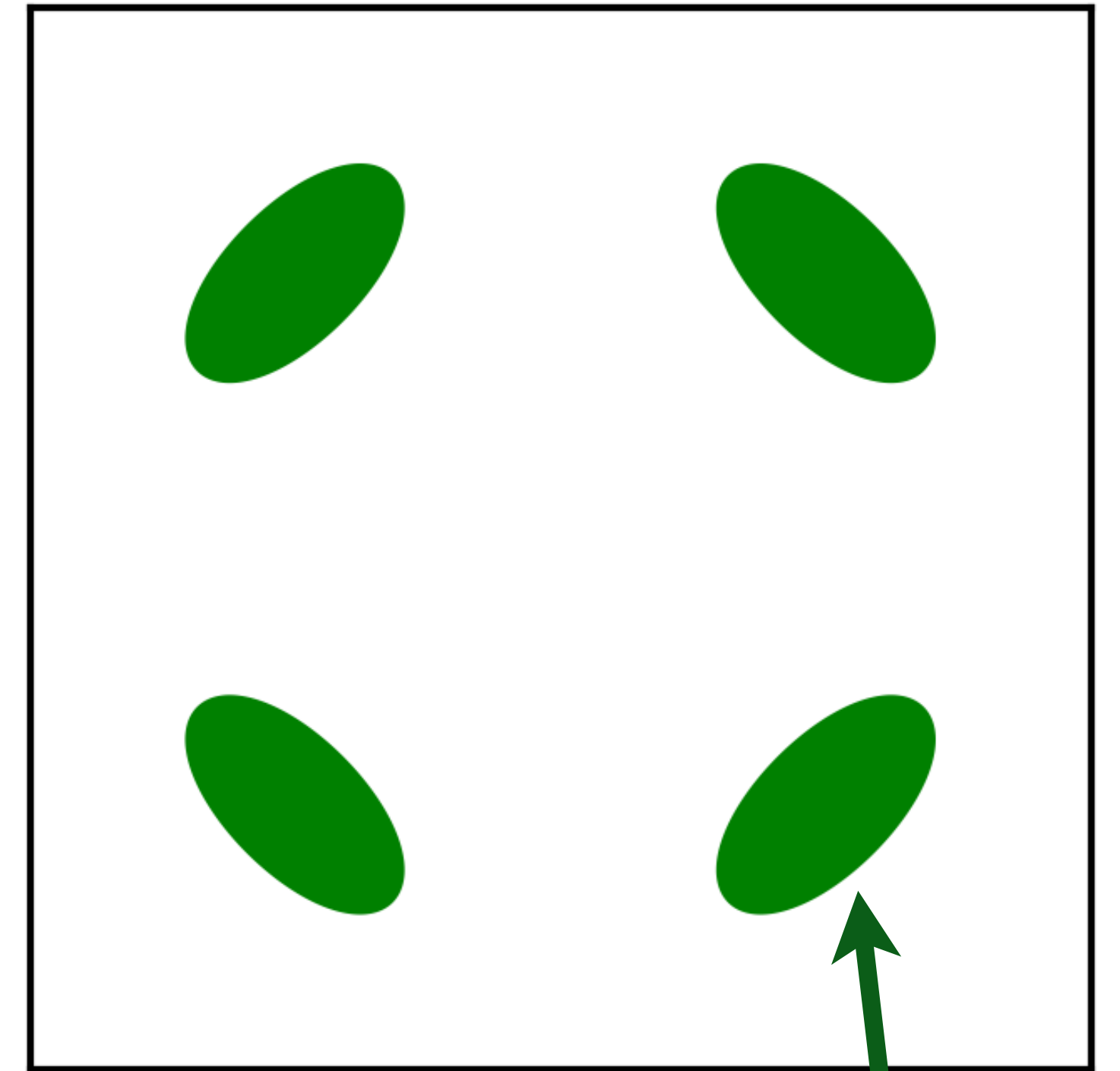
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non-Luttinger area.
Spin liquid

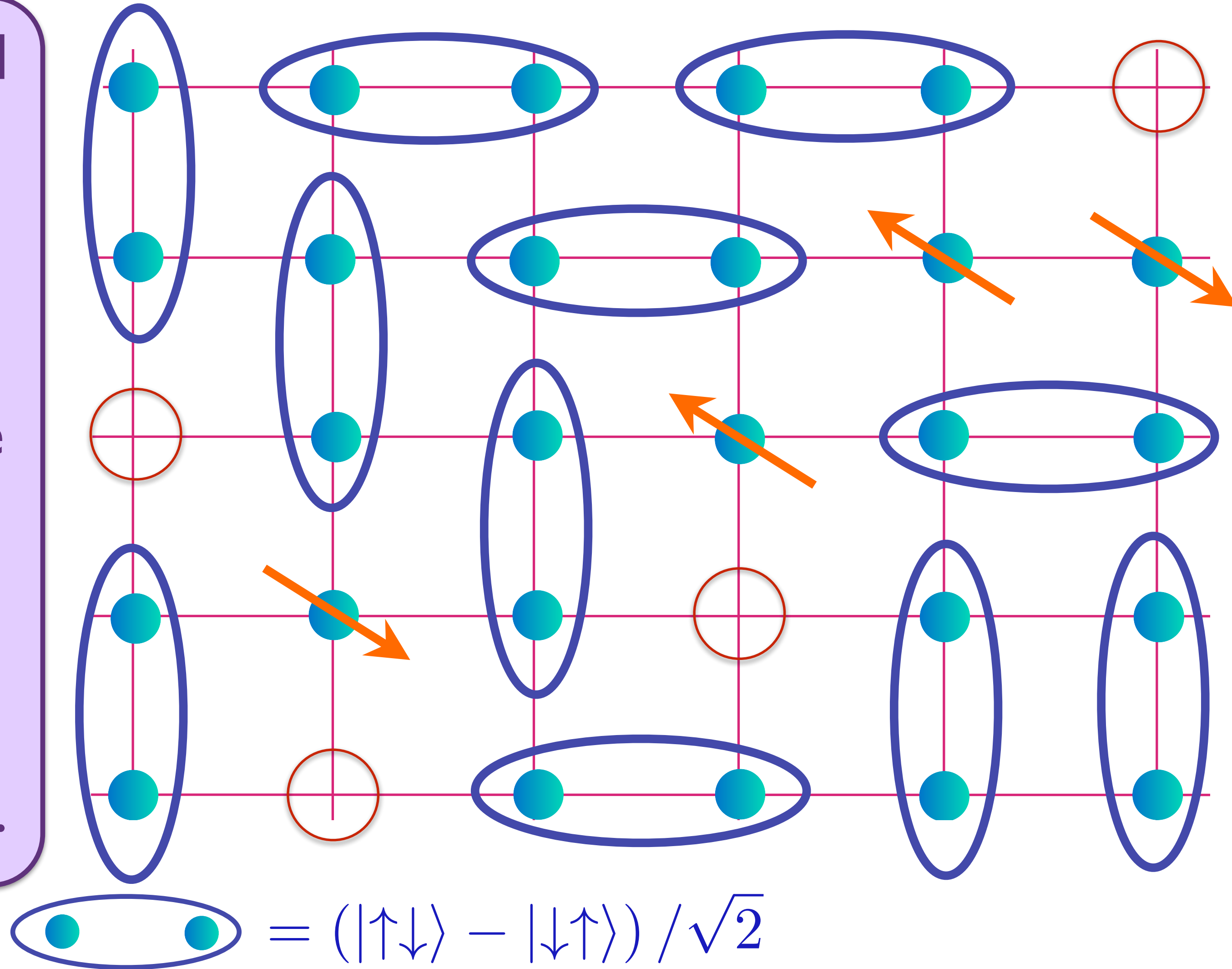


Area $p/4$

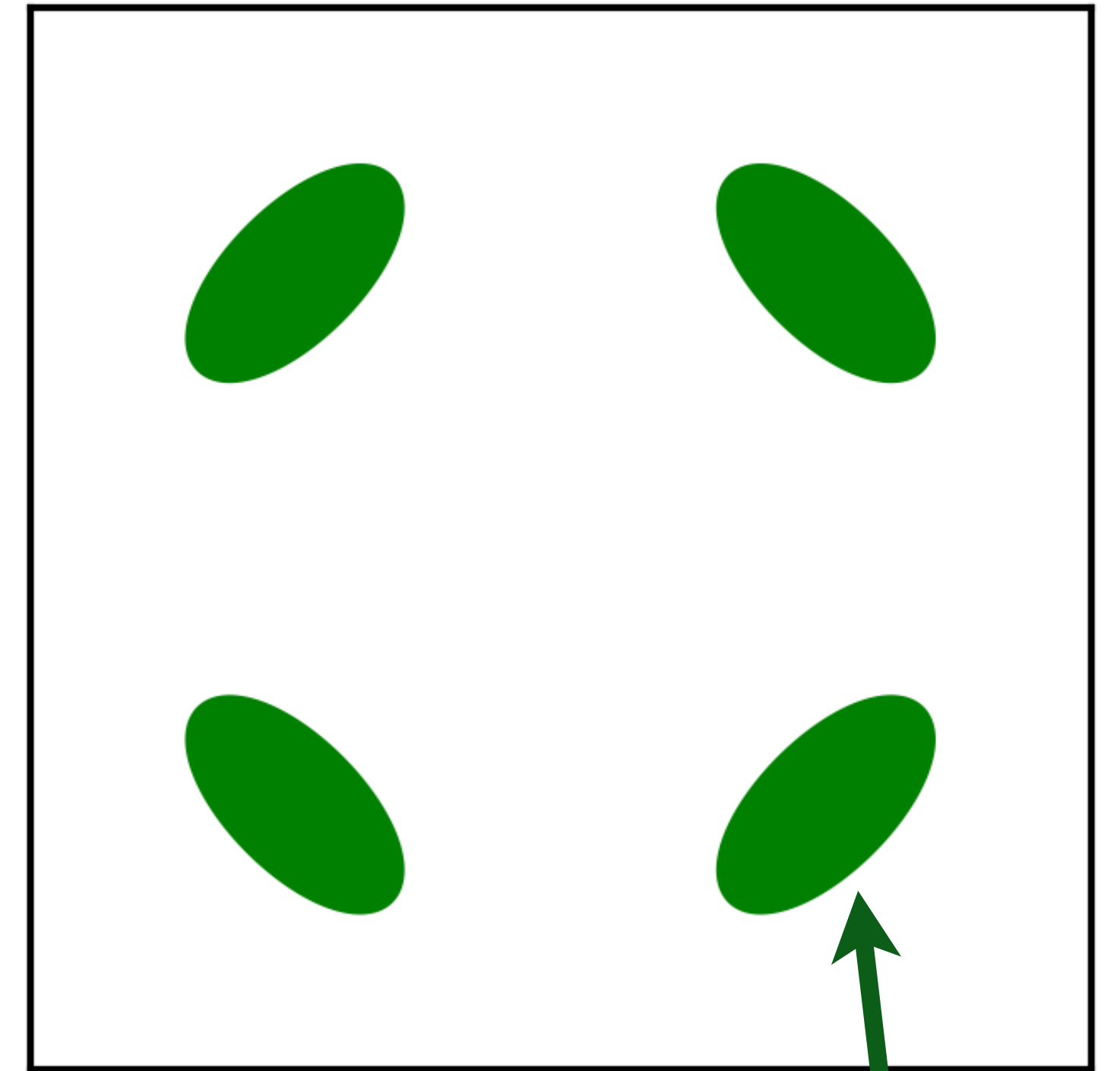
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non-Luttinger area.
Spin liquid

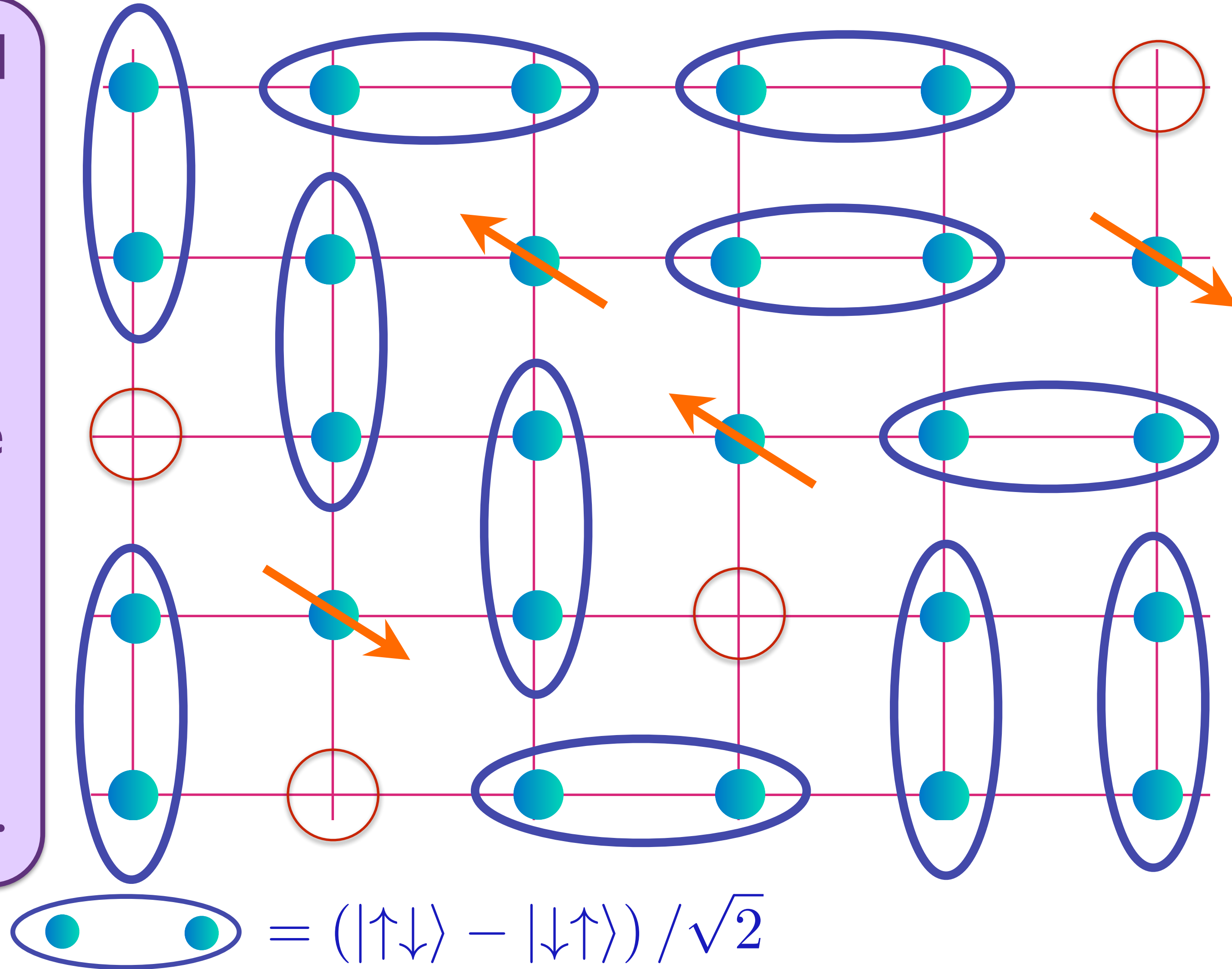


Area $p/4$

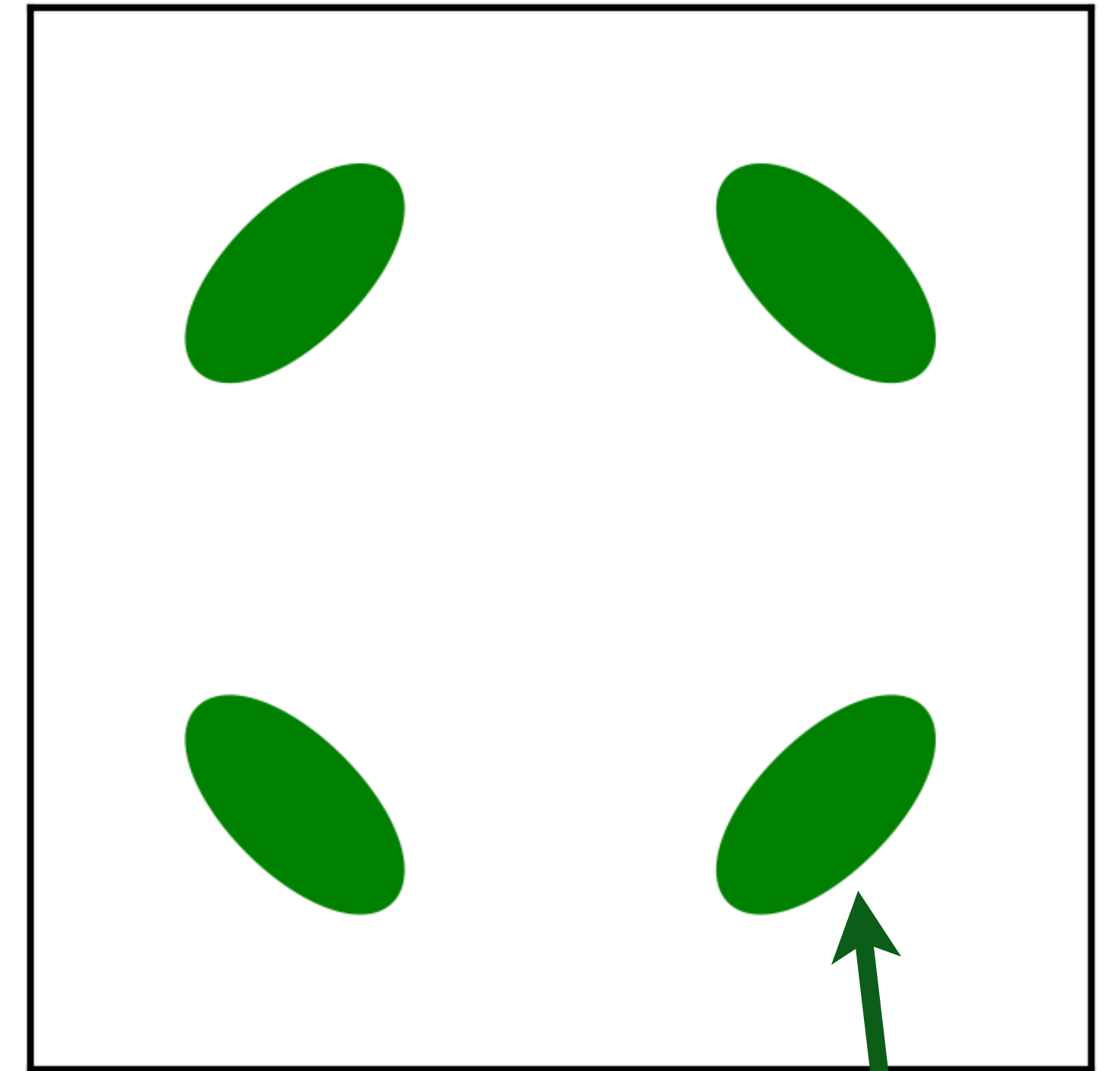
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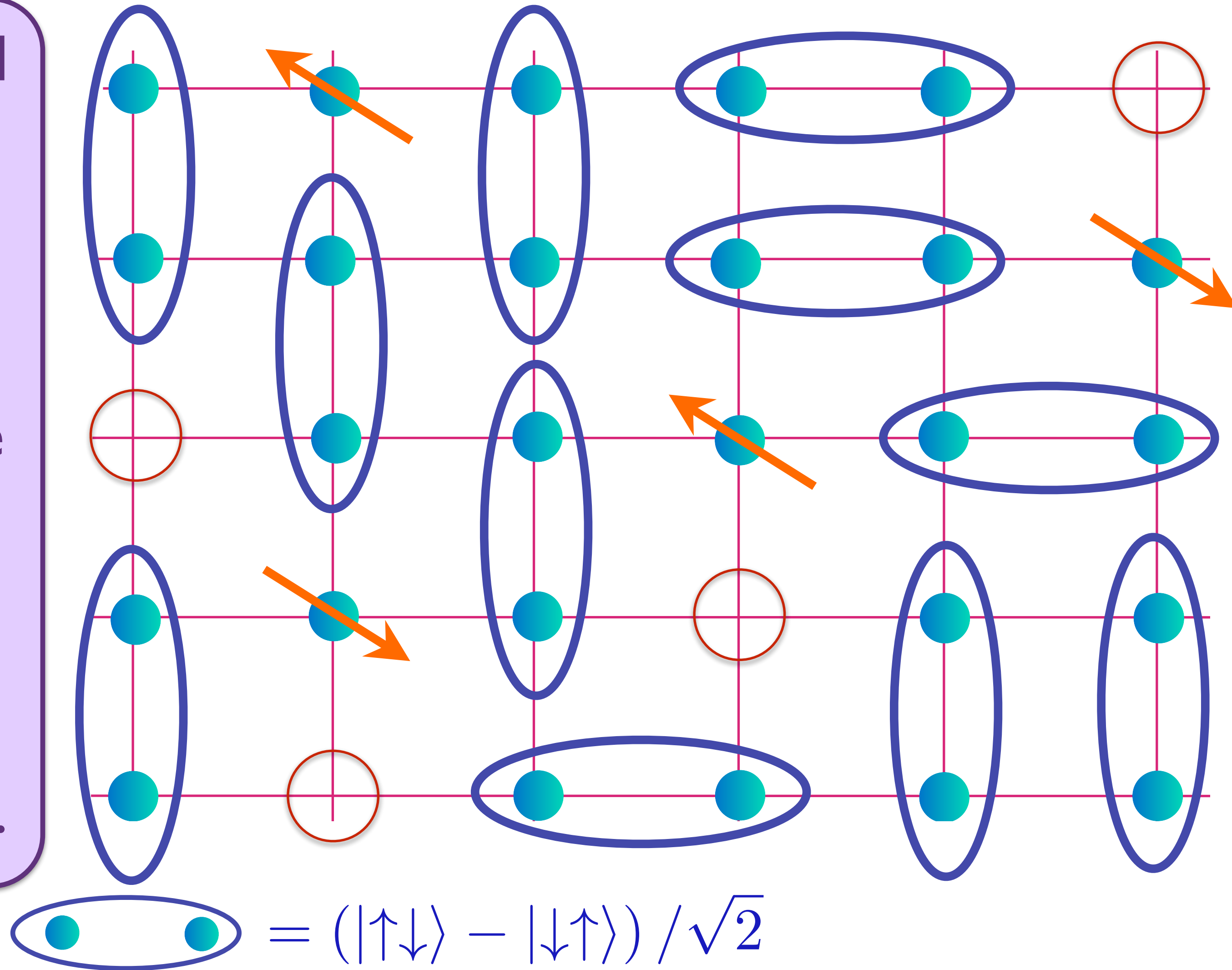


Area $p/4$

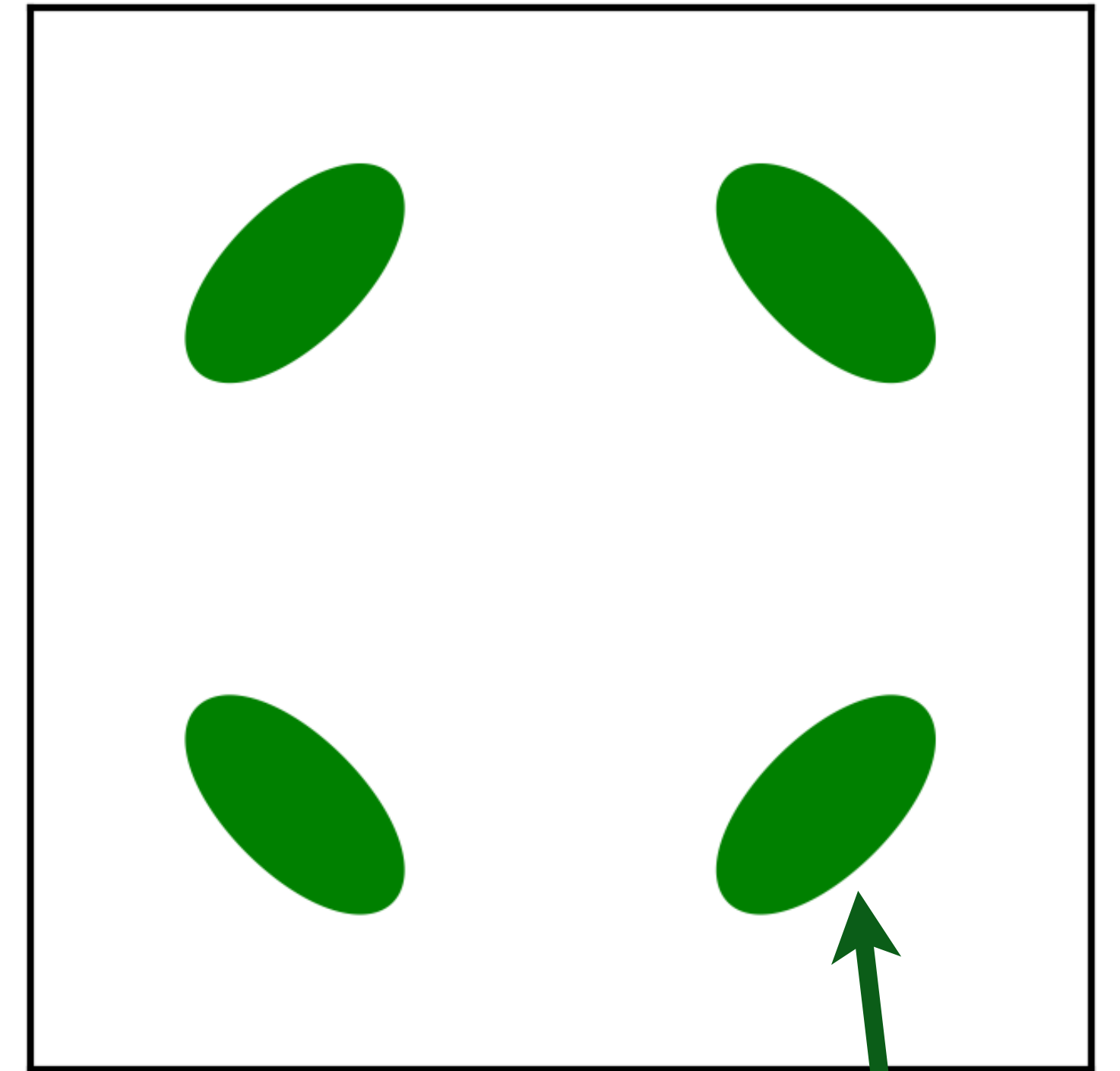
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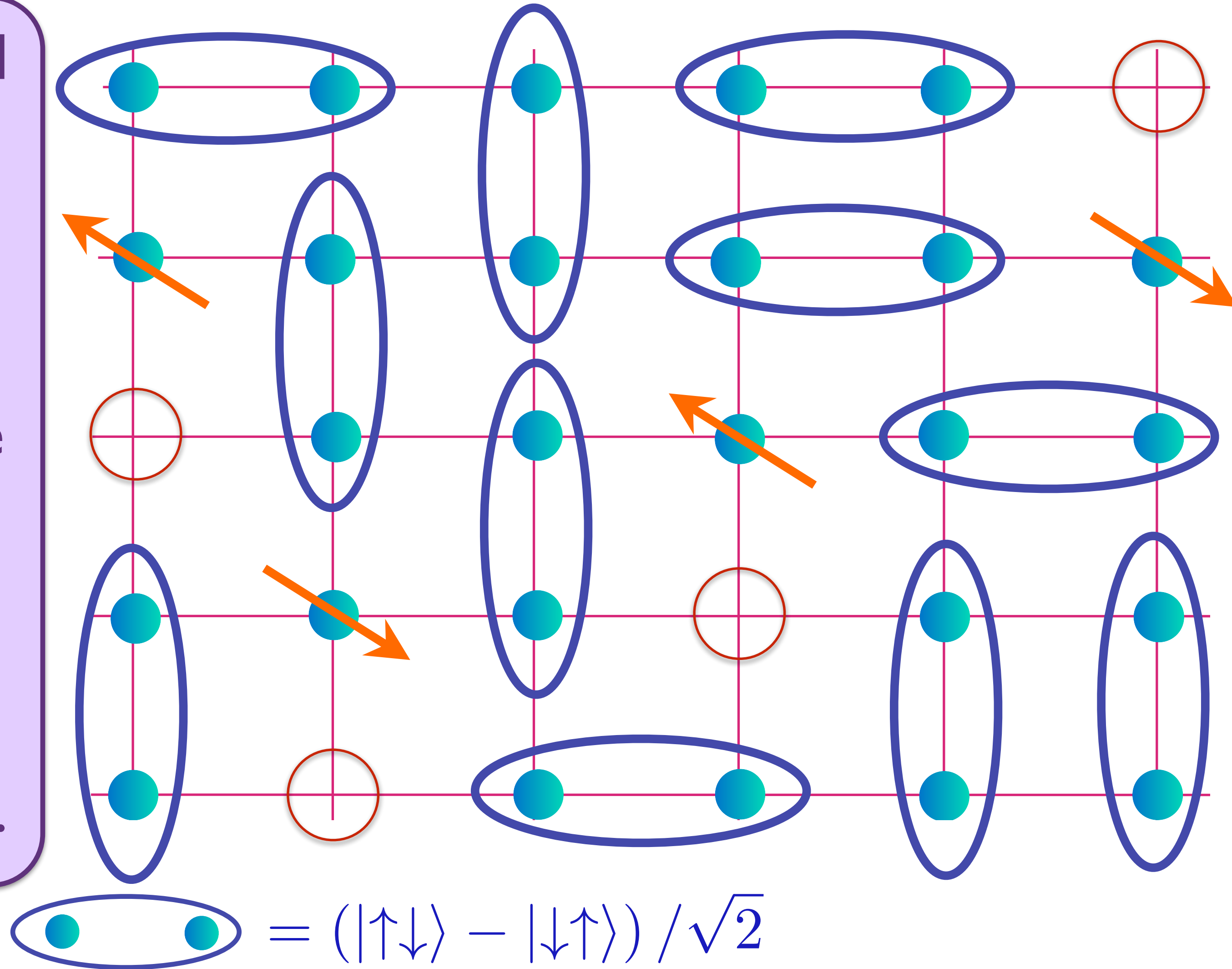


Area $p/4$

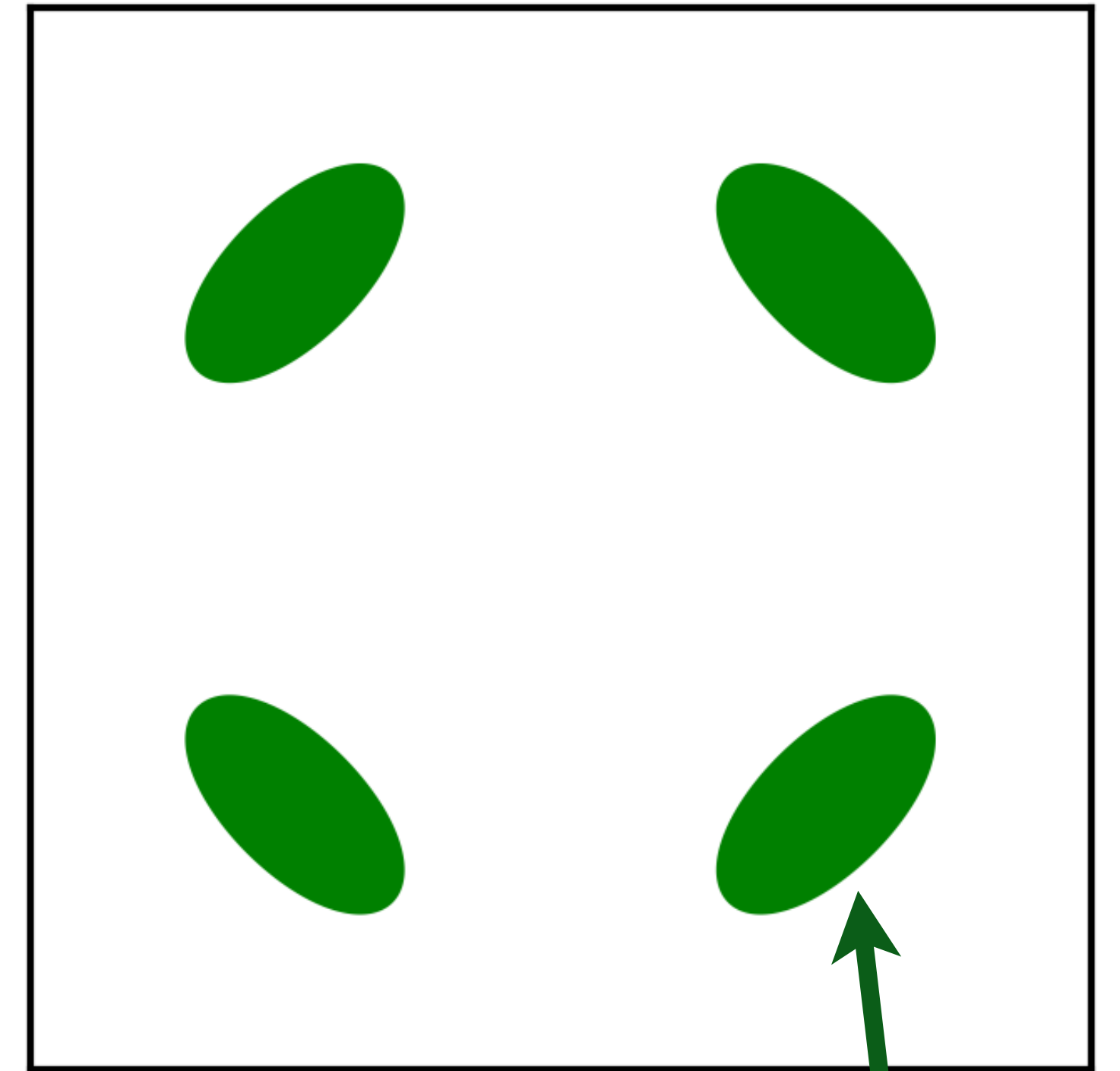
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non-Luttinger area.
Spin liquid

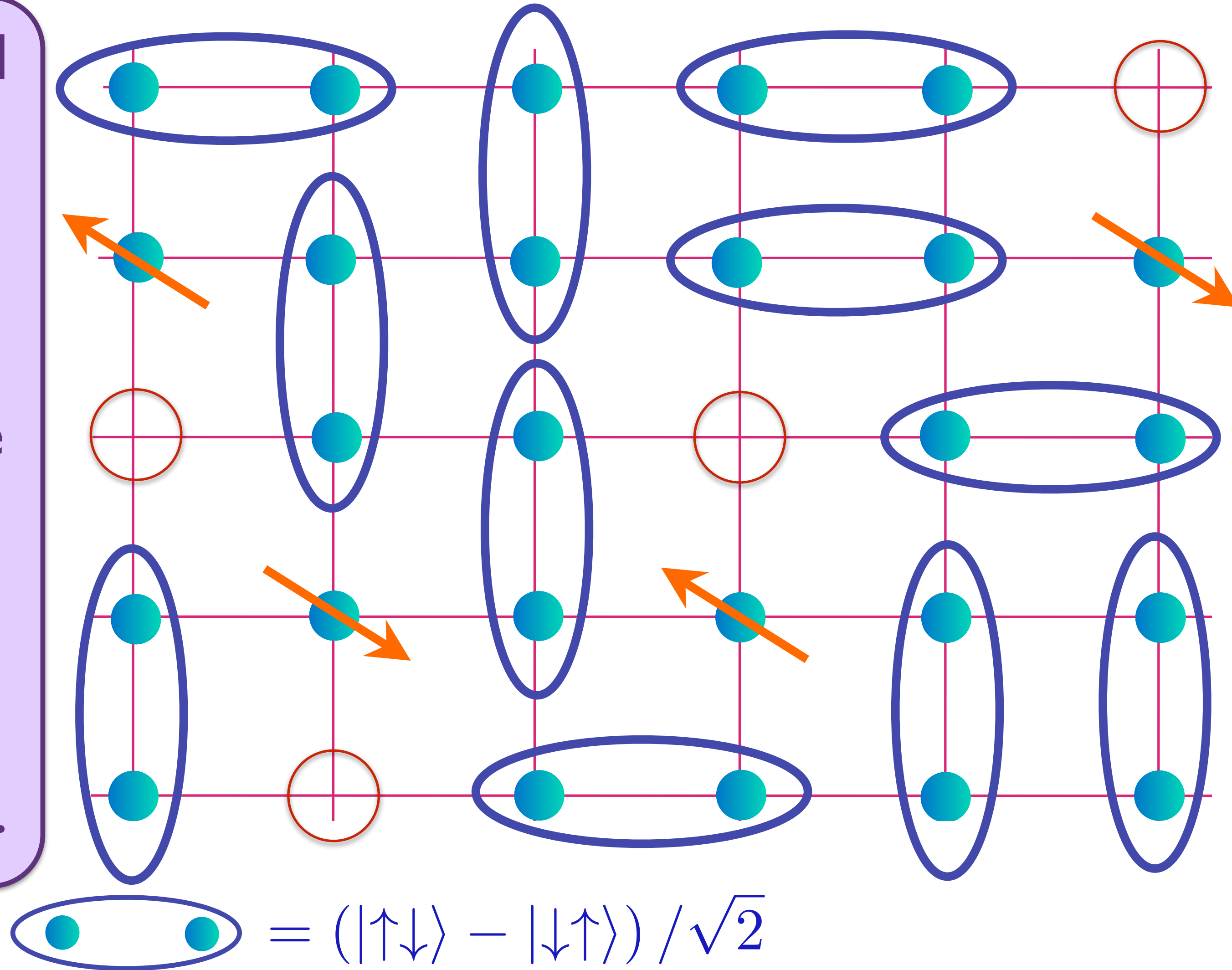


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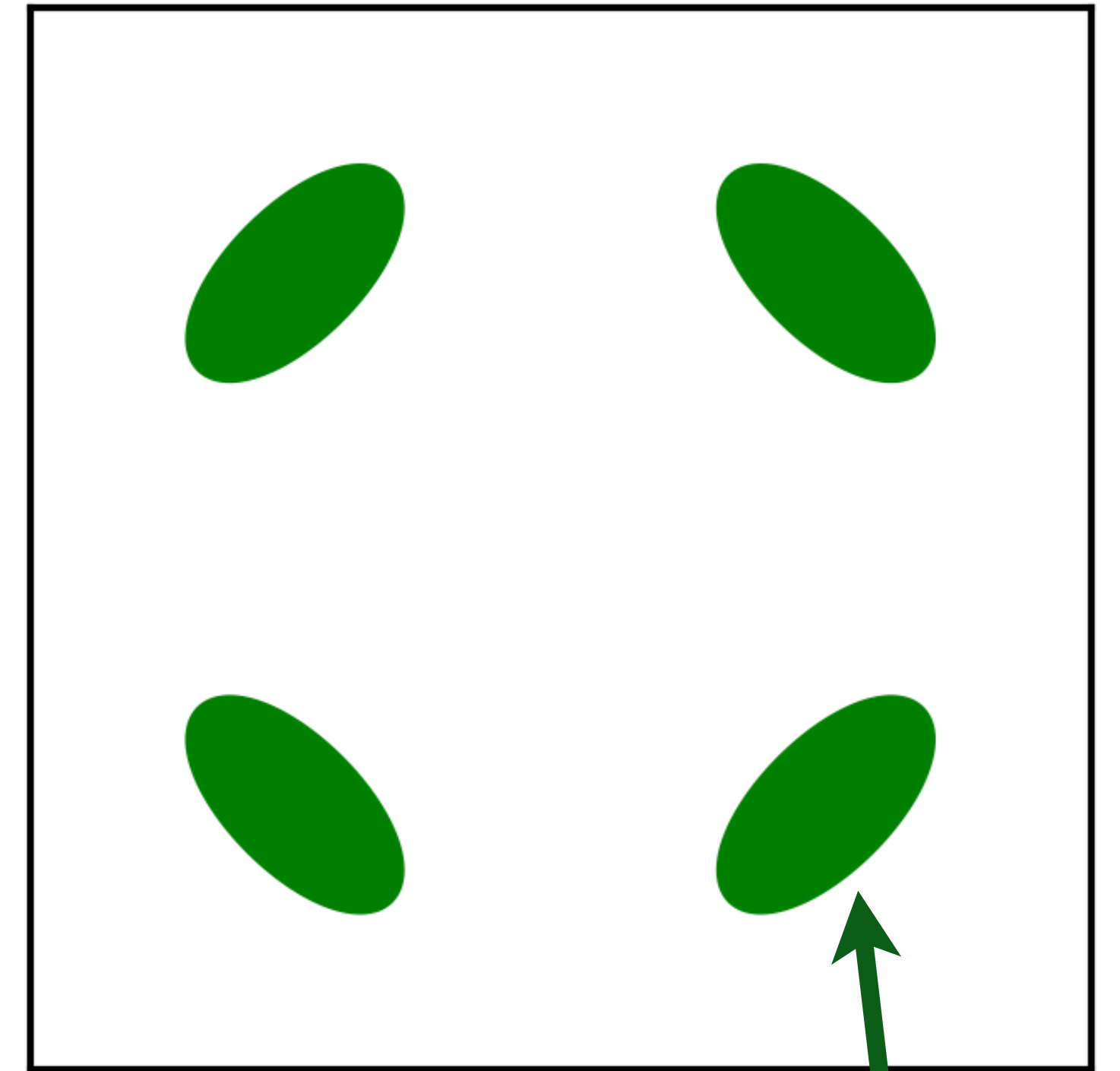
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Spin liquid

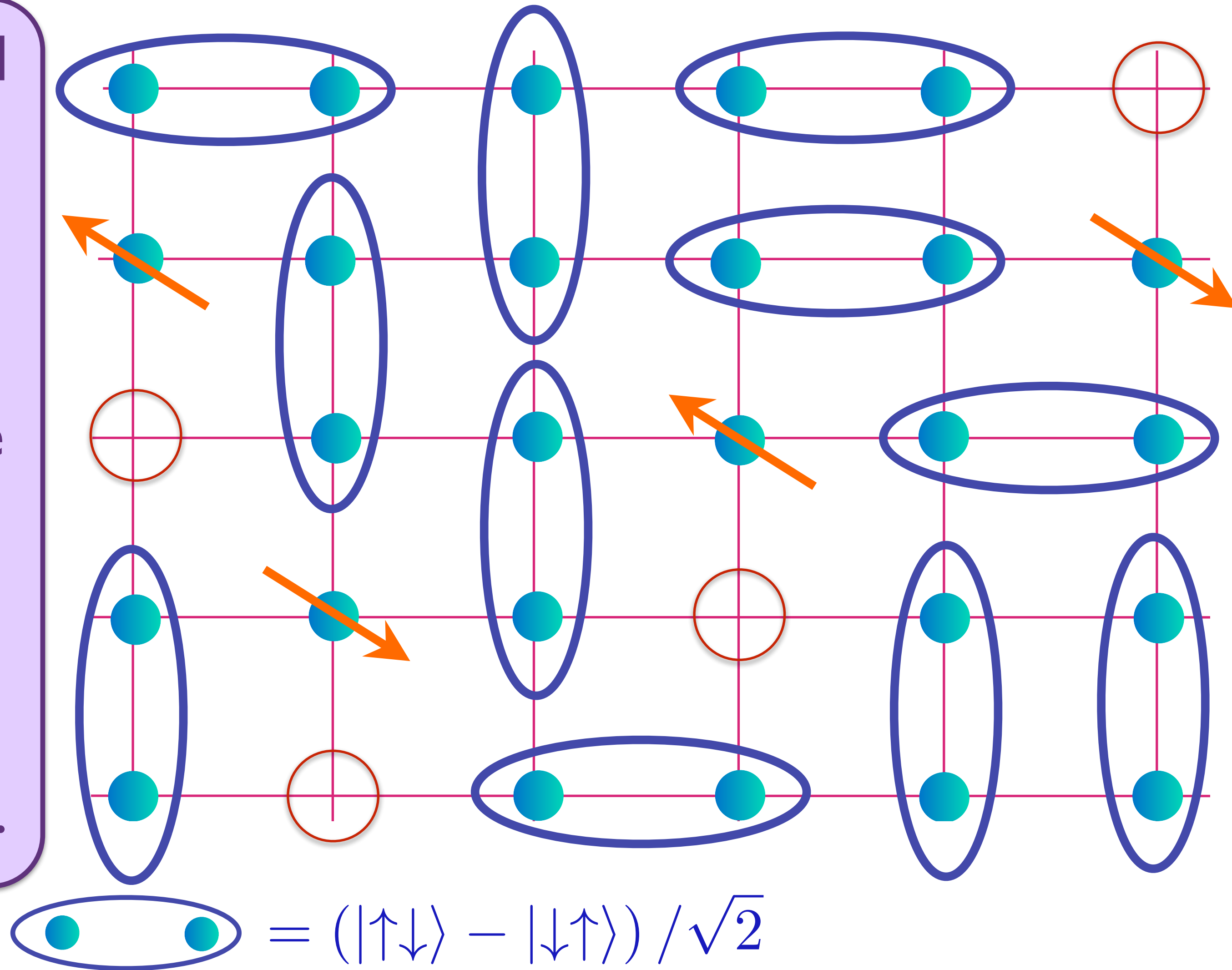


Area $p/4$

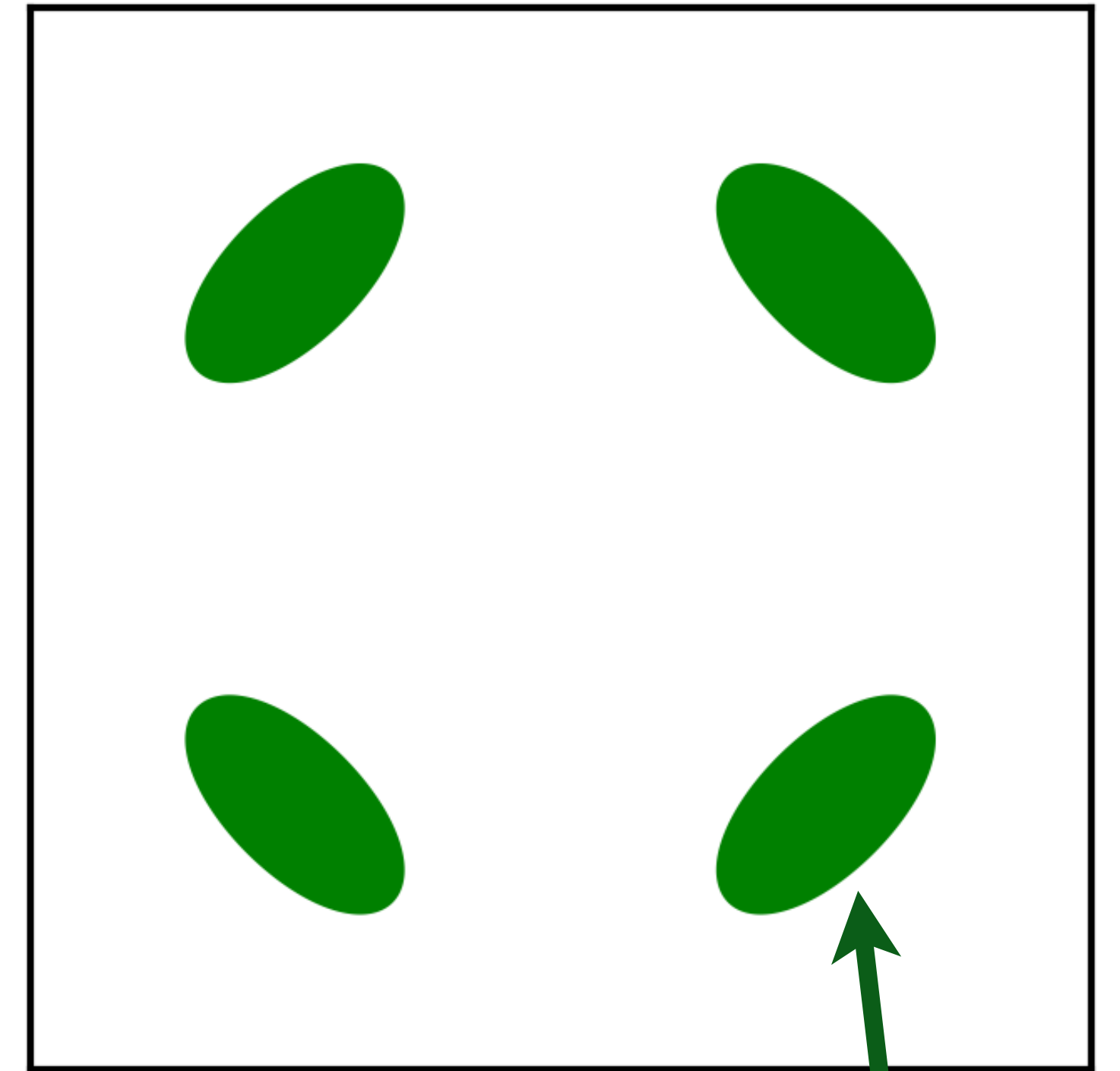
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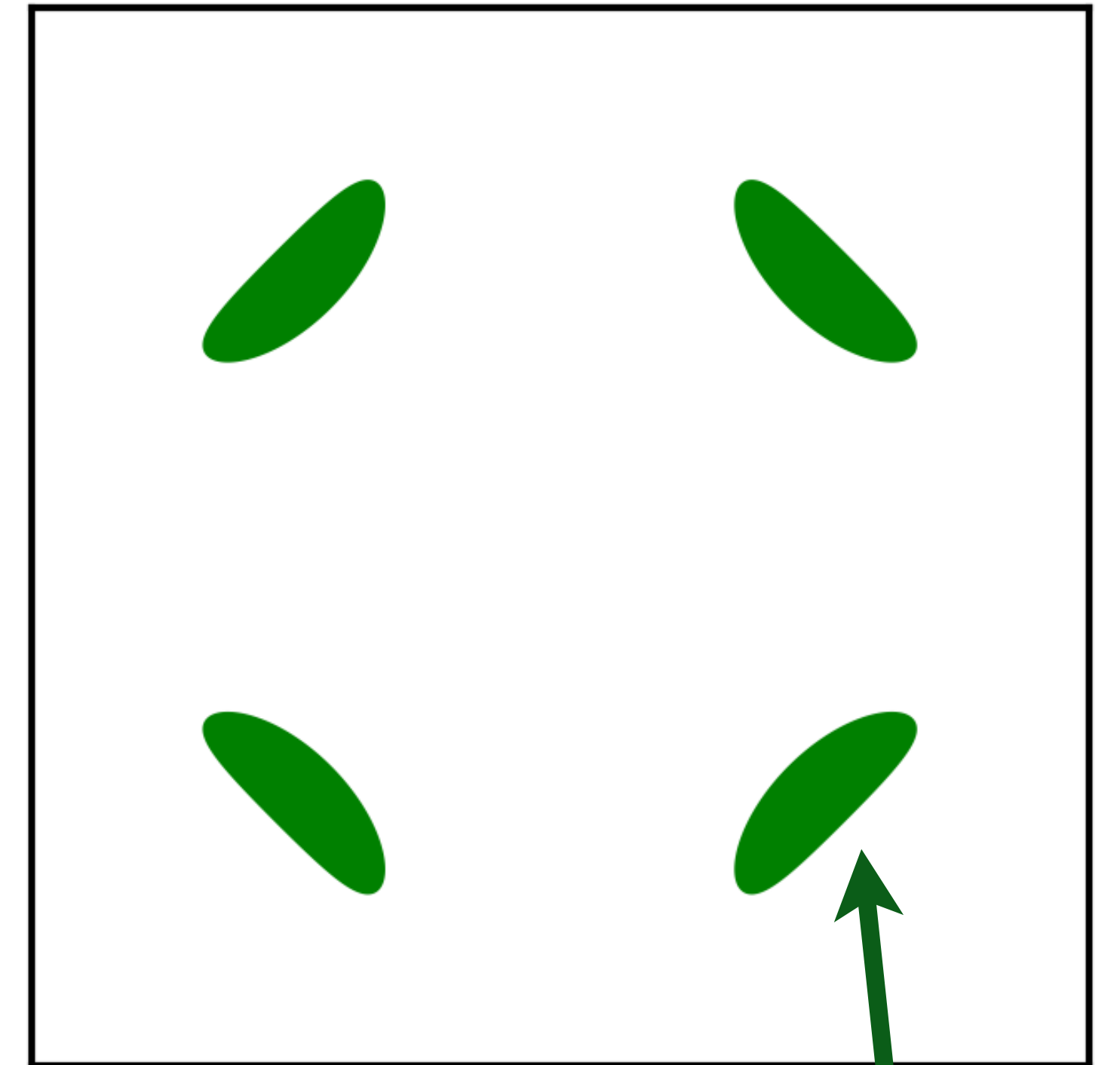
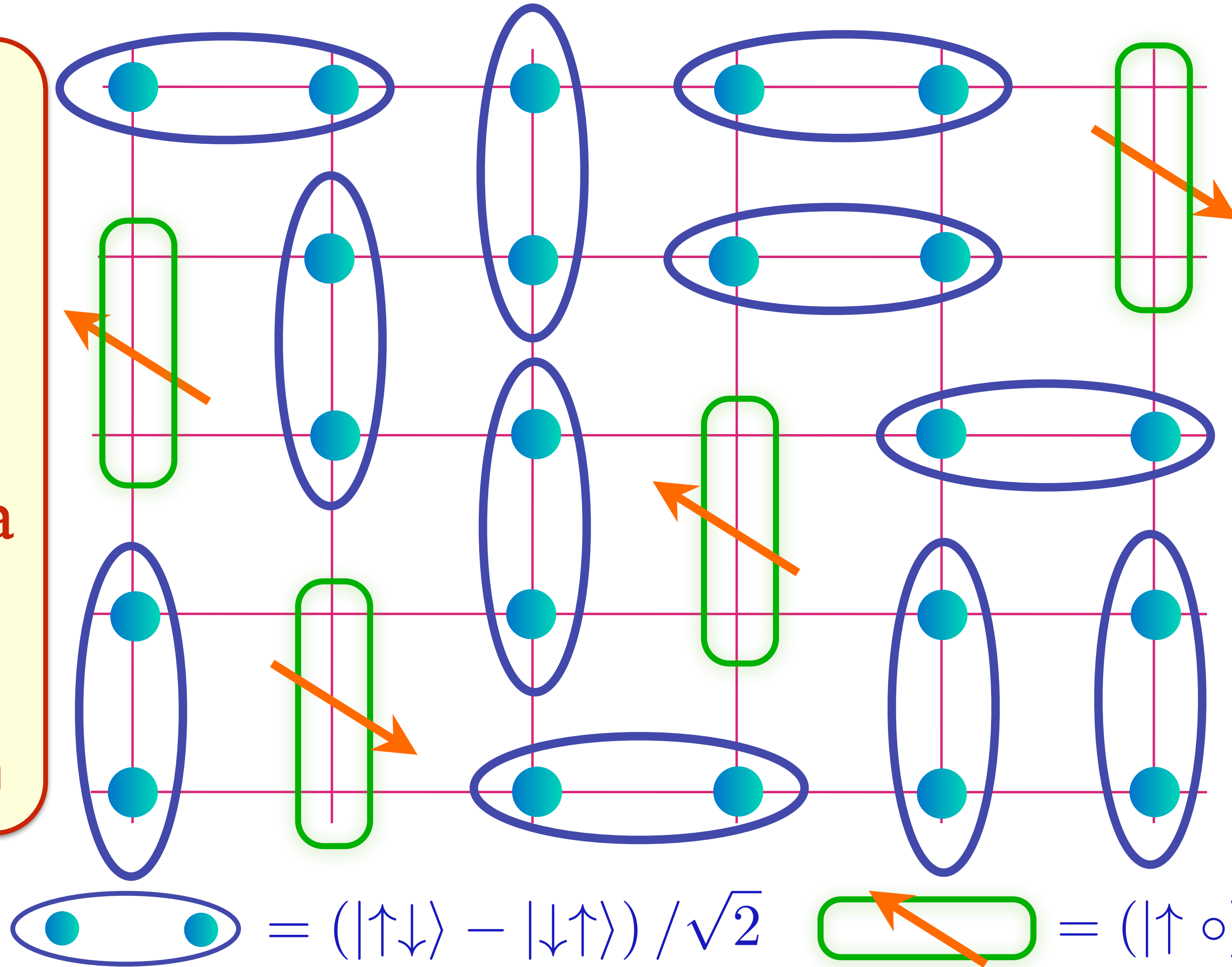
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

FL*

non-Luttinger area.
Spin liquid

Each green “dimer” is a bound state of a vacancy and a free spin



Area $p/8$

$$\text{Blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \quad \text{Green rectangle} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)

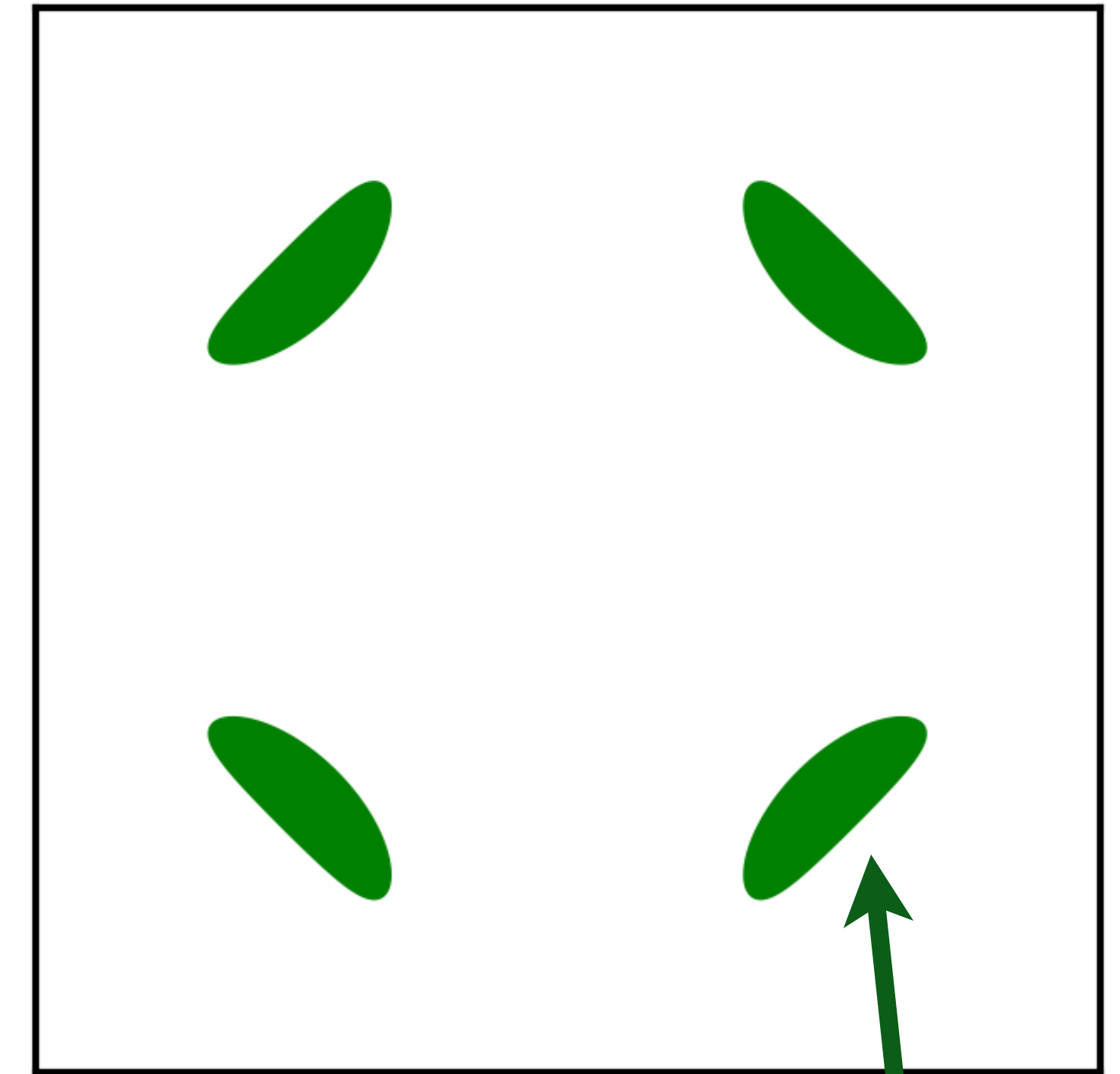
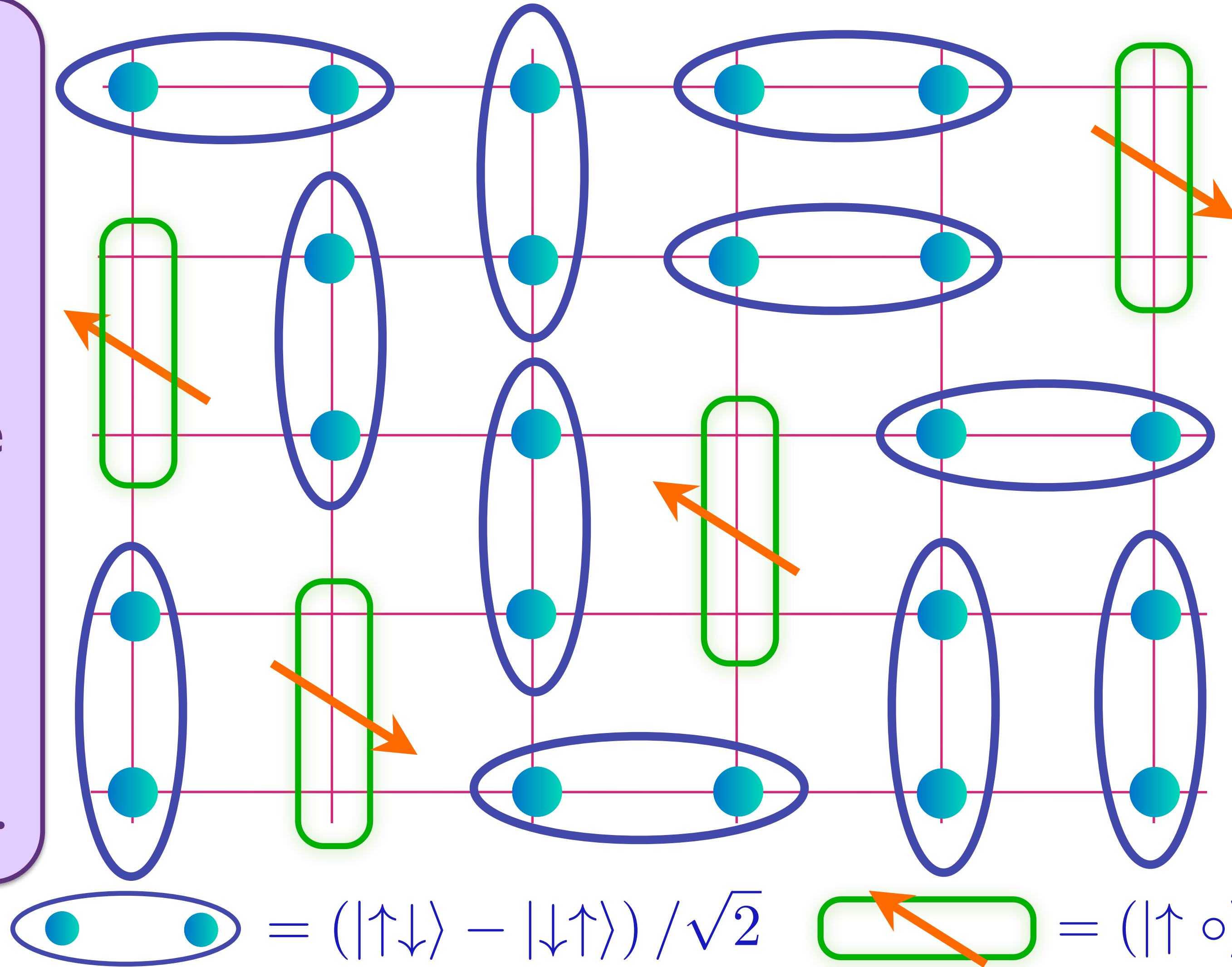
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

FL*

non-Luttinger area.
Spin liquid

Metal
with
density p
of
spin-1/2,
charge $+e$
“holes”
and
charge 0
spin-1/2
“spinons”.



Area $p/8$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \quad \begin{array}{|c|} \hline \bullet \\ \hline \end{array} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

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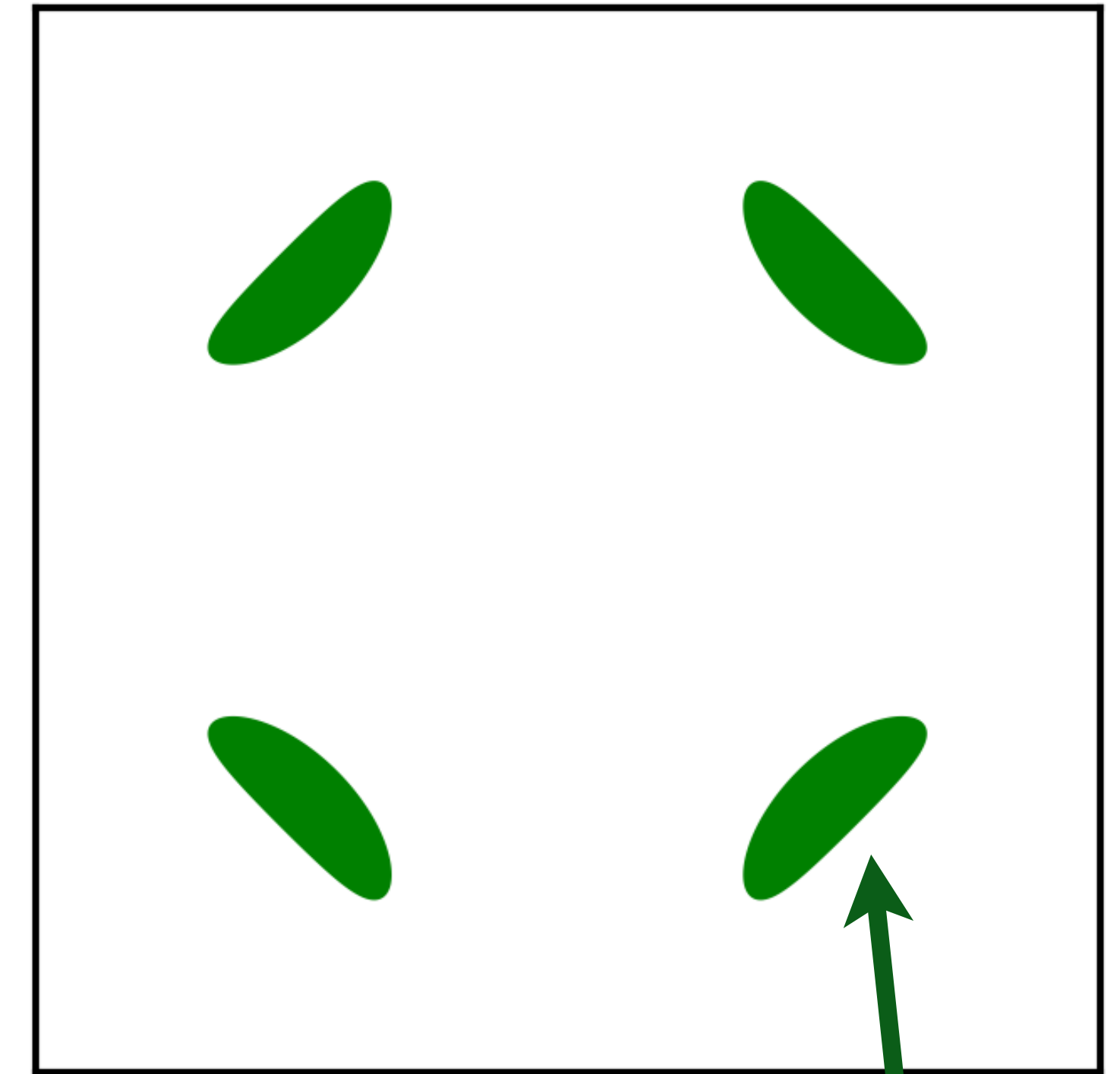
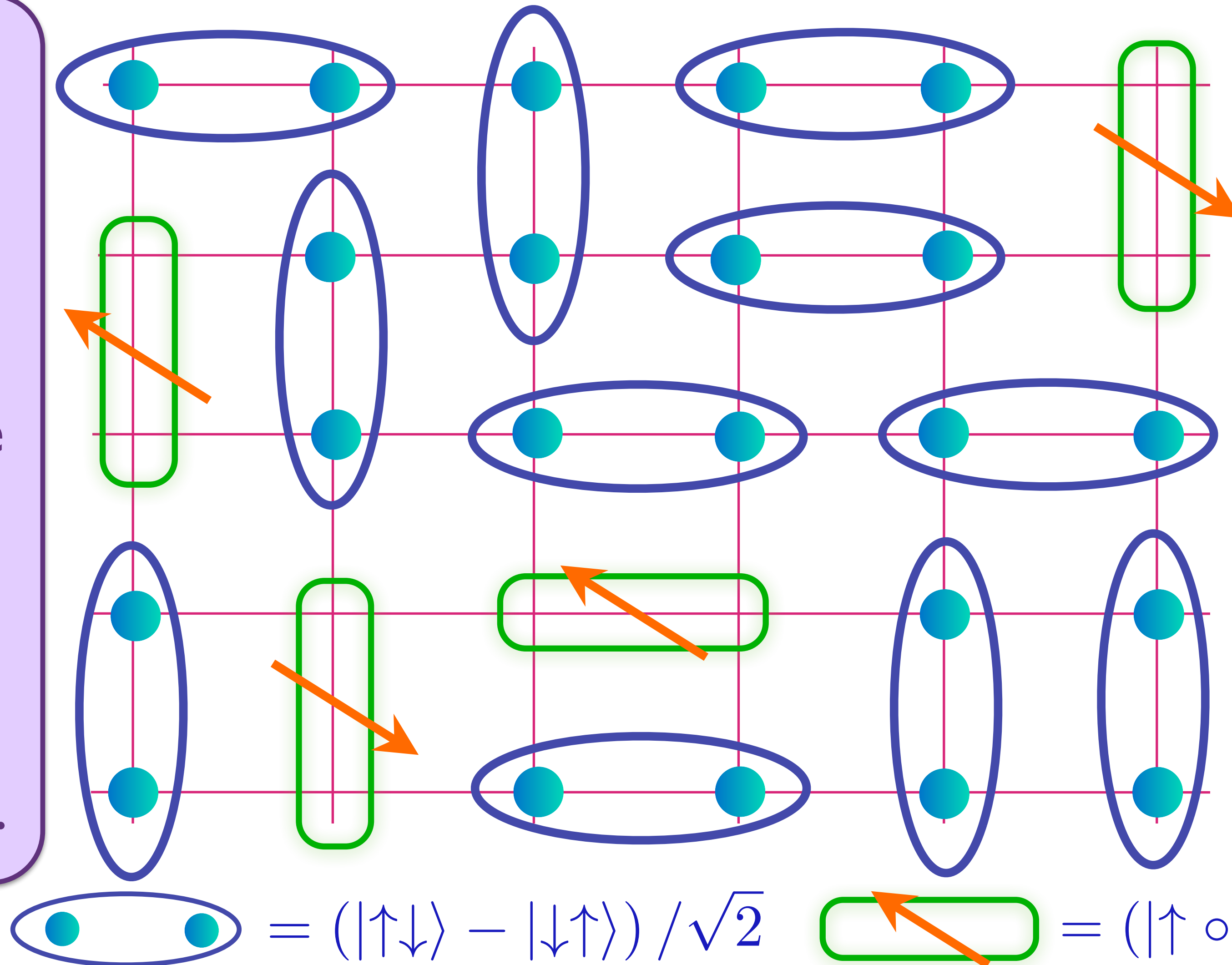
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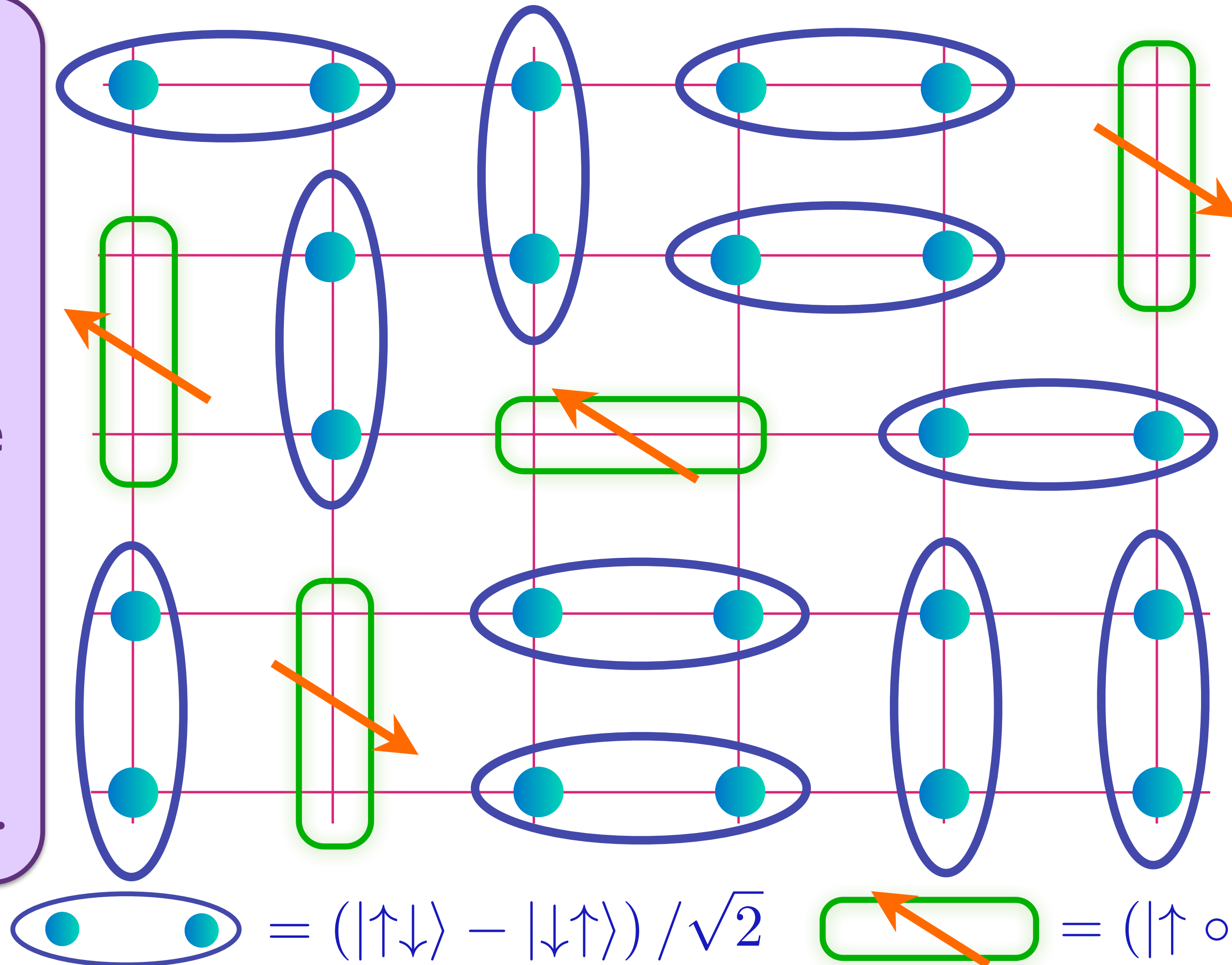
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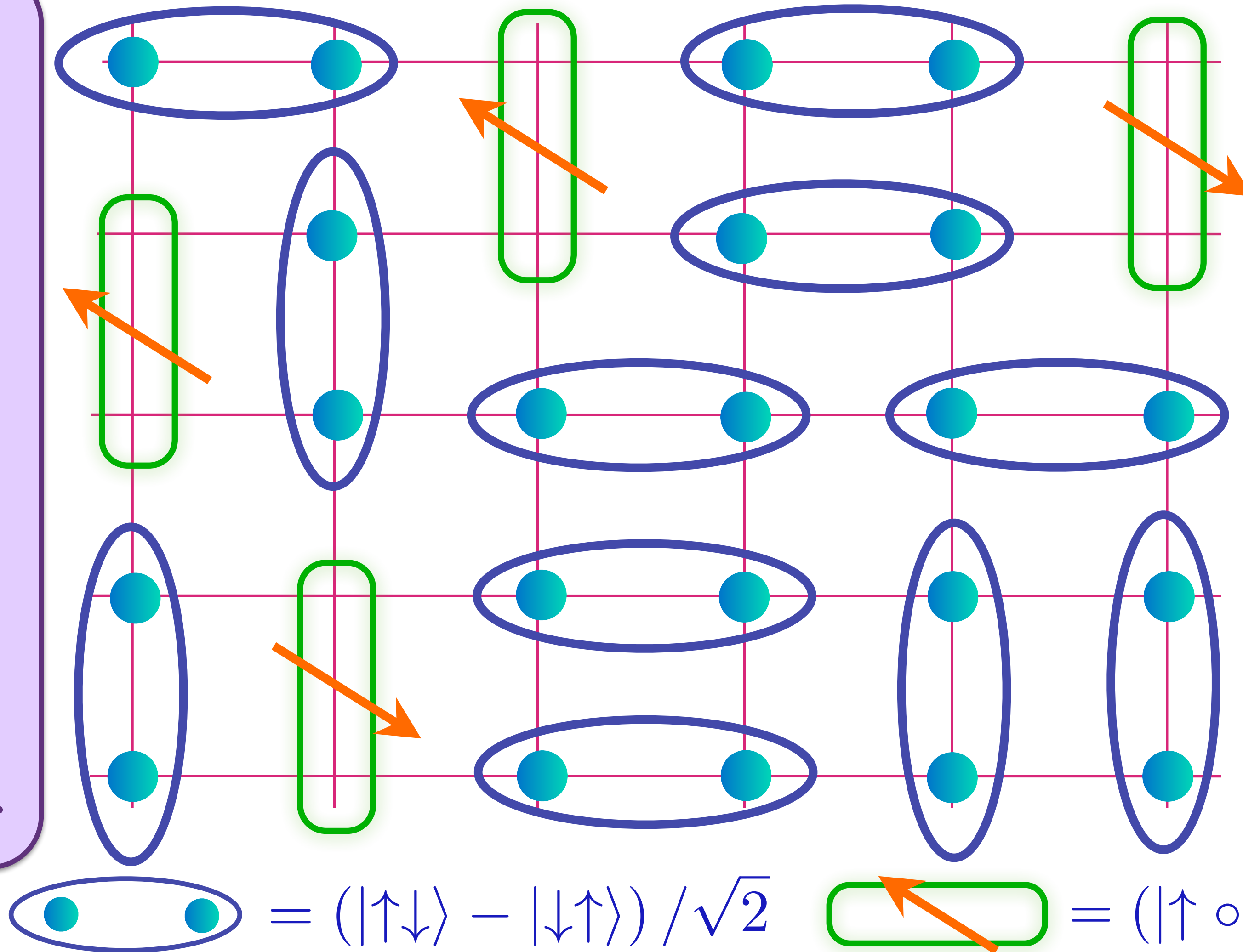
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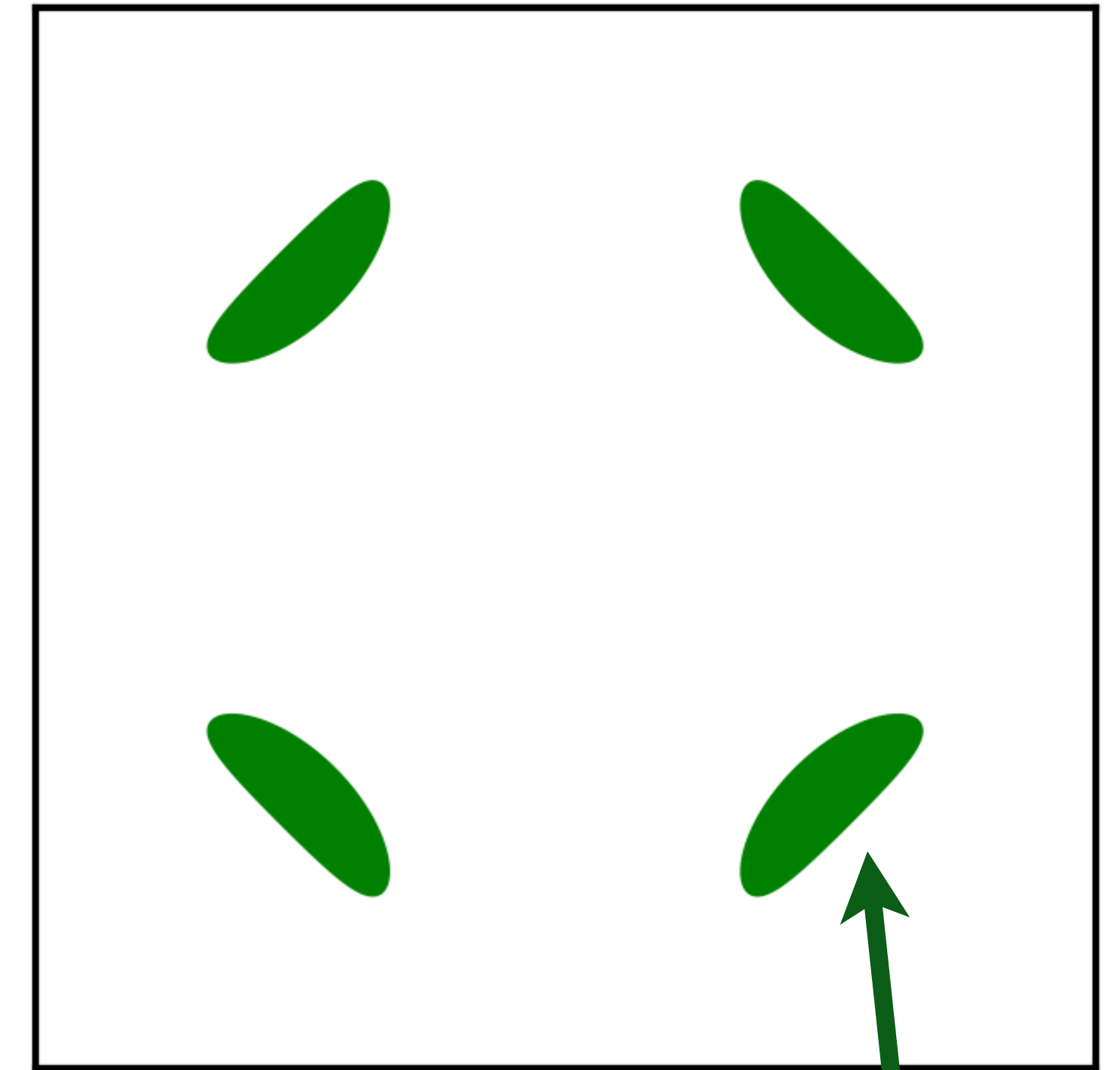
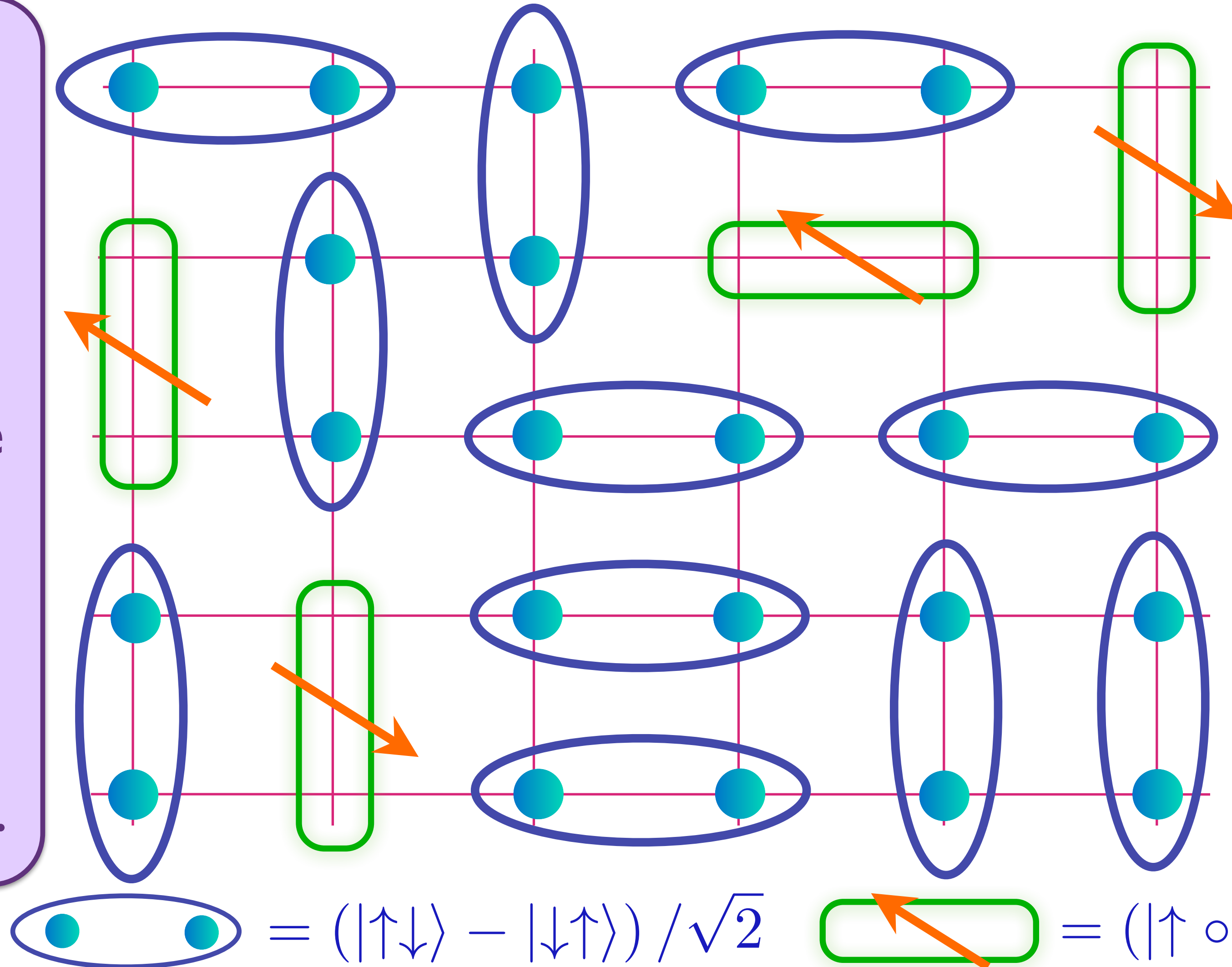
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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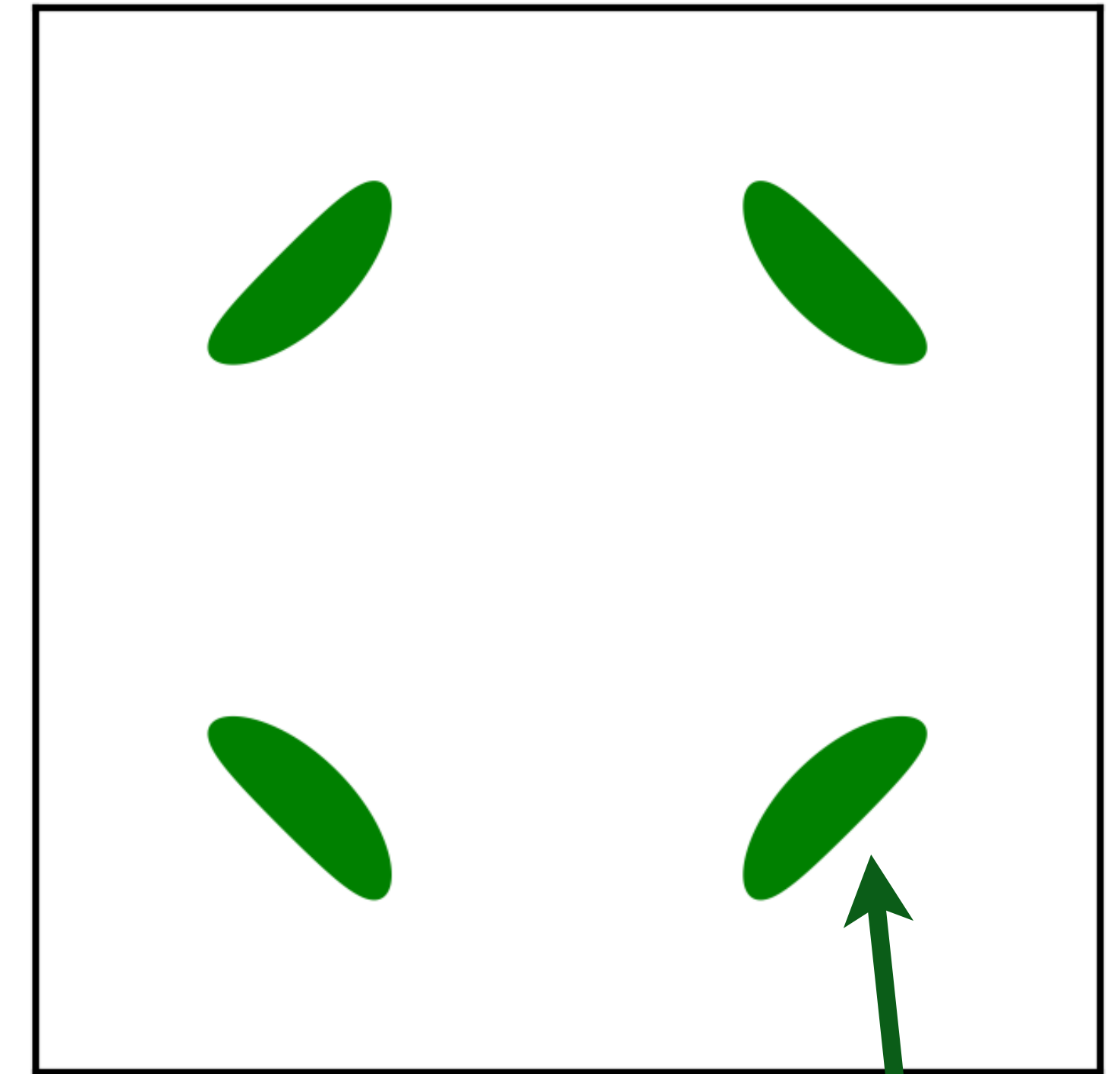
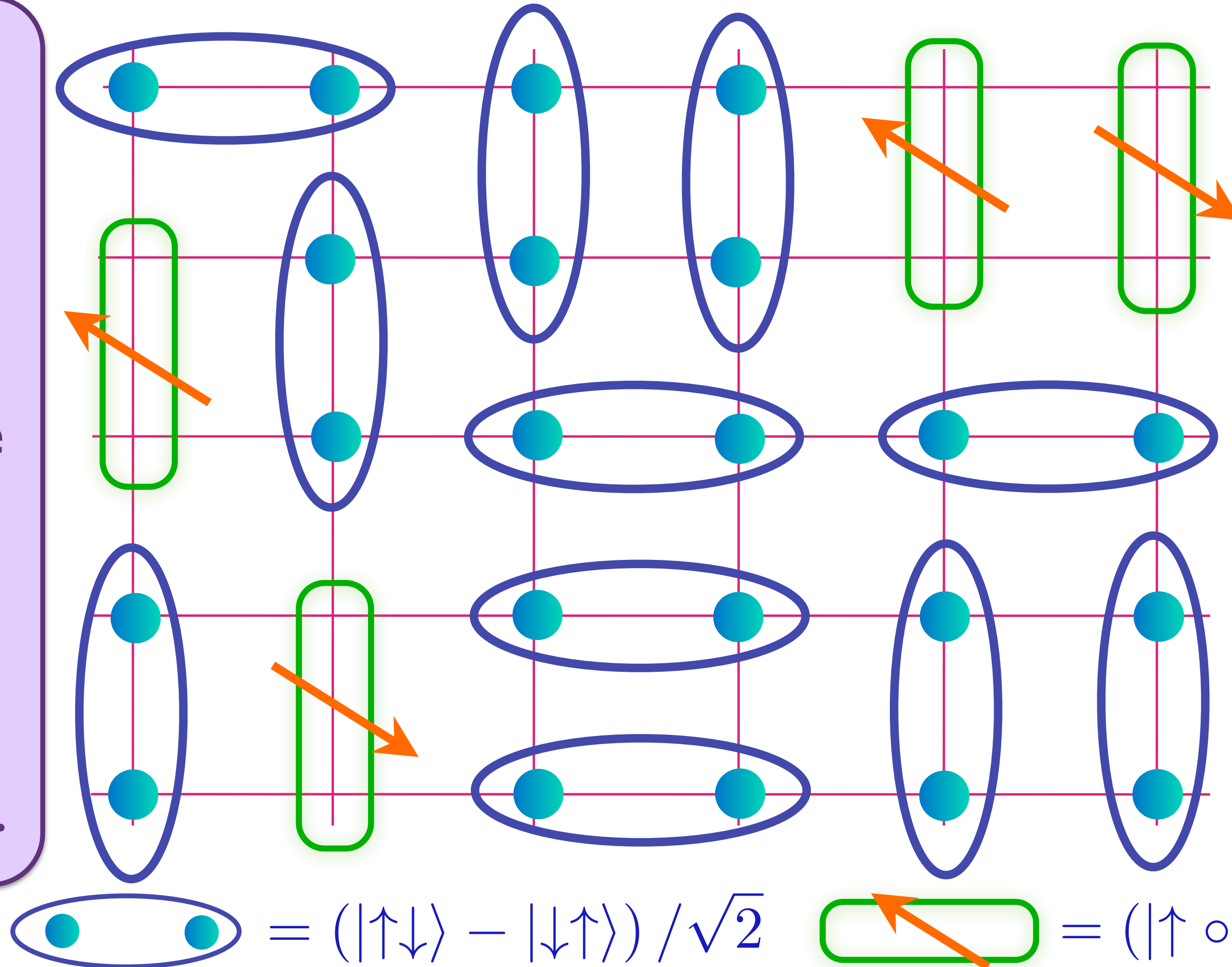
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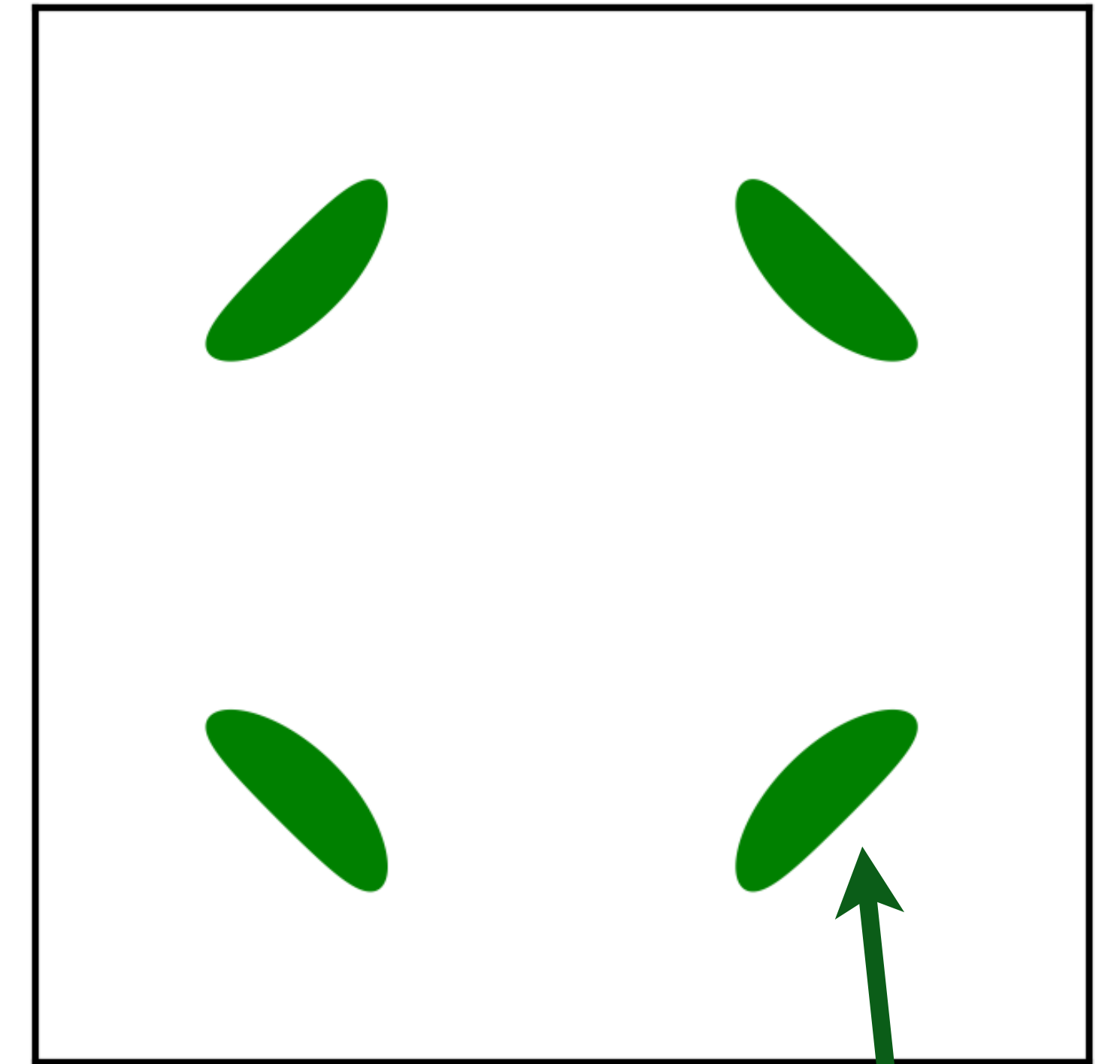
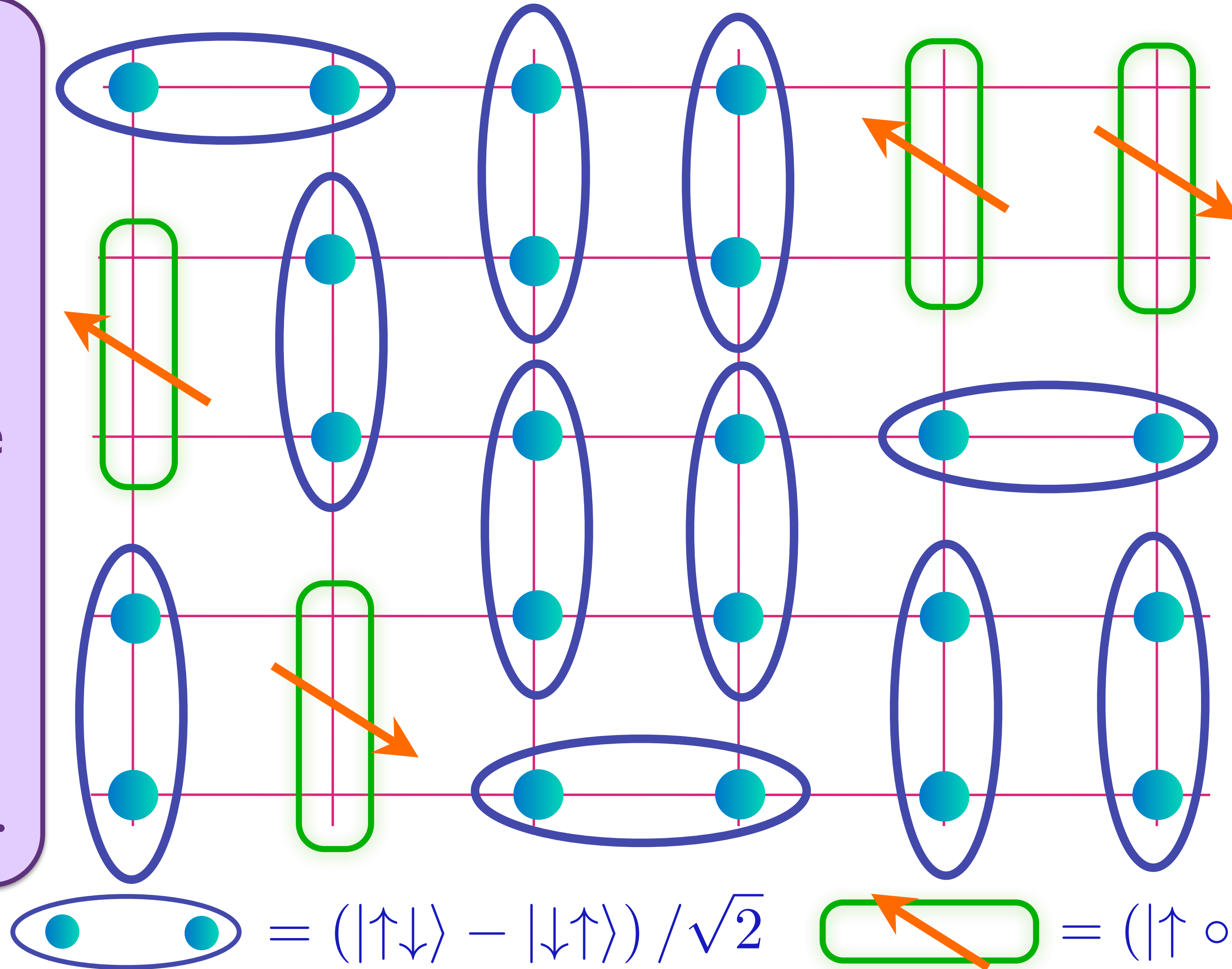
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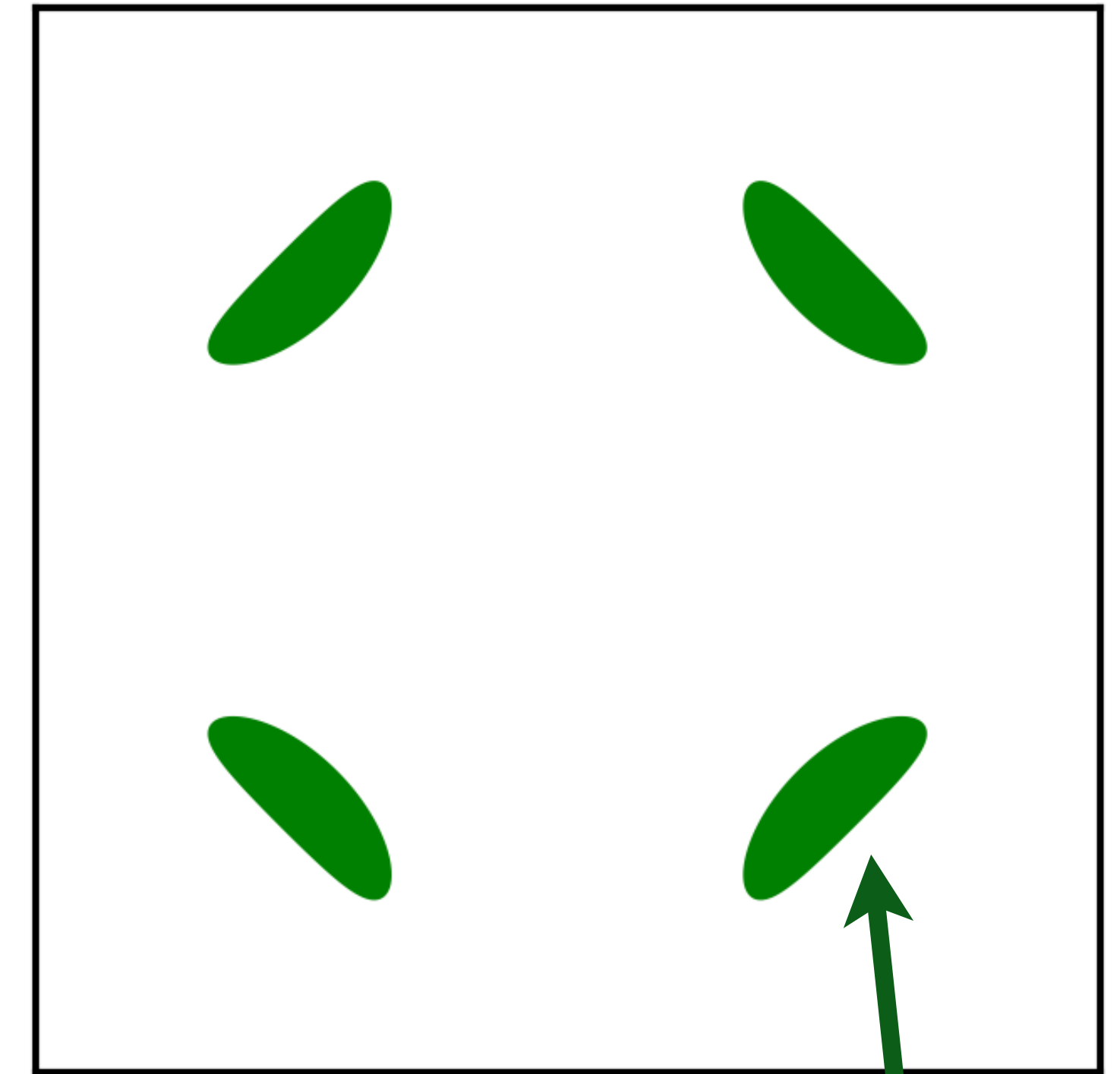
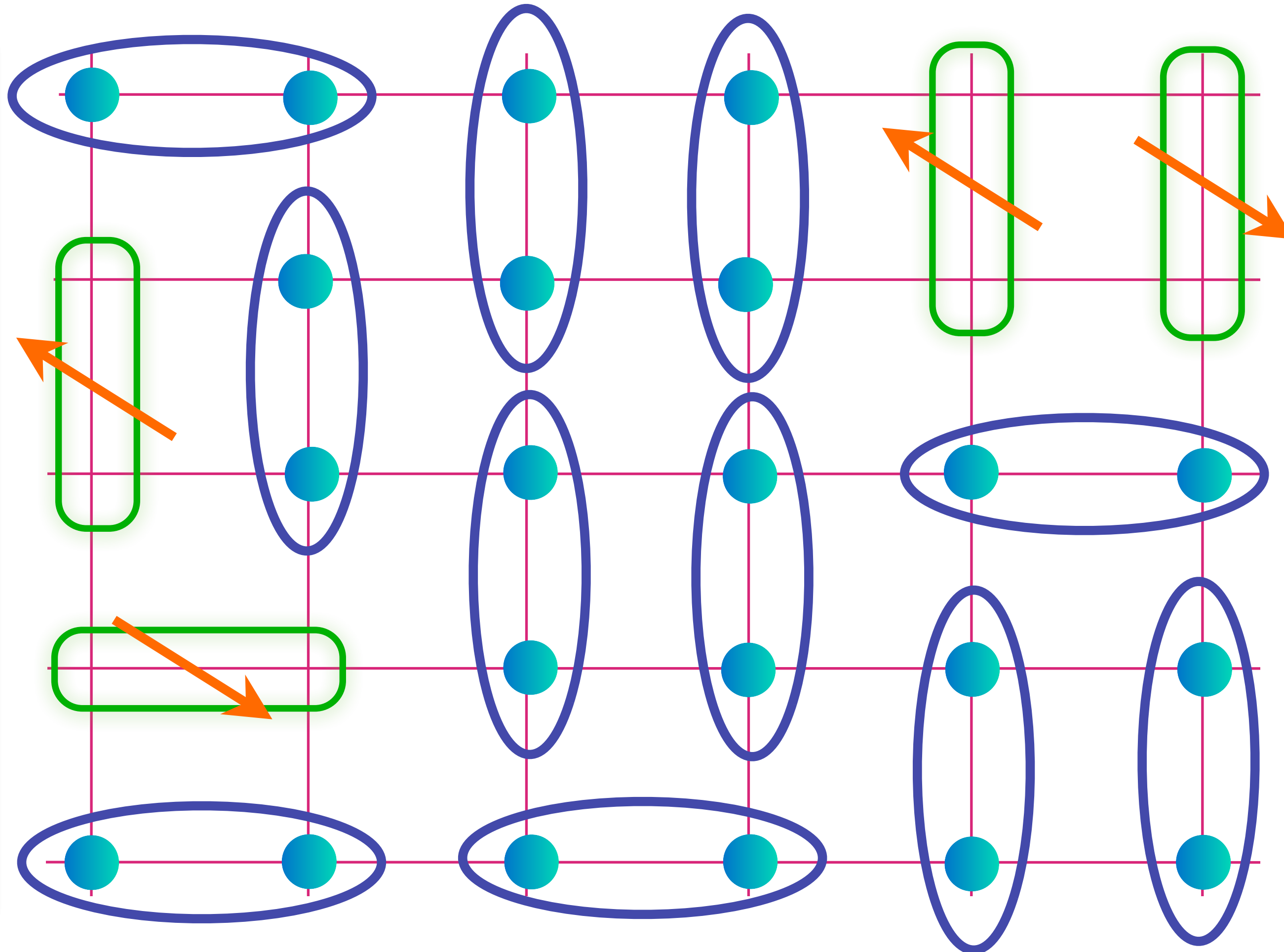
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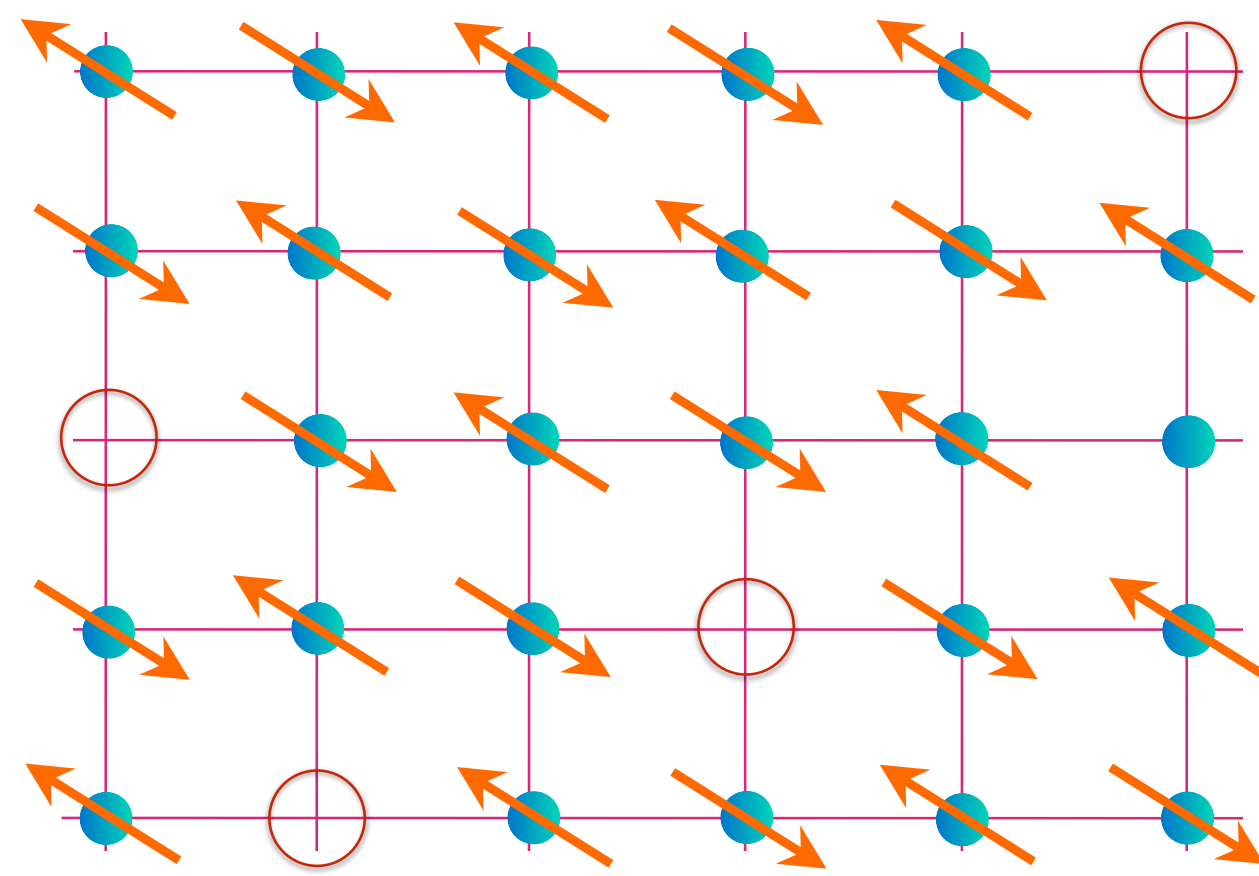
$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \quad \begin{array}{|c|} \hline \text{orange arrow} \\ \hline \end{array} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Area $p/8$

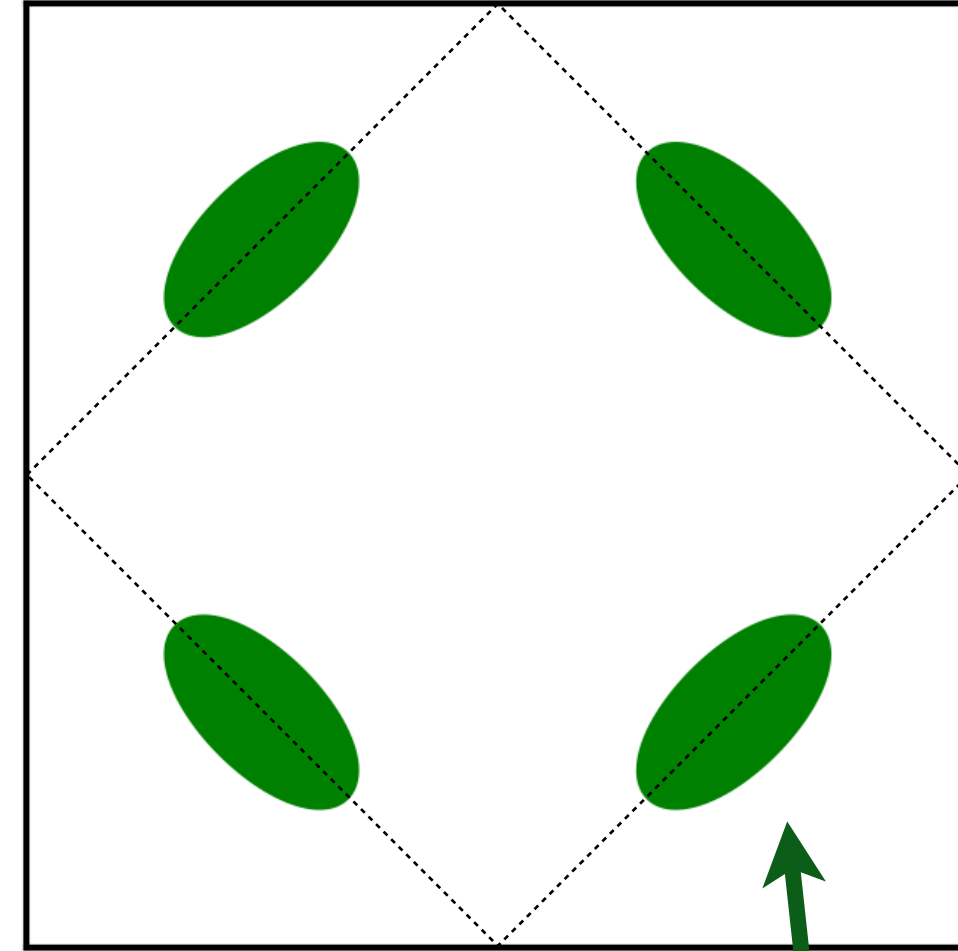
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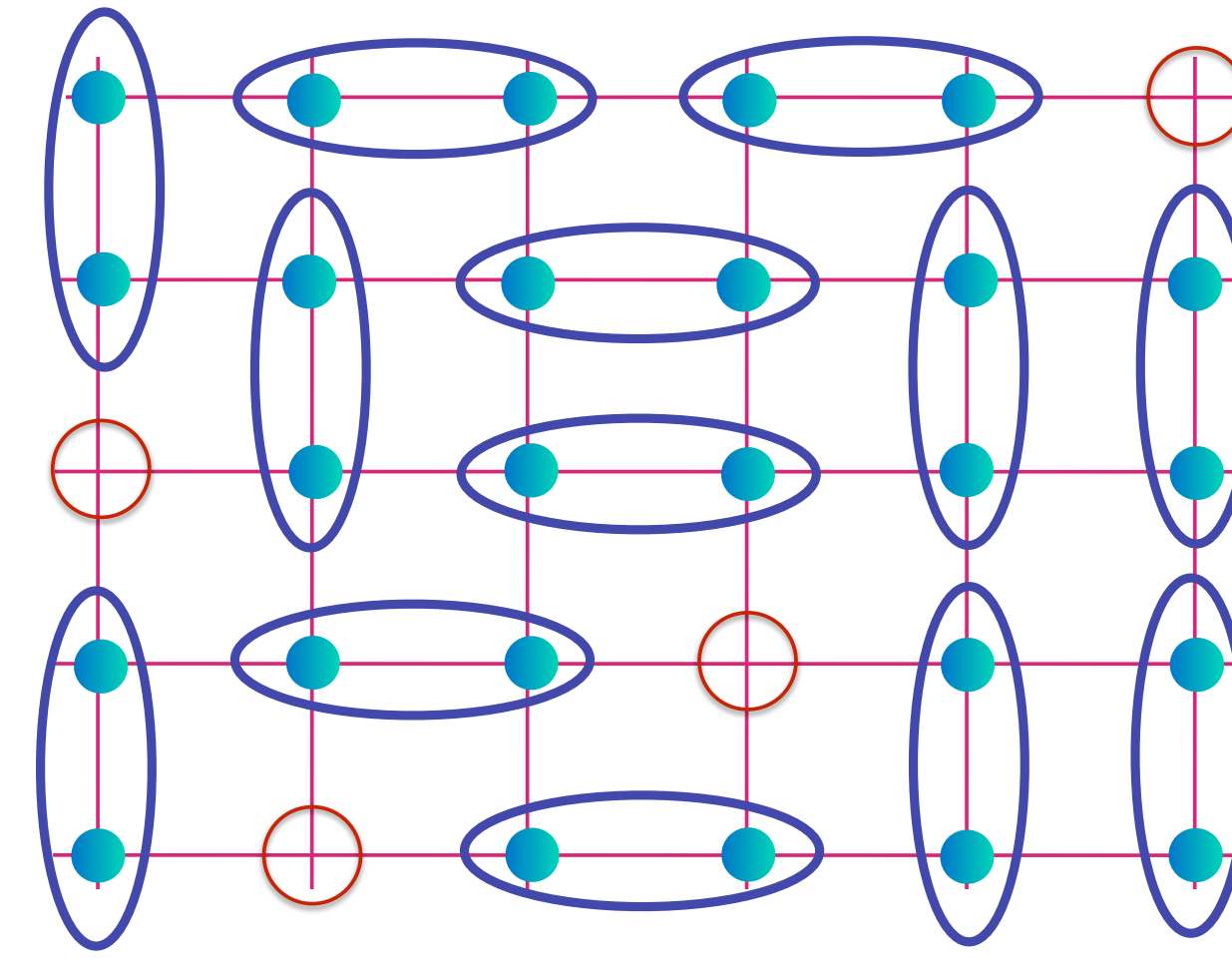
Doping an insulating antiferromagnet with holes of density p



AF metal

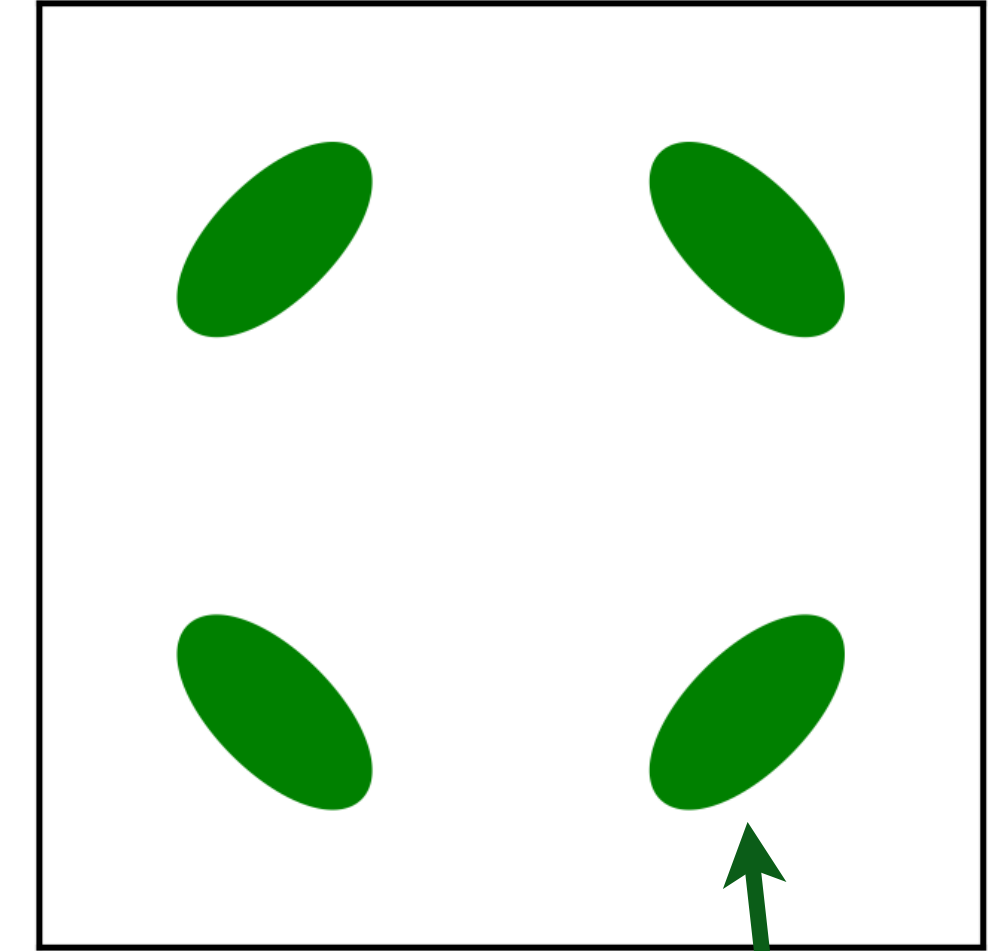


Area $p/4$



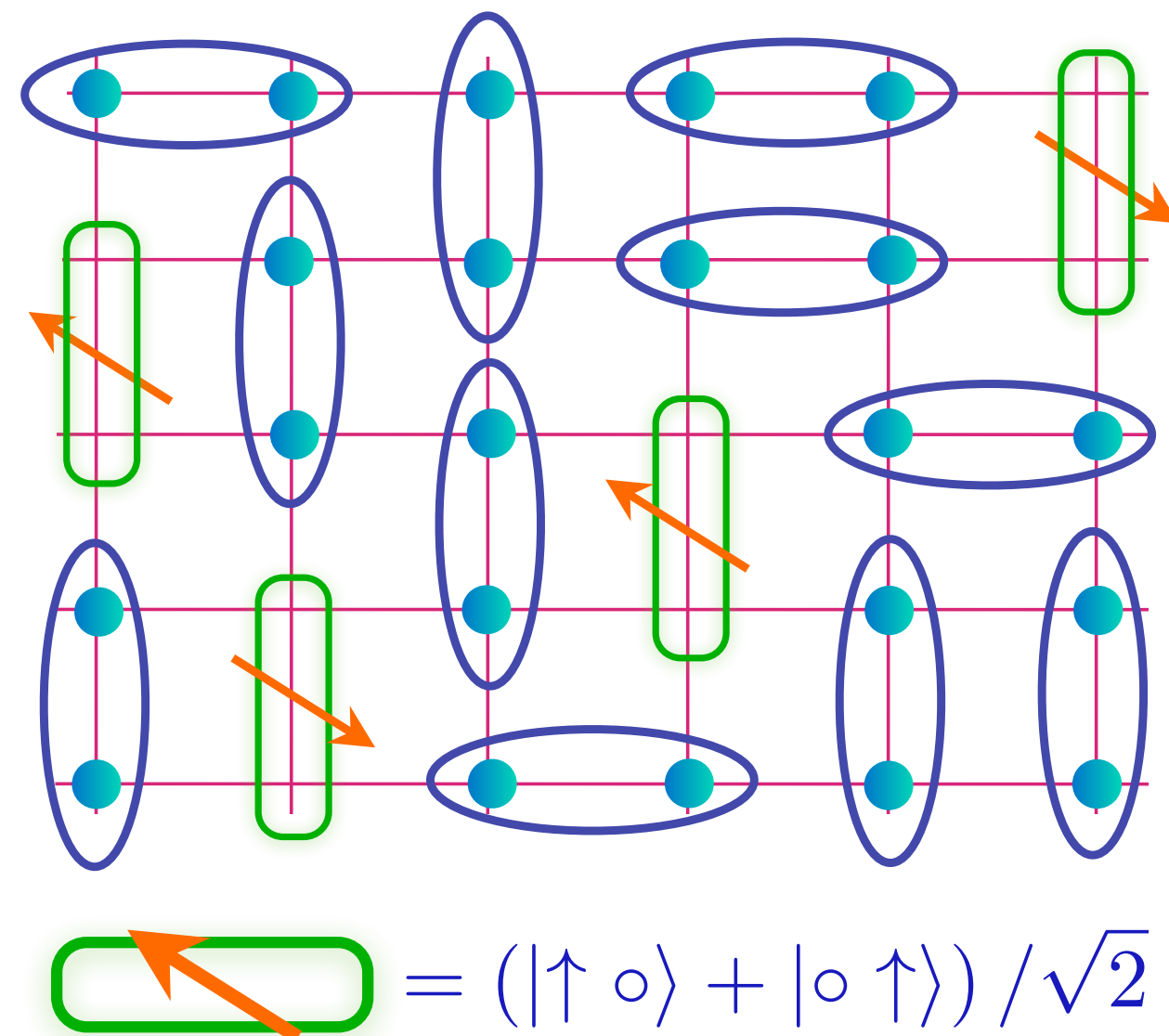
$$\text{blue ellipse} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

Holon metal



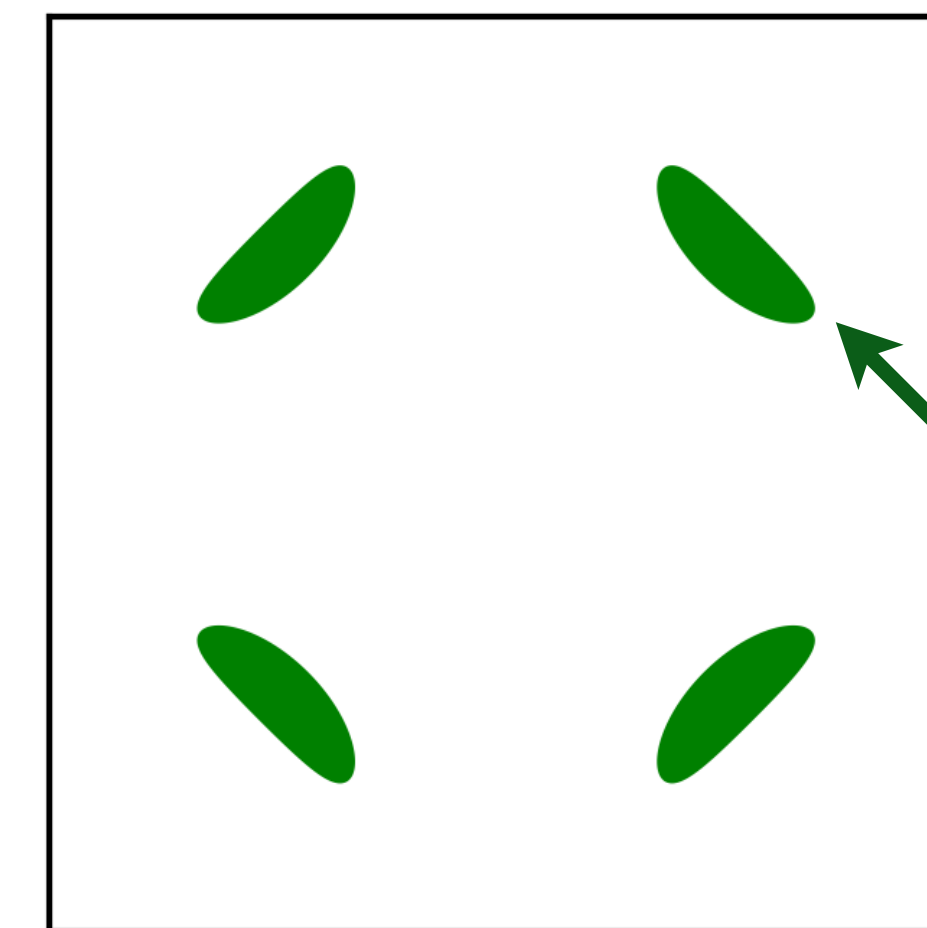
Area $p/4$

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 R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
 M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
 E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



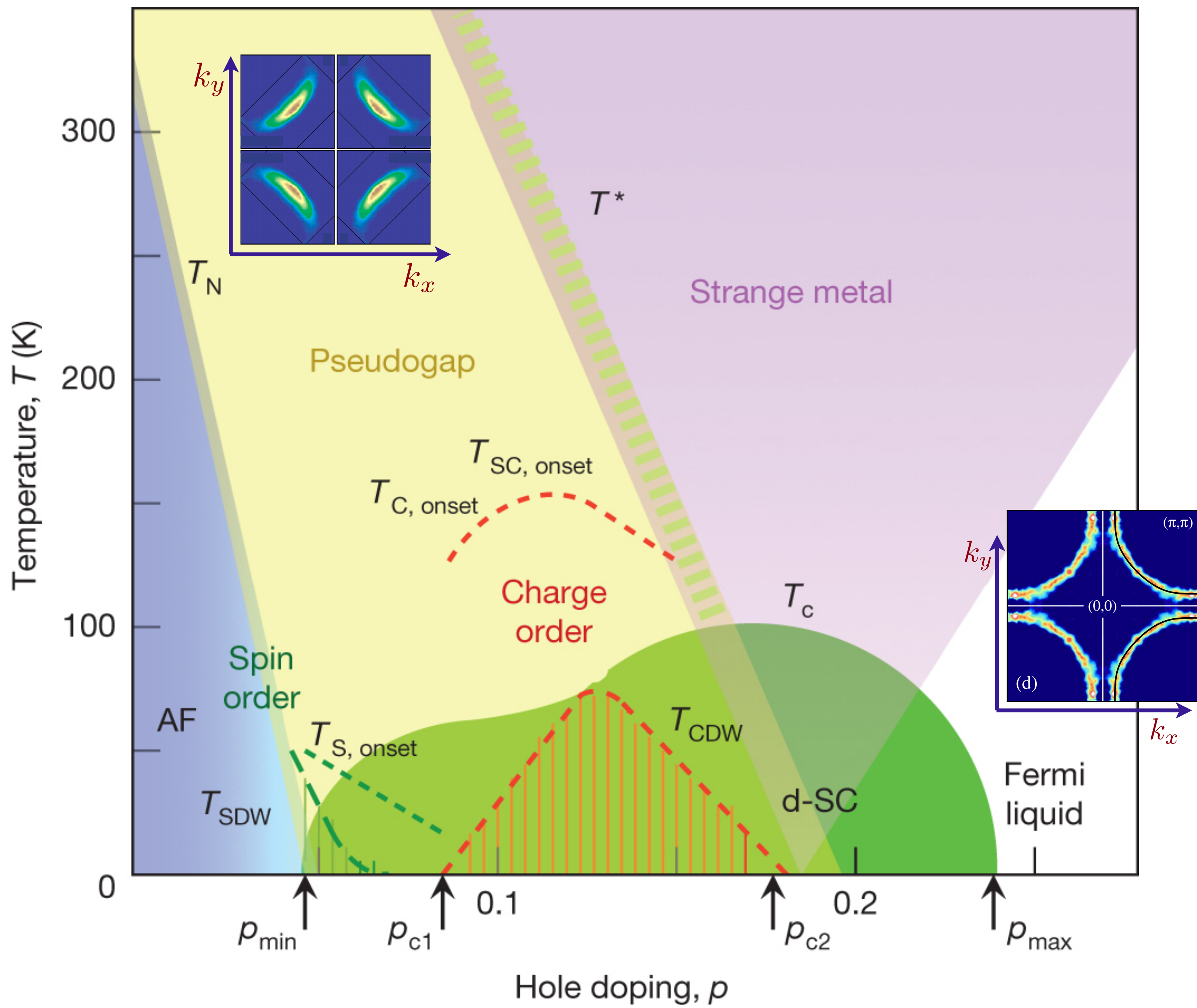
$$\text{green rectangle} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

FL*



Area $p/8$

FL* prediction for Fermi
surface size:
observed by the Yamaji
effect in the cuprates

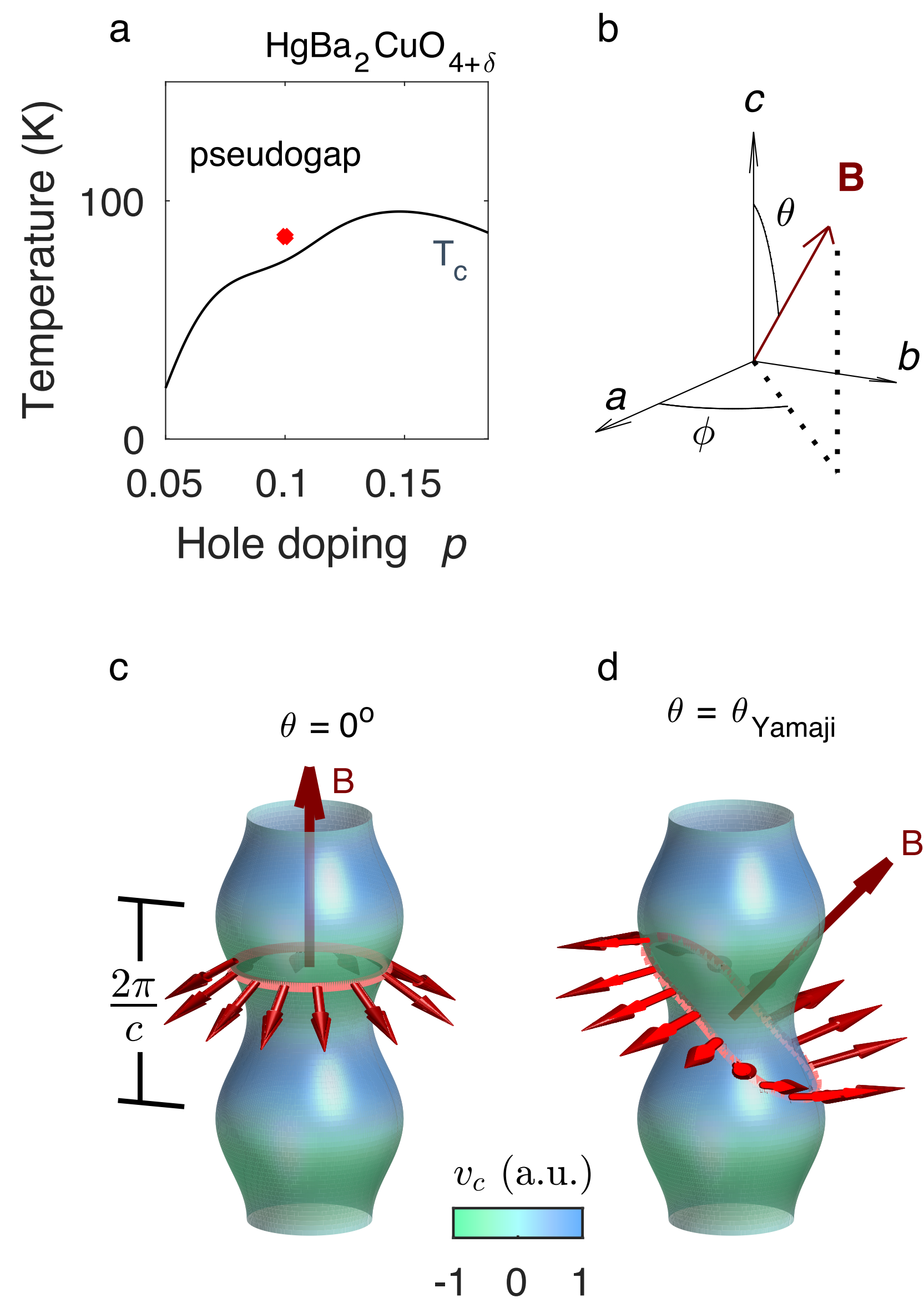


Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,^{1,*} Katherine A. Schreiber,¹ Oscar E. Ayala-Valenzuela,¹

arXiv:2411.10631

Eric D. Bauer,² Arkady Shekhter,¹ and Neil Harrison¹



At the Yamaji angle, the orbits in the plane orthogonal to B have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

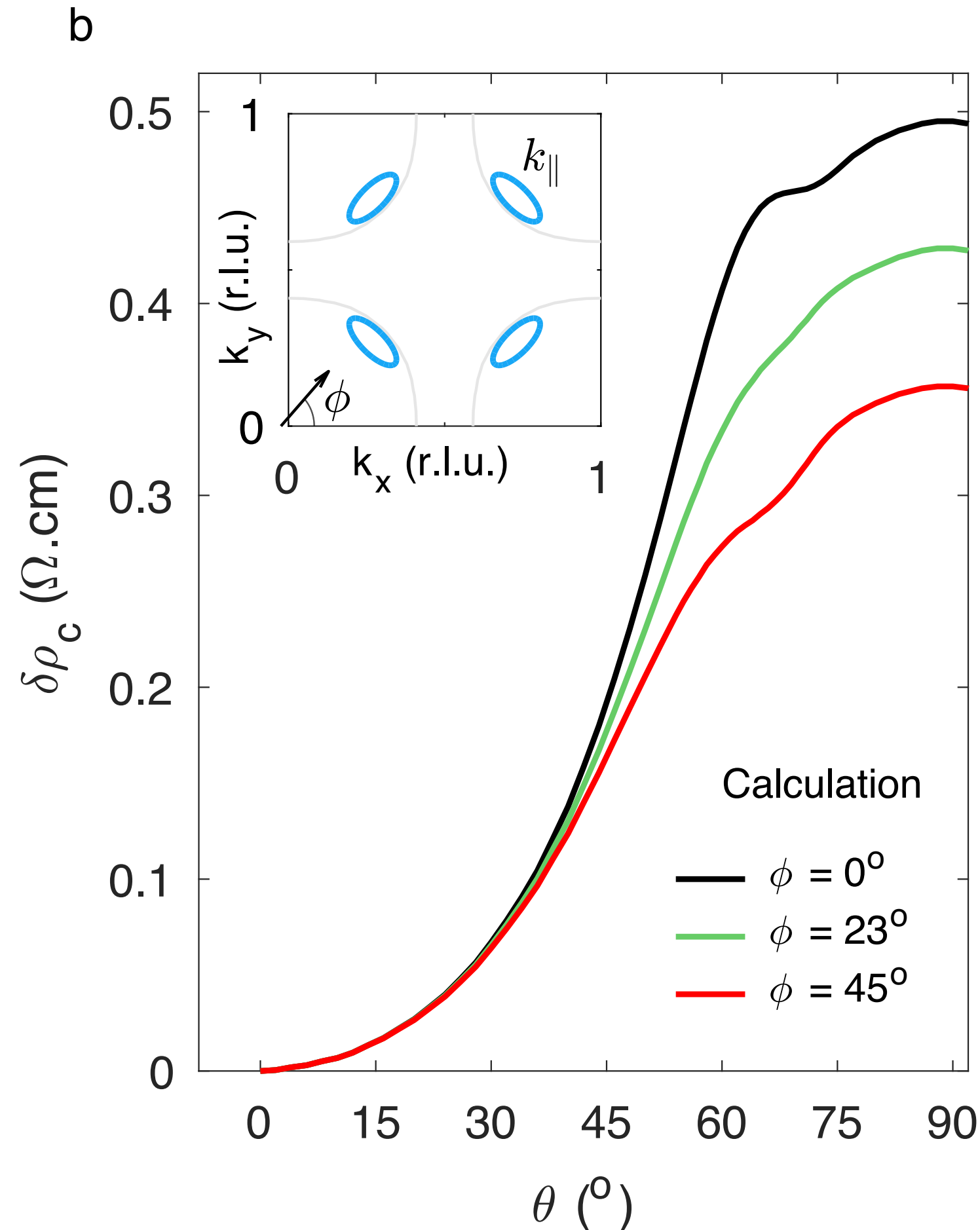
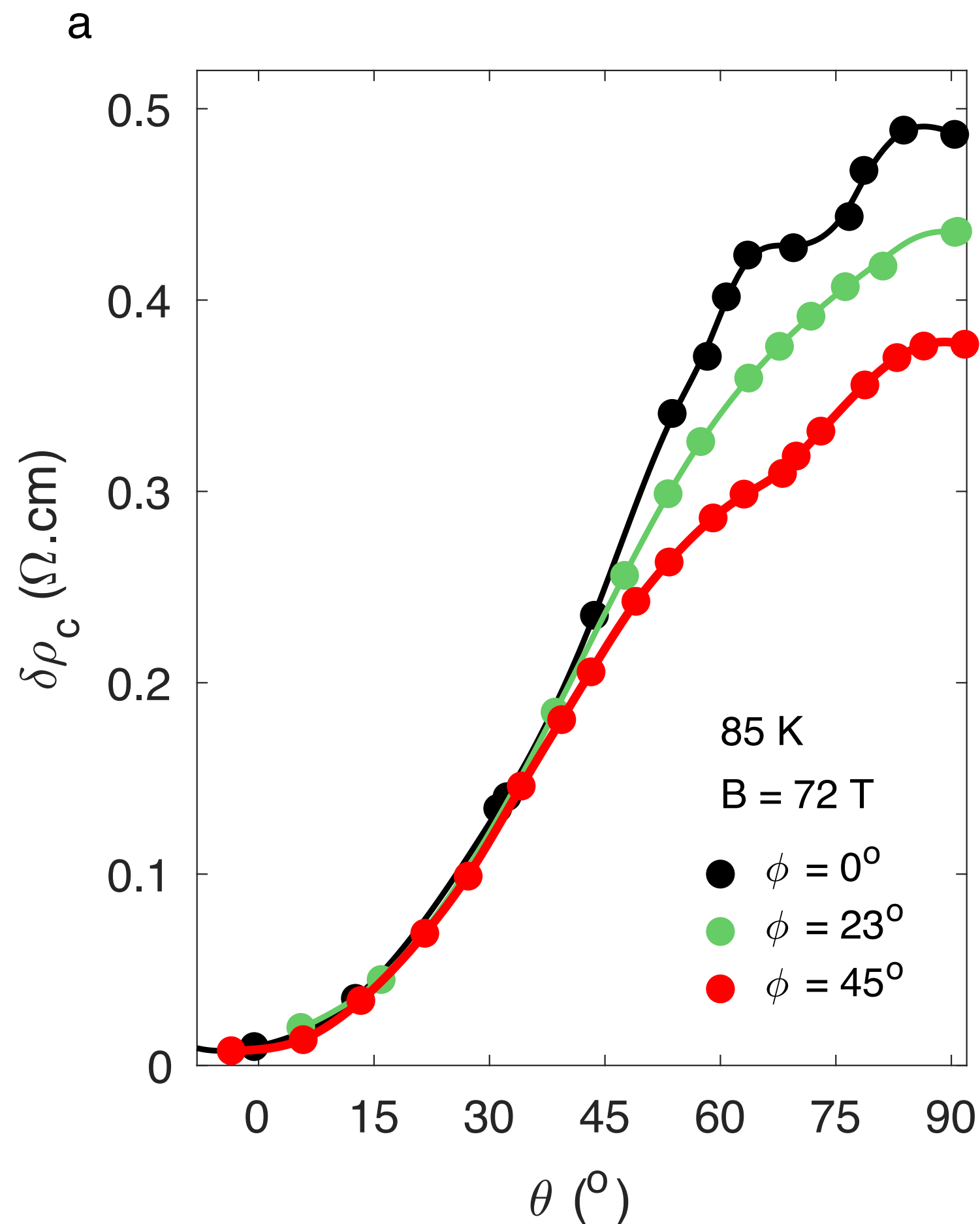
K.Yamaji JPSJ **58**, 1520 (1989)

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,^{1,*} Katherine A. Schreiber,¹ Oscar E. Ayala-Valenzuela,¹

Eric D. Bauer,² Arkady Shekhter,¹ and Neil Harrison¹

arXiv:2411.10631



“The small size of the pockets determined from the Yamaji effect is ... approximately 1.3% of the Brillouin zone area”

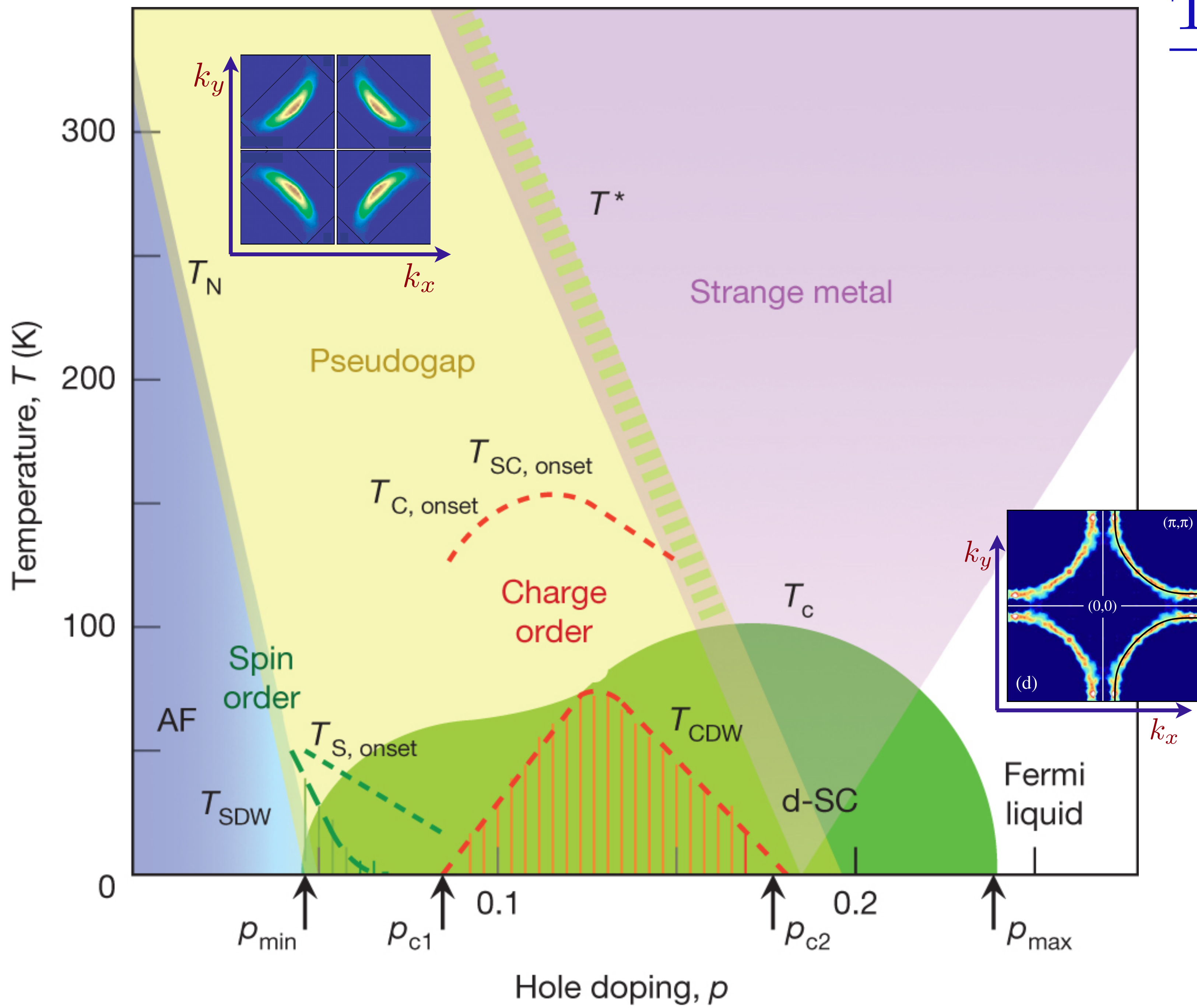
FL* pocket fraction
 $= p/8 = 1.25\%$!

Fluctuating AF metal fraction
 $= p/4 = 2.5\%$.

($p/8$ also in YRZ ansatz,
Peter Johnson photoemission,
and Jenny Hoffman
and Seamus Davis STMs)

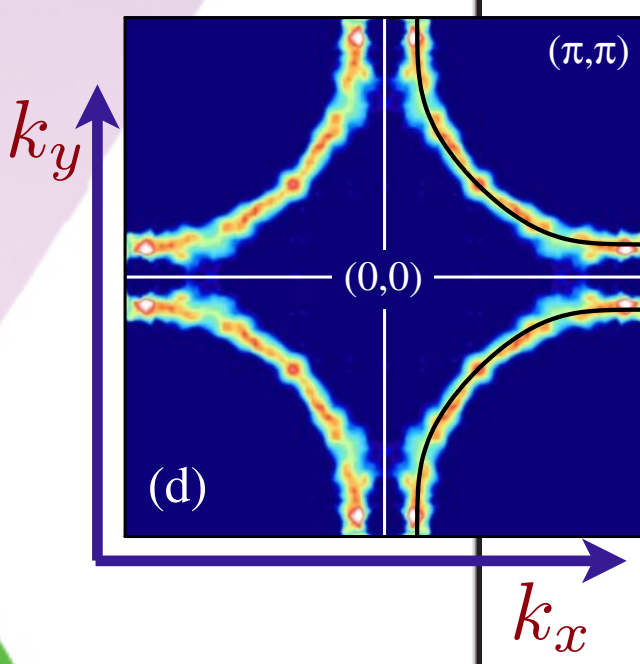
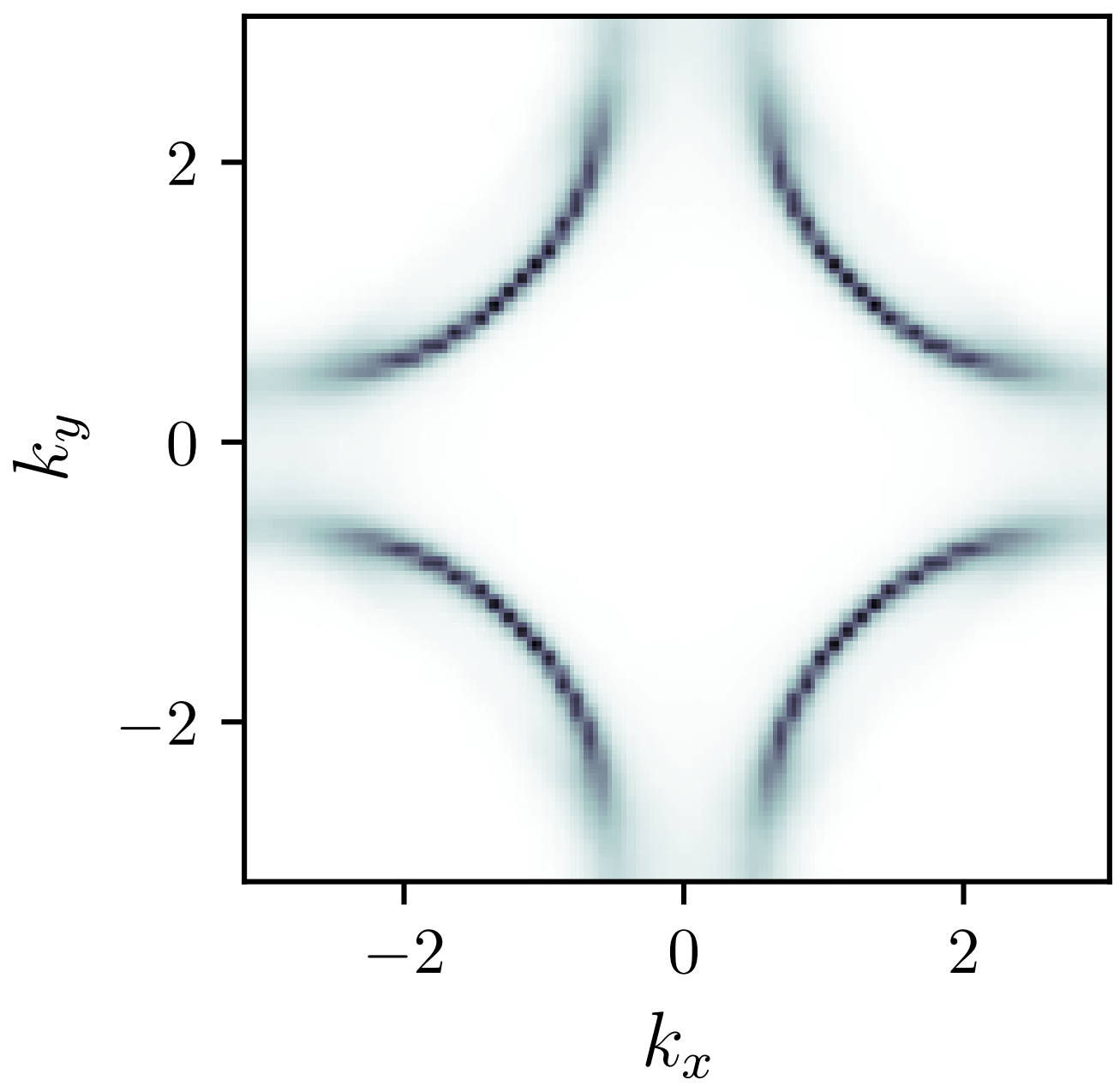
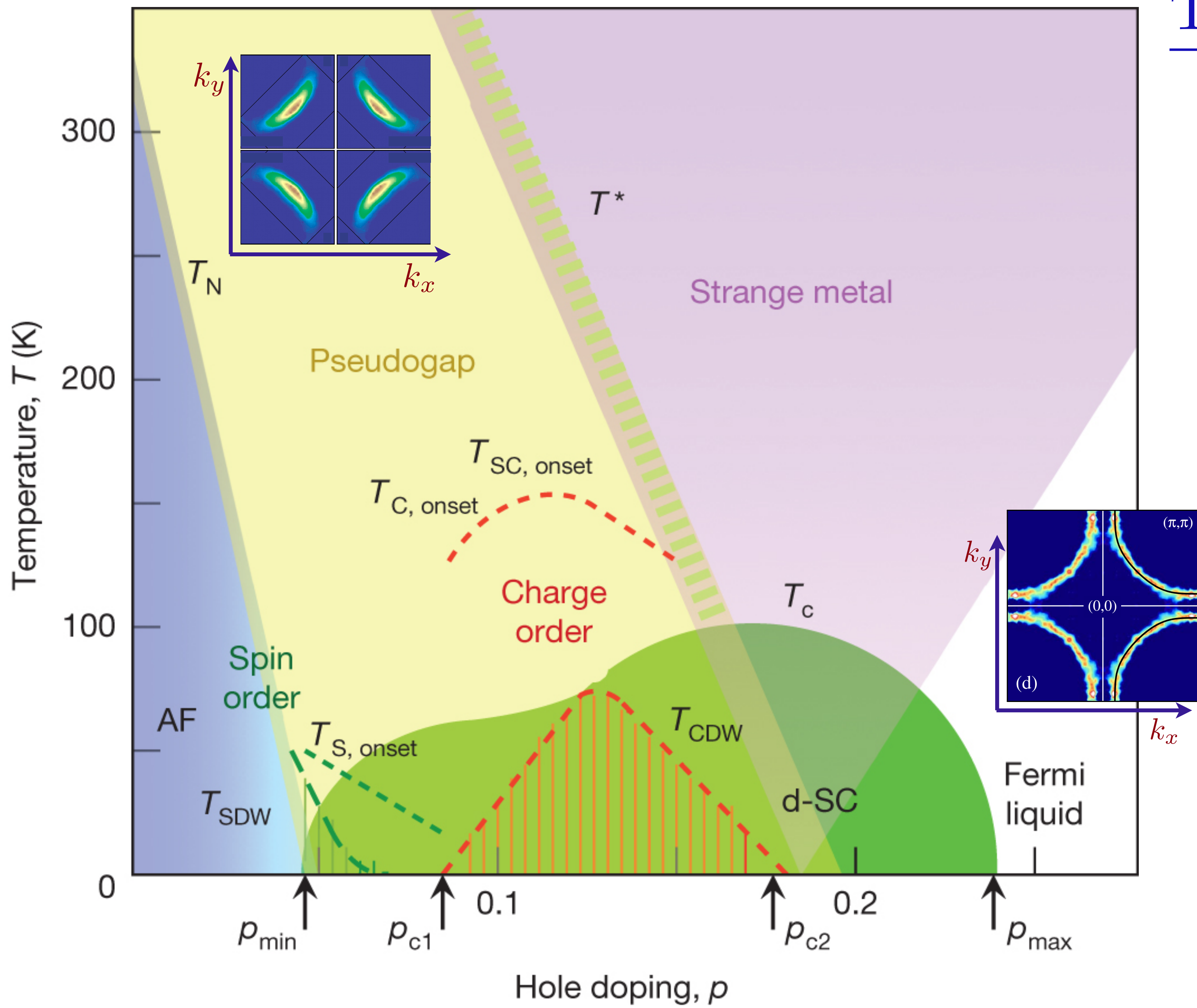
Theory of the FL* pseudogap
and its Higgs transition to a
BCS d-wave superconductor

Theory of the pseudogap



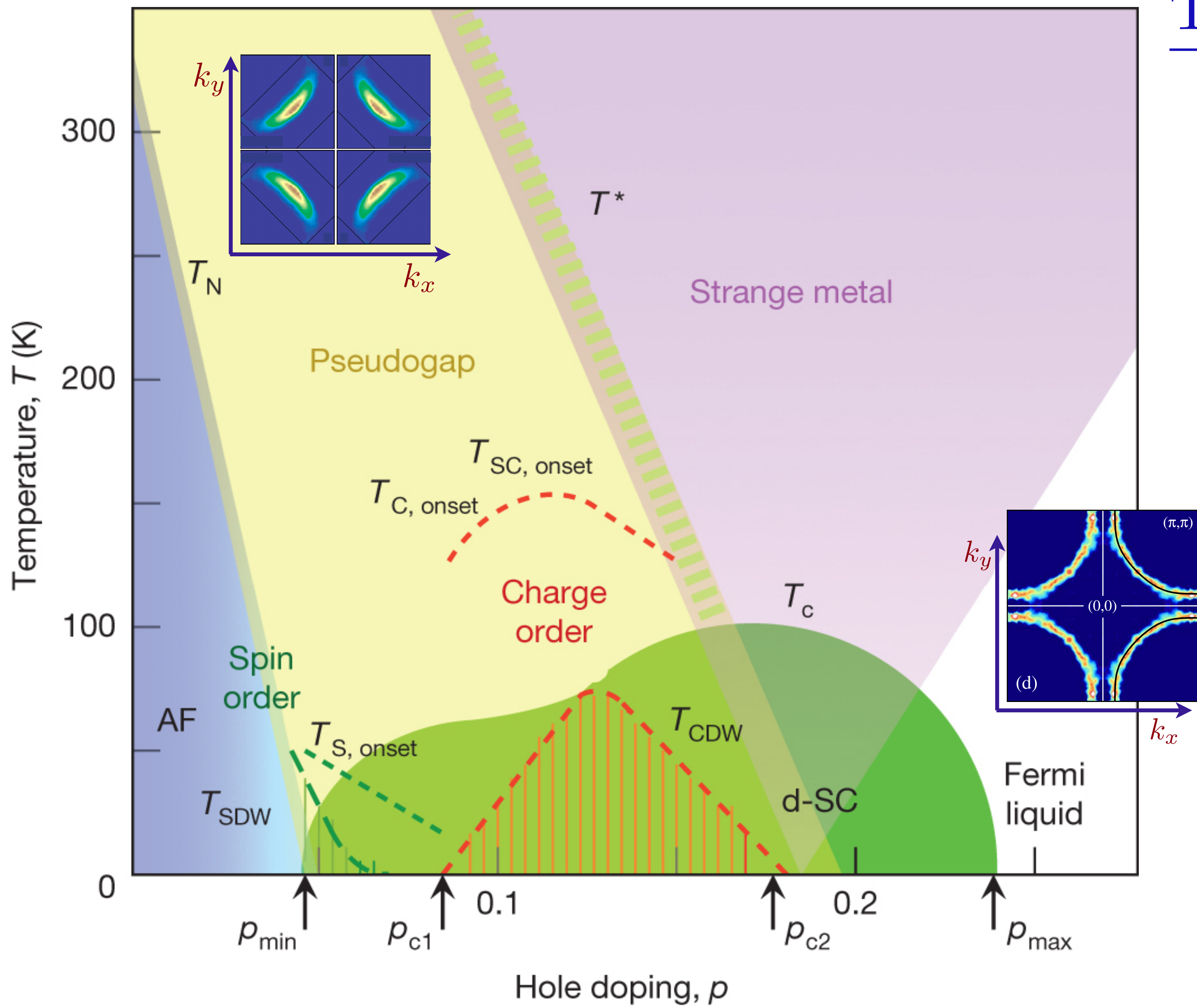
SU(2) gauge field U ,
and a
fundamental Higgs boson B
with charge e moving in a
background π -flux.
Yukawa couplings to
Fermi pockets and spinons.

Theory of the pseudogap

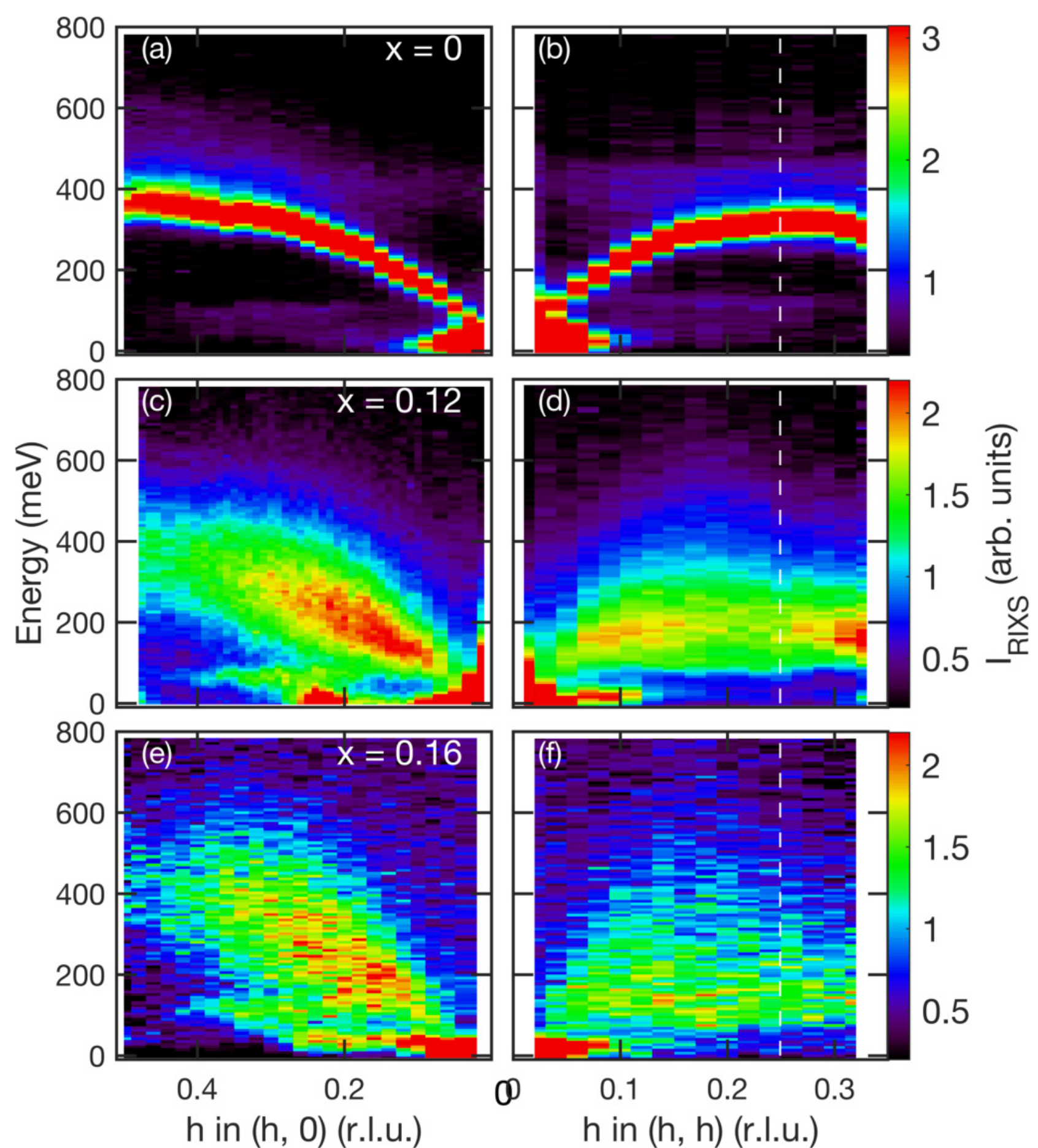


Born-Oppenheimer theory
with
thermal, classical B , U ,
and quantum fermions

P.M. Bonetti, M. Christos,
H. Pandey, R. Shanker, S.S.,
S. Sharma, to appear



Theory of the pseudogap

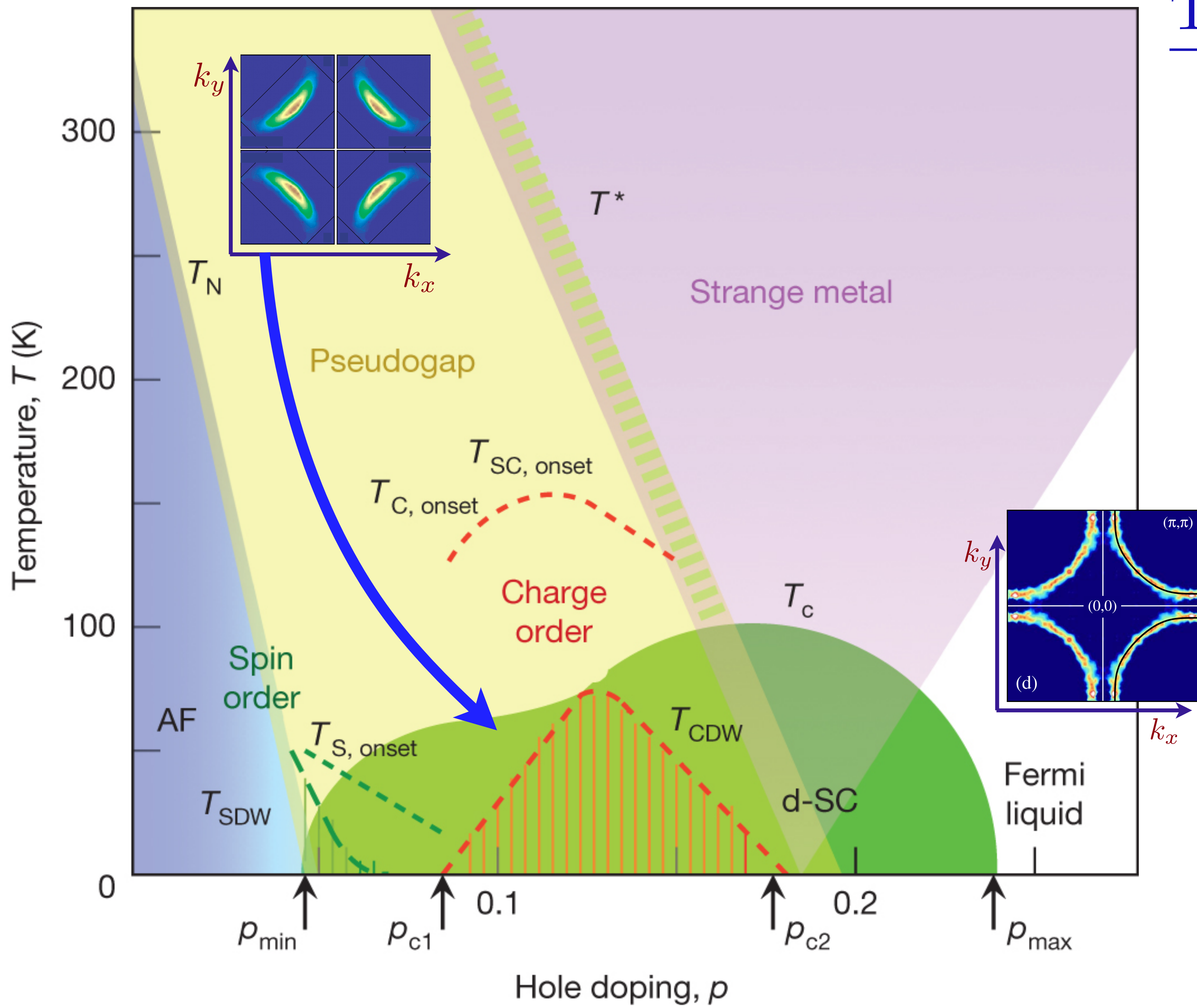


RIXS spectra (Robarts *et al.*,
PRB **100**, 214510 (2019))
match spinon continua
of π -flux spin liquid

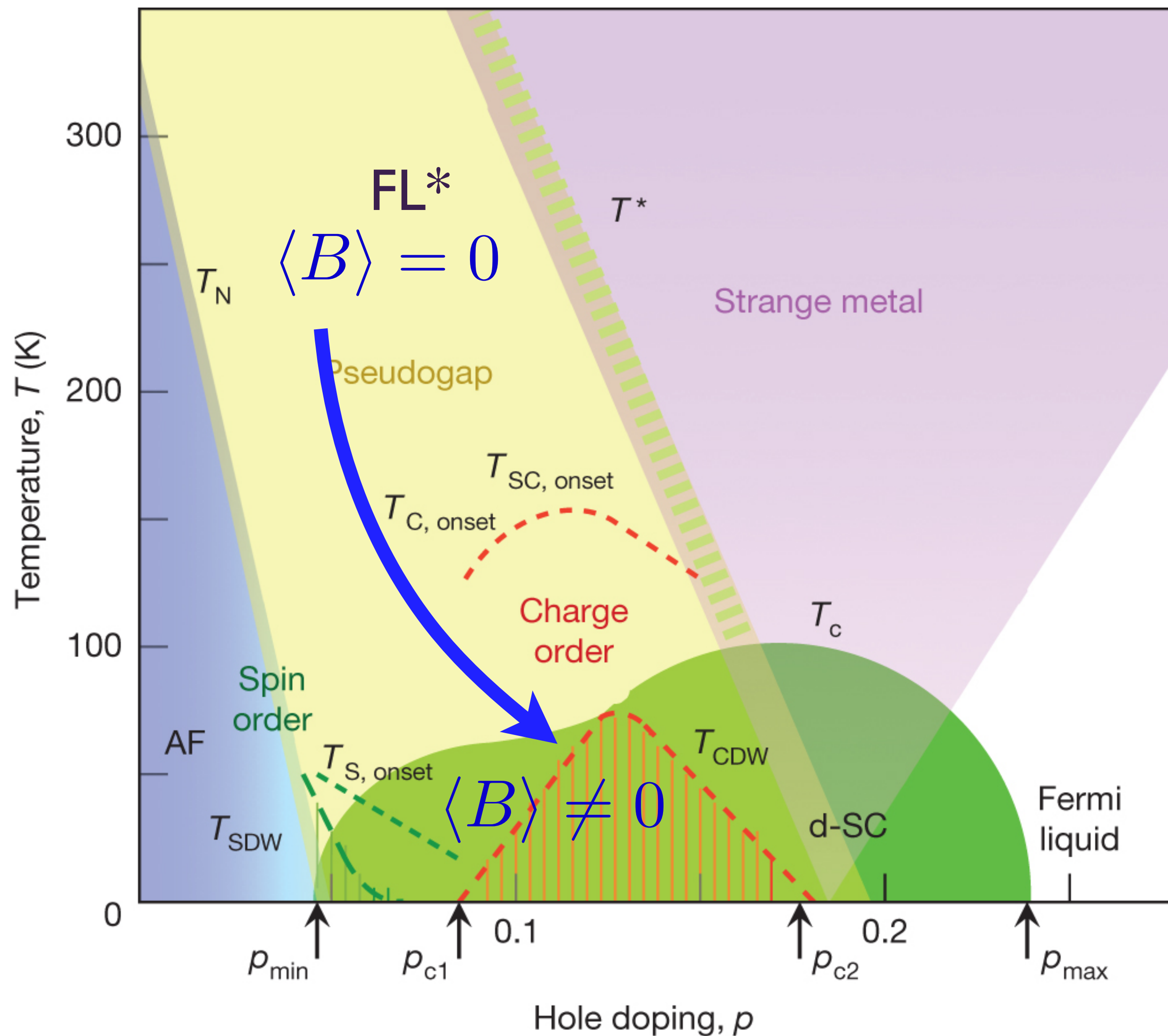
Theory of the pseudogap

Confinement/Higgs transition to a *d*-wave superconductor with $h/(2e)$ vortices and 4 nodal Bogoliubov quasiparticles with anisotropic velocities.

A non-BCS ‘mechanism’ for a BCS-type superconductor!



Theory of the pseudogap



- FL* pockets explain Yamaji effect.
- Thermal fluctuations of $B \Rightarrow$ theory of Fermi arcs in FL*.
- RIXS spectrum related to spinon continua of FL*
- 4 nodal Bogoliubov quasiparticles with anisotropic velocities.
- Fermi surfaces in charge ordered state explain quantum oscillations.
P. M. Bonetti, M. Christos, S. S. (BCS), PNAS **121**, e2418633121 (2024)
- BKT transition of $h/(2e)$ vortices to a d -wave superconductor.
- Halo of charge order in vortices.

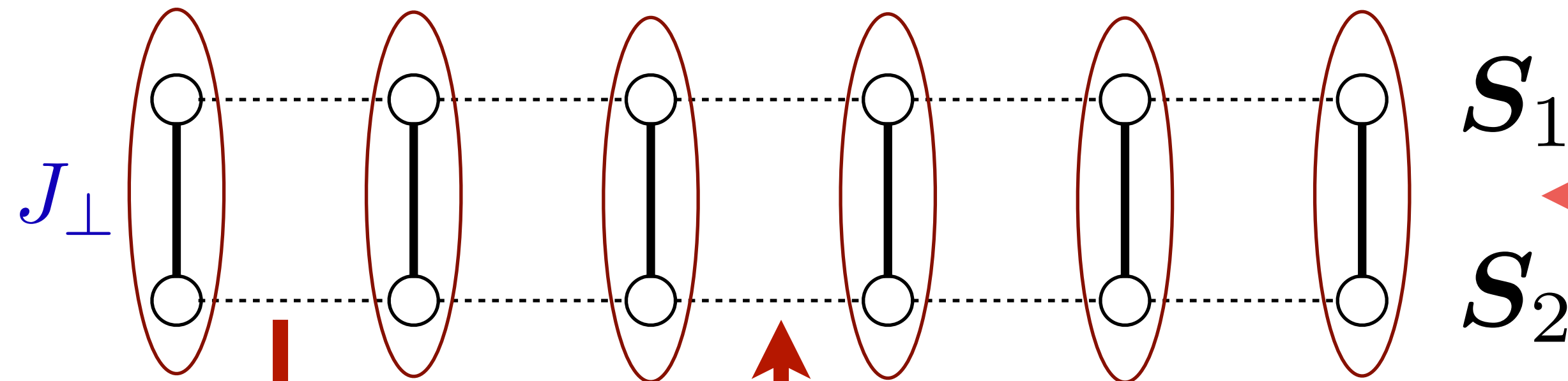
Ancilla theory of the
 FL^* pseudogap

Ancilla theory of the Hubbard model

(Luttinger-Oshikawa anomalies for dummies)

c_σ

Hubbard
model of
hole density
 $1+p$

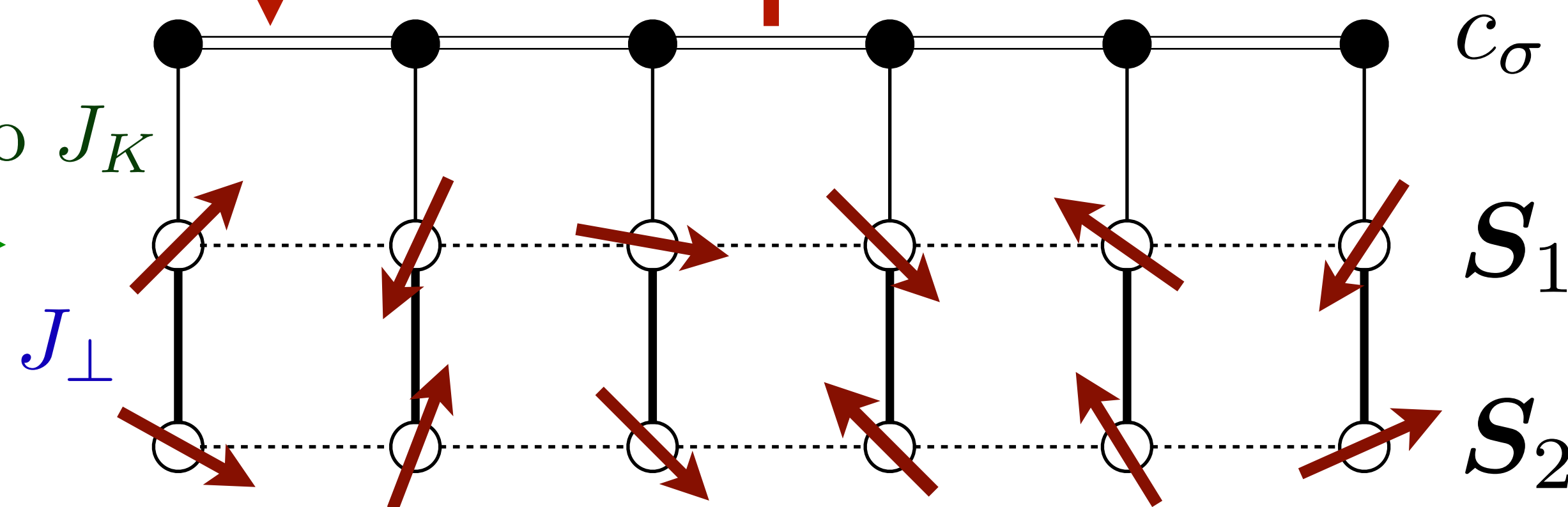


Ancilla qubits
in a “trivial”
gapped insulator

Hubbard-Stratonovich

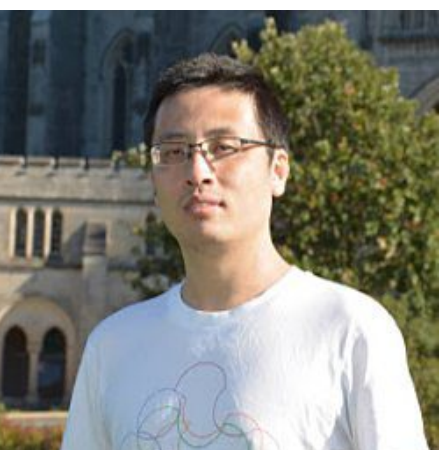
Schrieffer-Wolff (large $J_\perp$): $U = \frac{3J_K^2}{8J_\perp} + \frac{3J_K^3}{16J_\perp^2} + \dots$

Antiferromagnetic Kondo J_K



Free holes of
density
 $1+p$

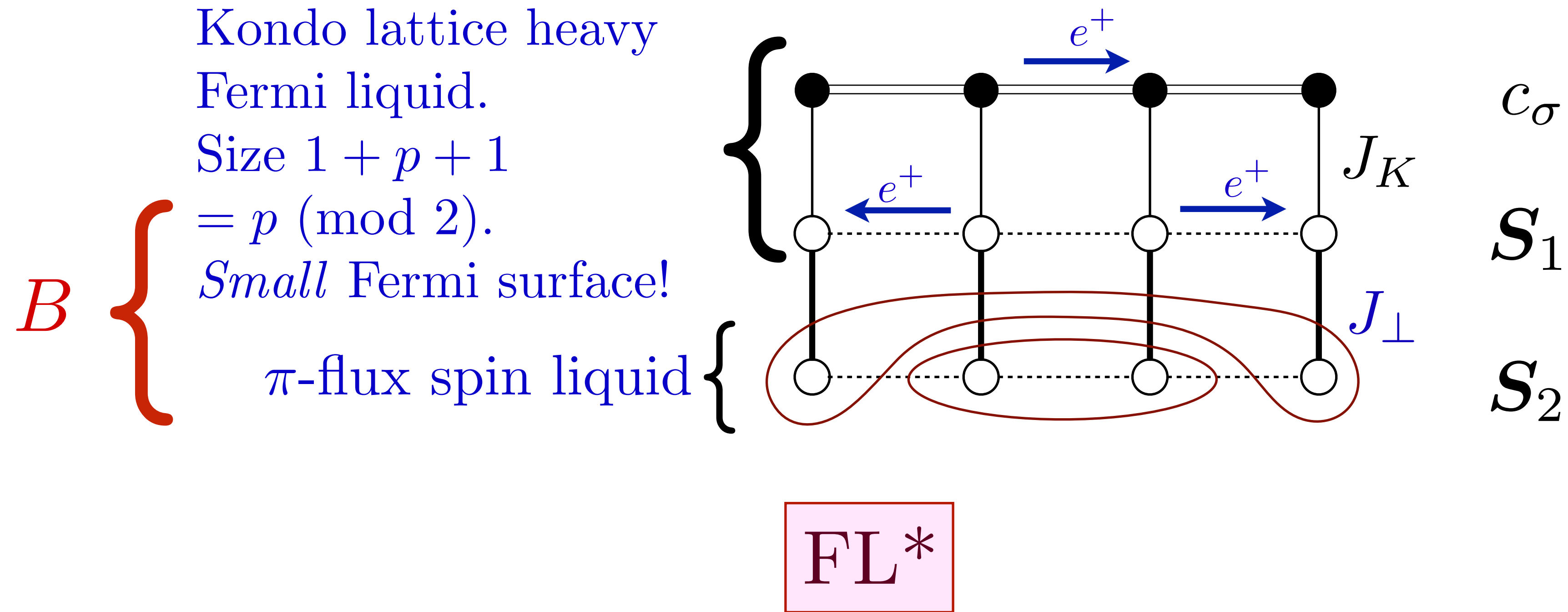
Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla theory of FL* and its confinement



Large J_K
and small p

Pseudogap metal =
 Kondo Lattice Heavy
 Fermi Liquid
 $\oplus$
 Spin Liquid

Also need a charge e ,
 SU(2) fundamental
 Higgs boson B
 which couples
 Kondo lattice
 and spin liquid

Similar boson in
 X.-G. Wen and
 P.A. Lee, PRL **76**,
 503 (1996) but
 applied to a
 different spin liquid

Ancilla theory of FL* and its confinement

$$H_{\text{Kondo lattice}} = \sum_{i,j} \left[-t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} \right] - \sum_i \Phi (c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma})$$

} Heavy Fermi liquid of electrons c, f_1 (Similar to YRZ)

$$H_{\text{spin liquid}} = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right); \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

} π -flux spin liquid of spinons f and SU(2) gauge field U

$$H_{\text{coupling}} = \sum_i \left(B_{1i}^* f_{1i\sigma}^\dagger f_{i\sigma} + B_{2i}^* \varepsilon_{\sigma\sigma'} f_{1i\sigma}^\dagger f_{i\sigma'}^\dagger + \text{H.c.} \right)$$

} Couple Kondo lattice and spin liquid by charge e , SU(2) fundamental Higgs boson B

$$H_{\text{Higgs}} = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + V(B)$$

} Higgs energy constrained by spin liquid