Essays in Financial Economics

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Abstract

This dissertation addresses the central issue of understanding how frictions to information flow distort the ability for prices to incorporate new information. In chapter 1, “Forgotten Portfolios”, I illustrate how the ability of a stock’s price to impound information can rely on the portfolios of its owners. I show that, in the presence of limited attention, investors rationally allocate their attention towards processing information that has a greater impact on their wealth. Chapter 2, “The Social Elite” (based on joint work with Alexander Chernyakov), examines how casual social interactions impact asset prices. Social networks play a vital role in the diffusion of information. I focus on fund managers and corporate officers of publicly traded firms and present evidence of information transfer at exclusive social gatherings. I find that when executives attend social gatherings their stock prices’ subsequent behavior directionally predicts upcoming earnings surprises. I show that fund managers who attend events that corporate officers from a particular firm also attend are more likely to purchase stock in that firm. I explore potential reasons for this tendency and find that fund managers demonstrably outperform when they decide to trade these socially-connected stocks. Further, socially-connected stocks that fund managers do not purchase subsequently underperform. In chapter 3, “Mean Reversion”, I propose a mechanism whereby learning from news jointly explains the patterns of short horizon momentum and long horizon reversals observed in equity prices. The model’s key departure from rationality is its assumption that investors underestimate the relative precision of news. Under mild assumptions, investors will exhibit a rational but perverse tendency to increase their belief in other private signals, regardless of whether or not the private signal is true.
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Chapter 1

Forgotten Portfolios

1.1 Introduction

There is a long history of psychology literature that documents the inability of individuals to allocate attention among multiple tasks. Early works such as Kahneman (1973) argue that, “attention is a scarce cognitive resource and attention to one task necessarily requires a substitution of cognitive resources from other tasks.”

In this essay I illustrate a mechanism whereby the rate at which information is impounded into a stock’s price is related to the portfolios of its owners. I show that in the presence of limited attention, investors rationally allocate their attention towards processing information that has a greater potential impact on their wealth.

From sell-side equity analysts to mutual fund portfolio managers, participants in modern financial markets are constantly inundated with information to disseminate each day. This has motivated the development of empirical asset-pricing models that explain the factors that impact how news enters prices. These frameworks have a common theme that involves information-processing, and price-updating, until markets reflect publicly available information.

The mechanism through which information is revealed is often the focal point when examining these issues. For example, more salient sources of news are impounded into prices
more quickly. Huberman and Regev (2001) provide a striking illustration of this idea. They examine the case of a biotechnology firm whose price increases four-fold after being featured in a news article on the cover of the Wall Street Journal. At the time the news was released, the firm’s price move was a response to a favorable assessment of recent innovations in cancer research. However, the article contained no actual new information; it was previously printed in the journal Nature several months before. At the time of the original publication, the news article resulted in a demonstrably smaller market response, which isn’t surprising since mostly Scientists and specialist investors read the journal Nature. As an information-revelation mechanism, the front page of the Wall Street Journal has a significantly greater influence on the marginal price setter.

The example by Huberman and Regev (2001) highlights the key characteristic of gradual information flow in asset markets. This essay provides a perspective on why information might be impounded into prices slowly, that is distinct to the information revelation mechanism. I examine how an investor’s level of distraction influences the responsiveness of prices to new information. Specifically, when investors own a particular stock, but that stock is for each of them only a small fraction of their portfolio, that stock is forgotten. For forgotten stocks, no single investor makes it a priority to disseminate its news.

In the presence of news, stocks that are a small proportion of an investor’s wealth will find it difficult to compete for attention. I quantify this tendency using a simple measure of the degree to which a stock’s largest investors are the investors for which they are a relatively small portfolio weight.

To better understand my approach, consider the following example. Table 1.1 summarizes all quantities discussed. Suppose there are two investors, one which I will refer to as “BIG” who has $50 of assets under management (AUM), and another referred to as “SML” who has $5 of AUM. Next, focus on two companies, AAPL and LULU, that both have a market capitalization of $5. Assume, that BIG owns $1 and $4 in AAPL and LULU respectively, and SML owns $4 and $1 in APPLE and LULU respectively. It follows that the percentages

1Investors in this context concretely refer to mutual funds.
Table 1.1: Measuring Investor Inattention, An Example

<table>
<thead>
<tr>
<th>Market Cap</th>
<th>Holdings (%)</th>
<th>Percentage of Portfolio Weight</th>
<th>Percentage of Attention</th>
<th>Attention</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAPL</td>
<td>5</td>
<td>BIG 1 SML 4</td>
<td>20% 80% 80% 2%</td>
<td>0.79</td>
</tr>
<tr>
<td>LULU</td>
<td>5</td>
<td>BIG 4 SML 1</td>
<td>80% 20% 20% 8%</td>
<td>0.37</td>
</tr>
<tr>
<td>AUM</td>
<td>50</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table provides the numerical values for an example that motivates my measure of investor inattention.

of each stock’s market capitalization owned by the investor is $20\% = \frac{\$4}{\$5}$ and $80\% = \frac{\$4}{\$5}$. Notice that the numbers imply that all of SML’s assets are invested between AAPL and LULU. On the other hand, between AAPL and LULU, BIG’s owns a combined $\$5$ and the remaining $\$45$ is assumed to be invested in other stocks. Other stocks will play no roll in this example.

BIG’s portfolio has $2\%$ invested in AAPL, $8\%$ invested in LULU. SML’s portfolio has $80\%$ invested in AAPL, $20\%$ invested in LULU. Suppose, each fund has 1 unit of attention which is distributed proportionally. If we consider LULU, then we see that it is allocated $0.28 = 8\% + 20\%$ units of attention. Of this total $0.28$ units, BIG is responsible for $29\% = \frac{0.08}{0.28}$ and SML is responsible for $71\% = \frac{0.20}{0.28}$. Table 1.1 show that the equivalent percentages of attention for AAPL are $2\%$ and $98\%$ for BIG and SML respectively.

I now pose my central question: *For each of, AAPL and LULU, taken individually, to what degree are its largest investors the investors for which they are a relatively small portfolio weight.*

Returning to the example, for LULU, BIG has $29\%$ of the “attention-outstanding” by holding $80\%$ of the shares outstanding, and SML has $71\%$ of the “attention-outstanding” by holding $20\%$ of the shares outstanding. I take the value-weighted average to obtain an attention measure of $0.37 = \sqrt{0.29 \cdot 0.8 + 0.71 \cdot 0.2}$. The analogous quantity for AAPL is $0.79$. There is more aggregate attention allocated to AAPL than LULU.

In this example, AAPL is the quintessential high-attention stock. The intuition here is that a single investor, SML, owns a relatively large quantity of AAPL ($80\%$ of its market
capitalization). At the same time, SML’s entire portfolio (80% portfolio weight) consists of AAPL. In this example, SML can be thought of as a highly specialized hedge fund manager; SML has a small and concentrated portfolio that is largely in a single stock. AAPL has a modest market capitalization in comparison to SML’s AUM, and so SML not only allocates a lot of attention to AAPL but also exerts price impact on AAPL when he trades. This measure forms the basis for my empirical strategy, and as this example demonstrates, my approach is not simply a function of a stock’s breadth of ownership or market capitalization.

To test my forgotten stock hypothesis, I examine the impact that investor inattention has on the robustness of post-earnings announcement drift (PEAD). I separate stocks into high-attention and low-attention groups each month. Within attention groups, stocks are further grouped to form PEAD portfolios on the basis of their most recent earnings surprise. The low-attention PEAD portfolio earns monthly factor adjusted-alphas of 153 basis points ($t=3.5$) whereas the high-attention PEAD portfolio is statistically insignificant and earns only -6 basis points ($t=-0.24$) in monthly factor adjusted alphas.

This effect of attention on PEAD is essentially monotonic across attention quintiles. I construct a portfolio that captures the pure inattention effect. Specifically, I construct a spread portfolio that hedges out “earnings momentum” by going long the low-attention PEAD portfolio and shorting its high-attention analog earns monthly factor-adjusted alphas of 160 basis points ($t=3.3$). Moreover, these economically significant alphas are not associated with large loadings on common risk factors. In particular, the magnitudes and t-statistics on risk factor loadings are small and insignificant.

I examine my mechanism in additional detail by illustrating that when investors’ attention constraints are binding then the effect is exacerbated. I combine news data and mutual fund holdings, in order to determine when investors face abnormally large news shocks about the stocks that they own. Then for each stock, I measure the degree to which that stock’s owners, on average, face an unexpected cognitive burden. I find that the underreaction pattern exhibited by forgotten stocks is most pronounced in stocks held by information-burdened funds. The spread between the low-attention PEAD portfolio and its high-attention analog
earns monthly factor adjusted alphas of 225 basis points \((t=2.14)\).

These results are true for both value-weighted and equal-weighted specifications of my test. I observe no sign of any return reversal in the subsequent six months after portfolio formation. Finally, I find that this effect is not driven by a few years in the sample nor does it attenuate in recent years.

The remainder of the essay is organized as follows. Section 1.2 of the essay provides a brief literature review and background on the topic. Section 1.3 describes the data used and reports a variety of summary statistics. Section 1.4 briefly outlines my hypothesis and presents the essay’s central results. Section 1.5 explores my underreaction mechanism in detail. Section 1.6 provides robustness checks and section 1.7 concludes.

1.2 The Context

This essay is integrally related to previous studies that analyze the effects of investors’ limited information processing capacity. The central theme of this strand of literature is that investors have limited capability to collect, consolidate, and interpret information, and therefore, we should expect asset prices to incorporate information slowly. More generally, there is a long history of literature that examines the inability of individuals to allocate attention among multiple tasks, such as early works like Kahneman (1973). Numerous theoretical models have since examined the effect of limited attention on market efficiency. For example, Merton (1987), Hong and Stein (1999), and Hirshleifer and Teoh (2003) argue that, in the presence of limited investor attention, delayed information revelation can generate return predictability that cannot be explained by traditional asset pricing models.

These theoretical foundations are largely supported by subsequent empirical studies. In particular, Huberman and Regev (2001), Barber and Odean (2006), DellaVigna and Pollet (2006), Hou (2006), Menzly and Ozbas (2006), Hong, Torous, and Valkanov (2007), and Cohen and Frazzini (2008) find that stock prices respond quickly to more salient information (e.g., news printed on the front page of the New York Times or information about stocks that have recently experienced extreme returns or trading volume). This essay also contributes
materially to a growing literature that focuses on the interplay between the media and capital markets. Antweiler and Frank (2004) examine messages in internet chat rooms focused on stocks and argue that message activity is related to trading volume and volatility. Tetlock, Saar-Tsechansky, Macskassy (2008) quantify the language used in financial news stories in an effort to predict firms’ accounting earnings and stock returns. They find that the fraction of negative words in firm-specific news stories forecasts low firm earnings, that firms’ stock prices briefly underreact to the information embedded in negative words and that the earnings and return predictability from negative words is largest for the stories that focus on fundamentals. Broadly speaking, I add to this literature by showing that the stock price reaction to even the most salient news will be delayed if investors’ attention is directed away from it towards other news that is more likely to affect their wealth.

My framework has novel implications for predictability by linked information events such as industry news. Previous works, such as Ramnath (2002), examine the correlation between earnings surprises of firms within in the same industry and finds that the first earnings surprise within an industry has information for both the earnings surprises and returns of other firms within the industry. Related to these findings I show that news about stocks with high investor attention predicts the subsequent returns of their low attention industry peers (but not the reverse).

Finally, this essay is also related to a general literature on the effects of institutional ownership on stock returns. Both conventional wisdom and empirical evidence suggest that increased investor sophistication is usually associated with less susceptibility to behavioral biases. Recent research, however, shows that institutional investors are certainly not impervious. For example, Frazzini (2006) shows that post-announcement price drift is most severe whenever capital gains and news events have the same sign. In a similar manner, Wermers (2003) shows that underperforming fund managers exhibit a strong disposition effect and Coval and Shumway (2000) provide evidence of loss aversion among professional market makers. Using the holdings of institutional investors, I provide evidence that one can identify a subset of stocks to which they pay less attention. This finding is consistent with
the previous evidence of institutional investors’ susceptibility to behavioral biases.

1.3 Data and Summary Statistics

The data in this study are collected from a variety of sources. Stock returns and firm characteristics are obtained from the CRSP and COMPUSTAT databases respectively. Sell-side analysts coverage is obtained from the Institutional Brokers Estimates System (I/B/E/S). Financial news data is obtained from the Wall Street Journal, spans January 2000 to December 2008 and covers all U.S. equities and ADRs. Mutual fund holdings are obtained from the Thomson Financial CDA/Spectrum Mutual Funds database.

Mutual funds are required to report their holdings semiannually. However, over half of the funds in my sample period file quarterly reports. Using these periodic portfolio snapshots I construct daily notional portfolios based on the last filing. Filings are assumed to be publicly available 30 days after the file date. Following Frazzini (2006) observations are omitted whenever:

1. the number of shares in a fund portfolio exceeds the total amount of shares outstanding

2. the value of the fund’s holding of a particular stock exceeds the total value of the fund as reported by Thomson\CDA.

3. the stock is reported as having zero shares outstanding

4. for funds for which we there is a link to the CRSP dataset, the total asset value of the fund reported by Thomson\CDA differs from that which is reported by CRSP by more than 100%.

Common stocks with prices below $5 are excluded.

Since I am examining my theory as it relates to equities, I exclude funds that are self-reported as being focused on Municipal Bonds, Bond & Preferred and Metals. Additionally, this assumption is rather conservative since in recent years filings are usually available on the next business day.
since I seek understand in effects induced by the marginal price setter I exclude funds believed to be index funds from the sample.\footnote{I exclude funds explicitly labeled as index fund in the CRSP mutual fund database. Additionally, since the links between the CDA and CRSP database is limited to only a subset of all funds I exclude funds whose name includes the words: “index”, “indx”, ”total market”, “REIT”, “nasdaq”, “QQQQ”, “S&P”, “russell 1000”, “russell 2000”, “russell 3000” and funds with more than 1,000 holdings.}

1.3.1 Aggregate Summary Statistics

Table 1.2 reports summary statistics for the stocks and mutual funds my sample. Panel A summarizes the coverage of funds and stocks present in the sample for select years. Columns 2 and 3 report the counts of the number of the funds and the number of stocks respectively. We see that subsequent to the year 2000, there are roughly 10,000 funds in the cross-section each year and this number is largely constant. It is about a half of that in the preceding 5 years and it approximately 2,500 between 1990 and 1995. The number of stocks in the sample has remained roughly constant over the entire time window. Column 3 reports the percentage of the universe of CRSP/COMPUSTAT common stocks present in my sample. As is the case with the counts of stocks, the percentage of stocks doesn’t move around notably except for in the most recent part of the sample. Column 4, reports then proportion of the total market capitalization of distinct stocks held by funds in my sample divided by the total market value of the CRSP/COMPUSTAT stock universe. Looking at the percentage of market value, one key takeaway is that although the percentage of stocks is closer to 50%, the stocks in the sample are larger than the median and comprise over 70% of both the total market capitalization of common stocks traded on the NYSE, AMEX and NASDAQ.

\footnote{Information on funds’ status as an index fund is obtained from the CRSP mutual fund database.}
### Table 1.2: Institutional Holdings, Summary Statistics

**Panel A. Mutual Fund Portfolios, Time Series**

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of Funds</th>
<th>Stocks Coverage</th>
<th>Percentage of Stocks</th>
<th>Percentage of Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>964</td>
<td>4,538</td>
<td>45%</td>
<td>65%</td>
</tr>
<tr>
<td>1990</td>
<td>1,335</td>
<td>4,678</td>
<td>44%</td>
<td>67%</td>
</tr>
<tr>
<td>1995</td>
<td>4,326</td>
<td>5,907</td>
<td>53%</td>
<td>71%</td>
</tr>
<tr>
<td>2000</td>
<td>10,086</td>
<td>5,835</td>
<td>60%</td>
<td>80%</td>
</tr>
<tr>
<td>2005</td>
<td>11,708</td>
<td>4,472</td>
<td>71%</td>
<td>87%</td>
</tr>
</tbody>
</table>

**Panel B. Pooled Fund-Year Characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>SD</th>
<th>P20</th>
<th>P80</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUM ($MM)</td>
<td>281</td>
<td>47</td>
<td>3</td>
<td>27,967</td>
<td>1,098</td>
<td>8</td>
<td>269</td>
</tr>
<tr>
<td>Number of holdings</td>
<td>75</td>
<td>45</td>
<td>9</td>
<td>1,829</td>
<td>125</td>
<td>21</td>
<td>92</td>
</tr>
</tbody>
</table>

**Panel C. Pooled Firm-Year Characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>SD</th>
<th>P20</th>
<th>P80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size percentile</td>
<td>62%</td>
<td>65%</td>
<td>29%</td>
<td>100%</td>
<td>24%</td>
<td>40%</td>
<td>86%</td>
</tr>
<tr>
<td>Book-to-market percentile</td>
<td>48%</td>
<td>47%</td>
<td>12%</td>
<td>100%</td>
<td>27%</td>
<td>21%</td>
<td>75%</td>
</tr>
<tr>
<td>Analyst Coverage Percentile</td>
<td>56%</td>
<td>61%</td>
<td>13%</td>
<td>100%</td>
<td>30%</td>
<td>20%</td>
<td>85%</td>
</tr>
<tr>
<td>Breadth of ownership</td>
<td>115</td>
<td>57</td>
<td>7</td>
<td>2,293</td>
<td>191</td>
<td>17</td>
<td>147</td>
</tr>
</tbody>
</table>

This table presents a summary of the stock and mutual fund data used in this paper. Panel A reports the number of mutual funds in my sample for select years and the coverage of the CRSP/Computstat universe of the stocks held by these funds. Panel B presents pooled fund-year statistics of the funds in my sample. Panel C presents pooled firm-year statistics of the stocks in my sample.

Panel B presents pooled fund-year sample statistics of the funds in my sample. This is useful in contextualizing magnitudes of inputs used to generate my inattention measure. Assets under management (AUM) is the market value of the equity holdings of a fund in millions. We see that over the entire sample the median fund has approximately $47 million in AUM. Funds usually have between $10 million and $300 million in AUM. Funds typically hold about 45 positions and usually have between 20 and 100 positions.

Panel C reports the sample characteristics compared to all stocks in the CRSP/COMPUSTAT universe. We see that the sample is slightly biased towards larger stocks (65th percentile), but close to the median in terms of book-to-market price (47th percentile). Stocks in the sample have slightly more analyst coverage and have a breadth of ownership of 57. Taken together this suggests that the baseline data sample is unlikely to be pathological in a manner...
that would bias the results.

1.4 Underreaction in Forgotten Stocks

In this section I examine return predictability in forgotten stocks and present my central results. This section has 3 distinct goals. I begin by describing the details of my attention measure and formally stating my hypothesis. Second, I present an example from the data and provide intuition regarding the magnitudes of various quantities. Third I test my main hypothesis; I use calendar time portfolios, as well a regression framework in which I can better control for several determinants of firm returns.

1.4.1 Measuring Investor Inattention

I proceed with some definitions and notation. For a stock $s$ and a mutual fund $f$, in each monthly time period $t$, I denote by $h_{s,f,t}$ the dollar value of $f$’s holding in $s$. I denote by $F_{s,t}$ the collection of funds that hold $s$ and I denote by $P_{f,t}$ the set of stocks held by $f$ (ie: fund $f$’s portfolio).

Let $AUM_{f,t}$ denote the dollar value of stock’s equity holdings\(^5\)

I define $MSHARE_{s,f,t}$, the share of a stock's aggregate mutual fund ownership that fund holds

\[
MSHARE_{s,f,t} = \frac{h_{s,f,t}}{\sum_{f_j \in F_{s,t}} h_{s,f_j,t}}
\]  

(1.1)

I define $\omega_{s,f,t}$, the portfolio weight of a fund’s stock holding, as the fund’s dollar holding divided by its AUM

\[
\omega_{s,f,t} = \frac{h_{s,f,t}}{AUM_{f,t}}
\]  

(1.2)

I define $\omega^*_{s,f,t}$, the proportion of aggregate portfolio weight attributed to a fund’s holding, as the fund’s portfolio weight in the stock divided by the sum of the portfolio weights in the stock across all funds

---

\(^5\)This is assets under management
\[ \omega^*_s,f,t \equiv \frac{\omega_s,f,t}{\sum_{f_j} \omega_{s,f_j,t}} \] (1.3)

My raw attention measure, is given by

\[ ATTN_{RAW,s,t} \equiv \sum_{f_j \in F_{s,t}} \left( \omega^*_s,f,t \cdot MFSHARE_{s,f_j,t} \right) \] (1.4)

Next, since the measure \( ATTN_{RAW,s,t} \) is likely to be related to a variety stock specific characteristics that are unrelated to my theory. I control for these stock-specific factors by performing monthly cross sectional regressions that orthogonalize the raw attention measure and produce my final attention measure.

The final attention measure, denoted by \( ATTN_{s,t} \), are the residuals from these monthly cross sectional regressions.

\[ ATTN_{s,t} \equiv \varepsilon_{s,t} \] (1.5)

where \( \varepsilon_{s,t} \) is obtain from the monthly cross-sectional regressions of the form

\[ ATTN_{RAW,s,t} = \alpha_t + \theta_t \cdot X_{s,t} + \varepsilon_{s,t} \] (1.6)

The terms \( \alpha_t \) and \( \theta_t \) are coefficient estimates. \( X_{s,t} \) is a panel of controls.

The cross sectional regressions control for beta, size, book-to-market, breadth of ownership, fraction of market cap owned by mutual funds, idiosyncratic volatility and past returns. These regressions are reported in table 1.3. BETA is the coefficient on market excess return from monthly regressions of daily excess returns on the three Fama and French factors and the Carhart momentum factor. BM is the log of the book-to-market ratio which is the Compustat book value of equity divided by the market value of equity; BM is computed as of the June of the previous calendar year following Daniel, Grinblatt, Titman, and Wermers (1997). SIZE is the log of the firm’s market value of equity as of the previous calendar month. \( RET_{t-1} \)
and $RET_{t-1,t-6}$ are the past one and six month stock returns respectively. $RET_{t-2,t-12}$ is the prior-year stock return excluding the past month. IDIOVOL is the standard deviation of the residuals from a monthly regression of daily excess returns on the three Fama and French factors and the Carhart momentum factor. TURN is the average turnover in the previous 12 months and NASD is a NASDAQ dummy. As in Frazzini (2008), I allow the coefficients to be different for NASDAQ stocks since turnover numbers have a different interpretation in a dealer markets. ANALYSTCOVER is the number of analysts that cover the stock as of the previous calendar month.

Table 1.3 reports Fama and MacBeth (1973) results from performing these cross sectional regressions. Specification 1 shows that $ATTN_{RAW,s,t}$ is negatively related to size and mutual fund ownership. Therefore the classification as a forgotten stock (low $ATTN_{RAW,s,t}$) is more likely in large-caps. In specification 2 I add recent returns. Here we see that stocks that have experienced large recent losses are more likely to be higher attention, with most of the effect coming from $RET_{t-2,t-12}$ (prior-year excluding the past month). In specification 3 I add idiosyncratic volatility. Higher IDIOVOL stocks are more likely to be high-attention stocks which makes sense. In specification 4 I add turnover (past year) and an interaction term between turnover and returns as a control. The results show that controlling for other size and idiosyncratic volatility especially, higher volume stocks tend to have lower attention. Here this is likely due to the interaction with BM, which goes from a t-stat of 2.57 to -0.45 between specifications 3 and 4. In specification 5 I add analyst coverage. I find the intuitive result, that more visible stocks, such as those with higher analyst coverage, tend to have higher $ATTN_{RAW,s,t}$. I use the residual attention measure derived from specification 1 to construct my residual attention measure.
### Table 1.3: Measuring Investor Inattention, Fama-Macbeth Regressions

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETA</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.006</td>
</tr>
<tr>
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<td>(-17.55)</td>
<td>(-16.41)</td>
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</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td>(3.10)</td>
<td>(1.64)</td>
<td>(2.57)</td>
<td>(-0.45)</td>
<td>(-0.08)</td>
</tr>
<tr>
<td>SIZE</td>
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<td>-0.06</td>
<td>-0.06</td>
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</tr>
<tr>
<td></td>
<td>(-44.25)</td>
<td>(-45.01)</td>
<td>(-45.32)</td>
<td>(-46.01)</td>
<td>(-48.69)</td>
</tr>
<tr>
<td>MFOWN</td>
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<td>-0.50</td>
<td>-0.50</td>
<td>-0.47</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td>(-23.35)</td>
<td>(-23.15)</td>
<td>(-23.09)</td>
<td>(-23.07)</td>
<td>(-23.70)</td>
</tr>
<tr>
<td>BREADTH</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(16.28)</td>
<td>(17.28)</td>
<td>(17.06)</td>
<td>(17.30)</td>
<td>(18.08)</td>
</tr>
<tr>
<td>$RET_{t-1}$</td>
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<td>-0.01</td>
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<td>0.00</td>
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</tr>
<tr>
<td></td>
<td>(-2.43)</td>
<td>(-1.75)</td>
<td>(0.92)</td>
<td>(0.85)</td>
<td></td>
</tr>
<tr>
<td>$RET_{t-2,t-12}$</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-29.19)</td>
<td>(-29.14)</td>
<td>(-25.57)</td>
<td>(-25.86)</td>
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</tr>
<tr>
<td>IDIOVOL</td>
<td>0.13</td>
<td>0.34</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.12)</td>
<td>(11.92)</td>
<td>(12.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TURN</td>
<td>-10.77</td>
<td>-10.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-12.92)</td>
<td>(-13.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASD·TURN</td>
<td>2.39</td>
<td>2.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.98)</td>
<td>(5.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ANALYSTCOVER</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INTERCEPT</td>
<td>1.35</td>
<td>1.40</td>
<td>1.39</td>
<td>1.38</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>(45.36)</td>
<td>(46.07)</td>
<td>(46.38)</td>
<td>(46.97)</td>
<td>(49.64)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.38</td>
<td>0.39</td>
<td>0.39</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

This table reports coefficients from Fama–MacBeth regressions of the raw attention measure, $ATTN_{RAW}$, on a set of firm-specific regressors.

### 1.4.2 Forgotten Stock Hypothesis and Underreaction

I describe my primary hypothesis and present an investment rule to construct the test portfolios. I postulate that in the presence of limited-attention, investors rationally allocate their cognitive resources towards processing information that has a larger impact on their wealth. Consequently, attention is directed away from stocks that have a smaller impact on investors’ wealth thereby generating post-event drift in these lower impact stocks.
HYPOTHESIS FS (FORGOTTEN STOCK): A stock’s price underreacts to information events when it’s largest investors are the ones that own it as a relatively small proportion of their portfolio. When investors’ attention constraints bind tightly, stocks experience additional underreaction to information events. When investors’ attention constraints are relaxed underreaction is less pronounced for all stocks.

1.4.3 Baseline PEAD Portfolios

The focus of my analysis is the effect of stock specific news on underreaction. As such, I use earnings surprise (the cumulative return from day t−1 to t+1 around the last earnings announcement) as my proxy for firm news. My first set of tests analyzes the returns to momentum after sorting according to by Attention Coefficients. I use a monthly post-earnings Announcement Drift (PEAD) strategy as the benchmark portfolio when presenting the results.

Table 1.4: PEAD Portfolios, Monthly Alphas

<table>
<thead>
<tr>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>L/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Spread</td>
<td>0.80%</td>
<td>1.03%</td>
<td>1.09%</td>
<td>1.22%</td>
<td>1.49%</td>
</tr>
<tr>
<td></td>
<td>(1.95)</td>
<td>(3.11)</td>
<td>(3.57)</td>
<td>(3.74)</td>
<td>(3.83)</td>
</tr>
<tr>
<td>CAPM</td>
<td>-0.43%</td>
<td>0.00%</td>
<td>0.15%</td>
<td>0.21%</td>
<td>0.30%</td>
</tr>
<tr>
<td></td>
<td>(-2.34)</td>
<td>(0.02)</td>
<td>(1.20)</td>
<td>(1.65)</td>
<td>(1.82)</td>
</tr>
<tr>
<td>FF 3-Factor</td>
<td>-0.51%</td>
<td>-0.11%</td>
<td>0.04%</td>
<td>0.12%</td>
<td>0.27%</td>
</tr>
<tr>
<td></td>
<td>(-4.70)</td>
<td>(-1.21)</td>
<td>(0.44)</td>
<td>(1.47)</td>
<td>(3.33)</td>
</tr>
<tr>
<td>4-Factor</td>
<td>-0.38%</td>
<td>-0.04%</td>
<td>0.08%</td>
<td>0.15%</td>
<td>0.30%</td>
</tr>
<tr>
<td></td>
<td>(-4.08)</td>
<td>(-0.44)</td>
<td>(0.92)</td>
<td>(1.91)</td>
<td>(3.85)</td>
</tr>
<tr>
<td>5-Factor</td>
<td>-0.38%</td>
<td>-0.04%</td>
<td>0.08%</td>
<td>0.16%</td>
<td>0.31%</td>
</tr>
<tr>
<td></td>
<td>(-3.97)</td>
<td>(-0.49)</td>
<td>(0.97)</td>
<td>(2.00)</td>
<td>(3.91)</td>
</tr>
</tbody>
</table>

This table reports raw and factor-adjusted alphas for cross sectional sorts of stocks based on their most earnings surprise.

In table 1.4 raw and factor-adjusted alphas for the baseline PEAD strategy. At the beginning of every calendar month, stocks are ranked in ascending order on the basis of their
most recent earnings surprise. The earnings-surprise ranked stocks are assigned to 5 quintile portfolios. The L/S reports results for going long the stocks in the 5th earnings surprise quintile and selling short the stocks in the 1st earnings surprise quintile. Stocks are value weighted within a given portfolio, and the portfolios are rebalanced every calendar month to maintain value weights. The dependent variable is the monthly excess return. I begin by reporting returns of this PEAD strategy. The last column in Table 1.4 confirms that there is significant earnings momentum in the full sample. The benchmark long-short portfolio generates monthly factor adjusted-alphas of 78 basis points ($t=7.5$). Negative (positive) earnings surprise stocks display negative (positive) return continuation, and the effect is monotonic across quintiles. My values are comparable to those reported in previous studies of PEAD.

### 1.4.4 Investor Inattention and PEAD Portfolios

Table 1.5 reports monthly alphas for the main test assets. At the beginning of every calendar month, stocks in the highest and lowest attention quintile are independently ranked in ascending order on the basis of their most recent earnings surprise. The earnings surprise ranked stocks are assigned to 5 quintile portfolios. The columns labeled L/S report results for going long the stocks in the 5th earnings surprise quintile and selling short the stocks in the 1st earnings surprise quintile. Stocks are rebalanced every calendar month to maintain value (or equal) weights. Separating stocks according to their attention quintiles induces large differences in subsequent PEAD.

Table 1.5 shows that a PEAD strategy that focuses solely on the trading low attention stocks earns large and significant returns, while a strategy that trades high-attention stocks does not. For example, the value-weighted PEAD portfolio in the low attention-quintile earns a three-factor alpha of 153 basis points per month ($t=3.49$), or almost 20% per year, while the high-attention momentum portfolio earns an insignificant -6 basis points per month ($t=-0.24$). These pronounced patterns are similar for both equal-weighted and value-weighted returns, suggesting that predictability is not limited to smaller firms.
Table 1.5: Effect of Fund Manager Attention on Monthly PEAD Alphas

<table>
<thead>
<tr>
<th>Panel A. PEAD Alphas by Attention Coefficient Quintiles (Equal Weighted)</th>
<th>High Attention</th>
<th>Low Attention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Bad News)</td>
<td>(Good News)</td>
</tr>
<tr>
<td>Raw Spread</td>
<td>0.96%</td>
<td>0.90%</td>
</tr>
<tr>
<td>(2.26)</td>
<td>(2.99)</td>
<td>(3.56)</td>
</tr>
<tr>
<td>CAPM</td>
<td>-0.27%</td>
<td>0.00%</td>
</tr>
<tr>
<td>(-1.24)</td>
<td>(-0.01)</td>
<td>(1.22)</td>
</tr>
<tr>
<td>FF 3-Factor</td>
<td>-0.37%</td>
<td>-0.13%</td>
</tr>
<tr>
<td>(-2.48)</td>
<td>(-1.00)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>4-Factor</td>
<td>-0.21%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>(-1.57)</td>
<td>(-0.20)</td>
<td>(1.04)</td>
</tr>
<tr>
<td>5-Factor</td>
<td>-0.23%</td>
<td>-0.05%</td>
</tr>
<tr>
<td>(-1.69)</td>
<td>(-0.47)</td>
<td>(0.92)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. PEAD Alphas by Attention Coefficient Quintiles (Value Weighted)</th>
<th>High Attention</th>
<th>Low Attention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Bad News)</td>
<td>(Good News)</td>
</tr>
<tr>
<td>Raw Spread</td>
<td>1.24%</td>
<td>1.14%</td>
</tr>
<tr>
<td>(2.90)</td>
<td>(3.31)</td>
<td>(2.64)</td>
</tr>
<tr>
<td>CAPM</td>
<td>0.03%</td>
<td>0.14%</td>
</tr>
<tr>
<td>(0.11)</td>
<td>(0.79)</td>
<td>(-0.68)</td>
</tr>
<tr>
<td>FF3 Factor</td>
<td>-0.15%</td>
<td>0.05%</td>
</tr>
<tr>
<td>(-0.69)</td>
<td>(0.28)</td>
<td>(-1.72)</td>
</tr>
<tr>
<td>4-Factor</td>
<td>0.00%</td>
<td>0.18%</td>
</tr>
<tr>
<td>(0.45)</td>
<td>(1.18)</td>
<td>(-1.04)</td>
</tr>
<tr>
<td>5-Factor</td>
<td>0.06%</td>
<td>0.13%</td>
</tr>
<tr>
<td>(0.32)</td>
<td>(0.88)</td>
<td>(-0.87)</td>
</tr>
</tbody>
</table>

This table reports raw and factor-adjusted alphas for PEAD sorts restricted to the highest and lowest attention quintiles. Panels A and B present the results of portfolios formed using equal and value weights respectively.
Table 1.6 presents the performance of the PEAD portfolios in table 1.5 for each of the attention quintiles.

Table 1.6: Alphas by Attention Coefficient Quintiles

<table>
<thead>
<tr>
<th>Panel A: Alphas by Attention Coefficient Quintiles (Equal Weighted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Attention</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Low Attention</td>
</tr>
<tr>
<td>L/S LOW</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>High Attention</td>
</tr>
<tr>
<td>L/S HIGH</td>
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<td>L/S LOW</td>
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<tr>
<td>High Attention</td>
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<tr>
<td>L/S HIGH</td>
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<tr>
<td>L/S HIGH</td>
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Panel B: Alphas by Attention Coefficient Quintiles (Value Weighted)

<table>
<thead>
<tr>
<th>High Attention</th>
<th>Q5</th>
<th>Q4</th>
<th>Q3</th>
<th>Q2</th>
<th>Q1</th>
<th>Raw Spread</th>
<th>CAPM</th>
<th>FF3 Factor</th>
<th>4-Factor</th>
<th>5-Factor</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.16%</td>
<td>-0.14%</td>
<td>-0.06%</td>
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<td></td>
<td></td>
<td>(-0.66)</td>
<td>(-0.56)</td>
<td>(-0.24)</td>
<td>(-0.90)</td>
<td>(-0.89)</td>
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<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
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<td>1.39%</td>
<td>1.40%</td>
<td>1.53%</td>
<td>1.40%</td>
<td>1.42%</td>
</tr>
<tr>
<td>L/S LOW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.55%</td>
<td>1.54%</td>
<td>1.58%</td>
<td>1.62%</td>
<td>1.63%</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>(3.35)</td>
<td>(3.23)</td>
<td>(3.31)</td>
<td>(3.34)</td>
<td>(3.35)</td>
</tr>
<tr>
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<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
<td>Q1</td>
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<td>1.40%</td>
<td>1.40%</td>
<td>1.40%</td>
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<tr>
<td>L/S HIGH</td>
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<td></td>
<td>1.55%</td>
<td>1.54%</td>
<td>1.58%</td>
<td>1.62%</td>
<td>1.63%</td>
</tr>
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<td></td>
<td>(3.35)</td>
<td>(3.23)</td>
<td>(3.31)</td>
<td>(3.34)</td>
<td>(3.35)</td>
</tr>
<tr>
<td>Low Attention</td>
<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
<td>Q1</td>
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<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
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<td>L/S LOW</td>
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<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
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</tr>
<tr>
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<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
</tr>
<tr>
<td>High Attention</td>
<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
<td>Q1</td>
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<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
</tr>
<tr>
<td>L/S HIGH</td>
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<td></td>
<td></td>
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<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
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</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
</tr>
<tr>
<td>Low Attention</td>
<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
<td>Q1</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
<td>1.42%</td>
</tr>
<tr>
<td>L/S LOW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
<td>1.55%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
<td>(3.35)</td>
</tr>
</tbody>
</table>

This table reports raw and factor-adjusted alphas for long-short PEAD portfolios within each attention quintile. Panels A and B present the results of portfolios formed using equal and value weights respectively.

Further, Table 1.6 shows that the returns to the PEAD strategy increase monotonically as the attention quintile decreases. Further, we can see from Figure II that the cumulative return to the high and low PEAD hedge portfolios exhibits no sign of any return reversal (or convergence) in the subsequent 6 months after the event date. These robust results support Hypothesis FS: A stock’s price underreacts to information events when most of its owners
hold it as a small proportion of their portfolio.

Figure 1.1 depicts the cumulative returns of the high- and low-attention earnings momentum portfolio as well as their spread. It is a graphical depiction of table 1.6. The key takeaway from figure 1.1 is that the alphas appear robust from a time-series perspective. The effect persists and returns accumulate in throughout the entire sample window.

![Figure 1.1: Inattention Effect, Cumulative Performance](image)

This figure depicts the raw cumulative return of the high- and low-attention earnings momentum portfolios. In addition, this figure includes the long-short portfolio strategy that goes long the low-attention earnings momentum portfolio and goes short the high-attention earnings momentum portfolio.

Table 1.7 reports Fama and French (1993) three-factor loadings and alphas for the monthly PEAD strategy in each attention quintile. In addition, I include the results to a pure inattention effect portfolio, $L/S_{LOW} - L/S_{HIGH}$. This is the portfolio that hedges out earnings momentum as a factor by going long the low attention portfolio and selling short the high attention portfolio.

$L/S_{LOW}$ ($L/S_{HIGH}$) is the alpha of a zero-cost long-short PEAD portfolio of low (high) attention stocks. $L/S_{LOW} - L/S_{HIGH}$ is the portfolio that hedges out earnings momentum as a factor by going long the low attention portfolio and going short the high attention portfolio.
Table 1.7: Five Factor Time-Series Regressions: Alphas and Factor Loadings

<table>
<thead>
<tr>
<th>Panel A: Factor Loadings (Equal Weighted)</th>
<th>High Attention</th>
<th>Low Attention</th>
<th>Inattention Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.46%</td>
<td>0.52%</td>
<td>0.53%</td>
</tr>
<tr>
<td>(3.11)</td>
<td>(3.59)</td>
<td>(3.26)</td>
<td>(3.50)</td>
</tr>
<tr>
<td>MKT</td>
<td>-0.02</td>
<td>-0.07</td>
<td>-0.08</td>
</tr>
<tr>
<td>(-0.76)</td>
<td>(-2.09)</td>
<td>(-2.36)</td>
<td>(1.52)</td>
</tr>
<tr>
<td>SMB</td>
<td>-0.17</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td>(-3.74)</td>
<td>(-0.02)</td>
<td>(-0.99)</td>
<td>(-3.02)</td>
</tr>
<tr>
<td>HML</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.11</td>
</tr>
<tr>
<td>(-0.45)</td>
<td>(-0.75)</td>
<td>(-1.91)</td>
<td>(0.95)</td>
</tr>
<tr>
<td>UMD</td>
<td>0.13</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>(3.98)</td>
<td>(4.95)</td>
<td>(3.52)</td>
<td>(5.15)</td>
</tr>
<tr>
<td>VWF</td>
<td>-0.04</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td>(-1.23)</td>
<td>(1.15)</td>
<td>(-0.20)</td>
<td>(0.83)</td>
</tr>
<tr>
<td>R²</td>
<td>0.10</td>
<td>0.13</td>
<td>0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Factor Loadings (Value Weighted)</th>
<th>High Attention</th>
<th>Low Attention</th>
<th>Inattention Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5</td>
<td>Q4</td>
<td>Q3</td>
<td>Q2</td>
</tr>
<tr>
<td>Alpha</td>
<td>-0.22%</td>
<td>0.05%</td>
<td>0.05%</td>
</tr>
<tr>
<td>(-0.89)</td>
<td>(0.26)</td>
<td>(0.20)</td>
<td>(1.40)</td>
</tr>
<tr>
<td>MKT</td>
<td>0.01</td>
<td>-0.11</td>
<td>-0.04</td>
</tr>
<tr>
<td>(0.23)</td>
<td>(-2.81)</td>
<td>(-0.61)</td>
<td>(-3.31)</td>
</tr>
<tr>
<td>SMB</td>
<td>-0.18</td>
<td>0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>(-2.36)</td>
<td>(0.23)</td>
<td>(-0.93)</td>
<td>(-0.87)</td>
</tr>
<tr>
<td>HML</td>
<td>-0.11</td>
<td>-0.06</td>
<td>-0.15</td>
</tr>
<tr>
<td>(-1.26)</td>
<td>(-0.88)</td>
<td>(-1.49)</td>
<td>(-1.37)</td>
</tr>
<tr>
<td>UMD</td>
<td>0.24</td>
<td>0.22</td>
<td>0.18</td>
</tr>
<tr>
<td>(4.58)</td>
<td>(5.67)</td>
<td>(3.12)</td>
<td>(3.59)</td>
</tr>
<tr>
<td>VWF</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>(-0.03)</td>
<td>(-0.30)</td>
<td>(-0.23)</td>
<td>(1.68)</td>
</tr>
<tr>
<td>R²</td>
<td>0.10</td>
<td>0.16</td>
<td>0.05</td>
</tr>
</tbody>
</table>

This table reports 5 factor loadings. That is, the results of a regression of monthly portfolio excess returns on the monthly returns from the three Fama and French (1993) factor-mimicking portfolios, Carhart’s (1997) momentum factor, and the Pastor, Stambaugh (2001) liquidity factor. Panels A and B present the results of portfolios formed from equal and value weights respectively.

The low-attention earnings momentum portfolios tend to have a slight negative beta and tend to be tilted towards growth stocks. The intercepts of the both the low-attention and
inattention effect portfolio are striking (142 basis points \(t=3.2\)) and 163 basis points \(t=3.35\) respectively).

Moreover, both inattention-effect portfolios have large returns despite negative loadings on HML, which, ceteris paribus, should decrease expected returns. All of the portfolios in the first 5 columns have a positive exposure to momentum. This is expected and reflects the fact that these portfolios are conditional momentum portfolios. The high-attention earnings momentum portfolio has a slight tilt towards large cap stocks and has the highest (albeit insignificant) market beta.

Neither of the inattention-effect portfolios have statistically significant factor loadings in the Fama & French framework, which is consistent with returns being driven by underreaction to the news content, rather than reflecting systematic risk.

1.4.5 Inattention Effect in Cross Sectional Regressions

Next I test my underreaction hypothesis in a regression framework, in which I can better control for other determinants of firm returns and isolate the marginal effect of my main variable, the Attention Coefficient.
This table reports Fama-MacBeth forecasting regressions of stock returns on earnings surprise and its interaction with my attention measure.

Specifically, my next set of tests employs monthly Fama and MacBeth (1973) forecasting regressions of one month ahead cumulative stock return on firm characteristics and indicators for past information events interacted with my attention variable. Since increased attention is associated with decreased underreaction I would expect the attention to decrease the effect of earnings surprise on returns. In performing my regression analysis, I restrict regressions to the month following the earnings announcement. After controlling for well-known determinants of stock returns, such as size, book-to-market ratio, idiosyncratic volatility and turnover I find that the coefficient on my interacted variable $ATTN_t \cdot \text{EARN}_S\text{URP}$ has a modest negative
effect, consistent with my predictions and portfolio results.

1.5 Information-Burdened Investors

In this section I identify a subset of stocks that are owned by investors with attention constraints that are likely to be binding. I test the hypothesis that investors react more sluggishly to news when they must divide their attention among many different sources of information. I posit that the inattention effect that I document should be larger (smaller) when a stock’s owners are faced with abnormally large (small) amounts of portfolio relevant news to sift through.

I combine data on mutual fund holdings with stock-level news counts to construct a proxy for news about a basket of stocks ($PORTNEWS_{f,t}$). It is the aggregate volume of monthly news about the stocks in a fund’s portfolio. If limited investor attention is driving the inattention effect, then for a given stock, ownership by funds with increased $PORTNEWS_{f,t}$ (controlling for other fund specific factors) should amplify the magnitude and significance of the result.

I expect funds to dynamically allocate their attention across time periods. In addition, I expect there to be heterogeneity in information processing capacity across funds. For example buy-side analysts may spend more hours in the office during earnings season. To control for this I add the variable EAMONTH proportion of a fund’s stocks that had earnings announcements in that month.

Cross sectional regressions are run every calendar month across the funds in my sample to orthogonalize the $PORTNEWS_{f,t}$ measure with respect to the past 3 months of lagged portfolio news $PORTNEWS_{f,t-k}$ $k \in \{1, 2, 3\}$, the proportion of a fund’s stocks that had earnings announcements in that month (EAMONTH) and the average market capitalization of the stocks in a fund’s portfolio (AVGSIZE).

$$PORTNEWS_{f,t} = \alpha_t + \theta_{f,t} \cdot X_{f,t} + \varepsilon_{f,t} \quad (1.7)$$
The fund-level information processing burden is the residual from the cross sectional regression $\varepsilon_{f,t}$. Funds with high $\varepsilon_{f,t}$ are faced with processing more news about their portfolio than expected.

Table 1.9 examines the determinants of $PORTNEWS_{f,t}$. Here I report the results from monthly fund-level cross sectional regressions $PORTNEWS_{f,t}$ on a set of controls. Although $PORTNEWS_{f,t}$ is highly persistent it does not have a unit root. We see in specification 4 that $PORTNEWS_{f,t}$ increases during earnings announcement months.

Table 1.9: Estimating Unexpected Portfolio News, Fama-Macbeth Regressions

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PORTNEWS_{f,t-1}$</td>
<td>0.99</td>
<td>0.68</td>
<td>0.63</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(53.13)</td>
<td>(20.79)</td>
<td>(18.12)</td>
<td>(53.21)</td>
<td>(52.78)</td>
<td>(52.81)</td>
<td>(18.40)</td>
</tr>
<tr>
<td>$PORTNEWS_{f,t-2}$</td>
<td>0.33</td>
<td>0.25</td>
<td></td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.45)</td>
<td>(6.70)</td>
<td></td>
<td>(6.66)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PORTNEWS_{f,t-3}$</td>
<td>0.12</td>
<td></td>
<td></td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.50)</td>
<td></td>
<td></td>
<td>(3.51)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EAMONTH</td>
<td>2.96</td>
<td>2.94</td>
<td>2.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.98)</td>
<td>(5.67)</td>
<td>(5.77)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVGSIZE</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.81)</td>
<td>(1.03)</td>
<td>(1.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INTERCEPT</td>
<td>0.66</td>
<td>0.56</td>
<td>0.51</td>
<td>0.36</td>
<td>-0.15</td>
<td>-0.66</td>
<td>-1.06</td>
</tr>
<tr>
<td></td>
<td>(8.04)</td>
<td>(7.49)</td>
<td>(7.23)</td>
<td>(4.11)</td>
<td>(-0.15)</td>
<td>(-0.65)</td>
<td>(-1.30)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.90</td>
<td>0.91</td>
<td>0.92</td>
<td>0.90</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
</tr>
</tbody>
</table>

This table reports the results from monthly fund-level cross sectional regressions of the form

$$PORTNEWS_{f,t} = \alpha_t + \theta_{f,t}X_{f,t} + \varepsilon_{f,t}$$

The dependent variable, $PORTNEWS_{f,t}$, the aggregate monthly news about the stocks in the fund’s portfolio.

Next, I consider the effect of a fund’s information processing burden on the ability for information to be quickly impounded into prices of it holdings. The burdened ownership of a stock ($BURDENOWN_{s,t}$) is the average information processing burden of a stock’s owners weighted by the share of mutual fund ownership of the stock by that fund.
\[ \text{BURDENOWN}_{s,t} = \sum_{f_j \in F_{s,t}} \left[ \left( \frac{h_{s,f_j,t}}{IO_{s,t}} \right) \varepsilon_{f_j,t} \right] \]

Table 1.10: Correlation between Burdened Ownership and Firm Characteristics

<table>
<thead>
<tr>
<th></th>
<th>BURDENOWN(_{s,t})</th>
<th>BURDENOWN(_{s,t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATT(_N_t)</td>
<td>2%</td>
<td>-10%</td>
</tr>
<tr>
<td>BM</td>
<td>-5%</td>
<td>1%</td>
</tr>
<tr>
<td>SIZE</td>
<td>9%</td>
<td>-10%</td>
</tr>
<tr>
<td>RET(_{t-2,t-12})</td>
<td>0%</td>
<td>1%</td>
</tr>
<tr>
<td>IDIOVOL</td>
<td>-4%</td>
<td>5%</td>
</tr>
<tr>
<td>TURN</td>
<td>4%</td>
<td>-2%</td>
</tr>
<tr>
<td>MFOWN</td>
<td>0%</td>
<td>-11%</td>
</tr>
<tr>
<td>BREADTH</td>
<td>8%</td>
<td>-7%</td>
</tr>
</tbody>
</table>

This table reports spearman rank correlation coefficients for \( \text{BURDENOWN}_{s,t} \) and a set of firm specific characteristics.

Table 1.10 presents Spearman rank correlation coefficients calculated over all months and stocks for \( \text{BURDENOWN}_{s,t} \) and a set of firm specific characteristics. We see that a stock’s burdened ownership is not highly correlated with any firm specific characteristics. As such, I do not expect portfolio sorts based on burdened ownership to be swamped by unrelated effects such as firm size, value or institutional ownership.
Table 1.11: Attention Alphas of Stocks Held by Burndened Funds

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Equal Weighted</th>
<th></th>
<th>Panel B: Value Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Attention</td>
<td>Low Attention</td>
<td>Inattention Effect</td>
</tr>
<tr>
<td></td>
<td>Q1</td>
<td>Q5</td>
<td>$L/S^{\text{LOW}} - L/S^{\text{HIGH}}$</td>
</tr>
<tr>
<td>High Burdened Ownership</td>
<td>0.11%</td>
<td>2.15%</td>
<td>2.04%</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(5.97)</td>
<td>(4.44)</td>
</tr>
<tr>
<td>Low Burdened Ownership</td>
<td>0.37%</td>
<td>1.07%</td>
<td>0.71%</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(2.74)</td>
<td>(1.64)</td>
</tr>
</tbody>
</table>

This table presents the effects of burdened ownership on PEAD attention portfolios. Panels A and B present the results of portfolios formed from equal and value weights respectively.

Under the null hypothesis that information burdened funds are less likely to allocate attention to their forgotten holdings we expect that

$$(L/S^{\text{LOW}} - L/S^{\text{HIGH}})_{\text{Burdened}} > (L/S^{\text{LOW}} - L/S^{\text{HIGH}})_{\text{Unburdened}}$$

That is, The spread between the momentum returns of low attention stocks versus high attentions stocks (the inattention effect) to be more pronounced for stocks that are held by burdened fund managers. Table 1.11 presents the effects of burdened ownership on high and low attention quintile PEAD portfolios formed using value and equal weights respectively. The monthly alpha from the value-weighted inattention effect portfolio for stocks with high burdened ownership is 204 basis points per month ($t = 4.44$). Meanwhile the inattention effect for stocks with low burdened ownership is an insignificant 107 basis points per month ($t = 2.74$).
1.6 Robustness Tests

In this section I perform a variety of additional robustness checks the results of which are depicted in Table 1.12 First, I show that the return predictability I document remains economically and statistically significant for stocks in a variety of information environments. Specifically, I show that the inattention effect is present in both the above- and below-median subsamples by analyst coverage. Next, I show that after orthogonalizing for firm size, stocks with high ownership by institutions (by breadth or by share of market cap) tend to exhibit a stronger inattention effect. This results because my measure has an increased signal-to-noise ratio when the stock has higher ownership and thus more information is dissipated by my sample of marginal price setters. Next, I show that the effect is more pronounced in smaller firms, which is consistent with increased frictions in the presence of tightly binding limits to arbitrage. Further, the inattention effect is exacerbated in growth stocks, consistent with increased information gathering requirements for firms with more intangible assets.
Table 1.12: Robustness & Variation in Inattention

<table>
<thead>
<tr>
<th>Panel A: Effect of Analyst Coverage and Fund Ownership on Inattention</th>
<th>MOM</th>
<th>L/S_{LOW}</th>
<th>L/S_{HIGH}</th>
<th>Inattention Effect</th>
<th>L/S_{LOW} - L/S_{HIGH}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.06%</td>
<td>1.65%</td>
<td>0.35%</td>
<td>1.30%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.02)</td>
<td>(4.68)</td>
<td>(1.06)</td>
<td>(2.80)</td>
<td></td>
</tr>
<tr>
<td>Higher Analyst Coverage</td>
<td>1.10%</td>
<td>1.89%</td>
<td>0.47%</td>
<td>1.68%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.64)</td>
<td>(3.92)</td>
<td>(0.99)</td>
<td>(2.41)</td>
<td></td>
</tr>
<tr>
<td>Lower Analyst Coverage</td>
<td>1.16%</td>
<td>1.65%</td>
<td>0.28%</td>
<td>1.14%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.73)</td>
<td>(3.16)</td>
<td>(0.52)</td>
<td>(1.51)</td>
<td></td>
</tr>
<tr>
<td>Higher Breadth of Ownership</td>
<td>1.10%</td>
<td>1.77%</td>
<td>0.14%</td>
<td>1.80%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.13)</td>
<td>(3.28)</td>
<td>(0.35)</td>
<td>(2.65)</td>
<td></td>
</tr>
<tr>
<td>Lower Breadth of Ownership</td>
<td>1.28%</td>
<td>2.03%</td>
<td>0.41%</td>
<td>1.43%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4.91]</td>
<td>[3.97]</td>
<td>[0.76]</td>
<td>[1.96]</td>
<td></td>
</tr>
<tr>
<td>Higher Mutual Fund Ownership</td>
<td>1.00%</td>
<td>2.17%</td>
<td>-0.04%</td>
<td>2.30%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.94)</td>
<td>(3.77)</td>
<td>(-0.09)</td>
<td>(3.45)</td>
<td></td>
</tr>
<tr>
<td>Lower Mutual Fund Ownership</td>
<td>1.22%</td>
<td>1.20%</td>
<td>0.69%</td>
<td>0.69%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.89)</td>
<td>(2.47)</td>
<td>(1.23)</td>
<td>(0.94)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Robustness Tests</th>
<th>MOM</th>
<th>L/S_{LOW}</th>
<th>L/S_{HIGH}</th>
<th>Inattention Effect</th>
<th>L/S_{LOW} - L/S_{HIGH}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larger Firms</td>
<td>0.88%</td>
<td>0.79%</td>
<td>0.48%</td>
<td>0.65%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.80)</td>
<td>(1.86)</td>
<td>(1.02)</td>
<td>(1.15)</td>
<td></td>
</tr>
<tr>
<td>Smaller Firms</td>
<td>1.29%</td>
<td>2.62%</td>
<td>0.01%</td>
<td>2.78%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.95)</td>
<td>(5.18)</td>
<td>(0.03)</td>
<td>(3.97)</td>
<td></td>
</tr>
<tr>
<td>Value Stocks</td>
<td>1.04%</td>
<td>1.40%</td>
<td>0.76%</td>
<td>0.71%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.58)</td>
<td>(2.80)</td>
<td>(1.41)</td>
<td>(0.92)</td>
<td></td>
</tr>
<tr>
<td>Growth Stocks</td>
<td>1.12%</td>
<td>1.34%</td>
<td>-0.27%</td>
<td>2.05%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.96)</td>
<td>(2.74)</td>
<td>(-0.53)</td>
<td>(2.99)</td>
<td></td>
</tr>
<tr>
<td>Higher Idio Vol</td>
<td>1.20%</td>
<td>1.72%</td>
<td>0.12%</td>
<td>1.64%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.37)</td>
<td>(3.40)</td>
<td>(0.24)</td>
<td>(2.42)</td>
<td></td>
</tr>
<tr>
<td>Lower Idio Vol</td>
<td>0.76%</td>
<td>1.61%</td>
<td>0.78%</td>
<td>0.89%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.55)</td>
<td>(4.03)</td>
<td>(1.88)</td>
<td>(1.54)</td>
<td></td>
</tr>
<tr>
<td>1985-1996</td>
<td>1.22%</td>
<td>1.51%</td>
<td>1.08%</td>
<td>0.44%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.92)</td>
<td>(2.56)</td>
<td>(2.29)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>1997-2008</td>
<td>1.09%</td>
<td>1.51%</td>
<td>-0.17%</td>
<td>1.68%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.92)</td>
<td>(3.19)</td>
<td>(-0.31)</td>
<td>(2.50)</td>
<td></td>
</tr>
<tr>
<td>DGTW</td>
<td>1.11%</td>
<td>1.76%</td>
<td>0.46%</td>
<td>1.30%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.83)</td>
<td>(5.22)</td>
<td>(1.45)</td>
<td>(2.94)</td>
<td></td>
</tr>
</tbody>
</table>

This table reports abnormal returns for baseline PEAD and my main test portfolios.

High (Low) Analyst coverage stocks are those with analyst coverage above (below) the
median of the IBES sample in that calendar month. High (low) breadth of ownership stocks are those with a breadth of ownership above (below) the median of the CDA Spectrum and CRSP/Compustat merge in that calendar month. Analyst coverage breadth of ownership and mutual fund ownership are orthogonalized with respect to firm size. Value (growth) stocks are those with book to market above (below) the median of the CRSP/Compustat sample in that calendar month. DGTW characteristic-adjusted returns are defined (following Daniel, Grinblatt, Titman, and Wermers (1997)) as raw monthly returns minus the returns on a value weighted portfolio of all CRSP firms in the same size, book-to-market, and one year momentum quintile.

For stocks with more binding limits to arbitrage, we should see a stronger inattention effect, as more arbitrageurs are less likely to engage in trading and fully update these firms’ prices. The proxy I use for limits to arbitrage is idiosyncratic volatility. As shown in rows 5 and 6 of Panel B in Table 1.12, the inattention effect is larger; for stocks with above median IDIOVOL the inattention effect yields 3 factor alphas of 164 basis points per month (t=2.42) however for stocks with below median IDIOVOL the effect reduces to 89 basis points per month (t=1.54). This is consistent with my prediction that firms that are more likely to expose arbitrageurs to noise trader risk should exhibit a stronger inattention effect.

In Rows 7 and 8 of Table 1.12 Panel B I partition my sample temporally into subsamples from 1985 to 1996 and from 1997 to 2008. The inattention effect yields a 3 factor alpha of 44 basis points per month (t=59) in the earlier subsample and 168 basis points per month (t=2.94) in the more recent subsample. Inspecting Figure 1.1 which depicts the annual performance of the inattention effect strategy we see that the poorer performance in the earlier subsample is likely to be biased downwards by 2 years (1985 and 1987). Nevertheless there is convincing evidence to suggest that the inattention effect has not suffered from arbitrage attenuation and has become stronger in the later part of the sample. Lastly, I show that my return predictability remains economically and statistically significant when using an alternative benchmark of expected returns. Specifically, I calculate characteristic-adjusted returns following Daniel et al. (1997). Using the same portfolio procedure described in Table
III, and characteristics-based adjustments for all firms, I report the resulting excess returns in the last row of Table X Panel B. Characteristic adjusted returns to the inattention effect portfolio remain large and statistically significant: returning value weighted 3 factor alphas of 130 basis points ($t=2.94$) per month.

### 1.7 Conclusion

I investigate a mechanism by which the rate at which information is impounded into a stock’s price is related to the portfolios of its owners. I show that in the presence of limited attention investors rationally allocate their attention toward information channels that have a large potential impact on their wealth. Specifically, I find convincing evidence that a stock’s price underreacts to information events when its largest owners hold it as a small proportion of their portfolio. Further, I show that stocks whose owners are attention constrained tend to experience additional underreaction to information events. In particular, the documented inattention effect is amplified in the subset of stocks held by attention constrained funds, yielding a value weighted three factor alpha of 225 basis points per month ($t=2.14$).

Consistent with investor inattention driving my results I find that the earnings news of high attention stocks predicts subsequent returns in their low attention industry peers (but not the reverse). I also find that the average earnings surprise of a stock’s high attention industry peers strongly predicts its future excess returns whereas the returns of the low attention peers does not.

A simple portfolio strategy that takes advantage of this return predictability yields economically and statistically significant risk adjusted profits of over 24% per year during the 23 year period from 1985 to 2008. These returns are distinct from previously known underreaction determinants, robust to different specifications and exhibit no return reversal in the long-run. In addition, the effect becomes more pronounced in recent years.

Most modern asset pricing frameworks have the common theme of information gathering and processing with the information revelation mechanism as the focal point. I show that in the presence of limited attention the rate at which information is impounded into a stock’s
prices is not only dependent on the information revelation mechanism but also intimately related to the attention constraints of the stock’s owners. Specifically, I show that a stock’s price will underreact to even the most salient news if its largest owners allocate their attention away from it.

Understanding how frictions arising from investors’ bounded rationality aggregate to impact price updating will provide a better picture of how capital responds to trading opportunities and a better understanding of what drives asset prices.
Chapter 2

The Social Elite\textsuperscript{6}

2.1 Introduction

Asset prices reflect investor beliefs, which are in turn driven by the flow of information. Many important channels supporting this flow of information are complex and difficult to observe. Because of this, the interplay between social networks and market efficiency has become a focal point in empirical asset pricing.

In this essay, we demonstrate that personal connections amongst the socio-economic elite has a significant impact on asset prices. By analyzing the timing and attendee composition of elite social gatherings, we provide evidence that casual social relationships are a conduit for information that affects investor behavior. Specifically, our analysis examines the interactions between equity fund managers and corporate officers of publicly traded firms. We study these two categories of individuals both jointly and in isolation. Looking specifically at firms, we show that when executives attend social gatherings their stock price’s subsequent behavior directionally predicts their upcoming earnings surprise. Studying funds and firms jointly, we show that fund managers that attend events that corporate officers from a particular firm also attend are more likely to purchase stock in that firm. In asking why this is the case we find that fund managers demonstrably \textit{outperform} when they decide to trade these "socially-connected" stocks. Further, the socially-connected stocks that they decide not to trade then

\textsuperscript{6}This chapter is based on Joint work with Alexander Chernyakov
underperform. Looking specifically at fund managers, we highlight a strong pattern where subsequent to these events, fund managers tend to increase their positions in stocks that were overweight by other fund managers.

To better understand our approach, consider the following example from our data\(^7\).

Figure 2.1: Fund Managers’ Holdings & Social Events, a stylized example

This figure depicts actual total returns of XYZ Corp. (anonymized name) during the first 2 quarters of 2013, the incidence of a social event and changes in several equity fund managers’ holdings.

At the beginning of 2013, a handful of funds reported having only de-minimis positions in XYZ Corp. But by the end of Q1 three funds reported via 13F filings that they had purchased substantial stock positions. These funds had never owned XYZ Corp during their prior 13F reporting history. Looking toward potential explanations for the 3 funds’ decisions to purchase XYZ Corp, our data reveals that several key portfolio managers employed by the 3 funds attended the 'Indwood House Gala’ on March, 13 2013, during the previous quarter. Also in attendance at that Gala was XYZ Corp’s CFO and his wife.

Figure 2.1 illustrates this time-line of events. Between the January 1, 2013 and May 23,\(^7\)Fund manager and firm names have been redacted
2013 XYZ’s stock price rises over 50%.

More generally, the 3 funds in this example are not the only funds employing investors who frequent the same events as XYZ’s CFO. Between 2007 and 2013 executives from XYZ Corp attended several social events like the one in this example. Looking at the price dynamics of XYZ Corp around these social events reveals a tendency for the average returns near event dates to produce superior forecasts of earnings surprises. Using a disparate collection of data sources we are able to cleanly test this hypothesis in generality.

Our approach is novel because unlike previous studies, that have focused on static social network dynamics and activities stemming from connections formed prior to information transfer (such as school ties or geographic and board interlock networks), we focus on unsystematic interactions that occur in casual social settings. In addition, because our focus is on fund managers and corporate officers, there is a clear incentive structure, on the part of fund-managers, that drives our hypotheses. On the one hand, fund managers have a strong desire to acquire information about firms in which they may want to invest, and on the other hand corporate officers are likely to possess valuable information about their firm. As such, we believe that these two classes of agents, taken together, provide an ideal basis for studying the impact that social networks have on information transfer.

The types of events we study are gatherings such as charity fundraisers, Wall Street centric socials, art gallery openings and high society galas within the United States. These events are typically invitation-only or have substantial attendance fees, and are almost exclusively attended by financial professionals, socialites, senior corporate officers and executives. Attendees are heavily skewed towards the very wealthy, educated and financially sophisticated, thus providing an appropriate setting for studying asset-pricing predictions since such individuals are more likely to participate in the stock market.

2.1.1 Intuition

We present a specific example of a real life case involving social events and information exchange that we believe captures the essence of our mechanism. We do not suggest that
information transferred at social events is necessarily illicit. Rather, this example serves to highlight an unambiguous instance of information exchange in our setting.

Edward Downe Jr. has as life story whose beginnings exemplify the American dream. He came from moderate means to become the head of a vast magazine empire. This initial entrepreneurial success marked the beginning of a relentlessly extravagant lifestyle. He became a socialite and immersed himself in the New York social scene and charity circuit. In 1980, he began a romantic relationship with Charlotte Ford, the great granddaughter of Ford Motor Company founder Henry Ford. And six years later they were married. He often threw galas at his and his wife’s beachfront homes in Southampton, NY. In 1992, the SEC argued that these events were a hub for the exchange of insider information and that its attendees used this information to reap millions of dollars in illegal stock market profits. In addition to Downe, the case implicated several other of high profile corporate officers, investors and Wall Street executives. The SEC asserted that between summer of 1987 and early 1989, various members of this social group exchanged confidential information about several companies and that some of them then traded in those stocks illegally. Ultimately, Downe was convicted of wire fraud and securities fraud, amongst other charges and was fined $11 million (Bill Clinton later pardoned the charge on his last day in office). The above case provides a unique perspective of our mechanism as a channel through which information flows within financial markets.

The remainder of the essay is organized as follows. Section 2.2 gives a brief background and literature review. Section 2.3 describes the social events and the market data used in the essay. Section 2.3 also presents a panel of summary statistics. Section 2.4 lays out our most basic result, evidence of information leakage at social events. Section 2.5 studies, in detail, the mechanism through which information leakage occurs and highlights the impact of social

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events on investor decision making. Section 2.6 extends our mechanism to interactions solely between fund managers and Section 2.7 concludes.

2.2 The Setting

Our research ties together a rich literature on the behavior of professional investors with an expanding literature on the interplay between financial markets and social networks. And relates closely to a branch of literature that focuses on how individuals’ membership in certain groups influences their behavior. Most similar to our work are the findings in Cohen, Frazzini and Malloy (2008). They find that mutual fund portfolio managers place larger bets on firms when connected to through shared education networks. They also find that these same fund managers perform significantly better on these connected holdings relative to their non-connected analogs. Cohen, Frazzini and Malloy (2010), finds that analysts outperform on stock recommendations on companies with which they have an educational link. Hong, Kubik, and Stein (2005), document word-of-mouth effects between same-city mutual fund managers with respect to their portfolio choices. They show that mutual fund managers are more likely to buy (or sell) a particular stock if other managers in the same city are buying (or selling) that same stock.

Prior research has focused on the impact of static networks on firm decisions and outcomes. For example, empirical finance literature on network sociology typically employs the use of corporate board linkages or board interlocks between firms as a measure of personal networks (Useem (1984), Mizruchi (1982, 1992), Hallock (1997), Larcker et al. (2005) and Conyon and Muldoon (2006)). Our approach is different in that we can observe directly the flow of information through our social network. Our research relates to several distinct threads of the literature on professional investors. Primarily, fund manager performance and informed/insider trading. Evidence of stock selection amongst fund managers is mixed. Some studies (Jensen (1968), Grinblatt and Titman (1989, 1993), Malkiel (1995), Grinblatt, Titman, and Wermers (1995), Gruber (1996), Daniel et al. (1997), Wermers (1997) and Carhart (1997)) find that mutual fund managers are successful in achieving superior returns while
others cast serious doubt on this assertion.

Works by several authors (Keown and Pinkerton (1981), Cao, Chen, and Gririn (2005),
Augustin, Brenner and Subrahmanyam (2014)) investigate informed trading activity in equity
and equity options prior to corporate events and document pervasive evidence of informed
trading prior to these events.

Our examination of information exchange between fund managers also builds on the em-
pirical evidence of investor herding by Wermers (1999) among others. The literature identifies
several reasons why managers may herd, such as due to reputational risk (Scharfstein and
Stein (1990)), correlated private information (Froot, Scharfstein, and Stein (1992)) or copy-
ing better informed managers (Bikhchandani, Hirshleifer, and Welch (1992)). This research
suggests that social interactions are a channel through which fund managers’ ideas diffuse.
Our ideas relate directly to prior work done on investor herding. For example, Froot, Scharf-
stein, and Stein (1992) point out that if the market is populated with short term investors,
then the perception of the truth, can sometimes be more important than the truth itself.
Social events are an ideal setting to gauge perceptions of other investors by exchanging pri-
vate information. We show that the portfolios of investors who attend a social event together
subsequently become more similar.

Finally, even though the literature suggests that active investment managers lack the
ability to generate alpha, recent work by Cohen et. al (2010) documents evidence to the
contrary. Specifically, they show that investment managers select a set of core "best-ideas"
but due to institutional incentives, also select stocks that do not have alpha. We contribute
to this literature by presenting evidence that these core "best-ideas" are disseminated when
investors casually socialize with each other and with corporate executives.

2.3 Data

The data in this study are collected from a multitude of sources and can be placed into three
distinct categories

1. Stock and mutual fund market data, such as prices, holdings and market capitalization
2. Event data, comprising the dates and attendees of social events

3. Biographical information, used to link unique individuals to equity funds or publicly traded firms

Data on fund holdings comes from the Thomson-Reuters Institutional Holdings Database, which includes all registered 13F institutions filing with the SEC. It provides common stock holdings as reported on Form 13F filed with the SEC. Specifically, subsequent to 1978 all institutions with more than $100 million under management are required to fill out 13F forms quarterly for all U.S. equity positions that have a market value of more than $200,000 or which constitute more than 10,000 shares. It should be noted that in contrast many academic studies that center on mutual fund holdings, our analysis focuses on active mutual funds as well as hedge funds. Daily and monthly stock returns are obtained from the Center for Research on Securities Prices (CRSP) via the New York Stock Exchange/American Stock Exchange (NYSE/AMEX) tape. Stock return data is matched with fundamental data taken from CRSP/COMPUSTAT merge database and we ensure that any accounting variables used are known to the market before the returns that they are used in conjunction with.

In order to bring together the specific set of gatherings covered in this study, we undertook an extensive look into the types of social events that are typically attended by influential individuals. We focus on individuals who can influence investment decisions in active equity funds and individuals in powerful corporate positions. Our goal is to create a broad representative panel of events that constitutes the social calendar of a quintessential corporate socialite. In order to better understand the ways in which the public media represents high-society events, we conducted a bottom-up examination of the various outlets that specialize in capturing this socialite lifestyle. We settled on following core group of data sources that we believe allow us to capture just that:

1. Patrick McMullan & Co

2. Billy Farrell Agency New York

3. Guest of a Guest
4. Panache-Privee

5. New York Social Diary

6. Bloomberg’s 'Scene Last Night' Column

7. CharityHappenings.org

Of these 7 data sources, the first 6 data sources explicitly contain events and attendees and CharityHappenings.org contains social events for major charities and non-profit institutions.

Patrick McMullan & Co is a photography firm focused exclusively on capturing highly exclusive events. It is run by the eponymous American photographer and socialite based in New York City. The extent to which Patrick McMullan & Co is a fixture in the New York social circuit is often said to be evidenced by the common use of the word “Patrick” as a verb in the same way that “Google” and “Facebook” are used as slang for obtaining information about an individual in the internet. Billy Farrell Agency New York is a digital photography agency that was established in 2010 by a group of former photographers from Patrick McMullan. Guest-of-a-Guest is another website similar to those described above and maintains a database of society events, people, and places. Panache Privee is a digital photo archive and blog which describes itself as "a leading interactive network of affluent and influential people, helping them connect and learn more about their peers". The New York Social Diary is public blog that publishes photographs of various socialites at a variety of past events and maintains a calendar of future events that they might attend. Bloomberg’s "Scene Last Night' Column, written by Amanda Gordon, provides more formal written coverage of events ranging from charity gala’s to birthday celebrations of hedge fund managers. All of these sources are public and are novel in this specific context in that for each event they publish the names of attendees of event. In most cases these sites also provide tagged photographs where individuals’ names have been mapped to unique ontological entities. The various event data sources are collected and cleaned using a combination of web-scraping, textual-analysis and hand-matching. Data collection and processing methodology is discussed in detail in the appendix.
Separately, we collect data on the senior board membership of major philanthropic organizations. We then associate with each charity a set of related events that are attended by its members. CharityHappenings.Org contains the dates of charity events. These charity events are then linked to the charity board’s executives and fund managers. Figures 2.2 depicts what the structure of

![Figure 2.2: Social Event Data Sources, An Example](image)

This figure is a screen-shot of the website for Patrick McMullan & Co, one of the data sources that was processed in order to create the social event and attendee information used.

Biographical information on various categories of individuals is obtained from BoardEx, a dataset produced by Management Diagnostics Limited, a research firm that specializes in social network data on company officials. Hedge funds and active mutual funds are hand matched, by name, to their respective corporate entities within Boardex, and publicly firms are identified by CUSIP number. For each corporate entity, Boardex then provides a list of key personnel. For example, for funds, Boardex provides biographical information about senior portfolio managers and traders, and for publicly traded firms provides information
about senior company officers such as the CEO, CFO, Chairman, etc. The data contains current and past roles of every individual with start and end dates. In addition, Boardex provides data on charity board membership. We map fund managers’ and corporate officers’ charity board membership information to a comprehensive set of dates of important fund-raising events for those charities (details discussed below). We then use the calendar of these events and the attendees implied by charities’ senior board members as a subclass of events in our study. Figure 2.1 provides a sense of what results from combining biographical data from Boardex with social event data from various sources. We are able to construct a clean picture of which fund manager and executives attended specific events and when these events transpired.

Table 2.1: Excerpt from Social Calender Extract

<table>
<thead>
<tr>
<th>Date</th>
<th>Event Name</th>
<th>Executives</th>
<th>Fund Managers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thursday, April 11, 2013</td>
<td>Public Art Fund 2013 Spring Benefit</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>Monday, May 13, 2013</td>
<td>Robin Hood Benefit 2013</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Tuesday, May 14, 2013</td>
<td>2013 High Line Spring Benefit</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>Tuesday, June 18, 2013</td>
<td>MoMA PS1 Benefit Gala 2013</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Saturday, July 20, 2013</td>
<td>Long House White Night Summer Benefit</td>
<td>4</td>
<td>19</td>
</tr>
</tbody>
</table>

This table presents an excerpt of the final social calendar data obtained by parsing the web-sites of New York Social Diary, Guest of a Guest, Patrick McMullan & Co., Billy Farrell Agency New York, Panache-Privée, Bloomberg’s Scene Last Night, and CharityHappenings.org.

Taken together our data sources are able to tell us, for example, that John Smith, a portfolio manager at Harvard Management Company attended the Robin Hood Benefit 2013 Gala on Monday, May 13, 2013. We are also able to crisply determine whether or not the CFO of XYZ Corp was also in attendance.

2.3.1 Aggregate Summary Statistics

Table 2.2 summarizes the attendance of events by fund managers and executives. Panel A summarizes the annual average number of distinct firms, corporate officers, funds and fund
managers present at each event. We see that on average an event has 11 firms and 2 funds in attendance. At first it might seem odd that the average number of corporate officers is roughly 9, which is smaller than the average number of firms. This is because some executives sit on the boards of several companies. When this occurs we attribute each of these board connections to that individual thereby resulting in a one-to-many mapping that frequently adds additional firms.
Table 2.2: Summary Statistics: Social Events, Firms & Funds

Panel A: Annual Event Composition

<table>
<thead>
<tr>
<th>Year</th>
<th>Firms</th>
<th>Corp Officers</th>
<th>Funds</th>
<th>Fund Managers</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>11.0</td>
<td>8.5</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>2008</td>
<td>11.3</td>
<td>8.9</td>
<td>1.7</td>
<td>1.6</td>
</tr>
<tr>
<td>2009</td>
<td>12.4</td>
<td>9.5</td>
<td>1.8</td>
<td>1.6</td>
</tr>
<tr>
<td>2010</td>
<td>12.4</td>
<td>9.7</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>2011</td>
<td>11.2</td>
<td>8.8</td>
<td>1.9</td>
<td>1.7</td>
</tr>
<tr>
<td>2012</td>
<td>11.7</td>
<td>9.1</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>2013</td>
<td>11.0</td>
<td>8.5</td>
<td>2.2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Panel B: Quarterly Event Attendance Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Events</td>
<td>291</td>
<td>272</td>
<td>137</td>
<td>563</td>
</tr>
<tr>
<td>Events Attended Per Firm</td>
<td>2.8</td>
<td>2.7</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Events Attended Per Fund</td>
<td>2.9</td>
<td>2.8</td>
<td>1.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Panel C: Pooled Fund-Quarter Characteristics

<table>
<thead>
<tr>
<th></th>
<th>At least 1 social event</th>
<th>No social events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funds</td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td></td>
<td>167</td>
<td>160</td>
</tr>
<tr>
<td>AUM Percentile</td>
<td>62%</td>
<td>62%</td>
</tr>
<tr>
<td>Number of holdings</td>
<td>222</td>
<td>224</td>
</tr>
</tbody>
</table>

Panel D: Pooled Stock-Quarter Characteristics

<table>
<thead>
<tr>
<th></th>
<th>At least 1 social event</th>
<th>No social events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td></td>
<td>526</td>
<td>532</td>
</tr>
<tr>
<td>Size percentile</td>
<td>48%</td>
<td>48%</td>
</tr>
<tr>
<td>Book-to-market percentile</td>
<td>53%</td>
<td>53%</td>
</tr>
</tbody>
</table>

This table presents a summary of the event, stock and fund data used in this paper. For each year in our sample Panel A summarizes the average number of distinct firms, corporate officers, funds and fund managers present at each event. Panels B - D provide time series properties (mean, median, minimum, maximum) of quarterly summary statistics. Panel B presents counts of the number of events, events attended per firm and events attended per fund each quarter. Panel C juxtaposes the characteristics of funds in our sample that have attended at least one social event with those that have attended none. Panel D juxtaposes the characteristics of firms whose executives have attended at least one social event in a given quarter with those that have not.

Panel B enumerates events, events per firm and events per fund each quarter; we see that there are about 280 events each quarter and conditional on a firm or fund attending any events they attend about 2.7 on average. We see in panel C that among funds we have roughly 6% coverage since 160 funds attend at least one social each quarter. Panel C also
juxtaposes the characteristics of funds that have attended at least one social event with those that have attended none; funds that attend at least one social event have slightly higher AUM (approx 60th vs 50th percentile) and hold slightly more positions (approx 220 v.s. 180). Finally, the characteristics of firms whose executives have attended at least one social event in a given quarter with those that have not. Here, percentiles are derived ex-ante from the entire CRSP/COMPUSTAT merged universe of stocks having share codes 10 and 11. Panel D highlights that among publicly traded firms those that attend social events are roughly balanced in size (if not slightly smaller) and have a slight value tilt.

2.4 Information Leakage in Casual Social Interactions

2.4.1 Earnings Predictability in Socially Connected Securities

We begin by presenting our canonical result that we believe serves as the driving force for the results in later sections. Specifically, we show that the average returns of stocks on days subsequent to social events attended by the firm’s executives predicts upcoming earning surprises.

Our first test approaches the issue by directly evaluating the forecasting ability of social events in OLS regressions. This approach allows us to cleanly control for any residual correlation that our event based instruments might contain. Our second test approaches the issue by asking if our mechanism creates distortions that are pronounced enough to be exploited by an arbitrageur. We test this hypothesis by constructing an equivalent trading rule which buys (sells) stocks that have been bid up (down) on the heels of social events.

Our empirical strategy is as follows. First, we compute Earnings surprise (EarnSurp), as the return of a stock in the three day window, centered around an earnings announcement adjusted for the market return in that window. This is completely orthodox. Next, we isolate the dates of each social event attended by a firm’s corporate officers during a given quarter. We then measure the price impact (\(PI_{s,t}\)) of each event as the 1 week excess return subsequent to, and including, the event date. We define the SocialEventReturn for a stock
each quarter as the equally weighted average of these 1 week excess returns.

To be precise, if there is a social event on date $t$ for stock $s$ then we define the variable

$$PI_{s,t} = \prod_{j=0}^{4} (1 + xret_{s,t+j})$$

as the price impact due to the social event, where $xret_{s,t+j}$ is the excess return over the value weighted market index.

We also define,

$$PI_{Prev_{s,t}} = \prod_{j=-5}^{-1} (1 + xret_{s,t+j})$$

and

$$PI_{Next_{s,t}} = \prod_{j=5}^{9} (1 + xret_{s,t+j})$$

as the analogous variables shifted by $\pm 1$ week.

Now, for a given earnings surprise we compute SocialEventReturn as the average $PI_{s,t}$, ensuring that the index $t + 5$ is strictly before the day prior to the earnings surprise (so there is no look-ahead). Averaging $PI_{Prev_{s,t}}$ and $PI_{Next_{s,t}}$ yields analogous terms for SocialEventReturnPrev and SocialEventReturnNext.

Table 2.3 reports pooled quarterly stock-level OLS regressions of earnings surprises on returns around social events. We see that there is a distinctly large and significant coefficient on SocialEventReturn. This is true even when controlling for a number of other factors. The $t$-statistic on SocialEventReturn ranges from 4.9 to 5.62.

Since the instruments SocialEventReturnPrev and SocialEventReturnNext are designed to act as placebo tests for SocialEventReturn, the true value of the social event instrument is seen when we compare its statistic to that of SocialEventReturnPrev and SocialEventReturnNext. It is notable that simply by shifting by a week in either direction we observe a pronounced decline in the $t$ statistics on social event returns.
This table reports pooled quarterly stock-level OLS regressions of earnings surprises on returns around social events. We test the hypothesis that returns immediately following a social event have predictive power for upcoming earnings surprises by estimating the following regression:

$$EarningsSurp_{s,t} = \alpha + \beta \cdot SocialEventReturn_{s,t} + \theta \cdot X_{s,t} + \varepsilon_{s,t}$$

The units of observation are stock-quarter. The dependent variable, Earnings surprise (EarningsSurp), is defined as the return of a stock in the three day window, centered around an earnings announcement adjusted for the market return in that window. Social Event Returns (SocialEventReturn) are defined as 5 day returns of firms subsequent to social events where an executive from that firm was present. The sample here consists only of firms that have at least one social event in a particular quarter. When a firm has more than one social event during that quarter we compute the equally weighted average of these 5 day returns $X_t$ denotes a set of controls.

We now construct a trading rule which buys (sells) stocks that have been bid up (down) on the heels of social events. Table reports daily calendar time excess returns of portfolios formed conditional on information embedded in price dynamics after social events. Test portfolios initiate positions 1 business day prior to a stock’s earnings announcement date and
liquidate positions 1 day after, holding positions for a total of 3 business days. Portfolios are formed as follows: at the close of each business day we begin by identifying the stocks that have an earnings announcement in the next 2 days. This is the widest set of stocks can be included in portfolio formation for today. We then identify the subset of stocks that have had at least one social event during the last 60 business days and compute Social Event Returns. Social Event Returns are defined as 5 day returns of firms subsequent to (and including) social events where an executive from that firm was present. When a firm has more than one social event during this 60 day signal formation period we compute the equally weighted average of these 5 day returns. We buy (sell) stocks that have experienced positive (negative) Social Event Returns. Buys and sells are hedged by taking an equal and opposite position in the value weighted market portfolio. As such, on any given day the net dollar exposure taken by this test portfolio is exactly zero and the gross exposure is either 0 or 1.

<table>
<thead>
<tr>
<th></th>
<th>Raw Return</th>
<th>4-Factor Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Event Returns</td>
<td>179.7</td>
<td>172.9</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Social Event (-1 week) Returns</td>
<td>47.5%</td>
<td>43.6</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Social Event (+ 1 week) Returns</td>
<td>-23.8</td>
<td>-22.0</td>
</tr>
<tr>
<td></td>
<td>-0.2</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

This table reports monthly calendar time excess returns of portfolios formed conditional on information embedded in price dynamics after social events. Although we report results for a daily rebalancing strategy, monthly alphas (in basis points) are presented to make quantities comparable throughout the paper.

Figure 2.3 depicts the cumulative returns of portfolios formed conditional on social event dynamics. We again buy (sell) stocks that have experienced positive (negative) Social Event Returns. Here we however do not force the gross exposure to be unity (by having active weights that sum to one). We instead construct the portfolio to have time-varying gross exposures (varying number of ‘bets in the book’). Although the strategy is now unconstrained in the number of bets that can be taken the net dollar exposure taken by this test portfolio
is still exactly zero. The return process in figure 2.3 has a sharpe ratio of approx 1.2.

Figure 2.3: Time Series of Earnings Anticipation, Abnormal Returns

This figure shows the cumulative returns of portfolios formed conditional on information embedded in price dynamics after social events. We buy (sell) stocks that have experienced positive (negative) Social Event Returns. Buys and sells are hedged by taking an equal and opposite position in the value weighted market portfolio. As such, on any given day the net dollar exposure taken by this test portfolio is exactly zero. The strategy is unconstrained in the number of bets that can be taken but it is normalized ex-post to an ex post standard deviation 10%.

These robust patterns deepen the question as to what mechanism is driving correlations we observe. This is precisely what we confront in the next section.

2.5 Mechanism: Social Influence & Investor Behavior

2.5.1 Measuring Social Connectivity

We are now in a position to turn our attention to the explicit joint interactions between corporate officers and fund managers. In this section we examine fund managers’ trading decisions with a focus on the role of social networks in the transfer of information to security
prices.

We begin by using shifts in fund manager portfolio weights as a tool for understanding their behavior. We compare purchases and sales in stocks where funds managers have recently had a social connection to those where they have not. Equity fund managers may trade in and out of securities for a proliferation of reasons. For example, some fund managers, though active, may be constrained in their ability to deviate from the weights of a benchmark or may be prohibited from becoming excessively concentrated in an industry. In addition to institutional details, fund managers may also trade as a result of cognitive biases, for example the disposition effect (Frazzini (2006)) or familiarity bias (Huberman (2001)).

In order to examine the impact of social connections on trading decisions we first concretely specify a notion of social connectivity. We define the variable SocialConnection which is an indicator variable that specifies whether or not a fund manager attended a social event that was also attended by a corporate officer for that firm during the past quarter. It’s worth noting that our hypotheses are contemporaneous.

So for example, suppose we are studying the impact of social events on the propensity of fund managers to increase their positions. If we are looking at changes in fund managers’ position changes during the second calendar quarter we designate the social events that transpired between April 1st and June 31st as part of our SocialConnection variable. In this example, the observed change in portfolio holding is based juxtaposing snapshots made on March 31st and June 31st. As such, the social events will be aligned with the changes. Later in the essay when we study how these socially induced trades perform we focus on the returns of stocks during the subsequent quarter. In this example, this would be the return of the stock from July 1st through September 30th.

All of the results in this and subsequent sections are robust to defining Social Connections using both the current and previous quarters instead of just the current quarter. In the appendix we also perform our tests using the variable SocialConnectionPast2QTRs, which is an indicator variable that specifies whether or not a fund manager attended a social event that was also attended by a corporate officer for that firm in the past 2 quarters.
2.5.2 Trading in Socially Connected Securities

Table 2.5 presents pooled OLS regressions of changes in fund manager portfolio weights on social event attendance variables, where the units of observation are stock-fund-quarter. We include quarter-fund fixed effects in all of our specifications and standard errors are adjusted for clustering at the quarter level. The independent variables of interest are those measuring the attendance of fund managers and executives at social events.

This table depicts our canonical result: all else equal, fund managers tend to increase their portfolio weight in stocks of firms subsequent to attending a social event with executives from that firm. Looking at specification 1 we see that fund managers increase their portfolio weight by roughly 27.5 basis points (t=3) in securities when they have a social connection during that quarter.

Table 2.5: Portfolio Rebalancing Subsequent to Social Events, OLS Regressions

<table>
<thead>
<tr>
<th></th>
<th>Dep Variable: Change in portfolio weight dW (BPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>SocialConnection</td>
<td>27.548</td>
</tr>
<tr>
<td></td>
<td>3.065</td>
</tr>
<tr>
<td>SocialFirm</td>
<td>-0.24</td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
</tr>
<tr>
<td>SocialFund</td>
<td>-1.15</td>
</tr>
<tr>
<td></td>
<td>-1.62</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Quarter</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Fund</td>
</tr>
</tbody>
</table>

This table reports pooled fund-stock-quarter OLS regressions of equity fund manager trading activity on fund and stock social event attendance characteristics. SocialConnection is an indicator variable that specifies whether or not an investor employed by fund f attended a social event that was also attended by a corporate officer affiliated with stock s during quarter t. SocialFirm is an indicator variable that specifies whether or not any corporate officers affiliated with stock s attended any social events during quarter t. SocialFund is an indicator variable that specifies whether or not any fund managers employed by fund f attended any social events during quarter t.

Our mechanism relies on fund managers and corporate officers attending the same events. One could argue that both funds and firms having employees that attend social events are
simply more visible institutions. This theory would imply that our social connection variable might just be a proxy for funds that are social, as opposed to the interaction effect that funds and firms attend the same events. The mechanism in aforementioned case could, for example, be that by being more visible social funds are more likely to attract new clients and have capital inflows. These inflows would induce correlation between our social connection variable and changes in portfolio weights. Similarly, this visibility theory also suggests that our social connection variable might just be a proxy for firms with visible executives, regardless of event co-attendance with specific fund managers. Again, via stocks, the mechanism in this case would be that by being more visible social firms are more likely to be attract more interest from institutions. These purchases would induce correlation between our social connection variable and changes in portfolio weights.

The empirical strategy in specifications 2 and 4 of table 2.5 disentangles our theory from these visibility hypotheses. In order to control for the possibility that funds and firms that attend any social events are different from those that do not attend social events we add two event-specific control variables, SocialFirm and SocialFund. SocialFirm is an indicator variable that specifies whether or not any corporate officers affiliated with that firm attended any social events during that quarter. SocialFund is an indicator variable that specifies whether or not any fund managers employed by that fund attended any social events during that quarter. The statistical significance of our SocialConnection variable (t=3) is largely unchanged when the SocialFirm and SocialFund controls are added. Furthermore, as can be seen in specification 2 the statistical significance of the controls are negligible.

In specifications 3 through 4 we include as variety of controls. We control for stock-level characteristics such as market capitalization, book-to-market ratio, past 12 month return, the number of positions held by the fund, breadth of ownership and the fraction of the stock’s market cap that is owned by active equity fund managers. Book to market is computed as of the June of the previous calendar year following Daniel, Grinblatt, Titman, and Wermers (1997). We also control for fund level characteristics such as the percentage of the fund’s total net assets invested in the style corresponding to the stock being considered (style is
calculated using the 5x5x5 SIZE,BM,MOM quintiles from the CRSP/Computstat universe as of the end of the previous month) and the average change in portfolio weight in that stock during that quarter across all funds. Once the control are added in specifications 3 and 4 we can no longer easily interpret the coefficients on SocialConnection as simple increase or decrease in portfolio weight. Nevertheless, we continue to observe a robust correlation between SocialConnection and changes in fund manager portfolio weights (t=5.1).

2.5.3 Returns to Socially Connected Trades

In this section we study the performance of fund managers’ trading in socially connected securities. The fact fund managers are more likely to purchase stocks of firms having executives with whom they have attended social events does not imply that this decision is advantageous. That is, the correlated trading documented in the previous section need not be due to superior information gathering by fund managers. It could instead be due to one of many cognitive biases, which could result in these trades being poor choices. For instance, given the empirical results thus far, a perfectly plausible hypothesis could be a “top of mind” effect in firms. That is, fund managers, upon interacting with firm executives at social events, subsequently seek out information about these firms but have no actual informational edge in assessing firm value or timing trades. This “top of mind” alternative hypotheses can explain the induced purchases that we see in the data but could not explain any outperformance on these purchases that might exist.

2.5.3.1 Types of Trades

In the analysis that follows we overlay a useful taxonomy on the types of trades that fund managers might perform. Before proceeding to the main analysis we discuss this issue briefly. We study the characteristics of stocks that are bought, initiated, picked-up, sold and liquidated by fund managers. Naturally, buys are stocks where a fund manager has increased holdings of the stock since the last report date and sells are stocks where a fund has decreased their holdings of a stock. Initiations are stocks where a fund went from a position
of zero to a positive position in the stock since the last report date. Liquidations are stocks
where a fund sold all of their holdings, going from a positive to a zero position in the stock
since the last report date. Pick-ups are stocks where a fund purchased the stock for the first
time over their entire recorded 13F filing history. Therefore, liquidations are a subset of sells,
initiations are a subset of buys, and pick-ups are a subset of initiations.

Table 2.6 provides some intuition for the proportions of each trade type. Column 1 says
that 49.5% of all trades are buys and 50.5% are sells; the balance between buys and sells
should not be surprising. Column 1 also says that about a third of buys are initiation and sell
are liquidations. Notably, taking all initiations, roughly 40% of the time the stock is being
initiated for the first time (i.e: is a pick-up).

<table>
<thead>
<tr>
<th>Table 2.6: Socially Connected v.s. Non-Connected Trading, Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of</td>
</tr>
<tr>
<td>All Trades</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>All Trades</td>
</tr>
<tr>
<td>Buys</td>
</tr>
<tr>
<td>Initiations</td>
</tr>
<tr>
<td>Pickups</td>
</tr>
<tr>
<td>Sells</td>
</tr>
<tr>
<td>Liquidations</td>
</tr>
<tr>
<td>Holdings</td>
</tr>
</tbody>
</table>

This table reports joint summary statistics on the types of trades made by investors and
our social connectivity measure. We count the proportion of stocks bought/initiated/picked
up/sold/liquidated/held by fund managers where a social interaction occurred with an execu-
tive and where one has not.

Columns 2 and 3 require some delicacy. First, the top row says that taking all trades
in our sample of fund managers 1 in every 500 (0.2%) is socially connected. Juxtaposing
columns 2 and 3 for the Buy row suggests that conditional on being connected, a trade is
not much more/less likely to be Buy (48% v.s. 49.5%). The same reasoning holds true for
the Sell row. Connected holding refer to stocks that are currently in fund portfolios and
were initiated during a quarter in which there was a social connection with that stock. The
inference for this category is empirically the same as that of All Trades.

2.5.3.2 Socially Connected v.s. Non-Connected Trading

We now use standard calendar time portfolios to test the hypothesis that fund managers outperform on trades induced by social connections. At the end of every calendar quarter stocks in each fund portfolio (based on the most recent 13F filing) are designated connected or not, on the basis of social events that transpired during that quarter. Test portfolios based on these connections are then formed at the start of the subsequent quarter.

So for example, suppose we are studying the subsequent performance of fund initiations that are coincident with social connectivity. If we are looking fund managers’ initiations during the second calendar quarter we designate the social events that transpired between April 1st and June 31st as part of our SocialConnection variable. In this example, the initiation is based on juxtaposing snapshots made on March 31st and June 31st. As such, the social events will be aligned with the interval during which the initiation transpires. A test portfolio that tracks the performance of these connected initiations will then establish a position from July 1st through September 30th (3rd calendar quarter). Test portfolios are rebalanced quarterly and are weighted by funds’ dollar holdings (weights always sum to 1).

Table 2.7 presents this section’s key result: fund managers outperform on purchases of stocks where they attended a social event with firm executives. We report calendar time excess returns in monthly basis-points. This outperformance is statistically and economically significant: socially connected purchases (buys/initiataions/pick-ups) earn between 100 and 200 basis-points monthly in raw returns. Whereas their non-connected analogs earn roughly half that. These returns are robust to controlling for common risk factors; they earn between 60 and 160 basis-points monthly in 4-factor alphas, which is the intercept on a regression of monthly portfolio excess returns on the monthly returns from the three Fama and French (1993) factor-mimicking portfolio, and Carhart’s (1997) momentum factor. The L/S column is a long short portfolio that is long connected portfolio in a particular category and short the analogous non-connected portfolio. The magnitude of the long-short test portfolio returns
are generally large and statistically significant: socially connected purchases outperform non-connected purchases by between 50 and 130 basis-points monthly in 4-factor alphas. Not only do connected purchases outperform, but as we would expect connected sells have more negative returns that their non-connected analogs.

Table 2.7: Socially Connected versus Non-Connected Trading, Abnormal Returns

<table>
<thead>
<tr>
<th></th>
<th>Raw Return</th>
<th>4-Factor Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Connected</td>
<td>Non-connected</td>
</tr>
<tr>
<td>Buys</td>
<td>75.52</td>
<td>113.00</td>
</tr>
<tr>
<td></td>
<td>1.28</td>
<td>1.74</td>
</tr>
<tr>
<td>Initiations</td>
<td>82.86</td>
<td>223.41</td>
</tr>
<tr>
<td></td>
<td>1.31</td>
<td>3.19</td>
</tr>
<tr>
<td>Pickups</td>
<td>83.29</td>
<td>214.71</td>
</tr>
<tr>
<td></td>
<td>1.32</td>
<td>3.02</td>
</tr>
<tr>
<td>Sells</td>
<td>73.48</td>
<td>35.82</td>
</tr>
<tr>
<td></td>
<td>1.33</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>0.67</td>
</tr>
<tr>
<td>Holdings</td>
<td>74.55</td>
<td>116.86</td>
</tr>
<tr>
<td></td>
<td>1.30</td>
<td>1.44</td>
</tr>
</tbody>
</table>

This table reports calendar time excess returns (in monthly basis points) on portfolios formed conditional on fund-firm social connections. At the end of every calendar quarter stocks in each fund portfolio (based on the most recent 13F filing) are designated as connected or not, on the basis of social events that transpired during that quarter. Test portfolios based on these connections are then formed at the start of the subsequent quarter. Test portfolios are rebalanced quarterly and are weighted by the funds’ dollar holdings. Connected companies are defined as firms where at least one corporate official attended a social event with a portfolio manager employed by the fund.

Notably, outperformance is more pronounced in the types of purchases that are likely triggered by the acquisition of fresh information. Our results are stronger for initiations, stocks where a fund went from a position of zero to a positive position in the security, and for pickups (first time initiations). Socially connected initiations outperform non-connected initiations by between 138 basis-points monthly in raw returns \((t=2.87)\) and by 147 basis-points monthly in 4-factor alphas \((t=3.05)\).

These economically large returns are unaccompanied by increased levels of risk: the annualized Sharpe Ratio of the portfolio of socially connected initiations is 1.2 compared to 0.5 for non-connected initiations.
Table 2.8 presents the results of a strategy that forms a long short portfolio that goes long fund managers’ buys and shorts fund managers’ sells. We then compare the connected and non-connected specifications of this zero cost portfolio. The key takeaway from table 2.8 is that the zero cost (long short) portfolios of connected Buys v.s. Sells and Initiations v.s. Liquidations earn statistically and economically significant returns. Focusing on Initiations v.s. Liquidations we see that this portfolio earns 172 basis-points monthly in 4-factor alphas ($t=2.86$). On the other hand, the long short portfolio that mimics all fund buys v.s sells is essentially flat, earning -0.39 basis-points monthly in 4-factor alphas ($t=-0.11$). Not surprisingly, the non-connected mimicking portfolio is flat also.

Table 2.8: Predictability of Socially Connected Portfolios, Abnormal Returns

<table>
<thead>
<tr>
<th>Raw Return</th>
<th>4-Factor Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
</tr>
<tr>
<td>Buys v.s.</td>
<td>2.04</td>
</tr>
<tr>
<td>Sells</td>
<td>0.37</td>
</tr>
<tr>
<td>Initiations v.s</td>
<td>2.53</td>
</tr>
<tr>
<td>Liquidations</td>
<td>0.25</td>
</tr>
</tbody>
</table>

This table reports calendar time excess returns on portfolios formed conditional on fund-firm social connections. This strategy forms a long short portfolio that goes long fund managers’ buys and shorts fund managers’ sells.

Table 2.9 reports the results of pooled time series regressions of monthly portfolio excess returns on the monthly returns from the three Fama and French (1993) factor-mimicking portfolio, and Carhart’s (1997) momentum factor. First focusing on the long short spread portfolios of buys v.s sells. The version of this portfolio that consists of all fund manager buys and sells has pronounced exposures to these common risk factors. Notably beta and momentum, which is not surprising if we believe that our sample of fund managers are value investors. That said, these factor exposures are absent in the connected version of this portfolio. The connected (and L/S) versions of the Initiations v.s. Liquidations portfolio has a slight size tilt, but its other exposures aren’t noteworthy.
Table 2.9: Socially Connected Portfolios, Four Factor Regression Loadings

<table>
<thead>
<tr>
<th></th>
<th>Buys v.s. Sells</th>
<th></th>
<th>Initiations v.s. Liquidations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Connected</td>
<td>Non-connected</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.0000</td>
<td>-0.11</td>
<td>0.0077</td>
</tr>
<tr>
<td></td>
<td>0.0077</td>
<td>1.96</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.0290</td>
<td>-0.0513</td>
</tr>
<tr>
<td></td>
<td>0.0077</td>
<td>3.51</td>
<td>-0.54</td>
</tr>
<tr>
<td>SMB</td>
<td>0.0321</td>
<td>0.0303</td>
<td>0.0856</td>
</tr>
<tr>
<td></td>
<td>0.1159</td>
<td>1.71</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>0.0303</td>
<td>0.0453</td>
<td>0.1007</td>
</tr>
<tr>
<td></td>
<td>0.0856</td>
<td>2.93</td>
<td>0.57</td>
</tr>
<tr>
<td>HML</td>
<td>0.0385</td>
<td>-0.0331</td>
<td>-0.1017</td>
</tr>
<tr>
<td></td>
<td>0.1460</td>
<td>-4.79</td>
<td>-1.29</td>
</tr>
</tbody>
</table>

This table reports 4 factor loadings. That is, the results of a regression of monthly portfolio excess returns on the monthly returns from the three Fama and French (1993) factor-mimicking portfolios, and Carhart’s (1997) momentum factor.

Given our empirical results thus far we believe that the Initiations v.s. Liquidations spread portfolio is that which is most representative of the phenomenon we are trying to understand. Figure 2.4 juxtaposes the time series of returns for the connected and non-connected specifications of this portfolio.
Looking at the large alphas particularly on the long side of our socially connected portfolios raises the obvious question: are socially connected stocks fundamentally different from the rest of the equity universe (eg: higher return)? For example, perhaps fund managers are randomly drawing from a static subset of the stock universe that consists of higher return stocks. If this is true then the portfolio of connected stocks that fund managers choose to hold should be distinct from those that they do choose not to.

Using the same portfolio construction approach as before, we compute value weighted returns on test portfolios of connected non-held stocks. Connected non-held stocks are those stocks where the fund manager had a social interaction with a firm executive during the quarter but maintained a zero position throughout the quarter. Table 2.10 presents the returns of test portfolios of connected stocks that fund managers choose not to hold. The portfolio of connected holdings and connected non-held positions are virtually identical and have as spread of 2 basis points monthly in 4 factor alphas (t=0.04). Nevertheless, those
connected stocks in which fund managers chose to initiate positions outperforms the those that they don’t trade by 120 basis points monthly in 4 factor alphas (t=2.51).

Table 2.10: Non-held Socially Connected Stocks, Abnormal Returns

<table>
<thead>
<tr>
<th></th>
<th>Raw Return</th>
<th>4-Factor Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connected Non-Held</td>
<td>78.29</td>
<td>32.95</td>
</tr>
<tr>
<td></td>
<td>1.52</td>
<td>2.29</td>
</tr>
<tr>
<td>Connected Non-Held v.s.</td>
<td>-34.69</td>
<td>-31.12</td>
</tr>
<tr>
<td>Connected Buys</td>
<td>-31.12</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>-1.21</td>
<td>-1.07</td>
</tr>
<tr>
<td>Connected Non-Held v.s.</td>
<td>-144.50</td>
<td>-124.24</td>
</tr>
<tr>
<td>Connected Initiations</td>
<td>-124.24</td>
<td>-2.51</td>
</tr>
<tr>
<td></td>
<td>-2.90</td>
<td>-2.51</td>
</tr>
<tr>
<td>Connected Non-Held v.s.</td>
<td>-136.47</td>
<td>-117.48</td>
</tr>
<tr>
<td>Connected Pickups</td>
<td>-117.48</td>
<td>-2.07</td>
</tr>
<tr>
<td></td>
<td>-2.36</td>
<td>-2.07</td>
</tr>
<tr>
<td>Connected Non-Held v.s.</td>
<td>-40.19</td>
<td>2.00</td>
</tr>
<tr>
<td>Connected Held</td>
<td>2.00</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>-0.73</td>
<td></td>
</tr>
</tbody>
</table>

This table reports calendar time excess returns (in monthly basis points) on portfolios formed conditional on fund-firm social connections. Connected non-held stocks are those stocks where the fund manager had a social interaction with a firm executive during the quarter but maintained a zero position throughout the quarter. Portfolios are value weighted by market capitalization.

2.6 Propagation of Fund Managers’ Best Ideas

In the previous sections, we studied the effect of social networks on executives and fund managers jointly. We now ask the obvious next question of whether the information flow that we document extends to the communication between fund managers. We examine the extent to which fund managers who attend the same social events tend to herd in their trading decisions, particularly on their highest conviction ideas.

We assume that when a fund manager attends a social event, that fund becomes connected to the other managers at that event. To test for herding in a fund manager’s trading of a particular stock, we define a measure of the extent to which a fund manager’s connected peers hold that stock as a best idea.

For each fund manager, we measure their social connectivity to a stock by counting the number of other distinct fund managers with whom they attended a social event and who
had that stock as a top holding. With our definition, the funds might have had these social interactions at different events during that quarter. Also, note that more than one individual might work for a specific fund. As long as employees of two distinct funds interact at a social event, then the two funds are designated as socially connected for that quarter. We define best ideas as holdings that are amongst the top 5 in a fund manager’s portfolio by dollar weight. We adopt a simple best idea aggregation scheme that assigns values proportional to the number of connected funds that have a stock as their best idea.\textsuperscript{13}

We present an example in order to give intuition for our methodological approach. For simplicity lets assume in this example that only the top 1 holding is considered to be a best idea. Consider a fund, which we will refer to as “XYZ Capital”. Suppose that after attending many social events throughout the quarter XYZ Capital has interacted with 20 other distinct funds. We say that XYZ is now socially connected to these 20 funds. Of the 20 funds with which XYZ is connected let’s assume that two funds’ best ideas are stocks A & B. Let’s assume that the remaining 18 funds all have a third stock, C, as their best idea. We assume that connected funds discuss their best ideas. And in this example, 90% of connected funds discussed stock C. From here we predict that XYZ Capital is likely to increase its position in stock C in the subsequent quarter.

2.6.1 Measuring cross-fund information transfer

We proceed by describing our empirical methodology in detail. For each fund manager $f$, we identify all other fund managers that attended the same social events. This yields fund $f$’s set of connected funds, which we denote by $K_{f,t}$. For each stock we measure of the degree to which it is held by these connected funds a best idea. This measure is denoted by $c_{s,f,t}$. Our best idea assignments are based on the funds’ dollar holdings as of the \textit{beginning of the quarter}. We denote by $BEST_{s,f,t}$ the indicator variable that fund $f$ holds stock $s$ as a best idea.

For fund $f$ and stock $s$, we denote by $\Sigma BEST_{s,f,t}$ the sum total number of instances

\textsuperscript{13}If, on the other hand, a fund has no social events in a particular quarter then our measure of social connectivity is set to zero for every stock its portfolio.
where \( s \) is a socially connected best idea.

\[
\Sigma_{\text{CONNECTEDBEST}}_{s,f,t} = \sum_{f_j \in K_{f,t}} \text{BEST}_{s,f_j,t}
\]

Its worth highlighting the distinction here that \( \text{BEST}_{s,f_j,t} \) is a property of a single stock that is 1 or 0 depending on the fund’s weight in that stock. Whereas \( \Sigma_{\text{CONNECTEDBEST}}_{s,f,t} \) is a property of the holdings of funds with which a fund is connected. In general, the larger the value of \( \Sigma_{\text{CONNECTEDBEST}}_{s,f,t} \) the more we expect fund \( f \) to increase its position in stock \( s \).

To obtain our final measure \( c_{s,f,t} \) we normalize \( \Sigma_{\text{CONNECTEDBEST}}_{s,f,t} \) by the total number of connected best ideas across all stocks.

\[
c_{s,f,t} = \frac{\Sigma_{\text{CONNECTEDBEST}}_{s,f,t}}{\sum_{s_k}(\Sigma_{\text{CONNECTEDBEST}}_{s_k,f,t})}
\]

When we study the effect of \( c_{s,f,t} \) on fund managers’ subsequent behavior we face the subtle identification problem of not knowing if our inferences about \( c_{s,f,t} \) are driven by the social events or by the tendency of \( c_{s,f,t} \) to correlate with stocks that have large weights in any fund’s portfolio. For this reason we also define an analogous measure across all funds (as opposed only connected funds). This measure is independent of our social connection data and serves as a benchmark for \( c_{s,f,t} \) by allowing us to correct for any bias that may result from a stock simply having a large weight in every fund’s portfolio. We define

\[
\Sigma_{\text{ALLBEST}}_{s,t} = \sum_{f_j} \text{BEST}_{s,f_j,t}
\]

and

\[
a_{s,t} = \frac{\Sigma_{\text{ALLBEST}}_{s,t}}{\sum_{s_k}(\Sigma_{\text{ALLBEST}}_{s_k,t})}
\]

We perform quarterly fund-stock Fama-McBeth OLS regressions of equity fund manager trading activity on social event and cross-fund portfolio characteristics. The dependent vari-
able is changes in fund manager portfolio weights. The key independent variable is $c_{s,f,t}$, our proxy for the degree to which a fund’s socially connected peers hold a stock as a best idea. Since the trading decisions of fund managers could be influenced by a variety of stock specific characteristics, we include several controls in our regressions. We control for stocks’ past returns and we control for the possibility that a common investing style may influence results by controlling for a stock’s market capitalization and book-to-market ratio (Barberis and Shleifer (2003), Froot and Teo (2008)). RETCurQTR is the return on the stock during the current quarter and RETPrevQTR is the return on the stock during the preceding quarter.

Table 2.11 presents our main result: when several fund managers attend the same social event they herd on each others’ “best ideas”

<table>
<thead>
<tr>
<th>Dep Variable: Change in portfolio weight $dW_{s,f,t}$ (BPS)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 5 Best Idea Socially Connected $c_{s,f,t}$</td>
<td>14.12</td>
<td>14.08</td>
<td>14.07</td>
</tr>
<tr>
<td></td>
<td>3.16</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>Top 5 Best Idea All Funds $a_{s,f,t}$</td>
<td>-74.05</td>
<td>-75.23</td>
<td>-82.24</td>
</tr>
<tr>
<td></td>
<td>-5.55</td>
<td>-5.66</td>
<td>-5.61</td>
</tr>
<tr>
<td>Change in IO $dIO_{s,t}$</td>
<td>0.97</td>
<td>1.01</td>
<td>4.40</td>
</tr>
<tr>
<td></td>
<td>4.07</td>
<td>4.54</td>
<td></td>
</tr>
<tr>
<td>SIZE</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BM</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RETCurQTR</td>
<td>-0.04</td>
<td>-2.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RETPrevQTR</td>
<td>-0.14</td>
<td>-6.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.02</td>
<td>0.01</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>5.48</td>
<td>0.83</td>
<td>-4.06</td>
</tr>
</tbody>
</table>

Notes: This table reports quarterly, fund-stock Fama-McBeth OLS regressions of equity fund manager trading activity on fund and cross-fund portfolio characteristics:

$$dW_{s,f,t} = \beta_0 + \beta_1 c_{s,f,t} + \beta_2 a_{s,t} + \theta X_{s,f,t} + \epsilon_{s,f,t}$$

The units of observation are fund-stock-quarter $(s, f,t)$.

The first specification demonstrates our key result, that fund managers’ trades are positively related to their socially connected best ideas. In the second specification we control
for the change in the stock's overall institutional ownership. Changes in overall institutional ownership are correlated as expected but does not affect the strength of our result. In the third specification, we control for SIZE, BM, and returns during the current and previous quarters. The magnitude and statistical significance of our result remains robust.

Previous research such as (Cohen, Malloy & Frazzini, 2008) has shown that fund managers that are connected to each other through shared educational backgrounds tend to herd in their trading decisions, particularly on stocks where they have a social connection to the firm. Our results hope to add an additional time dimension that is absent in the case of static links such as school ties.

Finally, to the extent that fund managers in the same industry have correlated holdings and also typically attend the same events, we would expect to observe a significantly positive point estimate on $c_{s,f,t}$. This however may have little to do with our measured social interactions per-se. We disentangle our proposed dynamic connection from potentially static connections by adding our connection variable lagged by a year as a regressor. If the connections being picked up by $c_{s,f,t}$ are static in the way that school ties are, the lagged social connection should still predict changes in portfolio weights. On the other hand, if the connections being picked up by $c_{s,f,t}$ are dynamic then lagging it by a year should remove any predictability. In unreported tests we confirm that this is the case by showing that our effect is robust to including the value of our social connection variable lagged by a year. This is important as it highlights the crucial time dimension of our result. It rules out alternative hypotheses where external persistent links drive the documented trading behavior.

### 2.7 Conclusion

Social networks play an important role in the diffusion of information. However, it is often difficult to measure, observe and quantify this process. Using a unique database of elite social events and their attendees, we document evidence of informed trading activity on the heels of exclusive gatherings. We illustrate that the returns of stocks subsequent to social events predicts their upcoming earnings surprises; this is true for precisely those events where the
firm’s corporate officers are in attendance.

This canonical result motivates a detailed inspection of event attendance in order to help understand the observed return predictability. We find that fund managers that attend events that are also attended by corporate officers are more likely to purchase stock in that firm. These fund managers on average increase their portfolio weight by an additional 20-50 basis points ($t=3$) after the event. We then explore potential reasons for this bias in fund managers’ trading in these socially connected stocks. We find that the difference in profitability between fund managers’ socially connected trades and their non-connected analogs is statistically and economically significant. Socially connected initiations outperform their non-connected analogs by over +150 basis points monthly ($t=+2.8$). The out-performance on these executive induced purchases suggests that, superior information gathering ability, linked to social events, explains fund managers’ bias toward buying socially connected stocks.

Even though fund managers exhibit a bias towards increasing their holdings in socially connected stocks, we find that when they unwind socially connected positions the decision is ex-post more profitable than in the case of their non-connected unwinds. Specifically, connected sells under-perform their non-connected analogs by -20 basis points monthly ($t=-0.78$).

A trading strategy that exploits the combination of fund managers’ rebalancing effects has non-trivial risk-return characteristics. The strategy that buys connected initiations and sells connected liquidations is economically and statistically significant. And the spread between the connected and non-connected versions of this strategy has a Sharpe Ratio of 1.2 and is uncorrelated with typical risk factors.

Lastly, we extend our mechanism by studying interactions taking place between fund managers. We find that event co-attendance by several fund managers precedes increases in portfolio-overlap, particularly in their best ideas. This further strengthens our thesis that casual personal connections are driving the trading patterns that we observe.

The organization of social networks is an important determinant of how information becomes reflected in stock prices. Our empirical analysis provides insight into the social
networks of certain market participants and its effect on their behavior. Taken together our results suggest that casual social interactions are central to the flow of information in financial markets and are, therefore, essential for building a complete understanding of price formation.
Chapter 3

Mean Reversion

3.1 Introduction

Traditional asset-pricing models, such as those based on rational-expectations, dictate that return predictability is wholly borne out of either risk premia or unanticipated shifts in liquidity. Theories such as the capital asset pricing model, arbitrage pricing theory and the Fama-French three-factor model require a tautological link between economically meaningful risk premia and any return predictability that might exist. Such mechanisms can hardly justify the rapidly increasing catalog of empirical anomalies found in asset price data, especially those relying entirely on price based signals. Behavioral models, such as those based on learning, information flow & investor disagreement have been a central development for the resolution of this apparent incongruence. Placing disciplined constraints on the potentially unbounded set of deviations from rationality upon which a behavioral model might be built is crucial. Beyond the generally desired qualities such as parsimony and an ability to generate new empirical implications, classical works such as DeBondt and Thaler (1995) advocate that behavioral finance theory should be founded on robust psychological evidence.

In this essay I propose a mechanism whereby information flow, and then subsequent learning explains the patterns of short horizon momentum and long horizon reversals that are observed in equity prices. The model’s key departure from rationality is its assumption
that investors are overconfident; they overestimate the relative precision of signals with which they are more closely involved. Overconfidence is both prevalent in, and relevant to, financial markets; DeBondt and Thaler (1995) assert that “perhaps the most robust finding in the psychology of judgment is that people are overconfident.”

My model features a risk-averse investor who begins with a prior distribution about an asset’s fundamental value that is a simple Gaussian. The investor combines exactly three sources of information with which she updates her beliefs: her prior distribution, a price-target for the asset, and news about the asset’s true value. At an initial date the investor receives a price-target about the value of the asset. Subsequently, the investor’s belief about the asset’s payoff distribution is shaped by the stream of news which she uses to update her initial prior distribution. The news is assumed to be a noisy but unbiased measurement of the asset’s true value. The investor is subject to a well-defined Bayesian filtering problem that, as I will show, can be profoundly affected by parameter mis-measurement. I show that if the investor over-estimates the price-target’s relative precision, then she will exhibit a rational tendency to initially increase her belief in its credibility regardless of whether or not it is true.

To understand my approach, consider the following example. Suppose that on the basis of fundamental analysis, an investor constructs a Gaussian prior for the distribution of the value of a stock. Now suppose a sell-side analyst gives the investor a price-target for the stock. The price-target, if true, deterministically pins down the value of the stock. As such, the investor’s prior distribution of the stock’s value goes from being a Gaussian to being a Gaussian mixture where the mixture components are her prior density and a point-mass at the price-target.14 If we assume that the price-target is optimistic (above the investor’s prior mean for the stock’s value) then observation of the price-target will increase the investor’s perceived value of the stock since she will add some weight to the state of the world in which the price-target is true. In each subsequent time period the investor observes news,

14The initial relative weight of the price-target component of the investor’s posterior distribution is exogenous - intuitively this will depend on the analyst’s reputation or simply how much the investor believes the price-target to be true.
measurements that are the asset’s true value combined with Gaussian noise.

What happens to the investor’s beliefs as she observes news revealing the stock’s true value? The investor performs model selection on two competing hypotheses: one where the asset’s value is exactly the price-target, so that the observed news is a noisy measurement of the price-target, and another model where the asset’s true value is drawn from a distribution given by the investor’s prior so that the observed news is a noisy measurement of the prior mean (with a higher variance). The investor then model-averages these hypotheses in order to form her final beliefs.

Strikingly, if the price-target isn’t too far above the investor’s prior mean then observing bad\textsuperscript{15} news can actually cause the investor to rationally increase her overall expectation of the asset’s value.

To see this, consider what happens when the investor observes news that the asset’s value is slightly below the investor’s prior mean. The investor updates her beliefs in two distinct ways: she updates her conditional posterior distribution for the asset’s value conditional on the price-target being false, and she reevaluates the weight placed on the price-target. The first part of her updating is easy to conceptualize, the investor forms a conditional posterior whose mean is a linear combination of her prior mean and the observed news. Thus, her conditional posterior mean is less than her prior mean if the observed news is less than her prior mean.

The second part of the investor’s updating asks, by which model the data is more likely to have been generated. If the price-target is close to the prior mean, then this amounts to juxtaposing the likelihoods of two Gaussian distributions with similar means but different variances. In this case the distribution with the lower variance is often the one more likely to be the data generating process. When the variance of the investor’s prior is large relative to the variance of the news, then she will update in favor of the price-target for a disproportionately large fraction of news that can be observed.

\textsuperscript{15} I refer to good (bad) news as realizations that are above (below) the mean of the investor’s prior distribution.
This figure depicts the likelihood of observed news. It compares the investor’s prior distribution with the likelihood function conditional on the asset’s value being the price-target.

Concretely, suppose the investor’s prior distribution has mean $\mu_z$ and variance $\nu_z$; and let $G$ denote the price-target and $\nu_e$ the variance of the normally distributed news shocks. Observing a news realization that is bad, but not too far below the prior mean, can cause the latter effect (up-weighting the optimistic price-target) to dominate the former effect (a lower conditional posterior mean), resulting in an overall increase in the model-averaged posterior expectation of the asset’s value. A visual depiction of this argument is presented in figure 3.1.

Perturbing this example makes this apparent violation of the representativeness heuristic even more clear. Suppose now that, instead of observing bad news, the investor observes news that is exactly equal to her prior mean. Given the assumptions and argument above, this would result in an even larger degree of up-weighting in favor of the price-target since the news is now closer to it than before. This new situation emphasizes the fact that the investor will interpret evidence that is exactly consistent with her prior distribution as instead being consistent with the price-target being true. I illustrate these points rigorously in section 3.3.
precise than it actually is, on average the investor will initially tend to increase her belief in the price-target, even if it is not true. Nevertheless, if the price-target is not true, then as the investor continues to observe increasingly more news about the stock, she eventually down-weights her belief in the price-target towards zero. The initial period of price-target up-weighting causes short horizon continuation, while the subsequent down-weighting leads to long horizon reversal as the effect of the price-target diminishes.

I test my theory by juxtaposing realized and option-implied volatility, and show that the overshooting effect predicted by the model is most pronounced when the market’s perception of a stock’s volatility consistently exceeds actual volatility. A momentum portfolio strategy that exploits this effect earns over 80 basis points per month (t=2.6) in factor adjusted alphas.

The remainder of the paper is organized as follows. Section 3.2 provides a brief background and literature review. Section 3.3 presents a canonical model of investor beliefs. Section 3.4 embeds these beliefs in an equilibrium pricing framework and section 3.5 derives & discusses my central theoretical results. Section 3.6 presents empirical validation of the model’s predictions and Section 3.7 concludes.

3.2 The Setting

This essay relates closely to a broad literature regarding market efficiency. The established idea of rationally priced securities, which reflect all publicly available information, has found considerable difficulty in explaining the mounting evidence of return predictability. For example, momentum, the tendency of assets to exhibit persistence in their price performance, has been observed not only in US equity markets (Jegadeesh and Titman (1993)), but also in European and emerging markets (Rouwenhorst (1998)) and a broad spectrum of asset classes (Asness, Moskowitz & Pedersen (2013)). The challenge for traditional risk-based asset-pricing models is further exacerbated by findings of mean reversion in equities at longer horizons of 3 to 5 years (DeBondt and Thaler, (1985, 1987)). As such, behavioral theories - approaches that include some sort of pathological behavior - have been advanced as a resolution to this

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16 Averaging over the less noisy reality
incongruence. In this vein, several distinct approaches have been set forth to jointly deliver both short-horizon continuation and long-horizon reversals. For instance, using an individual representative agent, Barberis, Shleifer, & Vishny (1998) and Daniel, Hirshleifer, & Subrahmanyam (1998) assume that price movements are driven by a small collection of cognitive biases. On the other hand, Hong and Stein (1999) build on the interactions between heterogeneous agents to explain the observed return predictability. My approach is somewhere in the middle, while it assumes a representative agent it is fundamentally more similar to the model of Hong and Stein (1999).

Empirical documentation of return predictability related to this model falls into two distinct categories: reversal and continuation. DeBondt and Thaler, (1985, 1987) find that stocks sorted based on their trailing 3 to 5 year performance tend to mean revert over the coming 3 to 5 years. There are also equivalent fundamental-reversion patterns observed in the time series of the aggregate market.

On the other hand, Jegadeesh and Titman (1993) document persistent out-performance of past winners over past losers by roughly 100 basis points per month over a horizon of 3 to 12 months. Relatedly, Hong, Lim, and Stein (1999) document that under-reaction in equity prices is most robust for stocks with low analyst coverage and is more pronounced for bad news. In terms of timing, Chan, Jegadeesh and Lakonishok (1996) show that the preponderance of the momentum effect is concentrated around subsequent earnings announcements. This essay pursues the goal of testing a new unified explanation for both momentum and reversal phenomena.

This work also relates to another aspect of market efficiency: the implications of asset volatility for future price dynamics. This is central to my empirical strategy and is discussed further in section 3.6 in the context of my empirical results.

To summarize, much of the previous work on explaining continuation and reversal patterns in asset prices are centered on issues such as trend-chasing by traders or fire-sale effects. My mechanism is distinct from other approaches to return predictability in that it focuses instead on the issue of an investor learning about the precision of competing signals. In addition
to explaining general empirical facts such as short horizon continuation and long horizon reversal, this model derives sharp predictions for when these anomalies should be relatively more or less robust.

3.3 A Simple Model With Learning

This model speaks to the expected price dynamics that result as an investor observes new information. The model’s insights are driven by peculiarities about the way that investors update their beliefs under certain conditions. A preliminary discourse on this belief updating is therefore a prerequisite for a coherent exposition of my central results.

This section has three principal goals. First, I present a simplified 2-period example of belief-updating and use it to orient the reader with some important notation. I will also rigorously justify the claim made in the introduction that: if the price-target isn’t too far above the mean of the investor’s prior, then observing bad news can actually cause the investor to rationally increase her overall expectation of the asset’s value. Second, I extend my notation and tools to accommodate multiple time periods. Third, I reformulate the investor’s signal-selection problem in such a way that the dependency on the model’s parameters becomes very transparent. In the later sections I will embed these assumptions about beliefs in an equilibrium pricing framework and study their effects on prices.

3.3.1 A Motivating Example

I proceed by describing a stylized example to motivate my approach. Consider an investor who begins with the prior that a stock’s fundamental value, \( z \), is normally distributed with mean \( \mu_z \) and variance \( v_z \), i.e. \( z \sim \mathcal{N}(\mu_z, v_z) \). A sell-side analyst tells the investor a price-target of \( G \), claiming that \( z = G \equiv \mu_z + \kappa \) (without loss of generality, we may assume that \( \kappa > 0 \)). In words, the price-target is optimistic and simply exceeds the investor’s prior mean by \( \kappa \). Initially, the investor believes that with probability \( \omega_0 \in (0, 1) \) the price-target is true,
but that with probability $1 - \omega_0$ the price-target is completely uninformative\(^{17}\).

The investor’s expectation of the stock’s fundamental value is weighted average of her prior and the price-target

$$
\mathbb{E}^I_0 [z] = \omega_0 G + (1 - \omega_0) \mu_z
$$

$$
= \omega_0 \kappa + \mu_z
$$

The investor then observes a noisy news signal $\psi = z + \varepsilon$ with $\varepsilon \sim \mathcal{N}(0, v_\varepsilon)$. The investor is a Bayesian, and she knows the distribution of $\varepsilon$. How do her beliefs change after she sees $\psi$? The investor’s updating task can be recast in the more familiar framework of model-selection, where “price-target is false” constitutes the first candidate model, and “price-target is true” constitutes the second. The investor updates her beliefs about the parameters in each model, and also updates the probabilities that she assigns each model.

From the investor’s perspective, under the assumption that the price-target is false, beliefs about $z$ update in the usual manner. She assigns $z$ a posterior normal distribution with the following moments:

$$
\mathbb{E}_1 [z | \psi] = \mu_z + \frac{v_\varepsilon}{v_z + v_\varepsilon} (\psi - \mu_z)
$$

$$
\mathbb{V}_1 [z | \psi] = \frac{1}{rac{1}{v_z} + \frac{1}{v_\varepsilon}} = \frac{v_z}{v_\varepsilon v_z + v_z}
$$

Note that I distinguish between the mean of the conditional posterior distribution $\mathbb{E} [\cdot]$ and the combined model average $\mathbb{E}^I [\cdot]$ by also using the superscript $I$.

Now, under the assumption that the price-target is true, $z$ has a degenerate prior, so no updating is necessary.

Next, the investor also updates her prior ($\omega_0$) concerning the probability that the price-

\(^{17}\) The initial weight $\omega_0$ can be thought of as the “reputation” of the individual or institution that is generating the price-target.
target is true to the posterior $\omega_1$.

The updating of $\omega$ lies at the heart of my work, and I explain it in detail. The investor is faced with a traditional Bayesian model selection problem, she forms a posterior probability that some model $M$ is true given data $D$. Bayes theorem dictates that

$$Pr(M|D) = \frac{Pr(D|M)Pr(M)}{Pr(D)}$$

In this context $M$ can be thought of as the model where the price-target is true, and $D$ represents the news. So this can be used by setting,

$$Pr(M) = \omega_0$$
$$Pr(M|D) = \omega_1$$
$$Pr(D) = Pr(D|True)\omega_0 + Pr(D|False)(1 - \omega_0)$$

Bayes rules therefore implies that investors update according to

$$\omega_1 = \frac{\omega_0 \left( \frac{Pr(D|True)}{Pr(D|False)} \right)}{\omega_0 \left( \frac{Pr(D|True)}{Pr(D|False)} \right) + (1 - \omega_0)}$$

The term given by $\frac{Pr(D|True)}{Pr(D|False)}$ is precisely the likelihood ratio or Bayes factor.

Now, if the target is true, then $\psi \sim \mathcal{N}(G, v_\varepsilon) \equiv \mathcal{N}(\mu_z + \kappa, v_\varepsilon)$ and so the likelihood of observing $\psi$ is

$$\mathcal{L}(\psi|true) = \frac{1}{\sqrt{2\pi v_\varepsilon}} \exp \left( -\frac{(\psi - \mu_z - \kappa)^2}{2v_\varepsilon} \right)$$

while if the target is false, $\psi \sim \mathcal{N}(\mu_z, v_z + v_\varepsilon)$ and the likelihood of observing $\psi$ is
\[ L(\psi|false) = \frac{1}{\sqrt{2\pi(v_z + v_\varepsilon)}} \exp\left(-\frac{(\psi - \mu_z)^2}{2(v_\varepsilon + v_z)}\right) \]

We define the likelihood ratio

\[ \Lambda \equiv \frac{L(\psi|True)}{L(\psi|False)} = \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp\left(\frac{(\mu_z - \psi)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi - \mu_z)^2}{2v_\varepsilon}\right) \] (3.2)

Applying Bayes’ rule yields

\[ \omega_1 = \frac{\Lambda \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \] (3.3)

My central results concern the manner in which the presence of the price-target alters the inferences that the investor makes from new, noisy signals about fundamental value.

Consider the case in which \( \psi = \mu_z \) (this both aids intuition and provides the starting point for my subsequent perturbation arguments). We begin by considering the implications for \( \omega_1 \). First, note that \( \omega_1 > \omega_0 \iff \Lambda > 1 \), so we are interested in the conditions that determine whether or not \( \Lambda \) is greater than unity.

\[ \Lambda = \frac{L(\psi = \mu_z|True)}{L(\psi = \mu_z|False)} = \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp\left(\frac{(\mu_z - \mu_z)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi - \mu_z)^2}{2v_\varepsilon}\right) \]

\[ = \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp\left(\frac{\kappa^2}{2v_\varepsilon}\right) \]

Consequently, routine algebraic manipulations show that

\[ \Lambda > 1 \iff \frac{v_z}{v_\varepsilon} > \exp\left(\frac{\kappa^2}{v_\varepsilon}\right) - 1 \] (3.4)

In other words, when the unconditional prior variance of \( z \) is sufficiently large relative to \( \kappa \).
and \( v_\varepsilon \), the signal \( \psi = \mu_z \) will lead the investor to strictly increase her belief in the price-target, that \( z = \mu_z + k \). Moreover, because \( \Lambda \) is monotone increasing with respect to \( \psi \) in a neighborhood of \( \mu_z \), when \( v_\varepsilon \) is large, there will be an interval of signals \( \psi < \mu_z \) that actually strengthen the investors belief in the price-target!

A Taylor expansion of the right-hand side of equation (3.4) provides an intuitive picture of what constitutes a “sufficiently large” \( v_\varepsilon \):

\[
\frac{v_z}{v_\varepsilon} > \exp\left(\frac{\kappa^2}{v_\varepsilon}\right) - 1 \\
\approx 1 + \frac{\kappa^2}{v_\varepsilon} - 1
\]

Therefore, to a first-order approximation (literally), the condition for the perverse reinforcement of the price-target is merely

\[ v_z > \kappa^2 \]

that the standard deviation of fundamental value exceeds \( \kappa \), the difference between the target and the original expectation. Stated differently, we need the price-target to be within a standard deviation of the prior mean.

Now, upon observing some \( \psi < \mu_z \), the investor lowers the value of \( z \) that she expects if the price-target is false. More rigorously,

\[
E_1 [z | \psi] = \mu_z + \frac{v_\varepsilon}{v_z + v_\varepsilon} (\psi - \mu_z)
\]

Although the investor may increase the weight she puts on the price-target being true, it is not obvious which effect will dominate in his expectation of \( z \) averaged across both models. By analogy to equation (3.1), this cross-model average is
\[ E_I^I[z] = \omega_1 (\mu_z + \kappa) + (1 - \omega_1) \left( \mu_z + \frac{v_z}{v_z + v_\epsilon} (\psi - \mu_z) \right) \]

It can be shown that the change in the overall expectation \( E_I^I[z] - E_I^0[z] \) is given by the following expression:

\[
E_I^I[z] - E_I^0[z] = \left( \frac{1 - \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \right) \left( \omega_0 \kappa (\Lambda - 1) + \frac{v_z}{v_z + v_\epsilon} (\psi - \mu_z) \right) \tag{3.5}
\]

If we fix some \( v_z \) satisfying \( \frac{v_z}{v_\epsilon} > \exp \left( \frac{\kappa^2}{v_\epsilon} \right) - 1 \) then it is assured that the entire term \( \omega_0 \kappa (\Lambda - 1) > 0 \). Now, let \( \psi \) tend towards \( \mu_z \) from below, it becomes clear that \( E_I^I[z] - E_I^0[z] \) will be greater than zero for some \( \psi < \mu_z \). This result reveals a very profound peculiarity that a little bit of bad news causes an investor to increase their overall valuation of the stock.

Figure 3.2 depicts the value of \( E_I^I[z] - E_I^0[z] \) for various values of \( \psi \). I assume that \( \mu_z = 50, G = 55, v_z = 200, v_\epsilon = 300 \) and \( \omega_0 = 0.5 \). The key takeaway from figure 3.2 is that for values of \( \psi \in [48.69, \mu_z] \) the investor’s model-averaged expectation increases (\( E_I^I[z] > E_I^0[z] \)). This is implied from the figure since the dashed green line (\( E_I^I[z] > E_I^0[z] \)) crosses above zero before \( \psi \) reaches \( \mu_z \).

I address the matter rigorously in section 3.5, but intuitively it is obvious that the preceding perturbation results can translate into a tendency for the investor to increase the weight that she places on the price-target in expectation over all news realizations.

Observe that since \( \psi \) is distributed symmetrically about \( \mu_z \) (integrate over \( z \)), and there exists some number \( \delta > 0 \) such that values of \( \psi \) in the interval \( (\mu_z - \delta, \mu_z) \) cause the investor to increase \( \omega \), it follows that the investor will increase \( \omega \) for more than half of the \( \psi \)’s drawn.

Establishing that the average initial change in \( \omega \) exceeds zero is more challenging, but as I show in section 3.5, this result will hold under some additional mild assumptions.

\[ \text{These are the same parameter values as those in the example in the introduction (figure 3.1).} \]
This figure depicts the change in the investor’s model-averaged expectation of the asset’s payoff for various values of $\psi$. Here I assume that $\mu_z = 50, G = 55, v_z = 200, v_\varepsilon = 300$ and $\omega_0 = 0.5$.

### 3.3.2 The Information Environment

The process by which investors refine their beliefs about asset payoffs is now considered in generality. I assume that investors have a constant stream of normally distributed news which they use to rationally update their estimates about $z$. Specifically, once $z$ is drawn (before the start of the model) there is a constant stream of news at dates $t \geq 1$, which is denoted by $\Psi_t$. It is assumed that $\Psi_t = z + \varepsilon_t$ where $\varepsilon_t \sim i.i.d. \mathcal{N}(0, v_\varepsilon)$ and $\varepsilon_t$ is independent of $z$ for all $t$. Let the realized value of $\Psi_t$ be denoted by $\psi_t$.

The two-period results in the previous section extend immediately to this multi-period setting through iteration. Only the notation changes. At time $t$, the analogues of $\mu_z$ and $v_z$ are $E_{t-1}[z]$ and $\nabla_{t-1}[z]$, respectively. For convenience, note that these quantities are updated upon the arrival of each new $\psi$ as follows:
\[ E_t[z] = \frac{v_e E_{t-1}[z] + \nabla_{t-1}[z] \psi_t}{v_e + \nabla_{t-1}[z]} \]
\[ \nabla_t[z] = \frac{v_e}{v_e + \nabla_{t-1}[z]} \nabla_{t-1}[z] \]

This is conveniently written as,

\[ E_t[z] = \mu_{\frac{\psi}{t}} + \frac{1}{t} \left( \sum_{s=1}^{t} \psi_s \right) v_z \]
\[ \nabla_t[z] = \frac{v_e v_z}{v_z + v_z} \]

To avoid confusion, I label the conditional likelihoods and the likelihood ratios with time subscripts hereafter.

Next, I examine the investor’s model selection problem. Consider the her information-set at the start of \textit{date-}t: if the signal is true then

\[ \Psi_t \sim \mathcal{N}(G, v_e) \]

however if it is not then

\[ \Psi_t \sim \mathcal{N}(E_{t-1}[z], v_e + \nabla_{t-1}[z]) \]

These hypotheses yield the following prior densities respectively,

\[ \mathcal{L}_t(\psi_t|\text{True}) = \frac{1}{\sqrt{2\pi v_z}} exp \left( -\frac{(\psi_t - G)^2}{2v_z} \right) \]
\[ \mathcal{L}_t(\psi_t|\text{False}) = \frac{1}{\sqrt{2\pi (v_e + \nabla_{t-1}[z])}} exp \left( -\frac{(\psi_t - E_{t-1}[z])^2}{2(v_e + \nabla_{t-1}[z])} \right) \]

The likelihood ratio is defined as,
\[ \Lambda_t = \frac{\mathcal{L}_t(\psi_t|True)}{\mathcal{L}_t(\psi_t|False)} \]

And in this case substitution yields,

\[ \Lambda_t = \sqrt{1 + \frac{1}{v_e} \mathbb{V}_{t-1}[z]} \exp \left( \frac{v_e (\psi_t - \mathbb{E}_{t-1}[z])^2 - (v_e + \mathbb{V}_{t-1}[z]) (\psi_t - G)^2}{2(v_e + \mathbb{V}_{t-1}[z])} \right) \]

Bayes rule implies that at date-\( t \) investors update their estimate that the signal is true according to,

\[ \omega_t = \frac{\Lambda_t \omega_{t-1}}{\Lambda_t \omega_{t-1} + (1 - \omega_{t-1})} \]  

(3.6)

It follows directly that

\[ \Lambda_t > 1 \iff \omega_t > \omega_{t-1} \]

### 3.3.3 Tractable Bayesian Updating

It is convenient to work with expressions that relate to key model parameters in a simple and intuitive way. The goal of this section is to establish an alternative expression for \( \omega_t \) that is convenient and depends transparently on the parameter \( \omega_0 \).

When faced with news, the investor uses equation 3.6 to determine \( \omega_t \) as a function of \( \omega_{t-1} \). This formula is inconvenient because \( \omega_0 \) is a parameter of the model but all subsequent values \( \omega_{t \geq 1} \) are determined recursively. It is useful to reformulate the Bayesian updating rule in equation 3.6 in a manner that is directly anchored to \( \omega_0 \).

First, observe that plugging in the expression for \( \omega_{t-1} \) yields

\[ \omega_t = \frac{\Lambda_t \Lambda_{t-1} \omega_{t-2}}{\Lambda_t \Lambda_{t-1} \omega_{t-2} + 1 - \omega_{t-2}} \]

Iterating equation 3.6 by repeated substitution (\( t \) times) yields,

\[ \omega_t = \frac{\prod_{k=1}^{t} \Lambda_{k-1} \omega_0}{\prod_{k=1}^{t} \Lambda_{k-1} \omega_0 + (1 - \omega_0)} \]  

(3.7)
Equation 3.7 is anchored at $\omega_0$ however without further simplification the term $\prod_{k=1}^{t} \Lambda_{k-1}$ makes the effect of news difficult to study. Although brute force algebra can yield a useful factorization of $\prod_{k=1}^{t} \Lambda_{k-1}$, I instead take a more intuitive approach to making the model more tractable.

The investor’s model-selection update rule can be reformulated by considering the problem faced at date-$t$ assuming that she has her date-$\emptyset$ prior distribution. After all, there is no path dependency in the determination of $\omega_t$; clearly an investor that observes a sequence of $\psi_t$ in-turn will arrive at the same value of $\omega_t$ as an investor that observes all of this data at once. In fact, the average value of the news realizations is a sufficient statistic for the sequence of realizations. Recall the assumption that the news $\Psi_t = z + \varepsilon_t$ where $\varepsilon_t \sim i.i.d. \mathcal{N}(0, v_{\varepsilon})$ and $\varepsilon_t$ is independent of $z$ for all $t$. Observing the cumulative average of the news realizations for the first time at date-$t$ simply results in a single dose of more precise news.

In this case the investor observes

$$\frac{1}{t} \sum_{s=1}^{t} \Psi_s = z + \left(\frac{1}{t} \sum_{s=1}^{t} \varepsilon_s\right)$$

where

$$\frac{1}{t} \sum_{s=1}^{t} \varepsilon_s \sim \mathcal{N}\left(0, \frac{v_{\varepsilon}}{t}\right)$$

I define $\Theta_t$ to be the random variable obtained by taking the cumulative time-average of the publicly available news up to and including date-$t$.

$$\Theta_t = \frac{1}{t} \sum_{s=1}^{t} \Psi_s$$

Let $\theta_t$ denote the realized value of $\Theta_t$. Since $\Psi_t = z + \varepsilon_t$ we have that $\Theta_t = z + \frac{1}{t} \sum_{s=1}^{t} \varepsilon_s$.

Now, consider the investor’s sequential updating problem through the lens of the “one-shot” updating problem at date-$t$ starting from date-$\emptyset$ information.

If the signal is true then $\Psi_t \sim \mathcal{N}(G, \frac{v}{t})$ which yields the following prior density for $\theta_t$
\[
\tilde{L}_t(\theta_t | \text{True}) = \frac{1}{\sqrt{2\pi} \left( \frac{v_z}{t} \right)} \exp \left( -\frac{(\theta_t - G)^2}{2 \left( \frac{v_z}{t} \right)} \right)
\]

If on the other hand, \( G \) is irrelevant, then \( \Psi_t \sim \mathcal{N}(\mu_z, \frac{v_z}{t} + v_z) \) which yields the following prior density for \( \theta_t \):

\[
\tilde{L}_t(\theta_t | \text{False}) = \frac{1}{\sqrt{2\pi} \left( \frac{v_z}{t} + v_z \right)} \exp \left( -\frac{(\theta_t - \mu_z)^2}{2 \left( \frac{v_z}{t} + v_z \right)} \right)
\]

This alternate construction yields a likelihood ratio \( \tilde{\Lambda}_t \) (anchored to \( \omega_0 \)) that is given by,

\[
\tilde{\Lambda}_t = \sqrt{1 + t \left( \frac{v_z}{v_e} \right) \exp \left( \frac{-v_z t \left( \theta_t - \left( \frac{v_z (G - \mu_z)}{v_z t} + G \right)^2}{2 v_e \left( v_e + v_z t \right)/t + (\mu_z - G)^2} \right)}
\]

I use the tilde \( \tilde{\cdot} \) above the terms \( \tilde{L}_t \) and \( \tilde{\Lambda}_t \) to distinguish between this construction, based on \( \Theta_t \), and the original updating construction based on \( \Psi_t \).

The final belief updating equation, anchored at \( \omega_0 \) is given by,

\[
\omega_t = \frac{\tilde{\Lambda}_t \omega_0}{\tilde{\Lambda}_t \omega_0 + (1 - \omega_0)} \tag{3.8}
\]

The values of \( \omega_t \) dictated by equation 3.6 are identical to those obtained by using equation 3.6.

### 3.4 An Equilibrium Pricing Framework

In this section, I embed my multi-period information environment in an equilibrium asset pricing framework. In its entirety my modeling approach consists of two key ingredients: a model of how investor beliefs evolve, and a theory of investor preferences. The former was developed in section 3.3 and I now turn to discussion of the latter.

I begin by first describing assets and investor preferences. I assume there are two assets, a risky stock and a risk-free asset. The stock is available in fixed supply which is normalized to unity and the risk-free asset is available in perfectly elastic supply with a return normalized
to zero.

The fundamental value $V$ of the stock is wholly determined by the random variable denoted by $z$.

$$ V = z $$

The value of $z$ is drawn at date-0, before the model begins, and is distributed normally with mean $\mu_z$ and variance $v_z$. That is, when evaluating the risky asset, the investor cares about only about a single stochastic quantity $z$, and that quantity has a payoff distribution that is the prior distribution described in section 3.3.

The market is comprised of a unit mass of individuals with constant absolute risk aversion (CARA) utility over the value of final wealth. This classic pairing of CARA utility with normally distributed returns has become the canonical model in the behavioral finance literature. It’s popularity is driven largely because it results in tractable demands and prices as each investor maximizes mean-variance utility over final wealth given by

$$ U(W_{t+1}) = \mathbb{E}_t[W_{t+1}] - \frac{\gamma^2}{2} \mathbb{V}_t[W_{t+1}] $$

(3.9)

Thus far, these asset-pricing assumptions are completely orthodox.

### 3.4.1 Risk & Return with CARA Normal Mixtures

As discussed in section 3.3, an investor observes a price-target $G$ that she incorrectly initially believes is informative about the value of the stock. I study the expected price dynamics that result as investors gradually realize that the price-target lacks predictive power. For simplicity I begin this section by focusing on the mixture distribution that the investor has at date-0 before she has observed any news however no generality is lost since at time $t$, the analogues of $\mu_z$ and $v_z$ are $\mathbb{E}_{t-1}[z]$ and $\mathbb{V}_{t-1}[z]$, respectively.

From the investor’s perspective, the distribution of $z$ is not normal but is instead a mixture of normals. This mixture distribution is comprised of a normal $\mathcal{N}(\mu_z, v_z)$ (the prior) and a
point-mass $\mathcal{N}(G, 0)$, where each has weights $1 - \omega_0$ and $\omega_0$ respectively. That is,

$$V = \begin{cases} 
G & \omega_0 \\
\mathcal{N}(\mu_z, v_z) & 1 - \omega_0 
\end{cases}$$

Unlike with normal distributions, when an asset-payoff distribution of this form is combined with CARA preferences it does not imply that investors maximize mean-variance utility and care only about the distribution’s first two moments. Instead investors maximize a more involved functional form for utility. This is explored below.

**Definition 3.1.** For a random variable, $X$, the moment-generating function $MGF(t)$ and cumulant-generating function $CGF(t)$ are defined by

$$MGF(t) = \mathbb{E}[\exp(tX)]$$

$$CGF(t) = \log(MGF(t))$$

$\forall t$ such that $\mathbb{E}[\exp(tX)] < \infty$.

Given the setup described above, the general problem of maximizing the expected value of terminal wealth can be written down as $\max_x \mathbb{E}[U(W_{t+1})]$ or $\mathbb{E}[U(W_t + x\varphi)]$ where $\varphi$ is the stochastic excess return generated by investing in the risky asset. CARA utility is given by $U(W_{t+1}) = -e^{-\gamma W_{t+1}}$ and as shown by Davila (2011), the objective function of an investor with CARA utility can be written as $\min_x CGF_x(-\gamma x)$ since,

$$\max_x \mathbb{E}[-e^{-\gamma W_{t+1}}] \iff \min_x \log \mathbb{E}[e^{-\gamma(W_t + x\theta)}]$$

$$\iff \min_x \log \mathbb{E}[-e^{-\gamma x\theta}]$$

$$\iff \min_x CGF(-\gamma x)$$

where $x$ denotes the number of shares of the risky asset demanded.

With these results in hand I now consider the $MGF(t)$ of this asset’s payoff distribution. Since the asset’s payoff distribution is a mixture, it follows that probability density function is a linear combination of functions.
It follows that,

\[
MGF(t) = \mathbb{E}[\exp(tz)] = \omega_0 \mathbb{E}[\exp(tG)] + (1 - \omega_0) \mathbb{E}[\exp(tN(\mu_z, v_z))]
\]

Intuitively, the moment-generating function is obtained by evaluating the expectation of

a point mass in proportion \( \omega_0 \)

\[
\omega_0 \mathbb{E}[\exp(tG)] = \omega_0 e^{tG}
\]

and a log-normal distribution in proportion \( 1 - \omega_0 \)

\[
(1 - \omega_0) \mathbb{E}[\exp(t \cdot N(\mu_z, v_z))] = (1 - \omega_0) e^{\mu_z t + \frac{1}{2} v_z t^2}
\]

Combining these terms yields

\[
MGF(t) = (1 - \omega_0) e^{\mu_z t + \frac{1}{2} v_z t^2} + \omega_0 e^{tG}
\]

From here the equilibrium price is obtained by evaluating the logarithm of the above
expression at \( t = -\gamma x \) and then maximizing with respect to the number of shares \( x \).

To start, set aside the delineation of the asset’s price from its expected payoff. The
investor minimizes the following quantity

\[
CGF(\gamma x) = \log (\mathbb{E}[\exp(-\gamma x X)])
\]

Substituting in the expression for the \( MGF(t) \) derived above yields an optimization prob-
lem that is given by,

\[
\min_x \log \left[ (1 - \omega_0) e^{-\mu_z \gamma x + \frac{1}{2} v_z x^2} + \omega_0 e^{-G \gamma x} \right]
\] (3.10)
Now, let the price of the asset be denoted by \( P_0 \). Imposition of the price is simply a mean-shift of the asset’s payoff distribution by \(-P_0\) and the optimization problem becomes,

\[
\min_x \log \left( (1 - \omega_0) e^{-(\mu_z - P_0)\gamma x} + \frac{\gamma^2 v_z x^2}{2} + \omega_0 e^{-(G - P_0)\gamma x} \right) 
\]

(3.11)

The first order condition is obtained by simply differentiating the minimand and setting the result to zero. The first order condition is given by

\[
\left( -\gamma (\mu_z - P_0) + \gamma^2 v_z x \right) (1 - \omega_0) e^{-(\mu_z - P_0)\gamma x} + \frac{\gamma^2 v_z x^2}{2} - (G - P_0) \gamma \omega_0 e^{-(G - P_0)\gamma x} = 0
\]

(3.12)

Equilibrium demand is obtained by re-arranging equation 3.12 and isolating terms involving \( x \). It follows that equilibrium demand is given by

\[
x = \frac{\mu_z - P_0}{\gamma v_z} + \frac{G - P_0}{\gamma v_z} \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\left[ \frac{\gamma^2 v_z}{2} \left( x - \left( \frac{\mu_z - G}{\gamma v_z} \right) \right)^2 - \frac{(\mu_z - G)^2}{2v_z} \right]} 
\]

(3.13)

Note that, unlike in the usual CARA normal case, this expression is non-linear since the number of shares demanded appears on both sides of this expression.

**What does this expression for equilibrium demand imply?**

This equation makes intuitive sense. First, observe that since \( 0 < \omega_0 < 1 \) we have

\[
\frac{\omega_0}{1 - \omega_0} \in [0, \infty]
\]

The term

\[
e^{-\left[ \frac{\gamma^2 v_z}{2} \left( x - \left( \frac{\mu_z - G}{\gamma v_z} \right) \right)^2 - \frac{(\mu_z - G)^2}{2v_z} \right]}
\]

may also be written as

\[
e^{-\left[ (G - \mu_z)\gamma x + \frac{\gamma^2 x^2 v_z}{2} \right]}
\]
This makes clear the fact that if $G > \mu_z$, then,

$$e^{-\frac{x^2}{2v_z}(x-(\frac{\mu_z-G}{\gamma v_z}))^2 - \frac{(\mu_z-G)^2}{2v_z}} = e^{-\left((G-\mu_z)\gamma x + \frac{\gamma^2 x^2 v_z}{2}\right)} \in [0, 1]$$

since

$$(G - \mu_z) \gamma x + \frac{\gamma^2 x^2 v_z}{2} > 0$$

This term can be thought of as an inflation factor relative to the CARA normal case.

As certainty in the price-target $\omega_0 \to 1$, demand $x \to \infty$ since the asset becomes risk-less and has a positive payoff of $G$. As certainty in the price-target $\omega_0 \to 0$ then the demand $x = \frac{\mu_z - P_0}{\gamma v_z}$ which is just the simple CARA normal case.

Assuming that $G = \mu_z$, the mean of the asset’s payoff distribution is $\mu_z$ and the variance is $(1 - \omega) v_z < v_z$. This yields,

$$x = \frac{\mu_z - P_0}{\gamma v_z} \left(1 + \left(\frac{\omega_0}{1 - \omega_0}\right) e^{-\frac{x^2}{2v_z}x^2}\right)$$

$$> \frac{\mu_z - P_0}{\gamma v_z}$$

Therefore this asset is strictly preferred to one whose payoffs are distributed normally with mean $\mu_z$ and variance $v_z$\textsuperscript{19}.

To better understand the influence that the price-target has on equilibrium demand, consider following example. Figure 3.3 plots demand for various values of the price-target while holding other parameters fixed. I assume that $P_0 = 50, \mu_z = 50, v_z = 100, \gamma = 0.001$. I plot values of $x$ (y-axis) for values of $G$ (x-axis) between 50 and 100. I repeat the exercise for $\omega_0 \in \{0.25, 0.5, 0.75\}$.

The central takeaway from figure 3.3 is that $x$ is non-monotonic and bounded in $G$. If the investors prior mean $\mu_z$ is kept fixed, then increasing $G$ does not boundlessly cause her demand to increase.

At high values of $G$ the investor demands what she would if she believed the asset pay

\textsuperscript{19}Intuitively, the tails of the mixture are thinner since it “places some extra mass” at the mean.
Figure 3.3: Effect of the Price-Target on Equilibrium Demand

Plot of the effect of the price-target on equilibrium demand. The parameters are $P_0 = 50$, $\mu_z = 50$, $v_z = 100$, $\gamma = 0.001$, $50 < G < 100$ and $\omega_0 \in \{0.25, 0.5, 0.75\}$.

off is distributed according to her prior distribution. That is, even though increasing $G$ unambiguously increases the investor’s expectation of the asset’s payoff, there is a point beyond which increases in $G$ cause the investor to demand less.

This non-monotonicity in $G$ occurs because at higher levels of $G$ the investor’s aversion to the even moments of the asset’s payoff distribution overwhelms her preference for increases in the mean and other odd moments. To see this intuition, consider the fact that the variance of the investor’s mixture distribution is given by

$$(1 - \omega_0) \left( v_z + (G - \mu_z)^2 \omega_0 \right)$$

and its mean is given by

$$(1 - \omega_0) \mu_z + \omega_0 G$$

For any linear combination of the two, having a negative weight on the variance and a positive weight on the mean, there exists a value of $G$ (or $\mu_z$) beyond which the linear
combination becomes negative as long as either $G$ or $\mu_z$ is kept fixed. Higher moments can be paired off in successive odd-even pairs to generalize this intuition. If $G$ and $\mu_z$ are increased simultaneously then the investor’s demand increases monotonically.

The equilibrium price is considered next. Since the supply of the risky asset is fixed at unity, the equilibrium price is obtained through market clearing by setting $x = 1$ and solving equation 3.13 for $P_0$. This yields,

$$P_0 = \frac{(\mu_z - \gamma v_z)(1 - \omega_0) + G \omega_0 e^{-\frac{\gamma^2 v_z}{2}(1 - (\frac{\mu_z - G}{\gamma v_z}))^2 - \frac{(\mu_z - G)^2}{2 v_z}}}{(1 - \omega_0) + \omega_0 e^{-\frac{\gamma^2 v_z}{2}(1 - (\frac{\mu_z - G}{\gamma v_z}))^2 - \frac{(\mu_z - G)^2}{2 v_z}}}$$

(3.14)

What does this expression for the equilibrium price imply?

It is worth pausing to comment on a few features of the expression for the equilibrium price. The price combines the price-target with the price if investors had CARA normal preferences based on her prior distribution. The weights on each component are dependent on both $\mu_z$ and $G$. This price is larger that the CARA normal case if the price-target is above it.

As the belief in the price target $\omega_0 \to 0$ it follows that

$$P_0 \to \mu_z - \gamma v_z$$

since the asset’s payoff distribution is again a simple Gaussian.

To better understand the influence that the price-target has on equilibrium prices, consider following example. Figure 3.4 plots the price for various values of the price-target while holding other parameters fixed. I again assume that $\mu_z = 50, v_z = 100, \gamma = 0.001$.

Because the equilibrium supply of the asset is normalized to 1, extreme values of $G$ must be plotted in order to illustrate its effect. I plot values of $P_0$ (y-axis) for values of $G$ (x-axis) between 50 and 5000. I repeat the exercise for $\omega_0 \in \{0.25, 0.5, 0.75\}$. Again, we see here that the investor’s aversion for even moments overwhelms when $\mu_z$ is kept fixed and $G$ is large.
Figure 3.4: Effect of the Price-Target on Equilibrium Prices

Plot of the effect of the price-target on equilibrium demand. The parameters are $\mu_z = 50, v_z = 100, \gamma = 0.001, 50 < G < 500$ and $\omega_0 \in \{0.25, 0.5, 0.75\}$

3.4.2 Price Dynamics

*What happens to prices as investors consume news?*

Now, equation 3.14 specifies the price at date-0 before any news is released. It is directly extended to determining the equilibrium price in future time periods by setting $\mu_z = \mathbb{E}_t [z]$ and $v_z = \mathbb{V}_t [z]$.

This yields,

$$P_t = \frac{(\mathbb{E}_t [z] - \gamma \mathbb{V}_t [z]) (1 - \omega_t) + G \omega_t e^{-\frac{1}{2} \gamma^2 \mathbb{V}_t [z] \left( 1 - \left( \frac{\mathbb{E}_t [z] - G}{\mathbb{V}_t [z]} \right)^2 - \frac{(\mathbb{E}_t [z] - G)^2}{2 \mathbb{V}_t [z]} \right)}}{(1 - \omega_t) + \omega_t e^{-\frac{1}{2} \gamma^2 \mathbb{V}_t [z] \left( 1 - \left( \frac{\mathbb{E}_t [z] - G}{\mathbb{V}_t [z]} \right)^2 - \frac{(\mathbb{E}_t [z] - G)^2}{2 \mathbb{V}_t [z]} \right)}} \quad (3.15)$$

To make notation more concise I write $P_t$ as

$$P_t = \frac{(1 - \omega_t) \pi_t + \omega_t \lambda_t G}{(1 - \omega_t) + \omega_t \lambda_t} \quad (3.16)$$

where
\[ \pi_t = E_t[z] - \gamma V_t[z] \]

and

\[
\lambda_t = e^{-\frac{\gamma^2}{2} V_t[z] \left(1 - \left(\frac{E_t[z] - G}{\gamma V_t[z]}\right)^2 - \frac{(E_t[z] - G)^2}{2 V_t[z]}\right)}
\]

I refer to the term \( \pi_t \) as the risk adjusted payoff and I refer to \( \lambda_t \) as simply a price inflation factor.

### 3.4.3 Absence of the Price-Target

Now that equilibrium pricing has been specified, I provide some intuition about the types of predictions made by the model that hinge on the presence of price-targets.

Since \( \omega_t = \frac{\tilde{\Lambda}_0 \omega_0}{\Lambda_t \omega_0 + (1 - \omega_0)} \) then \( \omega_0 = 0 \) implies that \( \omega_t = 0 \forall t \geq 1 \). Therefore, the influence of the price-target can be removed from the model by setting \( \omega_0 = 0 \). Under these conditions

\[ P_t = \pi_t = E_t[z] - \gamma V_t[z] \]

Since as shown in section 3.3

\[
E_t[z] = \frac{\mu_z}{p_t} \frac{v_z}{T} + \frac{1}{T} \left(\sum_{s=1}^{T} \psi_s \right) v_z
\]

\[
V_t[z] = \frac{v_z}{\frac{p_t}{T} + v_z}
\]

Direct substitution yields,
\[ P_t = \mu_z \frac{v}{t} + v_z + \left( \frac{1}{t} \sum_{s=1}^{t} \psi_s \right) \frac{v_z}{v} + v_z - \gamma \frac{v_z}{t} v_z \]

\[ = (\mu_z - \gamma v_z) \frac{v_z}{v} + v_z + \left( \frac{1}{t} \sum_{s=1}^{t} \psi_s \right) \frac{v_z}{v} + v_z \]

Therefore, in the absence of the price-target, the investor simply aggregates observed news \( \frac{1}{t} \sum_{s=1}^{t} \psi_s \) and combines it with the mean-variance \( \mu_z - \gamma v_z \) valuation driven by her prior.

**How do prices behave on average?**

In order study the behavior of \( P_t \) - on average - one needs to take expectations over news realizations. Only some terms are stochastic,

\[ P_t = \left( \mu_z - \gamma v_z \right) \frac{v_z}{v} + v_z + \left( \frac{1}{t} \sum_{s=1}^{t} \psi_s \right) \frac{v_z}{v} + v_z \]

Now since

\[ \frac{1}{t} \sum_{s=1}^{t} \psi_s \sim \mathcal{N}(z, \frac{v_z}{t}) \]

Conditional on \( z \), taking expectations over news \( (\psi) \) yields

\[ \mathbb{E}[P] = (\mu_z - \gamma v_z) \frac{v_z}{v} + v_z + z \frac{v_z}{v} + v_z \]

Now consider what is observed unconditionally. I assume that \( z \sim \mathcal{N}(\mu_z, v_z) \). Taking expectations over the fundamental value \( z \) yields,

\[ \mathbb{E}^z \left[ \mathbb{E}[P] \right] = \mu_z - \gamma \frac{v_z v_z}{v_z + v_z t} \]

Prices are low initially since this is when uncertainty is largest regarding the asset’s payoff distribution. As news is consumed, uncertainty is resolved. Since investors are risk averse
Plot of the unconditional expected path of prices, $E^Z \left[ E^\psi [P_t] \right]$, for parameter values $\mu_z = v_z = \gamma = 1$ and $v_\epsilon \in \{0.5, 1.0 \ldots 4.5, 5.0\}$.

prices will increase. The effects of the various parameters can be directly interpreted. The investor’s prior mean $\mu_z$ is simply a vertical shift in $P_t$. Increasing risk aversion $\gamma$ causes initial prices to be lower but results in a more pronounced increase in prices as uncertainty is resolved. Increasing $v_z$ has two effects: in the numerator it influences the price in the same way as risk aversion and in the denominator it is like increasing the effect of time.

The interesting parameter in this context is $v_\epsilon$. The role played by $v_\epsilon$ in this formula is intuitive: decreasing $v_\epsilon$ means that news is more precise, this causes prices to increase more rapidly once news is observed but unlike the other parameters it does not affect the initial price. To provide some intuition figure 3.5 illustrates the time-series dynamics of $P_t$ for a set of example parameters ($\mu_z = v_z = \gamma = 1$) and for a grid of $v_\epsilon \in \{0.5, 1.0 \ldots 4.5, 5.0\}$ equally spaced between 0.5 and 5 in 0.5 increments. That is, each line represents a different value of $v_\epsilon$ but all specifications have $\mu_z = v_z = \gamma = 1$.

Figure 3.5 and equation 3.17 both highlight the simple fact that in the absence of the price-target the equilibrium price never overshoots its long-run value $\mu_z$. Stated differently, in the absence of the price-target, the model never generates any negative serial correlation.
in returns at any horizon.
### 3.5 Expected Returns

Sections 3.3 and 3.4 develop a model that links news, investor beliefs and equilibrium prices. The question asked in this section is: What types of price regularities, if any, does the model embed? Or put differently, should an econometrician studying a market governed by this model expect to observe any return predictability?

In order to answer these questions I examine the behavior of the model’s equilibrium price, in expectation. I begin with a numerical example. To start, recall that, as a function of observed news, the equilibrium market price is given by,

\[ P_t = \frac{(1 - \omega_t) \pi_t + \omega_t \lambda_t G}{(1 - \omega_t) + \omega_t \lambda} \]

For simplicity, I restrict my attention to what occurs at date-1 when the investor observes news \( \psi_1 \). I begin with a simple numerical example to build intuition about how the investor’s estimates about volatility impacts prices. I assume that the price target is false and that \( G = 60, \mu_z = 50, \gamma = 0.06, v_z = 175, v_\varepsilon = 300 \) and \( \omega_0 = 0.7 \). These values are summarized in table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>60</td>
</tr>
<tr>
<td>( \mu_z )</td>
<td>50</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.06</td>
</tr>
<tr>
<td>( v_z )</td>
<td>175</td>
</tr>
<tr>
<td>( v_\varepsilon )</td>
<td>300</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 3.1: An Example of date-1 Equilibrium Prices, Parameters

First, I consider date-0. Under these assumptions we have that,

\[ \lambda_0 = e^{-\left[ (60-50)0.06 + \frac{(0.06)^2}{2} \right]} = 0.226 \]
\[ \pi_0 = 50 - 0.06 \cdot 175 = 39.5 \]
which implies

\[
P_0 = \frac{(1 - 0.7) \cdot 39.5 + 0.7 \cdot 0.226 \cdot 60}{(1 - 0.7) + 0.7 \cdot 0.226} = 45.93
\]

Next, consider prices at date-1. Figure 3.6 plots \( P_1 \) as a function of \( \psi_1 \) and the model parameters.

![Figure 3.6: Date-1 Prices as a Function of News](image)

This figure plots date-1 price as a function of news. Here I assume that \( G = 60, \mu_z = 50, \gamma = 0.06, v_z = 175, v_\epsilon = 300 \) and \( \omega_0 = 0.7 \).

We see that \( P_1 \) is monotonically increasing in \( \psi_1 \), which makes intuitive sense. We also see from figure 3.6 that if \( \psi_1 = \mu_z \), then \( P_1 > \mu_z \). In words, if the investor observes news that is exactly equal to the mean of her prior distribution, then the equilibrium price increases. This occurs for two reasons. The first is reason is that when the investor observes \( \psi_1 \) this resolves some uncertainty about the asset’s payoffs. As such, the asset becomes less risky from the investor’s perspective. The second and more important reason is that the investor actually increases her overall belief that the price target is true. Specifically, we have that

\[
\Lambda_0 = 1.065 > 1
\]
which implies that

\[
\omega_1 = \frac{1.065 \cdot 0.7}{1.065 \cdot 0.7 + 1 - 0.7} = 0.7133
\]

\[> 0.7\]

\[= \omega_0\]

In this case, both the reduction in uncertainty and the increased belief in the price target, tend to increase the price at date-1. In order to transparently see how these two forces influence the equilibrium price change between date-0 and date-1, I examine a single expression for the asset’s return.

Define price returns as

\[R_t = P_t - P_{t-1}\]

It can be shown that this expression for returns is given by \(^{20}\),

\[
R_t = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left[ \Delta \pi_t + (G - \pi_{t-1}) (\lambda_t \Lambda_{t-1} - \lambda_{t-1}) \frac{\omega_{t-1}}{1 - \omega_{t-1} (1 - \lambda_{t-1})} \right]
\]

(3.18)

where

\[\Delta \pi_t = \pi_t - \pi_{t-1}\]

and

\[\Delta \lambda_t = \lambda_t - \lambda_{t-1}\]

Setting \(t = 1\), equation 3.18 reduces to \(^{20}\) See appendix for proof. The term involving \(\Lambda_{t-1}\) appears in this equation by making use of \(\omega_{t+1} = \frac{\Lambda_t \omega_t}{\Lambda_t \omega_t + (1-\omega_t)} \iff \omega_{t+1} = \omega_{t-1} (\Lambda_{t-1} - 1)\)
\[ R_1 = \frac{1 - \omega_1}{1 - \omega_1 (1 - \lambda_1)} \left[ \Delta \pi_1 \right. \left. + (\lambda_1 \Lambda_0 - \lambda_0) \frac{(G - \pi_0) \omega_0}{1 - \omega_0 (1 - \lambda_0)} \right] \]

In equation 3.19 the term \( \Delta \pi_1 \) is given by

\[ \Delta \pi_1 = (\mathbb{E}_1 [z] - \gamma \mathbb{V}_1 [z]) - (\mathbb{E}_0 [z] - \gamma \mathbb{V}_0 [z]) \]

and captures changes in the asset’s risk-adjusted payoff, from the investor’s perspective, \textit{conditional on the price-target being false}. Looking at the second term, I note that

\[ \frac{(G - \pi_0) \omega_0}{1 - \omega_0 (1 - \lambda_0)} > 0 \]

Special attention should be paid to the expression \((\lambda_1 \Lambda_0 - \lambda_0)\), which reflect the degree to which the investor increases, \(\omega_0\), her belief that the price-target is true. I will also show later that in general \(\lambda_1 \approx \lambda_0\) so the sign of the term \((\lambda_1 \Lambda_0 - \lambda_0)\) literally reduces to the sign of \((\Lambda_0 - 1)\). To the extent that the price-target \(G\) exceeds the investor’s ex-ante risk-adjusted payoff \(\pi_0\), the price of the asset will definitely increase if \((\Lambda_0 - 1) > 0\). This is captured by the term \((G - \pi_0)\) in equation 3.19.

In this numerical example, the values of these two terms are

\[ \Delta \pi_1 \approx 3.86 \]

\[ (\lambda_1 \Lambda_0 - \lambda_0) \frac{(G - \pi_0) \omega_0}{1 - \omega_0 (1 - \lambda_0)} \approx 8 \]

Thus, in this example, when \(\psi_1 = \mu_z\) the price change is indeed driven by an increase in beliefs in the price-target.

Next, I consider expectation of the price at date-1. Since the conditional distribution of \(\psi_1\) is
\[ \psi_1 \sim \mathcal{N}(z, \varepsilon) \]

and the distribution of fundamentals is

\[ z \sim \mathcal{N}(\mu_z, \nu_z) \]

it follows that the unconditional distribution of \( \psi_1 \) is given by

\[ \psi_1 \sim \mathcal{N}(\mu_z, \nu_\varepsilon + \nu_z) \]

The unconditional distribution of \( \psi_1 \) is symmetric and has a mean of \( \mu_z \). And since \( P_1 \) is monotonically increasing in \( \psi_1 \) and \( P_1 (\psi = \mu_z) > \mu_z \) it follows that in this example for more than a half of the \( \psi_1 \) drawn an increase in \( P_1 \) will be observed. To further build intuition, in figure 3.7, I re-plot figure 3.6 and juxtapose the values of \( P_1 \) with the probability density of \( \psi_1 \) plotted on the right-hand axis.

**Figure 3.7: Probability of Observing News and Prices**

This figure plots date-1 price as a function of news. Here I assume that \( G = 60, \mu_z = 50, \gamma = 0.06, \nu_z = 175, \nu_\varepsilon = 300 \) and \( \omega_0 = 0.7 \).

Now consider the numerical values from the example. When the distribution of \( \psi_1 \) is
\[ \mathcal{N}(50, 175 + 300), \] as assumed, numerical integration shows that the expectation of the date-1 price is given by,

\[ \mathbb{E}_1^\psi [P_1] \approx 49.64 \]

Let \( v_{Total} = v_\varepsilon + v_z \) denote the total unconditional variance of \( \psi_1 \). Looking at figure 3.7, it is clear that a decrease in \( v_{Total} \) corresponds graphically to an increase in the “peakedness” of the news distribution around \( \mu_z \). Since \( P_1 > \mu_z \) at \( \psi_1 = \mu_z \), there is a (small) value of \( v_{Total} > 0 \) such that under that assumption of total variance, the unconditional expectation of \( P_1 \) is greater than \( \mu_z \).

As an example, suppose the actual distribution of the news shocks has a variance \( v_\varepsilon|True \) that is a quarter of what the investor believes (\( v_\varepsilon|True = \frac{v_\varepsilon}{6} = 50 \)). This yields,

\[ \mathbb{E}_1^{\psi(v_\varepsilon|True)} [P_1] \approx 50.72 > \mu_z \]

The price is above the investor’s prior mean.

Now consider how the expected price changes in later periods. As the investor consumes an arbitrarily large amount of news, over many time periods, \( \mathcal{V}_t[z] \to 0 \) and \( \mathbb{E}_t[z] \to \mu_z \) therefore \( \pi_t \to \mu_z \). If \( G \) is assumed to be false, then \( \omega_t \to 0 \) and taken together this means that \( P_t \to \mu_z \). In this particular example, \( \mu_z = 50 \). This implies, that this example, where \( v_\varepsilon|True = 50 \) and \( v_z = 300 \), prices unconditionally overshoot their long-term value, in expectation. After initially overshooting, prices revert toward \( \mu_z \) as additional information is observed. The effect on price changes caused by the investor’s beliefs about the variance of the news, compared to reality, is made transparent in this example.

Finally, if the investor’s assumption about \( v_\varepsilon \) is then shifted from 300 to be 50 (now in line with reality) and expectations are taken where \( v_\varepsilon|True = v_\varepsilon = 50 \), then the expectation of \( P_1 \) would go back down to \( \approx 49.65 \). In the next section I discuss the precise conditions that lead to expected overshooting in prices.
3.5.1 General Return Dynamics

I now provide some general insight into the conditions under which the documented overshooting effect is most pronounced. I illustrate that if investors believe that the unconditional variance of the news \((v_z + v_z)\) is larger than it is in actuality, then prices will generally overshoot their long term values. To start, I continue to focus on date-1 and to assume that \(G > \mu_z\).

Now, the price inflation factor \(\lambda_t\) plays a minor role - recall that it is given by

\[
\lambda_t = e^{-\left[(G - E_t[z])\gamma + \frac{\gamma^2 v_t[z]}{2}\right]}
\]

and so when investors are near risk neutral the value of this term is always approximately 1. For convenience I make one simplifying assumption, that the price inflation factor \(\lambda_1, \lambda_0 \approx 1\) since, as I will later explain, has little effect on time series dynamics.

Next, I rewrite equation 3.23 as

\[
E_1^\psi [P_1 - \mu_z] = E_1^\psi [\omega_1] (G - \mu_z) + E_1^\psi [(1 - \omega_1) (\pi_1 - \mu_z)] \quad (3.20)
\]

The first term, \(E_1^\psi [\omega_1] (G - \mu_z)\), is unambiguously positive. Next, if the true variance of the news distribution (that we are taking expectation over) tends toward zero then we have

\[
E_1^\psi [(1 - \omega_1) (\pi_1 - \mu_z)] = \left[(1 - \omega_1) (\pi_1 - \mu_z)\right]_{\psi = \mu_z}
\]

\[
= -(1 - \omega_1 (\psi = \mu_z)) \frac{\gamma v_z v_z}{v_z + v_z}
\]

which is small if investors are near risk neutral (i.e.: \(\gamma\) small).

Therefore,

\[
E_1^\psi [P_1 - \mu_z] \geq 0
\]
when investors overestimate the variance of the news distribution.

Next, I establish that this overshooting effect will still be observed event if the price-target is only correct a proportion \(1 - \omega_0\) of the time. Conditional on a date-0 draw of \(z\), the distribution of \(\psi_1 \sim \mathcal{N}(z, v_z)\). Now, suppose some value of \(z\) is drawn. Conditionally, the long term price is ultimately \(z\). If the price target is correct, then \(G = z\), and so the difference between the date-1 price and its long term value is

\[
\mathbb{E}_1^\psi [P_1 - z \mid z] = \underbrace{\mathbb{E}_1^\psi [\omega_1 (z - z) \mid z]}_{=0} + \mathbb{E}_1^\psi [(1 - \omega_1) (\pi_1 - \mu_z) \mid z] \tag{3.21}
\]

If the price target is not correct, then the difference between the date-1 price and its long term value is

\[
\mathbb{E}_1^\psi [P_1 - z \mid z] = \mathbb{E}_1^\psi [\omega_1 (G - z) \mid z] + \mathbb{E}_1^\psi [(1 - \omega_1) (\pi_1 - \mu_z) \mid z] \tag{3.22}
\]

I continue to assume, without loss of generality, that when the price-target is incorrect that \(G > \mu_z\) and that \(\gamma\) is small. Taking the two cases in equations 3.21 and 3.22, in proportions \(\omega_0\) and \((1 - \omega_0)\) respectively, and substituting in for \(\pi_1\), yields a combined expectation of

\[
(1 - \omega_0) \mathbb{E}_1^\psi [\omega_1 (G - z) \mid z] + \mathbb{E}_1^\psi \left[ (1 - \omega_1) \frac{v_z}{v_z + v_z} (z - \mu_z - \gamma v_z) \right] \mid z \right]
\]

This combined expectation is a function of the realized draw of \(z\). The extend of price overshooting is described by taking the expectation of this expression over \(z \sim \mathcal{N}(\mu_z, v_z)\). If the final unconditional variance of \(\psi\) is unexpectedly small in actuality, then this expectation approaches the case where \(\psi = \mu_z\). And with \(G > \mu_z\) this expectation is positive. Therefore, when the actual unconditional variance of \(\psi\) is sufficiently small, prices overshoot in aggregate.
3.5.2 A Heuristic for Return Predictability

A useful heuristic for the intensity of overshooting is the investor’s expected increase in $\omega_t$.

To see this intuition observe that the expectation of the price minus its long term value can also be written as

$$E_1^\psi [P_1 - \mu_z] = E_1^\psi [\omega_1 (G - \pi_1)] - \gamma \frac{v_\epsilon v_z}{v_\epsilon + v_z}$$  (3.23)

Next, observe that $(G - \pi_1) \geq (G - \pi_0) \geq (G - \mu_z) \geq 0$. Therefore, we see from equation 3.23 that the sign of $E_1^\psi [P_1 - \mu_z]$ is influenced by investor’s expected change in beliefs, $E_1^\psi [\omega_1]$.

Next, I examine the factors influencing the expected change in the investor’s belief that price-target is true.

In the discourse that follows I drive a wedge between what investors believe about news uncertainty and what news uncertainty actually is. Recall that investors are assumed to believe that the news shocks $\psi$ are unconditionally distributed normally with mean $\mu_z$ and variance $v_\epsilon + v_z$. I now consider what happens if the news shocks actually have variance$^{21}$ $v_\epsilon|\text{True} \neq v_\epsilon$. This assumption changes the variance assumed when taking expectations, not the variance assumed inside the investor’s likelihood function. Recall that $\omega_1 = \frac{\Lambda_0^\psi \omega_0}{\Lambda_0^\psi \omega_0 + (1 - \omega_0)}$, where $\Lambda_0$ depends on $\psi_1$.

Taking expectations over news shocks, it follows trivially that

$$E_1^\psi \left[ \frac{\omega_1}{\omega_0} \right] > 1 \iff E_1^\psi \left[ \frac{\Lambda_0}{\Lambda_0^\psi \omega_0 + (1 - \omega_0)} \right] > 1$$

Under what conditions is $E_1^\psi \left[ \frac{\omega_1}{\omega_0} \right] > 1$?

Since functions of the form $f(x) = 1/(ax+b)$ are convex, if the expression above is instead rewritten as

$$E_1^\psi \left[ \frac{1}{\omega_0 + (1 - \omega_0) \left[ \frac{1}{\Lambda_0^\psi} \right]} \right]$$

---

$^{21}$The “|True” subscript in $v_\epsilon|\text{True}$ is unrelated to the truth or falsity of the price-target $G$. It refers to the truth in that it represents the actual variance of the news shocks (whereas $v_\epsilon$ is the variance assumed by investors).
then Jensen’s Inequality implies that

$$E^\psi_1 \left[ \frac{\Lambda_0}{\Lambda_0 \omega_0 + (1 - \omega_0)} \right] > \frac{1}{\omega_0 + (1 - \omega_0) E^\psi_1 \left[ \frac{1}{\Lambda_0} \right]}$$

The lower bound on the right hand side exceeds 1 if

$$E^\psi_1 \left[ \frac{1}{\Lambda_0} \right] < 1$$

This expectation is given by\textsuperscript{22}

$$E^\psi_1 \left[ \frac{1}{\Lambda_0} \right] = \frac{v_\varepsilon}{v_\varepsilon^2 + v_z \left( v_\varepsilon - v_{\varepsilon|\text{True}} - v_z \right)} \exp \left( \frac{1}{2} \frac{(\mu_z - G)^2 \left( 2v_z + v_\varepsilon + v_{\varepsilon|\text{True}} \right)}{v_\varepsilon^2 + v_z \left( v_\varepsilon - v_{\varepsilon|\text{True}} - v_z \right)} \right)$$

From here it is clear that the conditions for $E^\psi_1 \left[ \frac{1}{\Lambda_0} \right] < 1$ are

$$\frac{(\mu_z - G)^2}{v_z} < \frac{v_\varepsilon^2 + v_z \left( v_\varepsilon - v_{\varepsilon|\text{True}} - v_z \right)}{2v_z + v_\varepsilon + v_{\varepsilon|\text{True}}} \log \left( \frac{v_\varepsilon^2 + v_z \left( v_\varepsilon - v_{\varepsilon|\text{True}} - v_z \right)}{v_\varepsilon} \right)$$

and $v_\varepsilon - v_{\varepsilon|\text{True}} > v_z$. In words, this means that if there is a sufficiently large gap between how noisy investors believe the news is and how noisy the news actually is, and the price-target is not too far from the prior mean, then $E^\psi_1 \left[ \frac{\omega_1}{\omega_0} \right] > 1$.

3.5.3 A Numerical Simulation

To get oriented with the phenomena that this model captures, consider the following illustrative numerical example. Consider how conditional price dynamics look when news is more precise than assumed.

\textsuperscript{22}See appendix for proof
Table 3.2: An Example of Equilibrium Price Dynamics, Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>2.25</td>
</tr>
<tr>
<td>$\mu_z$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.15</td>
</tr>
<tr>
<td>$v_z$</td>
<td>4.5</td>
</tr>
<tr>
<td>$v_\varepsilon$</td>
<td>5</td>
</tr>
<tr>
<td>$v_{\varepsilon</td>
<td>True}$</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Suppose the mean of the fundamental value $\mu_z$ and the value drawn $z$ are both equal to 1.5, and that the variance of the fundamental value $v_z$, is 4.5. Let the price-target $G$ be 2.25. The assumed variance of the news $v_\varepsilon$, is believed to be 5 but $v_{\varepsilon | True}$ is in fact 0.5 and investors initially believe that the price-target has a 40% chance of being true ($\omega_0 = 0.4$). Risk aversion is $\gamma$ is 0.15. Taken together, these assumptions result in an initial price of $\approx 0.9$. To simulate what happens on average, news streams $\psi_t = 1 + \varepsilon_t$ are repeatedly drawn according to $\varepsilon_t \sim N(0, 0.5)$ and the dynamics of the average price path is then studied. Given the parameters described above and in Table (3.2) the resulting expected price dynamics are depicted Figure (3.8).
Figure 3.8: An Example of Equilibrium Price Dynamics

We see that the path of the expected belief in the price-target ($\omega_t$) is non-monotonic. Values are set start out at 0.4 but increases to over 0.6 within the first 20 time periods. As more news is released, beliefs fall and $\omega_t$ eventually dips below 0.56 by about 50 time periods. The equilibrium price starts off below its long-term value $\mu_z$ because of risk aversion. The initial pronounced increase in $\omega_t$ causes $P_t$ to overshoot its long term value before finally decreasing towards it.

A crucial assumption driving the price-overshooting in this numerical example is the wide gap between $v_\varepsilon$ and $v_\varepsilon|\text{True}$ that was selected. In the next section I demonstrate that this dynamic is observable in stock prices. By mapping the parameters of the model to real world quantities I isolate portfolios that are more likely to exhibit these patterns of return predictability.
3.6 Empirical Results

In this section I test several empirical predictions of the model. The theoretical results outlined in the previous section suggest that the difference between realized return variation and the market’s expectation of return variation is central to understanding when return predictability is strongest. Specifically, my model dictates that the wedge between actual news uncertainty ($v_{\varepsilon|\text{True}}$) and what investors believe about news uncertainty ($v_{\varepsilon}$) is of central interest for studying return predictability.

Linking these ideas to reality, the parameters assumed by the investor in the model can be thought of as the views of the marginal price setter. In complete arbitrage-free markets a unique risk-neutral measure exists which equates asset prices to the discounted expected value of their future payoffs. Under this measure, the probabilities assigned to states of the world capture the risk preferences of the marginal price setter. To the extent that market participants believe that news variance is $v_{\varepsilon} \neq v_{\varepsilon|\text{True}}$ the magnitude of an asset’s volatility implied by the risk-neutral measure will reflect this belief. However, in reality market participants are subject to news shocks with their own properties, not what investors believe. Therefore identifying when realized volatility (physical-measure) diverges from the assumed level of volatility (risk-neutral measure) provides the ability to determine when and for which stocks the model’s predictions should hold.

Stock options are both highly liquid and have a well understood theory linking their prices to the volatilities of their underlying stocks. The difference between the expected risk-neutral variance and physical variance is often referred to as the variance risk premium. This essay relates to a strand of the literature focused on the extent to which option markets contain information that is distinct from what is reflected in stock prices. For example, Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) study the information content of the equity variance risk premium at an aggregate level. At the level of single stocks, Carr and Wu (2008) and Buraschi, Trojani and Vedolin (2014) document cross sectional variation. Of these, the one most closely related to this essay is Buraschi, Trojani and Vedolin (2014). In studying the cross-sectional and time-series variations in the variance
risk premia of individual stocks, they link the variance risk premium to belief disagreement among investors. In addition, they derive testable implications for this relationship. Their conceptualization of “belief disagreement” is closely connected to my notion of overconfidence. As such, equity options provide an ideal setting for extracting risk-neutral volatility estimates to study the model’s implications.

The theoretical results in section 3.5 illustrate that stocks for which the marginal investor’s wedge in beliefs is wide will have a tendency to exhibit short-term momentum and long-term reversion. In this sense the model dictates that momentum and reversion are driven by investors who do not update sufficiently to news. Even if investors initially have a good sense of how often price-targets are correct, they further increase their belief that it is true, in expectation.

This insight motivates my empirical strategy. I proxy for the gap in investor beliefs and in doing so identify a subset of stocks for which return predictability should be most robust. To test this prediction I use the spread between option-implied and realized volatility as a separating variable when constructing momentum portfolios. I show that portfolios where this spread is wider (narrower) produce relatively larger (smaller) factor-adjusted momentum alphas. Further, as predicted by the model, the larger alphas subsequently reverse more violently as prices revert to fundamentals.

Heuristically, both option-implied and realized volatility account for additional variation associated with an asset’s fundamental uncertainty \(v_{\varepsilon}\). However the difference between option-implied and realized volatility provide a clean link to the notion captured by \(v_{\varepsilon} - v_{\varepsilon|\text{True}}\) within the model.

I proxy for \(v_{\varepsilon} - v_{\varepsilon|\text{True}}\) using the spread between option-implied and realized volatility \((IV_t - RV_t)\). I am interested in isolating stocks that consistently rank among the highest in terms of \(IV_t - RV_t\). While there is evidence suggesting that stock volatility is predictable, views on how this predictability should be modeled is decidedly mixed (see Bollerslev, et al

\(^{23}RV_t\) is measured over the past 30 days and \(IV_t\) is computed by interpolation across calls and puts and over the first 4 standardized tenors (30,60,180,240 days). However, the final measure is completely unaffected by these choices.
While investors might be able to predict a stock’s volatility on over long horizons, it is plausible that over a horizon of several months they may have substantial forecasting errors. I design my measure to capture deviations a horizon of roughly 3 years. I create a simple (trailing) monthly measure that captures precisely this. Each day I sort the stocks in the cross section according to $IV_t - RV_t$ to obtain daily percentiles. I then compute the trailing average daily percentile over the past month for a stock$^{24}$. Wherever I refer to “$IV_t - RV_t$ percentile” I am referring to this trailing 1 month average.

The measure described above is likely to be correlated with a variety stock-specific characteristics that are unrelated to my theory. As such, I additionally control for firm size, book to market, and several other factors by performing monthly cross sectional regressions that orthogonalize the raw measure to these controls. I perform my main analysis of studying momentum portfolios using both the raw and the orthogonalized (residual) measure.

### 3.6.1 Data & Summary Statistics

Before proceeding further it is useful to review the data used to perform my empirical tests. I use data obtained from several sources. Stock returns and firm characteristics between January 1990 and December 2013 are obtained from the CRSP and COMPUSTAT database respectively. Sell-side analyst coverage is taken from the Institutional Brokers Estimates System (I/B/E/S). Option-implied volatility is obtained from Optionmetrics.

Table 3.3 presents summary statistics on the stocks contained in my sample. The primary bottleneck for data availability is the existence implied-volatility data for stocks in Optionmetrics. In Panel A I report the coverage of the firms in my data as a fraction of the universe of CRSP common stocks. On average the universe of stocks in this study comprises roughly 40% of the stocks traded on the NYSE, AMEX and NASDAQ between 2000 and 2012. Nevertheless, stocks that do have options are skewed towards larger firms. As such, my stock universe on average contains over 90% of the total market capitalization of common stocks traded on the NYSE, AMEX and NASDAQ.

$^{24}$The averaging over 3 years excludes the most recent month to mitigate any potential predictability that might arise from having very timely data. This does not affect my results.
This table presents summary statistics for stocks covered in my sample.

Panel B presents sample firm characteristics compared to all NYSE stocks. We see that the sample is not biased in any particular direction of the momentum universe but is skewed towards large cap glamor stocks.

3.6.2 Measuring Overestimation of News Uncertainty

I proceed by presenting empirical results that provide a sense of how $IV_t - RV_t$ correlates with several important stock characteristics. Table 3.4 reports coefficients from Fama–MacBeth regressions of raw $IV_t - RV_t$ percentiles on a set of firm-specific regressors.
Table 3.4: Measuring News Uncertainty Overestimation, Fama-Macbeth Regressions

<table>
<thead>
<tr>
<th>Dep Variable: $IV_t - RV_t$ percentile</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>BETA</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(-6.84)</td>
<td>(-6.81)</td>
<td>(-6.65)</td>
<td>(-7.44)</td>
<td>(-7.39)</td>
</tr>
<tr>
<td>BM</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(-54.79)</td>
<td>(-50.09)</td>
<td>(-52.01)</td>
<td>(-48.63)</td>
<td>(-49.37)</td>
</tr>
<tr>
<td>SIZE</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>(-94.92)</td>
<td>(-93.55)</td>
<td>(-94.59)</td>
<td>(-77.36)</td>
<td>(-78.66)</td>
</tr>
<tr>
<td>$RET_{t-1,t-6}$</td>
<td>-0.03</td>
<td>-0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-13.59)</td>
<td>(-12.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RET_{t-1}$</td>
<td></td>
<td>-0.01</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.27)</td>
<td>(-1.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RET_{t-2,t-12}$</td>
<td>-0.02</td>
<td>-0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-12.03)</td>
<td>(-11.70)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IDIOVOL</td>
<td>0.60</td>
<td>0.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(19.51)</td>
<td>(19.22)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TURN</td>
<td>-4.63</td>
<td>-3.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-14.25)</td>
<td>(-12.03)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NASD*TURN</td>
<td>4.49</td>
<td>4.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.50)</td>
<td>(10.80)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ANALYSTCOVER</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.05)</td>
<td>(5.31)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INTERCEPT</td>
<td>1.55</td>
<td>1.55</td>
<td>1.56</td>
<td>1.52</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>(136.21)</td>
<td>(134.26)</td>
<td>(135.28)</td>
<td>(111.99)</td>
<td>(113.38)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
<td>0.37</td>
<td>0.37</td>
</tr>
</tbody>
</table>

This table reports coefficients from Fama–MacBeth regressions of $IV_t - RV_t$ percentile on a set of firm-specific regressors.

In table 3.4 BETA is the coefficient on market excess return from monthly regressions of daily excess returns on the three Fama and French factors and the Carhart momentum factor. BM is the log of the book-to-market ratio which is the Compustat book value of equity divided by the market value of equity; BM is computed as of the June of the previous calendar year following Daniel, Grinblatt, Titman, and Wermers (1997). SIZE is the log of the firm’s market value of equity as of the previous calendar month. $RET_{t-1}$ and $RET_{t-1,t-6}$ are the past one and six month stock returns respectively. $RET_{t-2,t-12}$ is the prior-year stock return excluding the past month. IDIOVOL is the standard deviation of the residuals from
a monthly regression of daily excess returns on the three Fama and French factors and the Carhart momentum factor. TURN is the average turnover in the previous 12 months and NASD is a NASDAQ dummy. ANALYSTCOVER is the number of analysts that cover the stock as of the previous calendar month. Cross-sectional regressions are run every month. t-statistics are shown below the coefficient estimates. Specifications that include $RET_{t-1}$, $RET_{t-6}$ are set to exclude $RET_{t-1}$ and $RET_{t-2,t-12}$ since the latter 2 regressors roughly span the former.

Table 3.4 tells us that higher values of $IV_t - RV_t$ are generally associated with smaller, lower beta stocks that have lower book to market values. This is consistent with the literature on variance risk premia which highlights a tendency for variance swap strikes to exceed realized variance in more idiosyncratic and obscure firms.25

### 3.6.3 Baseline Momentum Portfolios

I begin by examining the performance of a canonical momentum strategy restricted to the dates and stocks for which I am able to compute $IV_t - RV_t$ percentiles. This will serve as a baseline for further results. For momentum, following Jegadeesh and Titman (2001) I use the simple measure of the past 6 month cumulative raw return on the asset.26

Table 3.5 presents factor adjusted alphas and annualized Sharpe ratios for rolling portfolios computed using 6 month returns. Specifically, at the beginning of every calendar month, stocks are ranked in ascending order on the basis of their previous 6 month’s cumulative raw returns. The return ranked stocks are assigned to 5 quintile portfolios (columns Q1 through Q5). Stocks are value weighted within a given portfolio, and the portfolios are rebalanced every calendar month to maintain value weights. The column L/S presents the results for going long the stocks in the 5th momentum quintile and selling short the stocks in the 1st momentum quintile.

To obtain these alphas regressions are performed where the dependent variable is the

---

25 The root cause is however generally attributed to risk based explanations.
26 Results are slightly weaker but qualitatively similar to those obtained using the 12-month cumulative raw return on the asset and skipping the most recent month’s return as in Jegadeesh and Titman (1993)
Table 3.5: Momentum Portfolios, Monthly Alphas 2000-2013

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Spread</td>
<td>107.05</td>
<td>94.50</td>
<td>90.08</td>
<td>88.66</td>
<td>94.78</td>
</tr>
<tr>
<td></td>
<td>(1.52)</td>
<td>(1.90)</td>
<td>(2.15)</td>
<td>(2.26)</td>
<td>(1.90)</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>71.23</td>
<td>68.08</td>
<td>67.66</td>
<td>67.84</td>
<td>72.08</td>
</tr>
<tr>
<td></td>
<td>(2.11)</td>
<td>(3.43)</td>
<td>(4.27)</td>
<td>(4.35)</td>
<td>(2.33)</td>
</tr>
<tr>
<td>Fama-French Alpha</td>
<td>52.82</td>
<td>43.24</td>
<td>38.58</td>
<td>34.21</td>
<td>30.28</td>
</tr>
<tr>
<td></td>
<td>(1.57)</td>
<td>(2.33)</td>
<td>(3.08)</td>
<td>(3.08)</td>
<td>(1.38)</td>
</tr>
<tr>
<td>Four Factor Alpha</td>
<td>63.26</td>
<td>47.98</td>
<td>40.06</td>
<td>32.87</td>
<td>25.79</td>
</tr>
<tr>
<td></td>
<td>(2.80)</td>
<td>(3.25)</td>
<td>(3.32)</td>
<td>(3.08)</td>
<td>(1.34)</td>
</tr>
<tr>
<td>Five Factor Alpha</td>
<td>53.24</td>
<td>38.00</td>
<td>32.16</td>
<td>26.88</td>
<td>26.38</td>
</tr>
<tr>
<td></td>
<td>(2.36)</td>
<td>(2.66)</td>
<td>(2.74)</td>
<td>(2.56)</td>
<td>(1.35)</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.41</td>
<td>0.51</td>
<td>0.58</td>
<td>0.61</td>
<td>0.51</td>
</tr>
</tbody>
</table>

This table reports annualized Sharpe ratios and factor adjusted momentum alphas for unconditional cross sectional sorts restricted to the test universe.

monthly excess return of the rolling strategy defined above. The explanatory variables are the monthly returns from the Fama and French (1993) mimicking portfolios, the Carhart momentum factor and the Pastor Stambaugh liquidity factor. The holding period for the rolling strategy is one month. Alphas are in monthly basis points, t-statistics are shown in parenthesis below the coefficient estimates.

Table 3.5 highlights the basic fact that the momentum “factor”, as defined above, performed poorly (raw spread of -12 basis points (t=-0.23)) during the sample window (2000-2013) for stocks that have traded options.

These momentum portfolios establish a baseline for test assets. I proceed to my primary analysis concerning news uncertainty overestimation which predicts that stocks for which $IV_t - RV_t$ ranks highest (lowest) in the cross section should exhibit more (less) pronounced momentum.

### 3.6.4 Conditional Momentum Portfolios

Table 3.6 reports monthly alphas for the main test assets. At the beginning of every calendar month stocks are first ranked according to their trailing 3 year average $IV_t - RV_t$ percentiles.
Subsequent to this pre-ranking, stocks in each $IV_t - RV_t$ quintiles are then independently ranked in ascending order on the basis of their most recent 6 month cumulative raw returns (momentum). The momentum ranked stocks are assigned to 5 quintile sub-portfolios. Panels A and B of table 3.6 consider each the 5 return-based quintile sub-portfolios for the highest (Q5) and lowest (Q1) quintiles by $IV_t - RV_t$. Portfolios are rebalanced every calendar month to maintain value weights.

The columns labeled L/S report the result of going long the stocks in the 5th momentum quintile (winners) and selling short the stocks in the 1st momentum quintile (losers).

Table 3.6: Effect of Raw News Uncertainty Overestimation on Momentum Alphas

<table>
<thead>
<tr>
<th>Panel A: High Raw $IV_t - RV_t$ percentile</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>L/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Spread</td>
<td>112.83</td>
<td>69.98</td>
<td>64.86</td>
<td>109.18</td>
<td>126.06</td>
<td>13.23</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>76.12</td>
<td>38.66</td>
<td>36.04</td>
<td>83.35</td>
<td>99.49</td>
<td>23.37</td>
</tr>
<tr>
<td>Fama-French Alpha</td>
<td>58.82</td>
<td>15.06</td>
<td>4.64</td>
<td>43.72</td>
<td>62.68</td>
<td>3.85</td>
</tr>
<tr>
<td>Four Factor Alpha</td>
<td>68.65</td>
<td>20.96</td>
<td>7.58</td>
<td>42.98</td>
<td>57.77</td>
<td>-10.88</td>
</tr>
<tr>
<td>Five Factor Alpha</td>
<td>65.94</td>
<td>7.11</td>
<td>2.48</td>
<td>44.81</td>
<td>63.54</td>
<td>-2.40</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.40</td>
<td>0.30</td>
<td>0.31</td>
<td>0.53</td>
<td>0.54</td>
<td>0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Low Raw $IV_t - RV_t$ percentile</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>L/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Spread</td>
<td>93.18</td>
<td>96.83</td>
<td>80.47</td>
<td>70.59</td>
<td>52.28</td>
<td>-40.90</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>62.59</td>
<td>75.23</td>
<td>61.49</td>
<td>53.22</td>
<td>32.85</td>
<td>-29.74</td>
</tr>
<tr>
<td>Fama-French Alpha</td>
<td>47.25</td>
<td>58.61</td>
<td>37.75</td>
<td>30.45</td>
<td>-5.22</td>
<td>-52.47</td>
</tr>
<tr>
<td>Four Factor Alpha</td>
<td>56.18</td>
<td>61.43</td>
<td>38.77</td>
<td>29.06</td>
<td>-9.98</td>
<td>-66.16</td>
</tr>
<tr>
<td>Five Factor Alpha</td>
<td>45.20</td>
<td>57.00</td>
<td>35.13</td>
<td>24.57</td>
<td>-6.66</td>
<td>-51.86</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.42</td>
<td>0.63</td>
<td>0.60</td>
<td>0.56</td>
<td>0.33</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

This table reports annualized Sharpe ratios and factor-adjusted alphas for momentum portfolios formed conditional on raw $IV_t - RV_t$ percentiles.
Separating stocks according to their $IV_t - RV_t$ quintiles results in notable differences in subsequent momentum. Table 3.6 illustrates that a momentum strategy that focuses solely on the trading of high $IV_t - RV_t$ stocks earns above average returns of $+24$ basis points monthly in 5 factor alphas ($t=0.66$), while a strategy that trades low $IV_t - RV_t$ stocks earns a negative 5 factors alpha of $-45$ basis points monthly ($t=-1.35$). This is precisely the variation in return that my model predicts.

As highlighted earlier, the $IV_t - RV_t$ percentile measure is likely to be related to a variety stock specific characteristics that are unrelated to my theory. Table 3.4 removes correlation with various controls. Specifically, specification 4 in table 3.4 performs monthly regressions of raw $IV_t - RV_t$ percentiles on market beta, book-to-market size, trailing 6 month returns, idiosyncratic volatility turnover and analyst coverage. Using the residuals from this regression specification I repeat the exercise performed in table 3.6, of constructing portfolios each month. Table 3.4 examines the effect of residual, as opposed to raw, values of $IV_t - RV_t$ percentiles on the performance of the baseline momentum strategy.
Table 3.7: Effect of Residual News Uncertainty Overestimation on Momentum Alphas

<table>
<thead>
<tr>
<th></th>
<th>Panel A: High Residual $IV_t - RV_t$ percentile</th>
<th></th>
<th>Panel B: Low Residual $IV_t - RV_t$ percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
</tr>
<tr>
<td>Raw Spread</td>
<td>60.24</td>
<td>67.69</td>
<td>65.48</td>
</tr>
<tr>
<td></td>
<td>(0.81)</td>
<td>(1.21)</td>
<td>(1.36)</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>23.19</td>
<td>38.92</td>
<td>40.28</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(1.53)</td>
<td>(2.01)</td>
</tr>
<tr>
<td>Fama-French Alpha</td>
<td>11.01</td>
<td>16.82</td>
<td>12.52</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.71)</td>
<td>(0.70)</td>
</tr>
<tr>
<td></td>
<td>(0.78)</td>
<td>(1.12)</td>
<td>(0.85)</td>
</tr>
<tr>
<td>Five Factor Alpha</td>
<td>11.11</td>
<td>10.83</td>
<td>11.38</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.22</td>
<td>0.33</td>
<td>0.37</td>
</tr>
</tbody>
</table>

This table reports annualized Sharpe ratios and factor-adjusted alphas for momentum portfolios formed conditional on residual $IV_t - RV_t$ percentiles.

In this set of tests stocks are first ranked on based on their residual $IV_t - RV_t$ percentiles each month. Stocks in each residual $IV_t - RV_t$ quintile are then independently ranked in ascending order on the basis of their most recent 6 month cumulative raw returns (momentum). The primary sorting variable is now cross-sectionally uncorrelated with firm size, book to market and several other factors by performing cross sectional regressions that orthogonalize the raw measure.

Notably, this new decreased correlation with the auxiliary regressors increases the performance of the conditional momentum strategy. Separating stocks according to their residual...
IV_t – RV_t quintiles induces economically and statistically significant differences in subsequent momentum. Table 3.7 illustrates that a momentum strategy that focuses solely on the trading of high residual IV_t – RV_t stocks earns above average returns of +31 basis points monthly in 5 factor alphas (t=0.84), while a strategy that trades low IV_t – RV_t stocks earns a negative 5 factors alpha of -45 basis points monthly (t=-1.35). These effects on returns also align with the theory and are notably more pronounced.

Given these results it is more convenient to isolate just a handful of individual portfolios that characterize core effect. That is precisely what is performed in the next set of results. Table 3.8 reports monthly alphas for portfolios obtained by going long the high IV_t – RV_t momentum portfolio and short the low IV_t – RV_t momentum portfolio. In terms of the results up to now, this is equivalent to buying the L/S column in panel A and selling the L/S column in panel B in tables 3.6 or 3.7.

Table 3.8: Net Effect of Variation in News Uncertainty Overestimation

<table>
<thead>
<tr>
<th>Factor Adjusted Alpha of L/S^{HIGH} – L/S^{LOW}</th>
<th>Raw IV_t – RV_t percentile</th>
<th>Residual IV_t – RV_t percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Spread</td>
<td>54.13</td>
<td>71.10</td>
</tr>
<tr>
<td></td>
<td>(1.59)</td>
<td>(2.37)</td>
</tr>
<tr>
<td>CAPM Alpha</td>
<td>53.10</td>
<td>72.41</td>
</tr>
<tr>
<td></td>
<td>(1.55)</td>
<td>(2.41)</td>
</tr>
<tr>
<td>Fama-French Alpha</td>
<td>56.32</td>
<td>69.35</td>
</tr>
<tr>
<td></td>
<td>(1.60)</td>
<td>(2.24)</td>
</tr>
<tr>
<td>Four Factor Alpha</td>
<td>55.28</td>
<td>67.75</td>
</tr>
<tr>
<td></td>
<td>(1.57)</td>
<td>(2.19)</td>
</tr>
<tr>
<td>Five Factor Alpha</td>
<td>49.46</td>
<td>62.98</td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(2.01)</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.43</td>
<td>0.64</td>
</tr>
</tbody>
</table>

This table reports annualized Sharpe ratios and factor-adjusted alphas for a momentum-hedged strategy that buys and sells momentum portfolios conditional on raw or residual IV_t – RV_t percentiles.

Unsurprisingly the core effect has highly statistically significant alphas using both residual and raw IV_t – RV_t and under various factor models. For example, column 2 of table 3.8 illustrates that a “hedged” momentum strategy that buys high residual IV_t – RV_t momentum
portfolios and hedges out its momentum exposure by selling the low residual analog earns returns of over +70 basis points monthly in factor adjusted alphas (t=+2.4). Juxtaposing columns 1 and 2 of table 3.8 also demonstrates that this effect is robust to the extent that it persists with or without orthogonalization.
Figure 3.9 depicts the cumulative returns of momentum-hedged portfolios formed conditional on raw and residual $IV_t - RV_t$ percentiles, my proxy for news-uncertainty overestimation by investors. This is a graphical depiction of table 3.8. Portfolios are formed by purchasing high raw or residual $IV_t - RV_t$ momentum portfolios and hedging out their momentum exposures by selling their low residual analogs. The key takeaway from figure 3.9 is that these alphas appear robust from a time-series perspective. Returns do not accumulate in short bursts, which if they did, could serve as evidence alternatives to our theory.

Figure 3.9: Cumulative Performance of “News Uncertainty Overestimation” Hedged Portfolios

This figure depicts the cumulative returns of momentum portfolios formed conditional on both raw and residual $IV_t - RV_t$ percentiles.
3.6.5 Reversion to Fundamentals

If the momentum patterns observed in the previous sections are in fact driven by news uncertainty overestimation then I would expect these profits to reverse sharply after several months.

Figure 3.10: Event Time Cumulative Returns to Momentum Portfolios

This figure depicts the cumulative event time returns for momentum portfolios based on sorts conditional on residual $IV_t - RV_t$ percentiles.

Figure 3.10 depicts the cumulative event time returns over the first 12 months for the
high and low residual ($IV_t - RV_t$ percentile) momentum portfolios. In line with the model’s predictions, we see that the momentum portfolio formed based on high $IV_t - RV_t$ reverses after about 10 months whereas the low analog is relatively unchanged in its downward trajectory\footnote{Recalling that the baseline momentum returns are negative on this window, this is expected.}.

### 3.7 Conclusion

In this essay I link the existence of mental-models of price-targets to asset return predictability. I present a behavioral explanation for the patterns of continuation and reversion observed in equity prices. My model features in investor who begins with a simple Gaussian prior distribution and a price-target about the value of the stock. The price-target is either true, in which case it resolves all uncertainty about the asset’s value, or it is not true, it which case it is completely unrelated to the value of the asset. Uncertainty is then subsequently resolved as a stream of news is consumed.

The central insight of the model is that overconfidence, mistakes about the amount of uncertainty in one’s information environment, creates a perverse tendency to initially update in favor of mental-models that are inconsistent with observed data. Specifically, I demonstrate that even if the price-target is useless, given its binary true/false nature there is a tendency to up-weight it initially which causes equilibrium prices to overshoot their long term value. This overshooting results in short term positive auto-correlation followed by negative return auto-correlations at long horizons.

In addition to explaining general empirical facts such as short-horizon continuation and long-horizon reversal, this model derives sharp predictions for when these anomalies should be relatively more or less robust. The model’s predictions are tested by juxtaposing realized and option-implied volatility in single stocks. A momentum portfolio strategy that exploits this effect earns over $+70$ basis points per month in factor adjusted alphas ($t=2.6$).

Taken together these results illustrate that mis-calibration about an investor’s information-environment can lead to inefficiencies in the path taken when learning whether or not one possess private information, and in turn significantly distort the way that news is impounded.
into securities prices.
Chapter 4

Appendices

4.1 Chapter 1 Appendix

4.1.1 Asymmetric Reactions to Common Information Shocks

I investigate how limited attention impacts the way in which the same piece of information is incorporated into the prices of different firms. To do this I consider two sets of firms that are both subject to common information shocks. The only difference is that investors pay more attention to one set of firms than another. Using this differential in investor attention, I demonstrate that attention constraints can result in substantial cross-asset return predictability. In particular, I examine information events that affect an entire industry. If investor inattention is driving the return predictability we observe then we would expect news that affects an entire industry to be impounded into the prices of high attention stocks first. As such, the price response of high attention stocks to (the industry component of) earnings news should predict subsequent returns in low attention industry peers.

To test my approach I decompose industry news to obtain an information rich component that is able to predict subsequent stock returns. Each month, for each industry $j$, I construct a measure of aggregate industry news, $INDNEWS_{j,t}$, by taking a value weighted average of the earnings surprise of the stocks in that industry.
Table 4.1: Industry-level Return Predictability, Pooled Regressions

<table>
<thead>
<tr>
<th>Decomposition of Industry Momentum</th>
<th>Dep Var:</th>
<th>( \text{INDRET}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \text{INDRET}_{t-1}^{\text{HIGH ATTN}} )</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(5.33)</td>
<td>(3.97)</td>
</tr>
<tr>
<td>( \text{INDRET}_{t-1}^{\text{LOW ATTN}} )</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(4.04)</td>
<td>(1.91)</td>
</tr>
<tr>
<td>( \text{INDRET}_{t-1} )</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.77)</td>
<td></td>
</tr>
<tr>
<td>( \text{INDNEWS}_{t-1}^{\text{LARGE}} )</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(5.32)</td>
<td>(5.41)</td>
</tr>
<tr>
<td>INTERCEPT</td>
<td>0.005</td>
<td>0.0014</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports pooled forecasting regressions of industry returns. Units of observation are industry and month. The dependent variables are the value weighted average industry return of the all stocks in the industry (columns 1-4), stocks with above median attention (columns 5-8) and stocks with below median attention (columns 9-12).

Panel A reports pooled forecasting regressions of industry returns. \( \text{INDRET}_{j,t}^{\text{HIGH ATTN}} \) (\( \text{INDRET}_{j,t}^{\text{LOW ATTN}} \)) is the lagged value weighted average excess return of the high attention (low attention) stocks in the industry of the stock. The dependent variable is the value weighted average industry return of the all stocks in the industry.
Every month this measure is constructed for each of the 48 industries in the Fama and French classification scheme. Using my proxy for investor attention I am then able to decompose industry news into a predictive component based on high attention stocks and an uninformative component based on low attention stocks.

\[
\text{INDNEWS}_{j,t} = \sum_{s \in \text{IND}_j} \text{EARNSURP}_{s,t} v_{w,s,j}
\]

\[
\text{INDRET}_{j,t} = \sum_{s \in \text{IND}_j} \text{XRET}_{s,t-1} v_{w,s,j} + \sum_{s \in \text{IND}_j} \text{XRET}_{s,t-1} v_{w,s,j}
\]

Informative industry momentum

Uninformative industry momentum

Here I present monthly Fama and MacBeth (1973) forecasting regressions of one month ahead excess return on decomposed industry news and a set of firm specific characteristics. Specifically, I control for well-known determinants of stock returns, such as size, book-to-market ratio, momentum, 1-month reversal, idiosyncratic volatility and turnover.

I find that the value weighted average earnings surprise of a stock’s high attention industry peers strongly predicts its future excess returns whereas the returns of the low attention peers does not. This is consistent with my hypothesis that investors’ limited information processing capacity can cause delays in revelation of common information in assets prices, generating cross asset predictability.

Panel B presents pooled forecasting regressions of industry portfolio returns on decomposed industry news, lagged industry portfolio returns and a set of industry aggregates of stock characteristics. Columns 1 through 4 show the results of regressions in which the dependent variable is the value weighted average return of the entire industry. We see that consistent with my predictions, only the high attention industry portfolio predicts future industry returns.

Columns 5 through 12 present 2 important facts consistent with my underreaction story.
First, the low attention industry portfolio exhibits far more predictability than the high attention industry portfolio. This is observed by the larger and statistically significant coefficients on $INDRET_{t-1}$ and $INDNEWS_t^{HIGHATTN}$ in column 12 compared to the smaller and statistically insignificant coefficients columns 8. Additionally, the high attention news component predicts the low attention portfolio however the low attention news component does not predict the high attention portfolio.

Panel C further demonstrates that this predictability is also generated by attention decompositions of past industry returns.

$$INDNEWS_{j,t} = \sum_{s \in IND_{j}^{HIGHATTN}} EARNSURP_{s,t} w_{s,j} + \sum_{s \in IND_{j}^{LOWATTN}} EARNSURP_{s,t} w_{s,j}$$

Informative industry news

$$= INDNEWS_{j,t}^{HIGHATTN} + INDNEWS_{j,t}^{LOWATTN}$$

4.2 Chapter 3 Appendix

4.2.1 A Motivating Thought Experiment

Likelihood ratio calculations

$$\Lambda \equiv \frac{L(\psi|true)}{L(\psi|false)}$$

$$= \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp \left( \frac{(\psi - \mu_z)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi - \mu_z - \kappa)^2}{2v_\varepsilon} \right)$$

Inequality conditions and approximation

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\[ \Lambda = \frac{\mathcal{L}(\psi|true)}{\mathcal{L}(\psi|false)} \]
\[ = \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp\left(\frac{(\psi - \mu_z)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi - \mu_z - \kappa)^2}{2v_\varepsilon}\right) \]
\[ 1 < \sqrt{\frac{v_z + v_\varepsilon}{v_\varepsilon}} \exp\left(-\frac{\kappa^2}{2v_\varepsilon}\right) \]
\[ 1 > \frac{v_\varepsilon}{v_\varepsilon + v_z} \exp\left(\frac{\kappa^2}{v_\varepsilon}\right) \]
\[ v_\varepsilon + v_z > v_\varepsilon \exp\left(\frac{\kappa^2}{v_\varepsilon}\right) \]
\[ v_z > v_\varepsilon \left(\exp\left(\frac{\kappa^2}{v_\varepsilon}\right) - 1\right) \]
\[ \frac{v_z}{v_\varepsilon} > \exp\left(\frac{\kappa^2}{v_\varepsilon}\right) - 1 \]
\[ \approx 1 + \frac{\kappa^2}{v_\varepsilon} - 1 \]
\[ \approx v_z > \kappa^2 \]

Investor's expectations, averaged across models

\[
\begin{align*}
E_0^f[z] &= \omega_0 S + (1 - \omega_0) \mu_z \\
&= \omega_0 \kappa + \mu_z \\
E_1^f[z] &= (\omega_1 - \omega_0) (\mu + \kappa) + (\omega_1 - \omega_0) \mu_z + (1 - \omega_1) \frac{v_z}{v_z + v_\varepsilon} (\psi - \mu_z) \\
E_1^f[z] - E_0^f[z] &= (\omega_1 - \omega_0) \kappa + (1 - \omega_1) \frac{v_z}{v_z + v_\varepsilon} (\psi - \mu_z) \\
&= \omega_0 (1 - \omega_0) \left(\frac{\Lambda - 1}{\Lambda \omega_0 + (1 - \omega_0)}\right) \kappa + \frac{1 - \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \left(\frac{v_z}{v_z + v_\varepsilon}\right) (\psi - \mu_z) \\
&= \left(\frac{1 - \omega_0}{\Lambda \omega_0 + (1 - \omega_0)}\right) \left(\omega_0 \kappa (\Lambda - 1) + \frac{v_z}{v_z + v_\varepsilon} (\psi - \mu_z)\right)
\end{align*}
\]

Algebraic manipulations of \( \omega_0 \) and \( \omega_1 \)
\[ \omega_1 = \frac{\Lambda \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \]

\[ \omega_1 - \omega_0 = \frac{\Lambda \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} - \omega_0 \]

\[ = \omega_0 \left( \frac{\Lambda}{\Lambda \omega_0 + (1 - \omega_0)} - 1 \right) \]

\[ = \omega_0 \left( \frac{\Lambda - \Lambda \omega_0 + \omega_0 - 1}{\Lambda \omega_0 + (1 - \omega_0)} \right) \]

\[ = \omega_0 \left( \frac{\Lambda - 1 - \omega_0 (\Lambda - 1)}{\Lambda \omega_0 + (1 - \omega_0)} \right) \]

\[ = \omega_0 (1 - \omega_0) \left( \frac{\Lambda - 1}{\Lambda \omega_0 + (1 - \omega_0)} \right) \]

\[ = \omega_0 (1 - \omega_0) \left( \frac{\Lambda - 1}{\Lambda \omega_0 + 1 - \omega_0} \right) \]

\[ = \omega_0 (1 - \omega_0) \left( \frac{\Lambda - 1}{1 + \omega_0 (\Lambda - 1)} \right) \]

\[ 1 - \omega_1 = 1 - \frac{\Lambda \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \]

\[ = \frac{\Lambda \omega_0 + (1 - \omega_0) - \Lambda \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \]

\[ = \frac{1 - \omega_0}{\Lambda \omega_0 + (1 - \omega_0)} \]

4.2.2 The cumulant generating function of normal mixtures

The moment generating function for a random variable is defined as

\[ MGF(t) = \mathbb{E}[\exp(tX)] \]

In the case of the normal mixture being considered, the expectation can be broken into 2 components having weights \(1 - \omega_0\) and \(\omega_0\). This observation allows us to write the \(MGF\) as

\[ MGF(t) = (1 - \omega_0) \mathbb{E}[\exp(tX_1)] + \omega_0 e^{(G - P)t} \]
where \( X_1 \sim \mathcal{N}(\mu_z - P, v_z) \)

Now, the random variable given by \( \exp(tX_1) \) is a log-normal, its expectation is therefore \( e^{(\mu_z - P)t + \frac{t^2 v_z}{2}} \) and so the MGF is given by

\[
MGF(t) = (1 - \omega_0) e^{(\mu_z - P)t + \frac{t^2 v_z}{2}} + \omega_0 e^{(G - P)t}
\]

Finally, since \( CGF(t) = \log(MGF(t)) \) we have that

\[
CGF(t) = \log \left( (1 - \omega_0) e^{(\mu_z - P)t + \frac{t^2 v_z}{2}} + \omega_0 e^{(G - P)t} \right)
\]

### 4.2.3 The first order condition for CARA utility and the mixture of a normal and a point mass

Investors minimize the following objective function with respect to \( x \)

\[
\min_x \log \left[ (1 - \omega_0) e^{-(\mu_z - P)\gamma x + \frac{\gamma^2 v_z}{2} x^2} + \omega_0 e^{-(G - P)\gamma x} \right]
\]

The first order condition is obtained by differentiating with respect to \( x \) and setting the result equal to zero. This yields,

\[
\frac{(-\gamma (\mu_z - P) + \gamma^2 v_z x) (1 - \omega_0) e^{-(\mu_z - P)\gamma x + \frac{\gamma^2 v_z}{2} x^2} - (G - P) \gamma \omega_0 e^{-(G - P)\gamma x}}{(1 - \omega_0) e^{-(\mu_z - P)\gamma x + \frac{\gamma^2 v_z}{2} x^2} + \omega_0 e^{-(G - P)\gamma x}} = 0
\]

The terms in the denominator are strictly positive. Therefore the first order condition is given by

\[
\left( -\gamma (\mu_z - P) + \gamma^2 v_z x \right) (1 - \omega_0) e^{-(\mu_z - P)\gamma x + \frac{\gamma^2 v_z}{2} x^2} - (G - P) \gamma \omega_0 e^{-(G - P)\gamma x} = 0
\]
4.2.4 Equilibrium demand

\[
x = \frac{(\mu_z - P) \gamma (1 - \omega_0) e^{-\mu_z \gamma x + \frac{\gamma^2}{2} v_z x^2} + (G - P) \gamma \omega_0 e^{-(\mu_z - P) \gamma x + \frac{\gamma^2}{2} v_z x^2}}{\gamma^2 v_z (1 - \omega_0) e^{-(\mu_z - P) \gamma x + \frac{\gamma^2}{2} v_z x^2}}
\]

\[
= \frac{(\mu_z - P)}{\gamma v_z} + \frac{(G - P)}{\gamma v_z} \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\frac{\gamma^2}{2} v_z \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}}
\]

\[
= \frac{(\mu_z - P)}{\gamma v_z} + \frac{(G - P)}{\gamma v_z} \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\frac{\gamma^2}{2} \left( v_z \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 - \frac{(\mu_z - G)^2}{2 v_z} \right)}
\]

4.2.5 Equilibrium price

Setting supply, which is assumed to be 1, equal to demand, \( x \) yields

\[
1 = \frac{\mu_z - P}{\gamma v_z} + \frac{(G - P)}{\gamma v_z} \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\frac{\gamma^2}{2} \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}} - 1
\]

\[
P = \frac{\mu_z + G \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\frac{\gamma^2}{2} \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}} - \gamma v_z}{1 + \left( \frac{\omega_0}{1 - \omega_0} \right) e^{-\frac{\gamma^2}{2} \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}}}
\]

\[
P = \frac{\mu_z - \gamma v_z \left( 1 - \omega_0 \right) + G \omega_0 e^{-\frac{\gamma^2}{2} v_z \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}}}{1 - \omega_0 \left( 1 - e^{-\frac{\gamma^2}{2} v_z \left( 1 - \frac{\mu_z - G}{\gamma v_z} \right)^2 + \frac{(\mu_z - G)^2}{2 v_z}} \right)}
\]
We consider the investor’s information-set at the start of date-$t$: if the signal is true then

$$\Psi_t \sim \mathcal{N}(G, v_\varepsilon)$$

however if it is not true then

$$\Psi_t \sim \mathcal{N}(E_{t-1}[z], v_\varepsilon + \sqrt{V_{t-1}[z]})$$

These hypotheses yield the following prior densities respectively,

$$L_t(\psi_t|True) = \frac{1}{\sqrt{2\pi v_\varepsilon}} e^{\exp\left(-\frac{(\psi_t - G)^2}{2v_\varepsilon}\right)}$$

$$L_t(\psi_t|False) = \frac{1}{\sqrt{2\pi (v_\varepsilon + \sqrt{V_{t-1}[z]})}} e^{\exp\left(-\frac{(\psi_t - E_{t-1}[z])^2}{2(v_\varepsilon + \sqrt{V_{t-1}[z]})}\right)}$$

Define the likelihood ratio,

$$\Lambda_t = \frac{L_t(\psi_t|True)}{L_t(\psi_t|False)}$$

$$= \frac{1}{\sqrt{2\pi v_\varepsilon}} e^{\exp\left(-\frac{(\psi_t - G)^2}{2v_\varepsilon}\right)} \cdot \frac{1}{\sqrt{2\pi (v_\varepsilon + \sqrt{V_{t-1}[z]})}} e^{\exp\left(-\frac{(\psi_t - E_{t-1}[z])^2}{2(v_\varepsilon + \sqrt{V_{t-1}[z]})}\right)}$$

$$= \sqrt{\frac{2\pi (v_\varepsilon + \sqrt{V_{t-1}[z]})}{2\pi v_\varepsilon}} e^{\exp\left(-\frac{(\psi_t - G)^2}{2v_\varepsilon} - \frac{(\psi_t - E_{t-1}[z])^2}{2(v_\varepsilon + \sqrt{V_{t-1}[z]})}\right)}$$

$$= \sqrt{1 + \frac{1}{v_\varepsilon} \sqrt{V_{t-1}[z]}} e^{\exp\left(\frac{v_\varepsilon (\psi_t - E_{t-1}[z])^2 - (v_\varepsilon + \sqrt{V_{t-1}[z]}) (\psi_t - G)^2}{2(v_\varepsilon + \sqrt{V_{t-1}[z]})}\right)}$$
4.2.7 Tractable Bayesian Updating

Consider the investor’s information-set at the start of date-\( t \) but having the date-0 prior. If the signal is true then \( \Psi_t \sim \mathcal{N}(G, \frac{v}{t}) \) which yields the following prior density for \( \theta_t \)

\[
\tilde{L}_t(\theta_t|\text{True}) = \frac{1}{\sqrt{2\pi(\frac{v}{t})}} e^{\frac{-(\theta_t - G)^2}{2(\frac{v}{t})}}
\]

If on the other hand, \( G \) is irrelevant, then \( \Psi_t \sim \mathcal{N}(\mu_z, \frac{v}{t} + v_z) \) which yields the following prior density for \( \theta_t \)

\[
\tilde{L}_t(\theta_t|\text{False}) = \frac{1}{\sqrt{2\pi(\frac{v}{t} + v_z)}} e^{\frac{-(\theta_t - \mu_z)^2}{2(\frac{v}{t} + v_z)}}
\]

Now define the likelihood ratio,

\[
\tilde{\Lambda}_t = \frac{\tilde{L}_t(\theta_t|\text{True})}{\tilde{L}_t(\theta_t|\text{False})}
\]

\[
= \frac{\frac{1}{\sqrt{2\pi(\frac{v}{t})}} e^{\frac{-(\theta_t - G)^2}{2(\frac{v}{t})}}}{\frac{1}{\sqrt{2\pi(\frac{v}{t} + v_z)}} e^{\frac{-(\theta_t - \mu_z)^2}{2(\frac{v}{t} + v_z)}}}
\]

\[
= \sqrt{1 + \frac{t (v_z)^2}{v_e}} e^{\frac{v_e (\theta_t - \mu_z)^2}{2(\frac{v}{t} + v_z)} - \frac{(\theta_t - G)^2}{2v_e}}
\]

\[
= \sqrt{1 + \frac{t (v_z)^2}{v_e}} e^{\frac{v_e (\theta_t - \mu_z)^2}{2(\frac{v}{t} + v_z)} - \frac{(\theta_t - G)^2}{2v_e}}
\]

\[
= \sqrt{1 + \frac{t (v_z)^2}{v_e}} e^{\frac{v_e (\theta_t - \mu_z)^2}{2(\frac{v}{t} + v_z)} - \frac{(\theta_t - G)^2}{2v_e}}
\]
4.2.8 Condition for an expected increase in $\omega_t$, conditional on $z$

\[
\frac{1}{\omega_0 + (1 - \omega_0)E_t^q \left[ \frac{1}{\Lambda_t} \right]} > 1
\]

\[
1 > \omega_0 + (1 - \omega_0)E_t^q \left[ \frac{1}{\Lambda_t} \right]
\]

\[
1 - \omega_0 > (1 - \omega_0)E_t^q \left[ \frac{1}{\Lambda_t} \right]
\]

\[
1 > E_t^q \left[ \frac{1}{\Lambda_t} \right]
\]
4.2.9 Unconditional expectation of the likelihood ratio

\[ \mathbb{E}^z \left[ \mathbb{E}^{\psi_1} [\Lambda_1 | z] \right] = 1 \]

Note explicitly that the likelihood ratio as a function of \( \psi \).

The first expectation is the integral given by

\[ \mathbb{E}^{\psi_1} [\Lambda_1 | z] = \int_{-\infty}^{\infty} \Lambda_1 (\psi) dP^{\psi_1} \]

The second integral is given by

\[ \mathbb{E}^z \left[ \mathbb{E}^{\psi_1} [\Lambda_1 | z] \right] = \int_{-\infty}^{\infty} \mathbb{E}^{\psi_1} [\Lambda_1 | z] dP^z \]

Combining these into an explicit double integral yields

\[ \mathbb{E}^z \left[ \mathbb{E}^{\psi_1} [\Lambda_1 | z] \right] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Lambda_1 (\psi_1) dP^{\psi_1} dP^z \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Lambda_1 (\psi_1) f^{\psi} (\psi_1) f^z (z) d\psi_1 dz \]

Where \( f^{\psi} (\psi_1) \) and \( f^z (z) \) are the probability density functions of \( \psi \) and \( z \) respectively.

\[ f^z (z) = \frac{1}{\sqrt{2\pi v_z}} \exp \left( -\frac{(\mu_z - z)^2}{2v_z} \right) \]

\[ f^{\psi} (\psi_1) = \frac{1}{\sqrt{2\pi v_\varepsilon}} \exp \left( -\frac{(\psi_1 - z)^2}{2v_\varepsilon} \right) \]

The product of the terms inside the double integral yields,
\[ \Lambda_1 (\psi_1) f^\psi (\psi_1) f^g (z) = \exp \left( -\frac{(\psi - \mu_z)^2}{2v_\varepsilon} + \frac{(\psi - \mu_z)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi_1 - z)^2}{2v_z} - \frac{(\mu_z - z)^2}{2v_z} \right) \]

\[ = \frac{\exp \left( -\frac{(\psi - \mu_z)^2}{2v_\varepsilon} + \frac{(\psi - \mu_z)^2}{2(v_\varepsilon + v_z)} - \frac{(\psi_1 - z)^2}{2v_z} - \frac{(\mu_z - z)^2}{2v_z} \right)}{\sqrt{2\pi \left( \frac{2\pi v_z (v_\varepsilon)^2}{v_\varepsilon + v_z} \right)}} \]

This is simply the probability density function of a bivariate normal centered at

\[ G = \mu_z + \kappa \]

and

\[ \mathbb{E}_1 [z | \psi] = \frac{\mu_z v_\varepsilon + \psi_1 v_z}{v_\varepsilon + v_z} \]

The mass under a well defined pdf is unity therefore

\[ \mathbb{E}^2 \left[ \mathbb{E}^{\psi_1} [\Lambda_1 | z] \right] \equiv 1 \]
4.2.10 Expected value of exponential of the quadratic of a Normal

\[ E^D_t \left[ \exp \left( A (x - B)^2 \right) \right] = \int_{-\infty}^{\infty} \exp \left( A (x - B)^2 \right) \frac{1}{\sigma \sqrt{2 \pi}} \exp \left( -\frac{(x - \mu)^2}{2\sigma^2} \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( Ax^2 - 2ABx + AB^2 + \frac{x^2}{2\sigma^2} + \frac{\mu x}{\sigma^2} - \frac{\mu^2}{2\sigma^2} \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( -Ax^2 + 2ABx - AB^2 + \frac{x^2}{2\sigma^2} + \frac{\mu x}{\sigma^2} + \frac{\mu^2}{2\sigma^2} \right) \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( \frac{1 - 2A\sigma^2}{2\sigma^2} \right) x^2 + \left( \frac{2AB\sigma^2 - \mu}{\sigma^2} \right) x \right. \\
\left. + \left( \frac{\mu^2 - 2AB^2\sigma^2}{2\sigma^2} \right) \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( \frac{1 - 2A\sigma^2}{2\sigma^2} \right) \left( x - \left( \frac{\mu - 2AB\sigma^2}{1 - 2A\sigma^2} \right) \right)^2 \right. \\
\left. + \left( \frac{\mu^2 - 2AB^2\sigma^2}{1 - 2A\sigma^2} \right) \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( \frac{1 - 2A\sigma^2}{2\sigma^2} \right) \left( x - \left( \frac{\mu - 2AB\sigma^2}{1 - 2A\sigma^2} \right) \right)^2 \right. \\
\left. + \left( \frac{1 - 2A\sigma^2}{2\sigma^2} \right) \left( \frac{\mu^2 - 2AB^2\sigma^2}{(1 - 2A\sigma^2)^2} - \left( \frac{\mu - 2AB\sigma^2}{1 - 2A\sigma^2} \right)^2 \right) \right) dx \]

\[ = \frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( \frac{1 - 2A\sigma^2}{2\sigma^2} \right) \left( x - \left( \frac{\mu - 2AB\sigma^2}{1 - 2A\sigma^2} \right) \right)^2 \right. \\
\left. - \frac{2A(B - \mu}\sigma^2}{1 - 2A\sigma^2} \right) dx \]

\[ = \frac{\exp \left( \frac{2A(B - \mu)^2}{2\sigma^2} \right)}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{\infty} \exp \left( -\left( \frac{\mu - 2AB\sigma^2}{1 - 2A\sigma^2} \right)^2 \right) dx \]

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Now, notice that

\[
\frac{1}{\left(\frac{\sigma}{\sqrt{1-2A\sigma^2}}\right)\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left(-\frac{(x - \left(\frac{\mu - 2AB\sigma^2}{1-2A\sigma^2}\right))^2}{2 \left(\frac{\sigma}{\sqrt{1-2A\sigma^2}}\right)^2}\right) \, dx = 1
\]

since it is the integral over the real line for the pdf of a normal distribution with mean \(\frac{\mu - 2AB\sigma^2}{1-2A\sigma^2}\) and standard deviation \(\frac{\sigma}{\sqrt{1-2A\sigma^2}}\).

Therefore we have that

\[
\mathbb{E}_t^D \left[ \exp \left( (A \cdot (x - B))^2 \right) \right] = \frac{1}{\sqrt{1 - 2A\sigma^2}} \exp \left( \frac{2A (B - \mu)^2 \sigma^2}{2\sigma^2 (1 - 2A\sigma^2)} \right)
\]

\[
= \frac{1}{\sqrt{1 - 2A\sigma^2}} \exp \left( \frac{A (B - \mu)^2}{1 - 2A\sigma^2} \right)
\]
4.2.11 Conditional expectation of $\frac{1}{\Lambda_t}$

Utilizing lemma 1, set

$$A = \frac{v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t}$$

$$B = \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G$$

$$m = z$$

$$s^2 = \frac{v_\varepsilon |\text{True}}{t}$$

Therefore,

$$(m - B)^2 = \left( z - \left( \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \right) \right)^2$$

$$1 - 2As^2 = 1 - 2 \frac{v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t} \frac{v_\varepsilon |\text{True}}{t}$$

$$= 1 - \frac{v_z v_\varepsilon |\text{True} t}{v_\varepsilon (v_\varepsilon + v_z t)}$$

$$= \frac{v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon |\text{True} t}{v_\varepsilon (v_\varepsilon + v_z t)}$$

$$\frac{A}{1 - 2As^2} = \left( \frac{v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t} \right) / \left( \frac{v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon |\text{True} t}{v_\varepsilon (v_\varepsilon + v_z t)} \right)$$

$$= \frac{v_z t}{2 \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon |\text{True} t \right) / t}$$

So that,

$$\frac{A(m - B)^2}{1 - 2As^2} = \frac{v_z t \left( z - \left( \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \right) \right)^2}{2 \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon |\text{True} t \right) / t}$$
Therefore,

\[
E_t^\theta \left[ \frac{1}{\lambda_t} \right] = \frac{\exp \left( -\frac{(\mu_z - G)^2}{2v_z} \right)}{\sqrt{1 + t \left( \frac{v_z}{v_z} \right)}} \cdot \exp \left( \frac{v_z t \left( \theta_t - \left( \frac{v_z(G - \mu_z)}{v_z t} + G \right) \right)^2}{2v_z (v_z + v_z t) / t} \right)
\]

\[
= \frac{\exp \left( -\frac{(\mu_z - G)^2}{2v_z} \right)}{\sqrt{1 + t \left( \frac{v_z}{v_z} \right)}} \cdot \frac{1}{\sqrt{v_z (v_z + v_z t) - v_z v_z |True_t}} \cdot \exp \left( \frac{v_z t \left( z - \left( \frac{v_z(G - \mu_z)}{v_z t} + G \right) \right)^2}{2v_z (v_z (v_z + v_z t) - v_z v_z |True_t) / t} \right)
\]

\[
= \frac{v_z \exp \left( -\frac{(\mu_z - G)^2}{2v_z} \right)}{\sqrt{v_z (v_z + v_z t) - v_z v_z |True_t}} \cdot \exp \left( \frac{(v_z t)^2 \left( z - \left( \frac{v_z(G - \mu_z)}{v_z t} + G \right) \right)^2}{2v_z (v_z (v_z + v_z t) - v_z v_z |True_t)} \right)
\]

This is valid when,

\[
\frac{v_z t}{2v_z (v_z + v_z t) / t} < \frac{1}{2s^2},
\]

\[
\frac{v_z |True_t v_z t}{v_z (v_z + v_z t)} < \frac{1}{2v_z |True_t t},
\]

\[
v_z |True_t < v_z (v_z + v_z t)
\]

\[
v_z |True_t < v_z \left( \frac{v_z}{v_z t} + 1 \right)
\]

which is always satisfied when \( v_z |True_t < v_z \) since \( \frac{v_z}{v_z t} \geq 0 \)
4.2.12 Conditional expectation of $\tilde{\Lambda}_t$

Utilizing lemma 1, set

\[
A = \frac{-v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t} \\
B = \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \\
m = z \\
s^2 = \frac{v_\varepsilon |\text{True}|}{t}
\]

Therefore,

\[
(m - B)^2 = \left( z - \left( \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \right) \right)^2
\]

\[
1 - 2A s^2 = 1 + 2 \frac{v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t} \frac{v_\varepsilon |\text{True}|}{t}
\]

\[
= 1 + \frac{v_z v_\varepsilon |\text{True}|}{v_\varepsilon (v_\varepsilon + v_z t)}
\]

\[
= \frac{v_\varepsilon (v_\varepsilon + v_z t) + v_z v_\varepsilon |\text{True}| t}{v_\varepsilon (v_\varepsilon + v_z t)}
\]

\[
\frac{A}{1 - 2A s^2} = \left( \frac{v_z t}{2v_\varepsilon (v_\varepsilon + v_z t)/t} \right) / \left( \frac{v_\varepsilon (v_\varepsilon + v_z t) + v_z v_\varepsilon |\text{True}| t}{v_\varepsilon (v_\varepsilon + v_z t)} \right)
\]

\[
= \frac{v_z t}{2 \left( v_\varepsilon (v_\varepsilon + v_z t) + v_z v_\varepsilon |\text{True}| t \right) / t}
\]

So that,

\[
\frac{A (m - B)^2}{1 - 2A s^2} = \frac{-v_z t \left( z - \left( \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \right) \right)^2}{2 \left( v_\varepsilon (v_\varepsilon + v_z t) + v_z v_\varepsilon |\text{True}| t \right) / t}
\]
Therefore,

\[
\mathbb{E}_t^\theta \left[ \tilde{\Lambda}_t \right] = \sqrt{1 + t \left( \frac{v_z}{v_\varepsilon} \right)} \exp \left( \frac{(\mu_z - G)^2}{2v_z} \right) \mathbb{E}_t^\theta \left[ \exp \left( \frac{-v_z t \left( \theta_t - \left( \frac{v_\varepsilon(G - \mu_z)}{v_z} + G \right) / t \right)}{2v_\varepsilon(v_\varepsilon + v_z t) / t} \right) \right]
\]

\[
= \sqrt{1 + t \left( \frac{v_z}{v_\varepsilon} \right)} \exp \left( \frac{(\mu_z - G)^2}{2v_z} \right) \exp \left( \frac{-v_z t \left( z - \left( \frac{v_\varepsilon(G - \mu_z)}{v_z} + G \right) / t \right)}{2v_\varepsilon(v_\varepsilon + v_z t) + v_z v_\varepsilon |True t / t} \right)
\]

\[
= \sqrt{\frac{(v_z + v_z t)^2}{v_\varepsilon(v_\varepsilon + v_z t) + v_z v_\varepsilon |True t}} \exp \left( \frac{(\mu_z - G)^2}{2v_z} \right) \exp \left( \frac{-v_z t \left( z - \left( \frac{v_\varepsilon(G - \mu_z)}{v_z} + G \right) / t \right)}{2v_\varepsilon(v_\varepsilon + v_z t) + v_z v_\varepsilon |True t / t} \right)
\]

This is valid when,

\[
A < \frac{1}{2s^2} \quad \frac{v_z t}{2v_\varepsilon(v_\varepsilon + v_z t) / t} < \frac{1}{2v_\varepsilon |True t / t} \quad v_\varepsilon |True t < v_\varepsilon (v_\varepsilon + v_z t) \quad v_\varepsilon |True t < v_\varepsilon \left( \frac{v_\varepsilon}{v_z t} + 1 \right)
\]

which is always satisfied when \( v_\varepsilon |True t < v_\varepsilon \) since \( \frac{v_\varepsilon}{v_z t} \geq 0 \)
4.2.13 Unconditional expectation of $\frac{1}{\Lambda_t}$

Utilizing lemma 1, set

\[
A = \frac{(v_z t)^2}{2 v_z \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t \right)}
\]

\[
B = \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G
\]

\[
m = \mu_z
\]

\[
s^2 = v_z
\]

Therefore,

\[
(m - B)^2 = \left( \mu_z - \left( \frac{v_\varepsilon (G - \mu_z)}{v_z t} + G \right) \right)^2
\]

\[
= \left( 1 + \frac{v_\varepsilon}{v_z t} \right)^2 (G - \mu_z)^2
\]

\[
1 - 2 A s^2 = 1 - 2 \frac{(v_z t)^2}{2 v_z \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t \right)} v_z
\]

\[
= 1 - \frac{(v_z t)^2}{v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t - (v_z t)^2}
\]

\[
\frac{A}{1 - 2 A s^2} = \frac{(v_z t)^2}{2 v_z \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t \right)} / \left( \frac{v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t - (v_z t)^2}{v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t} \right)
\]

\[
= \frac{(v_z t)^2}{2 v_z \left( v_\varepsilon (v_\varepsilon + v_z t) - v_z v_\varepsilon | T_{true} | t - (v_z t)^2 \right)}
\]
So that,

$$\frac{A(m - B)^2}{1 - 2As^2} = \frac{(v^2t)^2 \left( 1 + \frac{v}{v^2t} \right)^2 (G - \mu)^2}{2v \left( v^2 (v^2t + v^2z) - v^2v^2 \right)}$$

Therefore,

$$E^M [\mathcal{Y}(z)] = \frac{v \exp \left( -\frac{(G - \mu)^2}{2v} \right)}{\sqrt{v^2 (v^2t + v^2z) - v^2v^2}} \left[ \exp \left( \frac{(v^2t)^2 \left( 1 + \frac{v}{v^2t} \right)^2 (G - \mu)^2}{2v \left( v^2 (v^2t + v^2z) - v^2v^2 \right)} \right) \right]$$

$$= \frac{v \exp \left( -\frac{(G - \mu)^2}{2v} \right)}{\sqrt{v^2 (v^2t + v^2z) - v^2v^2}} \exp \left( \frac{(v^2t)^2 \left( 1 + \frac{v}{v^2t} \right)^2 (G - \mu)^2}{2v \left( v^2 (v^2t + v^2z) - v^2v^2 \right)} \right)$$

This is valid when,
The above condition is satisfiable when \(# (v\varepsilon)^2 + v\varepsilon v_z t - (v_z t)^2 > 0 \#\), which requires \(0 < (v\varepsilon)^2 + v\varepsilon v_z t - (v_z t)^2 \). This is only possible if,

\[
v\varepsilon > -v_z t + \sqrt{(v_z t)^2 + 4 (v_z t)^2} \]
\[
= v_z t \left( \frac{\sqrt{5} - 1}{2} \right)
\]

This condition is the same as \(\frac{1 + \sqrt{5}}{2} > \frac{v_z t}{v\varepsilon} \).

### 4.2.14 Derivation of Returns

Recall that,

\[
P_t = \frac{\lambda \omega_t G + (1 - \omega_t) \pi_t}{1 - \omega_t (1 - \lambda_t)}
\]

Returns are therefore given by,
\[ R_t = P_t - P_{t-1} \]
\[ = \frac{\lambda_t \omega_t G + (1 - \omega_t) \pi_t}{1 - \omega_t (1 - \lambda_t)} - \frac{\lambda_{t-1} \omega_{t-1} G + (1 - \omega_{t-1}) \pi_{t-1}}{1 - \omega_{t-1} (1 - \lambda_{t-1})} \]
\[ = \frac{(1 - \omega_t) \Delta \pi_t}{1 - \omega_t (1 - \lambda_t)} + \frac{\lambda_t \omega_t G + (1 - \omega_t) \pi_t}{1 - \omega_t (1 - \lambda_t)} - \frac{\lambda_{t-1} \omega_{t-1} G + (1 - \omega_{t-1}) \pi_{t-1}}{1 - \omega_{t-1} (1 - \lambda_{t-1})} \]
\[ = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left( \Delta \pi_t + \frac{\lambda_t \omega_t G + (1 - \omega_t) \pi_t}{1 - \omega_t} \right) \]
\[ = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left( \Delta \pi_t + \frac{(G - \pi_{t-1}) (\lambda_t \omega_t (1 - \omega_t (1 - \lambda_t)) - \lambda_{t-1} \omega_{t-1} (1 - \omega_t (1 - \lambda_t)))}{1 - \omega_{t-1} (1 - \lambda_{t-1}) (1 - \omega_t)} \right) \]
\[ = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left( \Delta \pi_t + \frac{(G - \pi_{t-1}) (\lambda_t \omega_t (1 - \omega_t (1 - \lambda_t)) - \lambda_{t-1} \omega_{t-1} (1 - \omega_t (1 - \lambda_t)))}{1 - \omega_{t-1} (1 - \lambda_{t-1}) (1 - \omega_t)} \right) \]
\[ = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left( \Delta \pi_t + \frac{(G - \pi_{t-1}) (\lambda_t (\omega_t - \omega_{t-1}) + \Delta \lambda_t \omega_{t-1} (1 - \omega_t))}{1 - \omega_{t-1} (1 - \lambda_{t-1}) (1 - \omega_t)} \right) \]

And since,
\[ \omega_t = \frac{\Lambda_{t-1} \omega_{t-1}}{\Lambda_{t-1} \omega_{t-1} + (1 - \omega_{t-1}) (1 - \omega_t)} \iff \omega_{t-1} (\Lambda_{t-1} - 1) = \omega_t - \omega_{t-1} \]

we have that
\[ R_t = \frac{1 - \omega_t}{1 - \omega_t (1 - \lambda_t)} \left( \Delta \pi_t + \frac{(G - \pi_{t-1}) (\lambda_t (\Lambda_{t-1} - 1) + \Delta \lambda_t) \omega_{t-1}}{1 - \omega_{t-1} (1 - \lambda_{t-1})} \right) \]
Bibliography


