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A Matter of Perspective: An Exploratory Study of the Relationship Between the Early Math Skills and Social Competence of Children from Low-Income Families

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A Thesis Presented to the Faculty
of the Graduate School of Education of Harvard University
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Education

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ABSTRACT

The U.S. is calling for expansion of preschool to help close the well-documented income-based achievement gap. Children from low-income families often enter kindergarten academically behind their higher income peers and recent findings indicate gaps in social-emotional aspects of school readiness as well, illustrating how early these gaps emerge and raising questions about cross-domain relationships. Therefore, this two-study dissertation explores the relationship between children's social competence and their early math development. Study 1 uses longitudinal growth modeling to explore within- and cross-domain relationships between children's a) interpersonal, social problem-solving skills and b) early math skills during a preschool year. Participants (*N*=76) were recruited from a MA preschool serving mostly children from low-income and minority families. Results show that children have positive, linear math learning trajectories that vary by age when not accounting for children's social competence. Children's development of flexibility in social problem-solving is associated with changes in the rates at which children learn math skills across a preschool year, controlling for baseline child demographics with no evidence of differential learning trajectories by age other than observed differences in math skills at preschool entry. Children's adaptive social problem-solving strategies show positive non-linear growth trajectories. Importantly, these adaptive problem-solving strategies *from the previous time period* have the potential (p =.12) to positively predict children's growth in early math skills during the preschool year. Study 2 draws a subsample (*N*=3485) from the *Head Start Impact Study*, (U.S. DHHS, 2010) a large, nationally representative study of Head Start, to investigate the potential mediating role of children's social competence on early math skills for children randomly assigned to Head Start. Results from a confirmatory factor analysis indicated good model fit for the latent construct with positive social skills and teacher-child relationships as indicators of social competence. Moreover, children's social competence was positively related to math achievement during the Head Start year. Taken together, results from these studies suggest that children's social competence may play an important role in promoting children's early math skills and may warrant more attention in preschool curricula especially as greater attention is paid to increasing implementation of challenging, developmentally-focused math curricula.

A Matter of Perspective: An Exploratory Study of the Relationship Between the Early Math Skills and Social Competence of Children from Low-Income Families

Overview

The U.S. is calling for expansion of preschool to help close the well-documented socio-economic achievement gap. Children from low-income families often enter kindergarten behind their higher income peers in academic skills, including math (Duncan & Magnuson, 2011). Recent findings also indicate gaps in social-emotional aspects of school readiness (e.g., eagerness-to-learn, peer interactions) that predict later academic achievement (Duncan & Magnuson, 2011). While gaps exist in both cognitive and social-emotional realms, little work focuses on understanding whether and how those two skills are related. While there are currently calls to promote children's early math skills and social-emotional development in early childhood care and education (Stipek et al., 2017), even less work has examined these cross-domain relationships during early childhood to understand their inter-play. Gaining a better understanding of the relations between academic and social-emotional skills early on in development can help inform current efforts to expand and improve the quality of preschool programming.

Providing a personally meaningful and relevant purpose is an important feature of effective early childhood mathematical content-based instruction (NAEYC/NAECS/SDE, 2003). Social settings provide important opportunities in which children can learn important math concepts (Frye et al., 2014; NAEYC/NAECS/SDE, 2003) and can often be enhanced through repeated and authentic interactions (Frye et al., 2014; NAEYC/NAECS/SDE, 2003). For example, children begin to learn that five minutes is a relatively short time-frame when they are engaged in a favorite activity (e.g., when a teacher announces, "Five more minutes until clean- up"), though they also realize that this same time-frame can feel like an incredibly long time when imposed as a barrier to something desired (e.g., when a parent insists that they must wait five more minutes until a sibling relinquishes a favorite book). At the same time, as children's concepts of number and time develop, children learn that five minutes is a longer time frame than two minutes. While not exclusive to social settings, it is often social interactions that provide children with greater opportunities for independent practice to utilize these mathematical concepts as negotiation strategies between others.

For example, when Child A wants to play with the toy that Child B has, Child B may offer a prosocial response, by saying Child A can have the toy in five minutes. Child A, not wanting to wait quite that long, may counter with a shorter time-frame (e.g., That's a long time…Can I have the toy in two minutes?). In doing so, Child A has indicated that she not only has the social competence to indicate her continued interest (and skills to) negotiate with Child B, but is also demonstrating greater number sense and concepts of time, thus illustrating the ways in which greater social competence can often coincide with application of math skills in social contexts.

When asked, "If Maya has three cookies and she gets two more, how many cookies does she have in all?" many 3-5 year olds—once they have a stable counting order—can figure out these solutions if they are also offered manipulatives. However, children with less social competence—and less perspective taking, in particular—have difficulty with these types of math story problems, saying for example, "I don't know Maya" or first inquiring who Maya is before attending to the math component. Indeed, many early childhood curricula present these types of problems in such vignettes (see for example, Big Math for Little Kids, Greenes et al., 2004; Building Blocks, Clements & Sarama, 2007aa) suggesting there is a generally accepted practice for embedding math content in social contexts as a foundation that must first be provided so that young children can better understand and apply these math concepts.

Concepts of more and less are important, yet can also be difficult for young children to master both with comparable acuity. While related, these are distinct concepts (e.g., 5 blocks is *more than* 3 blocks, while 5 blocks is *less than* 7 blocks) that children must master as they develop number sense. Children often acquire the concept of "more" sooner and, it is quite likely the frequency of use during social—or other—interactions that promotes this greater use and understanding (e.g., Child A says, "He has more than me" while we rarely hear Child A say, "I have less than him"). Again, social interactions provide a unique opportunity for children to engage in independent practice of these scripts they have heard, whether from adults or from their peers. For example, suppose Child A is playing with 10 toy cars. When Child B wants to play with the toy cars that Child A has, Child A says she will share them and gives Child B four cars. Child B counts his cars, then counts Child A's cars and says, "That's not fair, you have more than me!" The two children then each count their respective shares of the cars and redistribute them, so that they each have five cars. In this scenario, the two children have demonstrated a level of social competence that enabled them to continue engaging in a difficult social situation—that became more difficult when not initially resolved to Child B's satisfaction—to find a fair (to both children) and agreeable resolution (i.e., so that each have the same number of cars). In doing so, they also had to enact several mathematical skills, including counting, cardinality, number concepts of more/less, direct comparison of quantity, distribution and redistribution of sets and part-part-whole relationships—and perhaps others—that were secondary to the social context.

For young children, the greater emphasis is often on the social context—and aspects of social fairness—rather than the logical mathematical component. Specifically, in the previous example outlined above, fairness, rather than equality per se, is the more salient attribute, such that some children will insist that the cars be redistributed until each child has the same number *of blue cars* and the same number *of red cars* each even though each already have the same total number. For example, if Child A has five cars (all five of which are blue) and Child B has five cars (all five of which are red), many children will continue redistributing these cars on the basis that the current distribution is not fair, despite the fact that the children each have the same quantity of cars. Indeed, they may first acknowledge that they each have the same amount demonstrating they have one-to-one correspondence and cardinality, yet that distribution does not seem fair *because* these children have different colors within each subset. Because each child cannot have the same number of red and blue cars with this particular set (without breaking two of the cars in half), many young children will decide that it is "more fair" to redistribute the cars in such a way that Child A and Child B should each have two blue cars and two red cars and simply discard the other two cars (i.e., four cars each), choosing to only play with eight of the original cars instead. That is, the distribution, in the eyes of these young children, must be "fair" in the sense of being distributed in terms of both color *and* quantity, even if it means playing with fewer cars. Color is an irrelevant attribute to the counting and distributional process. If children were only or primarily focused on the mathematical component as the driving force of their social interactions—

rather than being driven by the social context (in this case, by the social element of fairness)—then they would always opt for solutions involving a logical mathematical argument, with considerations of fairness that supplant logical math arguments being secondary. In the example above, however, these children opted to play with fewer total cars, so that they would each have a “fair share” as defined by both color and quantity even though distribution of all 10 items was possible in this context (i.e., there was a total set of items that could be distributed into two equal subsets).

Children often invoke words of fairness when determining these problem-solving strategies (e.g., “one for her, one for me” until all objects are divided equally). While there are similarities between equality and fairness and fairness is often used as a rationale for children’s social and mathematical problem-solving strategies when creating equal sets, these are not synonymous terms. Ultimately, children will learn to distinguish fairness in the social context from equality in the mathematical context and formal instruction is important in helping to build this more formal mathematical understanding. This is important to distinguish because it is not only a naive error, but also provides insight into the potential directionality of this relation as it occurs for young children. Principles of equal distribution do not consider color or other irrelevant attributes; young children often do when considering fairness in distribution of goods during social interactions suggesting that it is the social interaction—and resolution—that is primary for these young children and the math that is secondary. Further, such a rudimentary understanding also suggests that, while social interactions provide opportunities for independent and facilitated practice to explore and experiment with mathematical concepts and ideas, young children need developmentally-focused, guided instruction to support and formalize their developing understanding of these mathematical concepts. These social contexts provide opportunities for independent practice and can also provide authentic opportunities for embedding math content and specialized vocabulary. By using social problem-solving contexts—which naturally occur in preschool classrooms frequently—as a more intentional source of integrating instructional math opportunities, children can be provided with scaffolded opportunities in which they learn to link these social interactions, many of which already include math, with more formal opportunities to learn (and ensure correct modeling and use of) math vocabulary and specialized math content.

In all these contexts, I have posited that the social context has been the driving force for children and math the secondary one. Certainly, others could make a different claim since children are clearly engaging in math in each of these social contexts, and it remains an important empirical question. While studies have investigated the relationship between children's social competence and literacy skills (e.g., Bierman et al., 2013) none have explored the link between social competence and math in early childhood settings. To address this lacuna, the two essays in this thesis provide a foundation for understanding whether and how young children's social competence relates to their mathematical reasoning and problem-solving skills during a preschool year. In the first Essay, an exploratory study (*N*=76), of the within and cross-domain relationships between children's a) interpersonal, social problem-solving skills and b) early math skills and reasoning during the preschool year are examined. Longitudinal growth modeling illuminates how these skills develop and relate to each other over time within and across children. Essay 2 then draws a subsample (*N*=3485) from the *Head Start Impact Study*

(U.S. Department of Health and Human Services, Administration for Children and Families, 2010), to investigate the potential mediating role of children's social competence in explaining the relationship between assignment to Head Start and early math skills during the preschool year. The Head Start Impact Study recruited a large, nationally representative sample of children attending Head Start.

Collectively, these studies are framed around the following research questions:

1. How do children's math skills develop during a preschool year?
2. How does children's social competence (i.e., interpersonal social problem-solving skills) develop during a preschool year?
    a. How do children's adaptive social problem-solving skills develop during a preschool year (as captured by the Challenging Situations Task)?
    b. How does children's flexibility in social problem-solving skills develop during a preschool year?
    c. How do children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised)?
3. Do children's math trajectories during a preschool year differ as a function of their social competence?
    a. Does children's development of flexibility in social problem-solving moderate their math trajectories?
    b. Does children's development of adaptive social problem-solving moderate their math trajectories?

4. Does social competence mediate the effects of Head Start assignment on early math skills?

Within each essay there is a discussion of the results with a consideration of the implications of the findings as well as limitations and directions for future research. In addition to laying the groundwork for understanding the relationship between these two domains that have long been considered important for promoting children's school readiness skills, these studies, taken together, have important implications for both policy and curricular decision-making in early childhood programs that serve children from low-income families.

Essay 1

Adaptive Flexibility: A Longitudinal Analysis of the Relation Between Social Competence and Early Math Skill Development of Children from Low-Income Families

Introduction

The Math Achievement Gap

Children from low-income families often enter kindergarten behind their higher income peers academically, particularly in math (Duncan & Magnuson, 2011). Children who have low math scores at kindergarten entry will, on average, continue to score lower than their higher-scoring peers throughout 8th grade, a trend that disproportionately affects low-income and minority children (Schoenfeld & Stipek, 2011). Math skills at 54 months predict classroom instruction type (i.e., basic skills or higher-order) in 3rd and 5th grade (Crosnoe et al., 2010), which may further perpetuate the achievement gap. International tests reveal math is a particular area of weakness for U.S. students, especially tasks that require students to *apply* math concepts; lower-income students face particular challenges on these tasks (OECD, 2015). Accordingly, there have been calls for more emphasis on STEM (science, technology, engineering and math) education, especially in early childhood (see, for example, Chesloff, 2013), and the new Common Core State Standards (CCSS) place greater emphasis on sophisticated math reasoning, and less on memorization and procedural tasks, than traditional state curriculum standards (Porter, McMaken, Hwang, & Yang, 2011).

Low-income status consistently increases one's chances of entering and exiting kindergarten scoring low on a range of math skills (counting, estimation; Jordan, Kaplan, Olah & Locuniak, 2006). Previous research has shown that children from low-income families, on average, have delayed math development upon school entry (Claessens & Engel, 2013) and experience less growth than their more advantaged peers during the early school years (Jordan, Kaplan, Olah & Locuniak, 2006).

The Social-Emotional Gap

Social-emotional development has long been considered an important aspect of school readiness (Shonkoff & Phillips, 2000). Recent findings also indicate gaps in aspects of school readiness (e.g., eagerness-to-learn, peer interactions) that are both important in their own right and also predict later academic achievement (Duncan & Magnuson, 2011). In a recent study of 4 year-olds, HS children were found to have different Social Emotional Learning (SEL) profiles, on average, than their peers in private childcare. Specifically, higher percentages of HS children were in the "SEL risk" group (i.e., lower self-regulation, more aggressive behaviors and fewer prosocial problem-solving skills) compared to children in private childcare (Denham et al., 2012). Moreover, children in the "SEL risk" group were then rated by their kindergarten teachers as having less adaptive behaviors (e.g., more aggressive behaviors, less positive relationships with teachers), in addition to lower language, literacy and math skills when compared to the SEL "competent-restrained" (i.e., children whose direct assessments indicate competency in these skills, yet demonstrate few observed positive behaviors) and the SEL "competent-expressive children" (i.e., children who demonstrate many prosocial and self-regulatory skills in multiple contexts). African-American boys may face particular risks; an analysis of the Early Childhood Longitudinal Study-Birth Cohort (ECLS-B) data showed that African-American boys have lower social-emotional outcomes than their peers even when controlling for SES (Aratani, Wight & Cooper, 2011). Shockingly, PreK teachers expel preschoolers at a rate more than three times the national rate of expulsion for K-12 students, with higher rates for African-American children and boys (Gilliam, 2005). Such findings suggest at least two issues that call for further investigation. First, teachers—and policy makers—may need more data to better understand the range of "typical" development that occurs in the social-emotional domain for children this age. Second, preschoolers—particularly those served in publicly funded programs—need more support in developing effective social problem-solving strategies, so they can employ more prosocial (and fewer aggressive) behaviors.

Early Math Skill Development

Promoting Mathematical Thinking

Research has shown that young children learn many important math skills when taught according to developmental progressions, which help support concept mastery (Sarama & Clements, 2009). For example, helping children to develop strong number sense provides an essential foundation for the acquisition of more complex math concepts and skills (National Research Council, 2009; Sarama & Clements, 2009). Such topics include stable counting order, cardinality (i.e., understanding the last number counted in a set represents that quantity), subitizing (i.e., recognizing a small set, such as a set of three items, without having to count the items), counting with one-to-one correspondence and comparing sets with more/less among other concepts. Moreover, developing a strong number sense can support children's development of more complex problem-solving and application, such as simple addition and subtraction, even before symbol knowledge has been introduced (Sarama & Clements, 2009). While there is debate about which—and how many—precise topics should be taught in preschool classrooms and curricula available for purchase focus on different math topics, the most prominent early childhood math curricula (e.g., Big Math for Little Kids, Greenes et al., 2004; Building Blocks, Clements & Sarama, 2007a) are developmentally-focused and include number sense suggesting that there is consensus about its importance for young children's early math skill development.

Yet too often, preschool math instruction focuses on discrete skills (e.g., counting, matching numbers to sets) and researchers have recently expressed concern about pushing down poor instructional practices from the elementary years—that emphasize rote learning—to the preschool curriculum (Stipek et al., 2017). These discrete skills, though foundational, are insufficient for the true concept mastery required for innovative problem-solving (Clements & Sarama, 2007a; Ginsburg, Inoue & Seo, 1999). Research has long suggested that young children learn best through hands-on, active experiences that promote exploration and build on children's strengths, instructional needs and interests (Bredekamp & Copple, 1997). Developmentally appropriate instructional practices capitalize on children's natural curiosities and developing knowledge bases (NAEYC & NAECS/SDE, 2003), helping children formalize their mathematical thinking. Formal math instruction, then, provides children with common language (e.g., stable counting order) enabling participation in focused math activities (e.g., addition with manipulatives). Math can also, however, be integrated into daily activities that promote oral language and literacy skills that are already regular components of many early childhood classrooms. Embedding math into regular routines can help children make real-world connections, providing opportunities for exploration and experimentation with math concepts (Ginsburg et al., 1999). Math in preschool classrooms has the potential to begin—not end—with stable counting order and matching numbers to sets and can incorporate much more meaningful, complex applied problem-solving.

The National Council of Teachers of Mathematics (NCTM, 2000; 2010) has, for many years, established problem-solving as an important goal for young children's mathematical foundations, based upon the research that shows that young children need a rich foundation of these early math concepts—on which they can build later formal concepts—taught in ways that are developmentally appropriate, yet conceptually sound. Researchers (Ginsburg 2006; Starkey, Klein & Wakeley, 2004) point out that children's early math interests are rather broad, including "number, shape, pattern, space and measurement" (Ginsburg, 2006, p. 17); thus we should not limit instruction to numeracy alone. One recent randomized control trial (Sophian, 2004) provides encouraging evidence of the potential for incorporating conceptually rich math curricula (e.g., number sense, measurement) to promote complex thinking in Head Start (henceforth HS), classrooms. After a year of teacher trainings, treatment HS children scored higher on both basic foundational skills and the more complex skills targeted in the intervention (Sophian, 2004).

While integration of mathematical thinking into daily activities is important, focused small-group instruction across a range of math topics (e.g., geometry, reasoning) is also key. Incorporation of research-based, focused math activities (with other supports) in a mixed SES preschool (including HS classes) increased children's math skills. Though middle-income children continued to outscore lower-income children, the gap narrowed (Starkey et al., 2004) suggesting that low-income children—such as those enrolled in publicly funded preschool programs, including HS—may particularly benefit from focused math instructional activities. Of particular relevance to the studies conducted here, differences were noted in the *types* of errors children made. While raw scores indicated no change in concept mastery per se, qualitative analysis revealed developmental shifts from lower-level errors in the fall to more sophisticated errors in the spring (Starkey et al., 2004), providing evidence of *growth in conceptual understanding.* This finding has particular implications for the ways we measure young children's math understanding during preschool years. Multiple waves of data collected from one group of children enables a more nuanced understanding of how children's mathematical understanding changes across a preschool year.

Many studies have shown that playing games can be an important mechanism for helping young children develop foundational math skills. For example, researchers have found that playing linear board games can help children develop concepts such as improved counting, number identification (Ramani & Siegler, 2008) and improved number line estimation (Siegler & Ramani, 2008). More recent evidence has shown that such informal math activities, in particular, can reduce the math knowledge/skills gap between higher and lower income children (Ramani & Siegler, 2011). Taken together, these findings suggest that such informal learning—in which math is integrated into engaging activities—can play an important role in developing key math concepts, particularly among children from low-income families who may especially need these opportunities in their early care and educational programs.

More recently, these positive math impacts have been replicated in HS classrooms with additional positive impacts on children's engagement and discussions during the math-based game times (Ramani, Siegler & Hitti, 2012). Specifically, when these activities were implemented with small groups of HS children (as opposed to individually with a researcher as in previous studies), results showed that engagement (e.g., attending to the game) between children increased as they played math games compared to peers who played games that did not have a math focus. In addition, children's comments (e.g., "I have two more to go" as well as non-math statements such as "I'm going to win") only increased when playing the math-based games (Ramani, Siegler & Hitti, 2012).

Many research-based math curricula embodying a child-centered framework have shown promising results, indicating that successful math instructional programs can be both developmentally appropriate and instructionally sound. For example, *Big Math for Little Kids (BMLK*, Greenes, Ginsburg & Balfanz, 2004) is a preschool and kindergarten curriculum that is designed for the early care and educational context. Lessons are linked to transitions and daily routines and reflect children's interests, in a well-designed way that allows mathematical ideas to be thoroughly developed, explored and reflected upon in early childhood classrooms. A recent RCT of *Big Math for Little Kids (BMLK;* Lewis, Clements, Ginsburg & Ertle, 2015*)* showed that intervention children outscored their peers on a global math assessment (ECLS-B). Because this assessment was not directly linked to the intervention curriculum, such findings suggest that intervention children gained transferrable math skills equivalent to an additional 2.9 months of schooling between fall of the preschool year through spring of the kindergarten year. In this sample of primarily low-income children from single-parent homes, intervention children also demonstrated better math language skills (e.g., better able to name geometric shapes and explain their reasoning). *Building Blocks* (Sarama & Clements, 2004) is another preschool curriculum that is based upon an analysis of how children naturally learn math concepts during the preschool years. Focusing the content on developmentally appropriate math domains (e.g., geometry, counting on), children's daily activities (e.g., art, songs, puzzles) are "mathematize[d]" (Sarama & Clements, 2004, p. 183). A summative evaluation of *Building Blocks* showed that intervention children developed more sophisticated problem-solving strategies and aspects of number sense (e.g., counting, use of tracking strategies, simple addition problems; Clements & Sarama, 2007a).

Recent research has provided important insights about the predictive utility of the specific math topics as children progress through the early elementary school years for children from low-income families, in particular. Specifically, researchers found that low-income preschoolers' knowledge of non-symbolic quantity (i.e., discriminating between sets with more and less) was predictive of 5th grade math achievement (Rittle-Johnson et al., 2016). In contrast, it was symbolic mapping knowledge (i.e., matching numerals to the correct number in addition to understanding their relative magnitudes from comparison of symbols alone, a more abstract and sophisticated skill) that was predictive by the time children had progressed to first grade. Such findings suggest that curricular development must consider both the developmental needs of those being served as well as the content area being implemented, to ensure that the content is being implemented in ways that promote concept mastery—rather than isolated skills—in order to promote long-term gains over time. Specifically, this research suggests that preschool curricula may need to focus on number sense concepts—more broadly—rather than explicitly, or exclusively, on numeral recognition (i.e., memorizing numerals without deeper conceptual understanding of quantities or the relationship between quantity and what the numeral represents) yet questions remain about how to best promote this concept development.

Additional questions—and challenges—remain regarding how to promote children's social-emotional development while implementing a developmentally-focused, yet academically rigorous math curriculum, particularly among those most often targeted with efforts to close the achievement gap and served in publicly funded preschool programs. While it is often recognized that both early math skills and social competence are important in the preschool context, this particular cross-domain relationship—which has been little explored to-date—can offer promising insights for promoting children's development in both areas of development during preschool.

Social-Emotional Development

Social competence, one aspect of social-emotional development, encompasses the effective use of various, interrelated social-emotional skills in social contexts. The extent to which one can coordinate and implement these different, yet complementary skills (e.g., social problem-solving skills, nature of peer interactions) can vary depending on the particular goal (e.g., negotiating conflicts, initiating friendships) and context (e.g., different times, situations; Rubin & Krasnor, 1992). Exhibiting more competent behaviors in social contexts indicates adaptive emotional well-being, whereas a lack of these skills can indicate maladjustment (Dodge, Pettit, Mcclaskey, Brown, & Gottman, 1986).

Interpersonal Social Problem-Solving Skills

Learning to take the perspective of others and resolve problems that arise is a typical part of daily interactions in early childhood classrooms. Selman (1980) differentiates social perspective coordination skills (i.e., interpersonal skills) from merely understanding that another may have a different point of view; i.e. developmentally, more adequate social perspective-taking is predicated on understanding how these different points of view "are *related* and *coordinated* with one another" (p. 22, emphasis in original). For example, when Child A builds a block tower, which is then knocked down by Child B, there are many possible causes and, thus, reactions by Child A (e.g., Child A might yell at Child B, cry, or tell the teacher) that depend on many factors (was Child B's action intentional or accidental? Does Child A understand there are different possible intentions and the different implications of each?). As young children learn to navigate these social situations and learn to take the perspective of others, they become better able to understand there are multiple strategies for negotiating effective, fair resolutions. It is through these frequent, informal interactions that they come to understand how different strategies can be simultaneously valid and can lead to just outcomes (e.g., Child B could apologize, Child B could help rebuild the block tower with Child A), and thus perhaps to adopt alternative problem-solving approaches. Through these social interactions, children learn to reconcile these initial perceived differences in perspective (e.g., accident vs. intention) and increasingly understand the ways in which each child's perspective is, ultimately, inter-related and connected to one another by an underlying common, unifying action (e.g., a block tower got knocked down). As they increasingly learn to take the perspective of others, children are better able to coordinate their peers' perspectives with their own to understand how these differing perspectives can be simultaneously legitimate (e.g., Child A exclaims his tower was knocked down intentionally by Child B; Child B says it happened accidentally), all of which has implications for the problem-solving strategy that children select in a given context. For example, initially, Child A may feel angry; as Child A learns to take intentionality into account (e.g., could this have been an accident?), this would likely change his problem-solving strategy (e.g., If I accidentally knocked down someone's tower, how would I want him to solve the problem with me?) and may even change how he feels in response (e.g., If it was an accident, would I want him to be angry at me?). This more nuanced complexity is inherent to more sophisticated social perspective-taking (Selman, 1980). As children learn to consider these alternate perspectives in social contexts—and contemplate potential social problem-solving strategies—I theorize that such consideration would have an impact on children's feelings about the act itself (e.g., sad about the situation instead of angry at the transgressor), particularly when this occurs with peers with whom Child A has sustained, positive relationships, as in many preschool classroom contexts. Accordingly, children would be more likely to engage in more nonaggressive (e.g., Child A cries and asks Child B to apologize as resolution) and/or prosocial (e.g., Child A asks Child B to help rebuild the tower) problem-solving strategies. As children learn to navigate these social situations in classrooms (e.g., observing what others do when their block towers collapse) and learn to take the perspective of others (Why did she solve it that way? How is that way (also) fair?), they come to understand there are multiple strategies for negotiating effective, fair resolutions. In sum, this greater understanding allows for more sophisticated skill development, yet children need opportunities for practice of these skills in order to solidify their understanding of these concepts.

Adaptive Social Problem-Solving Skills

Children's social problem-solving skills can be influenced by many factors (e.g., observational learning, instructional interventions). Much of the literature has focused on supporting the development of prosocial skills (e.g., Domitrovich et al., 2007; Denham et al., 2013) and minimizing aggressive behaviors (e.g., Shure & Spivak, 1980; Youngstrum et al., 2000; Webster-Stratton, 2001), particularly among children from low-income and minority backgrounds, typically considered “at risk”.

However, the resilience literature—which grew out of this research investigating “at risk” populations—describes the ways in which context must be considered when promoting developmental outcomes for “at-risk” children (Masten, Best & Garmezy, 1990), establishing the importance of considering individual differences when children are faced with adversity (Masten & Garmezy, 1985; Werner & Smith, 1982). Specifically, Masten and colleagues (1990) defined resilience as “the process of, capacity for, or outcome of successful adaptation despite challenging or threatening circumstances. Psychological resilience is concerned with behavioral adaptation, usually defined in terms of internal states of well-being or effective functioning in the environment or both” (p.426). Stated another way, we can see evidence of resilience when children change their behaviors—given the context of the situation—to promote their own well-being (when not to the detriment of another) and/or better interactions in their environments (e.g., homes, classrooms). Given the varied ways that children in these challenging circumstances showed evidence of resilience, researchers began considering the importance of using multiple measures and using them more flexibly to better capture specific indicators of children’s strengths (Kinard, 1998; Luthar, 1993). Given the low-income sample in this study, modifications have been made to each of the social problem-solving measures that capture “adaptive” social-problem solving skills, the basis for which is described in this section.

Another conceptual framework helpful to understanding the promotion of social competence is the literature on "Locus of Control," the extent to which one believes life events are determined by one's own actions (e.g., effort; internal locus of control) rather than by events external to the individual (e.g., good/bad luck). For example, research has long shown that internal locus of control is important for promoting social competence in young children, particularly when under stress. Children from high-risk backgrounds (e.g., family background, medical history) with an internal locus of control (ILOC) have been associated with problems in school compared to their peers with an external locus of control (see, for example, Werner & Smith, 1982; O'Grady & Metz, 1987). Results from a quasi-experimental elementary school-based intervention aiming to improve social skills and ILOC have shown that improved ILOC was associated with improved social competence (e.g., decreased acting out, decreased shy/anxious behaviors; DeMar, 1997). These findings illustrate the ways in which ILOC can help foster resilience, strengthening protective factors within some children who are at-risk due to challenging life circumstances.

Krasnor and Rubin (1981) noted that—for all children—flexibility in social problem-solving is a key indicator of social competence. Specifically, they argued that a child who continues to persist after an initial failure, within reason, is exhibiting an important awareness of the various components that inform effective social problem-solving (e.g., considering the specific circumstances of that situation). The resilience literature has noted that flexibility in perspective-taking may be a key skill for adaptive problem-solving, in particular (Masten et al., 1990). Thus, we might expect that resilient children may try to use various problem-solving strategies—and may attempt different strategies when the first does not work—in an effort to adapt to the environment as they are perceiving it. Specifically, as the child processes that the first strategy has not worked—which may or may not be a prosocial problem-solving strategy—he/she may adapt his/her approach to try a different problem-solving approach. Such an alternate approach might include retreating from the situation altogether or seeking out a different peer the next time (i.e., adaptive strategies may be employed in future endeavors, not necessarily immediately). Masten and colleagues (1990) specifically noted that resilient children were "resourceful and effective problem-solvers" (p. 432) and list finding escape routes and locating sources of help as key resilient skills, perhaps because these children are often keenly aware of danger when it is present and proactively seek ways to adapt if/when they feel their safety is threatened.

In the current study, I argue that there are many types of adaptive social problem-solving strategies and prosocial strategies are only one kind that children may utilize when faced with difficult or unanticipated circumstances in these social contexts. For example, if Child B approaches Child A and initiates an unprovoked attack (e.g., Child B hits Child A or Child B takes Child A's toy), the *ideal response in a lab setting* might be for Child A to tell Child B that it is not a nice thing to do and attempt to correct Child B's behavior (e.g., either by telling Child B not to hit or asking for the toy back). While some children may attempt such prosocial responses in classrooms at times, adaptive responses might also include quite different responses in classroom settings at other times. Specifically, such an unprovoked attack may conjure images of similar prior experiences for abused children, invoking "fight-or-flight" responses. In these cases, it is argued that an adaptive response would be to disengage from the aggressor and remove oneself from the dangerous situation. In this case, Child A might simply go play somewhere else rather than continue engaging with the aggressor, attempting to negotiate a prosocial outcome.

In the context of friendship initiation, similar scenarios may occur in which Child A attempts to initiate friendship or a shared activity with Child B and Child B declines (e.g., Child B says, "I don't want to play. I'm going to play with Child C"). A prosocial response would suggest that Child A persevere by asking Child B if he/she can join the duo (e.g., Can I play with you both?) However, some children may feel rebuffed—or simply overwhelmed by the thought of having to ask two children simultaneously—and may not feel comfortable asking to join Child B after he/she has indicated he/she does not want to play. In classrooms, there are often many potential playmates, so an adaptive response may be to simply ask a different child to play (e.g., Child D). In doing so, children may also be trying to respect Child B's expressed statement that he/she does not want to play right now, which is often difficult for many children of this age. Thus, when children are demonstrating respect for Child B's stated preference and seeks an alternate playmate, this should be perceived as an adaptive social problem-solving strategy in the context of a preschool classroom rather than an "avoidant" or "passive" behavior indicative of lesser social competence.

While the nature of social relationships and social interactions is very complex—even in early childhood—clear structure and clear alternatives must be scaffolded by the teacher. In early care and educational classrooms, these multiple types of strategies are often employed for a variety of reasons as a matter of routine. While in some cases this is a matter of practicality (i.e., it is not always possible to obtain a single object), it is also often an intentional authentic opportunity for helping children learn how to negotiate difficult social interactions. In many cases, children are explicitly (or implicitly) taught to "just get something else" in addition to teaching children prosocial approaches, in these classroom contexts. For example, when a child has a book that another child wants, he may be taught to ask for it (i.e., a prosocial response), though children are also taught to utilize the many other books in the classroom (i.e., "You can get a different book. We have many books."). Often, the number of children who can play in each "center" is limited (for a variety of reasons, including to promote positive social interactions). When the number of children at a particular center is exceeded and another child wants to enter, teachers often tell the child to choose a different center in the room. Thus, children may have already learned that to "go somewhere else" is an adaptive problem-solving strategy in these contexts. As such, Child A is readily able to engage in another activity elsewhere—as a matter of routine, in some cases—particularly in classroom contexts in which there are often multiple (good) options from which to choose.

In the context of friendship initiation, in particular, it is important to help children consider the feelings of all of those involved, which is an important aspect of perspective-taking. With this in mind, friendship initiation cannot be considered merely as a singular goal of that of the initiator—as it perhaps might be in a lab setting—when in a classroom context. It is important to consider both Child A's and Child B's feelings, particularly in a classroom context (i.e., as a classroom teacher who is charged with both children's well-being) in contrast to a lab setting in which only Child A's performance is being assessed. In early care and educational settings, it might be expected that teachers would encourage children to "go play with someone else" if one child has indicated he/she does not want to play since, in part, the teacher is likely trying to promote respect for all children's feelings.

Thus, it is important to take these different types of adaptive responses into account when measuring children's response—when children are in classrooms—compared to those in lab settings. These considerations led me to adapt the scoring procedures for both measures used in this study to capture children's social problem-solving to include "adaptive" problem-solving strategies (described in more detail in the Measures section).

The Role of Language

Early theorists, such as Vygotsky (1962) viewed language as an inherently social process, acquired from repeated interactions and observations with others over time. Specifically, he posited that young children learned to engage in discussions for the purpose of engaging the listener; to communicate his/her thoughts, feelings and ideas with another (Vygotsky, 1962). Thus, language plays an important role as children navigate social problem-solving dilemmas, attempting to communicate their respective viewpoints to the other. Research has shown that higher expressive and receptive language skills have been associated with higher social competence, particularly among low-income preschoolers (e.g., peer interactions during play) (Mendez, Fantuzzo & Dante, 2002). Likewise, research has demonstrated that, among low-income children, receptive language skills are important for effective social competence (e.g., teacher-reported social skills; Longoria, Page, Tait & Kennison, 2009).

Children's vocabulary is important during math instructional contexts for many reasons as well. In addition to general terms (e.g., context-specific vocabulary such as more, three, etc.) that represent distinct concepts, many of which are relative in nature (i.e., 5 blocks is *more than* 3 blocks, while 5 blocks is *less than* 7 blocks), math has specialized vocabulary that children must learn in order to adequately express their understanding and, in some cases, their lack of understanding of these concepts (e.g., to ask for clarification). For example, the early childhood math curriculum typically includes words such as "pattern" and "equal" which have specific definitions in these contexts, requiring considerable use of children's expressive and receptive language skills, even when children or teachers are demonstrating concepts with manipulatives. Of particular importance is a child's ability to explain how he/she has solved a problem, which can require quite sophisticated language skills. This is, in part, because it often requires retelling events in the past tense, which can often be more difficult for children of this age (Shipley, Maddox & Driver, 1991) and, particularly so for young English language learners (Nicoladis & Paradis, 2012; Washington & Iglesias, 2015). It is this process of retelling, however, that can be central to helping a child solidify his/her understanding and/or self-correct if he/she has made an initial error in problem-solving (Rowan & Bourne, 2001). Further, these language skills are central to being able to communicate effectively with one's peers to explain one's perspective, generally, and to explain one's mathematical problem-solving strategy, in particular (i.e., in response to instructional prompts such as, "Did anyone solve it a different way?") when math is embedded in group settings that promote collaborative problem-solving and discussion of multiple strategies for solving math problems.

A recent (2015) study showed that kindergarten children's phonological awareness (e.g., word production after deleting specific segments, for example, farm without /f/) was associated with their numeracy and applied problem solving skills (Foster, Anthony, Clements & Sarama, 2015). Such findings provide evidence that the relationship between language and math concept development is more complex than one's receptive or expressive language ability, suggesting it is unique aspects of more sophisticated language fluency, not merely vocabulary that contributed to children's early math skill development. This type of phonemic awareness development, an important component of phonological awareness, can be typically developed through songs, rhymes and other playful activities that one might expect to find in preschool classrooms, many of which tend to incorporate early numeracy concepts (e.g., Five Green and Speckled Frogs). Taken together, this body of evidence suggests that embedding math in social contexts that support children's language development may be an important way to incorporate important math vocabulary and phonemic awareness into existing classroom routines that naturally facilitate social interactions.

Cross-Domain Relationships

The Relationship Between Children's Social-Emotional and Cognitive Development

While little is known about the specific link between math and social competence, research has long shown that children's social-emotional and cognitive skills are interrelated and can be enhanced in settings that are both responsive and caring (Shonkoff & Phillips, 2000). As children transition from childcare to more formal classroom settings (e.g., preschool, kindergarten), the learning environment changes in many ways (e.g., more transitions, more formal instructional time). Children's social competence has been the basis for interventions to promote early academic skills, though these have—to date—been limited to the literacy domain. For example, a randomized evaluation of the

Research-based, Developmentally Informed (REDI) intervention has shown positive impacts on HS children's social competence (e.g., emotional understanding, social problem-solving) and literacy skills (e.g., vocabulary, phonological awareness; Bierman et al., 2013). Follow-up studies have shown sustained positive effects on children's developmental trajectories. HS children who received the REDI intervention were at reduced risk of showing aggression, experiencing peer rejection and having attention problems in 3rd grade, when they also had closer relationships with teachers and more engaged learning than non-REDI students (Nix et al., 2016), illustrating the ways in which targeting social-emotional skills in preschool can have lasting effects in both domains. None of these interventions, to my knowledge, have explored this link between social competence and math in early childhood.

A recent meta-analysis of 213 studies showed that Social-Emotional Learning (SEL) programs implemented in schools (K-12th grade) are effective in supporting both social and academic outcomes (Durlak et al., 2011). Specifically, SEL participants, overall, showed more social skills, positive attitudes toward self and others, and positive social behavior, as well as fewer conduct problems and less emotional distress. Importantly, when school staff (not researchers) implemented interventions, students showed improved achievement test scores and grades (Durlak et al., 2011) as well as the targeted SEL outcomes, illustrating the important role that teachers can play in promoting both social-emotional and academic outcomes in classroom contexts.[1]

Links between interpersonal and social problem-solving skills —as indicators of social competence—and early math skills have been less studied. Ginsburg (2006), while

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[1] Many studies in this meta-analysis support causal inferences

acknowledging the importance of instructional supports, identifies the social environment as a contributing factor to the development of mathematical knowledge. The Responsive Classroom (Ottmar, Rimm-Kaufman, Larsen & Merritt, 2011) provides some evidence, albeit with older children, as it incorporates multiple strategies to support both academic and social-emotional learning with 2nd-3rd graders. In a randomized trial, students who experienced a combination of more responsive teaching and stronger math instruction outscored their peers who experienced only one of these factors. These findings suggest that supportive classroom interactions may be instrumental in developing concept mastery in math. Thus, we might expect that more supportive (and less conflicted) teacher-child relationships and peer relationships (as evidenced by more Adaptive social problem-solving skills) in early childhood classrooms might promote better math outcomes.

Given the body of research that has demonstrated the importance of playing rule-based games (Ramani & Siegler, 2008), communication between peers (Ramani et al., 2012) and small-group instructional activities (Lewis et al., 2015; Clements & Sarama, 2007b) to facilitate effective math learning in early childhood classrooms, it seems that a critical set of skills that young children would also need in these contexts—in addition to effective instruction—would be effective interpersonal and social problem-solving skills. It cannot be assumed that children enter early childhood classrooms with social competence. Yet, if social competence is important for promoting effective math learning—which seems implicit in studies like those described above that use peer games to promote math skills—then it may be just as critical to promote the skills needed for effective interpersonal and social problem-solving, so that children can engage in and develop key social-emotional and academic learning with their peers throughout the preschool year.

Hypothesized Link Between Early Math Skills and Social Competence

There is often a great deal of “conflict” in early childhood classrooms, reflecting the developmental stage of this age group; learning to take the perspective of others and resolve problems is a typical part of daily interactions. Children who are supported in developing problem-solving skills and have close, supportive teacher-child relationships may have less conflict—or less prolonged conflict—in their classrooms. As children learn to navigate these social situations (e.g., observing what others do when problems arise) and learn to take the perspective of others (Why did she solve it that way? How is that way [also] fair?), they come to understand there are multiple strategies for negotiating effective, fair resolutions. These social problem-solving skills might then generalize to mathematical reasoning.

In these classrooms, where content is intentionally taught through songs, stories and pretend play, children who begin to consider alternate approaches to social problem-solving may develop a more complex—and complete—understanding of early math concepts (e.g., understanding the interrelatedness of 2+2=4 and 3+1=4, sans symbols). Daily routines can provide opportunities for flexible mathematical thinking. For example, when distributing an odd number (e.g., 15) of fruit snacks between two children, one child may suggest cutting the “last” snack in half so that each child has the same quantity (i.e., 7.5 fruit snacks), while the other child may suggest discarding the “last” snack so that each child has the same quantity (i.e., 7 fruit snacks). In these contexts, as children apply their interpersonal social problem-solving skills (Why did she solve it that way?

How is that way [also] fair?) that embed math (How/why is that problem-solving strategy [also] mathematically correct?), they learn to assimilate key components that are central to mathematical topics, possibly leading to a deeper mathematical conceptual understanding, which promotes transfer and allows application of mathematical concepts in novel situations. Thus I hypothesize that children with greater interpersonal problem-solving skills, if they also have access to teachers who provide appropriate instructional support, may develop more complex, sophisticated math reasoning and more flexible problem-solving. There has been extensive work relating early social-emotional outcomes to emergent literacy skills (Bierman et al., 2013; Barnett et al., 2008), yet none relating social competence to math, particularly during early childhood. This study explored whether the social competence advantage for literacy could be replicated for math.

Guiding Hypothesis

It was hypothesized that children's early math skills and adaptive social problem-solving skills would increase over time in a non-linear fashion during the preschool year. In contrast, it was hypothesized that children's flexibility in social problem-solving skills would develop over time in a linear fashion. For each of these skills, it was hypothesized that this rate of growth would be different for children of different ages (i.e., 3 year-olds would have different learning trajectories than 5 year-olds). Regarding the cross-domain relationships, I hypothesized that children with greater social competence (e.g., whose social competence increases at a faster rate) will have stronger, more positive rates of change in conceptual math understanding. Specifically, children who are better at resolving conflicts in prosocial and other nonaggressive ways (i.e., using adaptive and/or flexible social problem-solving strategies) will have steeper math learning trajectories (i.e., faster rates of learning) than their peers with less adaptive and/or less flexibility in social problem-solving.

Research Questions

In this study, the development of children's early math skills and indicators of social competence as well as the cross-domain relationships between these two domains were examined. The following specific question guided this work:

RQ1: How do children's math skills develop during a preschool year?

RQ2: How does children's social competence (i.e., interpersonal social problem-solving skills) develop during a preschool year?

RQ2a: How do children's adaptive social problem-solving skills develop during a preschool year (as captured by the Challenging Situations Task)?

RQ2b: How does children's flexibility in social problem-solving skills develop during a preschool year?

RQ2c: How do children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised)?

RQ3: Do children's math trajectories during a preschool year differ as a function of their social competence?

RQ3a: Does children's development of flexibility in social problem-solving moderate their math trajectories?

RQ3b: Does children's development of adaptive social problem-solving moderate their math trajectories?

Method

Participants

Participants (*N*=76) are a diverse sample of young children recruited from an early care and educational center in an urban Massachusetts area. The center has four participating preschool classrooms, two of which receive HS funding, though the entire center serves a majority (80%) of children from low-income families (annual incomes less than $24,300 for a family of four). The early care and educational center is NAEYC-accredited with a reported focus on the whole child (i.e., routines and curricula targeting children's social-emotional, cognitive and physical development). In the preschool classrooms that are part of this study, 78% of children are African or African-American and 20% are Hispanic/Latino (non-Black) and fewer than 3% are White or Asian children. Across classrooms, there is a high proportion of boys, with 63% male and 37% female. Approximately 80% of children in each preschool/HS classroom was consented into the study. The sample reflected the demographics of the center. Baseline age of children (i.e., the age of children when they took their first assessment) ranged from 34 to 70[2] months (*M*= 46.80, *SD*= 8.33). During preliminary analyses, one child was dropped from the analysis due to anomalous age at baseline (age=72 months in November of the preschool year). Two children were consented into the study, but never participated because they lost their voucher to attend the early care and educational center before assessments could commence and thus were no longer enrolled at the participating site.

[2] In the case of Baseline Age, children enrolled in preschool and the study at various points during this preschool year. As such, the child who was 70 months at baseline in this case was 70 months in June of the preschool year, which is much more typical than the child who was 72 months in November of the preschool year.

Efforts to collect data about children's home language though logistical challenges precluded collection of that information.

Procedures

Child assessments were conducted in the preschool during individual 30–45 minute sessions in a quiet area near the classroom. Each child was assessed up to 5 times over a 10-month period on both social competence (i.e., interpersonal social problem-solving skills) and math (mean number of waves per child=3.4). In addition, children's receptive vocabulary was assessed at the baseline and final waves (only baseline is analyzed here). Each wave of data was collected approximately every 60 days between November and June of the 2015-2016 preschool year. For late entrants into the study, an additional wave of data was collected in August of that preschool year since the early care and educational setting in this sample provides educational and care services year-round. Participating children received a small token (e.g., bookmark) after each assessment and consented families received a book.

Measures

Early Math Skills

At each wave, children's early math skills was measured using The Test of Early Mathematics Ability-3rd Edition (TEMA; Ginsburg & Baroody, 2003). TEMA is a 72-item standardized measure (with parallel forms) that captures both informal/formal math concepts (e.g., numeral identification, problem-solving) in young children (3-8 years). It has basal and ceiling levels to minimize test-taker fatigue and has been used in other studies with diverse preschoolers (Molfese et al., 2012). According to the TEMA scoring manual, the TEMA-3 has test-retest reliability of .82 and high internal consistency

(α=.92-.93) for 3- and 4- year-olds (Ginsburg & Baroody, 2003). To minimize fatigue, given the nature of this longitudinal study with multiple assessments and assessment waves within a single year, a modified ceiling rule was adopted. Specifically, within each wave, children were assessed until they reached a ceiling of three (rather than five) items. A similar modified testing approach was also used with this age group to minimize fatigue with the Head Start Impact Study (Puma et al., 2010).

Interpersonal Social Problem-Solving Skills

At each wave, children's interpersonal social problem-solving skills were captured using two different measures, each of which is described below.

Children's adaptive problem-solving skills were measured using a modified scoring of the Challenging Situations Task (CST; Denham, Bouril & Belouad, 1994). The CST has two parallel versions (each consisting of 6 situations) to assess young children's emotional and behavioral responses to unambiguous, hypothetical scenarios. The child is presented with picture cards and a verbal description of each scenario (e.g., "Mary was building a tower of blocks. Suddenly, Bobby knocked it down."). Children are then asked how they would feel (picture cards are presented; happy, sad, angry and "just OK") and what they would do; options are prosocial, aggressive, avoidant/passive, and crying. Children can respond verbally or by pointing to the picture. The CST has been previously used with HS children (Bierman et al., 2013) and, in that study, showed good internal consistency (α=0.68-0.77). The adaptive classification consists of combining the "Prosocial" and "Avoidant/Passive" categories. While the latter has been classified in this way by others, the present study views these actions as adaptive problem-solving behaviors (see Literature Review for more detail).

Children's social problem-solving strategy type (e.g., aggressive, adaptive) and flexibility in the use of problem-solving strategies were measured using The Social Problem-Solving Test-Revised (SPST-R; Rubin, 1988; 2016), a quantitative and qualitative measure. Presented with up to eight problem situations (e.g., object acquisition, friendship initiation), the child is asked 1) what the child could do or say to accomplish the desired goal and 2) what he/she would do in that situation. Three measures were derived from each of the two types of stories: (1) number of solutions (2) proportion of each solution type (e.g., adaptive, aggressive, etc.) and (3) flexibility in the use of problem-solving strategies. Across studies, the SPST-R has been shown to have good inter-rater agreement when implemented with diverse preschool and HS children (kappa's α=0.85-0.88; Jones Harden et al., 2000; Pettit et al., 1988). Scoring flexibility involves the comparison of the categories found among the child's responses. Specifically, two of the three child's responses for each vignette (i.e., the response with the most and the fewest categories) are compared to determine whether the child is utilizing different strategies across responses when attempting to solve the same problem. Importantly, the type of problem-solving strategy (e.g., whether prosocial or aggressive) is not examined for this code. The SPST-R was updated in 2016 to include alternate forms (Rubin, 2016). The SPST-R coding protocol was modified somewhat from the original and adaptations are described below. A portion (15-20%) of each wave was double-coded to ensure reliability (kappa's α=0.68-1.00 across Object Acquisition and Friendship Initiation tasks; weighted kappas ranged from 0.66-0.84 for flexibility) indicating substantial to near-perfect agreement in many cases.

Receptive Vocabulary

Given the important role that language plays in both social and academic contexts, particularly for young children, receptive language was measured at baseline and at the final data collection using The Peabody Picture Vocabulary Test (PPVT; Dunn & Dunn, 2007). The PPVT is a standardized, norm-referenced measure of receptive vocabulary used widely with diverse preschool populations (e.g., Raver et al., 2011). PPVT has two forms and basal/ceiling levels. The PPVT has test-retest reliability of .93 and high internal consistency for (α=.94; Dunn & Dunn, 2007). PPVT scores were grand mean centered when entered as a control in the analyses.

Assessment Protocols

Using a longitudinal data analytic method is key to addressing these research questions, as it will facilitate modeling of the math and social competence trajectories in the way that best fit the data (e.g., linear, curvilinear) to understand the nature of change over time in skills. Pre/post studies cannot separate measurement error from true change, whereas longitudinal studies help to parse out measurement error, thereby better isolating true change in proficiency (Singer & Willett, 2003) in both domains of development that are of interest in this study. The method utilized here enables description of the shape of individual children's growth trajectories that are key to appropriately addressing these research questions. Thus, the study proposed will provide much more nuanced information regarding the ways in which children's learning changes across a preschool year in each domain as well as the intersection between the two domains, helping us better understand the range of variation in existing preschool and HS classrooms.

In a longitudinal design such as this, careful attention has been paid to assessment administration. All measures have different test forms, which have been used in alternating waves of data collection to reduce the likelihood that the child has “learned” the test. This, combined with the basal and ceiling levels of the tests, minimizes the extent to which children are tested on the same content, even when the same form is used (i.e., on alternating occasions). Since children will typically have learned more by the subsequent data collection waves, both the corresponding basal and ceiling should result in mostly, if not entirely, different test items.

TEMA Assessment Procedures

Adjusted Ceiling Rule. The TEMA protocol is to use a 5-item ceiling, meaning that the test is to be discontinued once the child gets five consecutive items incorrect. However, many of the TEMA items have several sub-items within each “item” with some having as many as six sub-items. Given this, while implementing the original scoring rule, a child may actually complete an average of 15 consecutive tasks incorrectly before the test is discontinued (e.g., 5 items with 3 sub-items each). Because this assessment was one of several to assess multiple domains and was part of a regular assessment routine on a bi-monthly basis, a decision was made to shorten this ceiling rule to minimize child fatigue while still standardizing the testing protocol. Specifically, a 3-error ceiling rule was adopted, which was consistent with similar ceiling rule adjustments made when testing children of this age (see, for example, “Test Adaptations” in Puma et al., 2010).

Adjusted and Transferred Basal. During the first assessment, children were assessed according to TEMA directions, except with the adjusted ceiling rule. As such, they began the assessment according to their age (e.g., 4 year-olds began at the “Age 4” starting point). In subsequent assessment waves, an adjusted basal was adopted between assessment waves to further minimize fatigue on the children and to better assess the ways in which children's early math skills were developing across the preschool year. Specifically, between waves, scoring booklets were reviewed and each child's basal level—if one was achieved—was noted on the next assessment booklet. Then, in the subsequent assessment wave, the TEMA assessment resumed assessing two items prior to the previously established basal item. For example, if Item 8 was the previously established basal item, then the next assessment wave would begin with Item 6. This was done to provide a "warm-up" of a few items—on which the child had been successful—prior to having the children attempt what may be entirely new math content for the child (and may or may not have been beyond the child's age, depending upon the unique circumstances of each child). The TEMA scoring manual states that once a child achieves a basal, all prior items are scored correctly, even if individual items within that basal are incorrect (i.e., the incorrect items within a basal are ignored for scoring purposes once a basal is achieved). To ensure consistency in scoring, this scoring protocol prescribed by the TEMA manual was followed when using this adjusted basal as well. In some cases, children missed one of the warm-up items and/or the previously established basal item itself. Since this may have been due to idiosyncrasies of the test item (which may have been due to different test forms used in alternate assessment waves) or other causes of measurement error, the child was given credit for the previously established basal according to the TEMA scoring manual.[3] In these cases, additional items were included

[3] It should be noted that each TEMA item often has several sub-items. Therefore, when a child missed an item, they often were able to demonstrate mastery on several of the sub-

to be sure the child had really previously established this basal. Once this was established, the child's previous basal score moved forward to the current assessment. In many cases, however, children did not achieve a basal. In these cases, these children simply started at the beginning of the assessment with each assessment wave (even if they were older than 3 years old) and progressed through the assessment until they met the (adjusted) ceiling rule. This was done to document the ways in which children's *specific* early math skills were changing over time in addition to the overall TEMA score.

Use of Raw Scores. Raw scores were used in the analyses for four reasons. First, TEMA raw scores were somewhat skewed, so creating standardized z scores was not appropriate, particularly given the relatively small sample size. Second, the TEMA ceiling scoring rule was truncated as described above. In doing so, it was no longer appropriate to use norm scores when applying this truncated ceiling rule.[4] Three, it was important to select a modeling approach that would work with both RQ1 and RQ2 and, then, with RQ3, in which I address the intersection of the math and social-emotional constructs that are investigated separately in RQ1 and RQ2. Further, most studies utilizing developmental trajectories are conducted across multiple years with more than

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items suggesting it may have been the different testing forms that was problematic and not concept mastery per se.

[4] Specifically, the transformation rules (from raw scores to norm scores) that one would typically apply were voided since children could have theoretically scored higher with the prescribed 5-item ceiling rule. However, since this lower ceiling rule was applied uniformly within the sample, the modified implementation ensured the assessment was implemented in a standardized manner (i.e., with uniform implementation, procedures and scoring), introducing no bias into the sample. Thus, it is appropriate to view gains—or any other changes—as real changes over time. To compare these scores to a different sample, however (i.e., to a normed sample), *could* introduce bias since the normed population took that assessment under different conditions (specifically, with a different, higher ceiling rule). Thus, it would appear that the children in this sample have lower normed scores than may actually be the case had they taken TEMA with a 5-item ceiling.

one cohort, facilitating comparison of age groups across years, whereas the participants in this study are confined to participation within a single year. Thus, the children are different ages at the point of entry into the study and the children will never cycle up to become the next age (e.g., the entering 3 year olds will never become 4 year olds, as in a multi-year longitudinal analysis). In this context, comparing norm scores from one month to another (within a year) is not a reasonable metric within these domains. Rather, we would expect incremental growth—that can be best captured by raw scores on the same test (that utilizes the same scale)—in the span of a single preschool year. For all these reasons, raw scores are used to quantify the growth in early math skills across this preschool year.

SPST-R Assessment Procedures

The SPST-R was updated in 2016 and the coding protocol was modified somewhat from the original. Specifically, for the purpose of scoring flexibility, the response with the most and the least categories within a given prompt are compared. A skip rule was also implemented such that when a child received “Non-Response” score for the first item (in response to the prompt, “What could this child say or do?) the subsequent item (“If that didn’t work, then what could he/she say or do?”) was skipped given the illogical nature of asking such a follow-up in that context. In these cases, however, the child was given the opportunity to say what he/she would do (in response to, “What would you say/do?) and, as such, could receive a score for flexibility. In addition, a stop rule was implemented to minimize child fatigue while still standardizing the testing protocol. Specifically, the SPST-R was discontinued when a child asked that the assessment be discontinued or when a child did not respond—or received "Non-Response" scores—to four consecutive items, spanning two different story vignettes.

Data Entry and Consistency Checks

To ensure accuracy of the data scoring and entry, 20-25% of each wave were double scored and checked for data entry errors. Errors were found in less than 1% of the cases across all waves. When errors were found, the data were double-checked to verify the results. Specifically, the hard-copy form was pulled and examined to determine what corrections, if any, had to be made to the data. In some cases, the original score was correct and in that case, was maintained. In all other cases, the error was corrected on both the hard copy (noting an error had been made) and in the database. The corrected data was then imported into Stata for analysis.

Analytic Approach

To address the research questions in Essay 1, a person-period data set was created, such that each child contributed up to six rows of data—one row per wave of data that he or she contributed to the study during the preschool year (up to five waves per child) as well as the baseline score for each assessment—considered the most appropriate format for fitting multilevel models for change (Singer & Willett, 2003).

Below, I describe briefly each of the measures in this dataset as they relate to all research questions in Essay 1. In addition, several variables were constructed (e.g., PreK Start Date, Months in PreK) when creating the data set, which will also be described as they relate to each RQ.

Baseline Constructs. A series of baseline scores was constructed for each variable of interest (e.g., BaselineMath, BaselineAdaptive, BaselinePPVT) based upon the first time each assessment was administered to a given child. This allowed the time of year of first assessment to vary for each child, depending on when each child was enrolled into the study. All assessments were administered within the same one-month assessment window (and were typically administered on the same day). The earliest assessment date for each child is indicated for this variable for consistency. Raw scores were used in each.

PreK Start Date. A series of time-related variables was generated to calculate this variable. First, a "PreK start date" variable was generated based upon each child's individualized baseline test date (which varied for each child). Specifically, a formula was created in which September 15 was assumed to be the official start date of the preschool year (since this is the official enrollment date for these participating HS programs) and was assumed to be the start date for all children who were initially consented and began participating in Wave 1. For children who began participating in subsequent waves, it was assumed they enrolled in the preschool at later points in the preschool year and, thus, were assigned a later PreK start date. While this assumption will not always hold, it is more plausible that this is measurement error and not bias, given that the error is known to be in both directions (i.e., for some children, they definitely started the preschool year earlier than assumed, though for others, they definitely started later than what was assumed by the formula; on average, however, the assumption should hold in the population) and, unfortunately, administrative records and parental reports were beyond the scope of this study. Using that PreK Start Date, a continuous "Days in Preschool" variable was created, which also varied for each child. This variable was then transformed into "Months in Preschool" for greater model stability and ease of interpretation. This variable was not centered. Because many children from low-income families often enroll in preschool throughout the preschool year (e.g., when a child becomes age-eligible, when families move into the area, obtain a child care voucher, etc.), the unit of time has not been centered. By *not* centering this unit of time, however, the interpretation of the intercept now refers to the child's skills at preschool entry (i.e., each child's PreK start date), which has more practical and policy implications, both in this sample and in the broader population. For these reasons, this variable of time was not centered.

Outcome Variables:

MathRaw is a time-varying, child-level, interval outcome variable, with values entered into each row in the person-period dataset, for each child on each occasion, when available. In this sample, the MathRaw scale has a range of 0-25 with high scores indicating a child has many math skills whereas 0 indicates the child has none of these skills as measured by the TEMA assessment (M = 6.77, SD = 5.37).

MathRaw is the outcome of focus in the following three research questions:

- **Research Question 1:** How do children's math skills develop during a preschool year?
- **Research Question 3a:** Does children's development of flexibility in social problem-solving moderate their math trajectories?
- **Research Question 3c:** Does children's development of adaptive social problem-solving moderate their math trajectories?

Adaptive is a time-varying, child-level, interval outcome variable, with values entered into each row in the person-period dataset, for each child on each occasion, when available. The adaptive scale has a possible range of 0-6 with 6 indicating a child has many adaptive problem-solving skills whereas 0 indicates the child has none of these skills (*M* = 2.95, *SD* =1.82).

Adaptive is the outcome of focus in the following research question:

- **Research Question 2a:** How do children's adaptive social problem-solving skills develop during a preschool year?

Flexibility is a time-varying, child-level, continuous outcome variable, with values entered into each row in the person-period dataset, for each child on each occasion, when available. The flexibility scale has a continuous range of 0-3 with 3 indicating a child is very flexible in social problem-solving whereas 0 indicates the child is not at all flexible according to this measure (*M* = 1.45, *SD* = .77).

Flexibility is the outcome of focus in the following research question:

- **Research Question 2b:** How does children's flexibility in social problem-solving skills develop during a preschool year?

The child's individual flexibility scores for each vignette were averaged across vignette type (i.e., both object acquisition and friendship initiation). Because children did not necessarily complete all eight vignettes at each assessment, the average was calculated based upon the total number of vignettes (i.e., the denominator) completed by each child at each assessment wave. Thus, vignettes not attempted by the child do not represent missing data. The mean number of social problem-solving vignettes completed at each wave for all children in the sample was slightly lower at Wave 1, though was fairly comparable at all other time points (ranging from M_{W1}=6.19, SD_{WI}=2.36 to M_{W5}=7.60, SD_{W5}=1.54).

The following list of variables are the outcomes of focus in the following research question:

- **Research Question 2c:** How do children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised)?

Object Acquisition Tasks

AdaptiveSkills is a time-varying child-level outcome variable that has been included to describe the extent to which children report Adaptive problem-solving skills over the course of a preschool year. As a proportion, the Adaptive scale has a possible range of 0-1 with 1 indicating a child has many Adaptive problem-solving skills whereas 0 indicates the child has none of these skills (M = 0.77, SD = 0.28). Adaptive problem-solving strategies consist of prosocial efforts (e.g., asking or telling the other child to give him/her the object, references to sharing, waiting), seeking an authority (e.g., asking a teacher to intervene), and categories in which children mutually agreed upon a trade (e.g., you can have this object if you give me that object), loans (e.g., can I play with it for a little bit?) or planning for the future (e.g., When she lets go of it, I'll play with it). A new category was also created (Alternate Activity as described in the SPST-R Coding Manual), which is also considered an Adaptive problem-solving strategy.

ManipulativeSkills is a time-varying child-level outcome variable that has been included to describe the extent to which children report Manipulative problem-solving skills over the course of a preschool year. As a proportion, the Manipulative scale has a possible range of 0-1 with 1 indicating a child reports many Manipulative problem-solving skills whereas 0 indicates the child reports none of these skills (M = 0.02, SD =

0.06). Manipulative problem-solving strategies consist of responses in which the child reports using emotions in an attempt to get the object (e.g., I'll get mad if you don't give it to me or I won't like you anymore.) These responses can also include finagling to get the object (e.g., Saying "Look at that" and then taking the object.).

AggressiveSkills is a time-varying child-level outcome variable that has been included to describe the extent to which children report Aggressive problem-solving skills over the course of a preschool year. As a proportion, the Aggressive scale has a possible range of 0-1 with 1 indicating a child reports many Aggressive problem-solving skills whereas 0 indicates the child never reports these skills (*M* = 0.06, *SD* = 0.11). Aggressive problem-solving strategies consist of responses in which the child reports using force or threats of force, which can be verbal (e.g., Call him a mean name) or physical (e.g., I'd kick him) or might involve taking the object by force (e.g., I'll snatch it). Aggressive responses can also include damage to the object itself (e.g., I'd break it).

Non-Responses is a time-varying child-level outcome variable that has been included to describe the extent to which children do not respond with any problem-solving skills at all on the assessment. As a proportion, the Non-Response scale also has a possible range of 0-1 with 1 indicating a child has many Non-Responses (and reports few problem-solving strategies) whereas 0 indicates the child responds to all the items (*M* = 0 .25, *SD* = 0.56). Non-responses include responses that are off-topic (e.g., I like ice cream) and also responses that address the prompt, but do not identify a problem-solving strategy (e.g., He's not listening). These can include both verbal (I don't know) and non-verbal (e.g., child shrugs) responses.

Friendship Initiation Tasks

In these tasks, the target child is attempting to initiate a friendship (e.g., getting to know a child who is new to the setting). The problem-solving strategy used by the child is scored according to several categories, as described below. It is possible that children's responses can be coded in multiple ways (i.e., they are not mutually exclusive) according the scoring criteria below.

ProsocialandComplimentary is a time-varying child-level outcome variable that has been included to describe the extent to which children report Prosocial and Complimentary problem-solving strategies over the course of a preschool year. As a proportion, this scale has a possible range of 0-1 with 1 indicating a child reports many Prosocial and Complimentary problem-solving strategies whereas 0 indicates the child reports none of these strategies (*M* = 0.07, *SD* = 0.13). Prosocial and Complimentary problem-solving strategies consist of statements in which the child is nice to the new child (e.g., I'll share with you) or gives compliments to the new child (You're nice; I like your sandals).

AdultIntervention is a time-varying child-level outcome variable that has been included to describe the extent to which children report strategies to enlist the help of an adult and/or a third person (e.g., I'll ask the teacher what's his name) over the course of a preschool year. As a proportion, this scale has a possible range of 0-1 with 1 indicating a child reports many Adult Intervention problem-solving strategies whereas 0 indicates the child reports none of these strategies (*M* = 0.02, *SD* = 0.05).

ConversationOpeners is a time-varying child-level outcome variable that has been included to describe the extent to which children report personal questions (What's your name?) and/or polite or causal remarks (Hi. Nice to meet you.) over the course of a preschool year. As a proportion, this scale has a possible range of 0-1 with 1 indicating a child reports many Conversation Openers as problem-solving strategies whereas 0 indicates the child reports none of these strategies (M = 0.16, SD = 0.26).

IndirectInitiation is a time-varying child-level outcome variable that has been included to describe the extent to which children report non-assertive responses to seek friendship or prosocial acts (Can I be your friend?) from the other child. These responses can also simply include initiating physical proximity in order to get to know the new child (She could sit beside her). As a proportion, this scale has a possible range of 0-1 with 1 indicating a child reports many Indirect Initiation strategies whereas 0 indicates the child reports none of these strategies (M = 0.09, SD = 0.18).

DirectInitiation is a time-varying child-level outcome variable that has been included to describe the extent to which children report assertive responses to seek friendship (I'll be your friend) or initiate direct action towards the new child (Give him a hug), including offers to engage in a specific activity (Do you want to play with me?). As a proportion, this scale has a possible range of 0-1 with 1 indicating a child reports many Indirect Initiation strategies whereas 0 indicates the child reports none of these strategies (M = 0.17, SD = 0.21).

Non-Normative is a time-varying child-level outcome variable that has been included to describe the extent to which children report Non-Normative strategies to seek friendship, which can include conditional offers of friendship (I'll buy you a marker if you'll play with me) or responses that indicate verbal or physical attacks on the new child (I'll hit him.) As a proportion, this scale has a possible range of 0-1 with 1 indicating a child uses many of these strategies whereas 0 indicates the child reports none of these strategies (*M* = 0.03, *SD* = 0.12).

Non-Responses is a time-varying child-level outcome variable that has been included to describe the extent to which children do not report any problem-solving skills at all during the preschool year. As a proportion, the Non-Response scale also has a possible range of 0-1 with 1 indicating a child has many Non-Responses (and reports few problem-solving skills) whereas 0 indicates the child reports none of these types of responses (*M* = 0 .43, *SD* = 0.38). As with Object Acquisition tasks, non-responses include responses that are off-topic (e.g., I like ice cream) and also responses that address the prompt, but do not identify a problem-solving strategy (e.g., They're at the school.). These can include both verbal (I don't know) and non-verbal (e.g., child shrugs) responses.

Question Predictors

Months is a time-varying variable that counts the number of months that have elapsed since the beginning of the preschool year (i.e., September 15) and the collection of each subsequent wave of data. Thus, by its coding, it accounts for the different spacing of assessments, by wave, across children. As described previously, Months has not been centered. As such, the PreK start date can be interpreted as the child's "true initial status" (rather than the first assessment date).

MonthsxFlexibility is a time-varying child-level predictor that has been included to describe the extent to which development of flexibility in social problem-solving skills is associated with different children's math trajectories (i.e., faster or slower rates of change) over the course of a preschool year. The flexibility scale has a continuous range of 0-3 with 3 indicating a child is very flexible in social problem-solving whereas 0 indicates the child has none of this skill (*M* = 1.45, *SD* = .77).

MonthsxAdaptive is a time-varying child-level predictor that has been included to describe the extent to which adaptive problem-solving skills moderate children's math trajectories (i.e., faster or slower rates of change) over the course of a preschool year. The Adaptive scale has a possible range of 0-6 with 6 indicating a child has many adaptive problem-solving skills whereas 0 indicates the child has none of these skills (*M* = 2.95, *SD* =1.82).

MonthsxAdaptive^2. Because *Adaptive* has a curvilinear relationship with time, a quadratic term is added to these models. Given that the Adaptive scale has a possible range of 0-6, the Adaptive^2 scale, therefore, has a possible range of 0-36 (*M* = 11.93, *SD* = 11.71). *MonthsxAdaptive*^{2} is a time-varying child-level predictor that has been included to describe the extent to which Adaptive problem-solving skills moderate children's math trajectories (i.e., faster or slower rates of change) over the course of a preschool year.

Covariates. An important set child of child-level covariates was included in this set of models, in order to increase the precision of the estimation and the statistical power of the analyses. These covariates included: dichotomous indicators of individual children's sex and race/ethnicity, age at baseline assessment (reported in months) and baseline PPVT scores. As reported in the demographics above, 80% of participating children are from low-income families, negating the need for an income-based control in these models. In Table 1, summary statistics are presented for all the covariates examined in this study. In the equations presented below, this vector of covariates is represented with the symbol ***Z.***

Missing Data

There was very little missing data across all data collection waves. One child was missing baseline PPVT and one child was missing an adaptive score as measured by the Challenging Situations Task (in Wave 3, April of the preschool year, the only time period in which any child was missing data on this measure). Seven children were missing values on the SPST-R measure for flexibility. Specifically, two children were missing data at Wave 1, two children were missing data at Wave 2, two children were missing data at Wave 4 and 1 child was missing data at Wave 5 (no children were missing data at Wave 3). Nine children had their TEMA assessments conducted with administrative errors that resulted in missing data (i.e., unusable data). Specifically, three children in Wave 1 (November), four children in Wave 2, and two children in Wave 3 had their TEMA assessments conducted with administrative errors that resulted in missing data (i.e., unusable data).

Missing data were assumed to be missing at random (MAR). Correlations and logit regression models indicated that, while there were no correlations between gender, missingness on the adaptive and flexibility measures was positively related to some variables in the sample, including baseline age and PPVT as well as race/ethnicity confirming that data was not missing completely at random (MCAR). In a multilevel model framework, the assumption is that outcome variables are assumed to be missing at random, conditional on covariates, negating the need for imputation for outcome variables of interest (i.e., for RQ1 and RQ2). Given the very small amount of missingness (e.g., <1% for adaptive social problem-solving, 2.7% for flexibility and 3.4% for TEMA scores) and the fact that all of the children who had missing values had data in other waves on these variables, full case analysis was used if data was missing for any of the relevant predictor variables (i.e., listwise deletion was used in cases of missingness) in the cross-domain models used to address RQ3a and RQ3b.

PPVT Mean Imputation for Missing Value

One child was missing a baseline PPVT score. Because this was only one case, a simple linear regression model was adopted to impute this single score. A series of models was tested, though the following model was the most parsimonious model with predictive utility and was fit to represent children's receptive language skills at baseline (i.e., at preschool entry):

$$PPVTRaw = \beta_0 + \beta_1 cBaseAge + \beta_2 Black$$

This child's specific values were then entered into the above equation and that value was used as this child's baseline PPVT score. While there is likely to be some error that has not been accounted for, particularly given the simplicity of the model used, it is likely to be negligible given that this procedure was adopted for only one case in the entire sample.

Analytic Plan for Research Question 1 and 2

To address the RQs in Essay 1 and to account for the longitudinal nature of this data, 2-level multilevel models for change were fit, in which time was nested within children. These models were drawn from a person-period data set in which each child contributes a row for each time that s/he has data for math and/or social problem-solving skills, allowing one to account for the lack of independence of the child-level responses across measurement instances within children (Singer & Willett, 2003). Therefore, all children in the dataset have between one to five rows of data, one for each wave of data collected, plus a baseline score for the "first" assessment of each kind (e.g., math, social problem-solving skills, etc.) at the point of entry into the study. Multilevel models were fit using the "mixed" command in STATA 14.0 software (Stata Corporation, 2015) to account for the multi-level structure of the dataset. Because all four of these classrooms were in the same early care and education center and, thus, were under the same center director, they were likely implementing similar curriculum and daily routines for the children. To be thorough, models were fit with and without a fixed effect for classrooms to account for this clustering and results of both will be presented. In such models, the intercept can be interpreted as the estimated skills at preschool entry for children in the reference classroom. Because each of these skills of interest (i.e., early math skills and social problem-solving skills) were measured a maximum of five times in the study period (*M*=3.4 times per child), it is possible to compare the fit of a linear growth model to models with higher order terms (e.g., quadratic, cubic). In doing so, it can be determined whether the linear specification of time adequately explains within-person variation over time by comparing the fit statistics of all models.

RQ1: How do children's math skills develop during a preschool year?

The following model was fit to represent development of children's early math skills as a function of time, for child *i* at time *j*:

Level-1 Model: $Math_{ij} = \pi_{0i} + \pi_{1i}Months_{ij} + \varepsilon_{ij}$ where $\varepsilon_{ij} \sim N(0, \sigma^2)$

Level-2 Model: $\pi_{0i} = \gamma_{00} + \gamma_{01}\mathrm{cBaseAge_i} + \mathrm{Z_i}\boldsymbol{\beta} + \zeta_{0i}$

$\pi_{1i} = \gamma_{10} + \gamma_{11}\mathrm{cBaseAge_i} + \zeta_{1i}$

Composite Model: $Math_{ij} = \gamma_{00} + \gamma_{10}Months + \gamma_{20}cBaseAge + \gamma_{30}MonthsxcBaseAge + Z_i\boldsymbol{\beta} + \varepsilon_{ij} + \zeta_{1i}Months_{ij} + \zeta_{1i}$ where

$$\varepsilon_{ij} \sim N(0, \sigma_\epsilon^2) \; and \; \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix} \right)$$

In this specification $MATH_{ij}$ represents early math skills for a given child, *i*, on occasion *j*. π_{0i} represents predicted early math skills of child *i* at the first time of assessment (i.e., the intercept) while π_{1i} represents the rate of change in children's early math skills across the preschool year in which these skills were measured (i.e., the slope). Participating children ranged in age from age from 34-70 months (*M* = 46.80, *SD* = 8.33) at baseline and therefore an adjustment in age is made in the equation ($BaseAge_{T1}$), and was centered at the baseline age for all children (i.e., the age at which they began the study). In this model, there are three types of error; ε_{ij} describes the within-person residual (i.e., variance of the individual) while ζ_{0i} describes the residual in initial status (i.e., variance in the intercept) whereas ζ_{0i} describes variance in the rate of change (i.e., variance in the slope).

In the hypothesized composite model, individual growth parameter π_{0i} represents the *i*th child's initial true level (i.e., the intercept) of early math skills at preschool entry (i.e., on September 15 or whatever later date is assumed), and π_{1i} represents the linear rate of true change (i.e., the slope) in the child's trajectory. The level-2 parameter γ_{00} represents the population average of the level-1 intercepts and the level-2 parameter γ_{10} represents the population average of the level-1 slopes. It was hypothesized that—overall—estimates of population averages γ_{00} and γ_{10} would be positive and statistically significant, indicating that children's development of early math skills was non-zero during the preschool year and increased over time in a non-linear fashion. It was also hypothesized that this rate of growth would be different for children of different ages (i.e., 3 year olds would have different learning trajectories than 5 year olds).

RQ2a: How do children's adaptive social problem-solving skills develop during a preschool year (as captured by the Challenging Situations Task)?

The following model was fit to represent development of children's Adaptive social problem-solving skills as a function of time, for child *i* at time *j*:

Level-1 Model: $Adaptive_{ij} = \pi_{0i} + \pi_{1i} Months_{ij} + \pi_{2i} {Months^2}_{ij} + \varepsilon_{ij}$

where $\varepsilon_{ij} \sim N(0, \sigma^2)$

Level-2 Model: $\pi_{0i} = \gamma_{00} + Z_i \boldsymbol{\beta} + \zeta_{0i}$

$\pi_{1i} = \gamma_{10} + \zeta_{1i}$

$\pi_{2i} = \gamma_{20}$

Composite Model:

$$Adaptive_{ij} = \gamma_{00} + \gamma_{10} Months + \gamma_{20} Months^2 + Z_i \boldsymbol{\beta} + \zeta_{0i} + \zeta_{1i} Months + \varepsilon_{ij}$$

$$\text{where } \varepsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix} \right)$$

In this specification $Adaptive_{ij}$ represents the adaptive social problem-solving skills for a given child, *i*, on occasion *j* while π_{0i} represents the predicted adaptive social problem-solving skills of child *i* at preschool entry. Because a non-linear rate of change has been hypothesized, π_{1i} represents the linear rate of change in children's adaptive social problem-solving skills while π_{2i} represents the quadratic rate of change in children's adaptive social problem-solving skills across a preschool year. Note that the level-2 model parameter π_{2i} (i.e., the quadratic term for time) has been specified with a fixed effect for the population, following methods suggested by other researchers (e.g., Rabe-Hesketh & Skrondal, 2012).

It was hypothesized that—overall—estimates of population averages γ_{00} and γ_{10} and γ_{20} would be positive and statistically significant, indicating that children's development of adaptive social problem-solving skills was non-zero during the preschool year and increased over time in a non-linear fashion. It was also hypothesized that this rate of growth would be different for children of different ages (i.e., 3 year olds would have different learning trajectories than 5 year olds).

RQ2b: How does children's flexibility in social problem-solving skills develop during a preschool year?

The following model was fit to represent development of children's flexibility in social problem-solving skills as a function of time, for child *i* at time *j*:

Level-1 Model: $$Flexibility_{ij} = \pi_{0i} + \pi_{1i}Months_{ij} + \varepsilon_{ij}$$

$$\text{where } \varepsilon_{ij} \sim N(0, \sigma^2)$$

Level-2 Model: $$\pi_{0i} = \gamma_{00} + \gamma_{01}\text{cBaseAge}_i + Z_i\boldsymbol{\beta} + \zeta_{0i}$$

$$\pi_{1i} = \gamma_{10} + \gamma_{11}\text{cBaseAge}_i + \zeta_{1i}$$

Composite Model:

$$Flexibility_{ij} = \gamma_{00} + \gamma_{10}Months + \gamma_{20}cBaseAge + \gamma_{30}MonthsxcBaseAge + Z_i\boldsymbol{\beta} + \varepsilon_{ij} + \zeta_{1i}Months_{ij} + \zeta_{1i} \quad \text{where } \varepsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix}\right)$$

In this specification $Flexibility_{ij}$ represents the flexibility in social problem-solving skills for a given child, *i*, on occasion *j*. π_{0i} represents the predicted flexibility in social problem-solving skills of child *i* at preschool entry while π_{1i} represents the rate of change in children's flexibility in social problem-solving skills across a preschool year. Participating children ranged in age from 34-70 months (M = 46.80, SD = 8.33) at baseline and therefore an adjustment in age is made in the equation (cBaseAge_{T1}), and was centered at the baseline age for all children (i.e., the age at which they began the study).

It was hypothesized that—overall—estimates of population averages γ_{00} and γ_{10} would be positive and statistically significant, indicating that children's development of flexibility in social problem-solving skills was non-zero during the preschool year and increased over time in a linear fashion. It was also hypothesized that this rate of growth would be different for children of different ages (i.e., 3 year olds would have different learning trajectories than 5 year olds). A statistically significant estimate for γ_{30} (in the composite model) would indicate that, on average, flexibility in social problem-solving skills differ for children of different ages as they progress through a preschool year.

RQ2c: How do children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised)?

RQ2c investigates the ways in which children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised). Changes will be discussed qualitatively by describing changes in the aggregate and describing patterns over time (i.e., from one data collection period to the next) for the entire group.

Analytic Plan for RQ3a

RQ3a: Does children's development of flexibility in social problem-solving moderate their math trajectories?

The following model was fit to represent the moderation of children's math learning trajectories as a function of their flexibility in social problem-solving skills, for child *i* at time *j*:

Composite Model:

$$Math_{ij} = \gamma_{00} + \gamma_{10}Months_{ij} + \gamma_{20}Flexibility_{ij} + \gamma_{30}Months_{ij} \times Flexibility_{ij} + Z_i\boldsymbol{\beta} + \left[\zeta_{0i} + \zeta_{1i}Months_{ij} + \varepsilon_{ij}\right]$$

$$\text{where } \varepsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ and } \begin{bmatrix}\zeta_{0i}\\ \zeta_{1i}\end{bmatrix} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma_0^2 & \sigma_{01}\\ \sigma_{01} & \sigma_1^2\end{bmatrix}\right)$$

RQ3a investigates the extent to which children's initial status and development of flexibility in social problem-solving might alter the children's math learning trajectories over the course of the preschool year. Specifically, it is addressed by estimating parameter γ_{20} which describes the difference in early math skills due to the extent to which children have flexibility in social problem-solving skills when they begin the preschool year (i.e., differences in intercepts). Statistically significant estimates for γ_{30} would indicate that children's math learning trajectories have a differential relationship with children's social competence that depends upon children's development of flexibility in social problem-solving skills during the preschool year. Specifically, parameter γ_{30} describes how the rate of change in children's math learning trajectories is shaped by this interaction term with time and development of flexibility in social problem-solving skills. A statistically significant estimate for γ_{30} would indicate that, on average, the rate at which children develop flexibility in social problem-solving skills alters the rate of change at which children learn early math skills during a preschool year. While flexibility is a time-varying predictor, the level-2 model parameters γ_{20} and γ_{30} have been specified with a fixed effect for the population as there is no substantive interest in the stochastic variation across participants for these parameters, following protocols used by other researchers (Rabe-Hesketh & Skrondal, 2012).

Analytic Plan for RQ3b

RQ3b: Does children's development of adaptive social problem-solving skills moderate their math trajectories?

The following model was fit to represent the moderation of children's math learning trajectories as a function of their adaptive social problem-solving skills, for child *i* at time *j*:

Composite Model:

$$Math_{ij} = \gamma_{00} + \gamma_{10}Months_{ij} + \gamma_{20}cBaseAge_i + \gamma_{30}Months_{ij} \times cBaseAge_i + $$

$$+ \gamma_{40}Adaptive_{ij-1} + \gamma_{50}Adaptive^2_{ij-1} + \gamma_{60}Months_{ij} \times Adaptive_{ij-1} +$$

$$\gamma_{70}Months_{ij} \times Adaptive^2_{ij-1} + \gamma_{80}Months_{ij} \times Adaptive_{ij-1} \times cBaseAge_j +$$

$$\gamma_{90}Months_{ij} \times Adaptive^2_{ij-1} \times cBaseAge_j + Z_i\boldsymbol{\beta} + \zeta_{0i} + \zeta_{1i}Months_{ij} + \varepsilon_{ij} \text{ where}$$

$$\varepsilon_{ij} \sim N(0, \sigma^2_\epsilon) \ and \ \begin{bmatrix}\zeta_{0i}\\ \zeta_{1i}\end{bmatrix} \sim N\left(\begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}\sigma^2_0 & \sigma_{01}\\ \sigma_{01} & \sigma^2_1\end{bmatrix}\right)$$

RQ3b investigates the extent to which children's initial status and development of adaptive social problem-solving might alter the children's math learning trajectories over the course of the preschool year. Specifically, it is addressed by estimating parameters γ_{40} and γ_{50}, which, together, describe the difference in early math skills (i.e., differences in intercepts) due to the extent to which children have adaptive social problem-solving skills when they begin the preschool year as well as γ_{60} and γ_{70} which, together, describe the difference in early math skills (i.e., differences in slopes) due to the extent to which children develop adaptive social problem-solving skills as they progress through the preschool year. Note that these are time-lagged variables, which draw from the previous data collection period in order to better establish a time-ordered sequence between adaptive social problem-solving skills and math skills. Statistically significant estimates for γ_{40} and γ_{50} would indicate that children's math learning trajectories have a differential relationship with children's social competence that depend upon children's adaptive social problem-solving skills when they begin the preschool year. Statistically significant estimates for γ_{60} and γ_{70} would indicate that, on average, development of adaptive social problem-solving skills predict a change in the rate at which children learn early math skills during a preschool year and that these predicted changes may vary for older and younger children. While adaptive social problem-solving is a time-varying predictor, the level-2 model parameters for γ_{40}, γ_{50}, γ_{60} and γ_{70} have been specified with a fixed effect for the population. Specifically, there is no substantive interest in the stochastic variation across participants for these parameters.

Results

The results are presented for each research question in order. However, a detailed description of the model building process will be provided for RQ1 while only the results of the final selected model will be provided for subsequent analyses. Random effects models with a fixed effect for each classroom have been fit to investigate the potential unobserved characteristics of the classroom that may have influenced the relationships being predicted. Descriptive statistics for the sample demographics and for each of the scales are reported in Table 1 (below).

Table 1. Descriptive statistics of analytic variables

	N	Mean	SD
*Outcome Scores at Baseline Assessment**			
PPVT Raw Score	75	52.28	21.95
Math Raw Score	76	5.13	4.50
Adaptive Social Problem-Solving Skills	76	2.74	1.70
Average Flexibility in Problem Solving	75	1.32	0.89
Outcome Scores at the November Assessment			
Math Raw Score	53	4.57	4.39
Adaptive Social Problem-Solving Skills	56	2.55	1.63
Average Flexibility in Problem Solving	52	1.44	0.83
Outcome Scores at the February Assessment			
Math Raw Score	57	5.84	4.88
Adaptive Social Problem-Solving Skills	61	2.80	1.81
Average Flexibility in Problem Solving	61	1.35	0.84
Outcome Scores at the April Assessment			
Math Raw Score	54	7.96	5.58
Adaptive Social Problem-Solving Skills	56	2.57	1.75
Average Flexibility in Problem Solving	57	1.47	0.75
Outcome Scores at the June Assessment			
Math Raw Score	62	9.11	5.55
Adaptive Social Problem-Solving Skills	61	3.30	1.90
Average Flexibility in Problem Solving	60	1.54	0.67
Outcome Scores at the August Assessment			
Math Raw Score	31	9.48	5.84
Adaptive Social Problem-Solving Skills	30	4.47	1.53
Average Flexibility in Problem Solving	34	1.47	0.79
Demographics at Baseline			
BaselineAge	76	46.80	8.33
Male	48	0.63	0.49
Black	59	0.78	0.42
Hispanic/Latino/Non-Black	15	0.20	0.40
White	1	0.01	0.11
Asian	1	0.01	0.11

*Baseline Assessment is the first time that child is assessed at any point in the preschool year

RQ1: How do children's math skills develop during a preschool year?

Prior to conducting analyses, visual inspections of the data and examination of the basic descriptive statistics revealed that all the necessary assumptions had been met. The distributions of math scores were somewhat skewed at almost every time period in which data was collected, which was to be expected given the low-income sample (i.e., a homogeneous population). During the final wave, math scores were approximately normally distributed. However, preliminary analyses indicated that skew was not sufficient to threaten assumptions of linear regression. There was a fairly strong, positive correlation (r = 0.63, p<0.001) between children's early math skills and their receptive language skills at the baseline assessment.

Math scores varied moderately, with steady increments in mean scores at each wave of data collection (i.e., every 60 days, on average) as well as increased variation. In fall of the preschool year (November-December), the mean raw score was 4.57 (*SD*=4.39), 5.84 (*SD*=4.88) in February of the preschool year, 7.96 (*SD*= 5.58) in April of the preschool year and 9.11 (*SD*=5.55) in June of the preschool year. For those children who were also assessed in August (i.e., those who enrolled into the study after the study began), mean math scores were 9.48 (*SD*= 5.84), illustrating a continuation of this trend. At each time point, math scores were strongly and positively related to one another (ranging from r = 0.82-0.95, p< 0.001).

Table 2. Correlations between math scores across time periods throughout the preschool year.

	MathRaw PreK03	MathRaw PreK06	MathRaw PreK08	MathRaw PreK10	MathRaw PreK12
MathRaw PreK03	1.00				
MathRaw PreK06	0.89 (0.00)	1.00			
MathRaw PreK08	0.85 (0.00)	0.95 (0.00)	1.00		
MathRaw PreK10	0.86 (0.00)	0.93 (0.00)	0.92 (0.00)	1.00	
MathRaw PreK12	0.84 (0.00)	0.82 (0.00)	0.84 (0.00)	0.91 (0.00)	1.00

Note: Significance levels indicated in parentheses.

Because this study aimed to better understand how early math skills change across a preschool year in mixed-age classrooms serving children from low-income families—without a specific instructional intervention—a sequence of models were fit to determine the average shape of the math development trajectory for the sample (full taxonomy is presented in Table 3a and Table 3b, below). Model 1 in Table 3a (below) shows the unconditional means model predicting TEMA raw scores. Results from this model indicate that there is systematic variation in the children's early math skills that warranted further exploration, with both variation in initial status and the rate of change. Model 2 then tested for the extent to which changes in children's math skills could be attributed to time (as measured by months in a preschool year). In this unconditional growth model, there was again statistically significant evidence of systematic variation between children's initial status and learning trajectories during the preschool year that was predicted by months in preschool measured as linear time. Because a non-linear growth model was hypothesized, Model 3 introduced a quadratic term for time, though it was not significant ($\beta_{Months^2} = -.016, SE = .015, p = 0.295$) and fit statistics indicated a worse fitting model (BIC increased from 1269 in M2 to 1273 in M3). As a result, it was dropped from subsequent models and the model with a linear term for time was considered the best fitting unconditional model for growth in math skills (Model 2).

Several potential covariates (e.g., race/ethnicity, gender) were included in models to investigate whether and how they may affect children's math trajectories over the course of the preschool year though none were significant and, ultimately, were excluded from the final model (see full taxonomy, below). Though ultimately excluded, it is noteworthy that because the sample was predominantly African/African-American and male participants, an interaction term was included (BlackxMale) to test whether the initial status in early math skills was different for Black males compared to other children in the sample. As indicated in Model 9, there was a marginally significant relationship ($\beta_{BlackxMale} = -3.10, SE = 1.85, p = 0.09$). Also in Model 9, children's receptive language skills (i.e., cBasePPVT) were added to the model given the literature regarding the relationship between children's early language development and math skills, which yielded a significant relationship ($\beta_{\text{cBasePPVT}} = 0.08, SE = .02, p < 001$). In Model 12, the final model, only the significant predictors are retained. In this final model, the intercept (i.e., initial math status for children in the reference classroom) indicates that the average-age female with average receptive language skills would begin preschool with about five points on the TEMA assessment of early math skills ($\beta = 5.31, SE = 1.01, p < .001$). In this final model, both the initial status ($\beta_{cBaseAge} = .13, SE = .07, p < .05$) and true rates of change are different by baseline age of children

($\beta_{MonthsxcBaseAge} = .02, SE = .01, p = .05$) controlling for receptive language skills and gender.

While the rates of change are marginally significant in this model with a fixed effect for each classroom, fit statistics show that this "final" model explains an additional 50% of the within-person variation in growth in early math skills (over and above the linear growth model), suggesting that there was much less error in the prediction of children's level-1 growth trajectories with the inclusion of these additional predictors. Moreover, removal of the age interaction yielded a worse fitting model by several indices (see Appendix A for full description of results). In addition, this model shows that males begin preschool with 1.5 fewer points, on average, compared to their female peers on the TEMA math assessment ($\beta_{Male} = -1.51, SE = .75, p < .05$) controlling for baseline age, receptive language, and the number of months in a preschool year.

Because interpretation of the constant in a model such as this (i.e., a random effects model with a fixed effect for each classroom) results in interpretation of early math skills for children in the reference classroom in this sample, a more representative explanation of the findings for children in the population is presented below. Figure 1 (below), displays the predicted early math skills trajectories for prototypical children (36- and 60-months at preschool entry), controlling for mean baseline PPVT scores and gender. The plotted values in the figure have been derived from the estimates of the parameters of Equation [12], which are listed in Table 3b. Notice three patterns in Figure 1. First, illustrated by the vertical distance between the fitted lines for each age-group, is that children of different ages have different math skills at preschool entry. Because time was not centered, initial math skills have been estimated for each age group, prior to when the data collection period began. Specifically, the figure shows that the prototypical 36-month old girl would, on average, have TEMA math scores of 1.87 ($p < .05$) whereas the prototypical 60-month old girl would be estimated to have an average score of 8.31 ($p<0.001$) at preschool entry. The second pattern evident in the figure is the increase in the size of the age-based gaps in early math skills ($\beta_{MonthsxcBaseAge} = .02, p = .05$) between the predicted math scores for children of different ages by the end of the preschool year.[5] For example, the predicted end-of-year TEMA math score for the prototypical girl who was 36-months old at preschool entry would, on average, be 7.38 points whereas for the prototypical girl who was 60-months at preschool entry would be predicted to have an average score of 17.54 by the end of the preschool year. The third pattern to note is that, for each of these trends, males have lower entry and end-of-year math scores compared to their female peers as evidenced in the dashed lines for each age group.

In summary, younger children, on average, begin preschool with fewer math skills than their older peers, and increase at a slower rate during the preschool year, controlling for gender and receptive language skills. Specifically, the prototypical entering 36-month old child would, on average, be expected to gain 5.51 points on the TEMA assessment across the preschool year, while the prototypical entering 60-month old child would be predicted to gain nearly double the points (i.e., 9.23 points) during the same time-frame.

[5] This model assumes a 10-month preschool year, although many preschool and Head Start programs offer full-year wrap-around services, which would result in even larger end-of-year gaps.

Table 3a. Early Math Skills Development as a function of time (in months), moderated by the baseline age of the child at preschool entry, controlling for gender and baseline receptive language with a fixed effect for each participating classroom (Models 1-6)

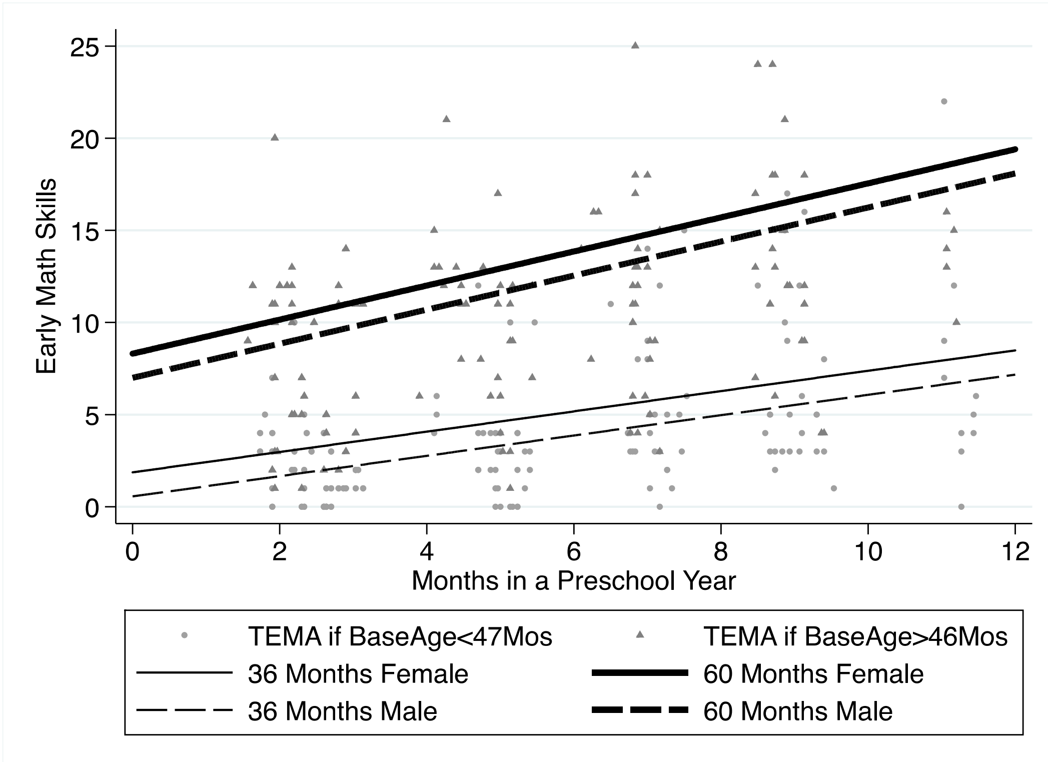
	(1) MathRaw	(2) MathRaw	(3) MathRaw	(4) MathRaw	(5) MathRaw	(6) MathRaw
Months		0.73^{***}	0.91^{***}	0.72^{***}	0.73^{***}	0.73^{***}
		(0.06)	(0.18)	(0.06)	(0.06)	(0.06)
$Months^2$			-0.02			
			(0.01)			
cBaseAge				0.31^{***}	0.26^{***}	0.25^{***}
				(0.06)	(0.06)	(0.06)
Class=1				0.00	0.00	0.00
				(.)	(.)	(.)
Class=2				-1.31	-1.30	-1.26
				(1.30)	(1.31)	(1.28)
Class=3				-2.12^{+}	-2.11^{+}	-1.82
				(1.27)	(1.27)	(1.26)
Class=4				-2.26^{+}	-2.24^{+}	-2.38^{+}
				(1.25)	(1.26)	(1.24)
Monthsx cBaseAge					0.02^{*}	0.02^{*}
					(0.01)	(0.01)
Male						-1.28
						(0.81)
Monthsx Male						
Black						
Monthsx Black						
Blackx Male						
cBasePPVT						
constant	4.53^{***}	3.28^{***}	2.86^{***}	4.83^{***}	4.79^{***}	5.53^{***}
	(0.51)	(0.51)	(0.65)	(0.98)	(0.98)	(1.08)
N	257	257	257	257	257	257
ll	-655.3	-618.1	-617.5	-595.3	-593.4	-592.2
AIC	1320.5	1248.2	1249.1	1210.6	1208.8	1208.3
BIC	1338.2	1269.4	1273.9	1246.1	1247.8	1250.9
$PseudoR^2$	0.00	0.10	0.10	0.50	0.51	0.52

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 3b. Early Math Skills Development as a function of time (in Months), moderated by the baseline age of the child at preschool entry, controlling for gender and baseline receptive language with a fixed effect for each participating classroom (Models 7-12)

	(7) MathRaw	(8) MathRaw	(9) MathRaw	(10) MathRaw	(11) MathRaw	(12) MathRaw
Months	0.81***	0.85***	0.84***	0.80***	0.71***	0.72***
	(0.10)	(0.16)	(0.16)	(0.10)	(0.06)	(0.06)
Months^2						
cBaseAge	0.25***	0.25***	0.13*	0.13*	0.13^{+}	0.13*
	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-1.27	-1.31	-0.80	-0.79	-0.78	-0.97
	(1.28)	(1.29)	(1.17)	(1.17)	(1.17)	(1.18)
Class=3	-1.82	-1.87	-0.62	-0.61	-0.61	-0.88
	(1.26)	(1.27)	(1.18)	(1.18)	(1.18)	(1.19)
Class=4	-2.39^{+}	-2.43^{+}	-1.87^{+}	-1.87	-1.86	-2.09^{+}
	(1.24)	(1.24)	(1.14)	(1.14)	(1.14)	(1.15)
Monthsx cBaseAge	0.01^{+}	0.01^{+}	0.01^{+}	0.01^{+}	0.02*	0.02^{+}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	-0.93	-0.94	1.40	1.38	0.89	-1.51*
	(0.88)	(0.89)	(1.74)	(1.74)	(1.68)	(0.75)
Monthsx Male	-0.13	-0.14	-0.14	-0.13		
	(0.12)	(0.13)	(0.13)	(0.13)		
Black		-0.21	1.58	1.42	1.44	
		(0.97)	(1.67)	(1.60)	(1.60)	
Monthsx Black		-0.05	-0.04			
		(0.14)	(0.14)			
Blackx Male			-3.10^{+}	-3.12^{+}	-3.13^{+}	
			(1.85)	(1.85)	(1.85)	
cBasePPVT			0.09***	0.09***	0.09***	0.08***
			(0.02)	(0.02)	(0.02)	(0.02)
constant	5.33***	5.54***	3.51^{+}	3.64*	3.92*	5.31***
	(1.09)	(1.41)	(1.87)	(1.82)	(1.81)	(1.01)
N	257	257	257	257	257	257
ll	-591.6	-591.5	-583.7	-583.8	-584.4	-586.3
AIC	1209.3	1213.0	1201.5	1199.6	1198.7	1198.6
BIC	1255.4	1266.2	1261.8	1256.4	1252.0	1244.8
PseudoR^2	0.52	0.52	0.62	0.62	0.62	0.60

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$



Predicted early math skills trajectories that differ by baseline age for prototypical children (36- and 60-months at preschool entry), controlling for baseline receptive language and gender. Figure illustrates steeper math trajectories for older (i.e., children who are 60 months at preschool entry) and shallower math trajectories for younger children (i.e., children who are 36 months at preschool entry).

RQ2a: How do children's adaptive social problem-solving skills develop during a preschool year (as captured by the Challenging Situations Task)?

Visual inspections of the data and examination of the basic descriptive statistics revealed that all the necessary assumptions had been met. Adaptive problem-solving skills scores were approximately normally distributed at almost every time period in which data was collected. The distribution was slightly left skewed at the final data collection period when the sample size was substantially smaller, indicating children had many more adaptive skills, on average. Since this measure provides children with pictures as response options there was only a moderate positive correlation (r = 0.37, *p*<0.001) between children's adaptive skills and their receptive language skills at the baseline assessment.

In general, across the sample, children showed steady gains across all time periods in adaptive problem-solving skills. In fall of the preschool year (November-December), the mean raw score was 2.55 (*SD* = 1.63) and had increased to an average of 4.47 (*SD* = 1.53) by the end (i.e., August) of the preschool year (see Table 1). For this set of skills, significant correlations only emerged from the second half of preschool forward. Specifically, during the initial few months of preschool (from November-February), there is a fairly low and nonsignificant correlation (*r* = .23, p =.11). From that point in the year forward, however, adaptive scores were quite strongly and positively related to one another (ranging from *r* = 0.46-0.48, *p*< 0.01 as shown in Table 4, below).

Table 4. Correlations among adaptive skills across time periods during a preschool year.

	Adaptive PreK03	Adaptive PreK06	Adaptive PreK08	Adaptive PreK10	Adaptive PreK12
Adaptive PreK03	1.000				
Adaptive PreK06	0.232 (0.11)	1.000			
Adaptive PreK08	0.230 (0.14)	0.477 (0.00)	1.000		
Adaptive PreK10	0.447 (0.00)	0.605 (0.00)	0.464 (0.00)	1.000	
Adaptive PreK12	0.311 (0.26)	0.396 (0.07)	0.345 (0.13)	0.472 (0.01)	1.000

Note: Significance levels in parentheses

To better understand how children's adaptive problem-solving skills changed across a preschool year in mixed-age classrooms, a sequence of models was tested, which is presented in Table 5a and Table 5b (below). Results from the unconditional means model indicated that there was systematic variation in the children's adaptive skills that warranted further exploration, with both variation in initial status and in the rate of change. Model 2 then tested for the extent to which changes in children's adaptive skills could be attributed to time (as measured by months in a preschool year). In this unconditional growth model, there was again statistically significant evidence of systematic variation between children's initial status and learning trajectories during the preschool year that could be predicted by the number of months in preschool. Because a non-linear growth model was hypothesized for adaptive skills, Model 3 introduced the quadratic term for time, which was significant ($\beta = .03, SE = .01, p < 0.001$) and was retained in all subsequent models.

Models 5-16 examine effects of predictors. In the final model (Model 16 in Table 5b, below), the initial status estimate (i.e., for children in the reference classroom) indicates that for the average-age child entering preschool, he/she will have an average of 3.26 adaptive problem-solving skills ($\beta = 3.26, SE = .52, p < .001$). Further, the data show that children's adaptive skills develop in a positive curvilinear way, on average ($\beta = 0.04, SE = .01, p < .05$) across a preschool year (see Figure 2, below). However, inspection of individual growth trajectories as well as Figure 2 (below) revealed substantial variation in the development of these growth patterns throughout the sample. Collectively, this data suggests that—without intervention—low-income children in preschool settings acquire adaptive problem-solving strategies in various forms. Evidence from this study also shows that age is positively related to the number of adaptive problem-solving strategies children have when they enter preschool ($\beta = .08, SE = .02, p < .001$) indicating that older children, on average, begin preschool with more adaptive problem-solving strategies than younger children. Again, because interpretation of the constant in a random effects model with a fixed effect for each classroom results in interpretation of the number of adaptive skills for children in the reference classroom in this sample, a more representative explanation of the findings for children in the population is presented below.

In Figure 2 (below), predicted adaptive skills trajectories for prototypical children (36- , 48- and 60-months at preschool entry) are displayed. The plotted values in the figure have been derived from the estimates of the parameters of Equation [16], which are listed in Table 5b. Specifically, it is estimated that a prototypical 36-month old child would, on average, be estimated to have 2.89 adaptive problem-solving skills at preschool entry, while a prototypical 48-month old child would, on average, be estimated to have 3.70 adaptive problem-solving skills at preschool entry whereas a prototypical 60-month old child would be predicted to have 4.51 adaptive problem-solving skills at preschool entry. By the end of the preschool year, however (as evidenced by the parallel lines in Figure 2, below) each of the prototypical children would be predicted to gain, on average, 0.24 points in adaptive social problem solving skills during a preschool year.

In summary, younger children are, on average, entering preschool with fewer of these social problem-solving skills than their older peers. However, all children—regardless of age—would be predicted to progress at the same rates when learning adaptive social problem solving skills during a preschool year.

Table 5a. Development of adaptive social problem-solving skills as a function of time (in months), moderated by the baseline age of the child at preschool entry (Models 1-8).

	(1) Adaptive	(2) Adaptive	(3) Adaptive	(4) Adaptive	(5) Adaptive	(6) Adaptive	(7) Adaptive	(8) Adaptive
Months		0.11***	-0.34*	-0.34*	-0.33*	-0.33*	-0.32*	-0.25
		(0.03)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.25)
$Months^2$			0.04**	0.04**	0.04**	0.04**	0.04**	0.03
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
cBaseAge				0.08***	0.07	0.07	0.07	0.07
				(0.02)	(0.05)	(0.05)	(0.05)	(0.05)
Class=1				0.00	0.00	0.00	0.00	0.00
				(.)	(.)	(.)	(.)	(.)
Class=2				0.40	0.39	0.39	0.44	0.45
				(0.50)	(0.50)	(0.49)	(0.49)	(0.49)
Class=3				0.02	0.02	0.12	0.17	0.17
				(0.49)	(0.49)	(0.49)	(0.49)	(0.49)
Class=4				0.23	0.22	0.15	0.19	0.19
				(0.49)	(0.49)	(0.48)	(0.48)	(0.48)
Monthsx					0.00	0.00	0.00	0.00
cBaseAge					(0.02)	(0.02)	(0.02)	(0.02)
$Months^2$x					-0.00	-0.00	-0.00	-0.00
cBaseAge					(0.00)	(0.00)	(0.00)	(0.00)
Male						-0.49	-0.44	-0.29
						(0.31)	(0.31)	(0.81)
Black							0.38	0.38
							(0.34)	(0.34)
Monthsx								-0.11
Male								(0.31)
$Months^2$x								0.01
Male								(0.03)
Monthsx								
Black								
$Months^2$x								
Black								
cBase								
PPVT								
constant	2.86***	2.36***	3.43***	3.26***	3.23***	3.53***	3.17***	3.06***
	(0.16)	(0.22)	(0.40)	(0.52)	(0.52)	(0.55)	(0.64)	(0.79)
N	266	266	266	266	266	266	266	266
ll	-513.7	-509.1	-504.1	-496.3	-496.1	-494.9	-494.3	-493.9
AIC	1037.3	1030.3	1022.2	1014.5	1018.3	1017.9	1018.7	1021.8
BIC	1055.2	1051.8	1047.3	1053.9	1064.9	1068.0	1072.5	1082.7
$PseudoR^2$	0.00	0.02	0.04	0.14	0.15	0.16	0.17	0.17

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 5b. Development of adaptive social problem-solving skills as a function of time (in months), moderated by the baseline age of the child at preschool entry with fixed effects for each participating classroom (Models 9-16).

	(9) Adaptive	(10) Adaptive	(11) Adaptive	(12) Adaptive	(13) Adaptive	(14) Adaptive	(15) Adaptive	(16) Adaptive
Months	0.07	0.08	-0.24	-0.33*	-0.34*	-0.34*	-0.34*	-0.34*
	(0.39)	(0.39)	(0.25)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)
Months^2	0.01	0.01	0.03	0.04**	0.04**	0.04**	0.04**	0.04**
	(0.03)	(0.03)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
cBaseAge	0.06	0.04	0.05	0.05	0.05*	0.05*	0.07***	0.08***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.03)	(0.03)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	0.42	0.42	0.45	0.44	0.45	0.41	0.40	0.40
	(0.49)	(0.48)	(0.48)	(0.48)	(0.48)	(0.48)	(0.49)	(0.50)
Class=3	0.13	0.28	0.32	0.32	0.32	0.30	0.12	0.02
	(0.49)	(0.49)	(0.49)	(0.49)	(0.49)	(0.49)	(0.49)	(0.49)
Class=4	0.20	0.22	0.21	0.21	0.22	0.19	0.15	0.23
	(0.48)	(0.47)	(0.47)	(0.47)	(0.47)	(0.47)	(0.48)	(0.49)
Monthsx	0.01	0.00	0.00	0.00				
cBaseAge	(0.02)	(0.02)	(0.02)	(0.02)				
Months^{2}x	-0.00	-0.00	-0.00	-0.00				
cBaseAge	(0.00)	(0.00)	(0.00)	(0.00)				
Male	-0.09	-0.07	-0.28	-0.49	-0.49	-0.53^{+}	-0.49	
	(0.84)	(0.84)	(0.81)	(0.31)	(0.31)	(0.31)	(0.31)	
Black	1.39	1.31	0.28	0.29	0.29			
	(0.92)	(0.92)	(0.34)	(0.34)	(0.34)			
Monthsx	-0.18	-0.21	-0.13					
Male	(0.32)	(0.32)	(0.31)					
Months^{2}x	0.02	0.02	0.02					
Male	(0.03)	(0.03)	(0.03)					
Monthsx	-0.36	-0.37						
Black	(0.34)	(0.34)						
Months^{2}x	0.02	0.02						
Black	(0.03)	(0.03)						
cBase		0.01	0.01	0.01	0.01	0.01^{+}		
PPVT		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)		
constant	2.19*	2.21*	3.09***	3.24***	3.26***	3.53***	3.55***	3.26***
	(1.10)	(1.09)	(0.79)	(0.63)	(0.63)	(0.54)	(0.55)	(0.52)
N	266	266	266	266	266	266	266	266
ll	-493.0	-491.9	-492.8	-493.2	-493.3	-493.7	-495.1	-496.3
AIC	1024.0	1023.7	1021.5	1018.5	1014.7	1013.4	1014.1	1014.5
BIC	1092.1	1095.4	1086.0	1075.8	1064.8	1060.0	1057.1	1053.9
PseudoR^2	0.18	0.19	0.18	0.18	0.18	0.18	0.16	0.14

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

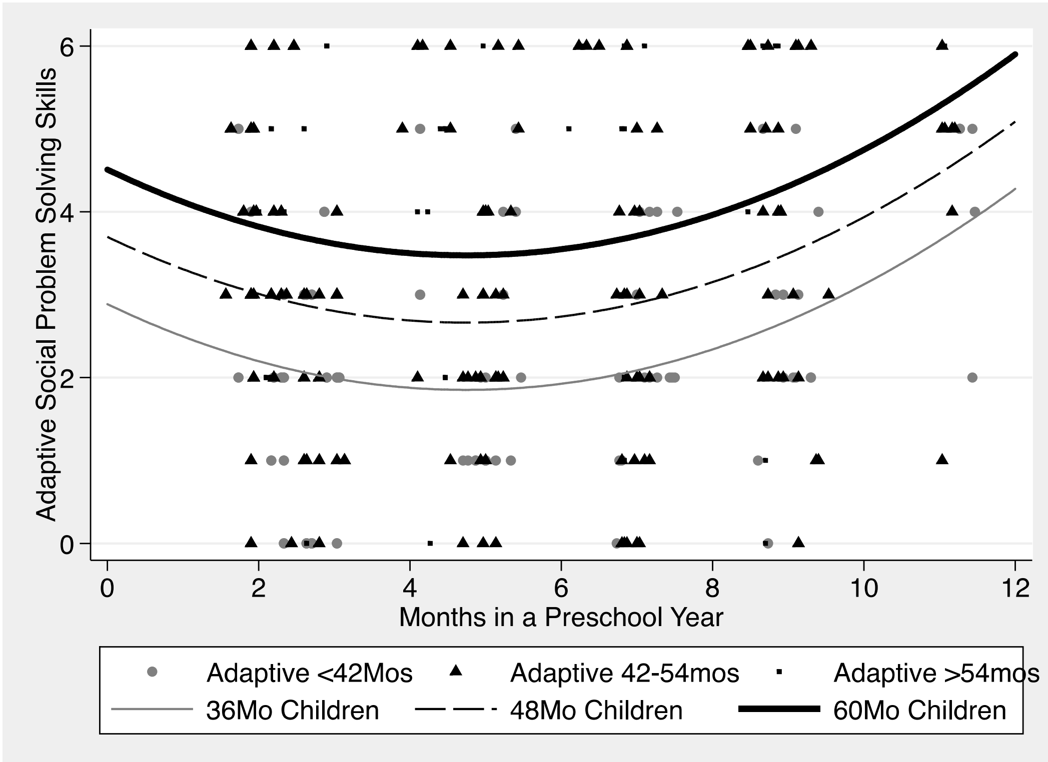


Figure 2. Adaptive skills trajectories controlling for baseline age. This figure shows prototypical trajectories for 36-, 48- and 60-month of children at preschool entry.

RQ2b: How does children's flexibility in social problem-solving skills develop during a preschool year?

Prior to conducting analyses, visual inspections of the data and examination of the basic descriptive statistics revealed that all the necessary assumptions had been met. The distributions of flexibility scores were approximately normally distributed at almost every time period in which data was collected, with somewhat less normal distribution in the fifth and final data collection period when the sample size was substantially smaller.

Flexibility scores, across the sample, stayed fairly constant across all time periods though the standard deviation decreased steadily at each period. This trend indicated that the sample was, on average, becoming more similar on this outcome, towards the end of the preschool year. In fall of the preschool year (November-December), the mean raw score was 1.44 (*SD* = .83) and slightly higher in June (*M* = 1.54, *SD* = .79; see Table 1 for full results). From one time point to the next, flexibility scores were fairly strongly and positively related to one another (ranging from r = 0.48-0.68, p< 0.01 as shown in Table 6, below).

Table 6. Correlations among flexibility in social problem-solving skills across time periods.

	AvgFlex PreK03	AvgFlex PreK06	AvgFlex PreK08	AvgFlex PreK10	AvgFlex PreK12
AvgFlex PreK03	1.00				
AvgFlex PreK06	0.68 (0.00)	1.00			
AvgFlex PreK08	0.66 (0.00)	0.66 (0.00)	1.00		
AvgFlex PreK10	0.62 (0.00)	0.64 (0.00)	0.59 (0.00)	1.00	
AvgFlex PreK12	0.53 (0.04)	0.26 (0.21)	0.53 (0.01)	0.48 (0.01)	1.00

Note: Significance levels in parentheses.

Because this study aimed to better understand way in which children's flexibility in problem-solving changed across a preschool year in mixed-age classrooms, a sequence of models was tested and is presented in Tables 7a and 7b (below).

Because this study aimed to better understand way in which children's flexibility in problem-solving changed across a preschool year in mixed-age classrooms, a sequence of models was tested. Model 1 in Table 7 (below) shows the unconditional means model predicting flexibility scores. Results from this model indicate that there is systematic variation in children's flexibility in social problem-solving skills that warranted further exploration, with both variation in initial status and in the rate of change. Model 2 then tested for the extent to which changes in children's flexibility could be attributed to time (as measured by months in a preschool year). In this unconditional growth model, there was again statistically significant evidence of systematic variation between children's initial status and learning trajectories during the preschool year that could be predicted by the number of months in preschool. In the context of flexibility, it was not specifically theorized that learning trajectories would be non-linear. To be thorough, however, a quadratic term for time was tested in Model 3, though it was not significant ($\beta = .003, SE = .004, p = 0.44$). As a result, it was dropped from subsequent models and a linear term for time was retained in all subsequent models for this outcome.

Models 4-12 examine effects of predictors. In the final model (Model 13 in Table 7b, below), the initial status estimate of flexibility in social problem solving skills (i.e., the intercept) shows that the average-age child with average receptive language skills begins preschool with an average score of 1.36 in flexibility in social problem-solving skills ($\beta_{cons} = 1.36, SE = .16, p < 0.001$) and that the number of months in a preschool year is positively related to learning flexibility in social problem-solving ($\beta_{Months} = .04, SE = .10, p < 0.001$). In this final model, both the initial status ($\beta_{cBaseAge} = .04, SE = .01, p < .001$) and true rates of change are different by baseline age of children ($\beta_{MonthsxcBaseAge} = .005, SE = .002, p < .01$) controlling for receptive language skills.

Because interpretation of the constant in this type of model results in interpretation of skills for children in the reference classroom in this sample, a more representative explanation of the findings for children in the population is presented next. In Figure 3 (below), predicted flexibility in social problem-solving skills trajectories for prototypical children (36- , 48- and 60-months at preschool entry), controlling for baseline receptive language scores, are displayed. The plotted values in the figure have been derived from the estimates of the parameters of Equation [13], which are listed in Table 7b.

Notice two patterns in Figure 3. First, illustrated by the vertical distance between the fitted lines for each age-group, is that children of different ages have differences in flexibility in social problem-solving skills at preschool entry. Because time was not centered, initial math skills have been estimated for each age group, prior to when the data collection period began. Specifically, the figure shows that (on a scale of 0-3) the prototypical 36-month old child would, on average, have flexibility in social problem-solving skills scores of 0.61 ($p < 0.001$) while the prototypical 48-month old child would be estimated to have an average score of 1.29 ($p<0.001$) whereas the prototypical 60-month old child would be estimated to have an average score of 2.12 ($p<0.001$) at preschool entry.

The second pattern evident in the figure is the increase in the size of the age-based gaps in flexibility in social problem-solving skills ($\beta_{MonthsxcBaseAge} = .005, SE = .002, p < .01$) between the predicted flexibility in social problem-solving skills for children of different ages by the end of the preschool year.[6] For example, the predicted end-of-year flexibility score for the prototypical entering 36-month old child would, on average, be 1.5 ($p<0.001$) points while the prototypical entering 48-month old child would be predicted to have an average score of 1.67 ($p<0.001$) whereas for the

[6] This model assumes a 10-month preschool year, although many preschool and Head Start programs offer full-year wrap-around services, which would result in even larger end-of-year gaps.

prototypical entering 60-month old child would be predicted to have an average score of 1.96 ($p<0.001$). Thus, the prototypical entering 36- and 48- month old children would, on average, be expected to gain flexibility in social problem-solving (0.89 and 0.38 points, respectively, on the SPST-R assessment) across a preschool year while the prototypical entering 60-month old child would be predicted to lose 0.44 points in flexibility of social problem-solving during the same time-frame.

In summary, the data show that there is a differential growth pattern by age such that younger children enter preschool with less flexibility in social problem solving skills, on average, and progress through the preschool year acquiring flexibility in social problem-solving skills at faster rates during the preschool year. Their older peers, in contrast, begin the preschool year with greater flexibility, on average, and their learning trajectories indicate less growth—and in some cases, no growth—as they progress through the preschool year.

Table 7a. Development of flexibility in social problem-solving skills as a function of time (in months), moderated by the baseline age of the child at preschool entry, controlling for receptive language of child at preschool entry with a fixed effect for each participating classroom (Models 1-7).

	(1) AvgFlex	(2) AvgFlex	(3) AvgFlex	(4) AvgFlex	(5) AvgFlex	(6) AvgFlex	(7) AvgFlex
Months		0.04^{**}	0.01	0.04^{**}	0.04^{***}	0.03	0.07^{*}
		(0.01)	(0.05)	(0.01)	(0.01)	(0.02)	(0.03)
Months^2			0.00				
			(0.00)				
cBaseAge				0.03^{***}	0.06^{***}	0.06^{***}	0.06^{***}
				(0.01)	(0.01)	(0.01)	(0.01)
Class=1				0.00	0.00	0.00	0.00
				(.)	(.)	(.)	(.)
Class=2				0.05	0.00	0.01	-0.00
				(0.19)	(0.19)	(0.19)	(0.20)
Class=3				-0.27	-0.33^{+}	-0.35^{+}	-0.36^{+}
				(0.19)	(0.19)	(0.19)	(0.20)
Class=4				-0.33^{+}	-0.42^{*}	-0.39^{*}	-0.41^{*}
				(0.19)	(0.20)	(0.20)	(0.20)
Monthsx cBaseAge					-0.00^{**}	-0.00^{**}	-0.00^{**}
					(0.00)	(0.00)	(0.00)
Male						-0.03	0.03
						(0.21)	(0.22)
Monthsx Male						0.02	0.01
						(0.03)	(0.03)
Black							0.28
							(0.24)
Monthsx Black							-0.05
							(0.03)
cBasePPVT							
constant	1.53^{***}	1.20^{***}	1.28^{***}	1.34^{***}	1.39^{***}	1.41^{***}	1.16^{***}
	(0.07)	(0.12)	(0.17)	(0.17)	(0.17)	(0.22)	(0.31)
N	260	260	260	260	260	260	260
ll	-243.4	-238.5	-238.3	-223.9	-219.5	-219.0	-217.7
AIC	496.8	489.0	490.5	467.8	460.9	463.9	465.5
BIC	514.6	510.4	515.5	503.4	500.1	510.2	518.9
PseudoR^2	0.00	0.01	0.01	0.30	0.31	0.31	0.31

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 7b. Development of flexibility in social problem-solving skills as a function of time (in months), moderated by the baseline age of the child at preschool entry, controlling for receptive language of child at preschool entry with a fixed effect for each participating classroom (Models 8-13).

	(8) AvgFlex	(9) AvgFlex	(10) AvgFlex	(11) AvgFlex	(12) AvgFlex	(13) AvgFlex
Months	0.07^{*}	0.08^{*}	0.08^{**}	0.04^{**}	0.04^{**}	0.04^{**}
	(0.03)	(0.03)	(0.02)	(0.01)	(0.01)	(0.01)
Months^2						
cBaseAge	0.06^{***}	0.04^{**}	0.04^{**}	0.04^{***}	0.05^{***}	0.04^{***}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-0.00	-0.02	-0.02	-0.01	-0.00	-0.01
	(0.20)	(0.18)	(0.18)	(0.18)	(0.18)	(0.18)
Class=3	-0.36^{+}	-0.24	-0.24	-0.23	-0.22	-0.21
	(0.20)	(0.18)	(0.18)	(0.18)	(0.18)	(0.18)
Class=4	-0.41^{*}	-0.40^{*}	-0.40^{*}	-0.38^{*}	-0.39^{*}	-0.40^{*}
	(0.20)	(0.18)	(0.18)	(0.18)	(0.18)	(0.18)
Monthsx cBaseAge	-0.00^{**} (0.00)	-0.00^{**} (0.00)	-0.00^{**} (0.00)	-0.00^{**} (0.00)	-0.00^{**} (0.00)	-0.00^{**} (0.00)
Male	0.03	0.02	0.01	0.03	0.05	
	(0.22)	(0.21)	(0.11)	(0.11)	(0.11)	
Monthsx Male	0.01 (0.03)	-0.00 (0.03)				
Black	0.28	0.24	0.23	-0.13		
	(0.24)	(0.23)	(0.23)	(0.12)		
Monthsx Black	-0.05 (0.03)	-0.05^{+} (0.03)	-0.05^{+} (0.03)			
cBasePPVT		0.01^{***}	0.01^{***}	0.01^{***}	0.01^{***}	0.01^{***}
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
constant	1.16^{***}	1.19^{***}	1.19^{***}	1.45^{***}	1.33^{***}	1.36^{***}
	(0.31)	(0.29)	(0.25)	(0.21)	(0.17)	(0.16)
N	260	260	260	260	260	260
ll	-217.7	-210.4	-210.4	-212.1	-212.7	-212.7
AIC	465.5	452.8	450.8	452.2	451.3	449.5
BIC	518.9	509.7	504.2	502.1	497.6	492.2
PseudoR^2	0.32	0.39	0.39	0.38	0.38	0.38

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

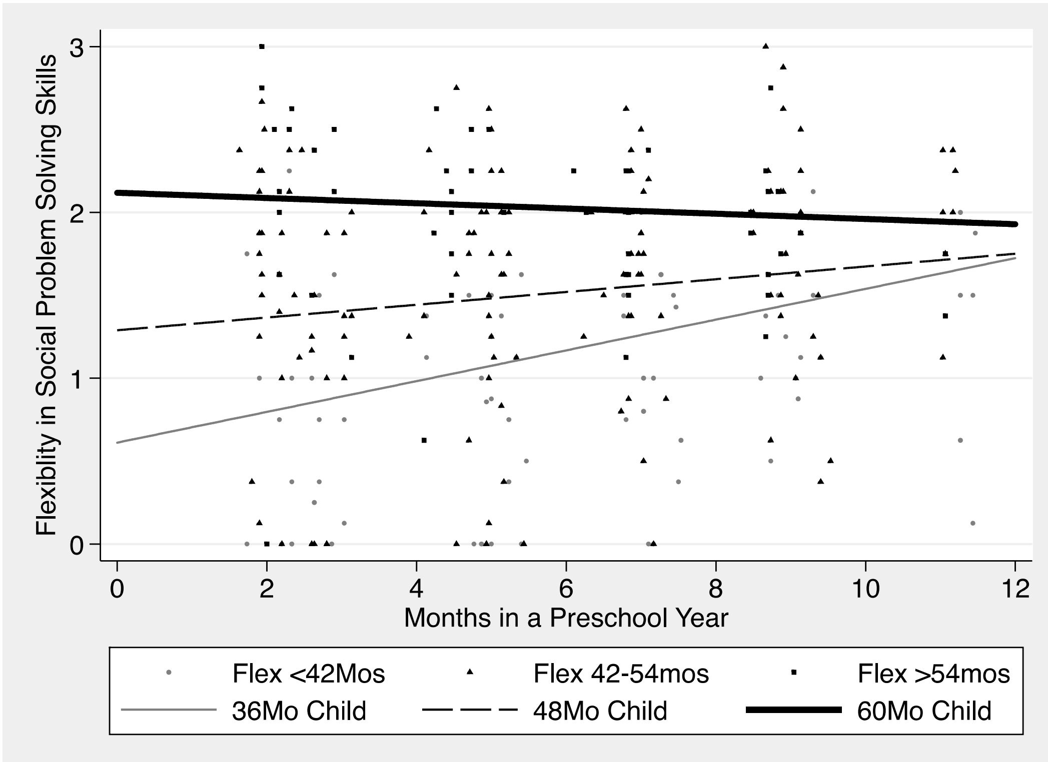


Figure 3. Moderating role of baseline age on development of flexibility in social problem-solving trajectories, controlling for baseline receptive language.

RQ2c: How do children's social problem-solving skills develop over the course of a preschool year (as captured by the Social Problem-Solving Skills Test-Revised)?

Object Acquisition

Qualitative review of the data and examination of the basic descriptive statistics revealed that children had very high proportions of adaptive problem-solving skills for object acquisition on the Social Problem Solving Skills Test-Revised at every time point. As indicated in Figure 4, below, children's aggregate reported problem-solving strategies remained relatively stable across the preschool year. In general, there are trends of non-responses decreasing, which would indicate children are generally better able to provide problem-solving strategies for Object Acquisition tasks as time in preschool progresses. In April of the preschool year (Wave 3), for some reason, the proportion of aggressive strategies increases—and the proportion of adaptive strategies decreases—though this is not a trend that is maintained. Interestingly, the same pattern of decreased adaptive strategies in April of the preschool year is noted in the Challenging Situations Task, suggesting that this is a true change in children's reported skills and not simply measurement error (or a metric effect).

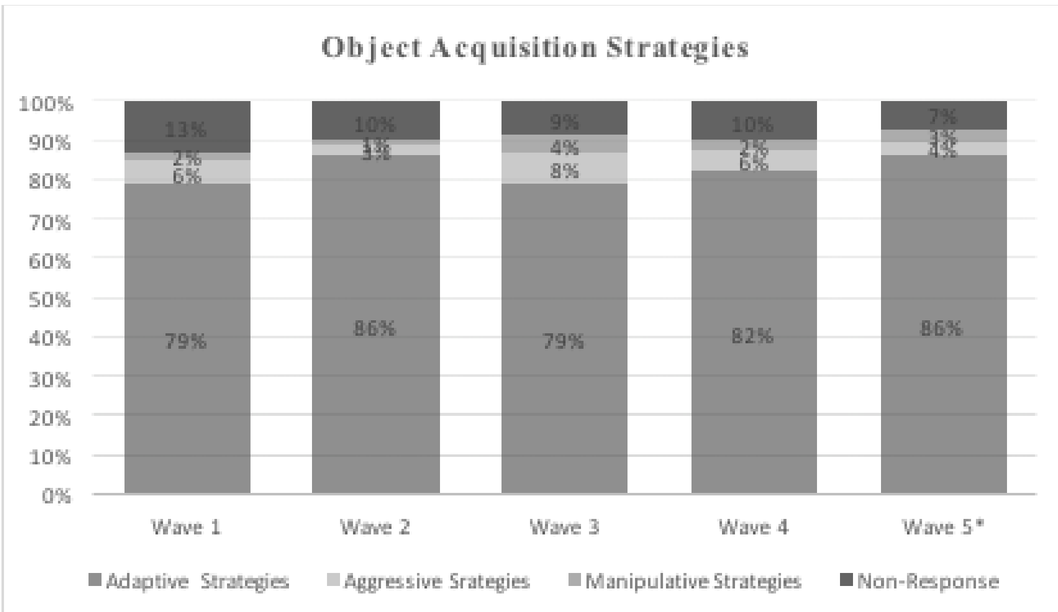


Figure 4. Proportion of Object Acquisition social problem-solving strategies used across a preschool year.

Friendship Initiation

As with the Object Acquisition tasks, children's aggregate reported problem-solving strategies for Friendship Initiation remained quite constant across the preschool year (note exception for Wave 5, discussed below) though there were some notable differences across time periods. Wave 5 is quite different for two reasons. First, it is comprised of a much smaller number of children (N=34) than the other data collection periods. Second, the composition of this smaller group may be different from the larger group in unobserved ways. For both of these reasons, the changes between time periods cannot directly be made from Wave 1-4 and Wave 5. For these reasons, it is excluded from the general discussion, which will focus on Waves 1-4.

In general (for Waves 1-4), Friendship Initiation was a much more difficult task for the children than the Object Acquisition tasks, as indicated by the high non-response rates at every data collection period (ranging from 37-41%). There is evidence of growth in Indirect Initiation skills (e.g., initiating physical proximity to the new child, asking the new child if he/she will be his/her friend) and Conversation Openers (e.g., saying "hi" and other similar statements). There is also evidence of decreases in Adult Intervention (e.g., asking an adult to initiate the friendship on behalf of the child). While this is not inherently an undesired strategy, seeing a decrease in this type of strategy—while seeing increases in other-child initiated strategies—suggests that these children are learning ways in which they can, themselves, initiate friendships—without having to rely on adults—as the preschool year progresses. Likewise, there is a similar pattern with Prosocial and Complimentary strategies. While this may seem problematic at first glance, these strategies are less mature—and often insufficient—if solely employed in this context. For example, if a child merely says, "I like your shoes" to a new child, this is insufficient to initiate a friendship though it is complimentary and may be well-meaning by the child to initiate good will. As the child learns more effective strategies for initiating friendship (e.g., saying "Do you want to be my friend?" which is an Indirect Initiation strategy), he/she may omit the fact that she likes the new child's shoes when trying to initiate friendship, especially if she has learned that this is not an effective strategy in this context. Therefore, we may see a decrease in these types of strategies *for* *this purpose* even though children may be becoming more socially competent (i.e., better understanding the context and purpose of various social interactions).

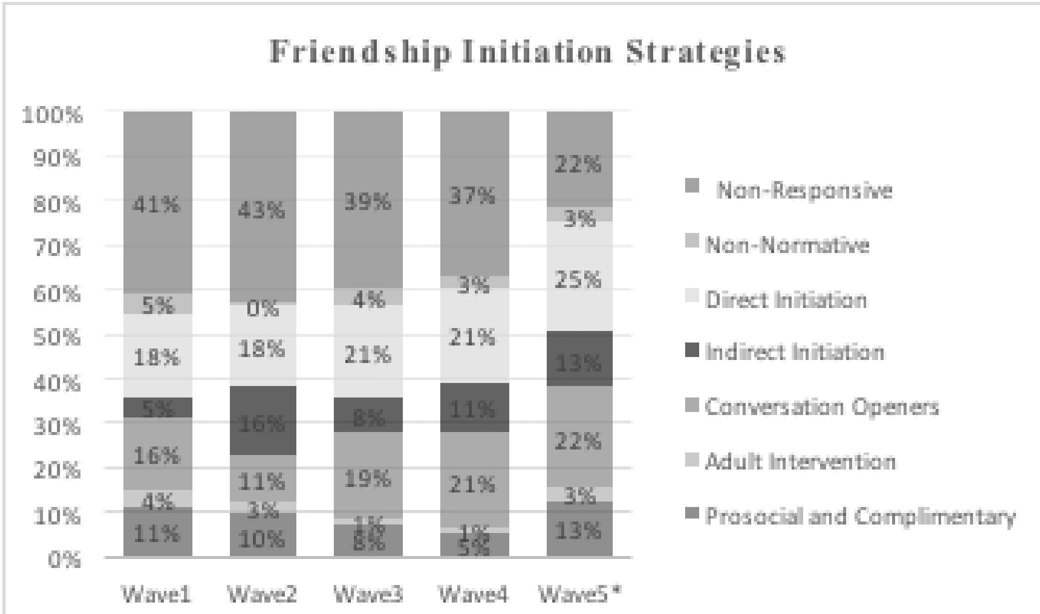


Figure 5. Proportion of Friendship Initiation social problem-solving strategies used across a preschool year.

Cross-Domain Relationships

Adaptive Social Problem-Solving and Early Math Skills

Overall, there was a general trend, across the data collection time periods, of moderate positive correlations between children's adaptive problem-solving skills and early math skills (r = 0.31-0.38, $p<0.10$) with the exception of one time period (April) in the middle of the preschool year. During April, for some reason, there was no significant correlation between these two skills (r = 0.10, $p=0.47$), which will be discussed further in the Discussion section. More importantly, however, each previous time period of adaptive skills is also moderately and positively correlated with early math skills (r = 0.29-.41, $p<0.05$), suggesting that there is a predictive relationship across domains, again, however, with the exception of April of this preschool year (see Table 8 for all correlations, below). It should be noted that the same lag time-period correlations are not present in the reverse direction (i.e., from Math to Adaptive Skills) in the same magnitude, nor in significance, suggesting that the relationship is indeed time-sequenced.

Table 8. Cross-domain and cross-time correlations among children's math and adaptive social problem solving skills during the preschool year.

	Adapt PreK03	Math PreK03	Adapt PreK06	Math PreK06	Adapt PreK08	Math PreK08	Adapt PreK10	Math PreK10	Adapt PreK12	Math PreK12
Adapt PreK03	1.00									
Math PreK03	0.38 (0.00)	1.00								
Adapt PreK06	0.23 (0.11)	0.25 (0.09)	1.00							
Math PreK06	0.41 (0.00)	0.89 (0.00)	0.34 (0.01)	1.00						
Adapt PreK08	0.23 (0.14)	0.12 (0.45)	0.48 (0.00)	0.08 (0.56)	1.00					
Math PreK08	0.28 (0.07)	0.85 (0.00)	0.29 (0.04)	0.95 (0.00)	0.10 (0.47)	1.00				
Adapt PreK10	0.43 (0.00)	0.22 (0.18)	0.62 (0.00)	0.31 (0.03)	0.49 (0.00)	0.24 (0.09)	1.00			
Math PreK10	0.39 (0.01)	0.86 (0.00)	0.31 (0.02)	0.93 (0.00)	0.13 (0.35)	0.92 (0.00)	0.32 (0.01)	1.00		
Adapt PreK12	0.31 (0.26)	0.44 (0.13)	0.40 (0.06)	0.27 (0.27)	0.33 (0.13)	0.14 (0.55)	0.47 (0.01)	0.35 (0.05)	1.00	
Math PreK12	0.08 (0.79)	0.84 (0.00)	0.56 (0.01)	0.82 (0.00)	0.02 (0.91)	0.84 (0.00)	0.40 (0.03)	0.91 (0.00)	0.31 (0.09)	1.00

Note: Significance levels in parentheses

Flexibility in Social Problem-Solving and Early Math Skills

As with adaptive skills, there was a similar general trend, across the data collection time periods for flexibility in social problem-solving. However, the observed relationships were even stronger with moderate to strong positive correlations between children's flexibility in problem-solving skills and early math skills (r = 0.36-.55, $p<0.05$). In this case, there was no exception for the time period of April. More importantly, however, children's flexibility in problem-solving skills at each previous time period is also moderately and positively correlated with early math skills (r = 0.41-.63, p= 0.03-p<.001), suggesting that there is a predictive relationship across domains (see Table 9 for all correlations, below). While there is also a predictive relationship in reverse (i.e., from math to flexibility over time), the relationship is less strong and is not significant at all time points (ranging from r = 0.31-.58, p=0.11 to p<.001).

Table 9. Cross-domain and cross-time correlations among children's math and flexibility in social problem solving skills during the preschool year.

	Math PreK03	AvgFlex PreK03	Math PreK06	AvgFlex PreK06	Math PreK08	AvgFlex PreK08	Math PreK10	AvgFlex PreK10	Math PreK12	AvgFlex PreK12
Math PreK03	1.00									
AvgFlex PreK03	0.55 (0.00)	1.00								
Math PreK06	0.89 (0.00)	0.63 (0.00)	1.00							
AvgFlex PreK06	0.58 (0.00)	0.70 (0.00)	0.49 (0.00)	1.00						
Math PreK08	0.85 (0.00)	0.58 (0.00)	0.95 (0.00)	0.47 (0.00)	1.00					
AvgFlex PreK08	0.48 (0.00)	0.64 (0.00)	0.56 (0.00)	0.63 (0.00)	0.51 (0.00)	1.00				
Math PreK10	0.86 (0.00)	0.59 (0.00)	0.93 (0.00)	0.46 (0.00)	0.92 (0.00)	0.55 (0.00)	1.00			
AvgFlex PreK10	0.37 (0.02)	0.57 (0.00)	0.48 (0.00)	0.63 (0.00)	0.41 (0.00)	0.59 (0.00)	0.47 (0.00)	1.00		
Math PreK12	0.84 (0.00)	0.56 (0.04)	0.82 (0.00)	0.42 (0.05)	0.84 (0.00)	0.71 (0.00)	0.91 (0.00)	0.41 (0.03)	1.00	
AvgFlex PreK12	0.35 (0.24)	0.33 (0.25)	0.14 (0.58)	0.45 (0.04)	0.20 (0.39)	0.30 (0.18)	0.31 (0.11)	0.46 (0.01)	0.36 (0.05)	1.00

Note: Significance levels in parentheses

RQ3a: Does children's development of flexibility in social problem-solving moderate their math trajectories?

This study aimed to better understand whether and how children's acquisition of flexibility in social problem-solving skills was related to their early math skills across a preschool year. Because these children were in mixed-age preschool classrooms serving children from low-income families—without a controlled intervention—a sequence of models was tested examining the role of flexibility and its relation with other predictors, and a full taxonomy is presented in Tables 10a and 10b (below). Note that these models build on earlier models presented in this analysis and, specifically, that Model 1 in Table 9a is the same as Model 12 in Table 2 presented earlier in the longitudinal math analysis (i.e., RQ1).

In the final model (Model 12 in Table 10b, below), the intercept (i.e., estimated initial math status for children in the reference classroom) indicates that the average-age female with average receptive language skills and no flexibility in social problem-solving would begin preschool with almost 7 points on the TEMA assessment ($\beta_{cons} = 6.91, SE = 1.21, p < 0.001$). Unlike in the math-only model (RQ1), the true rates of change are *not* different by baseline age of children; instead, they differ by extent to which children develop flexibility in problem-solving strategies during the preschool year ($\beta_{MonthsxAvgFlex} = .20, SE = .07, p < 0.05$). This finding indicates that children's development of flexibility in social problem-solving is associated with differences in the rates at child children learn math skills across the preschool year, when controlling for baseline receptive language, age and gender as evidenced in Figure 5 (below).

Specifically, children who develop greater flexibility in social problem-solving have steeper learning trajectories in math than their peers who develop these skills less readily during the preschool year. Figure 5a provides a helpful heuristic for modeling this relationship, though it is limited in its representation since it assumes that children will enter with some level of flexibility in social problem-solving (i.e., either high or low) which will remain constant over time. However, the relationship being modeled accounts for the ways in which children continue developing this skill as they interact with others in classroom settings during a preschool year. Thus, Figure 5b illustrates that while younger children have, on average, lower math skills at preschool entry than their older peers, their math learning trajectories are more parallel than in the math-only models that did not account for children's social competence. This figure also illustrates the ways in which math learning trajectories become steeper for children who develop more flexibility in social problem-solving during the preschool year compared to their same-aged peers who develop less flexibility in social problem-solving.

Table 10a. Cross-domain relationship between flexibility in social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 1-6).

	(1) MathRaw	(2) MathRaw	(3) MathRaw	(4) MathRaw	(5) MathRaw	(6) MathRaw
Months	0.72^{***}	0.73^{***}	0.45^{***}	0.45^{***}	0.52^{***}	0.58^{**}
	(0.06)	(0.06)	(0.13)	(0.13)	(0.15)	(0.20)
cBaseAge	0.13^{*}	0.11^{+}	0.15^{*}	0.14^{*}	0.15^{*}	0.15^{*}
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
Monthsx cBaseAge	0.02^{+}	0.02^{*}	0.01	0.01	0.01	0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	-1.51^{*}	-1.70^{*}	-1.74^{*}	-1.80^{*}	-1.35	-1.30
	(0.75)	(0.74)	(0.74)	(0.75)	(0.89)	(0.89)
cBasePPVT	0.08^{***}	0.08^{***}	0.08^{***}	0.09^{***}	0.09^{***}	0.09^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-0.97	-0.95	-1.01	-1.05	-1.06	-1.07
	(1.18)	(1.16)	(1.17)	(1.17)	(1.17)	(1.17)
Class=3	-0.88	-0.78	-0.89	-0.92	-0.92	-0.93
	(1.19)	(1.17)	(1.18)	(1.18)	(1.18)	(1.18)
Class=4	-2.09^{+}	-2.25^{+}	-2.51^{*}	-2.53^{*}	-2.53^{*}	-2.55^{*}
	(1.15)	(1.15)	(1.17)	(1.17)	(1.17)	(1.17)
AvgFlex		0.23	-0.72	-0.73	-0.71	-0.72
		(0.25)	(0.47)	(0.47)	(0.47)	(0.47)
Monthsx AvgFlex			0.18^{*}	0.18^{*}	0.18^{*}	0.17^{*}
			(0.07)	(0.07)	(0.07)	(0.07)
Black				-0.50	-0.51	-0.27
				(0.83)	(0.83)	(1.01)
Monthsx Male					-0.12	-0.13
					(0.13)	(0.13)
Monthsx Black						-0.06
						(0.14)
constant	5.31^{***}	4.99^{***}	6.56^{***}	7.02^{***}	6.74^{***}	6.54^{***}
	(1.01)	(1.07)	(1.26)	(1.47)	(1.50)	(1.57)
N	257	250	250	250	250	250
ll	-586.3	-570.3	-567.6	-567.4	-567.0	-566.9
AIC	1198.6	1168.7	1165.2	1166.8	1167.9	1169.7
BIC	1244.8	1218.0	1218.0	1223.2	1227.8	1233.1
PseudoR^2	0.60	0.61	0.62	0.62	0.62	0.62

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 10b. Cross-domain relationship between flexibility in social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 7-12).

	(7) MathRaw	(8) MathRaw	(9) MathRaw	(10) MathRaw	(11) MathRaw	(12) MathRaw
Months	0.58^{**}	0.47^{**}	0.45^{***}	0.45^{***}	0.45^{***}	0.41^{***}
	(0.20)	(0.17)	(0.13)	(0.13)	(0.13)	(0.12)
cBaseAge	0.14^{*}	0.14^{*}	0.14^{*}	0.14^{*}	0.15^{*}	0.18^{**}
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)
Monthsx cBaseAge	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	
Male	1.06	0.51	0.51	-1.80^{*}	-1.74^{*}	-1.74^{*}
	(1.79)	(1.71)	(1.71)	(0.75)	(0.74)	(0.74)
cBasePPVT	0.09^{***}	0.09^{***}	0.09^{***}	0.09^{***}	0.08^{***}	0.09^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-0.84	-0.82	-0.81	-1.05	-1.01	-1.05
	(1.16)	(1.16)	(1.16)	(1.17)	(1.17)	(1.17)
Class=3	-0.62	-0.62	-0.61	-0.92	-0.89	-0.96
	(1.18)	(1.18)	(1.18)	(1.18)	(1.18)	(1.18)
Class=4	-2.23^{+}	-2.22^{+}	-2.21^{+}	-2.53^{*}	-2.51^{*}	-2.62^{*}
	(1.17)	(1.17)	(1.17)	(1.17)	(1.17)	(1.16)
AvgFlex	-0.69	-0.71	-0.70	-0.73	-0.72	-0.92^{*}
	(0.47)	(0.47)	(0.47)	(0.47)	(0.47)	(0.42)
Monthsx AvgFlex	0.17^{*} (0.07)	0.18^{*} (0.07)	0.18^{*} (0.07)	0.18^{*} (0.07)	0.18^{*} (0.07)	0.20^{**} (0.07)
Black	1.82	1.65	1.57	-0.50		
	(1.71)	(1.70)	(1.61)	(0.83)		
Monthsx Male	-0.14 (0.13)					
Monthsx Black	-0.06 (0.14)	-0.02 (0.14)				
Blackx Male	-2.85 (1.87)	-2.82 (1.87)	-2.82 (1.87)			
constant	4.47^{*}	4.94^{*}	4.99^{*}	7.02^{***}	6.56^{***}	6.91^{***}
	(2.08)	(2.03)	(2.00)	(1.47)	(1.26)	(1.21)
N	250	250	250	250	250	250
ll	-565.7	-566.3	-566.3	-567.4	-567.6	-568.0
AIC	1169.5	1168.6	1166.6	1166.8	1165.2	1164.0
BIC	1236.4	1232.0	1226.5	1223.2	1218.0	1213.3
PseudoR^2	0.62	0.63	0.63	0.63	0.62	0.62

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

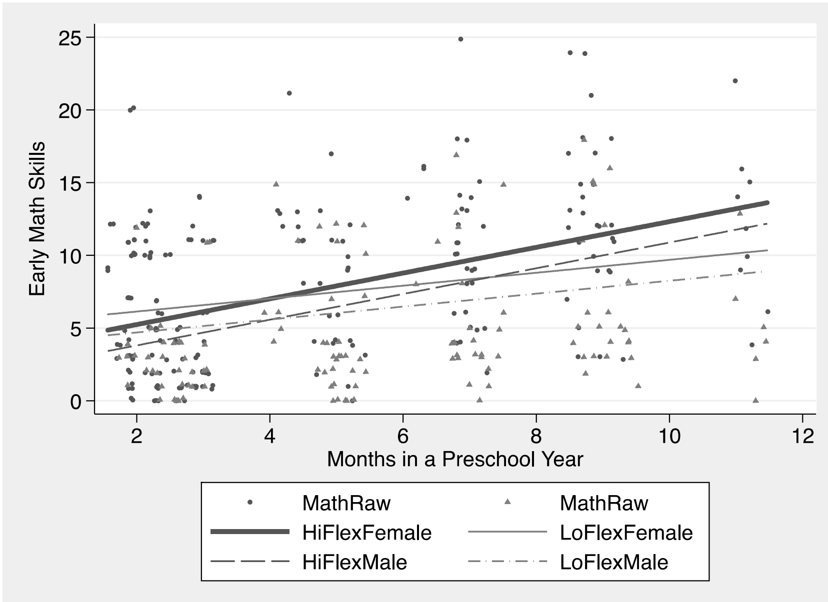


Figure 5a. Moderating role of development of flexibility in social problem-solving skills on early math skills trajectories, controlling for baseline receptive language, age and gender. Figure shows prototypical math trajectories for children with baseline flexibility in problem-solving at the 10th (Low) and 90th (High) percentiles at preschool entry; figure illustrates that males and females have similar trends over time, though males begin and end the preschool year with lower math skills than their female peers even when accounting for differences in flexibility in social problem-solving.

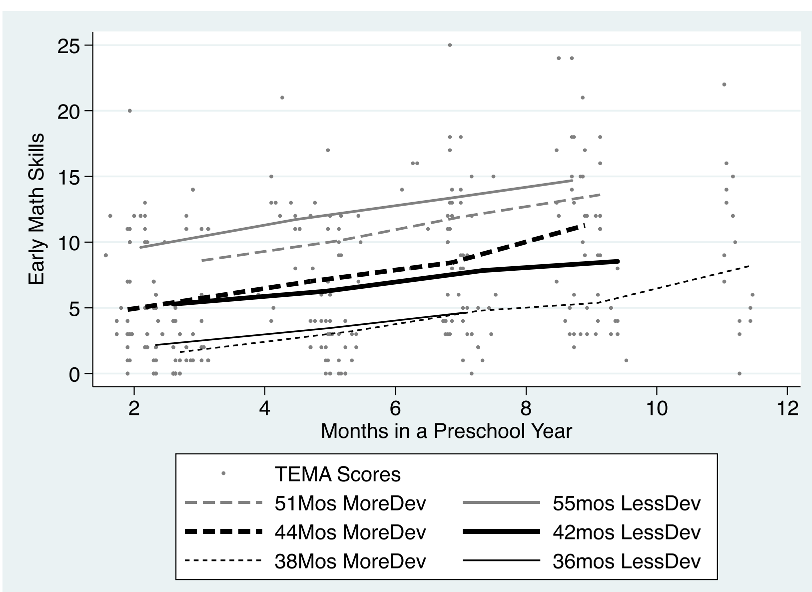


Figure 5b. Since children continue to develop flexibility in social problem-solving skills during the preschool year, this figure illustrates the predicted math learning trajectories of six selected children from the sample.

RQ3b: Does children's development of adaptive social problem-solving moderate their math trajectories?

Because children's adaptive social problem-solving skills are time-varying predictors and are internal states that are potentially changeable over time (i.e., a desired attribute) that are of material interest in this study as a predictor of children's early math skills, questions of reciprocal causation remain an issue in this cross-sectional observational dataset. While no causal claims can be made about these findings, an attempt was made to better ascertain the directional nature of this relationship. As such the following models utilize the adaptive skills that were measured in the data collection period temporally (i.e., on average, about 60 days) prior to the predicted math scores in these models in an effort to better establish a time-ordered sequence. Specifically, these models link each child's prior status on their adaptive social problem solving skills (i.e., the predictor) with their current status on their early math skills (i.e., the outcome). While this does not establish causality, it can help to better demonstrate a temporal ordering that would be necessary in establishing causality (Singer & Willet, 2003) given the hypothesized direction of this relationship and the nature of this exploratory study.

Prior to conducting analyses, the bivariate relationship between TEMA math scores and CST adaptive scores were examined, which showed curvature, violating the assumption of linearity. Given the curvilinear relationship that had been observed for adaptive scores and the linear relationship in growth in early math skills over time, a quadratic term for adaptive skills was entered into these models (Adaptive and $Adaptive^2$). A full taxonomy of models is presented in Table 11a and Table 11b (below).

Note that these models build on earlier models presented in this analysis and, specifically, that Model 1 in Table 11a is the same as Model 12 in Table 2 presented earlier in the longitudinal math analysis (i.e., RQ1).

Because this study is theory-building, the results of this final model include the theoretically motivated three-way interaction term between age, Lag Adaptive skills and time (p =.12), so that its theoretical importance can be discussed given the relatively small sample size.[7] In this final model (Model 12 in Table 11b, below), the intercept provides an estimate of children's early math skills at preschool entry quite consistent with those from other cross-domain models in this study, suggesting that the average-age female with average receptive language skills and no adaptive social problem-solving skills would begin preschool with about 6 points on the TEMA assessment ($\beta_{cons} = 6.00, SE = 2.01, p < 0.01$). And, consistent with the flexibility in social problem-solving models, these findings also indicate there is a difference in the rates at which younger and older children learn math skills across a preschool year that differ as a function of the rate at which their adaptive social problem-solving skills develop, when controlling for gender, the participating classroom and baseline receptive language at preschool entry ($\beta = -.002, SE = .001, p = 0.118$) as evidenced in Figure 6a and 6b (below).

While the results from RQ2a showed that, on average, children developed adaptive social problem-solving skills—regardless of age—on a trajectory with a positive curvilinear trend across the preschool year, there was considerable variation between and

[7] This p-value was p=.09 in the original set of models without a fixed effect for each classroom.

within children in this sample. In addition, older children tended to begin the preschool year with more adaptive skills than younger children, a trend that was also associated with higher math skills at preschool entry (see Model 8 in Table 13, presented in more detail in the Sensitivity Analysis). Given this, a three-way interaction term was entered into the lag-adaptive math cross domain models (MonthsxLagAdaptivexcBaseAge and MonthsxLagAdaptive^{2}xcBaseAge) to better understand the predictive relationship between adaptive social problem-solving skills and early math skills. Results show that, in this sample, among younger children, more growth in adaptive social problem-solving skills (from the previous time period) predicts more growth in early math skills compared to children who developed fewer of these adaptive social problem-solving skills during the preschool year. In contrast, older children who develop adaptive social problem-solving at faster rates have more similar (i.e., parallel) learning trajectories compared to their same-aged peers who develop adaptive social problem-solving at slower rates. However, it should be noted that, in this sample, many of the older children who showed such faster rates of adaptive social problem-solving began preschool with *very* high math skills, that this may account for some of the differences noted between age groups (which will be discussed in more detail the Discussion section). Specifically, these children already had such high math scores at preschool entry—which correlated with high adaptive social problem-solving skills—that while these older children were consistently predicted to have higher math scores than their same-aged peers who developed adaptive social problem-solving at slower rates, they did not have the same predicted gains in math compared to their younger peers who developed adaptive social problem-solving at similar rates across the preschool year.

Figures 6a and 6b shows predicted math trajectories for seven selected children from the sample to better illustrate these trends. Specifically, Figure 6a shows that for older children (see right panel in Figure 6a), predicted math trajectories are more similar (i.e., largely parallel). For children who are about 4 years-old at preschool entry (see middle panel in Figure 6a), we see somewhat different trends. Both solid lines show quite different math skills at preschool entry and represent children with low growth in adaptive skills. When compared to a 45 month-old child (at baseline, the dashed line in the middle panel) who began the preschool year with math skills that were in between these two children, Figure 6a shows that this child has a much steeper predicted math trajectory than either of the children of the same approximate age with low growth in adaptive skills. When looking at the youngest children in this sample (i.e., depicted in the left panel in Figure 6a) the trend is most striking; for the child with considerable growth in adaptive skills (as illustrated by the dashed line), his/her predicted math trajectory is quite steep as the year progresses while the child with limited growth in adaptive social problem-solving skills has a predicted math trajectory that levels off and starts to converge with the other child's despite having higher math skills at preschool entry.

While not significant at traditional levels, this finding is thought to have particular importance since using a lagged variable results in dropping an entire time period of observations (i.e., the final time period) in order to establish the time-sequence of events. In a sample this small, the power required to demonstrate this relationship may have been sacrificed, potentially increasing the likelihood of a Type II error. Specifically, in the RQ1 models, there were 75 children with 253 observations (mean number of waves per child is 3.4); yet in this set of models, using lagged variables, there are only 69 children with 182 observations (mean number of waves per child is 2.6). In an exploratory study such as this one, these results suggest a cross-domain relationship strong enough to justify testing the potential of social problem-solving in promoting children's early math skills.

Table 11a. Cross-domain relationship between (lagged) adaptive social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 1-6).

	(1) MathRaw	(2) MathRaw	(3) MathRaw	(4) MathRaw	(5) MathRaw	(6) MathRaw
Months	0.72***	0.68***	0.63**	0.67**	0.79**	0.79**
	(0.06)	(0.08)	(0.21)	(0.22)	(0.26)	(0.29)
cBaseAge	0.13*	0.15^{+}	0.09	0.08	0.08	0.08
	(0.07)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
Monthsx cBaseAge	0.02^{+}	0.00	0.01	0.01	0.01	0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	-1.51*	-1.68*	-1.62*	-1.83*	-1.23	-1.24
	(0.75)	(0.81)	(0.78)	(0.79)	(1.10)	(1.14)
cBasePPVT	0.08***	0.10***	0.10***	0.10***	0.10***	0.10***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-0.97	-1.20	-1.51	-1.70	-1.71	-1.71
	(1.18)	(1.25)	(1.21)	(1.21)	(1.20)	(1.20)
Class=3	-0.88	-0.41	-0.61	-0.70	-0.69	-0.68
	(1.19)	(1.29)	(1.25)	(1.24)	(1.24)	(1.24)
Class=4	-2.09^{+}	-2.65*	-2.80*	-2.89*	-2.89*	-2.89*
	(1.15)	(1.25)	(1.21)	(1.20)	(1.19)	(1.19)
LagAdaptive		-0.47	-1.71	-1.49	-1.28	-1.27
		(0.30)	(1.11)	(1.12)	(1.15)	(1.19)
LagAdaptive^2		0.10*	0.38*	0.35*	0.32^{+}	0.32^{+}
		(0.05)	(0.17)	(0.18)	(0.18)	(0.18)
Monthsx LagAdaptive			0.18	0.16	0.12	0.12
			(0.16)	(0.16)	(0.16)	(0.17)
Monthsx LagAdaptive^2			-0.04^{+}	-0.04	-0.03	-0.03
			(0.02)	(0.02)	(0.03)	(0.03)
Black				-1.12	-1.10	-1.12
				(0.89)	(0.89)	(1.27)
Monthsx Male					-0.13	-0.13
					(0.16)	(0.17)
Monthsx Black						0.00
						(0.19)
AvgFlex						
Monthsx Lag daptivex cBaseAge						
Monthsx LagAdaptive^{2}xcBaseAge						
constant	5.31***	5.94***	6.37***	7.12***	6.46**	6.47**
	(1.01)	(1.18)	(1.89)	(1.97)	(2.13)	(2.17)
N	257	182	182	182	182	182
ll	-586.3	-423.2	-420.9	-420.1	-419.8	-419.8
AIC	1198.6	876.3	875.7	876.2	877.6	879.6
BIC	1244.8	924.4	930.2	933.9	938.4	943.6
PseudoR^2	0.60	0.62	0.61	0.62	0.62	0.62

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 11b. Cross-domain relationship between (lagged) adaptive social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 7-12).

	(7) MathRaw	(8) MathRaw	(9) MathRaw	(10) MathRaw	(11) MathRaw	(12) MathRaw
Months	0.75^{*}	0.94^{**}	0.76^{**}	0.78^{**}	0.75^{**}	0.70^{**}
	(0.29)	(0.31)	(0.25)	(0.24)	(0.24)	(0.23)
cBaseAge	0.08	0.05	0.04	0.04	0.05	0.07
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
Monthsx cBaseAge	0.01	0.02	0.02^{+}	0.02^{+}	0.02^{+}	0.02^{+}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	-1.21	-1.02	-1.84^{*}	-1.83^{*}	-1.91^{*}	-1.69^{*}
	(1.15)	(1.14)	(0.81)	(0.81)	(0.80)	(0.78)
cBasePPVT	0.10^{***}	0.10^{***}	0.10^{***}	0.10^{***}	0.10^{***}	0.10^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-1.58	-1.49	-1.46	-1.47	-1.60	-1.42
	(1.22)	(1.22)	(1.23)	(1.22)	(1.21)	(1.21)
Class=3	-0.54	-0.33	-0.34	-0.36	-0.52	-0.43
	(1.25)	(1.26)	(1.27)	(1.26)	(1.25)	(1.26)
Class=4	-2.35^{+}	-2.23^{+}	-2.21^{+}	-2.22^{+}	-2.76^{*}	-2.68^{*}
	(1.24)	(1.25)	(1.25)	(1.25)	(1.20)	(1.21)
LagAdaptive	-1.09	-0.84	-0.96	-1.10	-1.34	-1.60
	(1.19)	(1.24)	(1.23)	(1.16)	(1.16)	(1.14)
LagAdaptive^2	0.28	0.26	0.28	0.30^{+}	0.34^{+}	0.37^{*}
	(0.18)	(0.19)	(0.19)	(0.18)	(0.18)	(0.17)
Monthsx LagAdaptive	0.10	0.03	0.05	0.07	0.11	0.14
	(0.17)	(0.17)	(0.17)	(0.16)	(0.16)	(0.16)
Monthsx LagAdaptive^2	-0.03	-0.02	-0.02	-0.02	-0.03	-0.03
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
Black	-1.21	-0.98	-1.34	-1.04	-1.13	
	(1.28)	(1.28)	(1.23)	(0.91)	(0.90)	
Monthsx Male	-0.11	-0.17				
	(0.17)	(0.17)				
Monthsx Black	0.04	-0.01	0.06			
	(0.19)	(0.18)	(0.17)			
AvgFlex	0.39	0.35	0.37	0.36		
	(0.31)	(0.31)	(0.31)	(0.30)		
Monthsx LagAdaptivex cBaseAge		0.01	0.01	0.01	0.01	0.01
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
MonthsxLag Adaptive^2x cBaseAge		-0.005^{+}	-0.005^{+}	-0.005^{+}	-0.005	-0.005
		(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
constant	5.80^{**}	4.87^{*}	5.73^{**}	5.74^{**}	6.70^{**}	6.00^{**}
	(2.21)	(2.29)	(2.14)	(2.14)	(2.08)	(2.01)
N	179	179	179	179	182	182
ll	-412.7	-409.1	-409.7	-409.7	-417.1	-417.9
AIC	867.4	864.3	863.3	861.5	874.2	873.8
BIC	934.3	937.6	933.5	928.4	938.3	934.7
PseudoR^2	0.62	0.63	0.63	0.63	0.63	0.62

Standard errors in parentheses

$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

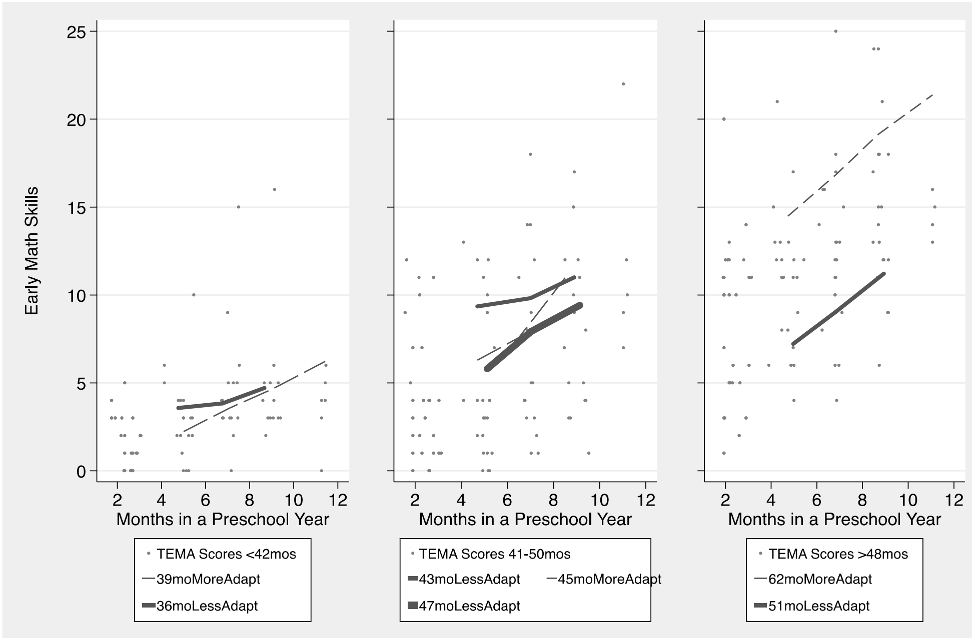


Figure 6a. This figure shows seven selected children from the sample, with their linear predictions plotted to illustrate the predicted relationship between change in (lag) adaptive skills during the preschool year and growth in early math skills. Each panel compares approximately same-aged peers.

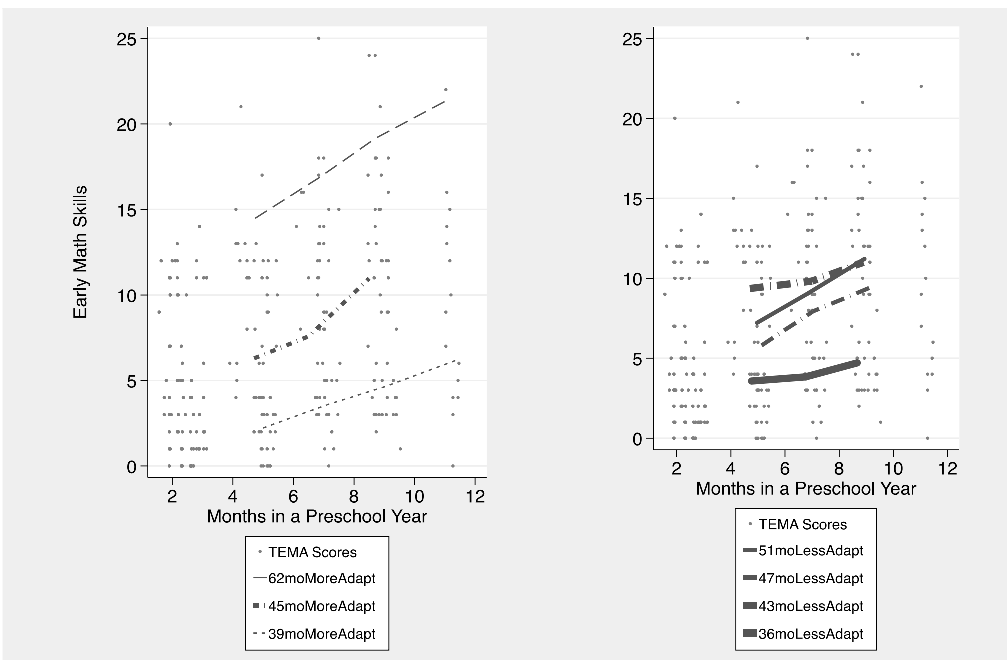


Figure 6b. This figure shows the same seven selected children from the sample with the same linear predictions plotted as in Figure 6a. Each panel shows children with similar rates of learning adaptive social problem-solving skills (i.e., “fast” or “slow”) during the preschool year, illustrating quite similar math learning trajectories even for children of different ages.

Predictive Utility of Baseline Adaptive Skills

Given that there was such little systematic growth in adaptive skills (i.e., an average gain of .24 points during the preschool year) and considerable variation both within and across children, a more parsimonious model was adopted to investigate the potential moderating role of children's adaptive social problem-solving skills at preschool entry on their math learning trajectories. A full taxonomy is presented in Table 11c and 11d (below). As indicated in the final model (Model 11 in Table 11d, below), the intercept (i.e., estimated initial math status for children in the reference classroom) indicates that the average-age child with average receptive language skills and no adaptive social problem-solving skills would begin preschool with about 4 points on the TEMA assessment ($\beta_{cons} = 4.06, SE = 1.16, p < 0.001$). As with previous models that have investigated the role of social competence, the true rates of change in early math skills are *not* different by baseline age of children; instead, they differ by the extent to which children have adaptive problem-solving strategies upon preschool entry ($\beta_{MonthsxBaseAdapt} = .07, SE = .04, p = 0.05,$ Model 11 in Table 11d below). This finding indicates that children's ability to engage in adaptive social problem-solving is associated with differences in the rates at which children learn math skills across the preschool year, when controlling for baseline receptive language and age as evidenced in Figure 6c (below). Specifically, children who begin preschool with more adaptive social problem-solving skills would have, on average, faster rates of math learning than children who have fewer of these skills. Figure 6c (below), displays the predicted early math skills trajectories for prototypical children (46.8 months at preschool entry), controlling for mean baseline PPVT scores. The plotted values in the figure have been derived from the estimates of the parameters of Equation [11], which are listed in Table 11d. Notice two patterns in Figure 6c. First, illustrated by the vertical distance between the fitted lines for each age-group, is that the children with more adaptive social problem-solving skills at preschool entry score higher on the TEMA math assessment than their peers with fewer adaptive social problem-solving skills. Because time was not centered, initial math skills have been estimated for each age group, prior to when the data collection period began. Specifically, the figure shows that the prototypical child with more adaptive social skills would, on average, have TEMA math scores of 5.12 ($p < .001$) whereas the prototypical child with fewer adaptive skills would be estimated to have an average score of 4.29 ($p<0.001$) at preschool entry. The second pattern evident in the figure is the increase in the size of the gaps in early math skills ($\beta_{MonthsxBaseAdapt} = .07, p = .05$) by the end of the preschool year.[8] For example, the predicted end-of-year TEMA math score for the prototypical child with more adaptive skills would, on average, be 12.99 points (i.e., a gain of 7.87 points) whereas for the prototypical child with fewer adaptive skills would be predicted to have an average score of only 9.45 by the end of the preschool year (i.e., a gain of only 5.16 points).

In summary, children with more adaptive social problem-solving skills at preschool entry, on average, begin preschool with more math skills and learn math at faster rates during the preschool year than their peers with fewer of these social problem-solving skills, controlling for receptive language skills.

[8] This model assumes a 10-month preschool year, although many preschool and Head Start programs offer full-year wrap-around services, which would result in even larger end-of-year gaps.

Table 11c. Cross-domain relationship between baseline adaptive social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 1-6).

	(1) MathRaw	(2) MathRaw	(3) MathRaw	(4) MathRaw	(5) MathRaw	(6) MathRaw
Months	0.72^{***}	0.72^{***}	0.58^{***}	0.66^{***}	0.66^{***}	0.71^{***}
	(0.06)	(0.06)	(0.12)	(0.17)	(0.17)	(0.19)
cBaseAge	0.13^{*}	0.12^{+}	0.13^{*}	0.14^{*}	0.13^{+}	0.13^{+}
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
Monthsx cBaseAge	0.02^{+} (0.01)	0.02^{*} (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)	0.01 (0.01)
Male	-1.51^{*}	-1.25^{+}	-1.24	-0.93	-1.04	-0.99
	(0.75)	(0.76)	(0.76)	(0.88)	(0.89)	(0.89)
cBasePPVT	0.08^{***}	0.07^{***}	0.07^{***}	0.07^{***}	0.08^{***}	0.08^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)	(.)
Class=2	-0.97	-1.14	-1.15	-1.15	-1.26	-1.29
	(1.18)	(1.17)	(1.17)	(1.17)	(1.16)	(1.16)
Class=3	-0.88	-0.82	-0.83	-0.83	-0.89	-0.91
	(1.19)	(1.17)	(1.18)	(1.18)	(1.17)	(1.16)
Class=4	-2.09^{+}	-2.06^{+}	-2.06^{+}	-2.06^{+}	-2.16^{+}	-2.17^{+}
	(1.15)	(1.13)	(1.13)	(1.13)	(1.12)	(1.12)
BaseAdapt		0.34	0.16	0.18	0.24	0.22
		(0.23)	(0.27)	(0.27)	(0.27)	(0.28)
Monthsx BaseAdapt			0.05 (0.04)	0.04 (0.04)	0.04 (0.04)	0.05 (0.04)
Monthsx Male				-0.09 (0.13)	-0.09 (0.13)	-0.10 (0.13)
Black					-1.17	-0.83
					(0.82)	(0.98)
Monthsx Black						-0.09 (0.14)
constant	5.31^{***}	4.23^{***}	4.74^{***}	4.47^{***}	5.35^{***}	5.16^{***}
	(1.01)	(1.24)	(1.30)	(1.36)	(1.48)	(1.51)
N	257	257	257	257	257	257
ll	-586.3	-585.2	-584.4	-584.2	-583.2	-583.0
AIC	1198.6	1198.5	1198.9	1200.4	1200.4	1202.0
BIC	1244.8	1248.2	1252.1	1257.2	1260.8	1265.9
PseudoR^2	0.60	0.61	0.61	0.61	0.62	0.62

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 11d. Cross-domain relationship between baseline adaptive social problem-solving skills and children's early math skills with a fixed effect for each participating classroom (Models 7-11).

	(7) MathRaw	(8) MathRaw	(9) MathRaw	(10) MathRaw	(11) MathRaw
Months	0.66***	0.58***	0.52***	0.52***	0.52***
	(0.17)	(0.12)	(0.12)	(0.12)	(0.12)
cBaseAge	0.13+	0.12+	0.16**	0.17**	0.19**
	(0.07)	(0.07)	(0.06)	(0.06)	(0.06)
Monthsx cBaseAge	0.01	0.01			
	(0.01)	(0.01)			
Male	-1.04	-1.37+	-1.36+	-1.23	
	(0.89)	(0.75)	(0.75)	(0.76)	
cBasePPVT	0.08***	0.08***	0.08***	0.07***	0.07**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Class=1	0.00	0.00	0.00	0.00	0.00
	(.)	(.)	(.)	(.)	(.)
Class=2	-1.26	-1.26	-1.30	-1.19	-1.26
	(1.16)	(1.16)	(1.16)	(1.17)	(1.19)
Class=3	-0.89	-0.89	-0.95	-0.88	-1.15
	(1.17)	(1.17)	(1.17)	(1.18)	(1.19)
Class=4	-2.16+	-2.16+	-2.22*	-2.12+	-1.98+
	(1.12)	(1.12)	(1.12)	(1.13)	(1.15)
BaseAdapt	0.24	0.22	0.14	0.08	0.17
	(0.27)	(0.27)	(0.27)	(0.26)	(0.26)
Monthsx BaseAdapt	0.04	0.05	0.07+	0.07+	0.07+
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Monthsx Male	-0.09				
	(0.13)				
Black	-1.17	-1.16	-1.17		
	(0.82)	(0.82)	(0.82)		
Monthsx Black					
Constant	5.35***	5.62***	5.91***	5.02***	4.06***
	(1.48)	(1.43)	(1.42)	(1.29)	(1.16)
N	257	257	257	257	257
Ll	-583.2	-583.5	-584.4	-585.4	-586.7
AIC	1200.4	1198.9	1198.8	1198.8	1199.3
BIC	1260.8	1255.7	1252.1	1248.5	1245.5
PseudoR^2	0.62	0.62	0.61	0.60	0.59

Standard errors in parentheses

+ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

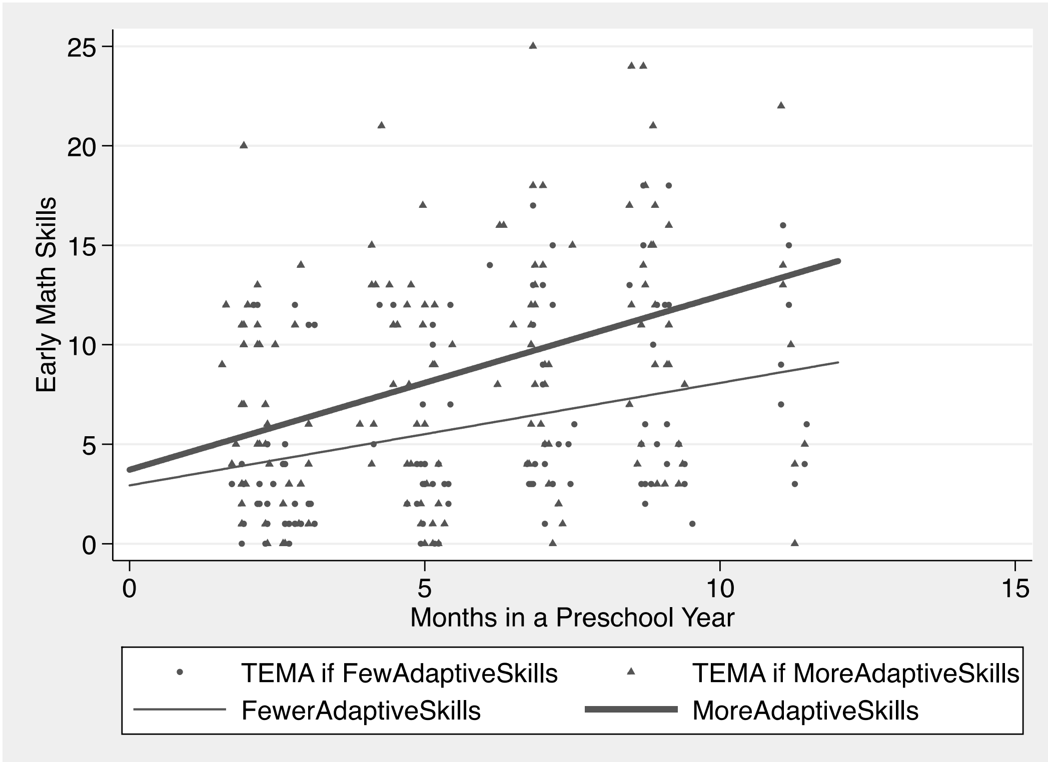


Figure 6c. Moderating role of adaptive social problem-solving skills on early math skills trajectories, controlling for baseline receptive language and age. Figure shows prototypical math trajectories for children with baseline adaptive social problem-solving skills at the 10^{th} (Low) and 90^{th} (High) percentiles at preschool entry.

Sensitivity Analysis

To address a potential critique that the "adaptive" social problem-solving skills are merely additive (by summing prosocial and avoidant categories within the same measure) rather than a theoretically driven construct, thus merely capture inflated gains (compared to what any single category could), a sensitivity analysis was conducted. Specifically, another composite—non-aggressive skills—was created, which consisted of summing prosocial, avoidant and crying responses in the Challenging Situations Task measure (i.e., all non-aggressive responses that the child could provide in a given prompt). Thus, this composite contains three possible responses at each time point, which, in theory, would inflate the scores to an even greater extent than the "adaptive" skills, if the latter were not theoretically driven by some underlying latent construct since both “crying” and “avoidant” responses have been described elsewhere as less desired responses in these social contexts. While crying is a non-aggressive response and is more desired than an aggressive response when children are faced with challenging social situations (e.g., being physically attacked or socially rejected by a peer), it is not considered an “adaptive” problem-solving strategy since it does not demonstrate that the child has and/or is able to utilize effective social problem-solving strategies when confronted with unanticipated social conflicts.

As with the previous set of cross-domain models with adaptive problem-solving skills, the bivariate relationship between TEMA math scores and CST “non-aggressive” scores showed curvature, violating the assumption of linearity. Given this, a quadratic term for non-aggressive skills was entered into these cross-domain models (NonAggressive and NonAggressive2). The full taxonomy is presented in Table 12 (below).

Therefore, in the final model (Model 8 in Table 12, below), the intercept (i.e., estimated initial math status for children in the reference classroom) indicates that the average-age child with average receptive language skills and report only aggressive social problem-solving skills would begin preschool with a little more than 3 points on the TEMA assessment ($\beta = 3.44, SE = 0.60, p < 0.001$). Unlike in the math-only model (i.e., RQ1) and similar to other models that have included indicators of children’s social competence in this study, the true rates of change are not different by baseline age of children. This analysis shows that early math skills at baseline entry (i.e., the intercept) can be predicted by non-aggressive skills ($\beta_{NonAggressive^2} = .08, SE = .04, p < .05$).

Since this composite has more (i.e., three of the four CST skills) of the possible responses, then the predictive utility should be greater than for adaptive problem-solving skills (which is a composite of two selected skills) if adaptive skills are not eliciting some underlying latent construct, yet adaptive social problem-solving skills predict more early math skills at preschool entry ($\beta_{Adaptive^2} = .11, SE = .04, p < .01$; see full taxonomy in Table 12, below). These models show that non-aggressive skills predict *less* about children's early math skills than adaptive skills at preschool entry—despite the fact that they capture more discrete skills overall—suggesting that adaptive social problem-solving skills may be drawing on a unique underlying element of children's social competence. While these starting points may seem negligible, it is important to point out that math skills at preschool entry are being reported as raw scores on the TEMA assessment. As such, each additional point represents concept mastery in a range of early math topics, so a 1-2 raw point difference at preschool entry actually represents considerable—and important—differences in children's early math knowledge (see Table 14, below, to help illustrate this point).[9] For example, a 4-year old with a TEMA raw score of 5 at preschool entry would have a TEMA ability score of 83. This corresponds to a percentile rank of 13, indicating this child outscores only 13% of other 4 year olds. If that 4-year old entered preschool with a TEMA raw score of 7, however, he would have a TEMA ability score of 89. This corresponds to a percentile rank of 23, indicating that he now outscores 23% of all other 4 year olds.

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[9] While the interaction with age is no longer significant with adaptive and/or non-aggressive skills in these models, TEMA raw scores must still be considered in context of the age of the child if/when transforming to math "ability" or norm scores in accordance with the TEMA manual.

These findings are especially important because prosocial skills alone did not predict children's math skills ($\beta_{Prosocial} = .01, SE = .11, p = 0.95$) even though, on average, children showed some evidence of acquiring these types of skills across the preschool year in this sample (see Table 1a and 1b, in Appendix B). Such findings provide additional evidence that there is more than one type of effective (i.e., adaptive) social problem-solving strategy that can be utilized in classroom settings that can promote children's social competence *and* promote children's early math skills, particularly among low-income children. Moreover, these types of social problem-solving strategies may be more appropriate in some contexts (e.g., they may be more relevant to the particular social context, they may be more familiar to some children who routinely experience a wider range of strategies in their daily routines, they may be important in the presence of high proportions of aggressive others).

Table 12. Early math skills development as a function of time (in months), predicted by non-aggressive skills of the child at preschool entry, controlling for gender and baseline receptive language and age with a fixed effect for each participating classroom.

	(1) Math Raw	(2) Math Raw	(3) Math Raw	(4) Math Raw	(5) Math Raw	(6) Math Raw	(7) Math Raw	(8) Math Raw
Months	0.72^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.68^{***}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
cBaseAge	0.17^{**}	0.16^{**}	0.16^{**}	0.15^{*}	0.16^{**}	0.16^{**}	0.18^{**}	0.23^{***}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)
Monthsx cBaseAge	0.02^{*} (0.01)	0.02^{*} (0.01)	0.02^{*} (0.01)	0.02^{*} (0.01)	0.02^{*} (0.01)	0.02^{*} (0.01)	0.02^{*} (0.01)	
Male	-1.31	-1.06	-1.17	1.65	-1.17	-1.06		
	(0.74)	(0.73)	(0.74)	(1.66)	(0.74)	(0.73)		
cBasePPVT	0.08^{***} (0.02)	0.08^{***} (0.02)	0.08^{***} (0.02)	0.08^{***} (0.02)	0.08^{***} (0.02)	0.08^{***} (0.02)	0.07^{***} (0.02)	0.07^{***} (0.02)
Non-Aggressive		-0.33 (0.27)	-0.33 (0.27)	-0.35 (0.27)	-0.33 (0.27)	-0.33 (0.27)	-0.32 (0.27)	-0.35 (0.27)
Non-Aggressive^2		0.08^{*} (0.04)	0.08^{*} (0.04)	0.08^{*} (0.04)	0.08^{*} (0.04)	0.08^{*} (0.04)	0.08^{*} (0.04)	0.08^{*} (0.04)
Black			-0.74	1.81	-0.74			
			(0.83)	(1.58)	(0.83)			
BlackxMale				-3.48 (1.83)				
constant	4.15^{***} (0.62)	4.05^{***} (0.77)	4.70^{***} (1.04)	2.55 (1.54)	4.69^{***} (1.04)	4.05^{***} (0.77)	3.34^{***} (0.60)	3.44^{***} (0.60)
N	257	256	256	256	256	256	256	256
ll	-588.1	-581.8	-581.4	-579.7	-581.4	-581.8	-582.8	-584.8
AIC	1196.3	1187.6	1188.8	1187.3	1188.8	1187.6	1187.7	1189.6
BIC	1231.8	1230.2	1234.9	1237.0	1234.9	1230.2	1226.6	1225.0

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 13. Early math skills development as a function of time (in months), predicted by adaptive skills of the child at preschool entry, controlling for the gender and baseline receptive language and age with a fixed effect for each participating classroom.

	(1) Math Raw	(2) Math Raw	(3) Math Raw	(4) Math Raw	(5) Math Raw	(6) Math Raw	(7) Math Raw	(8) Math Raw
Months	0.72^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.69^{***}	0.68^{***}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
cBaseAge	0.16^{**}	0.16^{**}	0.15^{*}	0.15^{*}	0.15^{*}	0.16^{**}	0.17^{**}	0.22^{***}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)
Monthsx cBaseAge	0.02^{*}	0.01^{+}	0.01^{+}	0.01^{+}	0.01^{+}	0.01^{+}	0.01^{+}	
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	
Male	-1.42^{+}	-1.32^{+}	-1.38^{+}	1.41	-1.38^{+}	-1.32^{+}		
	(0.73)	(0.71)	(0.72)	(1.64)	(0.72)	(0.71)		
cBase PPVT	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Adaptive		-0.48^{*}	-0.47^{*}	-0.49^{*}	-0.47^{*}	-0.48^{*}	-0.47^{+}	-0.52^{*}
		(0.24)	(0.24)	(0.24)	(0.24)	(0.24)	(0.24)	(0.24)
$Adaptive^2$		0.10^{**}	0.10^{**}	0.11^{**}	0.10^{**}	0.10^{**}	0.10^{**}	0.11^{**}
		(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Black			-0.43	2.06	-0.43			
			(0.83)	(1.55)	(0.83)			
Blackx Male				-3.42^{+}				
				(1.81)				
constant	4.13^{***}	4.41^{***}	4.78^{***}	2.68^{+}	4.78^{***}	4.41^{***}	3.58^{***}	3.67^{***}
	(0.61)	(0.68)	(0.97)	(1.48)	(0.97)	(0.68)	(0.51)	(0.51)
N	256	255	255	255	255	255	255	255
ll	-584.6	-579.0	-578.8	-577.1	-578.8	-579.0	-580.6	-582.2
AIC	1189.3	1181.9	1183.7	1182.2	1183.7	1181.9	1183.2	1184.5
BIC	1224.7	1224.4	1229.7	1231.8	1229.7	1224.4	1222.2	1219.9

Standard errors in parentheses
$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

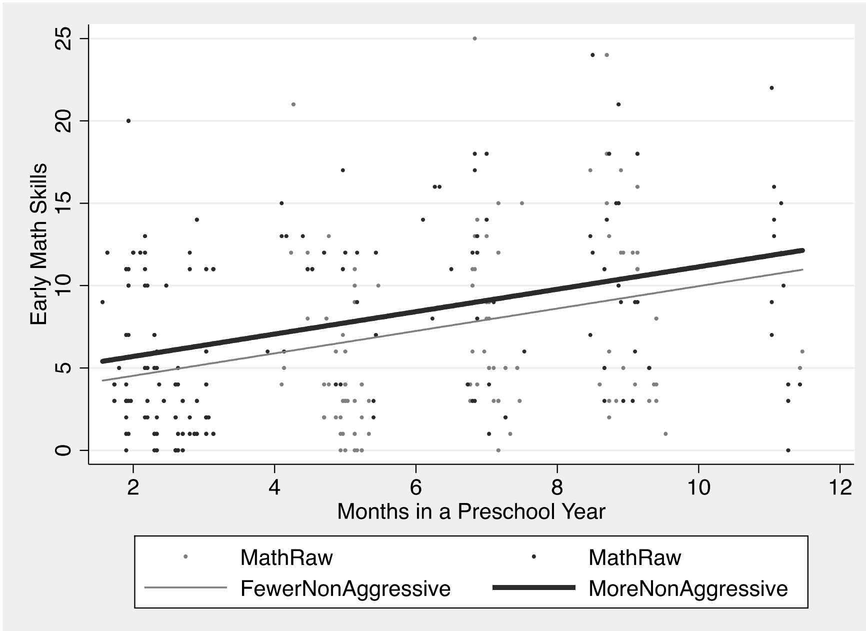

Figure 7a. Plot depicting the impact for prototypical children at the 10th and 90th percentile of "Non-Aggressive" social problem-solving skills at preschool entry on children's early math skills at preschool entry, controlling for baseline age receptive language.

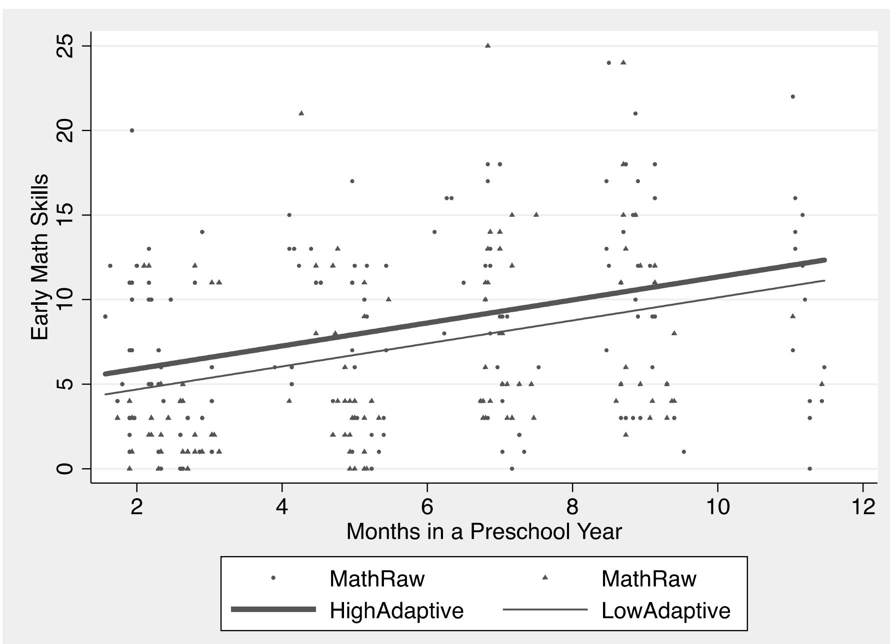

Figure 7b. Plot depicting the impact for prototypical children at the 10th and 90th percentile of "Adaptive" social problem-solving skills at preschool entry on children's early math skills at preschool entry, controlling for baseline age and receptive language. Note higher starting points for children with higher adaptive skills compared to non-aggressive skills at preschool entry.

Table 13. Table depicting relationship between
TEMA raw scores and TEMA math "ability" scores.

Age (in months)	TEMA Raw Score	TEMA Ability Score	Percentile Rank
36	3	94	35
	5	100	50
48	5	83	13
	7	89	23
60	6	74	4
	8	80	9

Note: Population distributions for TEMA ability scores have *M*=100, *SD*=15 and is a continuous scoring procedure. Percentile ranks can then be derived from the "ability" score. Because of the truncated ceiling rule used in this study, however, such comparisons cannot directly be made for the participants in this study and the above table is for illustrative purposes only.

Discussion

The aim of the present study was to better understand the ways in which low-income children's early math skills and various aspects of their social competence develop during a preschool year as well as to better understand the ways in which this development is inter-related.

Adaptive Skills. Adaptive social problem-solving strategies are a specific type of approach to resolving conflicts involving multiple types of problem-solving strategies utilizing key skills (e.g., perspective-taking). In the literature review, I argued there are alternate social problem-solving strategies (e.g., disengaging from an aggressor, engaging an alternate peer in play) that are adaptive in nature and complement prosocial problem-solving strategies, particularly in classroom contexts. While there is general consensus to minimize aggressive social problem-solving strategies during preschool, the findings from this study suggest that a broader range of social problem-solving strategies may be beneficial to children in early care and educational classrooms, in which children within the same approximate age range (3-6 years) have repeated interactions and sustained relationships with each other over time. In these contexts, children have repeated opportunities for interactions with individuals with whom they have shared histories and children often learn—intentionally or not—that some children in their classrooms are more aggressive (or are more receptive, etc.) than others and so they may need to utilize a greater range of social problem-solving strategies that are adaptive to the context, which may include, but may not be limited to prosocial problem-solving strategies as they learn to effectively navigate social dilemmas in context, which I argue is evidence of exhibiting greater social competence.

Decades ago, Selman and colleagues (Selman & Demorest, 1986; Selman & Schultz, 1989) argued that more sophisticated social competence was demonstrated by more reflective perspective-taking and cited motivation (in relation to the goal) as a key factor. While they argued that communication between peers was central to resolution, they also argued that children with more social competence would consider the specific circumstances with the person with whom there was conflict (e.g., the shared history, the unique context). Thus, I extend this argument given that, in many cases—especially when one has had previous interactions with aggressive or abusive others—it is often futile to try to reason with someone who has been aggressive for no reason (i.e., an unprovoked attack). For some children, confronting an aggressor may be too intimidating, which the prosocial act often requires. Depending on the context, a child who disengages from an unprovoked attack by a peer—with whom he otherwise has a relationship—may not be "merely" fleeing the scene, but rather carefully reflecting upon—and coordinating—both his and the other child's perspectives and, upon doing so, makes the conscious decision to transform himself rather than the other (a Level 2 orientation in Selman and Demorest's model), giving primary consideration to the other's thoughts and feelings. In this case, this child is demonstrating greater social competence and this act of walking away is an adaptive problem-solving skill *in this context*. Further, these children may be exercising their internal locus of control, rather than trying to exert control over the other child, particularly when the other child has already shown force against him/her in this case (e.g., by an unprovoked physical attack, physically grabbing a toy). In this context, this child has considered the circumstances and it could be argued that the child is attempting to preserve the relationship (a Level 3 action in Selman and Demorest's model) by not trying to change—or upset—the other child. Instead, the child is acquiescing something that is not that important (i.e., playing in the first location) in order to obtain his/her true goal (e.g., playing somewhere—even if somewhere else—without provocation). While it is not being argued that this is definitively Level 3 reasoning, previous research has shown that children as young as three years old can demonstrate Selman and Demorest's Level 3 reasoning in classroom settings (DeVries & Zan, 1994).

Likewise, children who relinquish a toy or play with someone else when initially rejected in these classroom contexts, may be better able to consider the feelings and perspectives of the other child—as well as him/herself—when contemplating potential social problem-solving solutions. In these cases, the child may come to understand that the other child must want the toy more since he took it so forcefully. As such, he/she acquiesces it, rather than ask for it back (i.e., focusing on only his/her perspective, a lower-level orientation). In the context of friendship initiation, rather than continuing to pursue only her perspective (i.e., a lower-level orientation) and again ask to play after being told the other child does not want to play with her, that child shows a higher-level orientation by being able to coordinate both her own and the other child's perspectives when she asks a different (third) child to play (i.e., she maintains motivation for the goal of playing with a peer, even if a different child than originally asked). Indeed, Selman and Demorest (1986) argue that greater social competence is evidenced by "putting emotions and cognitions in perspective" (p. 96) as the child's task is to coordinate his/her own feelings with those of the other child and then to enact a problem-solving strategy that reflects this coordination.

For children in classrooms who routinely engage with the same peers over time, it may be unreasonable to expect those children to continue employing *only* prosocial behaviors with a child who is repeatedly aggressive and/or uninterested in engaging in play with him/her. Indeed, if a child were to employ a single strategy merely as a matter of routine, this could indicate that this child is not considering the context of the situation, which is critical to demonstrating social competence. On the other hand, utilizing other social problem-solving strategies could be considered adaptive and more demonstrative of greater social competence when it indicates greater awareness of the specific context (i.e., what is happening in that specific time and place) and social cues of the other individuals in addition to the target child (i.e., the child is demonstrating more social perspective-taking) in addition to maintained focus on the desired outcome of the target child (i.e., ensuring the target child obtains what he/she wants).

Consistent with the guiding hypothesis, the data showed that low-income children's adaptive social problem-solving skills develop in a positive, non-linear way across a preschool year and that these patterns of growth are quite varied within and across children. Unlike with math, there were no differences noted in the adaptive learning trajectories by age, though there were differences in the number of adaptive social problem solving strategies upon preschool entry; younger children begin preschool with fewer adaptive social problem-solving strategies. Therefore, it may be important to include attention to social problem-solving strategies in the preschool curriculum to help children develop greater social competence and, in particular, to develop social problem-solving strategies. Evidence also suggests that, in all cases, children gained very few social problem-solving strategies across the preschool year, suggesting that this is a continuing need for many children as they progress through the preschool year. While some children may already utilize these strategies when they begin preschool, it is not surprising that we may not see substantial growth in use of these specific adaptive strategies without scaffolding and instruction from teachers to provide children with the tools (e.g., vocabulary, development of perspective-taking) to better and more effectively navigate social situations as they occur throughout the preschool year. While there was, on average, a significant positive curvilinear trend, the data shows evidence of both of gains and losses of these skills for many children, suggesting that children in classrooms may need explicit guidance to learn adaptive strategies for effectively resolving conflicts with others and, in addition, they need reinforcement that their use of strategies is sanctioned within classroom contexts so that they either continue to use (i.e., when adaptive in nature) or discontinue strategies when maladaptive to the context (e.g., aggressive strategies). Because competent use and selection of appropriate strategies is dependent upon the context, children are likely to need continued support from others—

especially trusted adults, such as classroom teachers with whom they are developing relationships—who can provide feedback and help them take the perspective of others so that they can learn to resolve conflicts independently. Preschool classrooms have traditionally provided this structure and adult scaffolding for development of children's social competence. This historical context combined with the data that significant cross-time correlations only emerged during the second half of this preschool year, with stronger correlations over time, suggests that preschools can be an important place—and the preschool age group an important developmental period—in which children can learn many of these skills. Group settings, such as preschools serving low-income families may be particularly well-suited to target instruction of adaptive social problem-solving strategies. Previous studies have shown that African-American boys have lower social competence (Aratani, Wight & Cooper, 2011). However, the present study found no evidence of either of these gaps, though it is possible that that this lack of finding was due to the disproportionately male and Black demographics of the sample or it could be due to the different indicators of social competence that were measured. It should be noted that, across both social problem-solving skills measures, children showed fewer adaptive skills at Wave 3 (April) than at other time points in the preschool year. Thus, it was not likely a metric effect and more likely indicative of true change in these preschoolers' social problem-solving skills across that preschool year. While it is not known what caused this change—perhaps a traumatic event occurred that caused the decrease in these adaptive strategies or perhaps the preschool ceased a social-emotional curricula they had been using prior to the dip—it was evidenced in both measures. Future investigations should include longitudinal data collection to better understand these trends across time.

Flexibility. Results from this study show that there were different learning trajectories by age in children's flexibility in social problem-solving across a preschool year. While older children (e.g., 5 year olds) enter preschool with more flexibility in social problem-solving, the data show that younger children do not enter preschool with much flexibility and, instead, acquire flexibility in social problem-solving during preschool at rates that allow them to nearly catch up to their older peers. In many cases, these data suggest that young children need opportunities to learn that there is more than one way to solve problems. Research has long endorsed mixed-age classrooms in support of children's development across domains (Katz, Evangelou & Hartman, 1990; Bredekamp & Copple, 1997) and these classrooms may have provided an environment in which older children were able to provide important modeling for the younger children. If so, that would be a rationale for continued implementation of mixed-age group classrooms to help children develop this aspect of social competence, particularly for younger children who may particularly benefit from their older peers as role models. As outlined in the previous section, younger children begin the preschool year with very few social problem-solving skills and thus have low flexibility. As they learn more social problem-solving skills in preschool, flexibility also increases.

In the current study, however, the older children, on average, did not gain skills in this aspect of social competence and some children (e.g., 5 year olds) had flat learning trajectories, despite the absence of any ceiling effect. It should be noted, however, that in this study, flexibility was measured by the SPST-R, which captures children's ability to utilize multiple, distinct problem-solving strategies when faced with problematic social situations and the type of problem-solving strategy (i.e., whether prosocial, aggressive, etc.) is not considered for this particular code. Thus, it is possible to note a lack of growth—or decrease for individual children—in flexibility across a preschool year when children learn to decrease negative solution strategies (i.e., offering fewer aggressive or manipulative responses) if they are not simultaneously learning other (e.g., prosocial, adaptive) social problem-solving strategies to compensate for the loss of the variety of strategies available to the child. In these cases, these children may have to subsequently rely only on the strategies he/she originally knew upon preschool entry. For these children, learning to decrease negative behaviors without learning additional social problem-solving strategies results in a net loss (or lack of growth, depending on the extent of change) of flexibility. Therefore, it may be important to intentionally embed social problem-solving skills into the preschool curriculum, so that preschool children of all ages can continue learning additional social problem-solving skills throughout the preschool year even as they decrease undesired (e.g., aggressive) behaviors and skills, ultimately yielding net increases in their flexibility in social problem-solving throughout the preschool year as well. This may be particularly important for older children who are enrolling in preschool programs, so that they too can develop flexibility in social problem-solving during a preschool year. While older children begin the year with more of these skills, on average, than their younger classmates, mixed-age classrooms may be particularly well-suited for helping children of all ages develop key interpersonal skills (e.g., perspective-taking; Katz, Evangelou & Hartman, 1990; Bredekamp & Copple, 1997) that promote flexibility in social problem-solving. While much of the research about mixed-age classrooms was conducted before 2000, this rich historical literature documented the ways in which mixed-age classrooms can help older children develop prosocial skills (Winsler, 1993) and better self-regulation (Lougee & Graziano, 1986 as cited in Katz, Evangelou & Hartman, 1990) among other benefits as they routinely interact with younger classmates. More recent research with a large sample of diverse HS children (N=4417) using longitudinal growth modeling showed that all children benefited in mixed-age classrooms on several school-readiness indicators (i.e., emergent literacy and numeracy, social-emotional skills, approaches to learning; Bell, Greenfield & Bulotsky-Shearer, 2013) suggesting there can be important benefits for children of all ages when a variety of school readiness outcomes are considered.

Early Math Skills. The data from the present study shows that growth in math skills has a positive, linear relationship with the number of months in a preschool year and that growth trajectories vary by the age at which children enter preschool when controlling for receptive language skills and gender when only this domain is examined. Previous studies have shown evidence of gender differences in the early school years with slight advantages to boys (Garcia, 2015; Liu & Barnard-Brak, 2015). The present study, however, finds the opposite trend in gender differences at preschool entry, with boys having 1.5 fewer points on the math assessment, on average, than girls. Note that these data come from a low-income and largely African-American male sample, and may be specific to this sample. More research is needed with low-income samples that are better balanced by race and gender to better understand this finding.

The findings from this study suggest somewhat more complex age-related relationships than the original academic-only model suggested, however, which have important implications for children in mixed-age classrooms, such as those being served in programs like HS. The baseline models investigating children's growth of math skills over time showed differences by age; specifically, that older children had faster rates of growth than younger children. However, these differences by age in the rates at which children learn math across the preschool year disappeared once some indicators of social competence were included in the models, suggesting that differences in the rates at which children learn math may be better explained by differences in social competence rather than by age per se. Specifically, the flexibility-math cross-domain model shows that, once we consider development of flexibility of social problem-solving skills, there are no longer differences by age in the rates at which children learn math across the preschool year; the variability in the rates at which children learn math can better be explained by children's growth in development of flexibility in social problem-solving skills. Such findings suggest that helping children learn how to flexibly negotiate social problem-solving situations during the preschool year may be an important skill that should be fostered, particularly for the role it may play in promoting early math skills.

Though suggestive, the findings from this study show that the development of adaptive social problem-solving skills and early math skills may be linked in ways that are not yet fully understood and warrant further investigation. Specifically, the (lagged) adaptive models illustrate that children who develop more adaptive social problem-solving skills during the preschool year acquire math skills at faster rates than their peers who develop fewer of these social problem-solving skills with a complex link to age, given children's skills—in both domains—at preschool entry. As noted previously, older children, on average, began preschool with more adaptive social problem-solving and early math skills. The lagged-adaptive math models showed that, for older children—regardless of the rate at which they learned adaptive social problem-solving strategies—

the predicted math trajectories were more similar (i.e., nearly parallel). Given the observational nature of this study, it is important to note, however, that a gap between their math skills persisted throughout the preschool year; the child with more adaptive skills at preschool entry also had more math skills at preschool entry, an advantage she maintained—in both domains—across the preschool year. As noted previously, in this sample, many of the older children who showed faster rates of adaptive social problem-solving began preschool with such high math skills, ceiling effects—given the curriculum taught—may have also played a role as these children simply have so many more early math skills at baseline and, though they continue learning more than their same-aged peers who develop fewer adaptive skills across the preschool year, we may see less acceleration in their math growth trajectories than in younger children who develop adaptive skills at similar accelerated rates. This is somewhat to be expected as the math concepts being tested in the TEMA assessment become quite sophisticated and may have been beyond what was taught in the preschool classrooms and/or what would be developmentally expected (e.g., abstract number line concepts, "number after" concepts with two-digit numbers). However, it is also possible that preschool teachers could be better supported in teaching some of these higher-level concepts—with more intentional professional development that takes children's learning trajectories into account—so that we might see greater evidence of faster rates of math learning for older children who develop more adaptive social problem-solving skills during the preschool year, more similar to the sharp increases evidenced in younger children in this sample. However, it is also possible that preschool teachers could be better supported in teaching some of these higher-level concepts—with more intentional professional development that takes children's learning trajectories into account—so that we might see older children develop social problem-solving skills during the preschool year as efficiently as the younger children in this sample. If the cross-domain relationship observed in this study holds up, this may then also lead to a reduced gap in early math skills.

In this observational sample, the number of children who showed systematic "faster" or "slower" rates of adaptive social problem-solving was limited. A more detailed review of the data shows that individual children in this sample tended to fluctuate in the number of adaptive social problem-solving skills across the preschool year, often gaining—then losing—and often re-gaining these skills, with an average gain of only .24 points in adaptive social problem-solving skills; the positive curvilinear relationship illustrates this dip over time for the aggregate as well. While this data show an intriguing relationship to later math growth and the prototypical children selected for Figure 6 do show children who represent the pattern in the fitted model (i.e., children in this sample with systematic "faster" and "slower" rates of adaptive social problem-solving, respectively), the results should be interpreted with caution given this broader context. Importantly, the finding that children's adaptive skills from the previous time period were related to math suggests a time-sequence that is consistent with a causal relation. While no causal claims can be made from this study, these findings suggest the value of investigating this particular cross-domain relationship with more controlled research designs. If this cross-domain relationship holds up, it suggests a mechanism for closing the math achievement gap among children from low-income families.

Summary Findings. Results from all of these models show that children enter preschool with different math skills, a trend that is likely to continue as we expand access to publicly funded preschool. Moreover, results from this study consistently showed that younger children, on average, begin preschool with fewer adaptive social problem-solving skills and less flexibility in social problem-solving at preschool entry and that each of these skills are associated, on average, with fewer math skills at preschool entry. The results from this study support the continuation of age-appropriate math learning objectives (i.e., 3 year-olds need different learning goals than 5 year-olds) as children of different ages progress through the preschool year at similar (i.e., largely parallel), though different learning rates relative to their ages (i.e., relative to their different initial starting points).

Since this was an observational study, it is not possible to know the ways in which the curriculum may have differed within and across these classrooms. It is possible that children within these mixed-age classrooms may have been exposed to different “doses” of math and/or social-emotional content. Because the math learning trajectories were similar once children’s social competence was taken into account, it seems likely that math may have been more routinely implemented in all of the classrooms in this sample. However, other results from this study suggest that social-emotional competencies may have been a larger focus with younger children. Such a differential curricular focus may help to explain some of the different patterns by age that emerged within this sample. If that were the case, it could explain why we see growth in flexibility of social problem-solving primarily for younger preschoolers. Likewise, we see clearer evidence of faster rates of math learning emerging for younger children who develop more adaptive social problem-solving as the preschool year progresses, which might also be consistent with a more balanced approach to instruction. For older children, we see evidence that they are being supported in their math skill development regardless of their math skills at preschool entry, demonstrating important progress towards a key kindergarten readiness skill. However, for these older children who enter preschool with less social competence, there is evidence that they do not gain—and sometimes lose—skills in social problem-solving during the preschool year. Further, it should be noted that a gap—in both social-emotional and academic skills—still remained between these older children at the end of the preschool year. Given the broader research that suggests all children of this preschool age can be supported in learning a developmentally challenging math curriculum and in ways that support children's social-emotional development, more research is needed with controlled studies that investigate this cross-domain relationship while supporting teachers to implement a well-balanced curriculum that aims to support children's early math skills and social competence so that these complex relationships can be better understood in practice.

Across various models presented in this essay, results have shown that multiple indicators of children's social competence (e.g., adaptive social problem-solving skills at preschool entry, development of flexibility in social-problem-solving across the preschool year) have the potential to play an important role in children's early math trajectories. While younger children generally enter preschool with fewer math skills than their older peers, as we would expect developmentally, results from all of these models show that children's math trajectories showed more acceleration when they experienced more growth in social problem-solving (i.e., adaptive and flexibility) than their same-aged peers with less development of these social skills, suggesting that age may be merely playing a developmental role in these different rates of math learning that have been observed (between children of different ages). Thus, with a developmentally appropriate preschool curriculum that incorporates challenging math content and also promotes key indicators of social competence, we might expect that children of all ages can excel in mixed-age classrooms, closing gaps among same-aged peers who enter preschool with different skills and promoting learning at similar rates for all children in these classrooms, across domains, as the cross-domain models in this study collectively suggest. If the instruction—or analysis—is unbalanced, however, (e.g., focusing only on the academic domain), we may see results that more closely mirror the math-only model, which may account for some of the previously published findings regarding differential effects by domain in mixed-age classrooms (e.g., Ansari et al., 2016; Guo et al., 2014).

As publicly funded preschool programs have expanded, there has been increased debate about providing the right balance between academic and social skills curricula. Recent evidence shows that some efforts to increase academic rigor have coincided with less developmentally-focused instructional practices in early grades. Using ECLS-K data, Bassok and colleagues (2016) noted that between 1998 and 2010, kindergarten had become increasingly focused on academic skills—and specifically in math—with decreases in music and art. Importantly, these researchers noted differences in the pedagogical approaches with more time devoted in kindergarten classrooms to more didactic (e.g., use of worksheets, workbooks) and whole-group instructional activities that more closely resembled first grade from the previous decade (Bassok, Latham, & Rorem, 2016). While they also noted increases in more child-centered approaches (e.g., using music to teach math concepts) that would be consistent with a net increase in time spent on math instruction, there were, “particularly large gains in time spent on rote, didactic tasks," (Bassok, Latham & Rorem, 2016, p.10) noted across this 12-year time-frame. These findings illustrate not only a shift in the curricular focus to one that is more academic, but also the ways in which these curricula are being taught (i.e., with less developmentally-focused instructional practices) which has occurred during a time of rapid expansion of early childhood programs (e.g., half-day to full-day kindergarten, expansion of preschool). RCTs from the past have shown that such direct instruction models can be more successful in yielding immediate academic gains, yet these often fade (Schweinhart et al., 2005). More importantly, children in these basic skills programs had less prosocial behavior and more antisocial behavior, which persisted over time (e.g., more felony arrests; Schweinhart & Weikart, 1997). In contrast, in the high-quality, child-centered Perry Preschool project, participants demonstrated better short term (e.g., higher academic gains, less grade retention) as well as long-term academic outcomes (e.g., increased high school completion, Schweinhart et al., 2005). Participants also had better health (e.g., less drug use) and life outcomes (e.g., less premature death, incarceration) suggesting that high-quality early childhood experiences have lasting effects well beyond the academic domain (Schweinhart et al., 2005).

The findings from the present study are consistent with developmentally-focused, comprehensive school-readiness models such as Michigan's Great Start Readiness (GSR) program that effectively promote low-income young children's social-emotional development (e.g., social relations) and their early academic outcomes (e.g., literacy skills) in efforts to close the achievement gap (Malofeeva, Daniel-Echols & Xiang, 2007). These initial trends continued into 4th grade, with GSR participants obtaining higher ratings in math, literacy and problem-solving compared to non-preschool participants (Malofeeva, Daniel-Echols & Xiang, 2007). Indeed, the present study showed that young children can be supported in developing important social problem-solving skills (i.e., adaptive skills and flexibility) which can help children to develop these social skills—important for promoting children's social competence—and this study also provides evidence that they are important for promoting faster rates of growth in early math skills during a preschool year.

Head Start was originally created to support children's cognitive, social-emotional and physical development and well-being (U.S. Department of Health, Education and Welfare, 1968; Cooke, 1965). While the program has evolved over time and has increased academic content, the current HS Program Standards continue to promote the well-being of the whole child (Head Start Performance Standards, 2015). Taken together, these findings suggest both adaptive problem-solving skills and flexibility in social problem-solving may be important contributors to children's math learning trajectories at both preschool entry and as they learn important math concepts across the year. Given the broader literature regarding lower social competence for African-American males (Aratani et al., 2011) and lower-income children (Denham et al., 2012) and the achievement gap, particularly in math, for low-income children (Claessens & Engel, 2013; Duncan & Magnuson, 2011; Jordan, Kaplan, Olah & Locuniak, 2006), these findings have particular importance for the curricular design and time allocation in preschool programs serving low-income families, including programs such as Head Start.

These findings also suggest that effective social problem-solving strategies must be considered in context—as is a child's social competence—and "adaptive" social problem-solving strategies, as they have been defined here have predictive utility for supporting low-income children's math trajectories during a preschool year. Such findings suggest that 1) there is an important cross-domain relationship that warrants greater attention in both research and practice and 2) there are multiple ways to effectively problem-solve that include—but may not always be limited to—"prosocial" problem-solving strategies in real-world contexts, such as classrooms that yield important and positive effects for children in both the social-emotional and cross-domain learning contexts. It should be noted that while prosocial behaviors, on average, also increased across the preschool year, they showed no predictive utility in this sample (see Table 1 in Appendix B for full model summary). Future research should investigate the potential long-term implication of such adaptive strategies for children's social competence as children progress into early elementary classrooms.

Importantly, however, one can only be flexible in social problem-solving if one has multiple social problem-solving strategies from which to choose when confronted with a dilemma. Thus, in combination with prior research regarding a well-rounded curriculum in support of the whole child (see, for example, Bredekamp & Copple, 1997) as well as previous intervention efforts from decades past (e.g., I Can Problem Solve, Shure & Spivak, 1980), these findings suggest that it is important to actively include social problem-solving in the preschool curricula so that children have a means for learning multiple social problem-solving strategies. Building upon the bounty of research that finds that prosocial strategies are important for developing social competence (Denham et al., 2012; Rubin & Krasnor, 1992; Dodge, Pettit, Mcclaskey, Brown, & Gottman, 1986), this research also suggests that children can be supported in developing both prosocial as well as alternate social problem-solving strategies that are adaptive to their environments, while at the same time minimizing aggressive and other unproductive strategies that do not actively promote social competence in the child (e.g., crying, manipulative behaviors). Given the quite varied individual learning trajectories that were observed for adaptive skills, these findings suggest that children need support in developing adaptive strategies during a preschool year as they encounter and learn to navigate difficult social situations.

While multiple predictors of academic achievement exist, some research has shown that early math skills (i.e., in kindergarten) are the strongest predictor of later academic achievement (Duncan et al., 2007). Despite showing this strong link, that meta-analysis was based upon several correlational studies and could not isolate a causal mechanism. Therefore, a critical question still remains regarding how to best promote children's early math skills. Importantly, the authors also acknowledged that most of the studies measured only a limited set of social-emotional constructs, with a focus on negative indicators (e.g., externalizing/internalizing behaviors; Duncan et al., 2007). There has been little investigation into the role of positive indicators of social-emotional development—and no investigation into the role of social competence (e.g., children's interpersonal problem-solving skills)—in predicting children's early math skills. Given the absence of research in this particular area, the current analyses were an attempt to better understand the ways in which low-income children's early math skills and positive indicators of social competence develop across a preschool year. While no causal inferences can be drawn from these findings, this study has generated a more nuanced understanding of this cross-domain relationship. Specifically, this study has provided an important foundation about the relations between several indicators of children's social competence and early math skills—and how these change during a preschool year—exploiting naturally existing variation across children attending preschool programs serving children from low-income families.

Limitations and Future Research

There are several limitations of this study's findings. A central challenge is the type of measures used to capture social competence. While two different measures were utilized, recognizing their inherent trade-offs, each is a measure of what children say they would do rather than observations of the social problem-solving skills children actually employ when negotiating difficult social dilemmas in their preschool classrooms. Thus, it is possible that there is a difference between what children have reported and what they would be observed doing. Nonetheless, the relationships observed reflect what children think is appropriate and/or expected of them when asked what they would do/say by adults, and the assessment calls on important perspective-taking skills. This type of direct assessment should continue to be used and should be supplemented with direct observations in future studies to better understand these cross-domain relationships. In addition, the stop rule that was implemented with the SPST-R included stopping the assessment by child request (in accordance with IRB protocols). The scoring of the flexibility was adjusted to take this into account to minimize bias and it was assumed that this was just one more facet of measurement error (e.g., children have good testing days and bad testing days) and thus, it was assumed that this error was random throughout the data collection period. However, it is possible that the children who indicated they wanted to stop the SPST-R assessment before completion of all eight vignettes were different in some ways and this should be considered in future analyses. Further, this analysis is based upon observational data. As such, children were not randomly assigned to classrooms. While attempts were made to adjust for the potential variation between classrooms, by including a fixed effect for each classroom, there is still a potential for omitted variable bias. Ideally, three-level nesting would be used to control for each classroom, which was not possible in a sample this small (i.e., there was inadequate variation over time with such small levels of nesting) and future, larger studies should further investigate this, a point that is expanded upon next.

Another set of limitations of the present study concerns both internal and external validity. Because this was an exploratory study, sample size was small and ultimately included a consented sample that was heavily skewed male and African/African-American, which may limit the extent to which findings can be generalized. Future studies investigating these cross-domain relationships should be replicated in larger and more representative samples. In addition, there is only a very limited set of demographic variables about the participating children in the present study. Given the low-income sample and geographic location in which the data was collected, it is likely that there were additional risk factors (e.g., single parent families, home instability) that may have been important to include, which were beyond the scope of the present study. In addition, HS mandates that at least 10% of all enrolled participants are children with special needs (Administration for Children and Families, n.d) to provide an inclusive learning environment for all children. However, the participating center indicated that none of the children enrolled in any of the preschool classrooms (which included two HS classrooms) had identified special needs. Thus, it is likely that this information has been under-reported and, therefore, could not be accounted for in these analyses. Future studies should account for this additional variation that is likely for children with special needs, so that we can better support all children in classrooms and better understand the reasons for the variation between children. In addition, attempts were made to collect data about other characteristics, such as the extent to which participating children spoke a language other than English in their homes, but administrative challenges hindered collection of this data. As such, children's receptive language is a proxy—though not a complete substitute—for this information. Having more complete data about children's home language could provide more information about the role that language plays in this cross-domain relationship, given that language is so important in each of these constructs independently.

The current study investigated the link between children's social competence and early math skills. While this study controlled for children's receptive language, it is possible that children's pre-literacy skills—other than receptive language—may also be contributing to the observed changes in children's math skills over time. More controlled studies that test this math-social skills link, including controls for children's early literacy skills, would help us better understand this relationship that has been observed and future studies should include each of these constructs.

Next Steps: Future Directions for Research

Positive emotional classroom climate has been associated with children's development of various indicators of social competence (Hamre & Pianta, 2001; Pianta, Steinberg & Rollins, 1999) and their early literacy skills (NICHD ECCRN, 2003) though little research has been done in the domain of early math skills. Research consistently shows that, on average, Head Start classrooms have positive emotional climate (*M*=5.3;

Moiduddin, Tarullo, West & Xue, 2012), yet low instructional scores (*M*=2.3; Moiduddin et al., 2012). In addition, research shows that, on average, Head Start programs have either null or negative impacts on children's social competence (U.S. Department of Education, 2015). Taken together, these findings suggest that, while early care and education providers in programs serving children from low-income families provide nurturing environments, they may need support in better understanding 1) how to support children's social competence when implementing a challenging academic curriculum and 2) how these two domains of development (i.e., cognitive and social-emotional development) may be related (i.e., they may develop in concert). This may be especially true given other evidence from those studies that shows that even within the dimension of Instructional Support, classrooms scored higher, on average, for Language Modeling (*M*=2.5) than for Concept Development (*M*=2.1; Moiduddin et al., 2012). The need for professional development may be even greater within the domain of math.

Given the limited research in this area and with this age group, in particular, an observational study was the first step in the scientific process to better understand how these relationships develop and vary over time within and across children and, importantly, across domains. Establishing a causal link between social problem-solving skills and early math skills would require an intervention study, in which helping children develop adaptive social problem-solving skills (i.e., various strategies responsive to the contexts) and flexibility in social problem-solving is targeted by the intervention and the impact on early math skills is measured.

Intervention Components

The intervention would consist of three components: social skill development (for children), professional development (for teachers) and coaching (for teachers). Treatment classrooms would participate in these intervention components while control classrooms would participate only in developmentally-appropriate, challenging math curriculum training, which would be the same as that targeted to the intervention teachers. Time would be allocated to making these potential links (e.g., it is possible to have a challenging, developmentally-appropriate math curricula *and* support children's social-emotional development) explicit for intervention teachers and providing concrete strategies for ways that teachers can support children's development in both domains in their classrooms. Other intervention activities would be allocated to the importance of flexibility in social problem-solving and strategies for supporting development of this skill in the classroom.

Taken together, these intervention components would capture several dimensions of children's development of social competence and early math skills over the course of a preschool year. Given the intervention design, the results would enable direct comparisons between those who participated in the intervention (i.e., those who were supported in challenging math while additionally being supported in developing social-emotional development and flexibility in social problem-solving) and those that only received math instruction, facilitating a better understanding of the mechanism that may be at play. Specifically, this intervention would have the potential to illuminate the causal role that the development of social problem-solving skill may play in promoting early math skills during the preschool year. As a further check on the specificity of the relationship between social skills and math, child participants would be assessed at the beginning and end of the intervention not just on social and math skills, but also on key language and literacy skills. I hypothesize that effects on math would be greater than effects on language/literacy. Of course, additional intervention models could be designed to illuminate a variety of possible hypothesized relationships between social skills and math.

Implications for Policy and Practice

Analyses of these growth trajectories facilitate a better understanding of both early math skills and social competence to better understand how these complementary, yet distinct aspects of school readiness change across a preschool year and to explore any interplay between the two. The findings from this study can provide a basis for professional development, guiding teachers to better understand how to build upon children's instructional needs—and strengths—using data that incorporates low-income children's learning trajectories across a preschool year. There is clear consensus that preschool teachers need continued professional development to more effectively implement conceptually-rich mathematics instruction (Schoenfeld & Stipek, 2011; Ginsburg et al., 2008). Research shows that professional development should promote an understanding of children's early math trajectories and pedagogical best practices (Ginsburg, 2006), yet recent efforts to increase preschool teachers' effectiveness have largely focused on assessing children, leading to teachers described by Deborah Stipek as "adept at diagnosing children's misunderstandings. But [they were] not at all sure what to do after they had identified a problem (Ginsburg, 2008, p.13)." Professional development, therefore, can be an opportunity to provide instructional strategies and coaching so teachers can support their young learners' development of foundational concepts; research suggests that understanding developmental trajectories is a key component of effective implementation of research-based, developmentally appropriate math curriculum (e.g., Building Blocks, Clements & Sarama, 2007a; Big Math for Little Kids, Balfanz, Ginsburg & Greenes, 2003). It is essential, however, that practitioners and policy makers alike understand the ways in which these trajectories develop—and vary—across a preschool year, particularly for children from low-income families, so that desired learning outcomes can be appropriately selected.

The cross-domain findings suggest that social competence—and specifically social problem-solving skills and flexibility in this skill—may help in promoting children's early math skills and thus should have more time devoted to it during the preschool day. The findings from this study suggest that children need both adaptive social problem-solving skills and flexibility in their use of multiple strategies when faced with challenging situations in social contexts. In classrooms, teachers can provide scaffolding for children when children are engaged in problematic social situations (e.g., helping children negotiate fair resolutions, encouraging children to ask the other child how he/she feels). Given the importance of flexibility in social problem-solving, teachers can also promote and/or accept more than one problem-solving strategy. In some cases, this may mean that some children opt to resolve a problem one way at one time and other children may resolve the same problem in a different way the next time; when these are productive (i.e., adaptive) problem-solving strategies (e.g., non-aggressive) and are perceived as fair to the children in the social context, different solutions can all be viewed as just.

Research has long shown that mixed-age classrooms have benefits for preschoolers of all ages (Katz et al., 1990; Bredekamp & Copple, 1997), though some research has shown the relationships are more complicated (see, for example, Winsler et al., 2002). More recently, some have argued that same-aged classrooms may better promote academic instructional gains in our efforts to close the achievement gap (see, for example, Ansari et al., 2016; Guo et al., 2014), yet many of these studies on which these calls are based did not include positive indicators of children's social-emotional competence (particularly as they have been defined here), thus limiting the extent to which their findings can be generalized to all aspects of school readiness. Given this, more research is needed with these different constructs of interest, including a broader range of positive indicators of social-emotional competence in PreK efficacy trials, to better understand the role of mixed-age classrooms in promoting social-emotional *and* academic outcomes in our efforts to close the achievement gap. The finding of differential math learning trajectories by age—which can be at times be better explained by indicators of children's social competence—has important implications for mixed-age classrooms and programs that utilize this approach, such as Head Start. Such programs and policies may need to consider the ways in—and rates at—which children develop academic concepts and social-emotional skills, so that appropriate—and differentiated—curricular goals can be made during the preschool year.

While there are attempts to support both academic and social-emotional development in many preschool classrooms, the implementation must be intentional and provide a balance that values both challenging, yet developmentally focused academic curricula and positive indicators of social competence. The findings from the present study suggest that supporting children's development of social-problem solving can promote greater social competence (i.e., more adaptive and less aggressive problem-solving skills and greater flexibility in these skills) as they progress through a preschool year. In turn, this better social competence can support faster acquisition of key math concepts. While more research is needed, this cross-domain relationship also suggests that—as children develop better social competence, in particular—math can be more effectively embedded in social contexts, facilitating more opportunities for children to "view their world mathematically" (Frye et al., 2014, p. 9). By using social problem-solving contexts—which naturally occur in preschool classrooms frequently—as a more intentional source of integrating opportunities for teaching math, children could be provided with additional scaffolded learning opportunities in which they learn to link these social interactions (many of which may already involve math) with more formal opportunities to learn math vocabulary and, when appropriate, math symbols and other more formal math concepts.

Essay 2

The Potential of Social Competence as a Mediator of Head Start Children's Early Math Skills: Evidence from the Head Start Impact Study

Introduction

Children from low-income families often enter kindergarten academically behind their higher income peers, particularly in math (Duncan & Magnuson, 2011). Children with lower math scores at kindergarten entry will, on average, continue to score lower than their higher-scoring peers throughout 8th grade, a trend that disproportionately affects low-income and minority children (Schoenfeld & Stipek, 2011). Evidence of better early math performance for boys also emerges during the early school years (Garcia, 2015; Liu & Barnard-Brak, 2015), posing additional barriers to overcoming achievement gaps. Recent findings also indicate gaps in important social-emotional aspects of school readiness (Duncan & Magnuson, 2011). Much literature suggests that children's social competence (i.e., how children interact with others) can play an important role in their academic achievement. For example, studies have investigated relationships between social competence and literacy skills (e.g., Bierman et al., 2013) and research with older students (2nd and 3rd graders) suggests that supportive classroom interactions—among peers and between students and teachers—may be important for developing math concept mastery (The Responsive Classroom; Ottmar, Rimm-Kaufman, Larsen & Merritt, 2011). Thus, we might expect more supportive classroom interactions to promote better math outcomes in young children. However, none have explored this link in early childhood settings.

Social-emotional development has long been considered an important aspect of school readiness and, increasingly, is associated with positive long-term academic outcomes (e.g., higher graduation rates; Garces, Thomas & Currie, 2002). Recent attention has shifted to promoting children's early math skills, given the association established with later academic achievement, even across domains (e.g., Duncan et al., 2007) yet the mechanism has not yet been identified.

Promoting Optimal Development of the Whole Child

In designing publicly-funded preschool programs, a persistent dilemma is providing the right balance between academic and social skills. RCTs from the past have shown that direct instruction models can be more successful in yielding immediate academic gains, yet these often fade. More importantly, children in these basic skills programs had less prosocial behavior and more antisocial behavior, which persisted over time (e.g., more felony arrests, Schweinhart & Weikart, 1997). In contrast, in the high-quality, child-centered Perry Preschool project, participants demonstrated better short term (e.g., higher academic gains, less grade retention) as well as long-term academic outcomes (e.g., increased high school completion). Participants had better health (e.g., less drug use) and life outcomes (e.g., less premature death, incarceration) suggesting that high-quality early childhood experiences have lasting effects well beyond the academic domain. It should be noted, however, that initial gains in IQ and achievement tests for Perry participants also faded by first and third grade, respectively (Schweinhart et al., 2005), suggesting that cognitive gains may not have mediated the long-term positive outcomes, and that the social-emotional component may have played a role.

Two critical constructs (i.e., social competence and math) have both been established as important predictors of better academic and life outcomes. It is not known, however, specifically how they relate to one another. Building on both bodies of work, the goal of the present study is to investigate social competence as a potential mediator of the impact of Head Start participation on early math skills.

Background

Head Start (henceforth, HS), a federally funded program, aims to promote school readiness for children in low-income families (median income of $22,714; Administration for Children and Families, ACF, 2014). Initiated during the War on Poverty as part of the Johnson administration in 1965, HS aimed to promote children's development and well-being. HS was originally created to support children's cognitive, social-emotional and physical development and well-being (U.S. Department of Health, Education and Welfare, 1968; Cooke, 1965). While the program has evolved over time and has increased academic content, the current HS Program Standards continue to promote the well-being of the whole child (HS Performance Standards, 2015). In addition, HS mandates that at least 10% of all enrolled participants be children with special needs (Administration for Children and Families, ACF, n.d) to provide an inclusive learning environment for all children.

Social competence, one aspect of social-emotional development, encompasses the effective use of various, interrelated social-emotional skills in social contexts. The extent to which one can coordinate and implement these different, yet complementary skills (e.g., teacher-child interactions, quality and nature of peer interactions) can vary depending on the particular goal (e.g., negotiating conflicts, initiating friendships) and context (e.g., different times, situations; Rubin & Rose-Krasnor, 1992). Exhibiting more competent behaviors in social contexts indicates adaptive emotional well-being, whereas a lack of these skills can indicate maladjustment (Dodge, Pettit, Mcclaskey, Brown & Gottman, 1986).

Learning to take the perspective of others and resolve problems that arise is a typical part of daily interactions in early childhood classrooms. Selman (1980) differentiates social perspective coordination skills (i.e., interpersonal skills) from merely understanding that another may have a different point of view; i.e. developmentally, more adequate social perspective-taking is predicated on understanding how these different points of view "are *related* and *coordinated* with one another" (p. 22, emphasis in original) suggesting that relationships with others are central to development of social competence. It is through frequent, informal interactions—such as those that occur in classrooms—that children learn to navigate social situations and take the perspective of others. Children's social skills and interactions with others, however, can be influenced by many factors. Selman and colleagues (Selman & Demorest, 1986; Selman & Schultz, 1989) argued that more sophisticated social competence was demonstrated by more reflective perspective-taking and cited motivation (in relation to the goal) as a key factor. While they argued that communication between peers was central to resolution, they also argued that children with more social competence would consider the specific circumstances relevant to the person with whom there was conflict (e.g., the shared history, the unique context; Selman & Demorest, 1986). Thus, we might expect that for children in classrooms for whom there are sustained interactions over the course of a year, these relationships with others—with both children and teachers—might also be a good indicator of children's social competence. For example, close teacher-child relationships indicate that children are able to recurrently maintain shared goals with the teacher and preserve the relationship with the teacher across different contexts, whereas more conflicted teacher-child relationships may result if children focus only on the child's immediate goal (e.g., continuing to play without listening to a teacher's directions). These children who fail to tend to the on-going relationship with the teacher (i.e., co-creating shared goals) will have more conflicted relationships and, thus, can be said to have lower social competence.

The Relationship Between Children's Social-Emotional and Cognitive Development

Research has long shown that children's social-emotional and cognitive skills are interrelated and can be enhanced in settings that are both responsive and caring (Shonkoff & Phillips, 2000). As children transition from childcare to more formal classroom settings (e.g., preschool, kindergarten), the learning environment changes in many ways (e.g., more transitions, more formal instructional time). Given this, classroom-based interventions, such as the Chicago School Readiness Project (CSRP), have been designed to promote children's self-regulation (e.g., Executive Functioning) as a mediator for academic achievement. CSRP preschoolers showed greater gains not only in self-regulation but also in receptive vocabulary, letter naming and early math skills, even though the intervention did not explicitly target pre-academic skills (Raver et al., 2011).

Children's social competence—as it is more broadly defined—has also been the basis for interventions to promote early academic skills, though these have—to date—been limited to the literacy domain. For example, a randomized evaluation of the Research-based, Developmentally Informed (REDI) intervention has shown positive impacts on HS children's social competence (e.g., emotional understanding, social problem-solving) and literacy skills (e.g., vocabulary, phonological awareness; Bierman et al., 2013). Follow-up studies have shown sustained positive effects. Specifically, HS children who received the REDI intervention were at reduced risk of showing aggression, experiencing peer rejection and having attention problems in 3rd grade, when they also had closer relationships with teachers and more engaged learning than non-REDI students (Nix et al., 2016), illustrating the ways in which targeting social-emotional skills in preschool can have lasting effects in both the cognitive and social-emotional domains. Thus, an important goal of the present study is to explore how such relationships might mediate academic outcomes—particularly in the domain of math—in HS classrooms.

Links between interpersonal and social problem-solving skills —as indicators of social competence—and early math skills have been less studied. Ginsburg (2006), while acknowledging the importance of instructional supports, declares the social environment as a contributing factor to the development of mathematical knowledge. A recent randomized control trial of middle- and upper-income preschoolers shows that social group membership (i.e., as members of an in-group) can improve performance and task persistence on math and spatial tasks compared to preschoolers who completed these same tasks independently (Master, Cheryan & Meltzoff, 2016). Specifically, Master and colleagues (2016) found that as "in-group" members,[10] children completed STEM tasks

[10] In social psychology, in-group/out-group membership is usually defined by race, class and other social groups by which one may identify. Thus, belonging is usually associated with being "in" one group while feelings of exclusion is associated when not "in" a particular group, particularly when a group is associated with a perceived higher social status. When such studies are conducted with young children, color teams are usually used as "in-groups" as was the case in the Master et al., 2016 study. In that study, all of the children completed the STEM tasks independently; what was manipulated was whether children were working as part of an in-group (to establish a sense of

with greater accuracy, interest, persistence and higher self-efficacy than control group children with no group affiliation, suggesting that feelings of belonging and connectedness among peers may be especially beneficial in math-related activities and instructional contexts. The Responsive Classroom (Ottmar et al., 2011) provides additional evidence, albeit with older children, as it incorporates multiple strategies to support both academic and social-emotional learning with 2nd-3rd graders. In a randomized trial, students who experienced a combination of more responsive teaching and stronger math instruction outscored their peers who experienced only one of these factors. These findings suggest that supportive classroom interactions may be important for developing concept mastery in math. Thus, one might expect that closer (and less conflicted) teacher-child relationships and positive peer relationships in early childhood classrooms might promote better math outcomes.

Much of the research that exists regarding the link between emotion and cognition in math academic task performance that *does* exist has been limited to the impact of negative emotion (e.g., math anxiety). For example, a recent study (Young, Wu & Menon, 2012) showed differences between children who expressed math anxiety among 7-9 year olds undergoing fMRI scans while solving basic addition and subtraction problems, even when controlling for gender, general math ability, working memory (a component of Executive Functioning), general intelligence and/or general anxiety. These findings suggest that emotion can play an important, though not yet fully understood role in math problem-solving as the children with math anxiety utilized different areas of their

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belongingness and group affiliation even though child was still working independently) or "independently" (i.e., no affiliation with an in-group or out-group).

brains while solving math problems, with increased activity in the amygdala (the area associated with fear) and decreased activity in the areas of the prefrontal cortex (the area associated with emotion regulation) as well as areas of the brain associated with working memory and numerical reasoning. These researchers found that not only were these related, but that this increased activity in the amygdala was *causing* the reduced activity in the other parts of the brain (Young et al., 2012) suggesting that emotion can play an important role in cognitive processing and, specifically, may interfere with a child's ability to process the cognitive content as we would typically expect (i.e., by solving addition and subtraction problems solely through activation of areas of the brain associated with numerical reasoning and/or working memory). Moreover, such evidence provides rationale for supporting children's development of closer teacher-child and peer relationships, so that such positive interactions can help to reduce children's fear and anxiety in classroom interactions, generally, and specifically during content-based instructional times when content-based anxiety may particularly play a role.

For children who enter formal schooling with low math skills, teacher-child relationships may be especially important. Crosnoe and colleagues (2010) found that teachers of children with lower math scores (at 54 months) reported more conflict and fewer close relationships than for higher-scoring children. Moreover, children, on average, scored better on grade-level math tests when they had less conflict with their teachers in both 3^{rd} and 5^{th} grade, demonstrating that it may not merely be initial skills that contribute to this trend (of skill begets skill; Heckman, 2008). Instead, the nature of teacher-child interactions may contribute to learning trajectories in ways not previously understood. The present study aimed to investigate this relationship further in the early childhood context to better understand the way in which teacher-child relationships—as one indicator of social competence—may mediate the effect of HS on early math skills.

Relationships with Others

In classrooms, children frequently interact with other children, yet their interactions are monitored—and often facilitated—by adults. Moreover, academic instruction is implemented by teachers, so the relationships that children have with the adults in their classrooms is critical to supporting both social competence and academic mastery, as development of both of these domains requires vulnerability and, at times, a willingness to be wrong and/or consider another's perspective and/or problem-solving strategy (in either/both domains). As such, this work builds upon The Bioecological Model, described by Bronfenbrenner and Morris (1998; 2006) which posits that development is influenced by multiple and contextualized factors—and the interaction between these factors—upon individuals. For children in early care and educational programs, the most proximal interactions (in addition to primary caregivers in the home) would include peers and teachers within the preschool program given the length of these program days. Thus, teacher-child relationships may be a critical mechanism for promoting children's learning across domains during a preschool year. However, these teacher-child relationships are bi-directional, so while the onus may fall upon the adult, it is worth investigating the extent to which these relationships—which are facilitated by a child's social skills with others—may mediate math achievement.

The Role of Language

Early theorists, such as Vygotsky (1962) viewed language as an inherently social process, acquired from repeated interactions and observations with others over time. Specifically, he posited that young children learned to engage in discussions for the purpose of engaging the listener; to communicate his/her thoughts, feelings and ideas with another (Vygotsky, 1962). Thus, language plays an important role as children navigate social problem-solving dilemmas, attempting to communicate their respective viewpoints to the other. Research has shown that higher expressive and receptive language skills have been associated with higher social competence (e.g., peer interactions during play), particularly among low-income preschoolers (Mendez, Fantuzzo & Dante, 2002). Likewise, research has demonstrated that, among low-income populations, receptive language skills are important for effective social competence (e.g., teacher-reported social skills; Longoria, Page, Tait & Kennison, 2009). While the type of special needs was not specified in the HSIS, studies have shown that preschoolers with language impairments have lower social competence (e.g., less appropriate emotion inferencing) compared to typically developing children (Ford & Milosky, 2008). Other research has also long shown that behavior problems and ADHD are associated with early academic skills (e.g., phonological awareness) among low-income preschoolers (Lonnigan et al., 1999).

Hypothesized Link Between Early Math Skills and Social Competence

Early childhood classrooms are the site of frequent interpersonal conflict; learning to take the perspective of others and resolve problems is a typical daily challenge. I hypothesize that children who are supported in developing problem-solving skills with peers and have close, supportive teacher-child relationships may have less conflict—or less prolonged conflict—in their classrooms. As children learn to navigate social situations (e.g., observing what others do when problems arise) and learn to take the perspective of others (Why did she solve it that way? How is that way [also] fair?), they come to understand there are multiple strategies for negotiating effective, fair resolutions.

Previous Findings of the Head Start Impact Study

The HSIS initial report indicated there were no advantages in early math skills for HS attendees (U.S. DHHS, 2010) though later analyses by other researchers—which combined the 3- and 4-year old cohorts—showed that there were gains in early math skills for children assigned to HS (Bloom & Weiland, 2015) suggesting the earlier analyses merely lacked statistical power to show this ITT effect. Evidence of HS effects on social competence was mixed in early HSIS reports and relied primarily on parental reports in the preschool years (U.S. DHHS, 2005). Rather than replicating the HSIS results, the present study investigates teacher-reported social skills and teacher-child relationships as indicators of children's social competence for children in HS and/or center-based care, given that 76% of 3 and 4 year-olds attend (non-parental) early care and education programs, the large majority of which (61%) are center-based settings (National Household Education Survey, 2012). In light of the rapid expansion of preschool programs and given HS's focus on the whole-child (i.e., both the social-emotional and cognitive domains), this study aims to address what may be "driving" these educational outcomes—specifically in early math skills—for children attending HS. Specifically, it asks, to what extent can children's social competence, as indexed by social skills and teacher-child relationships, help to explain the relation between HS assignment and early math achievement?

Research Question

In this study, I examine the potential mediating role of children's social competence in partially explaining the impact of an offer of HS enrollment on children's early math skills. The following specific question guided this work:

Does children's social competence (i.e., social skills, teacher-child relationships) mediate the effects of HS assignment on early math skills (as measured by Woodcock Johnson Applied Problems)?

Method

Participants

This study utilizes a subsample of the HS Impact Study, a large (*N*=4667) randomized control trial conducted to evaluate the efficacy of HS with a nationally representative sample of HS programs and children. In areas with enrollment waitlists, new HS applicants (3 and 4 year-olds) were randomly assigned to either treatment (i.e., assignment to HS; N=2783) or the control group (N=1884). The sample included 383 randomly selected HS centers in 84 randomly selected HS grantees/agencies across 23 states. In total, 4,667 newly entering children participated; 2,559 in the 3-year-old cohort and 2,108 in the 4-year-old cohort (U.S. DHHS, 2010). Many of the control group children still received some type of child care and, in many cases, also received HS services. Children in parental care (n=748) were not administered the social competence measures used in this analysis (by design of the HSIS), so they were excluded from this analysis. Because parental care comprised the majority (79%) of non-center based care and missing data patterns for the social competence measures were more prevalent for children in these types of care settings, which could bias the estimates of HS impacts on social competence, all (control group) children in non-center-based care (n= 952) and care type not specified (n=5) were dropped from the analysis reducing the number of participants to 3485. As a result, this study compares children assigned to HS with those in the control group who attended center-based early care and educational settings. (See Appendix C for a comparison of this analytic sample and those who were in non-center based early care and educational programs.)

Similar to methods used by other researchers (e.g., Bloom & Weiland, 2016; McCoy et al., 2015), this analysis combines the 3 and 4-year old cohorts, so participants ranged in age from two through five years old at the start of the preschool year, with the average child in the analytic sample being just over 3.5 years old. As shown in Table 1, below, the sample is ethnically diverse, with approximately 38% Hispanic children, 33% black children and 30% non-Hispanic white and/or children of other ethnicities and has a fairly equal distribution of boys and girls (49.6% girls). In addition, 13% of participating children were identified as having special needs (type not specified).

Procedures

Direct child assessments were conducted by trained assessors during Fall 2002 (baseline) and Spring 2003 (after 1 year of HS) for children in HS and the control group in the child's primary care setting. Teacher reports of children's social-emotional development were also included in Spring 2003 for children in HS and other non-parental care (e.g., center-based care, family-home providers). Demographic data (e.g., gender, race/ethnicity) was reported by the primary caregiver for children in both the HS (i.e., treatment) and the control group. For additional details on the procedures of the HS Impact Study, see Puma et al., 2010.

Measures

Outcome

Early Math Skills

The Woodcock-Johnson Applied Problems (WJAP; Woodcock, McGrew & Mather, 2001) is a standardized measure capturing math concepts and skills (e.g., problem-solving). Assessor reads prompts to child (with picture cue); child counts and performs math calculations. Analyses were based upon W scores, obtained by IRT models used in the HSIS (U.S. DHHS, 2010). WJAP has shown high reliability (0.8-0.9) across the 3 and 4 year-old cohorts in this study.

Predictor

Head Start Assignment

In this analysis, HS treatment represents random assignment to HS (i.e., the intent-to-treat (ITT) estimates of HS treatment). It does not account for actual participation or cross-overs (i.e., those who were assigned to treatment, but did not participate or those who were assigned to the control group, but managed to get into HS anyway).

Mediator

Social Competence: Children's Social Skills and Teacher-Child Relationships

Children's Social Skills

Two different measures were used to capture children's social competence, as indicated by children's social skills with their peers and their relationships with their teachers.

Child Observation Record (COR). HS teachers observed and assessed children using five selected items from the Initiative/Social Relations subscale of the COR. Teachers reported the quality and nature of children's efforts to engage in social problem-solving and interact with peers (e.g., "how well child makes friends"). Items were rated on a Likert-type scale (ranges 1-5) with lower numbers representing lower social skills. The COR has shown high reliability (α =0.75-0.82) and good inter-rater reliability (*r*=.69) for these items in prior work (High/Scope Educational Research Foundation, 2005).[11] In the HSIS, item-level data are available for COR.

Teacher-Child Relationships

Teacher-Child Relationship Scale (TCR). Children's relationships with their teachers were assessed using a composite of two subscales (i.e., Closeness and Conflict, reverse-coded) of the TCR scale, short-form (Pianta, 1996). The TCR captures a teacher's perception of closeness and conflict with specific children in her classroom as an indicator of children's social-emotional development. Specifically, the Closeness subscale uses a 5-point Likert scale (1=definitely does not apply, 5= definitely applies) and includes seven items with raw scores ranging from 7 to 35 (some items reverse-coded) that were summed to capture the degree to which the teacher-child relationship is supportive and/or effective (e.g., "When upset, this child will seek comfort from me"). In prior work this scale has shown test-retest reliability of 0.88 for Closeness and good internal consistency (α=0.86; Pianta, 2001).[1]

[11] Since the HSIS did not report outcome data for COR or the TCR scale for children during the HS year, psychometrics were not reported in the technical manual (Camilla Heid, personal communication, April 29, 2016); thus, psychometrics have been obtained from original documentation for each scale.

The Conflict subscale uses the same 5-point Likert scale and includes eight items with raw scores ranging from 8 to 40 (some items reverse-coded) that were summed to capture the degree to which the teacher-child relationship is negative and/or ineffective (e.g., "This child easily becomes angry with me"). In previous work, this subscale has shown high test-retest reliability (0.92) and good internal consistency (α=0.92; Pianta, 2001). Only subscale-level data are available for the TCR in the HSIS.

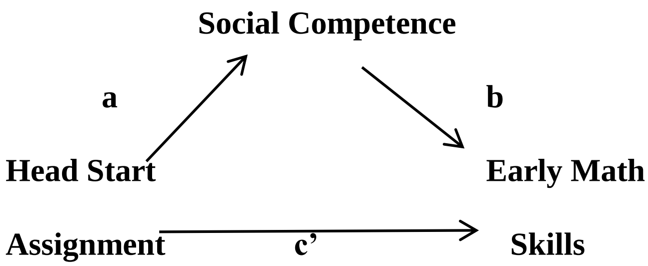
Control Variables

The Peabody Picture Vocabulary Test (PPVT-III). Given the important role that language plays in both social and academic contexts, particularly for young children who are rapidly acquiring language at this age, receptive language was included as a control variable as measured by the Peabody Picture Vocabulary Test (PPVT-III). The PPVT-III (Dunn & Dunn, 1997) is a standardized, norm-referenced measure of receptive vocabulary. In this assessment, children are presented with four pictures per page. The assessor states a word (e.g., penguin), and the child is asked to point to the corresponding picture. The HSIS used item response theory (IRT) to develop a shorter version of the PPVT-III to minimize test-taker fatigue. Spring W-scores (age-normed) are reported here (U.S. DHHS, 2010).

Child-Level Covariates. Given the rationale outlined in the literature review, a set of child-level covariates has been included in the analyses, including child gender, age at spring testing, special needs status (i.e., an indicator variable based upon parent report) and race (i.e., an indicator variable for Hispanic or black). These characteristics were reported by the primary caregiver in the fall. Baseline WJAP scores was also included to control for each child's math skills at preschool entry.

Analytic Plan

Figure 1. Path diagram of mediated model.



This analysis was conducted using Structural Equation Modeling (SEM) to test the direct and indirect relationships between variables of interest while accounting for children's nesting within classrooms. Specifically, a two-step modeling approach was used that included 1) developing and confirming a measurement model of social competence using Confirmatory Factor Analysis (CFA) and 2) composing a partially latent structural regression model. Robust standard errors were used to account for nesting of children within HS centers.

Confirmatory Factor Analysis. First, a CFA was conducted to test the extent to which the five COR items and two TCR subscales collectively capture the single latent construct of "social competence" using MPlus Version 7.31 (Muthen & Muthen, 2015). While both measures were assessed by the same teacher, it is likely that the COR and TCR were assessed on different days. That said, teachers likely assessed children on a given scale (e.g., assessing a child on all items related to teacher-child relationships on the TCR) on the same day. Given this, errors from the same measure (e.g., errors across TCR items) are likely to be more highly correlated than errors across measures (i.e., errors across items from TCR and COR). As such, the residuals for the indicators within each scale (i.e., across the 5 COR items and across the two TCR subscales) were allowed to covary.

Models are considered to be an adequate fit by several indicators. Specifically, the Comparative Fit Index (CFI) test indicates acceptable fit (>.90-.95 is acceptable, >0.95 considered ideal). The Root Mean Square Error of Approximation (RMSEA) adjusts for model parsimony (RMSEA<0.05-0.08 is considered acceptable, <0.06 considered ideal). The Standardized Root Mean Square Residual (SRMR) is an indicator of the proportion of residuals (<0.06-0.08 is considered acceptable, <0.08 considered ideal; Hu & Bentler, 1999). Each will be provided and discussed.

Structural Equation Modeling (SEM). To address the question of mediation, a structural equation modeling (SEM) framework was used to test the direct and indirect relationships between variables of interest while accounting for children's nesting within classrooms. Specifically, the TYPE = COMPLEX MPlus command was used to account for the clustering of children within classrooms. Robust standard errors were used (MLR) to estimate standard errors and confidence intervals for the generated estimates. Using SEM, direct (path c, in Fig.1 above), indirect (path axb) and total effects (path c'=[axb]+c) of the relationship between HS random assignment and math skills via the mediator of social competence were tested.

Missing data was noted for 20.92% of cases for spring math scores and 36.00% of cases were missing one or more of the social competence indicators (COR and TCR items; 31.00% were missing all seven indicators while all other social competence patterns were less than 1% each), 27.69% of cases across baseline (fall) math scores and 21.87% of cases across covariates were missing. Missing data were assumed to be missing at random (MAR). Correlations and logit models indicated that missingness on COR and TCR measures (which yield the social competence factors) was positively related to variables in the sample, including type of child care setting, age, child ethnicity, gender, fall PPVT score, confirming that data was not missing completely at random (MCAR). A multiple imputation strategy using chained equations (MICE) approach was used to account for missing data to better estimate the missing data patterns. Once it had been established the data were MAR, MICE generates multiple estimates for each missing value based upon other observed values in the dataset (Enders, 2010; Little & Rubin, 2002). Two-tailed t-tests did not show significant differences in non-missing data between children who had missing values on demographics and those who had valid data on these variables. It is possible, however, that the missing data depended on unobserved variables not available in the dataset since the MAR assumption can never fully be tested. Twenty imputed data sets were generated and included the noted correlated variables (described above) as auxiliary variables in analyses, which helps to attenuate bias (Enders, 2010; Allison, 2012) compared to other approaches used to address missing data (e.g., listwise deletion) and can help improve the likelihood of meeting the MAR assumption (Allison, 2012). Finally, several variables were re-scaled using linear transformation for analyses to make coefficients more interpretable. Specifically, age was centered at 0 and divided by 4 (i.e., reported in months instead of weeks) and the TCR subscales were divided by 10 so variance was more comparable to that of the indicators of COR to help model convergence.

Results

The results are presented in three parts, starting with descriptive statistics. Then, results from the measurement model examining the hypothesized latent construct, Social Competence, will be presented. Finally, results from the partially latent structural equation model examining the direct and indirect effects of Social Competence on children's early math skills will be presented.

Descriptive Statistics

Descriptive statistics for the sample demographics and for the Closeness and Conflict subscales are reported in Table 1 (below). Prior to conducting the CFA, the data were examined to ensure that the necessary assumptions had been met. Visual inspections of the data and examination of the basic descriptive statistics revealed some skew of all indicators of Social Competence and math scores, though the skew was considered acceptable in most cases (e.g., generally less than 2 for each). With the two subscales, Closeness and Conflict, however, kurtosis was considerable (i.e., 5.04 and 4.19 respectively), providing rationale for using Maximum Likelihood Robust (MLR) Estimation, which is a sandwich estimator that is robust to non-normality and non-independence of observations when used with TYPE=COMPLEX (Muthen & Muthen, 2015).

As expected, there are (see Table 2, below) moderate to strong, positive correlations among the Social Skills indicators (ranging from r = .42 to r = .55) with strong internal consistency for the entire scale ($\alpha = 0.80$). Similarly, the correlation is positive and moderate across the teacher-child relationship scales of Closeness and Conflict (reverse-coded, r=0.26). Across indicators, however, correlations are fairly weak though still positive (*r*=0.13 to *r*=0.27), which is also to be expected since these are capturing different aspects of children's social competence.

Table 1. Descriptive statistics of analytic variables

	Full Analytic Sample			Treatment Group			Control Group		
Spring Outcome Scores	N	Mean	SD	N	Mean	SD	N	Mean	SD
Spring WJAP	2756	385.97	27.86	2089	386.19	27.53	667	385.28	28.85
Child Solves Problems	2373	3.48	1.02	1929	3.47	1.00	444	3.55	1.08
How Child Does in Complex Play	2355	3.50	1.12	1919	3.49	1.11	436	3.53	1.16
How Well Child Makes Friends	2345	3.78	1.07	1911	3.79	1.06	434	3.74	1.11
How Well Child Works W/Children	2356	3.45	1.13	1914	3.45	1.12	442	3.43	1.16
How Well Child Expresses Feelings	2369	3.41	1.19	1924	3.39	1.19	445	3.51	1.17
TCR Closeness	2398	3.06	0.42	1943	3.06	0.42	455	3.06	0.42
TCR Conflict (reverse coded)	2391	3.12	0.61	1938	3.13	0.61	453	3.07	0.63
Fall Outcome Scores									
Fall PPVT	3433	91.03	8.58	2377	90.94	8.44	1052	91.22	8.88
Fall WJAP	2520	376.94	27.72	1735	377.05	27.49	785	376.59	28.25
Demographic Characteristics									
White/Other	3485	0.30	0.46	2414	0.30	0.46	1071	0.28	0.45
Black	3485	0.33	0.47	2414	0.32	0.47	1071	0.35	0.48
Hispanic	3485	0.38	0.48	2414	0.38	0.49	1071	0.37	0.48
Female	3485	0.50	0.50	2414	0.50	0.50	1071	0.48	0.50
Spring Test Age	2784	48.37	8.06	2104	48.35	7.96	676	48.48	8.36
Special Needs	3485	0.13	0.34	2414	0.14	0.35	1071	0.12	0.33

Preliminary Model: Confirmatory Factor Analysis (CFA)

A CFA was used to evaluate the structural validity of these indicators used in the HSIS data set (*N*=2421) to test whether these different aspects of children's social competence (i.e., positive social skills with peers and positive teacher-child relationships) represent one latent construct, Social Competence. After restricting the HSIS sample to those that had one or more indicators on COR and/or TCR, missing values (less than 8% of the CFA sample) were then filled in using Full Information Maximum Likelihood (FIML), the standard protocol in MPlus. Standardized factor loadings show moderate to strong, positive and significant correlations between the observed indicators and the latent construct of Social Competence ($e.g. for\ SocialSkills3: beta = .67, SE = .04, p < 0.001; Closeness: beta = .37, SE = .03, p < 0.001$). Several model fit indices indicated adequate model fit ($\chi^2(4), p < 0.001$, CFI = 0.99, RMSEA = 0.06 and the SRMR = 0.02).

Table 2. Correlations among variables of interest in study.

How well child…	Solves Problems	Does in Complex Play	Makes Friends	Works with Children	Expresses Feelings	TCR Closeness	TCR Conflict
Solves Problems	1.00						
Does in Complex Play	0.55	1.00					
Makes Friends	0.42	0.48	1.00				
Works With Children	0.48	0.47	0.47	1.00			
Expresses Feelings	0.45	0.44	0.47	0.56	1.00		
TCR Closeness	0.27	0.23	0.28	0.30	0.32	1.00	
TCR Conflict	0.18	0.13	0.20	0.32	0.30	0.28	1.00

Note: TCR Conflict is reverse coded.

Structural equation modeling

Results of the SEM structural model indicated adequate overall model fit by several model fit indices ($\chi^2(4), p < 0.001$, CFI = 0.95, RMSEA = 0.05 and the SRMR = 0.04). Within this model, evidence was mixed regarding the direct pathways, some of which support my hypothesis (as shown in Figure 2, below). Specifically, children's social competence was significantly and positively related to math achievement scores ($beta = 0.17, SE = 0.03, p < 0.001$ see Table 1 in Appendix D), controlling for HS random assignment, gender, receptive language skills, age and special needs status. Consistent with the broader literature, children randomly assigned to HS showed lower social competence ($beta = -0.02, SE = 0.02, p = 0.34$) than their peers assigned to the control group who were largely enrolled in center-based care programs, though this difference was not statistically significant. Given the lack of direct effect of HS assignment on social competence and math, there was no evidence for mediation in the present sample. Results of the test for an indirect pathway revealed no significant indirect relationship between assignment to HS and spring math outcomes via social competence ($beta = -0.24, SE = 0.25, p = 0.36$).

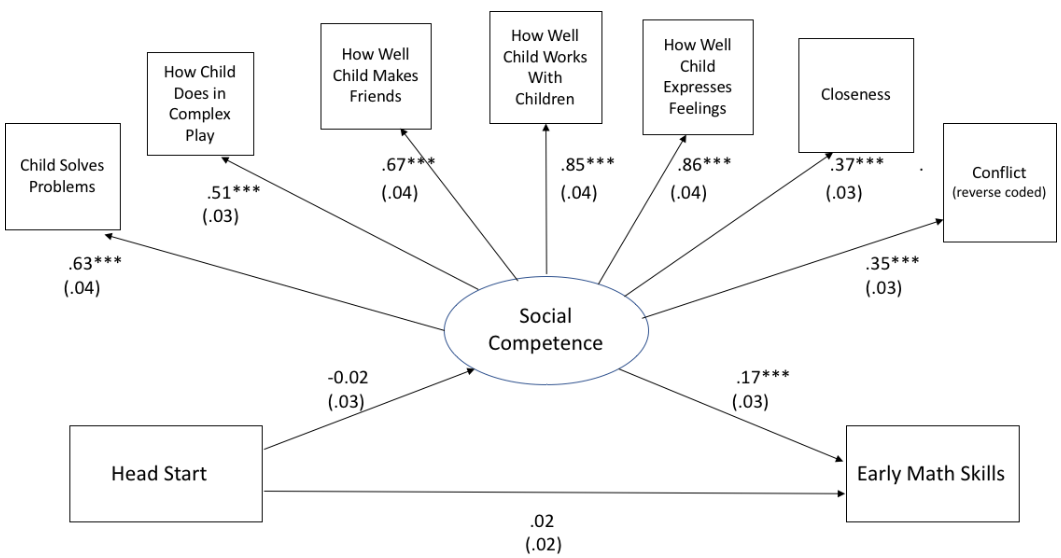


Figure 2. Standardized results of the partially-latent structural regression model. Notes: Residual variances between the COR indicators and TCR subscales correlated but not shown. Model controls for child gender, child race, child age, receptive language, special needs status and baseline WJAP; standardized estimates are reported. All standard errors adjusted to account for children's nesting within classrooms.

Discussion

The aim of the present study was to investigate whether children's social competence mediated the relationship between HS assignment and children's early math achievement. In order to address the primary research question of interest, a latent construct “Social Competence” was constructed using five items from COR that capture children’s social skills when interacting with other children and two subscales from the Teacher-Child Relationship scale, Closeness and Conflict, that capture the dynamics of the teacher-child relationship. There is a positive and significant predictive relationship between the latent construct identified in this study (Social Competence) and early math skills that is worthy of greater attention and research as these findings suggest that positive peer relationships and Close (and less Conflicted) teacher-child relationships may play an important role in predicting children’s early math skills during the HS year. Though links between interpersonal and social problem-solving skills —as indicators of social competence—and early math skills have not been widely documented, these findings are consistent with previous research. A recent meta-analysis of 213 studies, for example, showed that SEL programs implemented in schools (K-12th grade) are effective in supporting both social and academic outcomes, illustrating that gains in social-emotional outcomes may lead to indirect improvements in academic outcomes (Durlak, Weissberg, Dymnicki, Taylor & Schellinger, 2011). Specifically, SEL program participants, overall, showed more social skills, positive attitudes toward self and others, and positive social behavior, as well as fewer conduct problems and less emotional distress. Importantly, when school staff (as opposed to researchers) implemented interventions, students also showed improved achievement test scores and grades (Durlak et al., 2011), illustrating the important role that teachers can play in promoting both social-emotional and academic outcomes.[12]

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[12] Many studies in this meta-analysis support causal inferences

Instructional supports and use of research-based curricula are critical components of a high-quality early childhood care and educational setting (Clements & Sarama, 2007b; Ginsburg, Inoue & Seo, 1999), yet research has also shown the social environment contributes to the development of mathematical knowledge (Ginsburg, 2006). Moreover, interventions such as The Responsive Classroom (Ottmar, Rimm-Kaufman, Larsen & Merritt, 2011) show that the combination of more responsive teaching *and* stronger math instruction led to higher scores for students than merely experiencing only one of these factors. Such finding suggest that supportive classroom interactions—such as those that would promote children's social competence—may be instrumental in developing concept mastery in math and that a stronger academic focus absent these types of environments may not yield the desired academic outcomes. These results provide additional evidence that helping young children develop social competence and close teacher-child relationships can also support development of their early math skills as part of an enriching, high quality instructional learning environment that supports the whole child.

The present study, however, finds no evidence of a mediating role of social competence on children's early math skills. Despite the long line of research showing the inter-relatedness of children's social-emotional and cognitive skills, which can be enhanced in responsive and caring settings (Shonkoff & Phillips, 2000), there has been mixed evidence of mediation across domains in early childhood classroom contexts. It should be noted, however, that much of this work relating to early math skills has focused on the relationship with self-regulation. Self-regulation is the ability to regulate one's thoughts, behaviors and emotions and includes all developmental domains (i.e., cognitive, behavioral and social-emotional). In many cases, however, researchers use measures that do not tap into all domains of self-regulation; many studies only link children's regulation of the cognitive domain (i.e., working memory, Executive Functioning) to academic outcomes. Though this may be taken as evidence for a role for "social-emotional" development because cognitive regulation is one component of the broader construct of self-regulation, the skills being measured may all be within a single domain, i.e. cognitive skills. The link between Executive Functioning (EF) and early math skills is becoming particularly well-established, though it is not clear that this is actually a cross-domain transfer of skills given how EF is measured in these studies. In this context, EF has been associated with early math skills in early childhood (Blair & Razza, 2007; McClelland, Cameron, Connor, Farris, Jewkes & Morrison, 2007) and some studies have found evidence of a mediating relationship in early childhood settings (e.g., Raver, Jones, Li-Grining, Zhai, Bub & Pressler, 2011) while others have not yet demonstrated a mediating link (e.g., *Tools of the Mind*, Barnett, Jung, Yarosz, Thomas, Hornbeck, Stechuk & Burns, 2008) in these classroom contexts. Recent efforts to document the various ways that EF is measured in studies identified more than 40 unique terms (e.g., working memory, self-regulation) as indicators of EF (Jones, Bailey & Partee, 2015) illustrating the need for clearer definitions of this array of constructs to better understand the unique role of each in promoting key outcomes of interest. Importantly, Jones and colleagues (2015) also noted that regulation-related skills involving problem-solving and regulating emotions (e.g., emotion regulation) often utilize cognitive-based EF skills (e.g., goal-setting) as well skills from the social-emotional domain (e.g., recognizing emotions). Such findings suggest that children may need to utilize both types of skills in developing positive social skills to effectively interact with others, though more research is needed in this area to better understand how these skills work in concert.

While previous studies have shown links between EF and early math skills (Blair & Razza, 2007; McClelland et al., 2007), other researchers have begun to question the directionality of this relationship, suggesting that it may be early math skills—and related instructional activities—that promote EF skills in young children (Clements, Sarama & Germeroth, 2016) given that children actively problem-solve in ways quite different from adults (i.e., often relying on memory retrieval). Moreover, the research showing that the areas of the brain responsible for (respectively) EF and social-emotional processing are simultaneously active during math problem-solving (Young et al., 2012) further suggests that the association between EF and early math skills is in need of more investigation, so that these relationships can be more fully understood. The relationship between EF and early math skills thus far established may be spurious (i.e., if emotional processing was not measured in these other studies), if they are merely secondary to the primary emotional component that has not yet been fully investigated or, as Clements and colleagues (2016) have suggested, the relationship may very well be real but an effect of math on EF rather than EF on math.

Another important consideration is the extent to which HS may have negatively impacted children's development of social skills and close teacher-child relationships compared to those in other center-based early care and educational settings. This study showed a negative (though non-significant) impact on children's Social Competence for children who attended HS in this sample. As HS programs have become increasingly focused on academic outcomes over time, and particularly since 2001, (see, for example, "Getting a Head Start," 2001), it is possible that the focus on children's social competence has been compromised or minimized. While HS aims to promote development of the whole child, the findings from the present study provide no evidence of positive impacts on either children's social competence or early math skills.

As noted above, prior work using the HSIS dataset has been inconsistent in identifying a significant relationship between HS assignment and children's math outcomes (U.S. DHHS, 2010; Bloom & Weiland, 2015). In this study, there is no statistically significant effect of HS assignment on children's math. While it is possible that neither set of programs (i.e., HS or control group center-based programs) were implementing math instructional programs effectively, the null findings for impacts on math are more likely due to the adjusted control group used in this analytic sample. By excluding those in parental care and family-based care, children who were most likely to be different from the treatment group were also excluded. Specifically, it is more likely that children in center-based settings were participating in instructional routines more similar to those of HS programs than children in parental and family-based care. If this were the case, it would not be possible to detect a significant difference in the HS treatment effects on math, even if HS was implementing math programs effectively (or, more effectively than those in more typical control groups). Similarly, this study shows a non-significant impact on children's social competence. Much of the reported data from the HSIS regarding children's social-emotional development during the preschool years "relied solely on behavior reports from parents" (U.S. DHHS, 2005, p 6-1). It is not surprising, then, that this analysis yields findings that are different from those published in the initial reports (Puma et al., 2010; U.S. DHHS, 2010), since this analysis relies, instead, on information from teachers and caregivers other than parents. However, these findings are consistent with other randomized analyses of HS programs. Specifically, the What Works Clearinghouse has shown “no discernible effects” of HS participation on social-emotional development measures, i.e., percentage point gains (or decreases) ranging from -1 to +5. (U.S. Department of Education, 2015). If this is indeed the case among HS programs in the broader population, then a mediation would not be possible to model. HS programs may need support in developing more effective ways to promote social competence—specifically by developing better social skills among children as well as positive teacher-child relationships—and how to integrate these with more academically rigorous curricula. As with early math skills, however, the failure to find a mediating mechanism may reflect the changed comparison groups in this analysis. When more HS programs, on average, support the development of social competence in young children—and certainly many do—it may be possible to model this mediating path, if it indeed exists in the broader population.

Importantly, this study’s findings suggest that relationships with others may be important for supporting early math skills, particularly among children from low-income families. While perhaps not traditional indicators of children’s social competence by current metrics, the theoretical basis for considering the context and nature of children’s interactions with other children and their teachers in early care and educational classrooms is strong and the present study provides evidence (e.g., good model fit of the CFA, evidence of structural validity, etc.) that these indicators warrant further attention as positive indicators of children’s social competence.

While multiple predictors of academic achievement exist, Duncan and colleagues (2007) concluded from a meta-analysis that math skills at kindergarten entry are the *strongest* predictor of later academic achievement. That meta-analysis was based, however, upon several correlational studies and could not isolate a causal mechanism. Therefore, a critical question still remains regarding how to best promote children's early math skills. Importantly, Duncan and colleagues (2007) also acknowledged that most of the studies measured only a limited set of social-emotional constructs, with a focus on negative indicators (e.g., externalizing, internalizing behaviors). There has been little investigation into the role of positive indicators of social-emotional development—and no investigation into the role of social competence—in predicting children's early math skills. While this study also cannot isolate a causal mechanism, it has generated a more nuanced understanding of this cross-domain relationship. While no statistically significant mediation was found in this analysis, the findings from this study add to this body of literature in two important ways. First, the latent construct Social Competence comprised of these specific elements shows positive and significant utility in predicting children's early math skills. Second, these findings suggest that these particular social skills and relationships with others—and their combination as a unique latent construct—hold particular promise as a predictor of early math skills that warrants further exploration, so that they can be better understood for their role in early childhood care and educational settings serving children from low-income families.

Limitations and Future Research

There are several limitations of this study's findings. The current analyses were an attempt to better understand children's social competence as a mediating mechanism for explaining HS's impact on early math skills. While children were randomly assigned to HS, of course, social competence was not randomly assigned to children, limiting the extent to which inferences about causation can be made, even if mediation had been found. While causal claims about the relation between social competence and math skills cannot be made with this analysis, this mediation model helps us to understand the mechanisms that may be driving early math skills, which has policy and practice implications as associations were found between children's social competence and early math skills that warrant further investigation. Another key limitation is that Social Competence was measured at one time point only and only in the spring, which does not enable one to model change in children's social competence during the preschool year. Having fall (as a baseline) or fall and spring (to show change over time) would enable better modeling of the potential mediating relationship between HS and early math skills, if one indeed exists.

A central challenge is the non-random missing data patterns given the particular variables identified for this analysis (i.e., planned missingness for children in parental care and those whose care providers did not complete the teacher-reports). While this is a teacher-reported measure by design, having this information omitted from children in parental care severely limits the comparison group, decreasing statistical power and the external validity of study findings. While excluding this group of children enabled a more focused analysis of those in group-based child care settings, it may have compromised the original RCT design of the HSIS. More controlled studies may be needed to better understand this relationship given that a significant portion of the control group had to be discarded. Future research, therefore, might also conduct an analysis using a different analytic sample, so that randomization among children in center-based care can be the target population of inference, which is not possible in the HSIS.

Another limitation of the present study is that it does not explicitly include information on classrooms' use of specific curricula, which may include math, social-emotional content, both or neither. Considerable variation has been noted in the impacts of publicly funded early childhood programs and, specifically, in HS programs (e.g., Bloom & Weiland, 2015; Gilliam & Zigler, 2000) for both cognitive and social-emotional outcomes. Each HS program tailors its curriculum to match the needs of the children served (ACF, 2015) within the guidelines of the HS Performance Standards. While such adaptations aim to best suit the local populations, each can be a source of variation in program quality and, by extension, child-level outcomes. While TYPE=COMPLEX accounts for clustering of the children in classrooms, it does not fully account for the ways in which the specific characteristics of each center or HS grantee might play a role in how (and which) curricula are selected and implemented.

Although this study utilizes the Head Start Impact Study, allowing inclusion of a large number of children from HS classrooms across many contexts, these findings cannot be a strong basis for isolating the effects of HS for two reasons. First, since COR and TCR were not administered to children in parental care, those children were excluded from the analysis. Second, as a result of the first analytic decision, it was then not appropriate to use the sample weights. As such, these findings best generalize to preschool children from low-income families who are in center-based or family-based early care and educational settings.

Implications for Policy and Practice

Social-emotional development has long been considered an important aspect of school readiness and, increasingly, is being associated with positive long-term outcomes in both academic (e.g., reduced grade retention, higher graduation rates; e.g., see Garces, Thomas & Currie, 2002) and non-academic (e.g., lower mortality and delinquency rates, Ludwig & Miller, 2007; Schweinhart et al., 2010) domains. The accumulating evidence that children's social-emotional development may be an important predictor of academic achievement (e.g., Schweinhart et al., 2010; Nix et al., 2016) has important implications for the curricular focus of preschool programs.

This study has helped to generate a more nuanced understanding of this cross-domain relationship and has identified an important set of positive indicators of Social Competence (i.e., children's social skills and teacher-child relationships) that have a significant and positive predictive relationship for children's early math skills among children from low-income families. Results from this study can provide insight into potential mechanisms that can guide future investigations and yields important policy and practical implications to guide curricular decision-making and time-allocation in programs that serve children from low-income families.

Conclusion

As publicly funded preschool programs have become more numerous, multiple evaluations of their effectiveness have been conducted. While developing academic skills has long been considered one important goal in high-quality early childhood programs, academic preparation now appears to be the primary—and in some cases sole--focus of such programs. Of the recent efficacy trials of state-funded PreK programs (i.e., one particular type of publicly funded preschool program), only two studies measured social-emotional skills as outcomes of interest (Lipsey et al., 2013 and Peisner-Feinberg & Maris, 2005) despite the ample evidence that these skills are important aspects of school readiness. In many early childhood efficacy trials, when these metrics have been included, however, links continue to be established between negative indicators of social-emotional development (e.g., problem behaviors) or compliance-oriented skills in the social context of the classroom (e.g., following directions) rather than a broader conceptualization—and demonstration—of children's social competence.

While the scope of both of the studies presented here is somewhat limited (i.e., small sample size in Study 1, limited comparison groups in Study 2), they nonetheless make an important contribution to the literature in three ways. First, Study 1 provides a more nuanced understanding of the ways in which low-income children's development varies within and across these two domains during a preschool year. Second, the results from these studies introduce the possibility that children's social competence may promote math skill development during preschool. Third, the results from these studies illustrate the importance of investigating a broader range of indicators of social competence—including positive indicators of social competence—for a potential protective role in promoting academic achievement, especially among children from low-income families.

For many years, research has demonstrated the importance of implementing early childhood programs that promote children's development across domains (i.e., physical, cognitive and social-emotional development). As states increasingly implement publicly funded early care and educational programs—particularly in our efforts to close the achievement gap—it will be important to consider—and measure—the effectiveness of these programs across a broad range of outcomes (i.e., social-emotional and academic) and contexts (PreK as well as community-based settings) to better understand and quantify the effectiveness of these preschool programs in practice. Teaching and learning is a dynamic process that is best understood in context. Children acquire early math concepts as they are simultaneously interacting with other children and their teachers in classrooms, so we must more intentionally consider—and measure—the ways in which these constructs develop simultaneously and in conjunction with one another. Learning to skillfully navigate difficult situations as they emerge is a key 21st Century Learning skill that can help all children build the tools they need to critically think, reflect and effectively communicate their ideas and perspectives with others. We must begin this important work in preschool by better designing challenging, yet developmentally-focused instructional learning environments that support early learners as they become critical and flexible problem-solvers in both academic and social contexts.

References

(Essay 1)

Administration for Children and Families. (2015). Head Start program fact sheet fiscal year 2015. Retrieved May 15, 2016 from: https://eclkc.ohs.acf.hhs.gov/hslc/data/factsheets/2015-hs-program-factsheet.html

Ansari, A., Kelly Purtell, & Gershoff, E. (2016). Classroom Age Composition and the School Readiness of 3- and 4-Year-Olds in the Head Start Program Arya. *Psychological Science*, *27*(1), 53–63.

Aratani, Y., Wight, V. R., & Cooper, J. L. (2011). *Racial gaps in early childhood: Socio-emotional health, developmental, and educational outcomes among African-American boys*. Retrieved from http://www.nccp.org/publications/pdf/text_1014.pdf

Balfanz, R., Ginsburg, H. P., & Greenes, C. (2003). The Big Math for Little Kids early childhood mathematics program. Teaching Children Mathematics, 9(5), 264- 268.

Barnett, W. S., Jung, K., Yarosz, D. J., Thomas, J., Hornbeck, A., Stechuk, R., & Burns, S. (2008). Educational effects of the Tools of the Mind curriculum: A randomized trial. *Early Childhood Research Quarterly*, *23*(3), 299–313.

Bassok, D., Latham, S., & Rorem, A. (2016). Is Kindergarten the New First Grade? *AERA Open*, *1*(4), 1–31.

Bell, E. R., Greenfield, D. B., & Bulotsky-Shearer, R. J. (2013). Classroom age composition and rates of change in school readiness for children enrolled in Head Start. *Early Childhood Research Quarterly*, *28*(1), 1–10.

Bierman, K. L., Domitrovich, C. E., Nix, R. L., Gest, S. D., Welsh, J. A, Greenberg, M.

T., & Gill, S. (2013). Promoting academic and social-emotional school readiness: The Head Start REDI program, *79*(6), 1802–1817.

Bloom, H. S. & Weiland, C. (2015). Quantifying variation in Head Start effects on young children's cognitive and socio-emotional skills using data from the National Head Start Impact Study. Retrieved from: http://www.mdrc.org

Bredekamp, S. & Copple, C. (1997). *Developmentally appropriate practice in early childhood programs*. Washington, D.C.: National Association for the Education of Young Children.

Chesloff, J. D. (2013). Why STEM education must start in early childhood. *Education Week*, pp. 32–27.

Clements, D. H., & Sarama, J. (2007a). Building Blocks-SRA Real Math, Grade PreK. Columbus Ohio: SRA/ McGraw-Hill.

Clements, D. H., & Sarama, J. (2007b). Effects of a preschool mathematics curriculum: Summative research on the building blocks project. *Journal for Research in Mathematics Education*, *38*(2), 136–163.

Cooke, R. (1965). *Recommendations for a Head Start program*. U.S. Department Of Health, Education, And Welfare Office Of Child Development. Retrieved from http://eclkc.ohs.acf.hhs.gov/hslc/hs/about/docs/cooke-report.pdf

Crosnoe, R., Morrison, F., Burchinal, M., Pianta, R., & Keating, D. (2010). Instruction, teacher–student relations and math achievement trajectories in elementary school, *Journal of Educational Psychology,102*(2), 407–417.

Demar, B. J. (1997). A school-based group intervention to strengthen personal and social competencies in latency-age children. *Social Work in Education, 9*(4), 219–230.

Denham, S.A., Bouril, B., & Belouad, F. (1994). Preschoolers' affect and cognition about challenging peer situations. Child Study Journal. 1994; 24:1–21

Denham, S. A., Bassett, H., Mincic, M., Kalb, S., Way, E., Wyatt, T., & Segal, Y. (2012). Social-emotional learning profiles of preschoolers' early school success: A person-centered approach. *Learning and Individual Differences*, *22*(2), 178–189.

DeVries, R., & Zan, B. (1994). *Moral classrooms, moral children: Creating a constructivist atmosphere in early education*. New York: Teachers College Press.

Dodge, K. A, Pettit, G. S., Mcclaskey, C. L., Brown, M. M., & Gottman, J. M. (1986). Social Competence in Children. *Monographs of the Society in Social Competence*, *51*(2).

Duncan, G. J. & and Magnuson, K. (2011). The nature and impact of early achievement skills, attention skills, and behavior problems. In G. J. Duncan & R. Murnane (Eds.), *Whither opportunity? Rising inequality, schools, and children's life chances* (pp. 47-69). New York: Russell Sage Foundation.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446.

Dunn, L. M., & Dunn, L. M. (2007). Peabody Picture Vocabulary Test-Fourth Edition. Bloomington, MN: Pearson Assessments.

Durlak, J. A, Weissberg, R. P., Dymnicki, A. B., Taylor, R. D., & Schellinger, K. B. (2011). The impact of enhancing students' social and emotional learning: A meta-analysis of school-based universal interventions, 405- 432.

Foster, M. E., Anthony, J. L., Clements, D. H., & Sarama, J. H. (2015). Processes in the development of mathematics in kindergarten children from Title 1 schools. *Journal of Experimental Child Psychology*, *140*, 56–73.

Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2014). Teaching Math to Young Children, 165. Retrieved from http://whatworks.ed.gov

Garces, E., Thomas, D. & Currie, J. (2002, September). Longer-term effects of Head Start. *American Economic Review, 92* (4): 999-1012.

Garcia, E. (2015). *Inequalities at the Starting Gate: Cognitive and Noncognitive Skills Gaps between 2010-2011 Kindergarten Classmates*. Retrieved from http://www.epi.org/publication/inequalities-at-the-starting-gate-cognitive-and-noncognitive-gaps-in-the-2010-2011-kindergarten-class/

Gilliam, W. S. (2005). Prekindergarteners left behind: Expulsion rates in state prekindergarten programs (FCD Policy Brief No. 3). New York: Foundation for Child Development

Ginsburg, H. P. & Baroody, A. J. (2003). The test of early mathematics ability (3rd ed.). Austin, TX: Pro Ed.

Ginsburg, H. P., Inoue, N., & Seo, K.-H. (1999). Preschoolers doing mathematics: Observations of everyday activities. In J. Copley (Ed.), Mathematics in the early years (pp. 88-99). Reston,VA: National Council of Teachers of Mathematics

Ginsburg, H.P., Lee, J.S. & Boyd, J.S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report* (Vol. XXII). Retrieved from ERIC.

Ginsburg, Herbert P. (2006). Mathematical play and playful mathematics: A guide for early education. In D. Singer, R. M. Golinkoff & K. Hirsh-Pasek (Eds.), *Play = learning: How play motivates and enhances children's cognitive and social-emotional growth* (pp. 145-165). New York: Oxford University Press.

Guo, Y., Tompkins, V., Justice, L., & Petscher, Y. (2014). Classroom Age Composition and Vocabulary Development Among At-Risk Preschoolers. *Early Education & Development*, *25*(7), 1016–1034.

Hamre, B. K. & Pianta, R. C. (2001). Early teacher-child relationships and the trajectory of children's school outcomes through eighth grade. *Child Development*, *v72*(n2), p625-38.

Head Start Performance Standards, 45 C.F.R. Chapter XIII (2015).

Heckman, J. J. (2008). The case for investing in disadvantaged young children. *CESifo DICE Report* (Vol. 6). Retrieved from http://heckmanequation.org/content/resource/case-investing-disadvantaged-young-children

High/Scope Educational Research Foundation. (2005). *Information for Decision Makers*. Retrieved from http://www.onlinecor.net/file/CORDecisionMakerR08.08.pdf

Howes, C., & Hamilton, C. E. (1993). The changing experience of child care: Changes in teachers and in teacher-child relationships and children's social competence with peers. *Early Childhood Research Quarterly*, *8*, 15–32.

Jerome, E. M., Hamre, B. K., & Pianta, R. C. (2009). Teacher-child relationships from kindergarten to sixth grade: Early childhood predictors of teacher-perceived conflict and closeness. *Social Development*, *18*(4), 915–945.

Jones Harden, B., Winslow, M. B., Kendziora, K. T., Shahinfar, A., Rubin, K. H., Fox,

N. A., Zahn-Waxler, C. (2000). Externalizing problems in Head Start children: An ecological exploration. *Early Education & Development*, *11*(3), 357–385.

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at-risk for mathematics difficulties. *Child Development*, *77*(1), 153–177.

Katz, L., Evangelou, D., & Hartman, J. A. (1990). *The Case for Mixed-Age Grouping in Early Education*. *Journal of Chemical Information and Modeling* (Vol. 53). ERIC Clearinghouse on Elementary and Early Childhood Education, Urbana, Ill.; National Association for the Education of Young Children, Washington, D.C.

Kinard, E. M. (1998). Methodological issues in assessing resilience in maltreated children. *Child Abuse and Neglect*, *22*(7), 669–680.

Krasnor, L. R., & Rubin, K. H. (1981). Cognitive Assessment. In T. V. Merluzzi, C. R. Glass, & M. Genest (Eds.), *Cognitive Assessment*. New York.

Lachance, J. A., & Mazzocco, M. M. (2006). A longitudinal analysis of sex differences in math and spatial skills in primary school age children. *Learning and Individual Differences*, *16*(3), 195–216.

Lipsey, M. W., Hofer, K. G., Dong, N., Farran, D. C., & Bilbrey, C. (2013). Evaluation of the Tennessee Voluntary Prekindergarten Program: Kindergarten and First Grade Follow-Up Results from the Randomized Control Design (Research Report). Nashville, TN: Vanderbilt University, Peabody Research Institute.

Liu, X., & Barnard-Brak, L. (2015). Gender differences in mathematics and reading trajectories among children from kindergarten to eighth grade. *Research in Education*, *93*(93), 77–89.

Longoria, A. Q., Page, M. C., Tait, H., & Kennison, S. (2009). Relationship between kindergarten children's language ability and social competence social competence. *Early Child Development and Care*, *179*(7), 919–929.

Lonigan, C. J., Bloomfield, B. G., Anthony, J. L., Bacon, K. D., Phillips, B. M., & Samwel, C. S. (1999). Relations among emergent literacy skills, behavior problems, and social competence in preschool children from low- and middle-income backgrounds. *Topics in Early Childhood Special Education*, *19*(1), 40–53.

Ludwig, J. & Miller, D.L. (2007). Does Head Start improve children's life chances? Evidence from a regression–discontinuity design. *Quarterly Journal of Economics,122,*159-208.

Luthar, S. S. (1993). Annotation: Methodological and conceptual issues in research on childhood resilience. *Journal of Child Psychology and Psychiatry and Allied Disciplines*, *34*(4), 441–453.

Mendez, J. L., Fantuzzo, J., & Cicchetti, D. (2017). Profiles of social competence among low-income african american preschool children. Published by: Wiley on behalf of the Society for Research in Child Development

Molfese, V. J., Brown, E. T., Adelson, J. L., Beswick, J., Jacobi-Vessels, J., Thomas, L. & Culver, B. (2012). Examining associations between classroom environment and processes and early mathematics performance from pre-kindergarten to kindergarten. *Gifted Children*, *5*(2), Article 2.

Malofeeva, E.V., Daniel-Echols, M., & Xiang, Z. (2007). Findings From The Michigan School Readiness Program 6 To 8 Follow Up Study. High/Scope Educational Research Foundation. Ypsilanti, MI.

Moiduddin, E., Aikens, N., Tarullo, L., West, J., Xue, Y. (2012). Child Outcomes and Classroom Quality in FACES 2009. OPRE Report 2012-37a. Washington, DC: Office of Planning, Research and Evaluation, Administration for Children and Families, U.S. Department of Health and Human Services.

National Association for the Education of Young Children (NAEYC) & National Association of Early Childhood Specialists in State Departments of Education (NAECS/SDE). (2003). Early childhood curriculum, assessment, and program evaluation: Building an effective, accountable system in programs for children birth through age 8. Joint Position Statement. Washington, D.C.

National Center for Education Statistics (2012). *Early Childhood Program Participation Survey of the National Household Education Surveys Program (ECPP-NHES:2012).* Retrieved on February 16, 2017 from: https://nces.ed.gov/programs/digest/d15/tables/dt15_202.30.asp?current=yes

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity*. C. T. Cross, T. A. Woods, & H. Schweingruber, Eds. Center for Education, Division of Behavioral and Social Sciences. Washington, D.C: National Academies Press.

NICHD Early Child Care Research Network, & Duncan, G. J. (2003). Modeling the impacts of child care quality on children's preschool cognitive development. *Child Development*, *74*(5), 1454–1475. http://doi.org/10.1111/1467-8624.00617

Nicoladis, E., & Paradis, J. (2012). Acquiring Regular and Irregular Past Tense

Morphemes in English and French : Evidence From Bilingual Children, (March), 170–197.

Nix, R. L., Bierman, K. L., Heinrichs, B. S., Gest, S. D., Welsh, J. A., & Domitrovich, C. E. (2016). The randomized controlled trial of Head Start REDI: Sustained effects on developmental trajectories of social-emotional functioning. *Journal of Consulting and Clinical Psychology*, *84,(*4), 310-322.

OCED. (2015). *Program for International Student Assessment (PISA) Results from PISA 2012: United States Country Note*. Retrieved from https://www.oecd.org/pisa/keyfindings/

O'Grady, D., & Metz, J. R. (1987). Resilience in Children at High Risk for Psychological Disorder. *Journal of. Pediatric Psychology*, *12*(1), 3–23.

Ottmar, E. R., Rimm-Kaufman, S. E., Larsen, R., & Merritt, E. G. (2011). *Relations between mathematical knowledge for teaching, mathematics instructional quality, and student achievement in the context of the responsive classroom (RC) approach*. *Society for Research on Educational Effectiveness*. Retrieved from ERIC.

Peisner-Feinberg, E. S. & Maris, C. L. (2005). Evaluation of the North Carolina More at Four Pre-kindergarten Program: Year 3 Report (July 1, 2003-June 30, 2004). Chapel Hill, NC: FPG Child Development Institute.

Pettit, G. S., Dodge, K. a, & Brown, M. M. (1988). Early family experience, social problem solving patterns, and children's social competence. *Child Development*, *59*(1), 107–120.

Pianta, R. C., Steinberg, M. S. & Rollins, K. B. (1995). The first two years of school:

Teacher-child relationships and deflections in children's classroom adjustment. *Development and Psychopathology*, *7*(2), pp.295-312.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common Core Standards: The new U.S. intended curriculum. *Educational Researcher*, *40*(3), 103–116.

Rabe-Hesketh, S., & Skrondal, A. (2011). *Multilevel and Longitudinal Modeling Using Stata, Volume I: Continuous Responses* (Third Ed.). College Station, TX: SAGE Publications.

Ramani, G. B., Siegler, R. S., & Hitti, A. (2012). Taking it to the classroom: Number board games as a small group learning activity. *Journal of Educational Psychology*, *104*(3), 661–672.

Ramani, G. B., Siegler, R. S., Ramani, G. B., & Siegler, R. S. (2016). Promoting broad And stable improvements in low-income children's numerical knowledge through playing number board games, *79*(2), 375–394.

Raver, C. C., Jones, S. M., Li-Grining, C., Zhai, F., Bub, K., & Pressler, E. (2011). CSRP's impact on low-income preschoolers' preacademic skills: Self-regulation as a mediating mechanism. *Child Development*, *82*(1), 362–78.

Rowan, T., & Bourne, B. (2001). *Thinking Like Mathematicians: Putting the NCTM Standards Into Practice*. Portsmouth, NH: Heinemann.

Rubin, K. H., & Krasnor, L. R. (1992). Interpersonal Problem-Solving and Social Competence in Children. In V. B. Van Hasselt & M. Hersen (Eds.), *Handbook of Social Development: A Lifespan Perspective* (pp. 1–59). New York: Plenum.

Rubin, K. H. (1988). The Social Problem Solving Test-Revised. Unpublished manuscript. University of Waterloo. Waterloo, Ontario.

Rubin, K. H., Daniels-Beirness, T., & Hayvren, M. (1982). Social and social-cognitive correlates of sociometric status in preschool and kindergarten children. *Canadian Journal of Behavioural Science*, *14*(4), 338–349.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.

Schweinhart, L. J., Heckman, J. J., Malofeeva, L., Pinto, R. Moon, S., & Yavitz, A. (2010). *The cost-benefit analysis of the Preschool Curriculum Comparison Study*. Final Report to the John D. and Catherine T. MacArthur Foundation. Ypsilanti, MI: HighScope.

Schweinhart, L. J., Montie, J., Xiang, Z., Barnett, W. S., Belfield, C. R., & Nores, M. (2005). The High / Scope Perry Preschool Study Through Age 40 Summary, Conclusions, and Frequently Asked Questions. Ypsilanti, MI: High/Scope Press.

Schweinhart, L. J., & Weikart, D. P. (1997). The High/Scope preschool curriculum comparison study through age 23. *Early Childhood Research Quarterly, 12*(2), 117–143.

Selman, R.L. & Demorest, A.P. (1986). Putting thoughts and feelings into perspective: A developmental version of how children deal with interpersonal disequilibrium. In D.J. Bearison & H. Zimilies (Eds.), Thought and emotion (pp.93-128). Hillsdale, N.J.: LEA.

Selman, R.L. & Schultz, L.H. (1988). Interpersonal thought and action in the case of a troubled early adolescent: Toward a developmental model of the gap. In S. Shick (Ed.), Cognitive development and child psychotherapy (pp.207-246). New York: Plenum.

Shipley, K. G., Maddox, M. A., & Driver, J. E. (1991). Children's development of irregular past tense verb forms, *Language, Speech, and Hearing Services in Schools, 22*(July), 115–122.

Shonkoff, J. P. & Phillips, D. A. (2000). *From neurons to neighborhoods: The science of early childhood development.* National Research Council and Institute of Medicine Committee on Integrating the Science of Early Childhood Development. Washington, D.C.: National Academy Press.

Shure, M. B. & Spivak, G. (1982). Interpersonal problem-solving in young children: A cognitive approach to prevention. *American Journal of Community Psychology, 10*(3), 341–356.

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, *11*(5), 655–661.

Singer, J.D. & Willett, J.B. (2003). Applied longitudinal data analysis: Modeling change and event occurrence. Oxford University Press.

Schoenfeld, A. H. & Stipek, D. (2011). Math matters: Children's mathematical journeys start early. Report of a conference held November 7 & 8. Berkeley, CA

Sophian, C. (2004). Mathematics for the future: Developing a Head Start curriculum to support mathematics learning. *Early Childhood Research Quarterly*, *19*(1), 59–81.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99–120.

Stipek, D., Franke, M., Clements, D., Farran, D., & Coburn, C. (2017). *PK-3: What does it mean for instruction?* (Vol. 30). Retrieved from http://www.srcd.org/sites/default/files/documents/spr_30_2_final.pdf?utm_source=SRCD+Membership&utm_campaign=d052a57bc6-Social_Policy_Report_V30_2_PK–3_2_2_17&utm_medium=email&utm_term=0_a2f8196caa-d052a57bc6-293500189

U.S. Department of Education, Institute of Education Sciences, What Works Clearinghouse. (2015, July). Early Childhood Education intervention report: Head Start. Retrieved from http://whatworks.ed.gov

U.S. Department of Health, Education and Welfare, Office of Child Development. (1968). *Project Head Start 1968: The development of a program*. Washington, D.C.

U.S. Department of Health and Human Services, Administration for Children and Families (January 2010). Head Start Impact Study. Technical Report. Washington, D.C**.**

Vygotsky, L. (1962). Thought and language (A. Kozulin, Trans.). Cambridge, MA: MIT Press.

Washington, P. S., & Iglesias, A. (2015). Tense Shifting in Spanish-Speaking English Language Learners. *Communication Disorders Quarterly*, *36*(3), 152–161.

Werner, E. E. & Smith, R. S. (1982). *Vulnerable, but invincible: A longitudinal study of resilient children and youth*. New York, NY: McGraw-Hill.

Winsler, A. (1993). The social interactions and task activities of young children in

Mixed-age and same-age classrooms: An observational study. In *Paper presented at the Biennial Meeting of the Society for Research in Child Development (New Orleans, LA, March 25-28, 1993).* Retrieved from http://files.eric.ed.gov/fulltext/ED356074.pdf

Winsler, A., Caverly, S. L., Willson-Quayle, A., Carlton, M. P., Howell, C., & Long, G. N. (2002). The social and behavioral ecology of mixed-age and same-age preschool classrooms: A natural experiment. *Journal of Applied Developmental Psychology*, *23*(3), 305–330.

Youngstrom, E., Wolpaw, J. M., Kogos, J. L., Schoff, K., Ackerman, B., & Izard, C. (2000). Interpersonal problem solving in preschool and first grade: Developmental change and ecological validity. *Journal of Clinical Child Psychology*, *29*(4), 589–602.

References

(Essay 2)

Administration for Children and Families. (2014). Head Start program fact sheet fiscal year 2012. Retrieved January 22, 2015 from: https://eclkc.ohs.acf.hhs.gov/hslc/data/factsheets/docs/hs-program-fact-sheet-2014.pdf

Administration for Children and Families. (2015). Head Start program fact sheet fiscal year 2015. Retrieved May 15, 2016 from: https://eclkc.ohs.acf.hhs.gov/hslc/data/factsheets/2015-hs-program-factsheet.html

Allison, P. D. (2012). Handling Missing Data by Maximum Likelihood. *SAS Global Forum 2012 Statistics and Data Analysis*, 1–21.

Barnett, W. S., Jung, K., Yarosz, D. J., Thomas, J., Hornbeck, A., Stechuk, R., & Burns, S. (2008). Educational effects of the Tools of the Mind curriculum: A randomized trial. *Early Childhood Research Quarterly*, *23*(3), 299–313.

Bierman, K. L., Domitrovich, C. E., Nix, R. L., Gest, S. D., Welsh, J. A, Greenberg, M. T., & Gill, S. (2013). Promoting academic and social-emotional school readiness: The Head Start REDI program, *79*(6), 1802–1817.

Blair, C. & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child development*, *78*(2), 647–63.

Bloom, H. S. & Weiland, C. (2015). Quantifying variation in Head Start effects on young children's cognitive and socio-emotional skills using data from the National Head

Start Impact Study. Retrieved from: http://www.mdrc.org

Bronfenbrenner, U., & Morris, P. (2006). Handbook of child psychology. In W. Damon & R. M. Lerner (Eds.), *Child psychology in practice*. John Wiley & Sons. Hoboken, NJ.

Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly*, *36*, 79–90.

Cooke, R. (1965). *Recommendations for a Head Start program*. U.S. Department Of Health, Education, And Welfare Office Of Child Development. Retrieved from http://eclkc.ohs.acf.hhs.gov/hslc/hs/about/docs/cooke-report.pdf

Crosnoe, R., Morrison, F., Burchinal, M., Pianta, R., & Keating, D. (2010). Instruction, teacher–student relations and math achievement trajectories in elementary school, *Journal of Educational Psychology,102*(2), 407–417.

Dodge, K. A, Pettit, G. S., Mcclaskey, C. L., Brown, M. M., & Gottman, J. M. (1986). Social Competence in Children. *Monographs of the Society in Social Competence*, *51*(2).

Duncan, G. J. & and Magnuson, K. (2011). The nature and impact of early achievement skills, attention skills, and behavior problems. In G. J. Duncan & R. Murnane (Eds.), *Whither opportunity? Rising inequality, schools, and children's life chances* (pp. 47-69). New York: Russell Sage Foundation.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446.

Dunn, L.M., Dunn, L.L., & Dunn, D.M. (1997). Peabody Picture and Vocabulary Test-Third Edition. Circle Pines, MN: American Guidance Service.

Durlak, J. A, Weissberg, R. P., Dymnicki, A. B., Taylor, R. D., & Schellinger, K. B. (2011). The impact of enhancing students' social and emotional learning: A meta-analysis of school-based universal interventions, 405- 432.

Enders, C. K. (2010). *Applied Missing Data Analysis*. New York, NY: The Guilford Press.

Garces, E., Thomas, D. & Currie, J. (2002, September). Longer-term effects of Head Start. *American Economic Review, 92* (4): 999-1012.

Garcia, E. (2015). *Inequalities at the Starting Gate: Cognitive and Noncognitive Skills Gaps between 2010-2011 Kindergarten Classmates*. Retrieved from http://www.epi.org/publication/inequalities-at-the-starting-gate-cognitive-and-noncognitive-gaps-in-the-2010-2011-kindergarten-class/

Getting a Head Start: Is preschool too early for academic instruction? [Editorial]. *Education Next*, *1*(2). Retrieved from http://educationnext.org/gettingaheadstart/

Gilliam, W. S., & Zigler, E. F. (2000). A critical meta-analysis of all evaluations of state-funded preschool from 1977 to 1998: Implications for policy, service delivery and program evaluation. *Early Childhood Research Quarterly*, *15*(4), 441–473.

Ginsburg, Herbert P. (2006). Mathematical play and playful mathematics: A guide for early education. In D. Singer, R. M. Golinkoff & K. Hirsh-Pasek (Eds.), *Play = learning: How play motivates and enhances children's cognitive and social-emotional growth* (pp. 145-165). New York: Oxford University Press.

Head Start Performance Standards, 45 C.F.R. Chapter XIII (2015).

Heckman, J. J. (2008). The case for investing in disadvantaged young children. *CESifo DICE Report* (Vol. 6). Retrieved from http://heckmanequation.org/content/resource/case- investing-disadvantaged-young-children

High/Scope Educational Research Foundation. (2005). *Information for Decision Makers*. Retrieved from http://www.onlinecor.net/file/CORDecisionMakerR08.08.pdf

Hu, L. T., & Bentler, P. M. (1998). Fit indices in covariance structure modeling: Sensitivity to underparameterized model misspecification. *Psychological Methods*, *3*(4), 424–453.

Jones, S. M., Bailey, R., Barnes, S. P., & Partee, A. (2016). Executive Function Mapping Project: Untangling the Terms and Skills Related to Executive Function and Self-Regulation in Early Childhood. OPRE Report # 2016-88, Washington, DC: Office of Planning, Research and Evaluation, Administration for Children and Families, U.S. Department of Health and Human Services.

Kline, R.B. (2011). Principles and Practice of Structural Equation Modeling (3rd ed.). New York: Guilford Press.

Little, R.A. & Rubin, D.B. (2002). *Statistical analysis with missing data*. Hoboken, NJ: Wiley.

Liu, X., & Barnard-Brak, L. (2015). Gender differences in mathematics and reading trajectories among children from kindergarten to eighth grade. *Research in Education*, *93*(93), 77–89.

Ludwig, J. & Miller, D.L. (2007). Does Head Start improve children's life chances?

Evidence from a regression– discontinuity design. *Quarterly Journal of Economics,122*,159-208.

Master, A., Cheryan, S., & Meltzoff, A. N. (2016). Social group membership increases STEM engagement among preschoolers. *Developmental Psychology*.

McClelland, M. M., Cameron, C. E., Connor, C. M., Farris, C. L., Jewkes, A. M., & Morrison, F. J. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and math skills. *Developmental psychology*, *43*(4), 947–59.

Muthén, L. K., & Muthén, B. O. (2015). Mplus version 7.31. Los Angeles, CA: Muthén & Muthén.

Nix, R. L., Bierman, K. L., Heinrichs, B. S., Gest, S. D., Welsh, J. A., & Domitrovich, C. E. (2016). The randomized controlled trial of Head Start REDI: Sustained effects on developmental trajectories of social-emotional functioning. *Journal of Consulting and Clinical Psychology, 84,(*4), 310-322.

O'Grady, D., & Richard Metz, J. R. (1987). Resilience in Children at High Risk for Psychological Disorder. *Journal of Pediatric Psychology*, *12*(1), 3–23.

Ottmar, E. R., Rimm-Kaufman, S. E., Larsen, R., & Merritt, E. G. (2011). *Relations between mathematical knowledge for teaching, mathematics instructional quality, and student achievement in the context of the responsive classroom (RC) approach*. *Society for Research on Educational Effectiveness*. Retrieved from ERIC.

Pianta, R. C. (2001). *Student-Teacher Relationship Scale (STRS): Professional Manual*. Psychological Assessment Resources, Inc.

Peisner-Feinberg, E. S. & Maris, C. L. (2005). Evaluation of the North Carolina More at

Four Pre-kindergarten Program: Year 3 Report (July 1, 2003-June 30, 2004). Chapel Hill, NC: FPG Child Development Institute.

Raver, C. C., Jones, S. M., Li-Grining, C., Zhai, F., Bub, K., & Pressler, E. (2011). CSRP's impact on low-income preschoolers' preacademic skills: Self-regulation as a mediating mechanism. *Child Development*, *82*(1), 362–78.

Rubin, K. H., & Krasnor, L. (1992). Interpersonal Problem-Solving and Social Competence in Children. In V. B. Van Hasselt & M. Hersen (Eds.), *Handbook of Social Development: A Lifespan Perspective* (pp. 1–59). New York: Plenum.

Schweinhart, L. J., Heckman, J. J., Malofeeva, L., Pinto, R. Moon, S., & Yavitz, A. (2010). *The cost-benefit analysis of the Preschool Curriculum Comparison Study*. Final Report to the John D. and Catherine T. MacArthur Foundation. Ypsilanti, MI: HighScope.

Shonkoff, J. P. & Phillips, D. A. (2000). *From neurons to neighborhoods: The science of early childhood development.* National Research Council and Institute of Medicine Committee on Integrating the Science of Early Childhood Development. Washington, D.C.: National Academy Press.

U.S. Department of Education, Institute of Education Sciences, What Works Clearinghouse. (2015). Early childhood education intervention report: Head Start. Retrieved from http://whatworks.ed.gov

U.S. Department of Health, Education and Welfare, Office of Child Development. (1968). *Project Head Start 1968: The development of a program*. Washington, D.C.

U.S. Department of Health and Human Services, Administration for Children and

Families. (2005). *Head Start Impact Study First Year Findings*. Washington, D.C**.**

U.S. Department of Health and Human Services, Administration for Children and Families. (2010). Head Start Impact Study. Technical Report. Washington, D.C**.**

U.S. Department of Health and Human Services, Administration for Children and Families, Office of Head Start. (2015). Head Start Services Snapshot. Retrieved from http://eclkc.ohs.acf.hhs.gov/hslc/data/psr/2015/services-snapshot-all-programs-2014-2015.pdf

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). Woodcock-Johnson tests of achievement. Itasca, IL: Riverside Publishing.

Young, C.B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science, 29*(6), 997–1003.

Zhao, X., Lynch Jr., J. G., & Chen, Q. (2010). Reconsidering Baron and Kenny: Myths and truths about mediation analysis. *Journal of Consumer Research, 37*(2), 197–206.

Appendix A

In selection of the final model for RQ1, which investigated children's early math skills, in which a fixed effect for each classroom was included, the rates of change were marginally significant in the "final" model. However, this difference in growth rates was thought to have substantive importance and, given that there were four participating classrooms in a sample size of only 76 children, power may have played a role in reducing the interaction term to marginal significance. Given this, additional tests were conducted and are presented here along with a detailed summary of the fit statistics comparing these models. Specifically, this "final" model (Model 12, below) was compared to Model 13 (below) which removed the MonthsxcBaseAge interaction. As evidenced in the table below, removal of the age interaction yielded a worse fitting model by several indices. Specifically, AIC increased from 1198 (in Model 12) to 1200 (in Model 13) and the Pseudo R^2 decreased from 60.4% to 59.9% (from Model 12 to Model 13) indicating that the more simplistic model explained less of the within-child variation of the ways in which children's early math trajectories progress across a preschool year. Likelihood ratio tests also failed to reject the null hypothesis that the more parsimonious model—without the MonthsxcBaseAge interaction—was an equivalent fitting model. Thus, Model 12, which included the MonthsxcBaseAge interaction, was retained for both substantive and statistical reasons.

Table 1. Sensitivity analysis investigating model fit of inclusion of "MonthsxcBaseAge" interaction when predicting math skills in RQ1.

	(12) MathRaw	(13) MathRaw
Months	0.72***	0.71***
	(0.06)	(0.06)
cBaseAge	0.13*	0.18**
	(0.07)	(0.06)
Monthsx cBaseAge	0.02+	
	(0.01)	
Male	-1.51*	-1.49*
	(0.75)	(0.75)
cBasePPVT	0.08***	0.08***
	(0.02)	(0.02)
Class=1	0.00	0.00
	(.)	(.)
Class=2	-0.97	-1.03
	(1.18)	(1.18)
Class=3	-0.88	-0.95
	(1.19)	(1.19)
Class=4	-2.09+	-2.17+
	(1.15)	(1.14)
constant	5.31***	5.39***
	(1.01)	(1.01)
N	257	257
ll	-586.3	-588.2
AIC	1198.6	1200.4
BIC	1244.8	1243.0
LRtest_df		1
LRtest_chi^2		3.74
LRtest_p-value		.05
PseudoR^2	0.605	0.599

Standard errors in parentheses

$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Appendix B

Table 1a. Cross-domain relationship between prosocial social problem-solving skills and children's early math skills (Models 1-6).

	(1) MathRaw	(2) MathRaw	(3) MathRaw	(4) MathRaw	(5) MathRaw	(6) MathRaw
Months	0.72^{***}	0.72^{***}	0.72^{***}	0.72^{***}	0.75^{***}	0.84^{***}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.09)	(0.12)
cBaseAge	0.17^{**}	0.17^{**}	0.16^{**}	0.16^{*}	0.15^{*}	0.16^{*}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
Monthsx cBaseAge	0.02^{*}	0.02^{*}	0.02^{*}	0.02^{*}	0.02^{*}	0.02^{+}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	-1.31^{+}	-1.30^{+}	-1.42^{+}	1.45	1.47	2.00
	(0.74)	(0.74)	(0.75)	(1.70)	(1.70)	(1.76)
cBasePPVT	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Prosocial		0.01	0.01	0.01	0.12	0.15
		(0.11)	(0.11)	(0.11)	(0.24)	(0.24)
Black			-0.79	1.80	1.81	1.80
			(0.84)	(1.61)	(1.61)	(1.61)
Blackx Male				-3.52^{+}	-3.52^{+}	-3.52^{+}
				(1.87)	(1.86)	(1.86)
Monthsx Prosocial					-0.02	-0.02
					(0.04)	(0.04)
Monthsx Male						-0.14
						(0.13)
Monthsx Black						
constant	4.15^{***}	4.13^{***}	4.81^{***}	2.59^{+}	2.39	2.04
	(0.62)	(0.64)	(0.97)	(1.53)	(1.58)	(1.61)
N	257	256	256	256	256	256
ll	-588.1	-586.7	-586.2	-584.5	-584.4	-583.8
AIC	1196.3	1195.3	1196.5	1195.0	1196.8	1197.5
BIC	1231.8	1234.3	1239.0	1241.1	1246.4	1250.7
$PseudoR^2$						

Standard errors in parentheses

$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Table 1b. Cross-domain relationship between prosocial social problem-solving skills and children's early math skills (Models 7-12).

	(7) MathRaw	(8) MathRaw	(9) MathRaw	(10) MathRaw	(11) MathRaw	(12) MathRaw
Months	0.88^{***}	0.84^{***}	0.72^{***}	0.72^{***}	0.72^{***}	0.72^{***}
	(0.17)	(0.16)	(0.12)	(0.06)	(0.06)	(0.06)
cBaseAge	0.16^{*}	0.16^{**}	0.16^{*}	0.16^{*}	0.16^{**}	0.17^{**}
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
Monthsx cBaseAge	0.02^{*}	0.01^{+}	0.02^{*}	0.02^{*}	0.02^{*}	0.02^{*}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Male	2.03	1.99	1.45	1.45	-1.42^{+}	-1.30^{+}
	(1.76)	(1.76)	(1.70)	(1.70)	(0.75)	(0.74)
cBasePPVT	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}	0.08^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Prosocial	0.15	0.03	0.01	0.01	0.01	0.01
	(0.24)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
Black	1.96	1.95	1.83	1.80	-0.79	
	(1.68)	(1.69)	(1.68)	(1.61)	(0.84)	
Blackx Male	-3.51^{+}	-3.50^{+}	-3.51^{+}	-3.52^{+}		
	(1.86)	(1.86)	(1.87)	(1.87)		
Monthsx Prosocial	-0.02					
	(0.04)					
Monthsx Male	-0.15	-0.15				
	(0.13)	(0.13)				
Monthsx Black	-0.05	-0.05	-0.01			
	(0.14)	(0.14)	(0.14)			
constant	1.89	2.13	2.56	2.59^{+}	4.81^{***}	4.13^{***}
	(1.67)	(1.61)	(1.57)	(1.53)	(0.97)	(0.64)
N	256	256	256	256	256	256
ll	-583.7	-583.9	-584.5	-584.5	-586.2	-586.7
AIC	1199.4	1197.7	1197.0	1195.0	1196.5	1195.3
BIC	1256.1	1250.9	1246.6	1241.1	1239.0	1234.3
PseudoR^2						

Standard errors in parentheses

$^{+}$ $p < 0.10$, * $p < .05$, ** $p < .01$, *** $p < .001$

Appendix C

Comparison of Analytic Sample to Full Head Start Impact Study Sample

	Full Analytic Sample			Treatment Group			Control Group			Non-Center Based Care			Full HSIS Sample		
Spring Outcome Scores	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD
Spring WJAP	2756	385.97	27.86	2089	386.19	27.53	667	385.28	28.85	901	380.21	31.23	3661	384.53	28.86
Child Solves Problems	2373	3.48	1.02	1929	3.47	1.00	444	3.55	1.08	112	3.38	1.02	2485	3.48	1.02
How Child Does in Complex Play	2355	3.50	1.12	1919	3.49	1.11	436	3.53	1.16	112	3.46	1.13	2467	3.49	1.12
How Well Child Makes Friends	2345	3.78	1.07	1911	3.79	1.06	434	3.74	1.11	111	3.65	1.04	2456	3.78	1.07
How Well Child Works With Children	2356	3.45	1.13	1914	3.45	1.12	442	3.43	1.16	113	3.42	1.17	2469	3.45	1.13
How Well Child Expresses Feelings	2369	3.41	1.19	1924	3.39	1.19	445	3.51	1.17	113	3.66	1.19	2482	3.42	1.19
TCR Closeness	2398	3.06	0.42	1943	3.06	0.42	455	3.06	0.42	116	3.17	3.90	2514	3.07	4.18
TCR Conflict (reverse coded)	2391	3.12	0.61	1938	3.13	0.61	453	3.07	0.63	117	3.03	6.56	2508	3.11	6.17
Fall Outcome Scores															
Fall PPVT	3433	91.03	8.58	2377	90.94	8.44	1052	91.22	8.88	942	91.80	8.97	4375	91.19	8.67
Fall WJAP	2520	376.94	27.72	1735	377.05	27.49	785	376.59	28.25	709	375.70	28.63	3234	376.64	27.93
Demographic Characteristics															
White/Other	3485	0.30	0.46	2414	0.30	0.46	1071	0.28	0.45	952	0.41	0.49	4442	0.32	0.47
Black	3485	0.33	0.47	2414	0.32	0.47	1071	0.35	0.48	952	0.22	0.41	4442	0.30	0.46
Hispanic	3485	0.38	0.48	2414	0.38	0.49	1071	0.37	0.48	952	0.38	0.48	4442	0.38	0.48
Female	3485	0.50	0.50	2414	0.50	0.50	1071	0.48	0.50	952	0.50	0.50	4442	0.50	0.50
Spring Test Age	2784	48.37	8.06	2104	48.35	7.96	676	48.48	8.36	924	48.38	7.80	3708	48.37	7.99
Special Needs	3485	0.13	0.34	2414	0.14	0.35	1071	0.12	0.33	952	0.11	0.31	4442	0.13	0.33

Appendix D

Table 1. Standardized model results from multilevel structural regression model

Social Competence BY	Estimate	SE	Est/S.E	p-value	Missing
Child Solves Problems	0.67	0.03	21.95	0.00	0.24
How Child Does in Complex Play	0.61	0.03	24.41	0.00	0.41
How Well Child Makes Friends	0.67	0.03	19.77	0.00	0.46
How Well Child Works With Children	0.75	0.03	24.82	0.00	0.30
How Well Child Expresses Feelings	0.72	0.03	28.23	0.00	0.38
TCR Closeness	0.40	0.02	16.71	0.00	0.18
TCR Conflict (reverse coded)	0.34	0.03	12.29	0.00	0.31
SOC_COMP ON					
HS Treatment	-0.02	0.02	-0.95	0.34	0.33
Female	0.15	0.02	6.22	0.00	0.28
Fall PPVT	0.12	0.02	5.38	0.00	0.43
SpringTest Age	0.28	0.02	11.46	0.00	0.31
Special Needs	-0.15	0.02	-6.67	0.00	0.23
Spring WJAP ON					
Social Competence	0.17	0.03	6.84	0.00	0.52
Spring WJAP ON					
Female	0.01	0.02	0.79	0.43	0.24
Fall WJAP	0.44	0.02	19.76	0.00	0.49
Black	-0.11	0.02	-5.58	0.00	0.20
Hispanic	-0.12	0.02	-5.58	0.00	0.20
Fall PPVT	0.13	0.02	7.02	0.00	0.28
SpringTest Age	0.16	0.02	8.26	0.00	0.28
Special Needs	-0.03	0.01	-2.10	0.04	0.20
HS Treatment	0.03	0.02	1.56	0.12	0.30
Child Solves Problems WITH					
How Child Does in Complex Play	0.24	0.04	5.76	0.00	0.40
How Well Child Makes Friends	-0.07	0.07	-0.96	0.34	0.43
How Well Child Works With Children	-0.04	0.07	-0.61	0.54	0.30

How Well Child Expresses Feelings	-0.05	0.05	-0.98	0.33	0.28
How Child Does in Complex Play WITH					
How Well Child Makes Friends	0.11	0.05	2.38	0.02	0.45
How Well Child Works With Children	0.02	0.05	0.43	0.67	0.34
How Well Child Makes Friends WITH					
How Well Child Works With Children	-0.07	0.08	-0.92	0.36	0.37
How Well Child Expresses Feelings	-0.02	0.06	-0.42	0.67	0.34
How Well Child Works With Children WITH					
How Well Child Expresses Feelings	0.03	0.07	0.41	0.68	0.36
TCR Close WITH					
TCR Conflict (reverse coded)	0.14	0.03	5.29	0.00	0.43
Intercepts					
SpWJAP	5.38	0.35	15.42	0.00	0.24
Child Solves Problems	1.42	0.26	5.40	0.00	0.49
How Child Does in Complex Play	1.27	0.24	5.28	0.00	0.48
How Well Child Makes Friends	1.48	0.26	5.77	0.00	0.46
How Well Child Works With Children	0.76	0.26	2.94	0.00	0.50
How Well Child Expresses Feelings	0.71	0.21	3.37	0.00	0.41
TCR Closeness	6.10	0.23	26.60	0.00	0.20
TCR Conflict (reverse coded)	4.04	0.17	23.93	0.00	0.29
Residual Variances					
SpWJAP	0.57	0.02	33.20	0.00	0.31
Child Solves Problems	0.56	0.04	13.79	0.00	0.24
How Child Does in Complex Play	0.62	0.03	20.32	0.00	0.41
How Well Child Makes Friends	0.55	0.05	11.86	0.00	0.46
How Well Child Works With Children	0.44	0.05	9.66	0.00	0.30
How Well Child Expresses Feelings	0.48	0.04	13.10	0.00	0.38
TCR Closeness	0.84	0.02	43.57	0.00	0.19
TCR Conflict (reverse coded)	0.89	0.02	47.72	0.00	0.30
Social Competence	0.87	0.02	47.68	0.00	0.31