Vector Boson Excitations Near Deconfined Quantum Critical Points

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Accessibility
Vector boson excitations near deconfined quantum critical points

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We show that the Néel states of two-dimensional antiferromagnets have low energy vector boson excitations in the vicinity of deconfined quantum critical points. We compute the universal damping of these excitations arising from spin-wave emission. Detection of such a vector boson will demonstrate the existence of emergent topological gauge excitations in a quantum spin system.

Quantum spin systems are expected to display ground states with novel fractionalized and topological gauge excitations which have no analogs in the band insulators of Bloch theory [1]. The theoretical arguments for the existence of such excitations are convincing, but, so far, the excitations have not been detected unambiguously in any experiment, or even in numerical studies of semi-realistic model systems in two spatial dimensions. In experiments, the best candidate so far is the kagome antiferromagnet, and its neutron scattering spectrum displays strong evidence for fractionalization [2], but the specific excitations have not been identified. In numerics, there has been positive evidence for exotic physics in the topological entanglement entropy of the kagome antiferromagnet [3, 4], but this does not directly identify the excitation spectrum. Sandvik [5] has obtained convincing evidence of an emergent U(1) symmetry near the quantum critical point of a spin system, and this is strong, but indirect, evidence of an emergent, topological gauge excitation of a deconfined quantum critical point [6]; however, this does not yield any information on the excitation spectrum of the gauge boson.

In this paper we propose that the antiferromagnetically ordered (i.e., Néel) phase of a quantum spin system near a deconfined quantum critical point has an emergent vector boson excitation which should be detectable in numerical studies, and possibly eventually in experiments. This vector boson is the analog of the W and Z vector bosons of the standard model of particle physics, and is similarly a characteristic signature of the gauge structure of the underlying theory. In two spatial dimensions, the vector boson is universally damped by emissions of multiple spin-wave (i.e., Goldstone boson) excitations of the Néel phase, and the present paper will provide a quantitative computation of this damping. The Néel phase also has a universally damped Higgs excitation [7], and this mode has recently been detected in experiments [8] and numerics [9, 10] near a conventional quantum critical point. We argue here that similar methods can allow positive identification of a vector boson excitation in the Néel phase near a deconfined quantum critical point. Note that, while the Higgs mode is present for both conventional and deconfined critical points, the vector boson mode appears only in the latter case.

We note that there is a debate [11–19] in the literature of whether the gauge theory of the antiferromagnet ultimately describes a second-order quantum phase transition in antiferromagnets with a global SU(2) spin rotation symmetry. Our method of detecting the vector boson sidesteps this delicate issue, because the vector boson should exist even if the transition out of the Néel phase is weakly first-order. Its observation would be a direct signature that the theory of deconfined criticality with emergent gauge excitations describes the spectrum of the antiferromagnet at low energies.

For quantum antiferromagnets with SU(N) global symmetry, the deconfined critical theory [6, 20] is the CP^{N-1} field theory of relativistic complex scalars (‘spinons’) z_{\alpha} (\alpha = 1 \ldots N) minimally coupled to a U(1) gauge field A_{\mu} with partition function Z = \int Dz_{\alpha} D\tau DA_{\mu} e^{-S} with

\[ S = \int dz_{\alpha} \left( \frac{N}{8} |\partial_{\mu} - iA_{\mu}| z_{\alpha}|^2 + i\lambda(\{|z_{\alpha}|^2 - 1\}) \right), \]

where the integration is over 2+1 dimensional space and (imaginary) time, and \lambda is a Lagrange multiplier which implements the constraint \sum_{\alpha=1}^{N}|z_{\alpha}|^2 = 1. For N = 2, the Néel order parameter, \mathbf{n}(\mathbf{x}), is related to the spinons via

\[ \mathbf{n} = \sigma_{\alpha\beta} \mathbf{z}_{\alpha} \mathbf{z}_{\beta}, \]

where \sigma is a vector of Pauli matrices.

We are interested here in the spectrum of A_{\mu} excitations in the Néel phase at zero temperature, which appears for g < g_c with \langle \mathbf{n}(\mathbf{x}) \rangle \neq 0. We will detect this spectrum via correlations of the staggered vector spin chirality

\[ B_{\mu} = \epsilon_{\mu\alpha\beta} \partial_{\alpha} A_{\beta} = \frac{1}{4} \epsilon_{\mu\alpha\beta} \mathbf{n} \cdot (\partial_{\alpha} \mathbf{n} \times \partial_{\beta} \mathbf{n}); \]

the last term specifies how B_{\mu} can be related to the operators of the underlying antiferromagnet, and identifies it as the Skyrmion current: the spatial integral of its temporal component B_{t} is the Skyrmion number of the texture of the Néel order parameter underlying the topological nature of the vector boson. Indeed, correlations of \epsilon_{\mu\alpha\beta} \mathbf{n} \cdot (\partial_{\alpha} \mathbf{n} \times \partial_{\beta} \mathbf{n}) were measured recently by Fritz et al. [21] in quantum Monte Carlo. They can also be measured in Raman scattering [22, 23] if the light couples preferentially to one sublattice of the antiferromagnet.

In the vicinity of a deconfined critical point, we show that the existence of an emergent gauge field means that the correlations of B_{\mu} obey the universal scaling form

\[ \langle B_{\mu}(-p)B_{\nu}(p) \rangle = \left( \delta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right) \rho_s \mathcal{F}(p/\rho_s), \]

where p is a Euclidean 3-momentum, \rho_s is the ‘spin stiffness’ (or ‘helicity modulus’) of the Néel phase, and \mathcal{F} is a completely universal scaling function (including its overall scale).
The stiffness vanishes as $\rho_s \sim (g - g_c)^\nu$ upon approaching the deconfined critical point, where $\nu$ is the correlation length exponent. We also note that $\rho_s$ is analogous to the $f^2$ constant of the chiral Lagrangian of particle physics. Here, $\rho_s$ serves as the low energy scale controlling the excitations of the Néel phase.

The correlator in Eq. (4) is purely transverse; lattice models will also have a longitudinal component, but this is suppressed near the deconfined critical point. In contrast, for a conventional critical point, the longitudinal and transverse components both vanish rapidly with the same large power $\sim \rho_s^{2\Delta_u - 3}$ (and non-universal prefactor), where $\Delta_u \approx 3.4$ is the scaling dimension of the staggered spin chirality (compare to the linear $\sim \rho_s$ power in Eq. (4)).

This paper will obtain numerous properties of the scaling function $F$ for the deconfined critical point in the $1/N$ expansion. It is useful to first present the form of these gauge-field correlators in the large $N$ limit. After continuing the result to Lorentzian frequencies at zero spatial momentum ($p \to -i\omega$) we have

$$\rho_s F(\omega/\rho_s) = \frac{1}{N} \frac{-16\omega^2}{(-i\omega + 64\rho_s/N)};$$

where we have separated equivalent contractions into numerical factors $a_i$. The explicit expressions are given in the Supplemental Material. In the past, a reliable evaluation of such tensor-valued Feynman diagrams in momentum space for vector correlation functions has been almost intractable. Already for the much simpler case of the (conformally invariant) fixed point of the $O(N)$ vector model, a computation of vector correlation functions to $1/N$ is an intricate matter, necessitating the use of conformal field theory methods in real space. In momentum space, Cha et al. have succeeded to compute the current-current correlator of the $O(N)$ model to $1/N$ but needed to supplement their calculation with results from other computations; and their method seems hard to generalize to more complicated situations.

FIG. 1: $N \to \infty$ contribution to the gauge propagator that generates purely dissipative dynamics by decaying into two complex Goldstone bosons leading to Eq. (5). Wiggly line refers to the gauge field and the solid line the $N$ complex Goldstone bosons $\pi$.

From a technical viewpoint, the enabling achievement of this paper is the reliable, numerically verifiable computation of tensor-valued momentum integral of multi-loop diagrams using our algorithm Tensoria (see Refs. 34, 35 for an application of similar methods to three-point functions of conformal field theories).

Including the 12 contractions of Fig. 2 we write the renor-
factors are given in the Supplemental Material.

As in Fig. 1 the double line represents the longitudinal \( \sigma \) field, the dotted line represents the Lagrange multiplier \( \lambda \) field, and the dot-dashed line the fermionic ghost field \( c \). The analytic expressions including factors are given in the Supplemental Material.

The analytic expressions including factors are given in the Supplemental Material.

FIG. 2: 1-, 2- and 3-loop diagrams that renormalize the gauge propagator \( D_{\mu \nu}(p) = N(A_{\mu}(-p)A_{\nu}(p)) \) as

\[
[D_{\mu \nu}(p)]^{-1} = \left(1 - \frac{1}{N}\right) \frac{p}{16} \left(\delta_{\mu \nu} - \frac{p_{\mu}p_{\nu}}{p^2}\right) + 2\sigma_0^2 \delta_{\mu \nu} - \Sigma_{\mu \nu}(p),
\]

where the terms not proportional to \( 1/N \) are in fact the \( N \rightarrow \infty \) contribution shown in Fig. 1 that give the gauge boson its dynamics in the first place.

Before describing the \( 1/N \)-corrections further, we note here that our calculations passed several consistency checks. First of all, we have checked by explicit computation that all diagrams that, individually, would generate a mass for the Goldstone bosons, instead cancel with each other. Moreover, we have computed the correlation length exponent \( \nu = 1 - 48/(N\pi^2) \) in agreement with previous work \([36, 37]\) by summing the prefactors of all logarithmic singularities \( \sim \log[\Lambda/\sigma_0] \sigma_0^2 \) (where \( \Lambda \) is a momentum cutoff) appearing in the 12 self-energy diagrams of Fig. 2. Finally, we computed the gauge field propagator and the current-current correlator also at the critical point (for \( \sigma_0 = 0 \)) \([26]\) and showed in particular that all (logarithmic and other) singularities as a function of momentum \( p \) cancel with each other. Thereby, we demonstrated that, as expected \([38, 39]\), these conserved quantities do not pick up anomalous dimensions (beyond the \( N \rightarrow \infty \) renormalization which is essentially a dimensional effect), and that they fulfill the expected Ward identities between self-energy and vertex corrections.

We now present our main quantitative results for the quantum dynamics of the vector gauge boson in the vicinity of the critical point. As announced earlier, we will compute the position of the pole of the gauge-field propagator on the real frequency axis at zero spatial momentum, and the spectral function which can also be extracted in numerical simulations as well as potentially in experiments. In the large \( N \) limit, the pole of the gauge-field propagator (zero in the transverse component of Eq. (7)) is located at \(-32\sigma_0^2\), or \( \omega = -32\sigma_0^2 \) in frequency with the Lorentzian time signature, corresponding to an overdamped mode of purely dissipative character. This is just the well known consequence of the decay of the gauge boson into the “particle and anti-particle” continuum of Goldstone bosons (as per Fig. 1) with the scale \( 32\sigma_0^2 \) provided by the Higgs mechanism.

However, after accounting for the self-energy induced shifts of the pole position to \( 1/N \), the quantum dynamics of the gauge field changes qualitatively and gains an oscillatory response characteristic of a finite lifetime excitation similar to the scattering resonances of the W- and Z boson in particle physics. To determine the shift, the \( \Sigma_{\mu \nu}(p) \) integral is calculated at \( p = -32\sigma_0^2 \) numerically, as described in Ref. [2] by analytically continuing the contour integral. We take \( k \to ke^{i\theta} \) and \( p \to pe^{i\theta} \), then take the limit of \( \theta \to -\pi \), rotating the contour around the lower half plane. The \( \theta \) variation is done gradually, to ensure that we do not cross any poles. For a few of the diagrams, the integral can be done analytically by putting \( p \) and \( k \) right below the negative real axis, and the analytic and numerical results agree.
Finally, the universal structure of the pole position is revealed by writing $\sigma_0^2$ in terms of the spin stiffness $\rho_s$; their relationship is easily computed at $N = \infty$, and using the scaling law $\rho_s \sim (g - g_c)^\nu$ and the value of $\nu$ quoted above, we obtain at order $1/N$

$$\rho_s = \frac{N\sigma_0^2}{2} \left( 1 + \frac{48}{N\pi^2} \ln \left( \frac{A}{32\sigma_0^2} \right) + \frac{C}{N} \right),$$

where $C$ is a constant of order unity. Then, in terms of Lorentzian frequencies $\omega$, the pole is at

$$\frac{\omega_{\text{pole}}}{\rho_s/N} = -64i + \frac{32}{N} \left[ 7.319 - (11.191 - 2C)/i \right] + O(1/N^2).$$

In order to calculate the vector boson spectral function, we expand the renormalized propagator, Eq. (7), to order $1/N$ by $D_{\mu\nu}(p) = D^{\mu\nu}_0(p) + D^{\mu\nu}_c(p) + D^{\mu\nu}_{\text{int}}(p)D^{\mu\nu}_0(p)$, and write

$$D_{\mu\nu}(p) = \mathcal{H}_T(p) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \mathcal{H}_L(p) \frac{p_\mu p_\nu}{p^2}.$$  

The transverse part determines the response in Eq. (4), $\rho_s F(\omega/\rho_s) = (p^2/N)\mathcal{H}_T(p)$, and its imaginary part

$$\mathcal{F}''(\omega/\rho_s) = \frac{1}{\rho_s} \text{Im} \left[ \frac{p^2}{N} \mathcal{H}_T(p) \right]_{p \to -i\omega}$$

determines the spectral properties of the vector boson. The Wick rotation of $\mathcal{H}_T$ to real frequencies has to be done numerically similarly to the procedure described above Eq. (9) except that now we rotate directly onto the imaginary axis at $\theta \sim -\pi/2$.

As can be observed from Fig. 3, the vector boson spectrum at $N \to \infty$ displays a suppressed, broad continuum indicative of the dissipative nature of the vector boson at that order.

For the physically relevant smaller values of $N$, a peak-like structure emerges, which becomes progressively better defined upon approaching the case $N = 2$ for deconfined quantum magnets. This indicates an enhanced lifetime of the vector boson. In real-time experiments, the vector boson response is that of a damped oscillator. As with the Higgs mode close to the superfluid-to-Mott insulator quantum phase transition [7,8,10], we expect this emergent vector boson “resonance” for smaller $N$ to be observable in direct numerical simulations of $SU(N)$ quantum spin models [17,40] using the observables of Ref. [21], and potentially also in experiments.

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The self energy correction at order $1/N$ is a sum over 12 diagrams $\Sigma_{\mu\nu}(p) = \sum_{i=1}^{12} a_i \Sigma_{\mu\nu}^{(i)}(p)$, where the $a_i$ are symmetry and multiplicity factors for each diagram not directly contained in the vertices and propagator Feynman rules. Before the momentum integrations, the expressions for each of the diagrams are

\[
\begin{align*}
\Sigma_{\mu\nu}^{(1)}(p) &= \frac{32\sigma_0^2}{N} \int_q \frac{1}{(q + 32\sigma_0^2)(p + q + 16\sigma_0^2)} \\
\Sigma_{\mu\nu}^{(2)}(p) &= \frac{8}{N} \int_q \int_k k^2(p + k)^2(q + 16\sigma_0^2) \left( \frac{1}{(k + q)^2} - \frac{1}{q^2} \right) \\
\Sigma_{\mu\nu}^{(3)}(p) &= \frac{8}{N} \int_q \int_k k^2(p + k)^2(2k + q)^2(p + k + q)^2(q + 16\sigma_0^2) \\
\Sigma_{\mu\nu}^{(4)}(p) &= \frac{16}{N} \int_q \int_k \frac{(2k + p)^2(p + q)^2}{k^2(p + k)^2(q + 16\sigma_0^2)} \frac{(2k + q)_l(2k + q)_l}{(k + q)^2} - \frac{q_4dq}{q^2} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_4 = 2 \\
\Sigma_{\mu\nu}^{(5)}(p) &= \frac{16}{N} \int_q \int_k \frac{(2k + q)_l(2k + q)_l}{k^2(p + k)^2(q + 16\sigma_0^2)} \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \delta_{p\nu} \delta_{\mu\lambda} (\mu \leftrightarrow \nu) a_5 = 1 \\
\Sigma_{\mu\nu}^{(6)}(p) &= \frac{16}{N} \int_q \int_k \frac{(2k + q)_l(2k + q)_l}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_6 = 4 \\
\Sigma_{\mu\nu}^{(7)}(p) &= \frac{16}{N} \int_q \int_k \frac{1}{(k + q)^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_7 = 4 \\
\Sigma_{\mu\nu}^{(8)}(p) &= \frac{256}{N} \int_q \int_k \frac{1}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \frac{\sigma_0^2}{(q + 32\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_8 = 2 \\
\Sigma_{\mu\nu}^{(9)}(p) &= \frac{256\sigma_0^2}{N} \int_q \int_k \frac{1}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_9 = 4 \\
\Sigma_{\mu\nu}^{(10)}(p) &= \frac{-128}{N} \int_q \int_k \frac{(2k + p)^2(2k + q)_l(2l + p)^2(2l + q)_l}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_{10} = 2 \\
\Sigma_{\mu\nu}^{(11)}(p) &= \frac{128}{N} \int_q \int_k \frac{1}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_{11} = 4 \\
\Sigma_{\mu\nu}^{(12)}(p) &= \frac{-128}{N} \int_q \int_k \frac{1}{k^2(p + k)^2(p + q + 16\sigma_0^2)} \left( \delta_{lp} + \frac{q_4dq}{32\sigma_0^2} \right) a_{12} = 4. \\
\end{align*}
\]

These integrals can be evaluated using our algorithm Tensoria as described in the text and in Ref. [26]. To extract the critical exponent $\nu$ from the logarithmic singularity and evaluate the finite terms to determine the pole position and spectral function, we pull out the momentum cutoff ($\Lambda$) dependent part and split the rest into transversal and longitudinal components:

\[
\Sigma_{\mu\nu}^{(i)}(p) = \frac{1}{N} \left[ \mu_T(p) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \mu_L(p) \frac{p_\mu p_\nu}{p^2} \right] + \frac{1}{N} \left[ \frac{14\Lambda}{3\pi^2} - \frac{96\sigma_0^2}{\pi^2} \log \frac{\Lambda}{32\sigma_0^2} \right] \delta_{\mu\nu}.
\]

Here the term proportional to $\Lambda$ is a consequence of the non-gauge-invariant momentum cutoff, but can be safely absorbed into a shift in the position of the critical point; such artifacts do not propagate to the universal constants computed here, which are gauge invariant. The logarithmic singularity yields the correlation length $\nu = 1 - 48/(N\sigma_0^2)$ described in the main text, and is absorbed after $\sigma_0^2$ is expressed in terms of $\rho_s$ via Eq. (8). For the remaining part, we evaluate the analytically continued pole contributions at the location of the $N \rightarrow \infty$ pole position ($p = -32\sigma_0^2$). The numerical integration yield to high accuracy (all
The constants below are in units of $\sigma_0^2$:

\begin{align*}
I^{(1)}_T &= -2.882 - 0.374i, & I^{(1)}_L &= -0.213 + 0.246i \\
I^{(2)}_T &= -0.548 - 0.265i, & I^{(2)}_L &= -0.270 + 0.318i \\
I^{(3)}_T &= 0.539 + 1.197i, & I^{(3)}_L &= 0 + 0.098i \\
I^{(4)}_T &= 0.476 + 0i, & I^{(4)}_L &= -0.050 + 0i \\
I^{(5)}_T &= 1.895 - 0.587i, & I^{(5)}_L &= -0.994 + 0i \\
I^{(6)}_T &= -0.454 + 0i, & I^{(6)}_L &= 0.548 + 0i \\
I^{(7)}_T &= -1.754 + 0i, & I^{(7)}_L &= -0.274 + 0i \\
I^{(8)}_T &= -0.629 - 0.148i, & I^{(8)}_L &= -0.426 + 0.492i \\
I^{(9)}_T &= 2.882 + 0.374i, & I^{(9)}_L &= 0.213 - 0.246i \\
I^{(10)}_T &= -0.606 - 3.098i, & I^{(10)}_L &= -0.635 + 0.492i \\
I^{(11)}_T &= 1.083 + 0.148i, & I^{(11)}_L &= -0.121 - 0.492i \\
I^{(12)}_T &= -1.128 - 0.374i, & I^{(12)}_L &= 0.061 + 0.246i.
\end{align*}

(14)

Combining these, we get

\begin{align*}
I^{\text{tot}}_T &= \sum_i a_i I^{(i)}_T = -9.191 - 7.319i \\
I^{\text{tot}}_L &= \sum_i a_i I^{(i)}_L = -2.906 + 1.720i.
\end{align*}

(15)